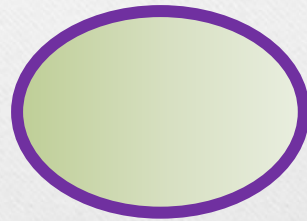


Neutrinos, Cosmological Phase-Transition and the Matter-Antimatter Asymmetry



ν



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- Why is there no anti-workshop at « sehcuoH seL »?
(pronunciation is left as an exercise to the reader)
- Almost only particles are observed at all scales (no anti-particles)
- The CMB gives us the ratio baryons/photons:



$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = 6 \cdot 10^{-10} \neq 0$$



Need for New Physics: **Sterile Neutrinos**

Outline

1. Motivating sterile Neutrinos
2. Phase-Transition
3. Tools and Results: Out-of-Equilibrium QFT

1. Motivating sterile Neutrinos: **light neutrinos and asymmetry**

ν



The Seesaw Mechanism

Light Standard Model neutrinos

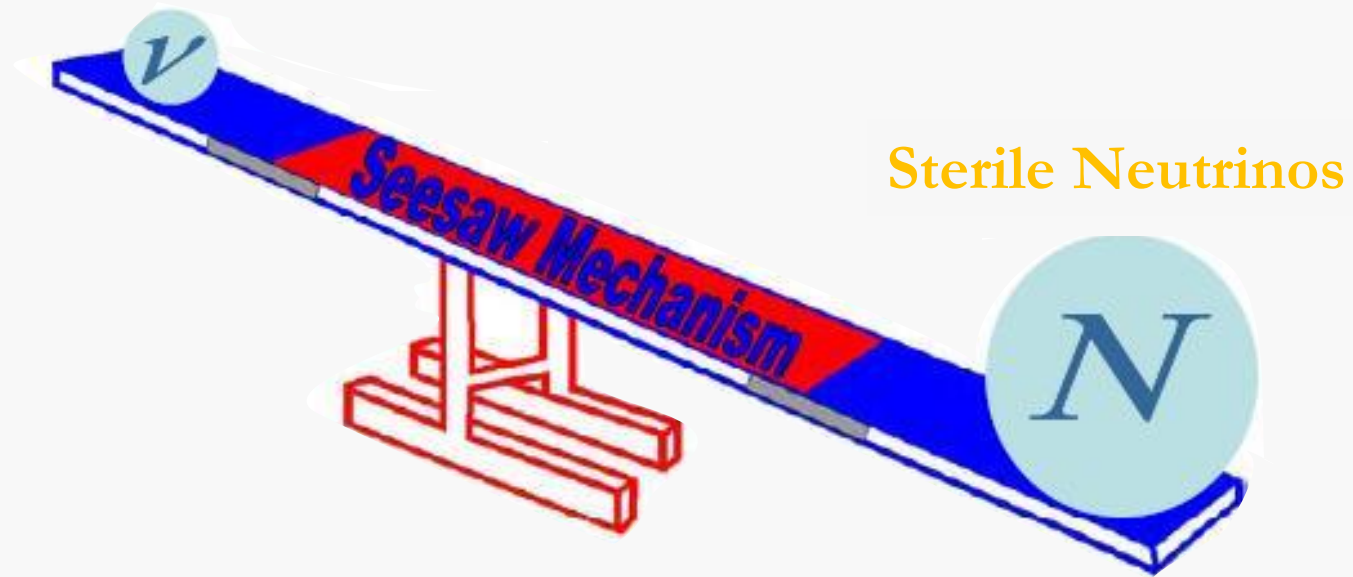


Figure by [Shaaban Khalil](#)

N : sterile Neutrinos

ϕ : Higgs field

l_α : lepton field, can be neutrino ν or electron e

L_{SM} : Lagrangian of the Standard Model

The Neutrinos interact
with the Standard Model

Violates **CP**
symmetry

$$L = L_{SM} + i\bar{N}\gamma^\mu\partial_\mu N - M_I\bar{N}_I^c N_I + Y_{I\alpha} N_i \bar{l}_\alpha \tilde{\phi} + h.c.$$

Massive Neutrinos
Majorana mass M

Violates **Lepton number**

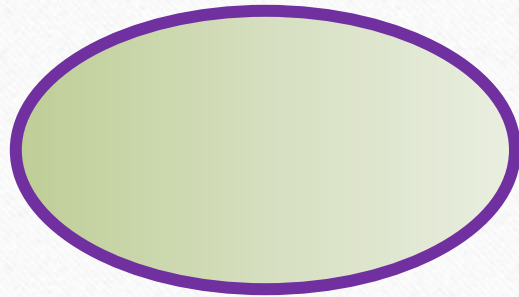
Sakharov conditions for generation of matter-antimatter asymmetry (1967):

- Baryon/Lepton number violation
- C and CP violation
- Out-of-Equilibrium



Andrei Sakharov

2. Phase Transition: going out of equilibrium



$$L = L_{SM} + i\bar{N}\gamma^\mu\partial_\mu N - M_I\bar{N}_I^c N_I + Y_{I\alpha}N_I\bar{l}_\alpha\tilde{\phi} + h.c.$$



[Rosauero-Alcaraz (2021)]

[Huang, Xie (2022)]

$$L = L_{SM} + L_S + i\bar{N}\gamma^\mu\partial_\mu N - \lambda_{NS}^I S\bar{N}_I^c N_I + Y_{I\alpha}N_I\bar{l}_\alpha\tilde{\phi} + h.c.$$

$$M_I = \lambda_{NS}^I S$$

New interactions, new
dynamics for the sterile sector

$$\langle S \rangle = 0 \rightarrow \langle S \rangle \neq 0$$

Typically, the scalar field S will experience a **phase-transition**.

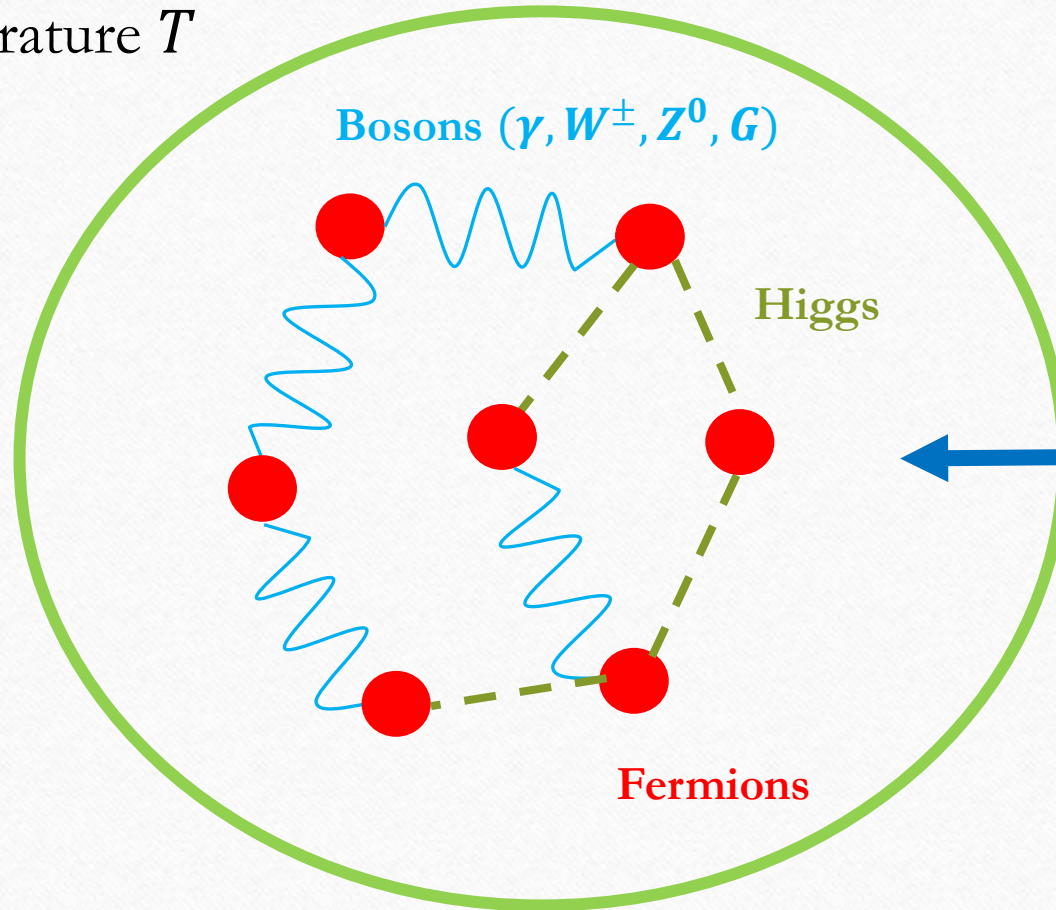
A (strongly) **first-order phase-transition** can be more interesting for two reasons:

1. Specific **gravitational wave** signature.
2. **Sudden change** in the value of the field, which allows for **non-equilibrium** dynamics.

Standard Model

New sector

Temperature T



Sterile Neutrinos

New scalar

Not in thermal equilibrium

« Active » sector

False vacuum

$$\langle S \rangle = 0$$

$$M = 0$$

True vacuum

$$\langle S \rangle \neq 0$$

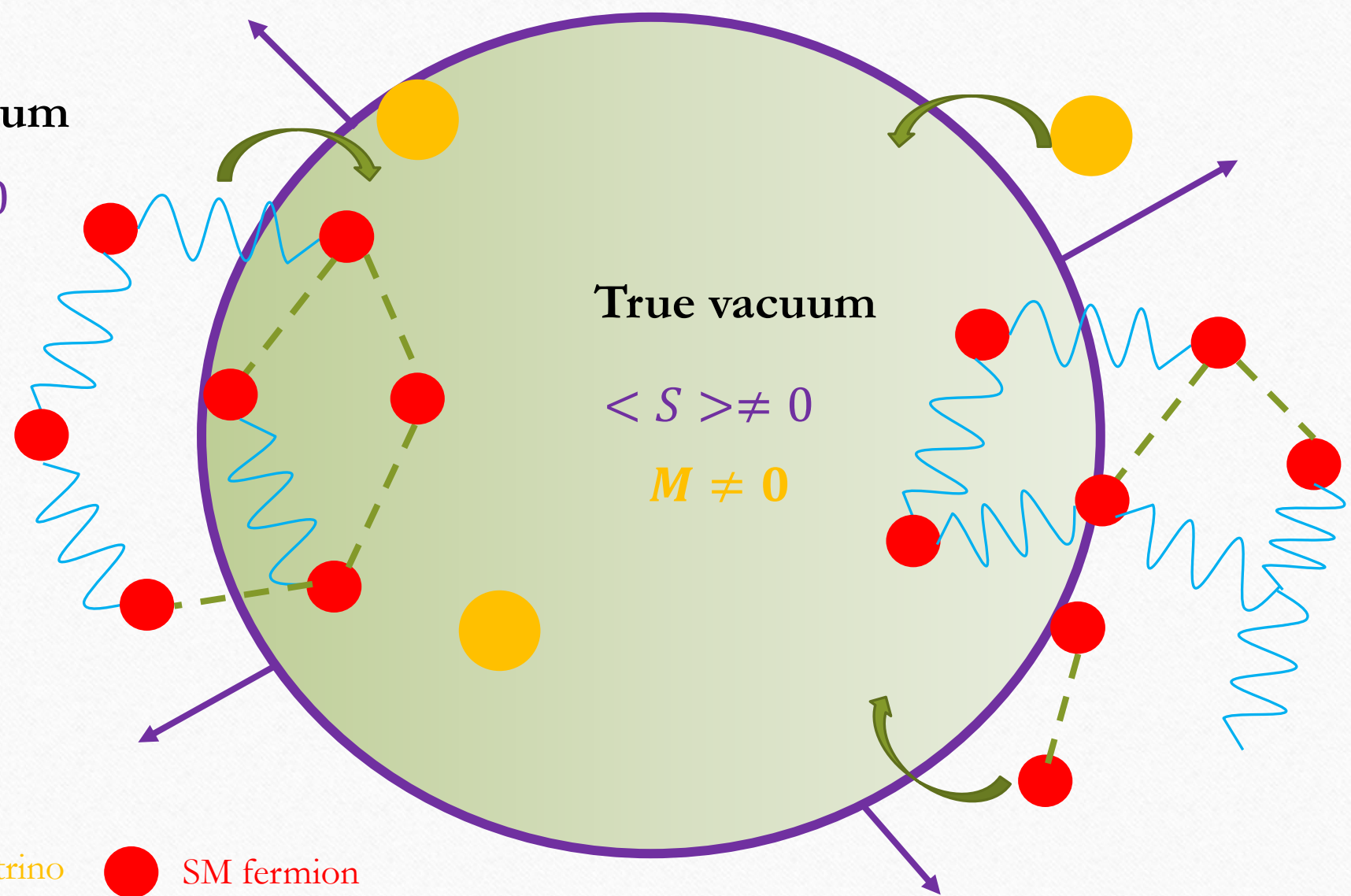
$$M \neq 0$$



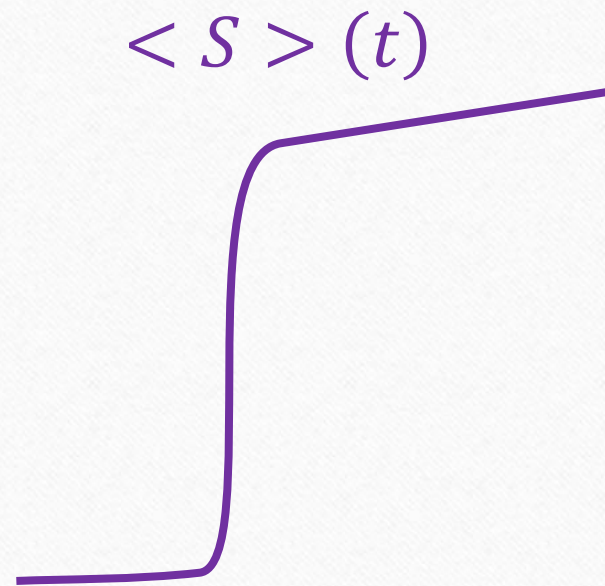
Sterile Neutrino



SM fermion



During the **phase-transition**, the masses of the Neutrinos are **time-dependent**. All quantities will have an explicit time-dependence along the **wall**.



$$M_I = \lambda_{NS}^I \langle S \rangle(t) = M_I(t)$$

3. Tools and Results: **Out-of-Equilibrium QFT**

Thermal Leptogenesis

[Anisimov, Buchmüller,
Drewes, Mendizabal (2010)]

Low-scale Leptogenesis

[Drewes, Garbrecht,
Gueter, Klaric (2010)]

Electroweak Baryogenesis

[Lee, Cirigliano, Ramsey-
Musolf (2005)]

...

Equilibrium QFT: Schwinger-Dyson Equation

$\Sigma =$ self-energy

Free propagator

$$S^{-1}(x - y) = i\delta(x - y)\gamma^\mu\partial_\mu - \Sigma(x - y)$$

Full propagator

Deviation due to interactions

$$i\gamma^\mu\partial_\mu S(x - y) - \Sigma \otimes S(x - y) = \delta(x - y)$$

(convolution)

Non-Equilibrium QFT: Schwinger-Dyson Equation

Σ = self-energy

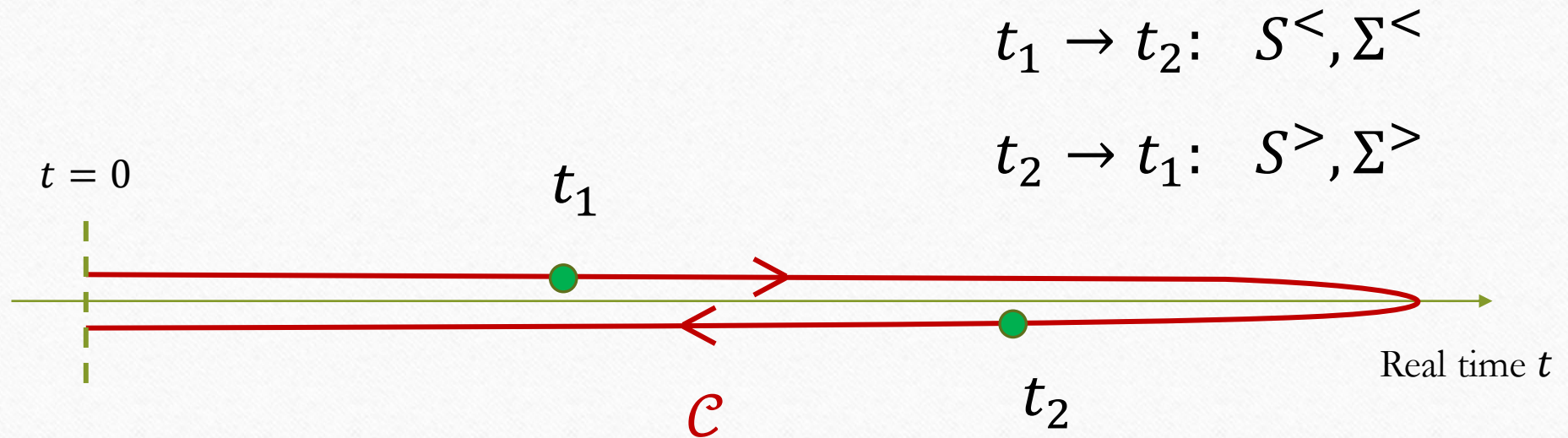
$$S^{-1}(x - y, \mathbf{t}) = i\delta(x - y)\gamma^\mu\partial_\mu - \Sigma(x - y, \mathbf{t})$$

$$i\gamma^\mu\partial_\mu S(x - y, \mathbf{t}) - \Sigma \otimes S(x - y, \mathbf{t}) = \delta(x - y)$$

\mathbf{t} -dependence: no longer time-translation invariant

[Keldysh (1965)]

Keldysh-Schwinger/CTP formalism



Initial value problem: we know the distribution only at $t = 0$ (~~$\langle f | S | i \rangle$~~)

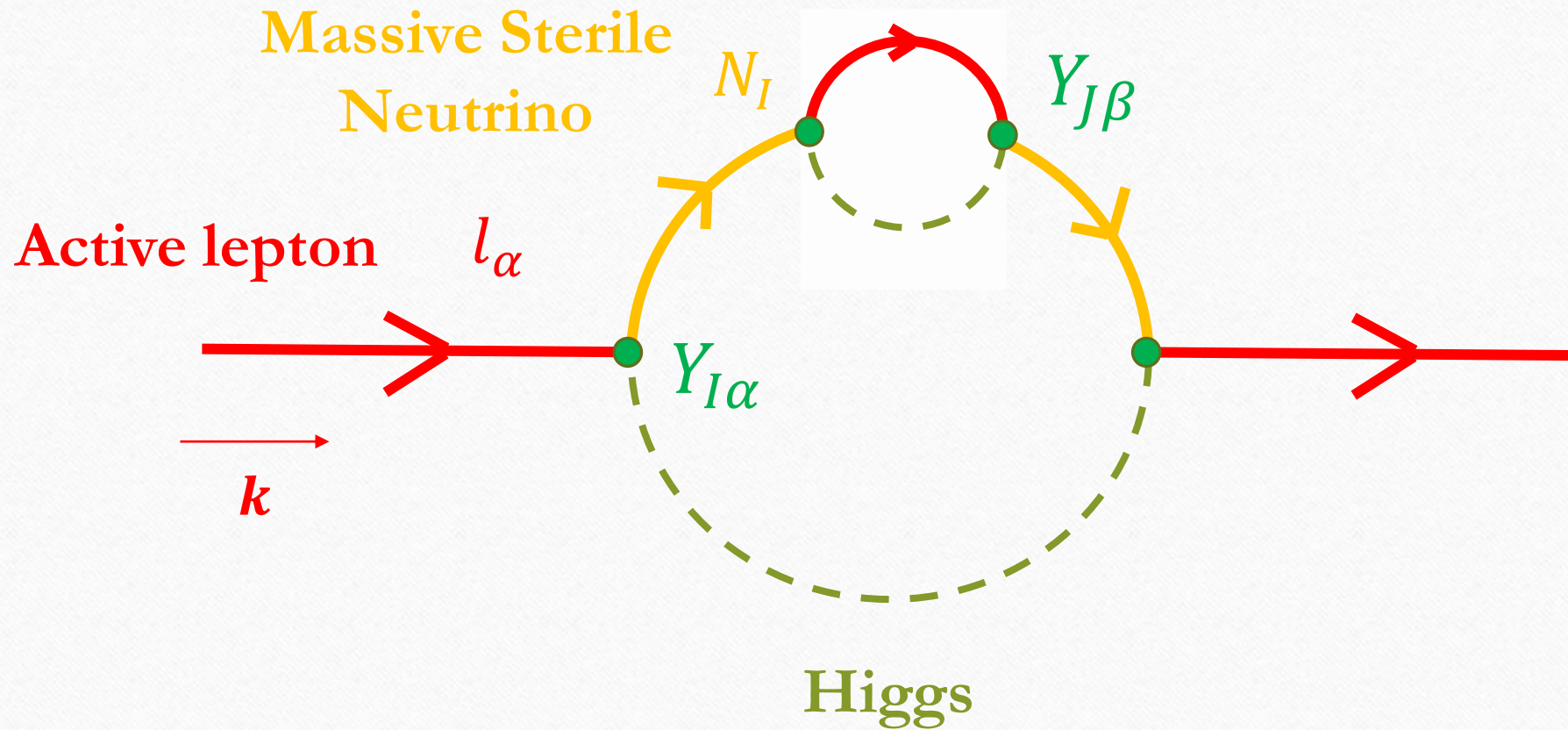
$$i \gamma^\mu \partial_\mu S(x - y, \mathbf{t}) - \Sigma \otimes S(x - y, \mathbf{t}) = \delta(x - y)$$

n_L = asymmetry between leptons and anti-leptons = density of lepton number

$j^\mu = -\text{Tr}(\gamma^\mu S_L)$ = quadri-current associated with the lepton number

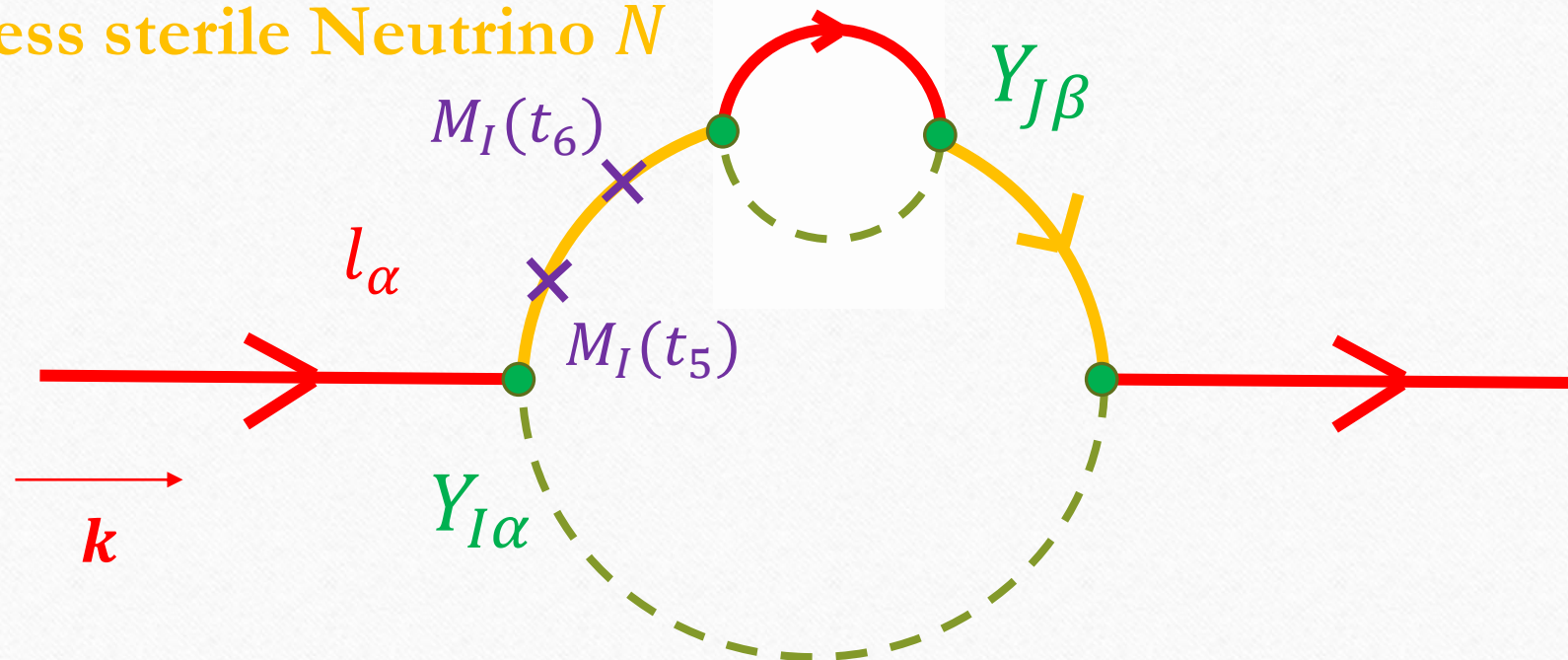
$$n_L = j^0 = -\text{Tr}(\gamma^0 S_L) = i \int dt_1 \text{Tr}(i \gamma^0 \partial_0 S_L)$$

$$= - \int dt_1 dt_2 (\text{Tr}(\Sigma^>(t_1, t_2) S_L^<(t_2, t_1)) - \text{Tr}(\Sigma^< S_L^>))$$



Two-loop self-energy Σ_k

Massless sterile Neutrino N



$$M_I \ll T$$

(expansion in M)

Higgs

Two-loop self-energy $\Sigma_{\mathbf{k}}$

Final (comoving) asymmetry

$$Y_L^\alpha = \frac{n_L^\alpha}{s} \propto \sum_{I,J} \frac{\text{Im} \left(Y_{I\alpha} Y_{J\alpha}^* (YY^\dagger)_{JI} \right)}{(YY^\dagger)_{II} (YY^\dagger)_{JJ}} \frac{M_I^2}{T^2} \times \text{phase space}$$

Majorana masses of the heavy Neutrino states (violate **lepton number**)

CP-violating term (differentiates particles and anti-particles)

Temperature of the phase transition (**out-of-equilibrium** process)

Casas-Ibarra parametrization

[Casas, Ibarra (2001)]

Observable at low-energy

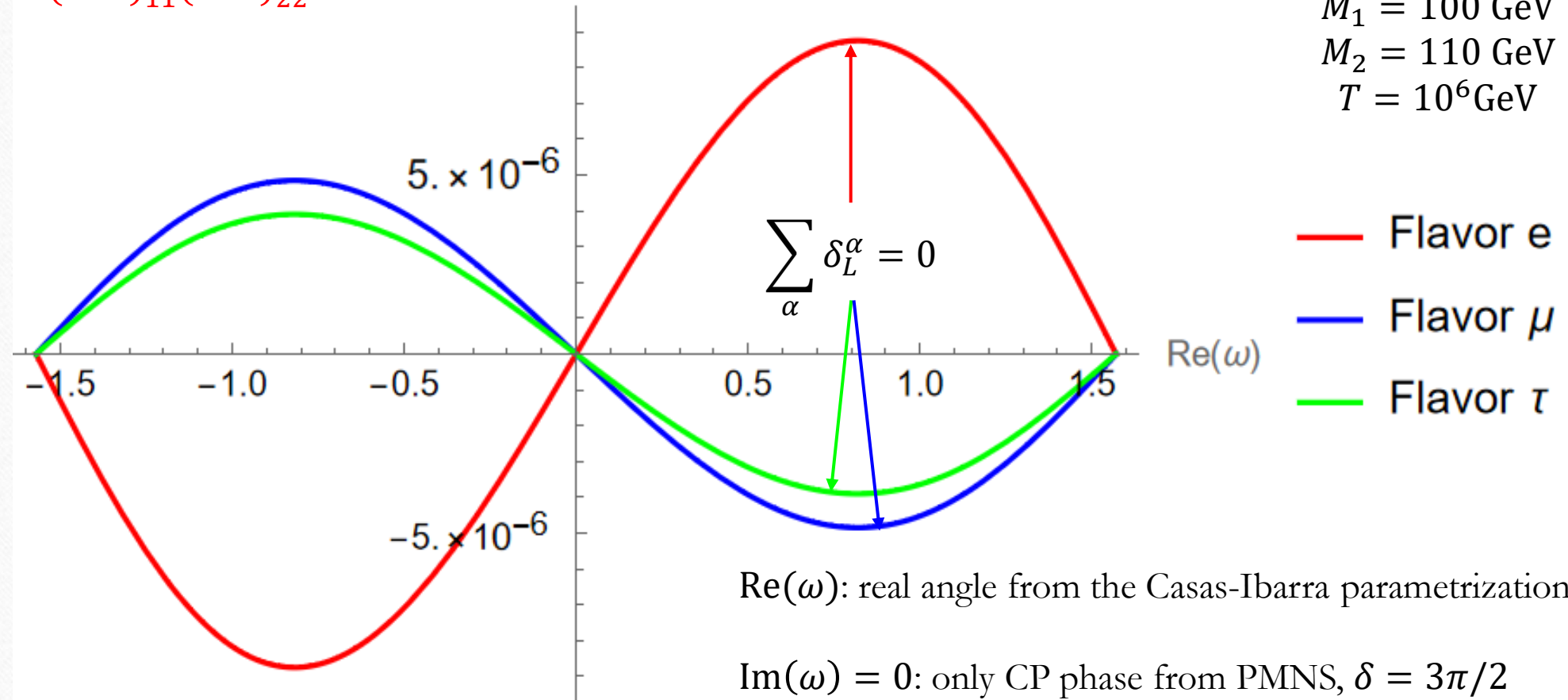
$$Y \equiv \sqrt{M} R \sqrt{m} U_{PMNS}^\dagger$$

$$R = \begin{pmatrix} \cos(\omega) & \sin(\omega) & 0 \\ -\sin(\omega) & \cos(\omega) & 0 \end{pmatrix} \quad m = \begin{pmatrix} m_3 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 \end{pmatrix} \simeq \begin{pmatrix} |\Delta m_{atm}| & 0 & 0 \\ 0 & |\Delta m_{sol}| & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(normal ordering)

Non-observable complex angle ω (parameter)

$$\frac{\text{Im} \left(Y_{1\alpha} Y_{2\alpha}^* (YY^\dagger)_{21} \right)}{(YY^\dagger)_{11} (YY^\dagger)_{22}} \frac{M_2^2 - M_1^2}{T^2} \equiv \delta_L^\alpha$$



This work can be compared to a work by Pascoli, Turner and Zhou, who derived a lepton asymmetry from a **varying Weinberg operator**.

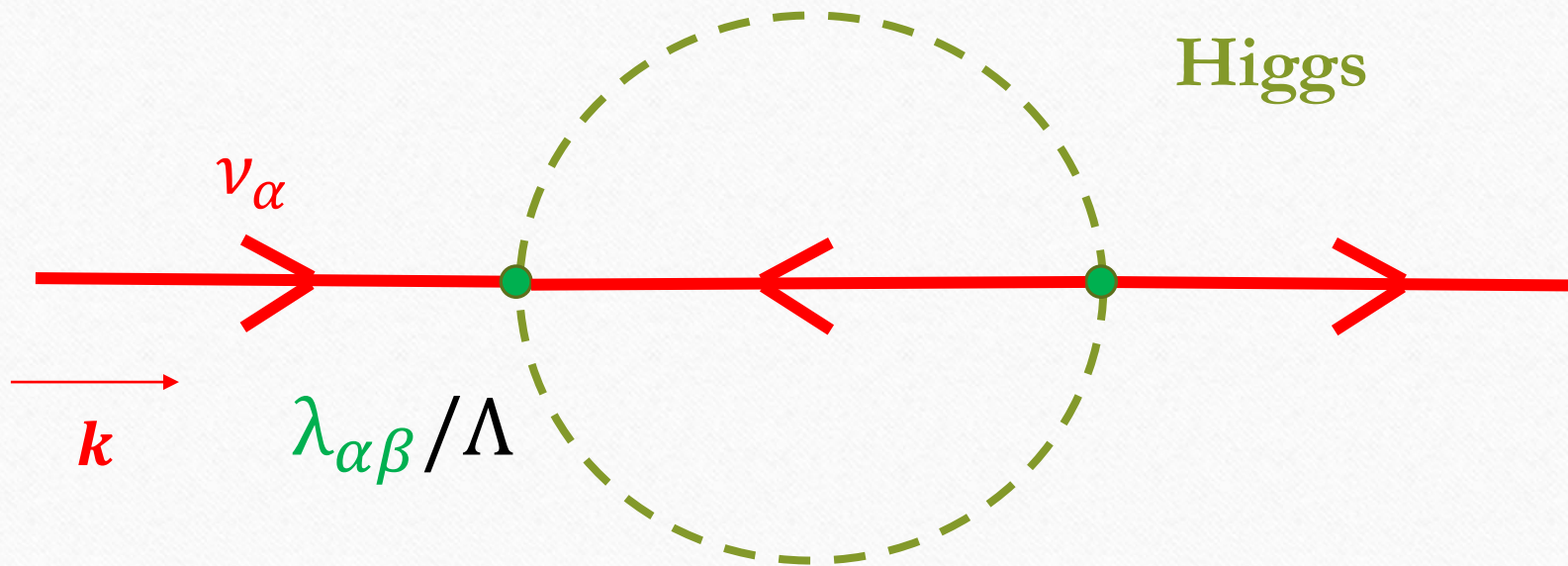
The coefficient of the operator is driven by a **phase-transition**.

$$L = L_{SM} + \frac{\lambda_{\alpha\beta}}{\Lambda} l_{\alpha} H C l_{\beta} H + h.c. = L_{SM} + \frac{\lambda_{\alpha\beta}(S)}{\Lambda} l_{\alpha} H C l_{\beta} H + h.c.$$

$$\lambda^0 \rightarrow \lambda, \quad m_{\nu}^0 \rightarrow m_{\nu}$$

(Active neutrino mass matrix)

$$L = L_{SM} + \frac{\lambda_{\alpha\beta}}{\Lambda} l_{\alpha} H C l_{\beta} H + h.c. = L_{SM} + \frac{\lambda_{\alpha\beta}(S)}{\Lambda} l_{\alpha} H C l_{\beta} H + h.c.$$



Two-loop self-energy $\Sigma_{\mathbf{k}}$

(Active) neutrino mass matrix before and after the PT (**CP and Lepton Number violation**)

v_H : Higgs vev

m_ν^0 : (active) neutrino mass matrix **before** PT
(parameter)

m_ν : (active) neutrino mass matrix **after** PT
(measurable)

$$Y_L \propto \frac{\text{Im}(\text{Tr}(m_\nu^0 m_\nu^*))}{v_H^4} T^2 \times \text{phase space}$$

Final (comoving) asymmetry

Temperature of the phase transition (**out-of-equilibrium** process)

Comparison of both models

- Asymmetry produced **during** the phase-transition (not via decay afterwards)
- Right-handed Neutrinos are **not** decoupled in our scenario
- Dependence on **low-energy observables**

Summary and conclusion

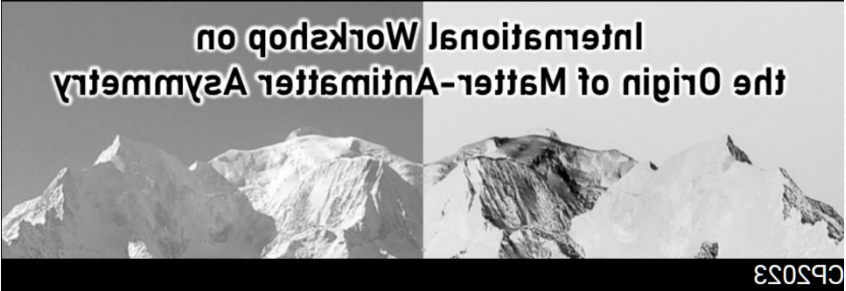
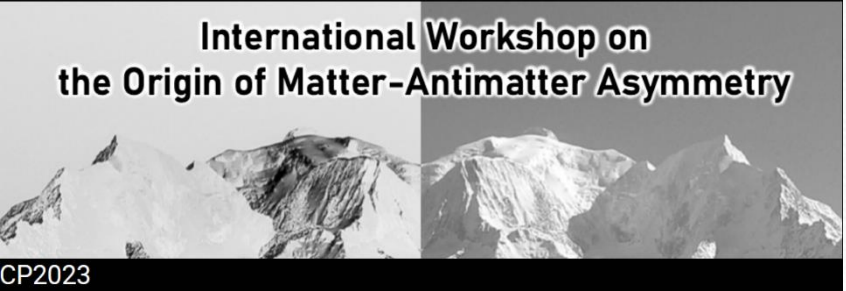
Work achieved

- We consider an original scenario where the Majorana Neutrinos acquire their mass and produce an asymmetry during a single phase-transition.
- We computed the asymmetry from first-principles (numerical work on progress).

True vacuum

ν

Thanks for your attention!



Leptogenesis via neutrino oscillations (ARS)

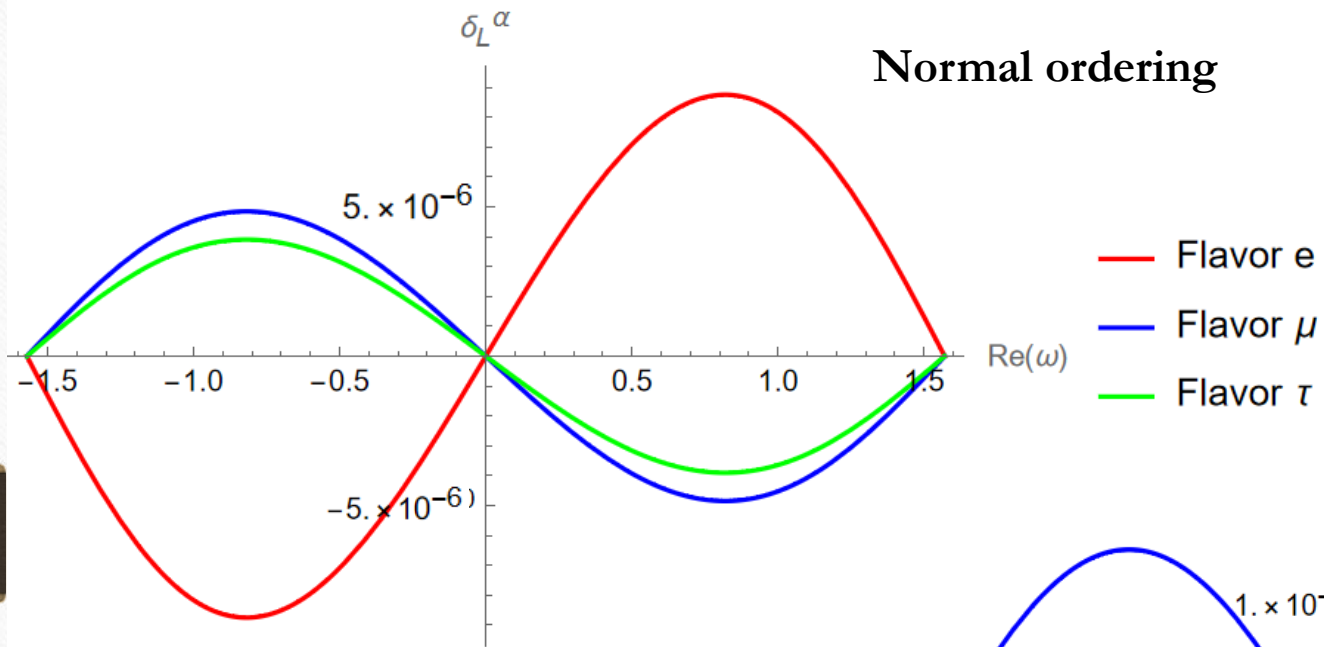
[See for example
Drewes, Garbrecht,
Gueter, Klaric (2010)]

$$T_{osc} \equiv \left(M_{Pl} \Delta M_{2,1}^2 \right)^{1/3}$$

$M_{Pl} \simeq 7 \times 10^{17} \text{ GeV}$
is the Planck Mass

$$T_{osc} = \left(7 \times 10^{17} \times (110^2 - 100^2) \right)^{1/3} \text{ GeV} \simeq 10^7 \text{ GeV} > T_{PT}$$

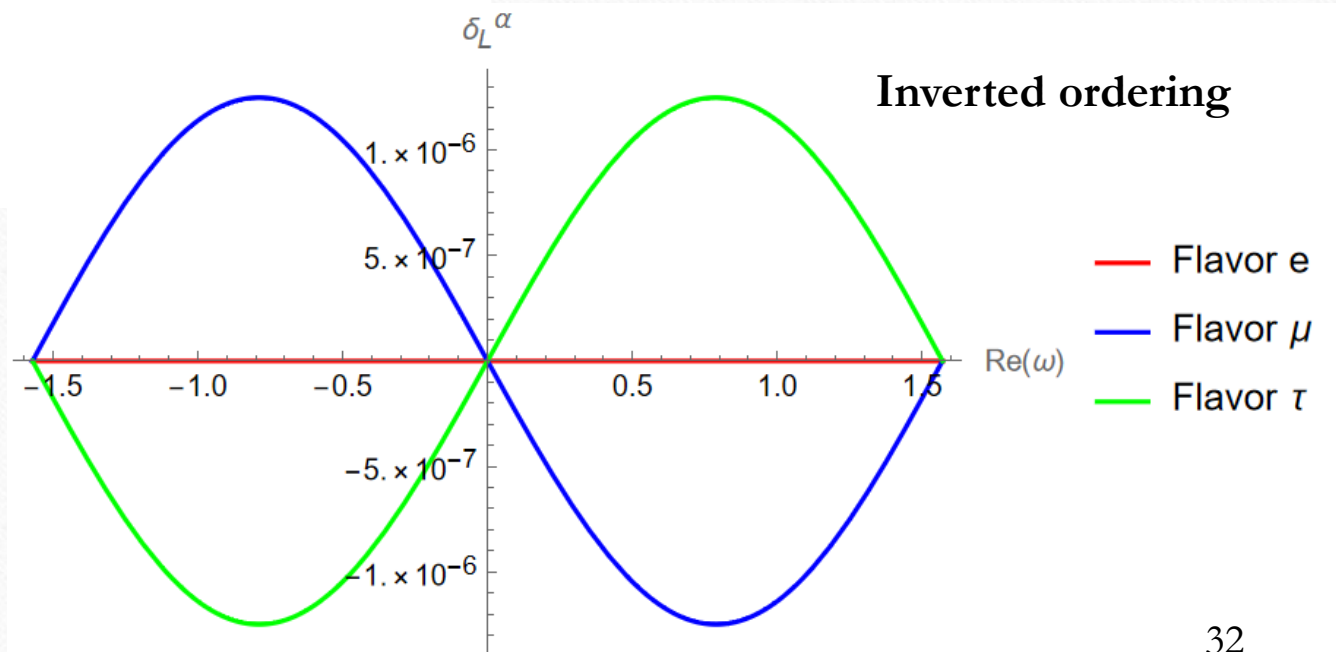
$$M_1 = 100 \text{ GeV} \quad M_2 = 110 \text{ GeV} \quad T_{PT} = 10^6 \text{ GeV}$$



$$\alpha_{21} = 0$$

$$\alpha_{31} = 2\pi$$

$$\delta = 3\pi/2$$



Further prospects

- Reflection of Sterile Neutrinos on the wall of the bubbles
- (ARS) Leptogenesis without Phase-transition with initial asymmetries – Washout of the asymmetries

The Weinberg operator could be derived from **right-handed Neutrinos**, if they decouple below some energy scale. The value of the Weinberg coefficient depends on the mass of these Neutrinos.

$$\lambda^0/\Lambda = Y^T M_{N0}^{-1} Y$$

$$\lambda/\Lambda = Y^T (M_{N0} + \lambda_{NS} v_S)^{-1} Y = Y^T M_N^{-1} Y$$

$$\Sigma_{\mathbf{k},\alpha}^{\leq}(t_1, t_2) = \int_{\mathcal{C}} dt_5 \int_{\mathcal{C}} dt_6 M_I(t_5) M_I(t_6) \times Y_{I\alpha} (Y_{I\beta}^* Y_{J\beta}) Y_{J\alpha}^*$$

$$\times \int_p \int_q \int_{\mathcal{C}} dt_3 dt_4 S_{N,p}(t_1, t_3) S_{L,k}(t_3, t_4) S_{N,p}(t_4, t_2) \times \Delta_H \Delta_H$$

Propagators for
the Higgs field



$$n_L^\alpha = - \int dt_1 dt_2 (\text{Tr}(\Sigma_\alpha^{\geq} S_L^{\leq}) - \text{Tr}(\Sigma_\alpha^{\leq} S_L^{\geq}))$$

$$\sim - \iiint_{\mathbf{k},p,q} \int dt_1 dt_2 (\text{Tr}(\Sigma_{\mathbf{k},\alpha}^{\geq} S_{L,\mathbf{k}}^{\leq}) - \text{Tr}(\Sigma_{\mathbf{k},\alpha}^{\leq} S_{L,\mathbf{k}}^{\geq})^*)$$

« phase space » = six-dimensional time-integrals
+ three momentum-integrations

$$I = \int_0^\infty dt_1 \dots \int_0^\infty dt_6 e^{-\gamma|t_1-t_2|} \dots e^{-\gamma|t_6-t_1|} e^{-\Gamma(t_1-t_2)} \dots e^{-\Gamma(t_6-t_1)} \\ \times \cos(k(t_1 - t_2)) \dots \cos(p(t_6 - t_1))$$

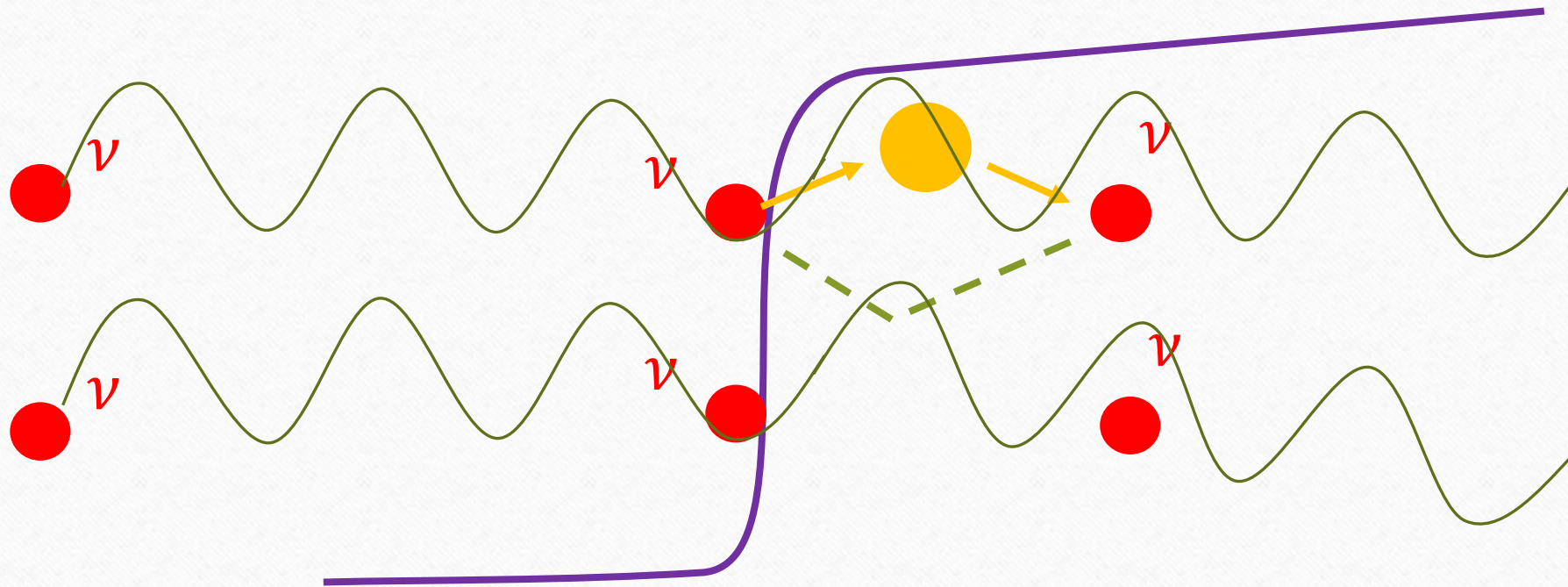
$$= \int_0^\infty dt_1 \int_{-t_1}^\infty dy_1 \dots \int_{-(t_1+y_1+\dots)}^\infty dy_5 e^{-\gamma|y_1|} \dots e^{-\gamma|y_5|} e^{-\Gamma y_1} \dots e^{-\Gamma y_5} \\ \times \cos(ky_1) \dots \cos(py_5)$$

Interferences between loops and the tree-level allow for a **distinction between particles and anti-particles** in the **probabilities** (square of the amplitude).

$$\begin{aligned}
 |\mathcal{M}|^2 &= |\mathcal{M}_0 + \mathcal{M}_1|^2 = |\mathcal{M}_0 + \mathcal{M}_1^{odd} + \mathcal{M}_1^{even}|^2 \\
 &= |\mathcal{M}_0|^2 + |\mathcal{M}_1|^2 + \mathcal{M}_0 \mathcal{M}_1^{odd*} + \mathcal{M}_0 \mathcal{M}_1^{even*} + c.c.
 \end{aligned}$$

CP conjugation

$$\begin{aligned}
 |\overline{\mathcal{M}}|^2 &= |\overline{\mathcal{M}}_0 + \overline{\mathcal{M}}_1|^2 = |\mathcal{M}_0^* - \mathcal{M}_1^{odd*} + \mathcal{M}_1^{even*}|^2 \\
 &= |\mathcal{M}_0|^2 + |\overline{\mathcal{M}}_1|^2 - \mathcal{M}_0 \mathcal{M}_1^{odd*} + \mathcal{M}_0 \mathcal{M}_1^{even*} + c.c.
 \end{aligned}$$



Quantum Interference

Mass of the sterile Neutrino $M = 0$

$M \neq 0$

General scheme

- Asymmetry in the **active** sector (Δ_α) is originally very small.
- Solve for the sterile sector only (**no « backreaction »**)
- Compute the source term.

Homogeneous and isotropic Universe

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2$$

Scale factor = « Size » of the Universe

Its evolution is driven by the **matter content**

$$f_h(\vec{p}, \vec{x}, t) = f_h(\vec{p}, t) \quad n_h = \int d^3\vec{p} f_h(\vec{p}, t) = n_h(t) = n^{eq} + \delta n_h$$

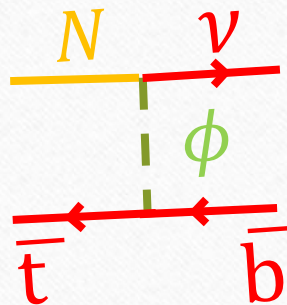
Quantum Boltzmann Equation

Hamiltonian: oscillations between different flavors

« Backreaction » of the asymmetry in active sector

$$\frac{d\delta n_h}{dt} = -\frac{i}{2} [H_{vac} + H_{th}, \delta n_h] - \frac{1}{2} \{\Gamma_h, \delta n_h\} + \frac{1}{2} \tilde{\Gamma}_\alpha \mathcal{A}_{\alpha\beta} \Delta_\beta$$

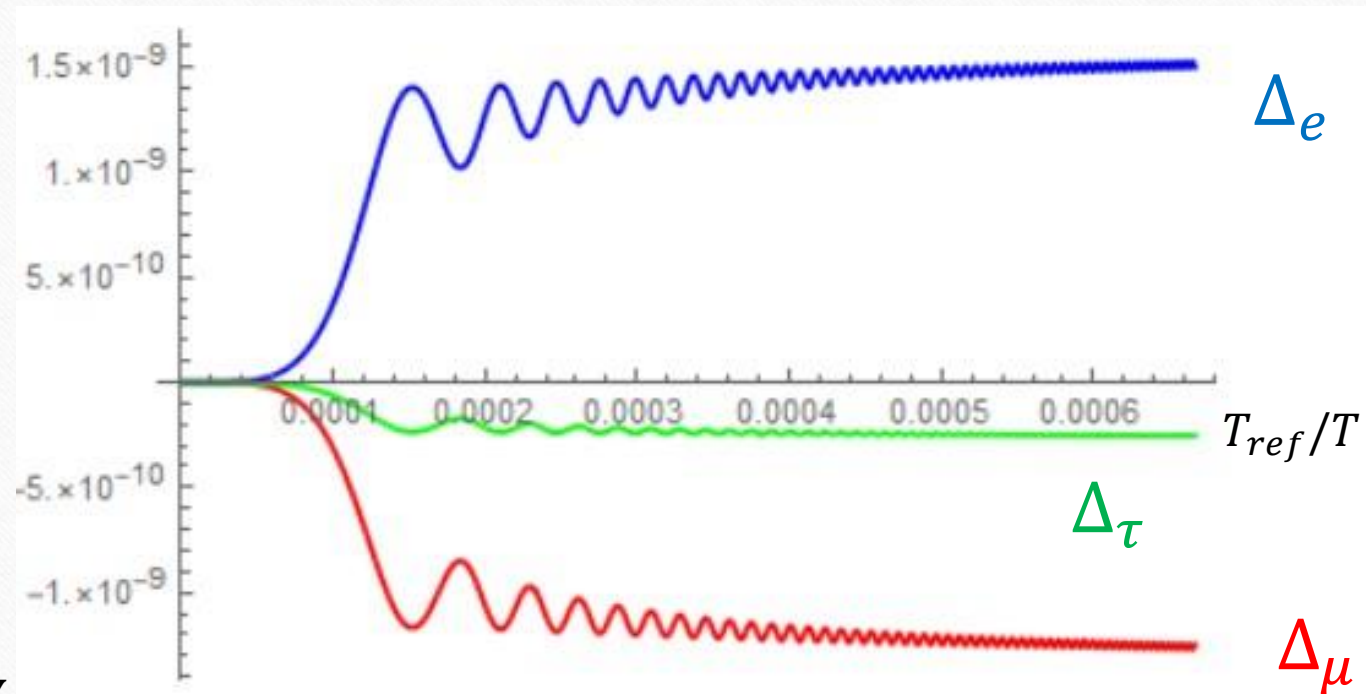
Decay rate: scattering that suppresses Neutrinos



Δ_β = asymmetry in the active sector for each flavor

$\mathcal{A}_{\alpha\beta}$ = linear coefficients

Example of an evolution of the asymmetry



$$M_1 = 50 \text{ GeV}$$
$$M_2 = M_1 + 10^{-3} \text{ GeV}$$