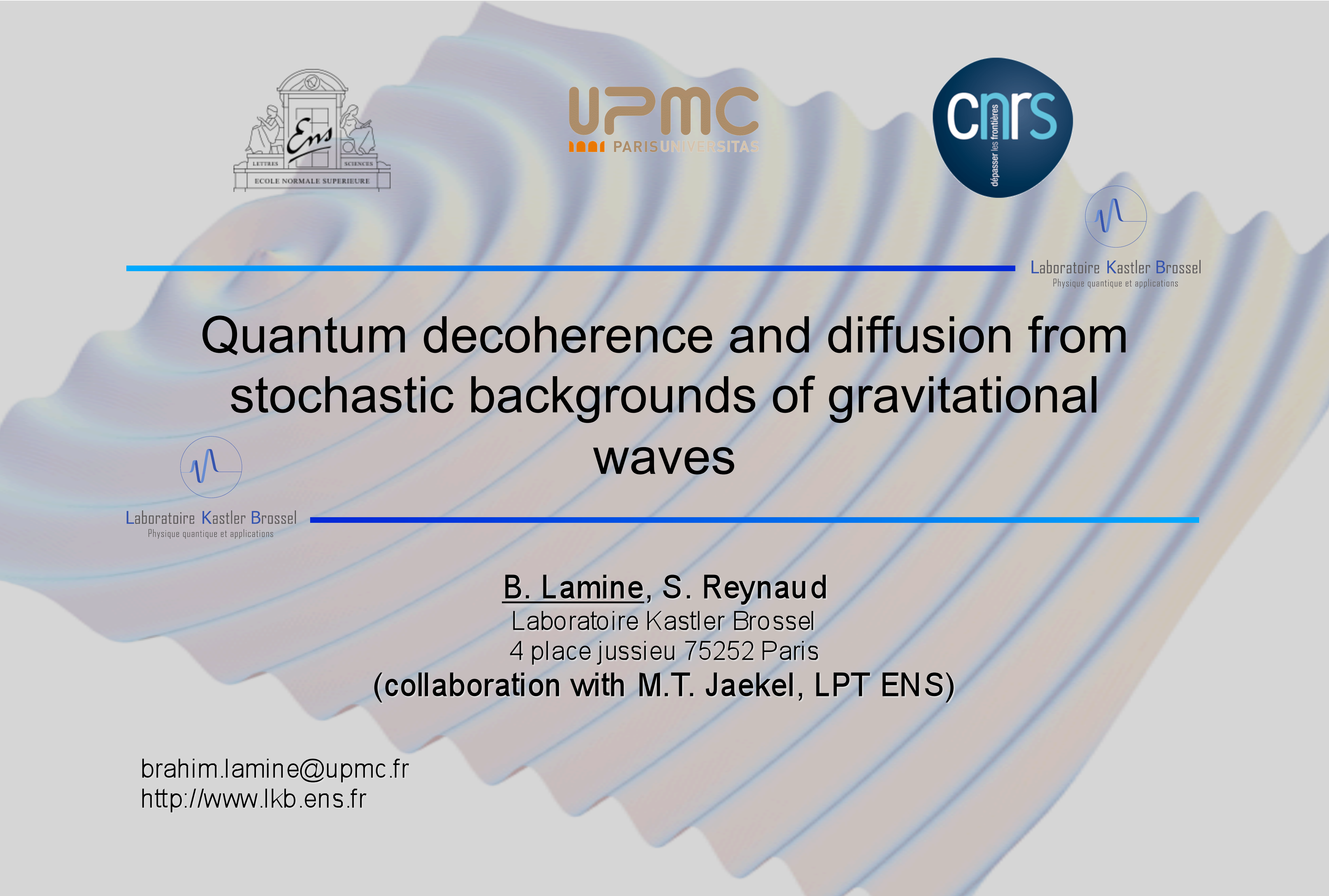

Laboratoire Kastler Brossel
Physique quantique et applications

Quantum decoherence and diffusion from stochastic backgrounds of gravitational waves

Laboratoire Kastler Brossel
Physique quantique et applications

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Some activities of the group

Laboratoire Kastler Brossel
Physique quantique et applications

Quantum Fluctuations and Relativity group

- CASIMIR effect => test of gravitation at small scale.
- Quantum-stimulated phenomenological extension of General Relativity => test of gravitation at large scale (satellite tests : ODYSSEY, SAGAS etc…).
- Quantum decoherence by gravitational waves backgrounds.
- Quantum localization observable in a curved spacetime.
  → effect of GW backgrounds on long-distance clock comparison.

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Quantum decoherence from stochastic backgrounds of gravitational waves

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Introduction

- « Large » systems do not exhibit coherence properties.
  - → Quantum decoherence : a quantum system interacting with an environment evolves towards a classical behavior.

- Gravitation is a universal environment.
  - → consequences are diffusion for classical systems and diffusion + decoherence for quantum systems
  - → implications for « large scale quantum experiments »

- Feynman *Lectures on Gravitation* (1962)
  - → Unavoidable interactions with our gravitational environment could be at the origin of the classical behavior of macroscopic systems.

$$t_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^5}} \simeq 5 \times 10^{-44}\mathrm{s} \qquad \ell_{\mathrm{P}} = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35}\mathrm{m}$$

→ But

$$m_{\mathrm{P}} = \sqrt{\frac{\hbar c}{G}} \simeq 22\mu\mathrm{g}$$

→ Also $m < m_{\mathrm{P}} \quad \Leftrightarrow \quad \ell_{\mathrm{C}} > \ell_{\mathrm{P}}$ $\qquad m > m_{\mathrm{P}} \quad \Leftrightarrow \quad \ell_{\mathrm{C}} < \ell_{\mathrm{P}}$

Gravitational decoherence

- Feynman *Lectures on Gravitation* (1962)
  - → Unavoidable interactions with our gravitational environment could be at the origin of the classical behavior of macroscopic systems.
  - → Planck scales argument.

$$t_\mathrm{P} = \sqrt{\frac{\hbar G}{c^5}} \simeq 5 \times 10^{-44}\mathrm{s} \qquad \ell_\mathrm{P} = \sqrt{\frac{\hbar G}{c^3}} \simeq 10^{-35}\mathrm{m}$$

  - → But $m_\mathrm{P} = \sqrt{\frac{\hbar c}{G}} \simeq 22\mu\mathrm{g}$ lies between microscopic and macroscopic masses.
  - → Is it accidental or is it a hint that space-time fluctuations are at the origin of some universal decoherence mechanism ?

- The Planck scale transition can be seen by comparing the Compton length $\ell_\mathrm{C} = \frac{\hbar}{mc}$ to the Planck length $\ell_\mathrm{P} = \frac{\hbar}{m_\mathrm{P} c}$

$$m < m_\mathrm{P} \quad \Leftrightarrow \quad \ell_\mathrm{C} > \ell_\mathrm{P} \qquad\qquad m > m_\mathrm{P} \quad \Leftrightarrow \quad \ell_\mathrm{C} < \ell_\mathrm{P}$$

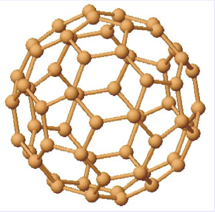
From quantum/classical border...

Quantum Domain

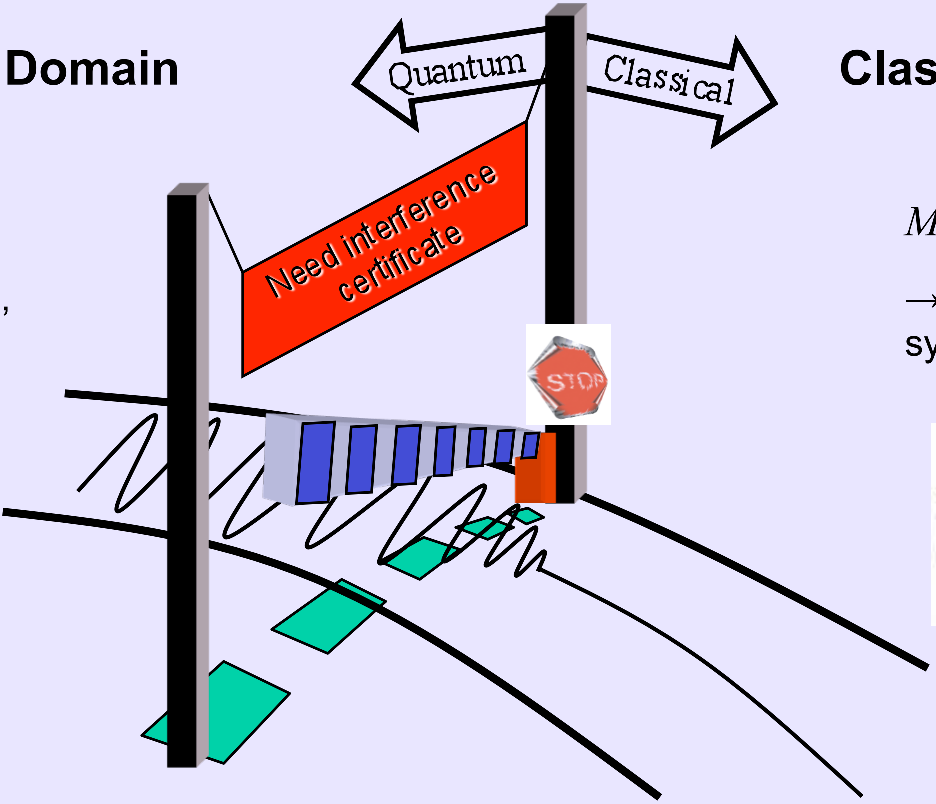
Microscopic objects

→ electrons, photons, atoms, molecules...

C_{60}



interferences



(from Zurek's original picture)

Classical Domain

Macroscopic objets

→ soccer ball, cats, planetary systems.

No interferences

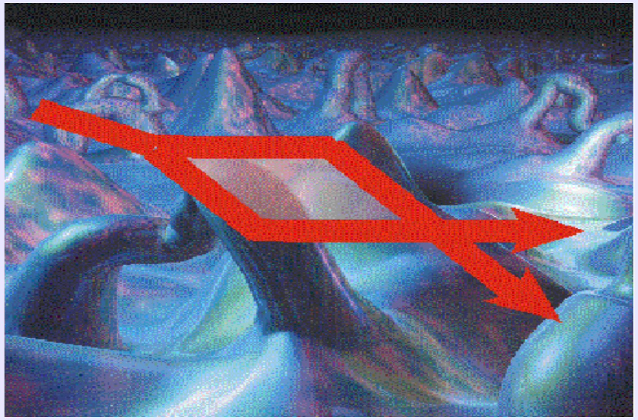
- Where is the borderline (in term of mass ? size ? etc...) ?

... to new gedanken experiments

- Would it be possible to prove the existence of intrinsic fluctuations of space-time by observing some universal diffusion ?
  - → situation analogous to that which allowed physicists to prove the existence of atoms by observing Brownian diffusion in the beginning of the 20th century.

F. Karolyhazy 1966, I.C. Percival 1997, G. Amelino-Camelia 2000,
Kok & Yurtsever 2003, R. Bingham & C. Wang 2006...

- Is this feasible with present state-of-the-art instruments such as atomic or optical interferometers ?
  - → Spatial projects : HYPER, MWXG, GAUGE, QUEST...



Decoherence in HYPER,
an artist's view (ESA, 2000)

Gravitational environment

- General relativity is the effective theory of gravity at experimentally accessible frequencies
  - → the associated intrinsic fluctuations are known, they are the gravitational waves (freely propagating solutions of linearized Einstein equations).
  - → perturbation of the metric : $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu} \ll 1$
  - → TT gauge : only h_{ij}

- GW backgrounds : collection of harmonic oscillators that can be quantized ($\hat{h}_{ij}$) even if we don't have a full theory of quantum gravity.

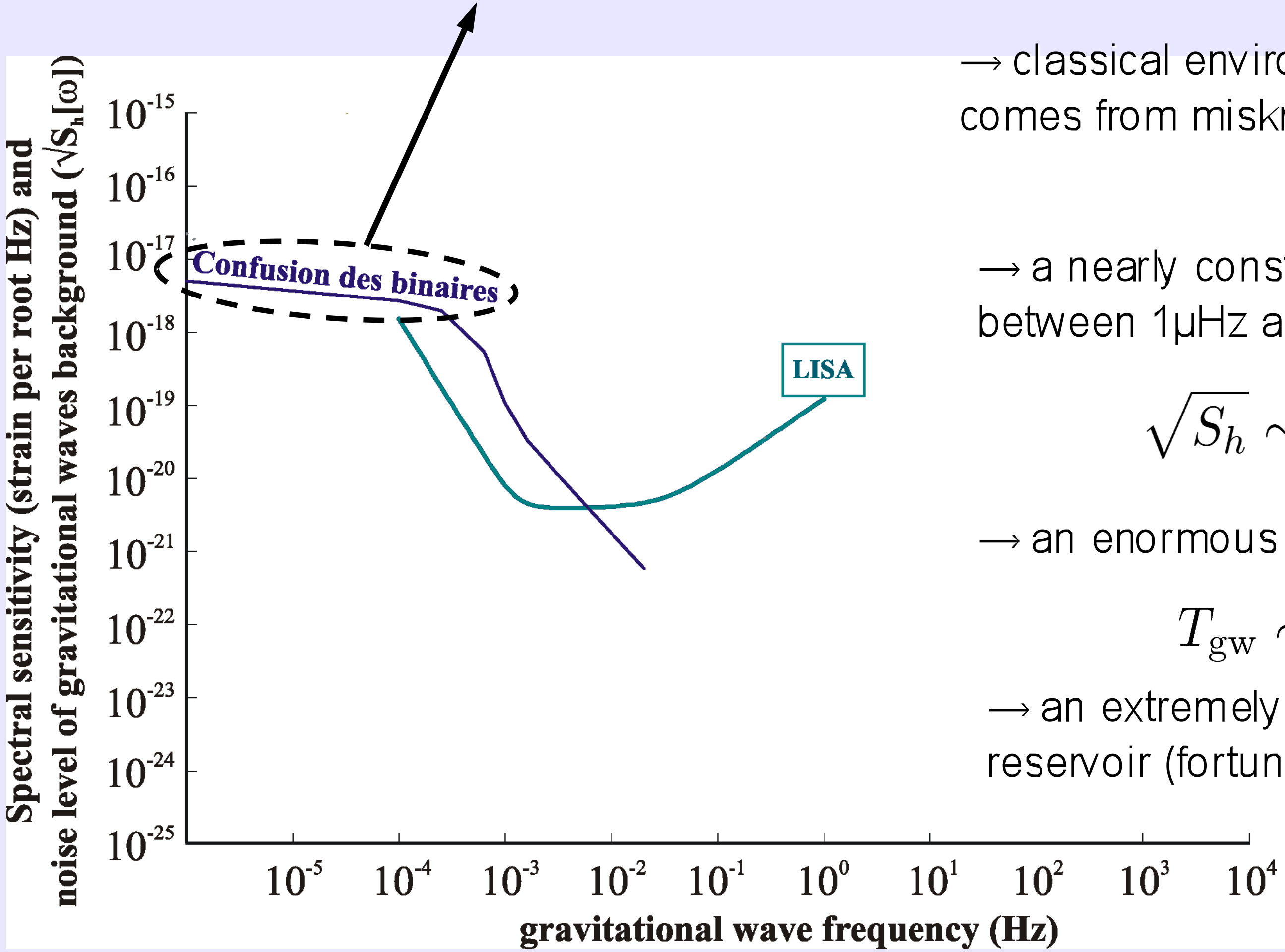
- Classical metric perturbations (anticommutator of $\hat{h}_{ij}$) are characterized by a noise spectrum $S_h[\omega]$ or equivalently a noise temperature (or a graviton number) :

$$S_h[\omega] = \frac{16G}{5c^5} k_\mathrm{B} T_\mathrm{gw}[\omega] = \frac{16G}{5c^5} n_\mathrm{gw}[\omega] \hbar\omega$$

- It is an ideal environment to create large decoherence without dissipation (G small, T_{gw} high).

Galactic backgrounds

Binary confusion background, generated by all unresolved binary systems in our Galaxy and its vicinity



B. Schutz *Class. Quant. Grav. 1999, gr-qc/9911034*

⟶ classical environment (stochastic nature comes from misknowledge of the sources)

⟶ a nearly constant spectrum between 1μHz and 1mHz

$$\sqrt{S_h} \sim 10^{-17}\mathrm{Hz}^{-\frac{1}{2}}$$

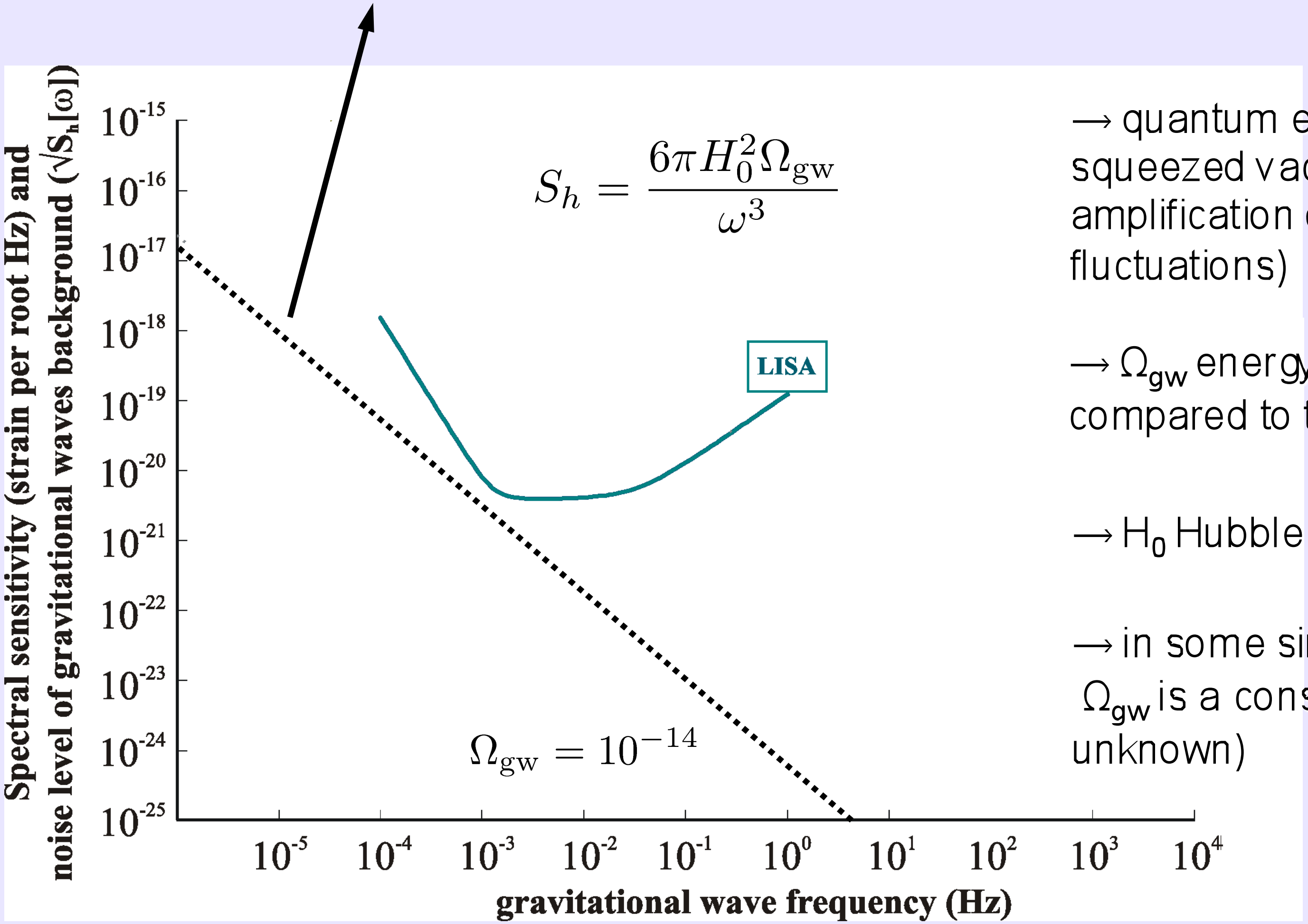
⟶ an enormous noise temperature

$$T_{\mathrm{gw}} \sim 10^{41}\mathrm{K}$$

⟶ an extremely weakly coupled reservoir (fortunately !), not thermal.

Cosmic backgrounds

Relic background with a primordial origin



→ quantum environment (highly squeezed vacuum stemming from amplification of initial vacuum fluctuations)

→ Ω_{gw} energy density of GW compared to the critical density

→ H_0 Hubble constant

→ in some simple models, Ω_{gw} is a constant (unfortunately unknown)

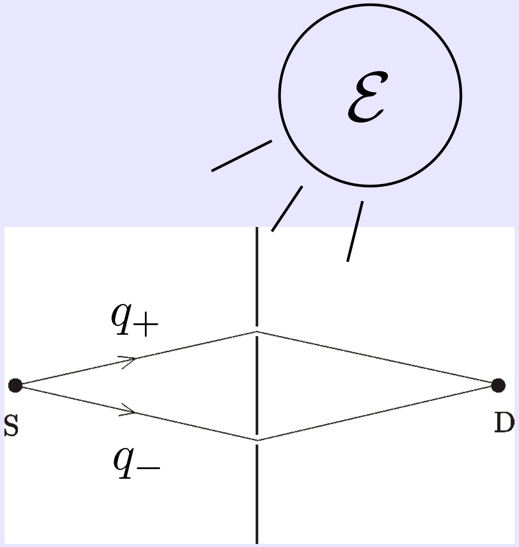
B. Schutz *Class. Quant. Grav. 1999, gr-qc/9911034*

Quantum decoherence

- Path integral interpretation : the quantum evolution is governed by interferences between quantum paths.
- Quantum coherences are characterized by the contrast of interferences between those quantum paths.

$$\mathcal{I} = |\mathcal{A}_+ + \mathcal{A}_-|^2 = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\mathrm{Re}(\mathcal{A}_+\mathcal{A}_-^*)$$

- In presence of an environment $\mathcal{E}$



$$\mathcal{I} = |\mathcal{A}_+|^2 + |\mathcal{A}_-|^2 + 2\mathrm{Re}(\mathcal{A}_+\mathcal{A}_-^*\mathcal{F}[q_+, q_-])$$

$$\langle\mathcal{E}_+|\mathcal{E}_+\rangle = \langle\mathcal{E}_-|\mathcal{E}_-\rangle = 1 \qquad \mathcal{F}[q_+, q_-] = \langle\mathcal{E}_+|\mathcal{E}_-\rangle$$

- $F[q_+(t),q_-(t)]$: Feynman-Vernon influence functional.

  → it gives the coherence between two paths $q_+(t)$ and $q_{\{}(t)$.

$$\mathcal{V} = |\mathcal{F}[q_+, q_-]| = |\langle\mathcal{E}_+|\mathcal{E}_-\rangle|$$

other approachs developped in R. Penrose 1996, Kok & Yurtsever 2003 ...

Application to atomic interferometers

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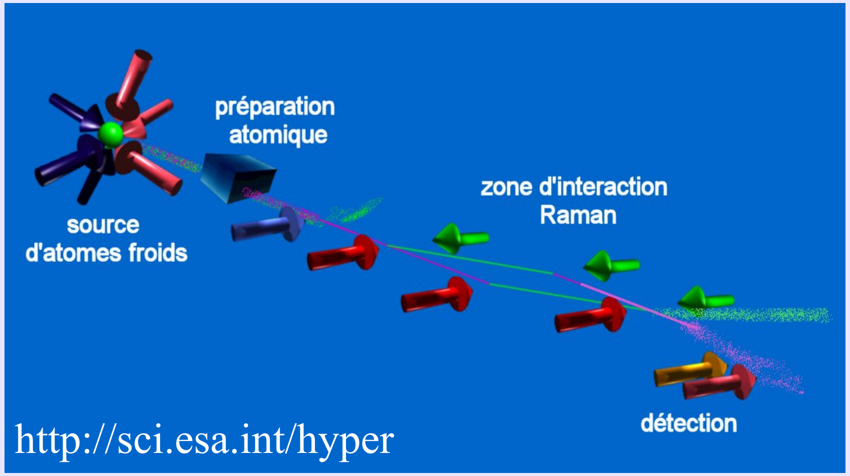
- HYPER project : the lasers used for stimulated Raman transition provide nearly freely falling mirrors and beam splitters for atoms.

$$\mathcal{V} = \exp\left(\boxed{i\Delta\varphi_{\mathrm{lin}}} - \boxed{\frac{1}{2}\Delta\varphi^2_{\mathrm{noise}}}\right)$$

diffusion

decoherence



http://sci.esa.int/hyper

- Linear term (sensitivity to GW)

$$\Delta\varphi_{\mathrm{lin}} = \frac{1}{2\hbar}\oint h_{\mu\nu}\frac{P^\mu P^\nu}{m}\,\mathrm{d}t$$

- Noise term (simplified white noise model)

$$\Delta\varphi^2_{\mathrm{noise}} \sim \left(\frac{\Delta E_k}{\hbar}\right)^2 S_h\,\tau$$

→ Brownian diffusion of the phase (τ the time of flight).

→ ΔE_k variation of kinetic energy (in comoving frame) by the beam splitter.

Discussion

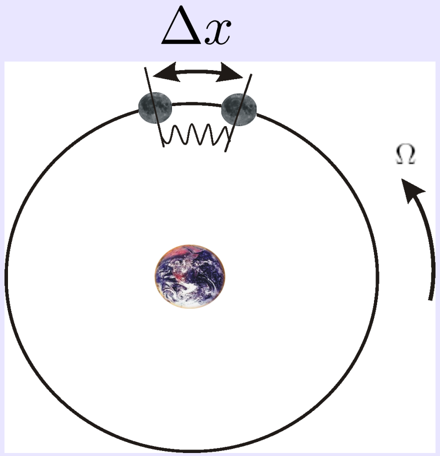
- Relevant parameters for the estimation of decoherence :
  - → kinetic energy (in comoving frame) of the probe (not its mass energy !)
  - → size of the interferometer (and separation angle)
  - → level of the noise spectrum (S_h)

link to Feynman argument on the Planck mass

- All existing interferometers verify $\Delta\varphi^2_{\mathrm{noise}} \lll 1$
  - → In HYPER, $\Delta\varphi^2_{\mathrm{noise}} \sim 10^{-12}$

- In the mean time, gravitational decoherence is the **dominant** mechanism for large macroscopic systems (planetary motions)

Reynaud, Maia Neto, Lambrecht and Jaekel, *EuroPhys. Lett.* **54** *(2001) 135*

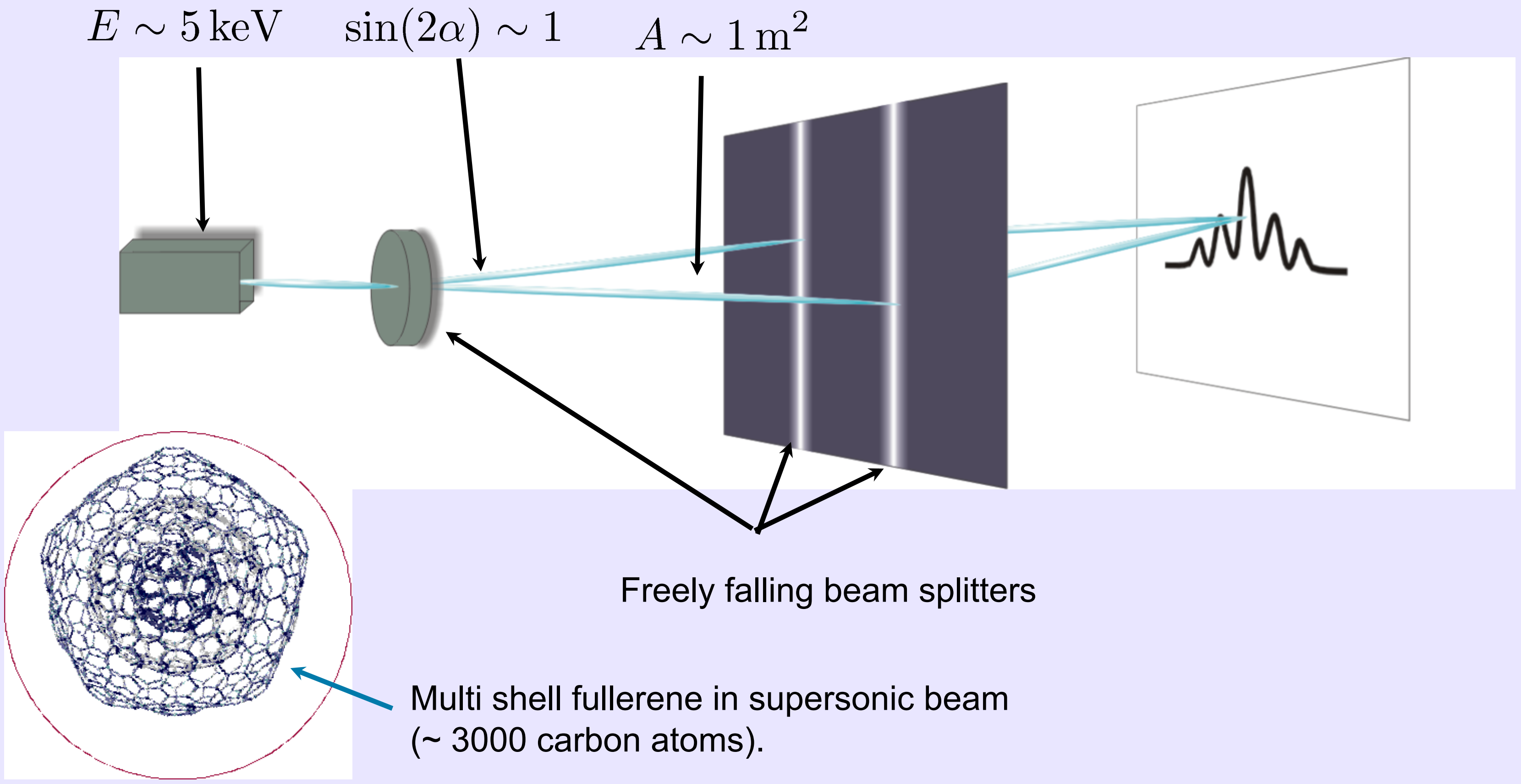


$$t_{\mathrm{dec}} \sim \frac{\hbar}{MR\Delta x\sqrt{\langle R_{0i0j}R_{0i0j}\rangle}} \sim 5\times 10^{-51}\,\mathrm{s}\left(\frac{10^{-14}}{\Omega_{\mathrm{gw}}}\right)^{1/2}\left(\frac{1\,\mathrm{m}}{\Delta x}\right)$$

- Challenge : would it be possible to design experiments allowing one to explore the transition on « mesoscopic » objects ?

« Challenge » for matter wave interferometry

B. Lamine, R. Hervé, A. Lambrecht, S. Reynaud, *PRL* **96** 050405 (2006)



$E \sim 5\,\mathrm{keV}$ $\sin(2\alpha) \sim 1$ $A \sim 1\,\mathrm{m}^2$

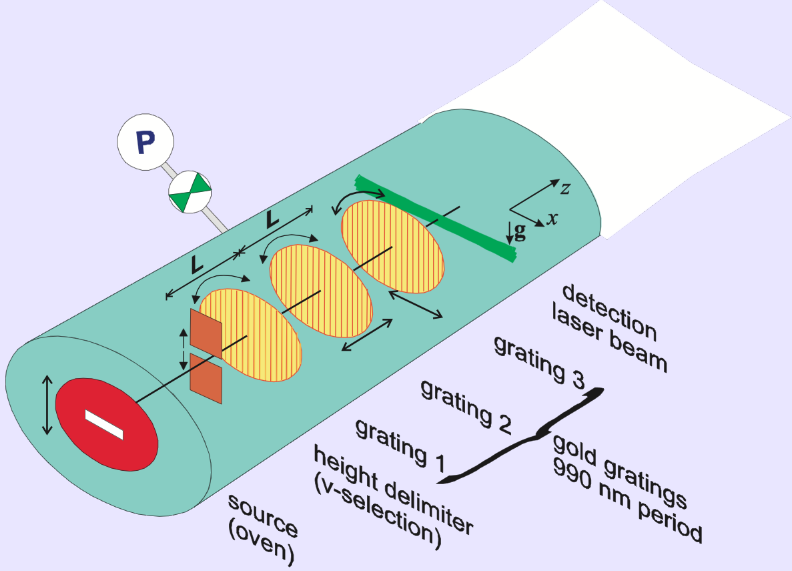
Freely falling beam splitters

Multi shell fullerene in supersonic beam
(~ 3000 carbon atoms).

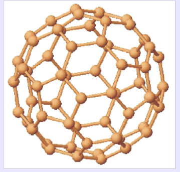
State of the art

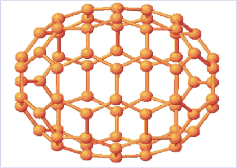
→ Interferences with large molecules.

Hacker-Müller *and al, Nature* **427** (2004) 711

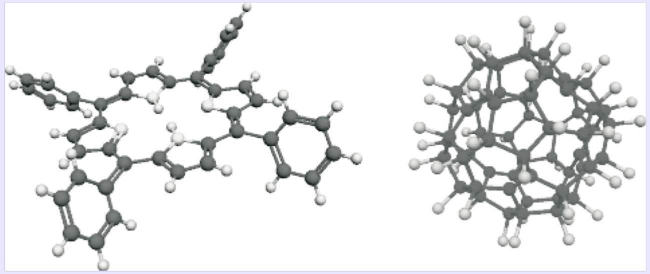


C_{60}





C_{70}



$C_{44}H_{30}N_4$ $C_{60}F_{48}$

→ Nevertheless, small separation angle, small kinetic energy and not freely falling beam splitters.

$E \sim 100\,\mathrm{meV}$ $\Delta\varphi^2_{\mathrm{noise}} \ll 1$

Optical interferometer

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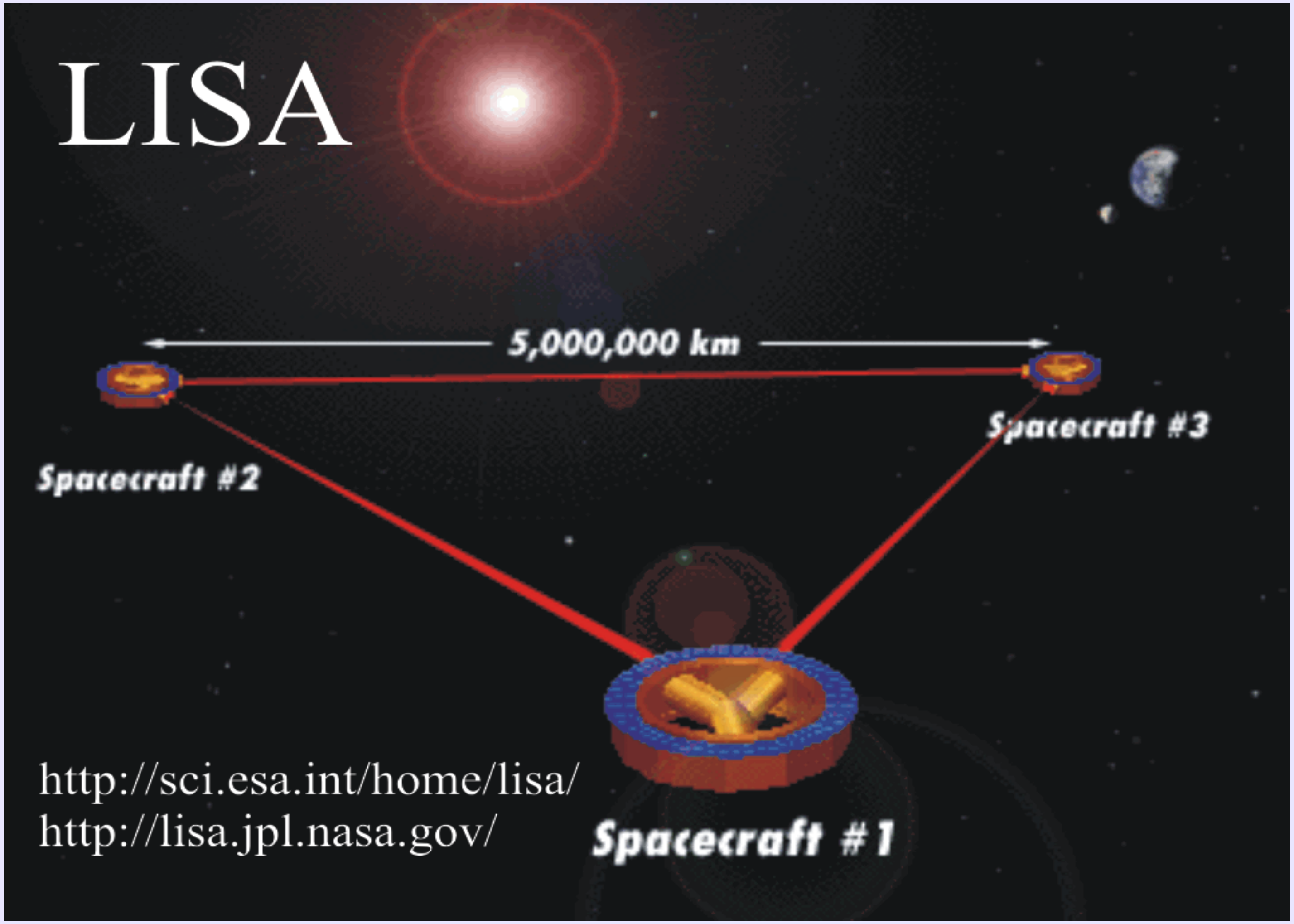


http://sci.esa.int/home/lisa/
http://lisa.jpl.nasa.gov/

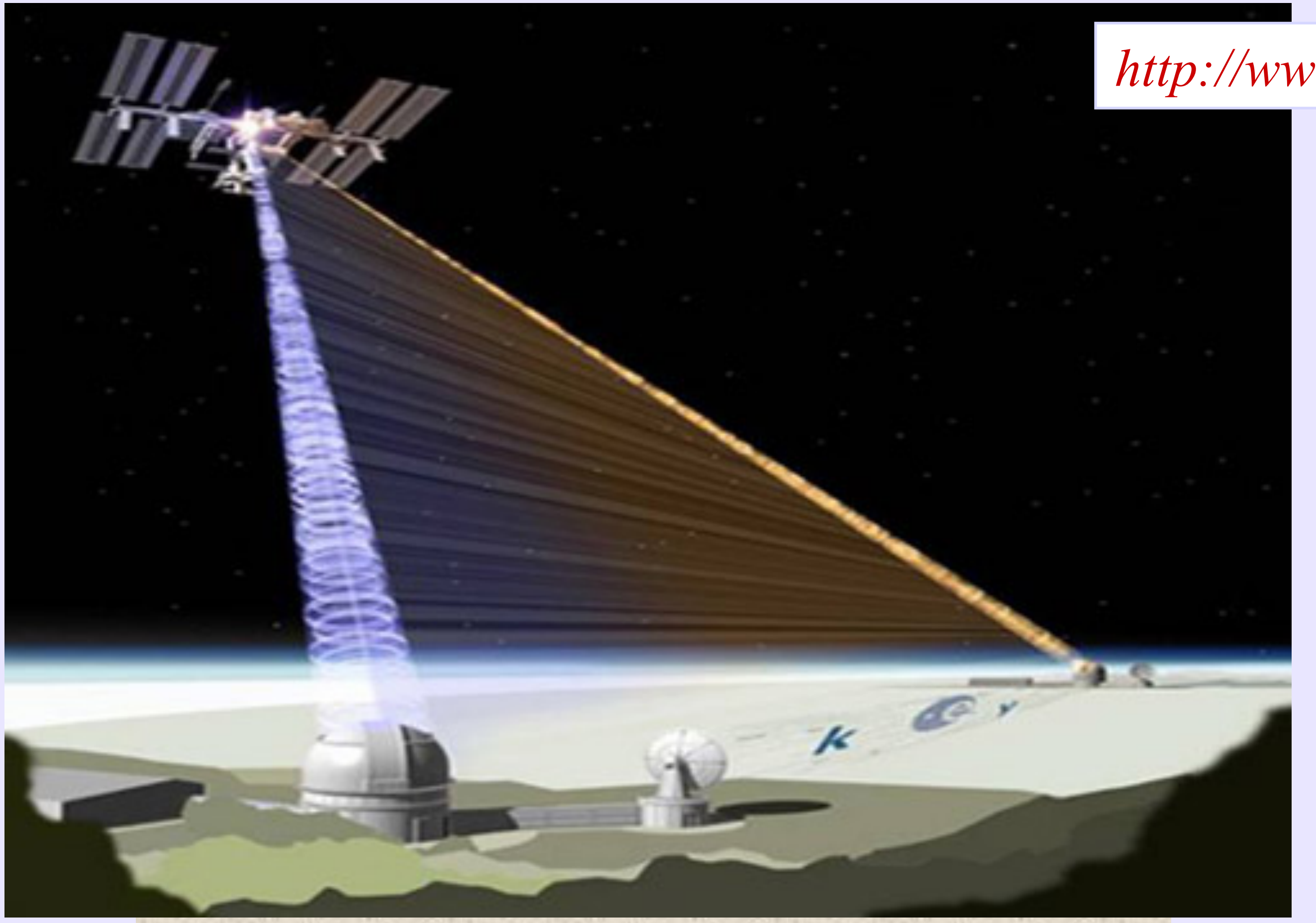
$$\Delta\varphi^2_{\text{noise}} \ll 1$$

LISA is microscopic !

Good news for the fringes…

EPR experiments on large distances

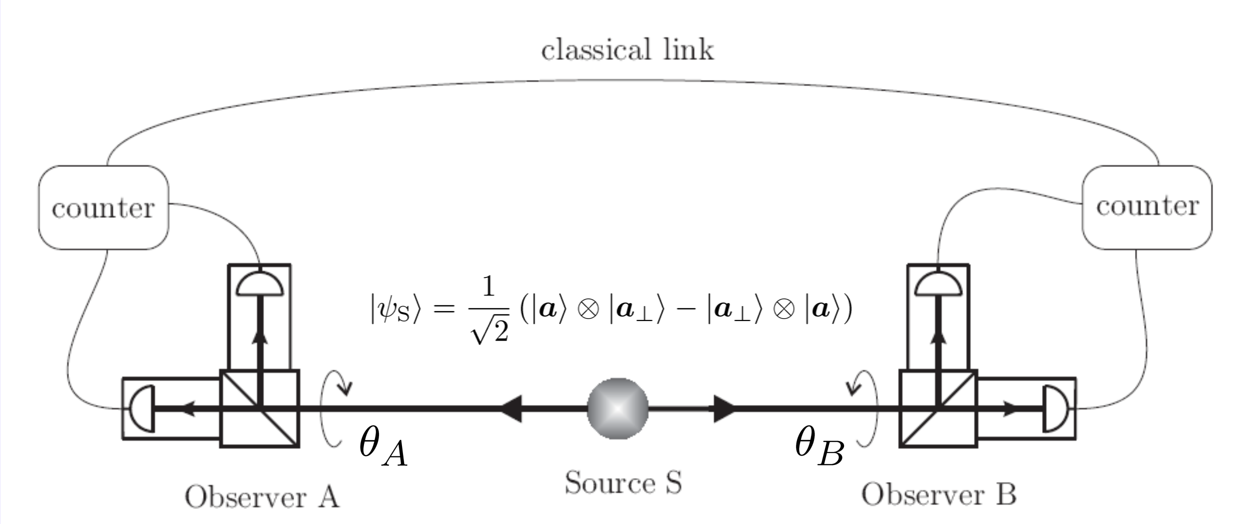
- Are there limits on the distance between two entangled quantum systems ?

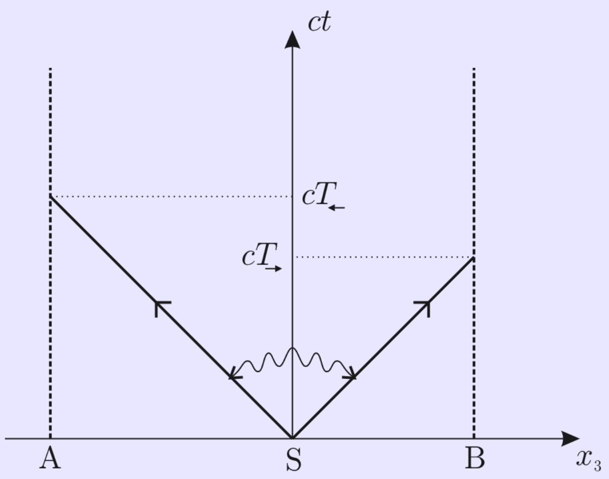


http://www.quantum.at/quest

T. Schmitt-Manderbach et al, *PRL* **98** 010504 (2007)

Bell-test





- Quantum correlation function

$$E(\varepsilon^A, \varepsilon^B) = C(\varepsilon^A, \varepsilon^B) + C(\varepsilon_\perp^A, \varepsilon_\perp^B) - C(\varepsilon_\perp^A, \varepsilon^B) - C(\varepsilon^A, \varepsilon_\perp^B)$$

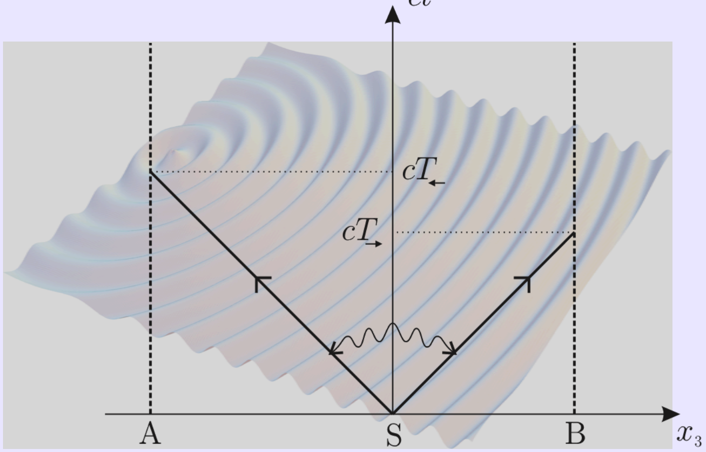
- Bell parameter $S = E(\varepsilon^A, \varepsilon^B) - E(\varepsilon^A, \varepsilon'^B) + E(\varepsilon'^A, \varepsilon^B) + E(\varepsilon'^A, \varepsilon'^B)$

- Clauser Horn Shimony Holt (CHSH) inequality $|S| \leq 2$
  → violated in QM with a maximum violation of $2\sqrt{2}$

Bell-test in presence of a GW

- In eikonal approximation, there is a rotation of polarisation (Strotsky effect or gravitational birefringence)

$$\alpha_{\leftrightarrows}(t) = \frac{1}{2}\int_{ct}^{ct+cT_{\leftrightarrows}} (\partial_1 h_{23} - \partial_2 h_{13})\mathrm{d}\sigma$$



- An EPR experiment can be viewed as an interferometric signal

$$E(\varepsilon^{\mathrm{A}}, \varepsilon^{\mathrm{B}})(t) = -\cos[2(\Theta - \alpha(t))]$$

$$\alpha = \alpha_{\rightarrow} - \alpha_{\leftarrow}$$

$$\Theta = \Theta_{\mathrm{B}} - \Theta_{\mathrm{A}}$$

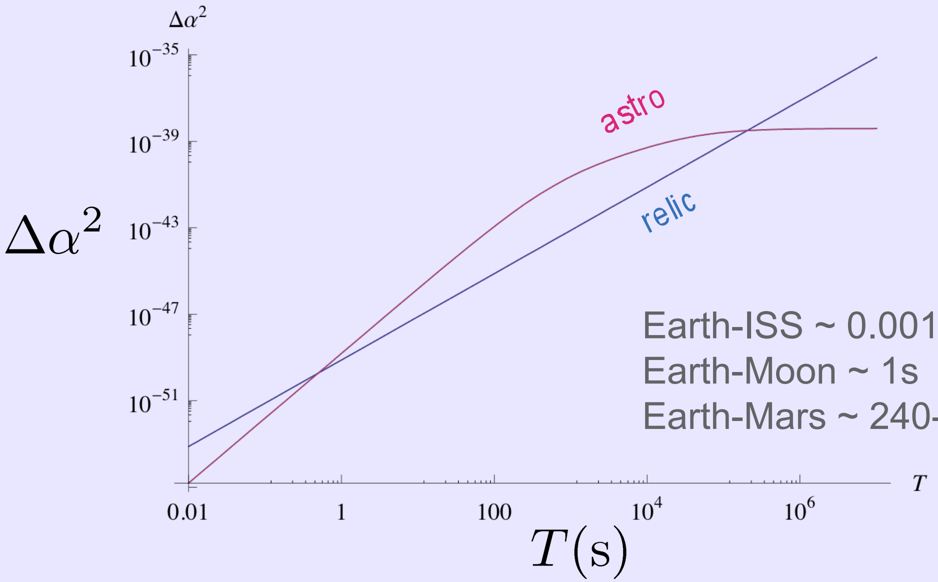
- A GW slightly changes the angles of maximal violation of the CSHS inequality.

EPR in stochastic GW backgrounds

- Quantum correlation function

$$E(\boldsymbol{\varepsilon}^{\mathrm{A}},\boldsymbol{\varepsilon}^{\mathrm{B}}) = -\langle\cos[2(\Theta-\alpha)]\rangle = \cos(2\Theta - 2\langle\alpha\rangle)\, e^{-2\Delta\alpha^2}$$

- Decrease of the CHSH Bell test parameter $S \leq 2\sqrt{2}\exp\left(-2\Delta\alpha^2\right)$



Earth-ISS ~ 0.001s
Earth-Moon ~ 1s
Earth-Mars ~ 240-1200 s

- At the largest cosmological scales, decoherence is governed by the value of the energy density of GW

$$\Delta\alpha^2 \sim \Omega_{\mathrm{GW}}$$

Conclusions

- Gravitational decoherence sets a natural border between classical and quantum objects (Feynman intuition was correct).

- Experimental challenge to monitor decoherence (new ideas ? BEC ? Quantum superposition of micro mechanical mirrors ? etc...).

- Quantum correlation based on polarisation can survive over the largest cosmological scales.

Influence functional

- The influence functional can be written as an exponential of linear and two quadratic kernels:

$$\mathcal{F}[q_+,q_-] = \exp\left(\frac{i}{4\hbar}\int_{t_i}^{t_f}(\ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)])\langle\hat{h}_{ij}(t)\rangle \mathrm{d}t\right)$$

$$i\Delta\varphi_{\mathrm{lin}}$$

$$-\frac{1}{2}\Delta\varphi^2_{\mathrm{noise}}$$

$$\times\exp\left(-\frac{1}{4\hbar}\int_{t_i}^{t_f}\mathrm{d}t\int_{t_i}^{t_f}\mathrm{d}s\,(\ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)])\sigma_{ijkl}(t-s)(\ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)])\right)$$

$$\times\exp\left(-\frac{1}{4\hbar}\int_{t_i}^{t_f}\mathrm{d}t\int_{t_i}^{t_f}\mathrm{d}s\,(\ddot{Q}^{ij}[q_+(t)] + \ddot{Q}^{ij}[q_-(t)])\xi_{ijkl}(t-s)(\ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)])\right)$$

$$-\frac{i}{2}\Delta\varphi^2_{\mathrm{diss}}$$

- With the following kernels :

$$\sigma_{ijkl}(\tau) = \frac{1}{4\hbar}\Big(\langle\hat{h}_{ij}(\tau)\cdot\hat{h}_{kl}(0)\rangle - \langle\hat{h}_{ij}(\tau)\rangle\langle\hat{h}_{kl}(0)\rangle\Big)$$

- ✖ Noise kernel.

$$\xi_{ijkl} = \frac{1}{4\hbar}\langle[\hat{h}_{ij}(\tau),\hat{h}_{kl}(0)]\rangle = i\delta_{ijkl}\int_0^\infty \mathrm{d}\omega\,\frac{2G\omega}{5\pi c^5}\,\sin(\omega\tau)$$

- ✖ Dissipation kernel (imaginary) (spontaneous emission of GW).

Gravitational diffusion in long distance clock comparison

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Tracking of remote spacecrafts

S. Reynaud *and al, Phys. Rev. D* **77** (2008) 122003

- Usual ranging observable

$$\tau_r = \frac{\tau_4 + \tau_1}{2} = \frac{\tau_u + \tau_d}{2}$$

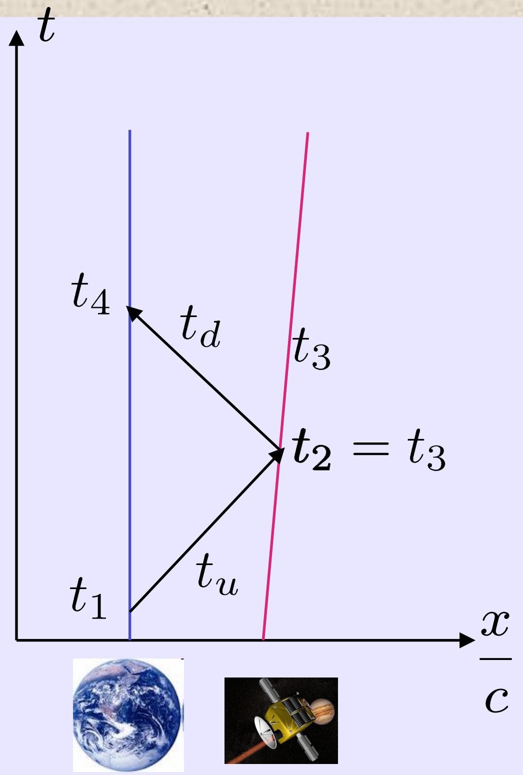
- More general observables for $t_2 \neq t_3$
  → useful only if very good on-board clock.

- Timing observable

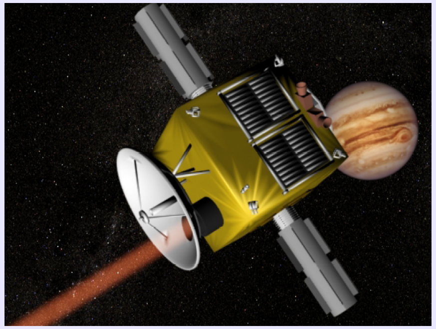
$$\tau_t = \frac{\tau_u - \tau_d}{2}$$

- Doppler and syntonization observables

$$y_r = \dot{\tau}_r \qquad\qquad y_t = \dot{\tau}_t$$



- SAGAS : Asynchronous two-way link between two ultra stable clocks (independant up and down link).

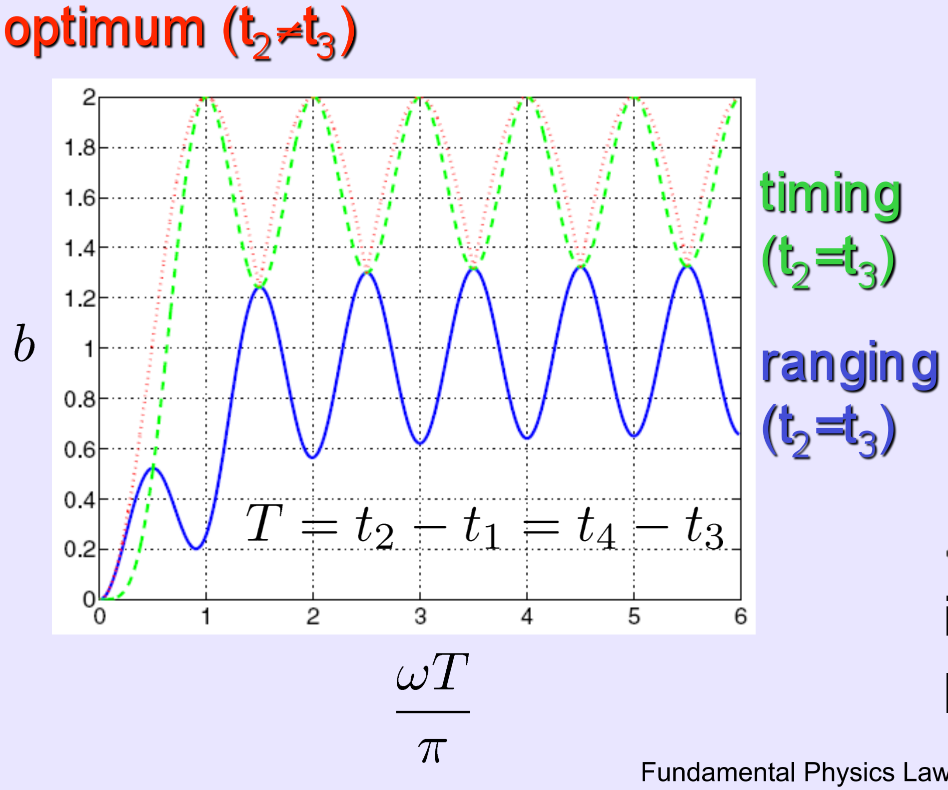


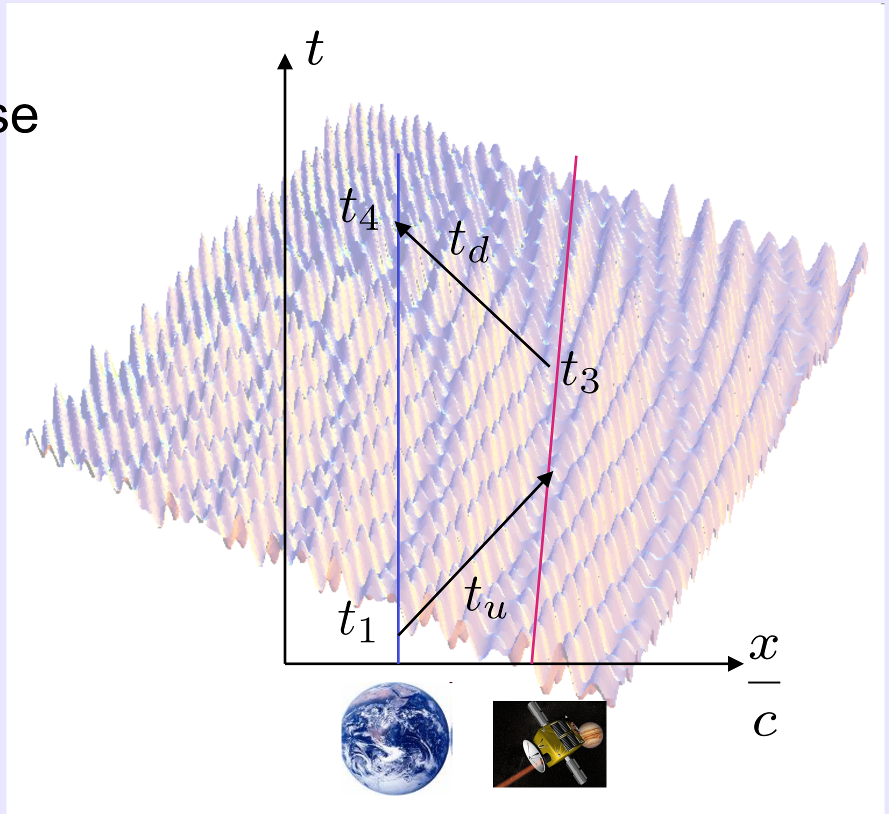
Effect of Gravitational Waves Backgrounds

- GW backgrounds induce a delay noise $\delta\tau_{u,d}$, therefore a frequency noise

$$S_y[\omega] = \frac{5}{8} b[\omega] S_h[\omega]$$

- Sensitivity function b[ω]



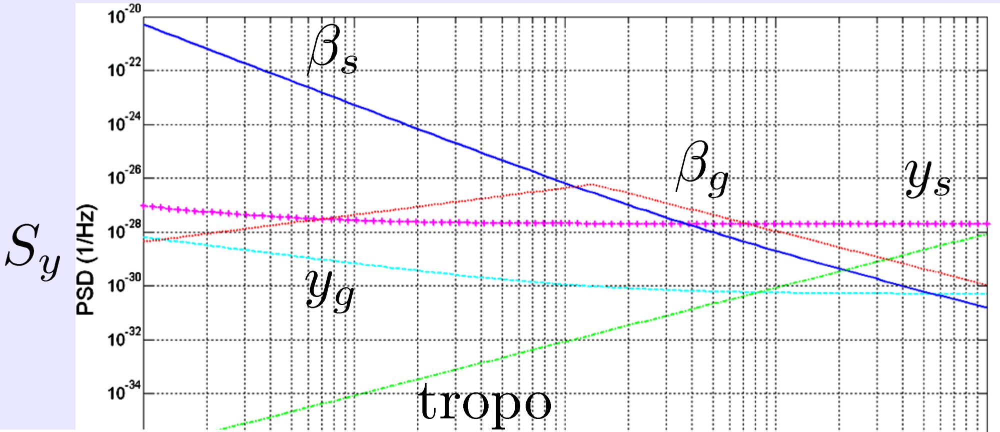


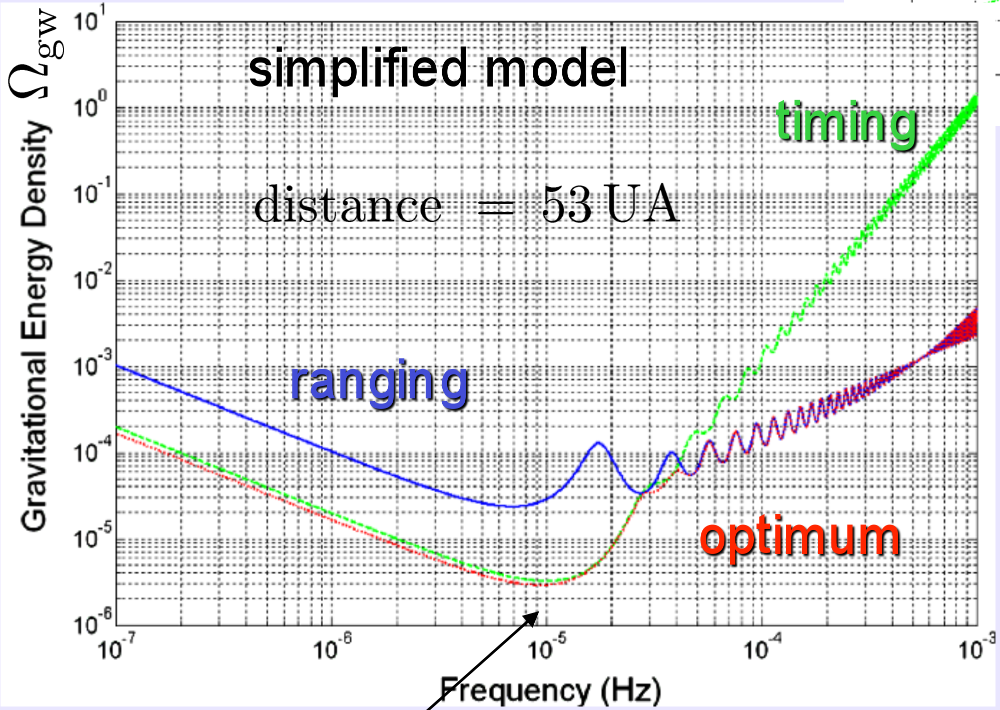
- The syntonization observable is intrinsically better coupled to GW at large distance.

Constraints on GW backgrounds

- measurement noises :
  - → acceleration noise
  - → clock noise
  - → tropospheric delays

$$S_h[\omega] = \frac{6\pi H_0^2 \Omega_{\mathrm{gw}}}{\omega^3} = \frac{8}{5}\,\frac{S_y[\omega]}{b[\omega]}$$





- Optimal observable obtained as a tradeoff between sensitivity and measurement noise.

Constraints on GW backgrounds

- Constraints on Ω_{gw}

- New limits expected with SAGAS