

Studying gluon TMDs via double J/ψ production in proton-proton collisions

Based on work of Florent SCARPA and Alice COLPANI SERRI

Jelle BOR

Heavy flavours from small to large systems, Institut Pascal (Orsay, France);
recent results and perspectives in hadron physics

October 17, 2022

- ① Introduction
- ② Gluon TMDs
- ③ Azimuthal Asymmetries
- ④ TMD Evolution
- ⑤ Numerical Results
- ⑥ Conclusions

1 Introduction

2 Gluon TMDs

3 Azimuthal Asymmetries

4 TMD Evolution

5 Numerical Results

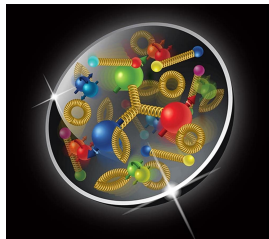
6 Conclusions

Introduction

Inclusive production of J/ψ pairs in pp collisions (via gluon fusion) gives azimuthal asymmetries

↔ understanding the internal structure of nucleons

↔ gluon dynamics poorly known



Results \iff (future) measurements at LHC (fixed-target) experiments \leftrightarrow Transverse Momentum Dependent PDFs (TMDs)

Transverse Momentum Dependent PDFs (1)

PDFs → great precision
Collinear QCD phenomenology

↔ only 1D information
↔ x dependence

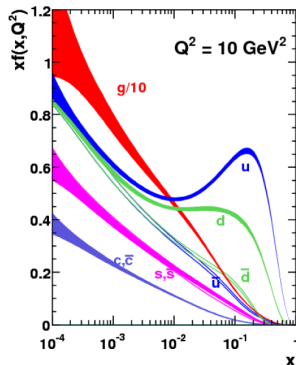


3D structure of the nucleon
Beyond collinear factorisation



Transverse dynamics

▷ Nucleon structure in terms of **TMDs**



Transverse Momentum Dependent PDFs (2)

TMDs → 3D structure of the nucleon

Correlations between k_T and the polarisation of the nucleon/parton

2 components ▷ collinear (x)

▷ transversal (\vec{k}_\perp) → **generate** q_T (final-state)

Quark TMDs extracted from data

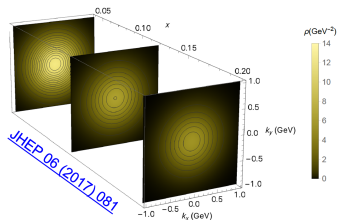
↔ SIDIS, DY processes

Gluon TMDs

↔ Extremely poorly known

↔ How to measure them?

Inclusive **quarkonium** production



A. Bacchetta et al. (JHEP 08 (2008) 023)

Quarkonium production processes

Experimental point of view:

- Quarkonium production observed in different experiments
- J/ψ : easy to produce and detect
 ↪ plenty of experimental data

Theoretical point of view:

- Not clear how to treat quarkonium production in general
- 3 common models → Colour Singlet Model (CSM)
 → Colour Octet Mechanism (COM)
 → Colour Evaporation Model (CEM)
- Not complete agreement with experimental data
- However, for J/ψ -pair production: **CSM** is the best!

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TMD factorisation

Study of gluon TMDs \rightarrow TMD factorisation ($q_T \ll Q$)

General factorised cross section

\hookrightarrow partonic scattering amplitude (*perturbative*)

\hookrightarrow k_T -dependent correlators (*non-perturbative*)

$$d\sigma = \int dx_1 dx_2 d^2\vec{k}_{T1} d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T) \\ \times \Phi_g^{\mu\nu}(x_1, \vec{k}_{T1}) \Phi_g^{\rho\sigma}(x_2, \vec{k}_{T2}) \left[\hat{\mathcal{M}}_{\mu\rho} \hat{\mathcal{M}}_{\nu\sigma}^* \right] \Big|_{\substack{k_1=x_1 P_1 \\ k_2=x_2 P_2}} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)$$

- In order to stay in TMD regime: $\max(q_T) = Q/2$

Gluon TMDs and correlators

TMD correlator parametrisation
for an unpolarised proton

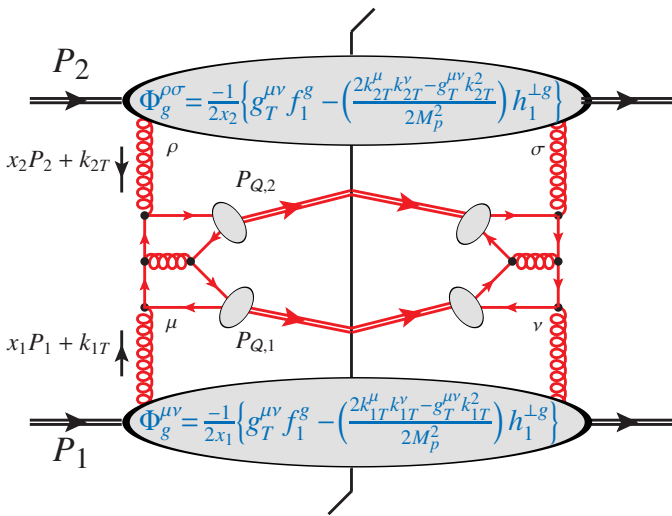
- ▷ unpolarised: $f_1^g \longrightarrow$
- ▷ linearly polarised: $h_1^{\perp g} \longrightarrow$

		Gluon		
		U	C	L
Nucleon	U	f_1		h_1^{\perp}
	L		g_{1L}	h_{1L}^{\perp}
	T	f_{1T}^{\perp}	g_{1T}	h_1, h_{1T}^{\perp}

$$\Phi_g^{\mu\nu}(x, \vec{k}_T) = -\frac{1}{2x} \left[g_T^{\mu\nu} f_1^g(x, \vec{k}_T^2) - \left(\frac{k_T^\mu k_T^\nu}{M_H^2} + g_T^{\mu\nu} \frac{\vec{k}_T^2}{2M_H^2} \right) h_1^{\perp g}(x, \vec{k}_T^2) \right]$$

↔ **Second term** goes to 0 if $k_T = 0$

P.J. Mulders and J. Rodrigues (Phys.Rev.D 63 (2001) 094021)

LO Feynman diagram for $p(P_1) + p(P_2) \rightarrow Q(P_{Q,1}) + Q(P_{Q,1}) + X$ 

Why di- J/ψ production?

- Single J/ψ production (CSM): a lot of data at low q_T ✓
↔ but gluon in the final state → presence of soft gluons (non-perturbative) between Initial State Interactions (ISIs) and Final State Interactions (FSIs) can be problematic
↔ **no** TMD factorisation ✗
- Single η_c production: no gluon in the final state ✓
↔ but **no** data at low q_T ✗
- Double J/ψ production:
 - ▷ data at low q_T ✓
 - ▷ no gluon in the final state ✓
 - ↔ gluon fusion: ISI can be encapsulated in the TMDs ✓
 - ↔ consider CSM: no FSIs ✓
 - **Safe TMD factorisation**

PhD Thesis F. Scarpa (10.33612/diss.128346301)

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Hadronic cross section

The general formula for the cross section of gluon fusion is:

$$\begin{aligned}d\sigma_{UU}^{gg} \propto & F_1 \times \mathcal{C}[f_1^g f_1^g] \\ & + F_2 \times \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}] \\ & + (F_3 \times \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F'_3 \times \mathcal{C}[w'_3 h_1^{\perp g} f_1^g]) \cos(2\Phi_{CS}) \\ & + (F_4 \times \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]) \cos(4\Phi_{CS})\end{aligned}$$

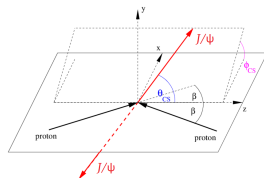
- First two members: azimuthally independent
- Third member: $\cos(2\Phi_{CS})$ -asymmetry
- Fourth member: $\cos(4\Phi_{CS})$ -asymmetry

Computation of azimuthal asymmetries (average)

The corresponding expressions for $\cos(2\Phi_{CS})$ and $\cos(4\Phi_{CS})$:

$$\langle \cos(2\phi_{CS}) \rangle = \frac{1}{2} \frac{F_3 \mathcal{C}[w_3 f_1^g h_1^{\perp g}] + F_3' \mathcal{C}[w_3' h_1^{\perp g} f_1^g]}{F_1 \mathcal{C}[f_1^g f_1^g] + F_2 \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$

$$\langle \cos(4\phi_{CS}) \rangle = \frac{1}{2} \frac{F_4 \mathcal{C}[w_4 h_1^{\perp g} h_1^{\perp g}]}{F_1 \mathcal{C}[f_1^g f_1^g] + F_2 \mathcal{C}[w_2 h_1^{\perp g} h_1^{\perp g}]}$$



- The hard-scattering coefficients (F_1 , F_2 , F_3 , F_3' , F_4) give the explicit dependence on $M_{\psi\psi}$ and θ_{CS} (given in backup slides)
- Modulations due to $h_1^{\perp g}$
- Set hard scale $Q \equiv M_{\psi\psi}$
- TMD evolution applied within the convolutions

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Introduction Evolution (1)

- Beyond tree level, the TMDs and hard factors F become scale dependent J. Collins (ISBN: 9781107645257)
- Implementing evolution is more easily done in impact parameter space (b_T), where convolutions become simple products:

$$d\sigma_{UU}^{gg} \propto \int d^2b_T e^{-ib_T \cdot q_T} \hat{W}(b_T, Q) + \mathcal{O}(q_T^2/Q^2)$$

$$\hat{W}(b_T, Q) = \hat{f}(x_1, b_T; \zeta_f, \mu) \hat{g}(x_2, b_T; \zeta_g, \mu) \mathcal{H}(Q; \mu).$$

- The convolutions are rewritten by Fourier transforming:

$$\begin{aligned} \mathcal{C}[w f g](x_1, x_2, \vec{q}_T) &= \int d^2\vec{k}_{T1} \int d^2\vec{k}_{T2} \delta^{(2)}(\vec{k}_{T1} + \vec{k}_{T2} - \vec{q}_T) \\ &\quad \times w_{n,m}(\vec{k}_{T1}, \vec{k}_{T2}) f(x_1, \vec{k}_{T1}) g(x_2, \vec{k}_{T2}) \\ &\Rightarrow \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) \hat{f}(x_1, b_T) \hat{g}(x_2, b_T) \end{aligned}$$

Introduction Evolution (2)

$$C[w f g](x_1, x_2, \vec{q}_T; Q) = \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) \\ \times e^{-S_A(b_T; Q^2, Q)} \hat{f}(x_1, b_T; \mu_b^2, \mu_b) \hat{g}(x_2, b_T; \mu_b^2, \mu_b)$$

- S_A contains $\ln Q b_T$
- Expressions (based on pQCD) are valid when:
 $b_0/Q \leq b_T \leq b_{T,\max}$
- At lower limit $\mu_b = b_0/b_T$ becomes larger than Q , i.e. evolution should stop ($S_A = 0$)
- At upper limit perturbation theory starts to fail, which is not exactly known. Common to take $b_{T,\max} = 0.5 \text{ GeV}^{-1}$ or $b_{T,\max} = 1.5 \text{ GeV}^{-1}$.
- This effectively boils down to a different resummation:
 $\mu_b(b_T)/Q \rightarrow \mu_b(b_T^*)/Q$

Introduction Evolution (3)

- We need to add a component that takes over as $b_T > b_{T,\max}$:

$$\hat{W}(b_T, Q) \equiv \hat{W}(b_T^*, Q) e^{-S_{NP}(b_T, Q)}$$

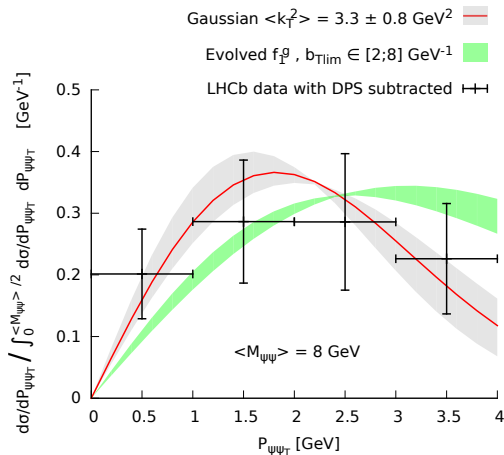
- There are different parameterizations for S_{NP} in the literature, but typically it is chosen to be a Gaussian:

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1 \text{ GeV}$$

- We obtain the following expression for the convolutions:

$$\begin{aligned} \mathcal{C}[w f g](x_1, x_2, \vec{q}_T; Q) &= \int_0^\infty \frac{db_T}{2\pi} b_T^n J_m(b_T q_T) e^{-S_A(b_T^*; Q^2, Q)} e^{-S_{NP}(b_T; Q)} \\ &\times \hat{f}(x_1, b_T^*; \mu_b^2, \mu_b) \hat{g}(x_2, b_T^*; \mu_b^2, \mu_b) \end{aligned}$$

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Results for $x_1 = x_2$: Florent

F. Scarpa et al. (Eur.Phys.J. C 80 no.2, (2020) 87)

R. Aaij et al. (JHEP06(2017)047)

$x_1 \neq x_2$; Master project Alice; co-supervised by me

Goal: phenomenological study of the azimuthal asymmetries for J/ψ pair production in pp collisions with $\rightarrow x_1 \neq x_2$

Implementation: **ex-novo code** in Python; faster and we implement use of LHAPDF package for PDFs (for perturbative tails TMDs)



Code validation: reproduced published results ($x_1 = x_2$)

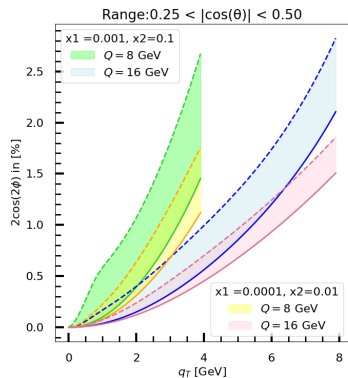


NEW: first studies with $x_1 \neq x_2$
(two sets of x_1, x_2 but same rapidity $y = \frac{1}{2} \ln \frac{x_1}{x_2}$)

Preliminary: predictions for $\cos(2\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: $[0.25; 0.50]$, $Q = M_{\psi\psi} = 8, 16$ GeV
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ contribution up to 3%

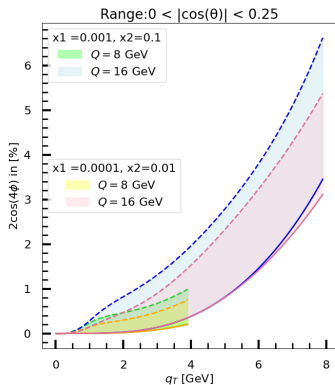
▷ big overlap in the low q_T region, not for large q_T

▷ \sim same magnitude for low and high Q

Preliminary: predictions for $\cos(4\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: $[0; 0.25]$, $Q = 8, 16$ GeV
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ max contribution 5 – 6%

▷ overlap $\forall q_T$

▷ much higher amplitude for high Q (at high q_T)

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Summary

- Double J/ψ production is a very promising process to investigate gluon TMDs
- Quarkonium q_T spectrum probes gluon transverse momenta
- Azimuthal asymmetries arise due to linear polarization of gluons inside unpolarized hadrons
- $x_1 \approx x_2$ seems to be favoured: lower azimuthal asymmetries for $\frac{x_1}{x_2} \neq 1$
- Further studies can be made considering polarised protons \rightarrow access to more gluon TMDs
- For $pp \rightarrow \eta_{c,b} X$:

A. Bacchetta et al. (arXiv:2208.06252) and proceedings

- For $ep \rightarrow J/\psi X$:

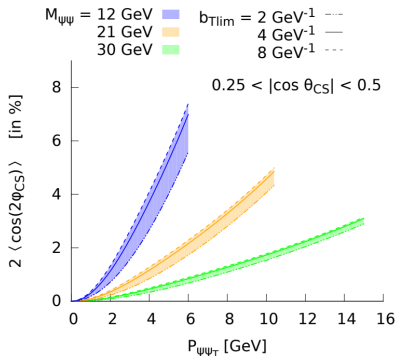
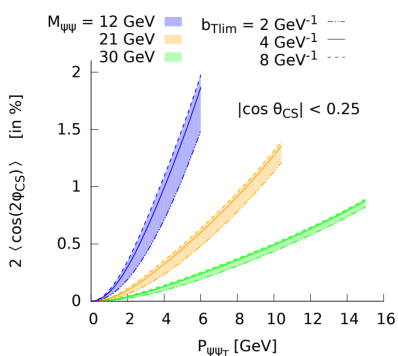
J. Bor and D. Boer (Phys.Rev.D 106 (2022) 1)

Backup slides

Results for $x_1 = x_2: \cos(2\Phi_{CS})$

Plots considering:

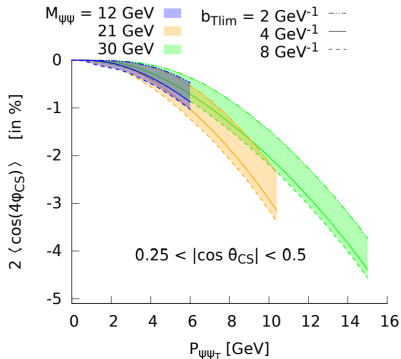
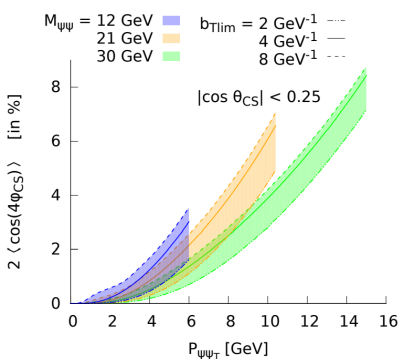
- Two ranges of $\cos(\theta_{CS})$: $[0; 0.25]$ and $[0.25; 0.50]$
- Three values for the invariant mass: 12, 21, 30 GeV; **$x_1=x_2$**



Results for $x_1 = x_2: \cos(4\Phi_{CS})$

Plots considering:

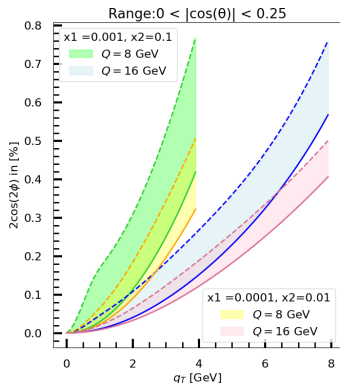
- Two ranges of $\cos(\theta_{CS})$: $[0; 0.25]$ and $[0.25; 0.50]$
- Three values for the invariant mass: 12, 21, 30 GeV; **$x_1=x_2$**



Results in for $x_1 \neq x_2$: $\cos(2\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: $[0; 0.25]$, $Q = M_{\psi\psi}$
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ contribution below 1%

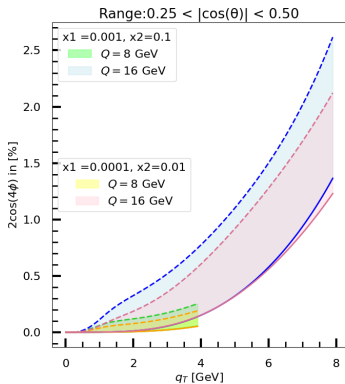
▷ big overlap in the low q_T region, not for large q_T

▷ \sim same magnitude for low and high Q

Results in for $x_1 \neq x_2$: $\cos(4\Phi_{CS})$

Plots considering:

- Range of $\cos(\theta_{CS})$: $[0.25; 0.50]$, $Q = M_{\psi\psi}$
- Two different sets $(x_1; x_2)$: $(10^{-3}; 10^{-1})$ and $(10^{-4}; 10^{-2})$



▷ contribution up to 3%

▷ higher amplitude for high Q (low Q negligible)

Hard scattering coefficients

$$F_1 = \frac{\mathcal{N}}{\mathcal{D}M_\Psi^2} \sum_{n=0}^6 f_{1,n}(\cos\theta_{CS})^{2n} \quad F_2 = \frac{2^4 3 M_\Psi^2 \mathcal{N}}{\mathcal{D}M_{\Psi\Psi}^4} \sum_{n=0}^4 f_{2,n}(\cos\theta_{CS})^{2n}$$

$$F'_3 = F_3 = \frac{-2^3(1-\alpha^2)\mathcal{N}}{\mathcal{D}M_{\Psi\Psi}^2} \sum_{n=0}^5 f_{3,n}(\cos\theta_{CS})^{2n}$$

$$F_4 = \frac{(1-\alpha^2)^2 \mathcal{N}}{\mathcal{D}M_{\Psi\Psi}^2} \sum_{n=0}^6 f_{4,n}(\cos\theta_{CS})^{2n}$$

with: $\alpha = \frac{2M_\Psi}{M_{\Psi\Psi}}$, $\mathcal{N} = 2^{11} 3^{-4} (N_c^2 - 1)^{-2} \pi^2 \alpha_s^4 |R_\Psi(0)|^4$,

$\mathcal{D} = M_{\Psi\Psi}^4 (1 - (1 - \alpha^2) \cos^2\theta_{CS})^4$ and $R_\Psi(0)$ is the J/ψ radial wave function at the origin and $N_c = 3$.

The Sudakov Factor and Scales

- The solution of the evolution equations results in:

$$\hat{f}_1^g(x_1, \mathbf{b}_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T; \zeta, \mu)} \hat{f}_1^g(x_1, \mathbf{b}_T; \mu_b^2, \mu_b)$$

$$\hat{h}_1^{\perp g}(x_1, \mathbf{b}_T; \zeta, \mu) = e^{-\frac{1}{2}S_A(b_T; \zeta, \mu)} \hat{h}_1^{\perp g}(x_1, \mathbf{b}_T; \mu_b^2, \mu_b)$$

- $\mu \sim Q$ avoids large logarithms in \mathcal{H}
- TMDs should be evaluated at their natural scale:
 $\sqrt{\zeta_0} \sim \mu_0 \ll \sqrt{\zeta} \sim \mu$
- \Rightarrow take $\sqrt{\zeta_0} \sim \mu_0 \sim \mu_b \equiv b_0/b_T$ (with $b_0 = 2e^{-\gamma_E}$), in order to minimize both logarithms of μb_T and ζb_T^2 in S_A , and then evolved up to $\sqrt{\zeta} \sim \mu \sim Q$

Perturbative tails

- The large transverse momentum perturbative tail of the TMDs can be written as:

$$\hat{f}_1^g(x, b_T; \mu_b^2, \mu_b) = f_{g/P}(x; \mu_b) + \mathcal{O}(\alpha_s) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

$$\hat{h}_1^{\perp g}(x, b_T; \mu_b^2, \mu_b) = -\frac{\alpha_s(\mu_b)}{\pi} \int_x^1 \frac{dx'}{x'} \left(\frac{x'}{x} - 1 \right) \left\{ C_A f_{g/P}(x'; \mu_b) + C_F \sum_{i=q, \bar{q}} f_{i/P}(x'; \mu_b) \right\} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(b_T \Lambda_{\text{QCD}})$$

P. Sun et al. (Phys.Rev.D 84 (2011) 094005)

b_T -Domains

- To ensure $b_0/Q \leq b_T$ we take:

$$b_c(b_T) = \sqrt{b_T^2 + \left(\frac{b_0}{Q}\right)^2}$$

- For $b_T \leq b_{T,\max}$:

$$b_T^*(b_c(b_T)) = \frac{b_c(b_T)}{\sqrt{1 + \left(\frac{b_c(b_T)}{b_{T,\max}}\right)^2}}$$

J. Collins et al. (Phys.Rev.D 94 (2016) 3, 034014)

The Non-perturbative Sudakov Factor

$$S_{NP}(b_T; Q) = A \ln \frac{Q}{Q_{NP}} b_T^2 \quad \text{with} \quad Q_{NP} = 1 \text{ GeV}$$

$b_{T,\text{lim}}$ (GeV ⁻¹)	r (fm $\sim 1/(0.2 \text{ GeV})$)	A (GeV ²)
2	0.2	0.64
4	0.4	0.16
8	0.8	0.04

Table 1: Values of the parameter A for $b_{T,\text{lim}}$ and r determined at $Q = 12$ GeV. A is defined at which $\exp(-S_{NP})$ becomes negligible ($\sim 10^{-3}$). To estimate the uncertainty associated with the S_{NP} we vary $b_{T,\text{lim}}$ spanning roughly from $b_{T,\text{max}} = 1.5 \text{ GeV}^{-1}$ to the charge radius of the proton. r is the range over which the interactions occur from the centre of the proton.

In this talk I will explain why double J/ψ production in proton-proton collisions is a promising process to study gluon TMDs. I will touch upon the azimuthal asymmetries that arise in this process and the TMD evolution that is included in the computations. To conclude I will present some numerical results from F. Scarpa and A. Colpani and refer to some other interesting studies.