PSEUDOSCALAR MESON FORM FACTORS
IN RELATIVISTIC QUANTUM MECHANICS:
CONSTRAINTS
FROM COVARIANT SPACE-TIME TRANSLATIONS

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MOTIVATIONS

Form factors of hadronic systems:

an important source of information on the quark dynamics,

especially at high momentum transfers $\rightarrow$ short-range dynamics

High momentum transfers: supposes a relativistic treatment

In relativistic quantum mechanics (RQM) approaches, however:

$\rightarrow$ strong sensitivity to the chosen implementation of relativity,

in the assumption of a one-body current (choice of the hypersurface)

Present work:

Look at the role of constraints related to space-time translations,

in the case of the pseudoscalar-meson form factors

CONSTRAINTS FROM POINCARÉ
COVARIANT SPACE-TIME TRANSLATIONS

A symmetry whose role is often ignored (beyond 4-momentum conservation)

$\longrightarrow$ implies relations such as:

$$e^{iP\cdot a}\ J^{\nu}(x)\ (S(x))\ e^{-iP\cdot a} = J^{\nu}(x+a)\ (S(x+a)),$$

and in particular:

$$J^{\nu}(x)\ (S(x)) = e^{iP\cdot x}\ J^{\nu}(0)\ (S(0))\ e^{-iP\cdot x}.$$

$\longrightarrow$ matrix element between eigenstates of P^{μ}

$$< i\ |J^{\nu}(x)\ (\text{or } S(x))|\ f >= e^{i(P_i-P_f)\cdot x}\ < i\ |J^{\nu}(0)\ (\text{or } (S(0))|\ f > .$$

$\longrightarrow$ 4-momentum conservation when combined with an external field $\propto e^{iq\cdot x}$ and assuming space-time translation invariance or integrating over x

$$(P_f - P_i)^{\mu} = q^{\mu}$$

FURTHER RELATIONS

Getting previous relations supposes many-body currents in RQM approaches

$\longrightarrow$ can be checked by considering further relations (Lev):

$$[P^{\mu}\,,\;J^{\nu}(x)] = -i\partial^{\mu}\,J^{\nu}(x), \quad [P^{\mu}\,,\;S(x)] = -i\partial^{\mu}\,S(x),$$

and especially

$$[P_{\mu}\,,[P^{\mu}\,,\;J^{\nu}(x)]] = -\partial_{\mu}\,\partial^{\mu}\,J^{\nu}(x), \quad [P_{\mu}\,,[P^{\mu}\,,\;S(x)]] = -\partial_{\mu}\,\partial^{\mu}\,S(x)\,.$$

$\longrightarrow$ between eigenstates of P^{μ}

$$<\;|q^2\;J^{\nu}(0)\;(\mathrm{or}\;S(0))|\;>=<\;|(p_i-p_f)^2\,J^{\nu}(0)\;(\mathrm{or}\;S(0))|\;>\;.$$

$\longrightarrow$ quite generally, in RQM approaches with the simplest single-particle current:

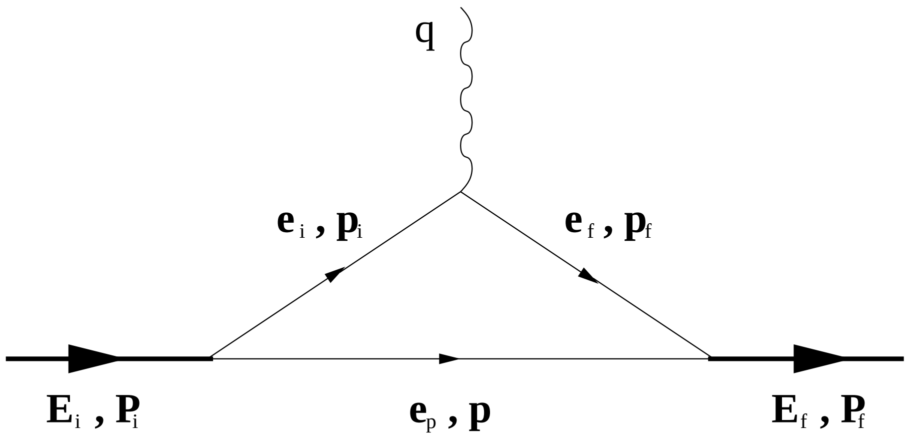
$$q^2 \neq (p_i-p_f)^2\,.$$

$\longrightarrow$ the squared momentum transferred to the constituents differs from the one transferred to the whole system: close relationship to discrepancies

IMPLEMENTING CONSTRAINTS FROM POINCARÉ SPACE-TIME TRANSLATION INVARIANCE

Observation:

the coefficient of Q,
in expressions of form factors,
involves a factor $\simeq \frac{2e_k}{M}$
that deviates from 1
by interaction effects
which are here or there
depending on the approach



Proposed:

- modify the coefficient of Q by a factor α so that to fulfill: "$(p_i - p_f)^2$" $= q^2$,
- was done numerically first, can now be done analytically in most cases

$\rightarrow$ **amounts to account for many-body currents at all orders in the interaction**

FURTHER DETAILS

Typical equation to be solved:

$$q^2 \;=\; \text{``}[(P_i - P_f)^2 + 2\,(\Delta_i - \Delta_f)\,(P_i - P_f)\cdot\xi + (\Delta_i - \Delta_f)^2\,\xi^2]\text{''}$$
$$\;=\; \alpha^2 q^2 - 2\,\alpha\,\text{``}(\Delta_i - \Delta_f)\text{''}\;q\cdot\xi + \text{``}(\Delta_i - \Delta_f)^{2}\text{''}\;\xi^2\,,$$

where Δ is proportional to an interaction effect.

To be noticed

$\longrightarrow$ No modification required for the standard front form ($q^+ = 0$), where the above equality is trivially fulfilled with $\alpha^2 = 1$ (using $q\cdot\xi = q\cdot\omega = 0$, $\xi^2 = \omega^2 = 0$)

$\longrightarrow$ one tractable solution has been found but there may be less tractable other ones

RESULTS INCORPORATING EFFECTS MOTIVATED BY SPACE-TIME TRANSLATION INVARIANCE

Sample of results:

Pion charge form factor for the following approaches:

- the front form with $q^+ = 0$ (“perpendicular” momentum configuration; F.F. (perp.))
- the instant form (I.F.) in the Breit frame
- the instant and front forms in a “parallel” momentum configuration (I.F.+F.F.(parallel))
- the “point-form” (Bakamjian, Sokolov; “P.F.”)

(the point-form (Dirac inspired; D.P.F.) is omitted here)

For illustration:

Gaussian wave function with a parameter obtained from the strength of the confinement force (Basdevant and Boukraa), and the quark mass fitted to reproduce the pion decay constant, f_π

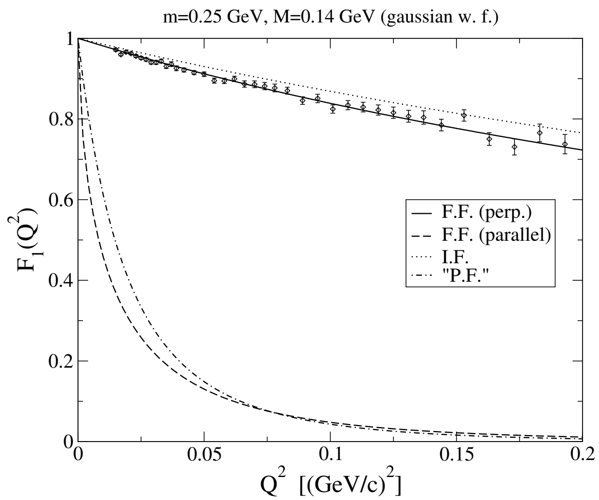
NUMERICAL RESULTS FOR THE PION CHARGE FORM FACTOR

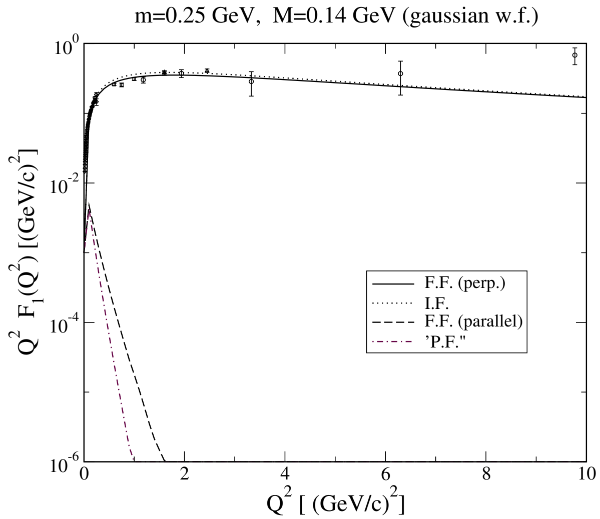
Uncorrected results

- Standard front and instant forms do well

- Role of M

- Lorentz invariance does not ensure good results



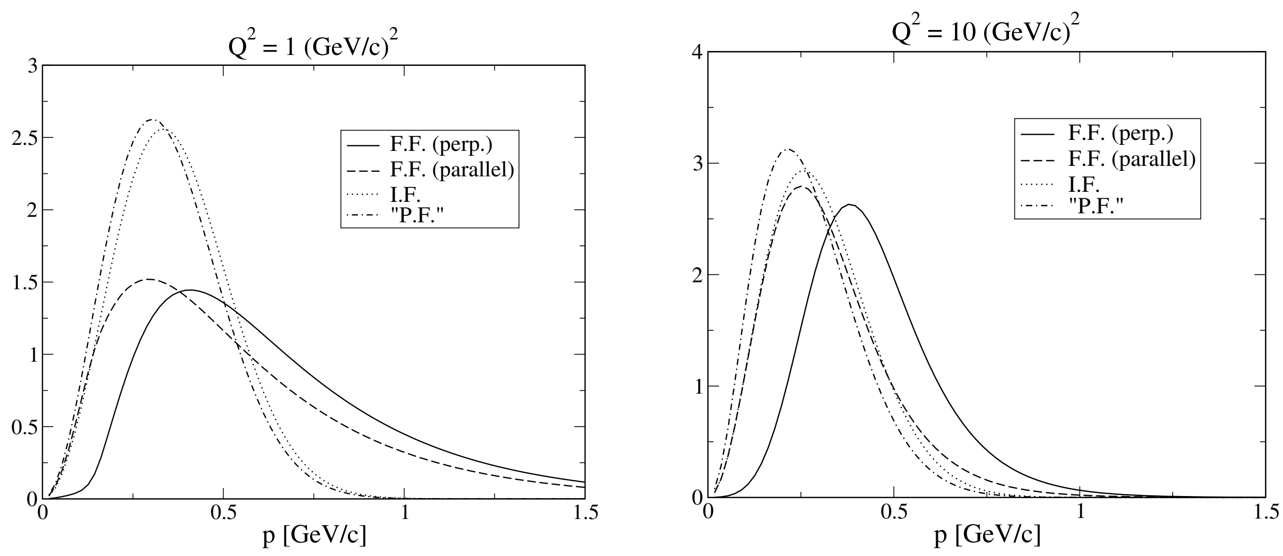


Corrected results

- all results can be made identical to the standard front-form one
- the effects due to a small value of M have disappeared
- some discrepancy with measurements
  $\rightarrow$ more complete mass operator (gluon exchange) and OGEC

EQUIVALENCE OF DIFFERENT APPROACHES:
A NON-TRIVIAL CHANGE OF VARIABLES

→ compare integrands in terms of the spectator-quark momentum p,
at 2 values of the momentum transfer Q^2 (case of the pion charge form factor).



A FEW OBSERVATIONS

- Similar results for the scalar pion form factor
  as well as the kaon meson form factors (smaller effects)

- Disappearance of some paradoxes ($r_{ch.} \rightarrow \infty$ with $M \rightarrow 0$)

- Support for the standard front-form results (unchanged)

- Dispersion-relation approach (Anisovich *et al.*, Krutov and Troitsky):
  convergence point for RQM ones

- Some space for extra many-body currents
  (required for the asymptotic pion charge form factor)

- Generalizations to non-zero spin systems, time-like momentum transfers, $\cdots$

CONCLUSION AND OUTLOOK

Large discrepancies between predicted form factors in different RQM approaches

- could be due to the underlying physics?
- or an incomplete implementation of relativity?

→ **present results ascribe them to missing properties from Poincaré covariant space-time translations**

→ **allow one to concentrate on the physics**

For a more complete comparison with measurements:

- accounting for the one-gluon exchange contribution to the mass operator (possible problem with non-perturbative effects: Cardarelli *et al.*, B.D.)
- accounting for the one-gluon exchange current ensuring the right asymptotics of the pion charge form factor