The σ and $f0(980)$ from $K_{e4} \oplus \pi\pi$ scattering data

X .G . Wang

School of Physics, Peking University
LPTA, University Montpellier II

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G. Mennessier, S. Narison, X. G. Wang, arXiv:1002.1402[hep-ph], to appear in PLB.

- Motivation
- Elastic $\pi\pi \longrightarrow \pi\pi$ scattering
- Inelastic $\pi\pi \longrightarrow \pi\pi/\bar{K}K$ scattering

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- The hadronic and diphoton couplings of light scalar mesons could provide important information about their nature.
- K-matrix model has been used to describe $\pi\pi - \bar{K}K$ strong processes. G. Mennessier, Z. Phys. **C16**(1983)241.
- We improve the model by introducing a form factor *shape function*.

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1 channel $\oplus$1" *bare*" *resonance*

We introduce a real analytic form factor *shape function*, G. Mennessier, S. Narison, W. Ochs, Phys. Lett. **B 665**(2008)205.

$$f_p(s) = \frac{s - s_{AP}}{s + \sigma_{DP}}, \quad P = \pi, K \tag{1}$$

It allows for an Alder zero $s = s_{AP}$ and a pole at $-\sigma_{DP} < 0$ to simulate left hand singularities.

The unitary $I = 0$ S wave $\pi\pi$ scattering amplitude is then written as

$$T_{PP} = \frac{g_P^2 f_P(s)}{s_R - s - g_P^2 \tilde{f}_P(s)} = \frac{g_P^2 f_P(s)}{\mathcal{D}_P(s)} \tag{2}$$

where

$$\mathrm{Im}\mathcal{D} = \mathrm{Im}(-g_\pi^2 \tilde{f}_P) = -(\theta\rho_P) g_P^2 f_P \ . \tag{3}$$

and hence

$$\mathrm{Im}(\tilde{f}_P) = (\theta\rho_P) f_P \ , \qquad \rho_P = \sqrt{1 - 4m_P^2/s}. \tag{4}$$

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The real part of $\tilde{f}_P$ is obtained from a dispersion relation with subtraction at $s = 0$. The *shape function is simple enough* so that we can get the analytic expression of the dispersion integral

$$\tilde{f}_P(s) = \frac{2}{\pi}(h_0(s) - h_0(0)) \ . \tag{5}$$

One can find the definition of $h_0(s)$ from G. Mennessier, S. Narison, W. Ochs, Phys. Lett. **B 665**(2008)205.

0 bare resonance$\equiv \lambda\phi^4$ model

In this case, one can introduce the shape function $f_2(s)$

$$T_{PP} = \frac{\Lambda f_2(s)}{1 - \Lambda \tilde{f}_2(s)} \ , \qquad f_2(s) = \frac{s - s_{AP}}{(s + \sigma_{D1})(s + \sigma_{D2})} \ . \tag{6}$$

where $\sigma_{D1} = \sigma_{D_\pi}$, and

$$\tilde{f}_2(s) = \frac{2}{\pi}[h_2(s) - h_2(0)] \ . \tag{7}$$

Fit Results

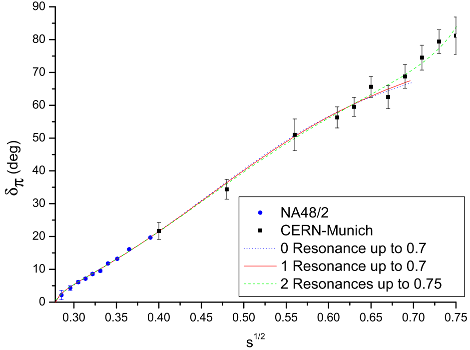

Figure: The fit result of $\pi\pi$ $I = 0$ S-wave phase shift.

Outputs	0 res.	1 res.	2 res.	Average
s_A	0.009(6)	0.0094(fixed)	0.0094(fixed)	
$\sigma_{D\pi}$	6.2(3.2)	1.41(7)	1.78(10)	
σ_{D2}	7.6(4.5)	-	-	
s_{Ra}	-	1.94(9)	26.97(1.54)	
Λ	108(34)	-	-	
$g_{\pi a}$	-	2.54(8)	10.42(30)	
s_{Rb}	-	-	0.61(31)	
$g_{\pi b}$	-	-	-0.39(8)	
$\chi^2_{d.o.f}$	$\frac{12.04}{14} = 0.86$	$\frac{11.73}{15} = 0.78$	$\frac{12.71}{16} = 0.79$	
M_σ	468(181)	456(19)	338(18)	*452(13)*
$\Gamma_\sigma/2$	261(211)	265(18)	260(19)	*259(16)*
$\lvert g_{\sigma\pi^+\pi^-}\rvert$	2.58(1.31)	2.72(16)	2.58(14)	*2.64(10)*

0 "bare" resonance model shows that the existence of the σ pole is *not an artifact* of the "bare" resonance entering the parametrization of T_{PP}.

2 channels $\oplus$2" *bare" resonances*

The generalization to couple channel is conceptually straightforward.
Consider two 2-body channels coupled to 2 "bare" resonances labeled a and b, with bare masses squared s_{Ra} and s_{Rb}. We shall work in the *minimal case* with only one shape function as approximation:

$$f(s) = \frac{s - s_A}{s + \sigma_D} \ . \tag{8}$$

The phase shifts and inelasticity η are defined by

$$\eta e^{2i\delta_P} = 1 + 2i\rho_P T_{PP} \ . \tag{9}$$

The experimental data sets we used:

- δ_π: NA48/2 on K_{e4}(below $0.4\mathrm{GeV}$) and CERN-Munich(above $0.4\mathrm{GeV}$) for all three fits;
- η: CERN-Munich(Set 1), Hyams(Set 2), Cohen(Set 3);
- $\delta_{\pi K} = \delta_\pi + \delta_K$:
  Etkin and Martin(Set 1), Cohen(Set 2), Cohen(Set 3)

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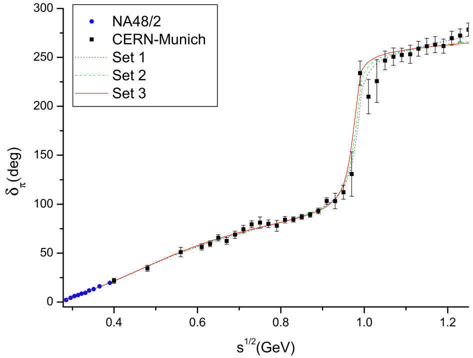

Figure: The fit result of $\pi\pi$ $I = 0$ S-wave phase shift.

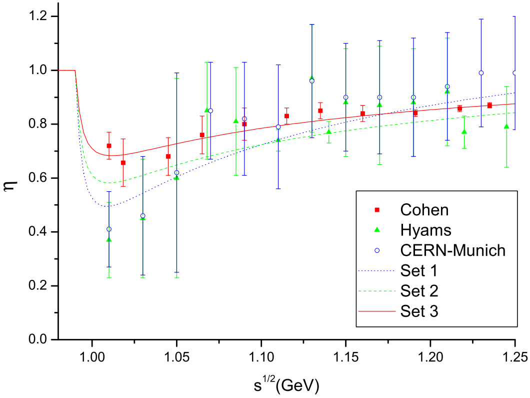

Figure: Inelasticity of $\pi\pi$ $I = 0$ S-wave.

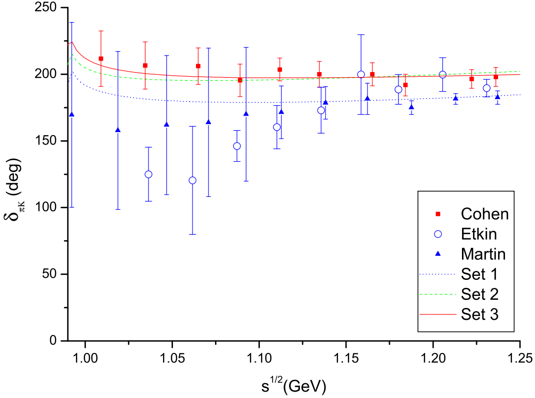

Figure: The phase shift $\delta_{\pi K} = \delta_\pi + \delta_K$.

With these choices, we expect to span *all possible regions* of the space of parameters.

Bare Parameters of the Model

Outputs	Set 1	Set 2	Set 3
s_A	0.016±0.004	0.013±0.006	0.010±0.006
σ_D	0.740±0.097	0.909±0.201	1.116±0.262
s_{Ra}	4.112±0.499	2.230±0.271	2.447±0.298
$g_{\pi a}$	-0.557∓0.177	0.864±0.391	0.997±0.516
g_{Ka}	3.191±0.499	1.458±0.262	1.684±0.363
s_{Rb}	1.291±0.062	1.187±0.094	1.354±0.149
$g_{\pi b}$	-1.562∓0.117	-1.527∓0.134	-1.756∓0.183
g_{Kb}	0.748±0.062	0.999±0.149	1.159±0.261
$\chi^2_{d.o.f}$	70.6/77=0.914	48.8/64=0.759	44.3/58=0.763

Physical quantities

Outputs	Set 1	Set 2	Set 3	Average
M_σ	435(74)	452(72)	457(76)	448(43)
$\Gamma_\sigma/2$	271(92)	266(65)	263(72)	266(43)
$\|g_{\sigma\pi^+\pi^-}\|$	2.72(78)	2.74(61)	2.73(61)	2.73(38)
$\|g_{\sigma K^+K^-}\|$	1.83(86)	0.80(55)	0.99(68)	1.06(38)
M_f	989(80)	982(47)	976(60)	981(34)
$\Gamma_f/2$	20(32)	18(16)	18(18)	18(11)
$\|g_{f\pi^+\pi^-}\|$	1.33(72)	1.22(60)	1.12(31)	1.17(26)
$\|g_{fK^+K^-}\|$	3.21(1.70)	2.98(70)	3.06(1.07)	3.03(55)

Table: the mass and width are in MeV, while the couplings are in GeV.

Final Results

For the σ:

Processes	$M_\sigma - i\Gamma_\sigma/2$	Refs.
Our work		
$K_{e4} \oplus \pi\pi \to \pi\pi$	452(13)-i259(16)	
$K_{e4} \oplus \pi\pi/K\bar{K}$	448(43)-i266(43)	
Average	*452(12)-i260(15)*	
Others		
$\pi\pi \to \pi\pi \oplus Roy \oplus ChPT$	$441^{+16}_{-8} - i272^{+9}_{-15}$	Caprini, PRL96
$\pi\pi \to \pi\pi/\bar{K}K \oplus Roy$	$461 \pm 15 - i(255 \pm 16)$	Kaminski, PRD77
$J/\psi \to \omega\pi\pi$	$541{\pm}39 - i(222 \pm 42)$	Ablikim, PLB598
$D^+ \to \pi^+\pi^-\pi^+$	$478{\pm}29 - i(162 \pm 46)$	Aitala, PRL86

- $|g_{\sigma\pi^+\pi^-}| = 2.65(10)\mathrm{GeV}$
  $r_{\sigma\pi K} \equiv \frac{|g_{\sigma K^+K^-}|}{|g_{\sigma\pi^+\pi^-}|} = 0.37(6)$
- The values of couplings confirm and improve the previous results. R. Kaminski, G. Mennessier, S. Narison, Phys. Lett. **B680**(2009)148
- The sizeable coupling of the σ to $\bar{K}K$ disfavours the usual $\pi\pi$ molecule and 4-quark assignment of the σ, where this coupling is expected to be negligible.

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For the $f_0(980)$:

- $M_f[\mathrm{MeV}] = 981(34) - i18(11)$
- $|g_{f\pi^+\pi^-}| = 1.17(26)\mathrm{GeV}$
- $r_{f\pi K} \equiv \frac{|g_{fK^+K^-}|}{|g_{f\pi^+\pi^-}|} = 2.59(1.34)$

Large value of $r_{f\pi K} \oplus$ narrow width $\Longrightarrow$ not pure $(u\bar{u} + d\bar{d})$
non-negligible width into $\pi\pi \Longrightarrow$ not pure $s\bar{s}$ or $K\bar{K}$ molecule.

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A large *gluonium component* mixed with a $q\bar{q}$ state of the σ and $f_0(980)$ seems to be necessary for evading the previous difficulties.

Summary

- An improved couple channel "K-matrix" model satisfying unitarity is used, taking into account Adler zero and *left hand singularities*.
- We use new K_{e4}(NA48/2)$\oplus$*different*$\pi\pi \rightarrow \pi\pi/K\bar{K}$ scattering data.
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Thanks for patience!