

# Recent results from JPAC collaboration



Miguel Albaladejo (IFIC)

Recent results and perspectives in hadron physics  
(Institute Pascal, Orsay, Oct. 17th, 2022)



# JPAC: Joint Physics Analysis Center

- Joint IU and JLab venture to extract physics results from [JLab12](#)
- Work in [theoretical/experimental/phenomenological](#) analysis
- Light/heavy meson [spectroscopy](#)
- Interaction with many [experimental collaborations](#): (GlueX, CLAS, BES, ...) and [LQCD groups](#)

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# Outline

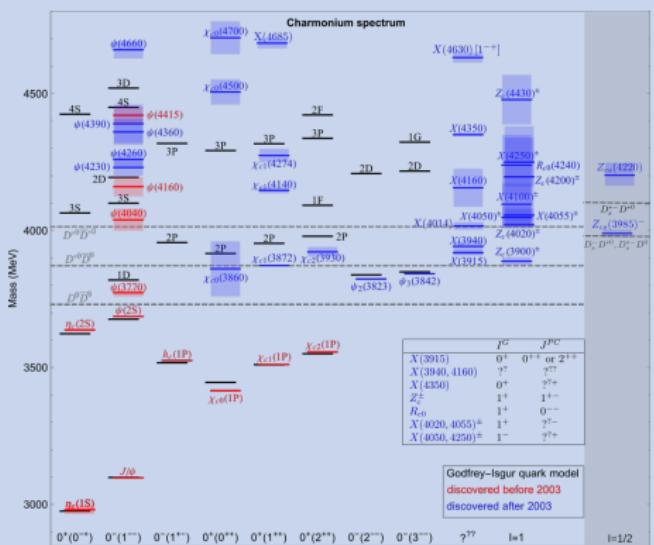
## ① XYZ photoproduction:

- Exclusive photoproduction [JPAC Collab., PR,D102,114010(20)]
- Inclusive photoproduction [JPAC Collab., 2209.05882 (accepted PRD)]

## ② Khuri-Treiman equations and $V \rightarrow 3\pi$ decays:

- $\omega \rightarrow 3\pi$  and  $\omega \rightarrow \gamma^* \pi^0$  [JPAC Collab., EPJ,C80,1107(20)]
- $J/\psi \rightarrow 3\pi$  [Work in progress...]

# XYZ states and photoproduction

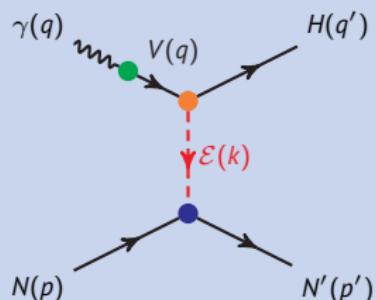


Taken from [Guo et al., 2020.07141]

- A new method to confirm or discard these new XYZ states
- In principle, photoproduction is free of triangle-singularities that can give rise to resonance-like effects
- Photoproduction framework has been used before

# Exclusive XYZ photoproduction amplitude

JPAC Collab., PR,D102,114010('20)

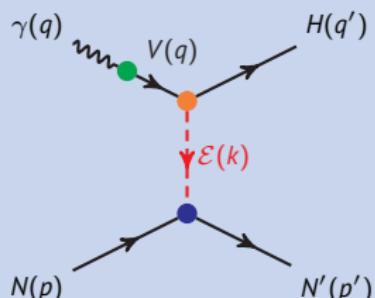


$$\langle \lambda_H \lambda'_N | T | \lambda_\gamma \lambda_N \rangle = \sum_{V, \epsilon} \underbrace{\frac{ef_V}{m_V}}_{VMD} \underbrace{T_{\lambda_V = \lambda_\gamma, \lambda_Q}^{\alpha_1 \dots \alpha_j}}_{VHE \text{ vertex}} \underbrace{\mathcal{P}_{\alpha_1 \dots \alpha_j; \beta_1 \dots \beta_j}}_{\text{propagator}} \underbrace{\mathcal{B}_{\lambda_N \lambda'_N}^{\beta_1 \dots \beta_j}}_{NNE \text{ vertex}}$$

- **VMD** couples  $V (= J/\psi, \Upsilon(nS))$  to photon  $\Gamma(V \rightarrow e^+ e^-) = 4\pi\alpha^2 \frac{f_V^2}{3m_V}$
- **Top vertex VHE**  $\Gamma(H \rightarrow V \epsilon) = \frac{1}{2J_H + 1} \frac{p}{8\pi m_H^2} \sum_{\text{pol.}} \left| T_{\lambda_V \lambda_H}^{\alpha_1 \dots \alpha_j} \epsilon_{\alpha_1 \dots \alpha_j}^*(k, \lambda_\epsilon) \right|^2$
- **Bottom vertex NNE** Taken from standard phenomenology (e.g.  $NN\pi \rightarrow g_{NN\pi}$ )

# Exclusive XYZ photoproduction amplitude

JPAC Collab., PR,D102,114010('20)



$$\langle \lambda_H \lambda'_N | T | \lambda_\gamma \lambda_N \rangle = \sum_{V, \mathcal{E}} \underbrace{\frac{ef_V}{m_V}}_{\text{VMD}} \underbrace{\mathcal{T}_{\lambda_V = \lambda_\gamma, \lambda_Q}^{\alpha_1 \dots \alpha_j}}_{\text{VHE vertex}} \underbrace{\mathcal{P}_{\alpha_1 \dots \alpha_j; \beta_1 \dots \beta_j}}_{\text{propagator}} \underbrace{\mathcal{B}_{\lambda_N \lambda'_N}^{\beta_1 \dots \beta_j}}_{\text{NN}\mathcal{E} \text{ vertex}}$$

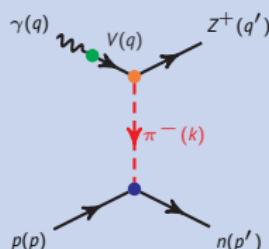
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- **Bottom vertex NN $\mathcal{E}$**  Taken from standard phenomenology (e.g.  $NN\pi \rightarrow g_{NN\pi}$ )

# $Z_{c,b}$ exclusive photoproduction

JPAC Collab., PR,D102,114010('20)



$Z$	$m_Z$ (MeV)	$\Gamma_Z$ (MeV)	$g_{\gamma Z \pi} (\times 10^{-2})$	$V$	$\mathcal{B}(Z \rightarrow V\pi)$ (%)	$g_{VZ\pi}$
$Z_c(3900)^+$	$3888.4(2.5)$	$28.3(2.5)$	$5.17$	$J/\psi$	$10.5 \pm 3.5$	$1.91$
$Z_b(10610)^+$	$10607.2(2.0)$	$18.4(2.4)$	$5.80$	$\Upsilon(1S)$	$0.54^{+0.19}_{-0.15}$	$0.49$
				$\Upsilon(2S)$	$3.6^{+1.1}_{-0.8}$	$3.30$
				$\Upsilon(3S)$	$2.1^{+0.8}_{-0.6}$	$9.22$
$Z'_b(10650)^+$	$10652.2(1.5)$	$11.5(2.2)$	$2.90$	$\Upsilon(1S)$	$0.17^{+0.08}_{-0.06}$	$0.21$
				$\Upsilon(2S)$	$1.4^{+0.6}_{-0.4}$	$1.47$
				$\Upsilon(3S)$	$1.6^{+0.7}_{-0.5}$	$4.80$

- Top vertex  $Z \rightarrow V\pi$ : Sizeable branching fractions

$$\mathcal{T}_{\lambda_V \lambda_Z} = \frac{g_{VZ\pi}}{m_Z} \varepsilon_\mu(q, \lambda_V) \varepsilon_\nu^*(q', \lambda_Z) [(q \cdot k) g^{\mu\nu} - k^\mu q^\nu] \quad \left[ g_{VZ\pi} = \sum_V \frac{ef_V}{m_V} g_{VZ\pi} \right]$$

- Bottom vertex  $NN\pi$ :

$$\mathcal{B}_{\lambda_N \lambda'_N} = \sqrt{2} g_{\pi NN} \beta(t) \bar{u}(p', \lambda'_N) \gamma_5 u(p, \lambda_N)$$

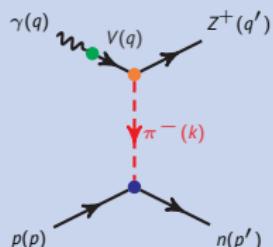
- $g_{\pi NN}^2 / (4\pi) \simeq 13.81(0.12)$  and  $\beta(t) = \exp\left(t'/\Lambda_\pi^2\right)$  with  $\Lambda_\pi = 0.9$  GeV

- Propagator:

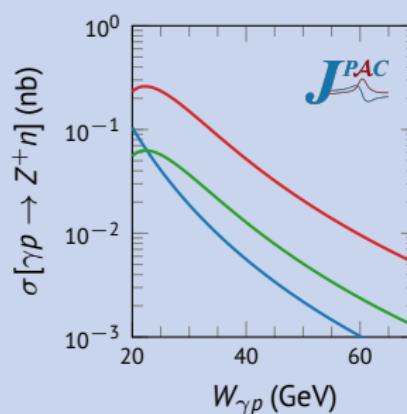
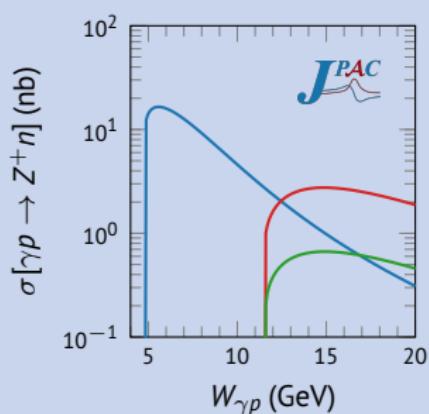
- Fixed spin up to  $W_{\gamma p} \lesssim E_{\text{th}} + 10$  GeV
- Reggeized pions:  $\alpha(t) = \alpha'(t - m_\pi^2)$  with  $\alpha' = 0.7$  GeV $^{-2}$

# $Z_{c,b}$ exclusive photoproduction

JPAC Collab., PR,D102,114010('20)

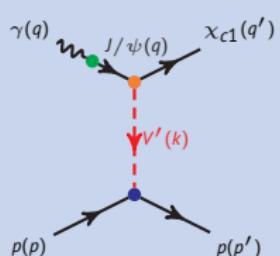


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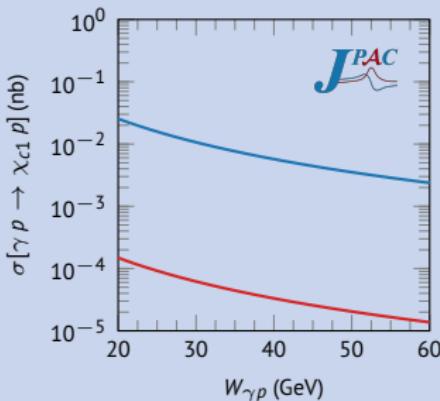
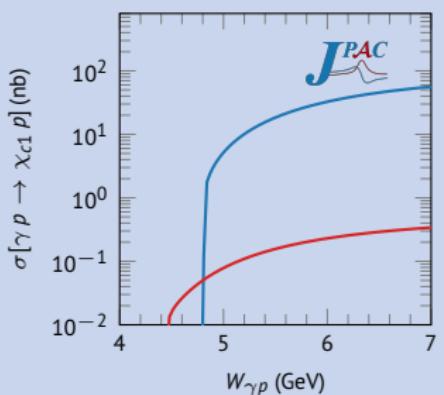


# $\chi_{c1}(1P)$ and $X(3872)$ photoproduction

JPAC Collab., PR,D102,114010('20)

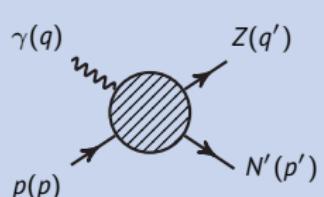


$X$	$m_X$ (MeV)	$\Gamma_X$ (MeV)	$V'$	$\mathcal{B}(X \rightarrow \gamma V')$ (%)	$g_{\gamma X V'} \cdot 10^{-3}$	
$\chi_{c1}(1P)$	3510.67(0.05)	0.84(0.04)	$\rho$	$2.16(0.17) \cdot 10^{-4}$	0.92	
			$\omega$	$6.8(0.8) \cdot 10^{-5}$	0.52	
			$\phi$	$2.4(0.5) \cdot 10^{-5}$	0.42	
			$J/\psi$	34.3(1.0)	$1.0 \cdot 10^3$	
$X$	$m_X$ (MeV)	$\Gamma_X$ (MeV)	$\mathcal{B}(X \rightarrow J/\psi \mathcal{E})$ (%)	$g_{\psi X \mathcal{E}}$	$g_{\gamma X \mathcal{E}} \cdot 10^{-3}$	
$X(3872)$	3871.69(0.17)	1.19(0.19)	$\rho$	$4.1^{+1.9}_{-1.1}$	0.13	3.6
			$\omega$	$4.4^{+2.3}_{-1.3}$	0.30	8.2



# Inclusive $Z_{c,b}$ photoproduction (I)

JPAC Collab., 2209.05882 (accepted PRD)

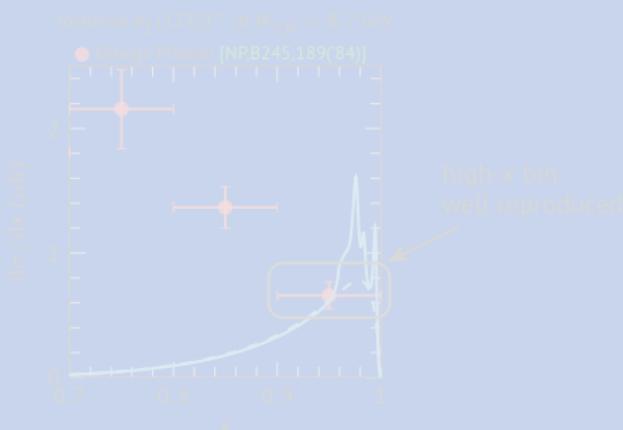


$$\begin{aligned}
 T_Z \frac{d^2\sigma}{d^2q'} &= K \sum_{|\Lambda|} \sum_{\lambda} \int \prod_{\lambda} \frac{d^2p_{\lambda}}{(2\pi)^2 2E_{\lambda}} \left[ A_{|\Lambda|}^{(H+2Q)^2} (2\pi)^4 \delta^4(q + p - q' - p_{\Lambda}) \right] \\
 &\sim 2K \sum_{|\Lambda|} \text{Disc } A_{|\Lambda|}^{(H)^2} \\
 &\approx \frac{1}{16\pi^2} \frac{\lambda^{1/2}(M_Q^2, t, M_Q^2)}{2E_N \sqrt{s}} |T_H(t) P_H(t, z)|^2 \sigma^{\pi^* H}(t, M_Q^2)
 \end{aligned}$$

- Generalized optical theorem to relate  $\gamma N \rightarrow Z Q$  with  $\gamma N \bar{Z}$  amplitude
- Pion-exchange model (as in exclusive) to write  $\sigma_{\gamma N \rightarrow Z Q}$  in terms of  $\sigma^{\pi^* H}(t, M_Q^2)$

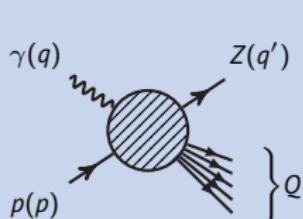
$$\frac{1}{t - m_Q^2} [T_H(t, 0) - \alpha'/T(-\alpha(t)) \frac{1 + e^{-im\alpha(t)}}{t}] \left( \frac{z}{M_Q^2} \right)^{\alpha(t)}$$

- Model benchmarked in  $b_1(1235)$  inclusive photoproduction [right plot]



# Inclusive $Z_{c,b}$ photoproduction (I)

JPAC Collab., 2209.05882 (accepted PRD)



$$\begin{aligned}
 E_Z \frac{d^3\sigma}{d^3q_f} &= \mathcal{K} \sum_{[\lambda]} \sum_Q \int \prod_n \frac{d^3p_n}{(2\pi)^3 2E_n} \left| A_{[\lambda]}^{\gamma N \rightarrow Z Q} \right|^2 (2\pi)^4 \delta^4(q + p - q' - p_Q) \\
 &= 2\mathcal{K} \sum_{[\lambda]} \text{Disc } A_{[\lambda]}^{\gamma N \bar{Z}} \\
 &\approx \frac{1}{16\pi^2} \frac{\lambda^{1/2} (M_Q^2, t, M_Q^2)}{2E_N \sqrt{s}} |T_\pi(t) T_\pi(t, z)|^2 \sigma^{\pi^+ \pi^-}(t, M_Q^2)
 \end{aligned}$$

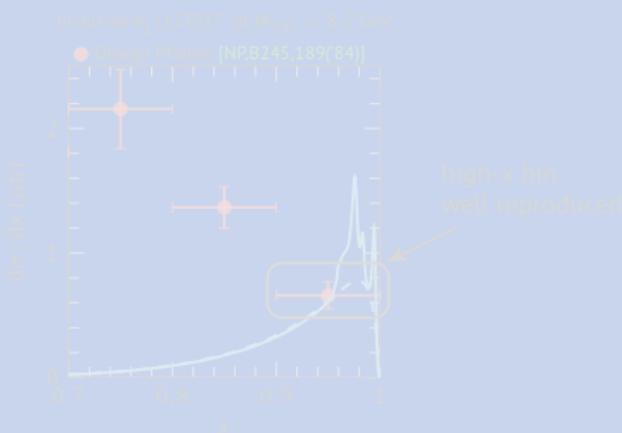
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$$\sigma_{\gamma N \rightarrow Z Q} \text{ in terms of } \sigma^{\pi^+ \pi^-}(t, M_Q^2)$$

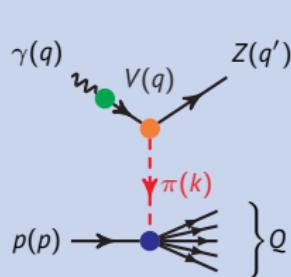
$$\frac{1}{t - m_Q^2} [T_\pi(t, z) - \alpha'(t) \Gamma(-\alpha(t)) \frac{1 + e^{-im\alpha(t)}}{t}] \left( \frac{z}{M_Q^2} \right)^{\alpha(t)}$$

- Model benchmarked in  $b_1(1235)$  inclusive photoproduction [right plot]



# Inclusive $Z_{c,b}$ photoproduction (I)

JPAC Collab., 2209.05882 (accepted PRD)



$$\begin{aligned}
 E_Z \frac{d^3\sigma}{d^3q_f} &= \mathcal{K} \sum_{[\lambda]} \sum_Q \int \prod_n \frac{d^3p_n}{(2\pi)^3 2E_n} \left| A_{[\lambda]}^{\gamma N \rightarrow Z Q} \right|^2 (2\pi)^4 \delta^4(q + p - q' - p_Q) \\
 &= 2\mathcal{K} \sum_{[\lambda]} \text{Disc } A_{[\lambda]}^{\gamma N \bar{Z}} \\
 &\simeq \frac{1}{16\pi^3} \frac{\lambda^{1/2}(M_Q^2, t, m_N^2)}{2E_\gamma \sqrt{s}} |T_\pi(t) \mathcal{P}_\pi(t, s)|^2 \sigma^{\pi^* N}(t, M_Q^2)
 \end{aligned}$$

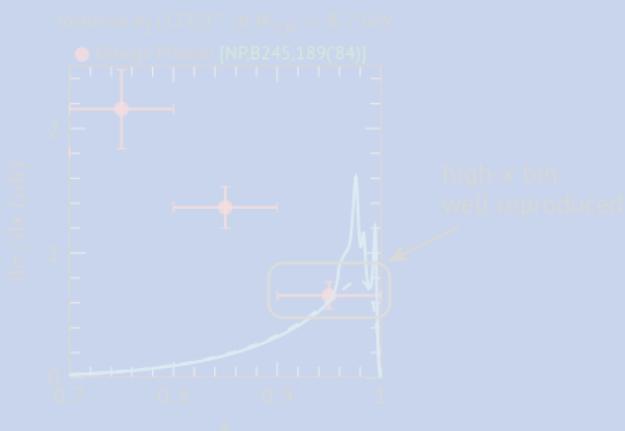
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$$\sigma_{\gamma N \rightarrow Z Q} \text{ in terms of } \sigma^{\pi^* N}(t, M_Q^2)$$

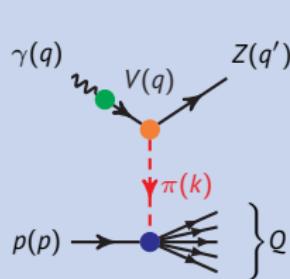
$$\frac{1}{t - m_\pi^2} P_{\pi\pi}(t, s) - \alpha' \Gamma(-\alpha(t)) \frac{1 + e^{-i\pi\alpha(t)}}{2} \left( \frac{s}{M_Q^2} \right)^{\alpha(t)}$$

- Model benchmarked in  $b_1(1235)$  inclusive photoproduction [right plot]



# Inclusive $Z_{c,b}$ photoproduction (I)

JPAC Collab., 2209.05882 (accepted PRD)



$$\begin{aligned}
 E_Z \frac{d^3\sigma}{d^3q_f} &= \mathcal{K} \sum_{[\lambda]} \sum_Q \int \prod_n \frac{d^3p_n}{(2\pi)^3 2E_n} \left| A_{[\lambda]}^{\gamma N \rightarrow Z Q} \right|^2 (2\pi)^4 \delta^4(q + p - q' - p_Q) \\
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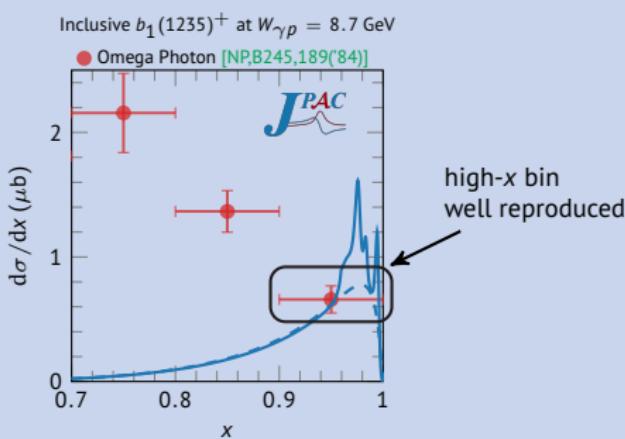
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- Pion-exchange model (as in exclusive) to write

$$\sigma_{\gamma N \rightarrow Z Q} \text{ in terms of } \sigma^{\pi^* N}(t, M_Q^2)$$

$$\frac{1}{t - m_\pi^2} P_{\pi\pi}(t, s) - \alpha' \Gamma(-\alpha(t)) \frac{1 + e^{-i\pi\alpha(t)}}{2} \left( \frac{s}{M_Q^2} \right)^{\alpha(t)}$$

- Model benchmarked in  $b_1(1235)$  inclusive photoproduction [right plot]

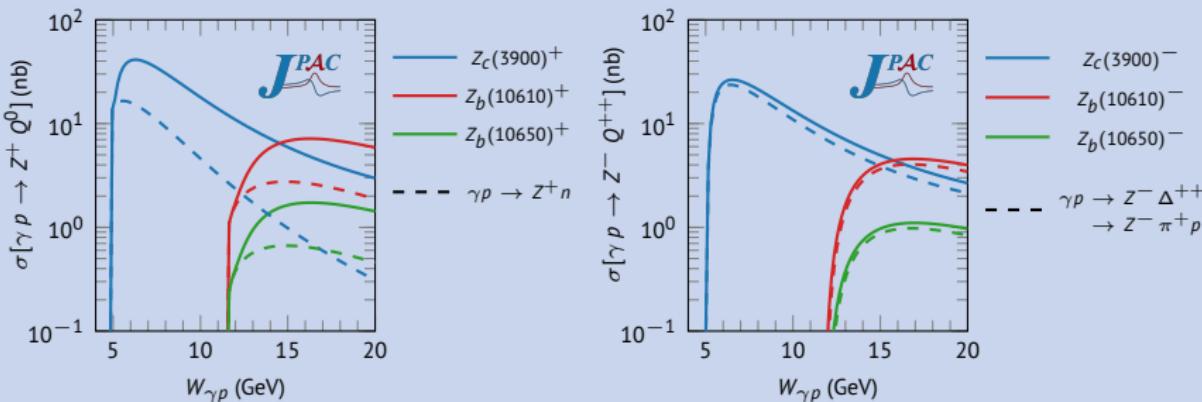


# Inclusive $Z_{c,b}$ photoproduction (II)

JPAC Collab., 2209.05882 (accepted PRD)

- Near threshold:

- Relevant contribution from inelastic for  $Z^+$  prod. [left plot]
- Dominant contribution from  $\Delta^{++}$  in  $Z^-$  prod. [right plot]



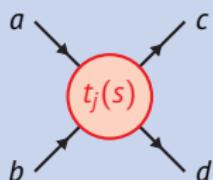
- High energy:

	$\sigma(\gamma p \rightarrow H^\pm Q) [\text{pb}]$			$\sigma(\gamma p \rightarrow H^+ n) [\text{pb}]$		
	$H$	30 GeV	60 GeV	90 GeV	30 GeV	60 GeV
$b_1(1235)$	$60 \cdot 10^{-3}$	$60 \cdot 10^{-3}$	$61 \cdot 10^{-3}$	43	2.3	$< 10^{-8}$
$Z_c(3900)$	187	146	140	19	1.0	$< 10^{-8}$
$Z_b(10610)$	163	15	5	150	10	$< 10^{-8}$
$Z_b(10650)$	40	4	1	37	2.4	$< 10^{-8}$

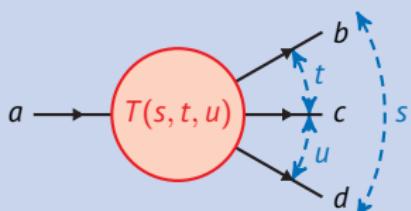
# Introduction: Khuri-Treiman equations in a nutshell

- Partial wave expansion **in the s-channel**:

$$T(s, t, u) = \sum_{\ell=0}^{\infty} (2\ell + 1) P_{\ell}(z_s) t_{\ell}(s)$$



- Two main (connected) problems:
  - Infinite number of PW
  - PW have RHC and LHC
- Only RHC: BS equation, K-matrix, DR,...
- Problem with “truncation”:  $t_{\ell}(s)$  only depends on  $s$ , so singularities in the  $t$ -,  $u$ -channel can only appear summing an infinite number of PW.



- In many decay processes one wants to take into account unitarity/FSI interactions in the three possible channels.

# Introduction: Khuri-Treiman equations in a nutshell

- Khuri-Treiman equations are a tool to achieve this **two-body unitarity** in the **three channels**  
[N. Khuri, S. Treiman, Phys. Rev. **119**, 1115 (1960)]
- Consider three ( $s$ -,  $t$ -,  $u$ -channels) **truncated** “isobar” expansions.
- Isobars  $f_\ell^{(s)}(s)$  have only RHC: amenable for **dispersion relations**.

$$\begin{aligned} T(s, t, u) &= \sum_{\ell=0}^{\infty} (2\ell + 1) P_\ell(z_s) t_\ell(s) \\ &= \sum_{\ell=0}^{n_s} (2\ell + 1) P_\ell(z_s) f_\ell^{(s)}(s) + \sum_{\ell=0}^{n_t} (2\ell + 1) P_\ell(z_t) f_\ell^{(t)}(t) + \sum_{\ell=0}^{n_u} (2\ell + 1) P_\ell(z_u) f_\ell^{(u)}(u) \end{aligned}$$

- $s$ -channel singularities appear in the  $s$ -channel isobar,  $t_\ell^{(s)}(s)$ .
- Singularities in the  $t$ -,  $u$ -channel are recovered!
- The LHC of the partial waves are given by the RHC of the crossed channel isobars

$$t_\ell(s) = \frac{1}{2} \int dz P_\ell(z) T(s, t', u') = f_\ell^{(s)}(s) + \frac{1}{2} \int dz Q_{\ell\ell'}(s, t') f_{\ell'}^{(t)}(t') .$$

# $\omega \rightarrow 3\pi$ amplitude. Phenomenology

JPAC Collab., EPJ C80, 1107 (2019)

- Amplitude:

$$\mathcal{M}_+(s, t, u) = \frac{\sqrt{\phi(s, t, u)}}{2} F(s, t, u) . \quad (\phi(s, t, u) = 4sp^2(s)q^2(s) \sin^2 \theta_s)$$

- Decay width:  $d^2\Gamma \sim \phi(s, t, u) |F(s, t, u)|^2$
- Dalitz plot parameters  $(\alpha, \beta, \gamma)$  “equivalent” to bins...  $(X, Y) \leftrightarrow (Z, \varphi) \leftrightarrow (s, t, u)$

$$|F(s, t, u)|^2 = |\mathcal{N}|^2 \left( 1 + 2\alpha Z + 2\beta Z^{\frac{3}{2}} \sin 3\varphi + 2\gamma Z^2 + \dots \right)$$

- Why revisit  $\omega \rightarrow 3\pi$ ?

	Bonn (2012)		JPAC (2015)		BESIII (2018)
	w/o KT	w KT	w/o KT	w KT	Exp.
$\alpha$	-0.0 ± 2	0	-0.0 ± 2	0	-0.0 ± 2
$\beta$	$31 \pm 2$	$26 \pm 2$	30	28	$29.5 \pm 8.0 \pm 5.3$

- One (or more) out of three is wrong...
  - i) Experiment?
  - ii) KT eqs., in general?
  - iii) Something particular?

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$$|F(s, t, u)|^2 = |\mathcal{N}|^2 \left( 1 + 2\alpha Z + 2\beta Z^{\frac{3}{2}} \sin 3\varphi + 2\gamma Z^2 + \dots \right)$$

- Why revisit  $\omega \rightarrow 3\pi$ ?

	Bonn (2012)		JPAC (2015)		BESIII (2018)	
	Eur. Phys. J. C72, 2014 (2012)	Phys. Rev. D91, 094029 (2015)	Phys. Rev. D98, 112007 (2018)	Exp.		
	w/o KT	w KT	w/o KT	w KT		
$\alpha$	$130 \pm 5$	$79 \pm 5$	125	84	$120.2 \pm 7.1 \pm 3.8$	
$\beta$	$31 \pm 2$	$26 \pm 2$	30	28	$29.5 \pm 8.0 \pm 5.3$	

- One (or more) out of three is wrong...
  - Experiment?
  - KT eqs., in general?
  - Something particular?

# $\omega \rightarrow 3\pi$ amplitude. Phenomenology

JPAC Collab., EPJ C80, 1107 (20)

- Amplitude:

$$\mathcal{M}_+(s, t, u) = \frac{\sqrt{\phi(s, t, u)}}{2} F(s, t, u) . \quad (\phi(s, t, u) = 4sp^2(s)q^2(s) \sin^2 \theta_s)$$

- Decay width:  $d^2\Gamma \sim \phi(s, t, u) |F(s, t, u)|^2$
- Dalitz plot parameters  $(\alpha, \beta, \gamma)$  “equivalent” to bins...  $(X, Y) \leftrightarrow (Z, \varphi) \leftrightarrow (s, t, u)$

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- One (or more) out of three is wrong...
  - Experiment?
  - KT eqs., in general?
  - Something particular?

# KT equations: DR, subtractions, solutions, and all that...

- PW decomposition:  $F(s, t, u) = \sum_{j \text{ odd}} P'_j(\cos \theta_s) [p(s)q(s)]^{j-1} f_j(s) = f_1(s) + \dots$
- KT/isobar decomposition: consider only  $j = 1$  ( $\rho$ ) isobar,  $F(s)$ :

$$F(s, t, u) = F(s) + F(t) + F(u)$$

- PW projection of the KT decomposition:

$$f_1(s) = F(s) + \hat{F}(s), \quad \hat{F}(s) = \frac{3}{2} \int_{-1}^1 dz_s (1 - z_s^2) F(t(s, z_s))$$

- Discontinuity:

$$\Delta F(s) = \Delta f_1(s) = \rho(s) t_{11}^*(s) f_1(s) = \rho(s) t_{11}^*(s) (F(s) + \hat{F}(s))$$

## Unsubtracted DR

$$F(s) = a F_0(s)$$

$$F_0(s) = \Omega(s) \left[ 1 + \frac{s}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{s'} \frac{\sin \delta(s') \hat{F}_0(s')}{|\Omega(s')|(s' - s)} \right]$$

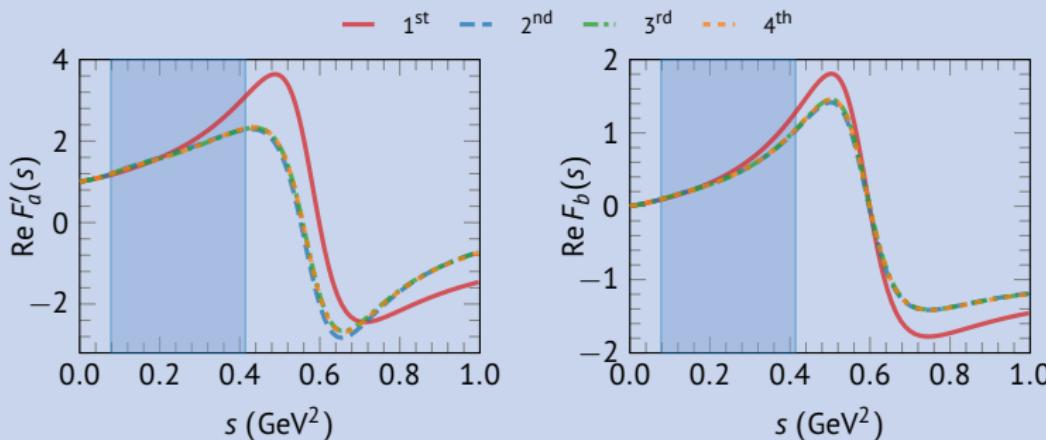
## Once-subtracted DR

$$F(s) = a (F'_a(s) + b F_b(s))$$

$$F'_a(s) = \Omega(s) \left[ 1 + \frac{s^2}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{s'^2} \frac{\sin \delta(s') \hat{F}'_a(s')}{|\Omega(s')|(s' - s)} \right]$$

$$F_b(s) = \Omega(s) \left[ s + \frac{s^2}{\pi} \int_{4m_\pi^2}^\infty \frac{ds'}{s'^2} \frac{\sin \delta(s') \hat{F}_b(s')}{|\Omega(s')|(s' - s)} \right]$$

# KT equations: DR, subtractions, solutions, and all that...



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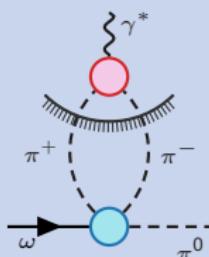
## $\omega \rightarrow \pi^0$ transition form factor

- The decays  $\omega(\rightarrow \pi^0 \gamma^*) \rightarrow \pi^0 l^+ l^-$  and  $\omega \rightarrow \pi^0 \gamma$  governed by the TFF  $f_{\omega\pi^0}(s)$ .

$$\mathcal{M}(\omega \rightarrow \pi^0 \ell^+ \ell^-) = f_{\omega\pi^0}(s) \epsilon_{\mu\nu\alpha\beta} \epsilon^\mu(p_\omega, \lambda) p^\nu q^\alpha \frac{ie^2}{s} \bar{u}(p_-) \gamma^\beta v(p_+) ,$$

$$\Gamma(\omega \rightarrow \pi^0 \gamma) = |f_{\omega\pi^0}(0)|^2 \frac{e^2 (m_\omega^2 - m_{\pi^0}^2)^3}{96\pi m_\omega^3} ,$$

- Dispersive representation:



$$f_{\omega\pi^0}(s) = f_{\omega\pi^0}(0) + \frac{s}{12\pi^2} \int_{4m_\pi^2}^{\infty} ds' \frac{q_\pi(s')^3}{s'^{\frac{3}{2}}(s'-s)} (F(s') + \hat{F}(s')) F_\pi^V(s')^*$$

- $f_{\omega\pi^0}(0) = |f_{\omega\pi^0}(0)| e^{i\phi_{\omega\pi^0}(0)}$
- Experimental information:  $F_{\omega\pi^0}(s) = \frac{f_{\omega\pi^0}(s)}{f_{\omega\pi^0}(0)}$
- Only the relative phase  $\frac{a}{f_{\omega\pi^0}(0)} = \frac{|a|}{|f_{\omega\pi^0}(0)|} e^{i(\phi_{\omega\pi^0}(0) - \phi_a)} .$

# Summary of amplitudes/free parameters/exp. input

## $\omega \rightarrow 3\pi$ amplitude $[F(s, t, u)]$

Free parameters:  $|a|, |b|, \phi_b$

Experimental input:

- $\Gamma_{3\pi}$
- Dalitz plot parameters

## $\omega \rightarrow \gamma^{(*)} \pi^0$ TFF $[f_{\omega\pi^0}(s)]$

Free parameters:  $|f_{\omega\pi^0}(0)|, \phi_{\omega\pi^0}(0)$   
 $(\oplus |a|, |b|, \phi_b)$

Experimental input:

- $\Gamma_{\gamma\pi^0}$
- $|F_{\omega\pi^0}(s)|^2$

# First analysis in three steps

JPAC Collab., EPJ C80, 1107 (20)

① Fix  $|b| \simeq 2.9$ ,  $\phi_b \simeq 1.9$  with the DP parameters.

② Fix  $|a| \simeq 280 \text{ GeV}^{-3}$ ,  $|f_{\omega\pi^0}(0)| \simeq 2.3 \text{ GeV}^{-1}$  from  $\Gamma_{\omega \rightarrow 3\pi}, \Gamma_{\omega \rightarrow \gamma\pi}$ .

③ You are left with  $\phi_{\omega\pi^0}(0)$  and the TFF Data.

$$\textcircled{1} \quad \chi_{\text{DP}}^2 = \left( \frac{\alpha(t) - \alpha(e)}{\sigma_\alpha} \right)^2 + \dots$$

$$\textcircled{2} \quad \chi_\Gamma^2 = \left( \frac{\Gamma_{3\pi}^{(t)} - \Gamma_{3\pi}^{(e)}}{\sigma_{\Gamma_{3\pi}}} \right)^2 + \left( \frac{\Gamma_{\gamma\pi}^{(t)} - \Gamma_{\gamma\pi}^{(e)}}{\sigma_{\Gamma_{\gamma\pi}}} \right)^2$$

$$\textcircled{3} \quad \chi_{\text{A2,NA60}}^2 = \sum_i \left( \frac{|F_{\omega\pi}(s_i)|^2 - |F_{\omega\pi}^{(i)}|^2}{\sigma_{F_{\omega\pi}^{(i)}}} \right)^2$$



- Two different minima (low and high  $\phi_{\omega\pi^0}(0)$ ) are found.
- Both have similar  $\chi^2$  of the TFF.

Make a **global, simultaneous**  
analysis

$$\chi^2 = N \left( \frac{\chi_{\text{DP}}^2}{N_{\text{DP}}} + \frac{\chi_\Gamma^2}{N_\Gamma} + \frac{\chi_{\text{NA60}}^2}{N_{\text{NA60}}} + \frac{\chi_{\text{A2}}^2}{N_{\text{A2}}} \right)$$

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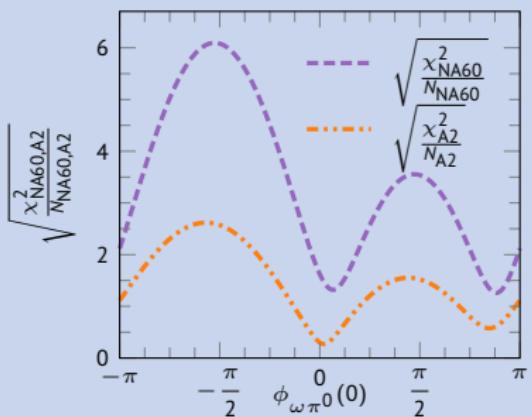
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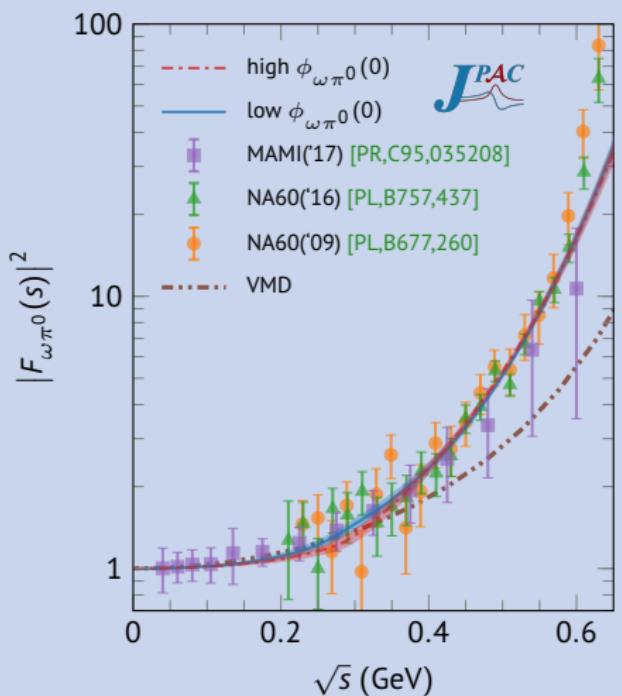
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# Results

JPAC Collab., EPJ C80, 1107 (20)



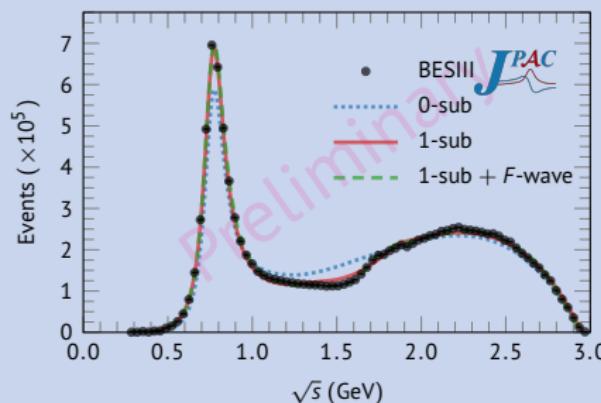
	$\alpha$	$\beta$	$\gamma$
BESIII	111(18)	25(10)	22(29)
low	112(15)	23(6)	29(6)
high	109(14)	26(6)	19(5)

Using once-subtracted DR for KT:

- Agreement is restored with DP parameters by BESIII
- One can also describe the  $\omega\pi^0$  TFF

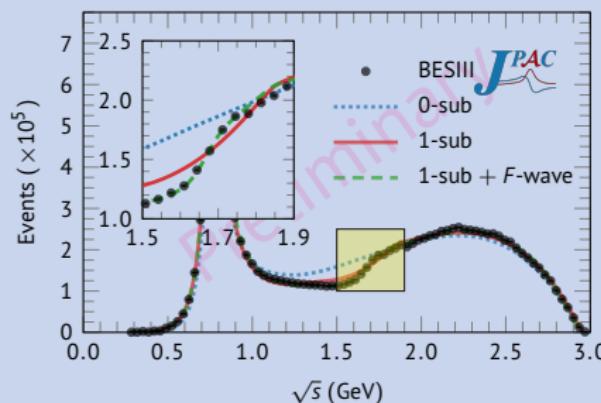
# $J/\psi \rightarrow 3\pi$ decays

- Completely analogous formalism ( $V$ )
- BESIII data [PLB710(12)]
- The decay is dominated by  $\rho$ , even if there is a larger phase space
- **0-sub** (prediction) get the basic features
- **1-sub** (fit) improves the description
- **1-sub + F-wave** [ $\rho_3(1690)$ ] describes better the movements above  $\gtrsim 1.5$  GeV.



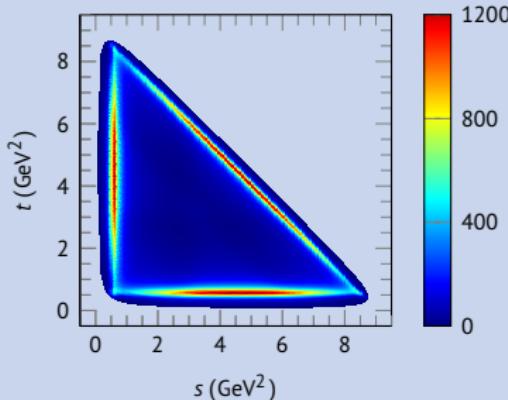
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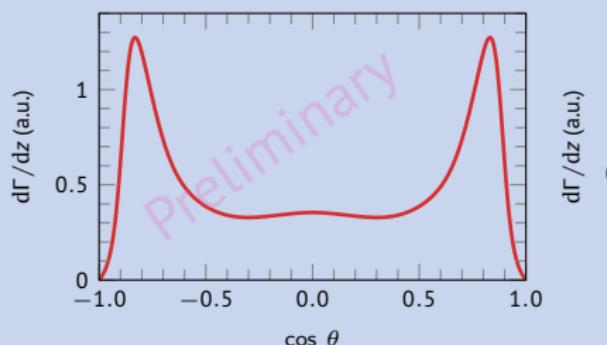


# $J/\psi \rightarrow 3\pi$ decays

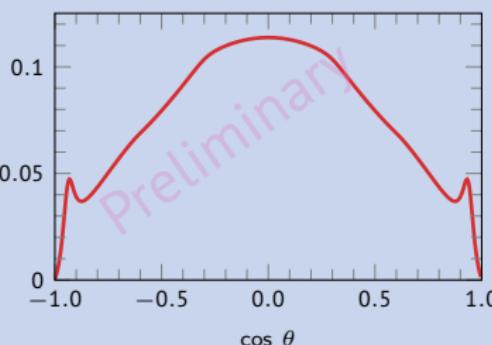
- Dalitz plot distribution similar to exp. one
- More statistics will allow to unveil other effects (resonances, interferences,...)
- Predictions can be done for angular [ $z = \cos \theta_s$ ] distributions, specially restricted to  $\rho$ -mass region.



Full  $\sqrt{s}$  range



$|\sqrt{s} - m_\rho| \leq 50$  MeV



# Summary

- JPAC very active in several hadron physics topics

- XYZ photoproduction

JPAC Collab., PRD102,114010(20)

JPAC Collab., 2209.05882 (accepted PRD)

- **Photoproduction of XYZ** offers the opportunity of investigating these enigmatic states in a new, perhaps cleaner, way.
- Exclusive photoproduction studied with quite **general formalisms** for both for low (fixed-spin) and high (reggeized)  $\gamma N$  energy
- Vertices extracted as much as possible from known experimental information and phenomenology.
- Inclusive reactions improves perspective (role of  $\Delta$ )
- Code can be found at <https://github.com/dwinney/jpacPhoto>

- KT equations and  $V \rightarrow 3\pi$ :

- KT equations are a powerful tool to study **3-body decays**
- They allow to implement **two-body unitarity** in all the **three channels** ( $s, t, u$ ).
- For  $\omega \rightarrow 3\pi$  decays:
  - Using once-subtracted DRs, we are able to reproduce the  $\omega \rightarrow 3\pi$  DP parameters,
  - and the  $\omega \rightarrow \pi^0 \gamma^*$  transition form factor data.
- For  $J/\psi \rightarrow 3\pi$  decays, good agreement with the data is found assuming elastic ( $P$ - and  $F$ -waves).

JPAC Collab., EPJ,C80,1107(20)

# Recent results from JPAC collaboration



Miguel Albaladejo (IFIC)

Recent results and perspectives in hadron physics  
(Institute Pascal, Orsay, Oct. 17th, 2022)

