# The W-boson mass and the strong interaction

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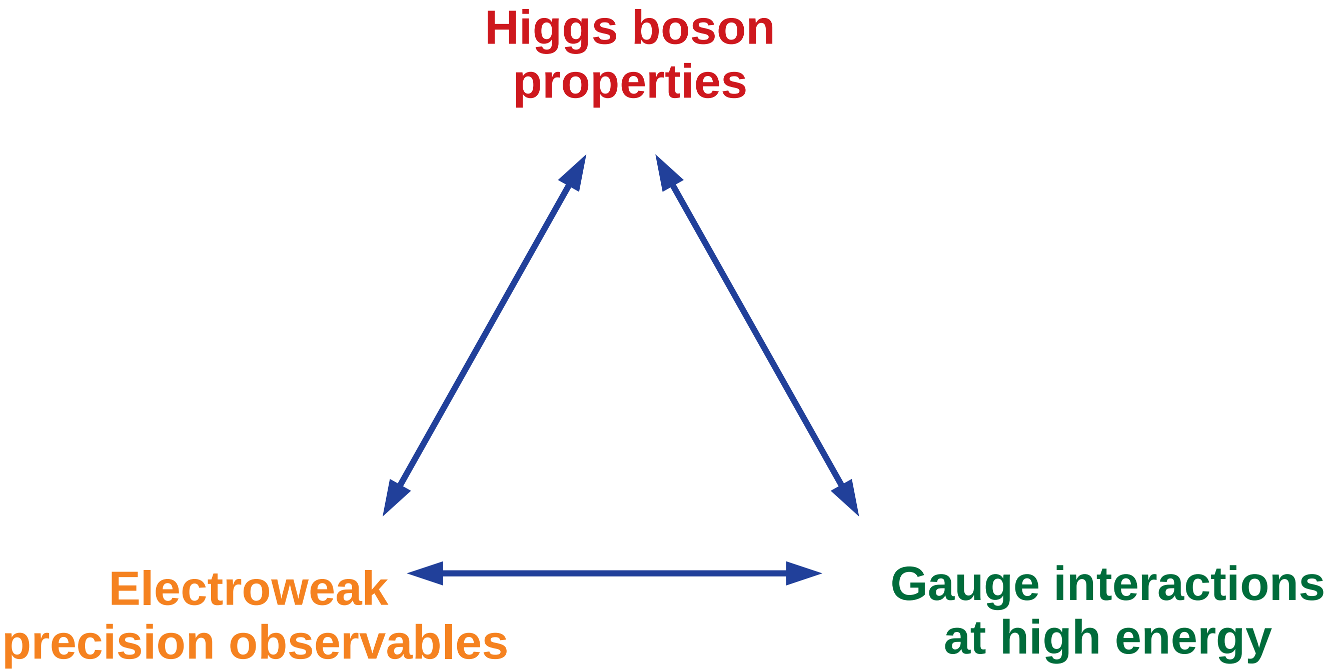
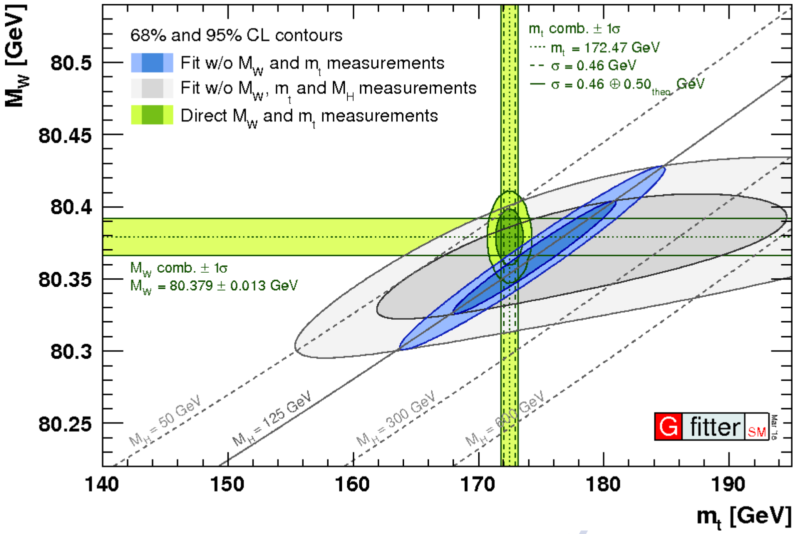
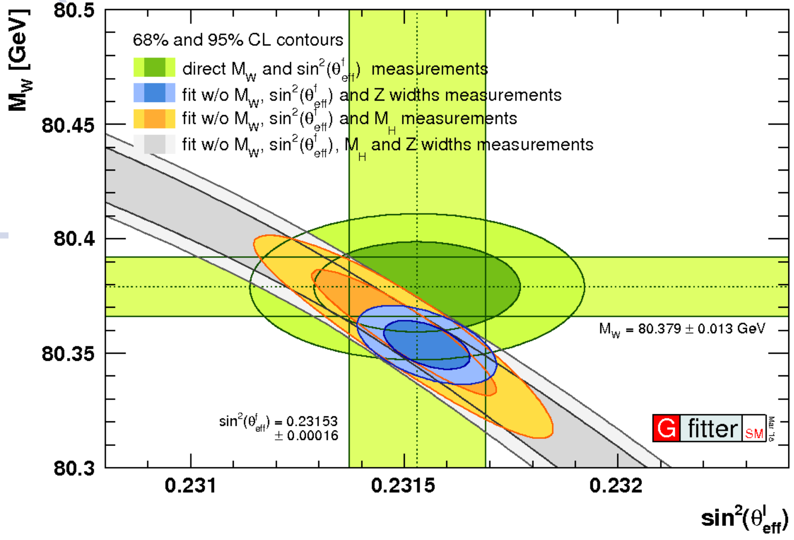
# Tests of the electroweak theory



# Tests of the electroweak theory




**Electroweak precision observables**

This talk

# Electroweak predictions in leading order

- The electroweak gauge sector of the SM is constrained by three precisely known parameters :
  - The electromagnetic coupling constant : $\alpha$ = 1/137035999206(11)
  - The muon decay constant : $G_\mu$ = 1.1663787(6) GeV$^{-2}$
  - The Z boson mass : $m_Z$ = 91.1876(21) GeV

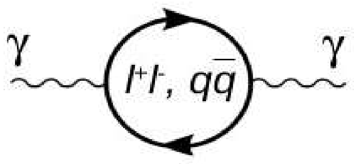
- The W boson mass is given by

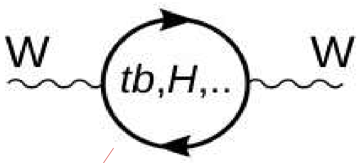
$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-4\frac{\pi\,\alpha}{\sqrt{2}\,G_\mu m_Z^2}}\right)$$

# Quantum corrections : $m_W$

- Higher-order corrections, predominantly the boson self-energies, modify the leading-order relations to

$$m_W^2 = \frac{m_Z^2}{2}\left(1+\sqrt{1-4\frac{\pi\alpha}{\sqrt{2}\,G_\mu m_Z^2}\,\frac{1}{1-\Delta r}}\right)$$

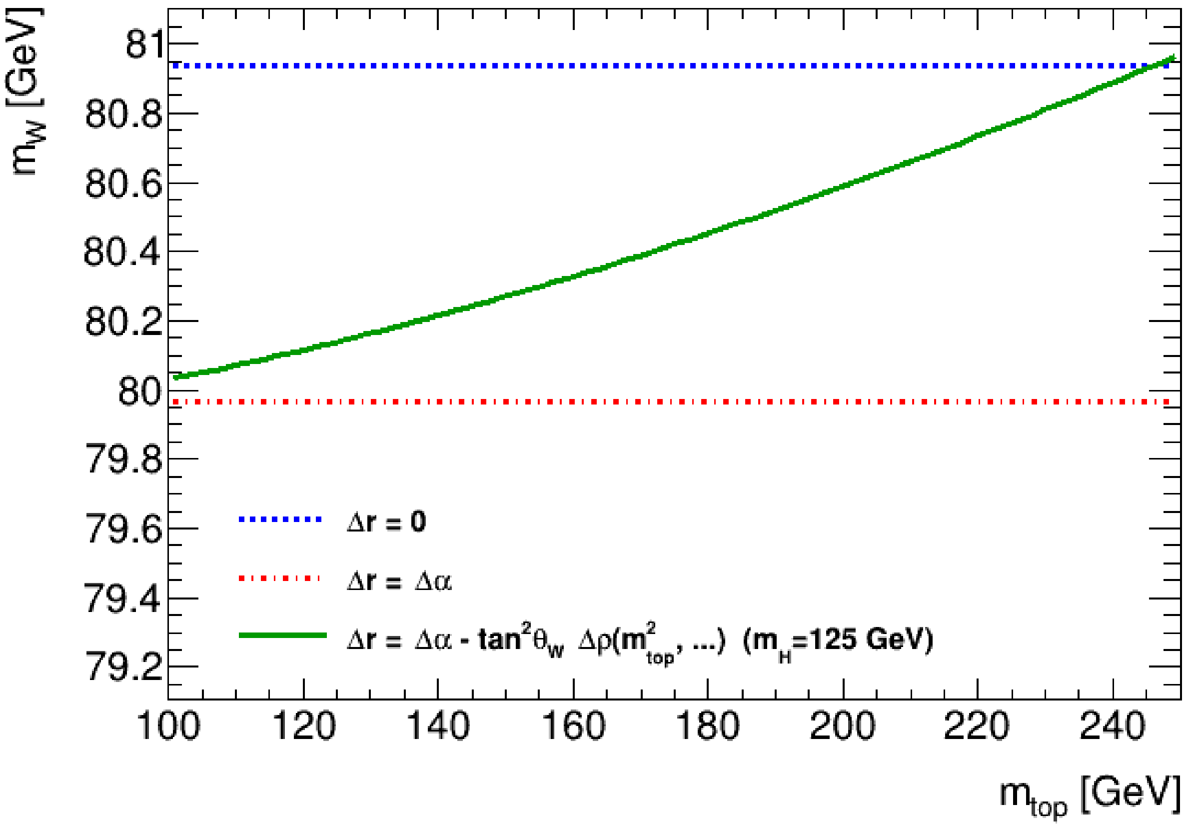



$$\Delta r = \Delta\alpha - \tan^2\theta_W\,\Delta\rho = \sim 0.059 - \frac{3G_\mu m_W^2}{8\sqrt{2}\,\pi^2}\left[\frac{m_{top}^2}{m_W^2}\cot^2\theta_W - \left(\ln\frac{m_H^2}{m_W^2}-\frac{5}{6}\right)+\dots\right]$$

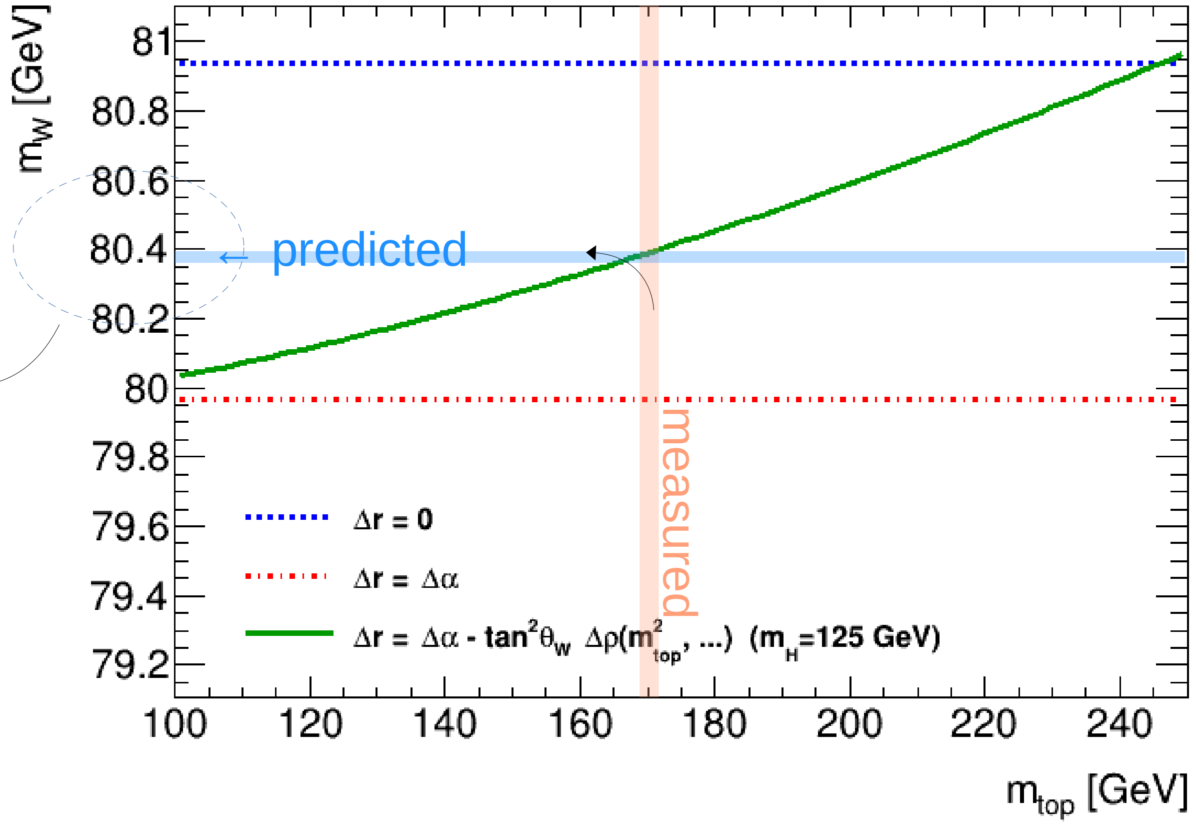
$\alpha(0) \sim 1/137.. \rightarrow \alpha(m_Z) \sim 1/128.9$

# Quantum corrections : $m_W$



# Quantum corrections : $m_W$

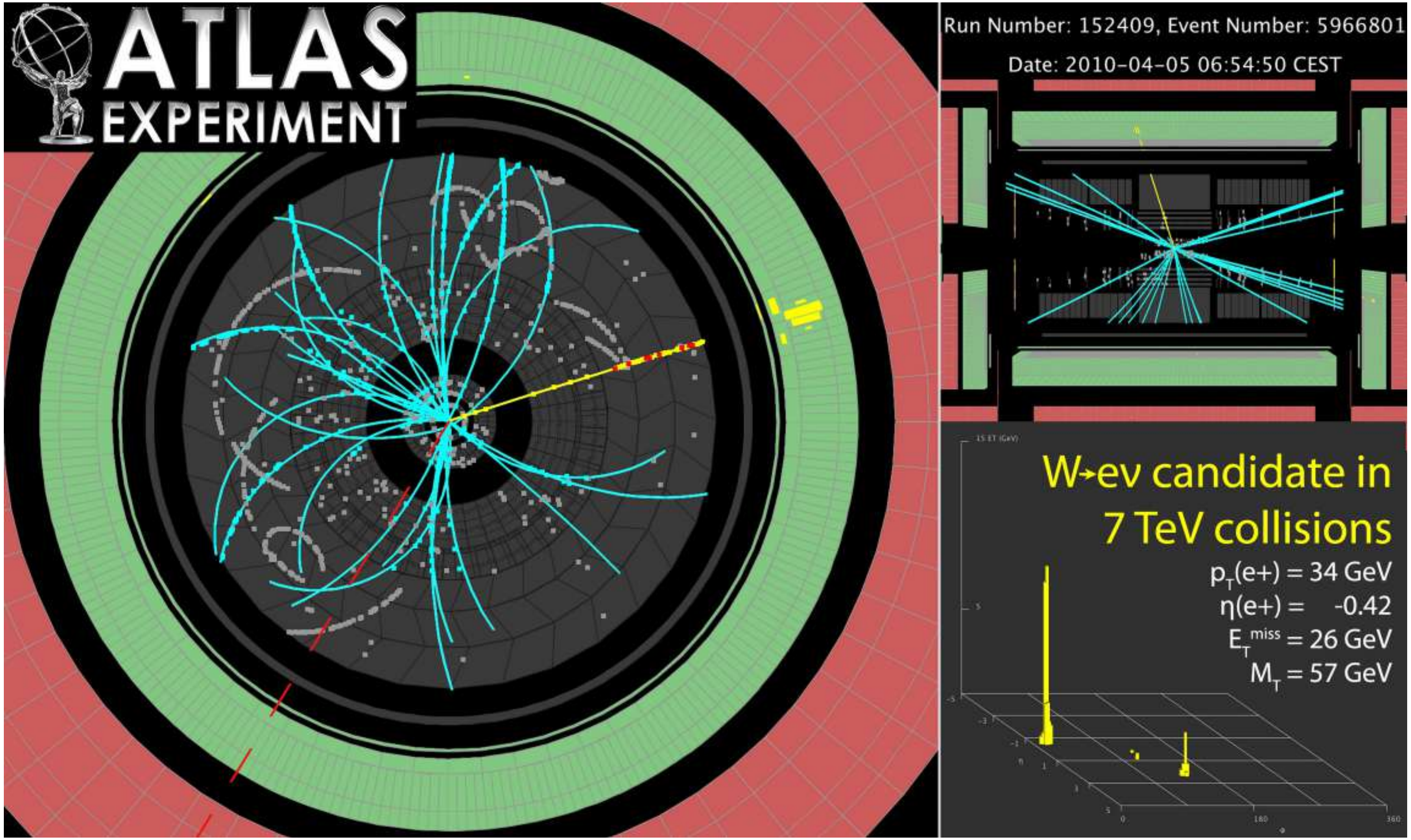
Measured value?




Nowadays:

- Inputs : $\delta m_{top} \sim 0.7$ GeV $\delta m_H < 0.2$ GeV
- Output : $m_W = 80.356 +/- 0.008$ GeV

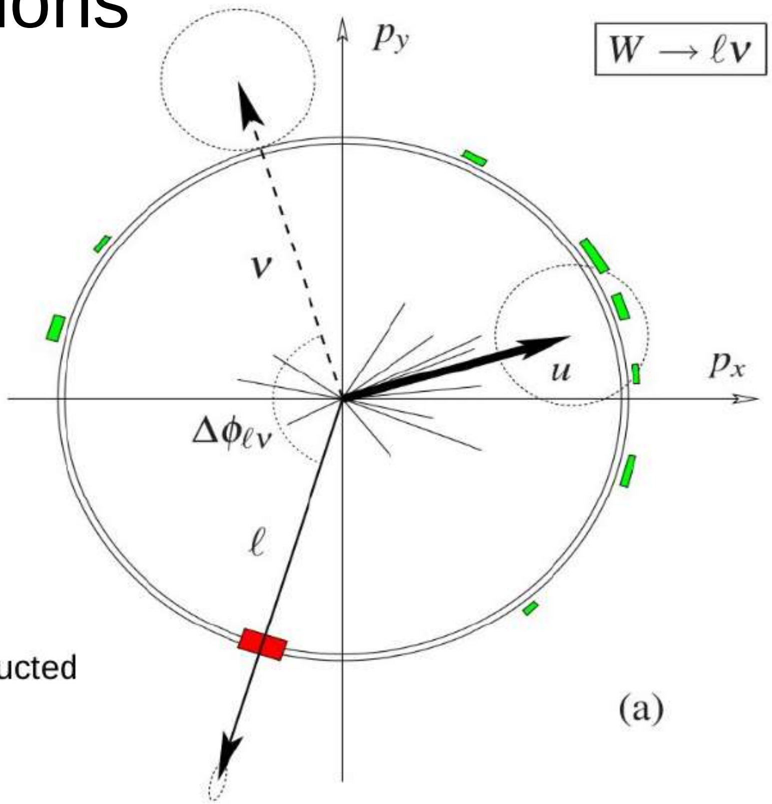
# The W boson mass in proton collisions



# The W boson mass in proton collisions

- Incomplete kinematics (missing neutrino!)
  → no invariant mass
  → rely on measured quantities, and exploit momentum conservation in the transverse plane

- Event representation :




- Main signature :
  single electron or muon $\vec{p}_T^{\,l}$ $p_T^l$

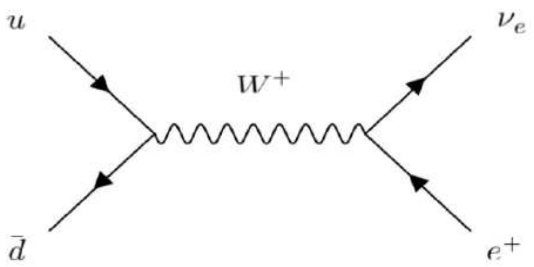
- Recoil : sum of "everything else" reconstructed in the calorimeters; a measure of $p_T^{W,Z}$

$$\vec{u}_T = \sum_i \vec{E}_{T,i}$$

- Derived quantities : $\vec{p}_T^{\,miss} = -\left(\vec{p}_T^{\,\ell} + \vec{u}_T\right)$ $m_T = \sqrt{2 p_T^{\ell} p_T^{miss}(1 - \cos\Delta\phi)}$

# The W boson mass in proton collisions

- The process at leading order, no width :




$$\hat{\sigma}_{u\bar{d}\to\ell^+\nu} = \frac{1}{3}\frac{|V_{ud}|^2}{3\pi}\left(\frac{G_F m_W^2}{\sqrt{2}}\right)^2 \delta(m^2 - m_W^2)$$

Unpolarized differential cross section (spin 1) :

$$\frac{d\hat{\sigma}_{u\bar{d}\to\ell^+\nu}}{d\cos\theta} \propto 1 + \cos^2\theta$$

$$p_{\mathrm{T}}^{\ell} = \frac{m_W}{2}\sin\theta$$

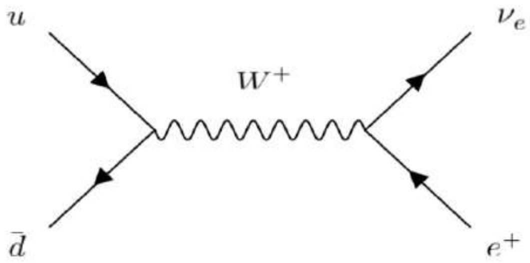
$\rightarrow$

$$\frac{d\hat{\sigma}_{u\bar{d}\to\ell^+\nu}}{dp_{\mathrm{T}}^{\ell}} \propto \frac{\left(1 - \frac{2p_{\mathrm{T}}^{\ell}}{m_{\mathrm{W}}^2}\right)}{\sqrt{1 - \frac{4p_{\mathrm{T}}^{\ell}}{m_{\mathrm{W}}^2}}}$$

→ the “Jacobian peak”

# The W boson mass in proton collisions

- The process at leading order, no width :

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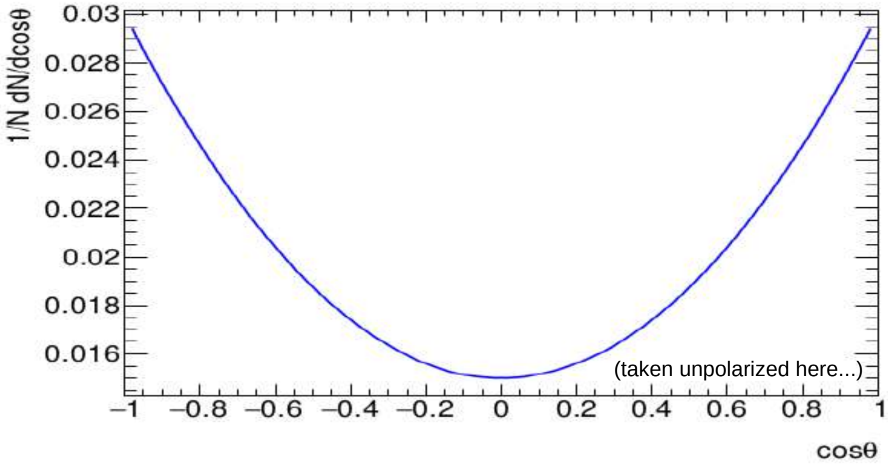
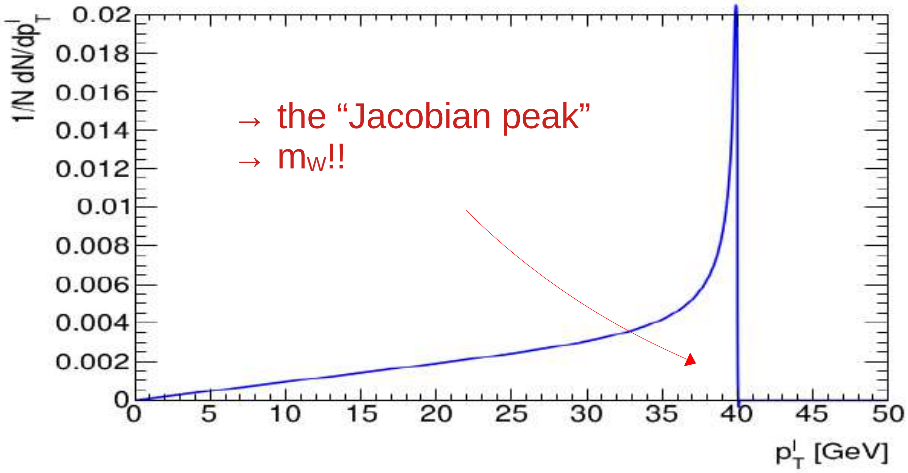
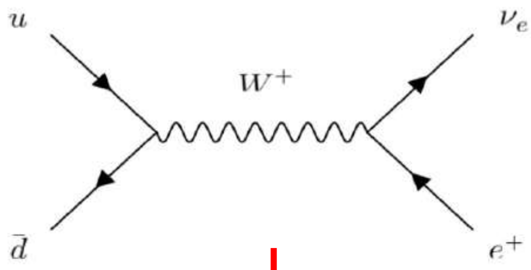


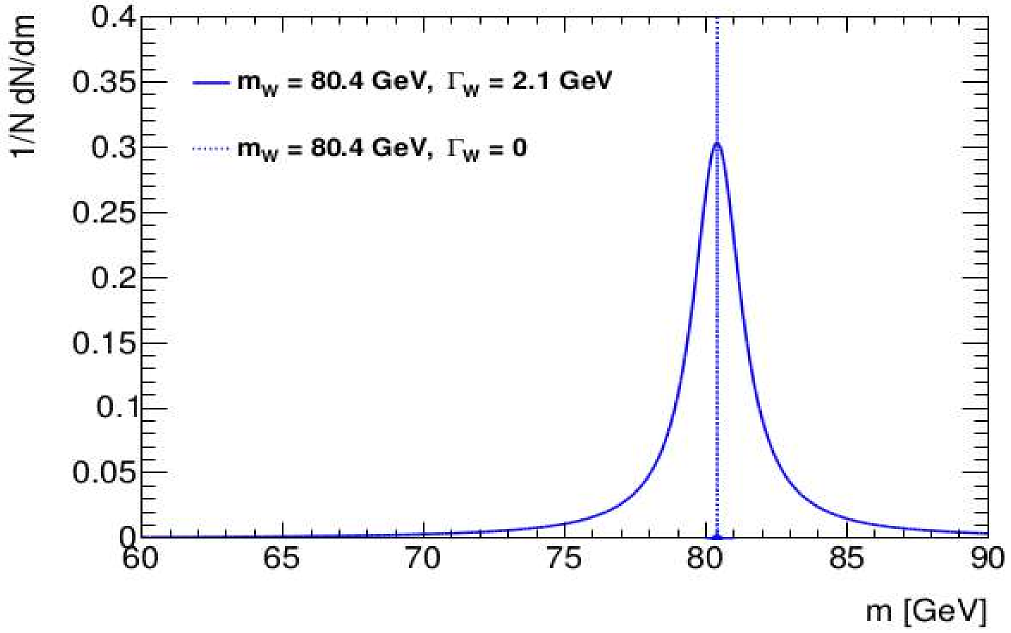



# The W boson mass in proton collisions

- Natural width :



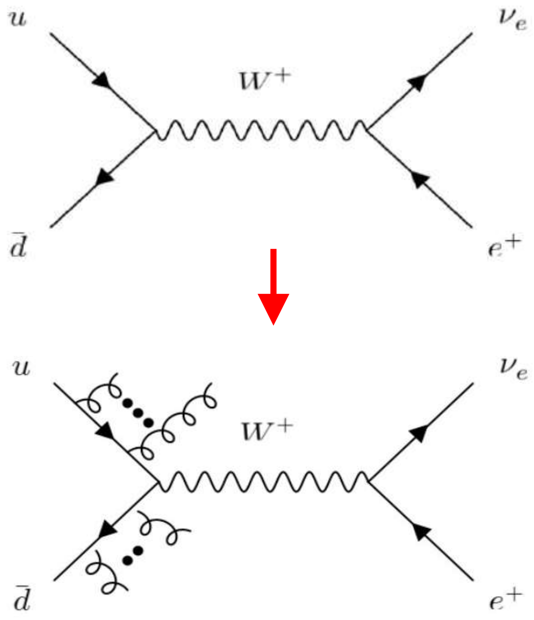

$$\delta(m^2 - m_W^2) \to \frac{m^2}{(m^2 - m_W^2)^2 + (m\Gamma_W/m_W)^2}$$

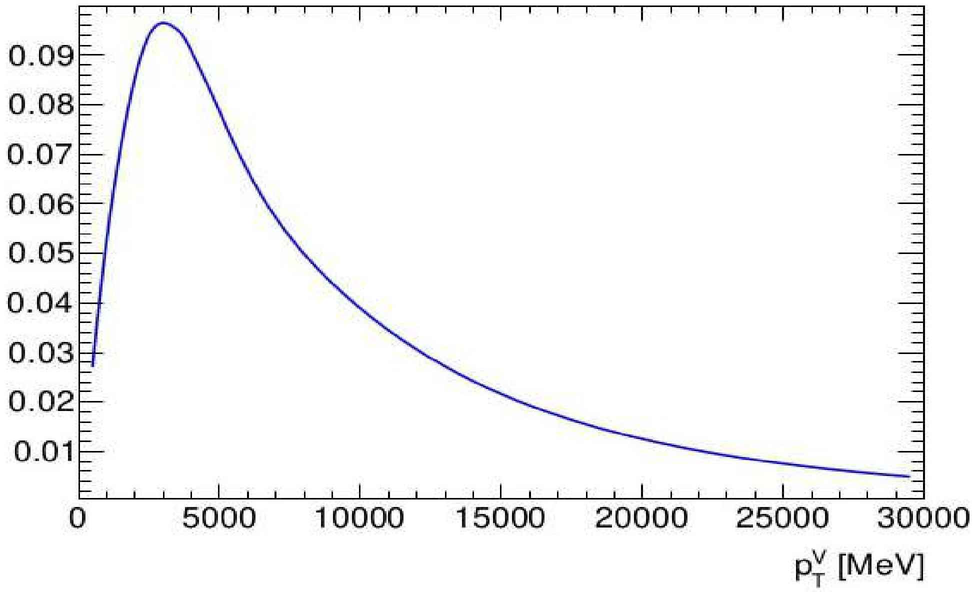


# The W boson mass in proton collisions

- Radiation in the initial state (QCD)

  → non trivial transverse momentum distribution

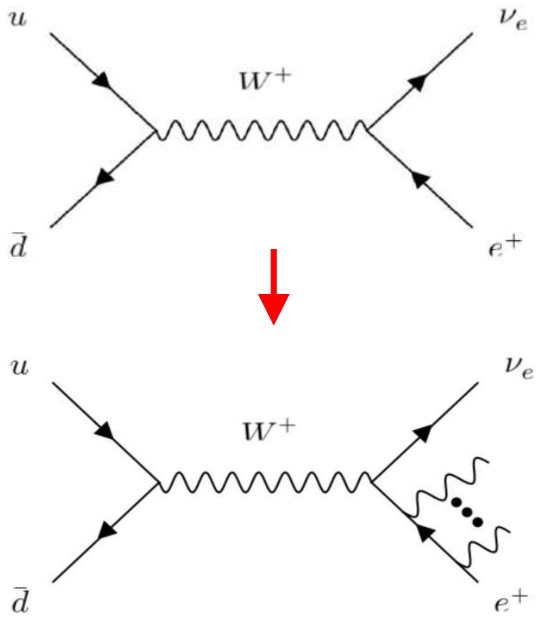


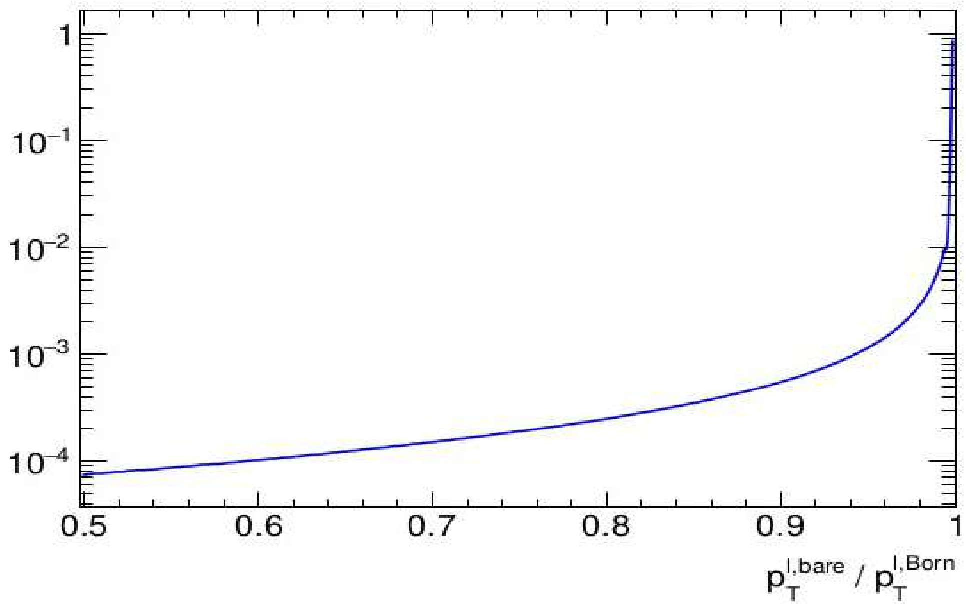



# The W boson mass in proton collisions

Radiation in the final state (QED)

→ decays leptons lose a fraction of their energy

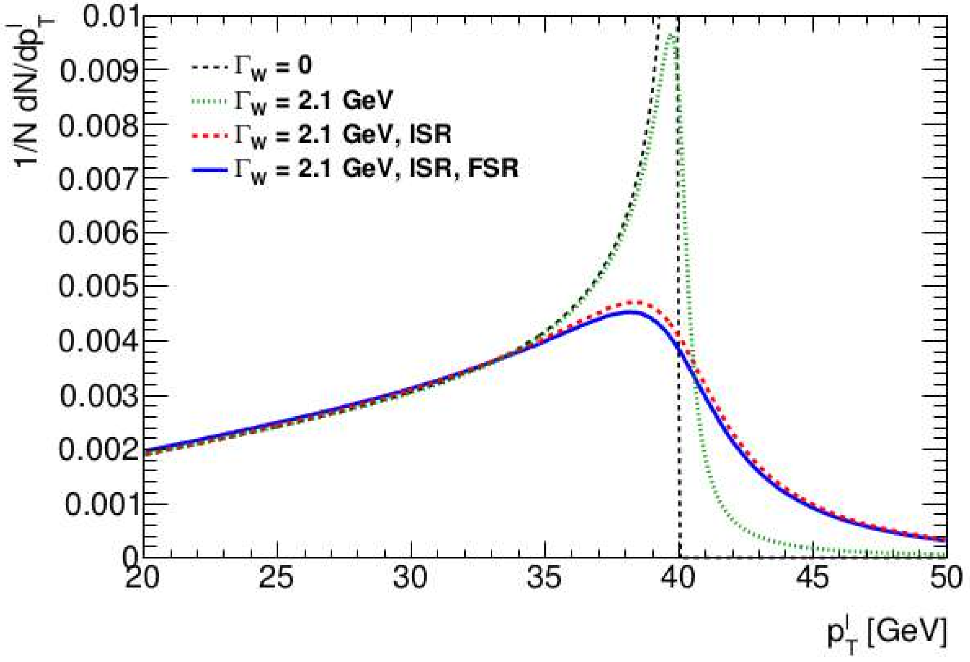


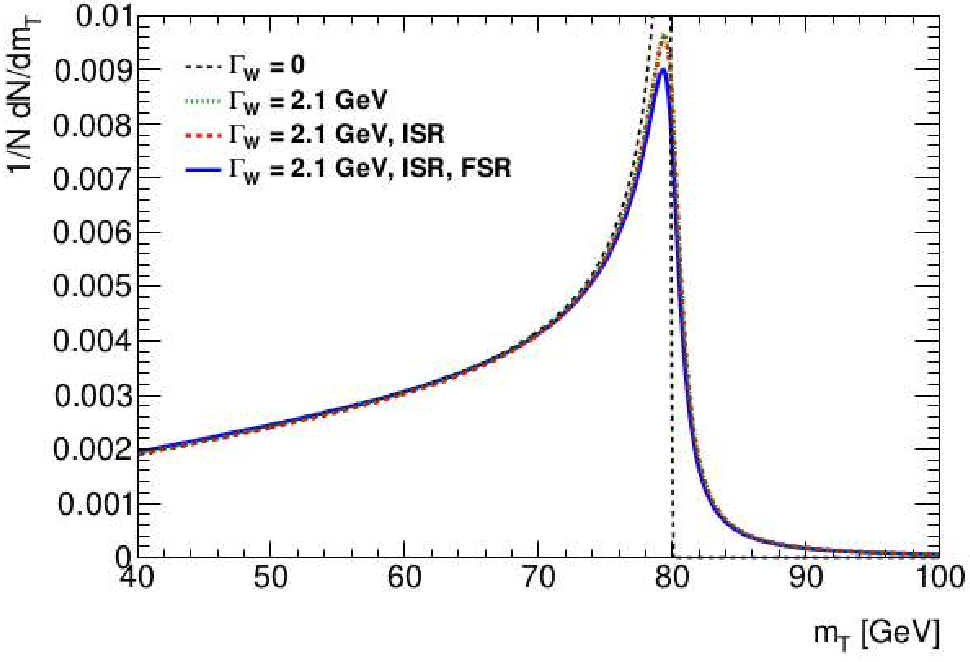



# The W boson mass in proton collisions

- Summary of physics effects

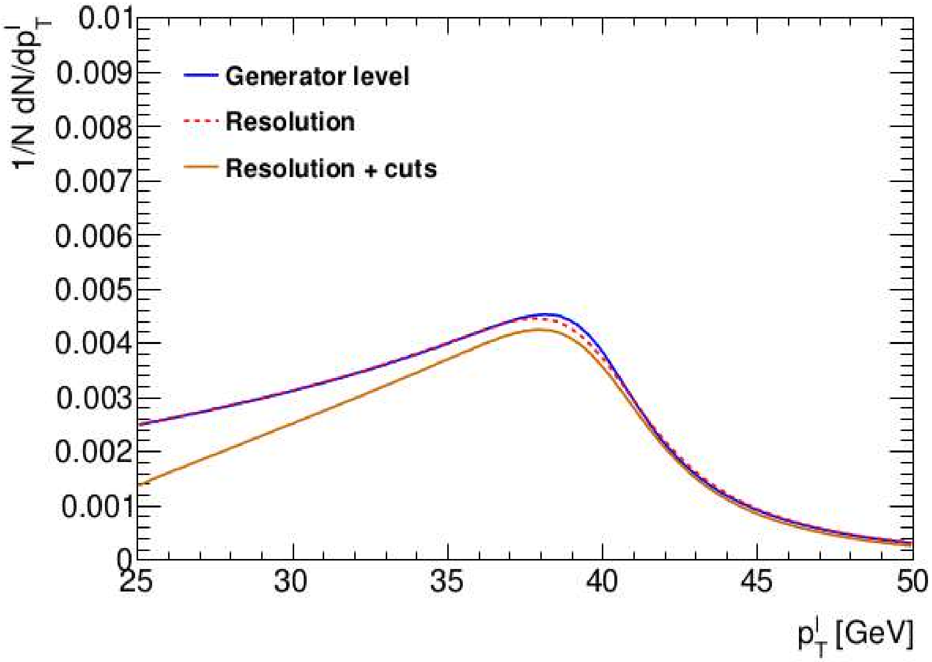
  → all carry uncertainties to be quantified!

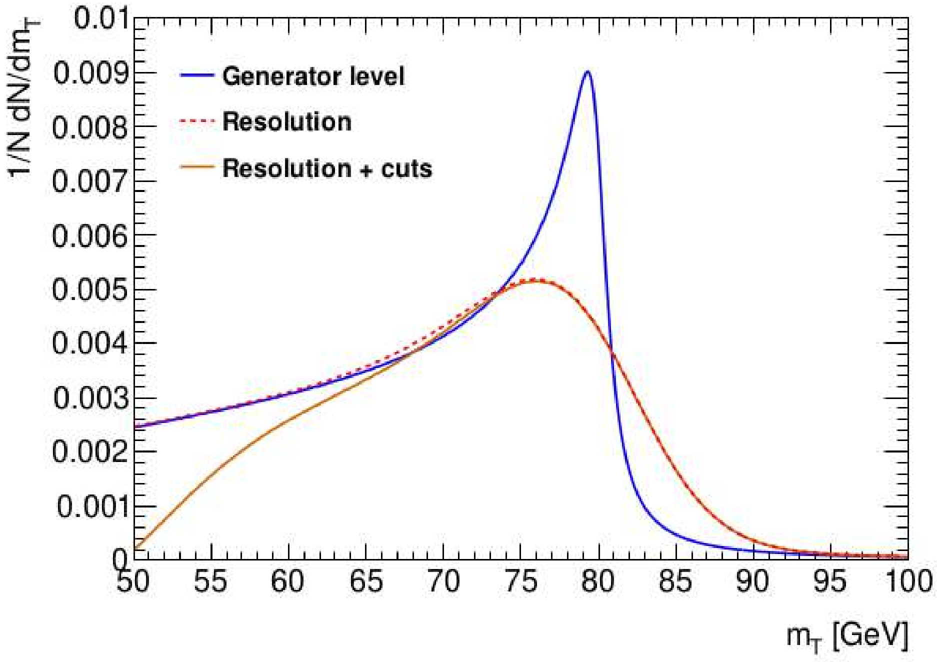


# The W boson mass in proton collisions

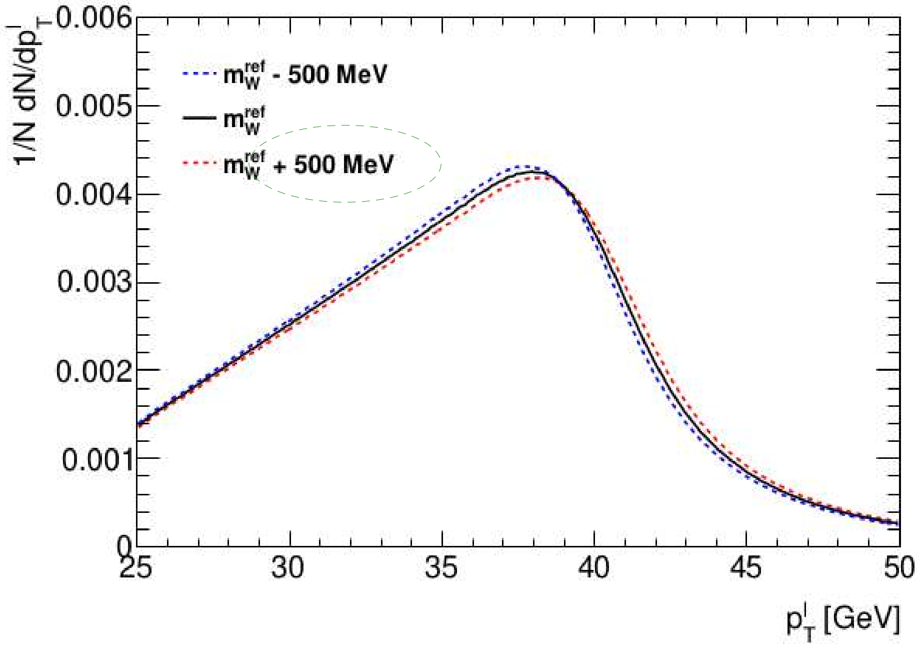
- Detector effects, also with uncertainties :
  - Lepton calibration and resolution; Missing $E_T$ resolution ~ 5 – 15 GeV
  - Efficiencies and acceptance ~15% (with non-trivial kinematic dependence!)

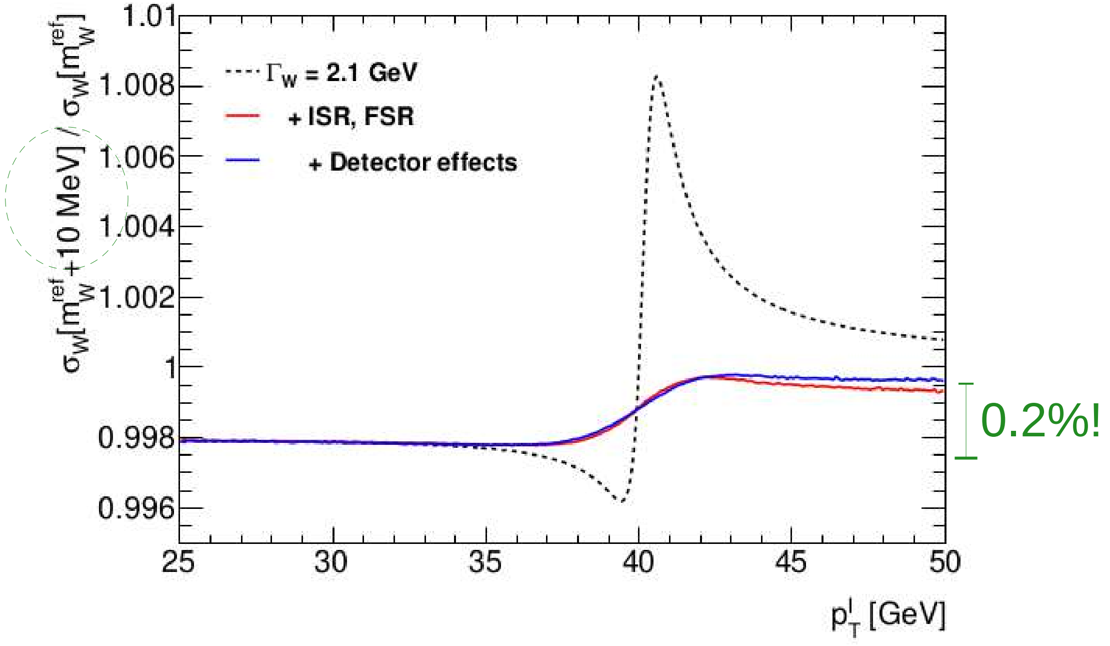


# The W boson mass in proton collisions

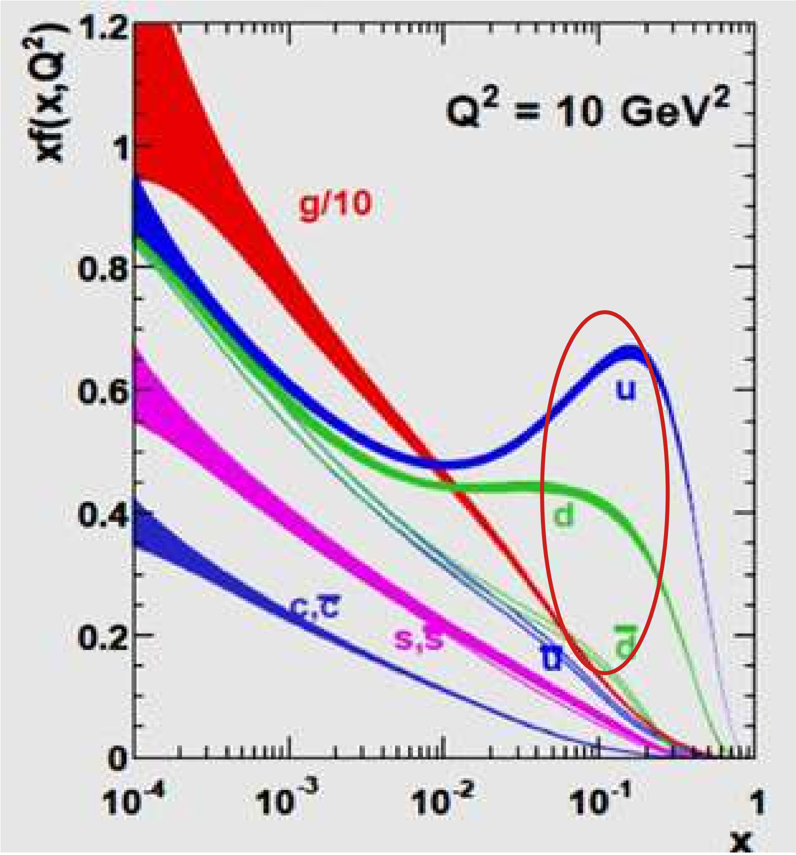
- Mass measurement : produce models ("templates") of the final state distributions for different mass hypotheses; compare to data



# Strong interaction effects

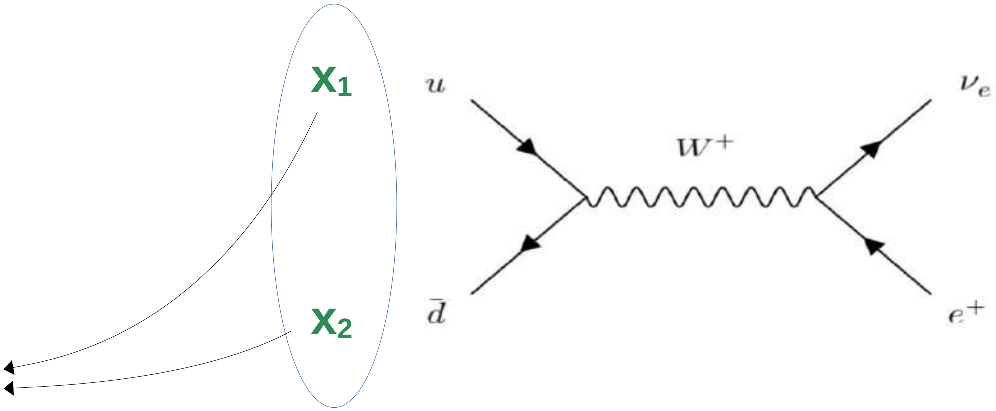
- PDFs




$m_W \sim 80$ GeV

$x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

| | | | | |
|---|---|---|---|---|
| Tevatron | $\sqrt{s} \sim 2$TeV | $p\bar{p}$ | $0<y<2$ | $x_{1,2} \sim 10^{-2} - 10^{-1}$ |
| ATLAS | $\sqrt{s} \sim 7$TeV | pp | $0<y<3$ | $x_{1,2} \sim 10^{-3} - 10^{-1}$ |
| LHCb | $\sqrt{s} \sim 13$TeV | pp | $y \sim 4$ | $x_{1,2} \sim 10^{-4} - 10^{-1}$ |



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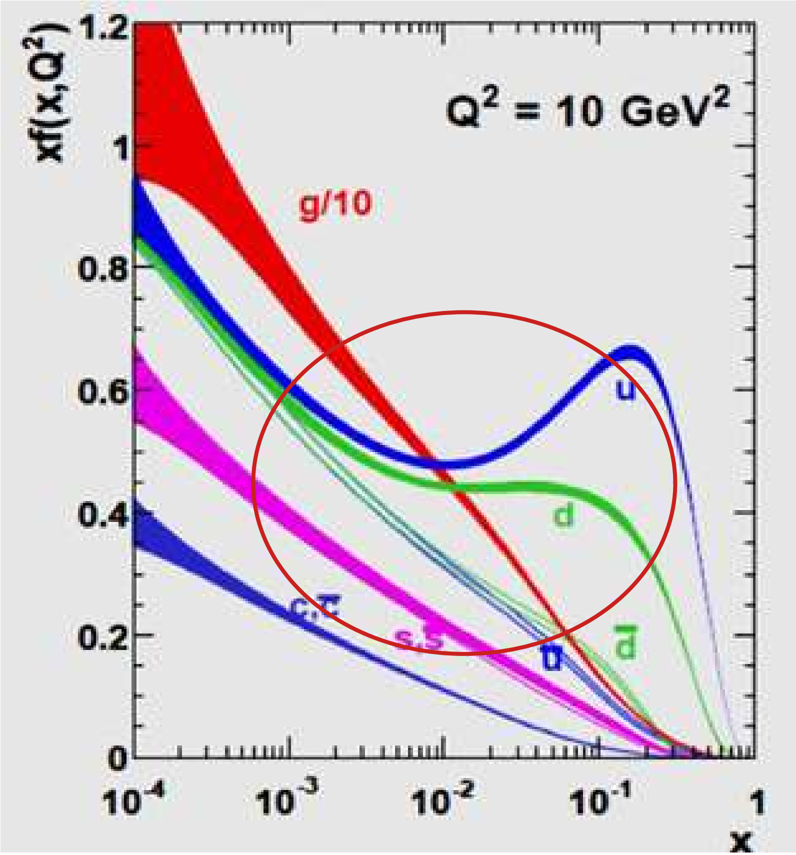


$m_W \sim 80$ GeV  $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

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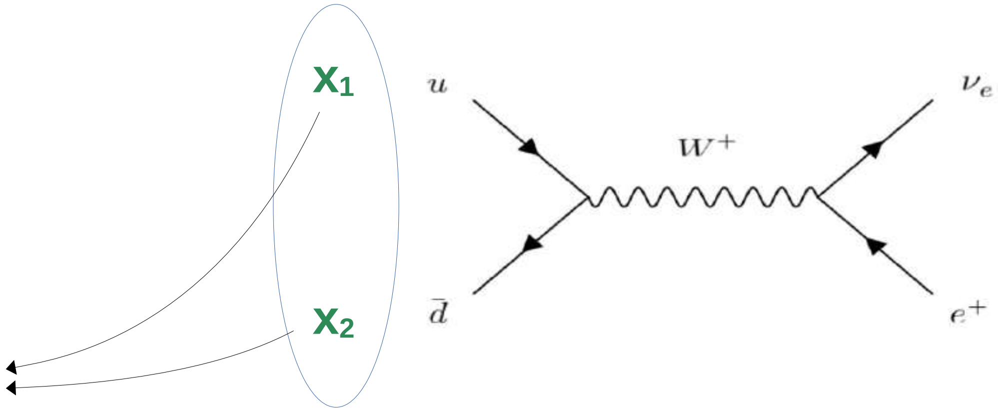

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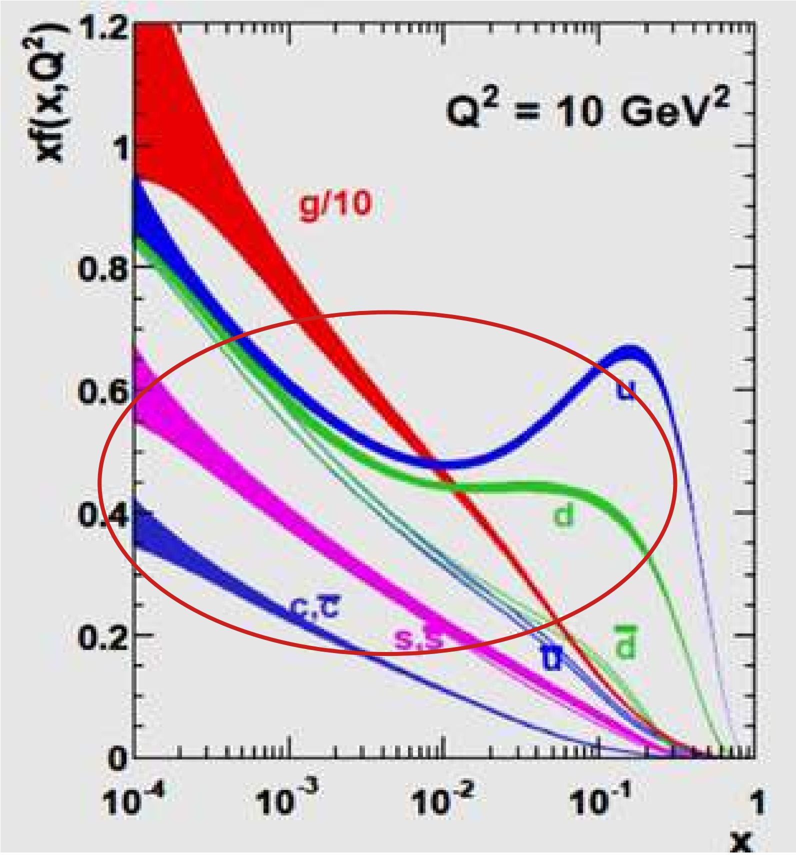


$m_W$ ~ 80 GeV $\quad$ $x_{1,2} = m/\sqrt{s}\, e^{\pm y}$

| | | | | |
|---|---|---|---|---|
| Tevatron | √s~ 2TeV | $p\bar{p}$ | 0<y<2 | $x_{1,2} \sim 10^{-2} - 10^{-1}$ |
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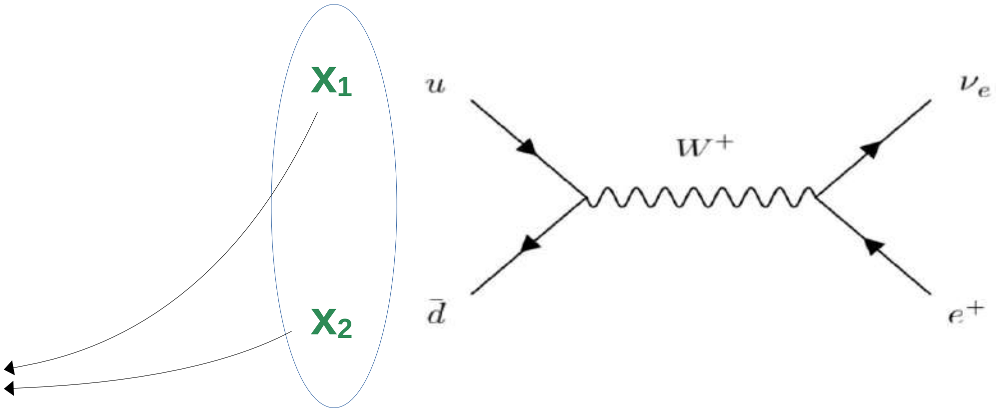
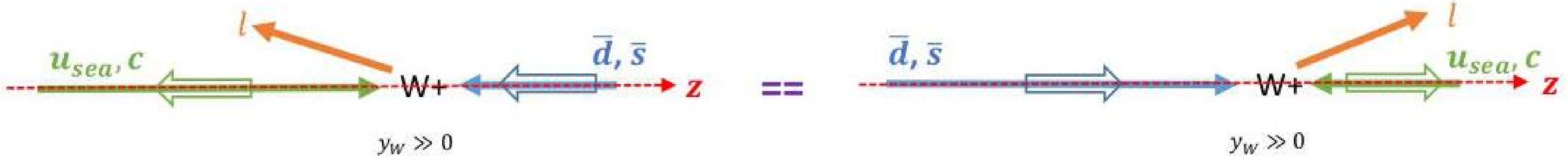


# Strong interaction effects

- PDFs
  - The valence and sea distributions
    - Determine the W-boson rapidity distribution → acceptance & fiducial distributions
    - The valence distributions polarize the W decay, with corresponding uncertainties.
    - For $W^+$ :




On average :

# Strong interaction effects

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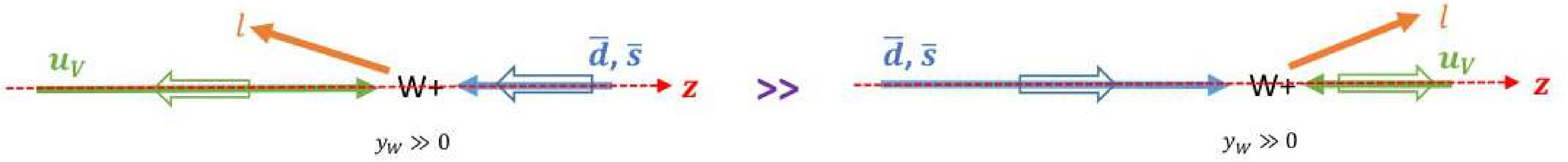
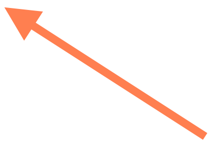



On average :



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  - The valence and sea distributions
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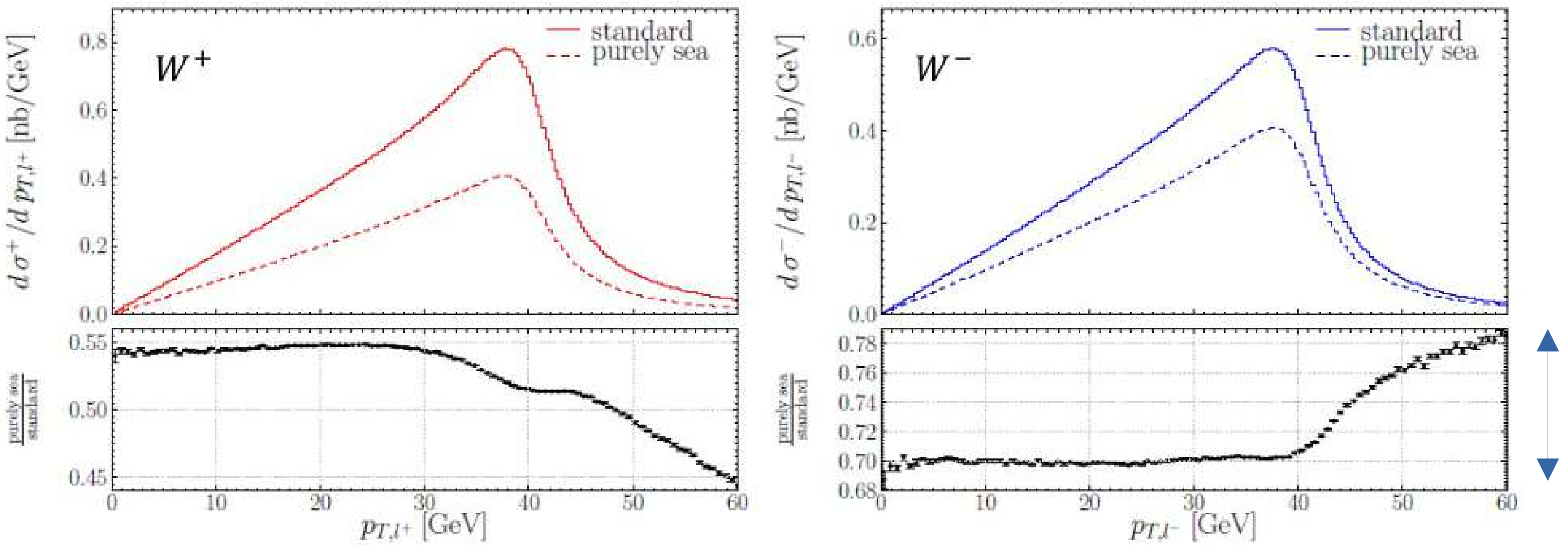
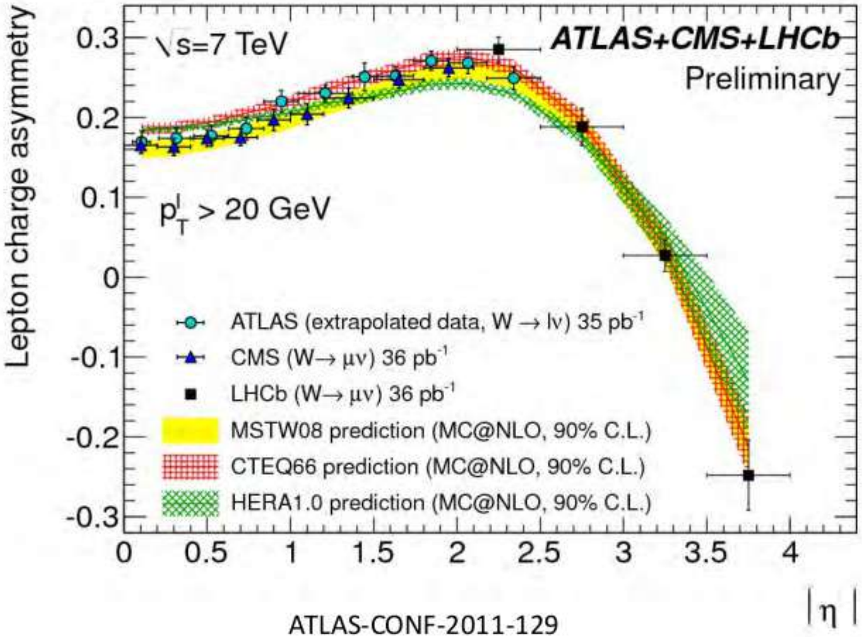



15% effect
Remember:
$\delta m_W$=10 MeV → 0.2%

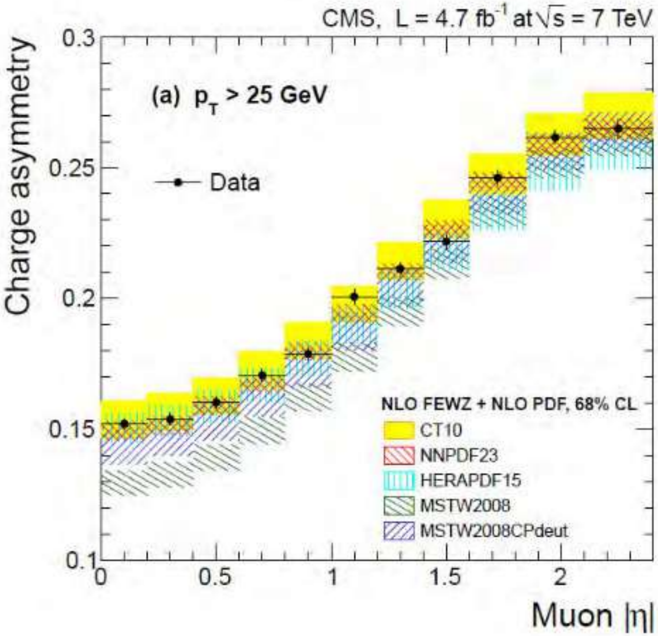
Eur.Phys.J.C69:379-397,2010

# Constraining PDFs: W charge asymmetry

- vs rapidity: $A(y) \approx \frac{u_V - d_V}{u_V + d_V + 2\,r_s c}$ ($r_s \approx \bar{s}/\bar{d}$ and assuming $\bar{u} \approx \bar{d}$ and $s \approx \bar{s}$).
- Experiments only access $\eta_{\text{lep}}$: effect blurred by V-A. Still very discriminating information: probes a mixture of $u_V/d_V$ and second generation quark PDFs



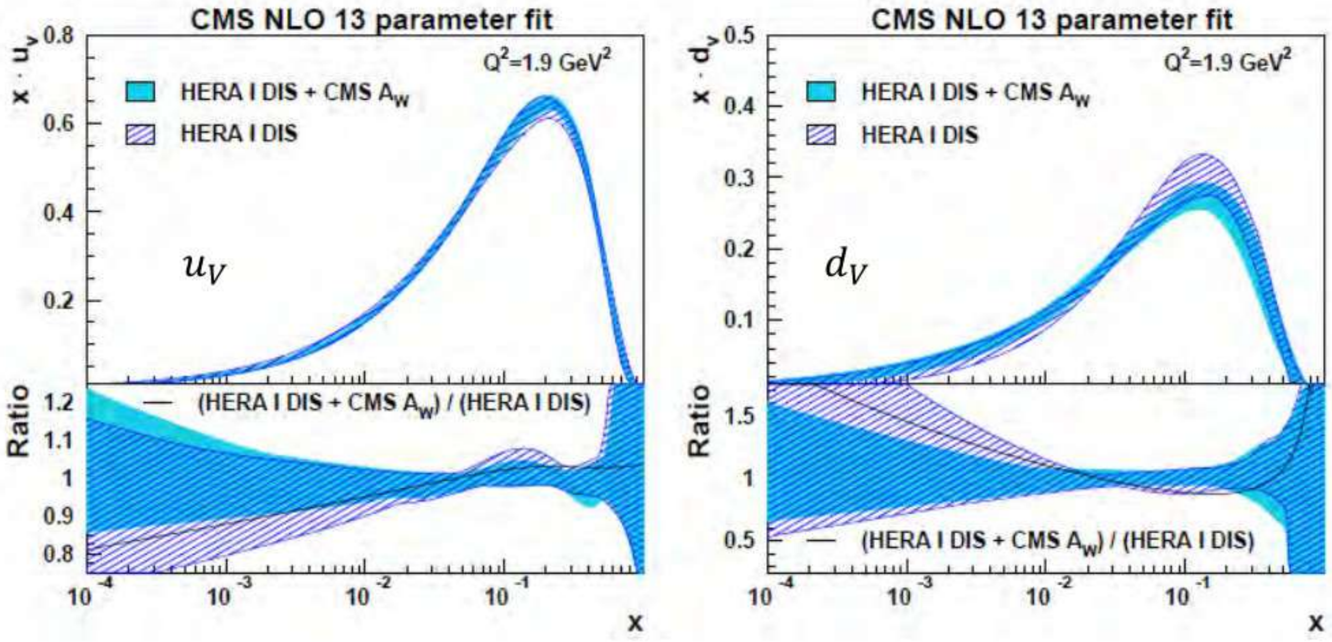

ATLAS-CONF-2011-129




Phys. Rev. Lett. 109 (2012) 111806

# Implications: Valence distributions

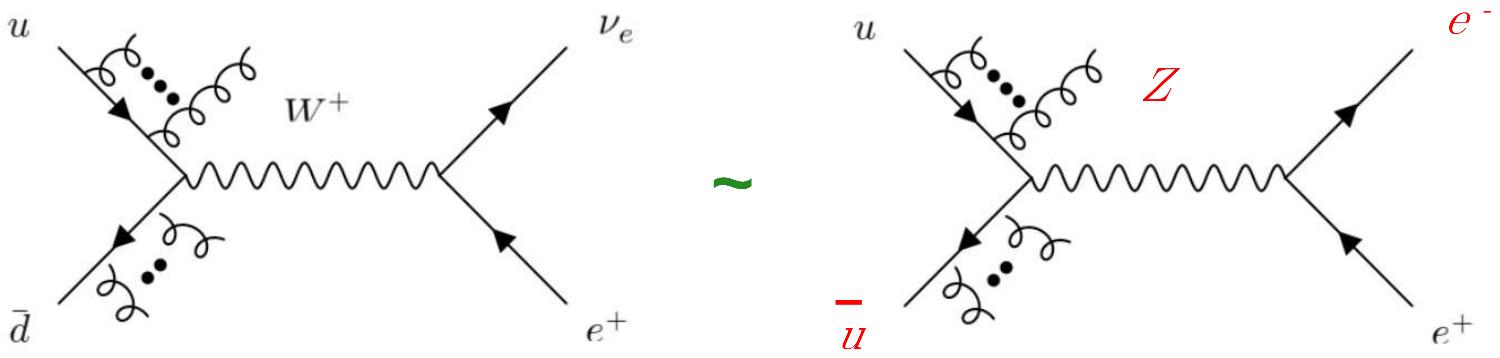
- Strategy (largely common to ATLAS and CMS): use HERA data by necessity; ad only collider data
  - Avoid data subject to larger theoretical uncertainty
- Impact of asymmetry measurement: most significant improvement in $d_V$
  - $d_V$ has more freedom as $u_V$ is better constrained by HERA data




Phys. Rev. Lett. 109 (2012) 111806

# Strong interaction effects

- Transverse momentum distribution
  - Initial state radiation involves large corrections, and is in part non-perturbative. W events are only partly measured (neutrino!)
  - Approach : adjust model parameters using Z events, which are close to W's and can be measured precisely; extrapolate to W production

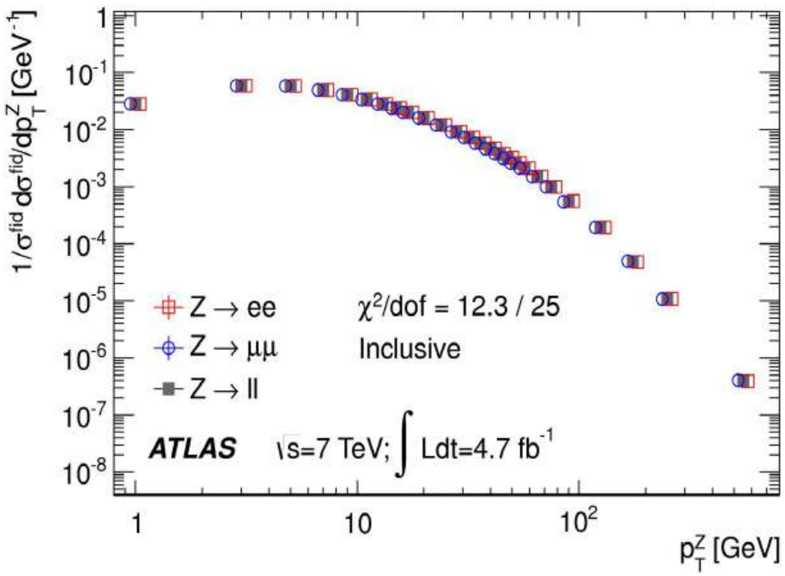


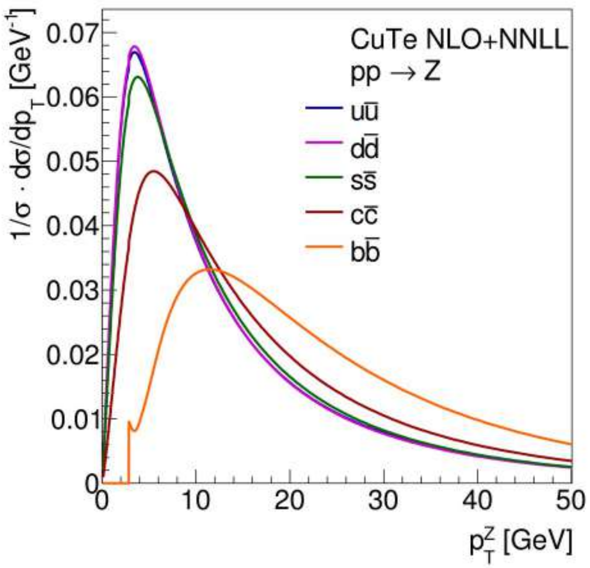
# Strong interaction effects

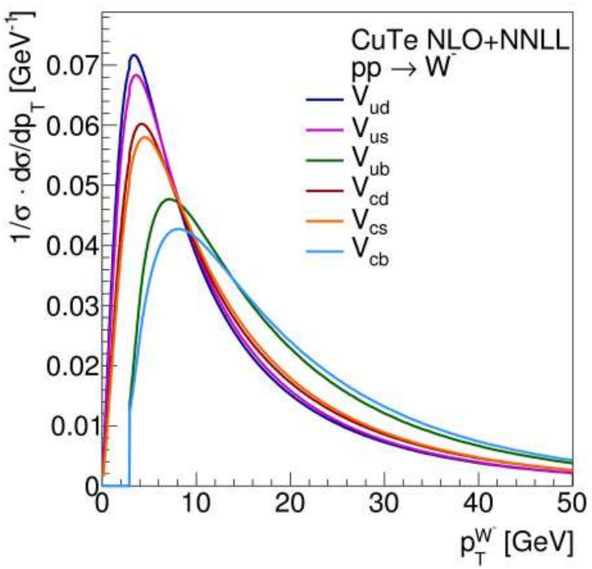
- Transverse momentum distribution
  - Z-based model tuning + Z → W extrapolation uncertainties
  - Problem : measurements are inclusive in initial parton configurations.
    Heavy-flavour contributions “kick” the $p_T$ distribution, and are different in W and Z

Measurement precision ~0.5%

ATLAS 2015

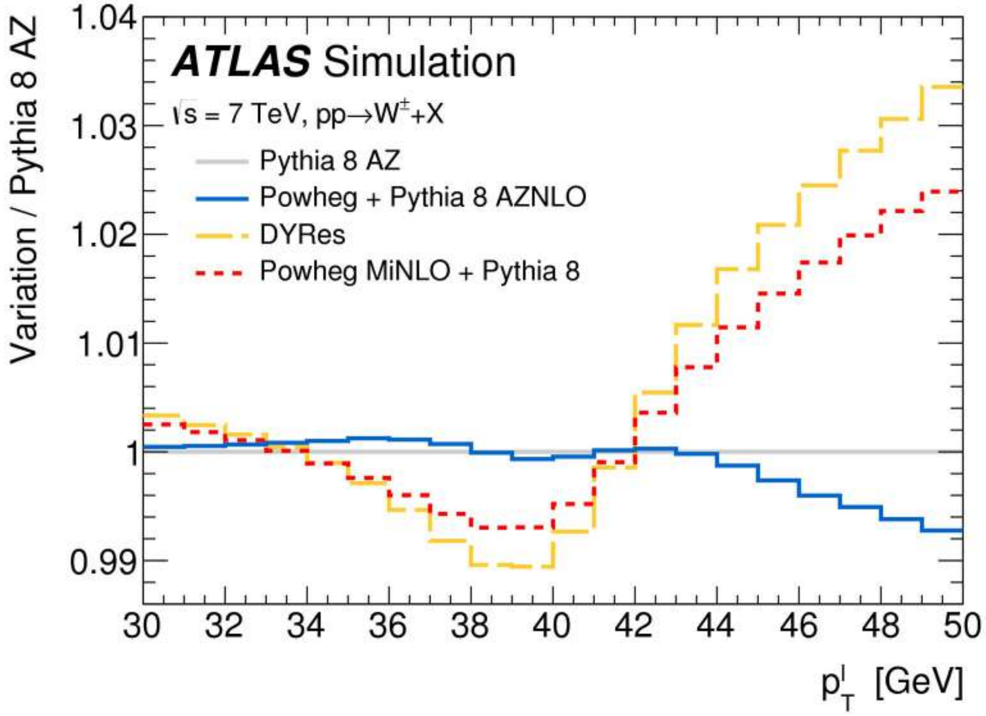


# Strong interaction effects

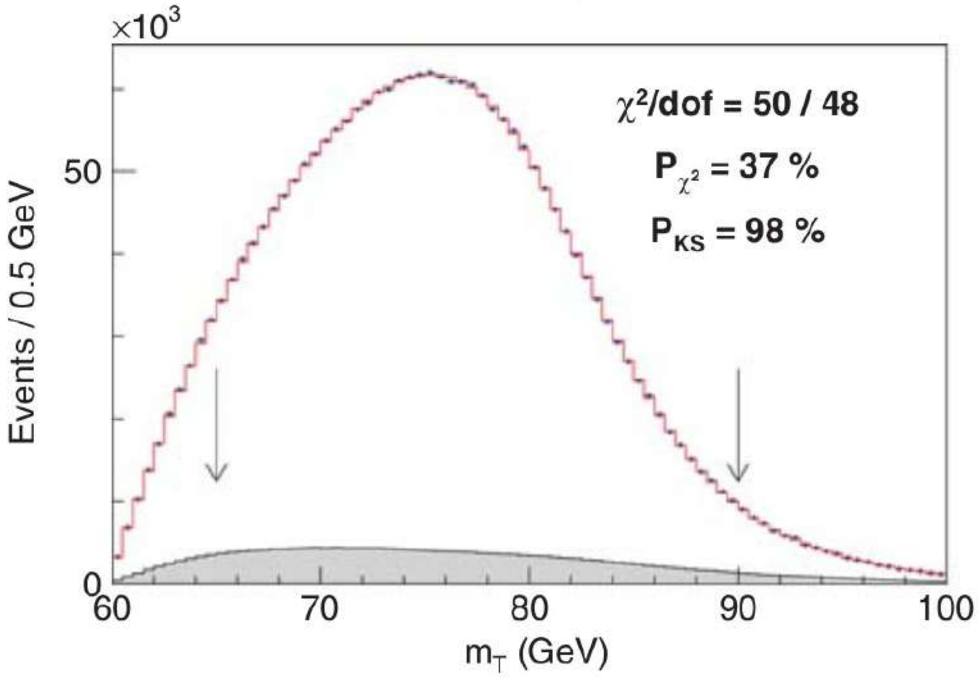
- Transverse momentum distribution
  - Comparison between selected theoretical predictions:

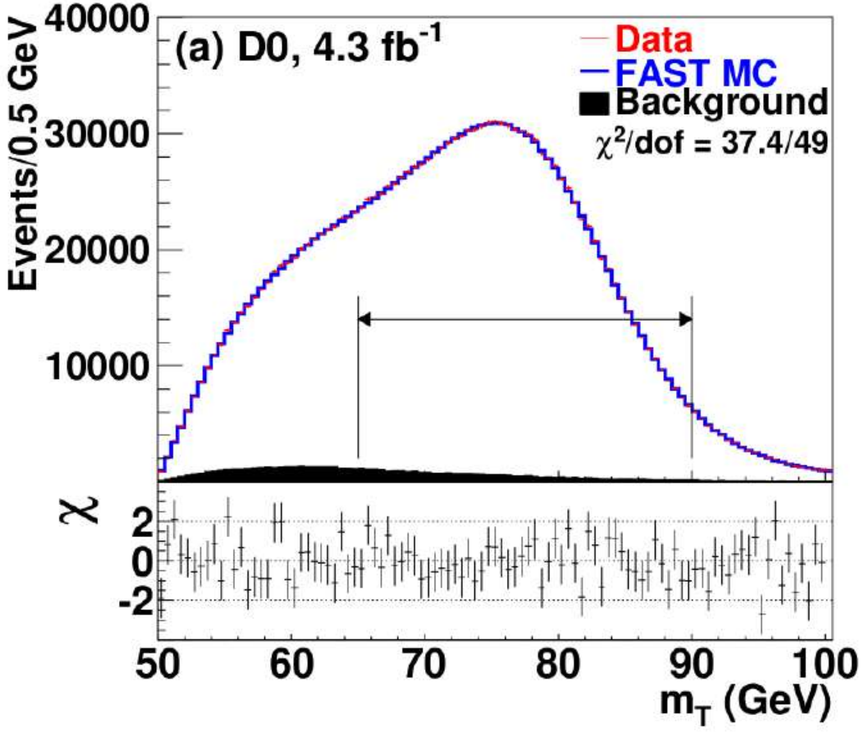



~3% effect
Remember:
$\delta m_W$=10 MeV → 0.2%

# After all is said and done...

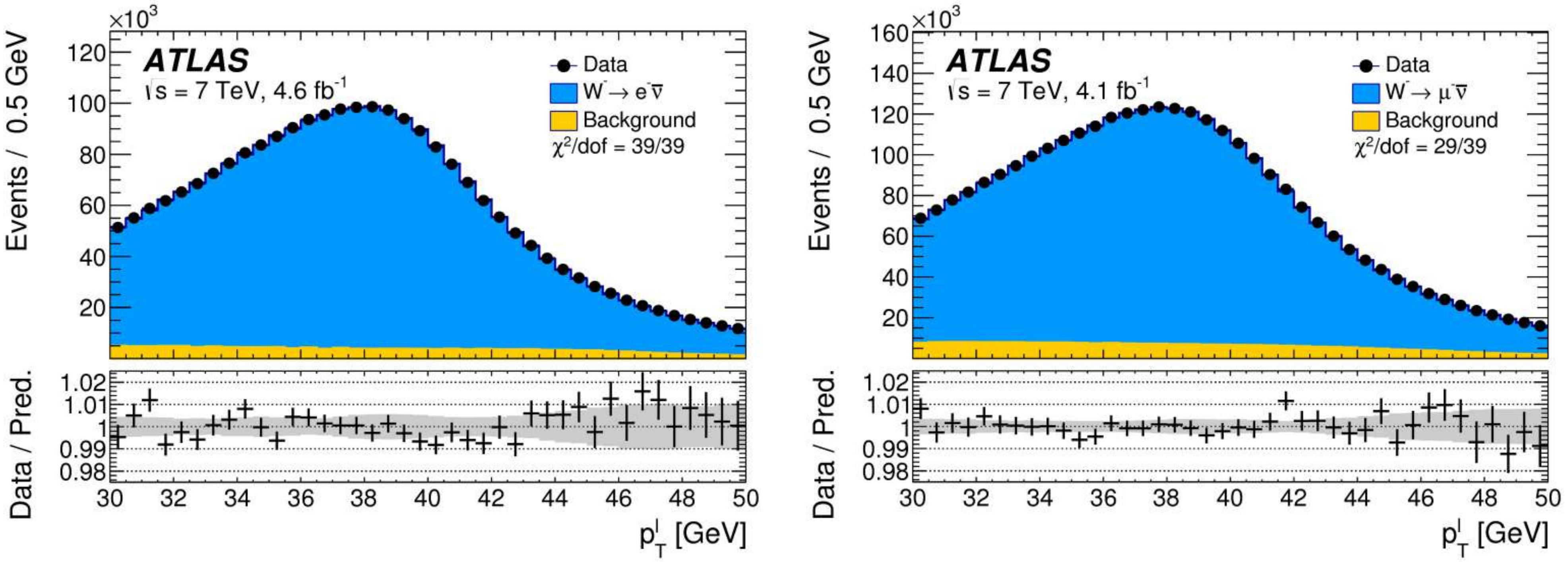
- CDF, D0



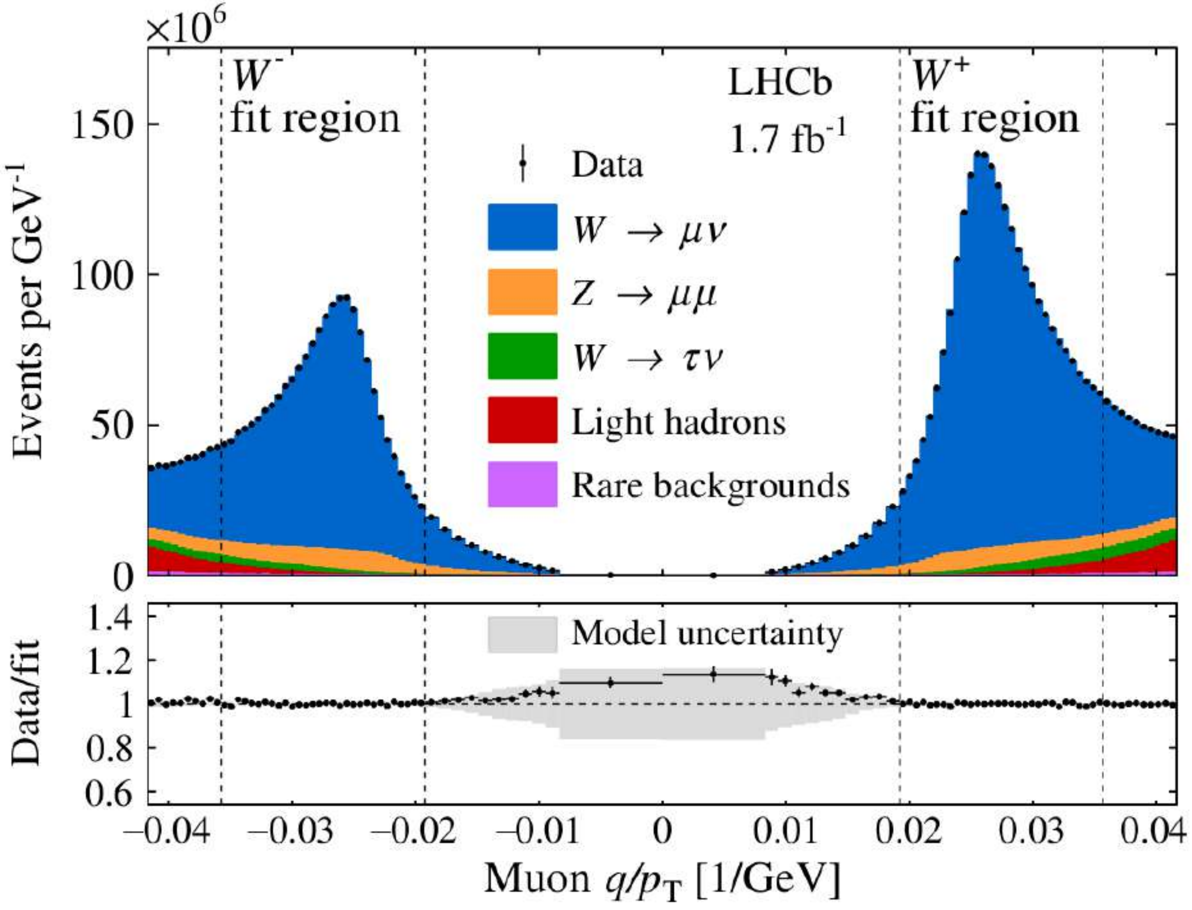
# After all is said and done...

- ATLAS

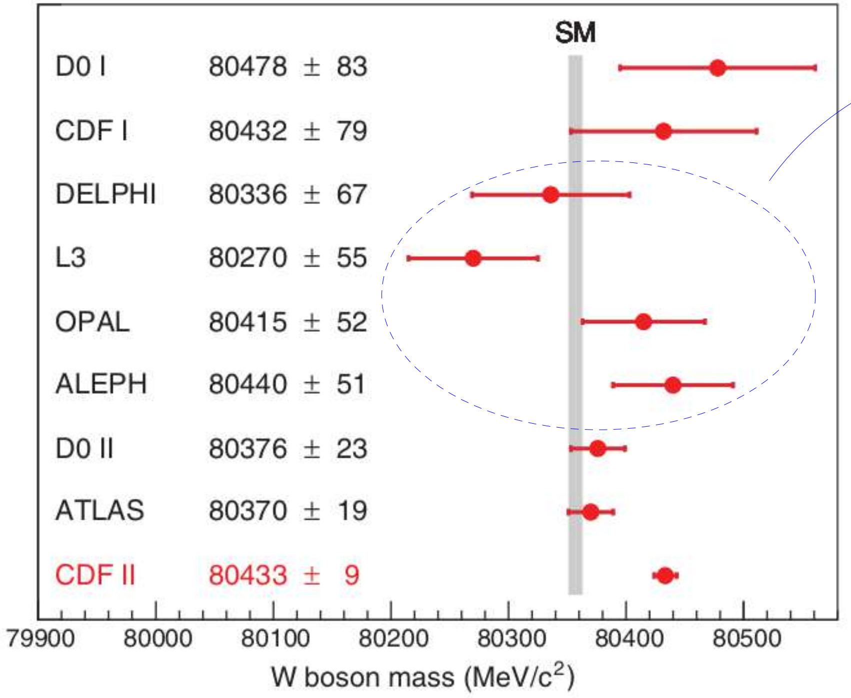


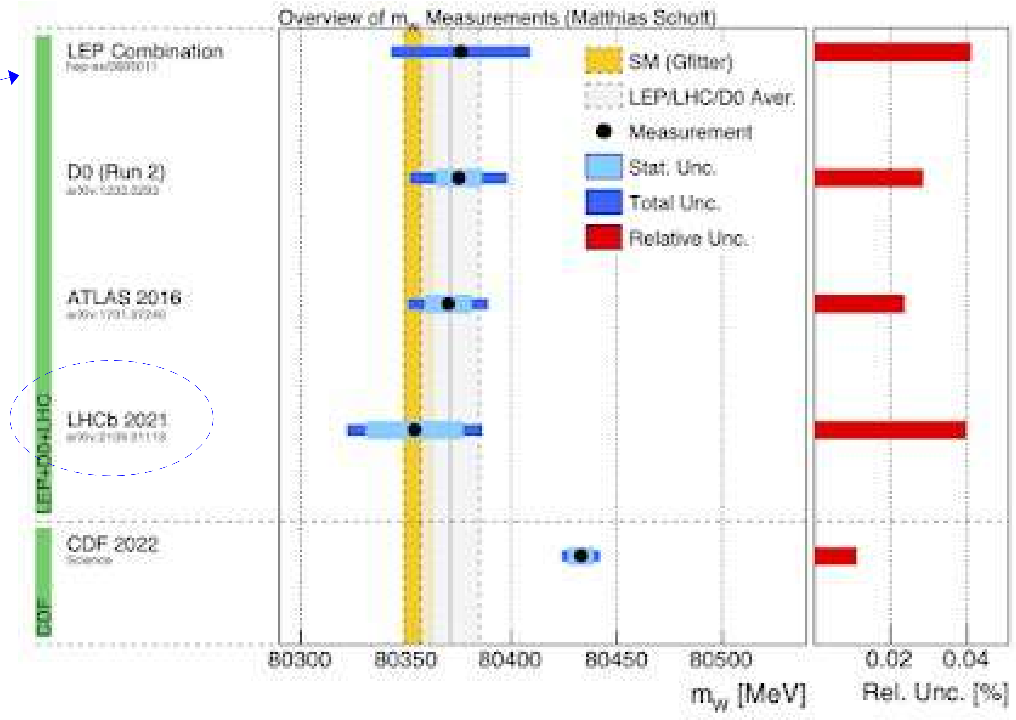
# After all is said and done...

- LHCb



# Experimental situation



# Experimental situation

- Last measurements:
  - ATLAS 2017

    $m_W$ = 80370 ± 7 (stat.) ± 11 (exp.) ± 10 (theory) ± 9 (PDF)
  - LHCb 2021

    $m_W$ = 80354 ± 23 (stat.) ± 10 (exp.) ± 17 (theory) ± 9 (PDF)
  - CDF 2022

    $M_W$ = 80433 ± 6.4 (stat.) ± 4.5 (exp.) ± 3.5 (theory) ± 3.9 (PDF)

# Conclusions

- The W boson mass is arguably the most difficult measurement in HEP
  - Partial event reconstruction, incomplete kinematics
  - Calibrations
  - Physics modelling
  - Precision goal

  → so mistakes can be made..

- The limit of these measurements is the limit of our understanding of QCD.
- Ultimate goals of ATLAS, CMS, LHCb ~10 MeV each, with different experimental conditions and methods