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Principles of Optical Interferometry

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Motivation: Large Aperture ⇒ High Resolution

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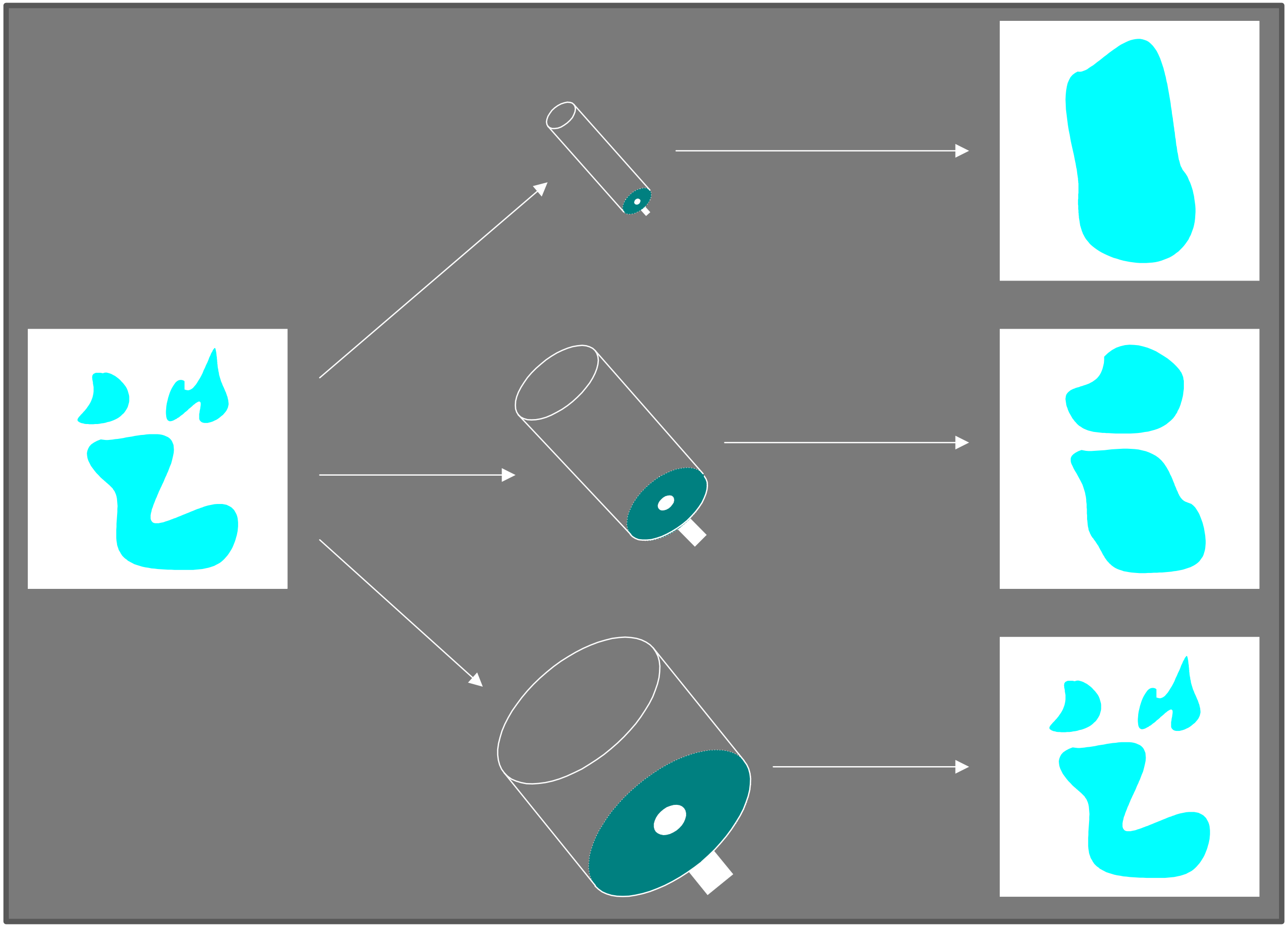

Point Spread Function of Telescopes / Interferometer

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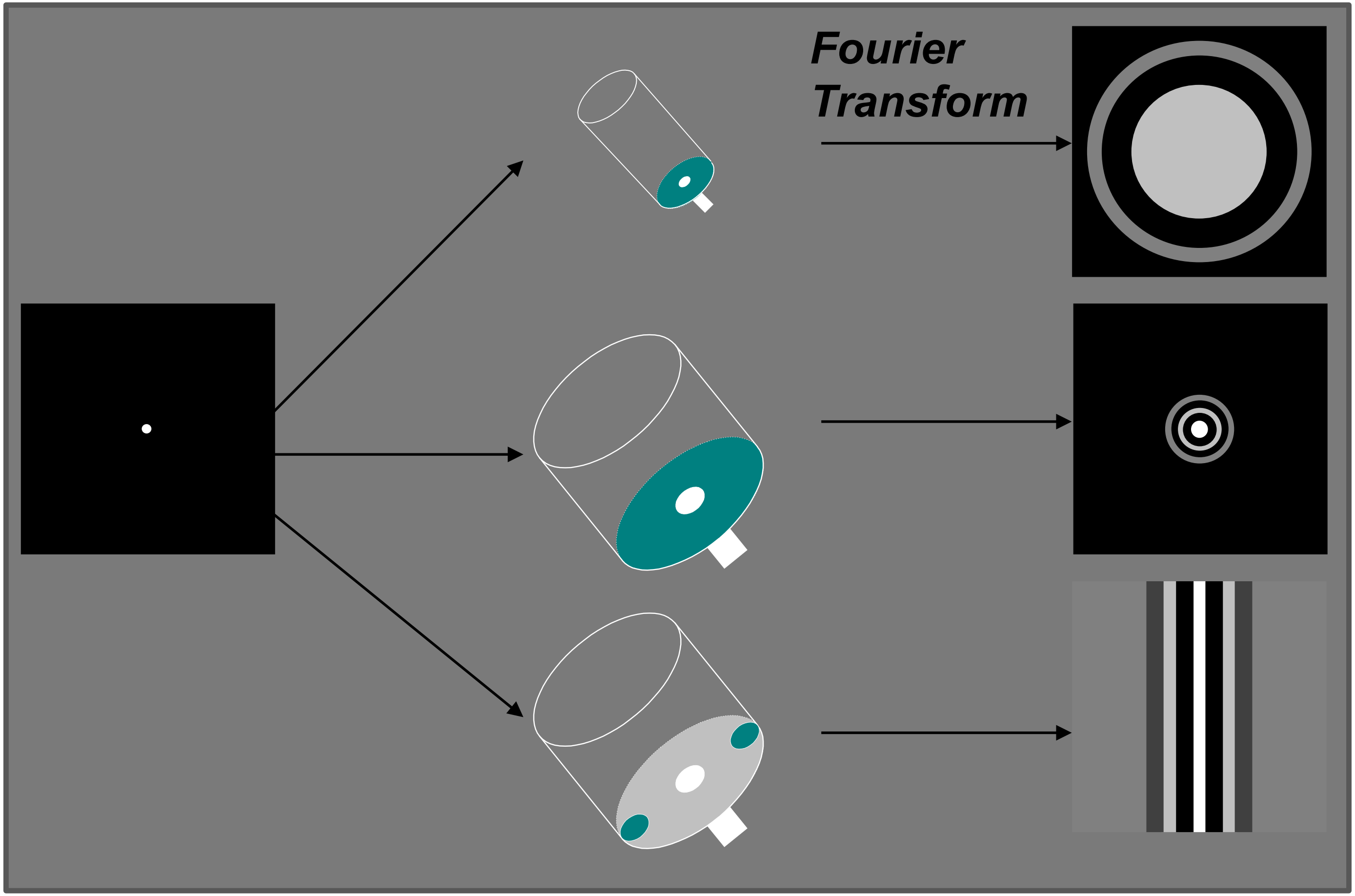
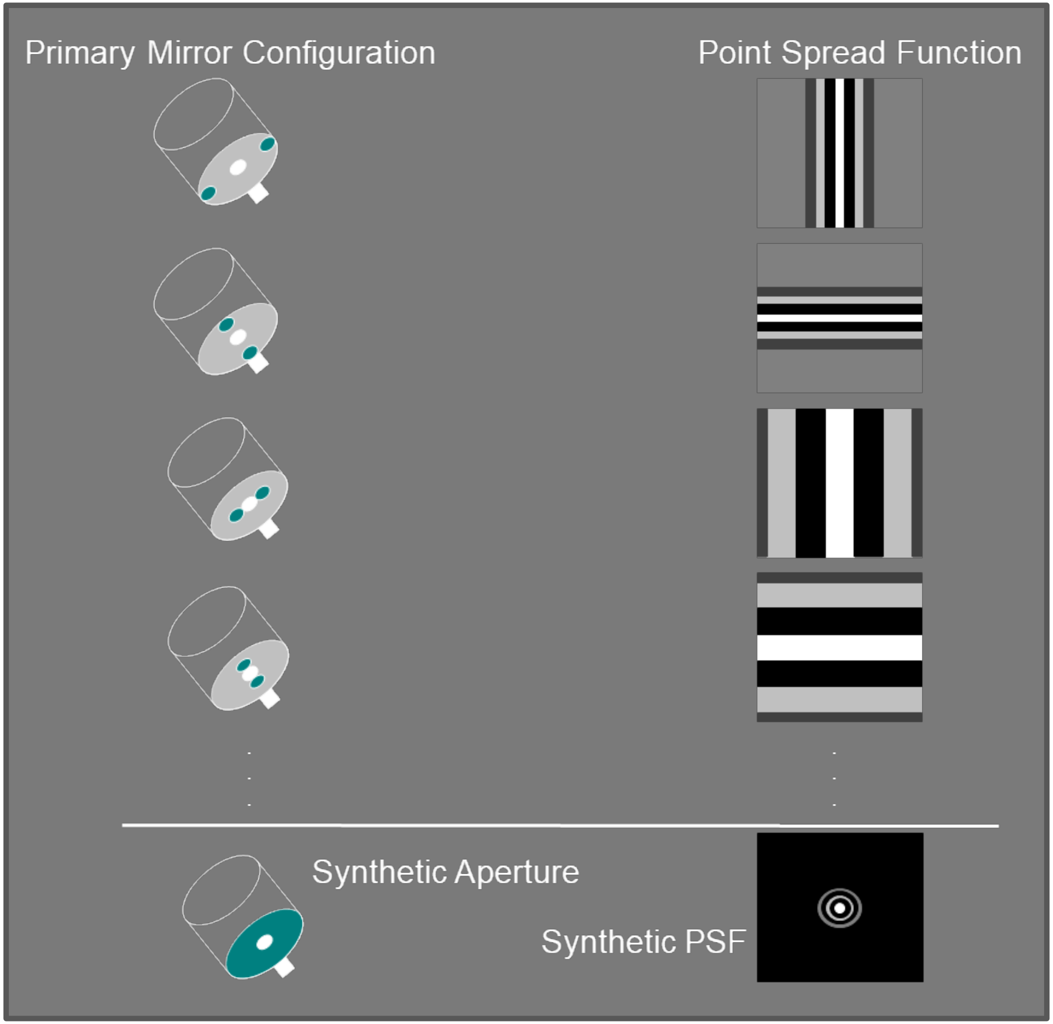


Synthetic Aperture Imaging with an Interferometer



Schematic Layout of Optical Michelson Interferometer

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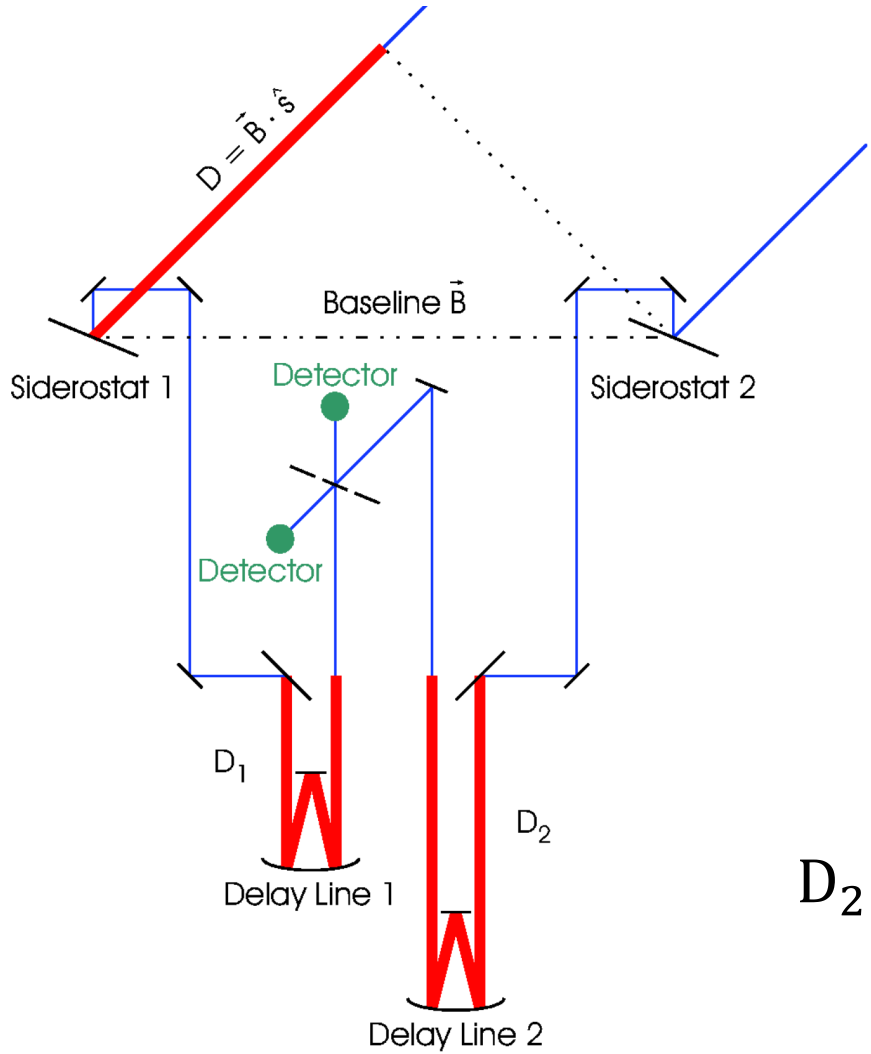

$$D_2 - D_1 \approx D$$

VLTI Delay Line System

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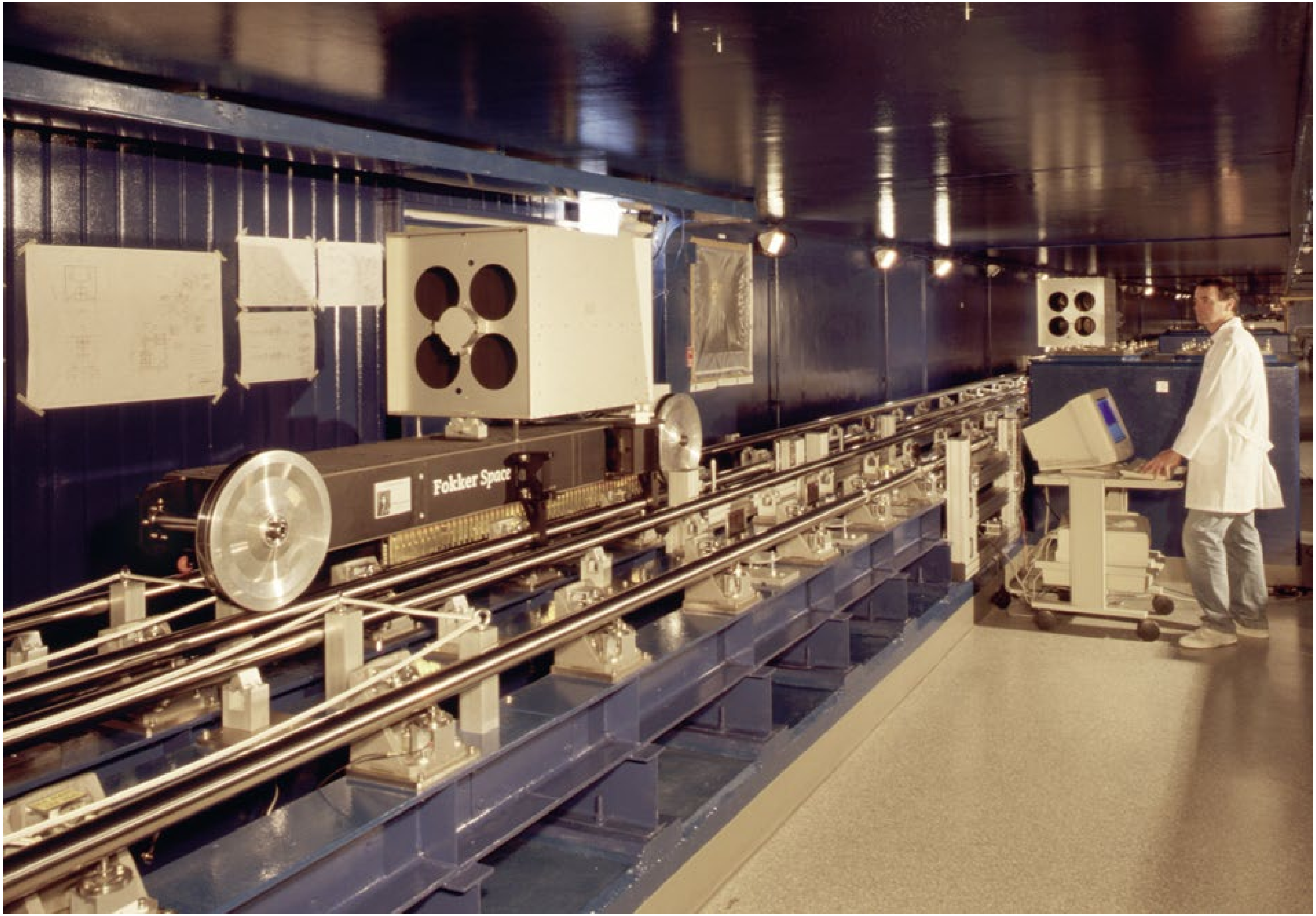

Basic Definition of Fringe Visibility (= Correlation Factor)

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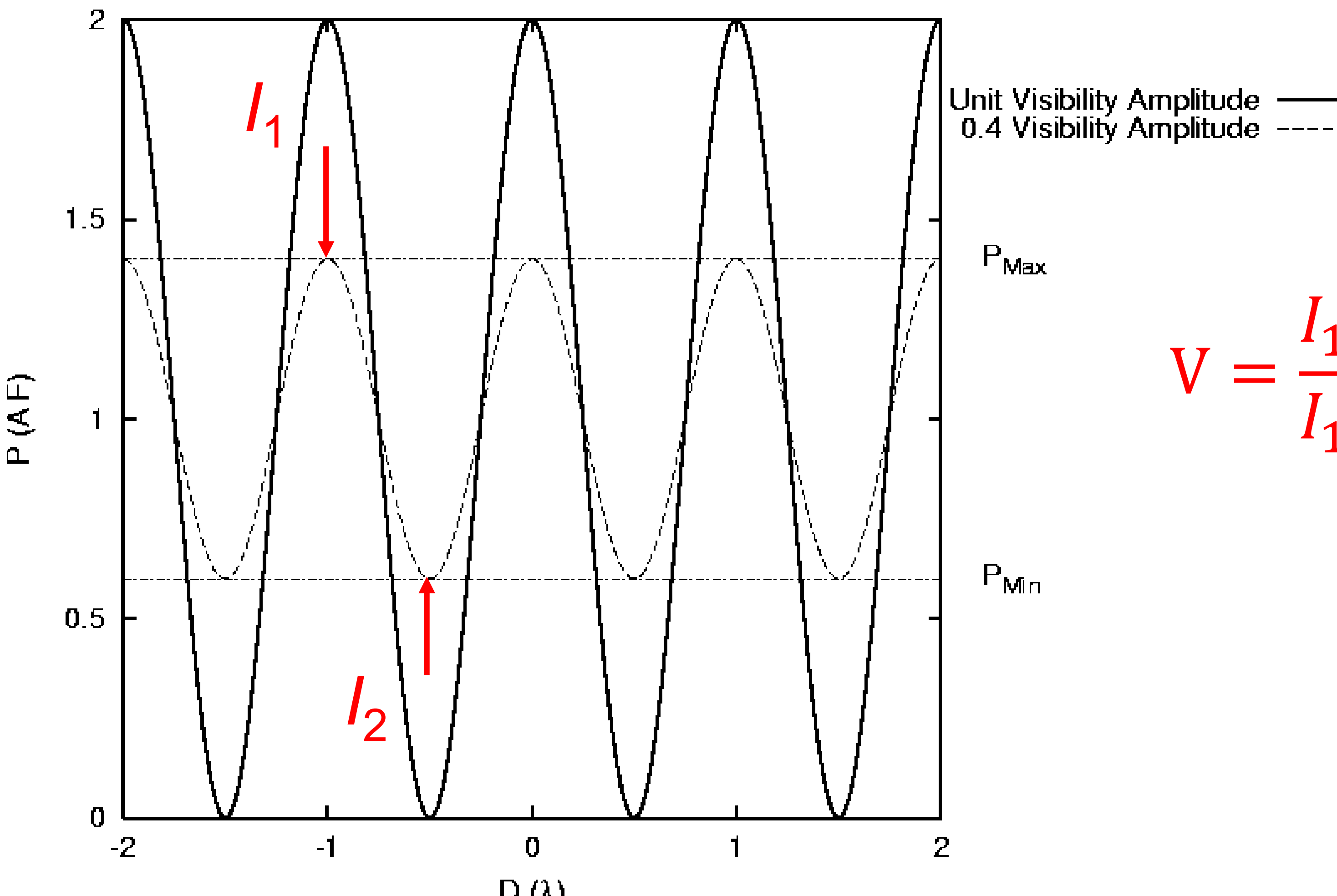

$$V = \frac{I_1 - I_2}{I_1 + I_2}$$

Complex Visibility and van Cittert-Zernike Theorem

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- The interferometer output at zero delay is called *complex visibility*.
- The complex visibility is the Fourier transform of the source brightness distribution:

  $$V = \Gamma(u, 0) = \int \left\langle |E(\xi)|^2 \right\rangle e^{-2\pi i \xi u} d\xi$$

  – $u =$ interferometer baseline, $\xi =$ sky coordinates
- Each observation on one baseline measures one Fourier component of the sky brightness distribution.

Coverage of the uv Plane

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- As for any signal, the Nyquist sampling theorem applies.
- The longest baselines determine the resolution of the observations.
- The shortest baselines determine the field-of-view that can be synthesized.
- All intermediate Fourier components have to be sampled adequately.

Seeing in Optical Astronomy

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- Turbulence above atmosphere leads to wavefront distortions.
- Lateral coherence length is described by the *Fried parameter* $r_0 \propto \lambda^{6/5}$.
  - The effective resolution of long exposures is the same as that with a telescope of diameter r_0.
  - Typical values at good site result in $0.5 \ldots 1''$ images.
- Coherence time $\tau_0 = r_0 / v_{\mathrm{wind}} \approx$ milliseconds
- Coherence angle $\theta_0 \propto \cos z \, r_0 / H \approx$ arcseconds

Consequences of Seeing for Interferometry

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- Single apertures have to be phased. For apertures larger than $\sim 3r_0$ adaptive optics is needed.
- Fringe tracking has to be performed with a servo bandwidth larger than $1/\tau_0$.
- Phase referencing is possible only over angles smaller than θ_0.
- The $\lambda^{6/5}$scaling of these quantities strongly favors operation at longer wavelengths.
  - #photons in coherence volume $\propto \lambda^{18/5}$

Phase Errors

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- Atmospheric turbulence corrupts phase above each telescope in interferometer array.
- The observed phase ϕ' is given by the sum of the true phase ϕ, and the phase errors ψ at the two telescopes (with correct signs):
  $\phi'_{12} = \phi_{12} + \psi_1 - \psi_2$
- The errors are frequently much larger than 1 radian, which makes phase data useless.

Closure Phases

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- Look at phase disturbance on triangle of baselines. The phase errors cancel in the sum:

$$\phi'_{12} = \phi_{12} + \psi_1 - \psi_2$$

$$\phi'_{23} = \phi_{23} + \psi_2 - \psi_3$$

$$\phi'_{31} = \phi_{31} + \psi_3 - \psi_1$$

$$\phi_{123} \equiv \phi'_{12} + \phi'_{23} + \phi'_{31} = \phi_{12} + \phi_{23} + \phi_{31}$$

- Closure phases contain useful information for imaging, uncorrupted by phase errors.

Amplitude and Intensity Interferometry

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- Amplitude Interferometry: combine light from two telescopes and detect

$$\langle I \rangle = \left\langle \left(E_1 + E_2 \right)^2 \right\rangle = E^2 \left\langle \left(\sin \omega t + \sin \left(\omega t + \varphi \right) \right)^2 \right\rangle$$

$$= E^2 \left(1 + \cos \varphi \right)$$

- Intensity Interferometry: detect light at two telescopes and compare signals

$$\langle I_1 I_2 \rangle = E^4 \left\langle \sin^2 \omega t \sin^2 \left(\omega t + \varphi \right) \right\rangle = E^4 \left(\tfrac{1}{4} + \tfrac{1}{8} \cos 2\varphi \right)$$

SNR in Amplitude and Intensity Interferometry

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- SNR in amplitude interferometry:

$$SNR_A = \sqrt{\alpha n_{ph} A}\ |\gamma_{ij}| \sqrt{T \Delta \nu}$$

- SNR in intensity interferometry:

$$SNR_I = \alpha n_{ph} A\ |\gamma_{ij}^2| \sqrt{T \Delta f}$$

- α: efficiency
- n_{ph}: photon flux
- A: collecting area
- γ_{ij}: coherence factor (visibility)
- T: observing time
- $\Delta\nu$: optical bandwidth
- Δf: electrical bandwidth

SNR in Amplitude and Intensity Interferometry

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- SNR in amplitude interferometry:

$$SNR_A = \sqrt{\alpha n_{ph} A}\ |\gamma_{ij}| \sqrt{T \Delta\nu}$$

- SNR in intensity interferometry:

$$SNR_I = \alpha n_{ph} A \left|\gamma_{ij}^2\right| \sqrt{T \Delta f}$$

- For 0$^{\mathrm{mag}}$ star: $n_{ph} \approx 10^{-4}\mathrm{m}^{-2}\mathrm{Hz}^{-1}\mathrm{s}^{-1}$
  - Note: 5 mag ≙ factor 100
- For 5$^{\mathrm{mag}}$, $\alpha = 0.1$, $A = 100\mathrm{m}^2$, $\Delta\nu = 10^{13}\mathrm{Hz}$, $|\gamma_{ij}| = 1$, $\Delta f = 1\mathrm{GHz}$, $T = 1\mathrm{hr}$:

$$SNR_A \approx 600{,}000,\ SNR_I \approx 20$$

SNR Scaling

- Amplitude Interferometry: $SNR \propto \sqrt{\alpha n_{ph} A \left|\gamma_{ij}^2\right|}$
- Intensity Interferometry: $SNR \propto \alpha n_{ph} A \left|\gamma_{ij}^2\right|$
- Aperture size and photon flux are much more important in intensity interferometry
- In interferometry, the SNR depends on $n_{ph}\left|\gamma_{ij}^2\right|$, not just on n_{ph}.
  - More strongly for intensity interferometry

The Problem of Low Coherence Factors

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- Most astrophysical measurements require data with low $|\gamma_{ij}|$
  - "There are good fringes, and there are useful fringes."
- Stellar diameters (location of first null of Airy function): $|\gamma_{ij}^2| \approx 0.1$
  - 2.5 mag for intensity interferometry
- Limb darkening (intensity of first Airy ring): $|\gamma_{ij}^2| \leq 0.0175$
  - 4.4 mag for intensity interferometry

SNR for Triple Correlation in Intensity Interferometry

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- Triple correlation yields closure phases → imaging
- $SNR_I^{(3)} = \left(\alpha n_{ph} A\right)^{3/2} \left|\gamma_{ij}\gamma_{jk}\gamma_{ki}\right| \Delta f \sqrt{\frac{T}{\Delta\nu}}$
  - Much lower than amplitude SNR → likely not useful for astronomy