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SMEFT and CP violation

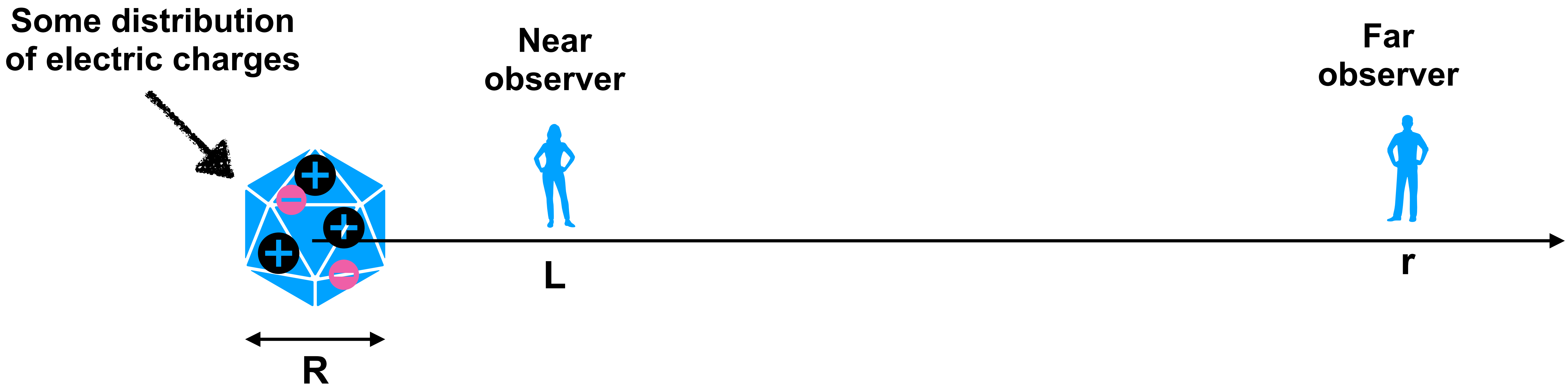
Lectures given at CP2023, International Workshop on the origin of matter-antimatter asymmetry

12-17 February 2023

Part 1

Brief Philosophy of EFT

Role of scale in physical problems



<u>Near observer</u>, L~R, needs to know the position of every charge to describe electric field in her proximity

<u>Far observer</u>, $r \gg R$, can instead use multipole expansion: $$V(\vec{r}) = \frac{Q}{r} + \frac{\vec{d}\cdot\vec{r}}{r^3} + \frac{Q_{ij} r_i r_j}{r^5} + \dots$$

$$\sim 1/r \qquad \sim R/r^2 \qquad \sim R^2/r^3$$

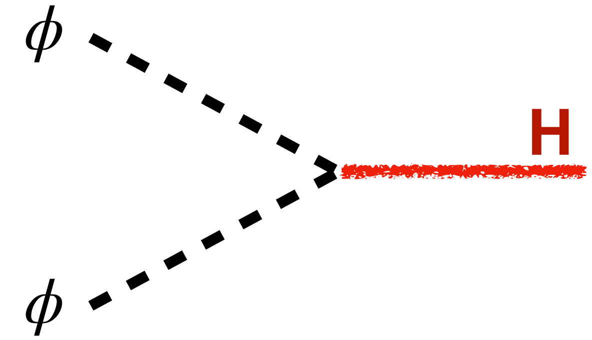
Higher order terms in the multipole expansion are suppressed by powers of the small parameter (R/r).
One can truncate the expansion at some order depending on the value of (R/r) and experimental precision

Far observer is able to describe electric field in his vicinity using just a few parameters:
the total electric charge Q, the dipole moment $\vec{d}$, eventually the quadrupole moment Q_{ij}, etc….

On the other hand, far observer can only guess the "fundamental" distributions of the charges, as many distinct distributions lead to the same first few moments

Far observer, like Molière's Mr. Jourdain,
discovers that he has been using EFT all his life

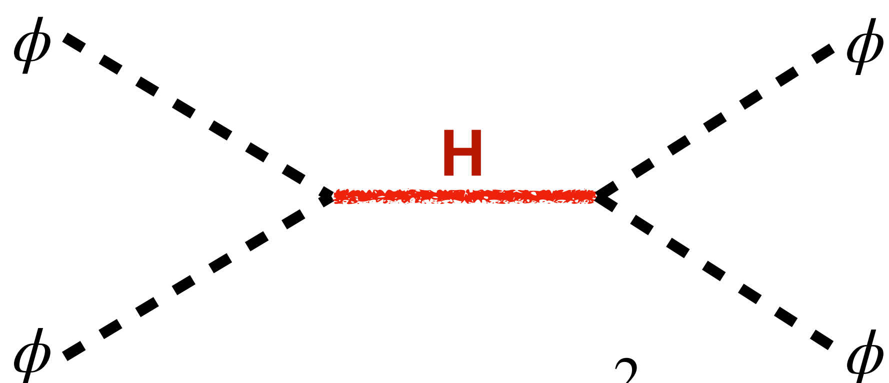
Role of scale in quantum field theory



Consider a theory of a light particle ϕ interacting with a heavy particle H

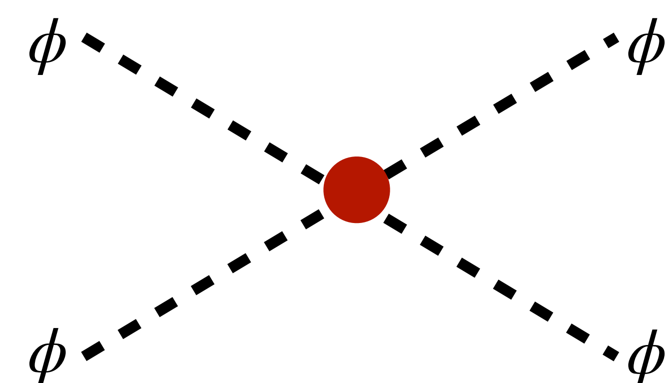
Heavy particle H propagator in momentum space:

$$P(p^2) = \frac{1}{p^2 - m_H^2 + i\epsilon} \approx \begin{cases} \dfrac{1}{p^2 + i\epsilon} & p^2 \gg m_H^2 \\ -\dfrac{1}{m_H^2} & p^2 \ll m_H^2 \end{cases}$$



$$\mathscr{M} \sim \frac{g^2}{p^2 + i\epsilon}$$

At large momentum scales, $p^2 >> m_H^2$, we see propagation of the heavy particle H. Long range force acting between light particles ϕ

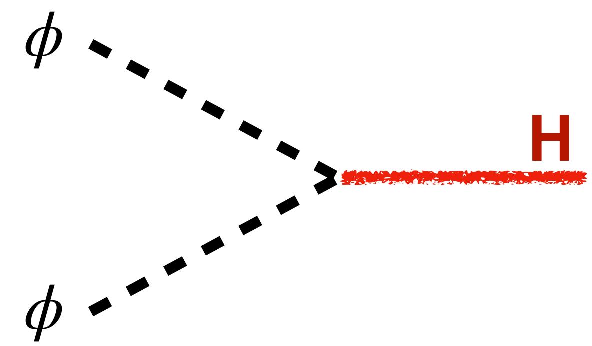


$$\mathscr{M} \sim \frac{g^2}{m_H^2}$$

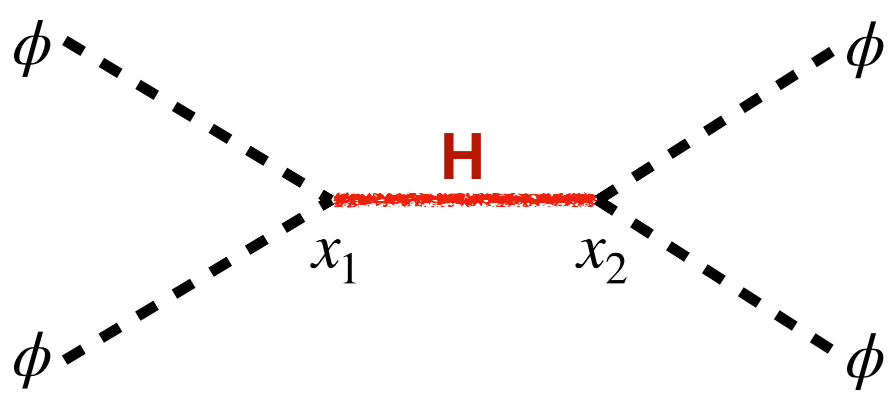
At small momentum scales, $p^2 << m_H^2$, propagation of the heavy particle H effectively leads to a contact interaction between light particles ϕ

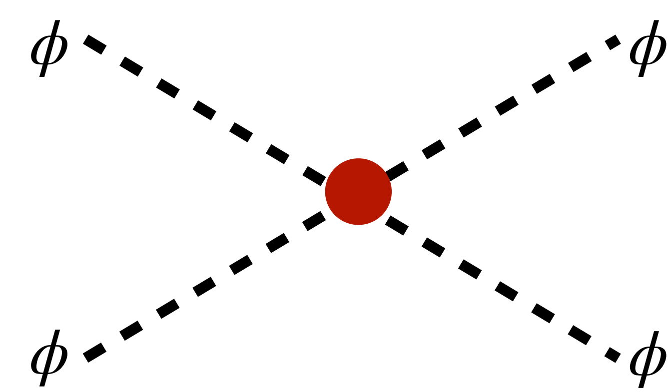
Role of scale in quantum field theory

Consider a theory of a light particle ϕ interacting with a heavy particle H



Heavy particle H propagator in coordinate space: $P(x_1, x_2) \sim \exp(-m_H |x_1 - x_2|)$





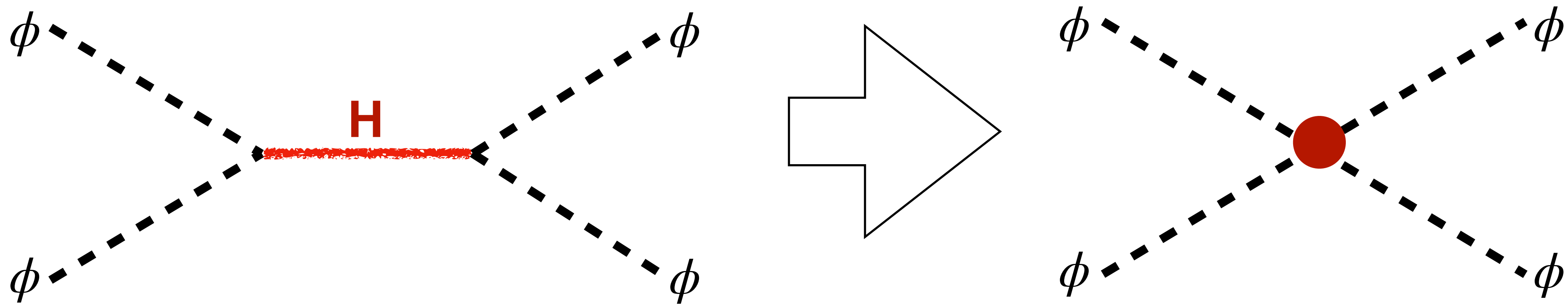
At small distance scales, $|x_1-x_2| \ll 1/m_H$, the heavy particle H propagates.
Force acting between light particles ϕ

At large distance scales, $|x_1-x_2| \gg 1/m_H$, propagation of the heavy particle H suppressed.
Interaction looks like a delta function potential

$$m_H \sim \Delta E \ll \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \ll 1$$

$$m_H \sim \Delta E \gg \frac{1}{|x_1 - x_2|} \sim \frac{1}{\Delta t} \quad \Rightarrow \quad \Delta E \Delta t \gg 1$$

Role of scale in quantum field theory



- Processes probing distance scales $L \gg m_H$, equivalently energy scales $E \ll m_H$, cannot resolve the propagation of H

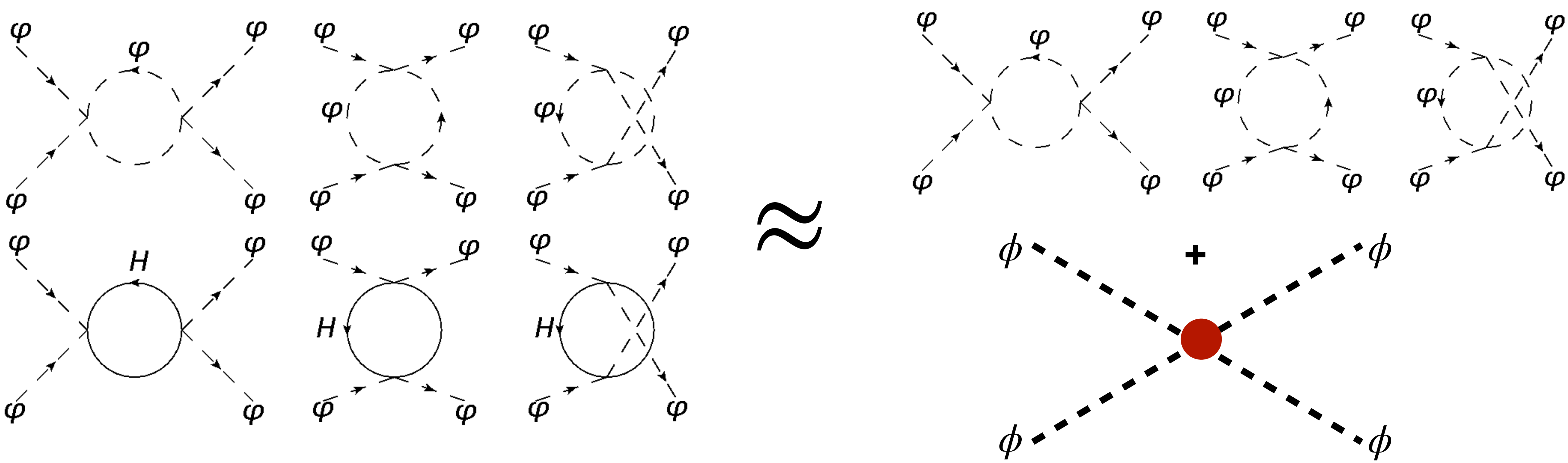
- Then, intuitively, exchange of heavy particle H between light particles ϕ should be indistinguishable from a contact interaction of ϕ

- In other words, the <u>effective theory</u> describing ϕ interactions should be well approximated by a local Lagrangian, that is, by a polynomial in ϕ and its derivatives

This is the generic way how the effective theory description arise in particle physics,

Role of scale in quantum field theory

Effective theory approach works beyond tree level



This works also for higher loops, and with both heavy and light particles in the loops

Effective field theory

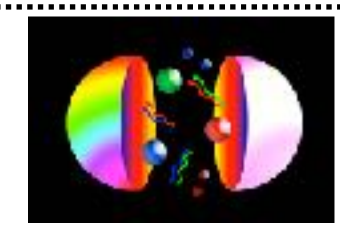
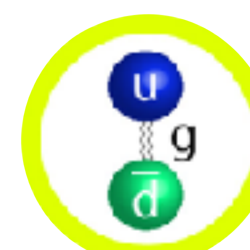
How to build an EFT

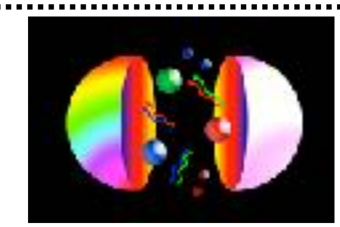
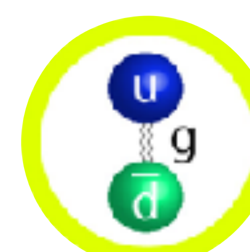
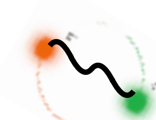
Bottom up

Top down

**Starting with a set of particles
we build the Lagrangian
describing all their possible interactions
obeying a prescribed set of symmetries
and organised in a consistent expansion**

**Starting with a given theory
(effective or fundamental)
we integrate out degrees of freedom
heavier than some prescribed mass scale**

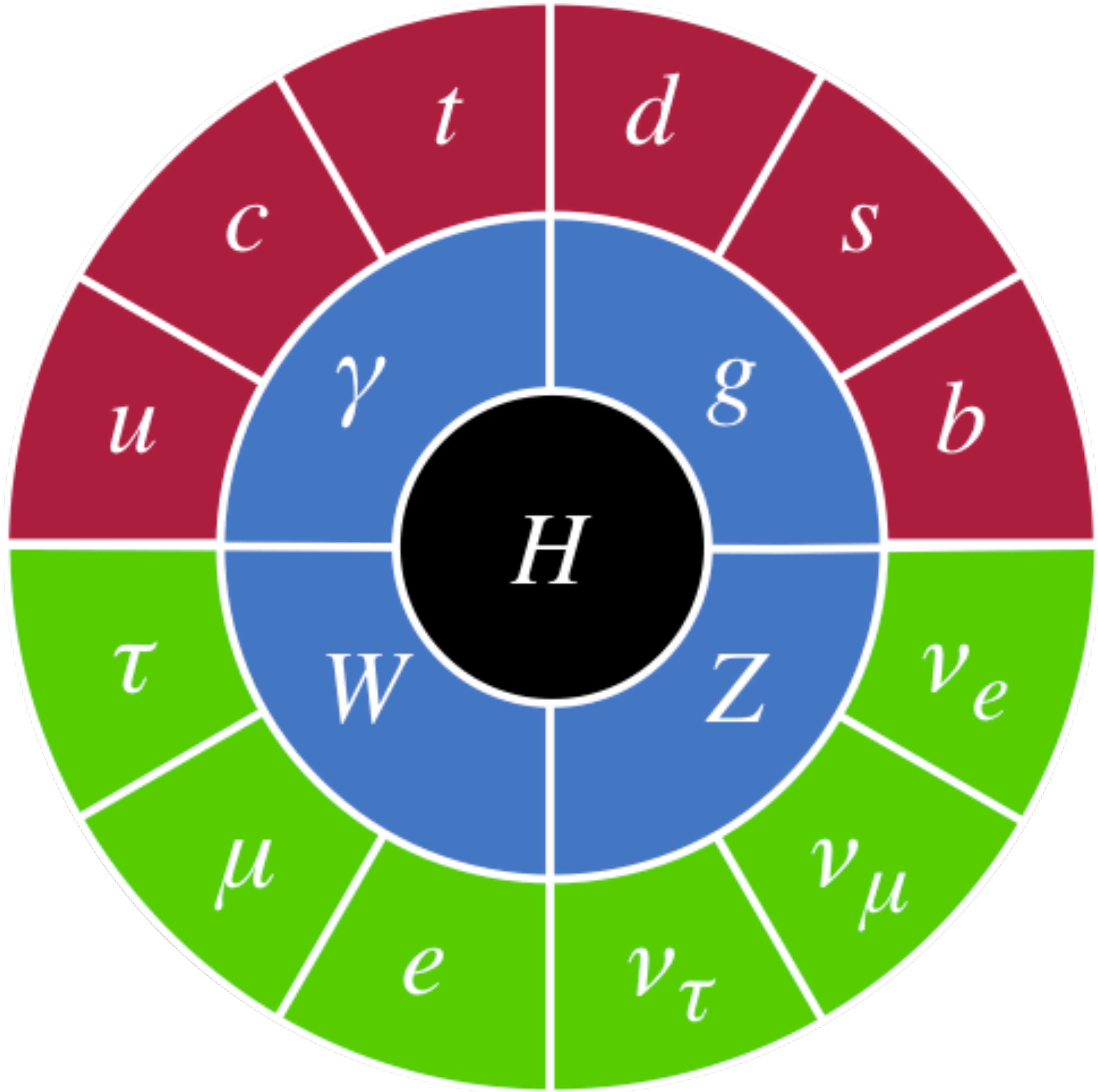
UV	10 TeV	**Dragons**	
SMEFT	100 GeV	$\gamma,\ g,\ W,\ Z,\ \nu_i,\ e,\ \mu,\ \tau$ + **u, d, s, c, b, t** + **h**	
WEFT5	5 GeV	$\gamma,\ g,\ \nu_i,\ e,\ \mu,\ \tau$ + **u, d, s, c, b**	
WEFT4	2 GeV	$\gamma,\ g,\ \nu_i,\ e,\ \mu,\ \tau$ + **u, d, s, c**	
ChRT	1 GeV	$\gamma,\ \nu_i,\ e,\ \mu$ + **hadrons**	
ChPT	100 MeV	$\gamma,\ \nu_i,\ e,\ \mu,\ \pi,\ K$	
QED	1 MeV	$\gamma,\ \nu_i,\ e$	
EH	0.01 eV	$\gamma,\ \nu_i$ γ	

UV	10 TeV	**Dragons** 
SMEFT	100 GeV	*These Lectures* $\gamma, g, W, Z, \nu_i, e, \mu, \tau$ + **u, d, s, c, b, t** + **h** 
WEFT5	5 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c, b** 
WEFT4	2 GeV	$\gamma, g, \nu_i, e, \mu, \tau$ + **u, d, s, c** 
ChRT	1 GeV	γ, ν_i, e, μ + **hadrons** 
ChPT	100 MeV	$\gamma, \nu_i, e, \mu, \pi, K$ 
QED	1 MeV	γ, ν_i, e 
EH	0.1 eV	γ, ν_i 
	0.01 eV	γ

Part 2

Introducing SMEFT

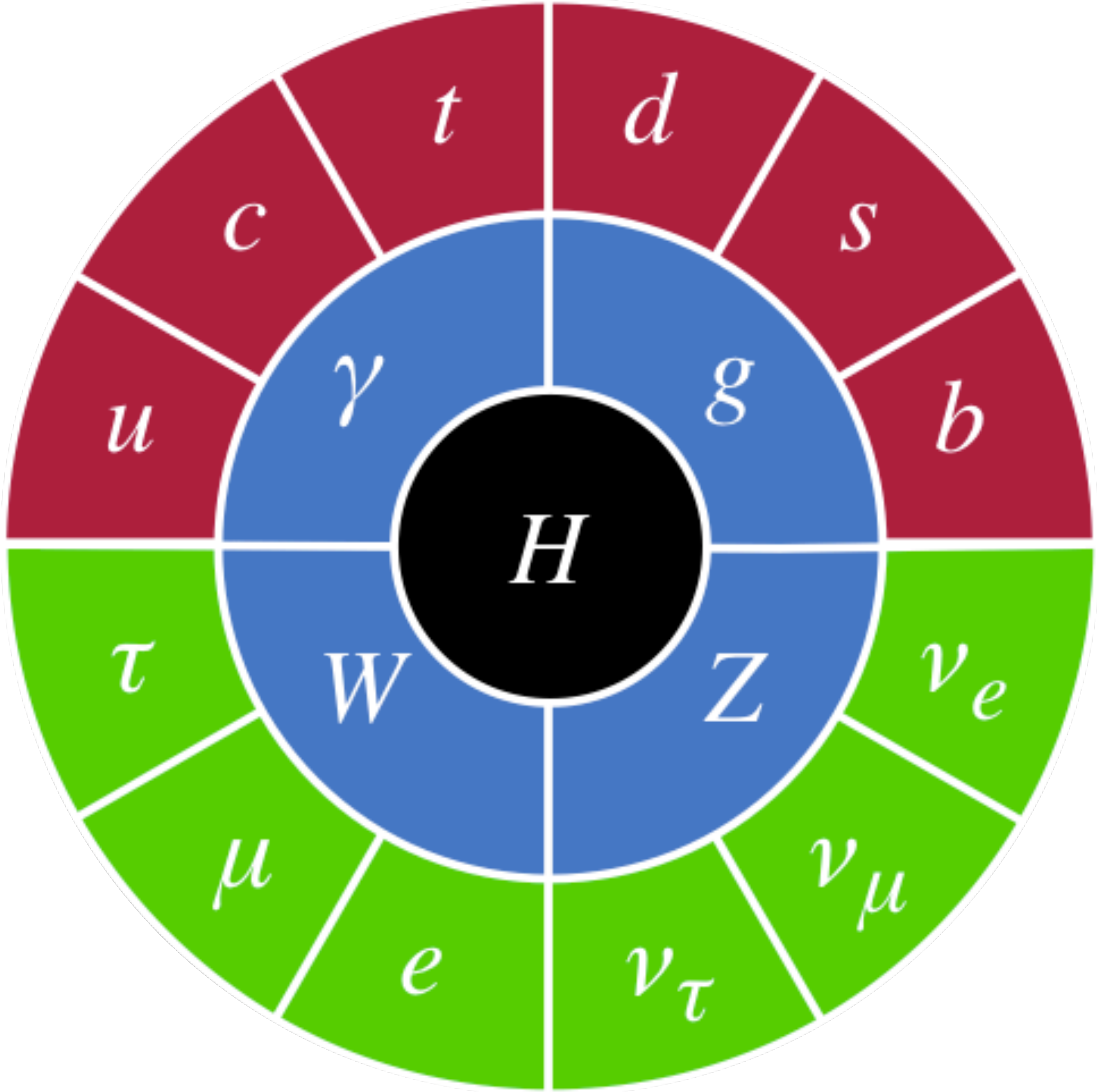
Elementary particles we know today



graviton

This set of particles are the propagating degrees of freedom (at least) right above the electroweak scale, that is at $E \sim$ 100 GeV - 1 TeV

Elementary particles we know today



In these lectures gravity is decoupled and ignored (good assumption in most of laboratory experiments). Otherwise the relevant EFT is called GRSMEFT.

SMEFT

SMEFT is an effective theory for these degrees of freedom:

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Name	Spin	Dimension
G^a_μ	$\mathbf{8}$	$\mathbf{1}$	0	Gluon	1	1
W^k_μ	$\mathbf{1}$	$\mathbf{3}$	0	Weak SU(2) bosons	1	1
B_μ	$\mathbf{1}$	$\mathbf{1}$	0	Hypercharge boson	1	1
Q	$\mathbf{3}$	$\mathbf{2}$	1/6	Quark doublets	1/2	3/2
U^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	-2/3	Up-type anti-quarks	1/2	3/2
D^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	Down-type anti-quarks	1/2	3/2
L	$\mathbf{1}$	$\mathbf{2}$	-1/2	Lepton doublets	1/2	3/2
E^c	$\mathbf{1}$	$\mathbf{1}$	1	Charged anti-leptons	1/2	3/2
H	$\mathbf{1}$	$\mathbf{2}$	1/2	Higgs field	0	1

incorporating certain physical assumptions:

1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions and spontaneously broken to SU(3)xU(1) by a VEV of the Higgs field**

Note on fermion conventions

I am using the 2-component spinor formalism

A Dirac fermion is described by a pair of spinor fields f and $\bar{f}^c$ **with the kinetic and mass terms**

$$\mathscr{L} = i\bar{f}\bar{\sigma}^\mu D_\mu f + i f^c \sigma^\mu D_\mu \bar{f}^c - m f^c f - m \bar{f}\bar{f}^c$$

$$\sigma^\mu = (1, \boldsymbol{\sigma})$$
$$\bar{\sigma}^\mu = (1, -\boldsymbol{\sigma})$$
$$\bar{f} \equiv f^*$$

To translate to 4-component Dirac notation use

$$F = \begin{pmatrix} f \\ \bar{f}^c \end{pmatrix}, \qquad \bar{F} = \begin{pmatrix} f^c & \bar{f} \end{pmatrix}, \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\bar{F} \equiv F^\dagger \gamma^0$$

For example

$$\bar{f}\bar{\sigma}^\mu \partial_\mu f = \bar{F}_L \gamma^\mu \partial_\mu F_L$$

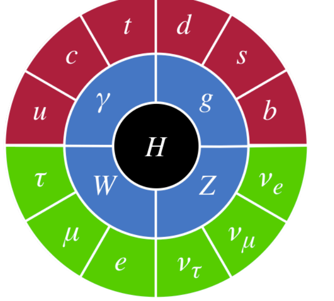
$$f^c \sigma^\mu \partial_\mu \bar{f}^c = \bar{F}_R \gamma^\mu \partial_\mu F_R$$

$$f^c f = \bar{F}_R F_L$$

$$\bar{f}\bar{f}^c = \bar{F}_L F_R$$

See the spinor bible [arXiv:0812.1594] for more details

SMEFT power counting



1. **Locality, unitarity, Poincaré symmetry**
2. **Mass gap: absence of non-SM degrees of freedom at or below the electroweak scale**
3. **Gauge symmetry: local SU(3)xSU(2)xU(1) symmetry strictly respected by all interactions**

We can organize the SMEFT Lagrangian in a dimensional expansion:

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Each $\mathscr{L}_D$ is a linear combination of SU(3)xSU(2)xU(1) invariant interaction terms (operators) where D is the sum of canonical dimensions of all the fields entering the interaction

Since Lagrangian has mass dimension $[\mathscr{L}] = 4$, by dimensional analysis the couplings (Wilson coefficients) of interactions in $\mathscr{L}_D$ have mass dimension $[C_D] = 4 - D$

Standard SMEFT power counting: $C_D \sim \dfrac{c_D}{\Lambda^{D-4}}$ **where** $c_D \sim 1$ **,**

and Λ is identified with the mass scale of the UV completion of the SMEFT,

In the spirit of EFT, each $\mathscr{L}_D$ should include a <u>complete</u> and <u>non-redundant</u> set of interactions

SMEFT power counting

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

SM Lagrangian

Higher-dimensional $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

At sufficiently high energies, such that we can ignore particle masses, amplitudes for physical processes take the form

$$\mathscr{M}_{\mathrm{SMEFT}} = \mathscr{M}_{\mathrm{SM}} + C_{D=5}E + C_{D=6}E^2 + C_{D=7}E^3 + C_{D=8}E^4 + \dots$$

$$\sim \mathscr{M}_{\mathrm{SM}} + \frac{c_5 E}{\Lambda} + \frac{c_6 E^2}{\Lambda^2} + \frac{c_7 E^3}{\Lambda^3} + \frac{c_8 E^4}{\Lambda^4} + \dots$$

Standard SMEFT power counting sets up the rules for expanding the amplitudes and observables in powers of the new physics scale Λ.

For $E \ll \Lambda$ expansion can be truncated at some D, depending on the desired precision

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

Only a single D=2 operator can be build from the SM fields:

$$\mathscr{L}_{D=2} = \mu_H^2 H^\dagger H$$

Philosophy of EFT: $\mu_H \sim \Lambda \gtrsim 1\ \text{TeV}$

Experiment: $\mu_H \sim 100\ \text{GeV}$

Unsolved mystery why $\mu_H^2 \ll \Lambda^2$,
which is called the hierarchy problem

From the point of view of EFT, the hierarchy problem is a breakdown of dimensional analysis

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

$$\mathscr{L}_{D=3} = 0$$

Simply, no gauge invariant operators made of SM fields exist at canonical dimension D=3

The absence of D=3 operators is a feature of SMEFT, but not a law of nature. E.g. in νSMEFT, where one also has singlet neutrino, one can write down

$$\mathscr{L}_{D=3}^{\nu\text{SMEFT}} = \frac{1}{2}\nu^c M_\nu \nu^c + \text{h.c.}$$

SMEFT

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=3} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

D=4 is special because it doesn't contain an explicit scale (marginal interactions)

$$\mathscr{L}_{D=4} = -\frac{1}{4}\sum_{V\in B,W^i,G^a} V_{\mu\nu}V^{\mu\nu} + \sum_{f\in Q,L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f\in U,D,E} if^c\sigma^\mu D_\mu \bar{f}^c$$
$$-\left(U^cY_u\tilde{H}^\dagger Q + D^cY_dH^\dagger Q + E^cY_eH^\dagger L + \mathrm{h.c.}\right) + D_\mu H^\dagger D^\mu H - \lambda(H^\dagger H)^2$$
$$+\tilde{\theta}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu},$$

$$\tilde{H}_a = \epsilon^{ab}H^*_b$$
$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc}V^b_\mu V^c_\nu$$
$$D_\mu f = \partial_\mu f + ig_s G^a_\mu T^a f + ig_L W^i_\mu \frac{\sigma^i}{2} f + ig_Y B_\mu Y f$$
$$\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta\ a}$$

$$U^c = \begin{pmatrix} u^c \\ c^c \\ t^c \end{pmatrix} \qquad Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{pmatrix}$$

$$D^c = \begin{pmatrix} d^c \\ s^c \\ b^c \end{pmatrix}$$

$$E^c = \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix} \qquad L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix}$$

Experiment: all these interactions at D=4 above have been observed, except for $\tilde{\theta}$

Strictly speaking, λ has not been observed directly. Its value is known within SM hypothesis, but not within SMEFT, without additional assumptions. Observation of double Higgs production (receiving contribution from cubic Higgs coupling) will be a direct proof that λ is there in the Lagrangian.

Note that $\theta_B B_{\mu\nu}\tilde{B}_{\mu\nu}$ has no physical consequences, while $\theta_W W^k_{\mu\nu}\tilde{W}^k_{\mu\nu}$ can be eliminated by chiral rotation

SMEFT at dimension-5

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Weinberg (1979)
Phys. Rev. Lett. 43, 1566

$$H \to \begin{pmatrix} 0 \\ \mathrm{v}/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{D=5} = (LH)C(LH) + \mathrm{h.c.} \to \frac{1}{2} \sum_{J,K=e,\mu,\tau} \mathrm{v}^2 C_{JK}(\nu_J \nu_K) + \mathrm{h.c.}$$

- At dimension 5, the only gauge-invariant operators one can construct are the so-called Weinberg operators, which break the lepton number

- After electroweak symmetry breaking they give rise to Majorana mass terms for the SM (left-handed) neutrinos with the mass matrix $M = -\mathrm{v}^2 C$

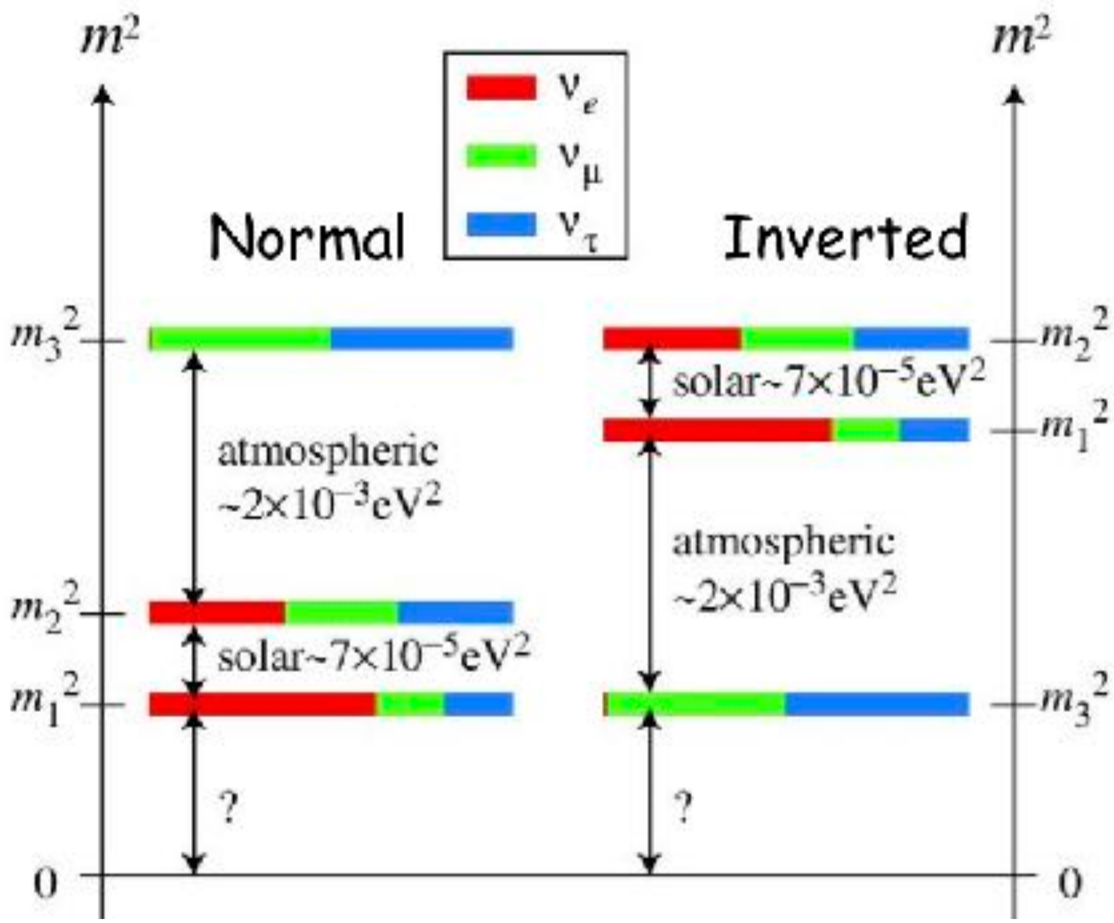
- Neutrino oscillation experiments strongly suggest that these operators are present (unless neutrino masses are of the Dirac type)

This is a huge success of the SMEFT paradigm:
corrections to the SM Lagrangian predicted at the next order in the EFT expansion, are indeed observed in experiment!

SMEFT at dimension-5

$$\mathscr{L}_{\mathrm{SMEFT}} \supset -\frac{1}{2}(\nu M \nu) + \mathrm{h.c.} \qquad M = -\mathrm{v}^2 C$$

Neutrino masses or most likely in the 0.01 eV - 0.1 eV ballpark (though the lightest neutrino may even be massless)

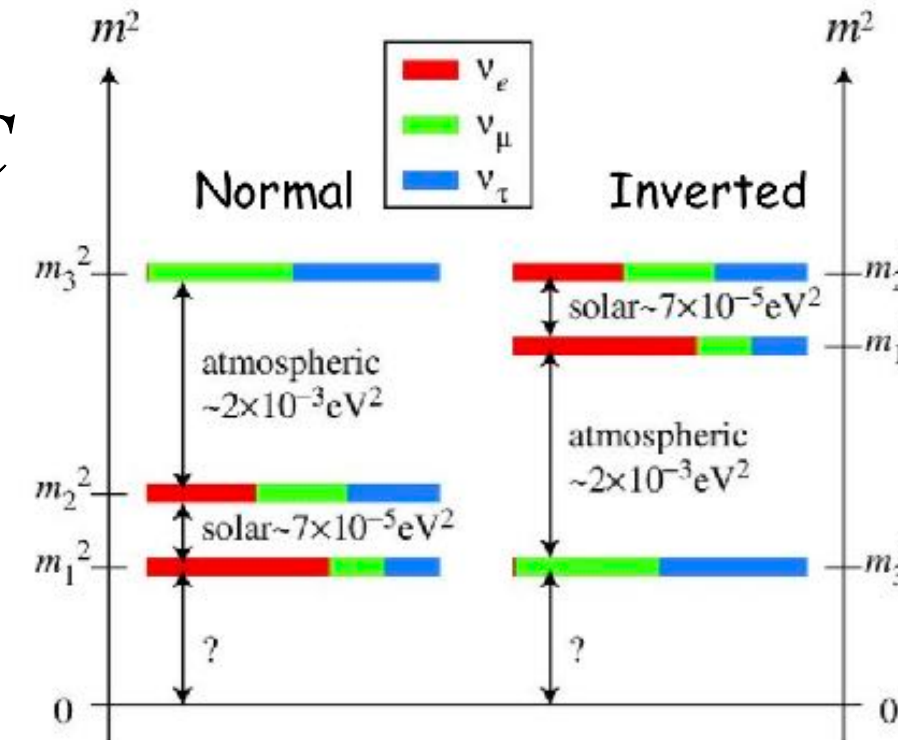


It follows that the dimension-5 Wilson coefficient is of order $C \sim \dfrac{1}{\Lambda}$ with $\Lambda \sim 10^{15}$ **GeV**

One one hand, that is perfect, because it suggests that the basic SMEFT assumption, $\Lambda \gg \mathrm{v}$, **is indeed satisfied**

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{1}{2}(\nu M \nu) + \text{h.c.} \quad M = -\,\mathrm{v}^2 C$$



However, $\Lambda \sim 10^{15}$ GeV leads to a *psychological* problem

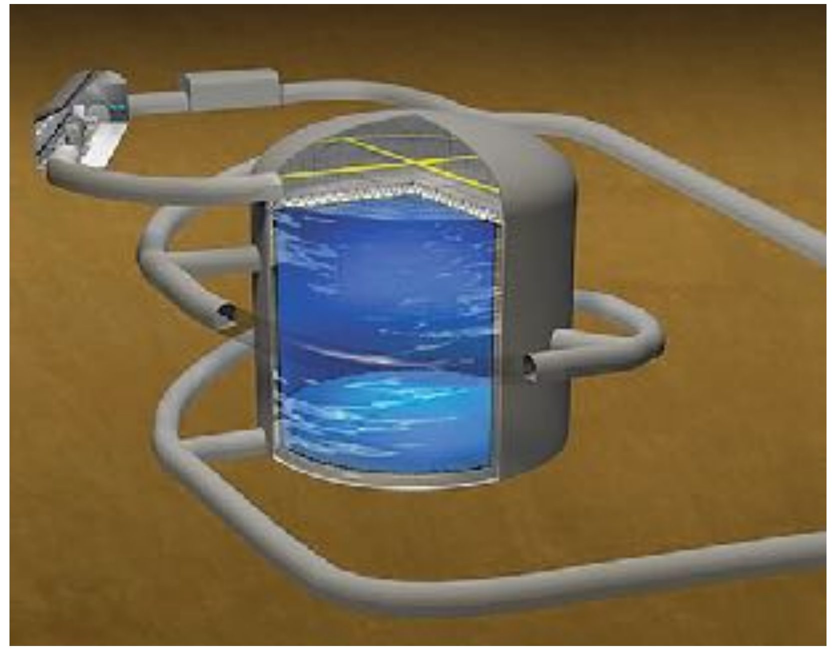
$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ then naive SMEFT counting suggest $\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}$, $\mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}$,

and so on

If this is really the correct estimate, then we will never see any other effects of higher-dimensional operators, except possibly of the baryon-number violating ones :/

Career opportunities



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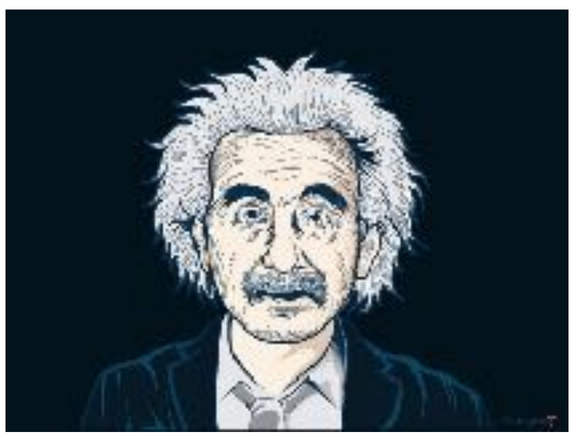
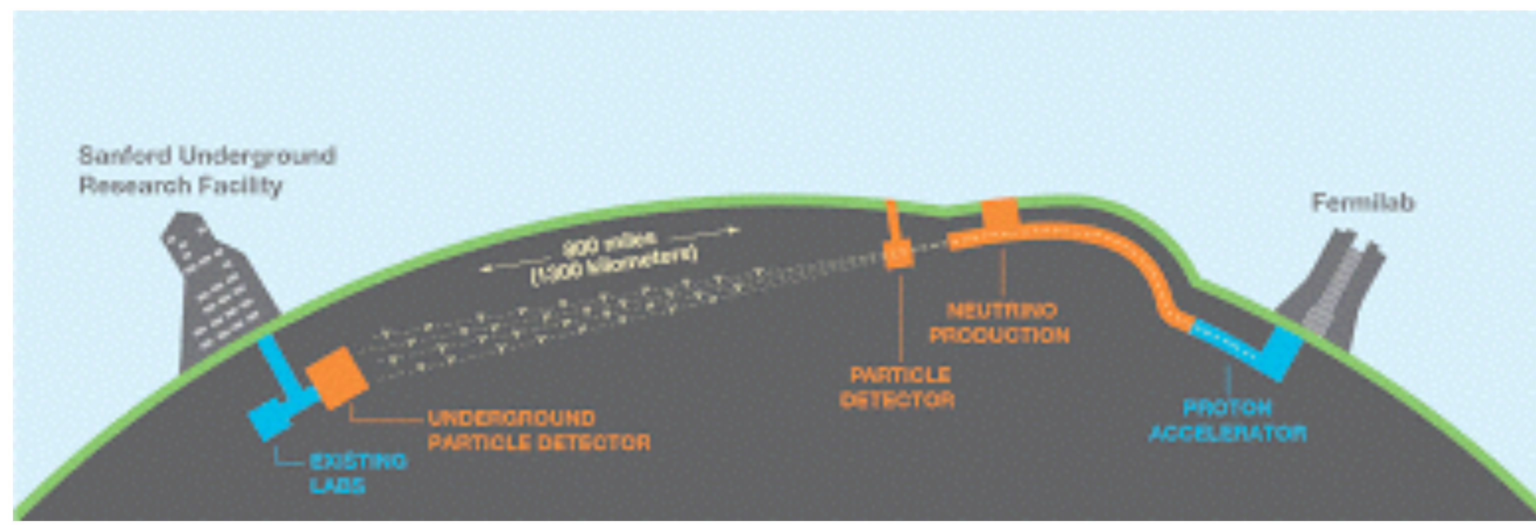
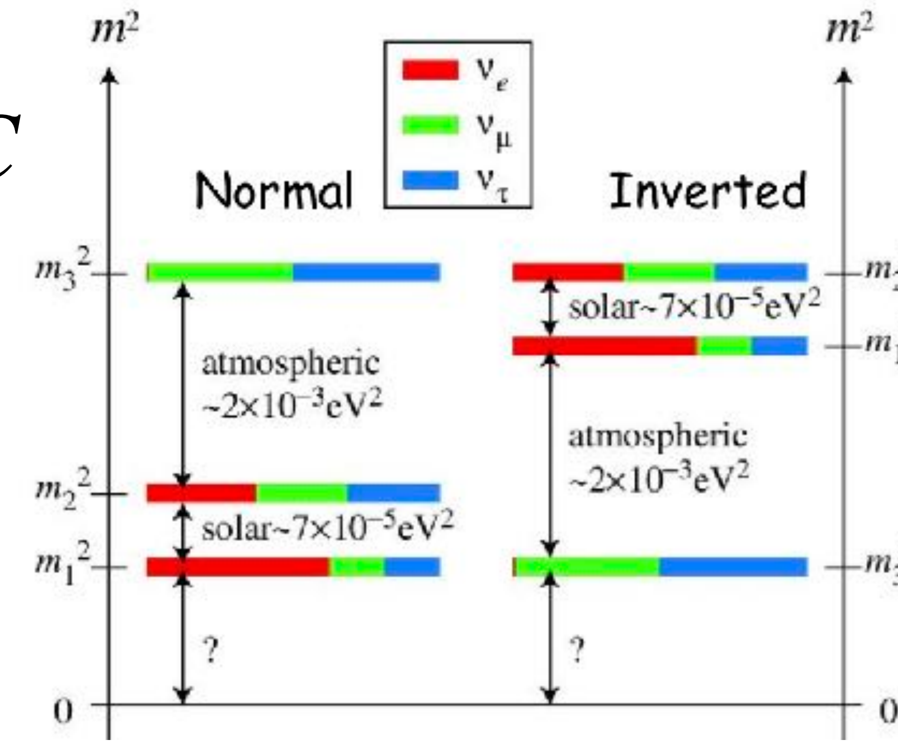




SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset -\frac{1}{2}(\nu M \nu) + \text{h.c.} \quad M = -\text{v}^2 C$$



$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

If $\mathscr{L}_{D=5} \sim \frac{1}{\Lambda}$ **then naive SMEFT counting suggest**

$$\mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda^3}, \ldots$$

However, this conclusion is not set in stone

It is possible that the true new physics scale is not far from TeV, but its coupling to the lepton sector is very small

Alternatively, it is possible (and likely) that there is more than one mass scale of new physics

Dimension-5 interactions are special because they violate lepton number L.

More generally, all odd-dimension SMEFT operators violate B-L

If we assume that the mass scale of new particles with B-L-violating interactions is Λ_L, **and there is also B-L-conserving new physics at the scale** $\Lambda \ll \Lambda_L$ **, then the estimate is**

$$\mathscr{L}_{D=5} \sim \frac{1}{\Lambda_L}, \mathscr{L}_{D=6} \sim \frac{1}{\Lambda^2}, \mathscr{L}_{D=7} \sim \frac{1}{\Lambda_L^3}, \mathscr{L}_{D=8} \sim \frac{1}{\Lambda^4},$$ **and so on**

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Grządkowski et al
arXiv:1008.4884

At dimension-6 all hell breaks loose

$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H(H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}|H^\dagger D_\mu H|^2 \\
& + C_{HWB}H^\dagger\sigma^k H\, W^k_{\mu\nu}B_{\mu\nu} + C_{HG}H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu} + C_{HW}H^\dagger H\, W^k_{\mu\nu}W^k_{\mu\nu} + C_{HB}H^\dagger H\, B_{\mu\nu}B_{\mu\nu} \\
& ++C_W\epsilon^{klm}W^k_{\mu\nu}W^l_{\nu\rho}W^m_{\rho\mu} + C_G f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu} \\
& + C_{H\widetilde{G}}H^\dagger H\,\widetilde{G}^a_{\mu\nu}G^a_{\mu\nu} + C_{H\widetilde{W}}H^\dagger H\,\widetilde{W}^k_{\mu\nu}W^k_{\mu\nu} + C_{H\widetilde{B}}H^\dagger H\,\widetilde{B}_{\mu\nu}B_{\mu\nu} + C_{H\widetilde{W}B}H^\dagger\sigma^k H\,\widetilde{W}^k_{\mu\nu}B_{\mu\nu} \\
& + C_{\widetilde{W}}\epsilon^{klm}\widetilde{W}^k_{\mu\nu}W^l_{\nu\rho}W^m_{\rho\mu} + C_{\widetilde{G}}f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L}HC_{eH}\bar{E}^c) + H^\dagger H(\bar{Q}\tilde{H}C_{uH}\bar{U}^c) + H^\dagger H(\bar{Q}HC_{dH}\bar{D}^c) \\
& + iH^\dagger\overleftrightarrow{D}_\mu H(\bar{L}C^{(1)}_{Hl}\bar{\sigma}^\mu L) + iH^\dagger\sigma^k\overleftrightarrow{D}_\mu H(\bar{L}C^{(3)}_{Hl}\bar{\sigma}^\mu\sigma^k L) + iH^\dagger\overleftrightarrow{D}_\mu H(E^cC_{He}\sigma^\mu\bar{E}^c) \\
& + iH^\dagger\overleftrightarrow{D}_\mu H(\bar{Q}C^{(1)}_{Hq}\bar{\sigma}^\mu Q) + iH^\dagger\sigma^k\overleftrightarrow{D}_\mu H(\bar{Q}C^{(3)}_{Hq}\bar{\sigma}^\mu\sigma^k Q) + iH^\dagger\overleftrightarrow{D}_\mu H(U^cC_{Hu}\sigma^\mu\bar{U}^c) \\
& + iH^\dagger\overleftrightarrow{D}_\mu H(D^cC_{Hd}\sigma^\mu\bar{D}^c) + \Big\{ i\tilde{H}^\dagger D_\mu H(U^cC_{Hud}\sigma^\mu\bar{D}^c) \\
& + (\bar{Q}\sigma^k\tilde{H}C_{uW}\bar{\sigma}^{\mu\nu}\bar{U}^c)W^k_{\mu\nu} + (\bar{Q}\tilde{H}C_{uB}\bar{\sigma}^{\mu\nu}\bar{U}^c)B_{\mu\nu} + (\bar{Q}\tilde{H}C_{uG}T^a\bar{\sigma}^{\mu\nu}\bar{U}^c)G^a_{\mu\nu} \\
& + (\bar{Q}\sigma^k HC_{dW}\bar{\sigma}^{\mu\nu}\bar{D}^c)W^k_{\mu\nu} + (\bar{Q}HC_{dB}\bar{\sigma}^{\mu\nu}\bar{D}^c)B_{\mu\nu} + (\bar{Q}HC_{dG}T^a\bar{\sigma}^{\mu\nu}\bar{D}^c)G^a_{\mu\nu} \\
& + (\bar{L}\sigma^k HC_{eW}\bar{\sigma}^{\mu\nu}\bar{E}^c)W^k_{\mu\nu} + (\bar{L}HC_{eB}\bar{\sigma}^{\mu\nu}\bar{E}^c)B_{\mu\nu} + \text{h.c.}\Big\} + \mathscr{L}^{4-\text{fermion}}_{D=6}
\end{aligned}$$

$|H|^2 B_{\mu\nu}\widetilde{B}_{\mu\nu}$
$|H|^2 G^a_{\mu\nu}\widetilde{G}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}\widetilde{W}^a_{\mu\nu}$
$|H|^2 W^a_{\mu\nu}W^a_{\mu\nu}$
$|H|^2 B_{\mu\nu}B_{\mu\nu}$
$G^a_{\mu\nu}G^a_{\nu\rho}\widetilde{G}^a_{\rho\mu}$
$|H|^6$
$|H|^2 G^a_{\mu\nu}G^a_{\mu\nu}$

SMEFT at dimension-6

$$\mathscr{L}_{\text{SMEFT}} \supset \sum_X C_X O_X$$

Bosonic CP-even operators

$$O_H = (H^\dagger H)^3$$

$$O_{H\Box} = (H^\dagger H)\,\Box\,(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

$$O_{HG} = H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$$

$$O_{HW} = H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu}$$

$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

SMEFT at dimension-6

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$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

These affect single Higgs boson couplings to SM gauge bosons. For example

$$C_{HG} H^\dagger H G^a_{\mu\nu} G^a_{\mu\nu} = C_{HG} \frac{(\mathrm{v}+h)^2}{2} G^a_{\mu\nu} G^a_{\mu\nu} \to \mathrm{v} C_{HG} h G^a_{\mu\nu} G^a_{\mu\nu}$$

For operators inducing couplings to photons and gluons bounds of order $|C| \lesssim \dfrac{1}{(10\ \mathrm{TeV})^2}$ **, while** $|C_{HD}| \lesssim \dfrac{1}{(\mathrm{TeV})^2}$ **from Higgs physics alone**

SMEFT at dimension-6

Bosonic CP-even operators

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$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

Peculiar effect...

Contributes to the kinetic term of the Higgs boson

$$C_{H\Box}(H^\dagger H)\,\Box\,(H^\dagger H) \to -\,\mathrm{v}^2 C_{H\Box}(\partial_\mu h)^2$$

Together with the SM kinetic term:

$$\mathcal{L}_{\mathrm{SMEFT}} \supset \frac{1}{2}(\partial_\mu h)^2\left(1 - 2\mathrm{v}^2 C_{H\Box}\right)$$

To restore canonical normalization, we need to rescale the Higgs boson field:

$$h \to h\left(1 + \mathrm{v}^2 C_{H\Box}\right)$$

All Higgs boson couplings present in the SM are modified in a universal way!

$$\frac{h}{\mathrm{v}}\left[2m_W^2 W^+_\mu W^-_\mu + m_Z^2 Z_\mu Z_\mu\right] \to \frac{h}{\mathrm{v}}\left(1 + \mathrm{v}^2 C_{H\Box}\right)\left[2m_W^2 W^+_\mu W^-_\mu + m_Z^2 Z_\mu Z_\mu\right]$$

$$\frac{h}{\mathrm{v}} m_f \bar{f} f \to \frac{h}{\mathrm{v}}\left(1 + \mathrm{v}^2 C_{H\Box}\right) m_f \bar{f} f$$

Bounds of order $|C_{H\Box}| \lesssim \dfrac{1}{(\mathrm{TeV})^2}$

SMEFT at dimension-6

Bosonic CP-even operators

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$$O_{H\Box} = (H^\dagger H)\,\Box\,(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

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$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

Affects cubic Higgs boson coupling

$$C_H (H^\dagger H)^3 = \frac{C_H}{8}(\mathrm{v}+h)^6 \to \frac{5\mathrm{v}C_H}{2} h^3$$

Currently weak bounds of order $|C_H| \lesssim \frac{1}{\mathrm{v}^2}$

SMEFT at dimension-6

Bosonic CP-even operators

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$$O_{H\Box} = (H^\dagger H)\,\Box\,(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

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$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

Induce anomalous triple gauge couplings
Bounds on the electroweak ones lead to

$$|C_W| \lesssim \frac{1}{(3\mathrm{TeV})^2},$$

bounds on the gluon ones much weaker

SMEFT at dimension-6

Bosonic CP-even operators

$$O_H = (H^\dagger H)^3$$

$$O_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

$$O_{HG} = H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$$

$$O_{HW} = H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu}$$

$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

These affect electroweak precision observables (W boson mass, Z branching fractions), which are measured at per-mille level at LEP

Bounds of order $|C| \lesssim \dfrac{1}{(10\ \mathrm{TeV})^2}$

SMEFT at dimension-6

Bosonic CP-even operators

$$O_H = (H^\dagger H)^3$$

$$O_{H\Box} = (H^\dagger H)\,\Box\,(H^\dagger H)$$

$$O_{HD} = |H^\dagger D_\mu H|^2$$

$$O_{HG} = H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$$

$$O_{HW} = H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu}$$

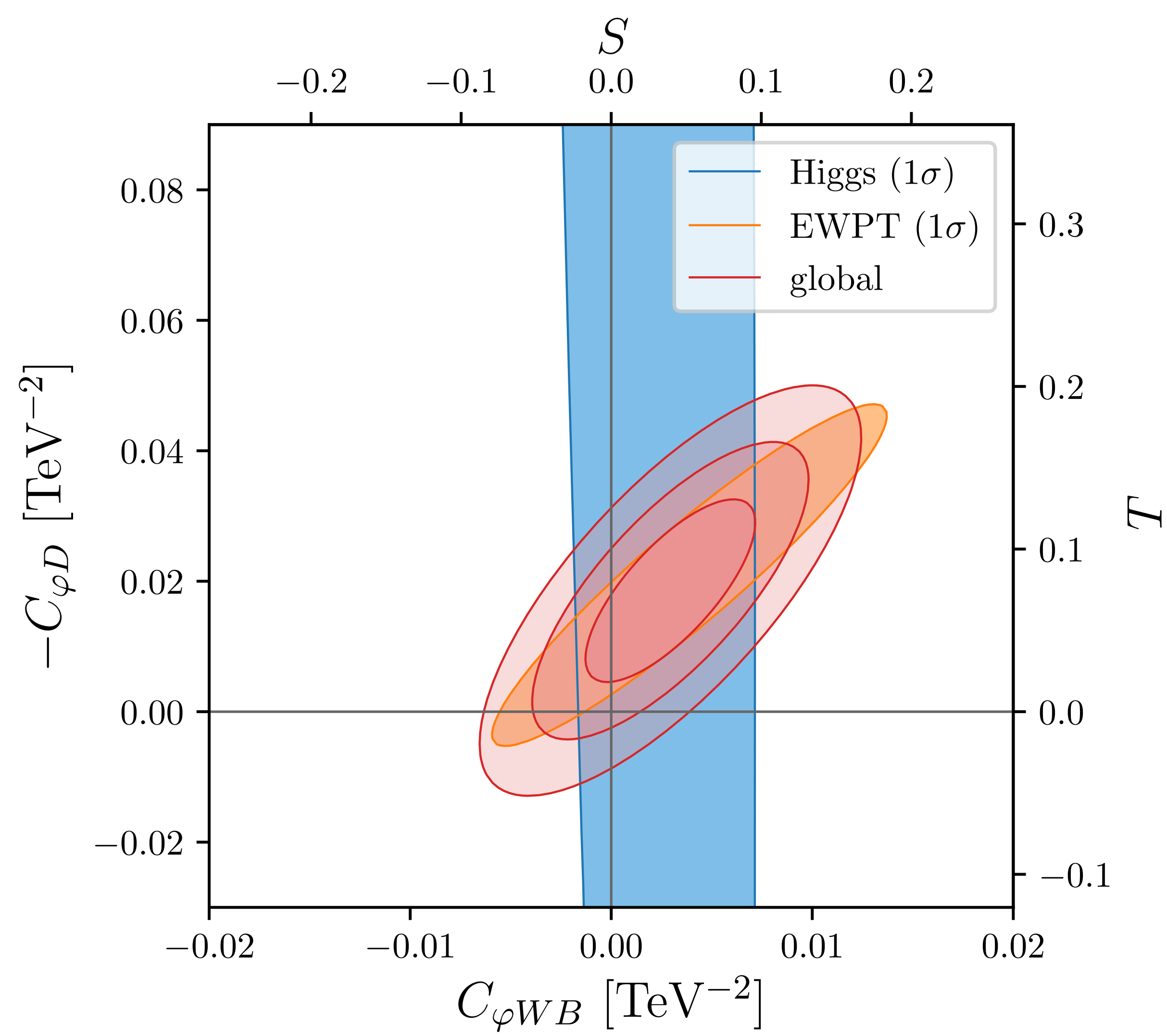
$$O_{HB} = H^\dagger H\, B_{\mu\nu} B_{\mu\nu}$$

$$O_{HWB} = H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu}$$

$$O_W = \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu}$$

$$O_G = f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}$$

Similar constraining power of Higgs and electroweak constraints on these particular operators Interesting synergy



SMEFT at dimension-6

$$+C_{H\widetilde{G}}H^\dagger H\,\widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}+C_{H\widetilde{W}}H^\dagger H\,\widetilde{W}^k_{\mu\nu}W^k_{\mu\nu}+C_{H\widetilde{B}}H^\dagger H\,\widetilde{B}_{\mu\nu}B_{\mu\nu}$$
$$+C_{H\widetilde{W}B}H^\dagger\sigma^k H\,\widetilde{W}^k_{\mu\nu}B_{\mu\nu}+C_{\widetilde{W}}\epsilon^{klm}\widetilde{W}^k_{\mu\nu}W^l_{\nu\rho}W^m_{\rho\mu}+C_{\widetilde{G}}f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu},$$

These affect single Higgs boson couplings to SM gauge bosons, and triple gauge couplings But also, via loop effects other CP observables, such as e.g. electron EDMs

SMEFT at dimension-6

$$\mathcal{L}_{\mathrm{SMEFT}} \supset \sum_{I,J=1}^{3} [O_{fH}]_{IJ}[C_{fH}]_{IJ} + \mathrm{h.c.}$$

Yukawa-like operators

$$O_{eH} = H^\dagger H(\bar{L}H\bar{E}^c)$$

$$O_{uH} = H^\dagger H(\bar{Q}\tilde{H}\bar{U}^c)$$

$$O_{dH} = H^\dagger H(\bar{Q}H\bar{D}^c)$$

These affect single Higgs boson couplings to SM fermions. Bounds depends on the flavor but typically don't exceed $|C| \lesssim \frac{1}{(1\ \mathrm{TeV})^2}$

SMEFT at dimension-6

Vertex-like operators

$$O_{Hl}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu L)$$

$$O_{Hl}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L}\bar{\sigma}^\mu \sigma^k L)$$

$$O_{He} = iH^\dagger \overleftrightarrow{D}_\mu H(E^c \sigma^\mu \bar{E}^c)$$

$$O_{Hq}^{(1)} = iH^\dagger \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu Q)$$

$$O_{Hq}^{(3)} = iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q}\bar{\sigma}^\mu \sigma^k Q)$$

$$O_{Hu} = iH^\dagger \overleftrightarrow{D}_\mu H(U^c \sigma^\mu \bar{U}^c)$$

$$O_{Hd} = iH^\dagger \overleftrightarrow{D}_\mu H(D^c \sigma^\mu \bar{D}^c)$$

$$O_{Hud} = i\tilde{H}^\dagger D_\mu H(U^c \sigma^\mu \bar{D}^c)$$

These affect electroweak precision observables (W boson mass, Z branching fractions), which are measured at per-mille level at LEP

Bounds of order $|C| \lesssim \frac{1}{(10\ \mathrm{TeV})^2}$

SMEFT at dimension-6

$$
\begin{aligned}
\mathcal{L}_{D=6}^{\text{dipole}} =& (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
&+ (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
&+ (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \text{h.c.} \qquad (
\end{aligned}
$$

These affect anomalous magnetic and electric moments of SM particles at tree level
Bounds depend on flavor and can be very strong, especially for the first generation

$$\sigma_{\mu\nu} = \frac{i}{2}\left[\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu\right] \qquad \bar{\sigma}_{\mu\nu} = \frac{i}{2}\left[\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu\right]$$

SMEFT at dimension-6

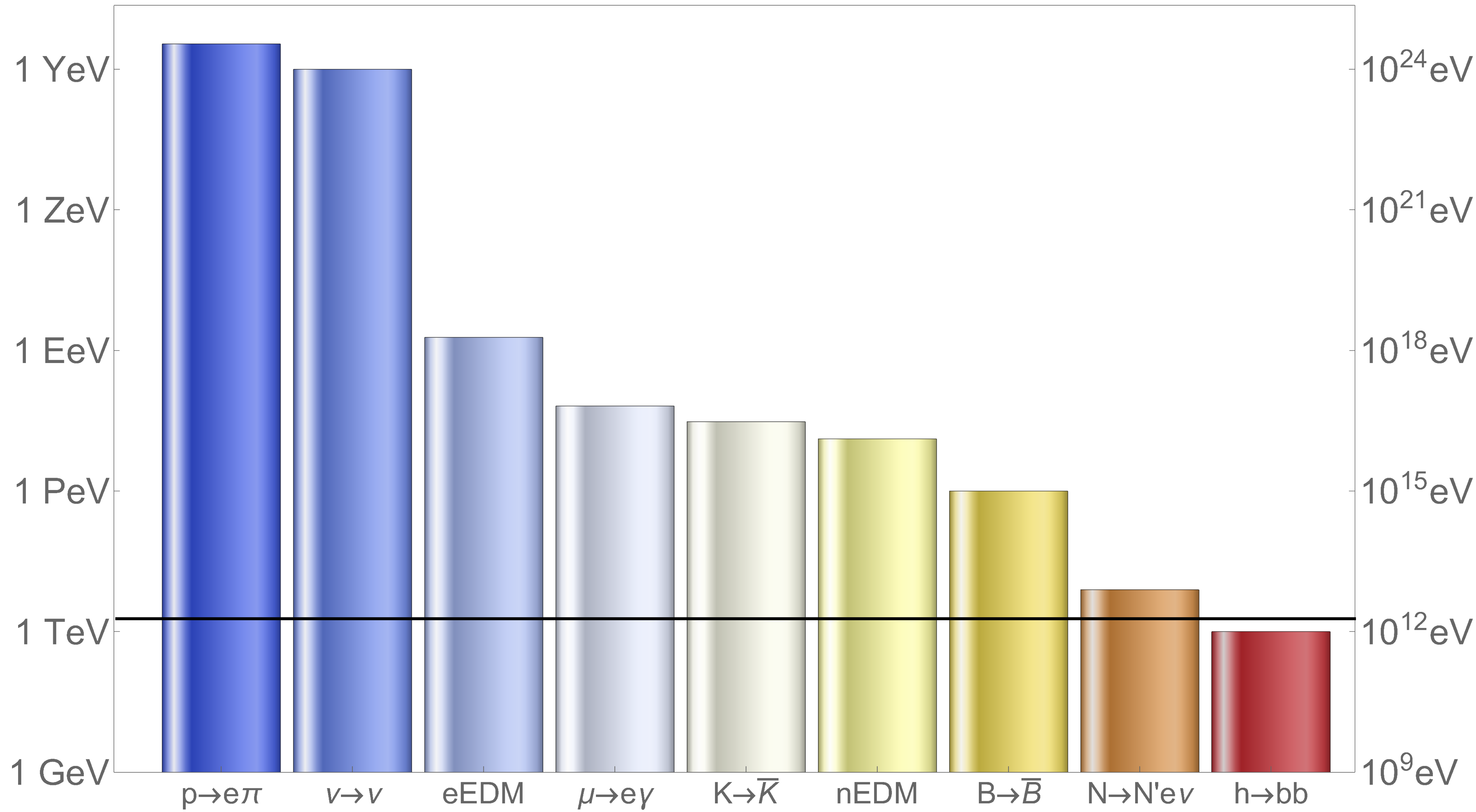
4-fermion operators

$$
\begin{aligned}
\mathscr{L}^{4-\text{fermion}}_{D=6} = & (\bar{L}\bar{\sigma}^\mu L)C_{ll}(\bar{L}\bar{\sigma}_\mu L) + (E^c\sigma_\mu \bar{E}^c)C_{ee}(E^c\sigma_\mu \bar{E}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{le}(E^c\sigma_\mu \bar{E}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C^{(1)}_{lq}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{L}\bar{\sigma}^\mu \sigma^k L)C^{(3)}_{lq}(\bar{Q}\bar{\sigma}_\mu \sigma^k Q) \\
& + (E^c\sigma_\mu \bar{E}^c)C_{eu}(U^c\sigma_\mu \bar{U}^c) + (E^c\sigma_\mu \bar{E}^c)C_{ed}(D^c\sigma_\mu \bar{D}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C_{lu}(U^c\sigma_\mu \bar{U}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{ld}(D^c\sigma_\mu \bar{D}^c) + (E^c\sigma_\mu \bar{E}^c)C_{eq}(Q\bar{\sigma}_\mu Q) \\
& + \left\{ (\bar{L}\bar{E}^c)C_{ledq}(D^c Q) + \epsilon^{kl}(\bar{L}^k\bar{E}^c)C^{(1)}_{lequ}(\bar{Q}^l\bar{U}^c) + \epsilon^{kl}(\bar{L}^k\bar{\sigma}^{\mu\nu}\bar{E}^c)C^{(3)}_{lequ}(\bar{Q}^l\bar{\sigma}^{\mu\nu}\bar{U}^c) + \mathrm{h.c.} \right\} \\
& + (\bar{Q}\bar{\sigma}^\mu Q)C^{(1)}_{qq}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{Q}\bar{\sigma}^\mu \sigma^k Q)C^{(3)}_{qq}(\bar{Q}\bar{\sigma}_\mu \sigma^k Q) \\
& + (U^c\sigma_\mu \bar{U}^c)C_{uu}(U^c\sigma_\mu \bar{U}^c) + (D^c\sigma_\mu \bar{D}^c)C_{dd}(D^c\sigma_\mu \bar{D}^c) \\
& + (U^c\sigma_\mu \bar{U}^c)C^{(1)}_{ud}(D^c\sigma_\mu \bar{D}^c) + (U^c\sigma_\mu T^a\bar{U}^c)C^{(8)}_{ud}(D^c\sigma_\mu T^a\bar{D}^c) \\
& + (Q^c\sigma_\mu \bar{Q}^c)C^{(1)}_{qu}(U^c\sigma_\mu \bar{U}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C^{(8)}_{qu}(U^c\sigma_\mu T^a\bar{U}^c)] \\
& + (Q^c\sigma_\mu \bar{Q}^c)C^{(1)}_{qd}(D^c\sigma_\mu \bar{D}^c) + (Q^c\sigma_\mu T^a\bar{Q}^c)C^{(8)}_{qd}(D^c\sigma_\mu T^a\bar{D}^c) \\
& + \left\{ \epsilon^{kl}(\bar{Q}^k\bar{U}^c)C^{(1)}_{quqd}(\bar{Q}^l\bar{D}^c) + \epsilon^{kl}(\bar{Q}^k T^a\bar{U}^c)C^{(1)}_{quqd}(\bar{Q}^l T^a\bar{D}^c) + \mathrm{h.c.} \right\} \\
& + \left\{ (D^c U^c)C_{duq}(\bar{Q}\bar{L}) + (QQ)C_{qqu}(\bar{U}^c\bar{E}^c) + (QQ)C_{qqq}(QL) + (D^c U^c)C_{duu}(U^c E^c) + \mathrm{h.c.} \right\}.
\end{aligned}
$$

These affect a wide range of physics.
Bounds can be very strong, especially for baryon-number violating operators and for certain flavor- or lepton-flavor-violating operators

SMEFT up to dimension-6

SMEFT Lagrangian up to dimension-6 provides a convenient framework for a bulk of precision physics happening today.
In particular, it allows one to quantify the strength of different observables



SMEFT up to dimension-6

SMEFT Lagrangian up to dimension-6 provides a convenient framework for a bulk of precision physics happening today.

Moreover, it leads to correlations between different observables, e.g. due to $SU(2)_W$ symmetry relating charged and neutral currents, and due to the interplay of tree- and loop-level contributions to observables

→ **Importance of global fits collecting results from different types of experiments !**

Global fits with SMEFT up to dimension-6

$$
\begin{pmatrix}
[\delta g_L^{Wl}]_{ee}\\
[\delta g_L^{Wl}]_{\mu\mu}\\
[\delta g_L^{Wl}]_{\tau\tau}\\
[\delta g_L^{Ze}]_{ee}\\
[\delta g_R^{Ze}]_{ee}\\
[\delta g_L^{Ze}]_{\mu\mu}\\
[\delta g_R^{Ze}]_{\mu\mu}\\
[\delta g_L^{Ze}]_{\tau\tau}\\
[\delta g_R^{Ze}]_{\tau\tau}\\
[\delta g_R^{Wq}]_{11}\\
[\delta g_L^{Zu}]_{11}\\
[\delta g_R^{Zu}]_{11}\\
[\delta g_L^{Zd}]_{11}\\
[\delta g_R^{Zd}]_{11}\\
[\delta g_L^{Zu}]_{22}\\
[\delta g_R^{Zu}]_{22}\\
[\delta g_L^{Zd}]_{22}\\
[\delta g_R^{Zd}]_{22}\\
[\delta g_L^{Zd}]_{33}\\
[\delta g_R^{Zd}]_{33}
\end{pmatrix}
=
\begin{pmatrix}
-1.8(2.6)\\
-0.6(2.2)\\
0.2(3.5)\\
-0.21(28)\\
-0.42(27)\\
0.2(1.2)\\
0.0(1.4)\\
-0.09(59)\\
0.61(62)\\
-3.8(8.1)\\
-7(22)\\
4(29)\\
-13(35)\\
10(120)\\
-1.5(3.6)\\
-3.3(5.3)\\
14(27)\\
34(46)\\
3.2(1.7)\\
22(8.8)
\end{pmatrix}
\times 10^{-3},
\qquad
\begin{pmatrix}
[c_{ll}]_{eeee}\\
[c_{le}]_{eeee}\\
[c_{ee}]_{eeee}\\
[c_{ll}]_{e\mu\mu e}\\
[c_{ll}]_{ee\mu\mu}\\
[c_{le}]_{e\mu\mu e}\\
[c_{le}]_{ee\mu\mu}\\
[c_{le}]_{\mu\mu ee}\\
[c_{ee}]_{ee\mu\mu}\\
[c_{ll}]_{e\tau\tau e}\\
[c_{ll}]_{ee\tau\tau}\\
[c_{le}]_{ee\tau\tau}\\
[c_{le}]_{\tau\tau ee}\\
[c_{ee}]_{ee\tau\tau}\\
[\hat{c}_{ll}]_{\mu\mu\mu\mu}\\
[c_{ll}]_{\mu\tau\tau\mu}\\
[c_{le}]_{\mu\tau\tau\mu}
\end{pmatrix}
=
\begin{pmatrix}
1.03(38)\\
-0.22(22)\\
0.19(38)\\
-0.56(80)\\
0.1(2.0)\\
11.4(6.8)\\
0.3(2.2)\\
-0.2(2.1)\\
0.2(2.3)\\
-0.60(68)\\
2(11)\\
-2.3(7.2)\\
1.7(7.2)\\
-1(12)\\
2(21)\\
1.5(1.9)\\
19(15)
\end{pmatrix}
\times 10^{-2},
$$

Ingredients

- e^+e^- collisions
- W boson mass and decays
- Drell-Yan at LHC and Tevatron
- Neutrino scattering on electrons
- Atomic parity violation
- Parity-violating electron scattering
- Nuclear beta decays
- Semi-leptonic decays of pions and kaons
- Trident muon production in ν scattering
- Leptonic and hadronic tau decays
- ν scattering on nuclei (coherent to not)

$$
\begin{pmatrix}
[c_{lq}^{(3)}]_{ee11}\\
[\hat{c}_{eq}]_{ee11}\\
[\hat{c}_{lu}]_{ee11}\\
[\hat{c}_{ld}]_{ee11}\\
[\hat{c}_{eu}]_{ee11}\\
[\hat{c}_{ed}]_{ee11}\\
[c_{lequ}^{(1)}]_{ee11}\\
[c_{ledq}]_{ee11}\\
[c_{lequ}^{(3)}]_{ee11}\\
[\hat{c}_{lq}^{(3)}]_{ee22}\\
[c_{lu}]_{ee22}\\
[\hat{c}_{ld}]_{ee22}\\
[c_{eq}]_{ee22}\\
[c_{eu}]_{ee22}\\
[\hat{c}_{ed}]_{ee22}\\
[\hat{c}_{lq}^{(3)}]_{ee33}\\
[c_{ld}]_{ee33}\\
[c_{eq}]_{ee33}\\
[c_{ed}]_{ee33}
\end{pmatrix}
=
\begin{pmatrix}
0.1(2.8)\\
-4(30)\\
-2.5(8.7)\\
-2(18)\\
-3.1(9.4)\\
-2(17)\\
-0.017(60)\\
-0.018(57)\\
0.023(66)\\
-61(32)\\
2.4(8.0)\\
-300(130)\\
-21(28)\\
-87(46)\\
250(140)\\
-8.5(8.0)\\
-1(10)\\
-3.1(5.1)\\
18(20)
\end{pmatrix}
\times 10^{-2}\,,
\qquad
\begin{pmatrix}
[c_{lq}^{(3)}]_{\mu\mu 11}\\
[c_{lq}]_{\mu\mu 11}\\
[c_{lu}]_{\mu\mu 11}\\
[c_{ld}]_{\mu\mu 11}\\
[\hat{c}_{eq}]_{\mu\mu 11}\\
\epsilon_P^{d\mu}(2\ \text{GeV})\\
[c_{lq}^{(3)}]_{\tau\tau 11}\\
[c_{lequ}^{(3)}]_{\tau\tau 11}\\
\epsilon_P^{d\tau}(2\ \text{GeV})
\end{pmatrix}
=
\begin{pmatrix}
3.0(3.5)\\
-0.2(5.8)\\
2.5(6.5)\\
5(24)\\
3(41)\\
-0.080(95)\\
-0.3(2.8)\\
-0.3(1.2)\\
0.93(85)
\end{pmatrix}
\times 10^{-2}.
$$

Breso-Pla et al
arXiv:2301.07036

Correlation matrix

Only 65 dimension-6 Wilson coefficients simultaneously constrained in this fit. Can do better :)

SMEFT at higher dimensions

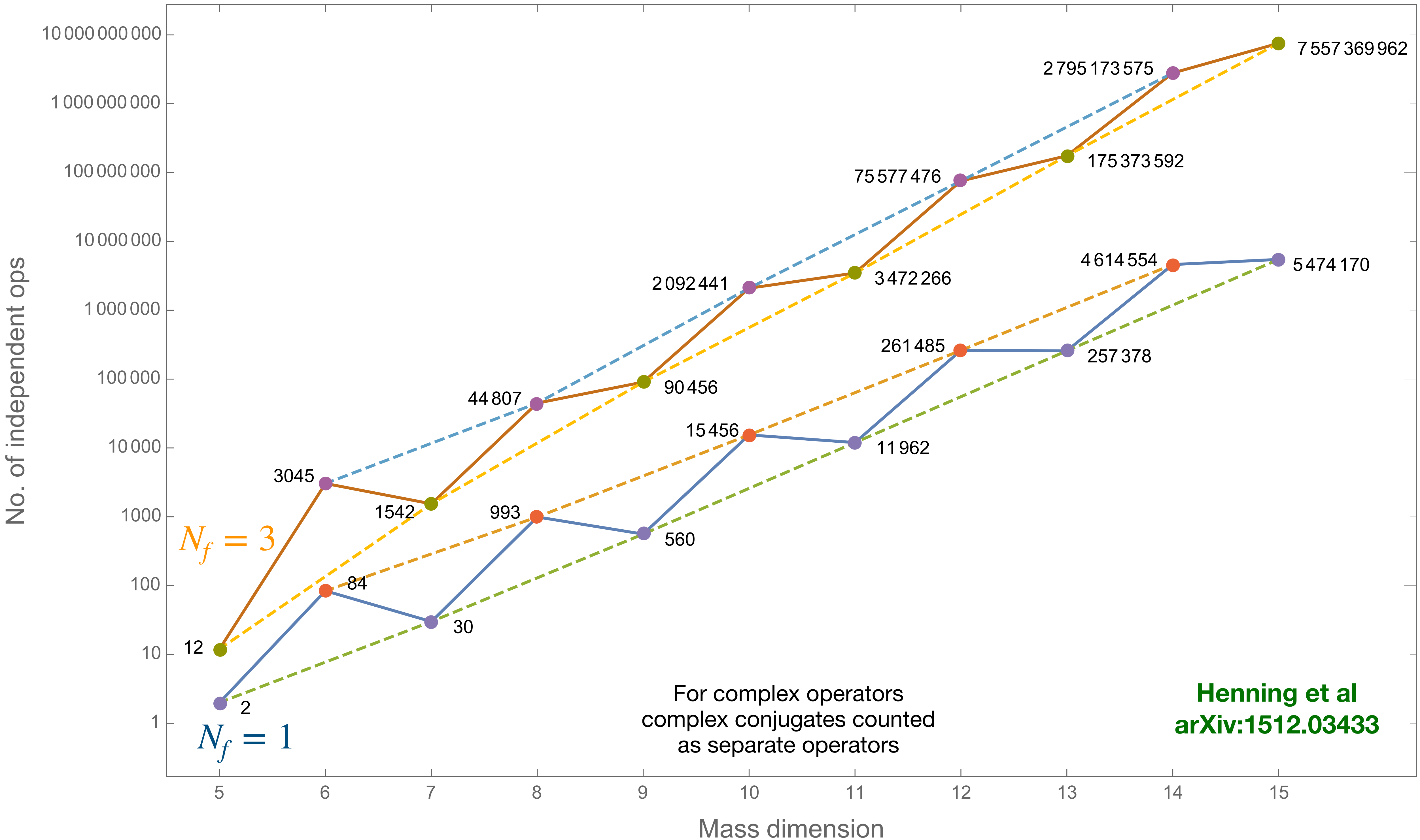
$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

Number of baryon-number-conserving operators as function of D and number of generations N_f

	$N_f=0$	$N_f=1$	$N_f=2$	$N_f=3$	...
Dimension-5	0	2	6	12	...
Dimension-6	15	76	582	2499	...
Dimension-7	0	22	212	948	...
Dimension-8	89	895	8251	36971	...
...	...	...	...	...	...

SMEFT at higher dimensions

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$



Exponential growth of the number of operators with the canonical dimension D

SMEFT at higher dimensions

SMEFT at dimension-5: Weinberg (1979) Phys. Rev. Lett. 43, 1566

SMEFT at dimension-6: Grzadkowski et al arXiv: 1008.4884

SMEFT at dimension-7: Lehman arXiv: 1410.4193

SMEFT at dimension-8: Li et al arXiv: 2005.00008

SMEFT at dimension-9: Li et al arXiv: 2012.09188

Code to generate a basis at arbitrary dimension in SMEFT: Li et al arXiv:2201.04639

Beyond dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

At tree level, light-by-light scattering receives contribution from dimension-8, which in some situations may compete with lower order loop contributions

$$\mathscr{L}_{D=8} \supset (B_{\mu\nu}B_{\mu\nu})^2 + \ldots$$

Neutron-antineutron oscillations arise at dimension-9

$$\mathscr{L}_{D=9} \supset \epsilon_{abc}\epsilon_{def}(\bar{d}_a\bar{d}_d)(q_b q_e)(q_c q_f) + \ldots$$

CP violating 3Z vertex in SMEFT from integrating out 2HDM arises via a dimension-12 operator!

$$\mathscr{L}_{D=12} \supset C_{12}[H^\dagger D^2(HH^\dagger H)]^2 + \mathrm{h.c.}$$

In all such cases however, you need to argue validity of your EFT and why you don't expect any larger effects of new physics from operators of lower dimensions

Beyond dimension-6

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

You need to be aware of the existence of higher-dimensional operators, whenever you need to argue validity of the EFT description

Moreover, a qualitatively new phenomenon may arise at higher dimensions

If experiment pinpoints a coefficient of some operators of dimension-6, then subleading dimension-8 operators will provide precious information

$$C_6 \sim \frac{g_*^2}{M^2} \qquad C_8 \sim \frac{g_*^2}{M^4}$$

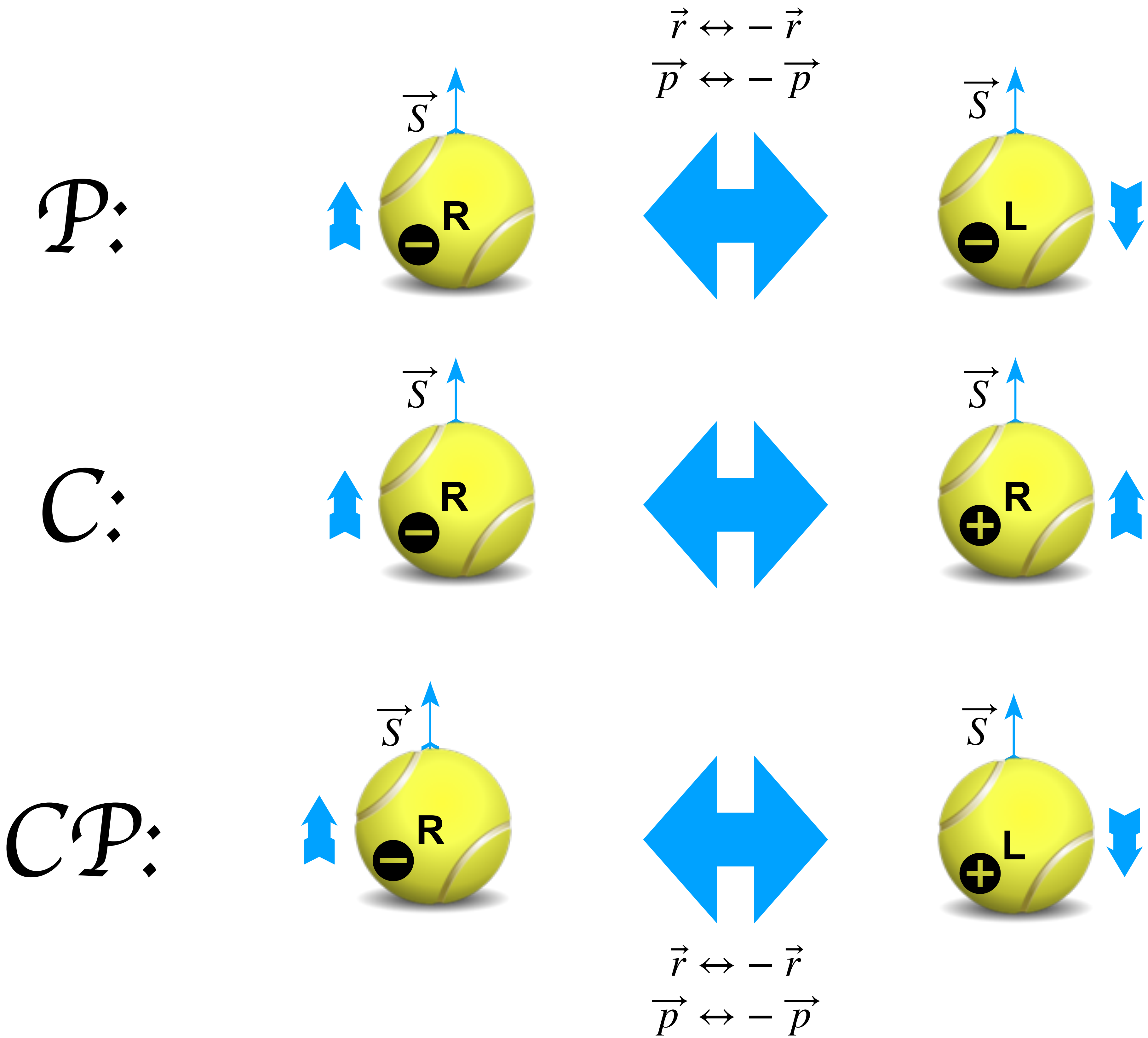
Only determines coupling over mass scala of new physics

May allow disentangle coupling and mass

Part 3

CP violation in SMEFT

What is CP



$\vec{r} \leftrightarrow -\vec{r}$
$\vec{p} \leftrightarrow -\vec{p}$
$\mathcal{P}$:
$\vec{S}$
R
L
C:
$\vec{S}$
R
R
$C\mathcal{P}$:
$\vec{S}$
R
L
$\vec{r} \leftrightarrow -\vec{r}$
$\vec{p} \leftrightarrow -\vec{p}$

CP formalism

CP on spin-0 scalars

annihilates particle — creates antiparticle

$$\Phi(x) = \int \frac{d^3k}{(2\pi)^3 2E_k}\left[a(k)e^{-ikx} + b^\dagger(k)e^{ikx}\right]$$

$$\Phi^\dagger(x) = \int \frac{d^3k}{(2\pi)^3 2E_k}\left[b(k)e^{-ikx} + a^\dagger(k)e^{ikx}\right]$$

annihilates antiparticle — creates antiparticle

Define charge conjugation as operator exchanging particles and antiparticles

$$Ca(k)C^{-1} = b(k)$$

$$Cb(k)C^{-1} = a(k)$$

$$\boxed{C : \quad C\Phi(x)C^{-1} = \Phi^\dagger(x)}$$

$$\boxed{P := P\Phi(t,\boldsymbol{x})P^{-1} \to \eta\Phi(t,-\boldsymbol{x})}$$

Parity act trivially on scalars, and flips sign for pseudo-scalars

$$CP : \quad (CP)\Phi(t,\boldsymbol{x})(CP)^{-1} = \eta\Phi^\dagger(t,-\boldsymbol{x})$$

In SMEFT

$$CP : \quad H(t,\boldsymbol{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} G_1(t,\boldsymbol{x}) + iG_2(t,\boldsymbol{x}) \\ v + h(t,\boldsymbol{x}) + iG_3(t,\boldsymbol{x})\end{pmatrix} \to H^\dagger(t,-\boldsymbol{x}) = \frac{1}{\sqrt{2}}\begin{pmatrix} G_1(t,-\boldsymbol{x}) - iG_2(t,-\boldsymbol{x}) \\ v + h(t,-\boldsymbol{x}) - iG_3(t,-\boldsymbol{x})\end{pmatrix}$$

In particular the Higgs boson is CP even

2-component fermions

4-component Dirac fermion $\Psi_a \quad a = 1\ldots4$ **describes a pair of spin 1/2 fermions**

Convenient for P and C conserving theories, like QED and QCD
Extremely inconvenient when Majorana fermions are involved, or when discrete symmetries are discussed

Split the Dirac fermion into halves:

$$\Psi_a = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^c_{\dot{\alpha}} \end{pmatrix} \qquad \begin{matrix} \alpha = 1,2 \\ \dot{\alpha} = 1,2 \end{matrix}$$

The 2 halves transform independently under the Lorentz symmetry.
The Lorentz algebra is equivalent to SU(2)xSU(2):
- upper 2-component spinor ψ **transforms under the first SU(2),**
- lower 2-component spinor $\bar{\psi}^c$ **transforms under the second SU(2)**

Thus the 2-component spinors are fundamental building blocks

In the 2-component language:

Dirac mass: $\mathscr{L} = m\,\psi^c\psi + m\,\bar{\psi}\bar{\psi}^c$

Majorana mass: $\mathscr{L} = M\,\psi\psi + M\,\bar{\psi}\bar{\psi}$

By convention, I'll be always working in the basis where the masses are real

Parity for 2-component fermions

4-component Dirac fermion $\Psi_a \qquad a = 1\ldots 4$

Convenient for P and C conserving theories, like QED and QCD
Extremely inconvenient when Majorana fermions are involved,
or when discrete symmetries are discussed

Split the Dirac fermion into halves:

$$\Psi_a = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^c_{\dot{\alpha}} \end{pmatrix} \qquad \begin{array}{l} \alpha = 1,2 \\ \dot{\alpha} = 1,2 \end{array}$$

Thus the 2-component spinors are fundamental building blocks

At high energies, E>>m,
ψ **describes spin 1/2 particle with negative helicity (left-handed),**
$\bar{\psi}^c$ **describes spin 1/2 particle with positive helicity (right-handed).**

$$P\psi_\alpha(t,\boldsymbol{x})P^{-1} = \bar{\psi}^{c\,\dot{\alpha}}(t,-\boldsymbol{x}),$$

$$P\psi^c_\alpha(t,\boldsymbol{x})P^{-1} = -\bar{\psi}^{\dot{\alpha}}(t,-\boldsymbol{x}),$$

exchanges left and right,
thus it corresponds to parity

2-component fermions

$$\Psi = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(k,h) u(k,h) e^{-ikx} + b^\dagger(k,h) v(k,h) e^{ikx} \right] \quad u = \begin{pmatrix} x \\ \bar{y} \end{pmatrix} \qquad v = \begin{pmatrix} y \\ \bar{x} \end{pmatrix}$$

4-component spinor wave functions

The same in terms of 2-component spinor

annihilates particle — **creates antiparticle**

$$\psi = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[a(k,h) x(k,h) e^{-ikx} + b^\dagger(k,h) y(k,h) e^{ikx} \right]$$

$$\psi^c = \sum_{h=\pm} \int \frac{d^3k}{(2\pi)^3 2E_k} \left[b(k,h) x(k,h) e^{-ikx} + a^\dagger(k,h) y(k,h) e^{ikx} \right]$$

annihilates antiparticle — **creates particle**

$$\Psi = \begin{pmatrix} \psi \\ \bar{\psi}^c \end{pmatrix}$$

$$C a(k) C^{-1} = b(k)$$
$$C b(k) C^{-1} = a(k)$$

$$\Rightarrow$$

$$C \psi(t, \boldsymbol{x}) C^{-1} = \psi^c(t, \boldsymbol{x}),$$
$$C \psi^c(t, \boldsymbol{x}) C^{-1} = \psi(t, \boldsymbol{x})\,.$$

CP on spin-1/2 fermions

$$P\psi_\alpha(t,\boldsymbol{x})P^{-1} = \bar{\psi}^{c\,\dot{\alpha}}(t,-\boldsymbol{x}),$$
$$P\psi^c_\alpha(t,\boldsymbol{x})P^{-1} = -\bar{\psi}^{\dot{\alpha}}(t,-\boldsymbol{x}),$$
$$C\psi(t,\boldsymbol{x})C^{-1} = \psi^c(t,\boldsymbol{x}),$$
$$C\psi^c(t,\boldsymbol{x})C^{-1} = \psi(t,\boldsymbol{x})\,.$$

CP:

$$(CP)\psi_\alpha(t,\boldsymbol{x})(CP)^{-1} = \bar{\psi}^{\dot{\alpha}}(t,-\boldsymbol{x})$$
$$(CP)\psi^c_\alpha(t,\boldsymbol{x})(CP)^{-1} = -\bar{\psi}^{c\,\dot{\alpha}}(t,-\boldsymbol{x})$$

Example of Yukawa interactions:

$$(CP)\int d^4x h\left[y\psi^c\psi + y^*\bar{\psi}\bar{\psi}^c\right](t,\boldsymbol{x})(CP)^{-1} \equiv (CP)\int d^4x h\left[y\psi^{c\,\alpha}\psi_\alpha + y^*\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{c\,\dot{\alpha}}\right](t,\boldsymbol{x})(CP)^{-1}$$
$$= \int d^4x h\left[y\bar{\psi}^c_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + y^*\psi^\alpha\psi^c_\alpha\right](t,-\boldsymbol{x}) = \int d^4x h\left[y\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{c\,\dot{\alpha}} + y^*\psi^{c\,\alpha}\psi_\alpha\right](t,\boldsymbol{x})$$
$$\equiv \int d^4x h\left[y^*\psi^c\psi + y\bar{\psi}\bar{\psi}^c\right](t,\boldsymbol{x})\,.$$

CP is violated if the Yukawa coupling y is complex
(in the basis where masses are real)

So CP violation is always associated with
phases in the Lagrangian, right ?

CP on spin-1 vectors

Consider CP acting on spin-1 vector fields $V^\mu = (V^0, V^i)$:

$$PV^0(t,\boldsymbol{x})P^{-1} = V^0(t,-\boldsymbol{x}),$$
$$PV^k(t,\boldsymbol{x})P^{-1} = -V^k(t,-\boldsymbol{x}),$$
$$CV^\mu(t,\boldsymbol{x})C^{-1} = -V^\mu(t,\boldsymbol{x})\,.$$

$$\Rightarrow$$

$$(CP)V^0(t,\boldsymbol{x})(CP)^{-1} = -V^0(t,-\boldsymbol{x}),$$
$$(CP)V^k(t,\boldsymbol{x})(CP)^{-1} = V^k(t,-\boldsymbol{x})\,.$$

It follows that the θ-like terms transform as

$$(CP)\int d^4x V_{\mu\nu}\tilde{V}^{\mu\nu}(x)(CP)^{-1} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}(CP)\int d^4x V_{\mu\nu}V_{\alpha\beta}(x)(CP)^{-1}$$

$$= 2\epsilon^{ijk}(CP)\int d^4x V_{0i}V_{jk}(x)(CP)^{-1} = 2\epsilon^{ijk}(CP)\int d^4x[\partial_0 V_i - \partial_i V_0][\partial_j V_k - \partial_k V_j](x)(CP)^{-1}$$

$$= 2\epsilon^{ijk}\int d^4x[\partial_0 V_i + \partial_i V_0][\partial_j V_k - \partial_k V_j](t,-\boldsymbol{x})$$

$$= -2\epsilon^{ijk}\int d^4x[\partial_0 V_i(t,\boldsymbol{x}) - \partial_i V_0(t,\boldsymbol{x})][\partial_j V_k(t,\boldsymbol{x}) - \partial_k V_j(t,\boldsymbol{x})] = -\int d^4x V_{\mu\nu}(x)\tilde{V}^{\mu\nu}(x)\,.$$

In particular, the QCD θ-term is CP-odd

CP on spin-1 vectors

CP violation is often associated with phases in the Lagrangian, but not always !

$$(CP)V^0(t,\boldsymbol{x})(CP)^{-1} = -V^0(t,-\boldsymbol{x}),$$
$$(CP)V^k(t,\boldsymbol{x})(CP)^{-1} = V^k(t,-\boldsymbol{x})\,.$$

By the same token:

$$\mathscr{L}_{\mathrm{SMEFT}} \supset C_{H\widetilde{B}} H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu}$$

is CP odd, even though it has no complex phase

$$\mathscr{L}_{\mathrm{SMEFT}} \supset C_{\widetilde{G}} f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} \widetilde{G}^c_{\rho\mu}$$

is CP odd, even though it has no complex phase

(yet another) Weinberg operator

CP on spin 1/2 and spin 1 interactions

Another example is dipole interactions

$$\mathscr{L}_{\text{SMEFT}} \supset [C_{eB}]_{11}(\bar{l}_1 H \bar{\sigma}^{\mu\nu} \bar{e}^c) B_{\mu\nu} + [C_{eB}]^*_{11}(e^c H^\dagger \sigma^{\mu\nu} l_1) B_{\mu\nu}$$

By the similar calculation as before

$$(CP)\int d^4x \left[[C_{eB}]_{11}(\bar{l}_1 H \bar{\sigma}^{\mu\nu} \bar{e}^c) + [C_{eB}]^*_{11}(e^c H^\dagger \sigma^{\mu\nu} l_1) \right] B_{\mu\nu}(x) (CP)^{-1}$$

$$= \int d^4x \left[[C_{eB}]^*_{11}(\bar{l}_1 H \bar{\sigma}^{\mu\nu} \bar{e}^c) + [C_{eB}]_{11}(e^c H^\dagger \sigma^{\mu\nu} l_1) \right] B_{\mu\nu}(x)$$

CP is violated if the Wilson coefficient C_{eB} is complex

Real part of d corresponds to anomalous **magnetic** moment of fermion ψ (CP conserving)

Imaginary part of d corresponds to anomalous **electric** moment of fermion ψ (CP violating)

CP violation at D=4

SMEFT at dimension-4

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{D=2} + \mathcal{L}_{D=3} + \mathcal{L}_{D=4} + \mathcal{L}_{D=5} + \mathcal{L}_{D=6} + \mathcal{L}_{D=7} + \mathcal{L}_{D=8} + \ldots$$

$$\mathcal{L}_{D=4} = -\frac{1}{4} \sum_{V \in B, W^i, G^a} V_{\mu\nu} V^{\mu\nu} + \sum_{f \in Q, L} i\bar{f}\bar{\sigma}^\mu D_\mu f + \sum_{f \in U, D, E} i f^c \sigma^\mu D_\mu \bar{f}^c$$
$$- \left(U^c Y_u \tilde{H}^\dagger Q + D^c Y_d H^\dagger Q + E^c Y_e H^\dagger L + \text{h.c.} \right) + D_\mu H^\dagger D^\mu H - \lambda (H^\dagger H)^2$$
$$+ \tilde{\theta} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu},$$

$$\tilde{H}_a = \epsilon^{ab} H_b^*$$

$$V^a_{\mu\nu} = \partial_\mu V^a_\nu - \partial_\nu V^a_\mu - g f^{abc} V^b_\mu V^c_\nu$$

$$D_\mu f = \partial_\mu f + i g_s G^a_\mu T^a f + i g_L W^i_\mu \frac{\sigma^i}{2} f + i g_Y B_\mu Y f$$

$$\tilde{G}^a_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} G^{\alpha\beta\ a}$$

$$U^c = \begin{pmatrix} u^c \\ c^c \\ t^c \end{pmatrix} \qquad Q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \\ \begin{pmatrix} c \\ s \end{pmatrix} \\ \begin{pmatrix} t \\ b \end{pmatrix} \end{pmatrix}$$

$$D^c = \begin{pmatrix} d^c \\ s^c \\ b^c \end{pmatrix}$$

$$E^c = \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix} \qquad L = \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix} \\ \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \\ \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \end{pmatrix}$$

CP violation at dimension-4

- After redefining away all the phases, 2 sources of CP violation remain in the D <= 4 part of the SMEFT

- One is the phase in the CKM matrix, describing charged current interactions between W and left-handed quarks.
  The effects of this phase have been observed in the B-meson, D-meson, and kaon systems. The value of this phase seems to be generic, that is it is consistent with order one phases in the quark Yukawa couplings

- The other is a combination of the $\tilde{\theta}$ parameter and the phase of the determinant of the quark matrix. This phase should lead to an EDM of the neutron and composite nuclei. The effects of this phase have not been observed so far and we have only stringent limits. It is a mystery why this phase, unlike the former one, does not take a generic value

CP violation at D=5

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$H \to \begin{pmatrix} 0 \\ \text{v}/\sqrt{2} \end{pmatrix}$$

$$\mathscr{L}_{D=5} = \sum_{\alpha,\beta=1}^{3}\left[\frac{c_{\alpha\beta}}{\Lambda_L}(L_\alpha H)(L_\beta H) + \frac{\bar{c}_{\alpha\beta}}{\Lambda_L}(H^\dagger \bar{L}_\alpha)(H^\dagger \bar{L}_\beta)\right]$$

$$L_\alpha \to \begin{pmatrix} \nu_\alpha \\ e_\alpha \end{pmatrix}$$

$$= \left(1 + \frac{h}{\text{v}}\right)^2 \frac{\text{v}^2}{\Lambda_L}\sum_{\alpha,\beta=1}^{3}\left[c_{\alpha\beta}\nu_\alpha\nu_\beta + \bar{c}_{\alpha\beta}\bar{\nu}_\alpha\bar{\nu}_\beta\right]$$

$$\text{CP}[\mathscr{L}_{D=5}] = \left(1 + \frac{h}{\text{v}}\right)^2 \frac{\text{v}^2}{\Lambda_L}\left[c_{\alpha\beta}\bar{\nu}_\alpha\bar{\nu}_\beta + \bar{c}_{\alpha\beta}\nu_\alpha\nu_\beta\right]$$

Neutrino masses can violate CP if they are complex!

SMEFT at dimension-5

QFT it is awkward to work with complex and off-diagonal masses, so we usually diagonalize the mass matrix and remove the phases by field redefinition

$$\mathscr{L}_{D=5} \supset \frac{\mathrm{v}^2}{\Lambda_L} \sum_{\alpha,\beta=1}^{3} \left[c_{\alpha\beta}\nu_\alpha\nu_\beta + \bar{c}_{\alpha\beta}\bar{\nu}_\alpha\bar{\nu}_\beta \right]$$

Unitary PMNS matrix

Rotate $\nu_\alpha \to U_{\alpha j}\nu_j$

$$c_{\alpha\beta}\nu_\alpha\nu_\beta \to U_{\alpha i}c_{\alpha\beta}U_{\beta j}\nu_i\nu_j$$

Choose $U^T c U = -\,\mathrm{diag}(c_1, c_2, c_3)$

$$\mathscr{L}_{D=5} \supset -\frac{\mathrm{v}^2}{\Lambda_L} \sum_{i=1}^{3} \left[c_i\nu_i\nu_i + \bar{c}_i\bar{\nu}_i\bar{\nu}_i \right]$$

$$m_{\nu_i} = c_i\frac{\mathrm{v}^2}{\Lambda_L} = |c_i| e^{i\phi_i}\frac{\mathrm{v}^2}{\Lambda_L}$$

Rephase $\nu_i \to P_i\nu_i, \quad P_i = e^{-i\phi_i}$

$$\mathscr{L}_{D=5} \supset -\sum_{i=1}^{3} m_{\nu_i} \left[\nu_i\nu_i + \bar{\nu}_i\bar{\nu}_i \right]$$

Masses are now real and all traces of CP violation vanish…

SMEFT at dimension-5

… not so fast

SMEFT Lagrangian at D=4 contains the CC interactions between leptons and W

$$\mathcal{L}_{D=5} \supset -\sum_{i=1}^{3} m_{\nu_i}\left[\nu_i\nu_i + \bar{\nu}_i\bar{\nu}_i\right]$$

$$\mathcal{L}_{D=4} \supset \frac{g_L}{\sqrt{2}} W_\mu^- \sum_{\alpha=1}^{3} \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_\alpha + \mathrm{h.c.}$$

$$\nu_\alpha \to \sum_{j=1}^{3} U_{\alpha j} P_j \nu_j \qquad P_i = e^{-i\phi_i}$$

$$\mathcal{L}_{D=4} \to \frac{g_L}{\sqrt{2}} W_\mu^- \sum_{\alpha,j=1}^{3} U_{\alpha j} P_j \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_j + \mathrm{h.c.}$$

CP-violating if U or P are complex

CP violation migrated from the neutrino mass matrix to charged-current interactions of leptons

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta}s_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$P_i = e^{i\phi}\begin{pmatrix} e^{i\alpha/2} & e^{i\beta/2} & 1 \end{pmatrix}$$

PMNS matrix U is totally analogous, to the CKM matrix for quarks (though numerically it is very different)

These are qualitatively new phases compared to the quark sector

The phase δ is called the Dirac phase

They are called the Majorana phases

Neutrino Oscillations

$$P(\nu_\alpha \to \nu_\beta) = \frac{\sum_{k,l=1}^{3} \exp\left(-i\frac{L(m_{\nu_k}^2 - m_{\nu_l}^2)}{2E_\nu}\right) \int d\Pi_P \mathscr{M}_{\alpha k}^{P} \mathscr{M}_{\alpha l}^{P*} \int d\Pi_D \mathscr{M}_{\beta k}^{D} \mathscr{M}_{\beta l}^{D*}}{\sum_{k,l=1}^{3} \int d\Pi_P |\mathscr{M}_{\alpha k}^{P}|^2 \int d\Pi_D |\mathscr{M}_{\beta l}^{D}|^2}$$

$$\mathscr{L}_{D=4} \to \frac{g_L}{\sqrt{2}} \sum_{\alpha,k=1}^{3} \left\{ W_\mu^- U_{\alpha k} P_k \left(\bar{\ell}_\alpha \bar{\sigma}_\mu \nu_k\right) + W_\mu^+ U_{\alpha k}^* P_k^* \left(\bar{\nu}_k \bar{\sigma}_\mu \ell_\alpha\right) \right\}$$

Neutrino production: $\mathscr{M}_{\alpha k}^{P} \sim U_{\alpha k}^* P_k^*$ $\qquad \mathscr{M}_{\alpha l}^{P*} \sim U_{\alpha l} P_l$

Neutrino detection: $\mathscr{M}_{\beta k}^{D} \sim U_{\beta k} P_k$ $\qquad \mathscr{M}_{\beta l}^{D*} \sim U_{\beta l}^* P_l^*$

$$P(\nu_\alpha \to \nu_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k}^* U_{\alpha l} U_{\beta k} U_{\beta l}^*$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k} U_{\alpha l}^* U_{\beta k}^* U_{\beta l}$$

$$\Delta_{kl}^2 \equiv m_{\nu_k}^2 - m_{\nu_l}^2$$

The Majorana phases cancel out in neutrino oscillations

CP violation in Neutrino Oscillations

$$P(\nu_\alpha \to \nu_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k}^* U_{\alpha l} U_{\beta k} U_{\beta l}^*$$

$$P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} U_{\alpha k} U_{\alpha l}^* U_{\beta k}^* U_{\beta l}$$

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 2i \sum_{k,l=1}^{3} e^{-i\frac{\Delta_{kl}^2}{2E_\nu}} \mathrm{Im}[U_{\alpha k}^* U_{\alpha l} U_{\beta k} U_{\beta l}^*]$$

In the usual parametrization of the PMNS matrix, for $\alpha \neq \beta$

$$P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = \pm\, s_{12} s_{13} s_{23} c_{12} c_{13}^2 c_{23} \sin \delta \left[\sin\left(\frac{\Delta_{21}^2 L}{2E_\nu}\right) - \sin\left(\frac{\Delta_{31}^2 L}{2E_\nu}\right) + \sin\left(\frac{\Delta_{32}^2 L}{2E_\nu}\right) \right]$$

while for $\alpha = \beta$, $P(\nu_\alpha \to \nu_\alpha) - P(\bar{\nu}_\alpha \to \bar{\nu}_\alpha) = 0$ **by CPT**

CP violation is hard…

- **At least 3 neutrinos must exist in nature**
- **All 3 mixing angles have to be non-trivial**
- **All 3 mass splittings have to be non-zero**
- **The Dirac phase needs to be different from 0 and from π**

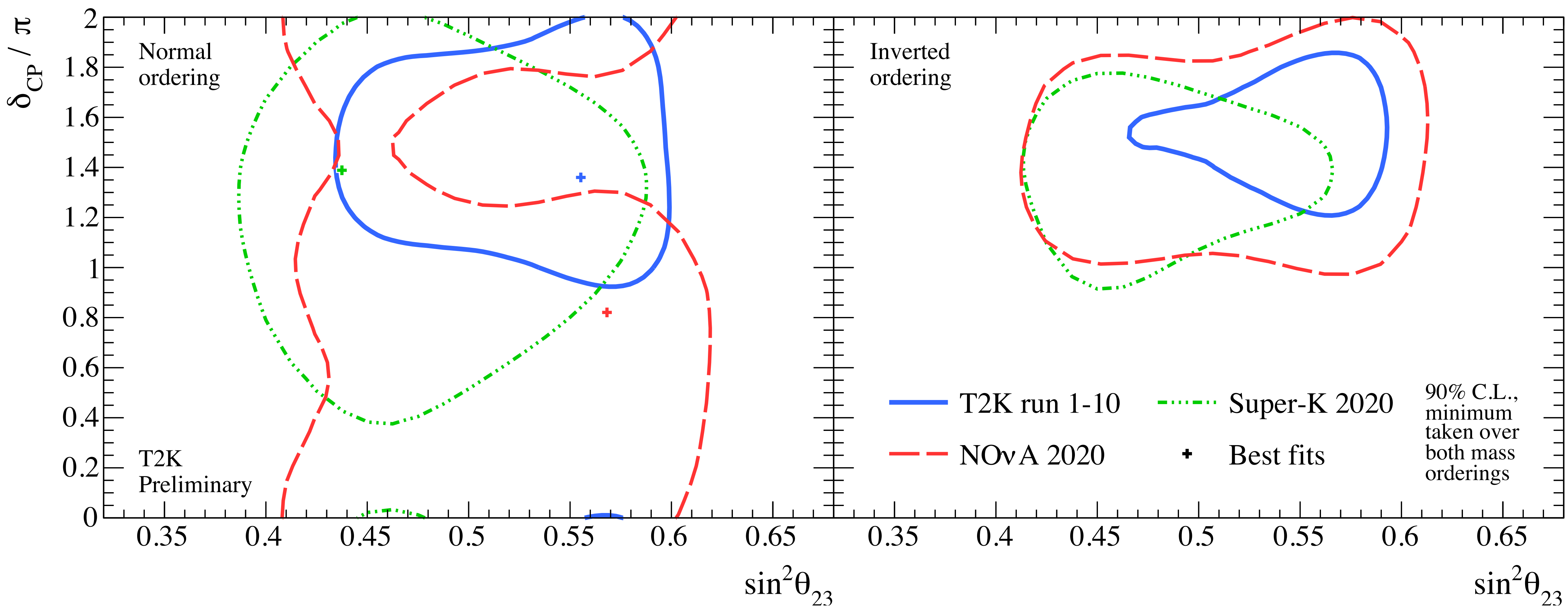
Fortunately, it seems that these conditions are fulfilled in the real world, barring confirmation about the Dirac phase…

CP violation in Neutrino Oscillations

T2K shows some mild preference for $\delta \sim -\pi/2$

Best fit

$$\delta = -1.97^{+0.97}_{-0.70}$$



plot from 2208.01164

Another triumph of SMEFT?
As expected by power counting arguments, CP violation first observed at D=4, then at D=5…

SMEFT at dimension-5

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{g_L}{\sqrt{2}} W^-_\mu \sum_{\alpha,j=1}^{3} U_{\alpha j} P_j \bar{\ell}_\alpha \bar{\sigma}_\mu \nu_j + \text{h.c.}$$

CP-violating if U or P are complex

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{-i\delta}s_{13} \\ -s_{12}c_{23} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}s_{13}c_{23} & -c_{12}s_{23} - e^{i\delta}s_{12}s_{13}c_{23} & c_{13}c_{23} \end{pmatrix}$$

$$P_i = e^{i\phi} \begin{pmatrix} e^{i\alpha/2} & e^{i\beta/2} & 1 \end{pmatrix}$$

PMNS matrix U is totally analogous, to the CKM matrix for quarks (though numerically it is very different)

These are qualitatively new phases compared to the quark sector

The phase δ is called the <u>Dirac phase</u>

They are called the <u>Majorana phases</u>

Almost there

Are these physical ???

Neutrino Antineutrino Oscillations

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} \frac{m_{\nu_k} m_{\nu_l}}{E_\nu^2} \exp\left(-i\frac{L(m_{\nu_k}^2 - m_{\nu_l}^2)}{2E_\nu}\right) \int d\Pi_P \mathscr{M}^P_{\alpha k} \bar{\mathscr{M}}^P_{\alpha l} \int d\Pi_D \mathscr{M}^D_{\beta k} \bar{\mathscr{M}}^D_{\beta l}$$

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{g_L}{\sqrt{2}} \sum_{\alpha,k=1}^{3} \left\{ W^-_\mu U_{\alpha k} P_k (\bar{\ell}_\alpha \bar{\sigma}_\mu \nu_k) + W^+_\mu U^*_{\alpha k} P^*_k (\bar{\nu}_k \bar{\sigma}_\mu \ell_\alpha) \right\}$$

Neutrino production: $\mathscr{M}^P_{\alpha k} \sim U^*_{\alpha k} P^*_k$ $\mathscr{M}^{P*}_{\alpha l} \sim U_{\alpha l} P_l$

Anti-Neutrino detection: $\mathscr{M}^D_{\beta k} \sim U^*_{\beta k} P^*_k$ $\mathscr{M}^{D*}_{\beta l} \sim U_{\beta l} P_l$

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U^*_{\alpha k} U_{\alpha l} U^*_{\beta k} U_{\beta l} (P^*_k)^2 (P_l)^2 \qquad \Delta^2_{kl} \equiv m^2_{\nu_k} - m^2_{\nu_l}$$

$$R(\bar{\nu}_\alpha \to \nu_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U_{\alpha k} U^*_{\alpha l} U_{\beta k} U^*_{\beta l} (P_k)^2 (P^*_l)^2$$

Majorana phases don’t cancel out!

Neutrino-Antineutrino Oscillations

$$R(\nu_\alpha \to \bar{\nu}_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U^*_{\alpha k} U_{\alpha l} U^*_{\beta k} U_{\beta l} (P^*_k)^2 (P_l)^2$$

$$R(\bar{\nu}_\alpha \to \nu_\beta) \sim \sum_{k,l=1}^{3} m_{\nu_k} m_{\nu_l} e^{-i\frac{\Delta^2_{kl}}{2E_\nu}} U_{\alpha k} U^*_{\alpha l} U_{\beta k} U^*_{\beta l} (P_k)^2 (P^*_l)^2$$

Take the limit for $s_{13} \to 0$ simplicity

$$R(\nu_e \to \bar{\nu}_\mu) - R(\bar{\nu}_e \to \nu_\mu) \sim m_{\nu_1} m_{\nu_2} c^2_{12} s^2_{12} \sin(\alpha - \beta) \sin\left(\frac{\Delta^2_{21} L}{2E_\nu}\right)$$

Majorana phases control CP violation in neutrino-antineutrino oscillations
The effect occurs even in the 2-neutrino oscillation limit

Unfortunately, the effect is very suppressed by the small neutrino masses, and may never be observed…

De Gouvea et al
hep-ph/0211394

CP violation at D= 6

SMEFT at dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \ldots$$

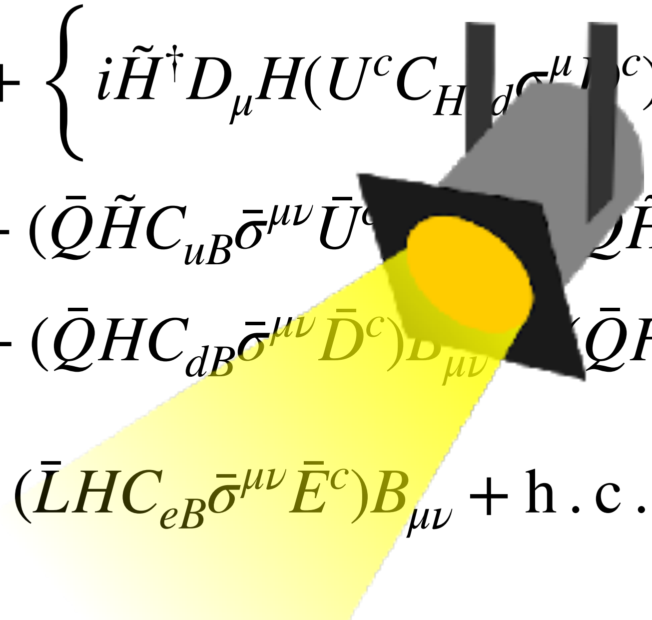
Grządkowski et al arXiv:1008.4884

$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H (H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}\,|H^\dagger D_\mu H|^2 \\
& + C_{HWB} H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} + C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} + C_{HW} H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} + C_{HB} H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \\
& + + C_W \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_G f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + C_{H\widetilde{G}} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu} + C_{H\widetilde{W}} H^\dagger H\, \widetilde{W}^k_{\mu\nu} W^k_{\mu\nu} + C_{H\widetilde{B}} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu} + C_{H\widetilde{W}B} H^\dagger \sigma^k H\, \widetilde{W}^k_{\mu\nu} B_{\mu\nu} \\
& + C_{\widetilde{W}} \epsilon^{klm} \widetilde{W}^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_{\widetilde{G}} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L} H C_{eH} \bar{E}^c) + H^\dagger H(\bar{Q}\tilde{H} C_{uH} \bar{U}^c) + H^\dagger H(\bar{Q} H C_{dH} \bar{D}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(\bar{L} C^{(1)}_{Hl} \bar{\sigma}^\mu L) + iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L} C^{(3)}_{Hl} \bar{\sigma}^\mu \sigma^k L) + iH^\dagger \overleftrightarrow{D}_\mu H(E^c C_{He} \sigma^\mu \bar{E}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(\bar{Q} C^{(1)}_{Hq} \bar{\sigma}^\mu Q) + iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q} C^{(3)}_{Hq} \bar{\sigma}^\mu \sigma^k Q) + iH^\dagger \overleftrightarrow{D}_\mu H(U^c C_{Hu} \sigma^\mu \bar{U}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(D^c C_{Hd} \sigma^\mu \bar{D}^c) + \Big\{ i\tilde{H}^\dagger D_\mu H(U^c C_{Hud} \sigma^\mu \bar{D}^c) \\
& + (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
& + (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
& + (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \mathrm{h.c.} \Big\} + \mathscr{L}^{4-\mathrm{fermion}}_{D=6}
\end{aligned}$$

SMEFT at dimension-6

$$\mathscr{L}_{\mathrm{SMEFT}} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$

$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H (H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}|H^\dagger D_\mu H|^2 \\
& + C_{HWB} H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} + C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} + C_{HW} H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} + C_{HB} H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \\
& + + C_W \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_G f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + C_{H\widetilde{G}} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu} + C_{H\widetilde{W}} H^\dagger H\, \widetilde{W}^k_{\mu\nu} W^k_{\mu\nu} + C_{H\widetilde{B}} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu} + C_{H\widetilde{W}B} H^\dagger \sigma^k H\, \widetilde{W}^k_{\mu\nu} B_{\mu\nu} \\
& + C_{\widetilde{W}} \epsilon^{klm} \widetilde{W}^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_{\widetilde{G}} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L} H C_{eH} \bar{E}^c) + H^\dagger H(\bar{Q}\tilde{H} C_{uH} \bar{U}^c) + H^\dagger H(\bar{Q} H C_{dH} \bar{D}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(\bar{L} C^{(1)}_{Hl} \bar{\sigma}^\mu L) + iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L} C^{(3)}_{Hl} \bar{\sigma}^\mu \sigma^k L) + iH^\dagger \overleftrightarrow{D}_\mu H(E^c C_{He} \sigma^\mu \bar{E}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(\bar{Q} C^{(1)}_{Hq} \bar{\sigma}^\mu Q) + iH^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q} C^{(3)}_{Hq} \bar{\sigma}^\mu \sigma^k Q) + iH^\dagger \overleftrightarrow{D}_\mu H(U^c C_{Hu} \sigma^\mu \bar{U}^c) \\
& + iH^\dagger \overleftrightarrow{D}_\mu H(D^c C_{Hd} \sigma^\mu \bar{D}^c) + \Big\{ i\tilde{H}^\dagger D_\mu H(U^c C_{Hud} \sigma^\mu \bar{D}^c) \\
& + (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
& + (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
& + (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \mathrm{h.c.} \Big\} + \mathscr{L}^{4-\mathrm{fermion}}_{D=6}
\end{aligned}$$



CP violation by electron EDM

Dimension-6 SMEFT Lagrangian contains:

$$\mathscr{L}_{D=6} \supset [C_{eB}]^*_{11} e^c \sigma_{\mu\nu} H^\dagger L B_{\mu\nu} + \mathrm{h.c.} \to \frac{[C_{eB}]^*_{11} \mathrm{v} \cos\theta_W}{\sqrt{2}} e^c \sigma_{\mu\nu} e F_{\mu\nu} + \mathrm{h.c.}$$

Compare it to

$$\mathscr{L}_{\mathrm{dipole}} = -\frac{\Delta\mu_e - i d_e}{4} F_{\mu\nu}(e_c \sigma^{\mu\nu} e) + \mathrm{h.c.}$$

Resulting tree-level contribution to electron EDM is

$$d_e = -2\sqrt{2}\cos\theta_W \mathrm{v}\, \mathrm{Im}\, [C_{eB}]_{11}$$

CP violation by electron EDM

Why this particular interaction is identified as EDM...

$$\mathscr{L} \supset i\bar{e}\bar{\sigma}^{\mu}\partial_{\mu}e + ie^{c}\sigma^{\mu}\partial_{\mu}\bar{e}^{c} - m_e\left[e^{c}e + \mathrm{h.c.}\right]$$
$$-q_e e A_{\mu}(\bar{e}\bar{\sigma}^{\mu}e) - q_e e A_{\mu}(e_c\sigma^{\mu}\bar{e}_c) - \left\{\frac{\Delta\mu_e - id_e}{4}F_{\mu\nu}(e_c\sigma^{\mu\nu}e) + \mathrm{h.c.}\right\}$$

Change of variables to non-relativistic degrees of freedom

$$e_{\alpha} = \frac{1}{\sqrt{2}}\left\{e^{-im_e t}\left(\psi + \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi\right)_{\alpha} - e^{im_e t}\left(\psi_c^{\dagger} - \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi_c^{\dagger}\right)_{\alpha}\right\} + \mathcal{O}(\nabla^2),$$
$$\bar{e}_c^{\dot{\alpha}} = \frac{1}{\sqrt{2}}\left\{e^{-im_e t}\left(\psi - \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi\right)_{\alpha} + e^{im_e t}\left(\psi_c^{\dagger} + \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi_c^{\dagger}\right)_{\alpha}\right\} + \mathcal{O}(\nabla^2).$$

where ψ is a non-relativistic electron field, and ψ_c is a non-relativistic positron field.

Plugging this change of variables into kinetic terms:

$$i\bar{e}\bar{\sigma}^{\mu}\partial_{\mu}e + ie^{c}\sigma^{\mu}\partial_{\mu}\bar{e}^{c} - m_e\left[e^{c}e + \mathrm{h.c.}\right] = i\psi^{\dagger}\partial_t\psi + \frac{1}{2m_e}\psi^{\dagger}\nabla^2\psi + i\psi_c^{\dagger}\partial_t\psi_c + \frac{1}{2m_e}\psi_c^{\dagger}\nabla^2\psi_c + \mathcal{O}(\nabla^3),$$

shows that ψ and ψ_c satisfy the Schrodinger equation

CP violation by electron EDM

Change of variables to non-relativistic degrees of freedom

$$e_\alpha = \frac{1}{\sqrt{2}}\left\{ e^{-im_e t}\left(\psi + \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi\right)_\alpha - e^{im_e t}\left(\psi_c^\dagger - \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi_c^\dagger\right)_\alpha \right\} + \mathcal{O}(\nabla^2),$$

$$\bar{e}_c^{\dot\alpha} = \frac{1}{\sqrt{2}}\left\{ e^{-im_e t}\left(\psi - \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi\right)_\alpha + e^{im_e t}\left(\psi_c^\dagger + \frac{i}{2m_e}\boldsymbol{\sigma}\cdot\nabla\psi_c^\dagger\right)_\alpha \right\} + \mathcal{O}(\nabla^2)\,.$$

Now plugging this change of variables into the interaction terms (ignoring positrons):

$$-q_e e A_\mu(\bar{e}\bar{\sigma}^\mu e) - q_e e A_\mu(e_c\sigma^\mu\bar{e}_c) - \left\{ \frac{\Delta\mu_e - id_e}{4}F_{\mu\nu}(e_c\sigma^{\mu\nu}e) + \mathrm{h.c.} \right\}$$

$$= -q_e e V\psi^\dagger\psi - \frac{iq_e e}{2m}A^k\psi^\dagger \overleftrightarrow{\nabla}_k\psi + \left(\frac{q_e e}{m_e} + \Delta\mu_e\right)B^k(\psi^\dagger\frac{\sigma^k}{2}\psi) + d_e E^k(\psi^\dagger\frac{\sigma^k}{2}\psi),$$

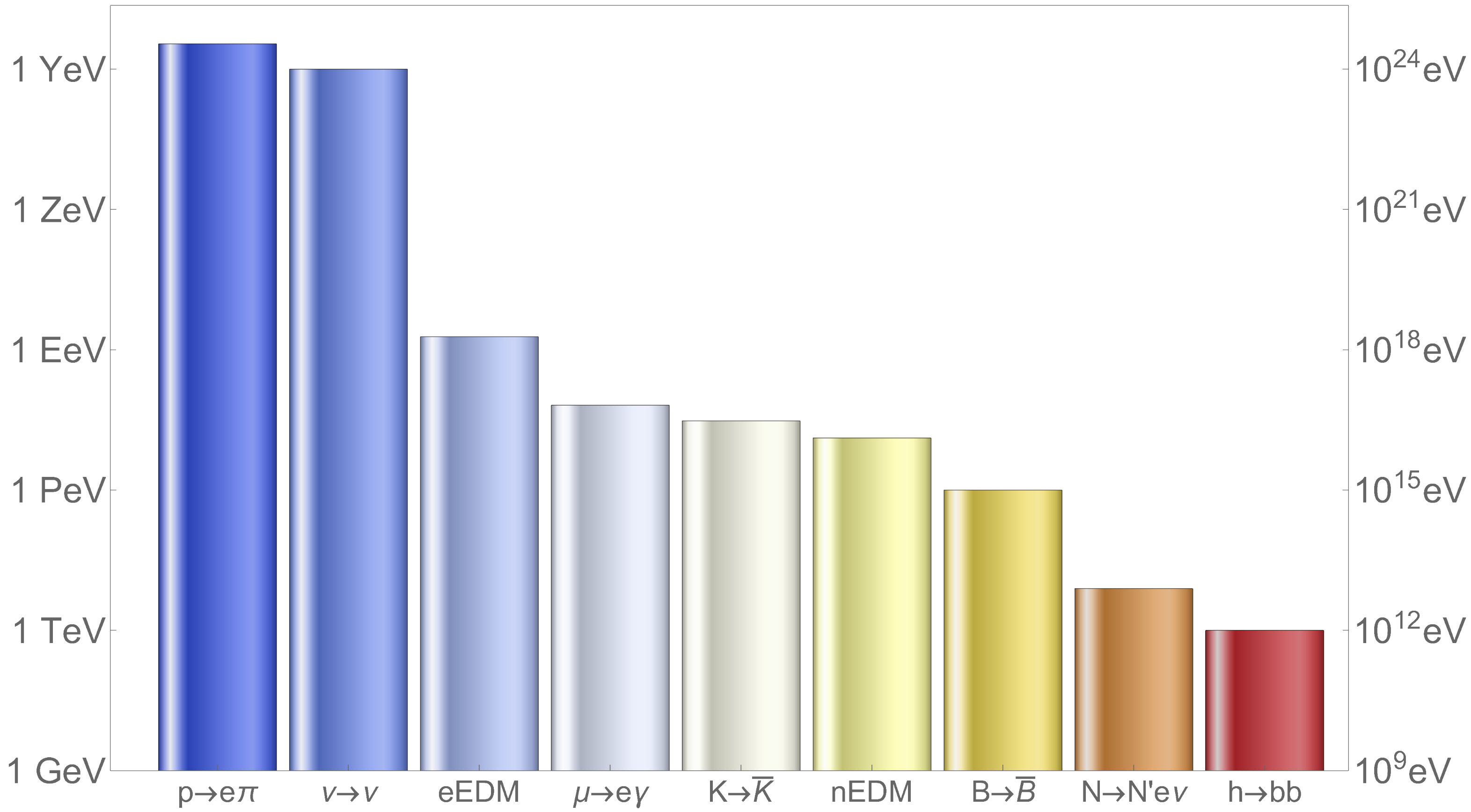
Show that the d_e parameter corresponds to interaction of the electric field with electron's spin

CP violation by electron EDM

$$d_e = -2\sqrt{2}\cos\theta_W \mathrm{v}\,\mathrm{Im}\,[C_{eB}]_{11}$$

ACME limit: $$|d_e| < 1.1\times 10^{-29} e\cdot cm = \frac{1.7\times 10^{-13}}{\mathrm{TeV}}$$

It follows $$\left|\mathrm{Im}\,[C_{eB}]_{11}\right| \leq \frac{1}{(1.9\ \mathrm{EeV})^2}$$



CP violation by electron EDM

$$\left| \mathrm{Im}\,[C_{eB}]_{11} \right| \leq \frac{1}{(1.9\ \mathrm{EeV})^2}$$

The reach of electron EDM depends on the hypothesis about the Wilson coefficient c_{eB}

1 $\mathrm{Im}\,[C_{eB}]_{11} \sim \frac{1}{\Lambda^2}$ $\Rightarrow$ $\Lambda \gtrsim 10^6\ \mathrm{TeV}$ **6 orders of magnitude above LHC!**

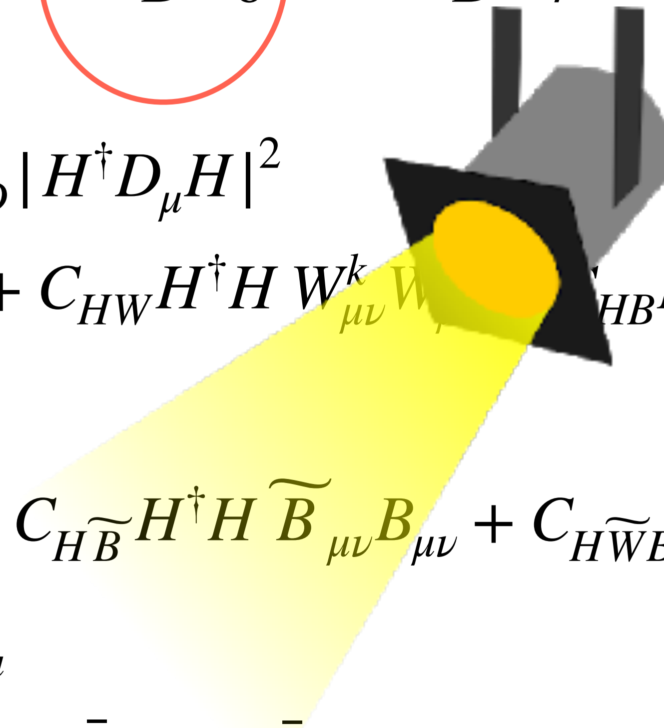
2 $\mathrm{Im}\,[C_{eB}]_{11} \sim \frac{y_e}{\Lambda^2}$ $\Rightarrow$ $\Lambda \gtrsim 10^3\ \mathrm{TeV}$ **3 orders of magnitude above LHC!**

3 $\mathrm{Im}\,[C_{eB}]_{11} \sim \frac{g_Y y_e}{16\pi^2\Lambda^2}$ $\Rightarrow$ $\Lambda \gtrsim 10^2\ \mathrm{TeV}$ **2 order of magnitude above LHC!**

Unlikely there is new physics below 100 TeV, because CP violation seems generic in nature and electron's EDM does not violate any other symmetry than CP and chiral symmetry

CP violation by electron EDM

$$\mathscr{L}_{\rm SMEFT} = \mathscr{L}_{D=2} + \mathscr{L}_{D=4} + \mathscr{L}_{D=5} + \mathscr{L}_{D=6} + \mathscr{L}_{D=7} + \mathscr{L}_{D=8} + \dots$$



$$\begin{aligned}
\mathscr{L}_{D=6} = {} & C_H (H^\dagger H)^3 + C_{H\Box}(H^\dagger H)\Box(H^\dagger H) + C_{HD}|H^\dagger D_\mu H|^2 \\
& + C_{HWB} H^\dagger \sigma^k H\, W^k_{\mu\nu} B_{\mu\nu} + C_{HG} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu} + C_{HW} H^\dagger H\, W^k_{\mu\nu} W^k_{\mu\nu} + C_{HB} H^\dagger H\, B_{\mu\nu} B_{\mu\nu} \\
& + + C_W \epsilon^{klm} W^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_G f^{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + C_{H\widetilde{G}} H^\dagger H\, \widetilde{G}^a_{\mu\nu} G^a_{\mu\nu} + C_{H\widetilde{W}} H^\dagger H\, \widetilde{W}^k_{\mu\nu} W^k_{\mu\nu} + C_{H\widetilde{B}} H^\dagger H\, \widetilde{B}_{\mu\nu} B_{\mu\nu} + C_{H\widetilde{W}B} H^\dagger \sigma^k H\, \widetilde{W}^k_{\mu\nu} B_{\mu\nu} \\
& + C_{\widetilde{W}} \epsilon^{klm} \widetilde{W}^k_{\mu\nu} W^l_{\nu\rho} W^m_{\rho\mu} + C_{\widetilde{G}} f^{abc} \widetilde{G}^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu} \\
& + H^\dagger H(\bar{L} H C_{eH} \bar{E}^c) + H^\dagger H(\bar{Q}\tilde{H} C_{uH} \bar{U}^c) + H^\dagger H(\bar{Q} H C_{dH} \bar{D}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(\bar{L} C^{(1)}_{Hl} \bar{\sigma}^\mu L) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{L} C^{(3)}_{Hl} \bar{\sigma}^\mu \sigma^k L) + i H^\dagger \overleftrightarrow{D}_\mu H(E^c C_{He} \sigma^\mu \bar{E}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(\bar{Q} C^{(1)}_{Hq} \bar{\sigma}^\mu Q) + i H^\dagger \sigma^k \overleftrightarrow{D}_\mu H(\bar{Q} C^{(3)}_{Hq} \bar{\sigma}^\mu \sigma^k Q) + i H^\dagger \overleftrightarrow{D}_\mu H(U^c C_{Hu} \sigma^\mu \bar{U}^c) \\
& + i H^\dagger \overleftrightarrow{D}_\mu H(D^c C_{Hd} \sigma^\mu \bar{D}^c) + \Big\{ i\tilde{H}^\dagger D_\mu H(U^c C_{Hud} \sigma^\mu \bar{D}^c) \\
& + (\bar{Q}\sigma^k \tilde{H} C_{uW} \bar{\sigma}^{\mu\nu} \bar{U}^c) W^k_{\mu\nu} + (\bar{Q}\tilde{H} C_{uB} \bar{\sigma}^{\mu\nu} \bar{U}^c) B_{\mu\nu} + (\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu} \bar{U}^c) G^a_{\mu\nu} \\
& + (\bar{Q}\sigma^k H C_{dW} \bar{\sigma}^{\mu\nu} \bar{D}^c) W^k_{\mu\nu} + (\bar{Q} H C_{dB} \bar{\sigma}^{\mu\nu} \bar{D}^c) B_{\mu\nu} + (\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu} \bar{D}^c) G^a_{\mu\nu} \\
& + (\bar{L}\sigma^k H C_{eW} \bar{\sigma}^{\mu\nu} \bar{E}^c) W^k_{\mu\nu} + (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \mathrm{h.c.} \Big\} + \mathscr{L}^{\rm 4-fermion}_{D=6}
\end{aligned}$$

CP violation by electron EDM

$$\mathscr{L}_{D=6} \supset \left\{ (\bar{L} H C_{eB} \bar{\sigma}^{\mu\nu} \bar{E}^c) B_{\mu\nu} + \mathrm{h.c.} \right\} + C_{HB} H^\dagger H B_{\mu\nu} B_{\mu\nu} + i C_{H\widetilde{B}} H^\dagger H B_{\mu\nu} \widetilde{B}_{\mu\nu}$$

This operators mix under renormalization group

$$\frac{dC_{eB}}{d\log\mu} = -\frac{g_Y}{8\pi^2}\left(C_{HB} + iC_{H\widetilde{B}}\right) Y_e + \dots$$

Solving it

$$\mathrm{Im}\, C_{eB}(m_Z) = \frac{g_Y}{8\pi^2} Y_e C_{H\widetilde{B}}(\Lambda) \log\left(\frac{\Lambda}{m_Z}\right) \Rightarrow$$

$$\mathrm{Im}\,[C_{eB}(m_Z)]_{11} = \frac{g_Y}{8\pi^2}\frac{\sqrt{2} m_e}{\mathrm{v}} C_{H\widetilde{B}}(\Lambda) \log\left(\frac{\Lambda}{m_Z}\right)$$

It follows

$$|C_{H\widetilde{B}}(\Lambda)| \lesssim \frac{1}{(200\ \mathrm{TeV})^2 \times \log\left(\Lambda/m_Z\right)}$$

This is a very strong constraint,
allowing CP violating corrections to Higgs coupling to weak gauge bosons of order 10^{-6}

CP violation by neutron EDM

Define WEFT Lagrangian at low scale as

$$\mathscr{L}_{\rm WEFT} \supset \Big\{ - C^{ij\,lm}_{1\,LR} (\bar{d}_m \bar{\sigma}_\mu u_l)(u^c_i \sigma^\mu d^c_j) - C^{ij\,lm}_{2\,LR} (\bar{d}_{m\,a} \bar{\sigma}_\mu u_{l\,b})(u^c_{i\,b} \sigma^\mu d^c_{j\,a})$$

$$-\frac{g_s}{2} \sum_{i,j=u,c} m_{u_j} C^{ij}_{gu} \bar{u}_i \sigma^{\mu\nu} T^a \bar{u}^c_j G^a_{\mu\nu} - \frac{g_s}{2} \sum_{i,j=u,c} m_{d_j} C^{ij}_{gd} \bar{d}_i \sigma^{\mu\nu} T^a \bar{d}^c_j G^a_{\mu\nu}$$

$$-\frac{e q_u}{2} \sum_{i,j=u,c} m_{u_j} C^{ij}_{\gamma u} \bar{u}_i \sigma^{\mu\nu} \bar{u}^c_j F_{\mu\nu} - \frac{e q_d}{2} \sum_{i,j=u,c} m_{d_j} C^{ij}_{\gamma d} \bar{d}_i \sigma^{\mu\nu} \bar{d}^c_j F_{\mu\nu} + \mathrm{h.c.} \Big\}$$

$$+\frac{g_s}{3} f^{abc} \hat{C}_{\widetilde{G}} \tilde{G}^a_{\mu\nu} G^b_{\mu\rho} G^c_{\nu\rho}$$

The neutron EDM is given in terms of these parameters as

$$d_n = \Big[(43 \pm 27)\mathrm{Im} C^{us\,us}_{1\,LR} + (210 \pm 130)\mathrm{Im} C^{us\,us}_{2\,LR} + (22 \pm 14)\mathrm{Im} C^{ud\,ud}_{1\,LR} + (110 \pm 70)\mathrm{Im} C^{ud\,ud}_{2\,LR}$$

$$-(0.93 \pm 0.05)\mathrm{Im} C^{uu}_{\gamma u} - (4.0 \pm 0.2)\mathrm{Im} C^{dd}_{\gamma d} - (0.8 \pm 0.9)\mathrm{Im} C^{ss}_{\gamma d}$$

$$-(3.9 \pm 2.0)\mathrm{Im} C^{uu}_{gu} - (16.8 \pm 8.4)\mathrm{Im} C^{dd}_{gd} + (320 \pm 260)\hat{C}_{\widetilde{G}} \Big] \mathrm{v}^2 \times 10^{-22} e\,\mathrm{cm},$$

Alioli et al.
1703.04751

nEDM experiment: $d_n = (0.0 \pm 1.1) \times 10^{-26} e\,\mathrm{cm}$

Neutron EDM as a lightning rod

Nuclear dipoles pick up many contributions from many CP violating SMEFT operators (and even more when RG running is taken into account)

At D=4

$\tilde{\theta} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$

$-U^c Y_u \tilde{H}^\dagger Q - D^c Y_d H^\dagger Q$

Liang et al. 2301.04331

$$d_n = -0.00148(14)(31)\left[\frac{16\pi^2}{g_s^2}\tilde{\theta} + \frac{1}{N_f}\arg\det M_q\right] e \ \mathrm{fm}$$

$i\tilde{H}^\dagger D_\mu H(U^c C_{Hud}\sigma^\mu \bar{D}^c)$

$\epsilon^{kl}(\bar{Q}^k T^a \bar{U}^c) C^{(8)}_{quqd}(\bar{Q}^l T^a \bar{D}^c)$

At D=6

$$d_n = \Big[(43 \pm 27)\mathrm{Im}C^{us\,us}_{1\,LR} + (210 \pm 130)\mathrm{Im}C^{us\,us}_{2\,LR} + (22 \pm 14)\mathrm{Im}C^{ud\,ud}_{1\,LR} + (110 \pm 70)\mathrm{Im}C^{ud\,ud}_{2\,LR}$$
$$-(0.93 \pm 0.05)\mathrm{Im}C^{uu}_{\gamma u} - (4.0 \pm 0.2)\mathrm{Im}C^{dd}_{\gamma d} - (0.8 \pm 0.9)\mathrm{Im}C^{ss}_{\gamma d}$$
$$-(3.9 \pm 2.0)\mathrm{Im}C^{uu}_{gu} - (16.8 \pm 8.4)\mathrm{Im}C^{dd}_{gd} + (320 \pm 260)\hat{C}_{\tilde{G}}\Big] v^2 \times 10^{-9} e\ \mathrm{fm},$$

$(\bar{Q} H C_{dB}\bar{\sigma}^{\mu\nu}\bar{D}^c)B_{\mu\nu}$

$(\bar{Q}\tilde{H} C_{uB}\bar{\sigma}^{\mu\nu}\bar{U}^c)B_{\mu\nu}$

$(\bar{Q}\tilde{H} C_{uG} T^a \bar{\sigma}^{\mu\nu}\bar{U}^c)G^a_{\mu\nu}$ $(\bar{Q} H C_{dG} T^a \bar{\sigma}^{\mu\nu}\bar{D}^c)G^a_{\mu\nu}$

$C_{\widetilde{G}} f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

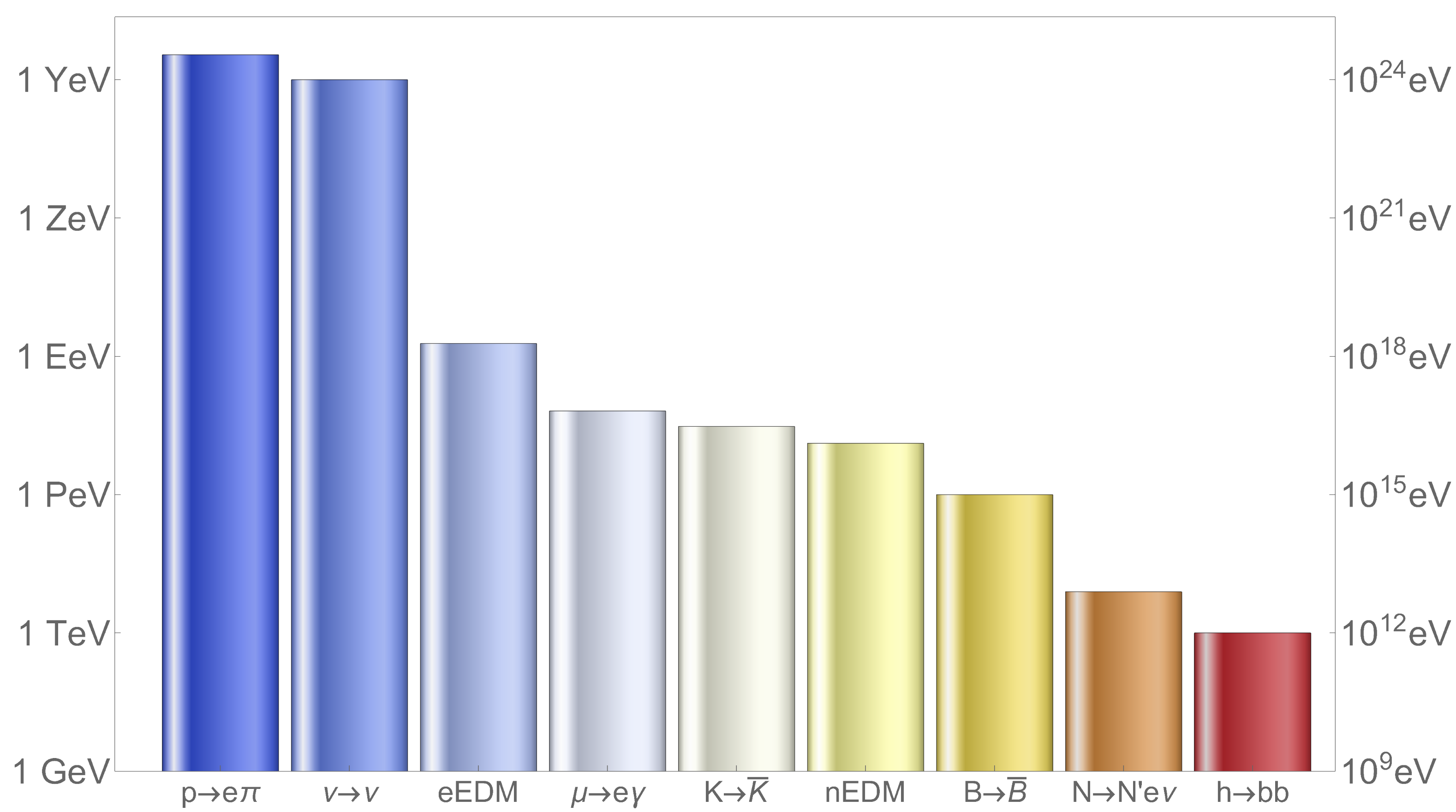
CP violation by neutron EDM

$$\mathscr{L}_{D=6} \supset (\bar{Q}\tilde{H}C_{uB}\bar{\sigma}^{\mu\nu}\bar{U}^c)B_{\mu\nu} + \mathrm{h.c.}$$

$$\mathscr{L}_{\mathrm{WEFT}} \supset \Big\{ - C^{ij\,lm}_{1\,LR}(\bar{d}_m\bar{\sigma}_\mu u_l)(u^c_i\sigma^\mu d^c_j) - C^{ij\,lm}_{2\,LR}(\bar{d}_{m\,a}\bar{\sigma}_\mu u_{l\,b})(u^c_{i\,b}\sigma^\mu d^c_{j\,a})$$
$$-\frac{g_s}{2}\sum_{i,j=u,c} m_{u_j}C^{ij}_{gu}\bar{u}_i\sigma^{\mu\nu}T^a\bar{u}^c_j G^a_{\mu\nu} - \frac{g_s}{2}\sum_{i,j=u,c} m_{d_j}C^{ij}_{gd}\bar{d}_i\sigma^{\mu\nu}T^a\bar{d}^c_j G^a_{\mu\nu}$$
$$-\frac{eq_u}{2}\sum_{i,j=u,c} m_{u_j}C^{ij}_{\gamma u}\bar{u}_i\sigma^{\mu\nu}\bar{u}^c_j F_{\mu\nu} - \frac{eq_d}{2}\sum_{i,j=u,c} m_{d_j}C^{ij}_{\gamma d}\bar{d}_i\sigma^{\mu\nu}\bar{d}^c_j F_{\mu\nu} + \mathrm{h.c.}\Big\}$$
$$+\frac{g_s}{3}f^{abc}\hat{C}_{\widetilde{G}}\tilde{G}^a_{\mu\nu}G^b_{\mu\rho}G^c_{\nu\rho}$$

Matching: $$C^{uu}_{\gamma u} = -\frac{3\cos\theta_W v}{\sqrt{2}em_u}[C_{uB}]_{11}$$

nEDM measurement implies $$|\,\mathrm{Im}[C_{uB}]_{11}\,| \lesssim \frac{1}{(13\ \mathrm{PeV})^2}$$ **at 95% CL**



CP violation by neutron EDM

Another less trivial example

$$\mathscr{L}_{D=6} \supset i\tilde{H}^\dagger D_\mu H(U^c C_{Hud}\sigma^\mu \bar{D}^c) + \mathrm{h.c.}$$

This operator induces (complex) W boson couplings to right-handed quarks

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{g_L}{\sqrt{2}} W^+_\mu \left[\bar{\nu}_e \bar{\sigma}^\mu e + V_{ud}\bar{u}\bar{\sigma}^\mu d + \frac{v^2}{2}[C_{Hud}]_{11} u^c \sigma^\mu \bar{d}^c \right] + \mathrm{h.c.}$$

Integrating out the W boson, the effective theory below the electroweak scale contains a certain 4-quark interaction

$$\mathscr{L}_{\text{WEFT}} \supset -V_{ud}[C_{Hud}]_{11}(\bar{d}\bar{\sigma}_\mu u)(u^c \sigma^\mu \bar{d}^c) + \mathrm{h.c.}$$

$$\begin{aligned}\mathscr{L}_{\text{WEFT}} \supset \Big\{ & - C^{ij\,lm}_{1\,LR}(\bar{d}_m \bar{\sigma}_\mu u_l)(u^c_i \sigma^\mu d^c_j) - C^{ij\,lm}_{2\,LR}(\bar{d}_{m\,a}\bar{\sigma}_\mu u_{l\,b})(u^c_{i\,b}\sigma^\mu d^c_{j\,a}) \\ & -\frac{g_s}{2}\sum_{i,j=u,c} m_{u_j} C^{ij}_{gu}\bar{u}_i \sigma^{\mu\nu}T^a \bar{u}^c_j G^a_{\mu\nu} - \frac{g_s}{2}\sum_{i,j=u,c} m_{d_j} C^{ij}_{gd}\bar{d}_i \sigma^{\mu\nu}T^a \bar{d}^c_j G^a_{\mu\nu} \\ & -\frac{eq_u}{2}\sum_{i,j=u,c} m_{u_j} C^{ij}_{\gamma u}\bar{u}_i \sigma^{\mu\nu}\bar{u}^c_j F_{\mu\nu} - \frac{eq_d}{2}\sum_{i,j=u,c} m_{d_j} C^{ij}_{\gamma d}\bar{d}_i \sigma^{\mu\nu}\bar{d}^c_j F_{\mu\nu} + \mathrm{h.c.} \Big\} \\ & + \frac{g_s}{3} f^{abc}\hat{C}_{\widetilde{G}}\tilde{G}^a_{\mu\nu}G^b_{\mu\rho}G^c_{\nu\rho}\end{aligned}$$

Matching:

$$C^{ud\,ud}_{1\,LR} = V_{ud}[C_{Hud}]_{11}$$

nEDM measurement implies

$$|\,\mathrm{Im}\,[C_{Hud}]_{11}| \lesssim \frac{1}{(100\ \mathrm{TeV})^2}$$

Probes scales of 100 TeV, e.g. in left-right symmetric models

CP violation by molecules

Tensor 4-fermion operators:

$$\mathscr{L}_{\mathrm{D}=6} \supset \epsilon^{kl}(\bar{L}^k \bar{\sigma}^{\mu\nu} \bar{E}^c) C^{(3)}_{lequ} (\bar{Q}^l \bar{\sigma}^{\mu\nu} \bar{U}^c) + \mathrm{h.c.}$$

These lead to charged current interactions (relevant e.g. for tensor contributions to beta decay), as well as to neutral current interactions

$$\mathscr{L}_{\mathrm{SMEFT}} \supset -[C^{(3)}_{lequ}]_{1111} V_{ud} (\bar{e}\bar{\sigma}^{\mu\nu}\bar{e}^c)(\bar{u}\bar{\sigma}_{\mu\nu}\bar{u}^c)$$

This in turn affect rotation frequency of paramagnetic molecules

$$\omega_{\mathrm{ThO}} \approx -16 \mathrm{Im}[C^{(3)}_{lequ}]_{1111} 10^8 \mathrm{TeV}^2 \,\mathrm{mrad/s}$$

Dekens et al 1810.05675

This leads to a strong constraint

$$|\mathrm{Im}[C^{(3)}_{lequ}]_{1111}| \lesssim \frac{1}{(40\ \mathrm{PeV})^2}$$

CP violation by meson mixing

Kaon states with definite strangeness:

$$|K^0\rangle \equiv \bar{s}d$$
$$|\bar{K}^0\rangle \equiv s\bar{d}$$

Kaon CP eigenstates

$$|K^0_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\bar{K}^0\rangle\right)$$

In a CP conserving theory one would have

$$K^0_+ \to 2\pi$$
$$K^0_- \to 3\pi$$

Kaon mass eigenstates

$$|K_S\rangle \simeq |K^0_+\rangle + \epsilon_K |K^0_-\rangle$$
$$|K_L\rangle \simeq |K^0_-\rangle + \epsilon_K |K^0_+\rangle$$

Instead, one observes

$$K_L \to \pi\pi$$

From experimental data one finds the CP violating parameter :

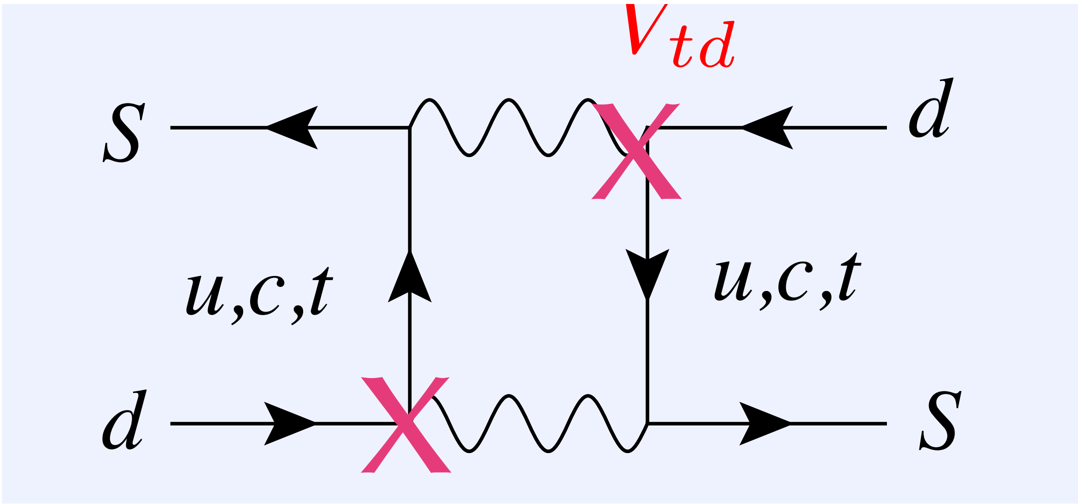
$$|\epsilon_K| = 2.228(11) \times 10^{-3}$$

CP violation by meson mixing

Integrating out W boson at one loop in the SM:

$$\mathscr{L}_{\mathrm{WEFT}} \supset c(V_{ts}^* V_{td})^2 \frac{m_W^2}{32\pi^2 v^4} (\bar{s}\bar{\sigma}_\mu d)(\bar{s}\bar{\sigma}^\mu d) + \mathrm{h.c.}$$

$$\approx \left(\frac{1}{(31\ \mathrm{PeV})^2} - \frac{i}{(28\ \mathrm{PeV})^2} \right) (\bar{s}\bar{\sigma}_\mu d)(\bar{s}\bar{\sigma}^\mu d) + \mathrm{h.c.}$$



SM prediction $|\epsilon_K^{\mathrm{SM}}| = 2.027(195) \times 10^{-3}$ $|\epsilon_K^{\mathrm{exp}}| = 2.228(11) \times 10^{-3}$

SMEFT has many similar 4-fermion operators violating strangeness by $\Delta S = 2$

$$\begin{aligned}
\mathscr{L}_{D=6}^{4-\mathrm{fermion}} = {} & (\bar{L}\bar{\sigma}^\mu L)C_{ll}(\bar{L}\bar{\sigma}_\mu L) + (E^c\sigma_\mu \bar{E}^c)C_{ee}(E^c\sigma_\mu \bar{E}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{le}(E^c\sigma_\mu \bar{E}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C_{lq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{L}\bar{\sigma}^\mu \sigma^k L)C_{lq}^{(3)}(\bar{Q}\bar{\sigma}_\mu \sigma^k Q) \\
& + (E^c\sigma_\mu \bar{E}^c)C_{eu}(U^c\sigma_\mu \bar{U}^c) + (E^c\sigma_\mu \bar{E}^c)C_{ed}(D^c\sigma_\mu \bar{D}^c) \\
& + (\bar{L}\bar{\sigma}^\mu L)C_{lu}(U^c\sigma_\mu \bar{U}^c) + (\bar{L}\bar{\sigma}^\mu L)C_{ld}(D^c\sigma_\mu \bar{D}^c) + (E^c\sigma_\mu \bar{E}^c)C_{eq}(Q\bar{\sigma}_\mu Q) \\
& + \Big\{ (\bar{L}\bar{E}^c)C_{ledq}(D^c Q) + \epsilon^{kl}(\bar{L}^k\bar{E}^c)C_{lequ}^{(1)}(\bar{Q}^l\bar{U}^c) + \epsilon^{kl}(\bar{L}^k\bar{\sigma}^{\mu\nu}\bar{E}^c)C_{lequ}^{(3)}(\bar{Q}^l\bar{\sigma}^{\mu\nu}\bar{U}^c) + \mathrm{h.c.} \Big\} \\
& + (\bar{Q}\bar{\sigma}^\mu Q)C_{qq}^{(1)}(\bar{Q}\bar{\sigma}_\mu Q) + (\bar{Q}\bar{\sigma}^\mu \sigma^k Q)C_{qq}^{(3)}(\bar{Q}\bar{\sigma}_\mu \sigma^k Q) \\
& + (U^c\sigma_\mu \bar{U}^c)C_{uu}(U^c\sigma_\mu \bar{U}^c) + (D^c\sigma_\mu \bar{D}^c)C_{dd}(D^c\sigma_\mu \bar{D}^c) \\
& + (U^c\sigma_\mu \bar{U}^c)C_{ud}^{(1)}(D^c\sigma_\mu \bar{D}^c) + (U^c\sigma_\mu T^a \bar{U}^c)C_{ud}^{(8)}(D^c\sigma_\mu T^a \bar{D}^c) \\
& + (Q^c\sigma_\mu \bar{Q}^c)C_{qu}^{(1)}(U^c\sigma_\mu \bar{U}^c) + (Q^c\sigma_\mu T^a \bar{Q}^c)C_{qu}^{(8)}(U^c\sigma_\mu T^a \bar{U}^c)] \\
& + (Q^c\sigma_\mu \bar{Q}^c)C_{qd}^{(1)}(D^c\sigma_\mu \bar{D}^c) + (Q^c\sigma_\mu T^a \bar{Q}^c)C_{qd}^{(8)}(D^c\sigma_\mu T^a \bar{D}^c) \\
& + \Big\{ \epsilon^{kl}(\bar{Q}^k\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^l\bar{D}^c) + \epsilon^{kl}(\bar{Q}^k T^a\bar{U}^c)C_{quqd}^{(1)}(\bar{Q}^l T^a\bar{D}^c) + \mathrm{h.c.} \Big\} \\
& + \Big\{ (D^c U^c)C_{duq}(\bar{Q}\bar{L}) + (QQ)C_{qqu}(\bar{U}^c\bar{E}^c) + (QQ)C_{qqq}(QL) + (D^c U^c)C_{duu}(U^c E^c) + \mathrm{h.c.} \Big\}.
\end{aligned}$$

$$\Rightarrow$$

$$\begin{aligned}
\mathscr{L}_{\mathrm{SMEFT}} \supset {} & [C_{qq}^{(1)}]_{2121}(\bar{q}_2\bar{\sigma}_\mu q_1)(\bar{q}_2\bar{\sigma}^\mu q_1) + [C_{qq}^{(3)}]_{2121}(\bar{q}_2\bar{\sigma}_\mu \sigma^k q_1)(\bar{q}_2\bar{\sigma}^\mu \sigma^k q_1) \\
& + [C_{qd}^{(1)}]_{2121}(\bar{q}_2\bar{\sigma}_\mu q_1)(s^c\sigma^\mu \bar{d}^c) + [C_{qd}^{(8)}]_{2121}(\bar{q}_2\bar{\sigma}_\mu T^a q_1)(s^c\sigma^\mu T^a \bar{d}^c) \\
& + [C_{dd}]_{2121}(s^c\sigma_\mu \bar{d}^c)(s^c\sigma^\mu \bar{d}^c) + \mathrm{h.c.}
\end{aligned}$$

CP violation by meson mixing

$$\mathscr{L}_{\rm SMEFT} \supset [C^{(1)}_{qq}]_{2121}(\bar q_2 \bar\sigma_\mu q_1)(\bar q_2 \bar\sigma^\mu q_1) + [C^{(3)}_{qq}]_{2121}(\bar q_2 \bar\sigma_\mu \sigma^k q_1)(\bar q_2 \bar\sigma^\mu \sigma^k q_1)$$
$$+[C^{(1)}_{qd}]_{2121}(\bar q_2 \bar\sigma_\mu q_1)(s^c \sigma^\mu \bar d^c) + [C^{(8)}_{qd}]_{2121}(\bar q_2 \bar\sigma_\mu T^a q_1)(s^c \sigma^\mu T^a \bar d^c)$$
$$+[C_{dd}]_{2121}(s^c \sigma_\mu \bar d^c)(s^c \sigma^\mu \bar d^c) + \mathrm{h.c.}$$

Translating this to ϵ_K

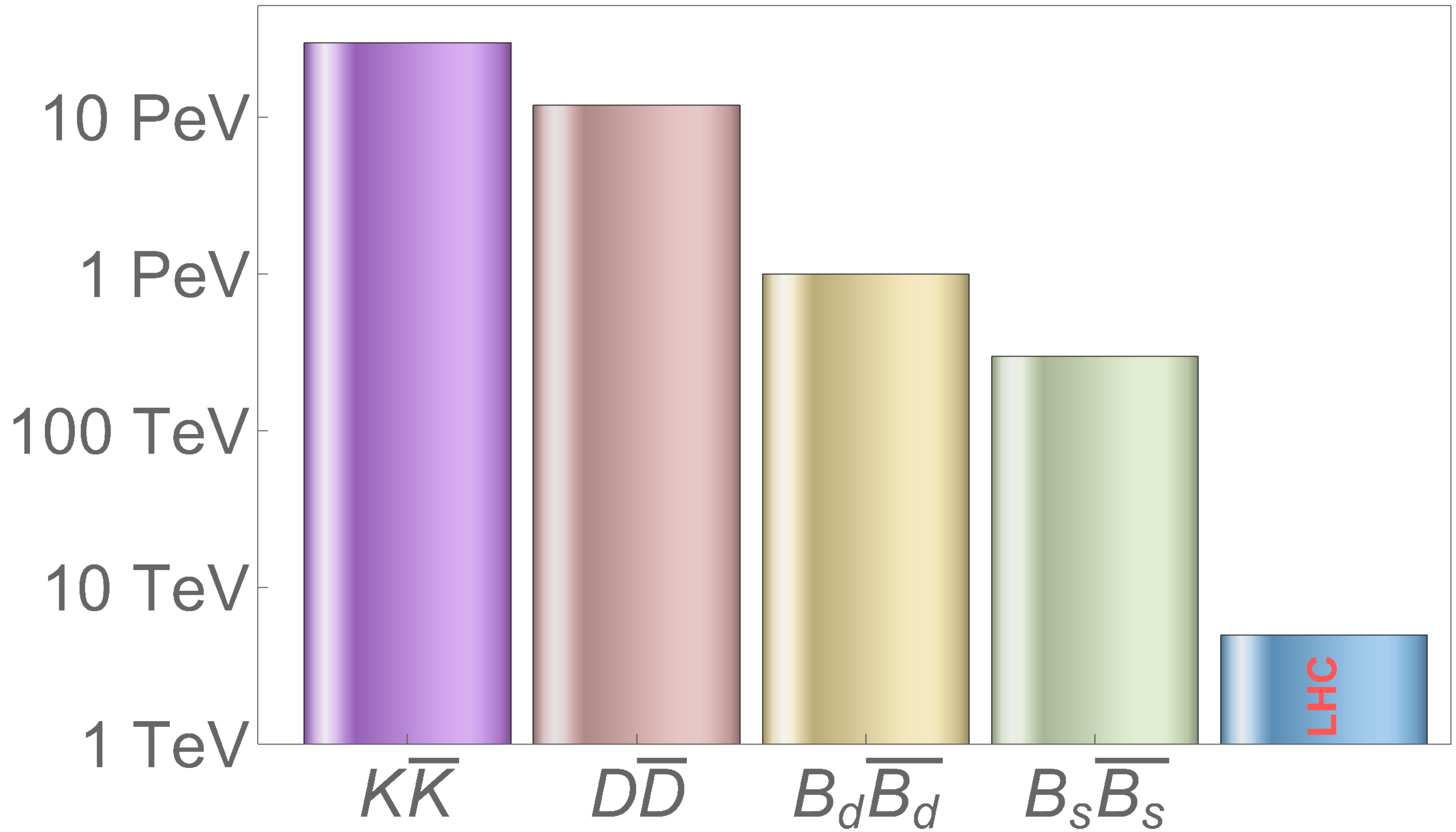
$$\frac{|\epsilon_K|}{|\epsilon_K^{\rm SM}|} = 1 + \mathrm{Im}\left\{ -(13.3\ \mathrm{PeV})^2[C^{(1)}_{qq} + C^{(3)}_{qq} + C_{dd}]_{2121} + (105\ \mathrm{PeV})^2[C^{(1)}_{qd}]_{2121} + (127\ \mathrm{PeV})^2[C^{(8)}_{qd}]_{2121} \right\}$$

One then derives the 95% CL constraints

$$-\frac{1}{(25\ \mathrm{PeV})^2} \lesssim \mathrm{Im}[C^{(1)}_{qq}, C^{(3)}_{qq}, C_{dd}]_{2121} \lesssim \frac{1}{(44\ \mathrm{PeV})^2},$$
$$-\frac{1}{(350\ \mathrm{PeV})^2} \lesssim \mathrm{Im}[C^{(1)}_{qd}]_{2121} \lesssim \frac{1}{(200\ \mathrm{PeV})^2},$$
$$-\frac{1}{(420\ \mathrm{PeV})^2} \lesssim \mathrm{Im}[C^{(8)}_{qd}]_{2121} \lesssim \frac{1}{(240\ \mathrm{PeV})^2},$$

CP violation by meson mixing

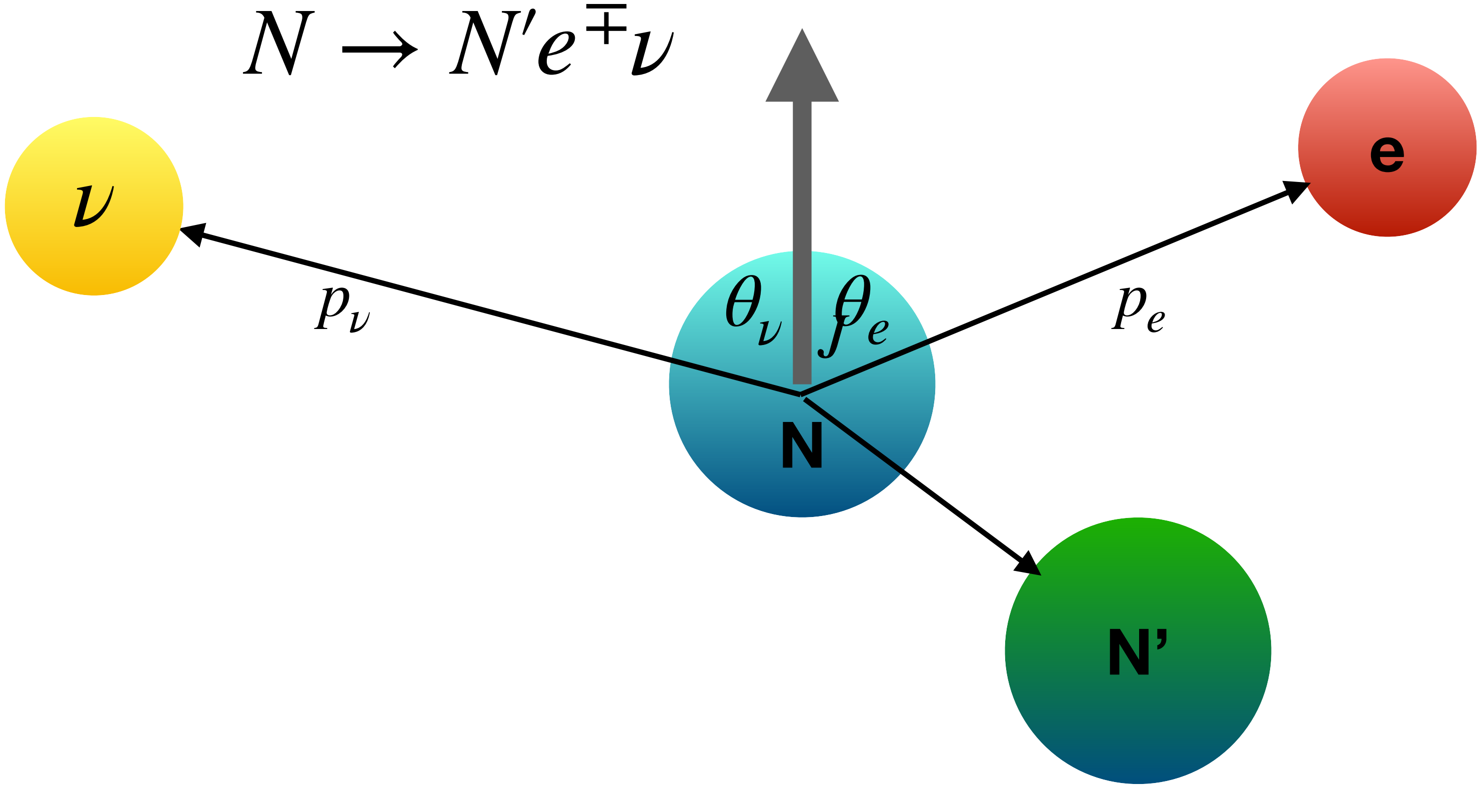
Similar logic leads to constraints from CP violation in other neutral meson systems



CP violation in beta decays

Effective Lagrangian describing allowed nuclear beta decays:

$$N \to N' e^{\mp} \nu$$



Electron energy/momentum

$$E_e = \sqrt{p_e^2 + m_e^2}$$

Neutrino energy

$$E_\nu = p_\nu = m_N - m_{N'} - E_e$$

Beta decay observables include lifetimes and correlations:

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = F(E_e)\left\{1 + b\frac{m_e}{E_e} + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e} + A\frac{\langle \boldsymbol{J} \rangle \cdot \boldsymbol{p}_e}{JE_e} + B\frac{\langle \boldsymbol{J} \rangle \cdot \boldsymbol{p}_\nu}{JE_\nu} \right.$$
$$\left. + c\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu - 3(\boldsymbol{p}_e \cdot \boldsymbol{j})(\boldsymbol{p}_\nu \cdot \boldsymbol{j})}{3E_eE_\nu}\left[\frac{J(J+1) - 3(\langle \boldsymbol{J} \rangle \cdot \boldsymbol{j})^2}{J(2J-1)}\right] + D\frac{\langle \boldsymbol{J} \rangle \cdot (\boldsymbol{p}_e \times \boldsymbol{p}_\nu)}{JE_eE_\nu}\right\}$$

Violates T, thus can be sensitive at tree level to CP violation in fundamental Lagrangian

CP violation in beta decays

$$\mathscr{L}_{\rm NR} \supset -(\bar\psi_p\psi_n)\Big[C_V^+(\bar e\bar\sigma^0\nu)+C_S^+(e^c\nu)\Big] + \sum_{k=1}^{3}(\bar\psi_p\sigma^k\psi_n)\Big[C_A^+(\bar e\bar\sigma^k\nu)+C_T^+(e^c\sigma^{0k}\nu)\Big]$$

At tree level

$$D\hat\xi = -2r\sqrt{\frac{J}{J+1}}\,{\rm Im}\Big\{C_V^+\bar C_A^+ - C_S^+\bar C_T^+\Big\} \qquad \hat\xi \equiv |C_V^+|^2 + |C_S^+|^2 + r^2\big[|C_A^+|^2 + |C_T^+|^2\big]$$

which is zero in SM

At loop level, Coulomb final-state interactions induce D proportional to real parts

$$D\hat\xi = \frac{(-q_e)Z\alpha m_e}{2p_e m_N}{\rm Re}\Bigg\{ r\sqrt{\frac{J}{J+1}}\Bigg(\Bigg[\frac{4m_e^2+p_e^2}{2m_e}C_V^+ + 2(2E_e - E_e^{\max})C_S^+\Bigg]\bar C_{WM}^+$$
$$+\frac{r^2}{2(J+1)}\Bigg(\Bigg[-\frac{4m_e^2+3p_e^2}{2m_e}C_A^+ - 2E_e^{\max}C_T^+\Bigg]\bar C_{WM}^+ + \Bigg[-\frac{4m_e^2+3p_e^2}{2m_e}C_A^+ - 2(2E_e - E_e^{\max})C_T^+\Bigg]\frac{\bar C_A^+}{A}\Bigg)\Bigg\}$$
$$+\frac{3q_e e\tilde\mu_{\mathcal{N}'}p_e}{8\pi}\Bigg[J\big(|C_V^+|^2 - |C_S^+|^2\big) - r\sqrt{\frac{J}{J+1}}{\rm Re}\big(C_V^+\bar C_A^+ - C_S^+\bar C_T^+\big) - r^2J\big(|C_A^+|^2 - |C_T^+|^2\big)\Bigg]$$

which is non-zero in SM $\quad D_{\rm SM}\sim 10^{-4}$

CP violation in beta decays

$$\mathcal{L}_{\rm NR} \supset -(\bar\psi_p \psi_n)\left[C_V^+(\bar e \bar\sigma^0 \nu) + C_S^+(e^c \nu)\right] + \sum_{k=1}^{3}(\bar\psi_p \sigma^k \psi_n)\left[C_A^+(\bar e \bar\sigma^k \nu) + C_T^+(e^c \sigma^{0k} \nu)\right]$$

$$D\hat\xi = -2r\sqrt{\frac{J}{J+1}}\,\mathrm{Im}\left\{C_V^+ \bar C_A^+ - C_S^+ \bar C_T^+\right\}$$

Can CP-violating new physics give contributions comparable to SM ?

$$C_T^+ \sim \mathrm{v}^2[C_{lequ}^{(3)}]_{1111}$$

$$\mathcal{L}_{\rm D=6} \supset \epsilon^{kl}(\bar L^k \bar\sigma^{\mu\nu} \bar E^c) C_{lequ}^{(3)} (\bar Q^l \bar\sigma^{\mu\nu} \bar U^c)$$

imaginary parts strongly constrained by ThO EDM

$$C_S^+ \sim \mathrm{v}^2[C_{lequ}^{(1)}]_{1111}$$

$$\mathcal{L}_{\rm D=6} \supset \epsilon^{kl}(\bar L^k \bar E^c) C_{lequ}^{(1)} (\bar Q^l \bar U^c)$$

Strongly constrained by pion decay

$$C_A^+ \sim \mathrm{v}^2 C_{Hud}$$

$$\mathcal{L}_{D=6} \supset i\tilde H^\dagger D_\mu H (U^c C_{Hud} \sigma^\mu \bar D^c) + \mathrm{h.c.}$$

Imaginary parts constrained by neutron EDMs

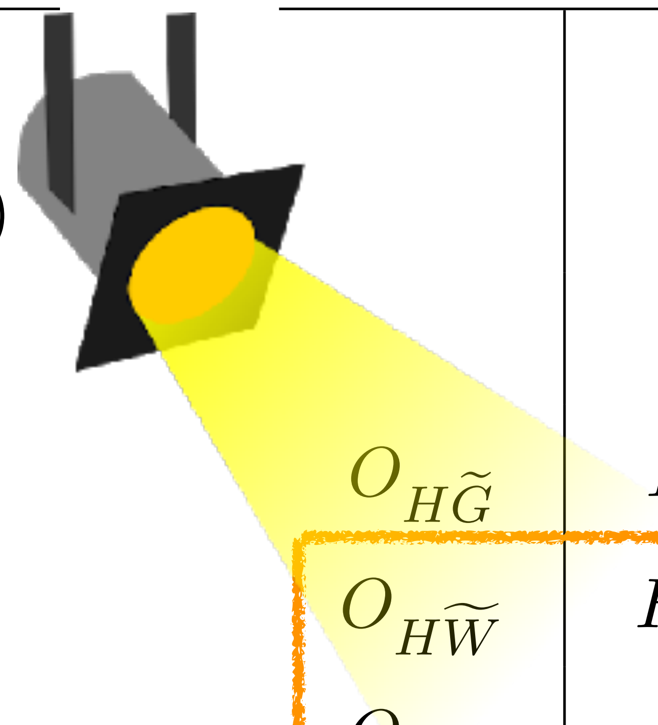
AA, Rodriguez-Sanchez [arXiv:2207.02161]

More detailed discussion concluding that $\Delta D \gtrsim 10^{-5}$ is difficult without fine-tuning

CP violation in Higgs sector

$$\mathscr{L}_{\text{SMEFT}} = \mathscr{L}_{\text{SM}} + \frac{1}{\Lambda_L}\mathscr{L}_{D=5} + \frac{1}{\Lambda^2}\mathscr{L}_{D=6} + \frac{1}{\Lambda_L^3}\mathscr{L}_{D=7} + \frac{1}{\Lambda^4}\mathscr{L}_{D=8} + \dots$$

$$v \ll \Lambda \ll \Lambda_L$$



Bosonic CP-even		Bosonic CP-odd	
O_H	$(H^\dagger H)^3$		
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$		
O_{HD}	$\left\lvert H^\dagger D_\mu H\right\rvert^2$		
O_{HG}	$H^\dagger H\, G^a_{\mu\nu}G^a_{\mu\nu}$	$O_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}^a_{\mu\nu}G^a_{\mu\nu}$
O_{HW}	$H^\dagger H\, W^i_{\mu\nu}W^i_{\mu\nu}$	$O_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}^i_{\mu\nu}W^i_{\mu\nu}$
O_{HB}	$H^\dagger H\, B_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B_{\mu\nu}$
O_{HWB}	$H^\dagger\sigma^i H\, W^i_{\mu\nu}B_{\mu\nu}$	$O_{H\widetilde{W}B}$	$H^\dagger\sigma^i H\, \widetilde{W}^i_{\mu\nu}B_{\mu\nu}$
O_W	$\epsilon^{ijk}W^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$	$O_{\widetilde{W}}$	$\epsilon^{ijk}\widetilde{W}^i_{\mu\nu}W^j_{\nu\rho}W^k_{\rho\mu}$
O_G	$f^{abc}G^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$	$O_{\widetilde{G}}$	$f^{abc}\widetilde{G}^a_{\mu\nu}G^b_{\nu\rho}G^c_{\rho\mu}$

CP violation in Higgs sector

$$\mathscr{L}_{D=6} \supset \frac{c_{H\tilde{B}}}{\Lambda^2} H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu} + \frac{c_{H\tilde{W}}}{\Lambda^2} H^\dagger H W^a_{\mu\nu} \widetilde{W}^a_{\mu\nu} + \frac{c_{H\tilde{W}B}}{\Lambda^2} H^\dagger \sigma^a H \widetilde{W}^a_{\mu\nu} B^{\mu\nu}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$
$$\widetilde{B}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} B_{\alpha\beta}$$

This leads to new CP violating interactions of the Higgs boson with electroweak vector bosons

$$\mathscr{L}_{\text{SMEFT}} \supset \frac{h}{\text{v}} \Big[2m_W^2 W^+_\mu W^-_\mu + m_Z^2 Z_\mu Z_\mu$$

SM interactions

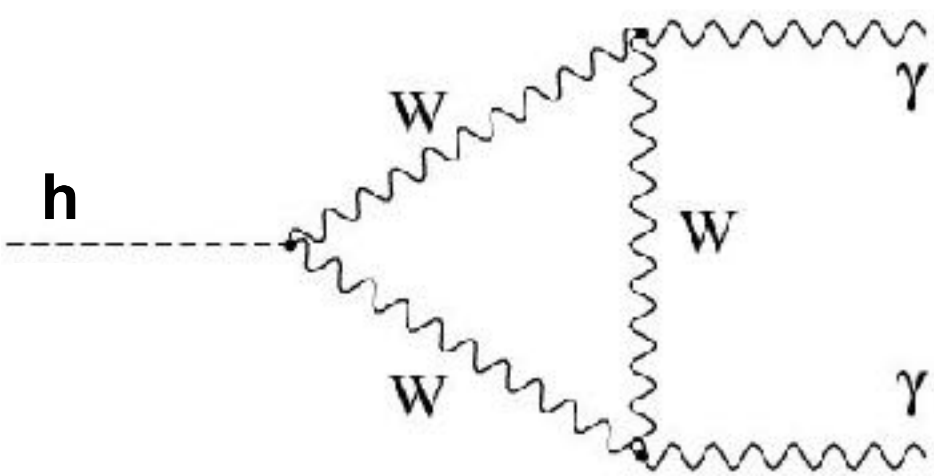
$$+ \tilde{c}_{ww} W^+_{\mu\nu} \widetilde{W}^-_{\mu\nu} + \tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu} + \tilde{c}_{z\gamma} F_{\mu\nu} \widetilde{Z}_{\mu\nu} + \tilde{c}_{zz} Z_{\mu\nu} \widetilde{Z}_{\mu\nu} \Big]$$

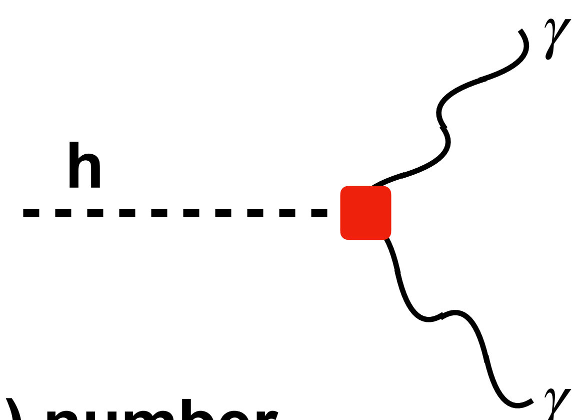
New CP violating interactions

4 couplings $\tilde{c}_{\text{vv}}$ **from 3 Wilson coefficients** $c_{H\tilde{B}}, c_{H\tilde{W}}, c_{H\tilde{W}B}$

Thus SMEFT predicts one relation between these CP violating couplings

These couplings will affect the Higgs production rates and decay width, e.g. Higgs decay to two photons





O(1) number

$$\Gamma(h \to \gamma\gamma) = \Gamma(h \to \gamma\gamma)_{\text{SM}} \left(1 + \#16\pi^2 |\tilde{c}_{\gamma\gamma}|^2\right)$$

CP violation in Higgs sector

$$\Gamma(h \to \gamma\gamma) = \Gamma(h \to \gamma\gamma)_{\text{SM}}\left(1 + \#16\pi^2 |\tilde{c}_{\gamma\gamma}|^2\right)$$

The Higgs branching ratio to photons is known at the 10% level

$$16\pi^2 |\tilde{c}_{\gamma\gamma}|^2 \lesssim 0.1 \qquad \Longrightarrow \qquad |\tilde{c}_{\gamma\gamma}| \lesssim 3 \times 10^{-2}$$

Translated to the scale of new physics

$$\mathscr{L}_{D=6} \supset \frac{c_{H\tilde{B}}}{\Lambda^2} H^\dagger H B_{\mu\nu} \widetilde{B}^{\mu\nu}$$

$$\tilde{c}_{\gamma\gamma} \sim c_{H\tilde{B}} \frac{\text{v}^2}{\Lambda^2} \qquad \Longrightarrow \qquad \Lambda \gtrsim 1.5 \text{ TeV} \sqrt{|c_{H\tilde{B}}|}$$

Only new physics close to the TeV scale can be probed
(this is the feature of all Higgs physics, not only for CP violating Higgs couplings)

Note that this observable cannot distinguish CP-violating and CP-conserving contributions

Higgs decays to either two positive or two negative helicity photons
and the relative phase between the two is affected by the CP violating coupling.
However, polarisation of high-energy photons is very difficult (impossible?) to observe

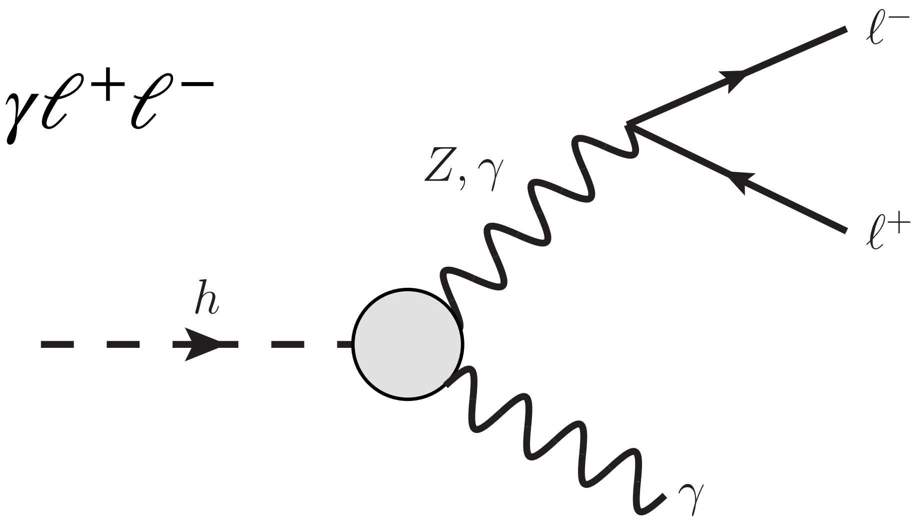
Bishara et al
1312.2955

CP violation in Higgs sector

CP violating observable can be constructed for 3- and more-body final states of Higgs decay

Process

$$h \to \gamma Z/\gamma^* \to \gamma \ell^+ \ell^-$$



$$\mathscr{L}_{\text{SMEFT}} \supset \frac{h}{\text{v}} \left[\tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu} + \tilde{c}_{z\gamma} F_{\mu\nu} \widetilde{Z}_{\mu\nu} \right]$$

All conditions for CP violation reunited:

- **weak phase due to CP violating couplings of photons and Z to the Higgs**
- **strong phase thanks to the relatively large Z width**
- **interference of different amplitudes with different weak and strong phases**
- **Polarization of Z/γ* can be probed by looking at the distribution of the lepton decay angle in the rest frame of intermediate Z/γ***

$$\frac{d\Gamma(h \to \gamma \ell^+ \ell^-)}{d\cos\theta} = (1 + \cos^2\theta) A_{\text{even}} + \cos\theta \, A_{\text{odd}}$$

$$A_{\text{odd}} \sim \frac{\Gamma_Z}{m_Z} \left(\#_1 \tilde{c}_{\gamma\gamma} + \#_2 \tilde{c}_{z\gamma} \right)$$

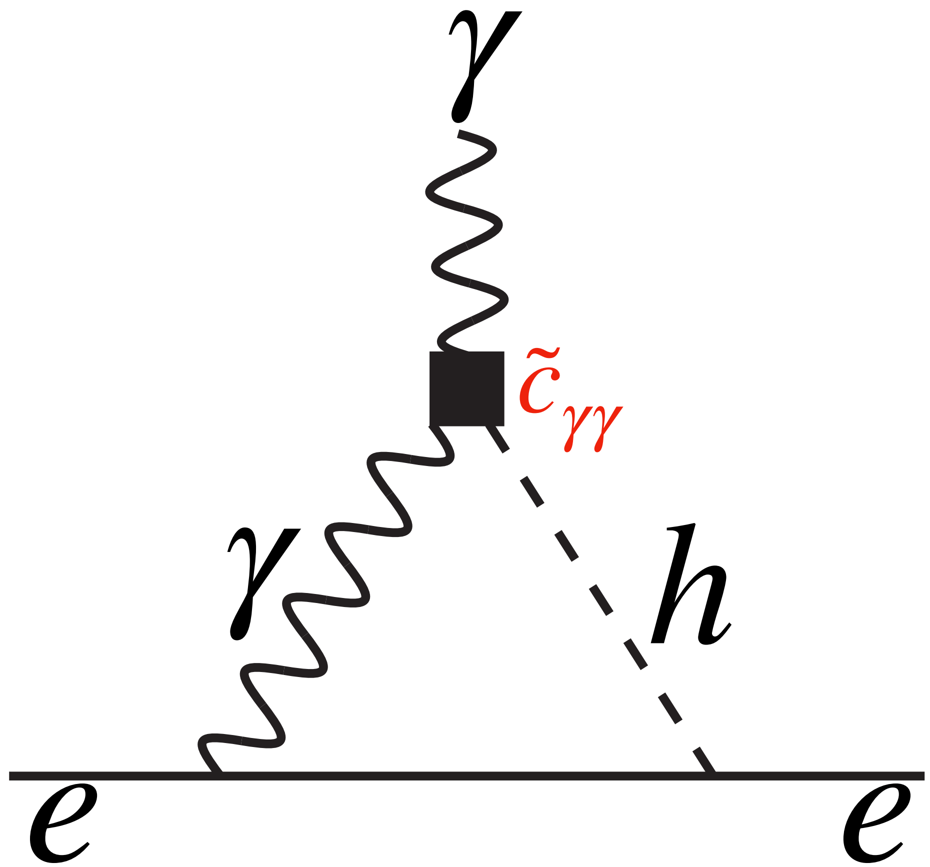
Chen et al. 1405.6723

Thus CP violation can be in principle observed in the Higgs sector (also in h→ ZZ→ 4l, h→ττ, …)
However, the sensitivity remains only for new physics around the TeV scale

CP violation in Higgs sector

$$\mathscr{L}_{\mathrm{SMEFT}} \supset \frac{h}{\mathrm{v}} \tilde{c}_{\gamma\gamma} F_{\mu\nu} \widetilde{F}_{\mu\nu}$$

$$\mathscr{L} \subset \frac{i}{2} d_e \left[e^c \sigma_{\mu\nu} e - \bar{e} \bar{\sigma}_{\mu\nu} \bar{e}^c \right] F_{\mu\nu}$$



By dimensional analysis: (or RG running)

$$d_e \sim \frac{\tilde{c}_{\gamma\gamma}}{16\pi^2} \frac{m_e e}{\mathrm{v}^2}$$

ACME limit $$|d_e| < \frac{1.7 \times 10^{-16}}{\mathrm{GeV}} \Rightarrow |\tilde{c}_{\gamma\gamma}| \lesssim 10^{-5}$$

Unless conspiracy, electron EDM limit exclude CP violating Higgs coupling to photon large enough to be ever observable at the LHC

CP violation in Higgs sector