Implications of A_4 modular symmetry on neutrino mass, mixing and leptogenesis with Linear seesaw

Rukmani Mohanta

University of Hyderabad, Hyderabad-500046, India

February 16, 2023

(Based on: Phys. Dark. Univ. **36**, 101027 (2022), with M. Behera et al.)

Neutrino Mass Generation: Quick Review

- Neutrino oscillation experiments provided compelling evidences for $m_\nu \neq 0$
- Massive neutrinos cannot be implemented in the SM through the Yukawa int, as there are no RH neutrinos.
- However, neutrino mass can arise at higher order: (dim-5 Weinberg operator)

$$\mathcal{O}_5 = \frac{Y^\nu_{ij}}{\Lambda}(\bar{L}_{L_i}\tilde{H})(\tilde{H}^T\bar{L}^C_{L_i}) + h.c. \Longrightarrow (m_\nu)_{ij} = \frac{Y^\nu_{ij}}{2\Lambda}v^2$$

- Shortcoming of Weinberg operator is that it violates L by two units, hence leading to Majorana mass term, but neutrinos could also be Dirac type
- One of the most viable theoretical frameworks used to yield neutrino masses is the see-saw mechanism.

Canonical Seesaw Mechanism

- The SM Lagrangian is extended with the addition of heavy RH Majorana neutrino singlets N_{R_i}
- New Yukawa interaction

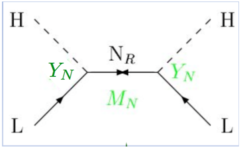
$$-\mathcal{L}_Y = Y_{ij}\overline{N_{R_i}}L_j H + \frac{1}{2}M_R\overline{N^c_{R_i}}N_{R_j} + \mathrm{h.c.}$$

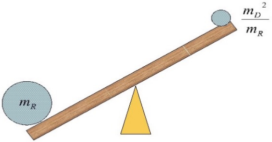
which gives the mass matrix in the $(\nu_L^c, N_R)^T$ basis

$$\mathbb{M} = \begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix}$$

- Upon diagonalization yields the light neutrino mass matrix as

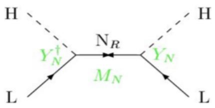
$$m_\nu = Y^T \frac{1}{M_R} Y\ v^2.$$





Seesaw Mechanisms

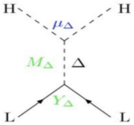
**Right-handed singlet:
(type-I seesaw)**



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$$

Minkowski; Gellman, Ramon, Slansky; Yanagida;Glashow; Mohapatra, Senjanovic

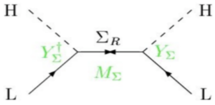
**Scalar triplet:
(type-II seesaw)**



$$m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$$

Magg, Wetterich; Lazarides, Shafi; Mohapatra, Senjanovic; Schechter, Valle

**Fermion triplet:
(type-III seesaw)**



$$m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

Foot, Lew, He, Joshi; Ma; Ma, Roy;T.H., Lin, Notari, Papucci, Strumia; Bajc, Nemevsek, Senjanovic; Dorsner, Fileviez-Perez;....

Courtesy: T. Hambye

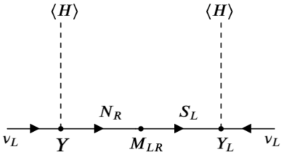
Modified Type-I Seesaw : Linear Seesaw

- The SM is extended by three RH (N_{R_i}) and three sterile (S_{Li}) singlet neutrinos
- The Yukawa interaction becomes

$$-\mathcal{L}_{linear} = Y\bar{N}_R\tilde{H}L + M_{LR}\bar{N}_R S_L + Y_L\bar{L}^C\tilde{H}S_L + h.c.$$

- The mass matrix for linear seesaw in the basis (ν_L, N_R, S_L^c)

$$M_{linear} = \begin{pmatrix} 0 & m_D & M_{LS} \\ m_D^T & 0 & M_{LR} \\ M_{LS}^T & M_{LR}^T & 0 \end{pmatrix}$$



- The mass formula for light neutrinos evolves from the above mass matrix for $M_{LR} \gg M_D, M_{LS}$

$$m_{linear} = m_D M_{LR}^{-1} M_{LS}^T + \text{transpose}$$

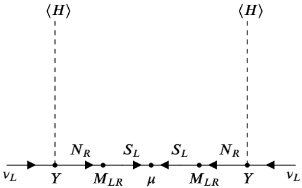
Inverse Seesaw

- The SM is extended by three RH (N_{R_i}) and three sterile (S_{Li}) singlet neutrinos
- The Yukawa interaction becomes

$$-\mathcal{L}_{linear} = Y\bar{N}_R LH + M_{LR}\bar{N}_R S_L + \mu \bar{S}_L^c S_L + h.c.$$

- The mass matrix for linear seesaw is

$$M_{linear} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_{LR} \\ M_{LS}^T & M_{LR}^T & \mu \end{pmatrix}$$



- The mass formula for light neutrinos for $\mu \ll m_D \ll M_R$

$$m_{inv} = m_D^T (M_{LR}^T)^{-1} \mu M_{LR}^{-1} m_D + h.c \sim \frac{v^2}{M_{LR}^2}\mu$$

A_4 Discrete Flavor Symmetry

- Since indication of CP violation in lepton sector has been observed at T2K and NOvA, we are in the era to develop Flavor theory of leptons
- A_4 models are attractive, because A_4 is the minimal group which has a triplet as irreducible reps: $(\mathbf{3}, \mathbf{1}, \mathbf{1}', \mathbf{1}'')$.
- It enables us to explain the flavour symmetry:
  $\mathbf{3} : (L_e, L_\mu, L_\tau), \quad \mathbf{1} : e_R, \quad \mathbf{1}' : \mu_R, \quad \mathbf{1}'' : \tau_R$
- However, it yields a vanishing reactor mixing angle θ_{13}
- Inclusion of simple perturbation by introducing the flavon fields (SM singlet scalars, but transform nontrivially under flavor group) can generate nonzero θ_{13}
- The flavons play a crucial role in determining the flavour structure due to their particular vacuum alignment.

A_4 Modular Symmetry

- The full modular group $\Gamma \equiv SL(2,Z)$ is the group of 2×2 matrices, with integer entries and determinant 1.
- The modular group $\bar{\Gamma}$ is the linear fractional transformation, acting on τ, varying in the upper half complex plane $\mathcal{H} = \{\tau \in \mathrm{Im}\tau > 0\}$, having the form:

$$\tau \longrightarrow \frac{a\tau + b}{c\tau + d}, \quad \{a, b, c, d \text{ integers}, \quad ad - bc = 1\}$$

- The modular group $\bar{\Gamma}$ can be generated by S and T:

$$S: \quad \tau \to -\frac{1}{\tau}, \quad \text{(duality)} \qquad T: \quad \tau \to \tau + 1, \quad \text{(discrete shift symmetry)}$$

which satisfy the multiplication rule

$$S^2 = (ST)^3 = \mathbb{1}$$

A_4 Modular Symmetry

- The *Principal congruence subgroup of level N*, (normal subgroup of Γ)

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,Z), \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

- We define $\bar{\Gamma}(N) = \Gamma(N)/\{\mathbb{1}, -\mathbb{1}\}$ for $N = 1, 2$, while $\bar{\Gamma}(N) = \Gamma(N)$ for $N > 2$
- The quotient group $\Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$ is the inhomogeneous finite modular group generated by two elements S and T :

$$S^2 = (ST)^3 = T^N = \mathbb{1}$$

- Γ_1 is the trivial group, $\Gamma_2 \cong S_3$, $\Gamma_3 \cong A_4$, $\Gamma_4 \cong S_4$, and $\Gamma_5 \cong A_5$.
- Holomorphic functions which transform as

$$f(\tau) \to (c\tau + d)^k f(\tau)$$

under the modular transformation are called modular forms of weight k.

Important features of A_4 Modular Symmetry

- The superpotential should be invariant under modular symmetry, i.e.,it should have vanishing modular weight
- The Yukawa couplings are functions of modulus τ, and transform nontrivially under the modular symmetry and can have nonvanishing modular weights
- The modular symmetry is broken by the vacuum expectation value of τ, i.e. at the compactification scale, which is of order of the Planck scale or slightly lower scale.
- The Dedekind eta function $\eta(\tau)$ is one of the famous modular forms written as

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \qquad q = e^{2\pi i \tau}.$$

Model Framework

- We consider modular A_4 symmetry to study the neutrino phenomenology
- A global $U(1)_X$ symmetry is imposed to avoid certain unwanted terms in the superpotential
- Particle spectrum enriched by six extra heavy fermion fields (N_{R_i} and S_{L_i}) and one weighton (ρ)
- The A_4 and $U(1)_X$ symmetries are considered to be broken at a scale much higher than the scale of EWSB

Fields	e_R^c	μ_R^c	τ_R^c	L_L	N_R	S_L^c	$H_{u,d}$	ρ
$SU(2)_L$	1	1	1	2	1	1	2	1
$U(1)_Y$	1	1	1	$-\frac{1}{2}$	0	0	$\frac{1}{2}, -\frac{1}{2}$	0
$U(1)_X$	1	1	1	-1	1	-2	0	1
A_4	1	$1'$	$1''$	$1, 1'', 1'$	3	3	1	1
k_I	1	1	1	-1	-1	-1	0	0

Table: Particle content of the model and their charges under $SU(2)_L \times U(1)_Y \times A_4$ where k_I is the modular weight.

Model Framework

- The importance of A_4 modular symmetry is that less no. of flavon fields are required, as Yukawa couplings have the non-trivial group transformation

Yukawa coupling	A_4	k_I
Y	**3**	**2**

- The Yukawa coupling $\mathbf{Y} = (y_1, y_2, y_3)$ with weight 2, which transforms as a triplet of A_4 can be expressed in terms $\eta(\tau)$ and its derivative

$$\begin{aligned}
y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\
y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right).
\end{aligned}$$

Masses for the Charged leptons

- The LH doublets $(L_{e_L}, L_{\mu_L}, L_{\tau_L})$ transform as $1, 1'', 1'$ under A_4 symmetry and are assigned $U(1)_X$ charge of -1, $k_l = -1$
- The RH charged leptons are $1, 1', 1''$ under A_4 with $U(1)_X$ charge $+1$ and $k_l = -1$
- The relevant superpotential term for charged leptons is given by

$$\mathcal{W}_{M_\ell} = y_\ell^{ee} L_{e_L} H_d \ e_R^c + y_\ell^{\mu\mu} L_{\mu_L} H_d \ \mu_R^c + y_\ell^{\tau\tau} L_{\tau_L} H_d \ \tau_R^c \, .$$

- The charged lepton mass matrix takes the form:

$$M_\ell = \begin{pmatrix} y_\ell^{ee} v_d/\sqrt{2} & 0 & 0 \\ 0 & y_\ell^{\mu\mu} v_d/\sqrt{2} & 0 \\ 0 & 0 & y_\ell^{\tau\tau} v_d/\sqrt{2} \end{pmatrix} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} .$$

Dirac and pseudo Dirac mass terms for Neutral fermions

- The RH N_{R_i}'s transform as triplets under A_4 modular group with $U(1)_X$ charge of 1 and modular weight -1.
- The Yukawa couplings to transform as triplets under the A_4 with modular weight 2, so one can write invariant Dirac superpotential as

$$\mathcal{W}_D = \alpha_D L_{e_L} H_u \, (Y N_R)_1 + \beta_D L_{\mu_L} H_u \, (Y N_R)_{1'} + \gamma_D L_{\tau_L} H_u \, (Y N_R)_{1''}.$$

- The resulting Dirac neutrino mass matrix is found to be

$$M_D = \frac{v_u}{\sqrt{2}} \begin{bmatrix} \alpha_D & 0 & 0 \\ 0 & \beta_D & 0 \\ 0 & 0 & \gamma_D \end{bmatrix} \begin{bmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{bmatrix}_{LR}.$$

Dirac and pseudo Dirac mass terms for Neutral fermions

- As we also have the extra sterile fermions S_{Li}, the pseudo-Dirac term is allowed, and the corresponding superpotential is given as

$$\mathcal{W}_{LS} = \left[\alpha'_D L_{e_L} H_u \, (Y S_L^c)_1 + \beta'_D L_{\mu_L} H_u \, (Y S_L^c)_{1'} + \gamma'_D L_{\tau_L} H_u \, (Y S_L^c)_{1''}\right] \frac{\rho^3}{\Lambda^3} \, ,$$

- The flavor structure for the pseudo-Dirac neutrino mass matrix takes the form,

$$M_{LS} = \frac{v_u}{\sqrt{2}} \left(\frac{v_\rho}{\sqrt{2}\Lambda}\right)^3 \begin{bmatrix} \alpha'_D & 0 & 0 \\ 0 & \beta'_D & 0 \\ 0 & 0 & \gamma'_D \end{bmatrix} \begin{bmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{bmatrix}_{LR} .$$

Mixing between heavy Neutral fermions N_R and S_L^c

- One can have the interactions leading to the mixing between these additional fermion field as

$$\mathcal{W}_{M_{RS}} = [\alpha_{NS} Y(S_L^c N_R)_{\mathrm{sym}} + \beta_{NS} Y(S_L^c N_R)_{\mathrm{Anti-sym}}]\rho$$

- Using $\langle\rho\rangle = v_\rho/\sqrt{2}$, the resulting mass matrix is

$$M_{RS} = \frac{v_\rho}{\sqrt{2}} \left(\frac{\alpha_{NS}}{3} \begin{bmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_1 & 2y_3 \end{bmatrix} + \beta_{NS} \begin{bmatrix} 0 & y_3 & -y_2 \\ -y_3 & 0 & y_1 \\ y_2 & -y_1 & 0 \end{bmatrix} \right).$$

- $\alpha_{NS}/3 \neq \beta_{NS}$, otherwise M_{RS} becomes singular
- The masses for the heavy fermions in the basis $(N_R, S_L^c)^T$, can be written as

$$M_{Hf} = \begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & 0 \end{pmatrix}.$$

- Therefore, one can have six doubly degenerate mass eigenstates for the heavy superfields upon diagonalization.

Linear Seesaw mechanism for light neutrino Masses

- Within the present model invoked with A_4 modular symmetry, the complete 9×9 mass matrix in the flavor basis of $(\nu_L, N_R, S_L^c)^T$ is

$$\mathbb{M} = \left(\begin{array}{c|ccc} & \nu_L & N_R & S_L^c \\ \hline \nu_L & 0 & M_D & M_{LS} \\ N_R & M_D^T & 0 & M_{RS} \\ S_L^c & M_{LS}^T & M_{RS}^T & 0 \end{array} \right).$$

- The linear seesaw mass formula for light neutrinos is given with the assumption $M_{RS} \gg M_D, M_{LS}$ as,

$$m_\nu = M_D M_{RS}^{-1} M_{LS}^T + \mathrm{transpose}.$$

- We numerically diagonalize the neutrino mass matrix through the relation $U^\dagger \mathcal{M} U = \mathrm{diag}(m_1^2, m_2^2, m_3^2)$, where $\mathcal{M} = m_\nu m_\nu^\dagger$ and U is an unitary matrix

$$\sin^2\theta_{13} = |U_{13}|^2, \quad \sin^2\theta_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2}.$$

Numerical Analysis

- To fit to the current neutrino oscillation data, we chose the following ranges for the model parameters:

$$\mathrm{Re}[\tau] \in [-0.5, 0.5], \quad \mathrm{Im}[\tau] \in [1, 2], \quad \{\alpha_D, \beta_D, \gamma_D\} \in 10^{-5}\ [0.1, 1],$$
$$\{\alpha'_D, \beta'_D, \gamma'_D\} \in 10^{-2}\ [0.1, 1], \qquad \alpha_{NS} \in [0, 0.5], \quad \beta_{NS} \in [0, 0.0001],$$
$$v_\rho \in [10, 100]\ \mathrm{TeV}, \quad \Lambda \in [100, 1000]\ \mathrm{TeV}.$$

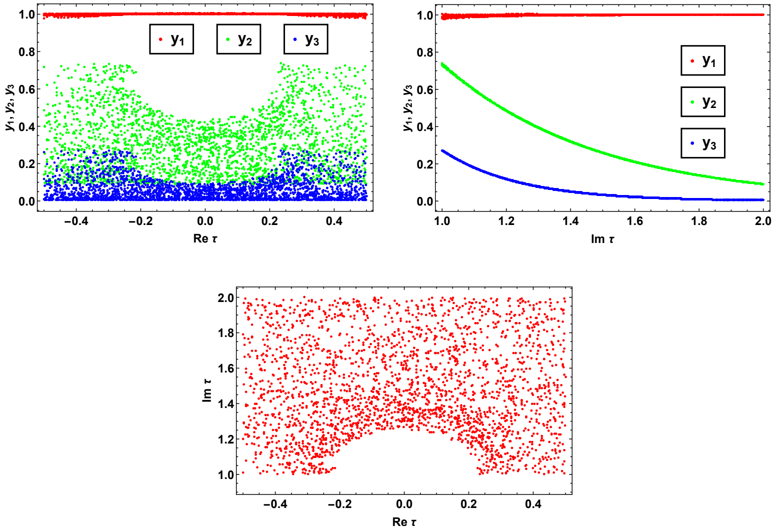
- The input parameters are randomly scanned over to obtain the allowed parameter space satisfying the neutrino oscillation data and $\sum m_i < 0.12$ eV
- The typical range of τ is found to be:

$$-0.5 \ \lesssim\ Re[\tau] \lesssim\ 0.5 \ \ \mathrm{and}\ \ 1\ \lesssim\ Im[\tau] \lesssim\ 2 \ \ \mathrm{for}\ \ \mathrm{NO}.$$

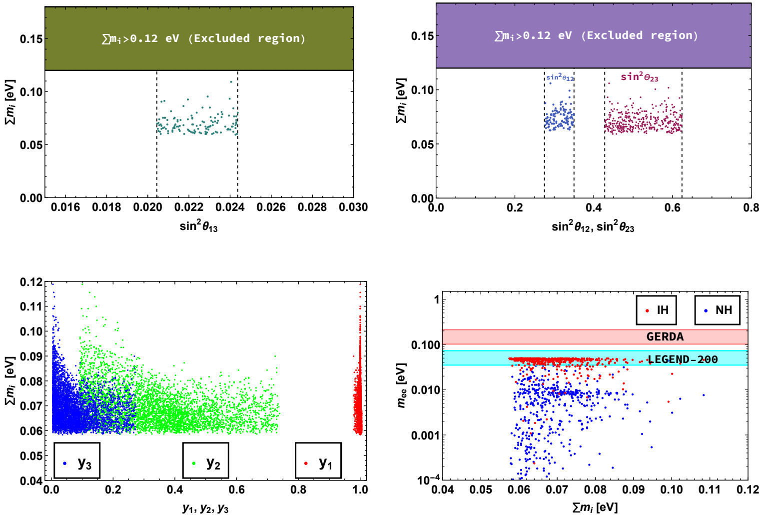
- Yukawa couplings as function of τ are found to vary in the region:

$$0.99\ \lesssim\ y_1(\tau) \lesssim\ 1, \qquad 0.1\ \lesssim\ y_2(\tau) \lesssim\ 0.8\ \ \mathrm{and}\ \ 0.01\ \lesssim\ y_3(\tau) \lesssim\ 0.3.$$

Some Results



Some Results



Baryon Asymmetry of the Universe

- Understanding the origin of neutrino mass and BAU are two major challenges in Particle Physics
- Leptogenesis plays a vital role in relating these two issues
- The BAU is parametrized in terms of the following quantity:

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = (6.04 \pm 0.08) \times 10^{-10}$$

- An alternative way to express matter-antimatter asymmetry is to use the ratio

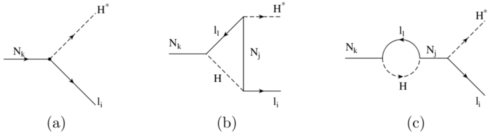
$$Y_B = \frac{n_b - n_{\bar{b}}}{s}$$

- The two formulations in terms of Y_B and η, at the present time, are easily related:

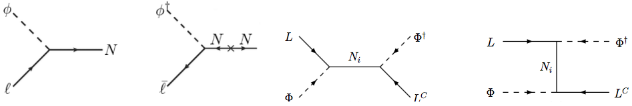
$$Y_B = \frac{n_\gamma^0}{s^0}\eta = 0.142\eta = (8.77 \pm 0.24) \times 10^{-11}.$$

Leptogenesis in 3 Basic Steps

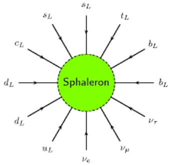
1. Generation of lepton asymmetry by the decay of heavy Majorana neutrino



2. Partial washout of the asymmetry due to inverse decays and scattering

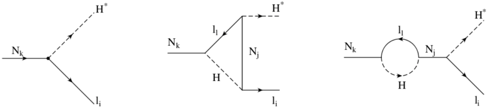


3. Conversion of the residual lepton asymmetry to baryon asymmetry



Resonant Leptogenesis

- For quasi-degenerate heavy Majorana neutrinos, i.e., $M_{N_i} - M_{N_j} \ll M_{N_i}$, the self energy ($\varepsilon$) contribution to the CP asymmetry becomes dominant



(a) (b) (c)

- Resonant leptogenesis occurs when $M_{N_i} - M_{N_j} \sim \Gamma_{N_i}$, in this case CP asymmetry can become very large (even order 1)
- The ε-type CP asymmetry,

$$\varepsilon_{N_i} = \frac{\mathrm{Im}(h^{\nu\dagger} h^{\nu})^2_{ij}}{(h^{\nu\dagger} h^{\nu})_{ii}(h^{\nu\dagger} h^{\nu})_{jj}} \frac{(m^2_{N_i} - m^2_{N_j}) m_{N_i} \Gamma^{(0)}_{N_j}}{(m^2_{N_i} - m^2_{N_j})^2 + m^2_{N_i} \Gamma^{(0)\,2}_{N_j}}$$

- $\mathcal{O}(1)$ CP asymmetries are possible when,

$$m_{N_2} - m_{N_1} \sim \frac{1}{2}\Gamma^{(0)}_{N_{1,2}}, \qquad \frac{\mathrm{Im}(h^{\nu\dagger} h^{\nu})^2_{ij}}{(h^{\nu\dagger} h^{\nu})_{ii}(h^{\nu\dagger} h^{\nu})_{jj}} \sim 1$$

Resonant Leptogenesis in the present framework

- Six heavy states with doubly degenerate masses for each pair, obtained by diagonalization of the heavy fermion mass matrix

$$M_{Hf} = \begin{pmatrix} 0 & M_{RS} \\ M_{RS}^T & 0 \end{pmatrix}$$

- But one can introduce a higher dimensional mass term for the heavy RH neutrinos (N_R) as

$$L_M = -\alpha_R Y N_R^c N_R^c \frac{\rho^2}{\Lambda}$$

- Thus, one can construct the right-handed Majorana mass matrix as

$$M_R = \frac{\alpha_R v_\rho^2}{6\Lambda} \begin{pmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_1 & 2y_3 \end{pmatrix}.$$

- α_R is extremely small to retain the linear seesaw structure of the mass matrix i.e., $M_D, M_{LS} \gg M_R$

Leptogenesis cont.

- Block diagonalization of the mass matrix yields:

$$M' = \begin{pmatrix} M_{RS} + \frac{M_R}{2} & -\frac{M_R}{2} \\ -\frac{M_R}{2} & -M_{RS} + \frac{M_R}{2} \end{pmatrix} \approx \begin{pmatrix} M_{RS} + \frac{M_R}{2} & 0 \\ 0 & -M_{RS} + \frac{M_R}{2} \end{pmatrix}.$$

- And the mass eigenstates ($N^{\pm}$) are related to N_R and S_L^c through

$$\begin{pmatrix} S_{Li}^c \\ N_{Ri} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} N_i^+ \\ N_i^- \end{pmatrix}.$$

- The mass eigenvalues for the new states N^+ and N^- can be expressed as

$$M_{RS} \pm \frac{M_R}{2} = \left(\frac{\alpha_{NS} v_\rho}{\sqrt{2}} \pm \frac{\alpha_R v_\rho^2}{4\Lambda} \right) \begin{pmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_1 & 2y_3 \end{pmatrix}.$$

- The lightest pair, assumed to be in the TeV scale, dominantly contribute to the CP asymmetry, i.e., the contribution from one loop self energy dominates over the vertex diagram.

One Flavor Approximation

- The evolution of lepton asymmetry can be deduced from the Boltzmann equations.
- Sakharov criteria demand the decay of parent fermion to be out of equilibrium to generate the lepton asymmetry.
- To impose this condition, one has to compare the Hubble rate with the decay rate

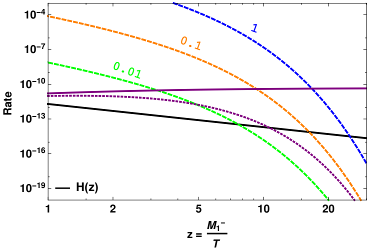
$$K = \frac{\Gamma_{N_1^-}}{H(T = M_1^-)} \, .$$

- The Boltzmann equations for the evolution of the number densities of RH fermions, in terms of yield parameter

$$\frac{dY_{N-}}{dz} = -\frac{z}{sH(M_1^-)}\left[\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}} - 1\right)\gamma_D + \left(\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}}\right)^2 - 1\right)\gamma_S\right],$$
$$\frac{dY_{B-L}}{dz} = -\frac{z}{sH(M_1^-)}\left[\epsilon_{N-}\left(\frac{Y_{N-}}{Y_{N-}^{\rm eq}} - 1\right)\gamma_D - \frac{Y_{B-L}}{Y_\ell^{\rm eq}}\frac{\gamma_D}{2}\right]$$

One Flavor Approximation

Figure: Interaction rates with Hubble expansion.



- Decay (Γ_D) in Purple solid line and inverse decay $\left(\Gamma_D \frac{Y^{\mathrm{eq}}_{N^-}}{Y^{\mathrm{eq}}_{\ell}}\right)$ dotted purple line with the coupling strength $\sim 10^{-6}$.
- The scattering rate $\left(\frac{\gamma_S}{s Y^{\mathrm{eq}}_{N^-}}\right)$ for $N_1^- N_1^- \to \rho\rho$ is projected for various set of values for coupling, consistent with neutrino oscillation study.

One Flavor Approximation

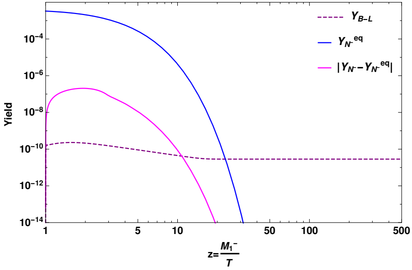


Figure: Evolution of Y_{B-L} (dashed) as a function of $z = M_1^- / T$.

- The obtained lepton asymmetry gets converted to the observed baryon asymmetry through sphaleron transition

$$Y_B = \left(\frac{8N_f + 4N_H}{22N_f + 13N_H} \right) Y_{B-L} \sim \mathcal{O}(10^{-10}).$$

Flavor Consideration

- One flavor approximation is reasonable at high scale ($T > 10^{12}$ GeV), where all the Yukawa interactions are out of equilibrium.
- But for temperatures below 10^{12} GeV, various Yukawa couplings come into equilibrium $\Longrightarrow$ flavor effects play a crucial role in generating the final lepton asymmetry.
- For temperatures below 10^{5} GeV, all the Yukawa interactions are in equilibrium and the asymmetry is stored in the individual lepton sector.
- The Boltzmann equation for generating lepton asymmetry in each flavor is

$$\frac{dY^{\alpha}_{B-L_{\alpha}}}{dz} = -\frac{z}{sH(M_1^-)}\left[\epsilon^{\alpha}_{N-}\left(\frac{Y_{N-}}{Y^{eq}_{N-}} - 1\right)\gamma_D - \left(\frac{\gamma^{\alpha}_D}{2}\right)\frac{A_{\alpha\alpha}Y^{\alpha}_{\mathrm{B-L}_{\alpha}}}{Y^{eq}_{\ell}}\right],$$

where

$$\gamma^{\alpha}_D = sY^{eq}_{N-}\Gamma^{\alpha}_{N-}\frac{K_1(z)}{K_2(z)}, \quad \gamma_D = \sum_{\alpha}\gamma^{\alpha}_D$$

Flavor Consideration

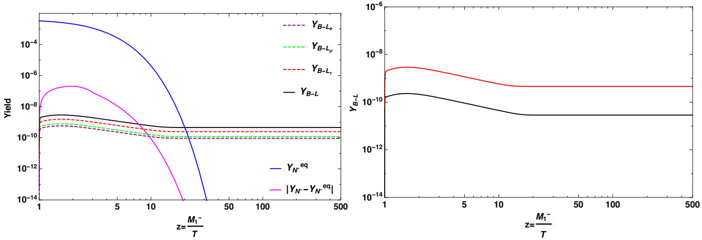


Figure: Left panel displays yield with inclusion of flavor effects. Right panel shows the enhancement in the yield due to three-flavor case over one-flavor approximation.

- The enhancement is because, in one flavor approximation the decay of heavy fermion to a specific lepton flavor final state ($N \to \ell_\alpha H$) can get washed out by the inverse decays of any flavor ($\ell_\beta + H \to N$) unlike the flavored case

Conclusion

- The modular A_4 flavor symmetry is quite successful in accommodating the observed neutrino oscillation data.
- The important aspect of modular symmetry is that the Yukawa couplings to transform non-trivially under modular A_4 group, which replaces the role of conventional flavon fields.
- Leptogenesis can be explained through the decay of lightest heavy fermion eigenstate
- It is found that the evolution of lepton asymmetry at TeV scale comes out to be of the order $\approx 10^{-10}$, which is sufficient to explain the present baryon asymmetry of the Universe.

Thank you for your attention !