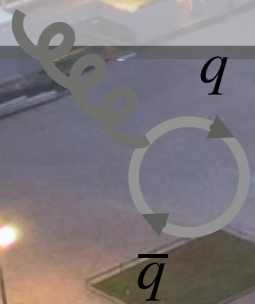
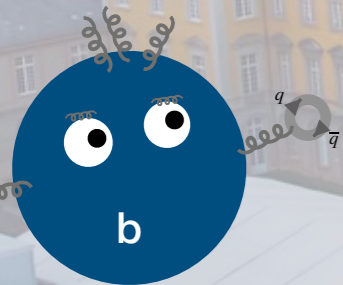




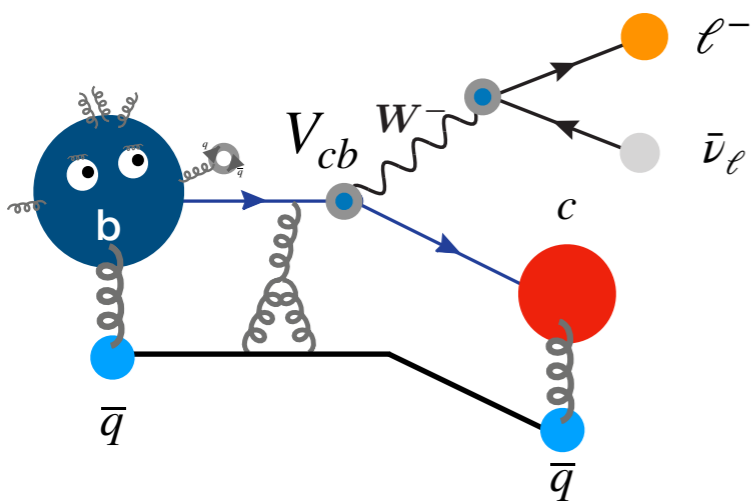
A guided tour on $b \rightarrow c\ell\bar{\nu}_\ell$ with $\ell = e, \mu, \tau$

Marseille *Mini* Theory - Experiment Workshop on Semileptonic Decays

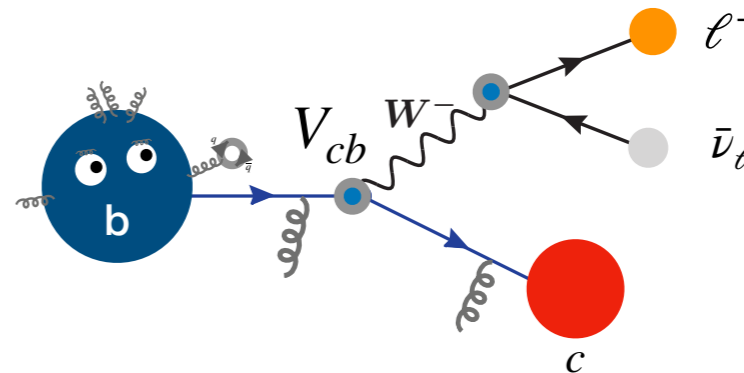


Rough Tour Guide for today

1. Exclusive $B \rightarrow D^{(*,**)} \ell \bar{\nu}_\ell$
 Measurements with $\ell = e, \mu$



2. Inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$
 Measurements with $\ell = e, \mu$



3. Ratio measurements with τ

$$R = \frac{\text{Signal } b \rightarrow q \tau \bar{\nu}_\tau}{\text{Normalization } b \rightarrow q \ell \bar{\nu}_\ell}$$

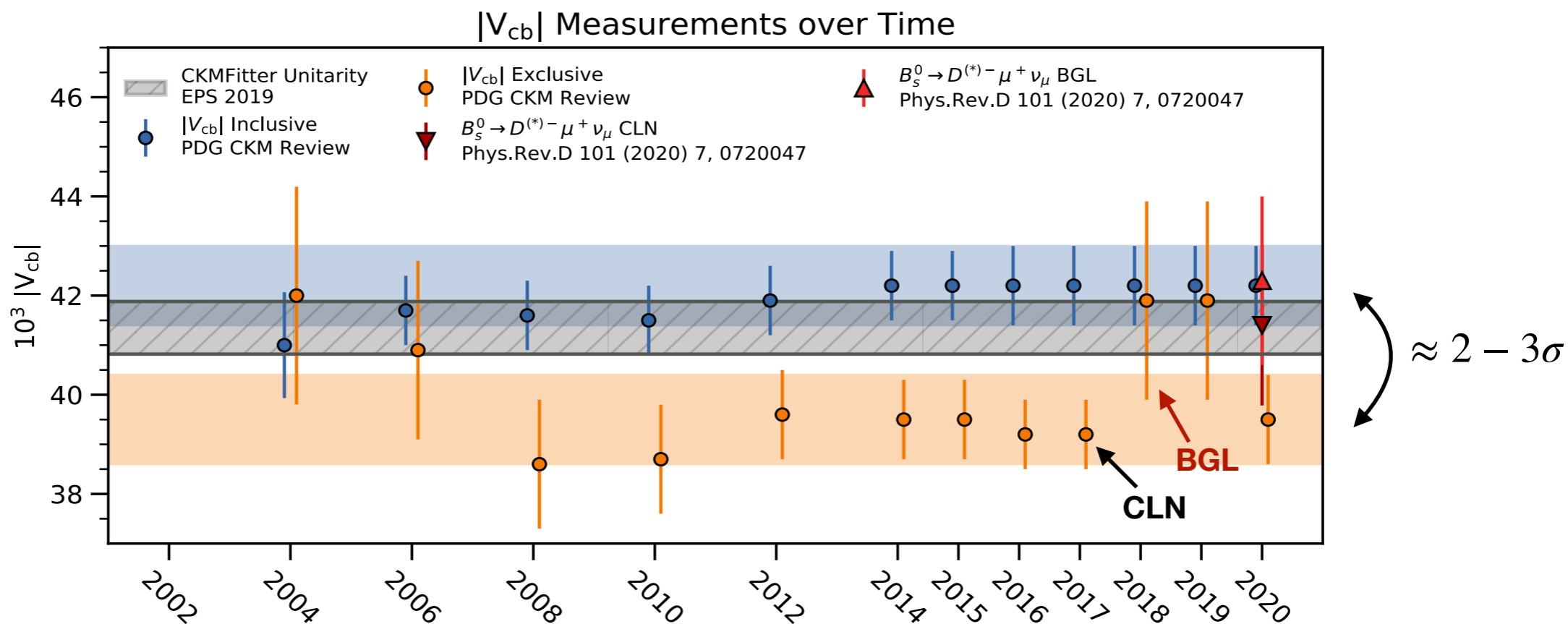
$\ell = e, \mu$

Backdrop of Inclusive vs. Exclusive

$|V_{cb}|$

Inclusive

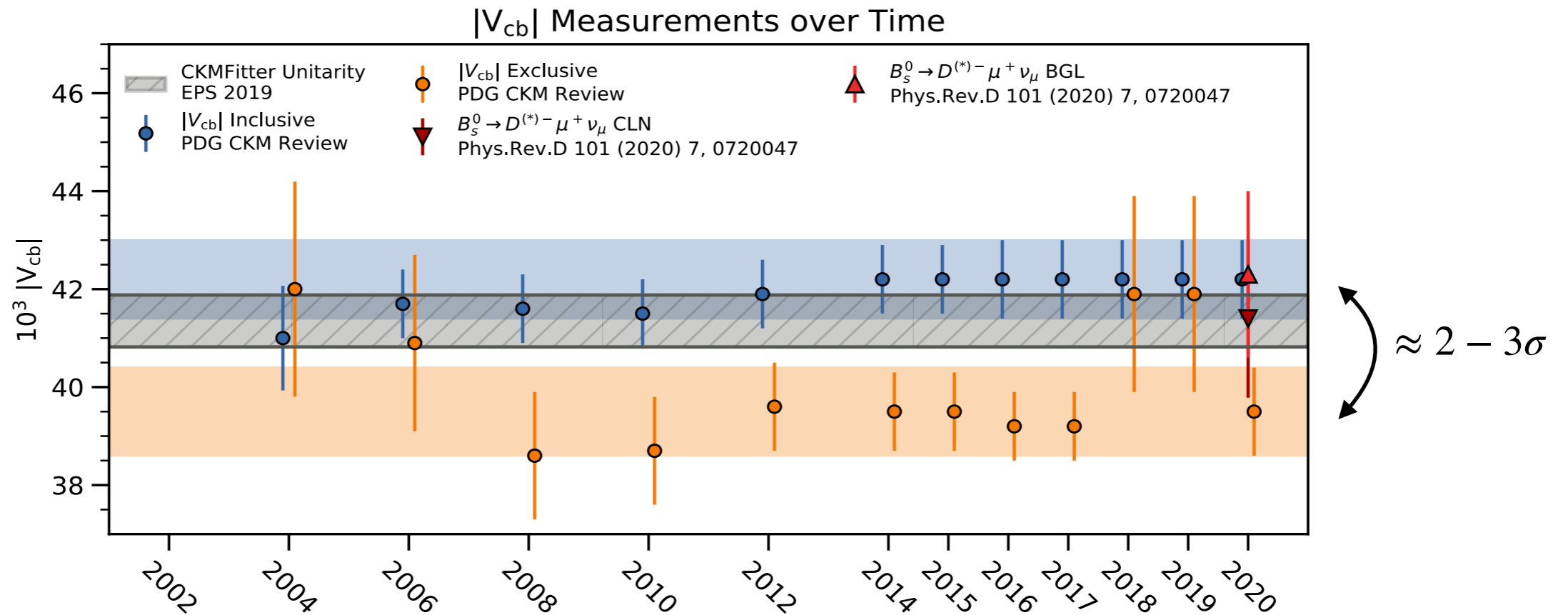
Exclusive



CLN

Experiment	$\eta_{EW} F(1) V_{cb} $ (rescaled) [10^{-3}]	ρ^2 (rescaled)	Correlation (stat/syst/total)	Parameters	Remarks
ALEPH	31.38 +/- 1.80 +/- 1.24	0.488 +/- 0.226 +/- 0.146	0.94/0.69/0.86	input parameters	Phys.Lett.B395:373-387,1997
CLEO	40.16 +/- 1.24 +/- 1.54	1.363 +/- 0.084 +/- 0.087	0.87/0.90/0.89	input parameters	Phys.Rev.Lett.89:081803,2002
OPAL excl	36.20 +/- 1.58 +/- 1.47	1.198 +/- 0.206 +/- 0.153	0.95/0.48/0.75	input parameters	Phys.Lett.B482:15-30,2000
OPAL partial reco	37.44 +/- 1.20 +/- 2.32	1.090 +/- 0.137 +/- 0.297	0.77/0.79/0.79	input parameters	Phys.Lett.B482:15-30,2000
DELPHI partial reco	35.52 +/- 1.41 +/- 2.29	1.139 +/- 0.123 +/- 0.382	0.94/0.68/0.71	input parameters	Phys.Lett.B510:55-74,2001
DELPHI excl	35.87 +/- 1.69 +/- 1.95	1.070 +/- 0.141 +/- 0.153	0.89/0.81/0.84	input parameters	Eur.Phys.J.C33:213-232,2004
BELLE	34.82 +/- 0.15 +/- 0.55	1.106 +/- 0.031 +/- 0.008	0.66/-0.30/0.09	input parameters	Submitted to Phys.Rev.D82
BABAR excl	33.77 +/- 0.29 +/- 0.97	1.182 +/- 0.048 +/- 0.029	0.27/0.02/0.08	input parameters	Phys.Rev.D77:032002,2008
BABAR D*0	34.55 +/- 0.58 +/- 1.06	1.124 +/- 0.058 +/- 0.053	0.90/0.48/0.60	input parameters	Phys.Rev.Lett.100:231803,2008
BABAR global fit	35.45 +/- 0.20 +/- 1.08	1.171 +/- 0.019 +/- 0.060	0.38/0.87/0.84	input parameters	Phys.Rev.D79:012002,2009
Average	35.00 +/- 0.11 +/- 0.34	1.121 +/- 0.014 +/- 0.019	0.50/0.33/0.34	chi2/dof = 42.2/23 (CL = 0.009)	eps_1 pdf_1 eps_2 pdf_2

Backdrop of Inclusive vs. Exclusive



$|V_{cb}|$ & HQE / form factor parameters are important input for:

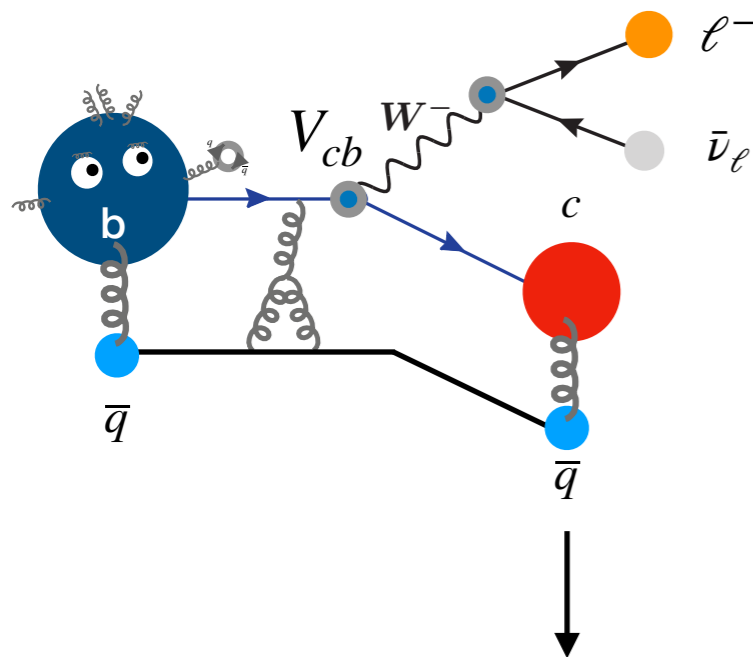
- ★ CKM UT Fit (& search for NP)
- ★ Prediction of SM processes (e.g. SL decays with τ , $B_{(s)} \rightarrow \mu\mu, \dots$)
- ★ Modelling of backgrounds for $b \rightarrow u\ell\bar{\nu}_\ell$ & $|V_{ub}|$

...



1. Exclusive

Overview



Mesons containing a heavy quark Q are made up of a heavy quark and a light antiquark \bar{q} (and gluons and $q\bar{q}$ pairs)

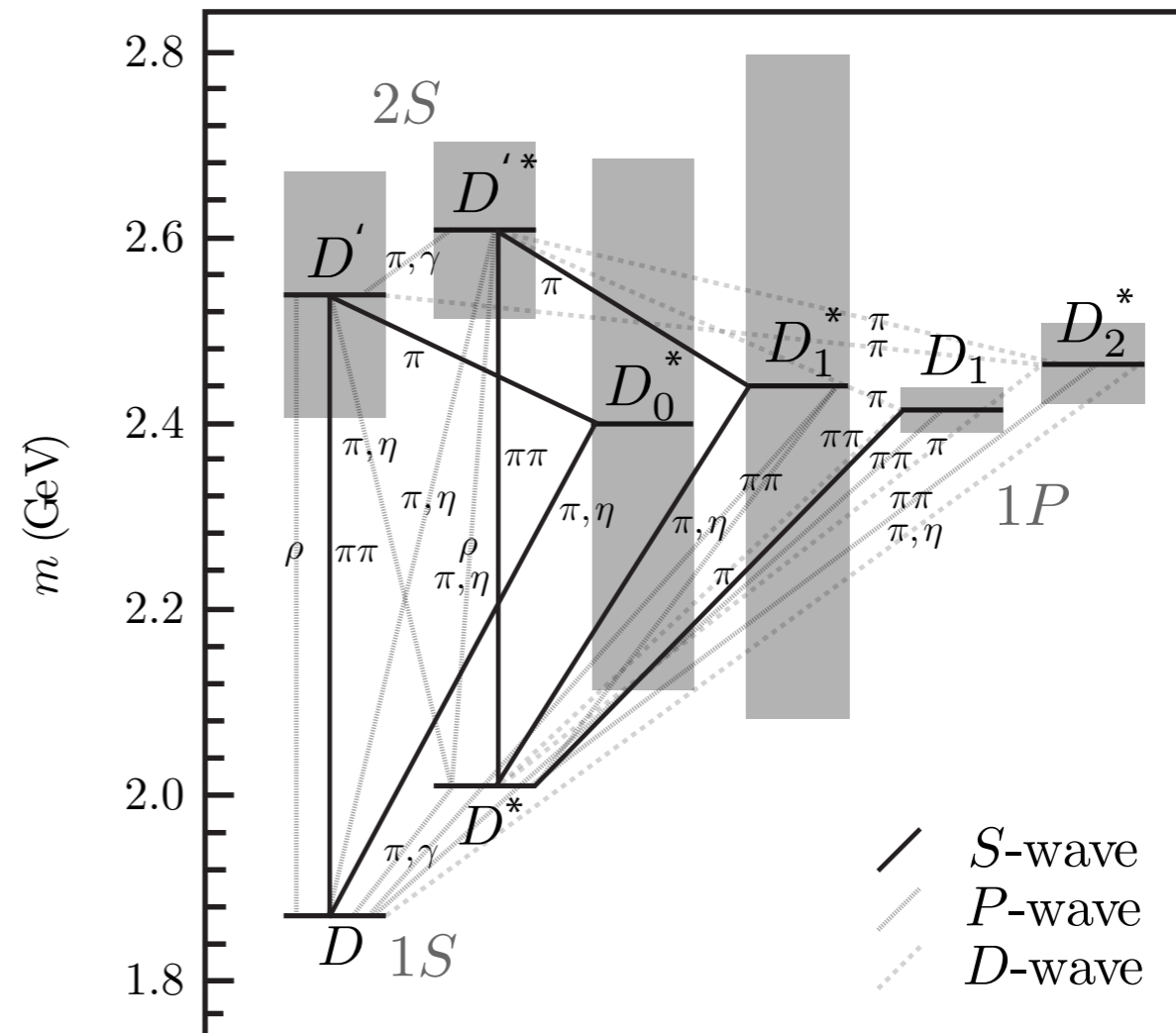
2 $L = 0$ ground states: D/D^*

4 $L = 1$ ground states: D_0, D'_1, D_1, D_2

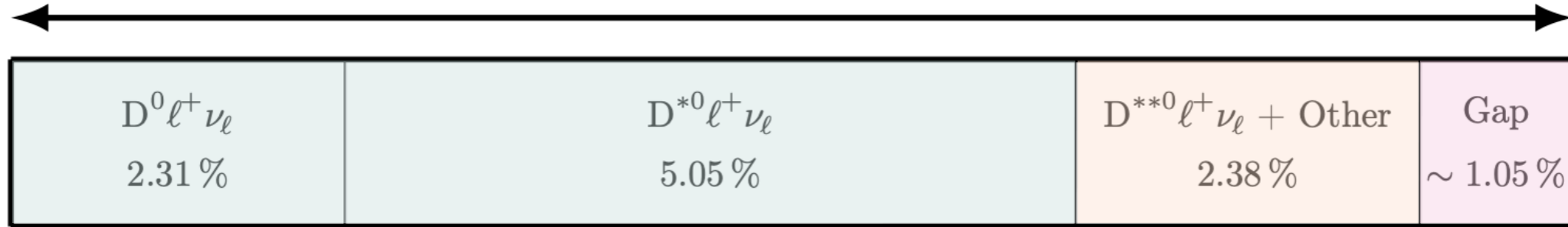
(or sometimes D_0^*, D_1^*, D_1, D_2^* or just D^{**})

D/D^* saturate **~75%** of the inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$ rate and are the **principal route** to V_{cb}

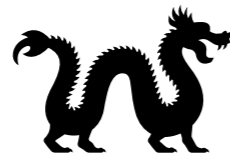
D^{**} saturate **~15%** of the inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$ rate, mostly are perceived as background



$$\mathcal{B}(B^+ \rightarrow X_c^0 \ell^+ \nu_\ell) \approx 10.79\%$$



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D \pi \pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$



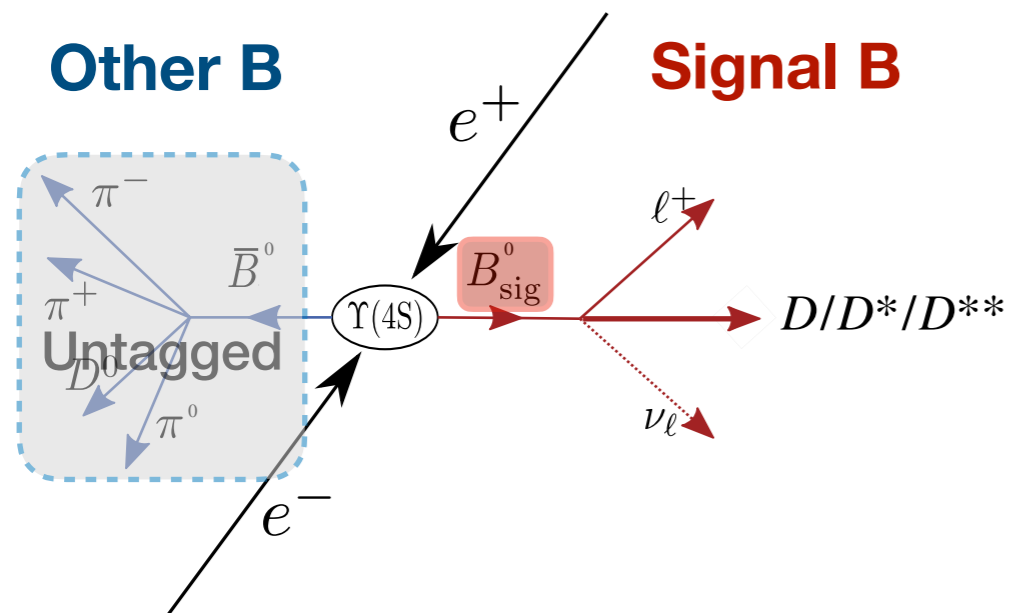
Fairly well known.
Some iso-spin tension.

Broad states based on
3 measurements.
(BaBar, Belle, DELPHI)

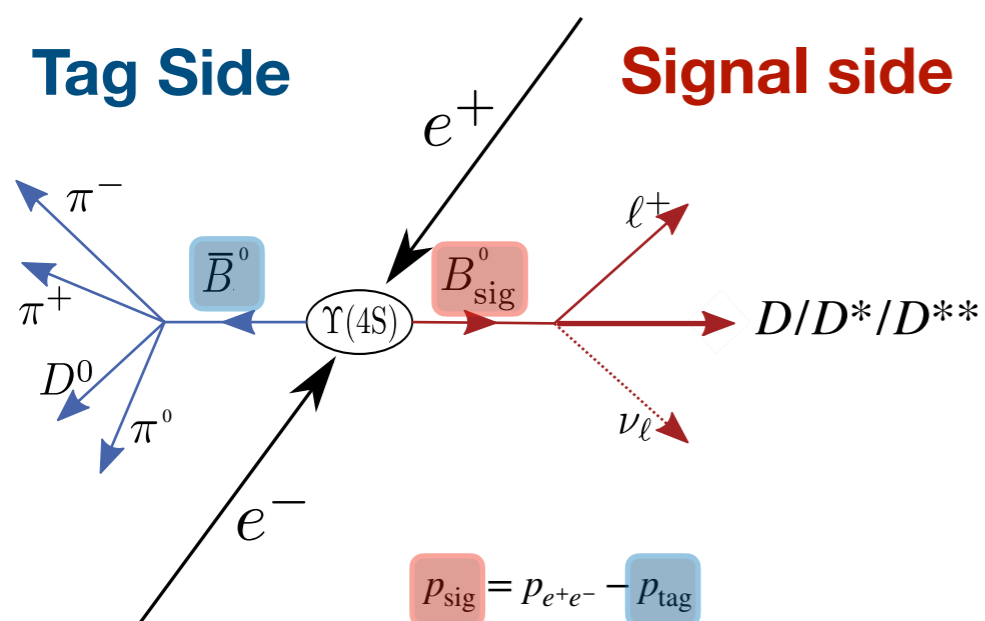
Some hints from
the BaBar result.

New result from Belle soon

Measurement Strategies (e^+e^- B-Factory)

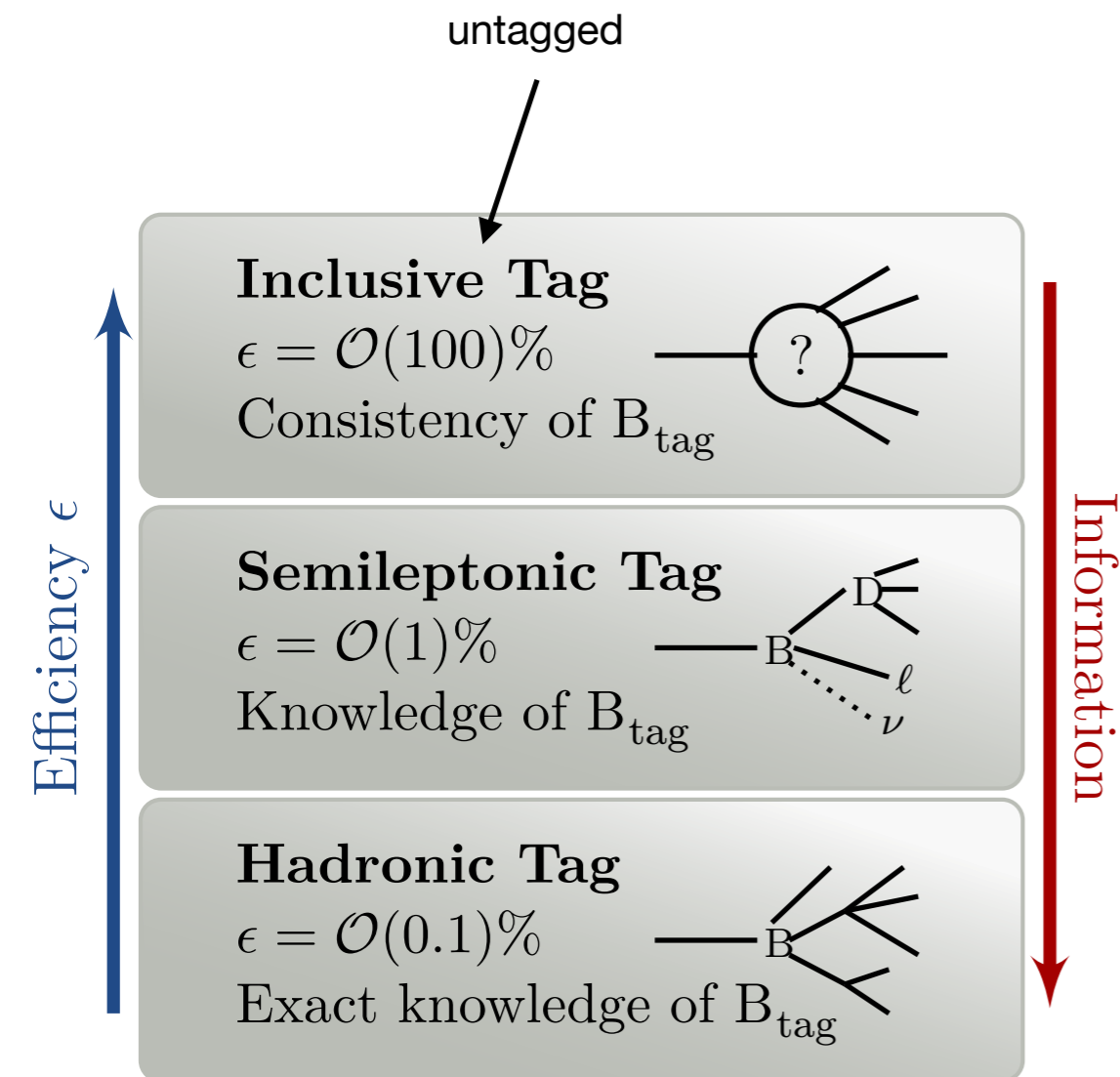


- + Very high efficiency
- + Measurement of absolute branching fractions straightforward (depends on total # of $N_{B\bar{B}}$, understanding efficiencies)
- Less experimental control, e.g. more background from $e^+e^+ \rightarrow q\bar{q}$
- Cannot directly access signal B rest frame, need tricks



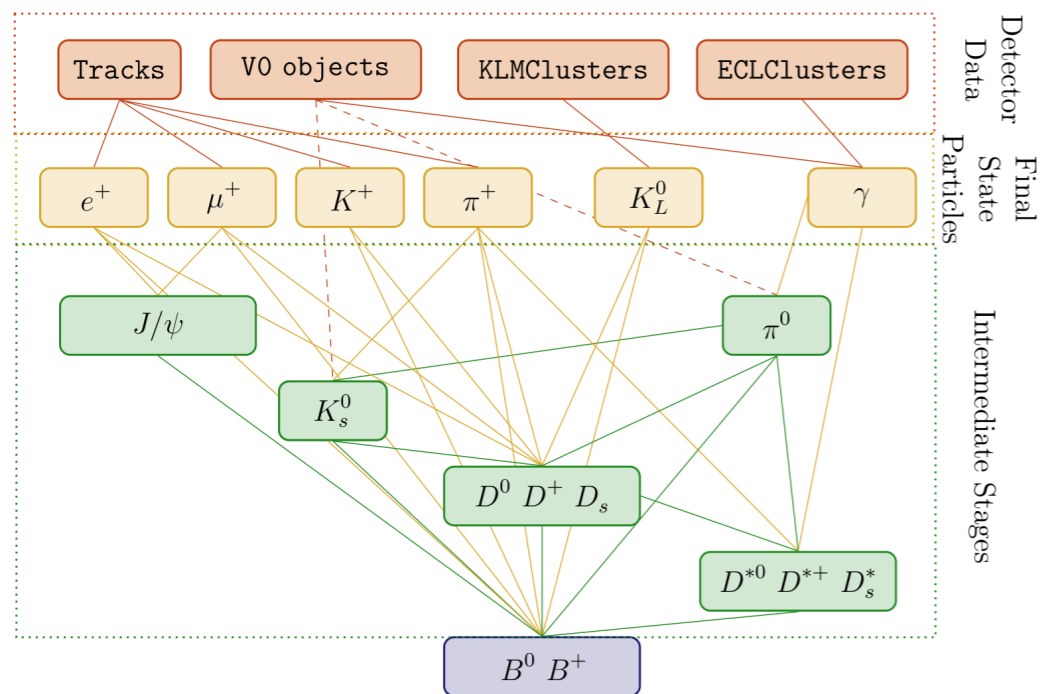
- + High degree of experimental control, e.g. can identify all final state particles with either the signal or the tag side
- + If hadronic modes for tagging are used, can reconstruct B rest frame
- Understanding efficiencies is difficult
- Low efficiency reduces the effective statistical power

Measurement Strategies (e^+e^- B-Factory)



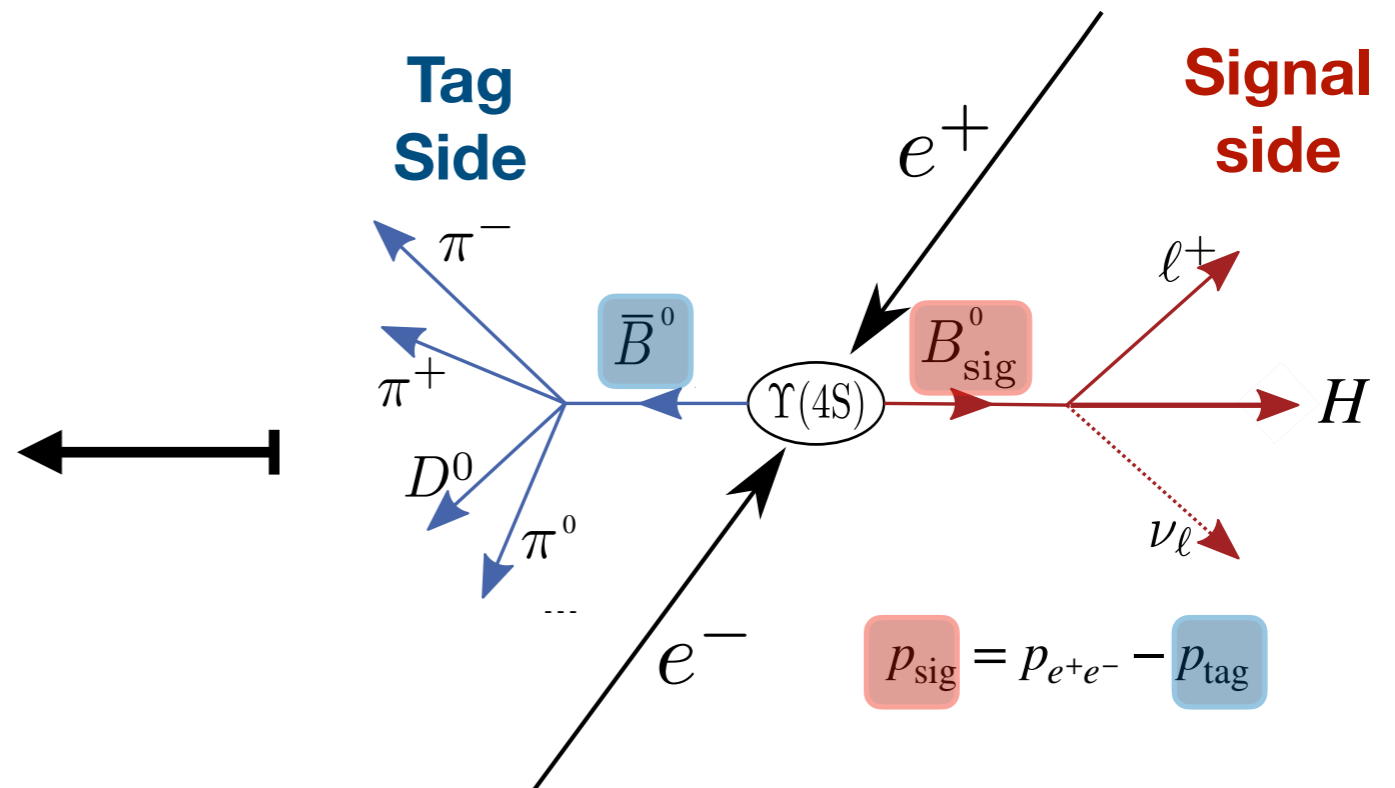
- + Very high efficiency
 - + Measurement of absolute branching fractions straightforward
 (depends on total # of $N_{B\bar{B}}$, understanding efficiencies)
 - Less experimental control, e.g. more background from $e^+e^+ \rightarrow q\bar{q}$
 - Cannot directly access signal B rest frame, need tricks
-
- + High degree of experimental control, e.g. can identify all final state particles with either the signal or the tag side
 - + If hadronic modes for tagging are used, can reconstruct B rest frame
 - Understanding efficiencies is difficult
 - Low efficiency reduces the effective statistical power

Tagging in a nutshell



Candidates reconstructed with **hierarchical** approach via e.g. **neural networks (FR)** or **boosted decision trees (FEI)**

Over 10'000 decay cascades with an **efficiency of 0.28% / 0.18%** for B^\pm and B^0/\bar{B}^0



E.g. train a classifier to identify correctly reconstructed electron candidates:

Input variables: all four momenta & particle identification scores

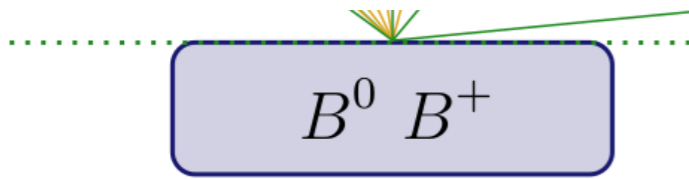
Output: Score \mathcal{O}_e

Apply mild selection on \mathcal{O}_e to reduce # of candidate particles

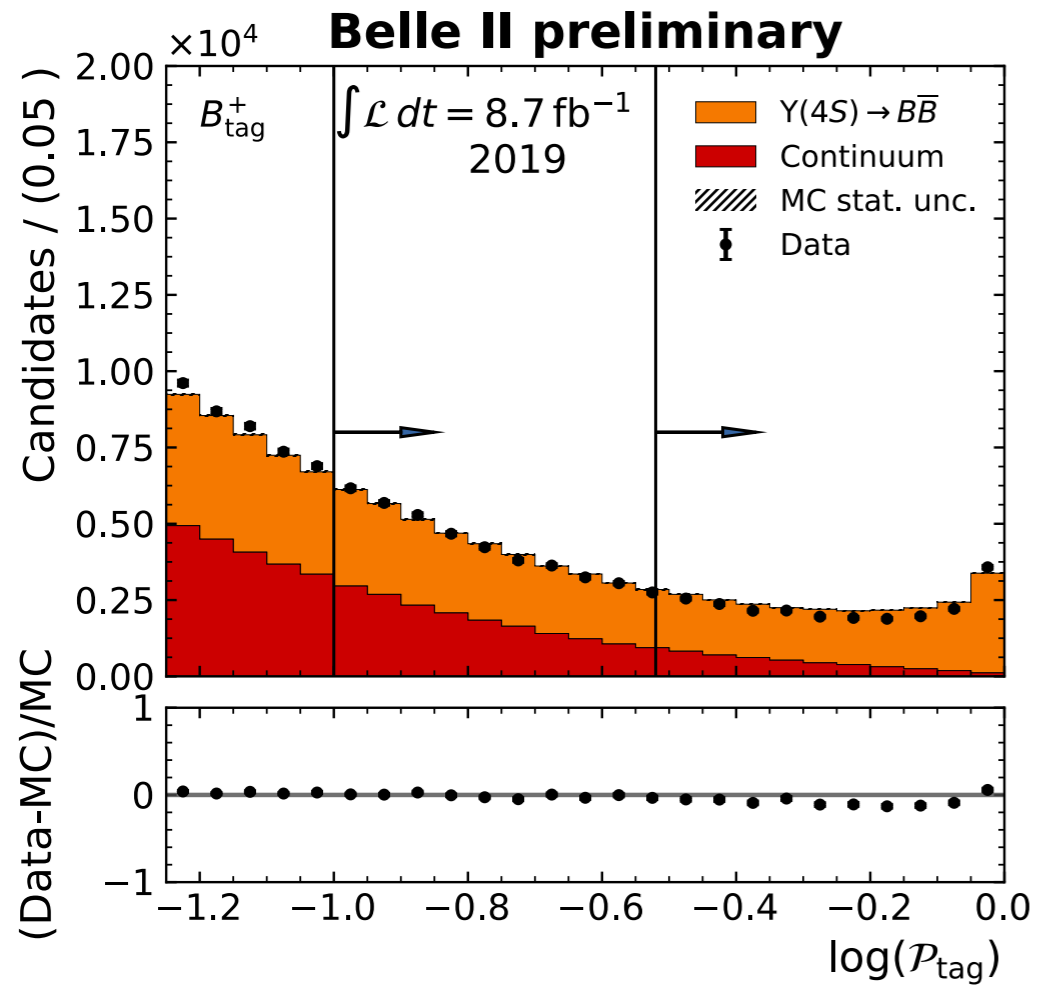
Then train a classifier to identify correctly reconstructed J/ψ candidates

Input variables: all four momenta and output scores of previous layer

Output variable: $\mathcal{O}_{J/\psi}$ [...]



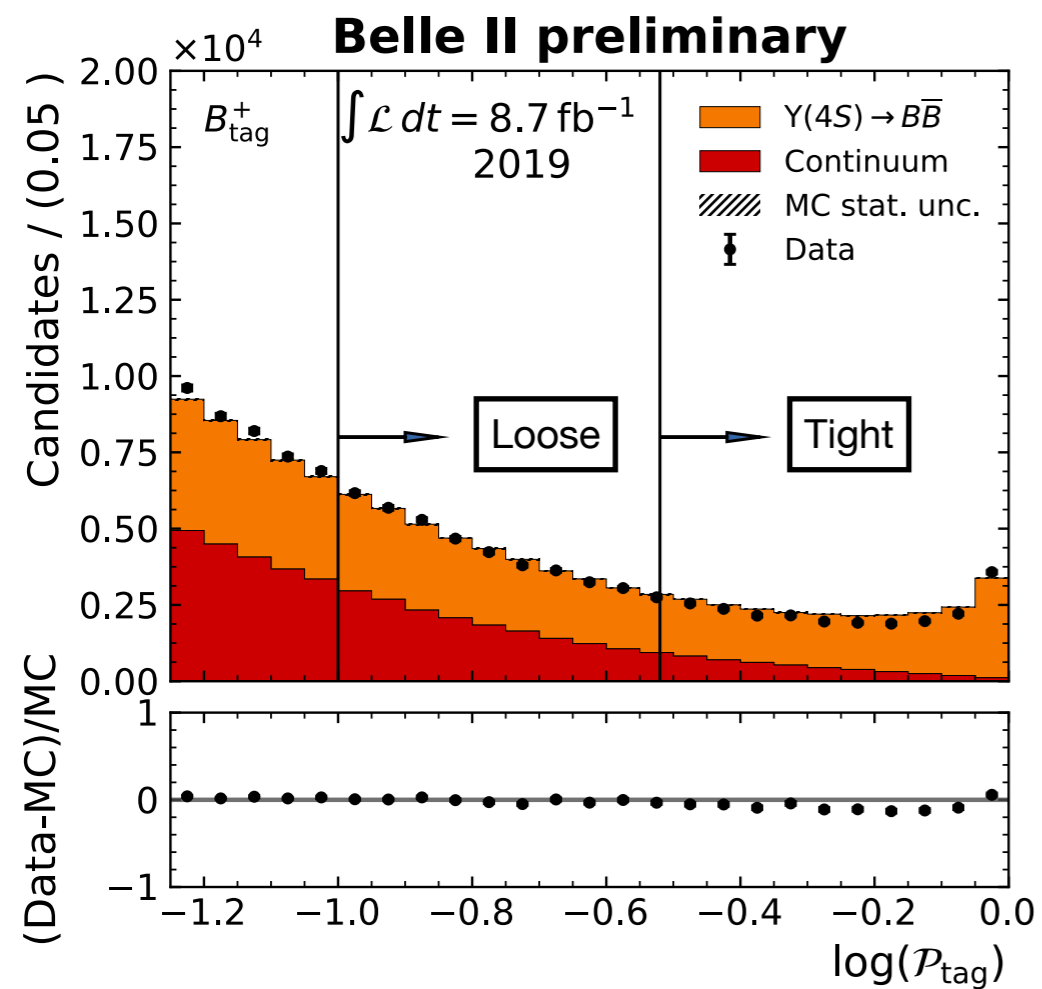
Output classifier = Measure of how well we reconstructed the B-Meson decay



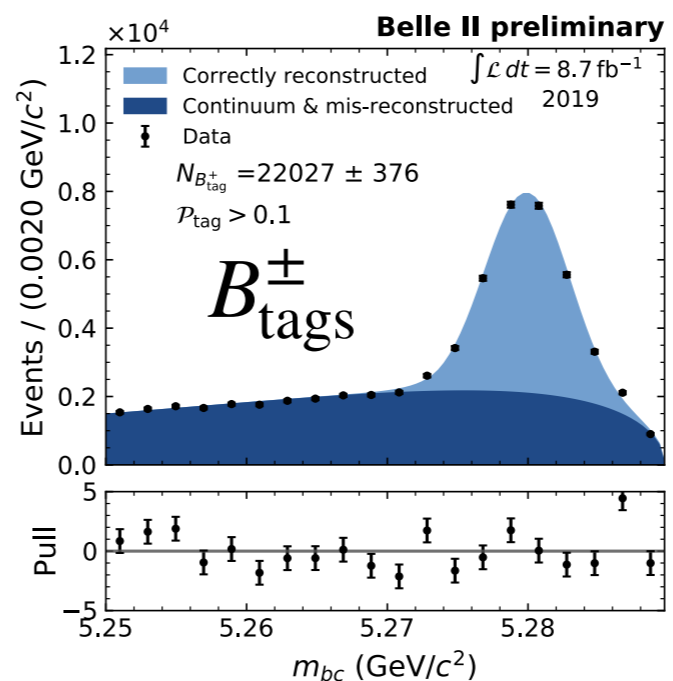
beam constrained mass

ca. 5.279 GeV

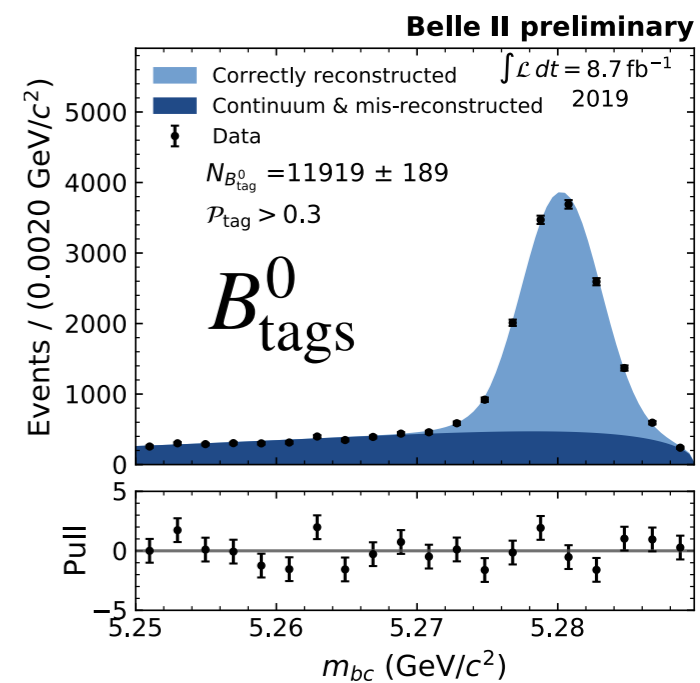
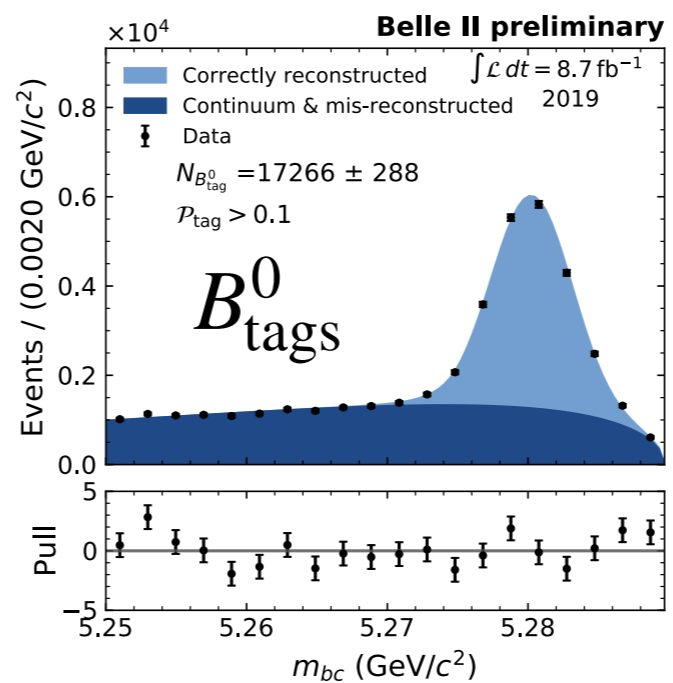
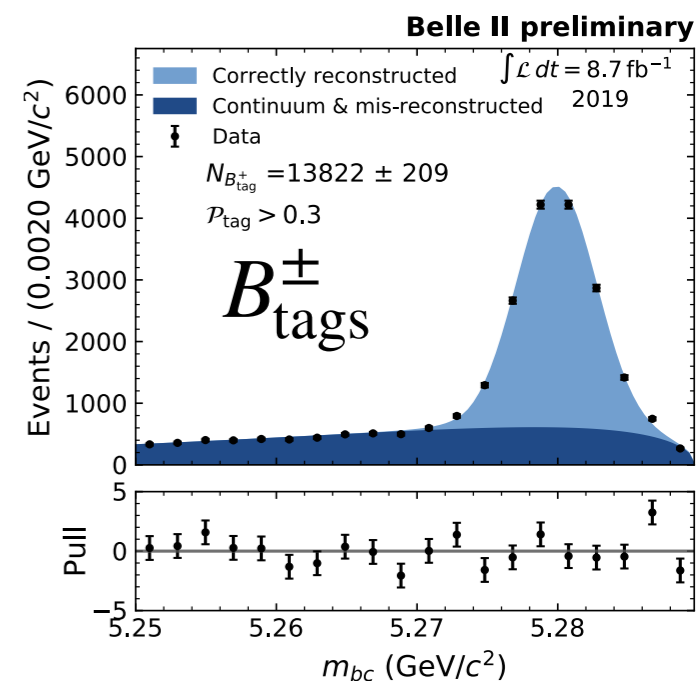
$$m_{bc} = \sqrt{E_{\text{beam}}^2/4 - |\vec{p}_{B_{\text{tag}}}|^2} \simeq m_B$$



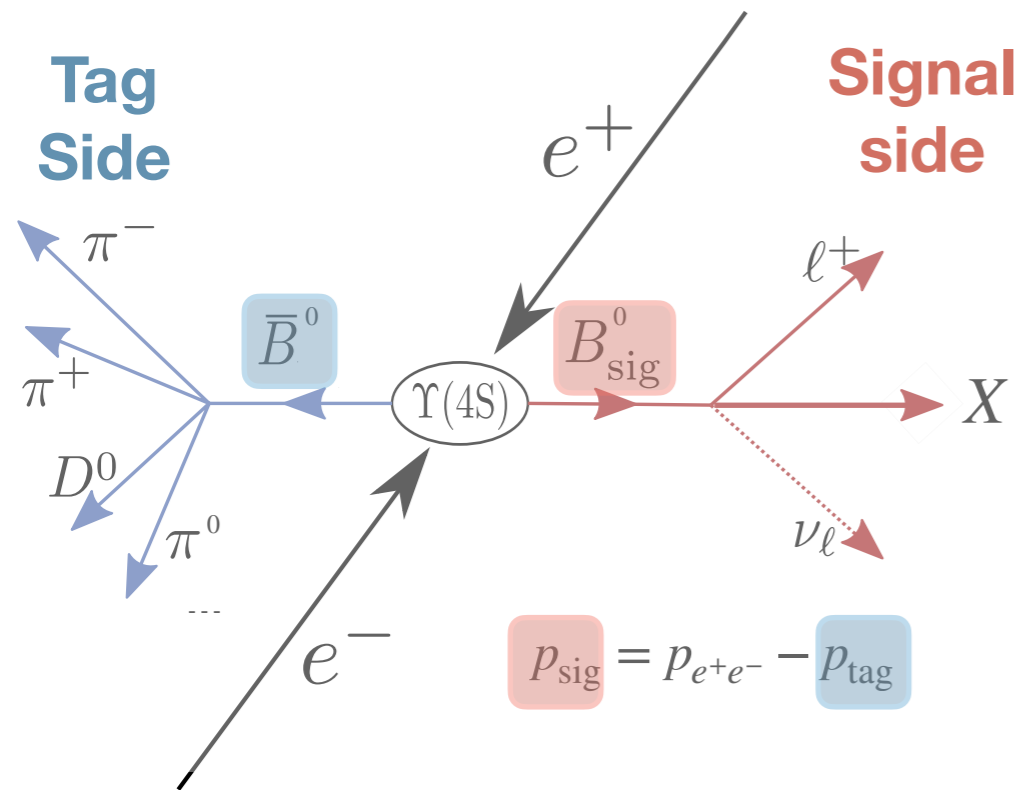
Loose Selection



Tight Selection



Efficiency can be calibrated,
but this has caveats

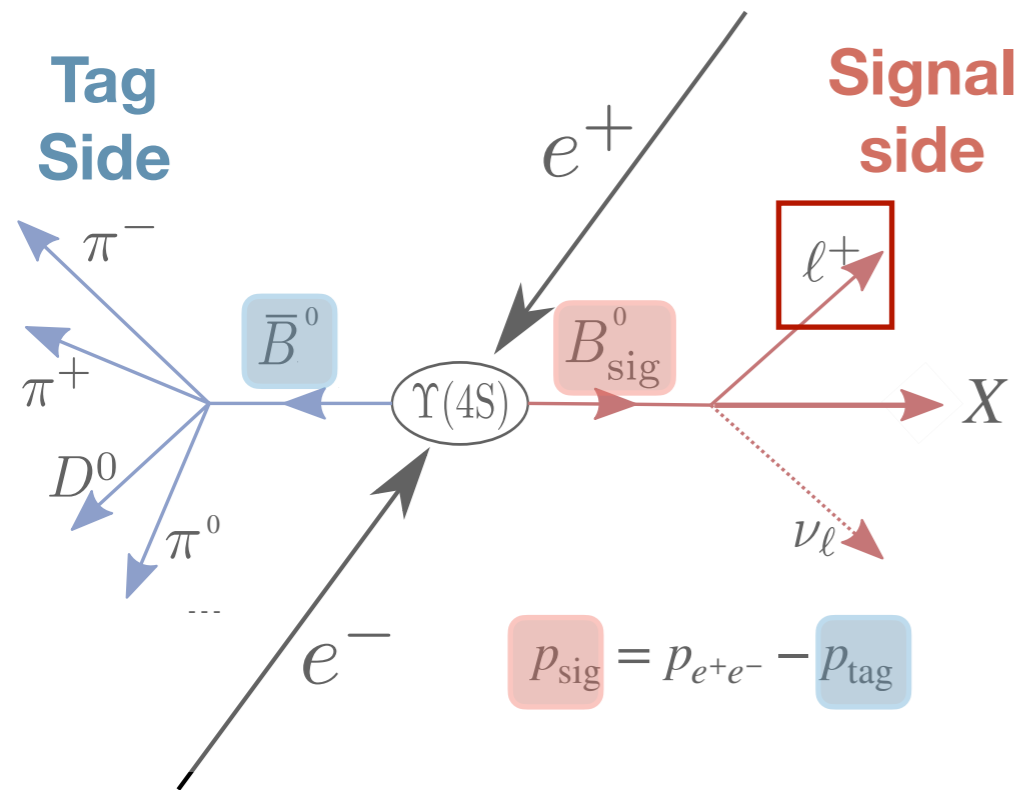


Why is the efficiency different? Use
10'000 different decays, use
uncalibrated detector information,
line-shapes differ in simulation
→ all aggregated in \mathcal{P}_{tag}

Strategy: use a well measured
process, add it to your MC with its
measured BF and compare

$$\frac{N_{X\ell\bar{\nu}_\ell}^{\text{Data}}}{N_{X\ell\bar{\nu}_\ell}^{\text{MC}}}$$

Efficiency can be calibrated,
but this has caveats

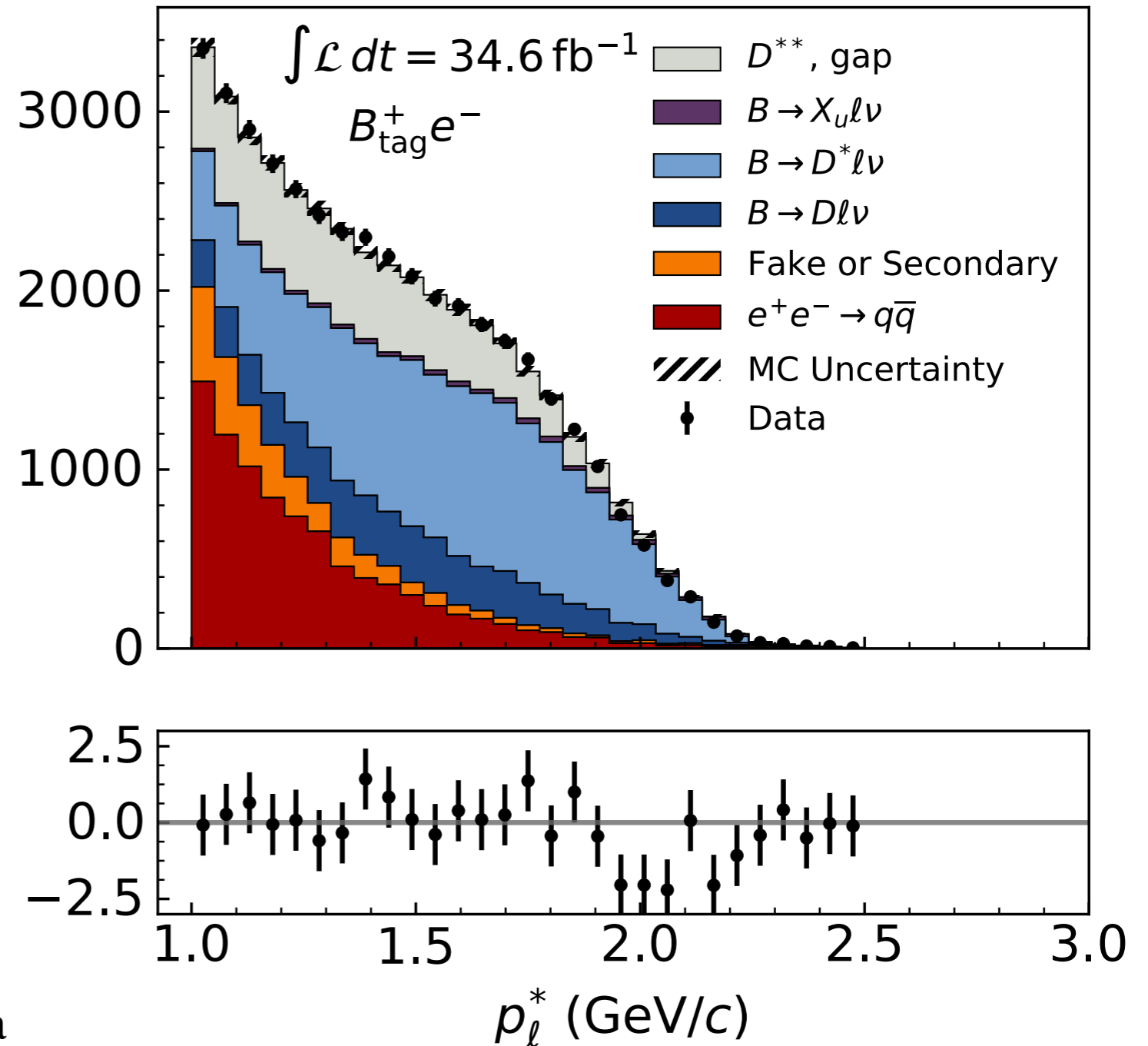


Why is the efficiency different? Use
10'000 different decays, use
uncalibrated detector information,
line-shapes differ in simulation
→ all aggregated in \mathcal{P}_{tag}

Strategy: use a well measured
process, add it to your MC with its
measured BF and compare

$$\frac{N_{Xl\bar{\nu}_l}^{\text{Data}}}{N_{Xl\bar{\nu}_l}^{\text{MC}}}$$

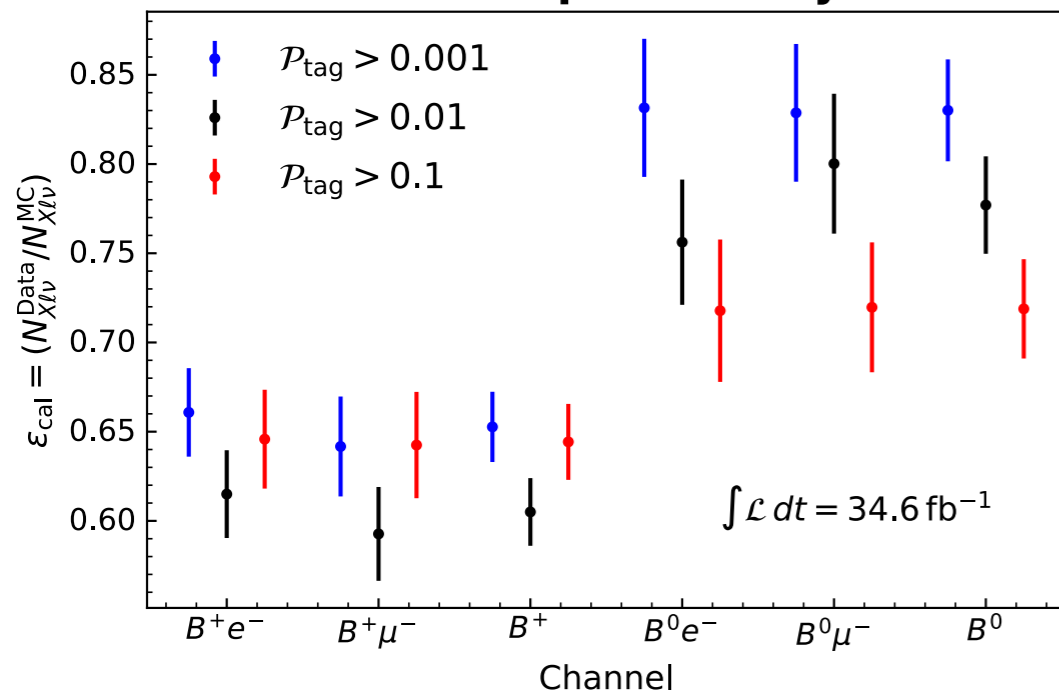
Belle II preliminary



Efficiency Calibration

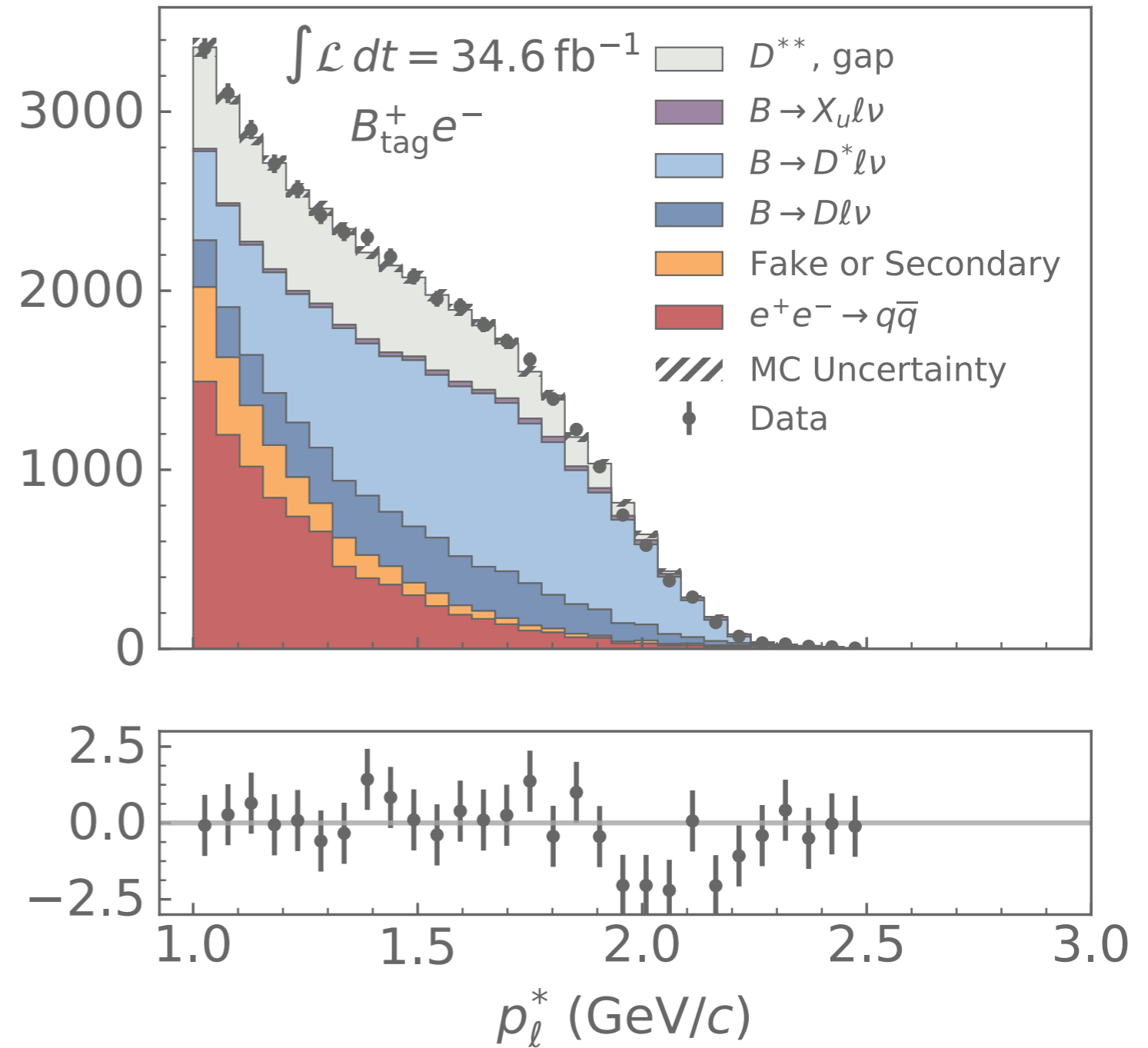
$$\epsilon_{\text{cal}} = \frac{N_{X\ell\bar{\nu}_\ell}^{\text{Data}}}{N_{X\ell\bar{\nu}_\ell}^{\text{MC}}}$$

Belle II preliminary



B^+		
$\mathcal{P}_{\text{tag}} >$	ϵ	uncertainty [%]
0.001	0.65 ± 0.02	3.0
0.01	0.61 ± 0.02	3.1
0.1	0.64 ± 0.02	3.3
B^0		
$\mathcal{P}_{\text{tag}} >$	ϵ	uncertainty [%]
0.001	0.83 ± 0.03	3.4
0.01	0.78 ± 0.03	3.5
0.1	0.72 ± 0.03	3.9

Belle II preliminary



Tagged measurements of $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$

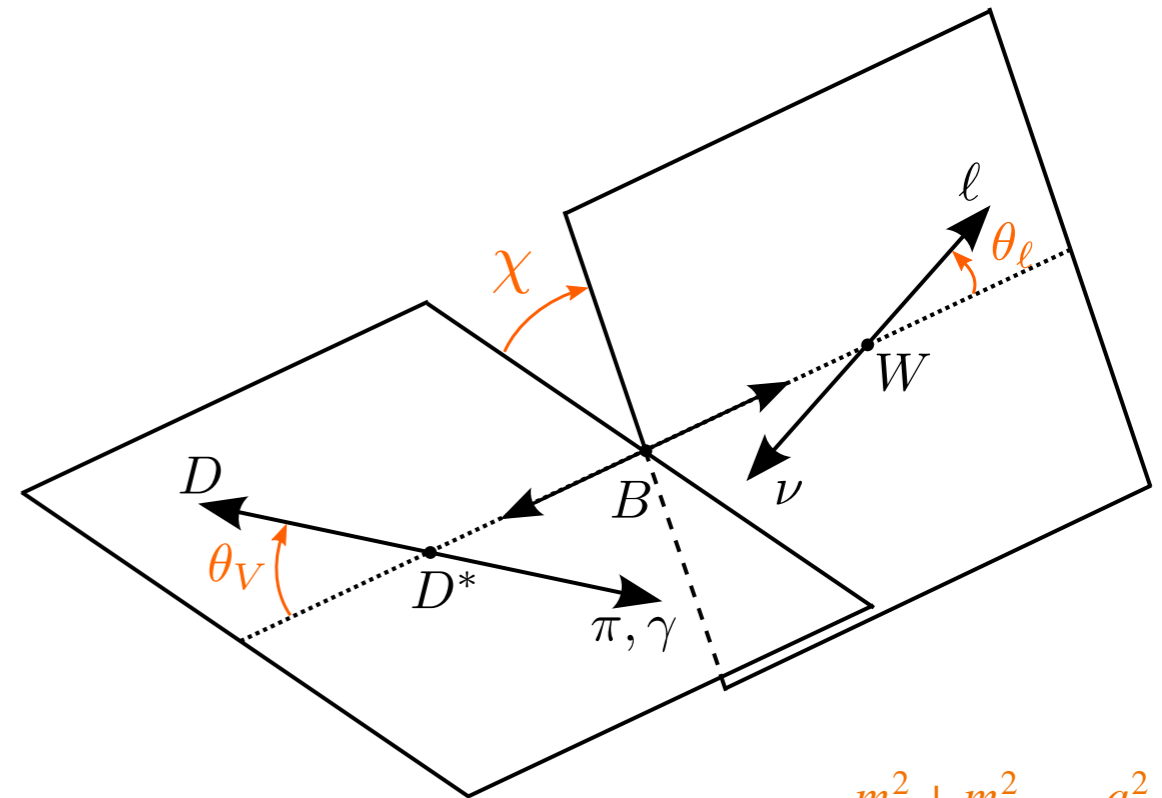
Target B^0 and B^+ and reconstruct D in many modes :

$$\begin{aligned}
 & D^+ \rightarrow K^- \pi^+ \pi^+, \quad D^+ \rightarrow K^- \pi^+ \pi^+ \pi^0, \\
 & D^+ \rightarrow K^- \pi^+ \pi^+ \pi^+ \pi^-, \quad D^+ \rightarrow K_S^0 \pi^+, \quad D^+ \rightarrow K_S^0 \pi^+ \pi^0, \\
 & D^+ \rightarrow K_S^0 \pi^+ \pi^+ \pi^-, \quad D^+ \rightarrow K_S^0 K^+, \quad D^+ \rightarrow K^+ K^- \pi^+, \\
 & D^0 \rightarrow K^- \pi^+, \quad D^0 \rightarrow K^- \pi^+ \pi^0, \quad D^0 \rightarrow K^- \pi^+ \pi^+ \pi^-, \\
 & D^0 \rightarrow K^- \pi^+ \pi^+ \pi^- \pi^0, \quad D^0 \rightarrow K_S^0 \pi^0, \quad D^0 \rightarrow K_S^0 \pi^+ \pi^-, \\
 & D^0 \rightarrow K_S^0 \pi^+ \pi^- \pi^0, \quad \text{and } D^0 \rightarrow K^- K^+.
 \end{aligned}$$

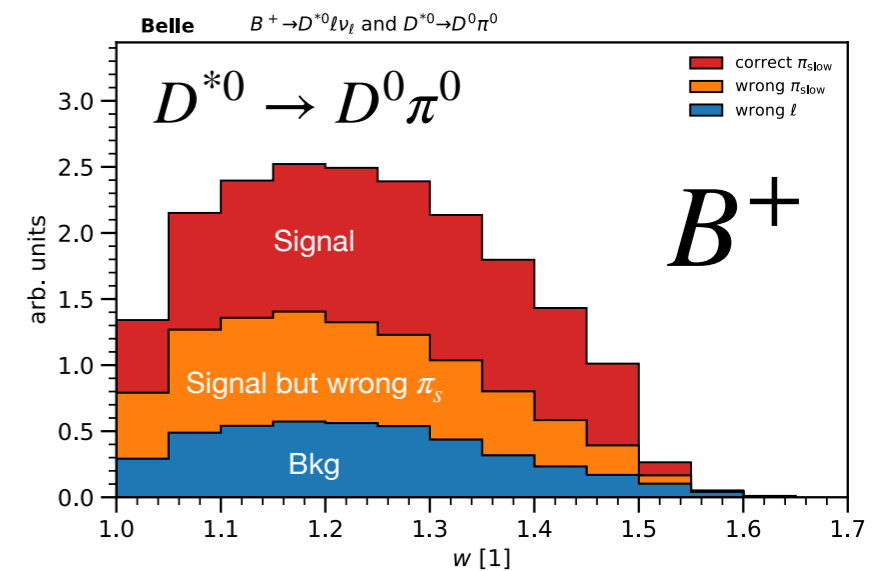
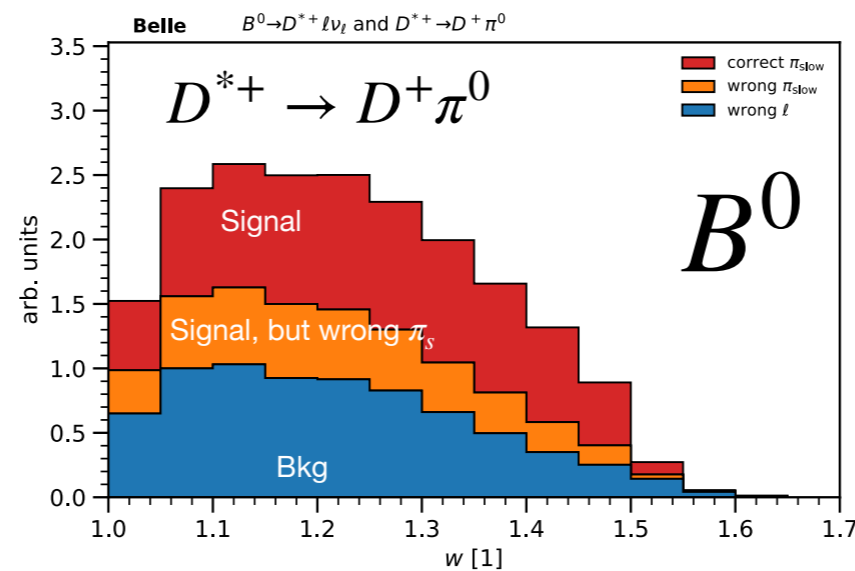
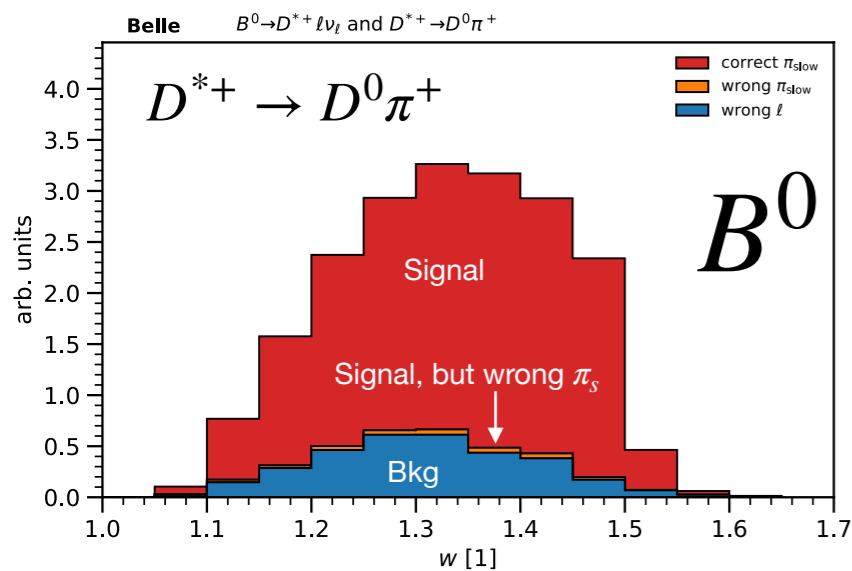
Reconstruct $D^{*+} \rightarrow D^0 \pi^+, D^{*+} \rightarrow D^+ \pi^0, D^{*0} \rightarrow D^0 \pi^0$

In principle also can do $D^{*0} \rightarrow D^0 \gamma$, but has different Lorentz structure & angular distributions

Tagged measurement can directly reconstruct **B** rest frame & access $\{w, \cos \theta_\ell, \cos \theta_V, \chi\}$



$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

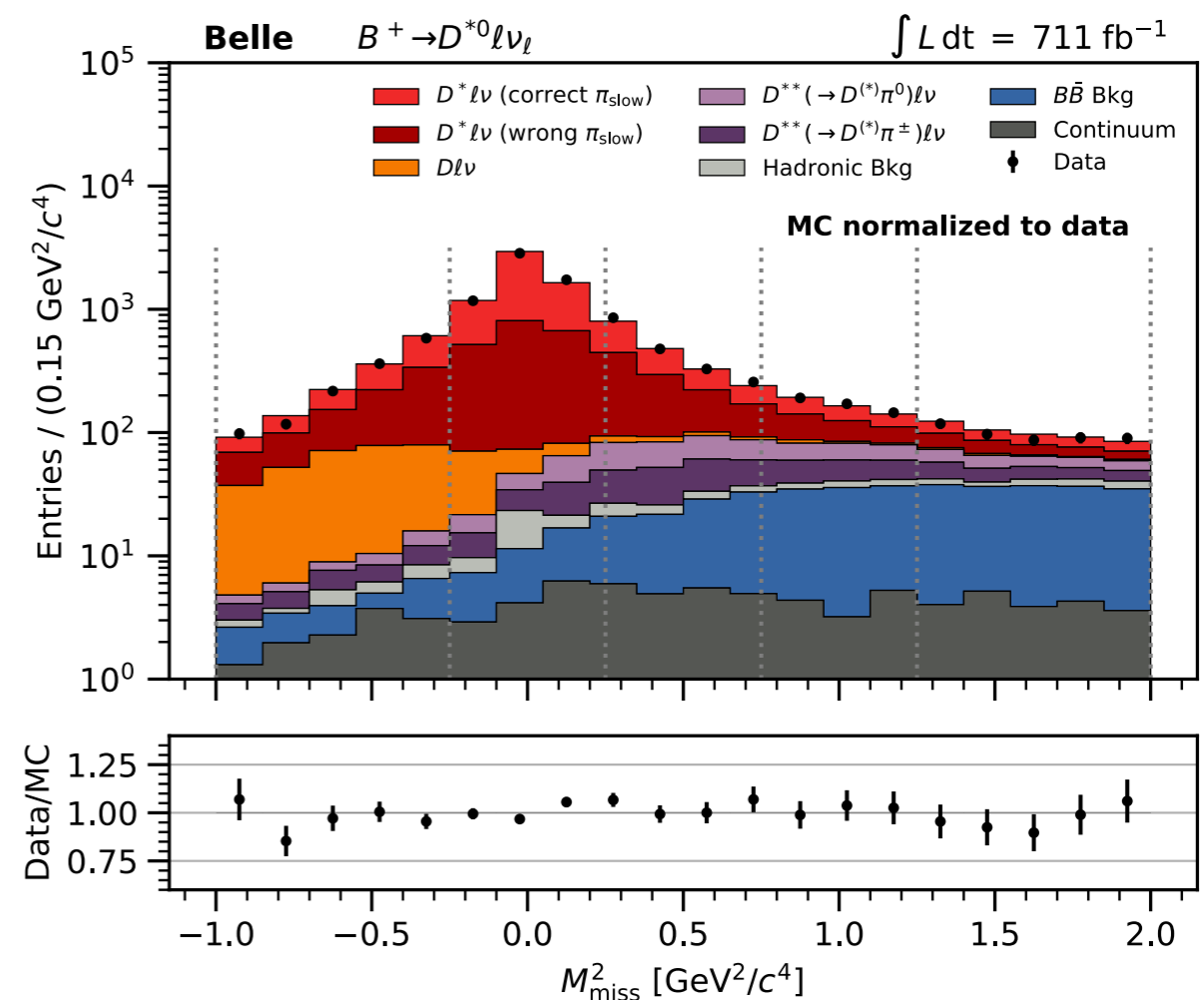
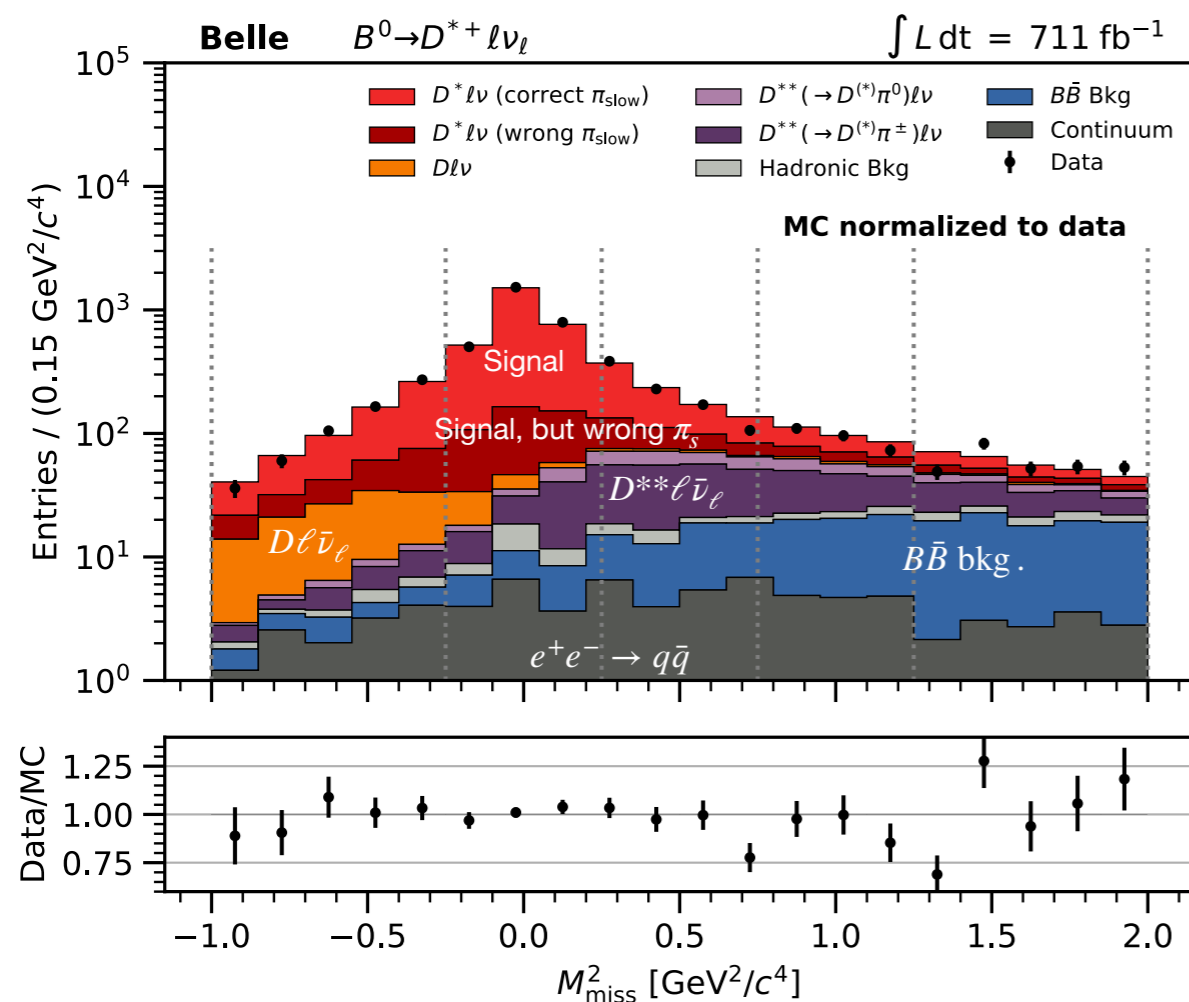


Background subtraction

Need to subtract residual **background** contributions:

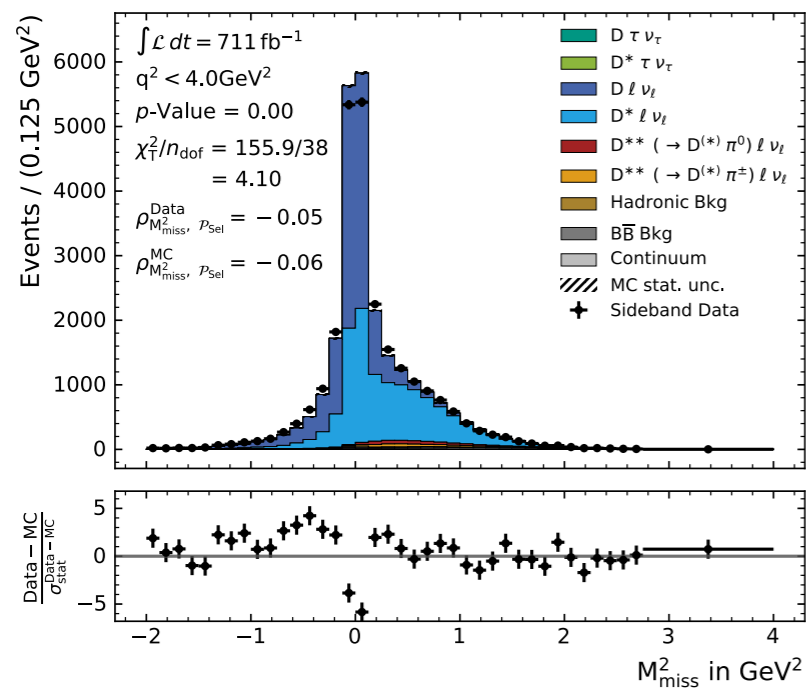
- From other SL decays ($B \rightarrow D^{**}\ell\bar{\nu}_\ell$ or $B \rightarrow D\ell\bar{\nu}_\ell$)
- From other **B decays** (with fake or real leptons)
- From Continuum ($e^+e^- \rightarrow q\bar{q}$)

Use: $0 = m_\nu^2 \simeq M_{\text{miss}}^2 = (E_{\text{miss}}, \mathbf{p}_{\text{miss}})^2 = (p_B - p_{D^*} - p_\ell)^2$ or $U = E_{\text{miss}} - |\mathbf{p}_{\text{miss}}|$



MC modelling of M_{miss}^2 challenging

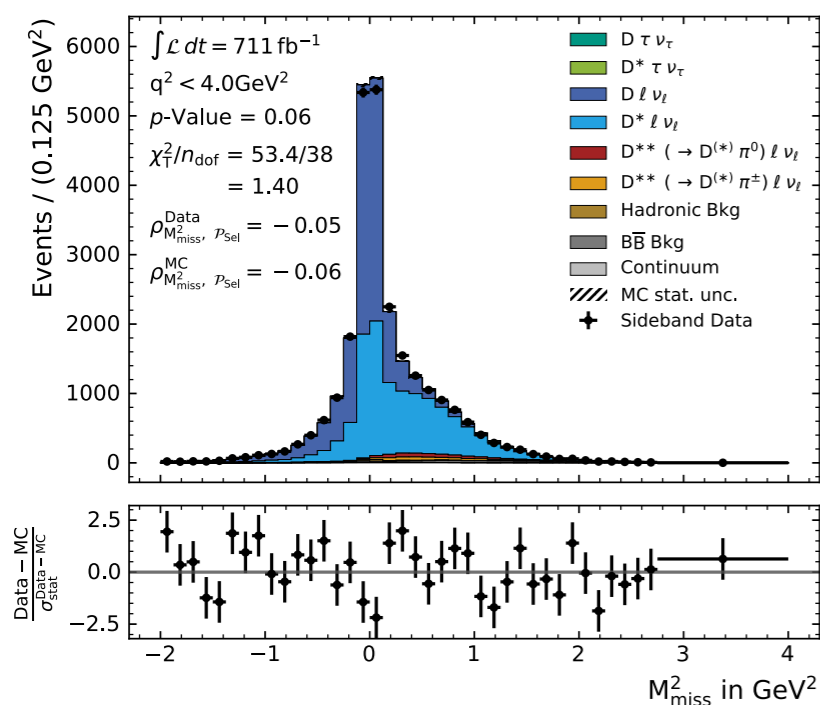
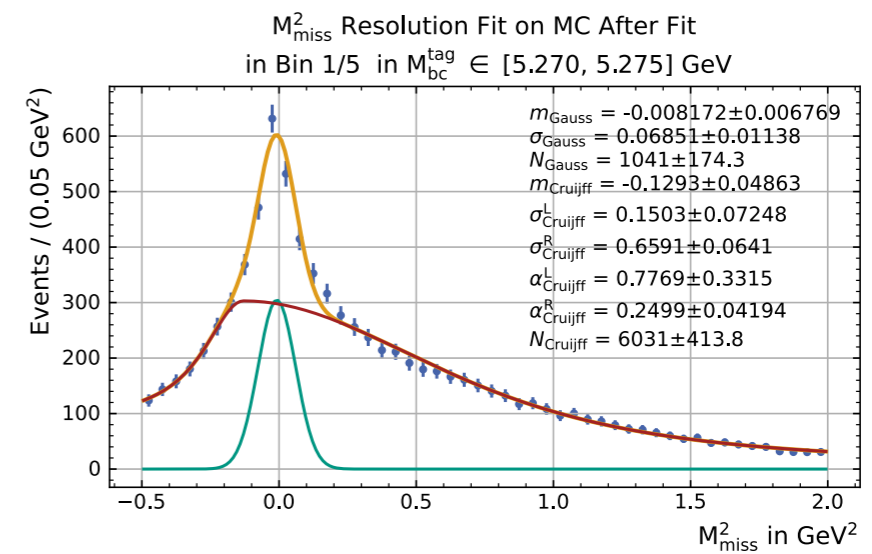
Need to apply additional smearing to match actual resolution



Use an appropriate smearing function

(e.g. asymmetric Laplace distribution and as a function of m_{bc})

$$f_{\text{AL}}(x; m, \lambda, \kappa) = \frac{\lambda}{\kappa + 1/\kappa} \begin{cases} \exp((\lambda/\kappa)(x - m)) & \text{if } x < m, \\ \exp(-\lambda\kappa(x - m)) & \text{if } x \geq m, \end{cases}$$



Fit in Bins of $\{w, \cos \theta_\ell, \cos \theta_V, \chi\}$

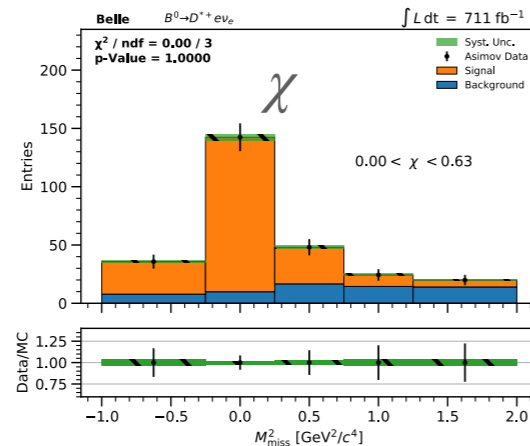
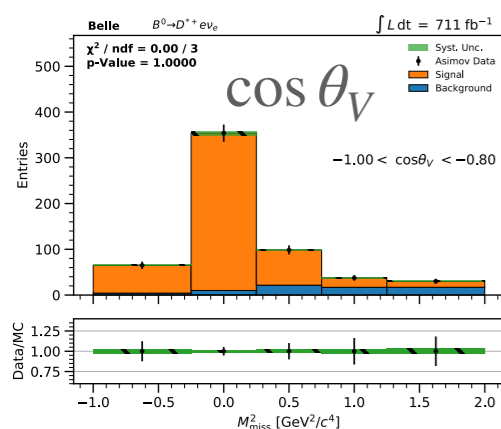
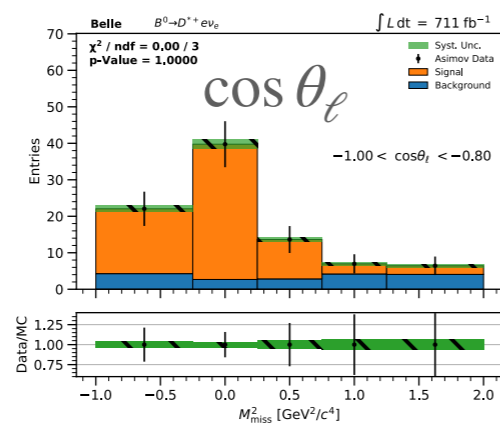
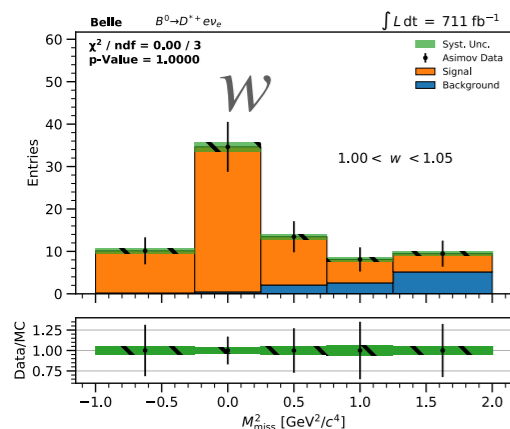
E.g. Can use **binned likelihood** fit to **1D distributions**

(good to use coarse binning to reduce modelling dependence (Bkg shape, resolution))

4D fit also possible; but binned approach suffers from curse of dimensionality

→ **better unbinned (but then need to worry about efficiency & migrations)**

Example **1D fits** to MC (Asimov fits)



Best approach: use folding to extract relevant information

$$\frac{d^4\Gamma}{dq^2 d\cos\theta^* d\cos\theta_\ell d\chi} = \frac{9}{32\pi} [(I_1^s \sin^2\theta^* + I_1^c \cos^2\theta^*) + (I_2^s \sin^2\theta^* + I_2^c \cos^2\theta^*) \cos 2\theta_\ell + I_3 \sin^2\theta^* \sin^2\theta_\ell \cos 2\chi + I_4 \sin 2\theta^* \sin 2\theta_\ell \cos \chi + I_5 \sin 2\theta^* \sin \theta_\ell \cos \chi + (I_6^c \cos^2\theta^* + I_6^s \sin^2\theta^*) \cos \theta_\ell + I_7 \sin 2\theta^* \sin \theta_\ell \sin \chi + I_8 \sin 2\theta^* \sin 2\theta_\ell \sin \chi + I_9 \sin^2\theta^* \sin^2\theta_\ell \sin 2\chi],$$

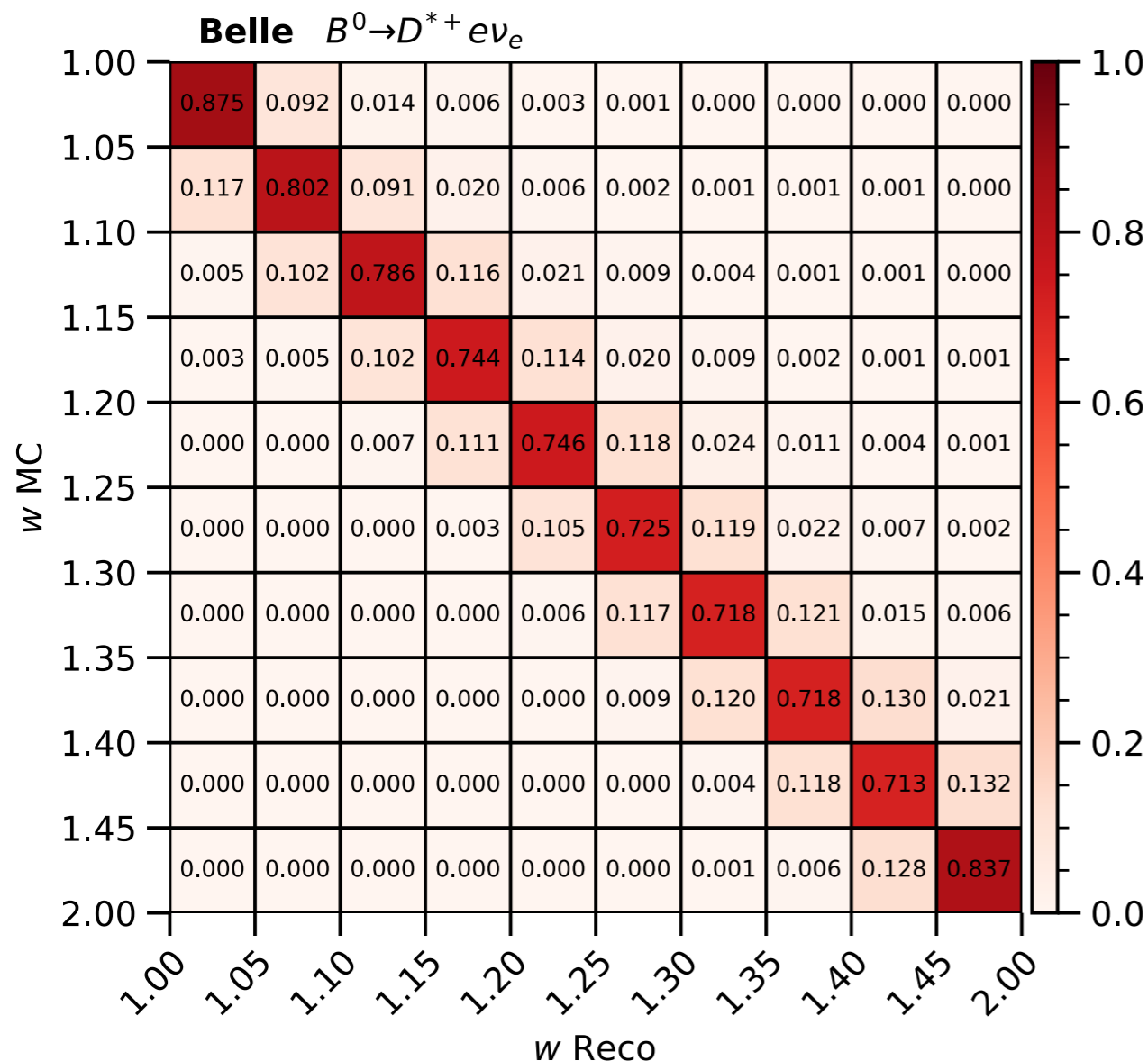
I.e. by building smart asymmetries, can project out the relevant 12 terms (integrated over a certain q^2 range)

Detector migrations

An event reconstructed in a given *bin i*, might not have had a “true” value corresponding to a *bin j*

Can be parametrized as a migration matrix:

$$\mathcal{M}_{ij} = \mathcal{P}(\text{reco. in bin } i \mid \text{true value in bin } j)$$



Can recover true values by “unfolding” determined yields, mapping reco \rightarrow true

Simplest version: migration matrix inversion

$$\mathbf{x}_{\text{true}} = \mathcal{M}_{ij}^{-1} \mathbf{x}_{\text{reco}}$$

Many approaches to dampen impact of increase in variance

(mostly a problem with large migrations \rightarrow true bin is then the sum of many reco bins with high weights)

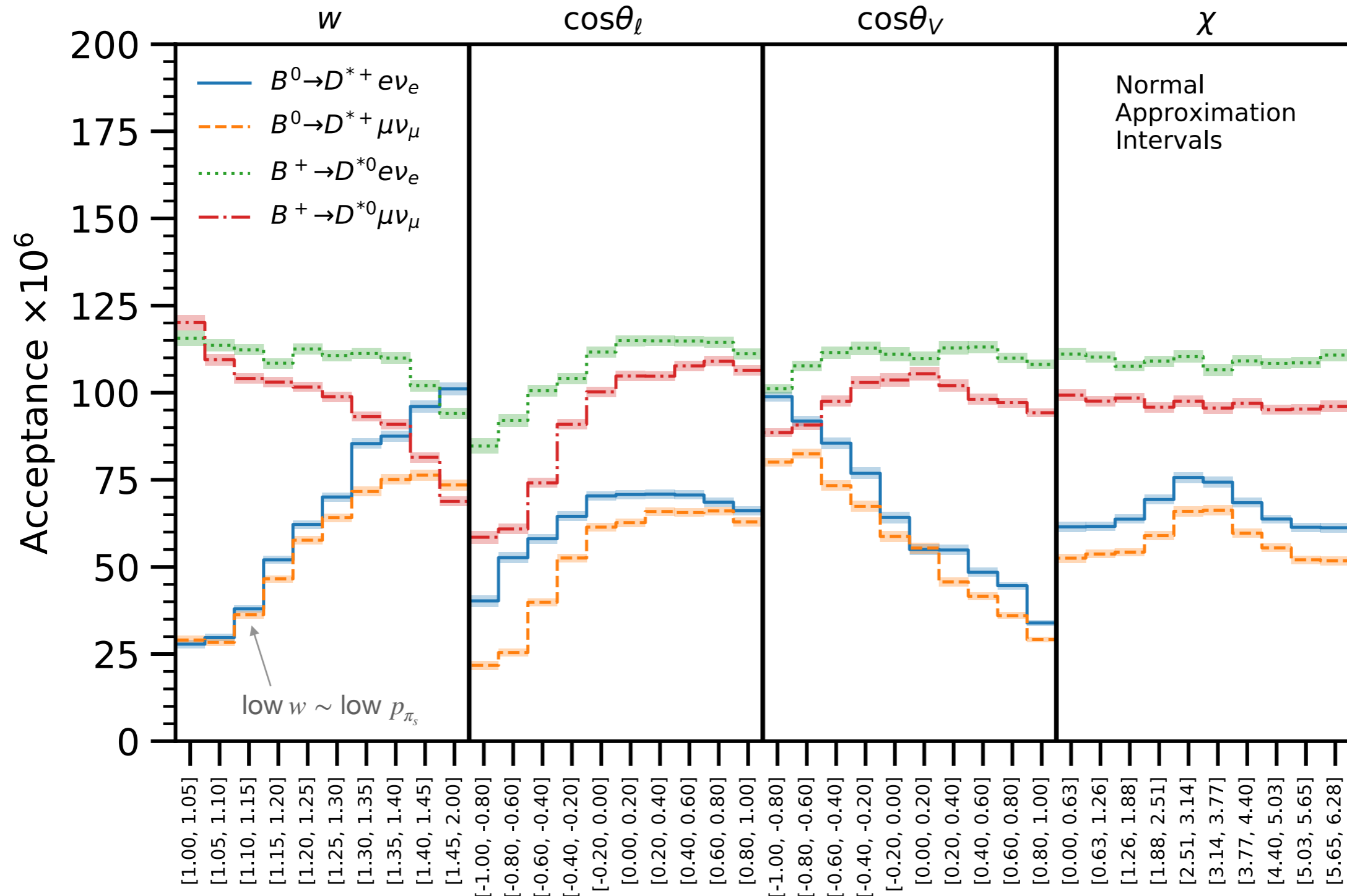
or to reduce impact of MC prior

(here less an issue; but Bayesian unfolding can propagate the observed shape to MC to minimize model dependencies)

Acceptance \times Efficiency

After migration effects are corrected, need to correct also for selection effects

(Acceptance \times Efficiency)



A word on Efficiencies

Efficiencies can be are a large source of uncertainties

Two examples relevant for this:

- Lepton Identification Uncertainty

Often based on a global likelihood (or a multivariate classifier) using individual likelihoods (or input features) to calculate a score how likely the identified particle is an electron or a muon

Symbolically:

$$\mathcal{L} = \mathcal{L}_{\text{CDC}} \times \mathcal{L}_{\text{ECL}} \times \mathcal{L}_{\text{TOP}} \times \mathcal{L}_{\text{KLM}}$$

Ionization energy loss

$E/|\vec{p}|$

Information from Cherenkov light angles

Matched KLM cluster hit?

Detailed description: The diagram shows the equation $\mathcal{L} = \mathcal{L}_{\text{CDC}} \times \mathcal{L}_{\text{ECL}} \times \mathcal{L}_{\text{TOP}} \times \mathcal{L}_{\text{KLM}}$. Below \mathcal{L}_{CDC} is the text 'Ionization energy loss' with an upward arrow. Below \mathcal{L}_{ECL} is the text ' $E/|\vec{p}|$ ' with an upward arrow. Below \mathcal{L}_{TOP} is the text 'Information from Cherenkov light angles' with an upward arrow. Below \mathcal{L}_{KLM} is the text 'Matched KLM cluster hit?' with an arrow pointing towards the term.

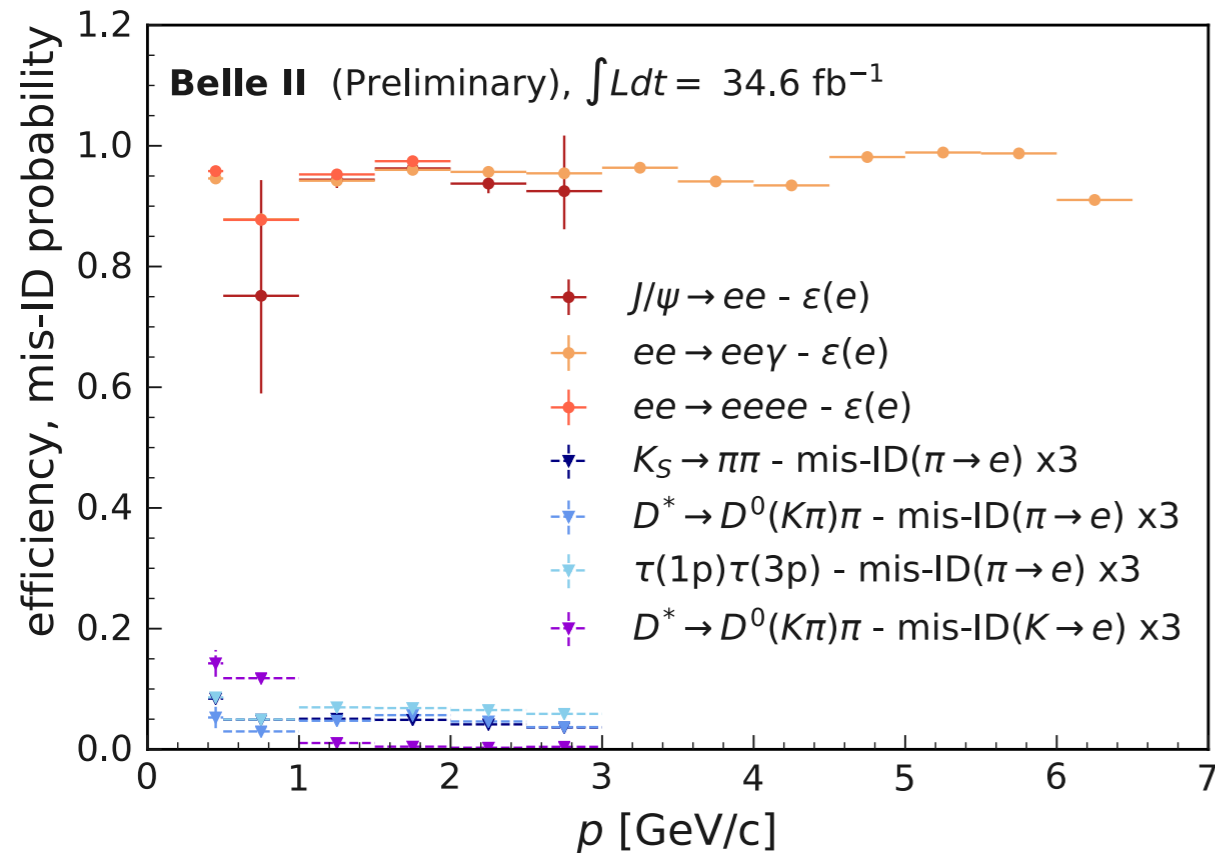
Use clean physics sample to correct MC efficiencies and fake rates

E.g. $e^+e^- \rightarrow \mu\mu\gamma, e^+e^- \rightarrow e^+e^-\gamma, J/\psi \rightarrow \ell\ell, \dots$

Construct likelihood ratio for Lepton ID: $\ell \text{ ID} = \mathcal{L}_\ell / [\mathcal{L}_e + \mathcal{L}_\mu + \mathcal{L}_\pi + \mathcal{L}_K + \mathcal{L}_p]$

Electrons

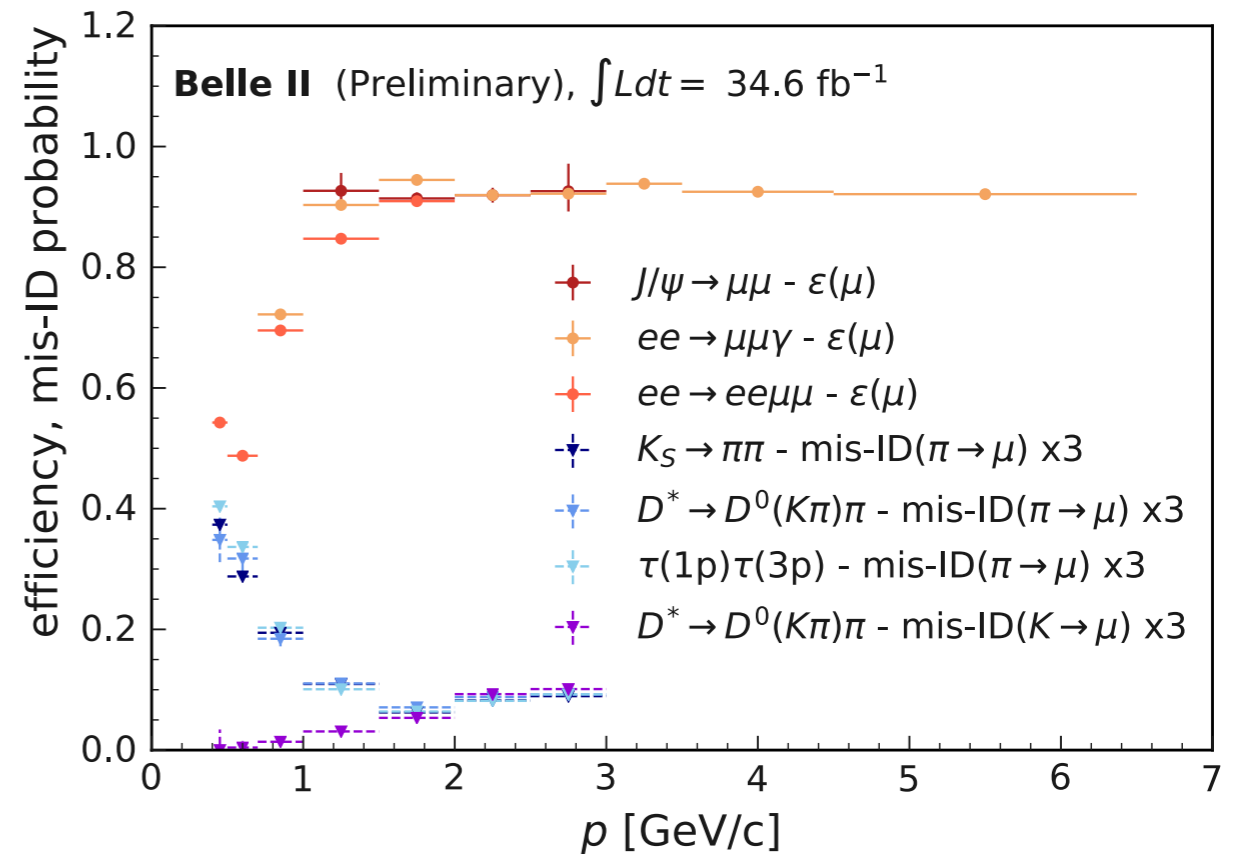
$1.13 \leq \theta < 1.57$ rad, electronID > 0.9



Momentum in lab frame

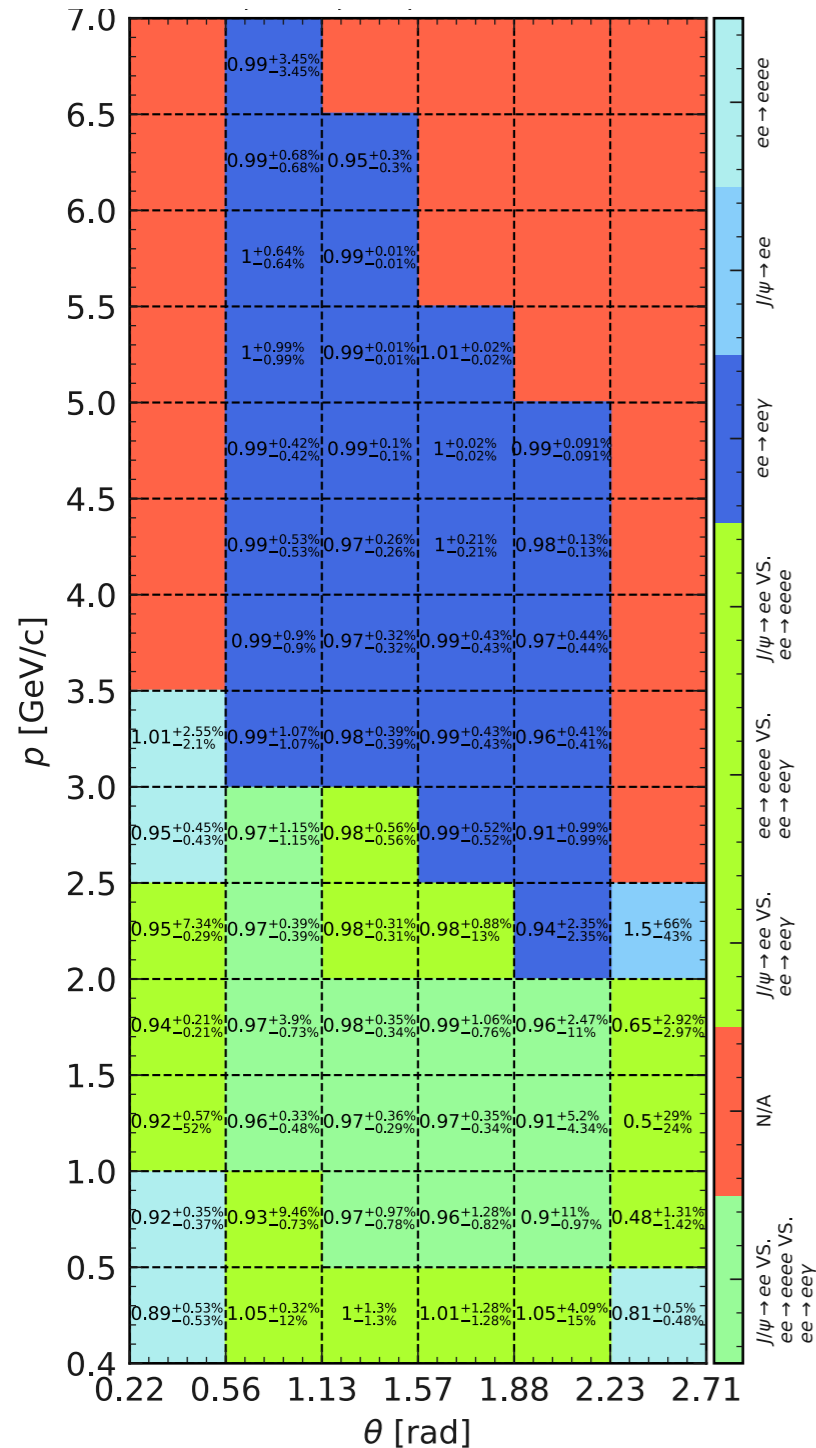
Muons

$0.82 \leq \theta < 1.16$ rad, muonID > 0.9



Construct correction tables of efficiency ratios $\frac{\epsilon_{\text{Data}}}{\epsilon_{\text{MC}}}$

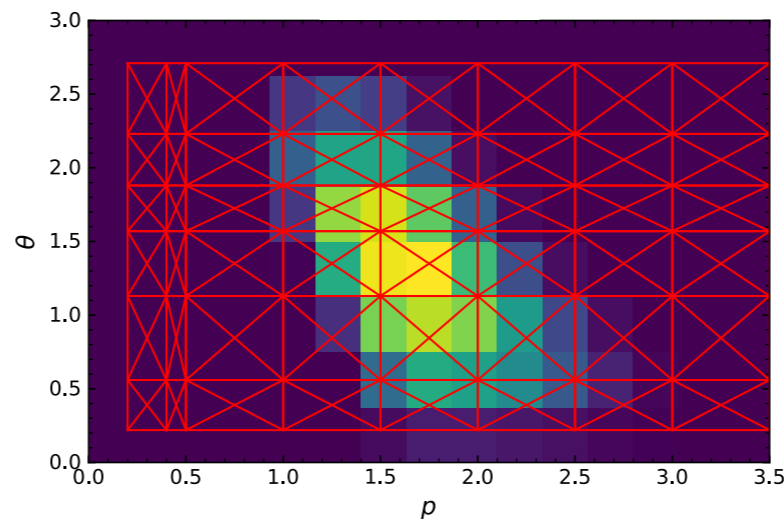
as a function of **lab momentum** and **detector position** (polar angle) to correct MC efficiencies



Precision limited by available control channel statistics (i.e. goes down by Lumi)

Non-closure between channels is added as extra uncertainty (limiting factor at very high luminosity)

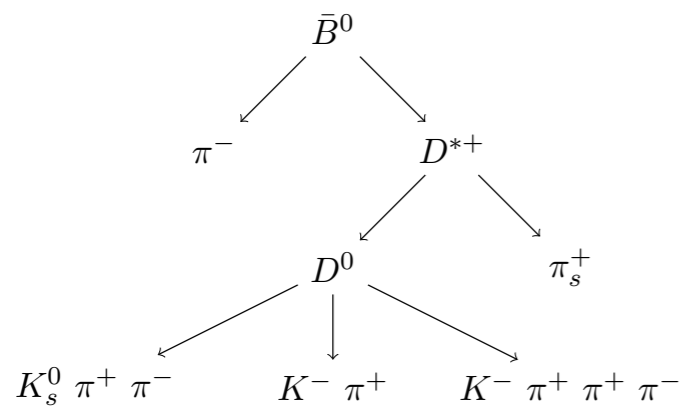
Coverage of control channels and signal are different, i.e. not all control channels have same relevance)



Second example:

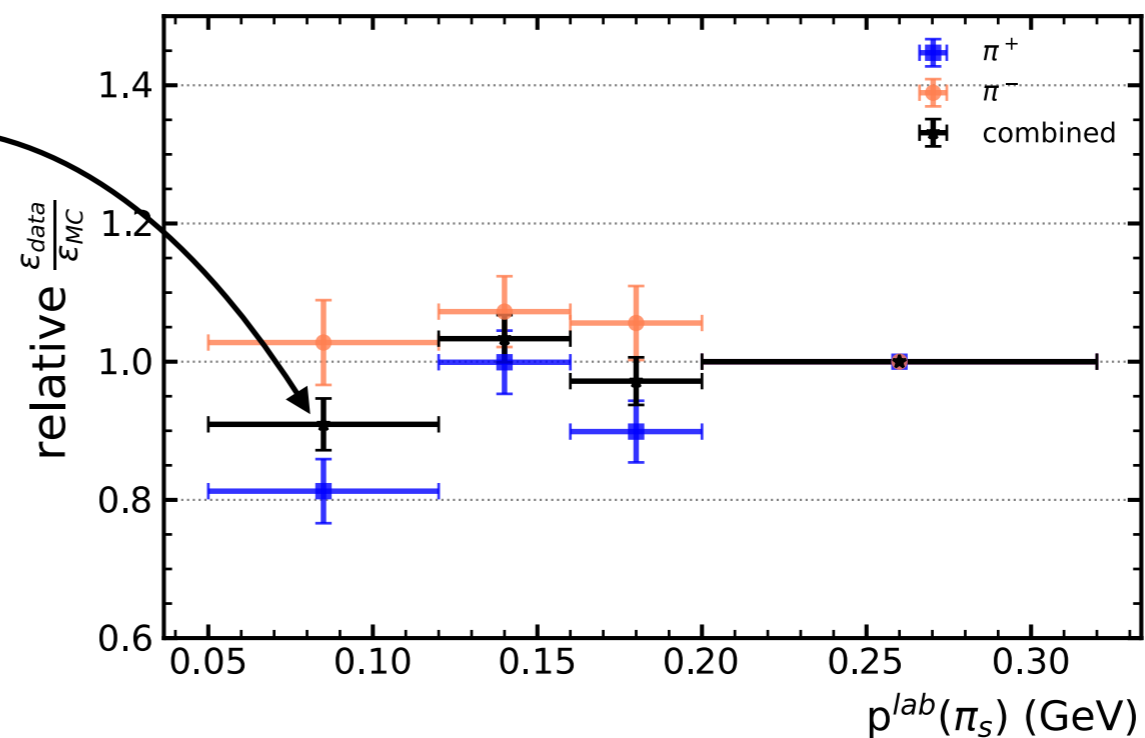
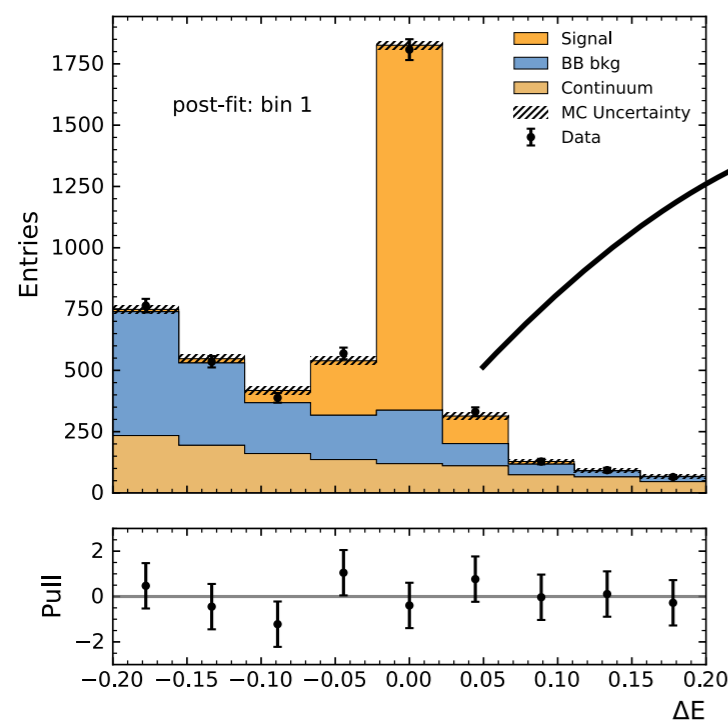
- Slow pion reconstruction efficiency

Also needs to be measured in data, e.g. via $B^0 \rightarrow D^{*+} \pi^-$ decays

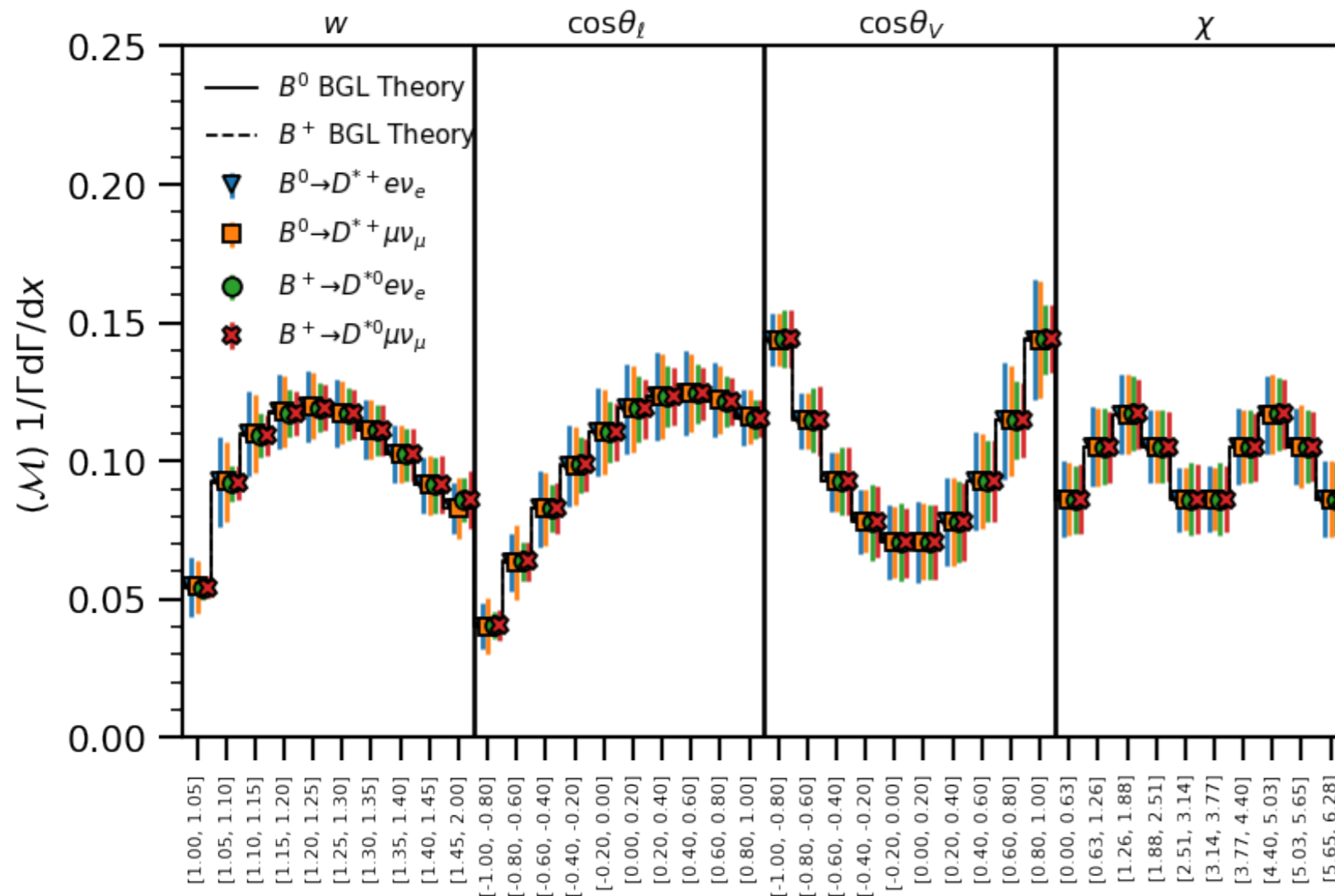


Extract signal in a fit to $\Delta E = \sqrt{s}/2 - E_B$
in bins of $p_{\pi_s}^{\text{lab}}$

Measure ratio efficiency ratio **relative** to
high-momentum region of $p_{\pi_s}^{\text{lab}} > 200 \text{ MeV}$



The final result (MC)



Note how the different channels are complementary in different regions of phase-space

(e.g. B^+ has much better precision at low w than B^0 , but both have equal precision at high w)

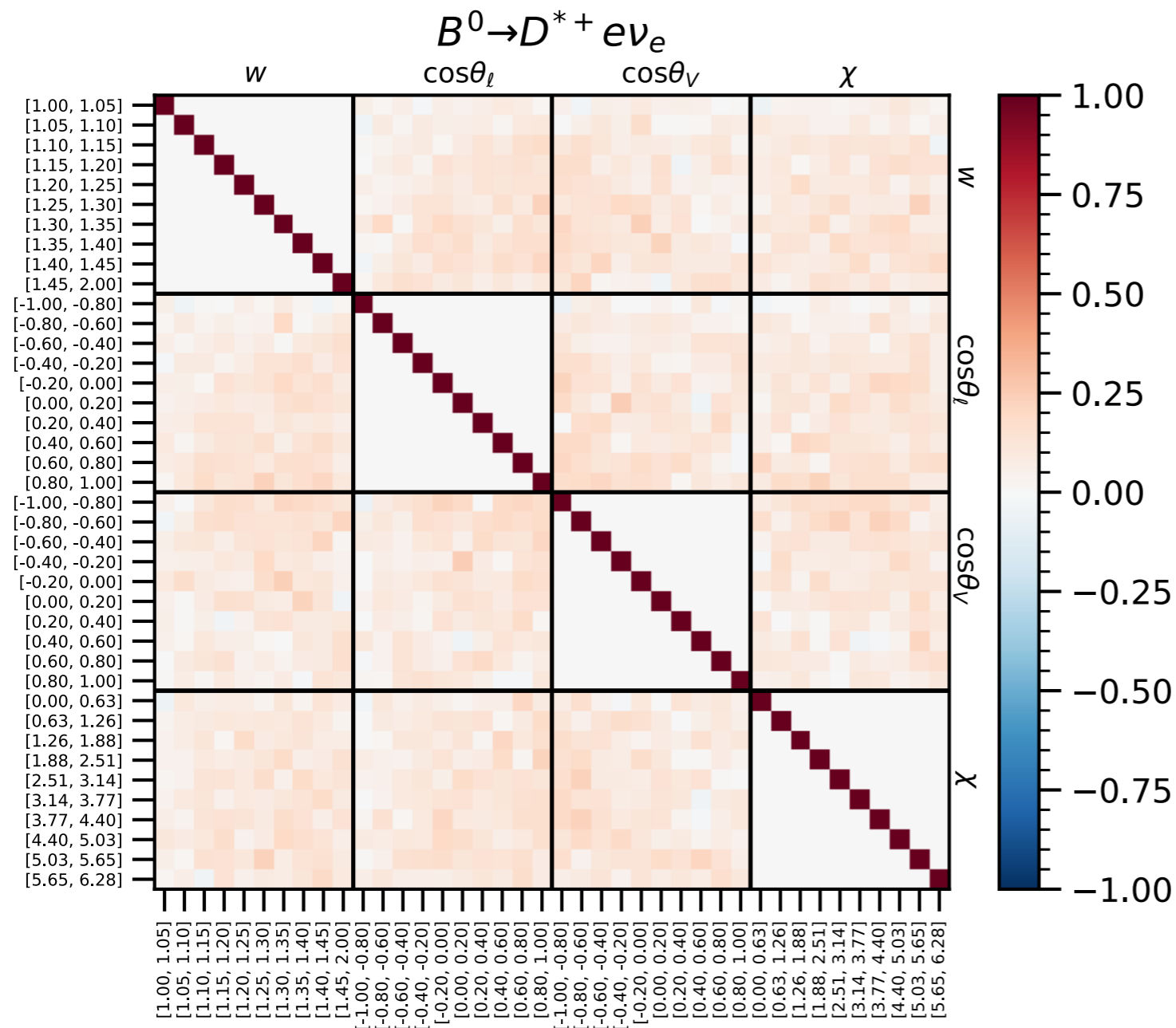
For a simultaneous analysis, need to determine correlations between different 1D projections → can be done using **bootstrapping**

Very simple: create a replica of your data set by sampling with replacement

Repeat full analysis chain of 4 x1D measurement for **each replica**

Pearson correlator of replica sample provides estimator for statistical correlation between bins:

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$



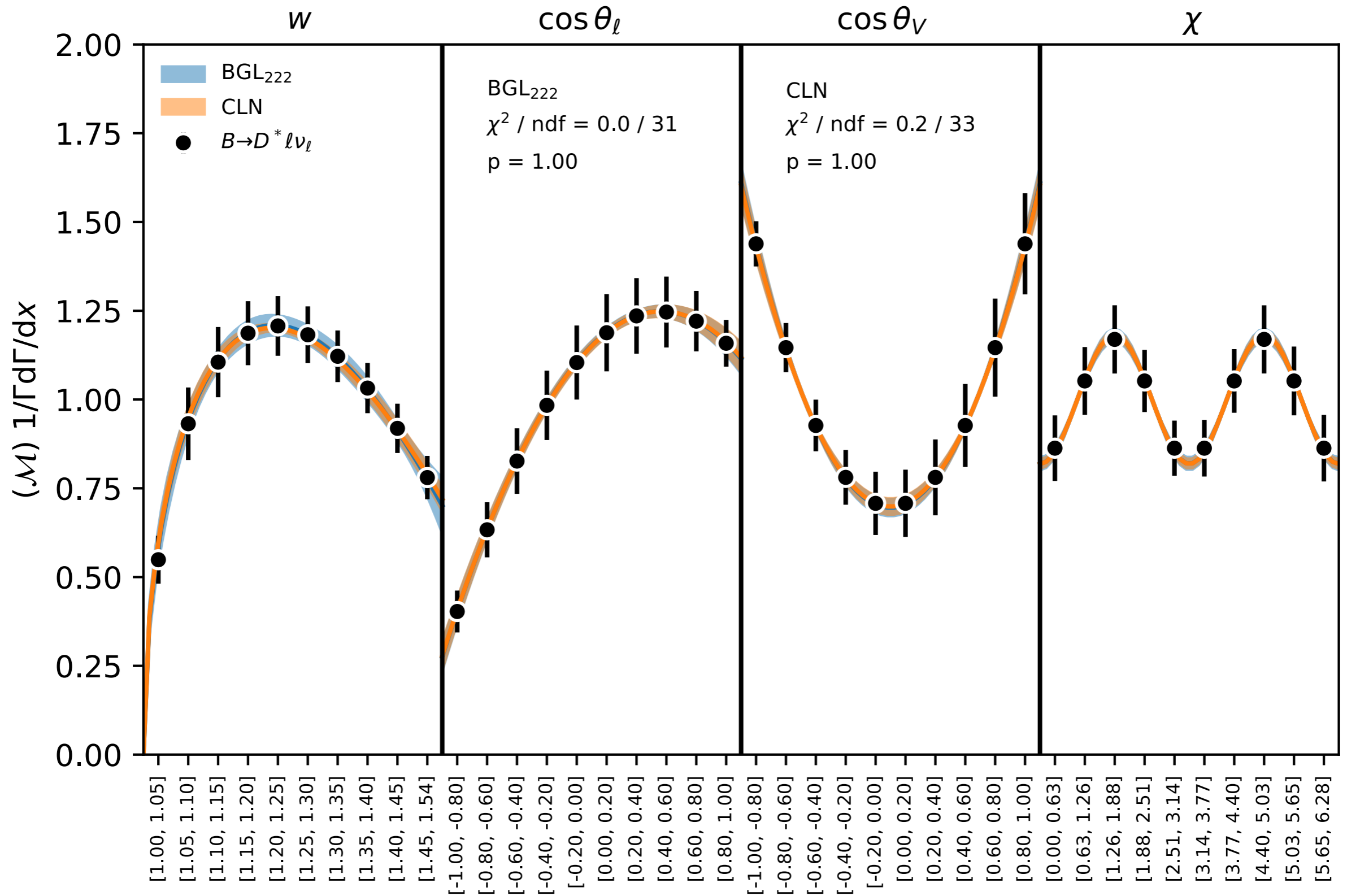
But since we measured projections of the same data, the effective **degrees of freedom** are not 40, but 37 (Jung, Van Dyk)

Best use of tagged data:

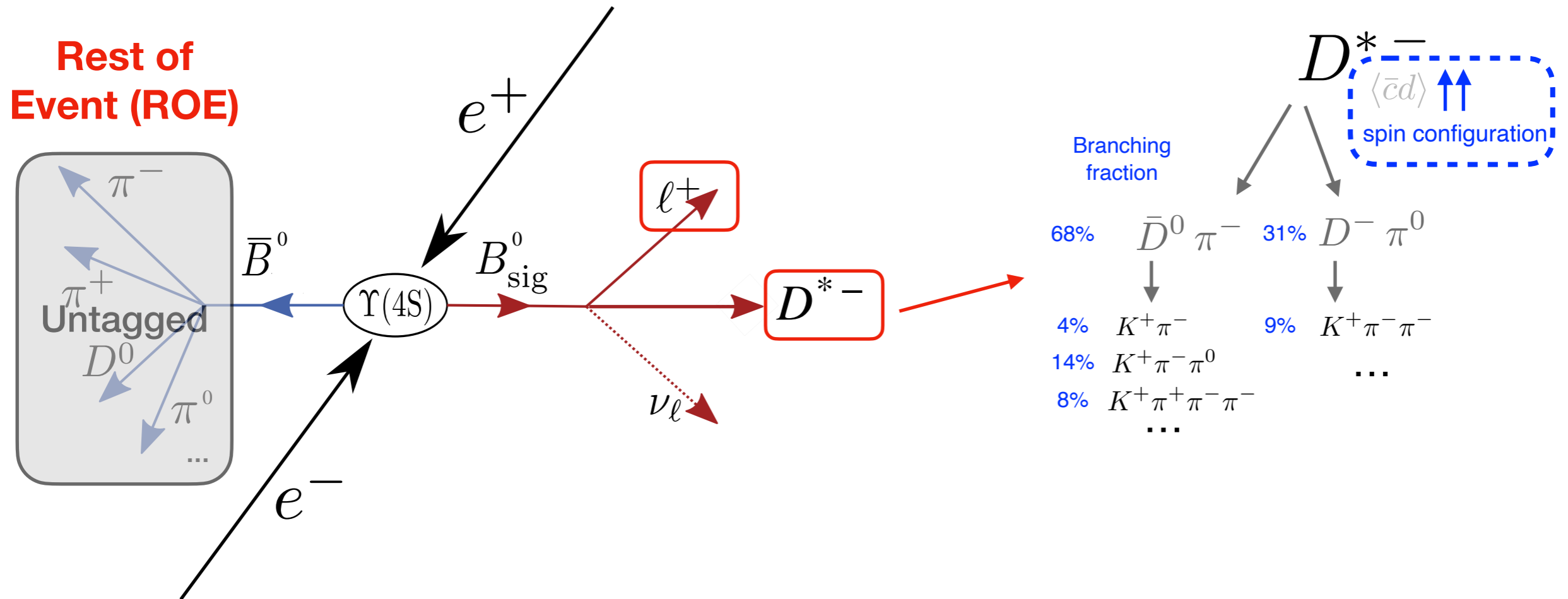
Fit normalized shapes (and if available total rate)

36 dof from shapes (4*9) and 1 from normalization

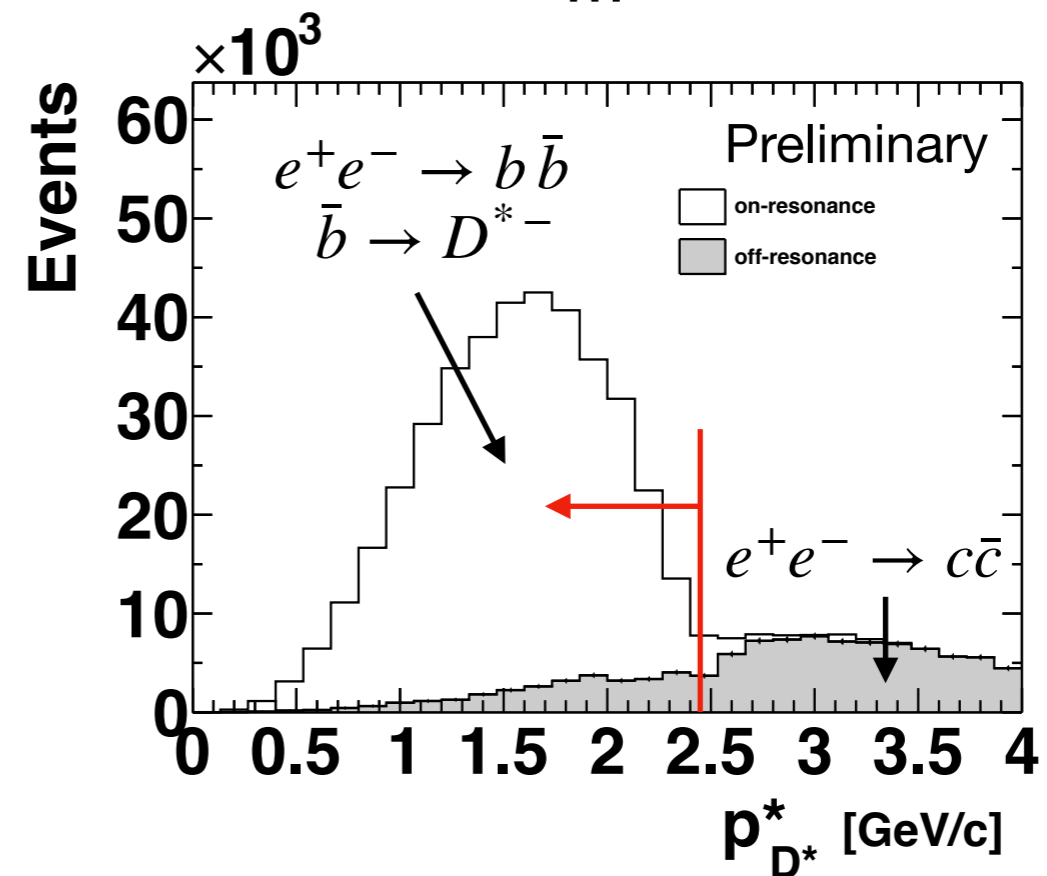
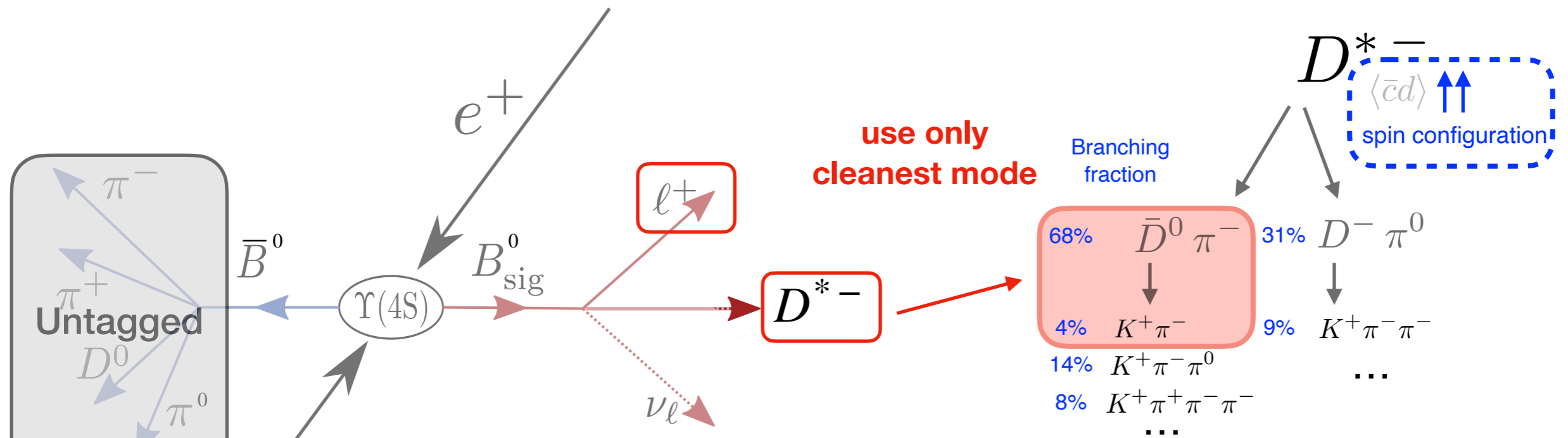
(Asimov again)



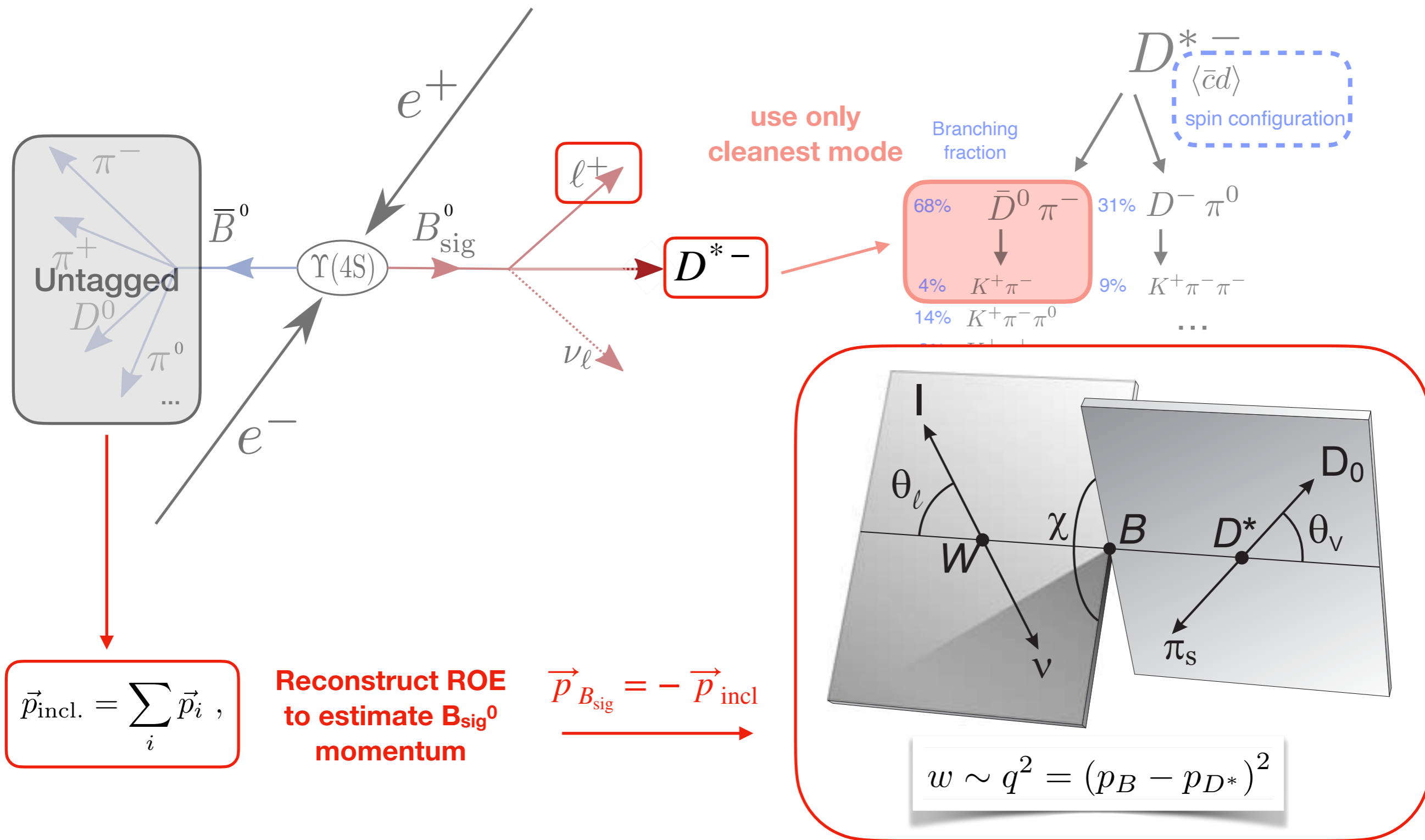
Untagged measurements of $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$



Untagged measurements of $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$



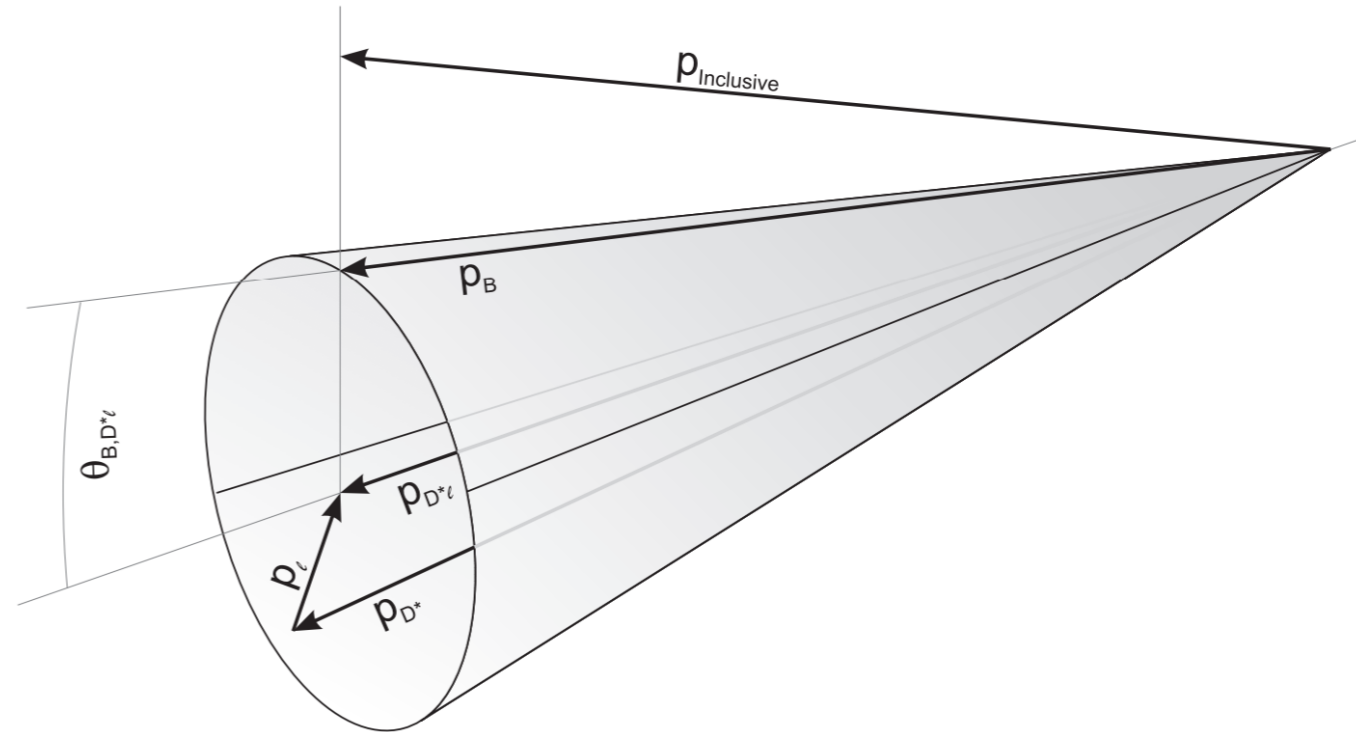
Untagged measurements of $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$



Alternative Reconstruction Methods

Can exploit that the B meson lies on a **cone**, whose opening angle is fully determined by properties of visible particles:

$$\cos \theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\mathbf{p}_B||\mathbf{p}_{D^*\ell}|}$$



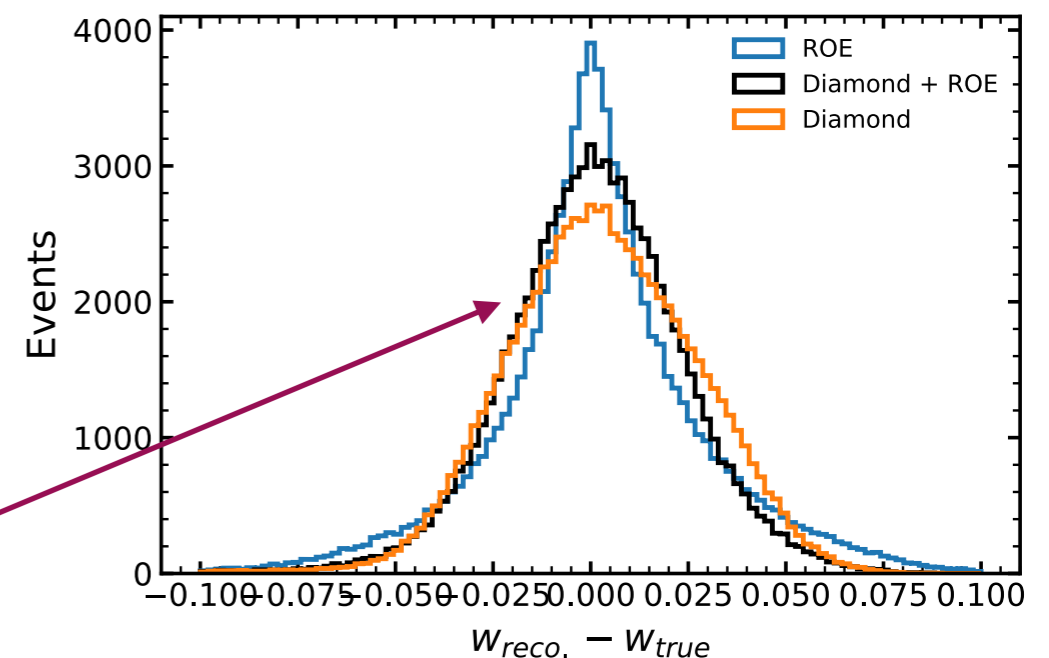
Can use this to estimate B meson direction building a weighted average on the cone

$$(E^B, p_B^x, p_B^y, p_B^z) = (\sqrt{s}/2, |\mathbf{p}_B| \sin \theta_{BY} \cos \phi, |\mathbf{p}_B| \sin \theta_{BY} \sin \phi, |\mathbf{p}_B| \cos \theta_{BY})$$

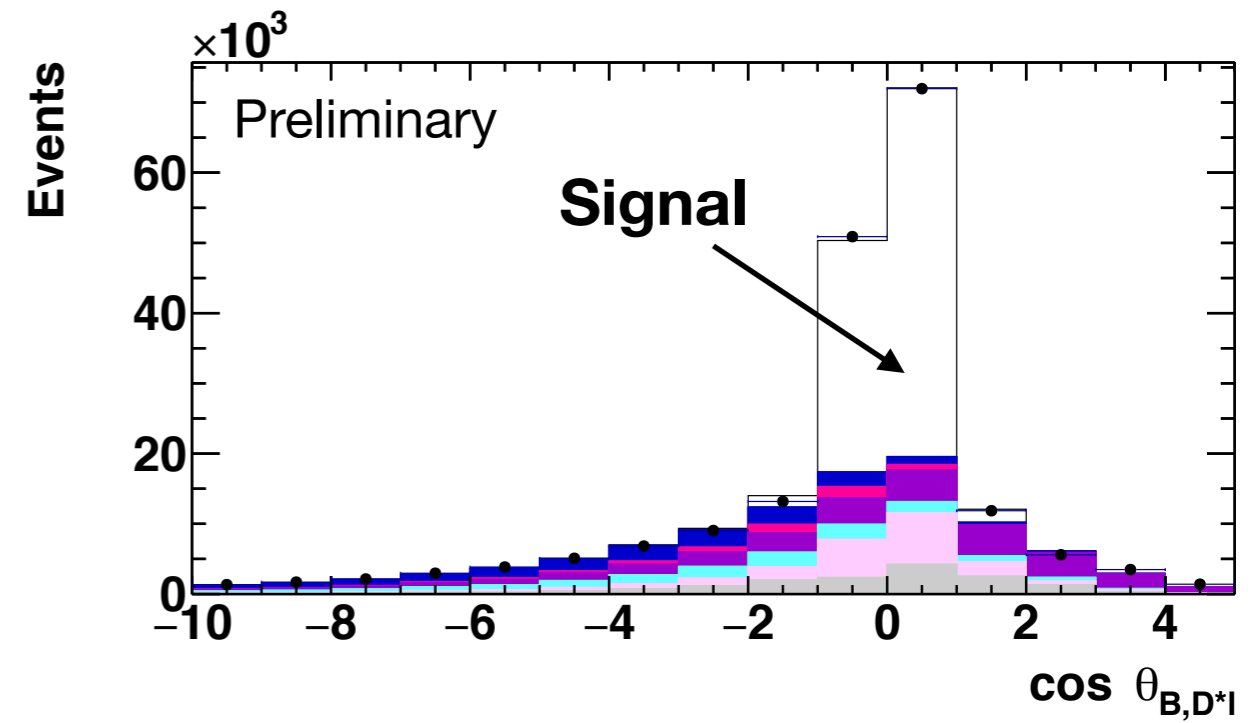
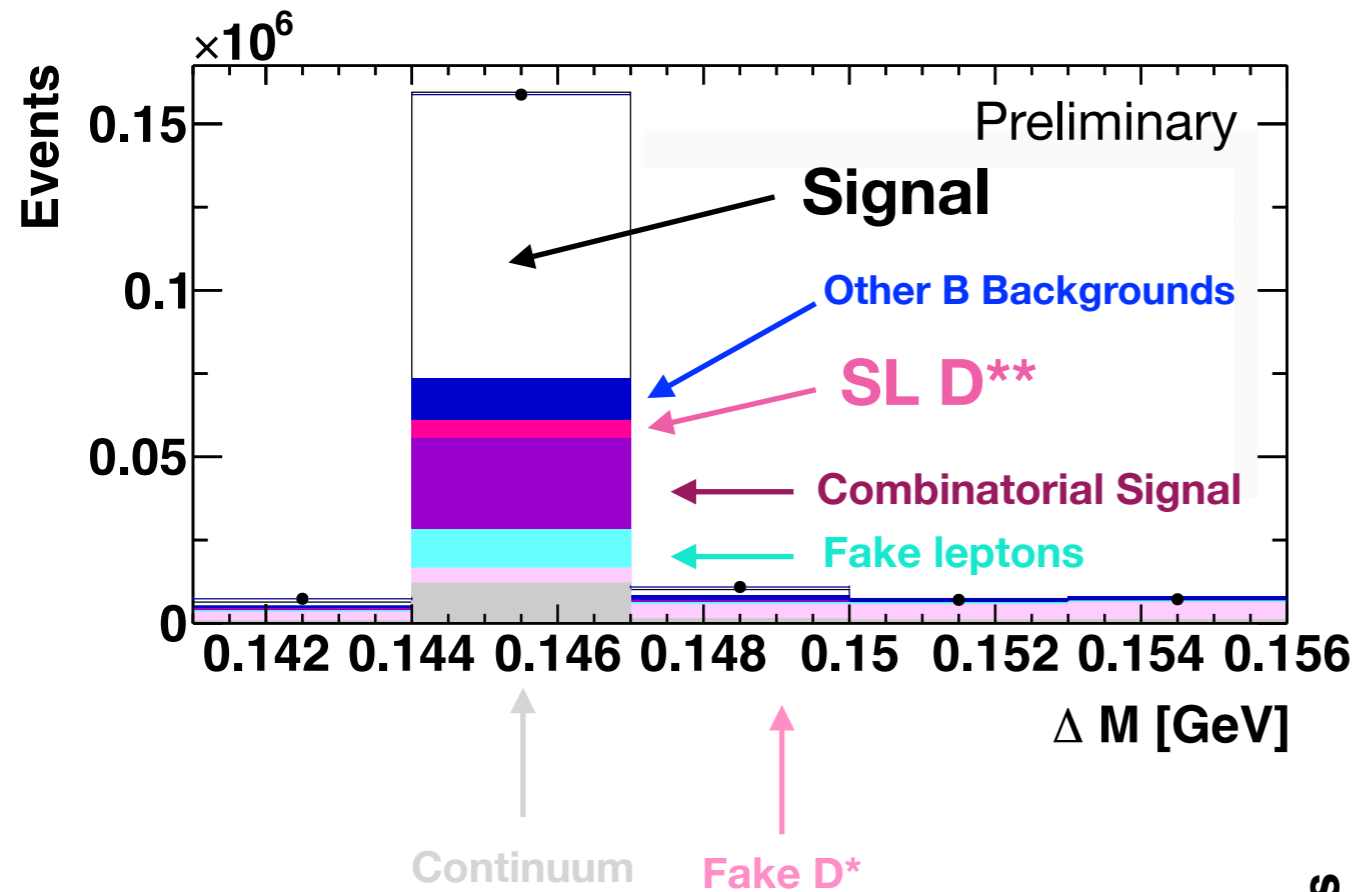
with weights according to $w_i = \sin^2 \theta_i$ with θ denoting the polar angle

(following the angular distribution of $\Upsilon(4S) \rightarrow B\bar{B}$)

One can also **combine** both estimates



Background Subtraction

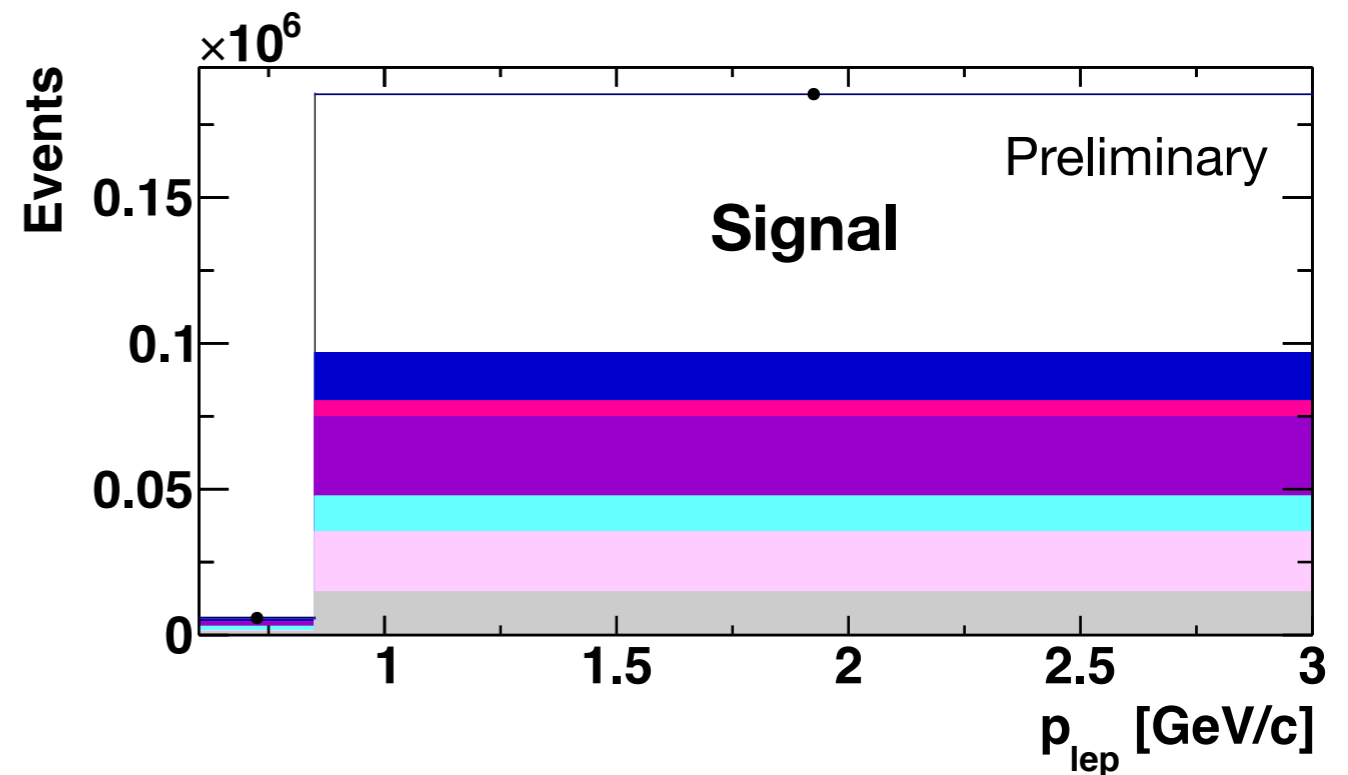


3 Variables used:

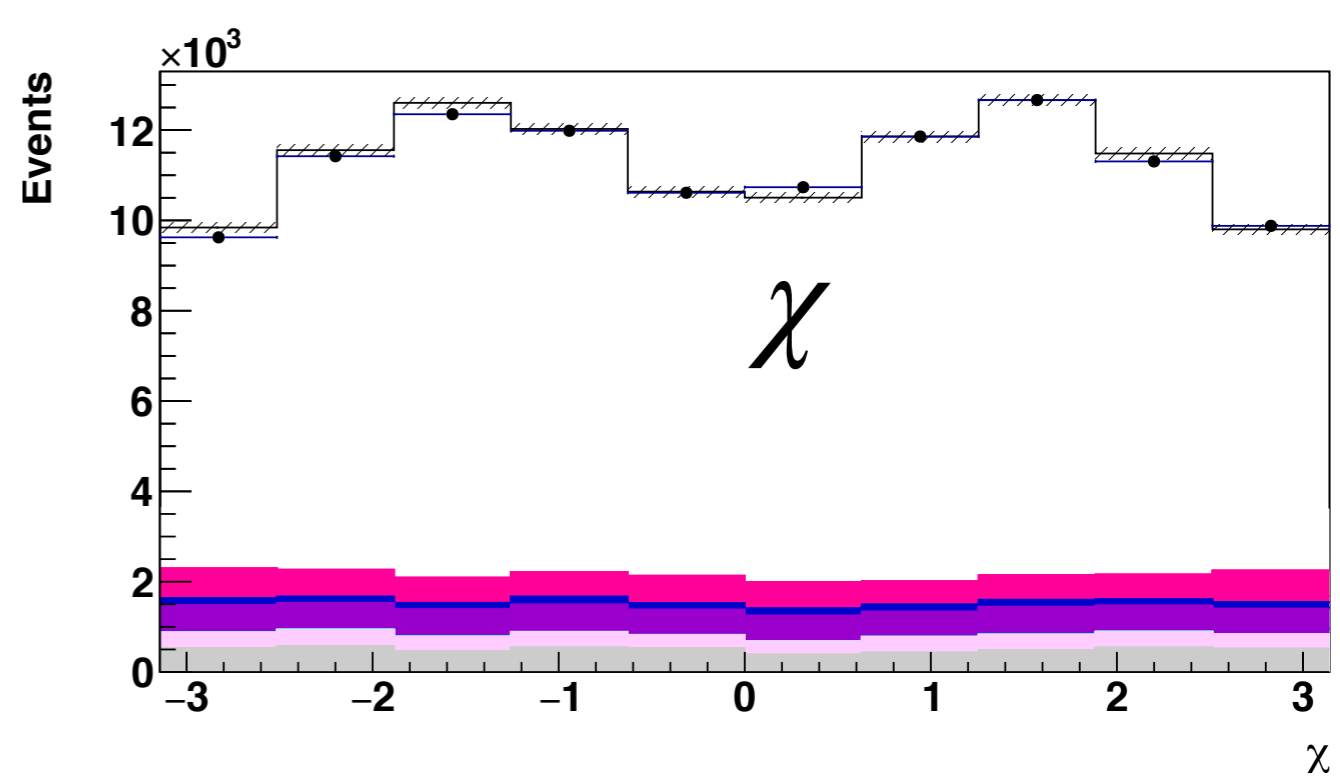
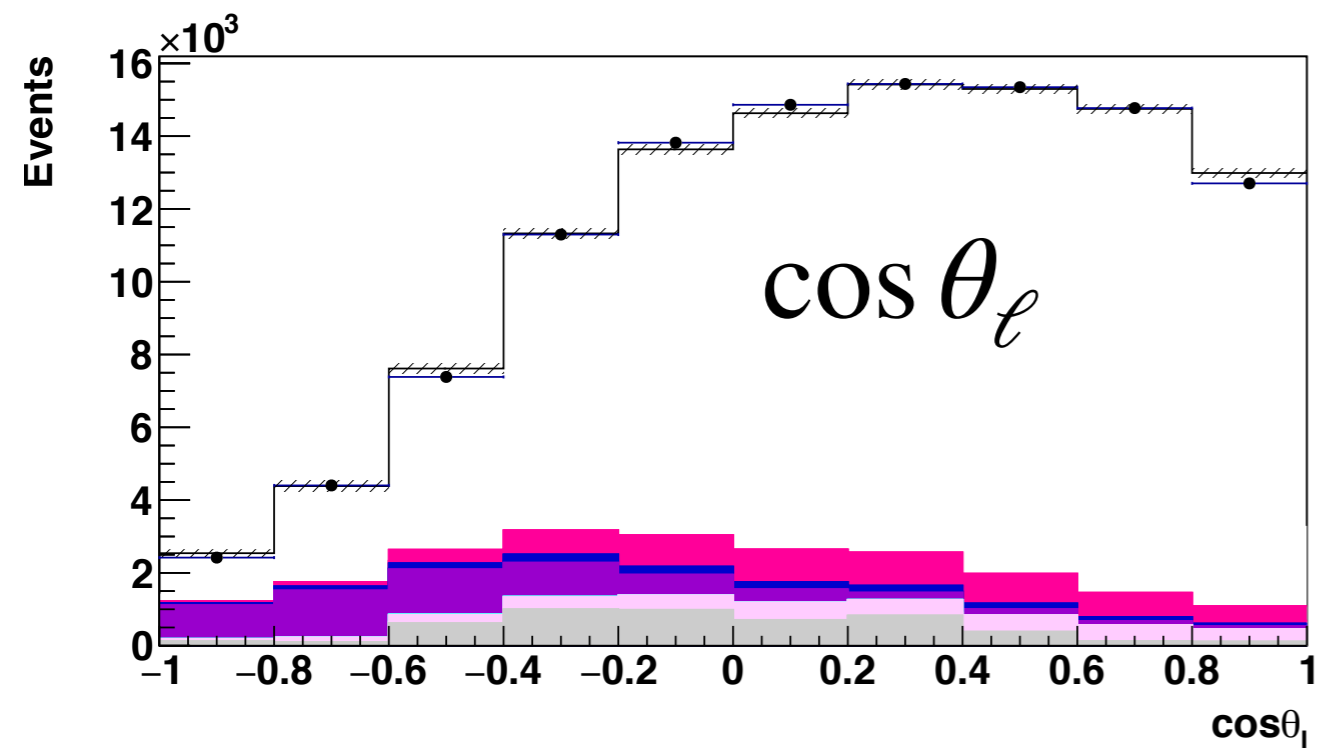
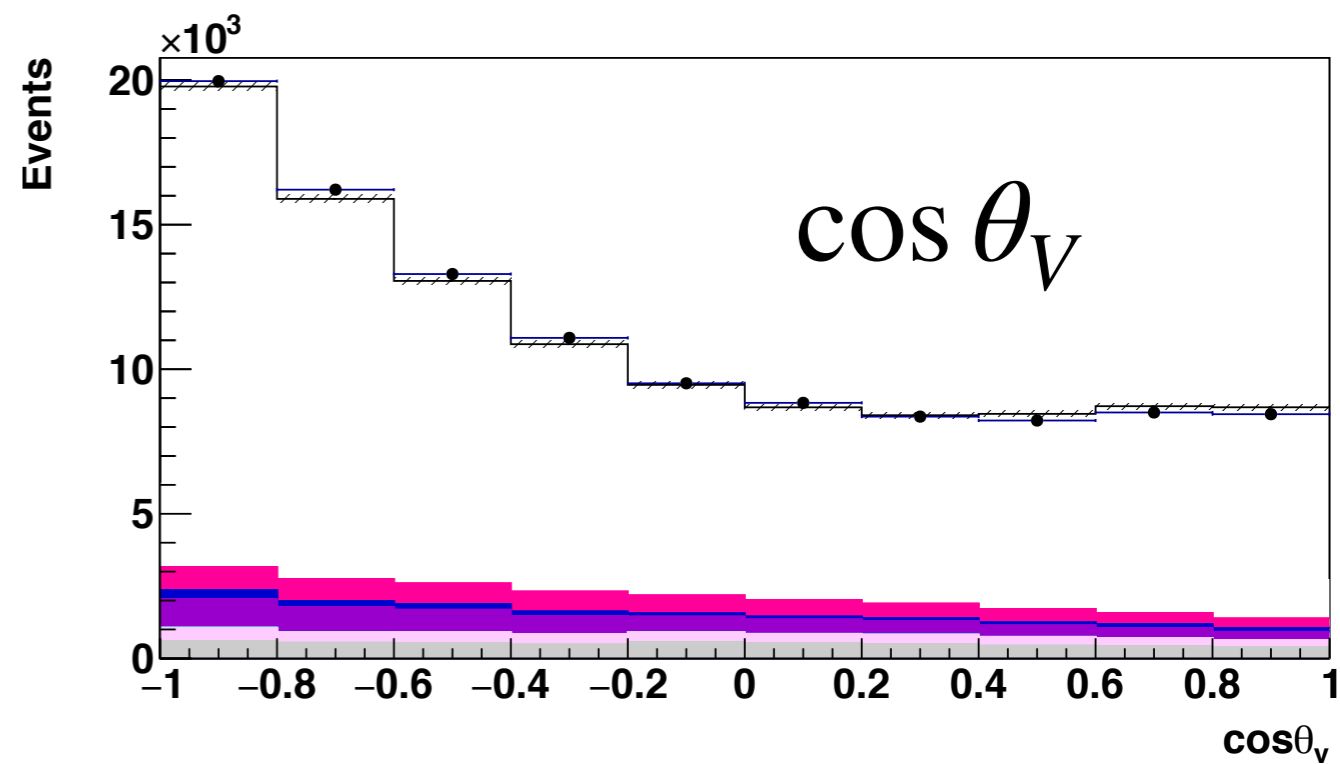
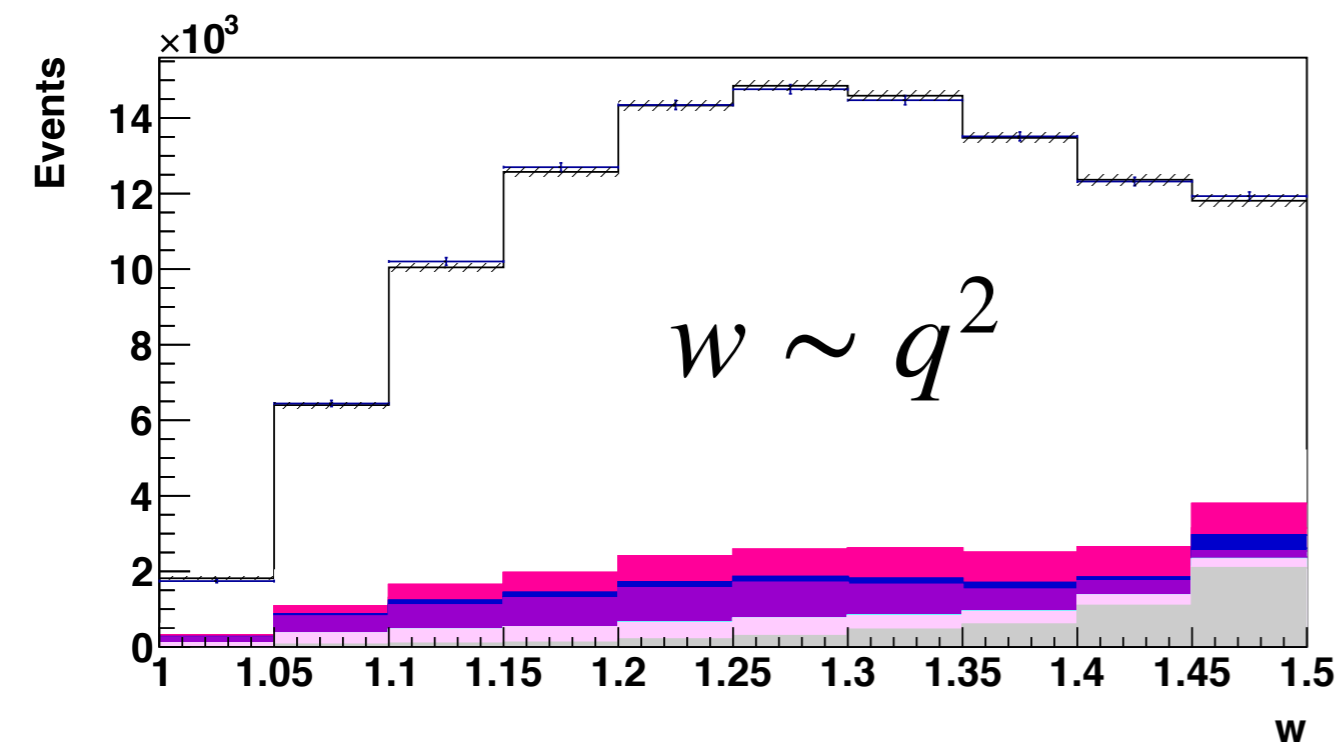
$$\Delta M = m_{D^*} - m_D$$

$$\cos \theta_{B,D^*\ell} = \frac{2E_B E_{D^*\ell} - m_B^2 - m_{D^*\ell}^2}{2|\vec{p}_B||\vec{p}_{D^*\ell}|}$$

$$p_{\text{lep}}$$



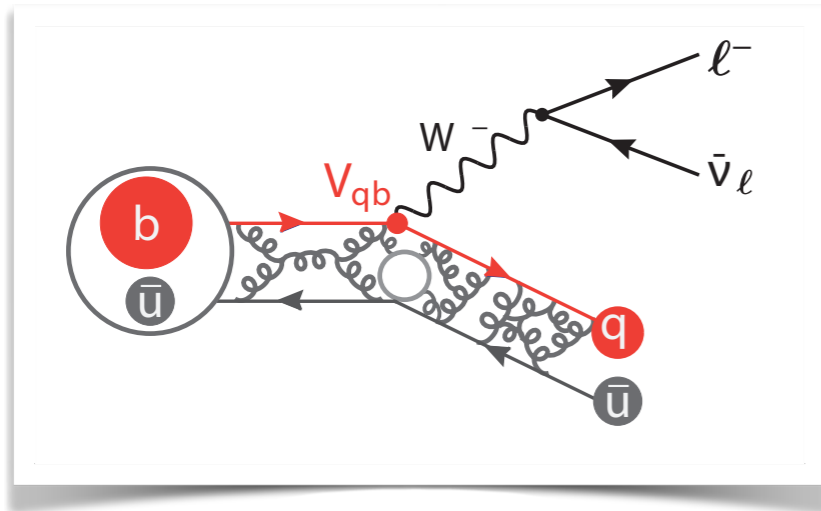
Kinematic Distributions





2. Inclusive

Overview



Inclusive $|V_{cb}|$

$$\bar{B} \rightarrow X_c \ell \bar{\nu}_\ell$$

Operator Product Expansion (OPE)

$$\mathcal{B} = |V_{qb}|^2 \left[\Gamma(b \rightarrow q \ell \bar{\nu}_\ell) + 1/m_{c,b} + \alpha_s + \dots \right]$$

Established approach: Use **spectral moments** (hadronic mass moments, lepton energy moments etc.) to determine non-perturbative matrix elements (ME) of OPE and extract $|V_{cb}|$

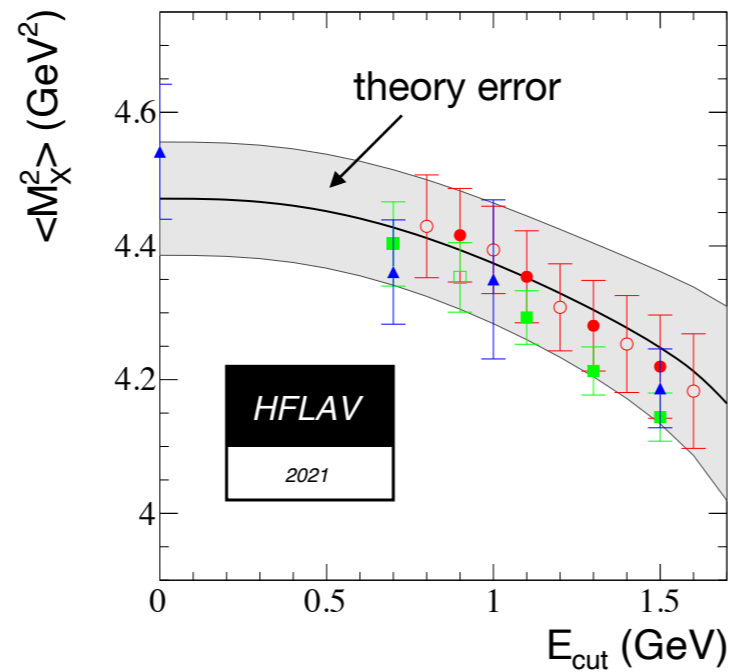
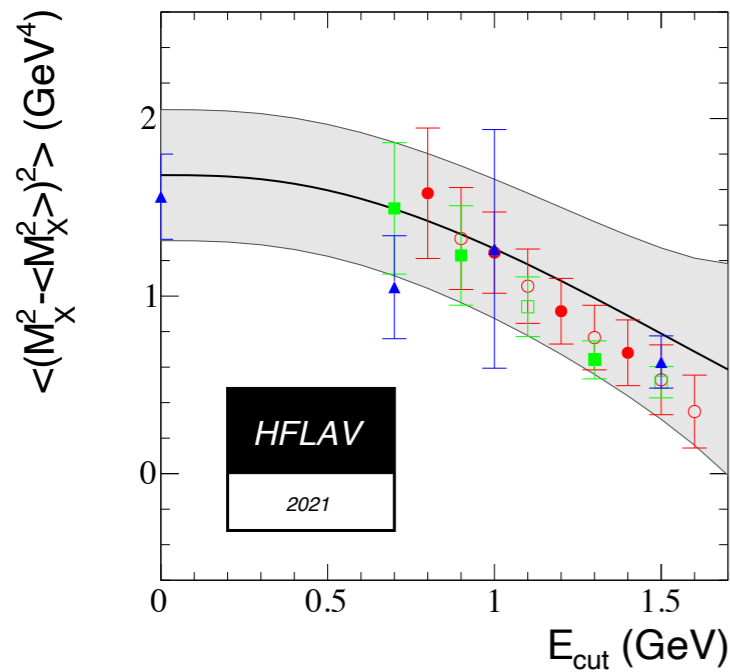
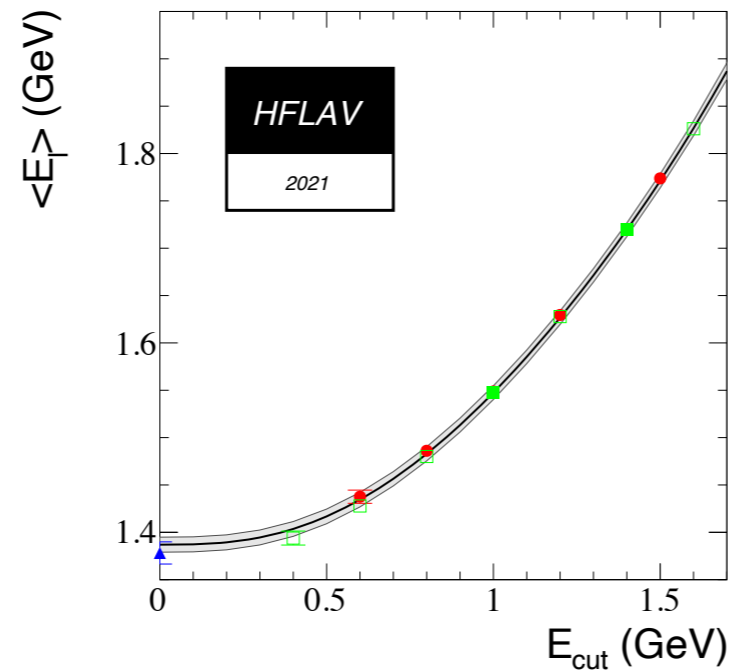
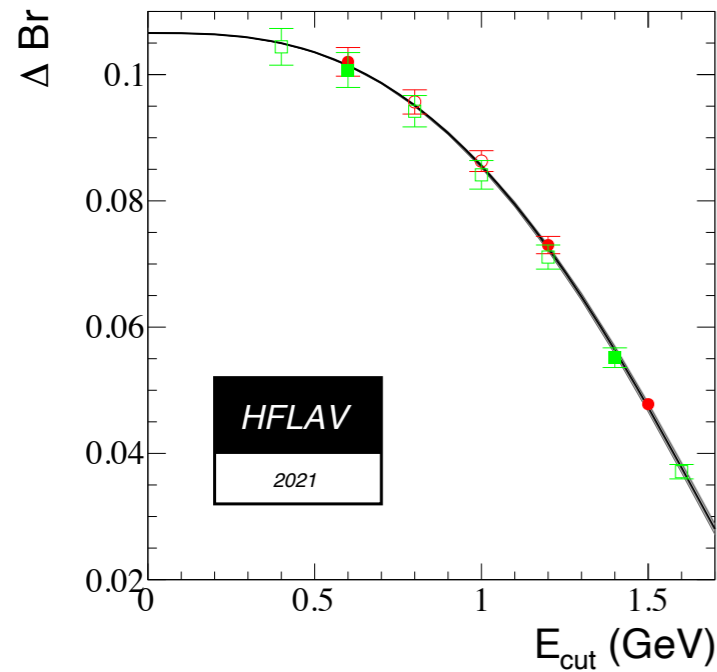
$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho_{LS}} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

- $d\Gamma_i$ are computed **perturbatively**
- The non-perturbative dynamics is enclosed into the HQE parameters: $\mu_\pi, \mu_G, \rho_D, \rho_{LS} \sim \langle B | \bar{b}_\nu iD^\mu \dots iD^\nu \Gamma_{\mu\dots\nu} b_\nu | B \rangle$
- HQE parameters are **extracted from data**.

Experiment	Hadron moments $\langle M^n_X \rangle$	Lepton moments $\langle E^n_l \rangle$	References
BaBar	n=2 c=0.9,1.1,1.3,1.5 n=4 c=0.8,1.0,1.2,1.4 n=6 c=0.9,1.3 [1]	n=0 c=0.6,1.2,1.5 n=1 c=0.6,0.8,1.0,1.2,1.5 n=2 c=0.6,1.0,1.5 n=3 c=0.8,1.2 [1,2]	[1] Phys.Rev. D81 (2010) 032003 [2] Phys.Rev. D69 (2004) 111104
Belle	n=2 c=0.7,1.1,1.3,1.5 n=4 c=0.7,0.9,1.3 [3]	n=0 c=0.6,1.4 n=1 c=1.0,1.4 n=2 c=0.6,1.4 n=3 c=0.8,1.2 [4]	[3] Phys.Rev. D75 (2007) 032005 [4] Phys.Rev. D75 (2007) 032001
CDF	n=2 c=0.7 n=4 c=0.7 [5]	.	[5] Phys.Rev. D71 (2005) 051103
CLEO	n=2 c=1.0,1.5 n=4 c=1.0,1.5 [6]	.	[6] Phys.Rev. D70 (2004) 032002
DELPHI	n=2 c=0.0 n=4 c=0.0 n=6 c=0.0 [7]	n=1 c=0.0 n=2 c=0.0 n=3 c=0.0 [7]	[7] Eur.Phys.J. C45 (2006) 35-59

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$

Spectral Moment Fit from HFLAV (Kinetic scheme)



χ^2 -fit of the spectral moments, which **includes theory uncertainties and correlations** based on a fixed correlation model

Constrains $m_c^{1S} = 0.986 \pm 0.013$ GeV

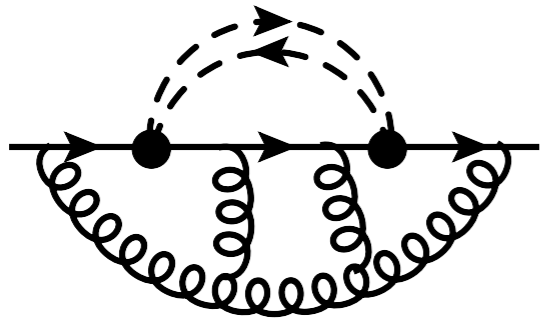
(Phys. Rev. D80 074010, 2009)

$\text{Br}(B \rightarrow X_c l \nu)$ (%)	$ V_{cb} $ (10^{-3})
10.65 \pm 0.16	42.19 \pm 0.78

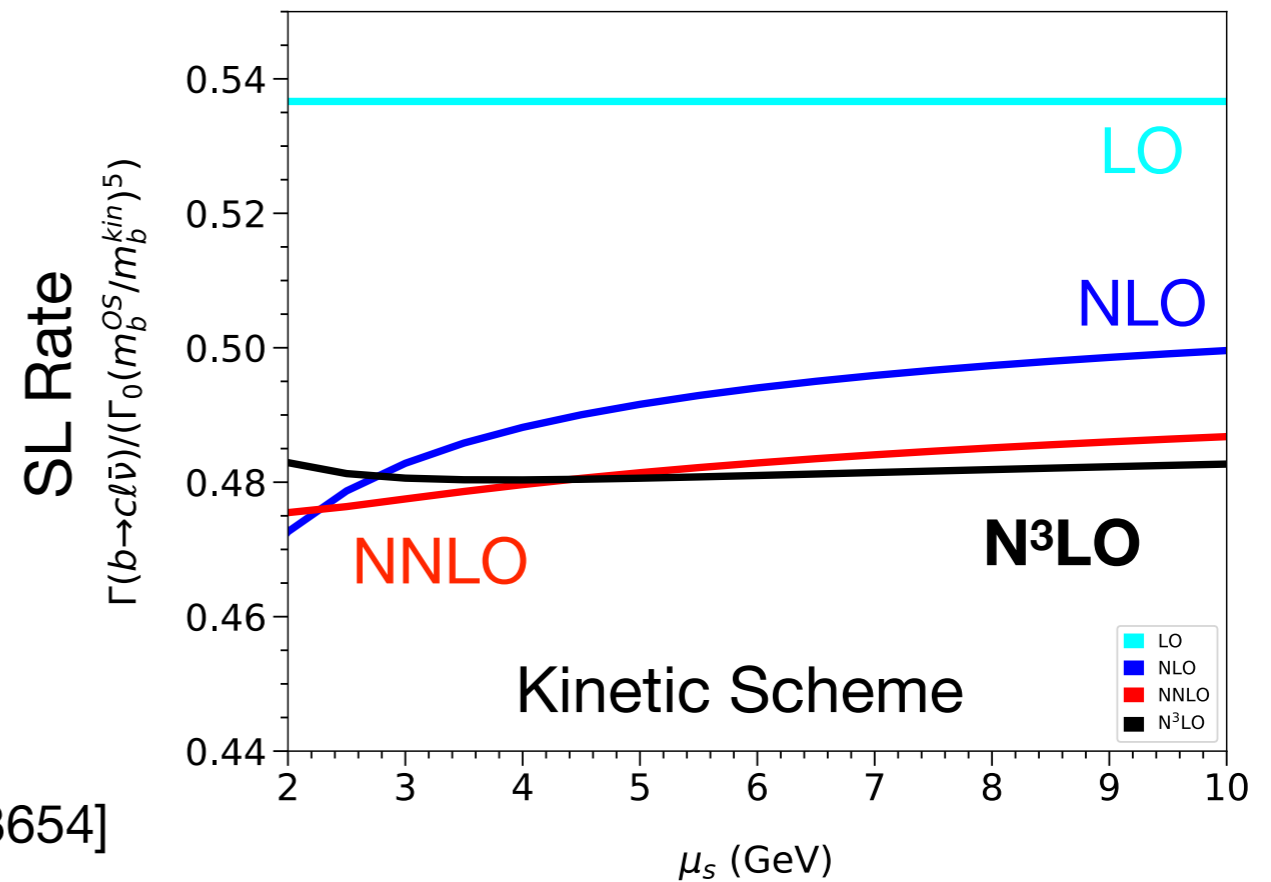
m_b^{kin} (GeV)	μ_{pi}^2 (GeV ²)
4.554 \pm 0.018	0.464 \pm 0.076

State-of-the-art

Fantastic progress on the theory side:
semileptonic rate @ N³LO!



M. Fael, K. Schönwald, M. Steinhauser
[Phys.Rev.D 104 (2021) 1, 016003, arXiv:2011.13654]



Renormalization scale

Updated inclusive fit to $\langle E_\ell \rangle$, $\langle M_X \rangle$ moments:

$$|V_{cb}| = 42.16(30)_{th}(32)_{exp}(25)_\Gamma \cdot 10^{-3}$$

$$\Delta |V_{cb}| / |V_{cb}| = 1.2\%!$$

M. Bordone, B. Capdevila, P. Gambino
[Phys.Lett.B 822 (2021) 136679, arXiv:2107.00604]

m_b^{kin}	$\bar{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{cl\nu}$	$10^3 V_{cb} $
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51
1	0.307	-0.141	0.047	0.612	-0.196	-0.064	-0.420
	1	0.018	-0.010	-0.162	0.048	0.028	0.061
		1	0.735	-0.054	0.067	0.172	0.429
			1	-0.157	-0.149	0.091	0.299
				1	0.001	0.013	-0.225
					1	-0.033	-0.005
						1	0.684

See also [Phys.Lett.B 829 (2022) 137068, 2202.01434] for very recent 1S fit finding $|V_{cb}| = (42.5 \pm 1.1) \times 10^{-3}$

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho LS} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$



Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472]
(M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting **reparametrization invariance**, but **not true for every observable**

Spectral moments :

$$\langle M^n[w] \rangle = \int d\Phi w^n(v, p_\ell, p_\nu) W^{\mu\nu} L_{\mu\nu}$$

$v = p_B/m_B$

- $w = (m_B v - q)^2 \Rightarrow \langle M_X^n \rangle$ Moments not RPI (depends on v)
- $w = v \cdot p_\ell \Rightarrow \langle E_\ell^n \rangle$ Moments not RPI (depends on v)
- $w = q^2 \Rightarrow \langle (q^2)^n \rangle$ Moments RPI! (does not depend on v)

$$d\Gamma = d\Gamma_0 + d\Gamma_{\mu\pi} \frac{\mu_\pi^2}{m_b^2} + d\Gamma_{\mu G} \frac{\mu_G^2}{m_b^2} + d\Gamma_{\rho D} \frac{\rho_D^3}{m_b^3} + d\Gamma_{\rho LS} \frac{\rho_{LS}^3}{m_b^3} + \dots$$

Bad news: number of these matrix elements increases if one increases expansion in $1/m_{b,c}$



Innovative idea from [JHEP 02 (2019) 177, arXiv:1812.07472]
(M. Fael, T. Mannel, K. Vos)

→ Number of ME reduce by exploiting **reparametrization invariance**, but **not true for every observable**

Measurements of q^2 **moments** of **inclusive** $B \rightarrow X_c \ell \bar{\nu}_\ell$ decays with hadronic tagging [PRD 104, 112011 (2021), arXiv:2109.01685]

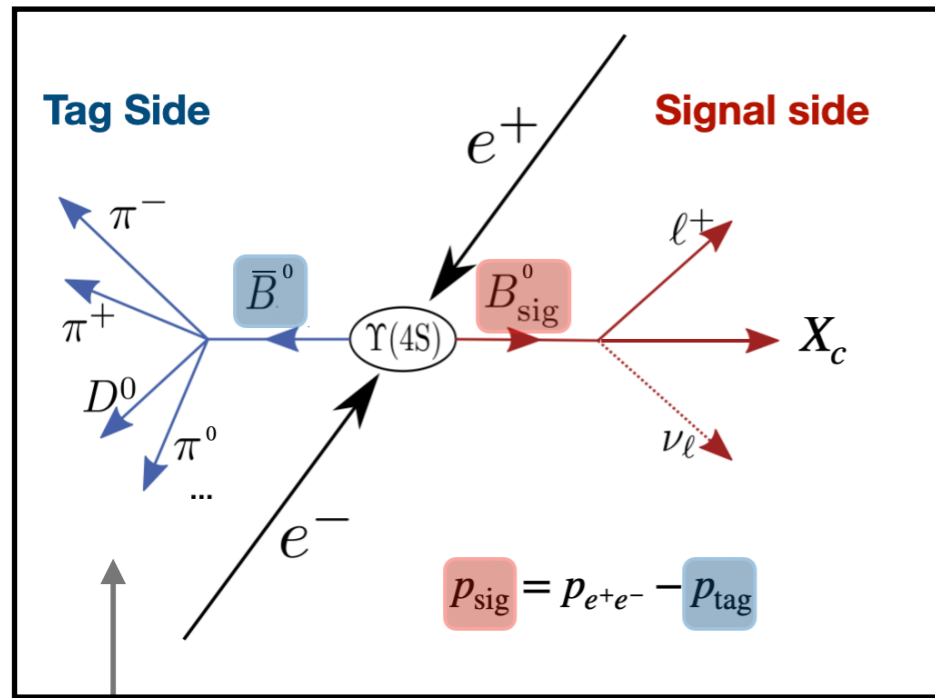


Measurements of Lepton **Mass squared moments** in **inclusive** $B \rightarrow X_c \ell \bar{\nu}_\ell$ Decays with the Belle II Experiment
[Under review by PRD, arXiv:2205.06372]



How to measure spectral moments

Key-technique: hadronic tagging



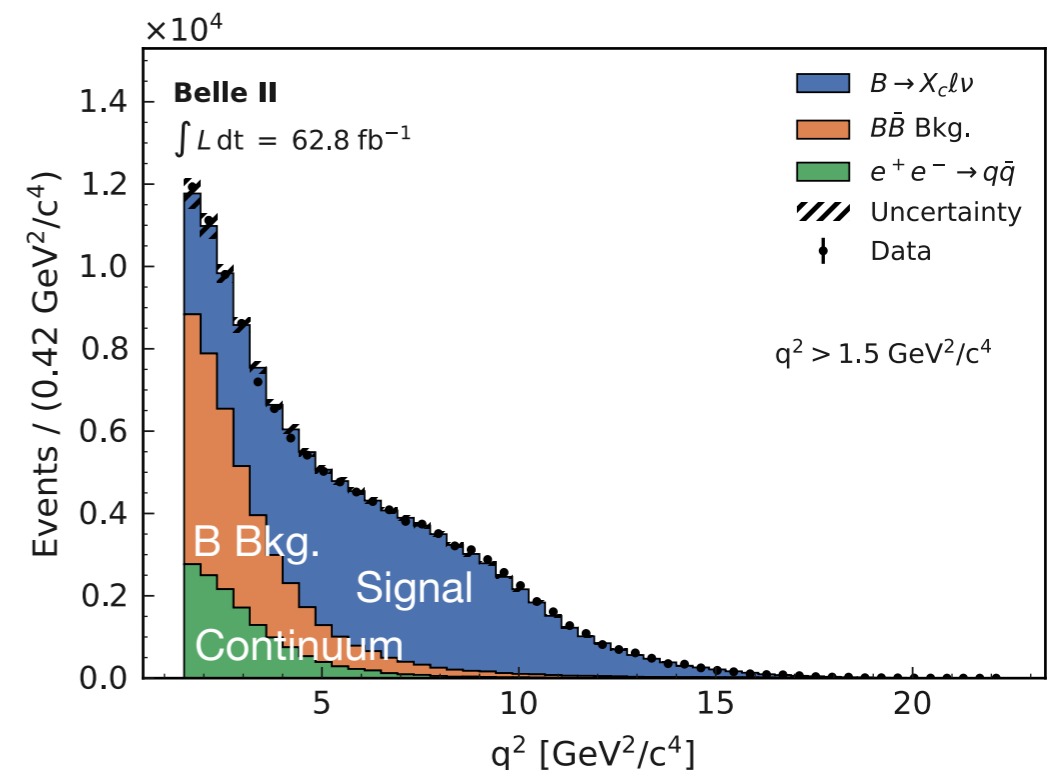
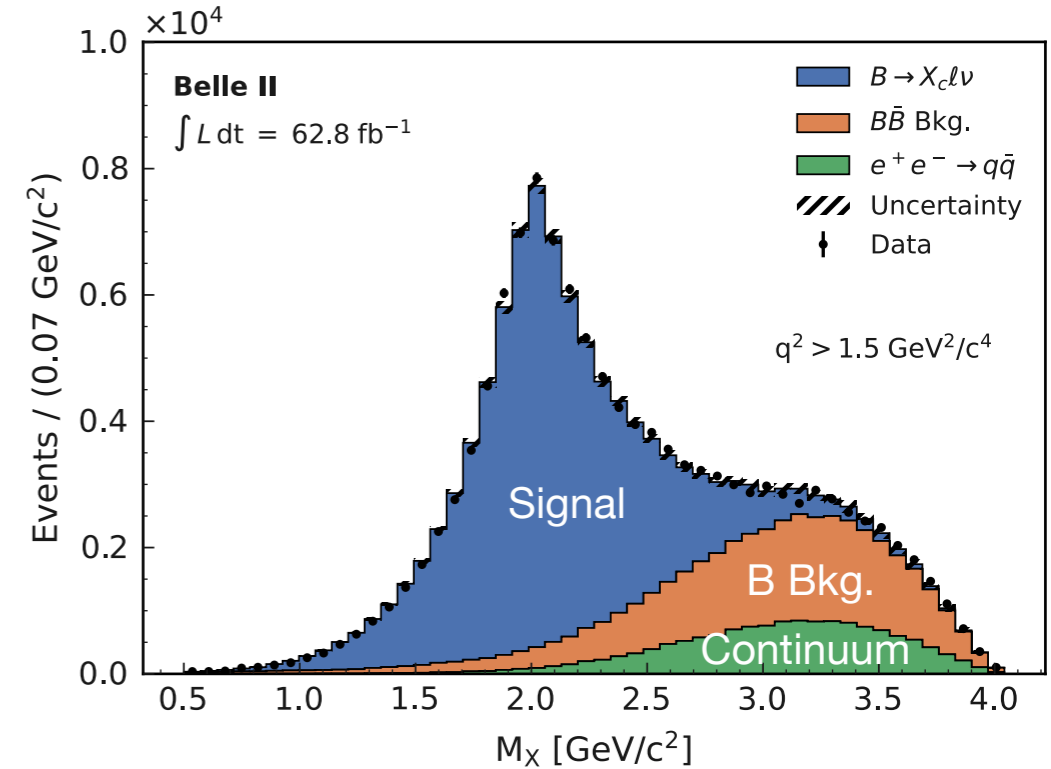
Can identify X_c constituents

$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$

$$q^2 = (p_{\text{sig}} - p_{X_c})^2$$

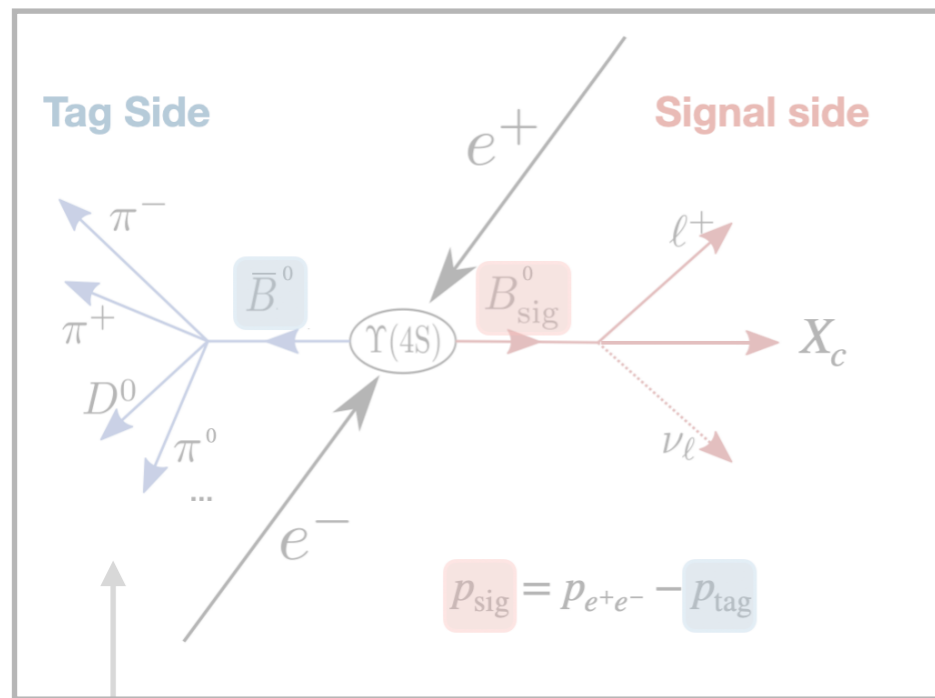
Hadronic Tagging with Belle II algorithm (FEI)

[Full Event Interpretation, T. Keck et al, Comp. Soft. Big. Sci 3 (2019), arXiv:1807.08680]



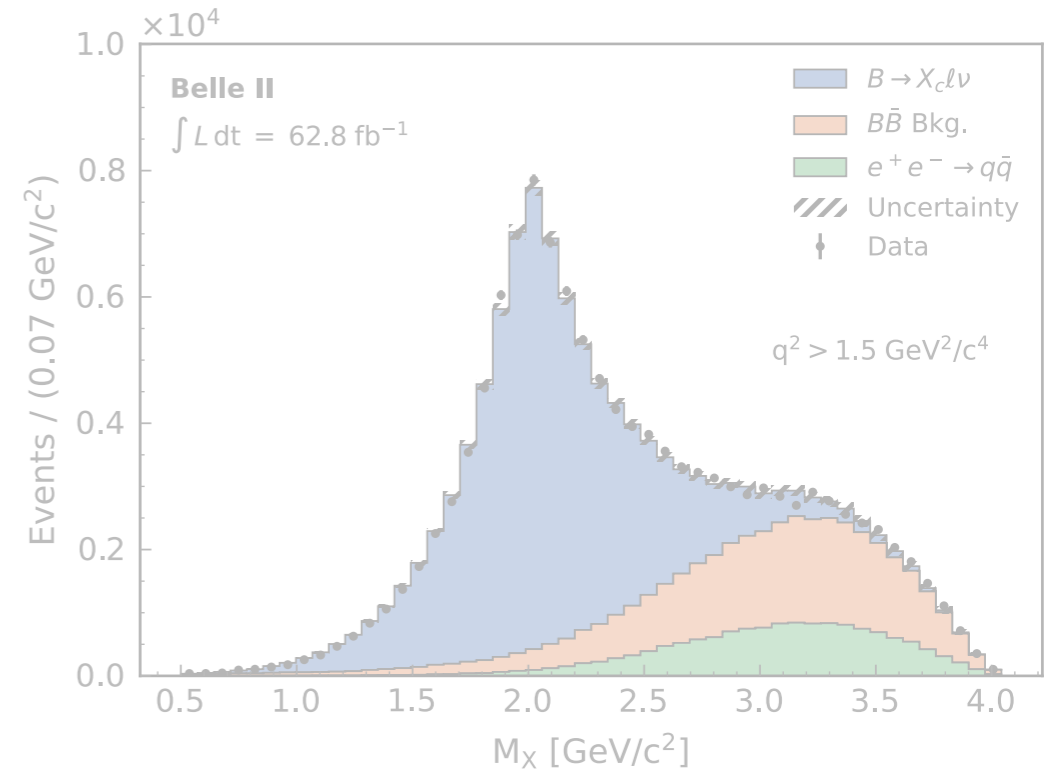
How to measure spectral moments

Key-technique: hadronic tagging



Can identify X_c constituents

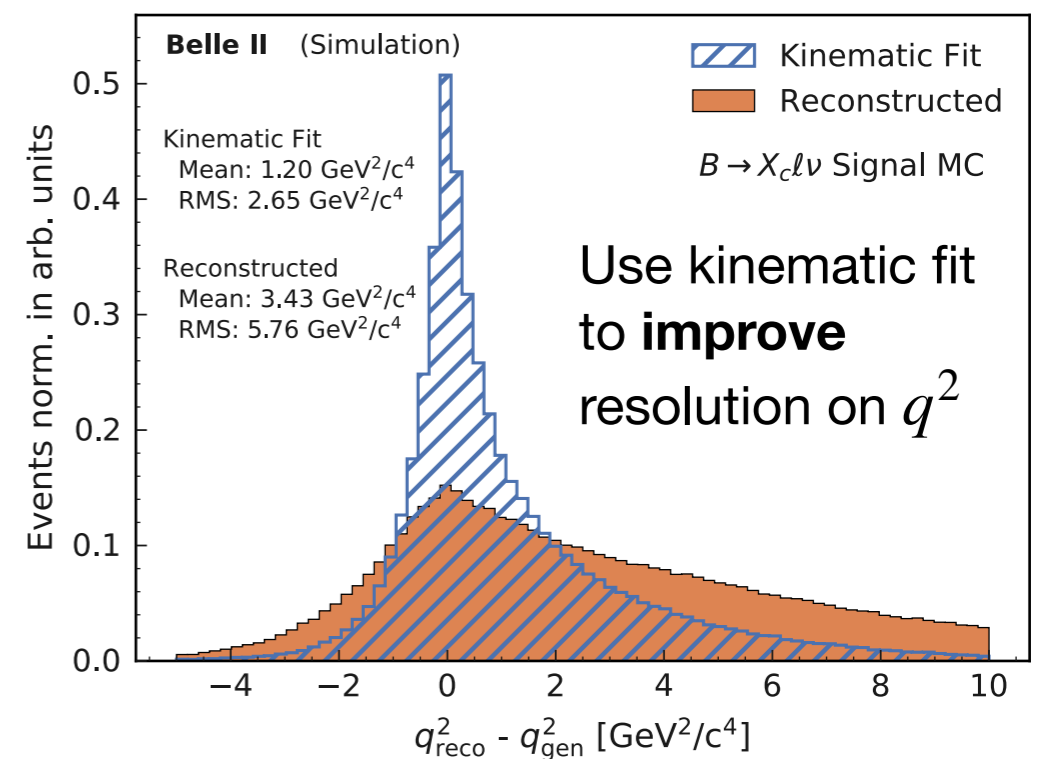
$$M_X = \sqrt{(p_{X_c})_\mu (p_{X_c})^\mu}$$



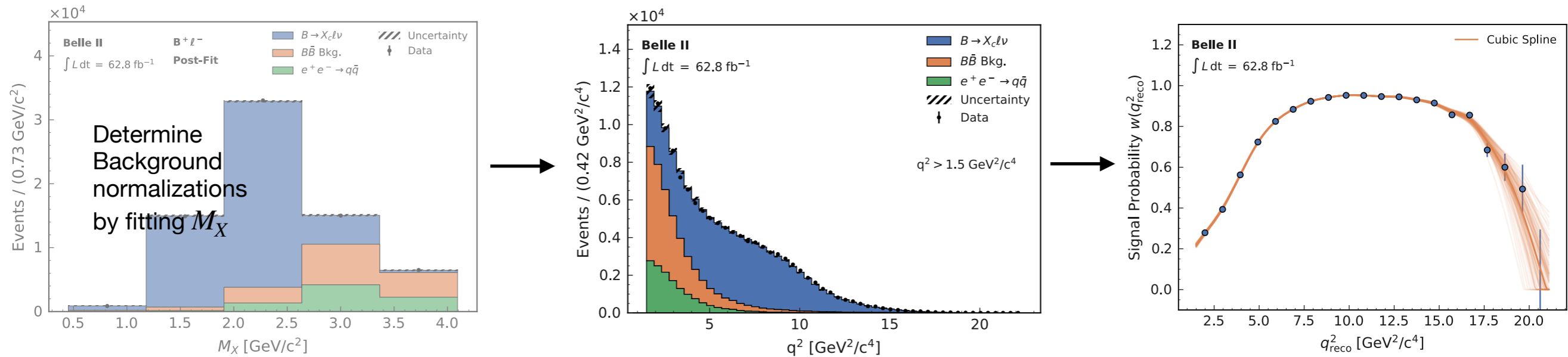
Improved Hadronic Tagging
using Belle II algorithm
(ca. 2 times more efficient)

[Full Event Interpretation, T. Keck et al,
Comp. Soft. Big. Sci 3 (2019),
arXiv:1807.08680]

$$q^2 = (p_{\text{sig}} - p_{X_c})^2$$



Measurement in a nutshell



Step #1: Subtract Background

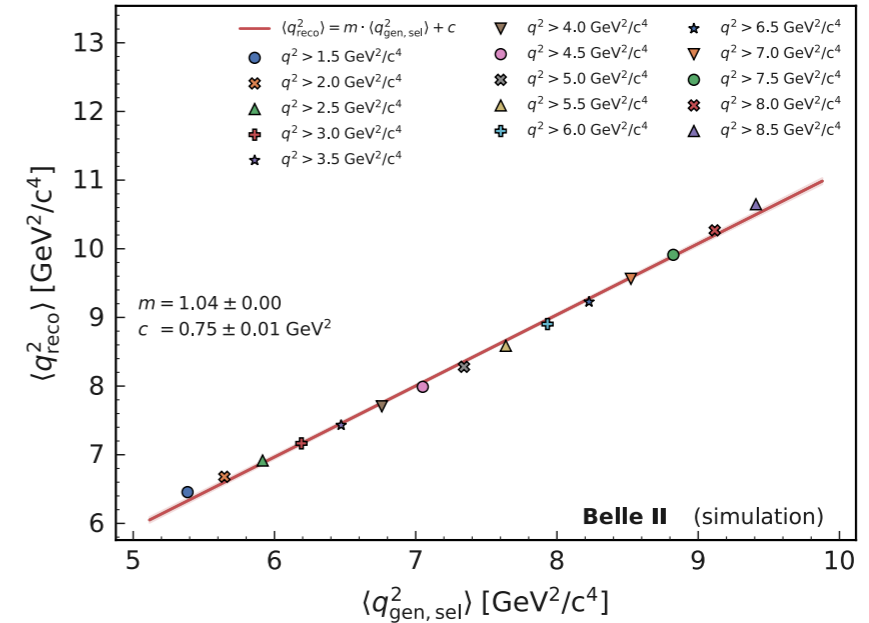
Event-wise Master-formula

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q_{\text{reco},i}^2) \times q_{\text{calib},i}^{2n}}{\sum_j^{N_{\text{data}}} w(q_{\text{reco},j}^2)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}},$$

Measurement in a nutshell

Exploit **linear** dependence
between rec. & true moments

$$q_{\text{cal } i}^{2m} = (q_{\text{reco } i}^{2m} - c) / m$$



Step #1: Subtract Background

Step #2: Calibrate moment

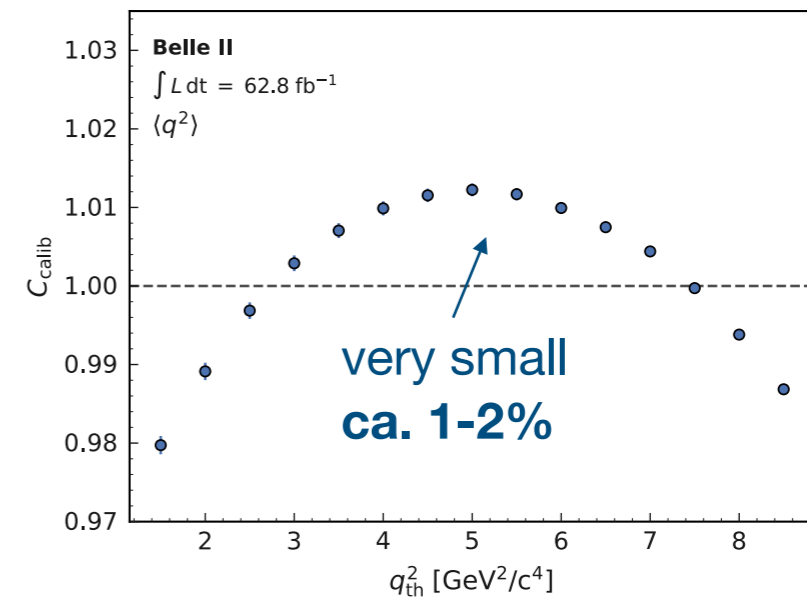
Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q_{\text{reco},i}^2) \times q_{\text{calib},i}^{2n}}{\sum_j^{N_{\text{data}}} w(q_{\text{reco},j}^2)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}},$$

Measurement in a nutshell



Very small deviation from linear behavior between reconstruct and true q^2



Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

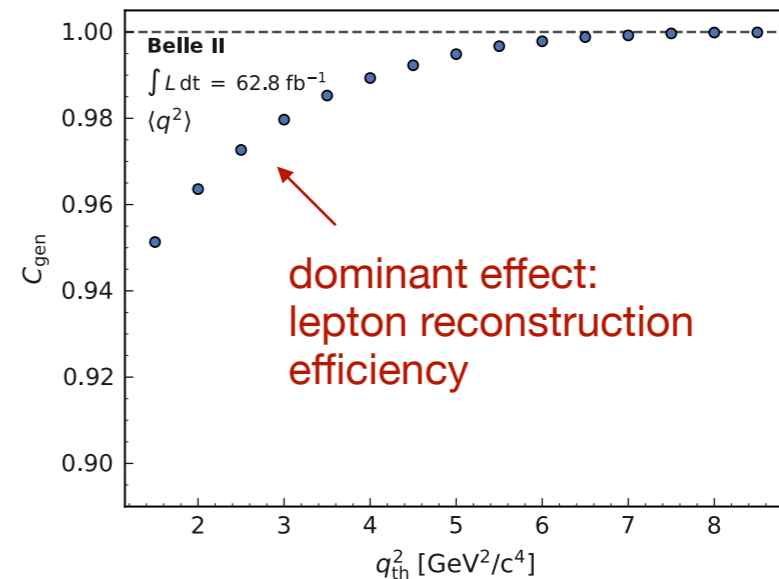
$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q_{\text{reco},i}^2) \times q_{\text{calib},i}^{2n}}{\sum_j^{N_{\text{data}}} w(q_{\text{reco},j}^2)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}},$$

Step #3: If you fail, try again

Measurement in a nutshell



Account for **efficiency & acceptance effects**



Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q_{\text{reco},i}^2) \times q_{\text{calib},i}^{2n}}{\sum_j^{N_{\text{data}}} w(q_{\text{reco},j}^2)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}},$$

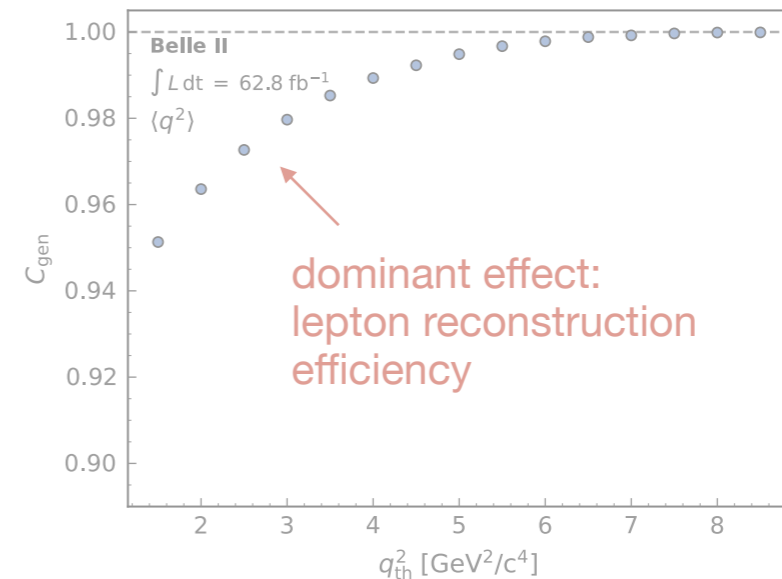
Step #3: If you fail, try again

Step #4: Correct for selection effects

Measurement in a nutshell



Account for **efficiency & acceptance effects**



Step #1: Subtract Background

Step #2: Calibrate moment

Event-wise **Master-formula**

$$\langle q^{2n} \rangle = \frac{\sum_i^{N_{\text{data}}} w(q_{\text{reco},i}^2) \times q_{\text{calib},i}^{2n}}{\sum_j^{N_{\text{data}}} w(q_{\text{reco},j}^2)} \times \mathcal{C}_{\text{calib}} \times \mathcal{C}_{\text{gen}},$$

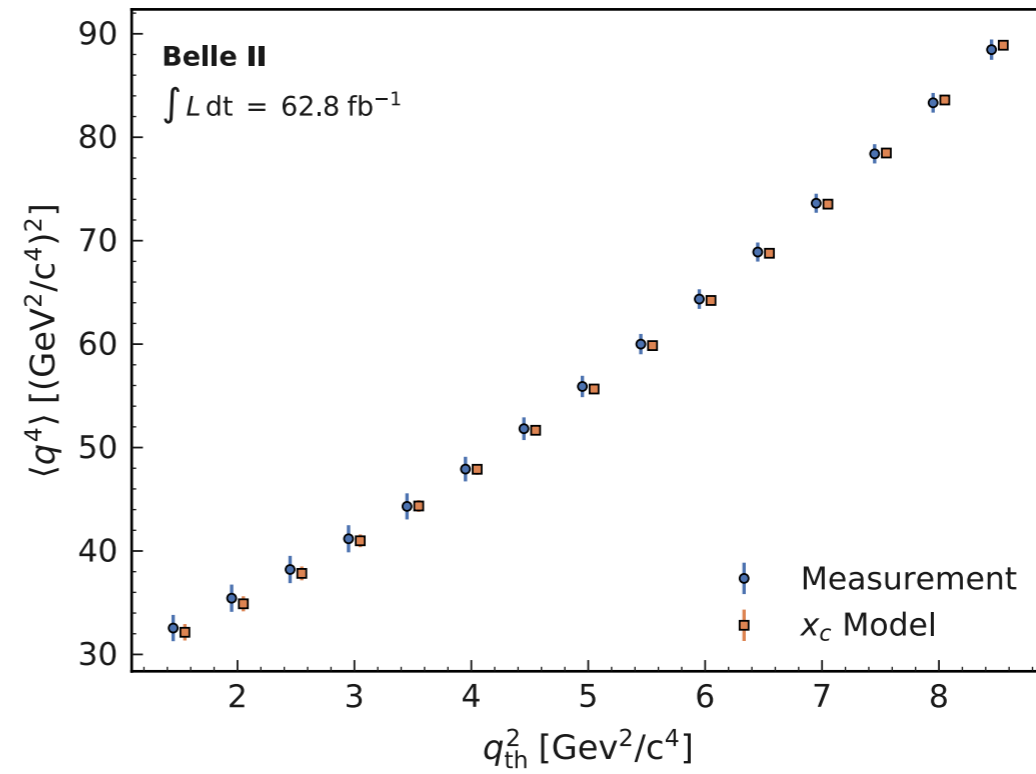
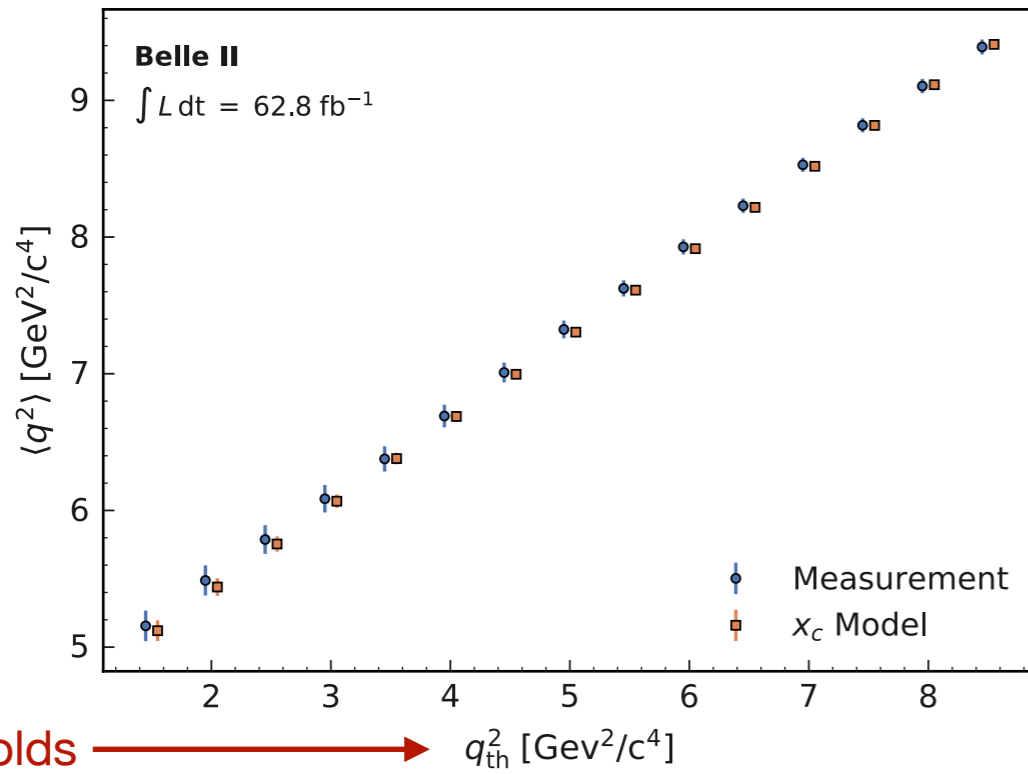
Step #3: If you fail, try again

Step #4: Correct for selection effects

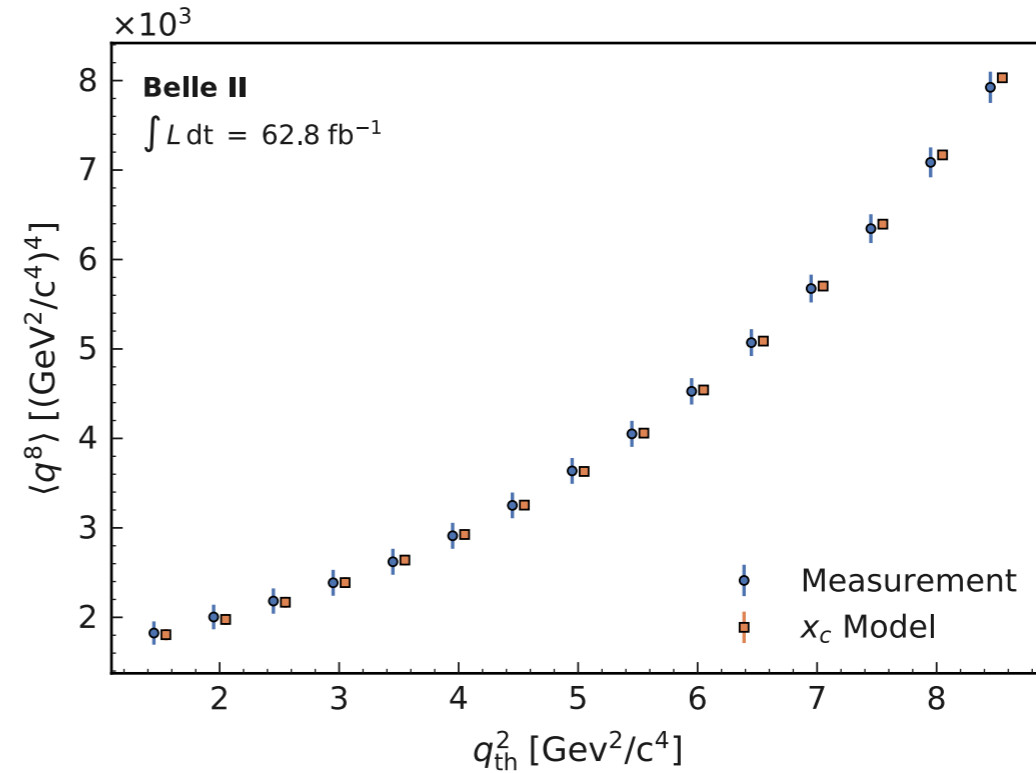
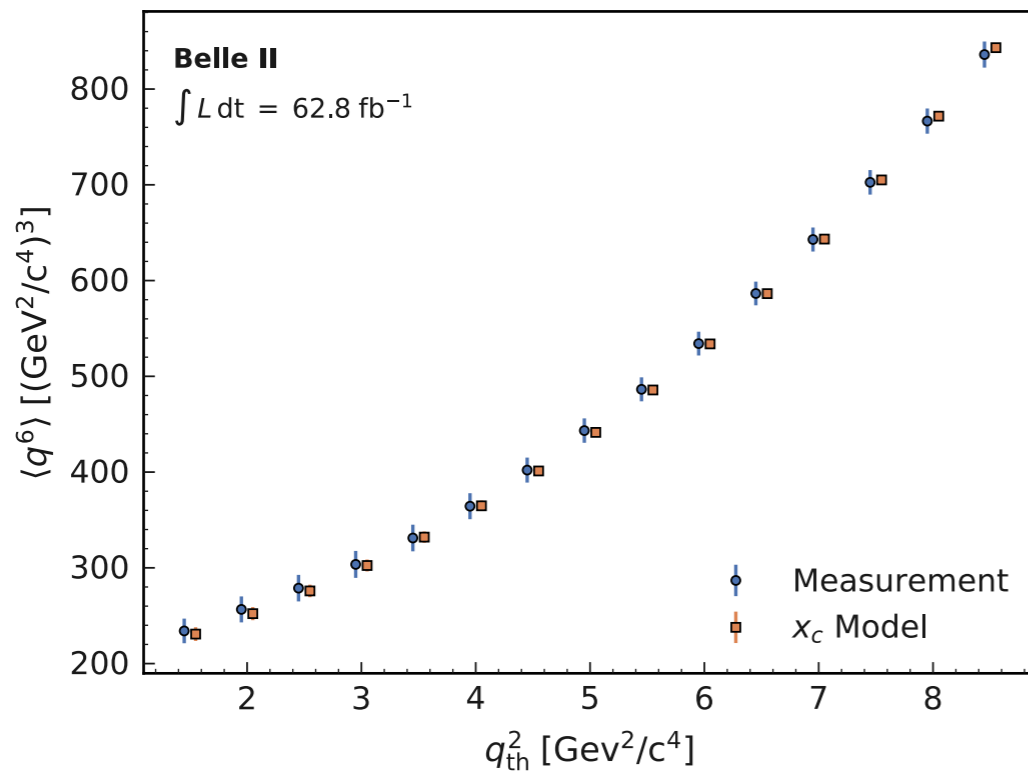


Repeat this for many **different thresholds cuts q_{th}^2**

Belle II q^2 spectral moments

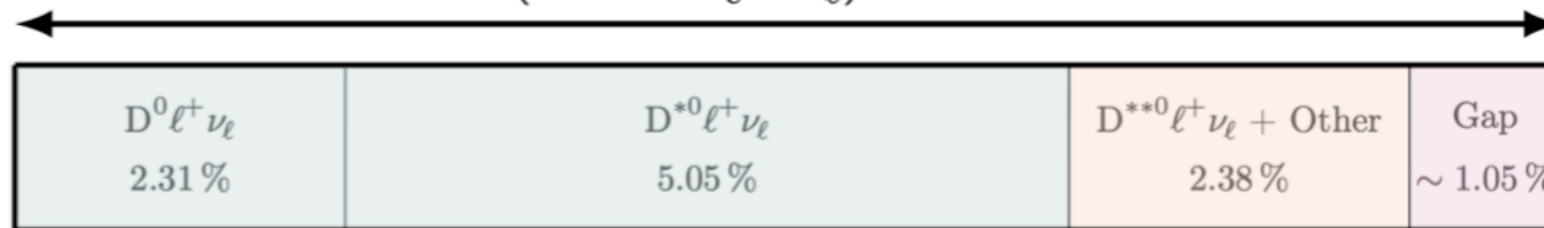



q^2 thresholds \longrightarrow q_{th}^2 [GeV²/c⁴]



Note: Measurements rely on MC

$$\mathcal{B}(B^+ \rightarrow X_c^0 \ell^+ \nu_\ell) \approx 10.79\%$$



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D \pi \pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
		
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Fairly well known.
Some iso-spin tension.

Broad states based on
3 measurements.
(BaBar, Belle, DELPHI)

Some hints from
the BaBar result.

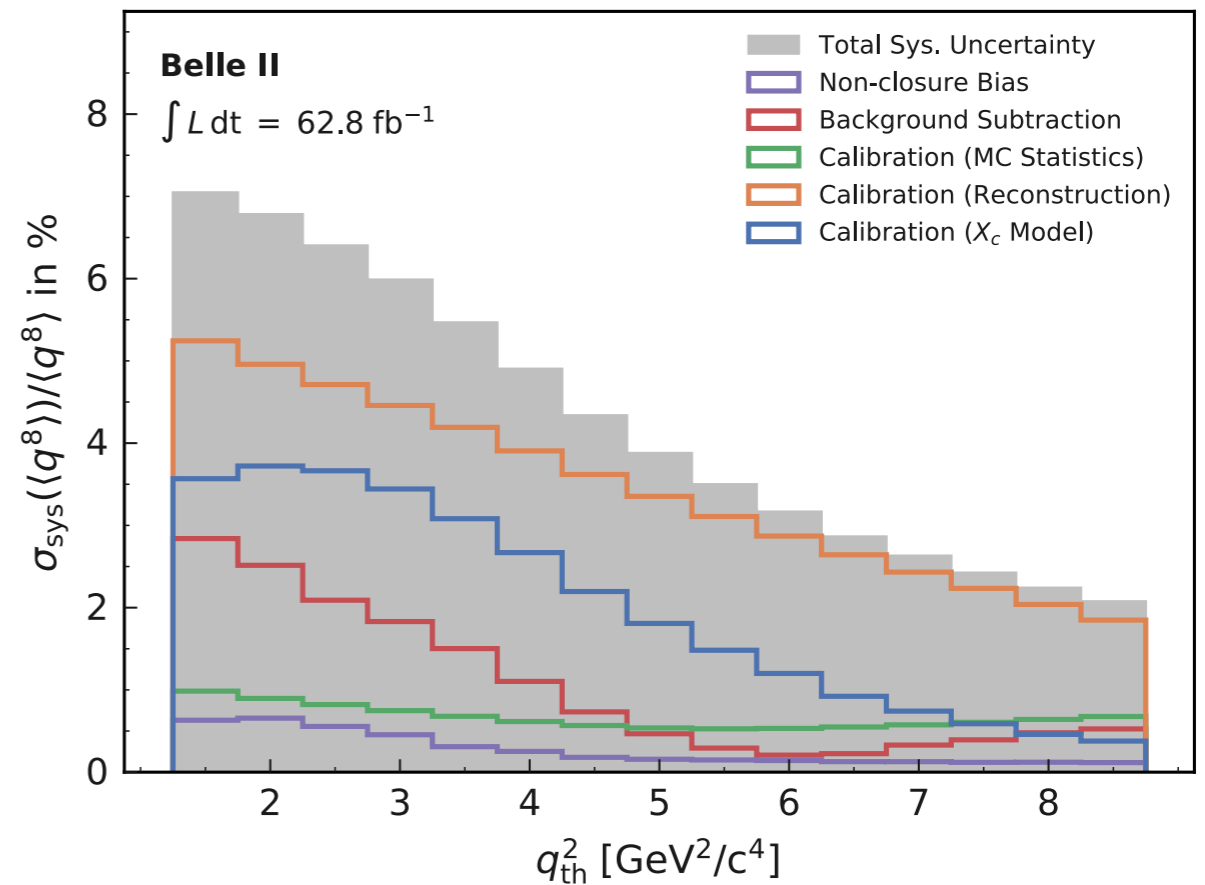
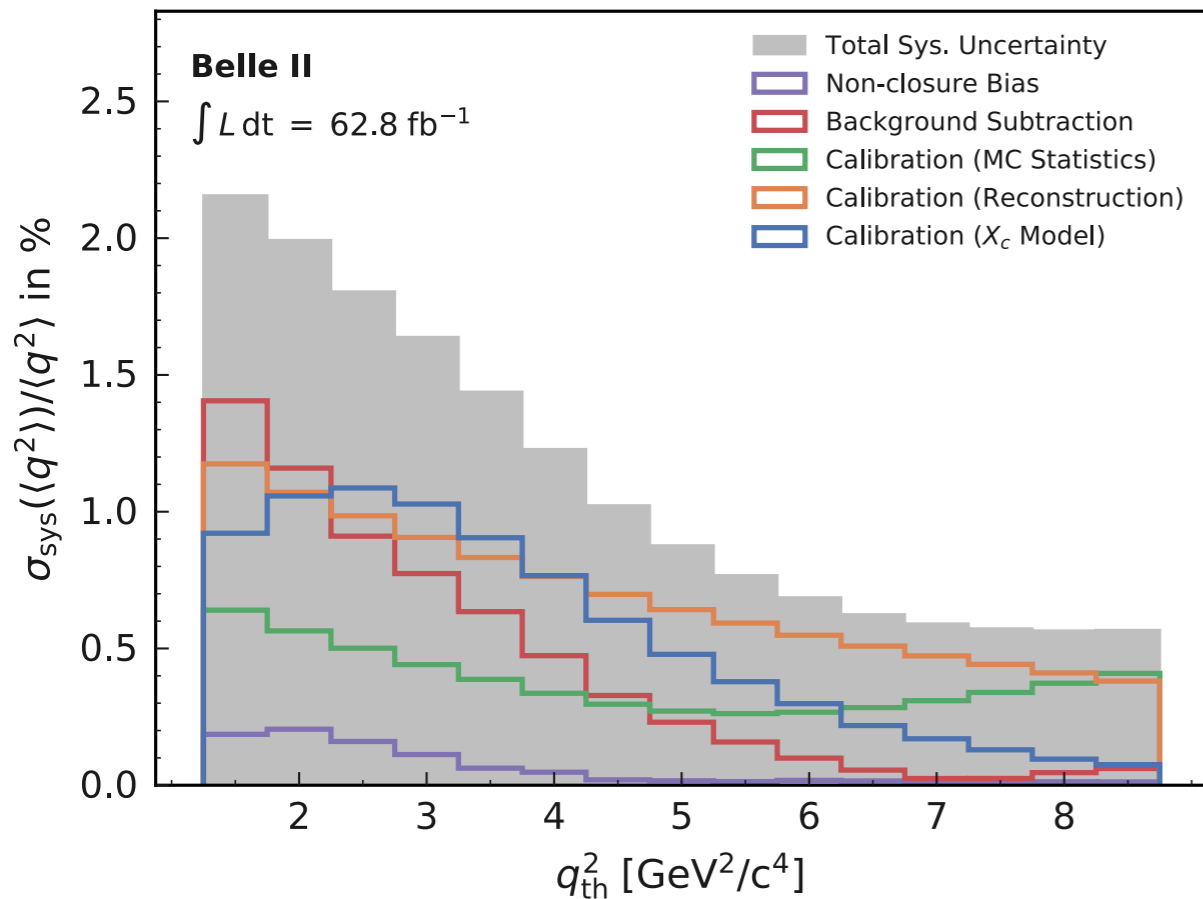
New result from Belle soon

Image credit: F. Metzner

Slide: R. Van Tonder

Belle II q^2 spectral moments

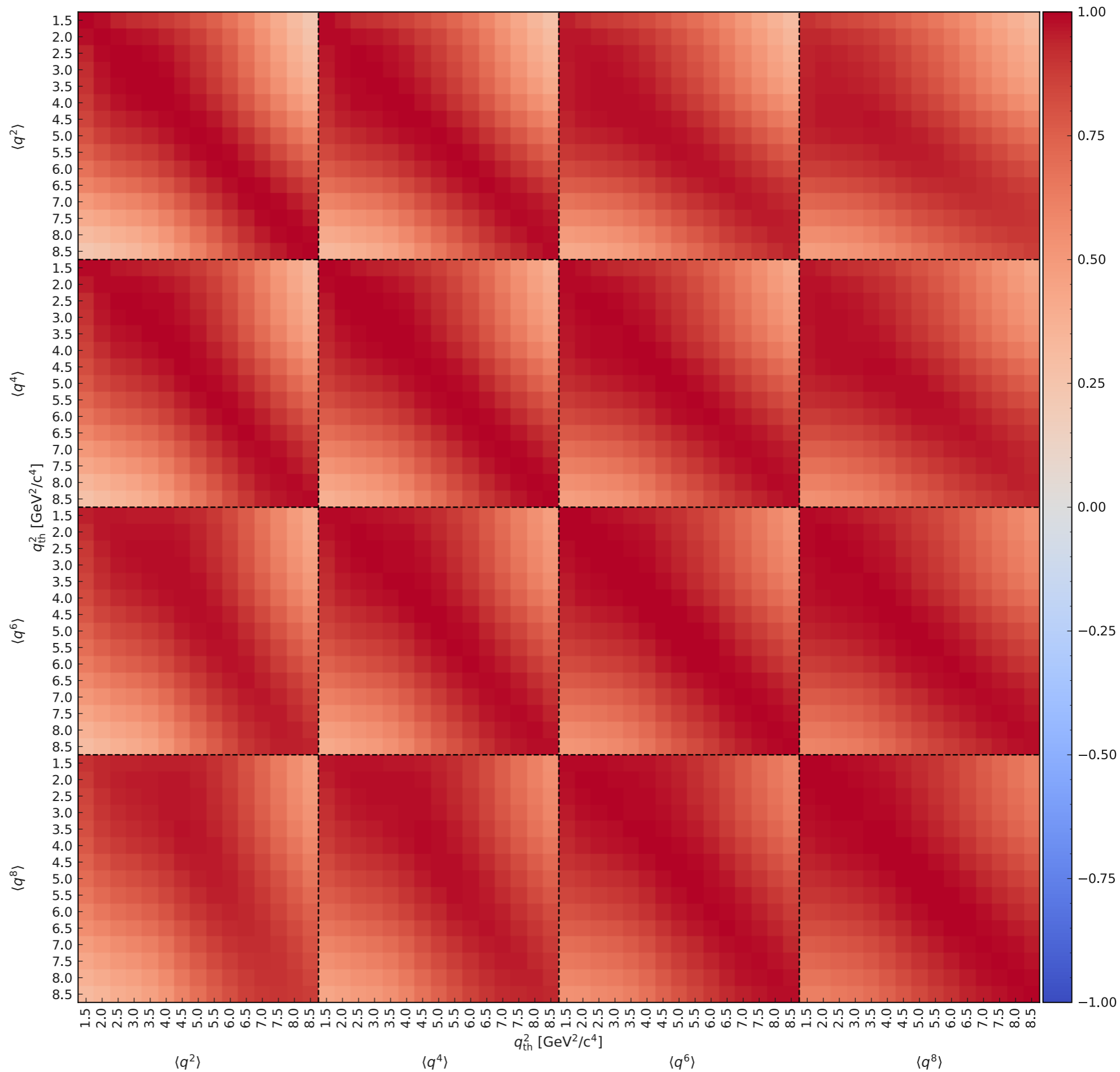
Largest uncertainty from reconstruction, background subtraction, X_c model



Belle II sensitivity similar to Belle already.

**Statistical plus
systematic
correlations**

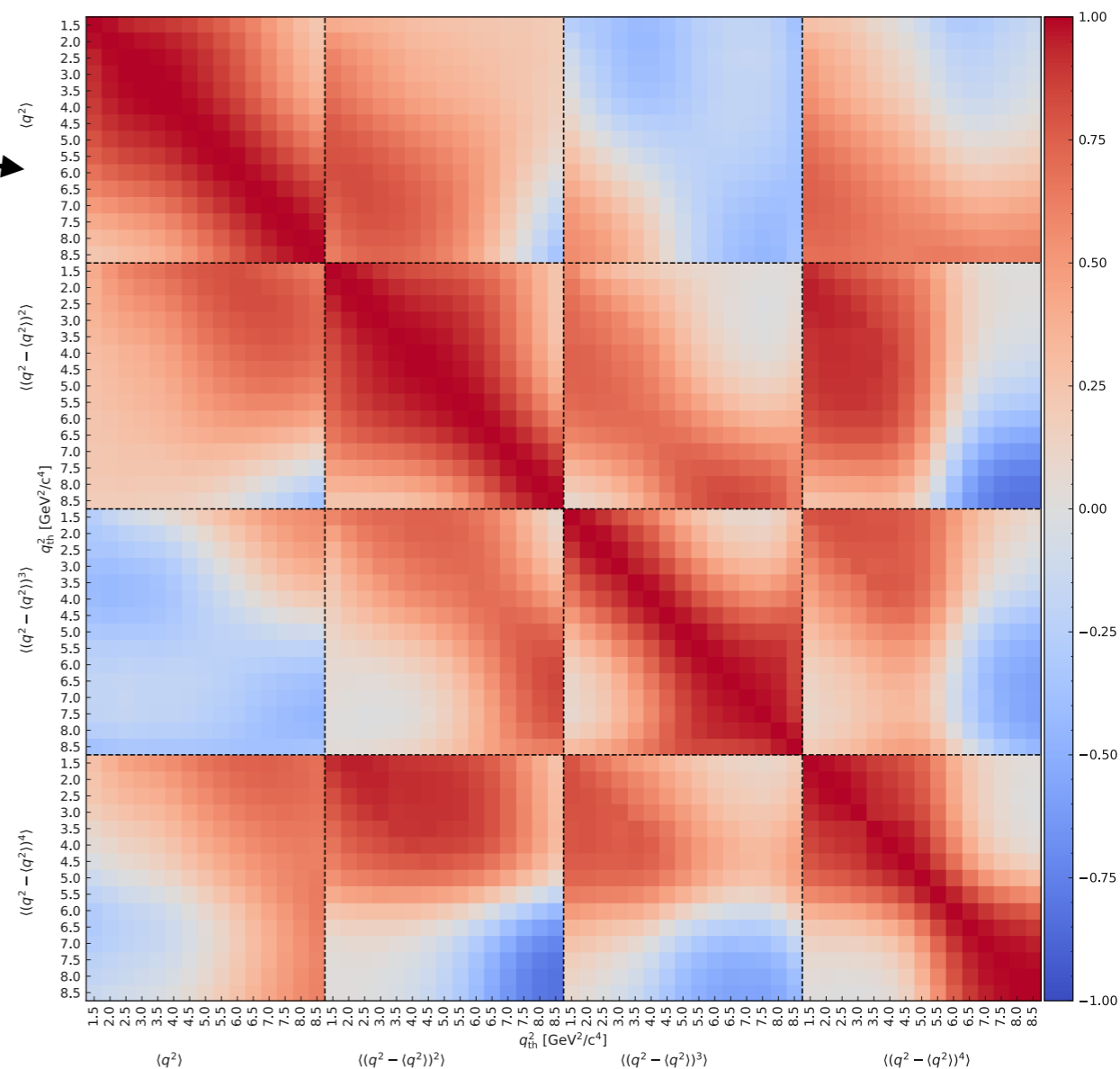
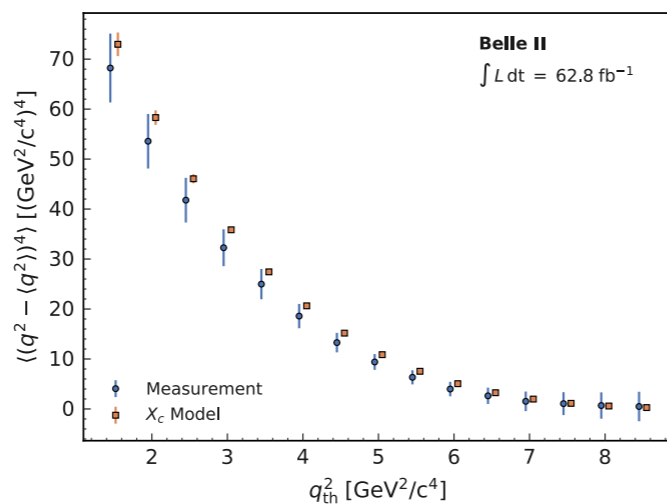
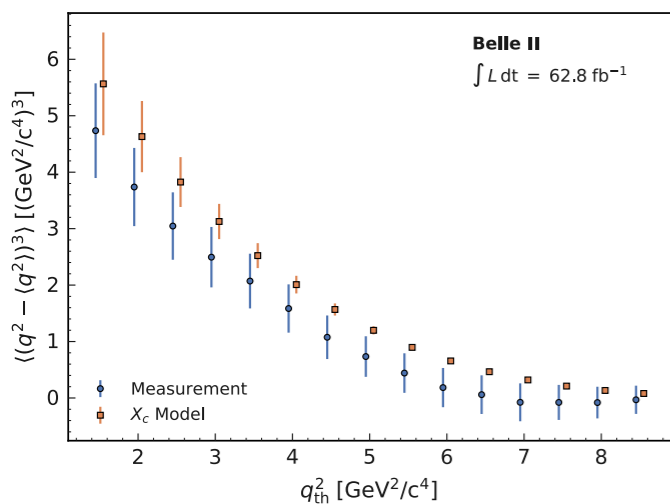
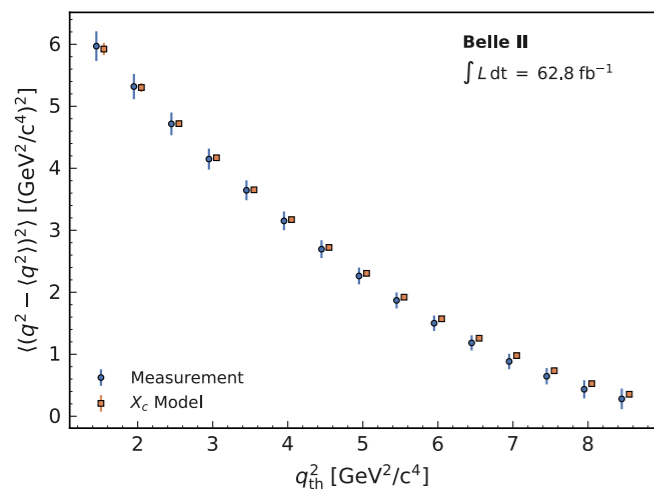
fairly



From moments to *central moments*

Central moments are **less** strongly correlated

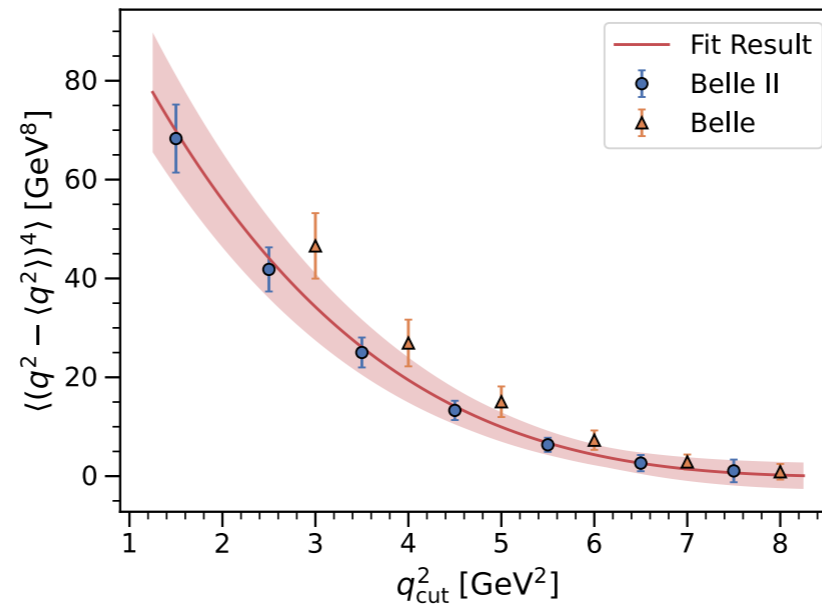
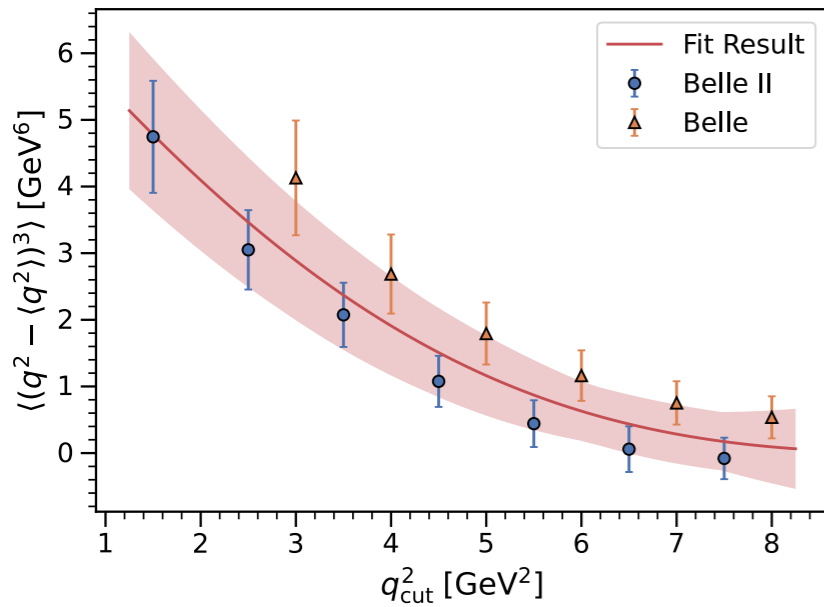
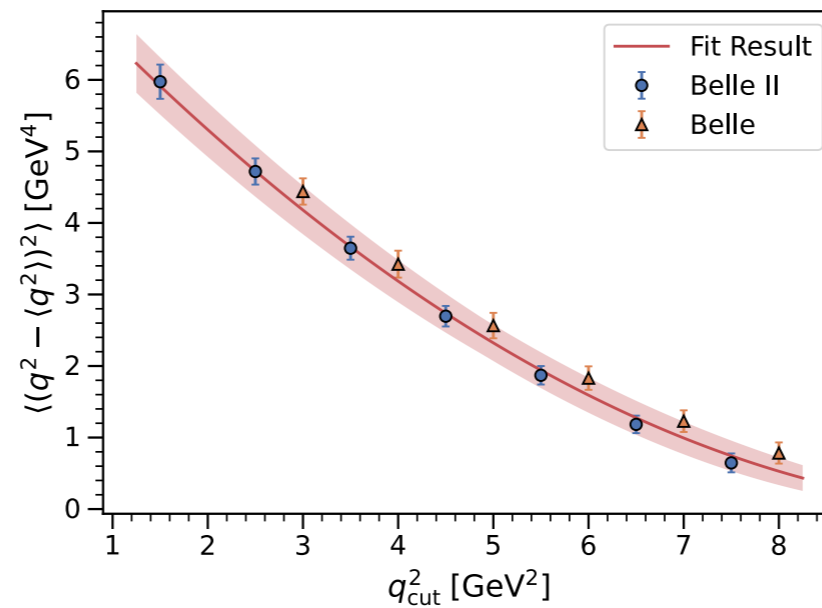
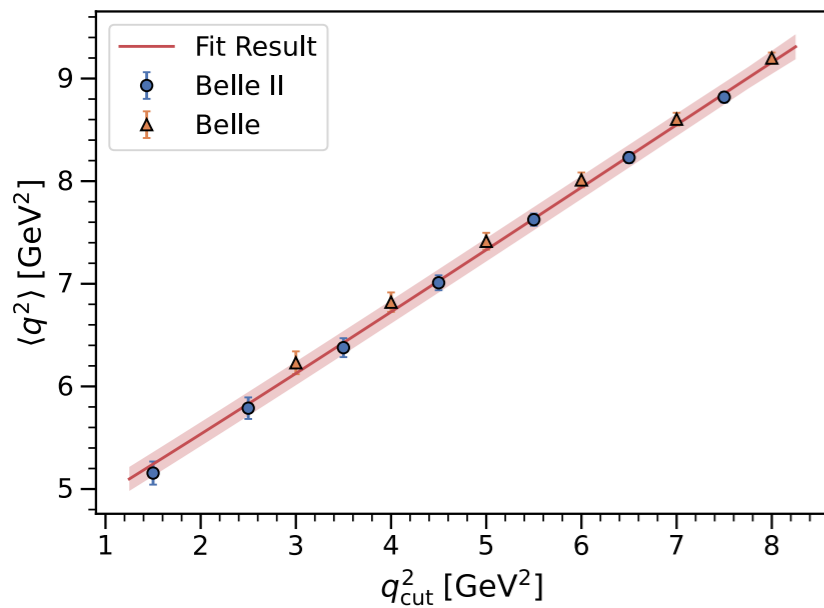
$$\begin{pmatrix} \langle q^2 \rangle \\ \langle q^4 \rangle \\ \langle q^6 \rangle \\ \langle q^8 \rangle \end{pmatrix} \rightarrow \begin{pmatrix} \langle q^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^2 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^3 \rangle \\ \langle (q^2 - \langle q^2 \rangle)^4 \rangle \end{pmatrix}$$



$|V_{cb}|$ from q^2 mom.

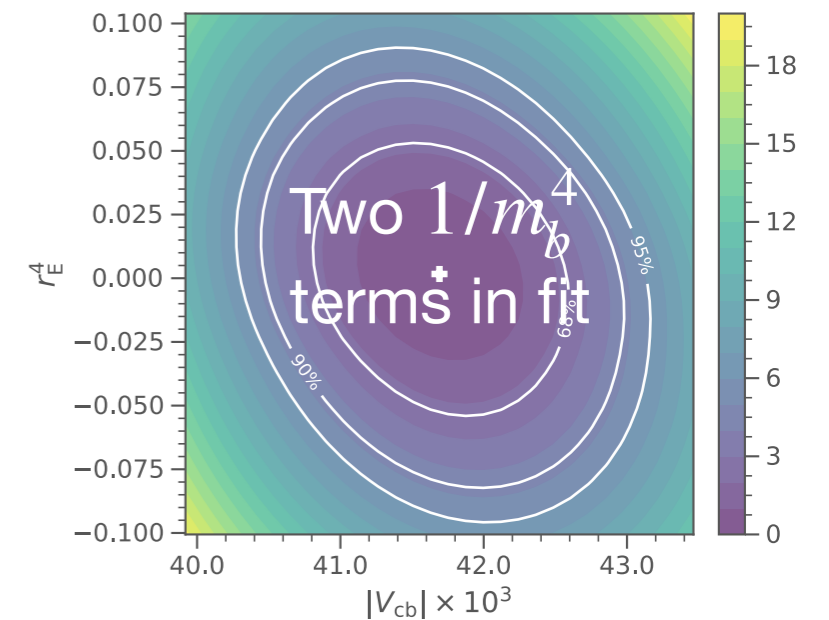
F. Bernlochner, M. Fael, K. Olschwesky, E. Persson,
R. Van Tonder, K. Vos, M. Welsch [arXiv:2205.10274]

Extraction of $|V_{cb}|$ from q^2 moments:



Included corrections
on the mom. predictions

$\langle (q^2)^n \rangle$	tree	α_s	α_s^2	α_s^3
Partonic	✓	✓		
μ_G^2	✓	✓		
ρ_D^3	✓	✓		
$1/m_b^4$	✓			



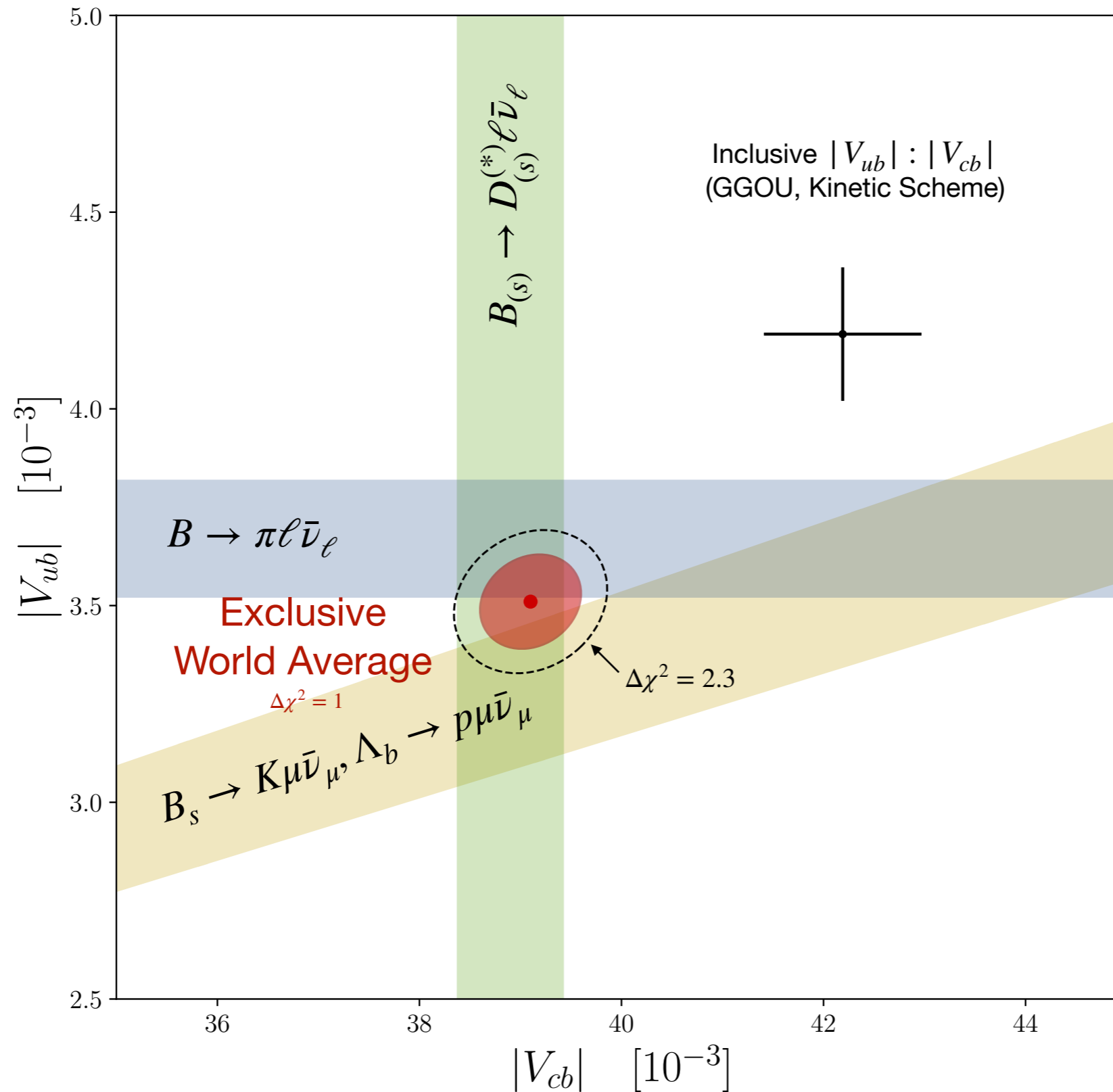
→ $|V_{cb}| = (41.69 \pm 0.59|_{\text{fit}} \pm 0.23|_{\text{h.o.}}) \cdot 10^{-3} = (41.69 \pm 0.63) \cdot 10^{-3}$



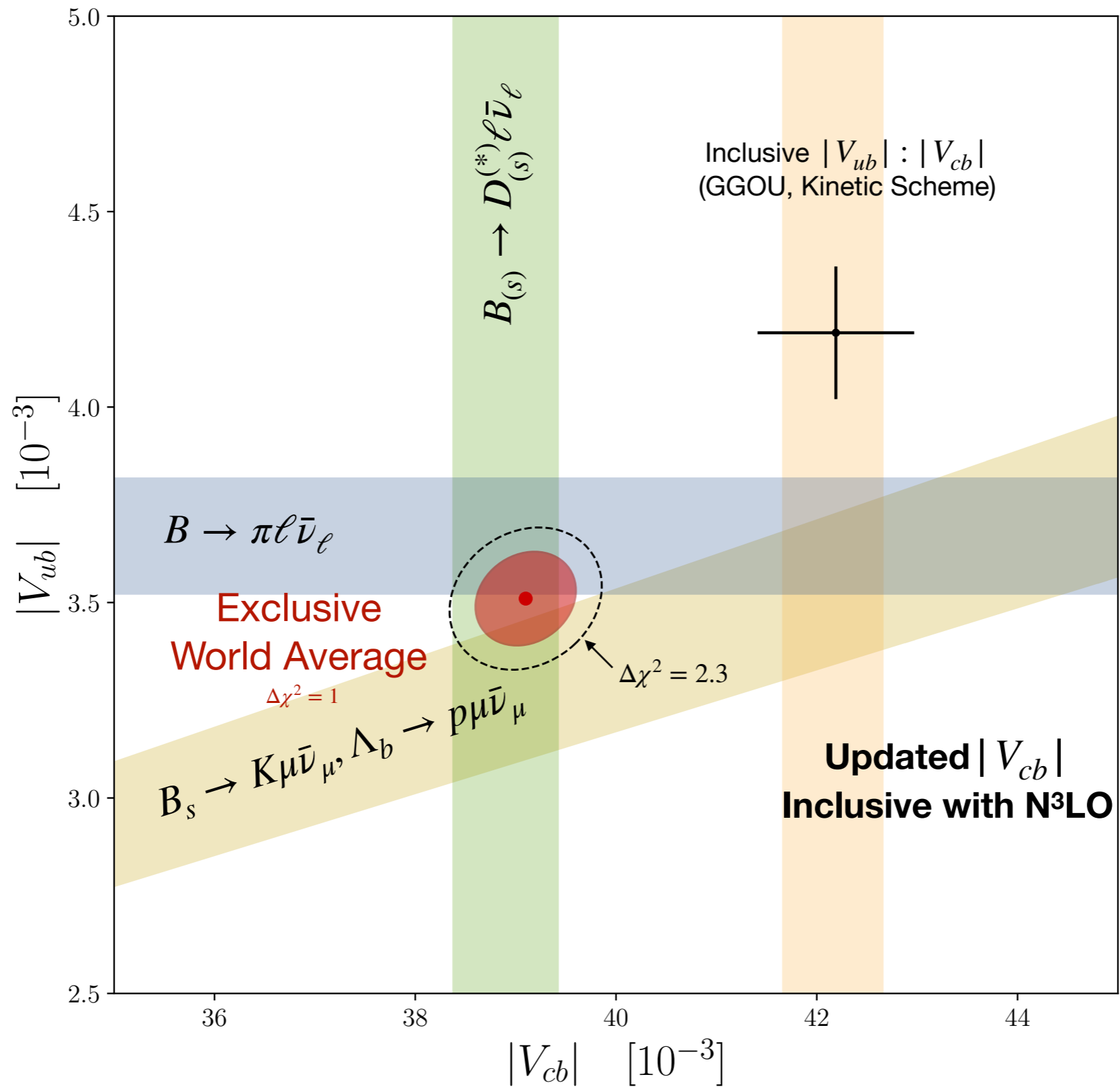
Summary on $|V_{cb}|$

Summary

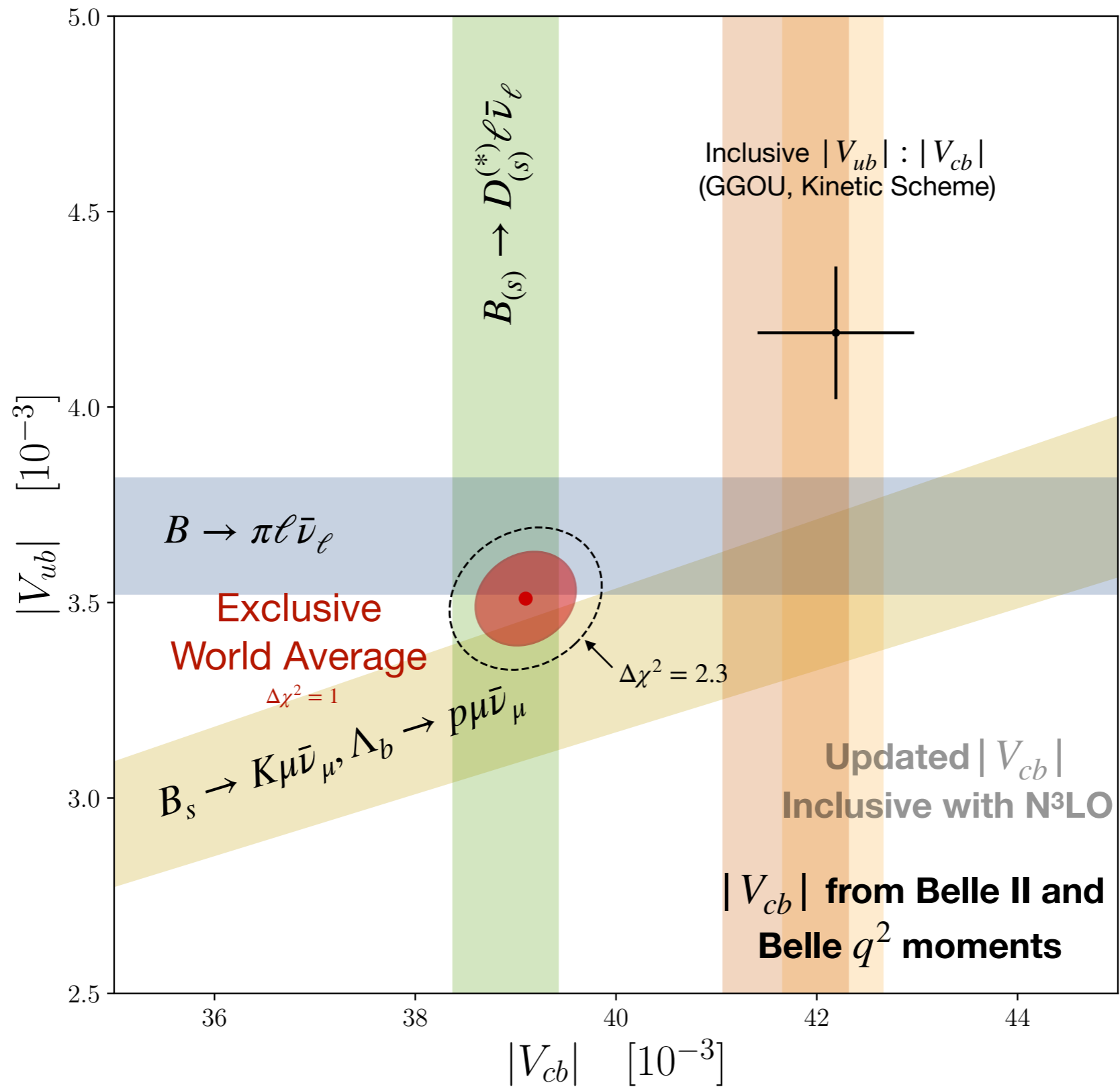
Numbers from new HFLAV 2021 report



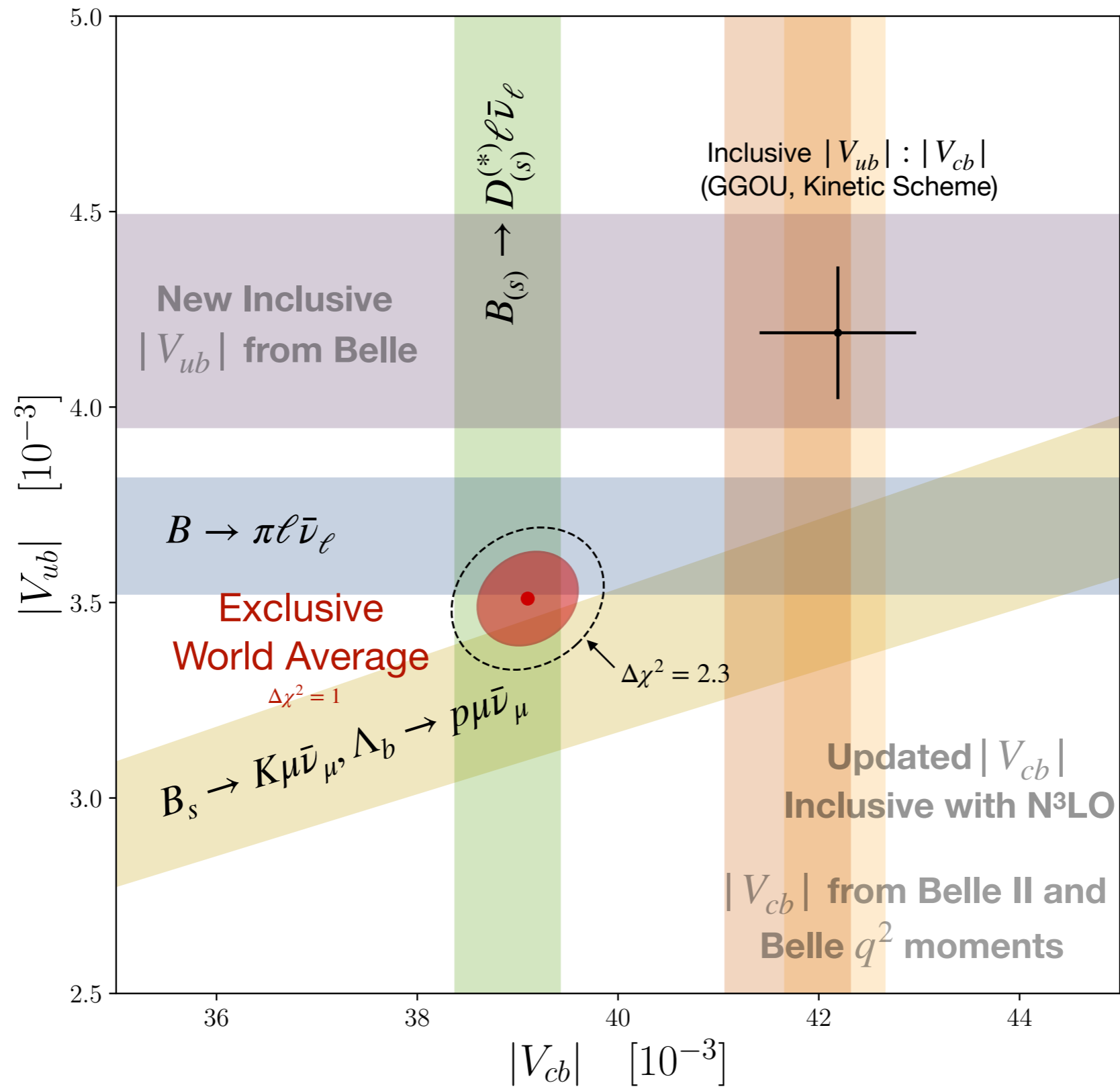
Summary



Summary



Summary





3. τ

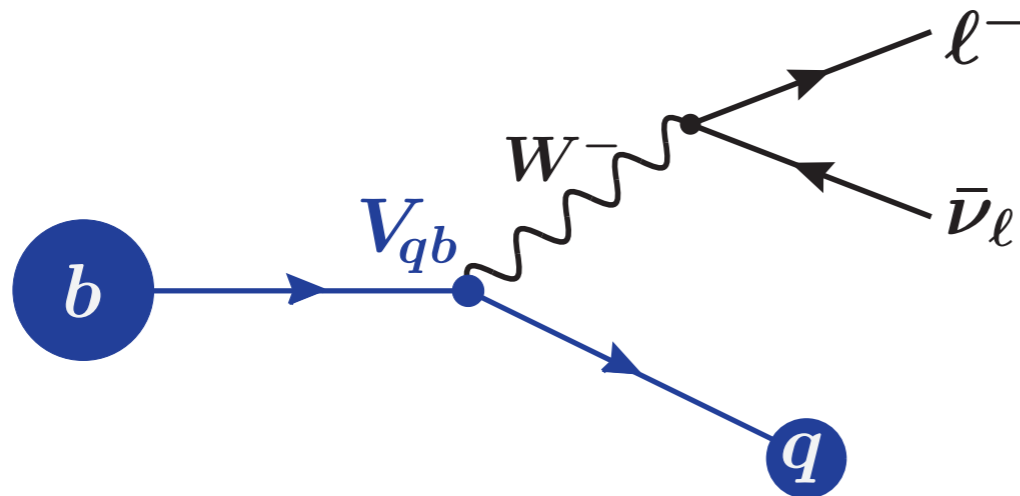
Measurement Strategies

$$R = \frac{\text{Signal } b \rightarrow q \tau \bar{\nu}_\tau}{\text{Normalization } b \rightarrow q \ell \bar{\nu}_\ell}$$

$\ell = e, \mu$

1. Leptonic or Hadronic τ decays?

Some properties (e.g. τ polarization) readily accessible in hadronic decays.



2. Albeit not necessarily a rare decay of O(%) in BF, TRICKY to separate from normalisation and backgrounds

LHCb: Isolation criteria, displacement of τ , kinematics

B-Factories: Full reconstruction of event (Tagging), matching topology, kinematics

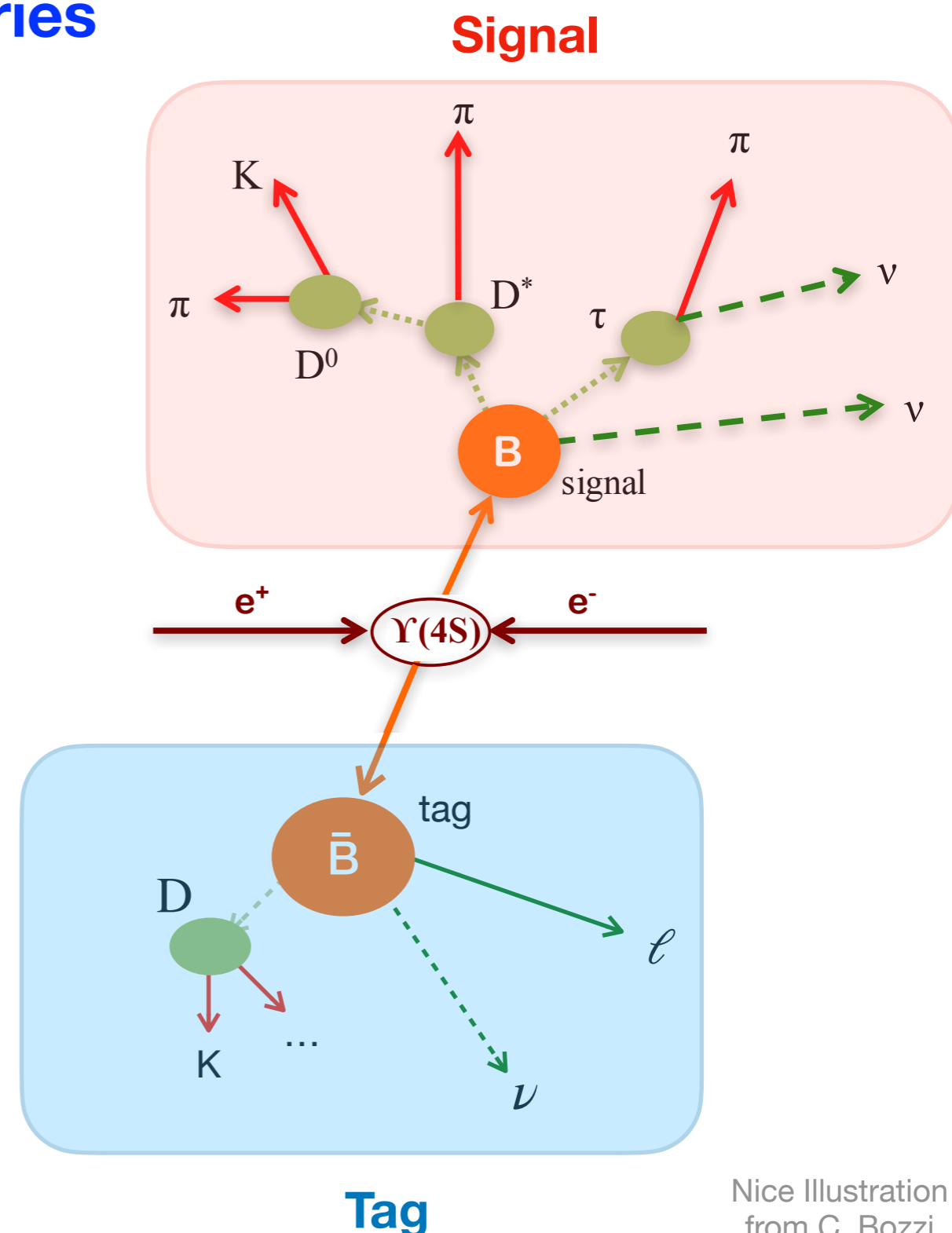
Measurement Strategies

3. Semileptonic decays at B-Factories

- ▶ e^+/e^- collision produces $Y(4S) \rightarrow B\bar{B}$
- ▶ Fully reconstruct one of the two B-mesons ('tag') → **possible to assign all particles** to either signal or tag B
- ▶ **Missing four-momentum (neutrinos)** can be reconstructed with high precision

$$p_{\text{miss}} = (p_{\text{beam}} - p_{B\text{tag}} - p_{D^{(*)}} - p_{\ell})$$

✓ **Small efficiency (~0.2-0.4%)**
compensated by large integrated
luminosity

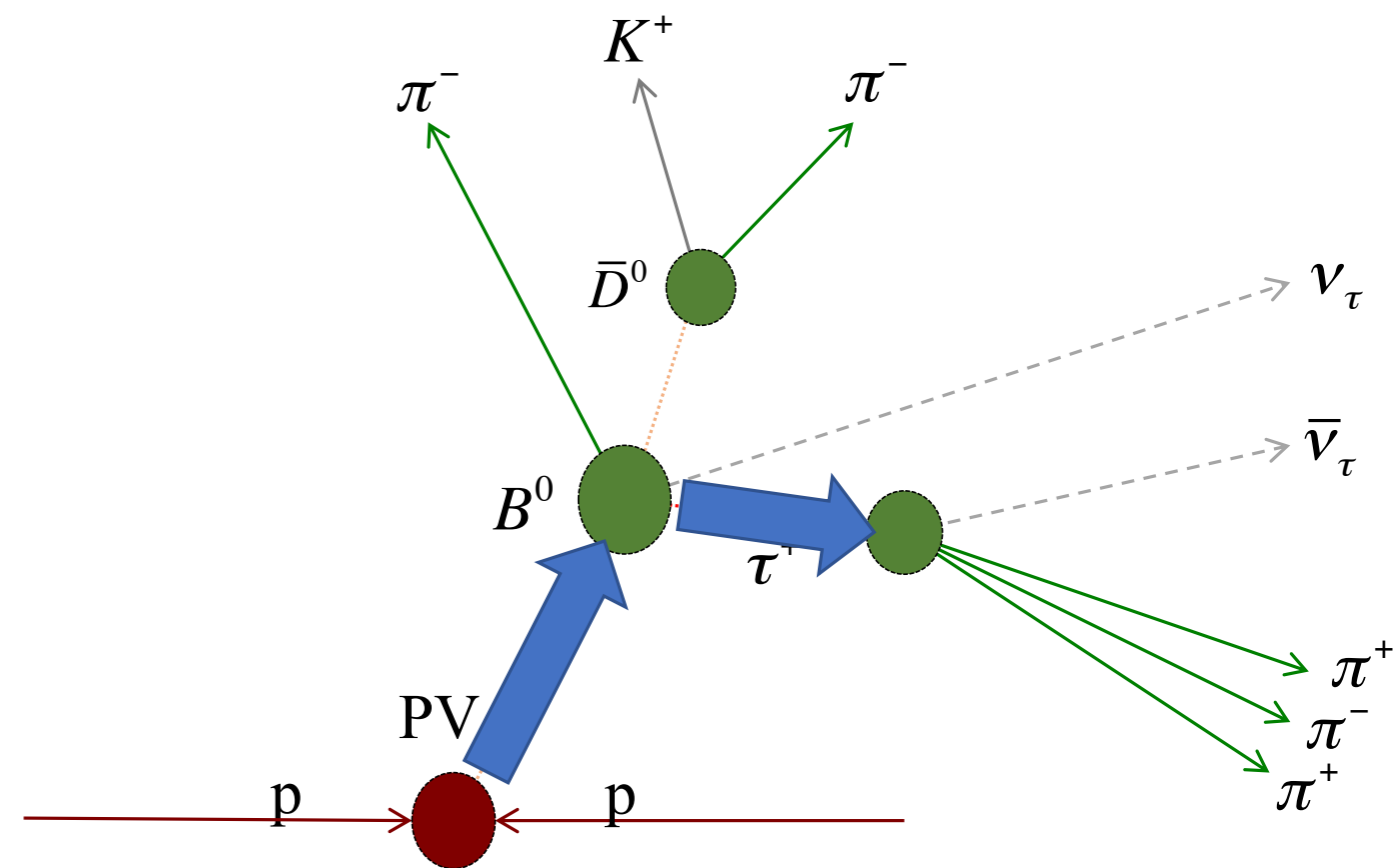


Measurement Strategies

4. Semileptonic decays at LHCb

- ▶ No constraint from beam energy at a hadron machine, **but..**
- ▶ **Large Lorentz boost** with decay lengths in the range of **mm**
- ✓ **Well-separated decay vertices**
- ✓ **Momentum direction of decaying particle is well known**
- ▶ With known masses and other decay products can even **reconstruct four-momentum transfer squared q^2** up to a two-fold ambiguity

$$q^2 = (p_{X_b} - p_{X_q})^2$$



Nice Illustration
from C. Bozzi

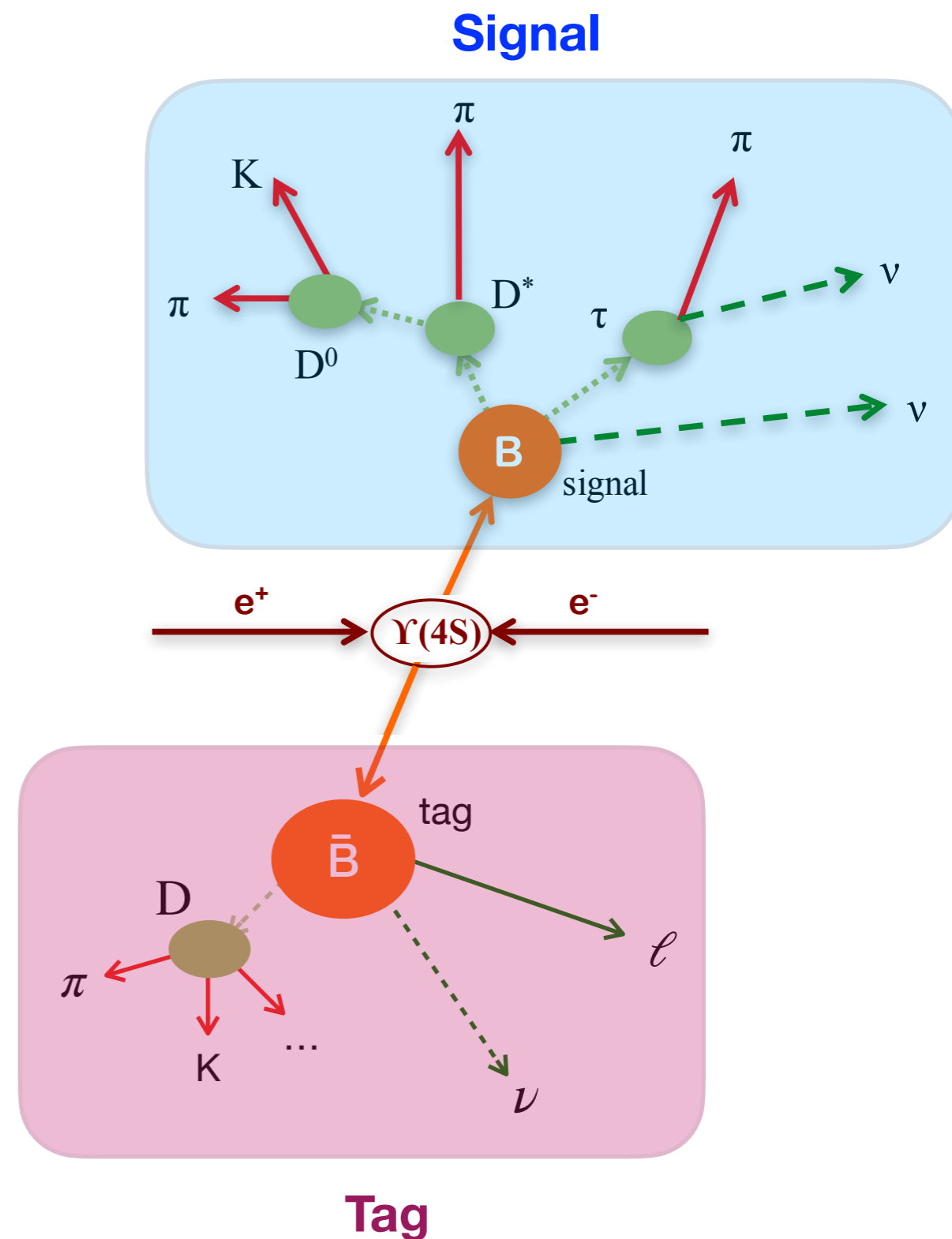
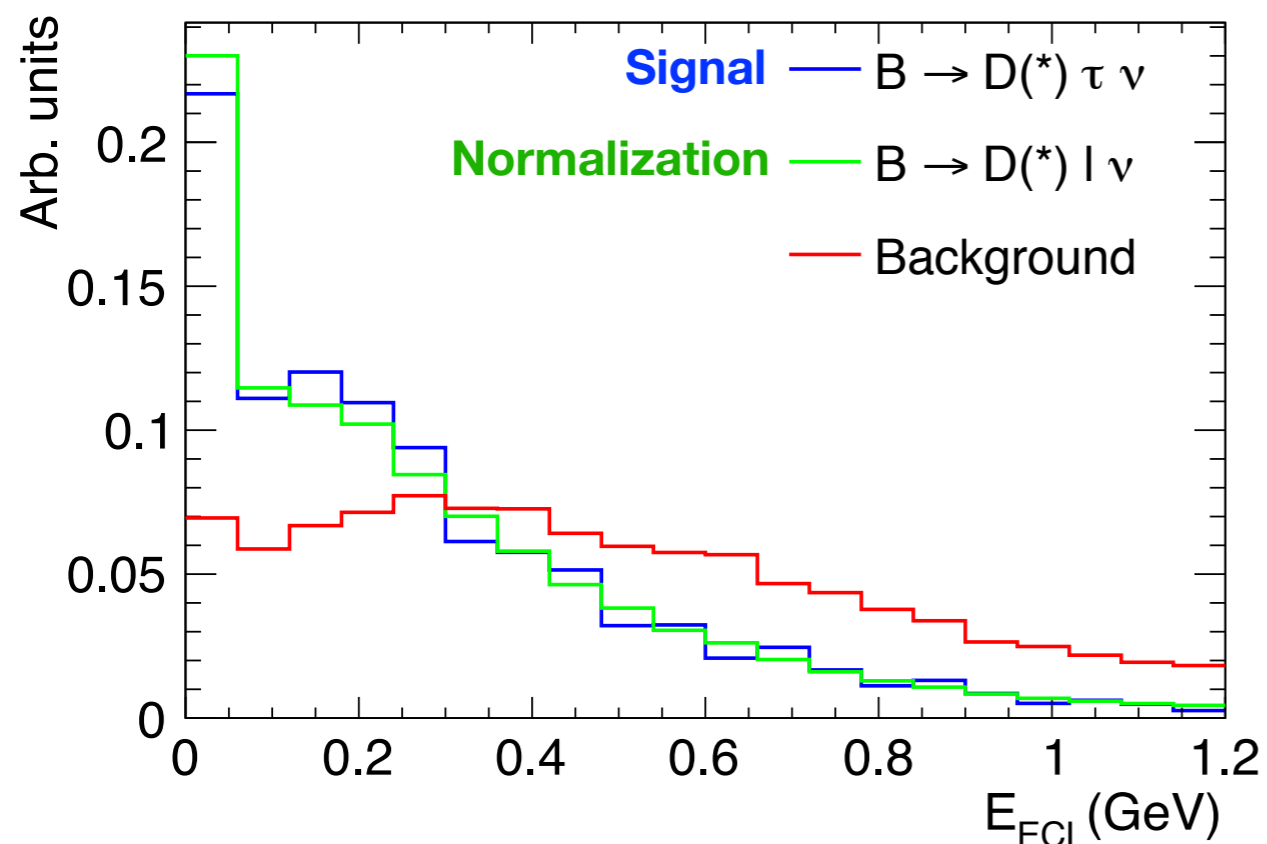
Even bit more complicated
for leptonic tau decays

Latest $R(D^{(*)})$ from Belle

G. Caria et al (Belle),
Phys. Rev. Lett. 124, 161803, April 2020
[arXiv:1904.08794]

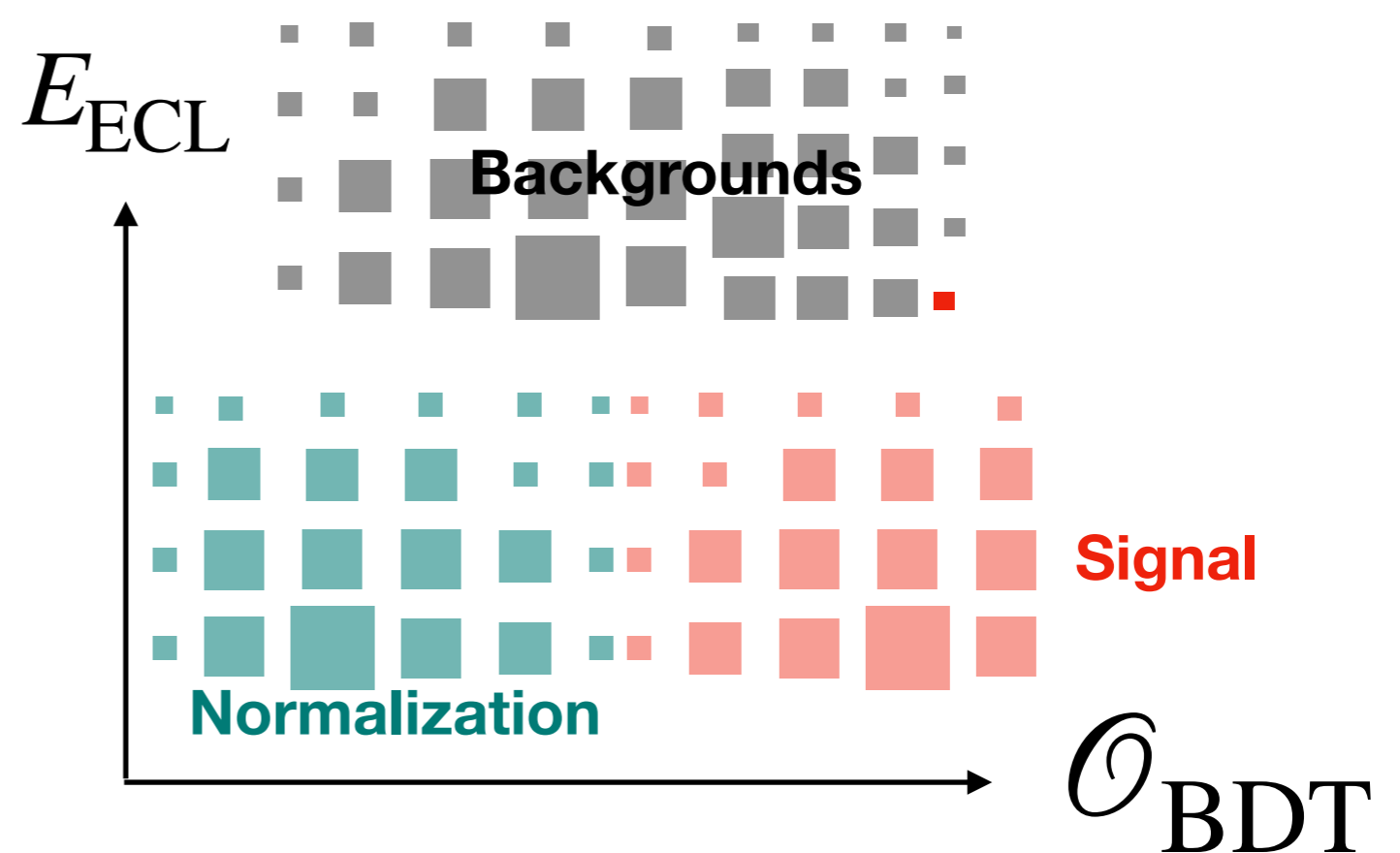
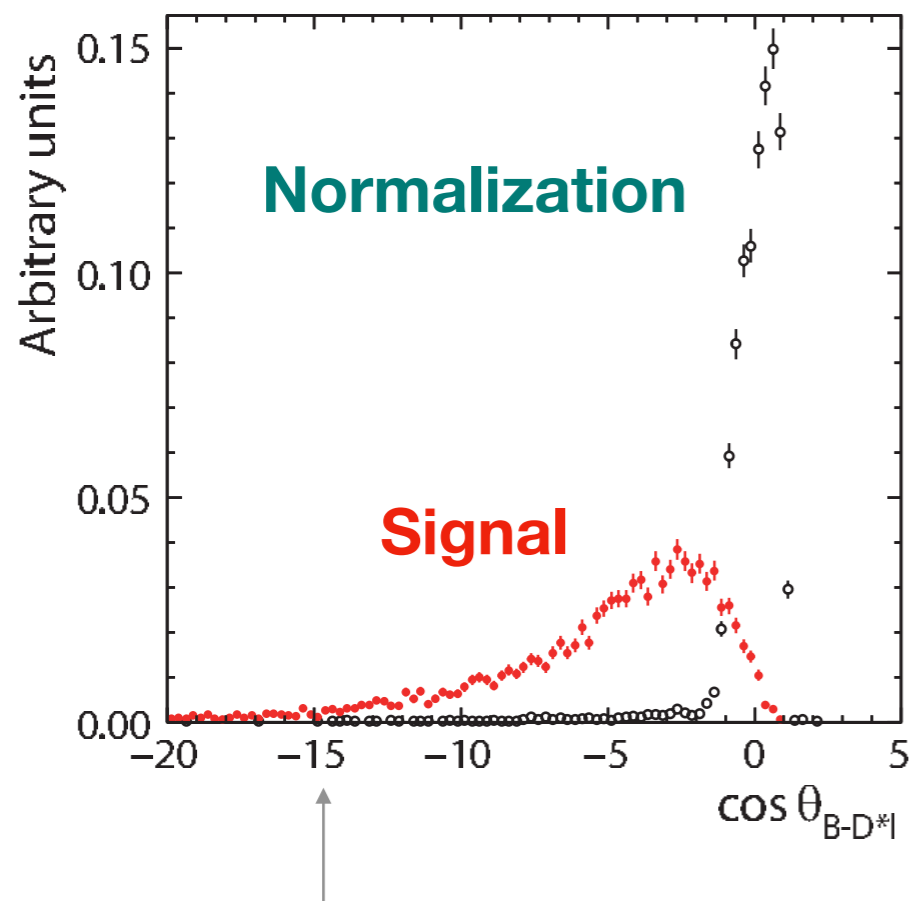
- ▶ Reconstruct one of the two B-mesons ('tag') in **semileptonic modes** → **possible to assign all particles in detector** to tag- & signal-side
- ▶ **Demand Matching topology** + **unassigned energy in the calorimeter** E_{ECL} to discriminate background from signal

$$E_{\text{extra}} = E_{\text{ECL}} = \sum_i E_i^\gamma$$



Separation of signal & normalization

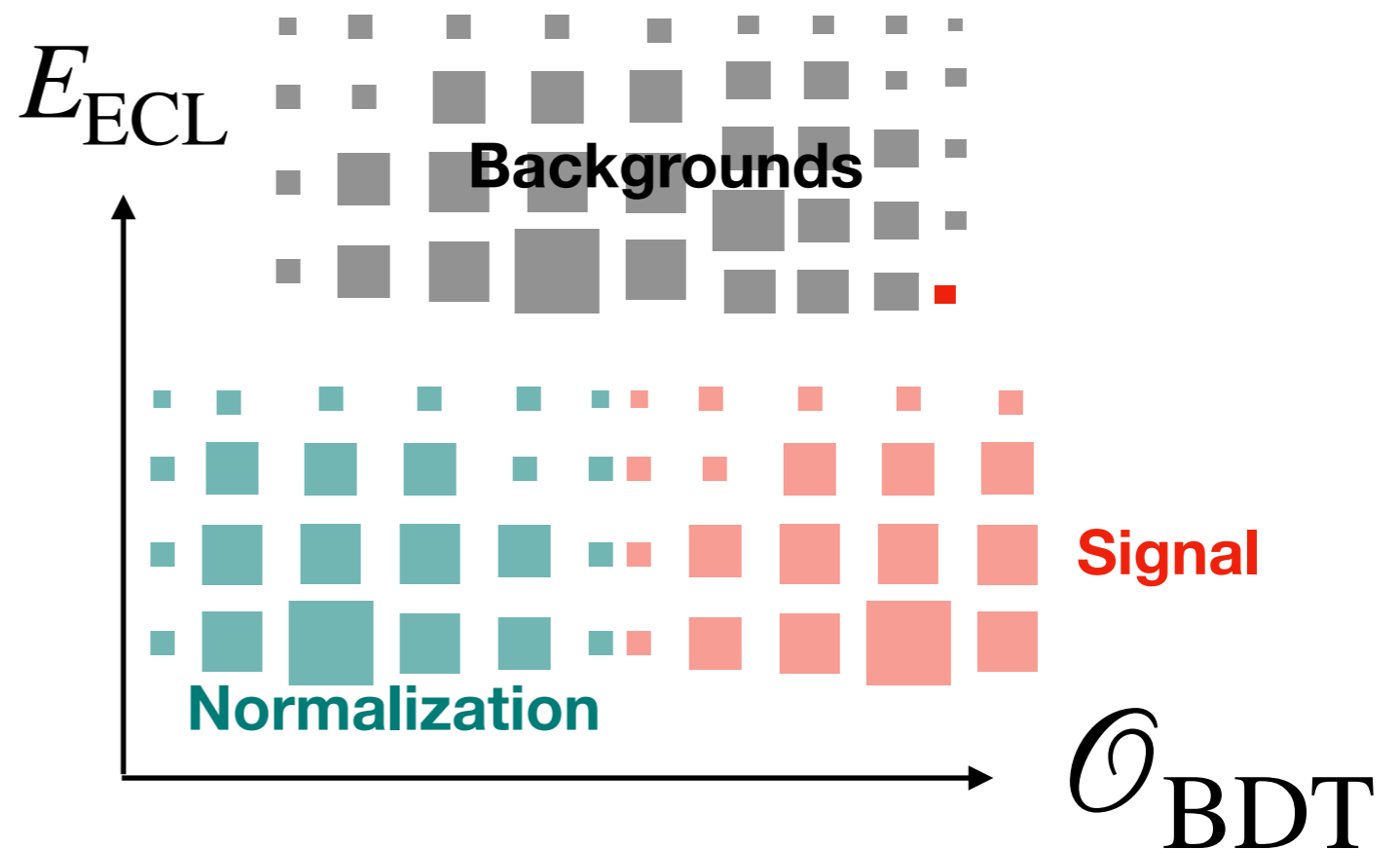
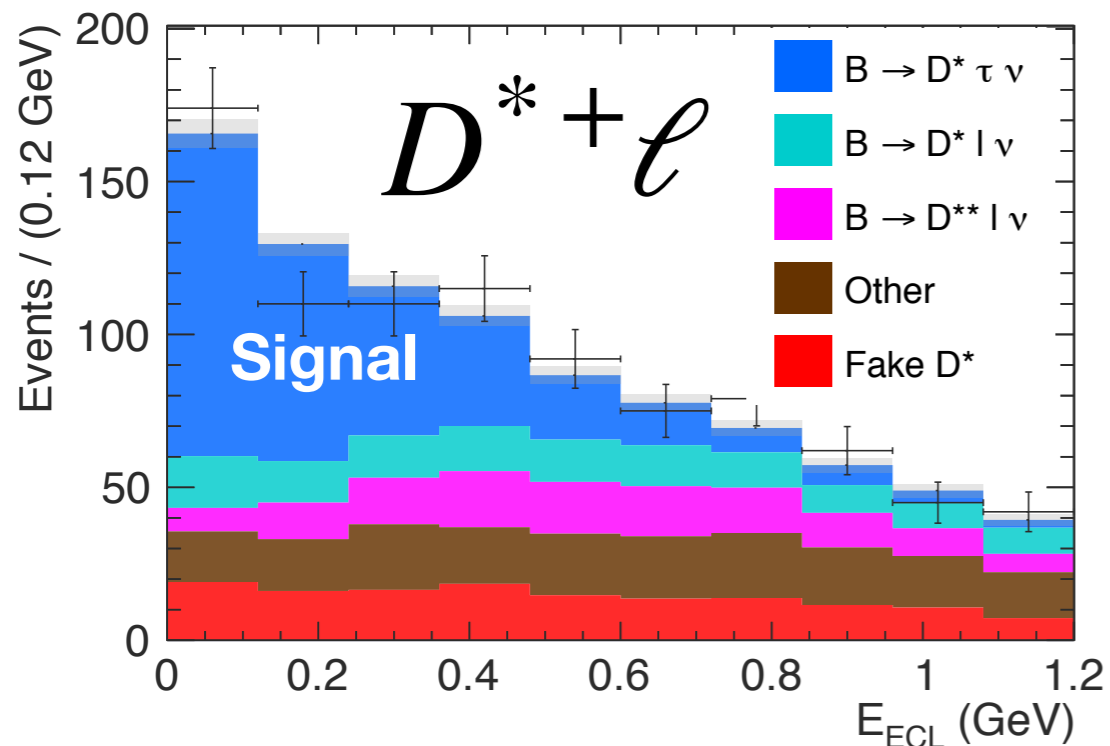
- ▶ Use kinematic properties to separate $B \rightarrow D^{(*)}\tau\nu$ signal from $B \rightarrow D^{(*)}\ell\nu$ normalization
- ▶ Construct BDT with 3 variables: $\cos \theta_{B-D^{(*)}\ell}$, E_{vis} , $m_{\text{miss}}^2 = p_{\text{miss}}^2$



In case you are wondering how a cosine can be outside $[-1,1]$: it's because the reconstruction uses measured energies and the definition assumes only a single missing neutrino

Separation of signal & normalization

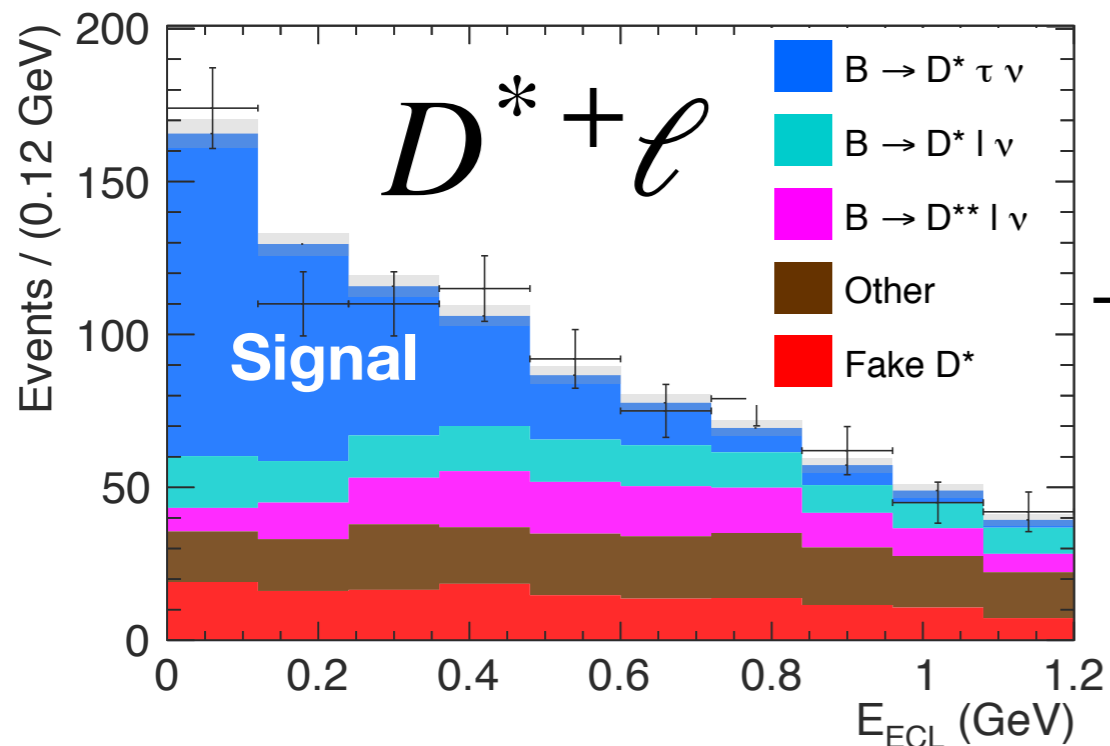
- ▶ Use kinematic properties to separate $B \rightarrow D^{(*)}\tau\nu$ signal from $B \rightarrow D^{(*)}\ell\nu$ normalization
- ▶ Construct BDT with 3 variables: $\cos\theta_{B-D^{(*)}\ell}$, E_{vis} , $m_{\text{miss}}^2 = p_{\text{miss}}^2$



Signal-enriched selection with cut on \mathcal{O}_{BDT}

Separation of signal & normalization

- ▶ Use kinematic properties to separate $B \rightarrow D^{(*)}\tau\nu$ signal from $B \rightarrow D^{(*)}\ell\nu$ normalization
- ▶ Construct BDT with 3 variables: $\cos\theta_{B-D^{(*)}\ell}$, E_{vis} , $m_{\text{miss}}^2 = p_{\text{miss}}^2$



$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$

$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

Most precise measurement to date

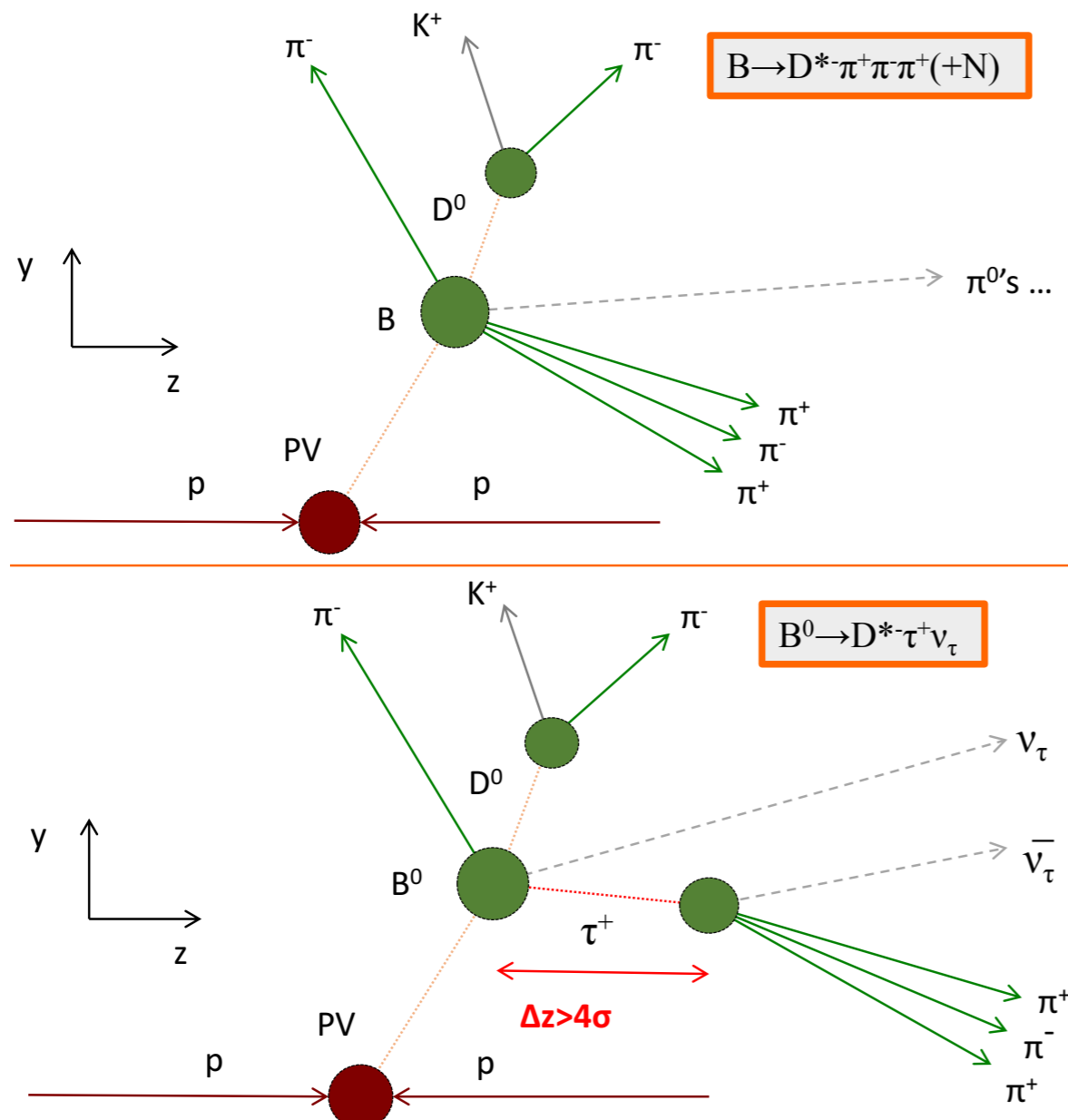
Signal enriched selection with cut on \mathcal{O}_{BDT}

LHCb Measurement of $R(D^*)$

R. Aaij et al (LHCb),
 Phys.Rev.Lett.120,171802 (2018) [arXiv:1708.08856]
 Phys.Rev.D 97, 072013 (2018) [arXiv:1709.02505]

- Tau reconstructed via $\tau \rightarrow \pi^+ \pi^+ \pi^- (\pi^0) \nu$, only two neutrinos missing

Although a semileptonic decay is studied, nearly no background from $B \rightarrow D^* X \mu \nu$



- Main background: prompt

$X_b \rightarrow D^* \pi \pi \pi + \text{neutrals}$

BF \sim 100 times larger than signal,
 all pions are promptly produced

- Suppressed by requiring minimum distance between X_b & τ vertices ($> 4 \sigma_{\Delta z}$)

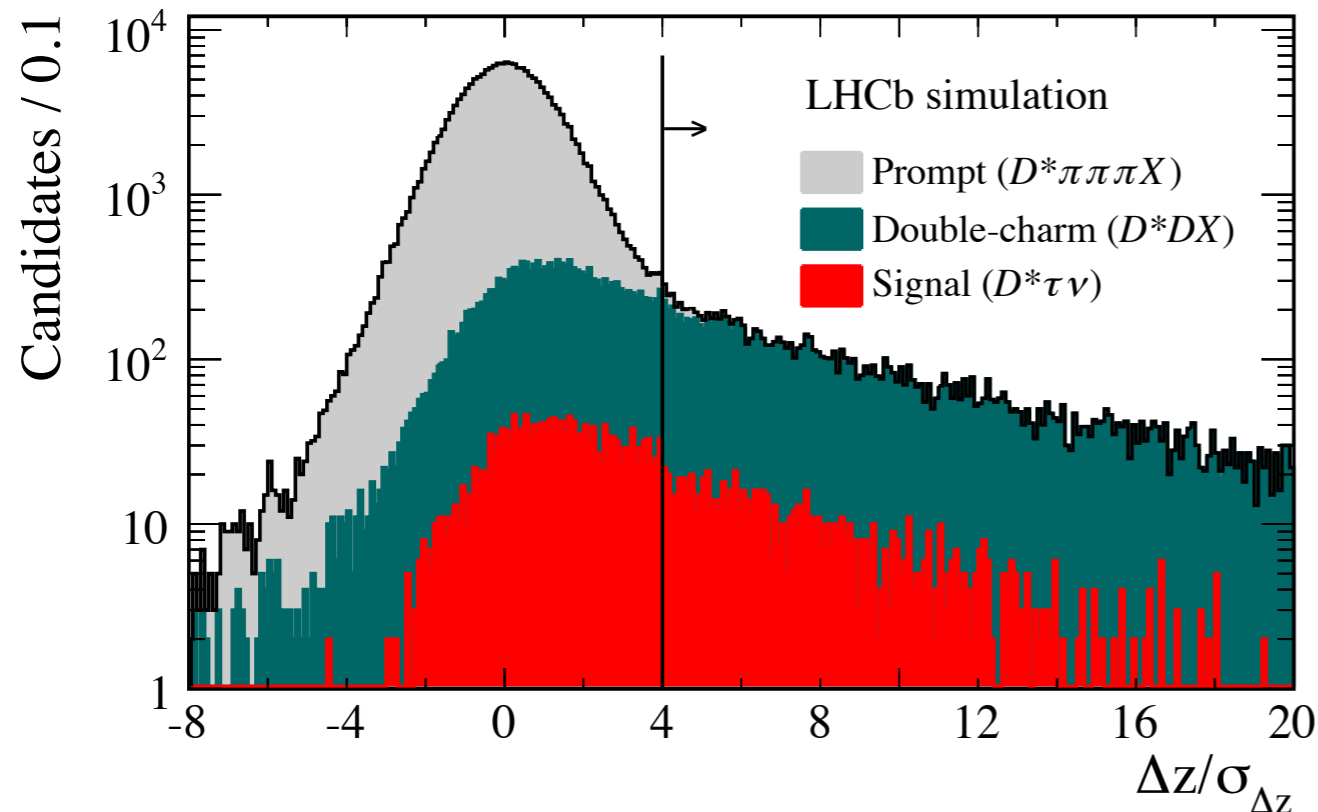
$\sigma_{\Delta z}$: resolution of vertices separation

- Reduces this background by three orders of magnitude

LHCb Measurement of $R(D^*)$

- ▶ Tau reconstructed via $\tau \rightarrow \pi^+ \pi^+ \pi^- (\pi^0) \nu$, only two neutrinos missing

Although a semileptonic decay is studied, nearly no background from $B \rightarrow D^* X \mu \nu$



- ▶ Main background: prompt



BF ~ 100 times larger than signal,
all pions are promptly produced

- ▶ Suppressed by requiring minimum distance between X_b & τ vertices ($> 4 \sigma_{\Delta z}$)

$\sigma_{\Delta z}$: resolution of vertices separation

- ▶ Remaining double charm bkg:



- ▶ Reduces this background by three orders of magnitude

LHCb Measurement of $R(D^*)$

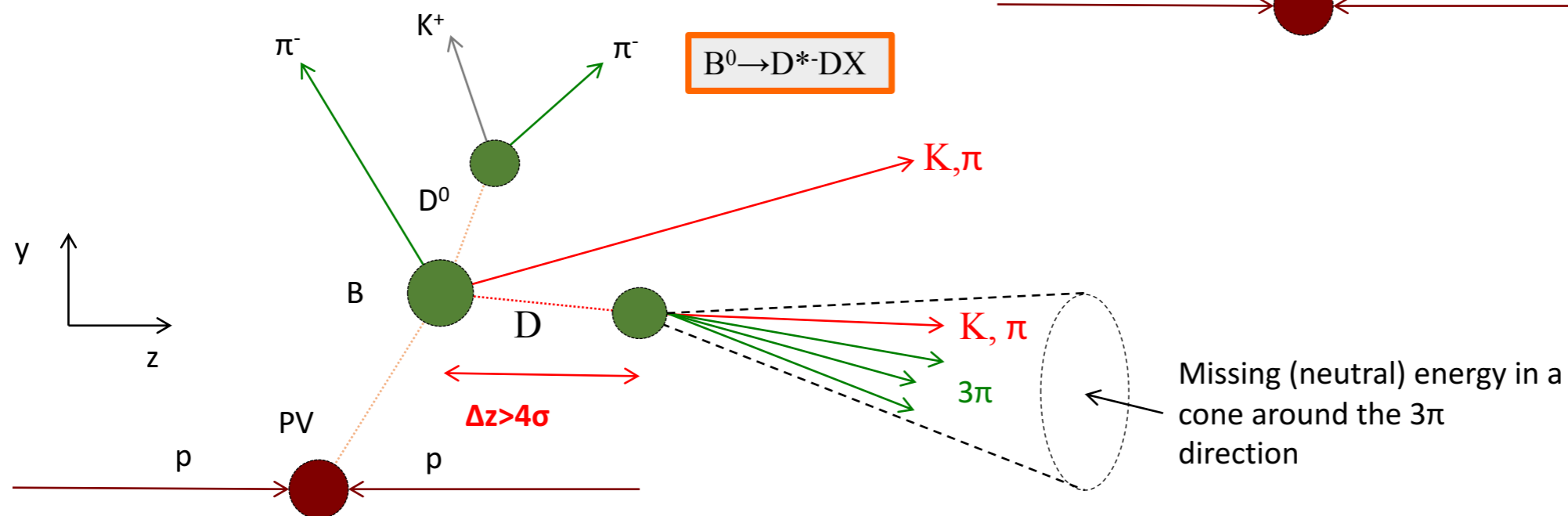
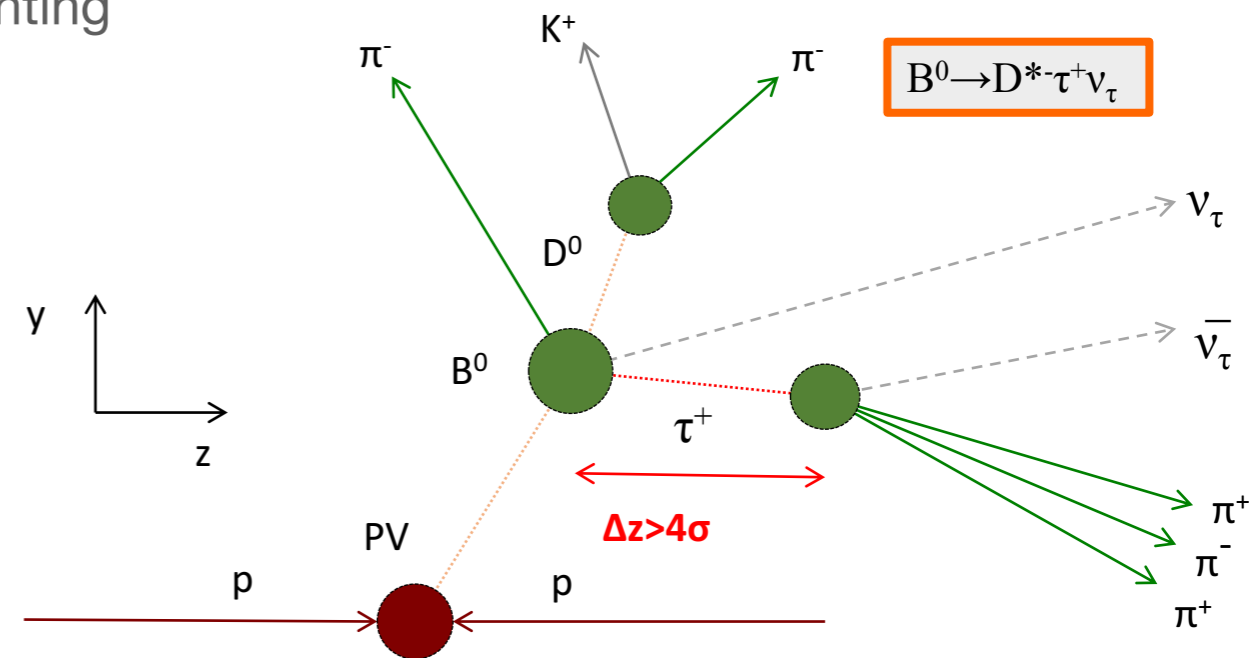
- ▶ Remaining backgrounds reduced via isolation & MVA

Require signal candidates to be **well isolated**

i.e. reject events with extra charged particles pointing to the B and/or τ

Events with additional neutral energy are suppressed with a MVA

More information about that in backup



LHCb Measurement of $R(D^*)$

► Extraction in **3D fit** to

MVA : q^2 : τ decay time

↑
Invariant masses of 3π system
Invariant mass of $D^*3\pi$ system
Neutral isolation variables

← q^2 reconstructed with some tricks (more in backup)

4 Bins 8 Bins 8 Bins

► Components:

1 Signal component for $\tau \rightarrow \pi^+\pi^+\pi^-(\pi^0)\nu$

11 Background components

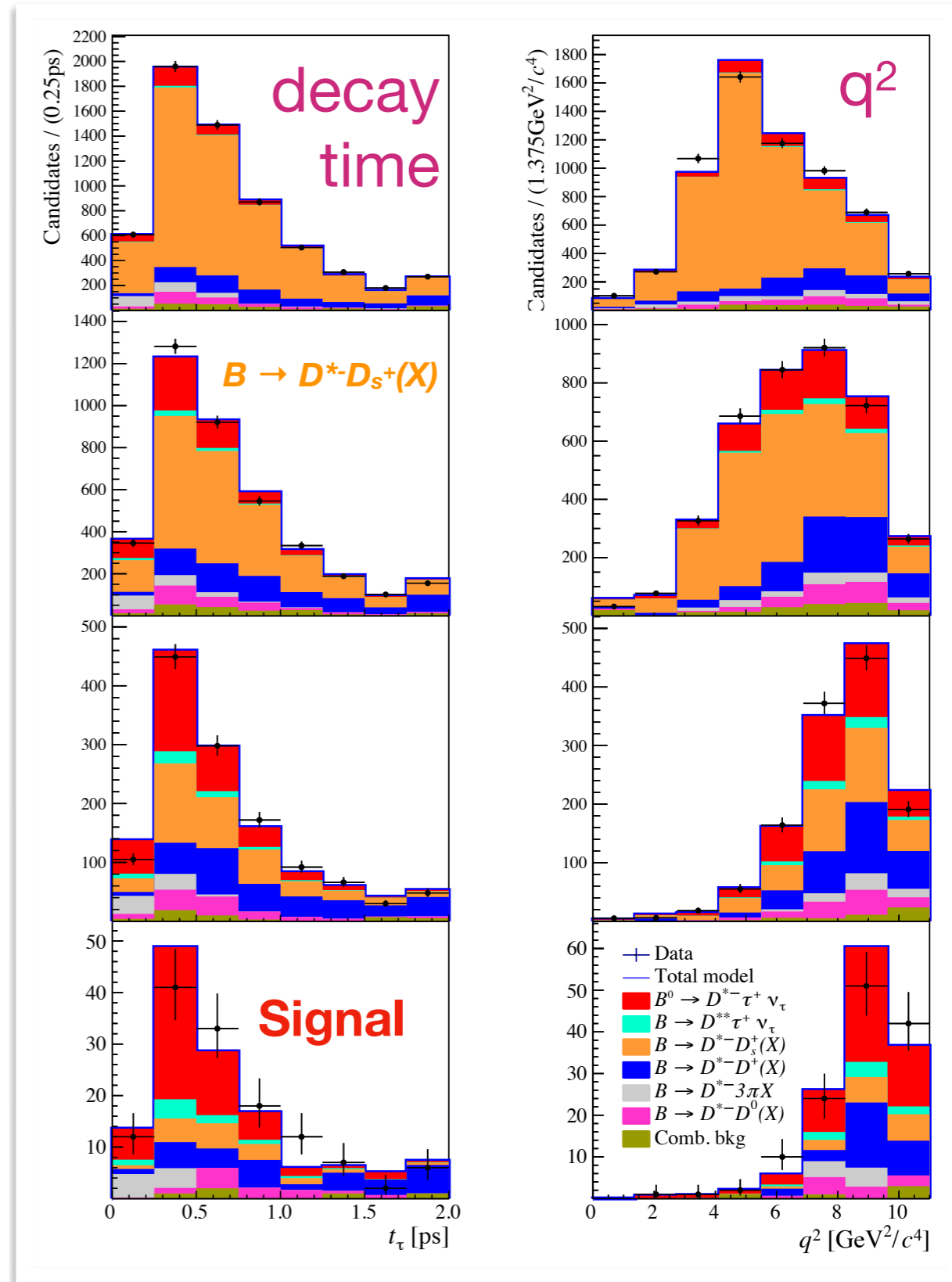
► $\sim 1296 \pm 86$ Signal events

► Using normalization mode and light lepton BFs:

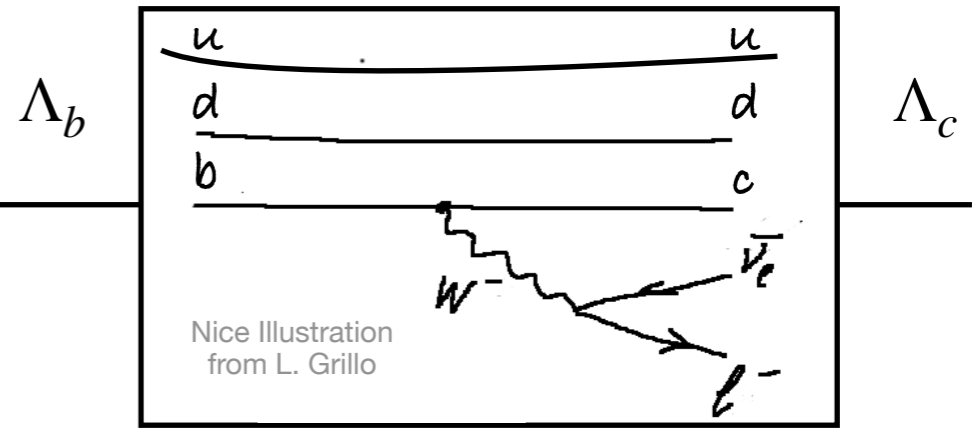
More information about normalization in backup

$$R(D^*) = 0.286 \pm 0.019 \text{ (stat)} \pm 0.025 \text{ (syst)} \pm 0.021 \text{ (norm)}$$

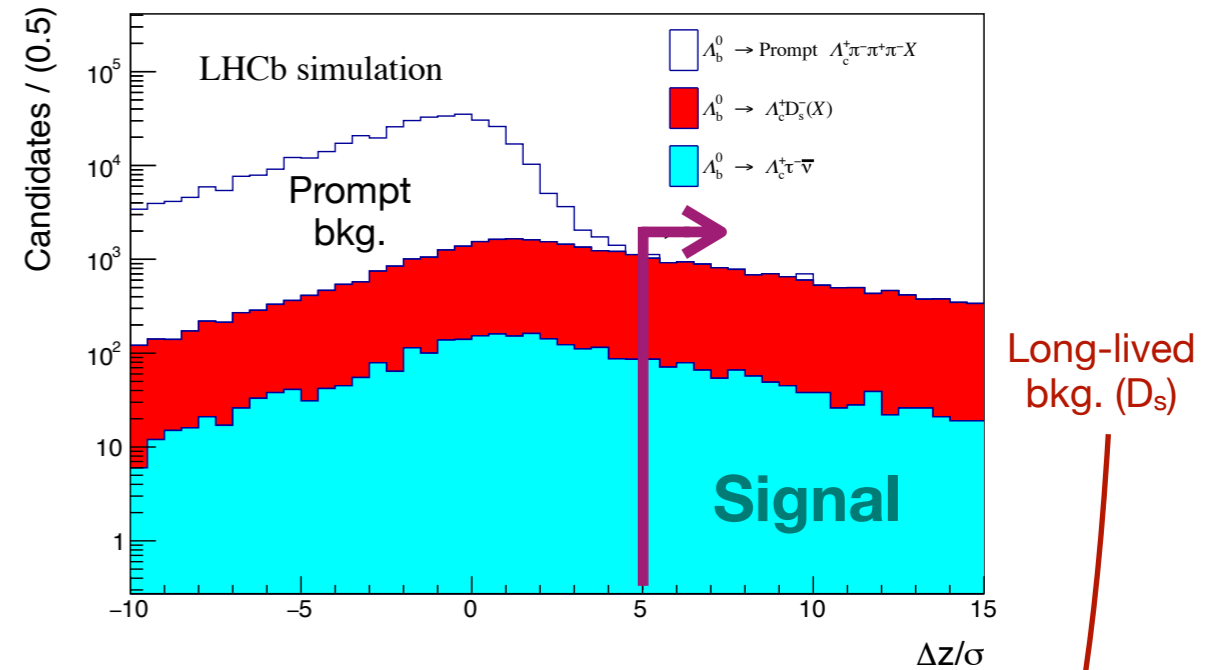
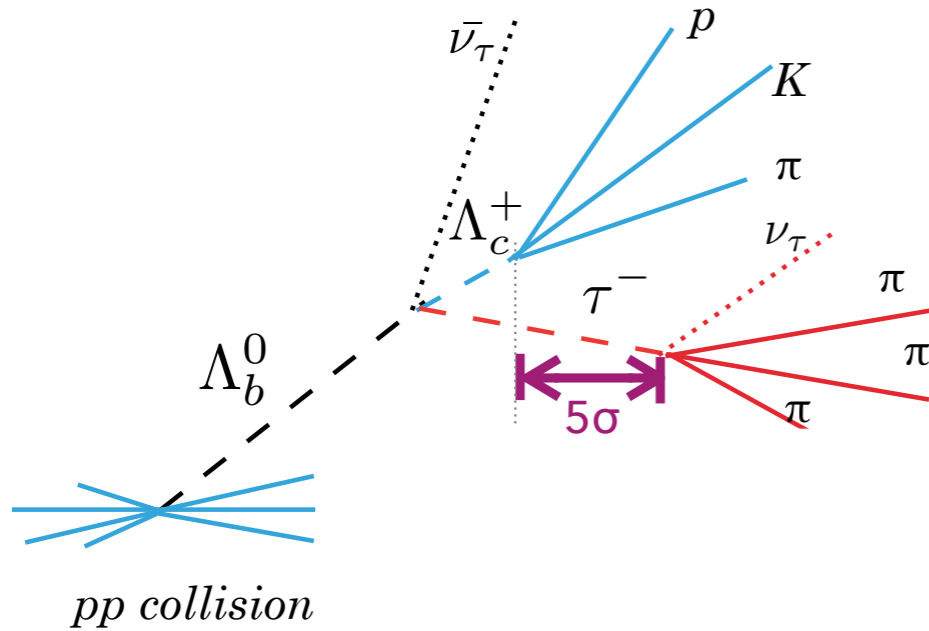
Purer MVA Selection



LHCb $R(\Lambda_c)$ Measurement



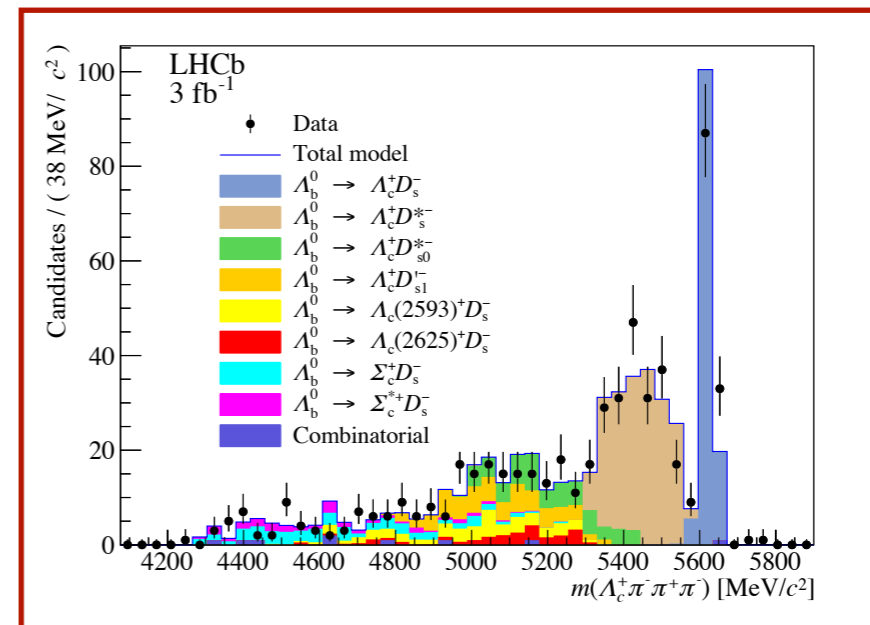
Same experimental Method: exploit vertex separation



$$m_{3\pi} \in [m_{D_s} - 45 \text{ MeV}, m_{D_s} + 45 \text{ MeV}]$$

Target ratio:

$$\begin{aligned} \mathcal{K}(\Lambda_c^+) &= \frac{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ 3\pi)} \\ &= \frac{N_{sig}}{N_{norm}} \times \frac{\epsilon_{norm}}{\epsilon_{sig}} \times \frac{1}{\mathcal{B}(\tau^- \rightarrow 3\pi(\pi^0)\nu_\tau)} \end{aligned}$$



Bkg. composition constrained by fit to $m_{3\pi}$

► Extraction in **3D fit** to
MVA : q^2 : τ decay time

Kinematic and angular information of 3π system, neutral energy in cone around 3π direction

$$N(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau^- \bar{\nu}_\tau) = 349 \pm 40$$

$$N(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^- (X)) = 2757 \pm 80$$

External input:

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ 3\pi) = (6.14 \pm 0.94) \times 10^{-3}$$

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau \bar{\nu}_\tau) = (1.50 \pm 0.16 \pm 0.25 \pm 0.23) \%$$

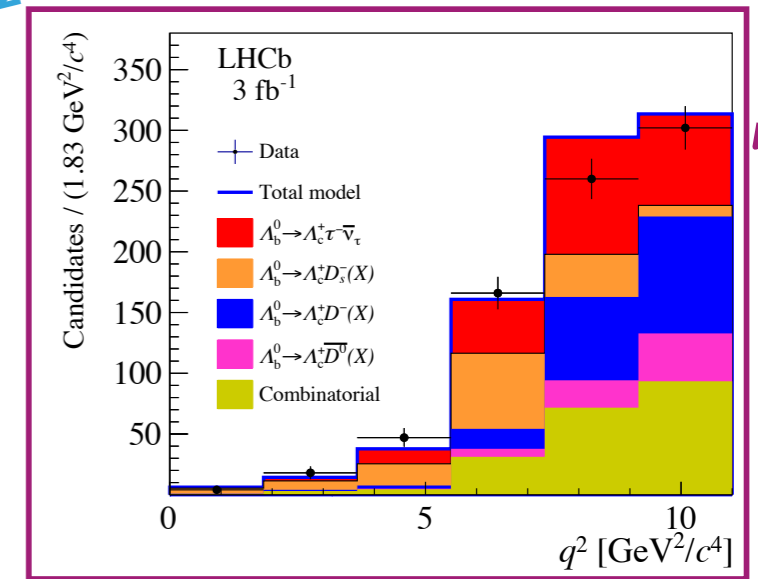
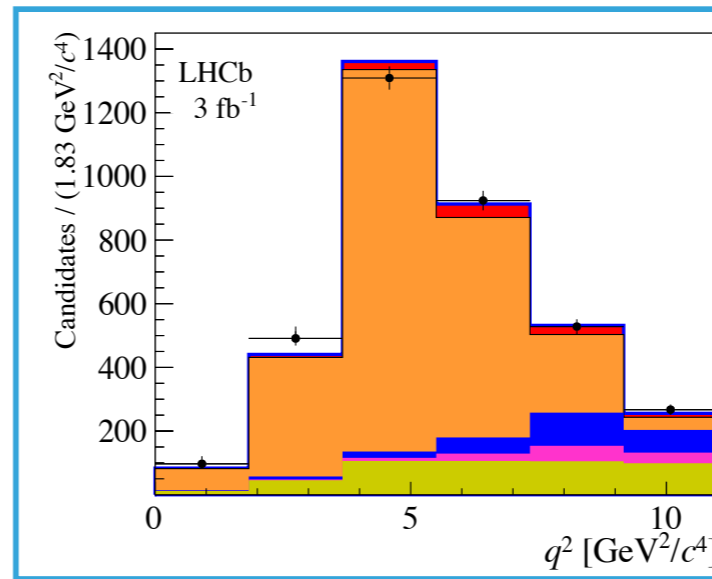
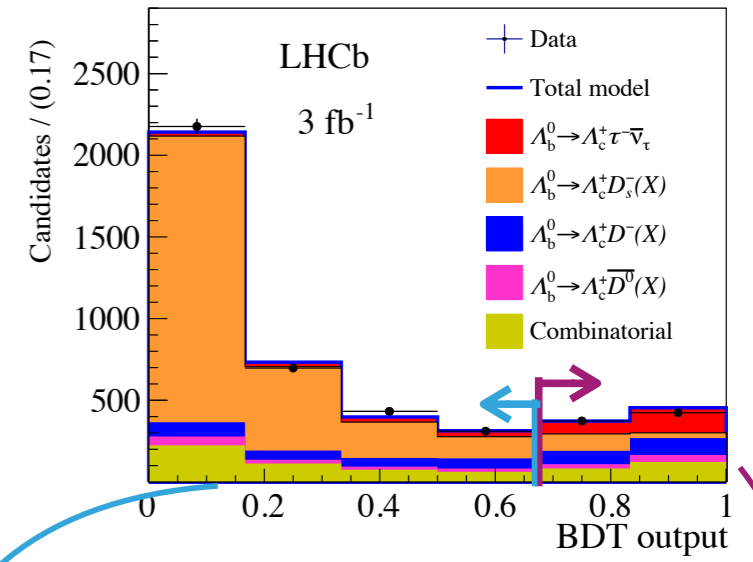
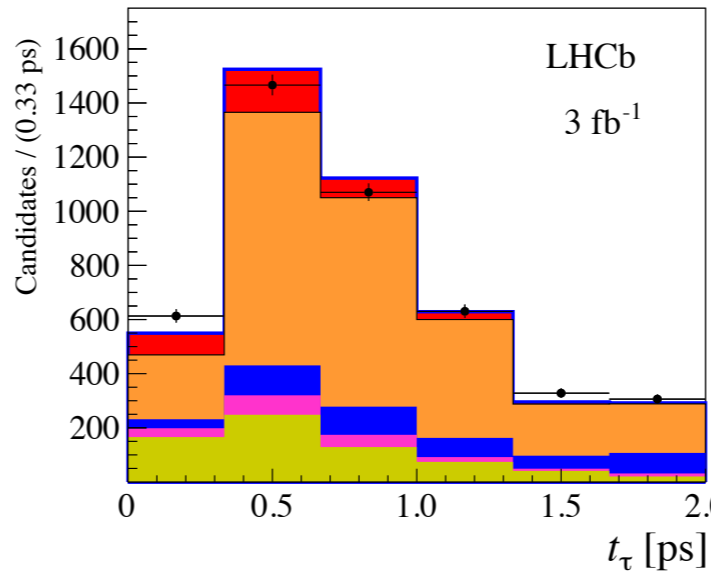
First observation with 6.1σ !

More external input:

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu \bar{\nu}_\mu) = (6.2 \pm 1.4) \%$$

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\text{stat}} \pm 0.040_{\text{syst}} \pm 0.059_{\text{ext}}$$

$$R(\Lambda_c^+) = 0.242 \pm 0.026_{\text{stat}} \pm 0.040_{\text{syst}} \pm 0.059_{\text{ext}}$$



Compatible with SM

$$R(\Lambda_c^+)_{\text{SM}} = 0.340 \pm 0.004$$

F. Bernlochner, Zoltan Ligeti, Dean J. Robinson, William L. Sutcliffe,
[arXiv:1808.09464], [arXiv:1812.07593]

► Extraction in **3D fit** to
MVA : q^2 : τ decay time

Kinematic and angular information of 3π system, neutral energy in cone around 3π direction

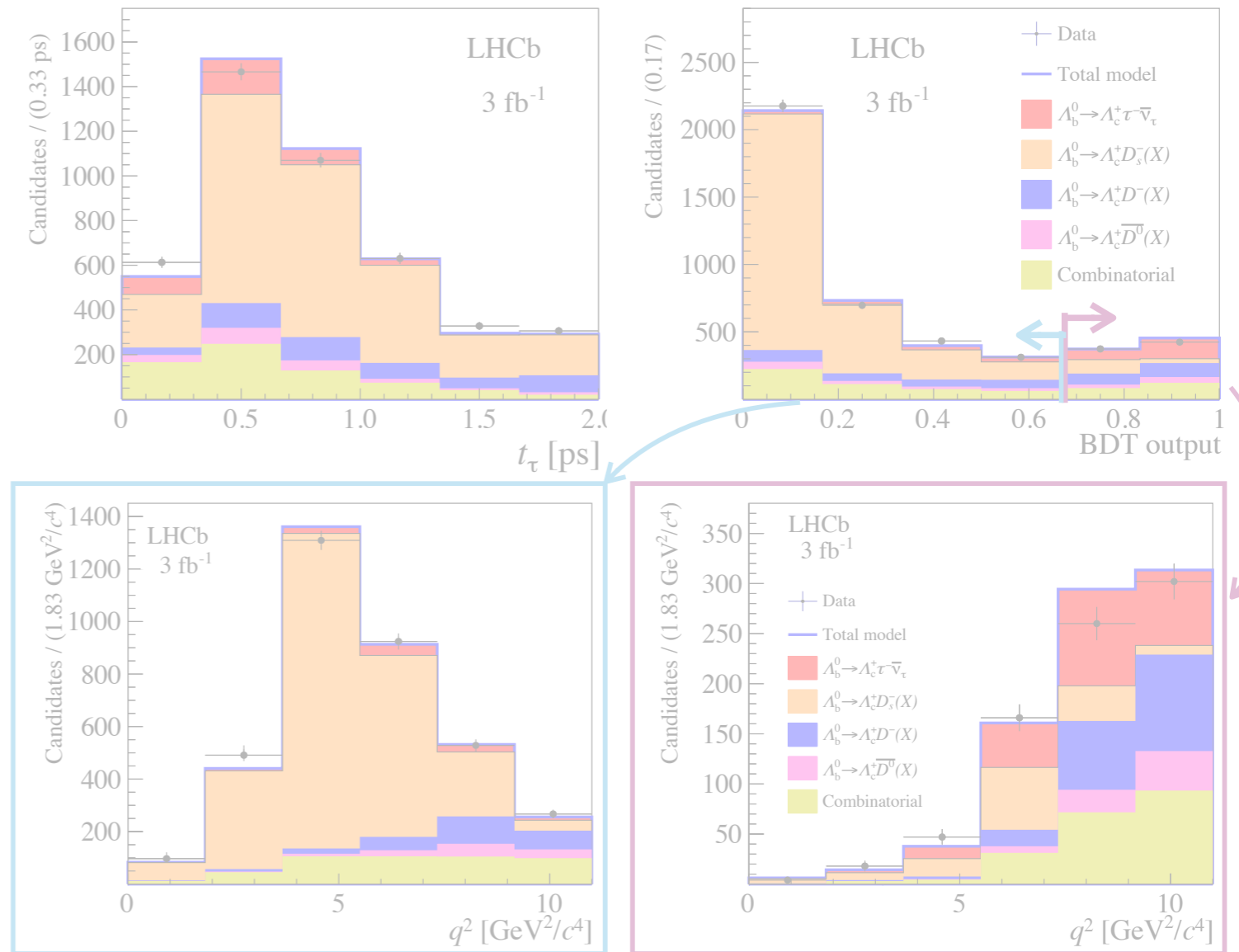
$$N(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau^- \bar{\nu}_\tau) = 349 \pm 40$$

$$N(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^- (X)) = 2757 \pm 80$$

External input:

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ 3\pi) = (6.14 \pm 0.94) \times 10^{-3}$$

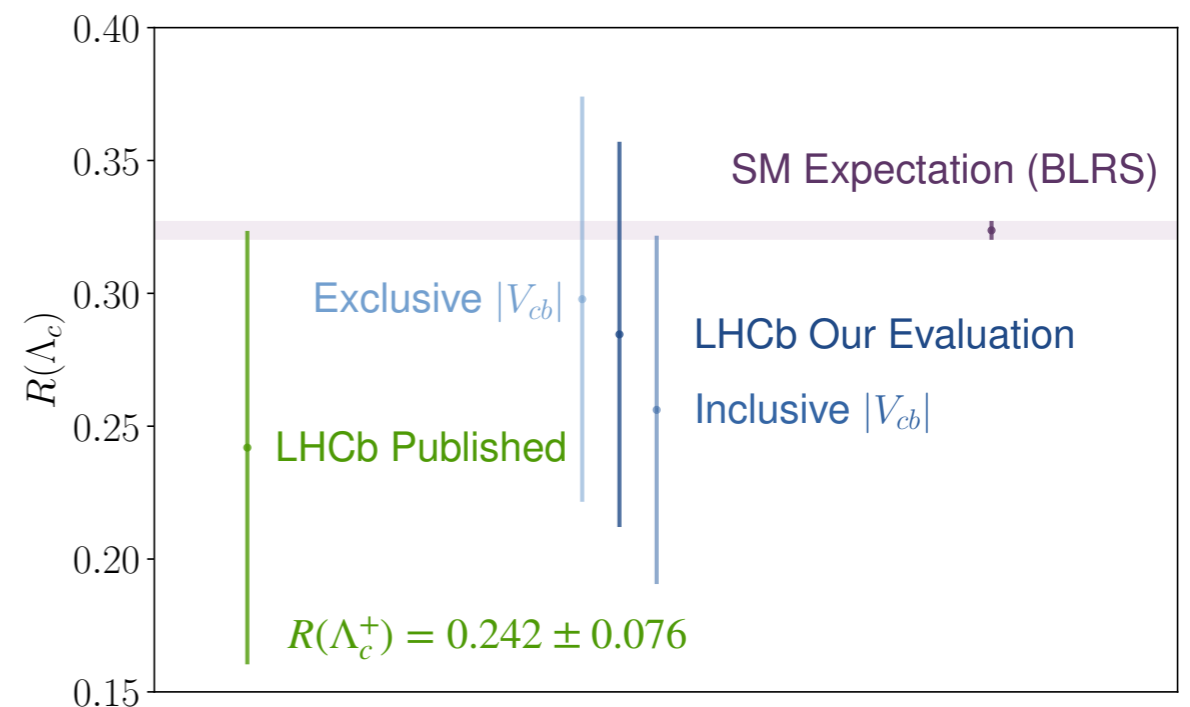
$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \tau \bar{\nu}_\tau) = (1.50 \pm 0.16 \pm 0.25 \pm 0.23) \%$$



Can also use SM prediction for $\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \mu \bar{\nu}_\mu)$ instead of LEP measurement

FB, Zoltan Ligeti, Michele Papucci, Dean Robinson, [arXiv:2206.11282 [hep-ph]]

$$R(\Lambda_c^+) = 0.285 \pm 0.073$$

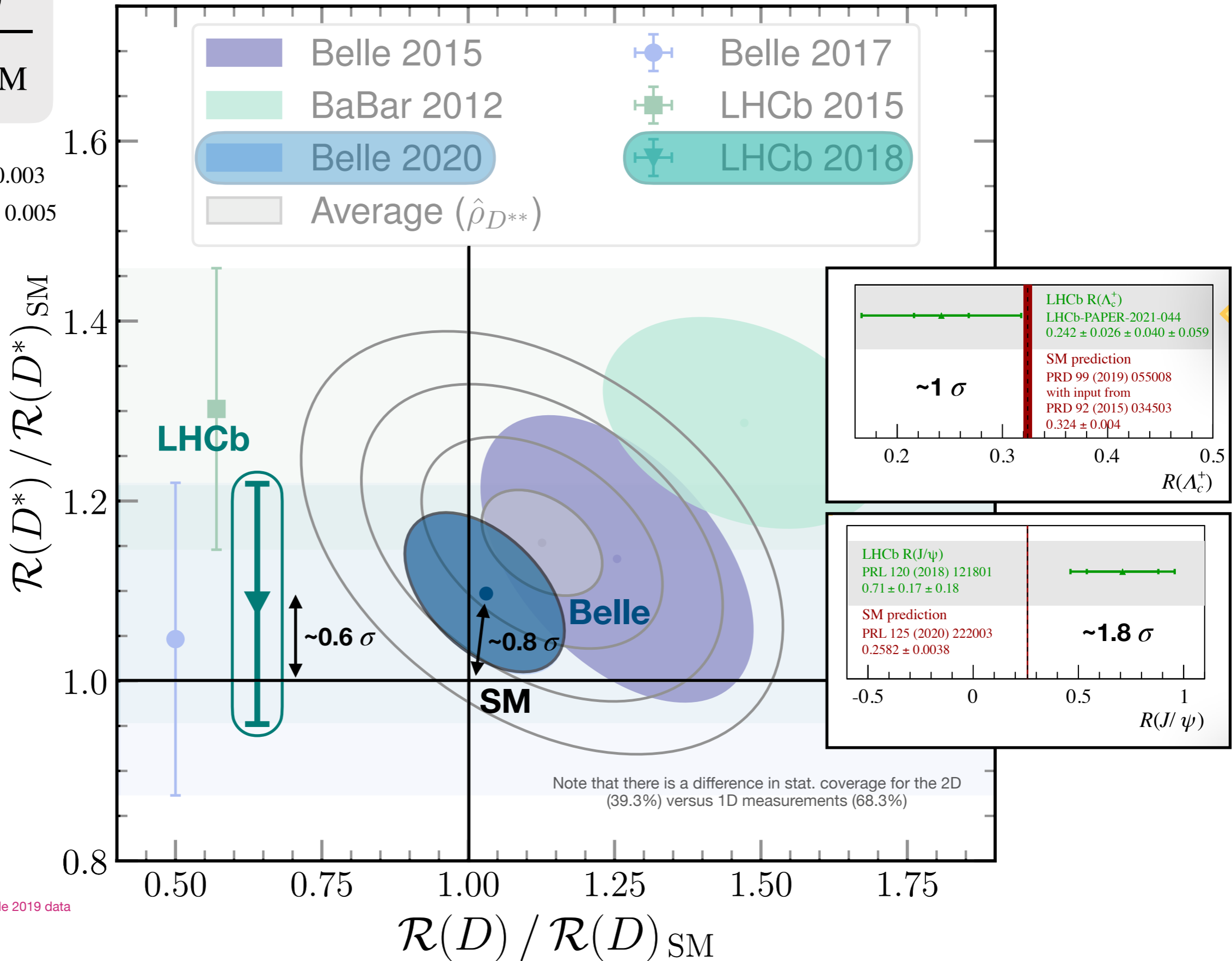


$$\frac{\mathcal{R}(D^{(*)})}{\mathcal{R}(D^{(*)})_{\text{SM}}}$$

$$\mathcal{R}(D)_{\text{SM}} = 0.299 \pm 0.003$$

$$\mathcal{R}(D^*)_{\text{SM}} = 0.258 \pm 0.005$$

HFLAV arithmetic average
 of SM Calculations



More Recent SM Calculations:

BaBar B- \rightarrow D*
<https://arxiv.org/abs/1903.10002>
 - $R(D^*)=0.253 \pm 0.005$

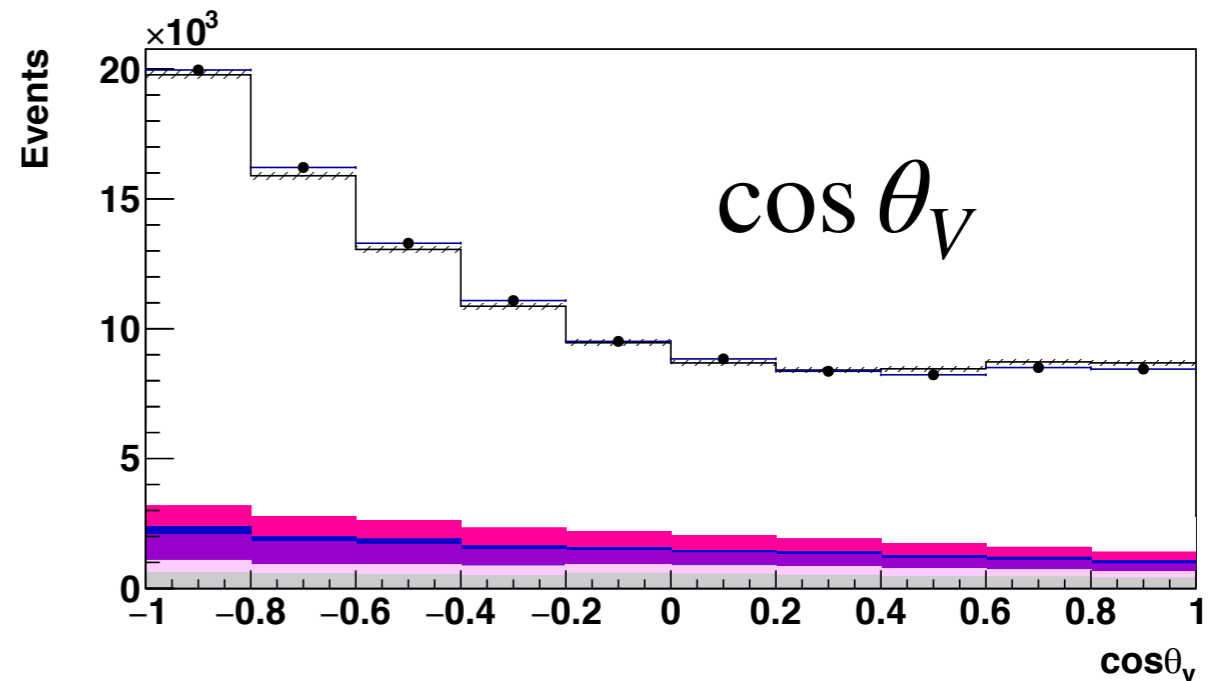
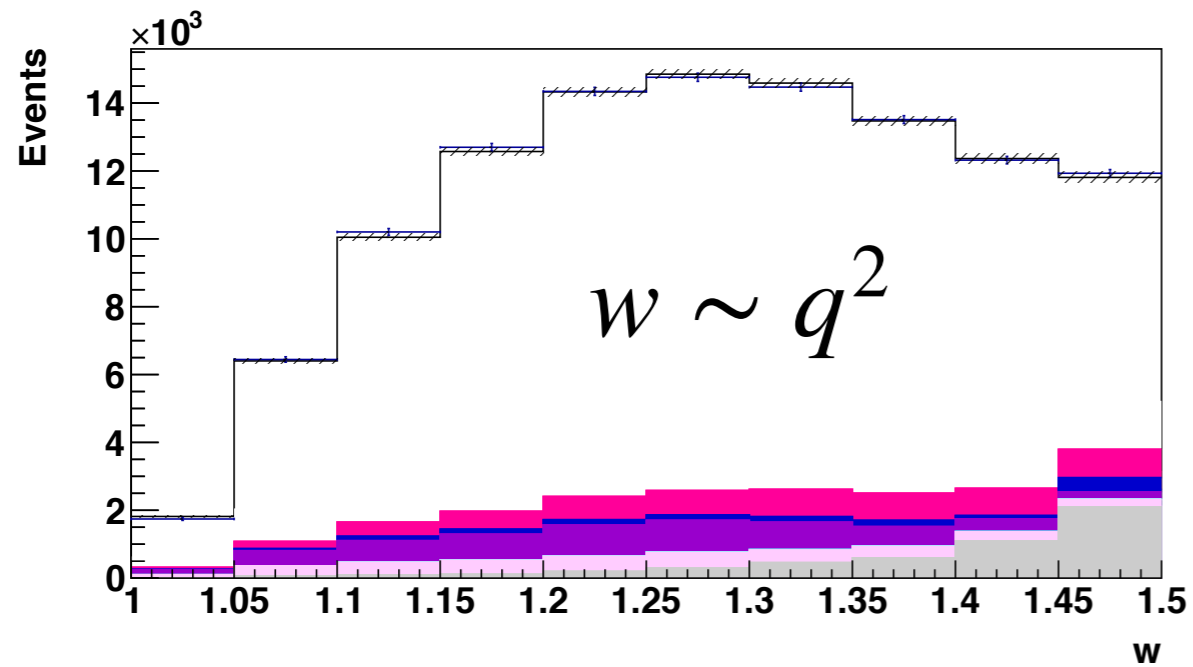
Gambino, Jung, Schacht using Belle 2019 data
<https://arxiv.org/abs/1905.08209>
 - $R(D^*)=0.254 \pm 0.007 \pm 0.006$

Bordone, Jung, van Dyk using Belle 2019 data
<https://arxiv.org/abs/1908.09398>
 - $R(D)=297 \pm 0.003$, $R(D^*)=0.250 \pm 0.003$

See also: <https://hflav-eos.web.cern.ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html>

An aerial night view of a university campus. The central focus is a large, multi-story yellow building with a grey roof and numerous windows. In front of it is a large, rectangular green courtyard. The surrounding city is visible in the background, with various buildings and a prominent church spire. The sky is dark with some clouds. The text "More Discussion Material" is overlaid in the center of the image.

More Discussion Material



Fit of 1D projections with correlations results in two very compatible values of $|V_{cb}|$ for CLN & BGL:

Caprini-Lellouch-Neubert Form Factors

$$|V_{cb}| = (38.4 \pm 0.2 \pm 0.6 \pm 0.6) \times 10^{-3}$$

Boyd-Grinstein-Lebed Form Factors

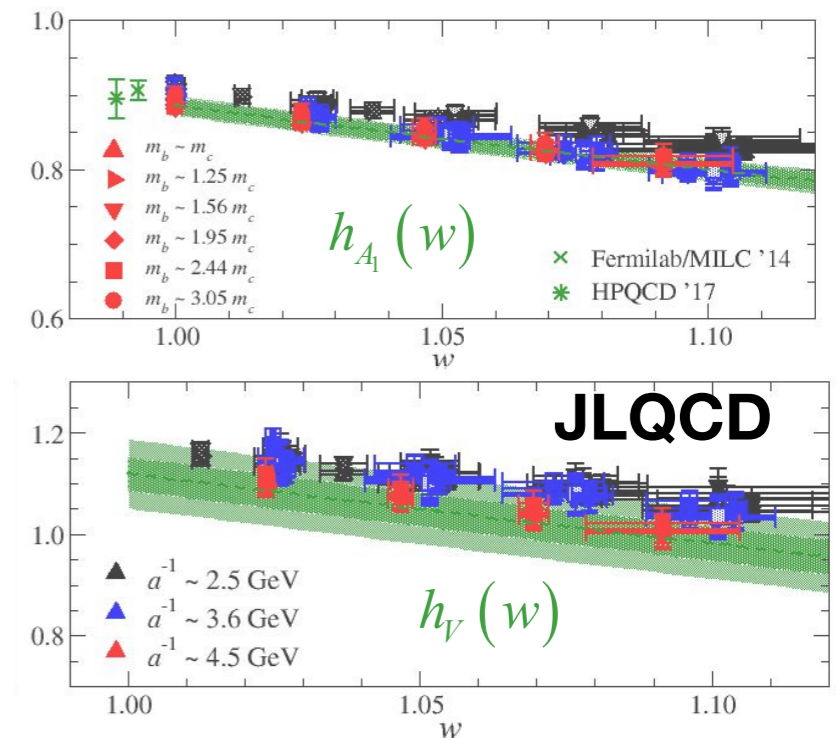
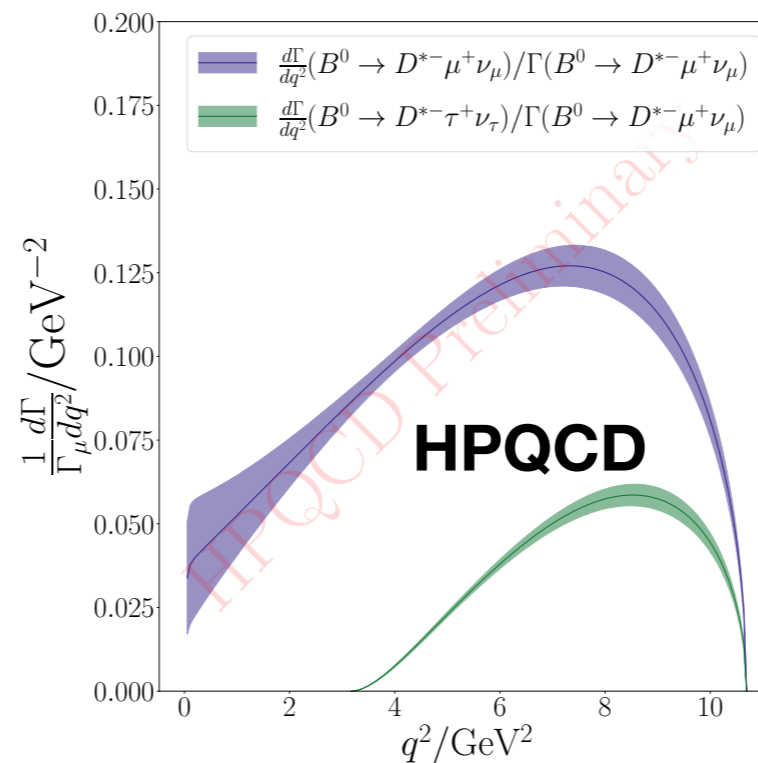
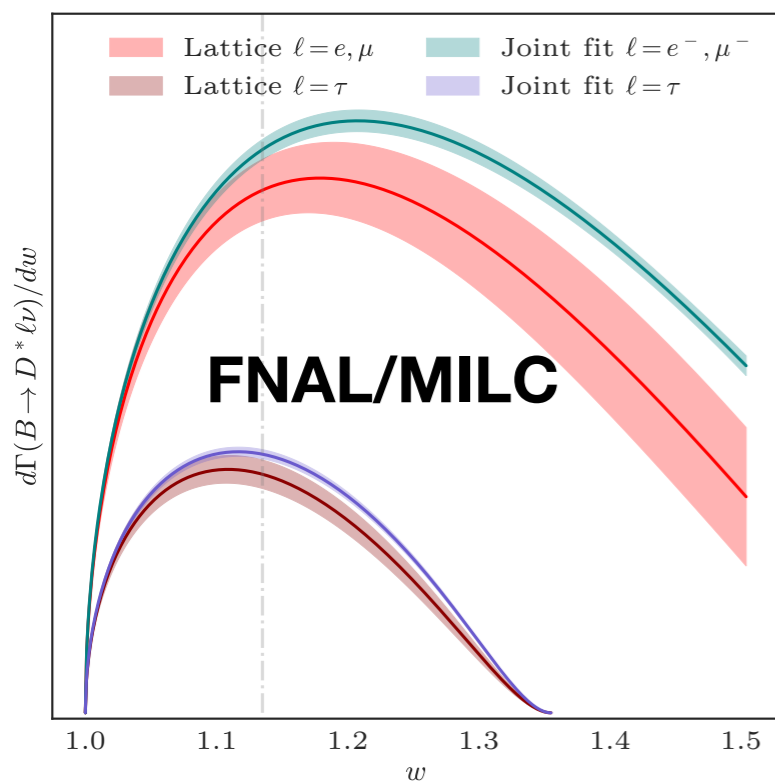
$$|V_{cb}| = (38.3 \pm 0.3 \pm 0.7 \pm 0.6) \times 10^{-3}$$

New Developments in exclusive $|V_{cb}|$

Very exciting times:

After more than 10 years in the making, we have first beyond zero recoil LQCD predictions beyond zero recoil for $B \rightarrow D^* \ell \bar{\nu}_\ell$:-)

One is finished, two are nearly finished:



A. Bazavov et al. [FNAL/MILC] [Under Review, arXiv:2105.14019]

Truncation Order

Martin will tell us more about form factors (FF) and how to determine from these distributions $|V_{cb}|$

One model independent way to parametrize FFs is the **BGL** parametrization (Boyd-Grinstein-Lebed, [arXiv:hep-ph/9705252])

$$g(z) = \frac{1}{P_g(z)\phi_g(z)} \sum_{n=0}^N a_n z^n, \quad f(z) = \frac{1}{P_f(z)\phi_f(z)} \sum_{n=0}^N b_n z^n, \quad \mathcal{F}_1(z) = \frac{1}{P_{\mathcal{F}_1}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^N c_n z^n,$$

One Problem you face as an experimentalist: where do you truncate?

Truncate too soon:

- Model dependence in extracted result for $|V_{cb}|$?

Truncate too late:

- Unnecessarily increase variance on $|V_{cb}|$?

Is there an **ideal** truncation order?

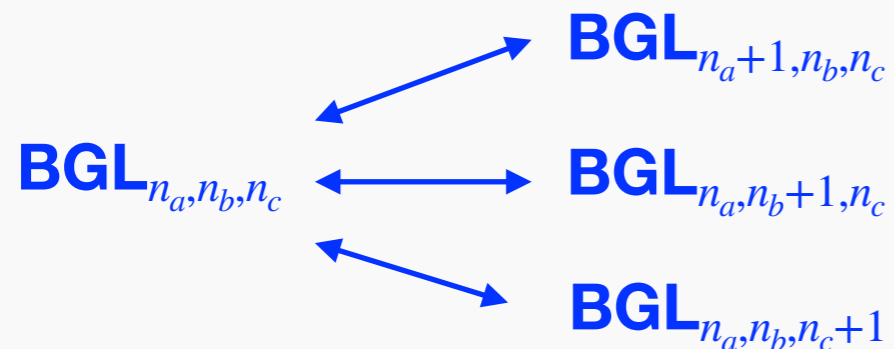
Nested Hypothesis Tests or Saturation Constraints

This work

[arXiv:1902.09553, PRD100,013005 (2019)]

Use a **nested hypothesis test** to determine optimal truncation order

Challenge nested fits



Test statistics & Decision boundary

$$\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 \quad \Delta\chi^2 > 1$$

Distributed like a χ^2 -distribution with 1 dof
(Wilk's theorem)

Gambino, Jung, Schacht

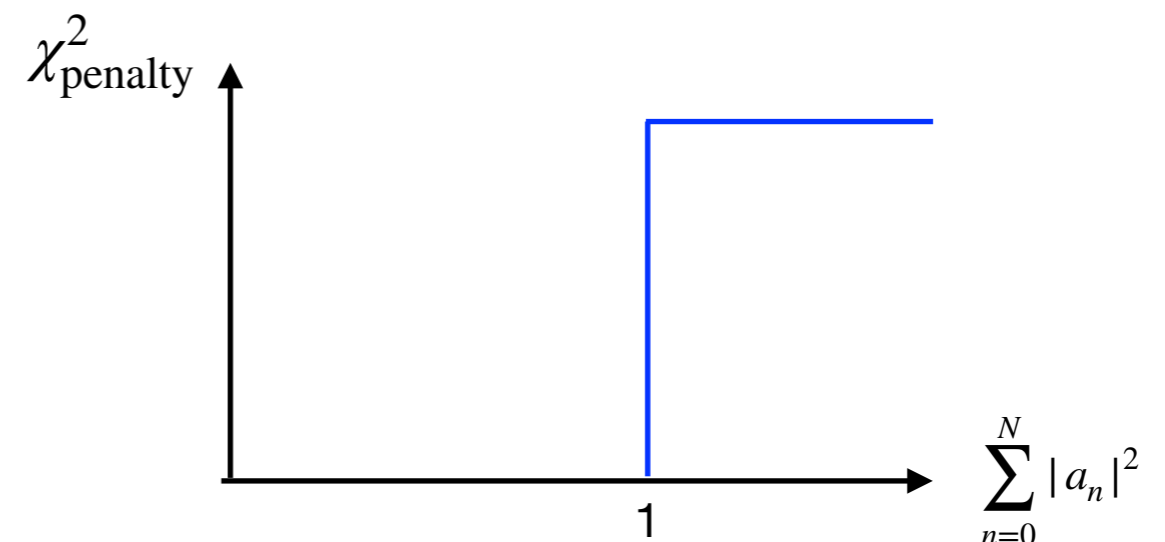
[arXiv:1905.08209, PLB]

Constrain contributions from higher order coefficients using **unitarity bounds**

$$\sum_{n=0}^N |a_n|^2 \leq 1 \quad \sum_{n=0}^N (|b_n|^2 + |c_n|^2) \leq 1$$

e.g.

$$\chi^2 \rightarrow \chi^2 + \chi_{\text{penalty}}^2$$



Nesting Procedure

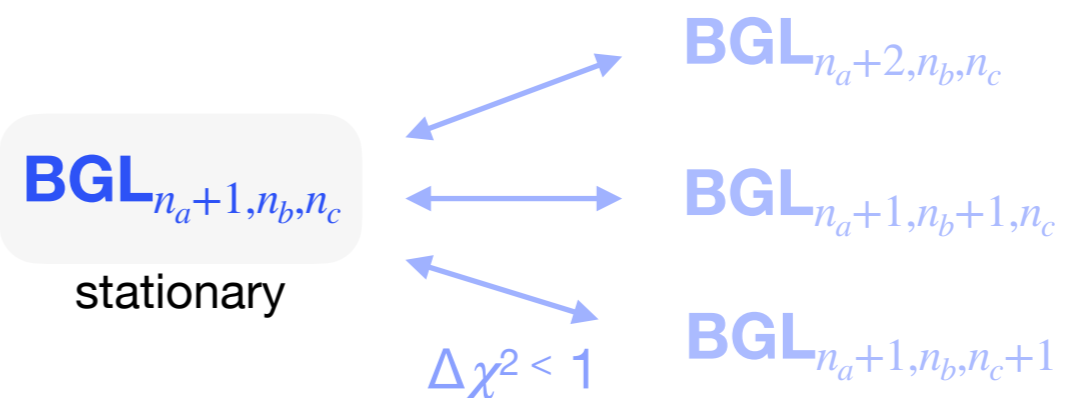
Steps:

1 Carry out nested fits with one parameter added

2 Accept descendant over parent fit, if $\Delta\chi^2 > 1$

3 Repeat 1 and 2 until you find **stationary** points

4 If multiple **stationary** points remain, choose the one with smallest N , then smallest χ^2



Toy study to illustrate possible bias

Use the central values of the **BGL₂₂₂** fit as a starting point to add **fine structure**

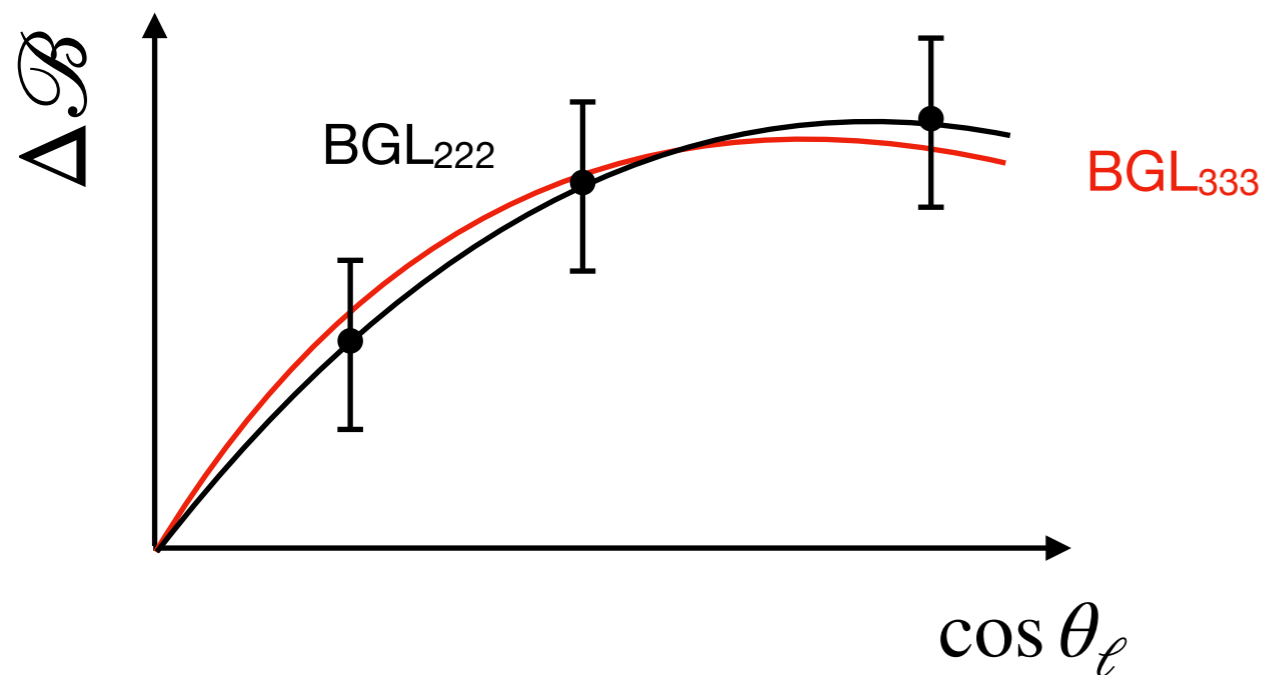


	'1-times'	'10-times'
Parameter	Value $\times 10^2$	Value $\times 10^2$
\tilde{a}_2	2.6954	26.954
\tilde{b}_2	-0.2040	-2.040
\tilde{c}_3	0.5350	5.350



Create a "true" higher order Hypothesis of order **BGL₃₃₃**

Has fine structure element the **current data cannot resolve**



Toy study to illustrate possible bias

Use the central values of the **BGL₂₂₂** fit as a starting point to add **fine structure**

	'1-times'	'10-times'
Parameter	Value $\times 10^2$	Value $\times 10^2$
\tilde{a}_2	2.6954	26.954
\tilde{b}_2	-0.2040	-2.040
\tilde{c}_3	0.5350	5.350

Toy Test

Produce **ensemble** of toy measurements using **untagged covariance** & **BGL₃₃₃** central values

Each toy is fitted to build the descendant tree and carry out a **nested hypo. test** to select its preferred **BGL_{n_an_bn_c}**

Create a "true" higher order Hypothesis of order **BGL₃₃₃**

Has fine structure element the **current data cannot resolve**

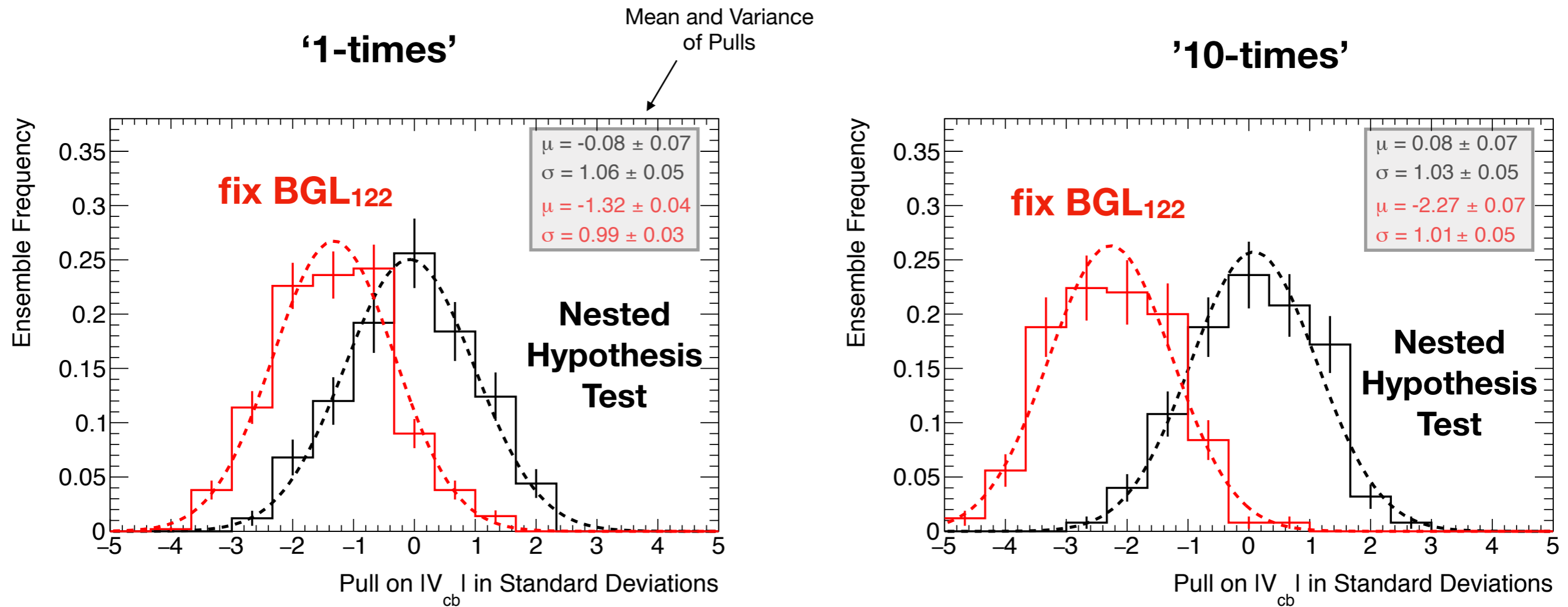
As calculated from selected BGL_{n_an_bn_c} fit of each toy

Construct Pulls

$$\text{Pull} = \frac{|V_{cb}|_{\text{true}} - |V_{cb}|_{\text{toy}}}{\Delta |V_{cb}|_{\text{toy}}}$$

If methodology unbiased, should follow a standard normal distribution (mean 0, width 1)

Bias



→ Procedure produces **unbiased** $|V_{cb}|$ values, **just picking a hypothesis (BGL₁₂₂) does not**

Relative Frequency of selected Hypothesis:

	BGL ₁₂₂	BGL ₂₁₂	BGL ₂₂₁	BGL ₂₂₂	BGL ₂₂₃	BGL ₂₃₂	BGL ₃₂₂	BGL ₂₃₃	BGL ₃₂₃	BGL ₃₃₂	BGL ₃₃₃
1-times	6%	0%	37%	27%	6%	6%	11%	0%	2%	4%	0.4%
10-times	0%	0%	8%	38%	14%	8%	16%	3%	4%	8%	1%

More on the Gap

Model 1:

Equidistribution of all final state particles in phase space

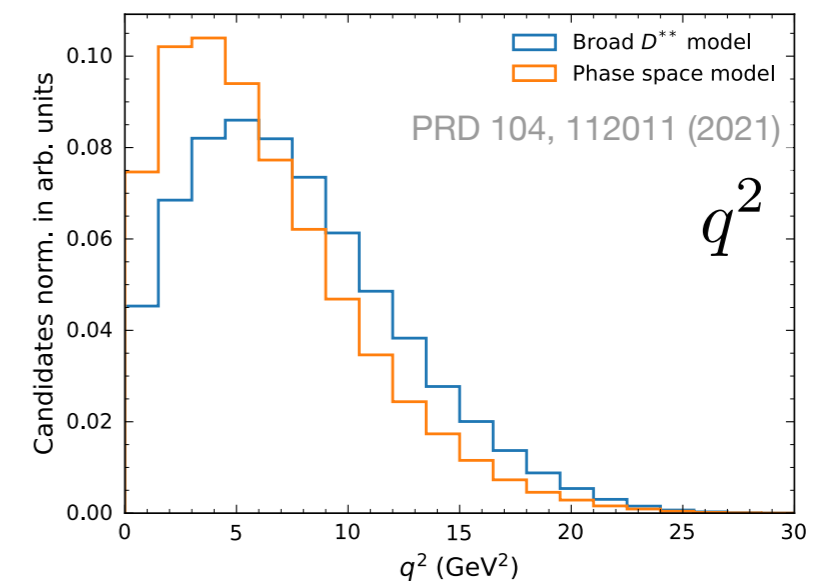
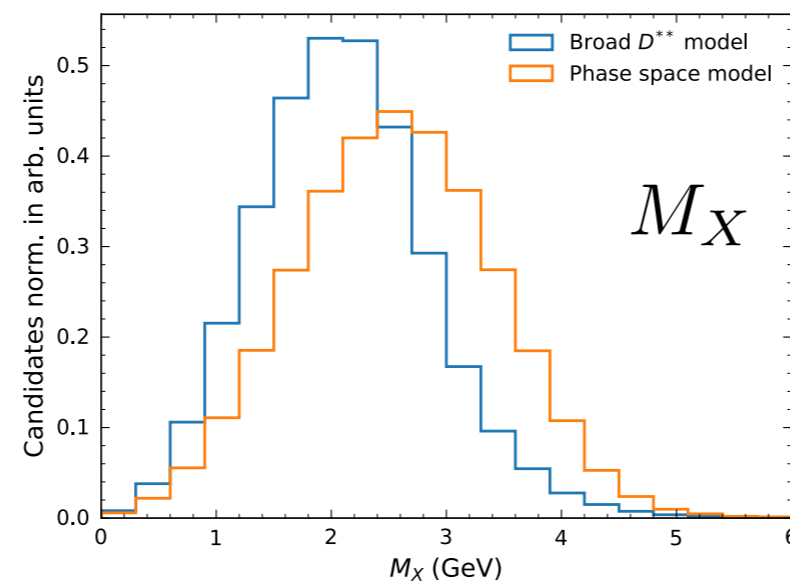
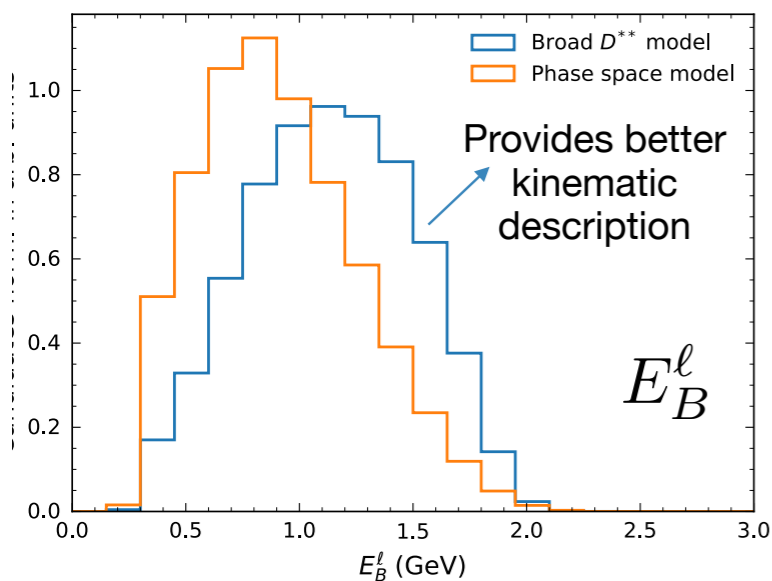
Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D \pi \pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \rightarrow D \eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow D^* \eta \ell^+ \nu_\ell$	$(4.0 \pm 4.0) \times 10^{-3}$	$(4.0 \pm 4.0) \times 10^{-3}$
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

Model 2:

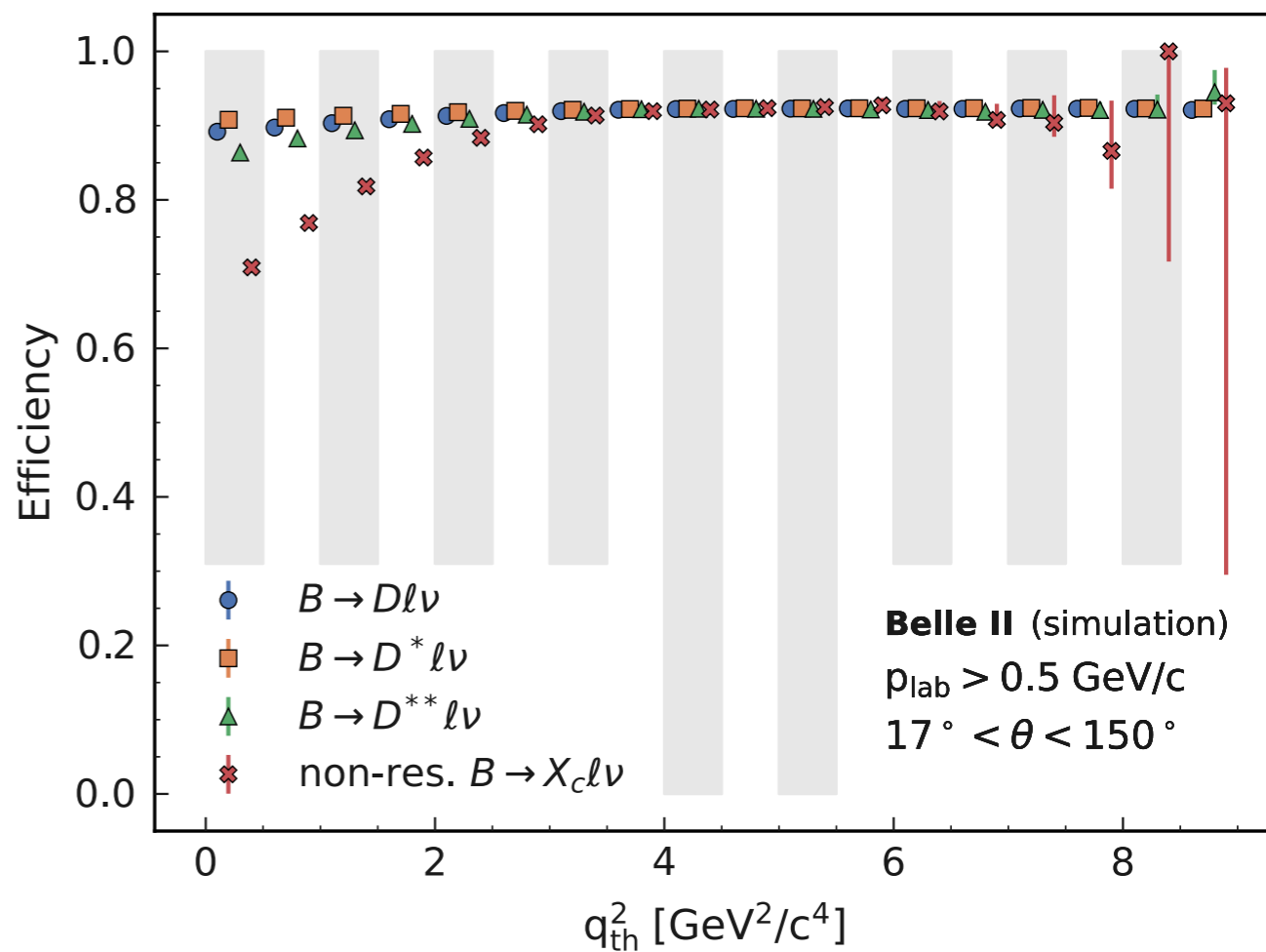
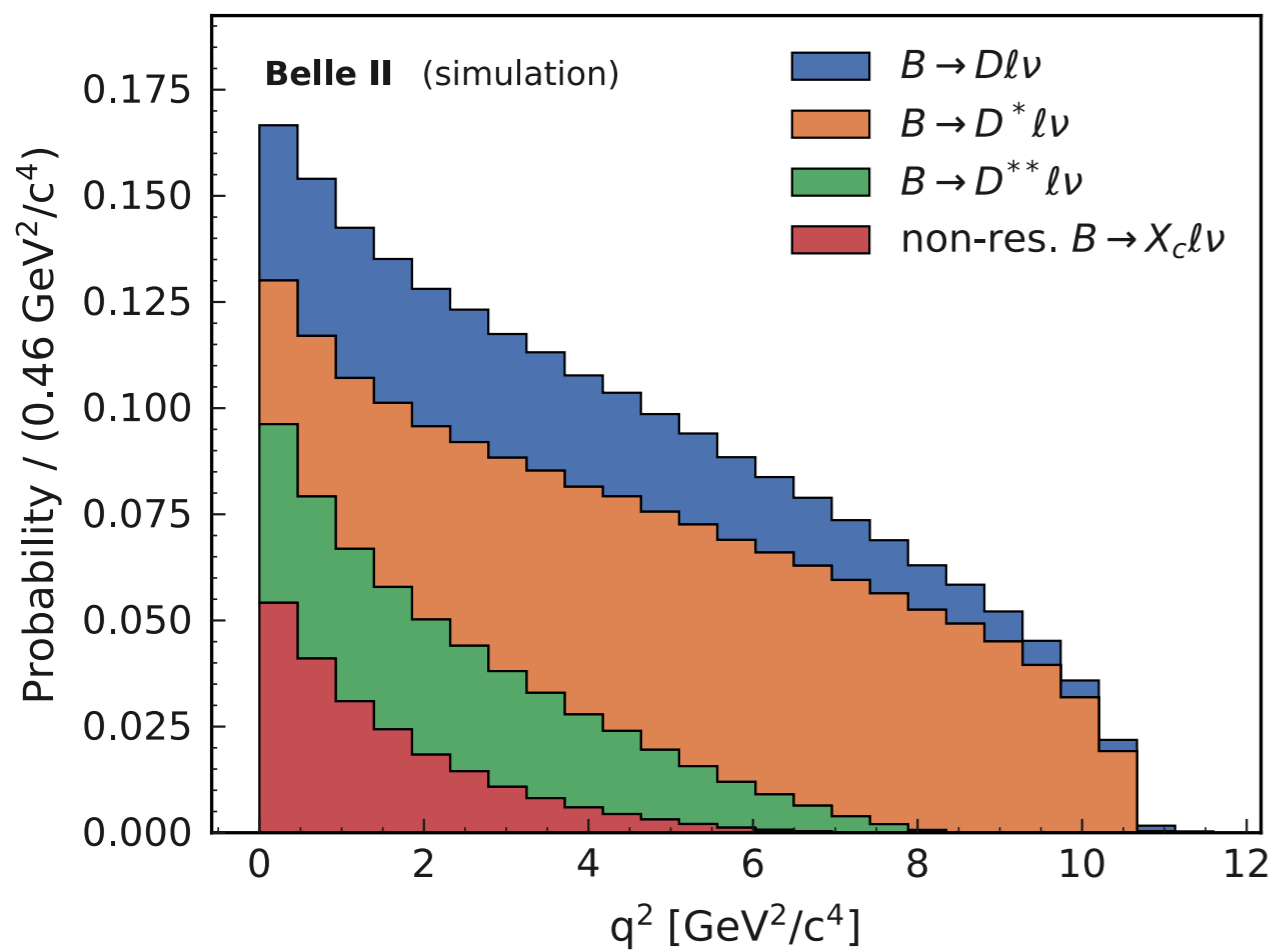
Decay via intermediate broad D^{**} state

Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D \pi \pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \rightarrow D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D \pi \pi$)	$(0.03 \pm 0.03) \times 10^{-2}$	$(0.03 \pm 0.03) \times 10^{-2}$
$B \rightarrow D_0^* \pi \pi \ell^+ \nu_\ell$ ($\hookrightarrow D^* \pi \pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \rightarrow D_1^* \pi \pi \ell^+ \nu_\ell$ ($\hookrightarrow D^* \pi \pi$)	$(0.108 \pm 0.051) \times 10^{-2}$	$(0.101 \pm 0.048) \times 10^{-2}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$ ($\hookrightarrow D \eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$
$B \rightarrow D_1^* \ell^+ \nu_\ell$ ($\hookrightarrow D^* \eta$)	$(0.396 \pm 0.396) \times 10^{-2}$	$(0.399 \pm 0.399) \times 10^{-2}$

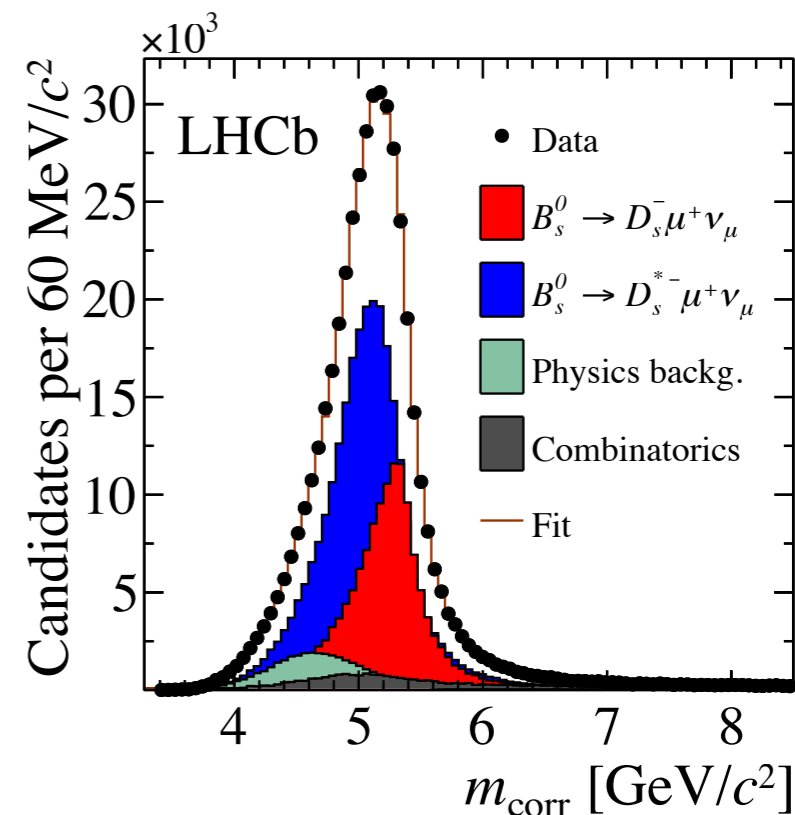
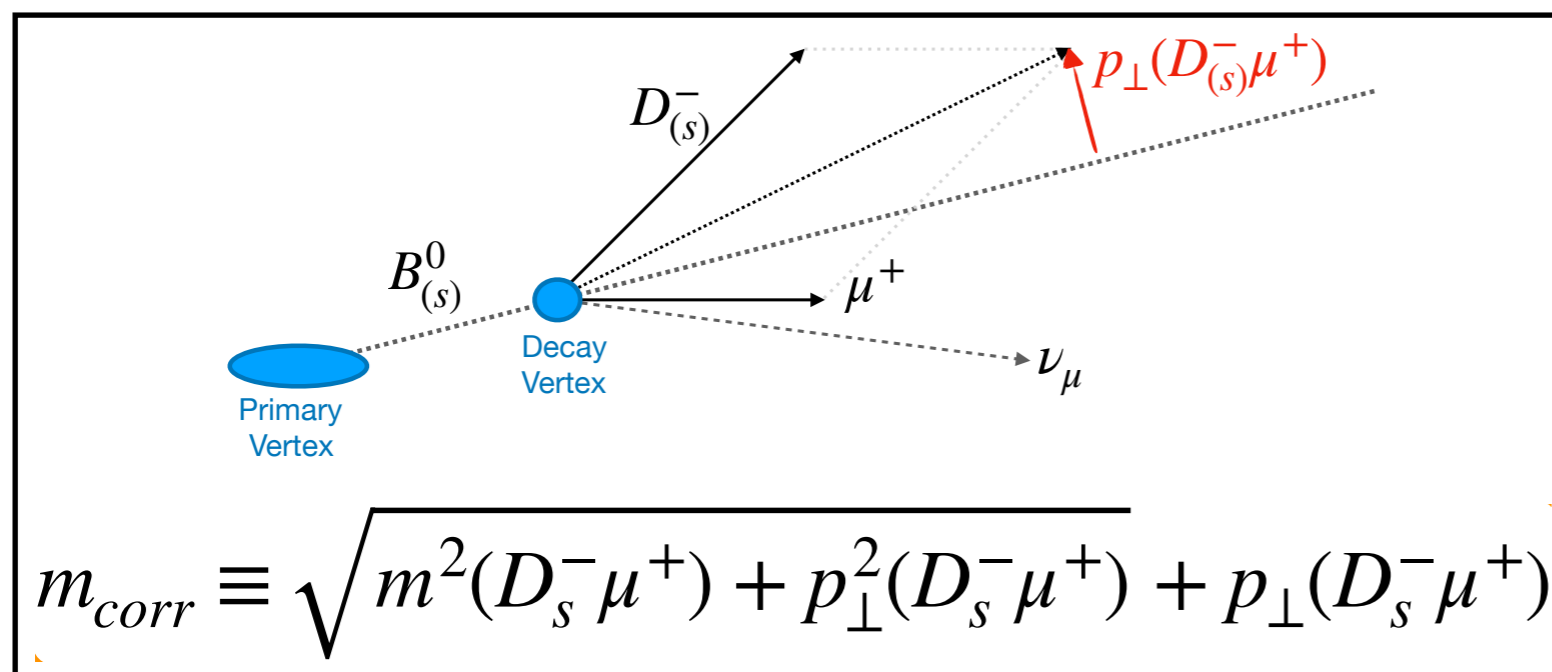
(Assign 100% BR uncertainty in systematics covariance matrix)



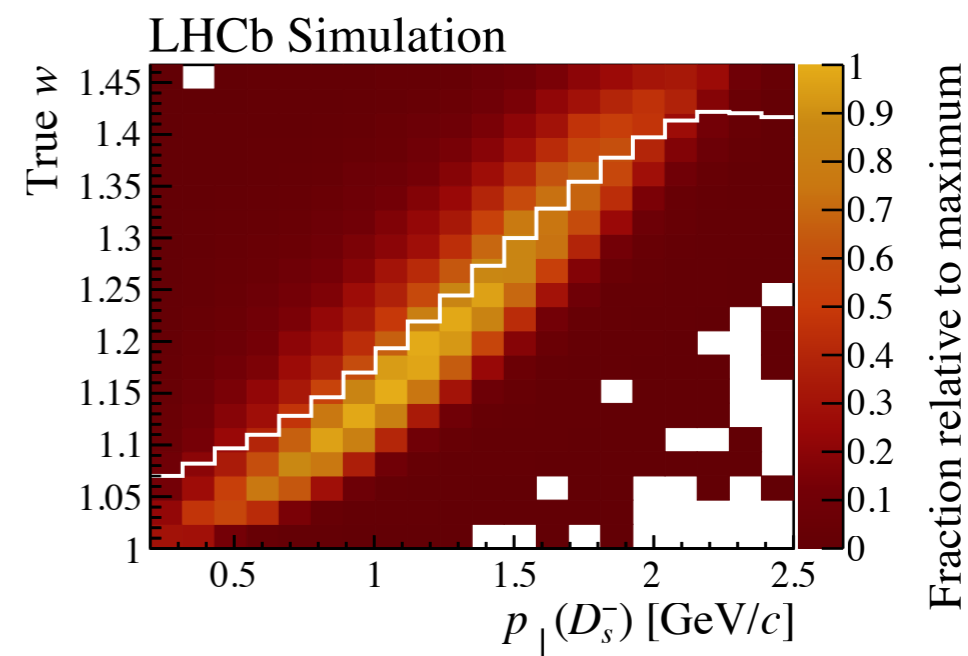
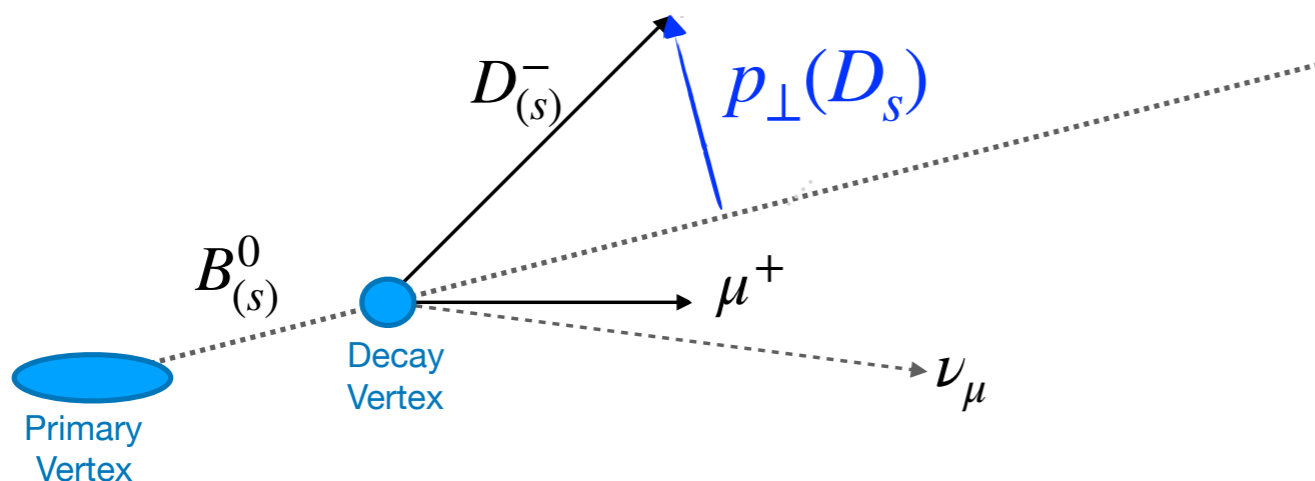
X_c Simulation



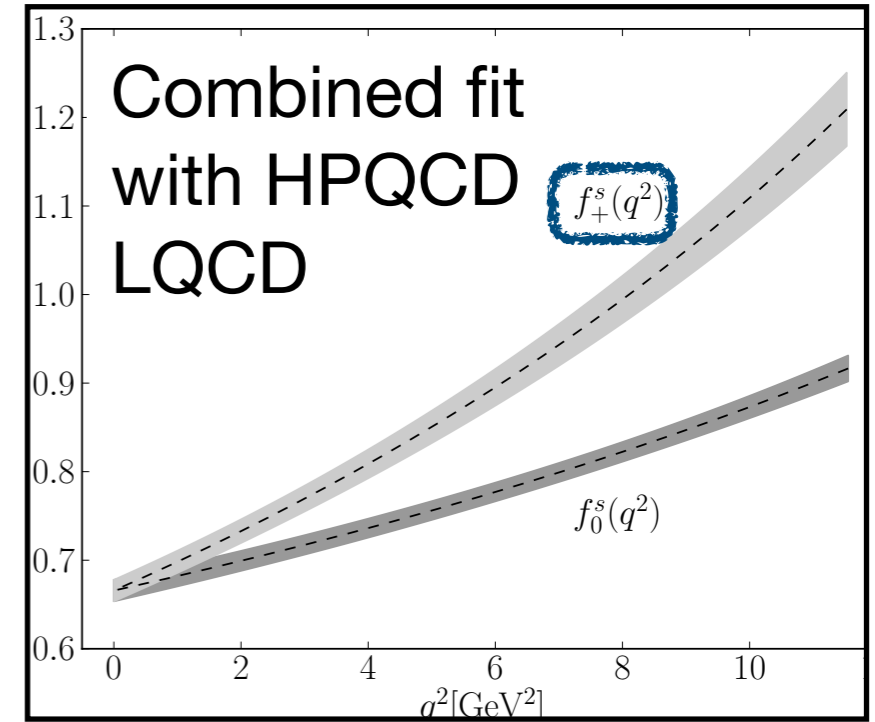
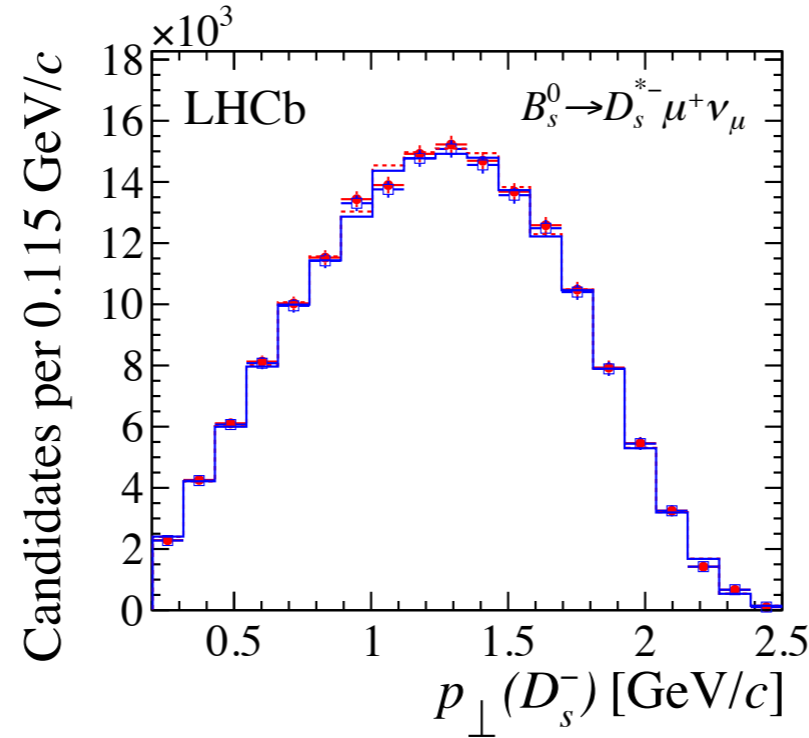
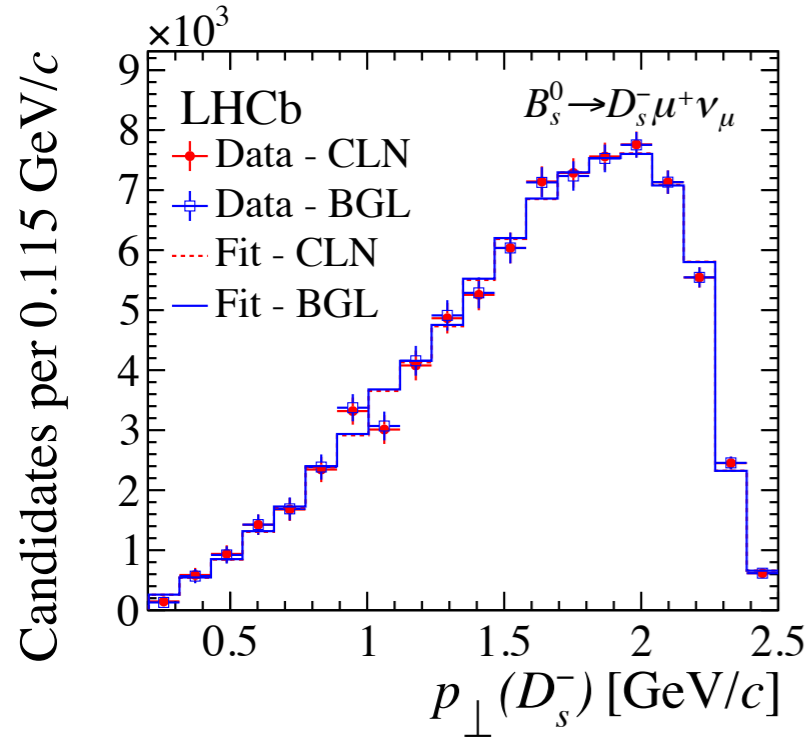
Leverage large **separation** of **decay vertex** from **primary vertex** to reconstruct B_s **flight direction**; reconstruct *corrected* mass m_{corr} :



Exploit $p_\perp(D_s)$ correlation with w to fit form factors

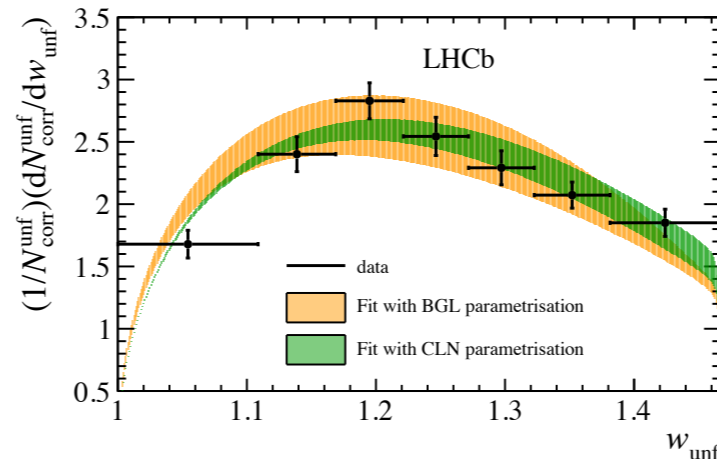


Background subtracted and fitted distributions:



→ $|V_{cb}|_{\text{BGL}} = (41.7 \pm 0.8(\text{stat}) \pm 0.9(\text{syst}) \pm 1.1(\text{ext})) \times 10^{-3}$

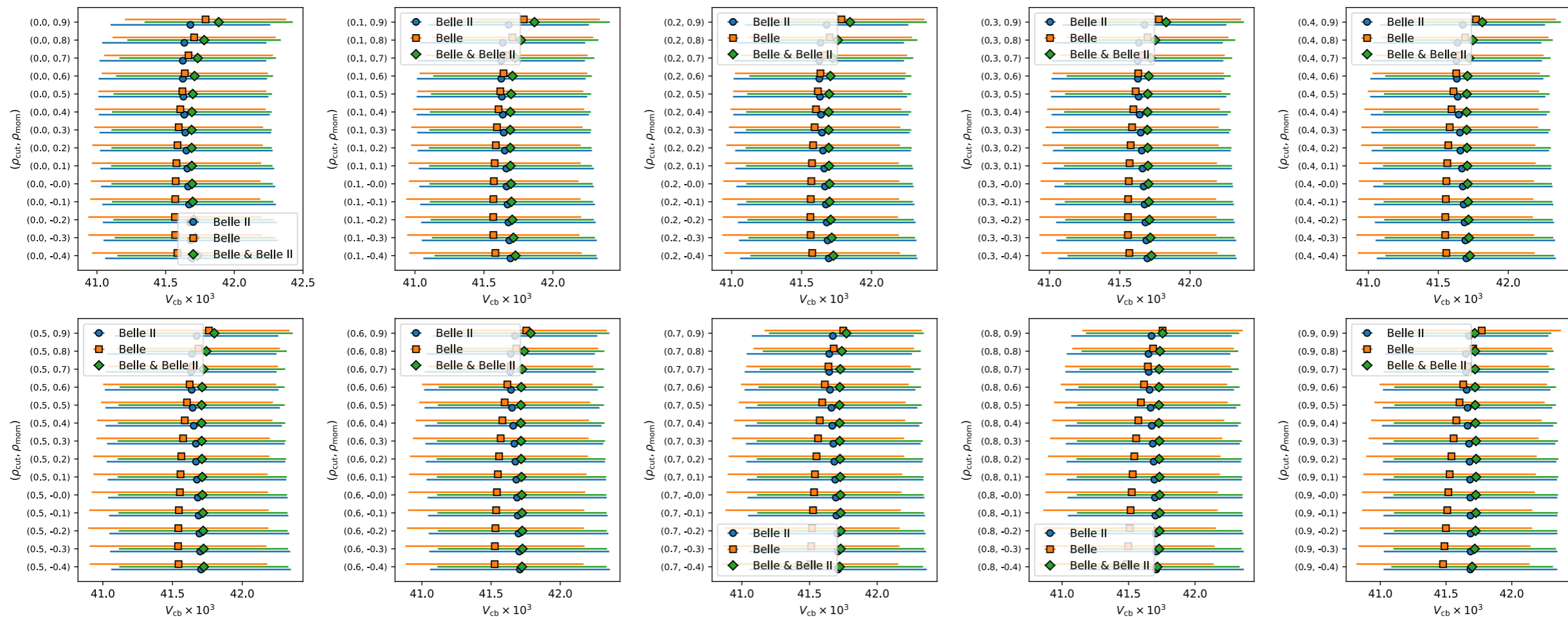
Also provide unfolded w spectrum for $B_s \rightarrow D_s^* \mu \bar{\nu}_\mu$

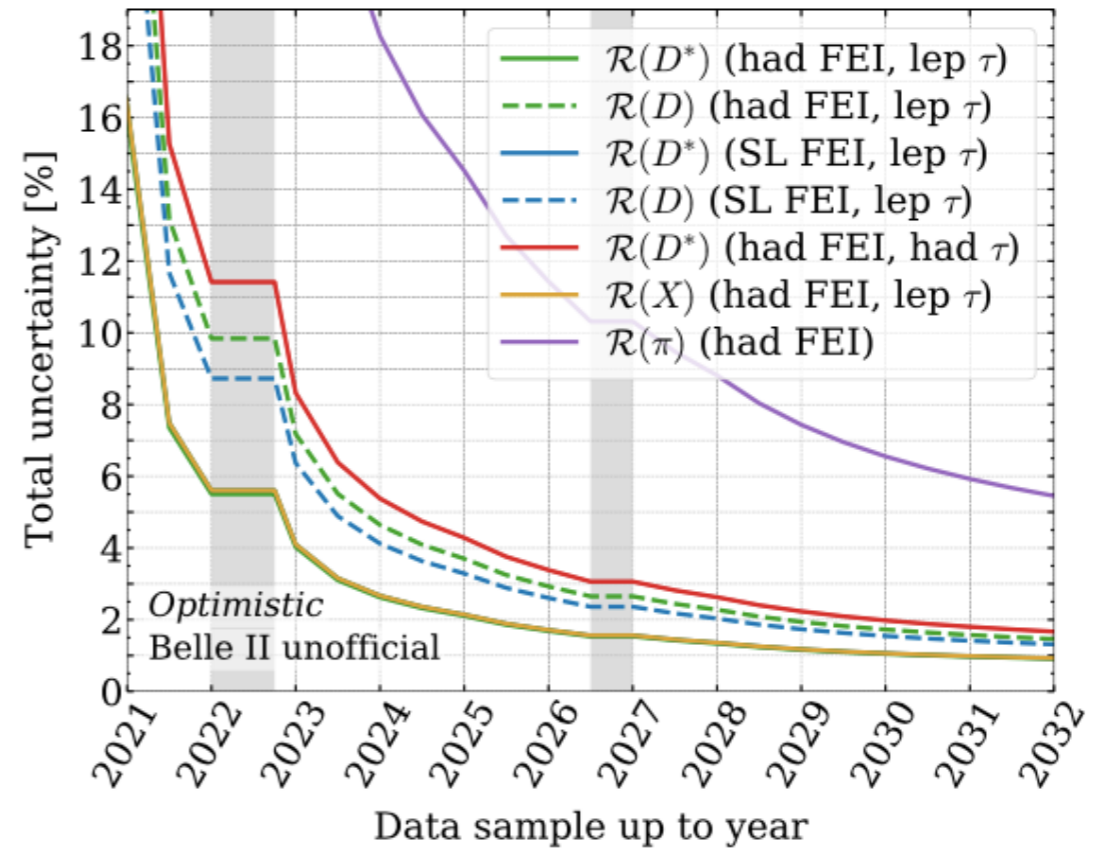
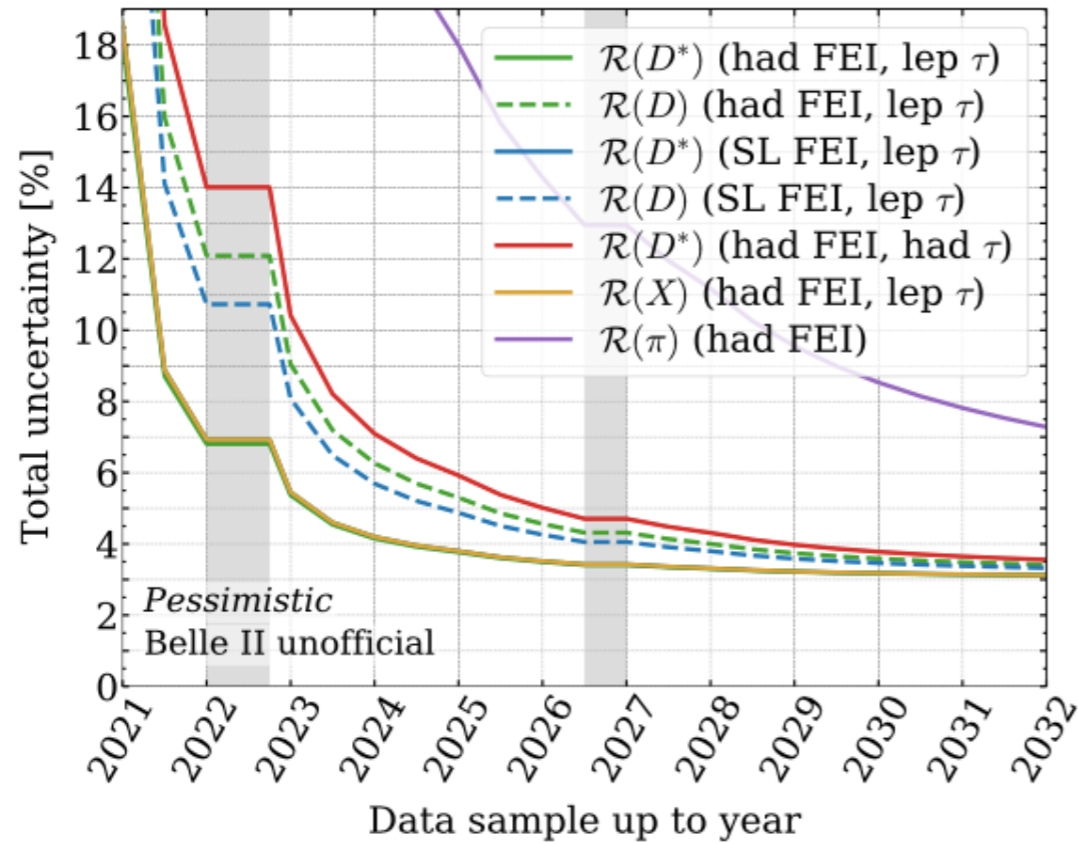
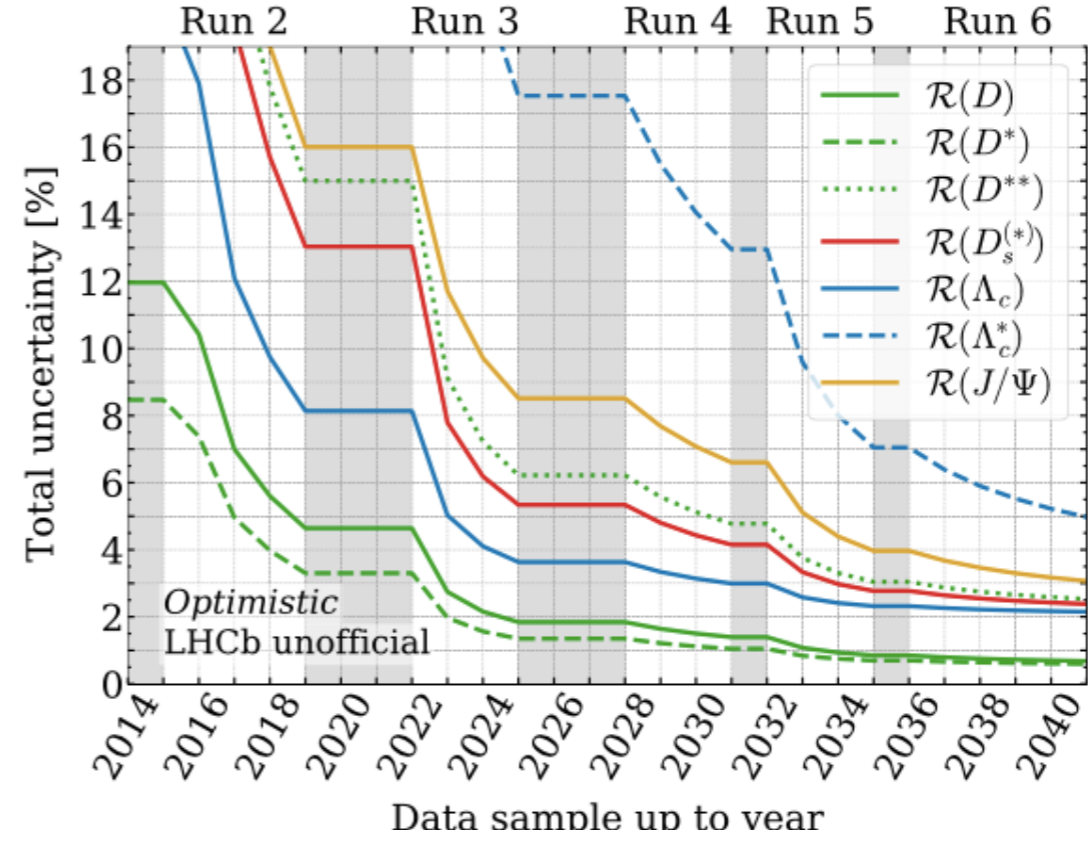
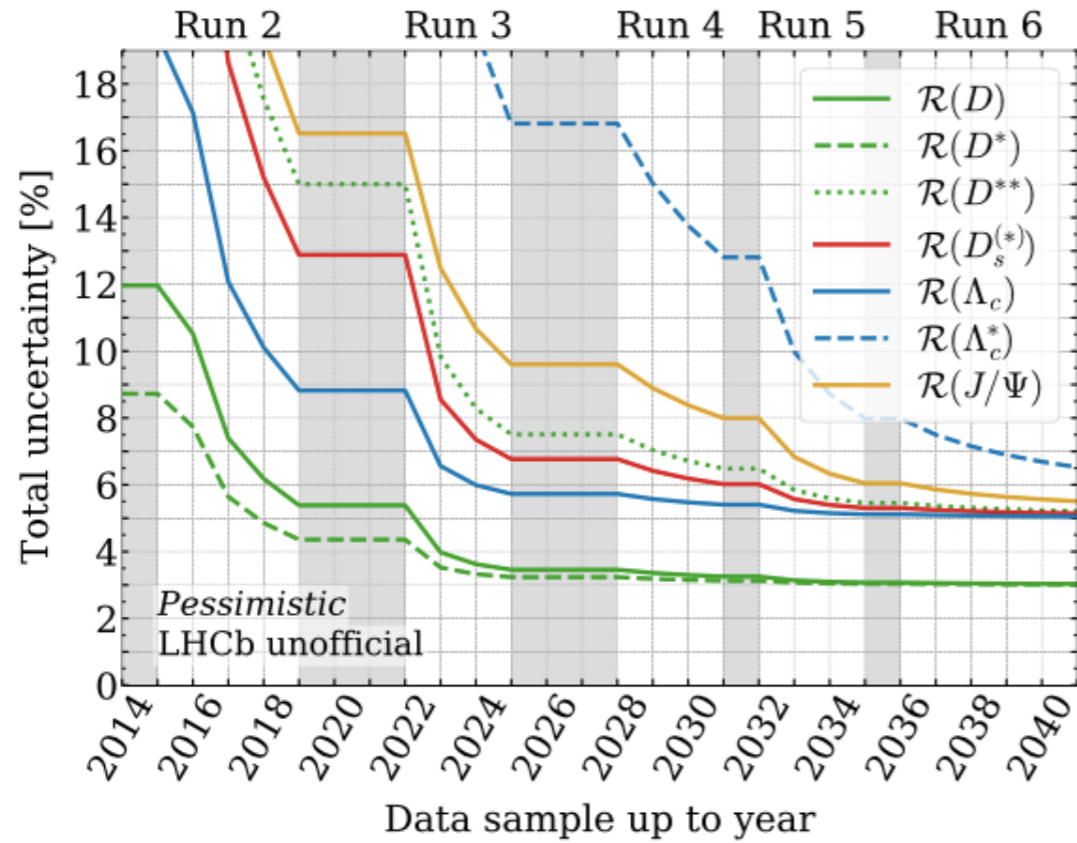


Theory Correlations in inclusive $|V_{cb}|$

$$\rho_n[q_n(q_A^2), q_n(q_B^2)] = \rho_{\text{cut}}^x \quad \text{with} \quad x = \frac{|q_A^2 - q_B^2|}{0.5 \text{ GeV}^2}.$$

$$\rho_{nm}[q_m(q_A^2), q_n(q_B^2)] = \text{sign}(\rho_{\text{mom}}) \cdot |\rho_{\text{mom}}|^{|m-n|} \cdot \rho_n(q_n(q_A^2), q_n(q_B^2)).$$





Meet the “Measurement Matrix”

Hadronic
or
inclusive
tagging

SL
tagging

Leptonic
 τ

Hadronic
 τ

✓	✓
✓	✗

Belle:
Phys.Rev.Lett.118,211801 (2017)
Phys. Rev. D 97, 012004 (2018)
(D* had tag)

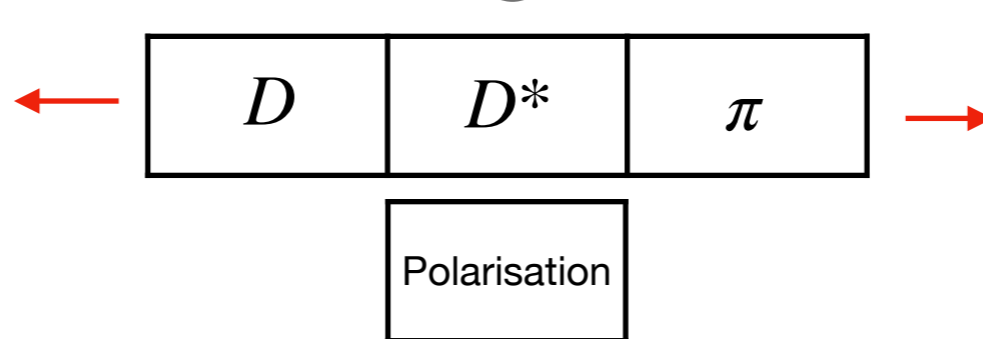


Polarisation

LHCb:
Phys.Rev.Lett.115,111803 (2015)
(D*, Leptonic τ)
Phys.Rev.D 97, 072013 (2018)
Phys.Rev.Lett.120,171802 (2018)
(D*, Hadronic τ)



$q^2 = (p_B - p_{D^{(*)}})^2$	$p_{D^*} \quad p_\ell$
-------------------------------	------------------------



Belle:
Phys.Rev.D 92, 072014 (2015)
(D/D* had tag, q^2)
Phys.Rev. D94,072007 (2016)
(D*, SL tag, p_{D^*} , p_l)

Belle:
Phys. Rev. D 93, 032007 (2016)
(π had tag)

BaBar:
Phys.Rev.Lett. 109,101802 (2012)
Phys.Rev.D 88, 072012 (2013)
(D/D* had tag, q^2)

Prel. Belle: <https://arxiv.org/pdf/1901.06380.pdf> (D*, incl. tagging)

& older work, e.g.
Belle:
Phys.Rev. D82 (2010) 072005
(D/D* incl. tag)

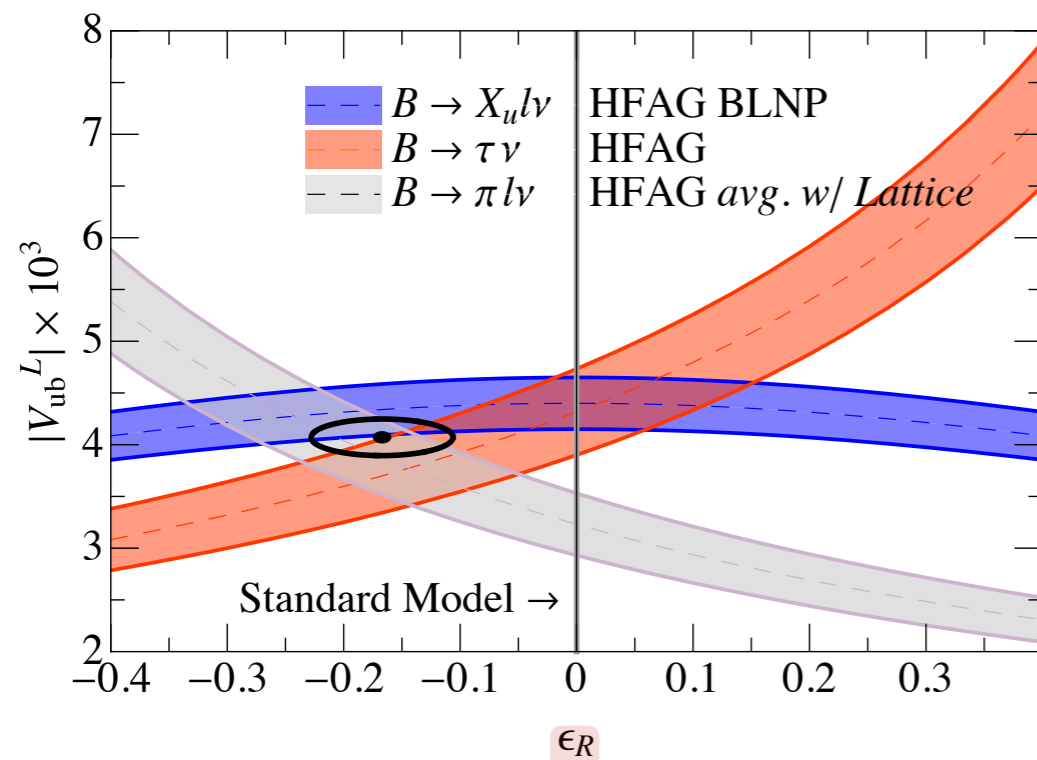
The two categories of measurements

1st Category

Measurements that have **no** or **trivial** or **negligible** dependence on parameter of interest

Example: **Right-handed currents** & $|V_{ub}|$

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ub}^L (\bar{u}\gamma_\mu P_L b + \epsilon_R \bar{u}\gamma_\mu P_R b) (\bar{\nu}\gamma^\mu P_L \ell) + \text{h.c.},$$



Decay	$ V_{ub} \times 10^3$	ϵ_R dependence
$B \rightarrow \pi \ell \bar{\nu}$	3.23 ± 0.30	$1 + \epsilon_R$
$B \rightarrow X_u \ell \bar{\nu}$	4.39 ± 0.21	$\sqrt{1 + \epsilon_R^2}$
$B \rightarrow \tau \bar{\nu}_\tau$	4.32 ± 0.42	$1 - \epsilon_R$

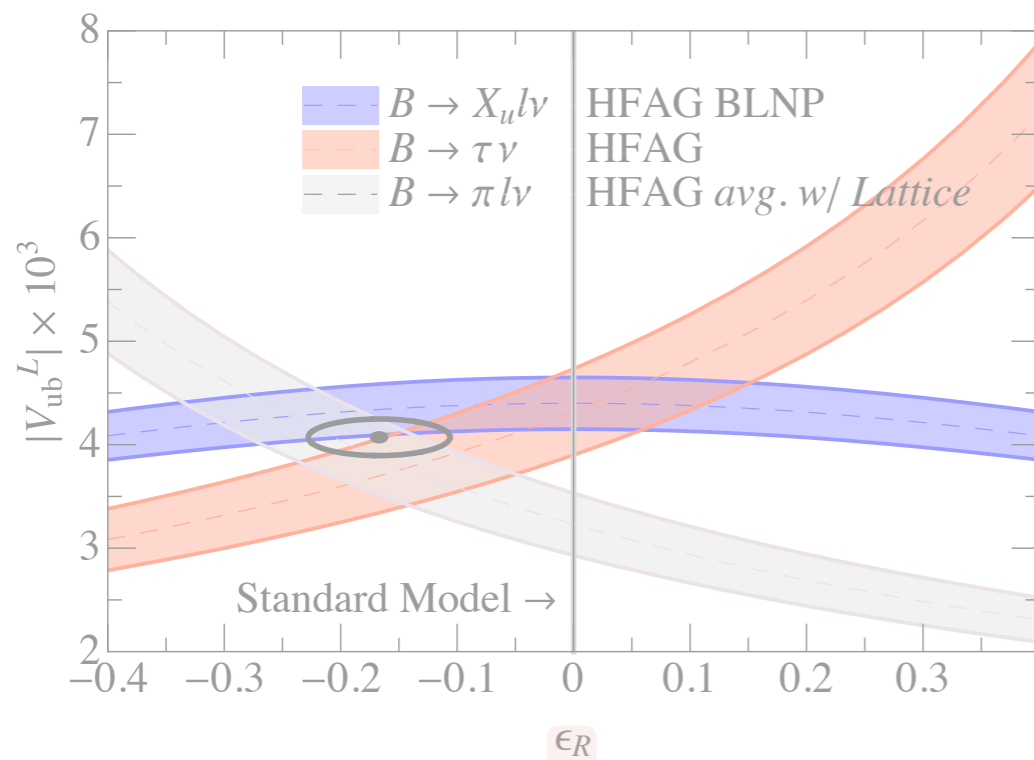
The two categories of measurements

1st Category

Measurements that have **no** or **trivial** or **negligible** dependence on parameter of interest

Example: **Right-handed currents & $|V_{ub}|$**

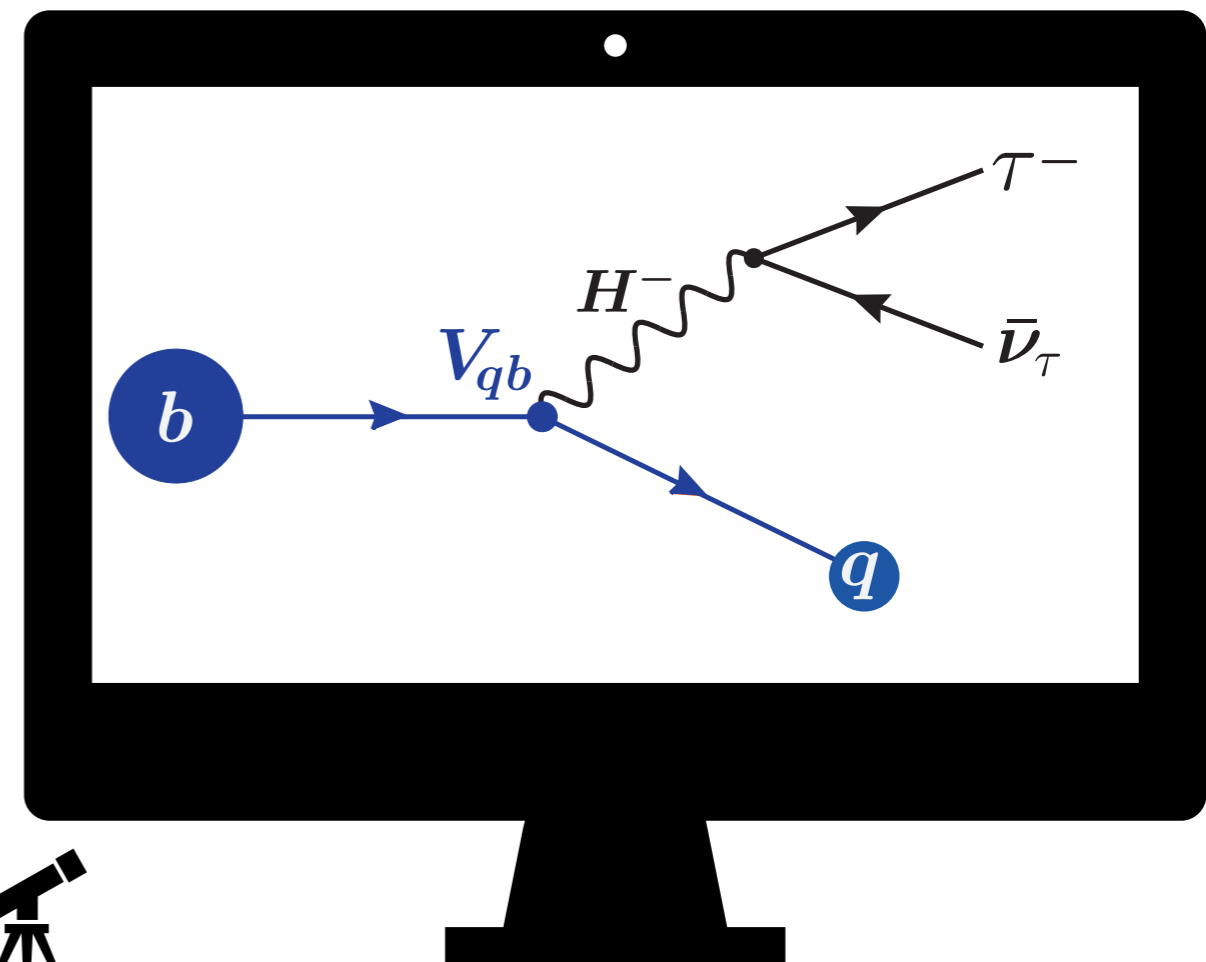
$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ub}^L (\bar{u}\gamma_\mu P_L b + \epsilon_R \bar{u}\gamma_\mu P_R b) (\bar{\nu}\gamma^\mu P_L \ell) + \text{h.c.},$$



Decay	$ V_{ub} \times 10^3$	ϵ_R dependence
$B \rightarrow \pi l \bar{\nu}$	3.23 ± 0.30	$1 + \epsilon_R$
$B \rightarrow X_u l \bar{\nu}$	4.39 ± 0.21	$\sqrt{1 + \epsilon_R^2}$
$B \rightarrow \tau \bar{\nu}_\tau$	4.32 ± 0.42	$1 - \epsilon_R$

2nd Category

Measurements that have **non-trivial** dependence on parameter of interest / other params.



- ▶ Let's say you want to use the **measured $R(D^{(*)})$ ratios** to learn something about the anomaly and **your favorite model** that could explain it!

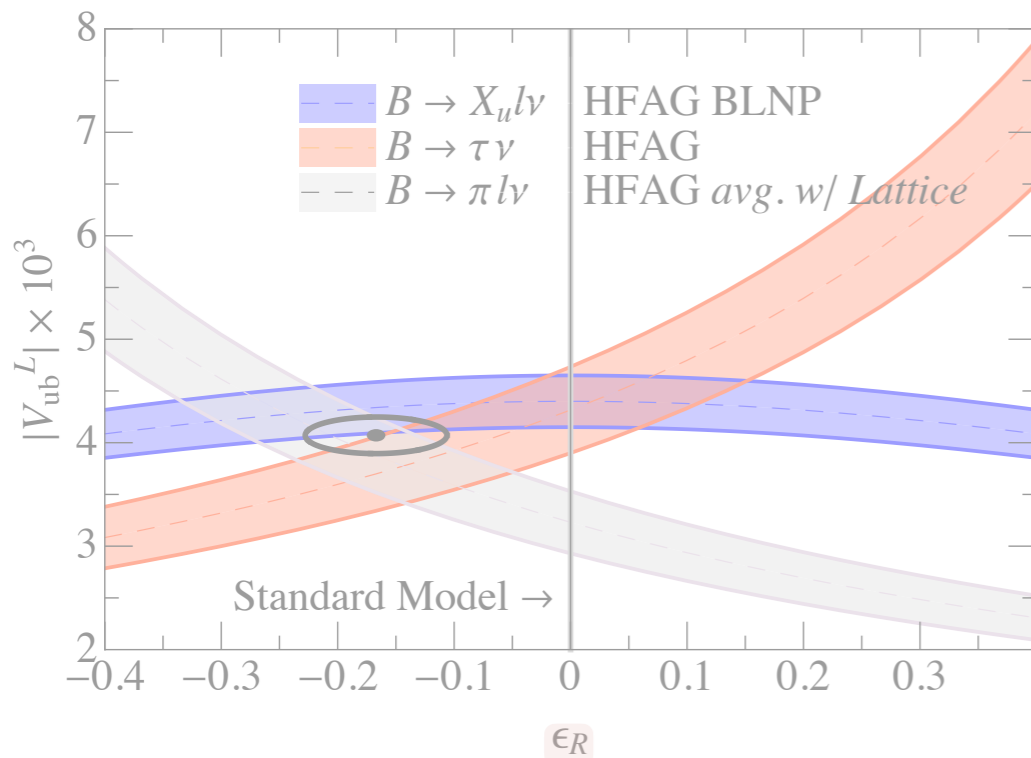
The two categories of measurements

1st Category

Measurements that have **no** or **trivial** or **negligible** dependence on parameter of interest

Example: **Right-handed currents & $|V_{ub}|$**

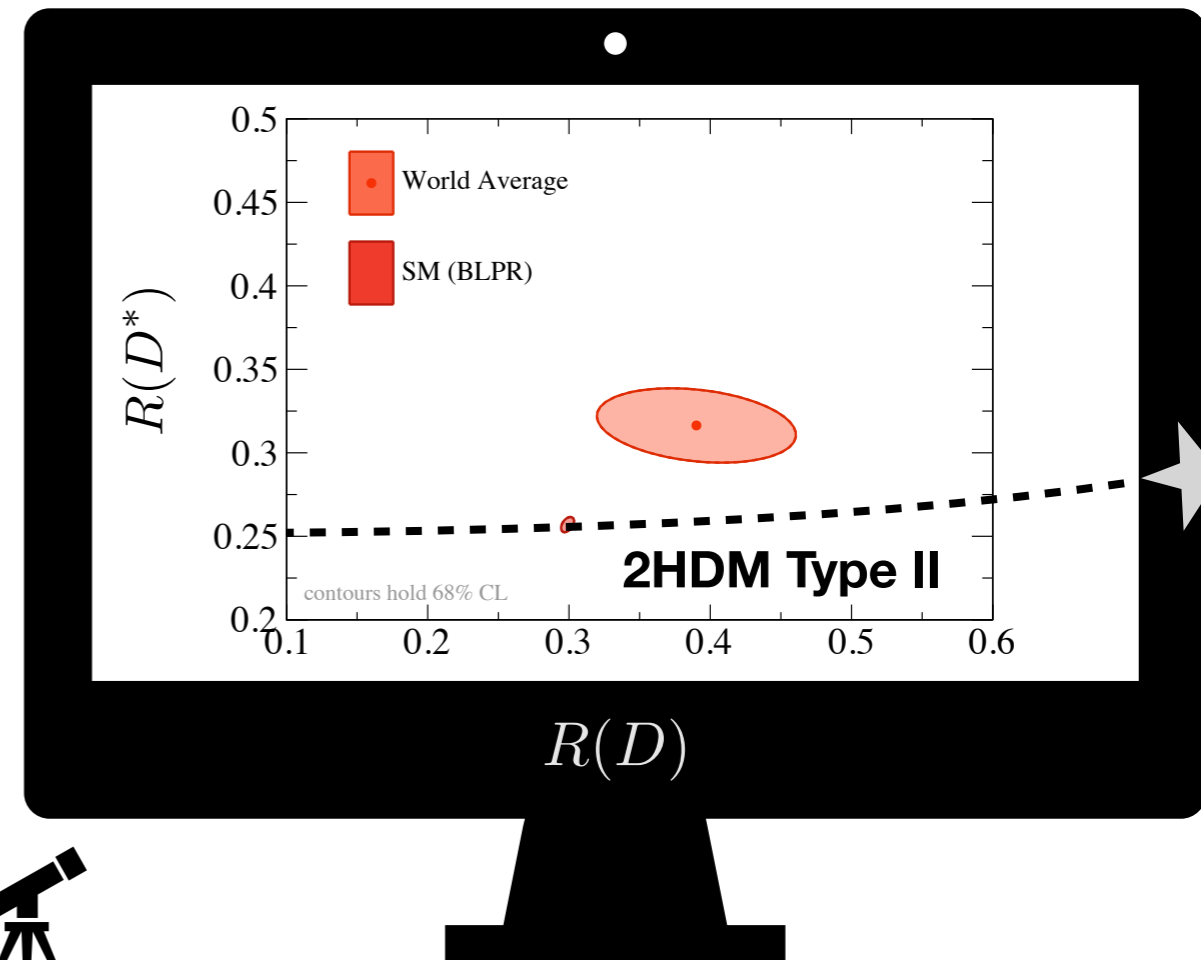
$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ub}^L (\bar{u}\gamma_\mu P_L b + \epsilon_R \bar{u}\gamma_\mu P_R b) (\bar{\nu}\gamma^\mu P_L \ell) + \text{h.c.},$$



Decay	$ V_{ub} \times 10^3$	ϵ_R dependence
$B \rightarrow \pi \ell \bar{\nu}$	3.23 ± 0.30	$1 + \epsilon_R$
$B \rightarrow X_{u\ell} \bar{\nu}$	4.39 ± 0.21	$\sqrt{1 + \epsilon_R^2}$
$B \rightarrow \tau \bar{\nu}_\tau$	4.32 ± 0.42	$1 - \epsilon_R$

2nd Category

Measurements that have **non-trivial** dependence on parameter of interest / other params.



- ▶ Let's say you want to use the **measured $R(D^{(*)})$ ratios** to learn something about the anomaly and **your favorite model** that could explain it!

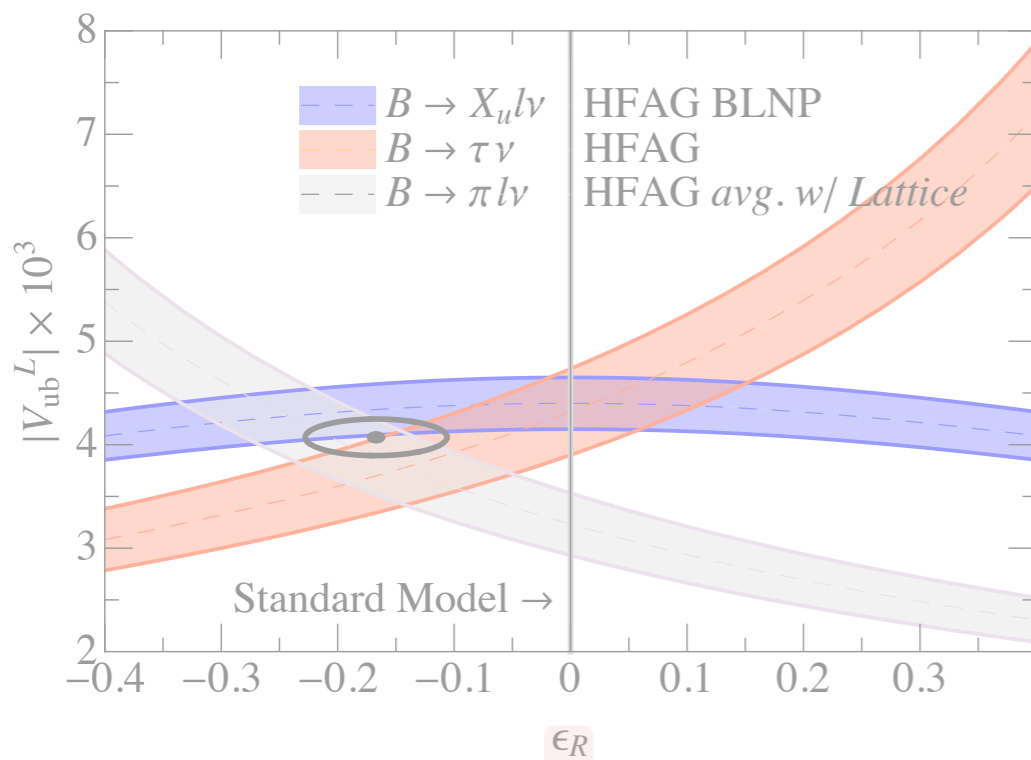
The two categories of measurements

1st Category

Measurements that have **no** or **trivial** or **negligible** dependence on parameter of interest

Example: **Right-handed currents & $|V_{ub}|$**

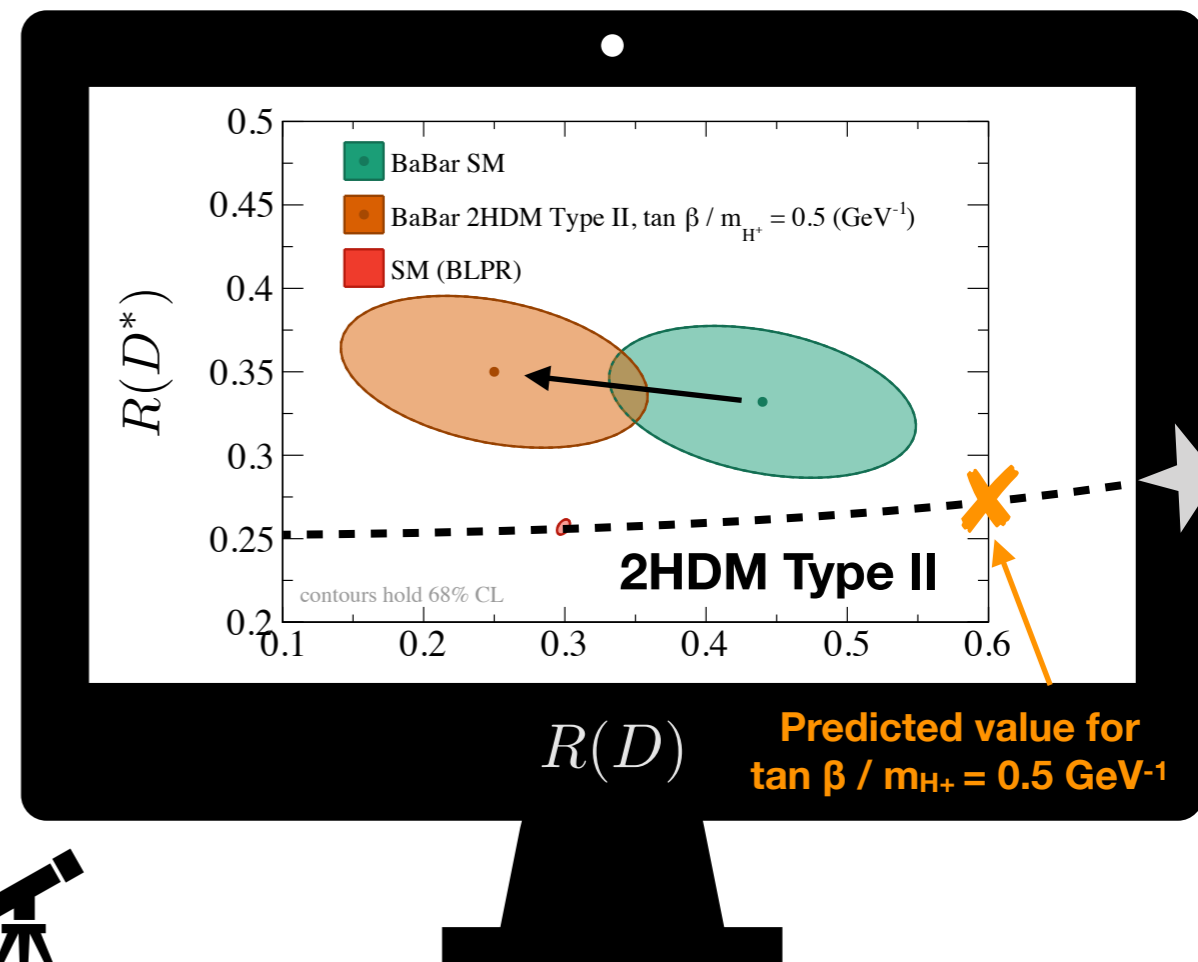
$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ub}^L (\bar{u}\gamma_\mu P_L b + \epsilon_R \bar{u}\gamma_\mu P_R b) (\bar{\nu}\gamma^\mu P_L \ell) + \text{h.c.},$$



Decay	$ V_{ub} \times 10^3$	ϵ_R dependence
$B \rightarrow \pi \ell \bar{\nu}$	3.23 ± 0.30	$1 + \epsilon_R$
$B \rightarrow X_{u\ell} \bar{\nu}$	4.39 ± 0.21	$\sqrt{1 + \epsilon_R^2}$
$B \rightarrow \tau \bar{\nu}_\tau$	4.32 ± 0.42	$1 - \epsilon_R$

2nd Category

Measurements that have **non-trivial** dependence on parameter of interest / other params.



- As it turns out, **not that easy** – the **measured points** themselves are **extracted assuming the SM** and kinematic distributions sensitive to the Pol are altering the measurement

NP Interpretation Strategies for $H_b \rightarrow H_c \tau \bar{\nu}$

What you
can do today

#1

Just fit ratios, hope that **bias** is small with respect to the current precision

Frankly a perfectly sane strategy; after all the experiments do not provide any other information one could use and not all measurements might have such a strong dependence as e.g. BaBar

What we should
allow you to do

#2

Fold your model into the MC simulation, directly confront the data

#3

Provide theorists with direct measurements of Wilson coefficients; these can be used to confront your favorite model

a fairly prominent problem

SciPost Physics

Submission

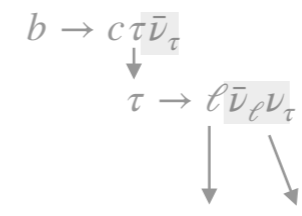
WORKING DRAFT

Publishing statistical models: Getting the most out of particle physics experiments

1
2
3
4 Kyle Cranmer^{1*}, Sabine Kraml^{2†}, Harrison B. Prosper^{3§} (editors),
5 Philip Bechtle⁴, Florian U. Bernlochner⁴, Itay M. Bloch⁵, Enzo Canonero⁶,
6 Marcin Chrzaszcz⁷, Andrea Coccaro⁸, Glen Cowan⁹, Matthew Feickert¹⁰, Nahuel
7 Ferreiro Iachellini^{11,12}, Andrew Fowlie¹³, Lukas Heinrich¹⁴, Alexander Held¹,
8 Thomas Kuhr^{12,15}, Anders Kvellestad¹⁶, Maeve Madigan¹⁷, Farvah Mahmoudi^{14,18},
9 Knut Dundas Morã¹⁹, Mark S. Neubauer¹⁰, Maurizio Pierini¹⁴, Juan Rojo⁸,
10 Sezen Sekmen²¹, Luca Silvestrini²², Veronica Sanz^{23,24}, Giordon Stark²⁵,
11 Riccardo Torre⁸, Robert Thorne²⁶, Wolfgang Waltenberger²⁷, Nicholas Wardle²⁸,
12 Jonas Wittbrodt²⁹

[to appear soon]

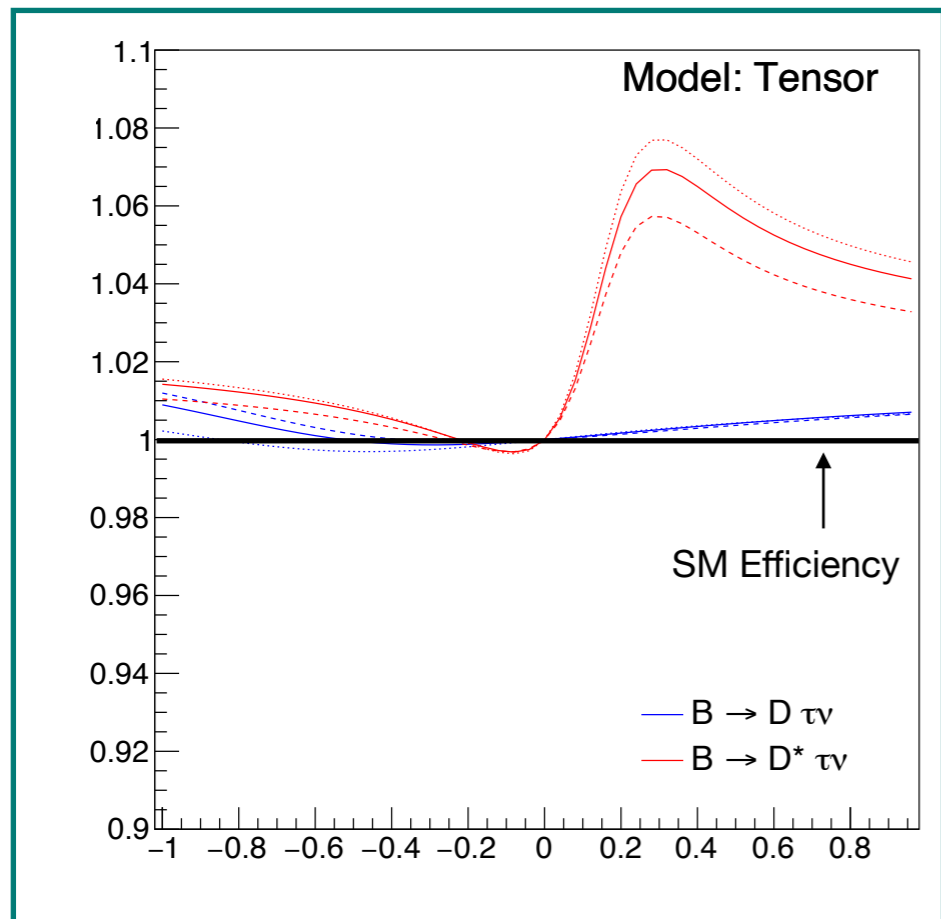
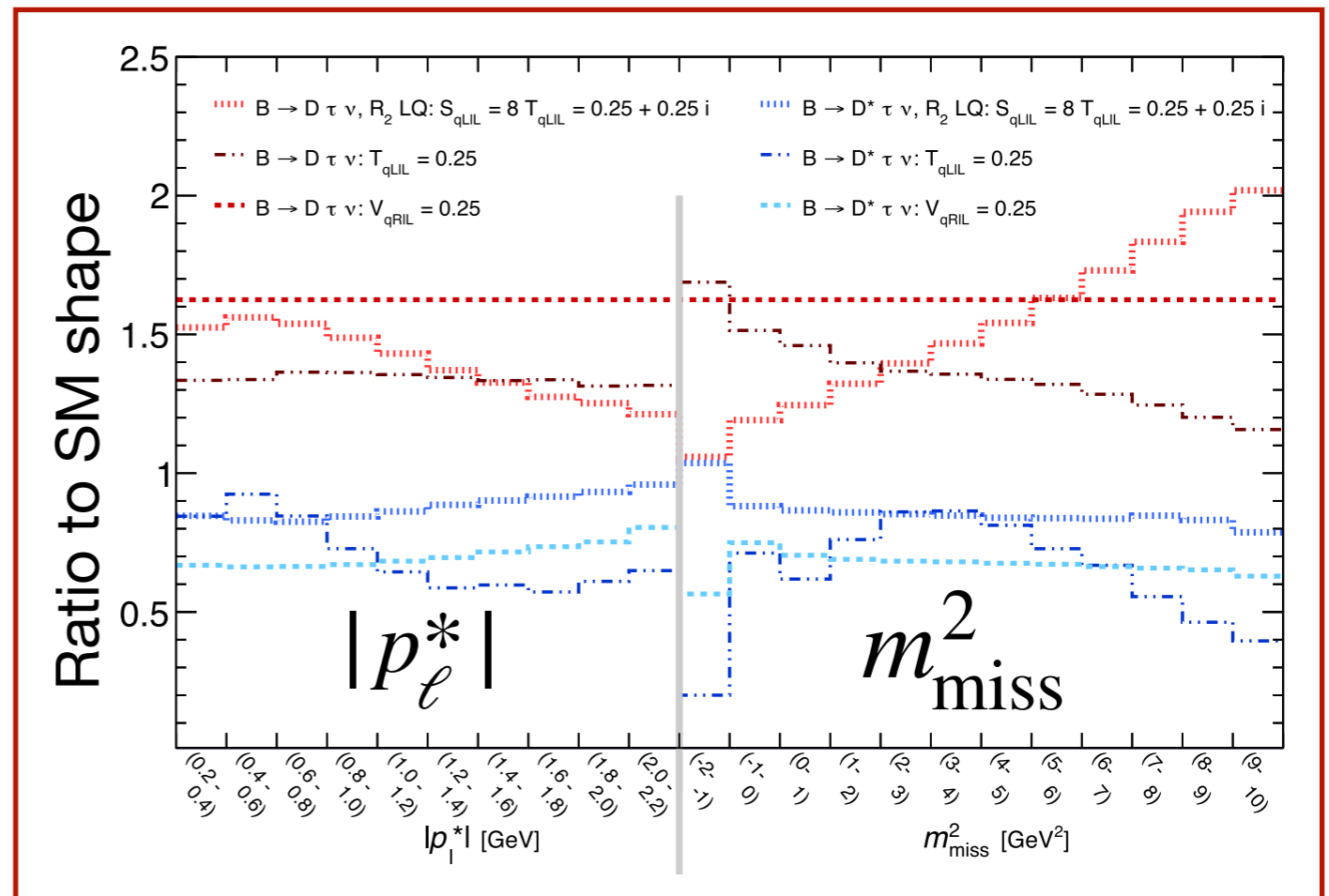
Benefit: no biases, more sensitivity as shape of **all** kinematic distributions help distinguish between models



Use **kinematic quantities** (e.g. $|p_\ell^*|$, m_{miss}^2 , q^2)
to **subtract background**

$$\mathcal{R}(D^{(*)}) = \frac{N_{\text{sig}}}{N_{\text{norm}}} \times \frac{\epsilon_{\text{norm}}}{\epsilon_{\text{sig}}}$$

Assume **SM** acceptance x efficiency



C_T

Slightly dramatic example of what could happen

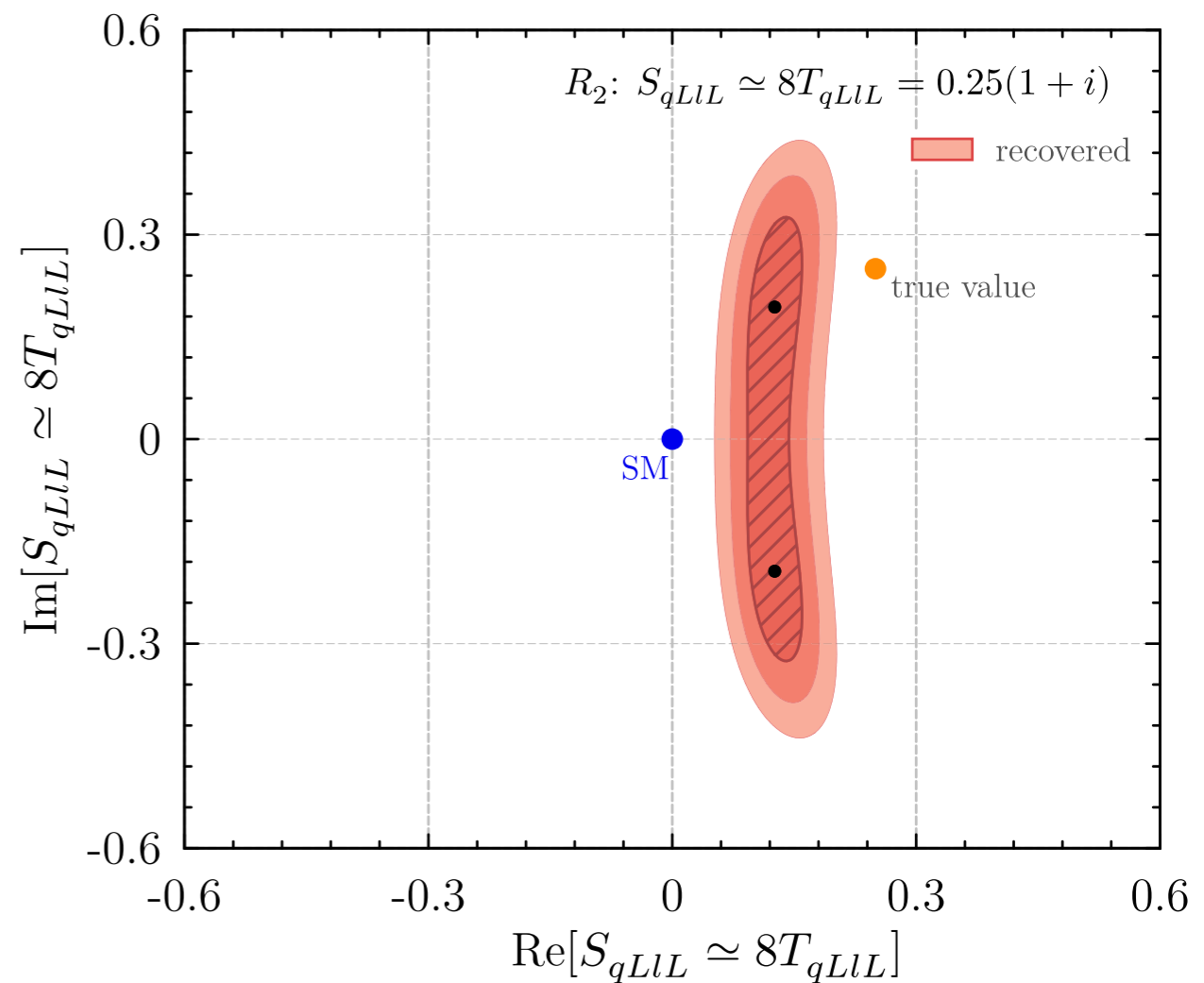
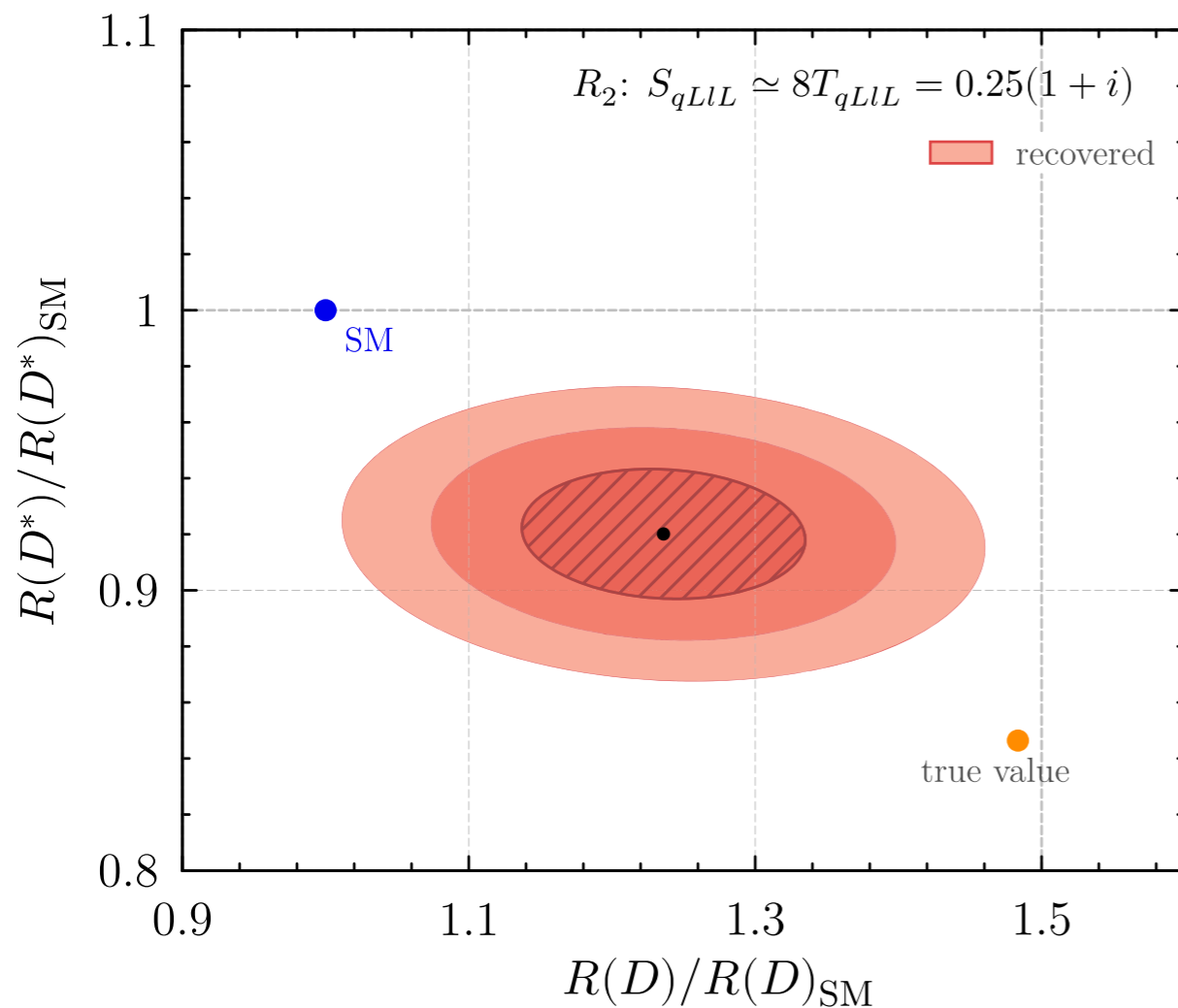
Produce fit shapes / eff.
with some NP



Determine $\mathcal{R}(D^{(*)})$
using SM shapes / eff.



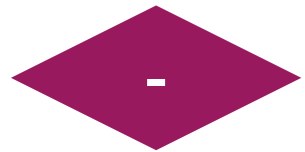
Determine NP couplings
from measured $\mathcal{R}(D^{(*)})$



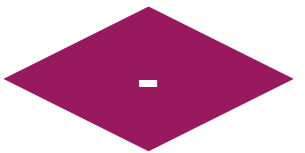
Note: the values were chosen intentionally not to reproduce the measured values to avoid the temptation to correct measured values..

HAMMER — a tool to correct $H_b \rightarrow H_c \tau \bar{\nu}$ to arbitrary NP

Challenge: Produce MC for each NP working point



Need a MC generator that incorporates **all NP effects** and **modern form factors**
(e.g. EvtGen does not)



Very expensive; MC statistics is already one of the largest systematic uncertainties on these measurements

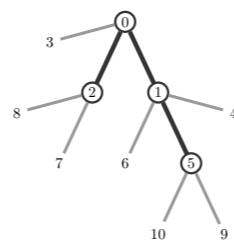


HAMMER offers a solution to these problems

SM or Phase-space MC can be corrected to NP or FFs via ratio of event weights

$$r_I = \frac{d\Gamma_I^{\text{new}} / d\mathcal{PS}}{d\Gamma_I^{\text{old}} / d\mathcal{PS}},$$

Helicity Amplitude Module
for Matrix Element Reweighting



To correct angular distributions one needs to do this for all D^* and τ decay products



$$\sum_{\alpha, i, \beta, j} c_{\alpha} c_{\beta}^{\dagger} F_i F_j^{\dagger} W_{\alpha i \beta j},$$

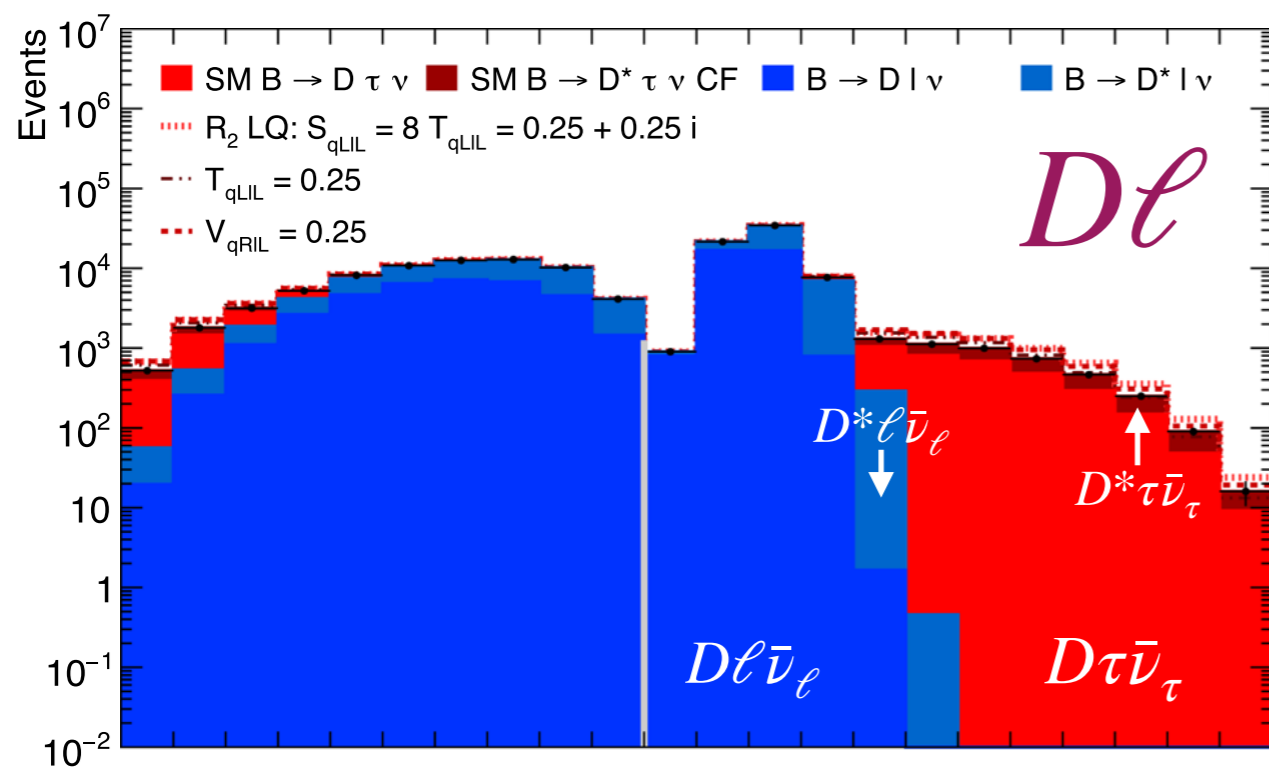
encode hadronic form factors

tensor that encodes amplitudes of given process

sum independent of Wilson coefficients c_{α}
→ can exploit this to create **fast predictions**

An illustrative Toy Example

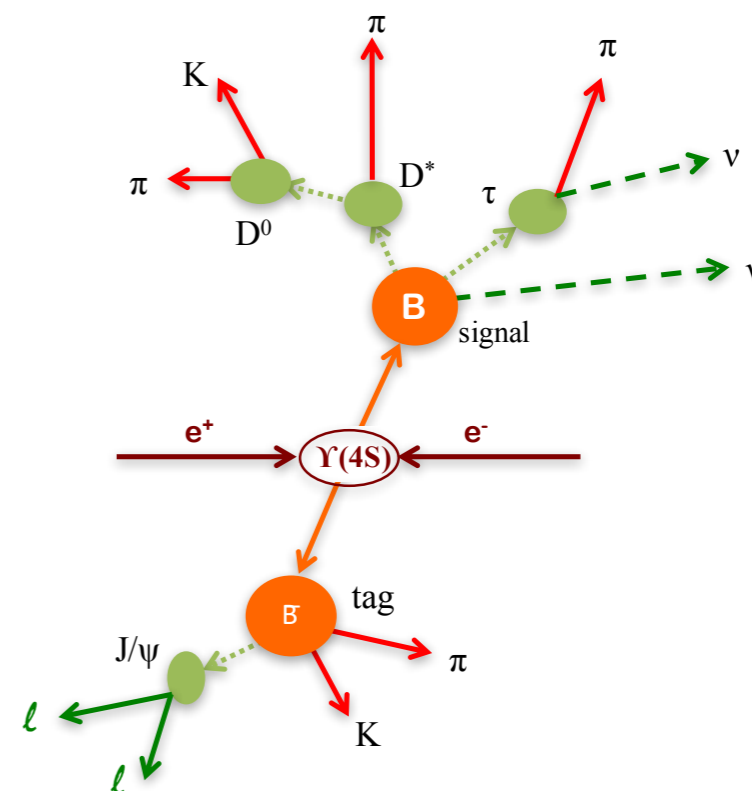
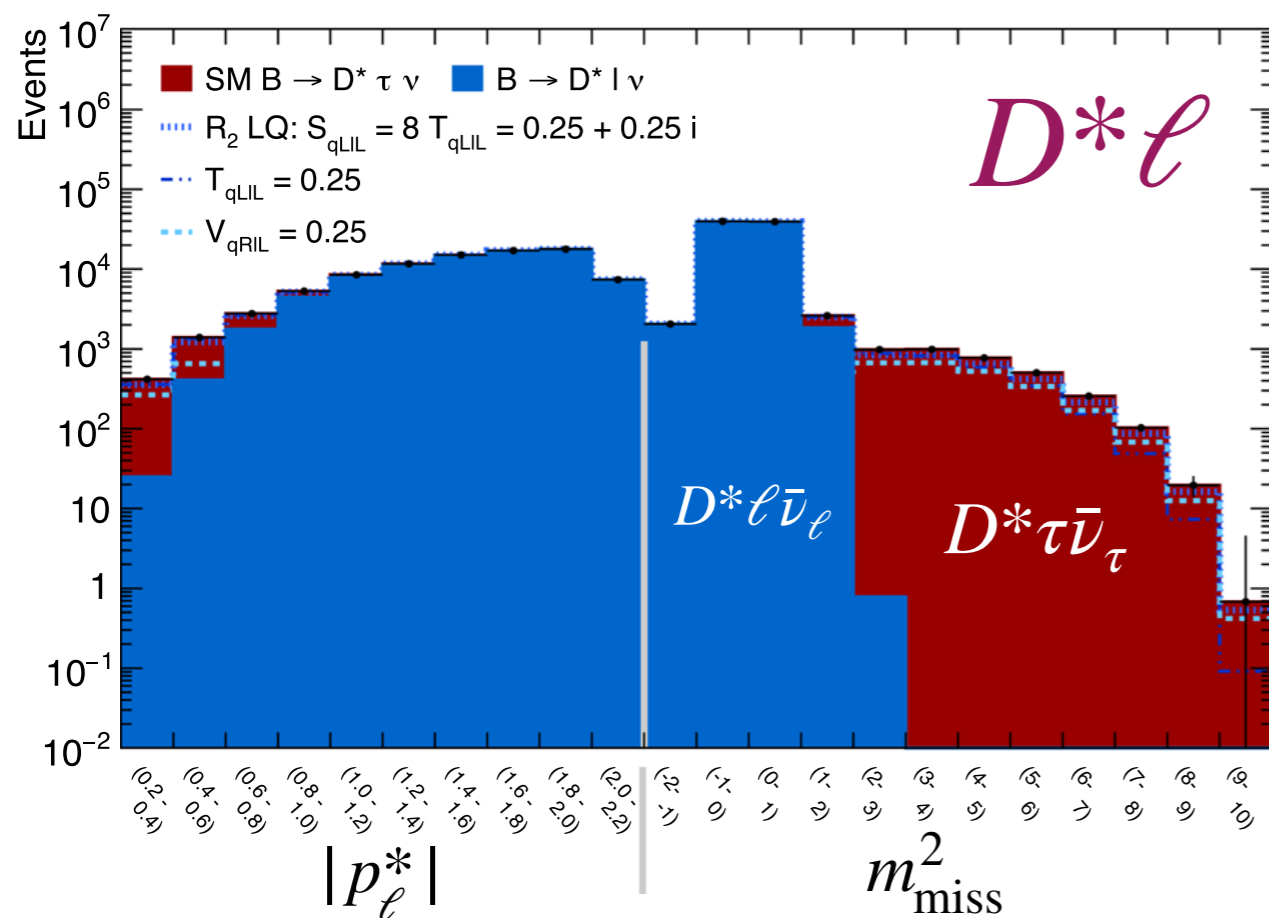
FB, S. Duell, Z. Ligeti, M. Papucci, D. Robinson
 Eur. Phys. J. C (2020) **80**: 883 [arXiv:2002:00020]



2 Categories: $D\ell, D^*\ell$

Binned 2D fit in $m_{\text{miss}}^2 : |p_\ell^*|$

Corresponds to a guesstimate of how an analysis with 5/ab of Belle II data could look like in a single channel



A toy example

