Strange baryon femtoscopy in ALICE

Raffaele Del Grande[1,*]

[1] Physik Department E62, Technische Universität München, 85748 Garching, Germany

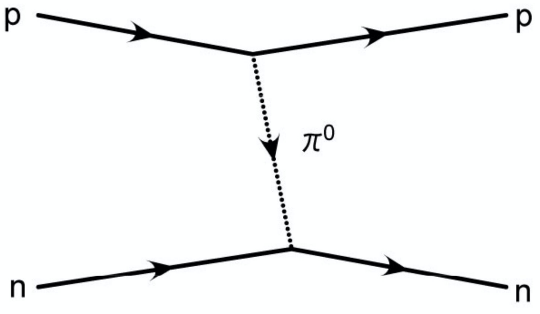
On behalf of the ALICE Collaboration

STRONG-2020 Annual Meeting (2022 edition)
Workshop: “Recent results and perspectives in hadron physics”

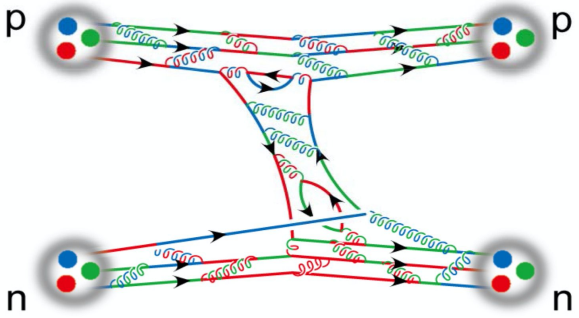
Orsay, 17 October 2022

***raffaele.del-grande@tum.de**

Residual strong interaction among hadrons



$$\mathcal{L}_{EFT}[\pi, N, \ldots; m_\pi, m_N, \ldots, C_i]$$



$$\mathcal{L}_{QCD}[q, \overline{q}, A; m_q, \alpha_s]$$

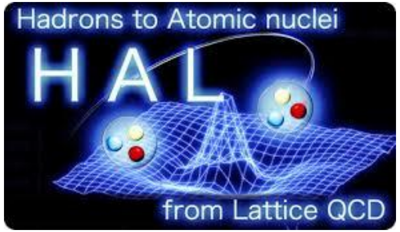
Effective Field Theories (EFT)

- **hadrons** as degrees of freedom
- low-energy coefficients **constrained by data**

Lattice QCD

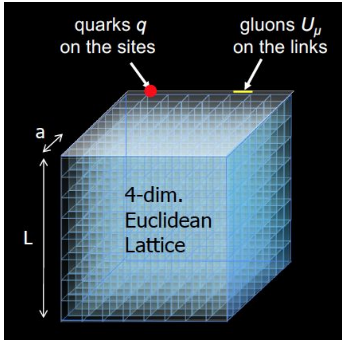
- Understanding of the interaction starting from **quarks and gluons**

Residual strong interaction from lattice



T. Hatsuda, K. Sasaki et al.

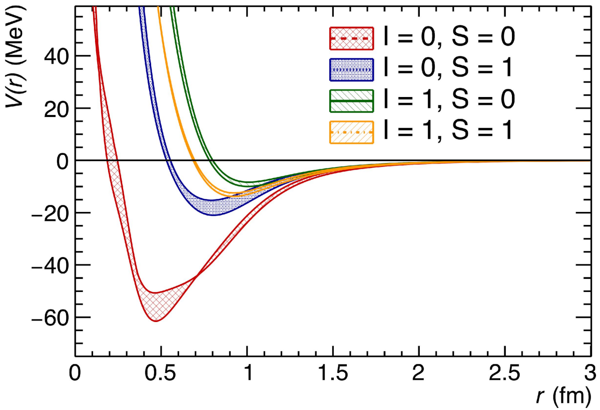
HAL QCD Coll. PLB 792 (2019)
HAL QCD Coll. NPA 998 (2020)
HAL QCD Coll. PRD 99 (2019)



a = 0.085 fm
L = 8.1 fm

$$m_\pi = 146\ \mathrm{MeV}/c^2$$
$$m_\mathrm{K} = 525\ \mathrm{MeV}/c^2$$

Local potentials for the nucleon-Ξ interactions



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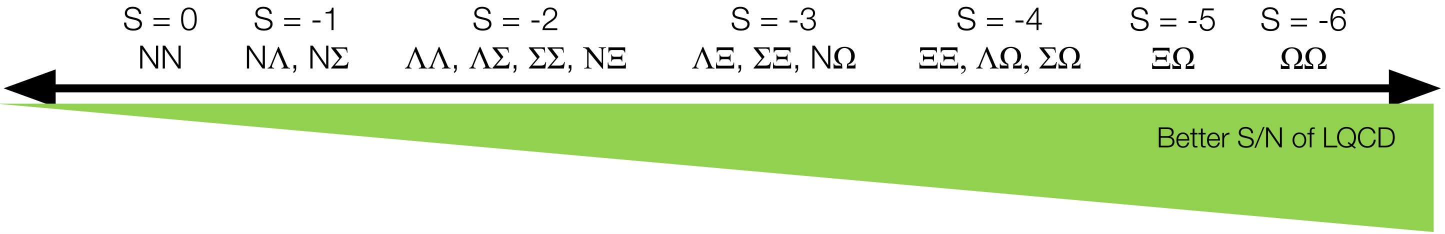


S = 0
NN

S = -1
NΛ, NΣ

S = -2
ΛΛ, ΛΣ, ΣΣ, NΞ

S = -3
ΛΞ, ΣΞ, NΩ

S = -4
ΞΞ, ΛΩ, ΣΩ

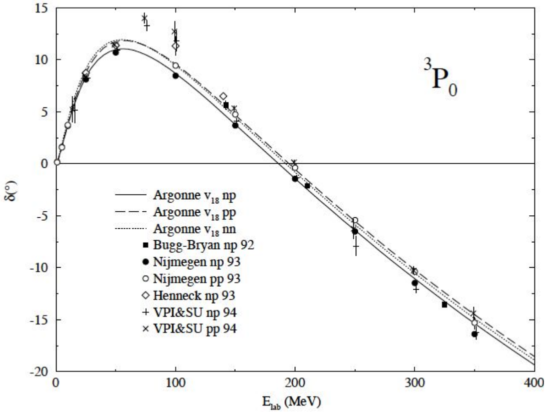
S = -5
ΞΩ

S = -6
ΩΩ

Better S/N of LQCD

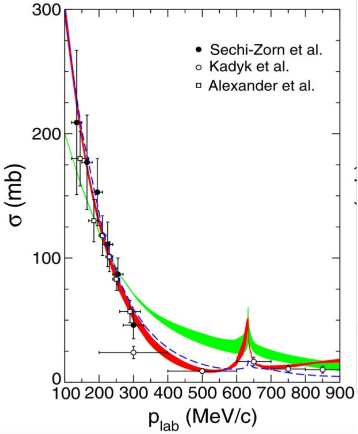
Experimental data for two-body interactions

N–N → N–N



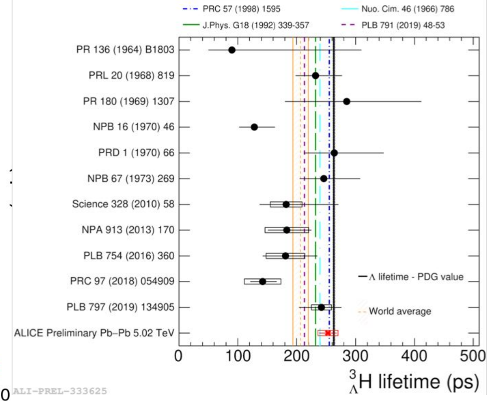
R. B. Wiringa et al. PRC 51 (1995)

p–Λ → p–Λ

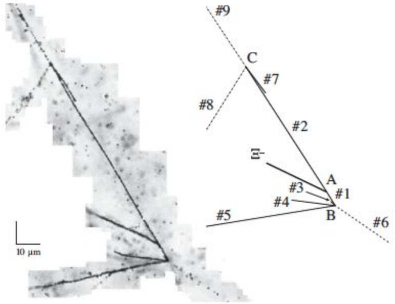


LO from H. Polinder et al. NPA 779 (2006)
NLO from J. Haidenbauer et al. NPA 915 (2013)

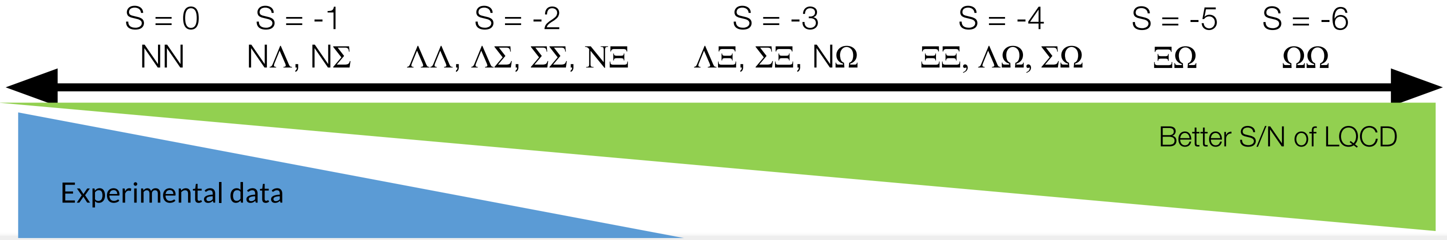
Hypertriton



Ξ hypernucleus



J-PARC E07 Coll. PRL 126 (2021)



S = 0	S = -1	S = -2	S = -3	S = -4	S = -5	S = -6
NN	NΛ, NΣ	ΛΛ, ΛΣ, ΣΣ, NΞ	ΛΞ, ΣΞ, NΩ	ΞΞ, ΛΩ, ΣΩ	ΞΩ	ΩΩ

Better S/N of LQCD

Experimental data

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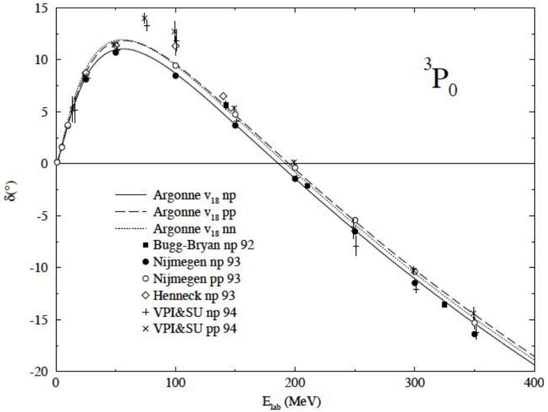
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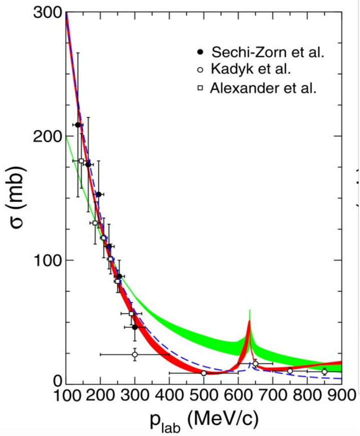
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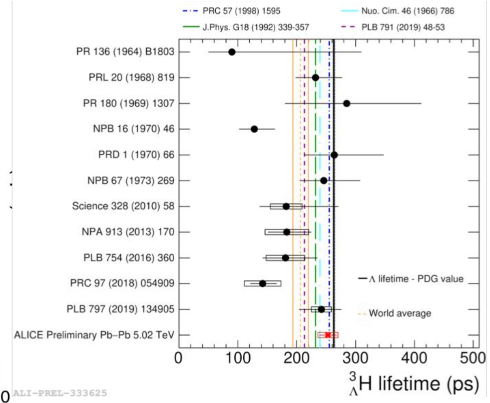
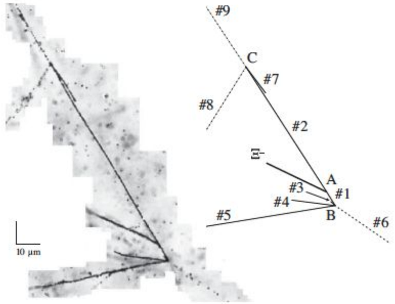

Hypertriton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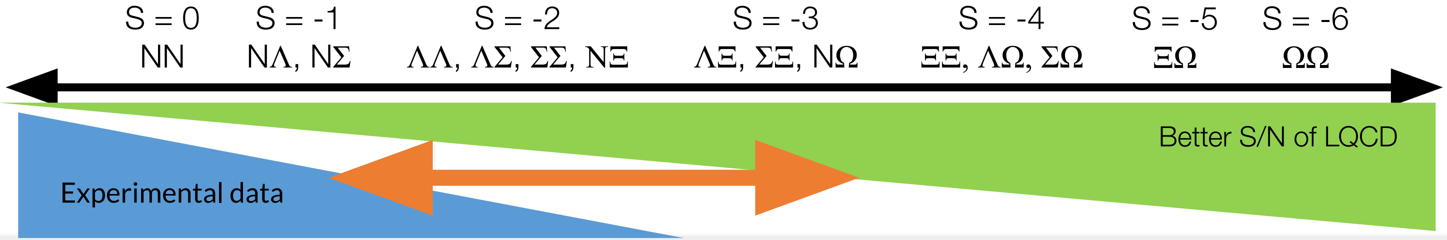

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ΩΩ

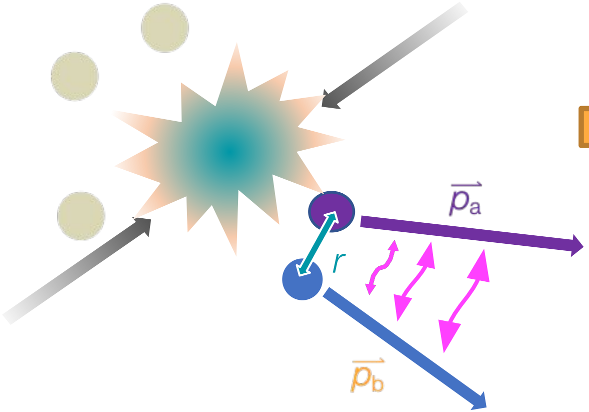
Better S/N of LQCD

Experimental data

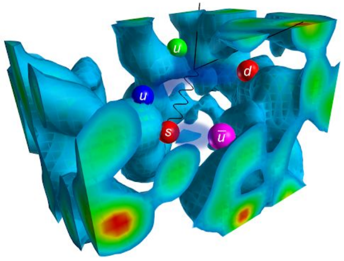
Investigating hadronic interactions at LHC

ALICE at the LHC

Hadron-hadron strong interactions

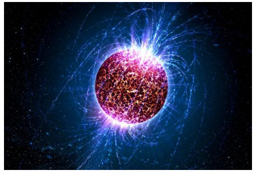


Test of Lattice QCD and χEFT calculations

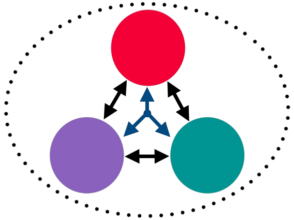


D. B. Leinweber/University of Adelaide

Impact on the Equation of State of Neutron stars



Three-body interactions



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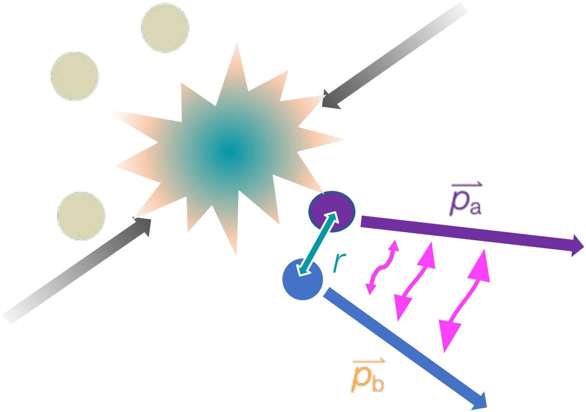
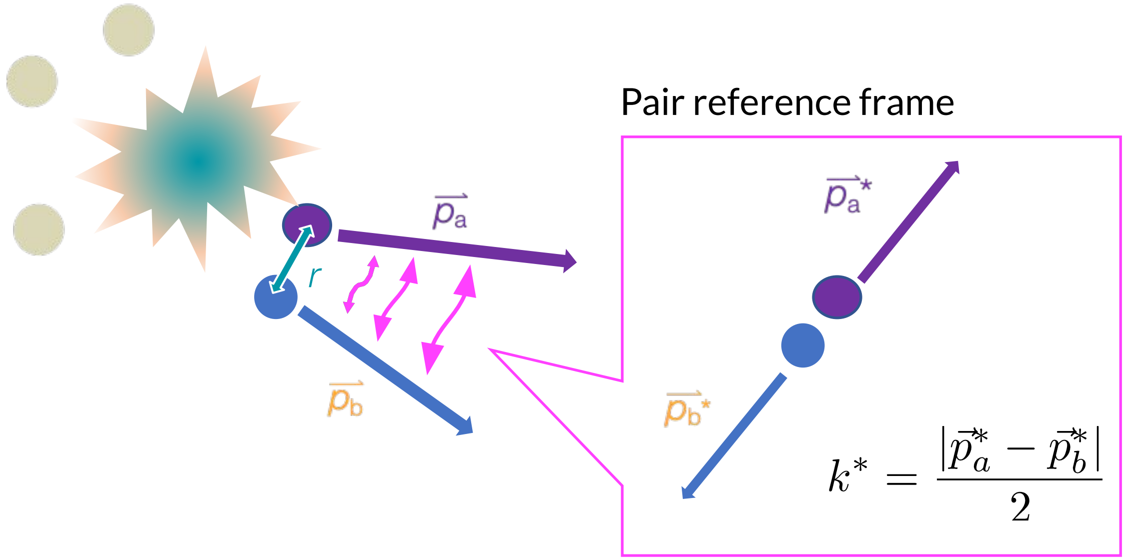


Femtoscopy technique

Two-particle correlation function

$$C(\mathbf{p}_a, \mathbf{p}_b) \equiv \frac{P(\mathbf{p}_a, \mathbf{p}_b)}{P(\mathbf{p}_a) \cdot P(\mathbf{p}_b)}$$

Femtoscopy technique



Pair reference frame

$$k^* = \frac{|\vec{p}_a^{\,*} - \vec{p}_b^{\,*}|}{2}$$

Correlation function:

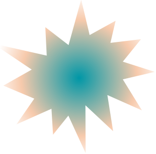
Two-particle wave function

$$C\left(k^*\right) = \mathcal{N} \cdot \frac{N_{\text{same}}\left(k^*\right)}{N_{\text{mixed}}\left(k^*\right)} = \int S(r) \left|\psi\left(k^*, r\right)\right|^2 \mathrm{d}^3 r$$

Emission source

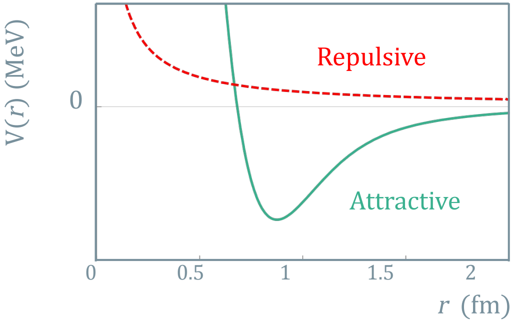
Femtoscopy technique

Source parameterization

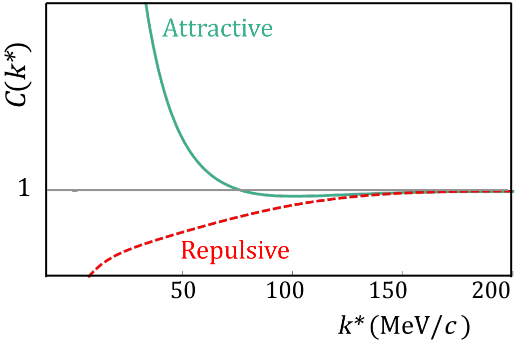


Gaussian source

Interacting potential



Correlation function



Schrödinger equation

CATS (Correlation Analysis Tool using the Schrödinger equation)

D. Mihaylov et al. EPJC 78 (2018)

Correlation function:

Two-particle wave function

$$C(k^*) = \mathcal{N} \cdot \frac{N_{\text{same}}(k^*)}{N_{\text{mixed}}(k^*)} = \int S(r) \left| \psi(k^*, r) \right|^2 \mathrm{d}^3 r$$

Emission source

- >1 if the interaction is attractive
- $= 1$ if there is no interaction
- <1 if the interaction is repulsive

Source determination

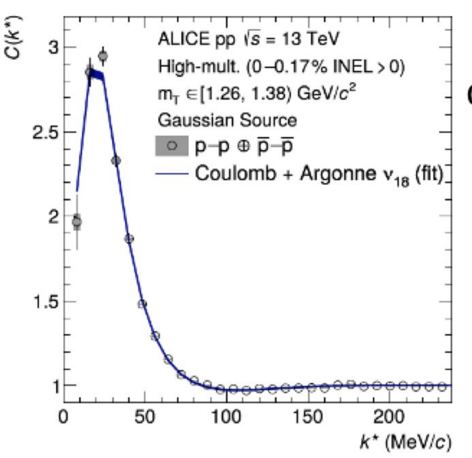
The first step is “traditional” femtoscopy: known interaction → determine source size

- p–p interaction: Argonne v18 potential
- crosscheck with p–Λ (χEFT)

$$C(k^*) = \int \boxed{S(r)} \left|\psi(\vec{k}^*,\vec{r})\right|^2 d^3r$$

[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

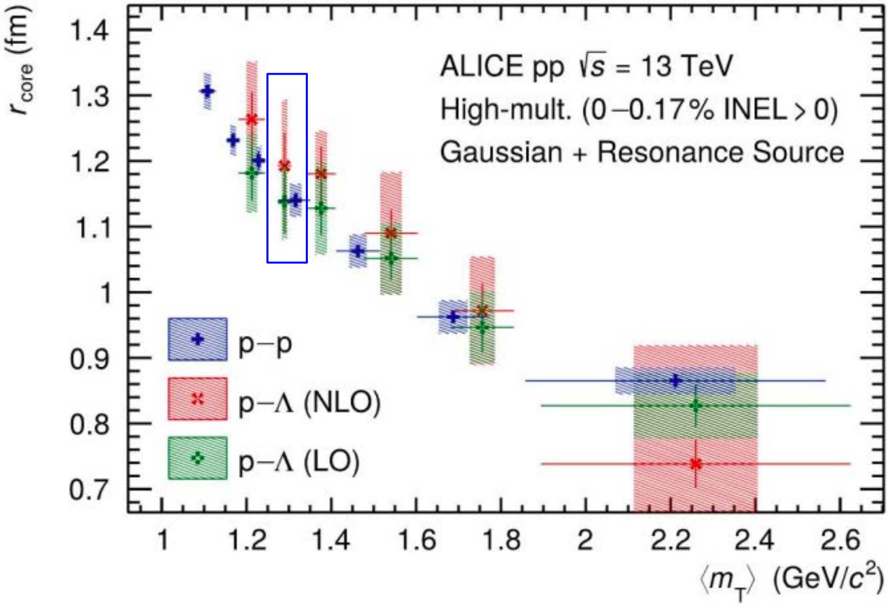
Source model



$$C(k^*) = \int \boxed{S(r)} \left|\psi(\vec{k}^*, \vec{r})\right|^2 d^3r$$

pp Correlation: AV18 + Coulomb potentials used to calculate $\psi(\vec{k}^*, \vec{r})$

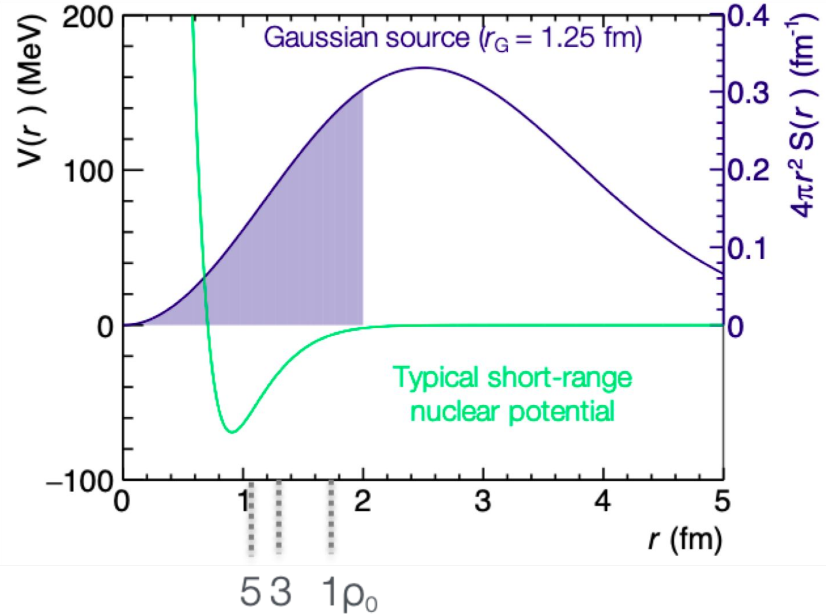
One universal source for all hadrons with strong resonance decays considered for each pair of interest

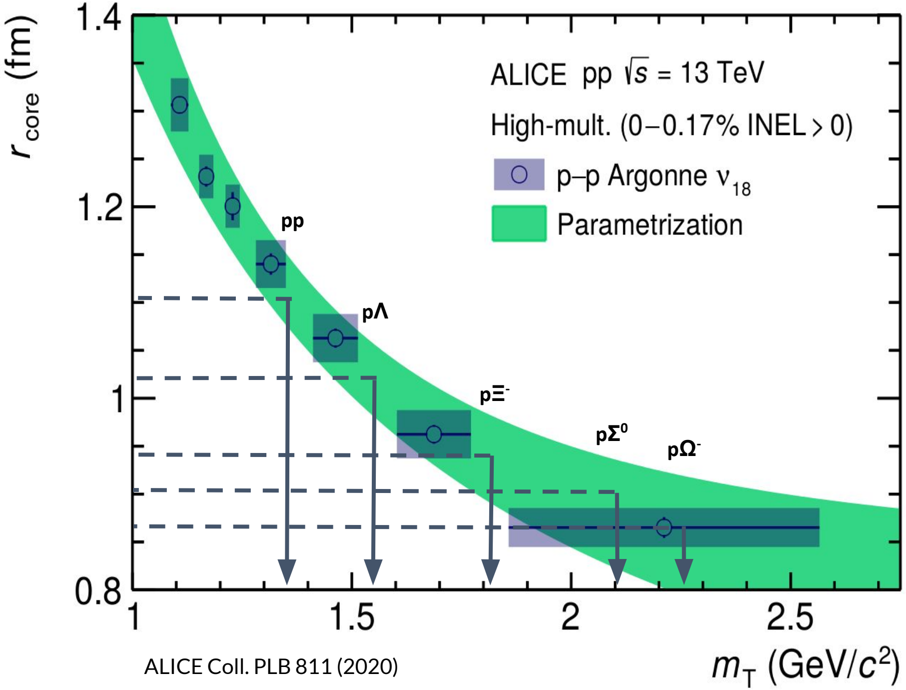


ALICE Coll. PLB 811 (2020)

Source model

Small particle-emitting source created in pp and p–Pb collisions at the LHC.



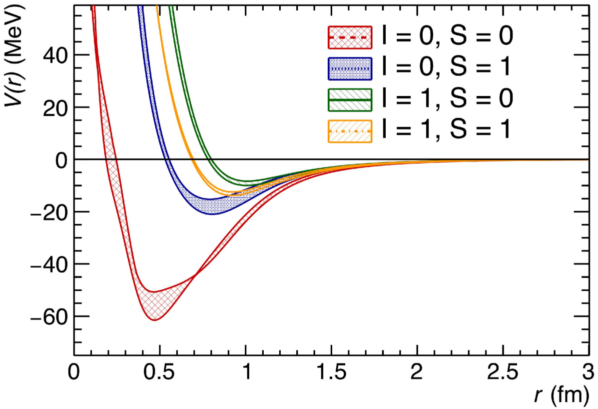


ALICE Coll. PLB 811 (2020)

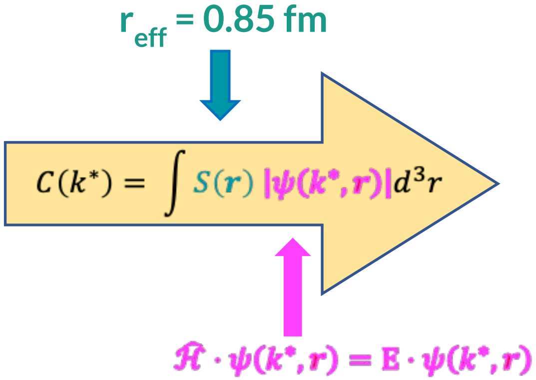
|S|=2 sector: p–Ξ⁻ interaction and first test of LQCD

Lattice QCD potentials from HAL QCD collaboration available

Local potentials for the nucleon-Ξ interactions



HAL QCD Coll. NPA 998 (2020)



r_{eff} = 0.85 fm

$$C(k^*) = \int S(r)\,|\psi(k^*,r)|\,d^3r$$

$$\mathcal{H}\cdot\psi(k^*,r) = \mathrm{E}\cdot\psi(k^*,r)$$

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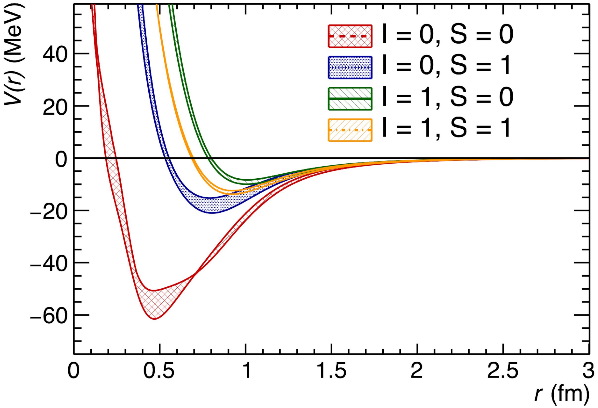
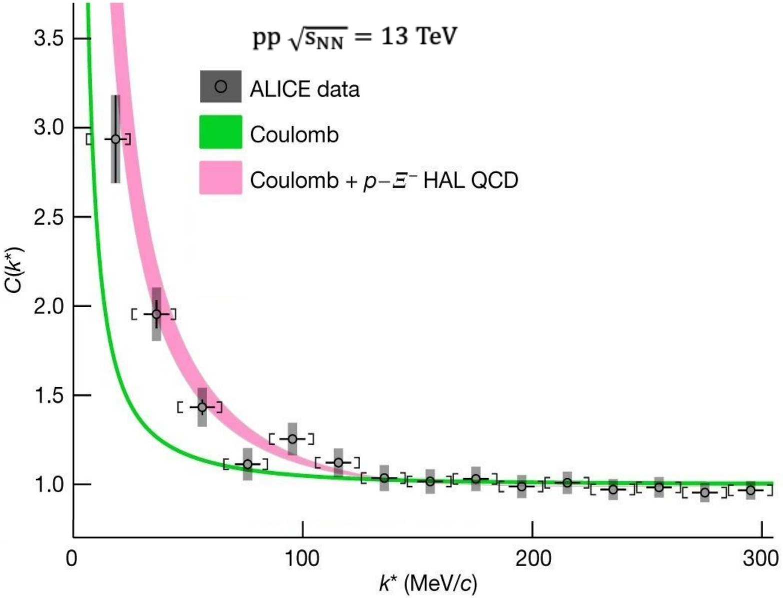


HAL QCD Coll. NPA 998 (2020)

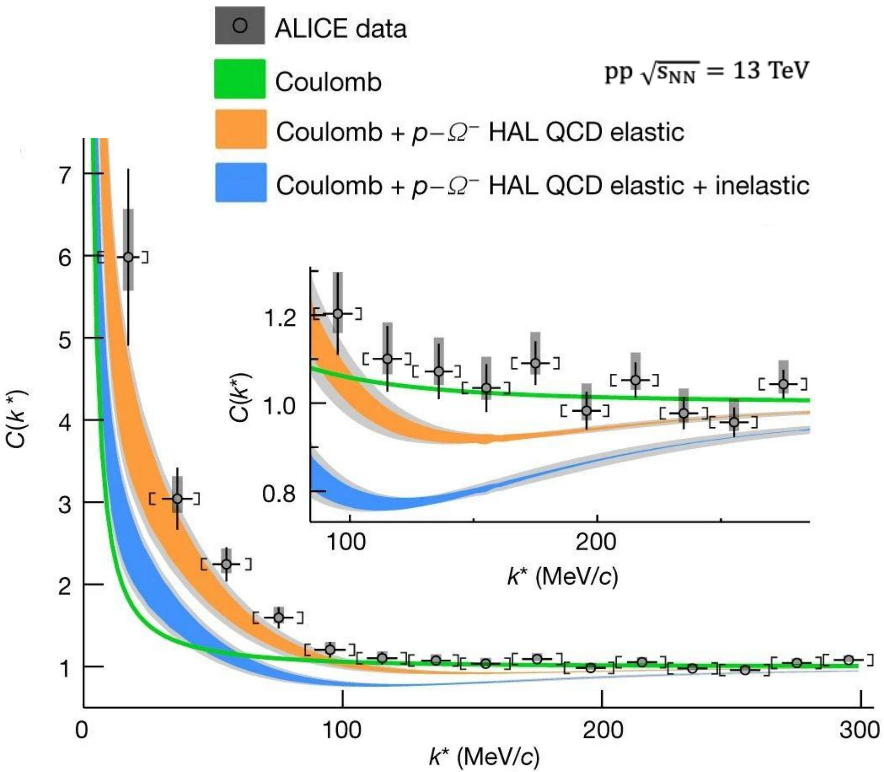
ALICE Coll. Nature 588 232–238 (2020)



Observation of a strong attractive interaction beyond Coulomb in agreement with lattice predictions

|S|=3: p–Ω⁻ correlation function in pp at 13 TeV

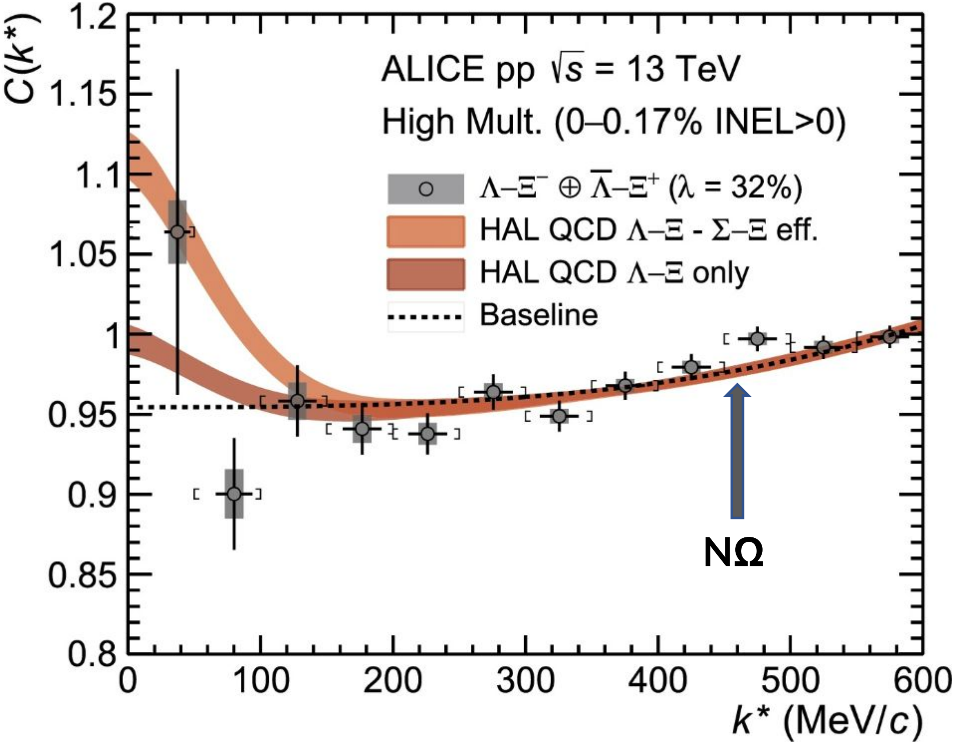
ALICE Coll. Nature 588 232–238 (2020)



- Enhancement above Coulomb
  → Observation of the strong interaction
- Attraction in $^5S_2$ results in the prediction of a bound state (Binding Energy = 1.54 MeV)
- Missing potential of the $^3S_1$ channel
  → Test of two cases:
  - Inelastic channels dominated by absorption
  - Neglecting inelastic channels
- Data more precise than lattice calculations
- So far, no indication of a bound state

$|S|=3$: Λ–Ξ^- interaction with femtoscopy

ALICE Coll. arXiv:2204.10258, Accepted by PLB



- Unknown contribution from coupled channels in Lattice QCD calculations

  → Coupling $\Lambda\Xi$–$\Sigma\Xi$ sizable in HAL QCD calculation

  → No sensitivity yet

  (“No coupling” 0.64 nσ VS “Coupling“ 1.43 nσ)

- No NΩ cusp visible

  → Hint to negligible NΩ-$\Lambda\Xi$ coupling

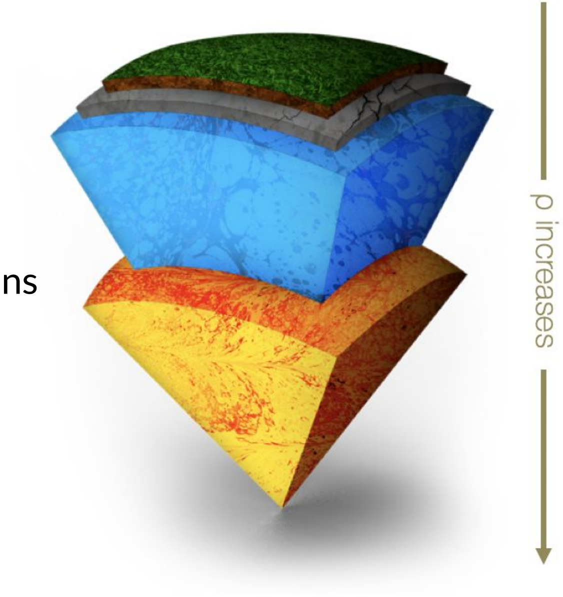
Impact on the Equation of State of neutron stars

Neutron stars

Dimensions
R ~ 10 – 15 km
M ~ 1.5 – 2.2 $M_\odot$

Outer Crust
Ions, electron gas, Neutrons

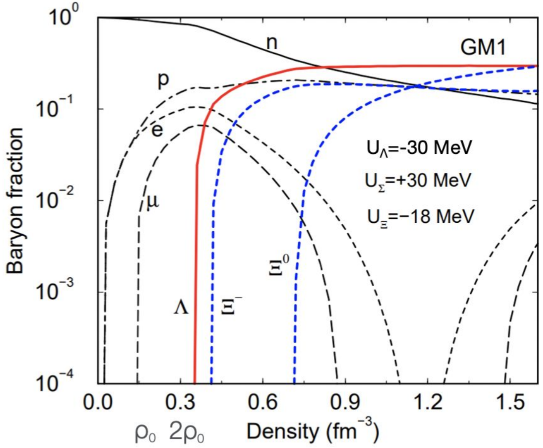
Inner Core
Neutrons?
Protons?
Hyperons?
Kaon condensate?
Quark Matter?



Neutron stars are very dense, compact objects

What is the Equation of State?
What are the constituents to consider?
How do they interact?

J. Schaffner-Bielich NPA 835 (2010) 279



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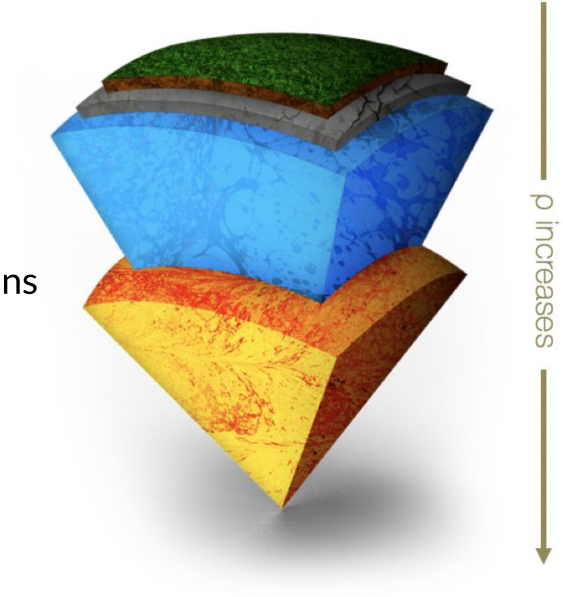

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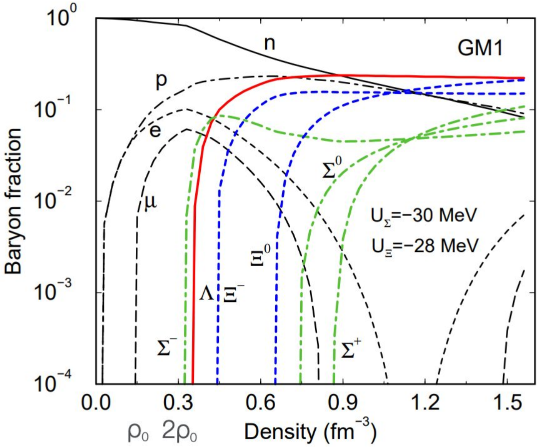
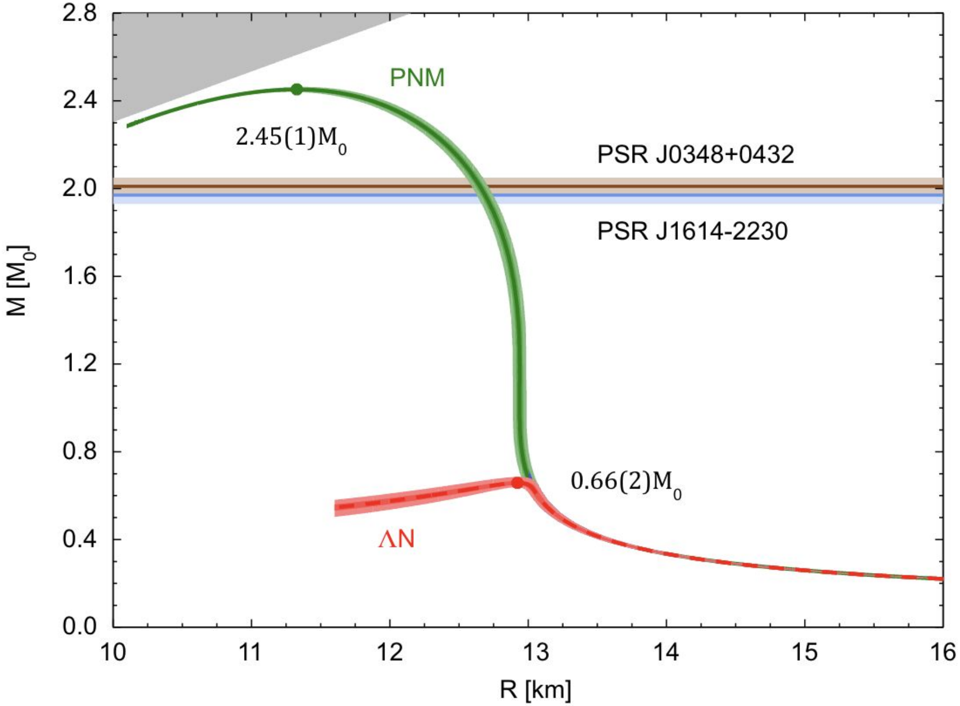


Hyperon appearance in neutron stars?

Adapted from D. Lonardoni et al. PRL 114 092301 (2015)



- Hyperons might appear in neutron stars since it is energetically favourable
- But the resulting equation of state might be too soft to explain heavy neutron stars

Hyperon appearance in neutron stars?

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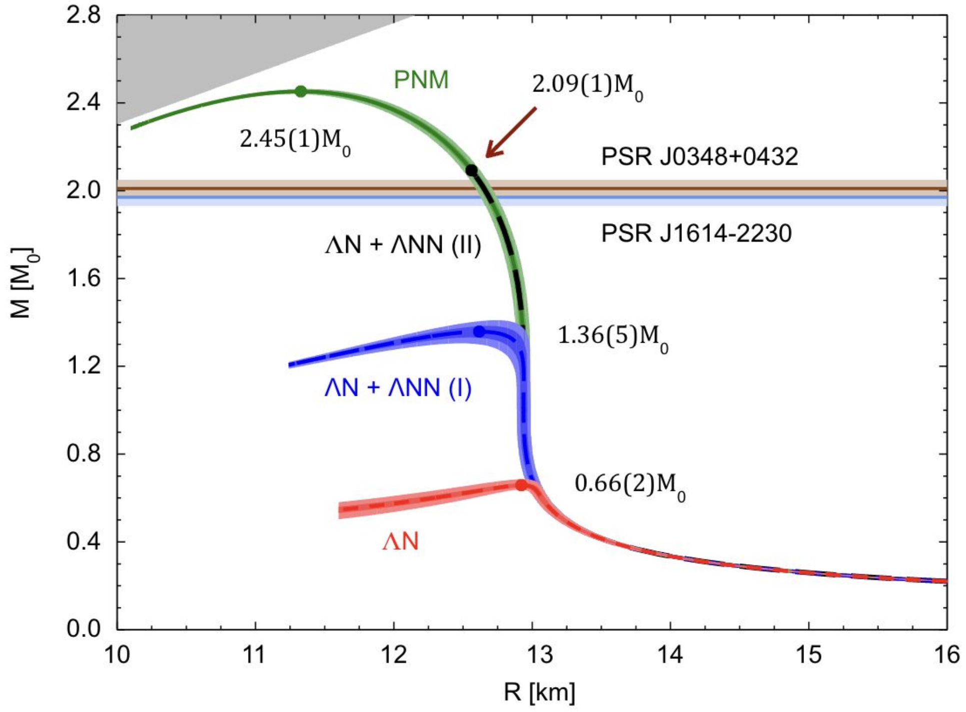


- Hyperons might appear in neutron stars since it is energetically favourable
- But the resulting equation of state might be too soft to explain heavy neutron stars
- Possible solution: repulsive three-body interaction

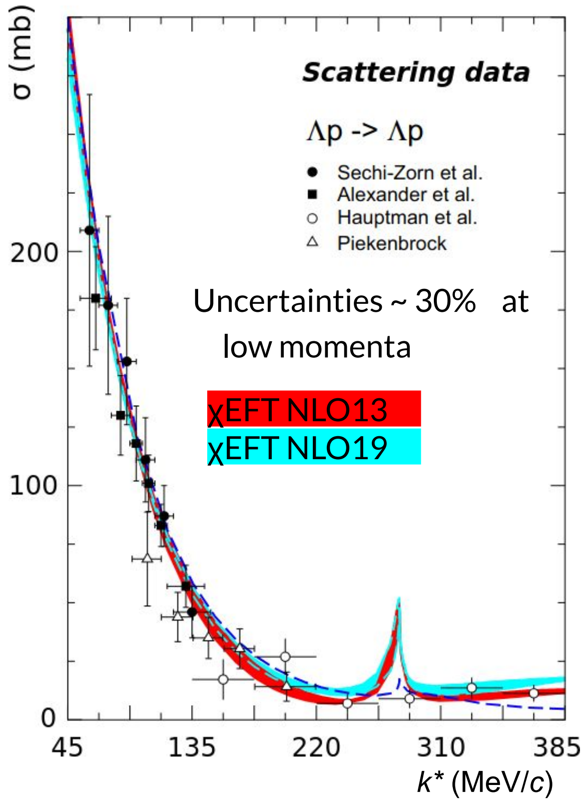
|S| = 1: p–Λ interaction

- Low statistics and not available at low momenta

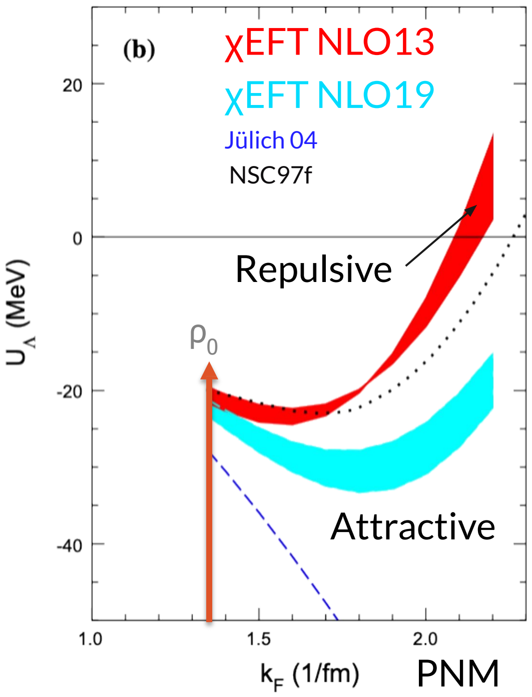
- ΛN-ΣN coupled system → two-body coupling to ΣN is not (yet) measured

- ΣN coupling strength relevant for EoS
  - Strongly affects the behaviour of Λ at finite density
  - Implications for ΛNN interactions

- NLO19 predicts weak coupling NΛ-NΣ
  - Attractive Λ interaction in neutron matter



Uncertainties ~ 30% at low momenta

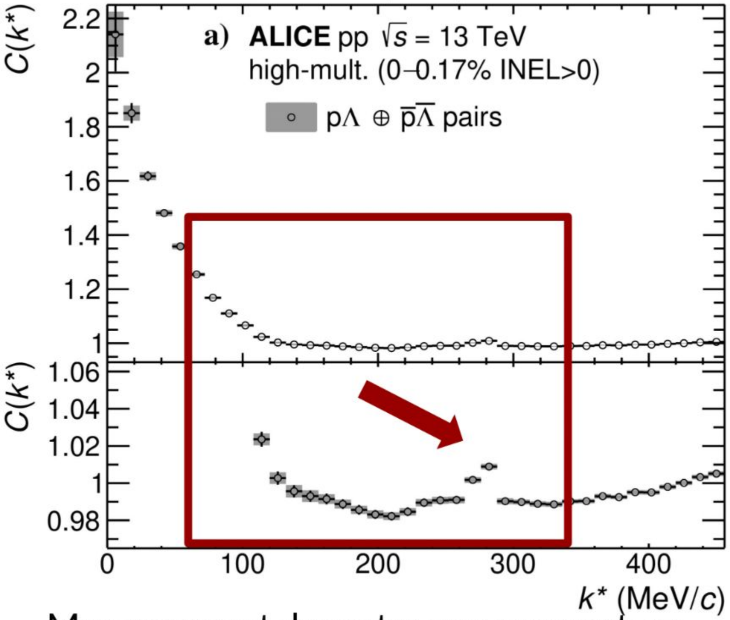


J.Haidenbauer, N.Kaiser et al. NPA 915 24 (2013)

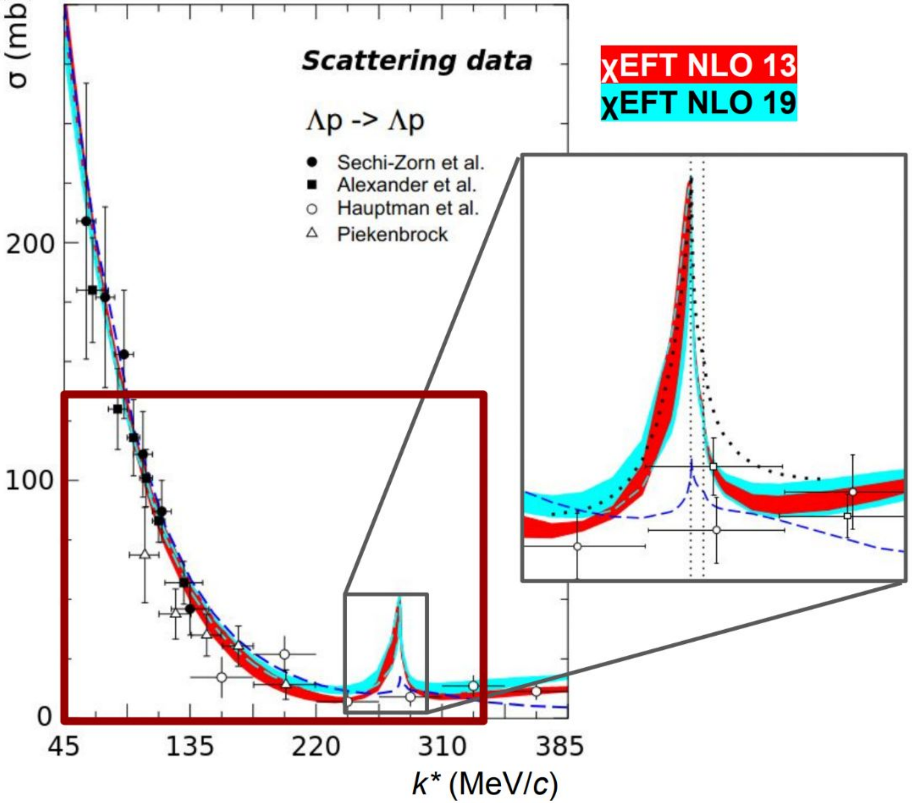
J.Haidenbauer, U. Meißner EPJA 56 (2020)

|S| = 1: p–Λ interaction

ALICE Coll. PLB 833 (2022), 137272



Eur.Phys.J.A 56 (2020) 3, 91



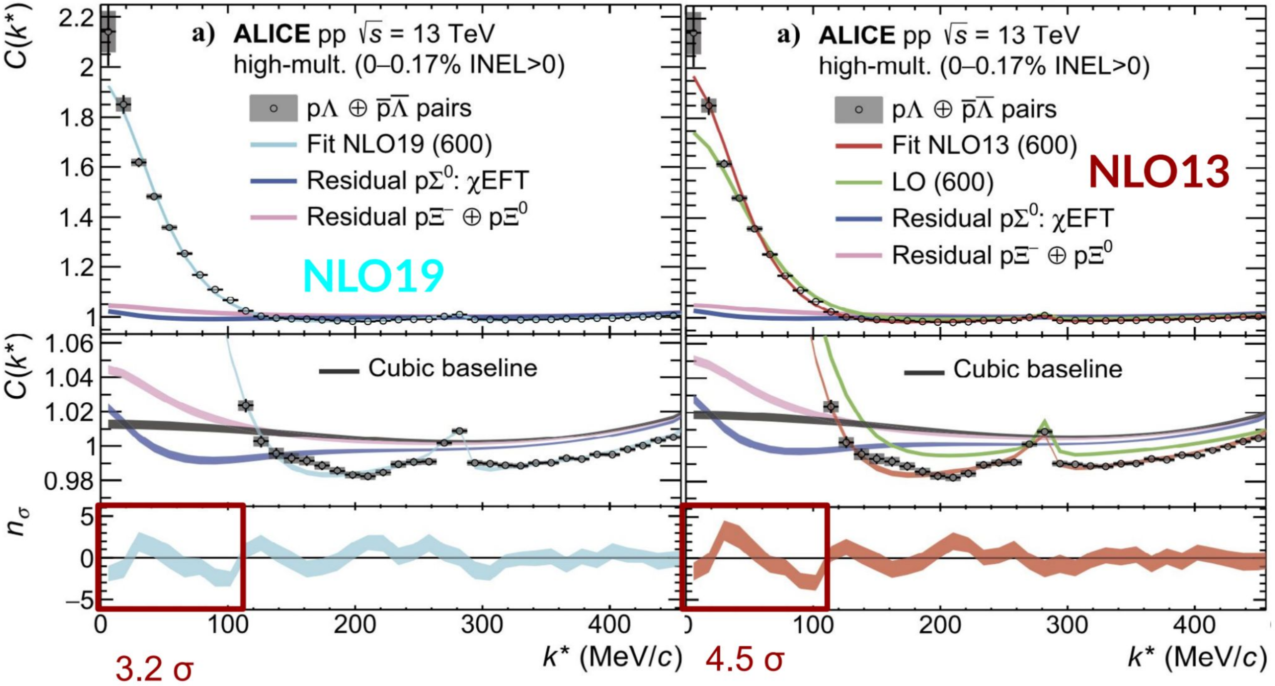
- Measurement down to zero momentum
- Factor 20 improved precision in data (<1%)
- First experimental evidence of ΣN cusp in 2-body channel

|S| = 1: p-Λ interaction

- Comparison with χEFT potentials

  -- Sensitivity to different ΣN coupling strength

  -- NLO19 favoured
  (n_σ = 3.2)
  → attractive interaction of Λ at large densities

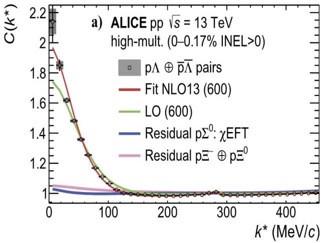


ALICE Coll. PLB 833 137272 (2022)

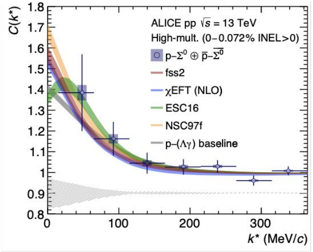
An example of Equation of State for neutron stars

Correlation = two-body interaction

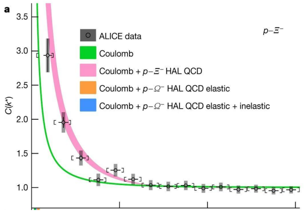
pΛ



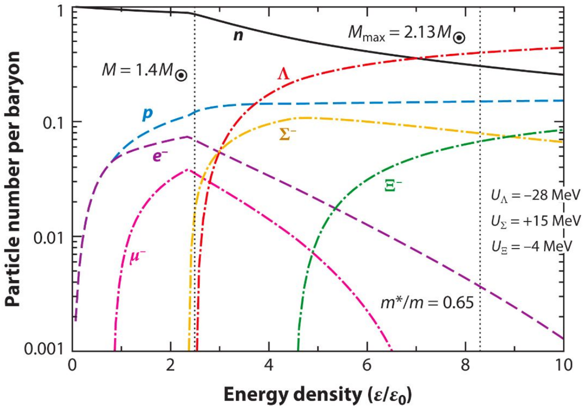
pΣ



pΞ

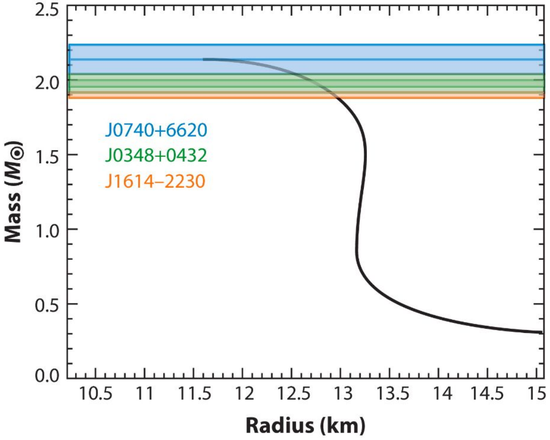


Single-particle potentials = Equation of State



Courtesy J. Schaffner-Bielich 2020

Mass-Radius diagram for hyperon stars

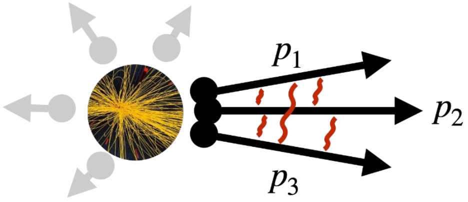


L. Fabbietti et al. Ann.Rev.Nucl.Part.Sci. 71 (2021)

What about the three-body strong interaction?

p-p-p and p-p-Λ correlation functions

Three-body interaction models can be tested using three-particle correlation functions



particle momenta are correlated due to

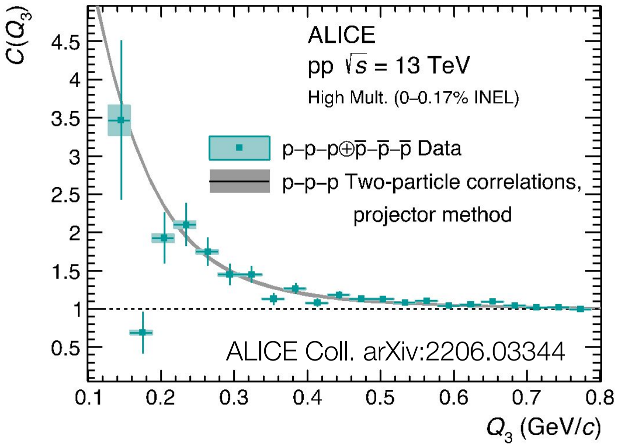
- **two-body interactions**
- **three-body interactions**

correlation function (the **observable**)

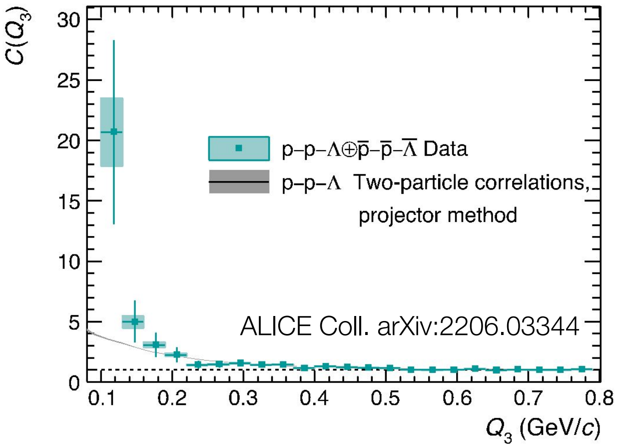
$$C(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = \iiint S_3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)\,|\psi_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)|^2 d^3x_1 d^3x_2 d^3x_3$$

wave function

p-p-p



p-p-Λ



Experimental correlation function

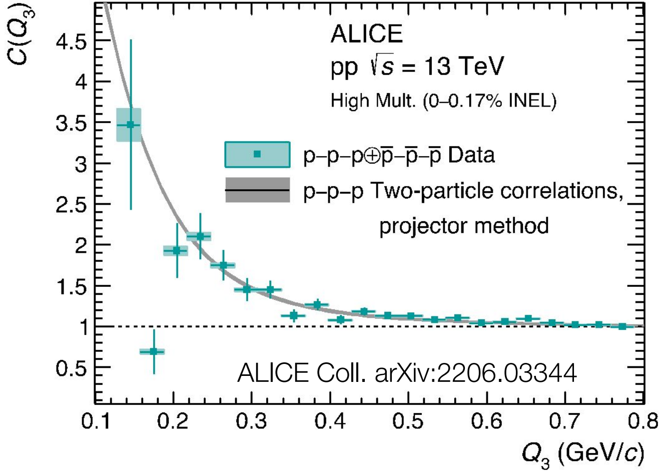
$$C(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) \equiv \frac{P(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)}{P(\mathbf{p}_1)\,P(\mathbf{p}_2)\,P(\mathbf{p}_3)} = \mathcal{N}\frac{N_{\mathrm{same}}(Q_3)}{N_{\mathrm{mixed}}(Q_3)}$$

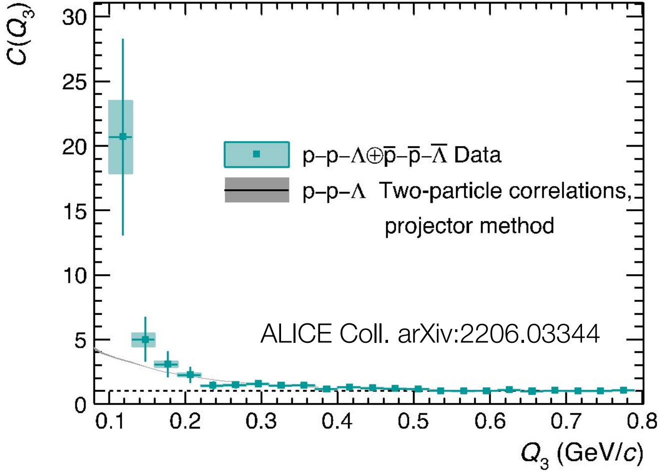
measured as function of the hyper-momentum Q_3

$$Q_3 = \sqrt{-q_{12}^2 - q_{23}^2 - q_{31}^2}$$

$$q_{ij}^{\mu} = 2\left(\frac{m_j E_i}{m_i+m_j} - \frac{m_i E_j}{m_i+m_j}, \frac{m_j}{m_i+m_j}\mathbf{p}_i - \frac{m_i}{m_i+m_j}\mathbf{p}_j\right)$$

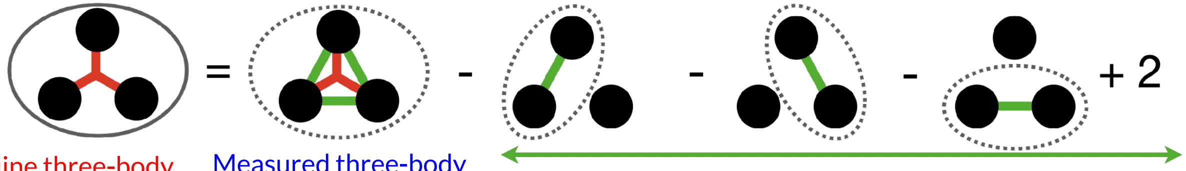
p-p-p and p-p-Λ correlation functions


p-p-p


p-p-Λ

Genuine three-particle correlations isolated using the Kubo's cumulant expansion method:

R. Kubo, J. Phys. Soc. Jpn. 177 (1962)



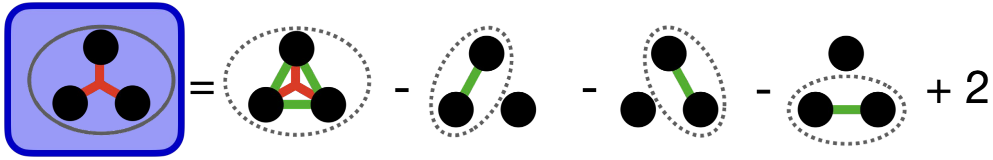
Genuine three-body correlations (cumulant)

Measured three-body correlation

Two-body correlations

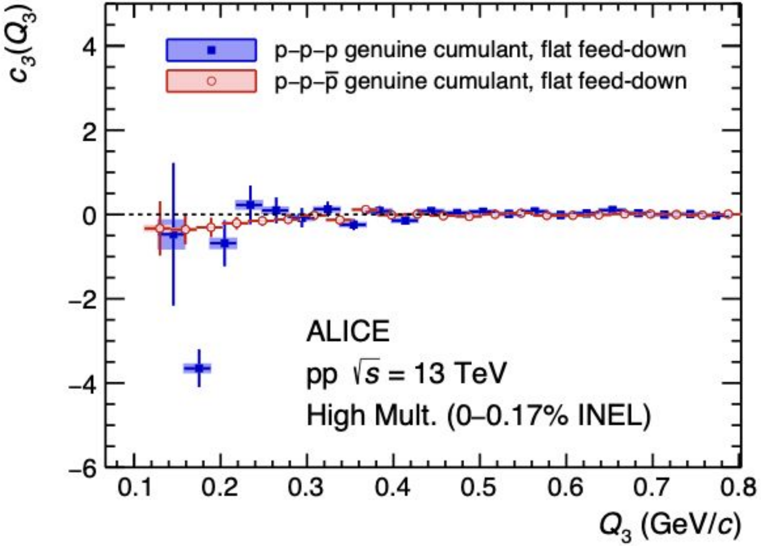
[R.D.G. et al. EPJC 82 (2022) 244]

p-p-p cumulant



cumulant

ALICE Coll. arXiv:2206.03344



Statistical significance

→ $n_\sigma = 6.7$ for $Q_3 < 0.4$ GeV/c

Conclusion

→ Evidence of a genuine three-body effect in the p-p-p system at the LHC

Possible interpretations

→ Pauli blocking at the three-particle level

→ Long-range Coulomb interaction effects

→ Three-body strong interaction

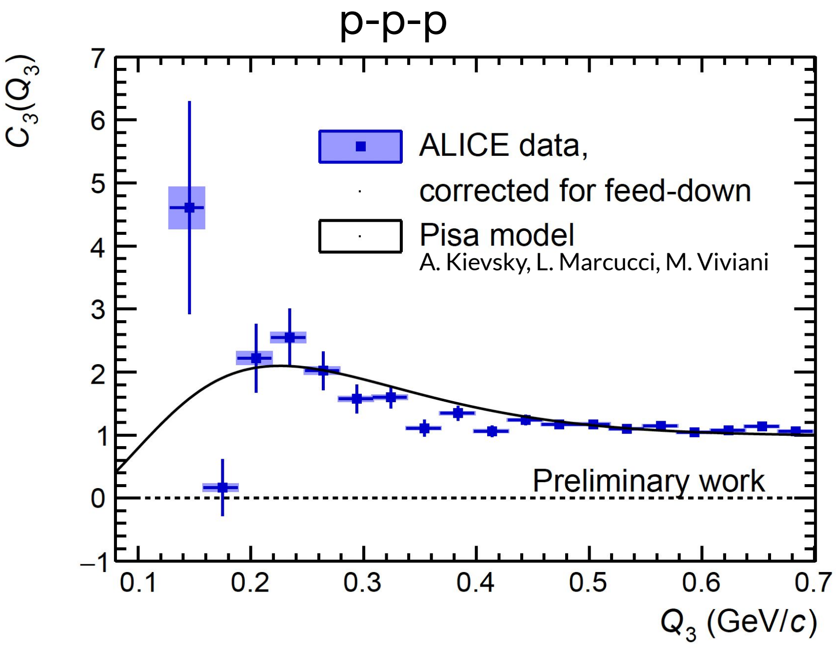
🟥 Test with mixed charge particles, cumulant negligible

p-p-p correlation function

Next step: use the full fledged three-body calculations to test the theoretical models

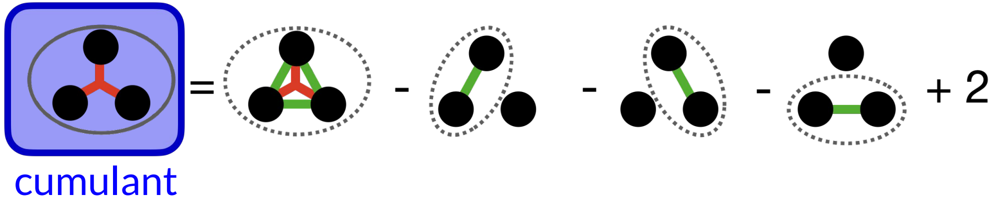
$$C(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) = \iiint S_3(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)|\psi_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3}(\mathbf{x}_1,\mathbf{x}_2,\mathbf{x}_3)|^2 d^3x_1 d^3x_2 d^3x_3$$

wave function obtained
using the Pisa model

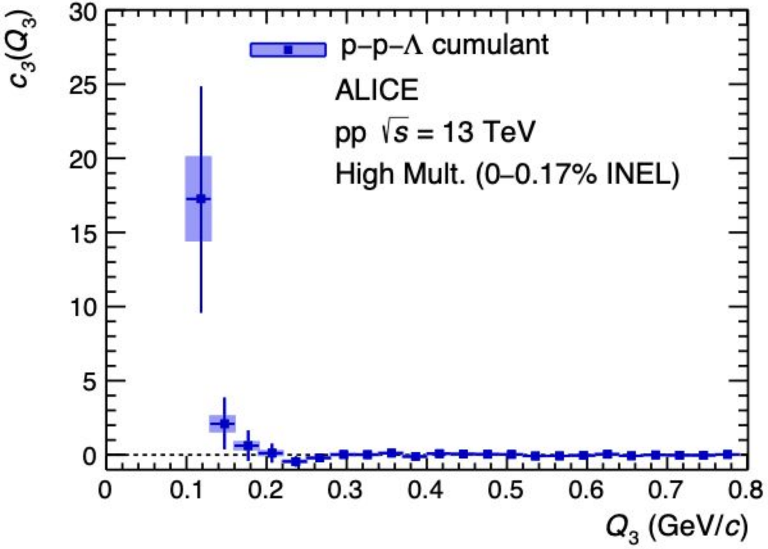


Preliminary result:
→ source size of 2 fm

p-p-Λ cumulant



ALICE Coll. arXiv:2206.03344



Statistical significance
→ n_σ = 0.8 for Q_3< 0.4 GeV/*c*

Conclusion
→ No significant deviation from the null hypothesis

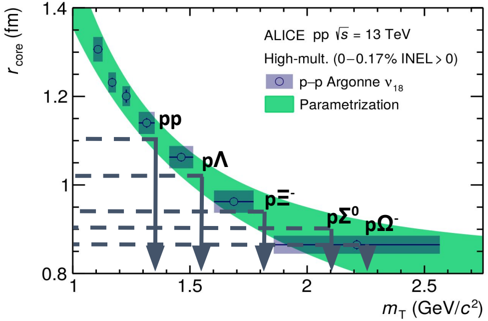
A factor 500 in statistics from the Run 3 data taking

→ Non-zero cumulant can be directly linked to the three-body strong interaction
→ Important measurement for neutron stars

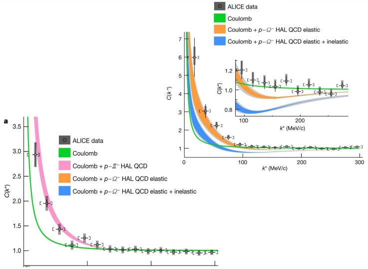
Summary

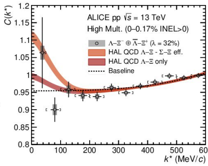
Femtoscopy in small systems

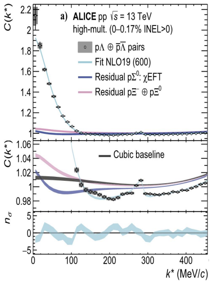
Universal source



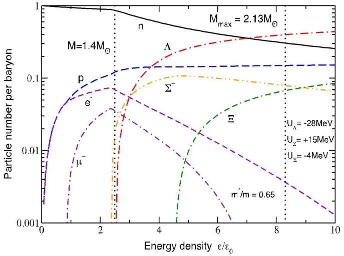
Test of hadron-hadron interactions from EFTs and Lattice QCD

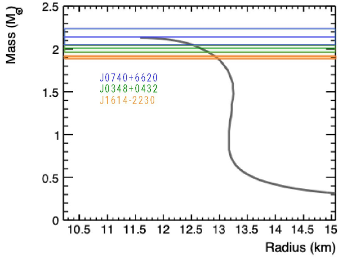




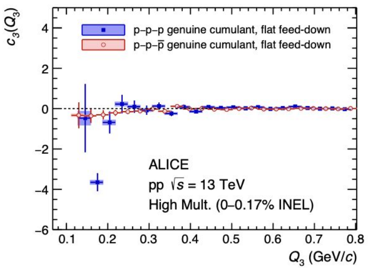


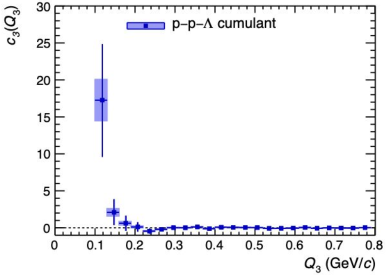
Equation of State for dense pure neutron matter containing hyperons can be improved





High-statistics measurements of three-particle correlations are at horizon





THANKS

Source determination

<u>The first step is “traditional” femtoscopy:</u> known interaction → determine source size

- p−p interaction: Argonne v18 potential
- crosscheck with p−Λ (χEFT)

Determine **gaussian “core” radius**
- As a function of pair <m_T>
- **Common to all hadron-hadron pairs**

✖

Effect of strong short-lived resonances

Adds **exponential tail** to the source profile

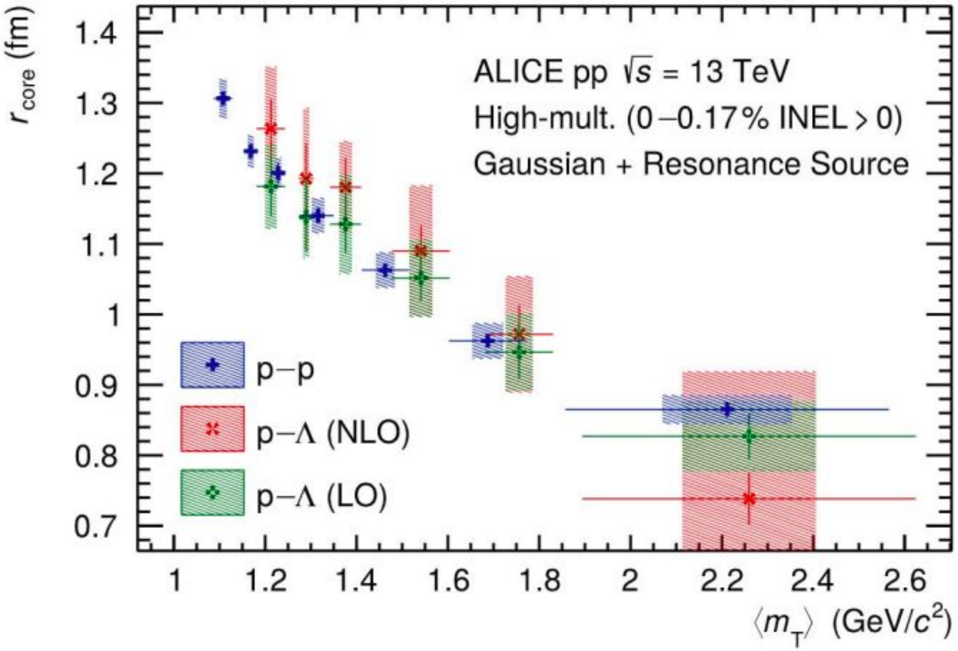
→ Angular distributions from EPOS

→ Production fraction from SHM

	Primordial	Resonances lifetime
p	35.8 %	1.65 fm
Λ	35.6 %	4.69 fm

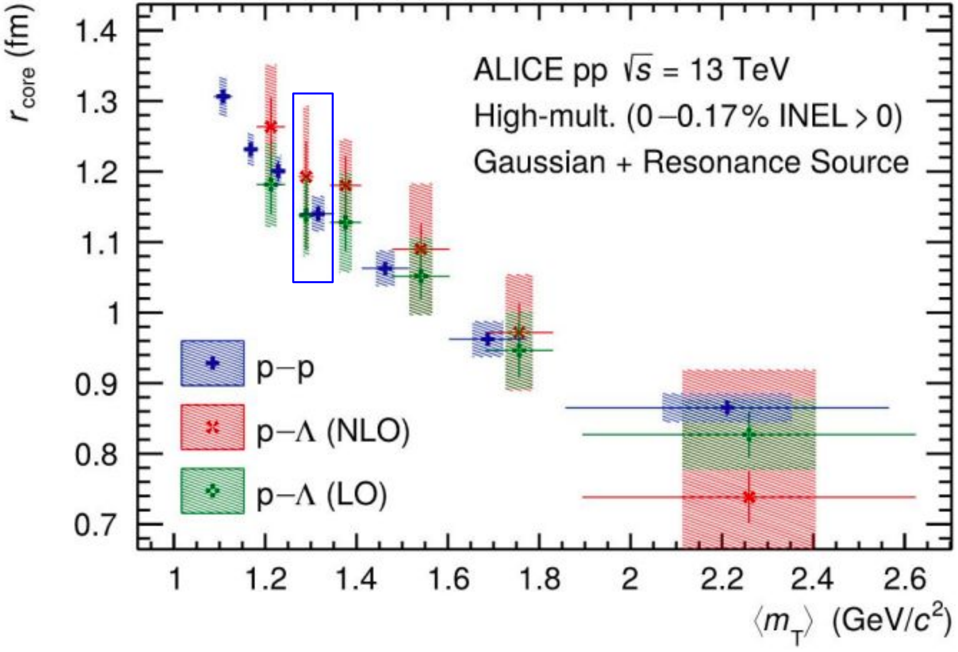
[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

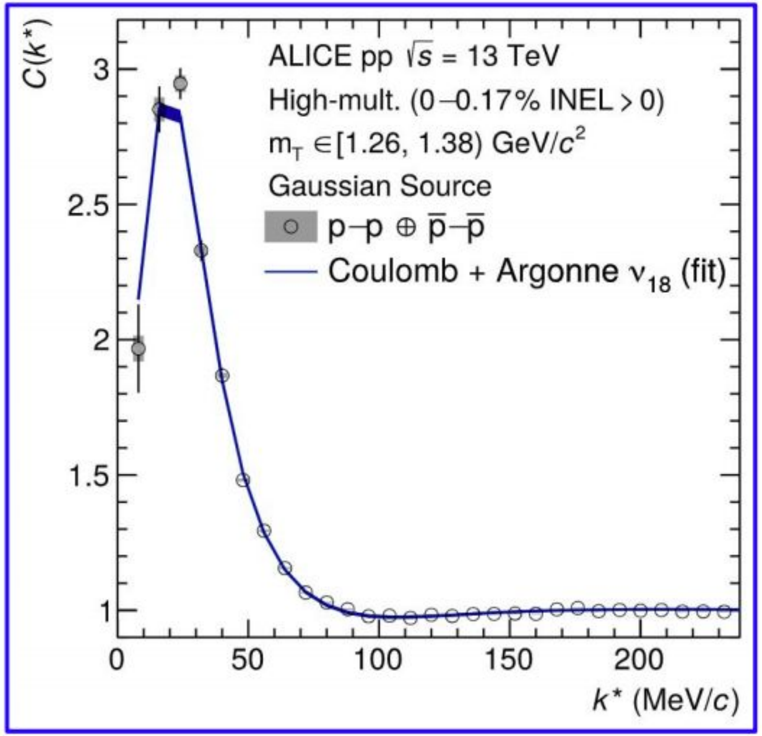
Source determination



[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

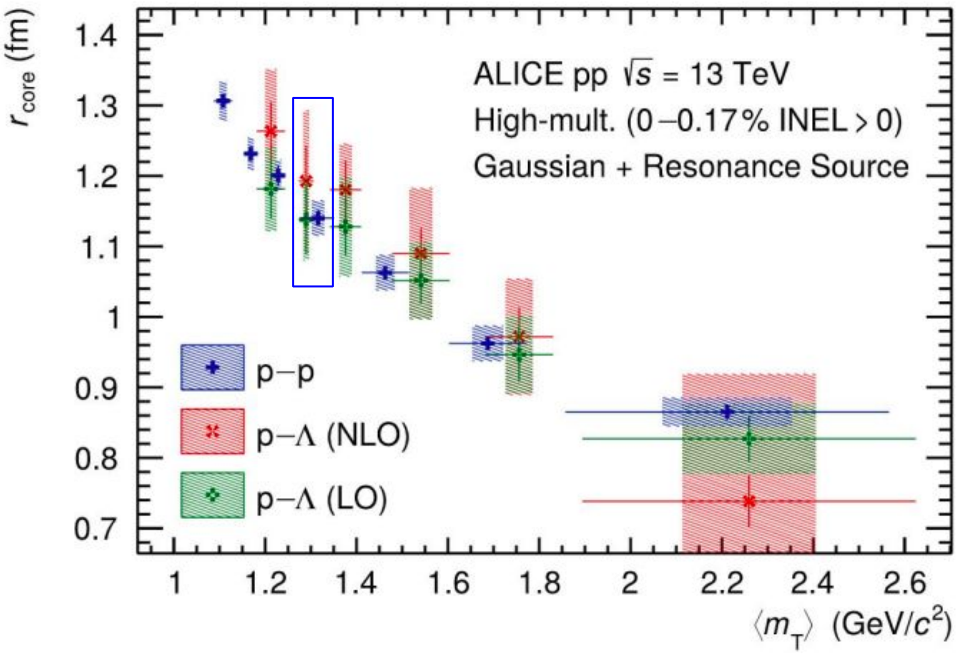
Source determination

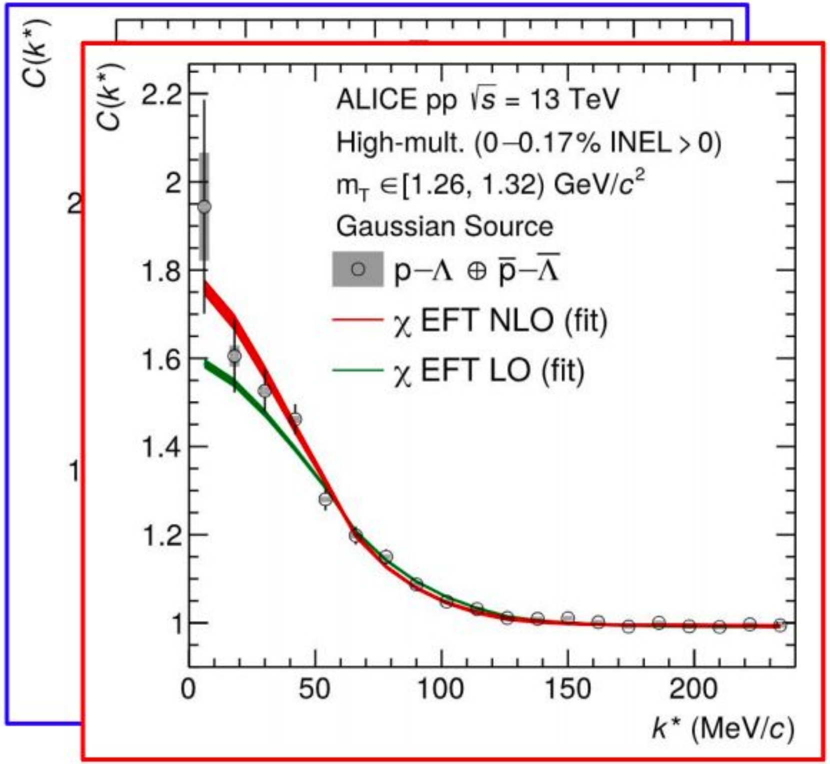




[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

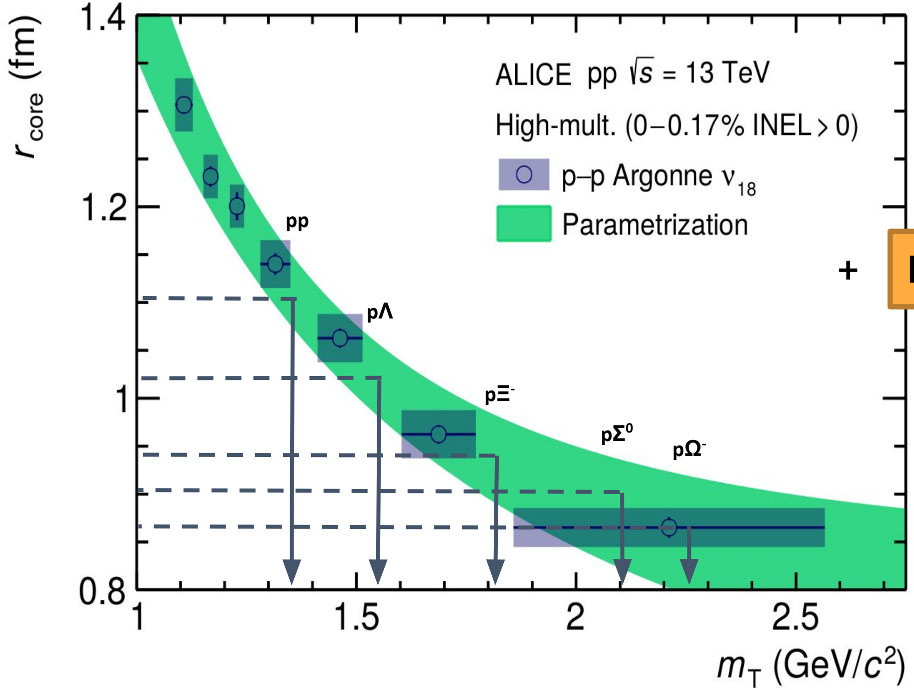
Source determination





[ALICE Coll., Phys. Lett. B 811 (2020) 135849]

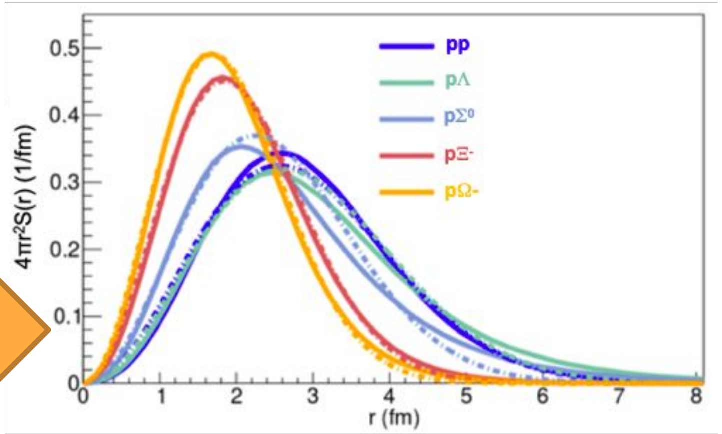
Gaussian source with resonances



ALICE Coll. PLB 811 (2020)

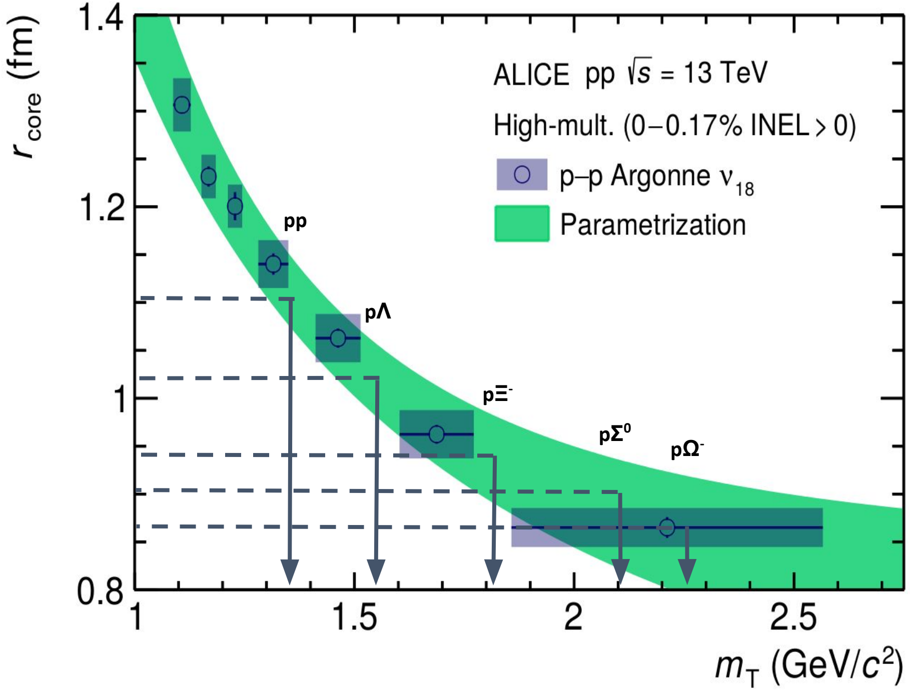
+

Resonances



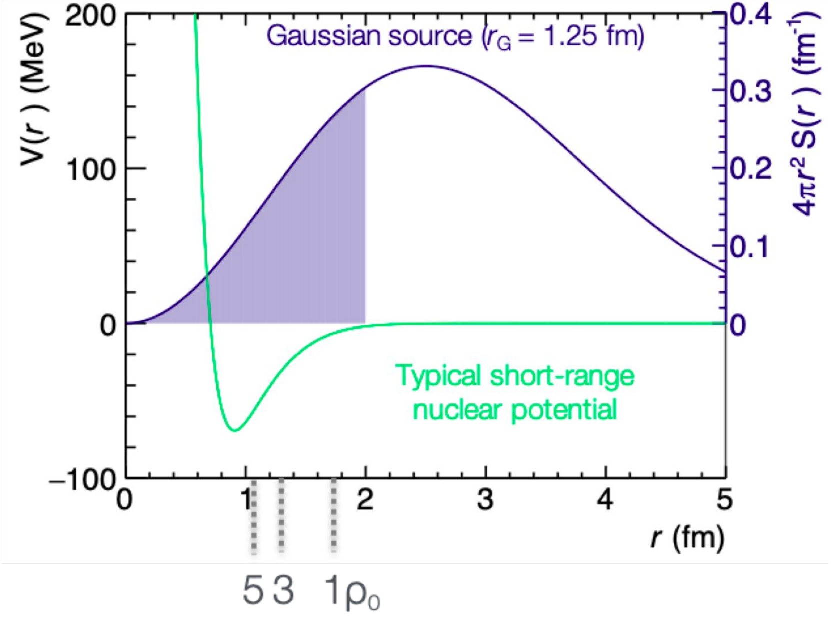
Pair	r_{Core} [fm]	r_{Eff} [fm]
p-p	1.1	1.2
p-Λ	1.0	1.3
p-Σ^0	0.87	1.02
p-Ξ^-	0.93	1.02
p-Ω^-	0.86	0.95

Small particle-emitting sources



ALICE Coll. PLB 811 (2020)

Small particle-emitting source created in pp and p−Pb collisions at the LHC.



Lower order contributions evaluation

Data-driven approach

Using the **same** and **mixed events** distributions:

$$C([\mathbf{p}_1,\mathbf{p}_2],\mathbf{p}_3) = \frac{N_2(\mathbf{p}_1,\mathbf{p}_2)\ N_1(\mathbf{p}_3)}{N_1(\mathbf{p}_1)\ N_1(\mathbf{p}_2)\ N_1(\mathbf{p}_3)}$$

The hyper-momentum Q_3 is calculated from the measured single particle momenta

$$(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \rightarrow Q_3$$

Projector method

[R. Del Grande, L. Serksnyte et al, to appear on the ArXiv (2021)]

Using the **two-body correlation function** of the pair (1,2).

A kinematic transformation from

$$k^*_{12}\ \text{(pair)} \rightarrow Q_3\ \text{(triplet)}$$

$$C(k^*_{12}) \rightarrow C(Q_3)$$

is performed.

For the pair i-j we have

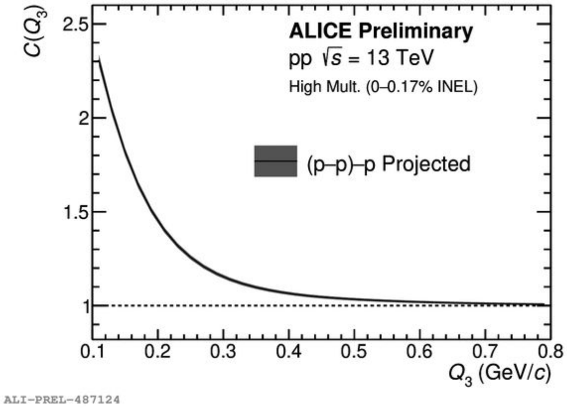
$$C_3^{ij}(Q_3) = \int \underbrace{\boxed{C_2(k^*_{ij})}}_{\text{two-body correlation function}} \underbrace{\boxed{W_{ij}(k^*_{ij}, Q_3)}}_{\text{projector}}\, dk^*_{ij}$$

Two-body correlations

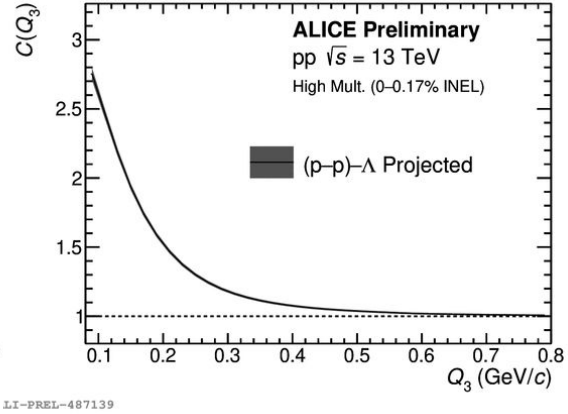
$$\boxed{C_{ij}(Q_3)} = \int \boxed{C_{ij}(k^*)} \cdot W_{ij}(k^*, Q_3)\ dk^*$$

Outputs:

(proton-proton)-proton

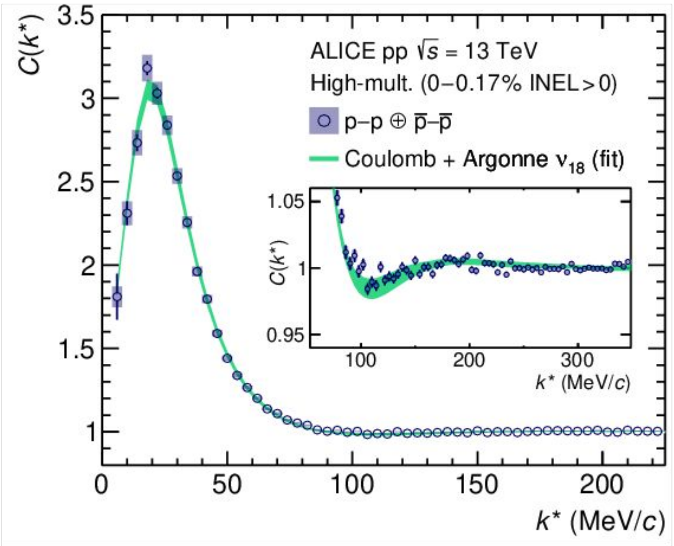


(proton-proton)-Λ



Input:

proton-proton



[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

Two-body correlations

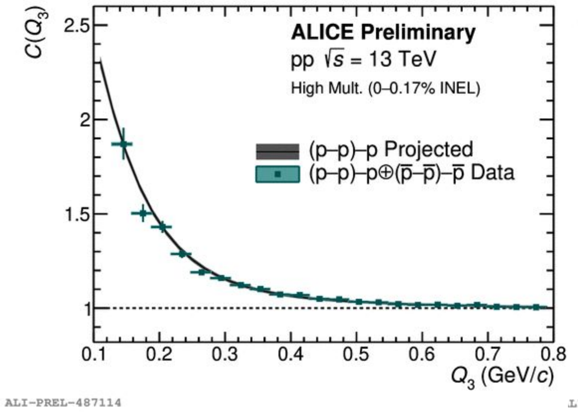
$$\boxed{C_{ij}(Q_3)} = \int \boxed{C_{ij}(k^*)} \cdot W_{ij}(k^*, Q_3)\ dk^*$$

Outputs:

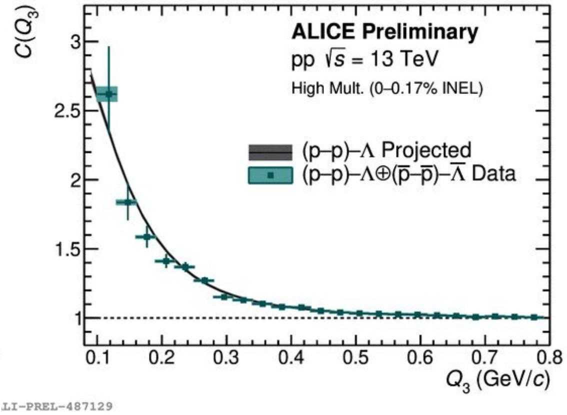
Input:

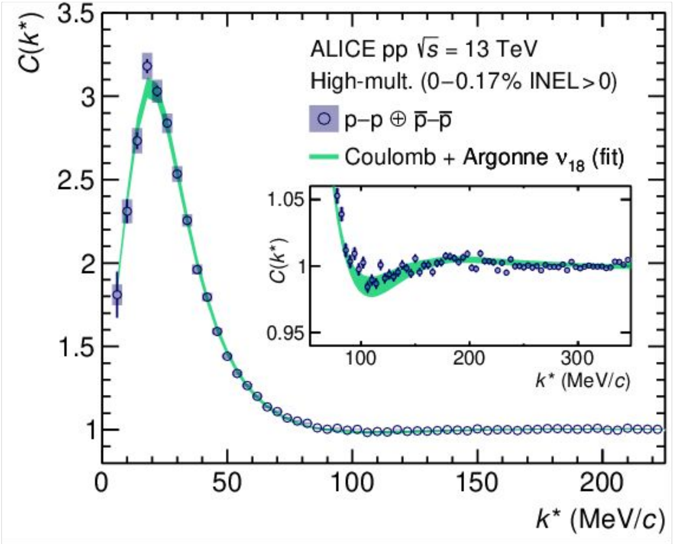
proton-proton

(proton-proton)-proton



(proton-proton)-Λ





Data-driven approach VS Projector method

[ALICE Collaboration / Physics Letters B 805 (2020) 135419]

Two-body correlations

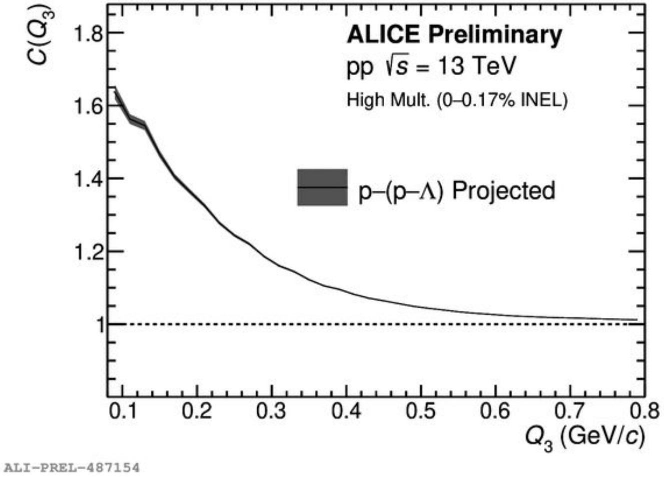
$$\boxed{C_{ij}(Q_3)} = \int \boxed{C_{ij}(k^*)} \cdot W_{ij}(k^*, Q_3)\ dk^*$$

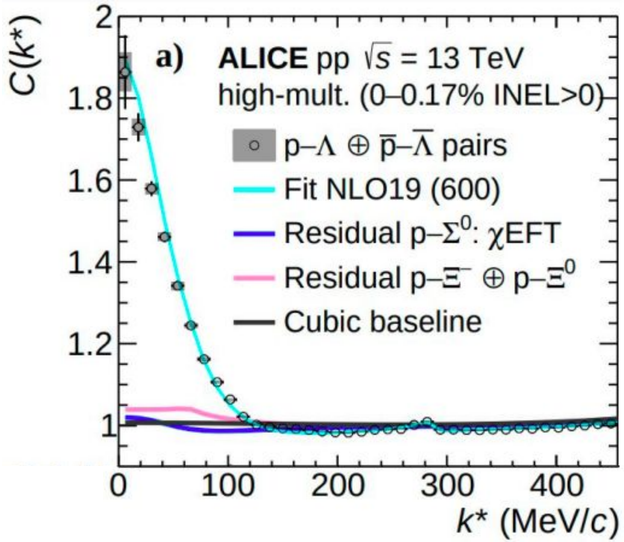
Output:

(proton-Λ)-proton

Input:

proton-Λ





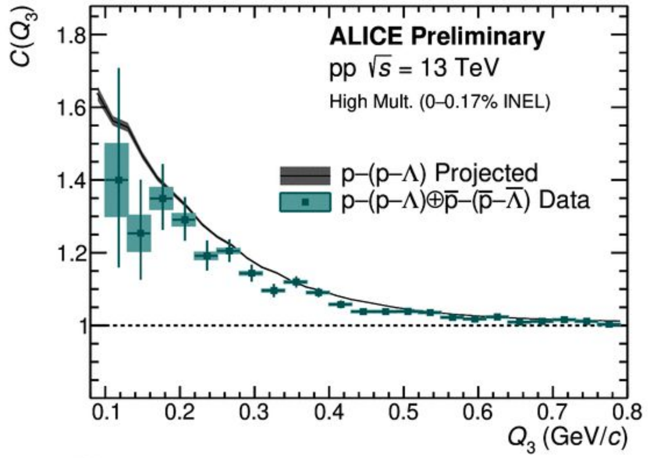
[ALICE Collaboration / arXiv:2104.04427 (submitted to PRL)]

Two-body correlations

$$\boxed{C_{ij}(Q_3)} = \int \boxed{C_{ij}(k^*)} \cdot W_{ij}(k^*, Q_3)\ dk^*$$

Output:

(proton-Λ)-proton

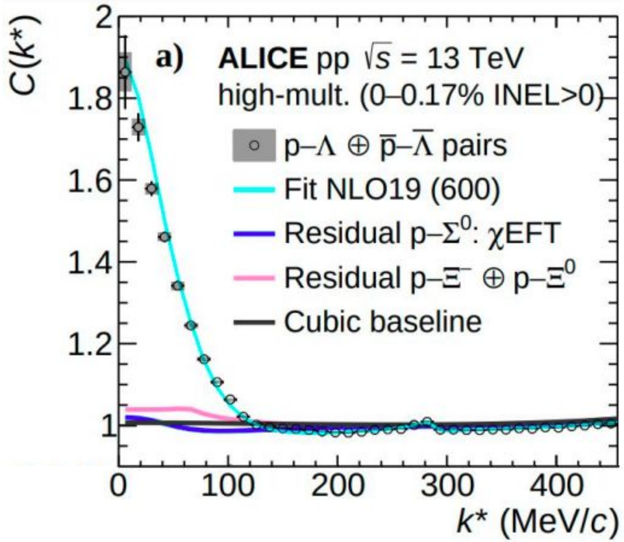


ALI-PREL-487144

Data-driven approach VS **Projector method**

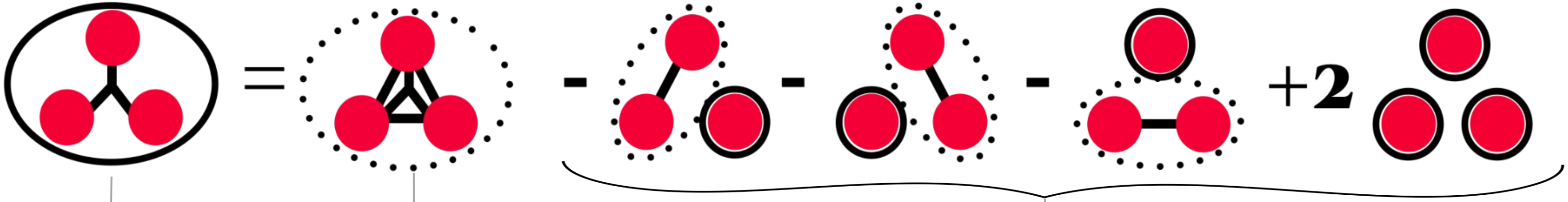
Input:

proton-Λ



[ALICE Collaboration / arXiv:2104.04427 (submitted to PRL)]

Kubo's Cumulant expansion method

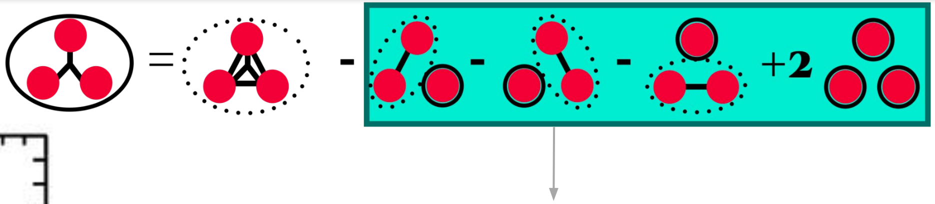


Measured correlation function

Lower order correlations

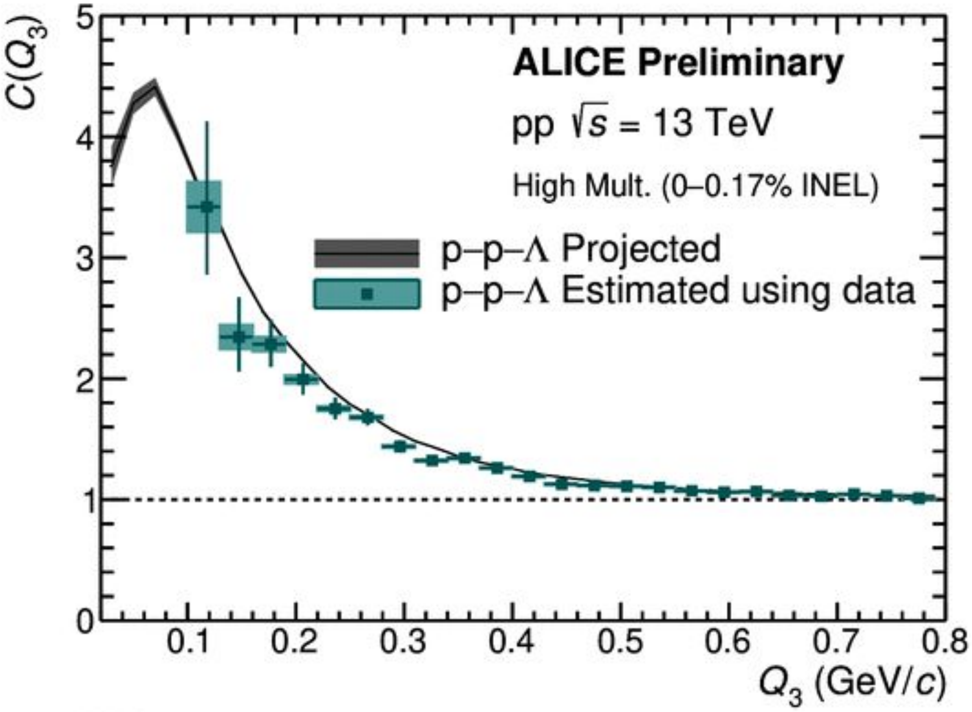
Three-body cumulant
(to be extracted)

p-p-Λ: two-body CF projected onto Q_3



Lower order contributions to the three-body cumulant

$$C^{\text{two}-\text{body}}_{pp\Lambda}(Q_3) = C_3^{pp}(Q_3) + 2\ C_3^{p\Lambda}(Q_3) - 2$$

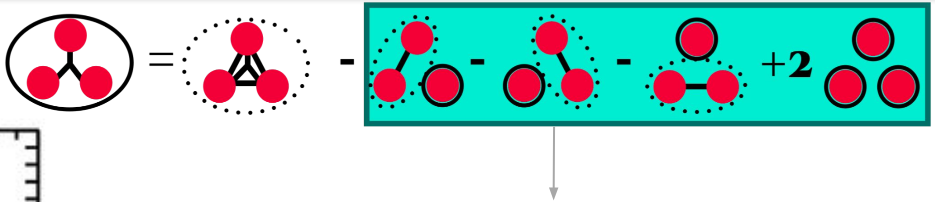


ALI-PREL-487165

Comparison:

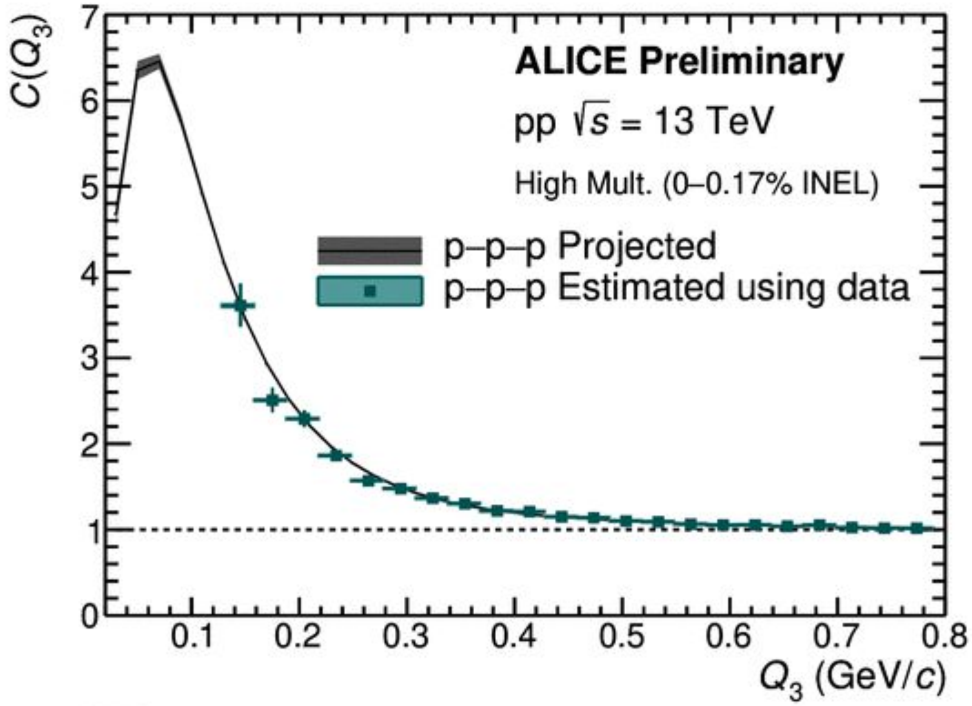
- Data-driven approach
- Projector method

p-p-p: two-body CF projected onto Q_3



Lower order contributions to the three-body cumulant

$$C_{ppp}^{\text{two}-\text{body}}(Q_3) = 3\ C_3^{pp}(Q_3) - 2$$



ALI-PREL-487159

Comparison:

- Data-driven approach
- Projector method

Kubo’s cumulant expansion method

- X_i denotes the general i-th stochastic variable
- The most general decomposition of 2-particle correlation is:

$$\langle X_1 X_2 \rangle = \langle X_1 \rangle \langle X_2 \rangle + \langle X_1 X_2 \rangle_c$$

- By definition, the 2nd term on the right is the 2-particle cumulant
- Cumulants cannot be measured directly, however:

$$\langle X_1 X_2 \rangle_c = \langle X_1 X_2 \rangle - \langle X_1 \rangle \langle X_2 \rangle$$

Kubo's cumulant expansion method

- The most general decomposition of 3-particle correlation is:

$$\begin{aligned}\langle X_1X_2X_3\rangle &= \langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\\ &+ \langle X_1X_2\rangle_c\langle X_3\rangle+\langle X_1X_3\rangle_c\langle X_2\rangle+\langle X_2X_3\rangle_c\langle X_1\rangle\\ &+ \langle X_1X_2X_3\rangle_c\end{aligned}$$

- Using the 2-particle cumulant: $\langle X_1X_2\rangle_c=\langle X_1X_2\rangle-\langle X_1\rangle\langle X_2\rangle$

- Working recursively from higher to lower orders, we have 3-particle cumulant expressed in terms of the measured 3-, 2-, and 1-particle averages:

$$\begin{aligned}\langle X_1X_2X_3\rangle_c &= \langle X_1X_2X_3\rangle\\ &- \langle X_1X_2\rangle\langle X_3\rangle-\langle X_1X_3\rangle\langle X_2\rangle-\langle X_2X_3\rangle\langle X_1\rangle\\ &+ 2\langle X_1\rangle\langle X_2\rangle\langle X_3\rangle\end{aligned}$$

Projection onto Q_3

- The projection onto Q_3 is performed as follows

$$C_3(Q_3) = \iiint_{(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\in\mathcal{D}} C_3(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3)\ \mathcal{N}\ d^3\mathbf{p}_1\ d^3\mathbf{p}_2\ d^3\mathbf{p}_3$$

$$\mathcal{D} = \{(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3) \in \mathcal{S} \mid Q_3 = \text{constant}\}$$

density of states in the phase space (uniform)

- In the case of two-body correlations, the projections turns to be

two-body correlation function

projector $W(k_1, Q_3)$ ----> phase space density at Q_3 = constant

$$C_3(Q_3) = \int_0^{\sqrt{\frac{\gamma}{\alpha\gamma-\beta^2}}\,Q_3} \boxed{C_2(k_1)}\ \boxed{\frac{16(\alpha\gamma-\beta^2)^{3/2}k_1^2}{\pi Q_3^4\gamma^2}\sqrt{\gamma Q_3^2-(\alpha\gamma-\beta^2)k_1^2}}\ dk_1$$

where α, β and γ are constants depending on the particles mass.

λ parameters

The measured correlation function includes also misidentified particles and and feed-down particles coming from decays of resonances. Total **measured** function thus is:

$$C(XYZ) = \sum_{i,j,k} \lambda_{i,j,k}(XYZ)C_{i,j,k}(XYZ) = \underbrace{\boxed{\lambda_{X_0,Y_0,Z_0}(XYZ)C_{X_0,Y_0,Z_0}(XYZ)}}_{\text{Correctly identified primary particles}} + \sum_{ijk!=X_0Y_0Z_0} \lambda_{i,j,k}(XYZ)C_{i,j,k}(XYZ)$$

- The cumulant is calculated with the measured correlation functions not accounting for the λ parameters.

$$\lambda_{i,j,k}(XYZ) = \mathscr{P}(X_i)f(X_i)\mathscr{P}(Y_j)f(Y_j)\mathscr{P}(Z_k)f(Z_k)$$

$$\overbrace{\boxed{c(XYZ)}}^{\text{Extracted from measurement}} = \sum_{i,j,k} \lambda_{i,j,k}(XYZ)c(X_iY_jZ_k) = \overbrace{\boxed{\lambda_{X_0Y_0Z_0}(XYZ)c(X_0Y_0Z_0)}}^{\text{What we are interested in}} + \overbrace{\boxed{\sum_{i,j,k\neq(X_0Y_0Z_0)} \lambda_{i,j,k}(XYZ)c(X_iY_jZ_k)}}^{\text{Feed-down and misidentified particle contribution}}$$

- The genuine three body interaction for the feed-down and misidentified particle contributions is currently not known.

λ parameters

- The λ parameters requires purity and the secondary fraction evaluation.
- The average Λ purity is 95.57% and for protons the purity is 98.34%.
- The fractions of secondaries are estimated using Monte Carlo simulations.

Some of the contributions with highest lambda parameters:

p-p-p	61.8%
p-p-p_{Λ} x3	19.6%
p-p-p_{Σ^+} x3	8.5%
p-p_{Λ}-p_{Λ} x3	0.69%
p-p_{Λ}-p_{Σ^+} x3	0.3 %
p-p_{Σ^+}-p_{Σ^+} x3	0.13%

p-p-Λ	40.5%
p-p-Λ_{Σ^0}	13.5%
p-p-Λ_{Ξ^0}	7.56%
p-p-Λ_{Ξ^-}	7.56%
p-p_{Λ}-Λ x2	8.56%
p-p_{Σ^+}-Λ x2	3.7%