(Un)conventional mesons between 1-2 GeV and the lightest hybrid nonet

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Outline

Symmetries of QCD

Conventional and unconventional mesons

From J=3 downwards

J=3: a well-established nonet

J=2: from ideal tensor to unknown axial-tensor mesons

J=2: pseudotensor mesons: an open question about them

J=1: excited vector mesons: an open question

J=1: toward a nonet of hybrid states?

(J=0: A new entry: the glueballonium)

Conclusions

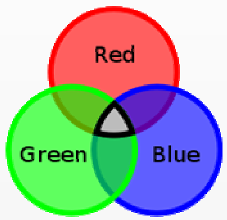
Symmetries of QCD

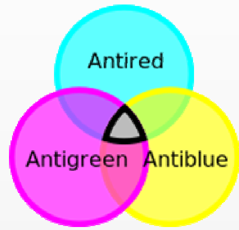
	Giuseppe Lodovico Lagrangia
Born	25 January 1736
	Turin
Died	10 April 1813 (aged 77)
	Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



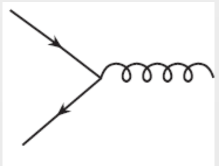


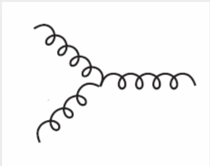
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s ,...$$

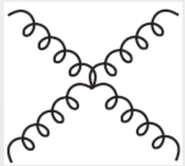
8 type of gluons ($R\bar{G}$,$B\bar{G}$,…)

$$A_\mu^a \; ; \; a = 1,...,8$$

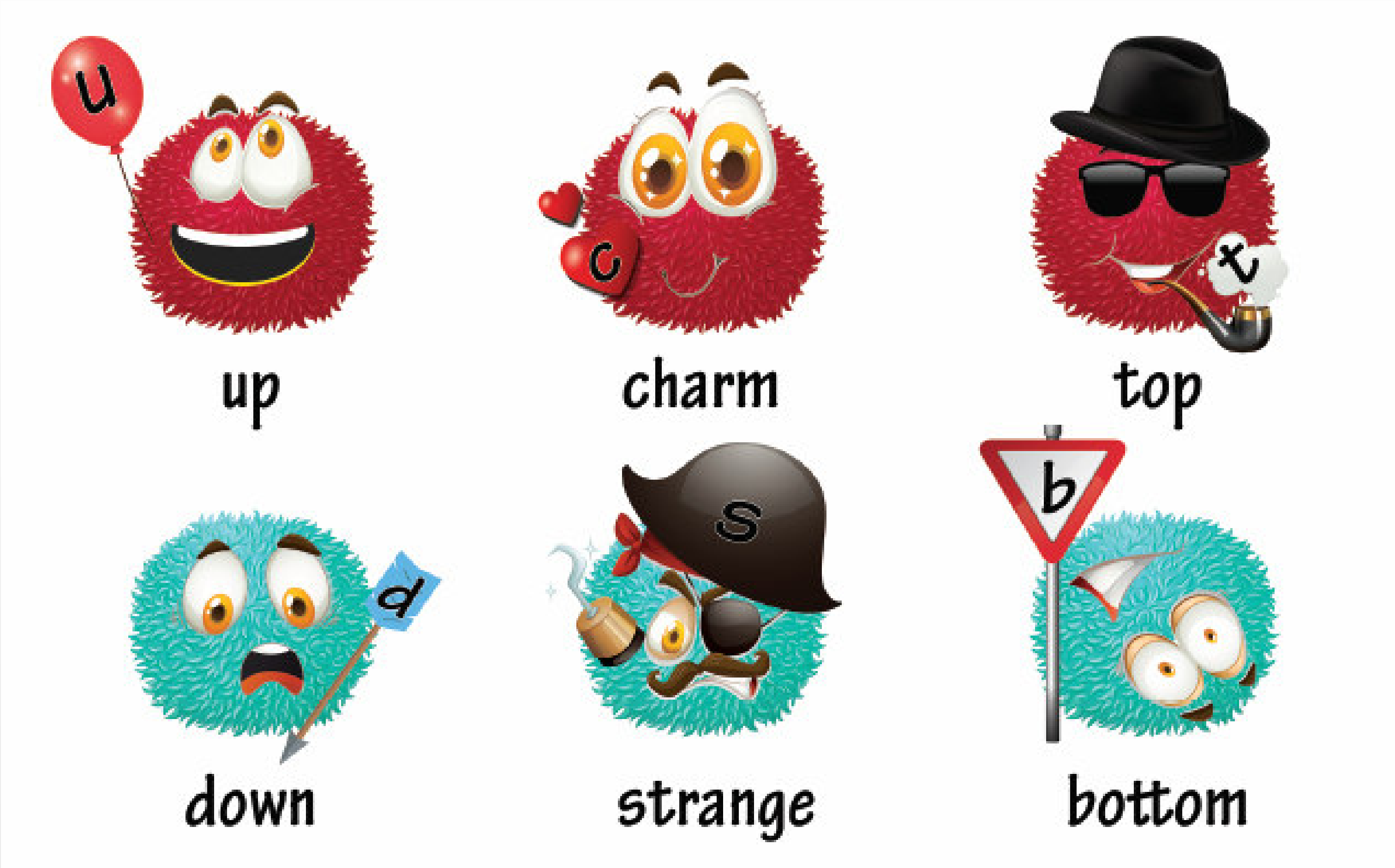
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$







Confinement: quarks never ‘seen’ directly. How they might look like ☺



Picture by Pawel Piotrowski

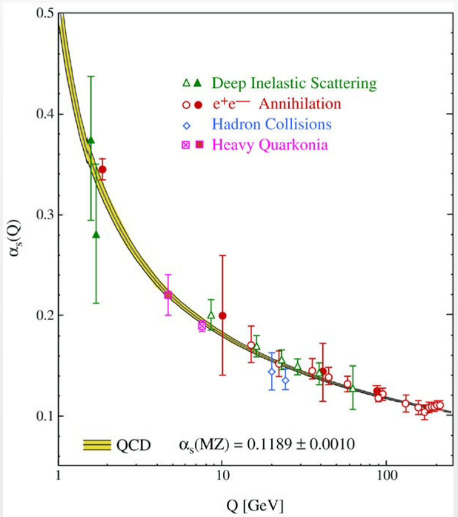
Trace anomaly:
the emergence of a dimension

Chiral limit: $m_{.} = 0$

$x^{\mu} \rightarrow x'^{\mu} = \lambda^{-1} x^{\mu}$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250 \text{ MeV}$

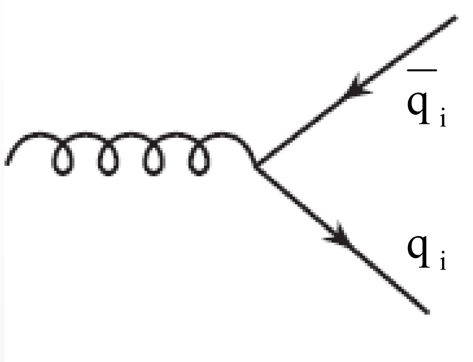
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \rightarrow m^{*}_{gluon} \approx 500 - 800 \text{ MeV}$

Gluon condensate: $\left\langle G^{a}_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



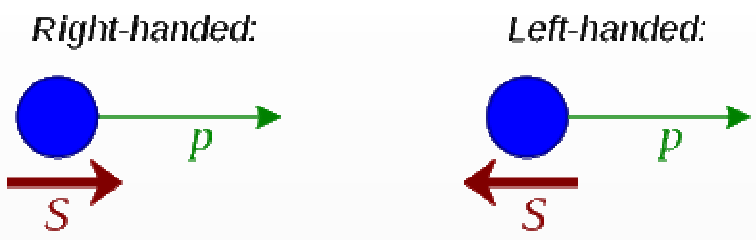
Gluon-quark-antiquark vertex

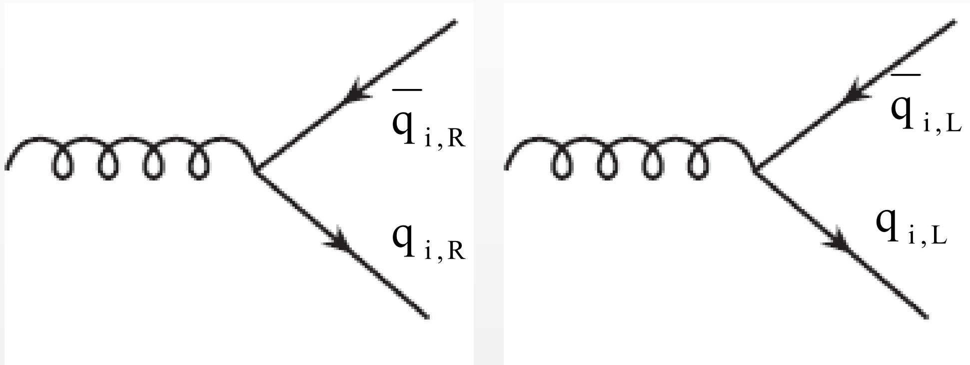
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \to U_{ij} q_j$$

$$U \in U(3)_V \to U^+ U = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

Chiral (or axial) anomaly: explicitely broken by quantum fluctuations

$$\partial^\mu (\bar{q}^i \gamma_\mu \gamma_5 q^i) = \frac{3g^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \mathrm{tr}(G_{\mu\nu} G_{\rho\sigma})$$

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**scale anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**chiral anomaly**)

Conventional mesons: quark-antiquark states

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are “white” and are called hadrons.

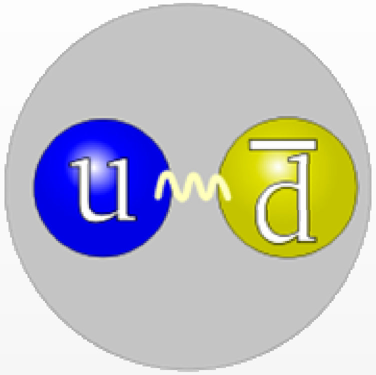
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Example of conventional quark-antiquark states: the ρ and the π mesons



$$|\rho^+\rangle \propto |u\bar{d}\rangle + \frac{1}{N_c}\left(|\pi^+\pi^0\rangle + ...\right)$$

where

$$|u\bar{d}\rangle = |\text{valence } u + \text{valence } \bar{d} + \text{gluons}\rangle$$

Rho-meson $m_{\rho^+} = 775 \ \text{MeV}$

Pion $m_{\pi^+} = 139 \ \text{MeV}$

$$m_u + m_d \approx 7 \ \text{MeV}$$

Mass generation in QCD is a nonpert. penomenon based on SSB
(mentioned previusly).

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	$\theta_{\rm quad}$ [°]	$\theta_{\rm lin}$ [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	−11.3	−24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	K_{1B} †	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	K_{1A} †	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)$ †	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Quark-antiquark mesons (PDG 2018)

$n\,^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f	θ_{quad} [°]	θ_{lin} [°]
$1\,^1S_0$	0^{-+}	π	K	η	$\eta'(958)$	-11.3	-24.5
$1\,^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$	39.2	36.5
$1\,^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^\dagger$	$h_1(1380)$	$h_1(1170)$		
$1\,^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$		
$1\,^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^\dagger$	$f_1(1420)$	$f_1(1285)$		
$1\,^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$	29.6	28.0
$1\,^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^\dagger$	$\eta_2(1870)$	$\eta_2(1645)$		
$1\,^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$		
$1\,^3D_2$	2^{--}		$K_2(1820)$				
$1\,^3D_3$	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\phi_3(1850)$	$\omega_3(1670)$	31.8	30.8
$1\,^3F_4$	4^{++}	$a_4(2040)$	$K_4^*(2045)$		$f_4(2050)$		
$1\,^3G_5$	5^{--}	$\rho_5(2350)$	$K_5^*(2380)$				
$1\,^3H_6$	6^{++}	$a_6(2450)$			$f_6(2510)$		
$2\,^1S_0$	0^{-+}	$\pi(1300)$	$K(1460)$	$\eta(1475)$	$\eta(1295)$		
$2\,^3S_1$	1^{--}	$\rho(1450)$	$K^*(1410)$	$\phi(1680)$	$\omega(1420)$		
$3\,^1S_0$	0^{-+}	$\pi(1800)$			$\eta(1760)$		

Some selected nonets

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Chiral partners

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

TABLE I. Chiral multiplets, their currents, and transformations up to $J = 3$. [* and/or $f_0(1500)$; **a mix of.] The first two columns correspond to the assignment suggested in the Quark Model review of the PDG [8], to which we refer for further details and references (see also the discussion in the text).

J^{PC}, $^{2S+1}L_J$	$I=1(\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}})$; $I=1(-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d)$; $I=0(\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**}$	Microscopic currents	Chiral multiplet	Transformation under $SU(3)_\mathrm{L} \times SU(3)_\mathrm{R} \times \times U(1)_\mathrm{A}$
0^{-+}, $^1S_0$	π; K; $\eta, \eta'(958)$	$P^{ij} = \frac{1}{2}\bar{q}^j \mathrm{i}\gamma^5 q^i$	$\Phi = S + \mathrm{i}P$ $(\Phi^{ij} = \bar{q}^j_\mathrm{R} q^i_\mathrm{L})$	$\Phi \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi U^\dagger_\mathrm{R}$
0^{++}, $^3P_0$	$a_0(1450)$; $K_0^*(1430)$; $f_0(1370), f_0(1710)^*$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		
1^{--}, $^1S_1$	$\rho(770)$; $K^*(892)$; $\omega(782), \phi(1020)$	$V^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L^{ij}_\mu = \bar{q}^j_\mathrm{L} \gamma_\mu q^i_\mathrm{L})$	$L_\mu \to U_\mathrm{L} L_\mu U^\dagger_\mathrm{L}$
1^{++}, $^3P_1$	$a_1(1260)$; $K_{1,A}$; $f_1(1285), f_1(1420)$	$A^{ij}_\mu = \frac{1}{2}\bar{q}^j \gamma^5 \gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R^{ij}_\mu = \bar{q}^j_\mathrm{R} \gamma_\mu q^i_\mathrm{R})$	$R_\mu \to U_\mathrm{R} R_\mu U^\dagger_\mathrm{R}$
1^{+-}, $^1P_1$	$b_1(1235)$; $K_{1,B}$; $h_1(1170), h_1(1380)$	$P^{ij}_\mu = -\frac{1}{2}\bar{q}^j \gamma^5 \overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi^{ij}_\mu = \bar{q}^j_\mathrm{R} \mathrm{i}\overleftrightarrow{D}_\mu q^i_\mathrm{L})$	$\Phi_\mu \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi_\mu U^\dagger_\mathrm{R}$
1^{--}, $^3D_1$	$\rho(1700)$; $K^*(1680)$; $\omega(1650), \phi(?)$	$S^{ij}_\mu = \frac{1}{2}\bar{q}^j \mathrm{i}\overleftrightarrow{D}_\mu q^i$		
2^{++}, $^3P_2$	$a_2(1320)$; $K_2^*(1430)$; $f_2(1270), f_2'(1525)$	$V^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\mu + \cdots) q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{L} (\gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i_\mathrm{L})$	$L_{\mu\nu} \to U_\mathrm{L} L_{\mu\nu} U^\dagger_\mathrm{L}$
2^{--}, $^3D_2$	$\rho_2(?)$; $K_2(1820)$; $\omega_2(?), \phi_2(?)$	$A^{ij}_{\mu\nu} = \frac{1}{2}\bar{q}^j (\gamma^5 \gamma_\mu \mathrm{i}\overleftrightarrow{D}_\nu + \cdots) q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R} (\gamma_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_\mathrm{R})$	$R_{\mu\nu} \to U_\mathrm{R} R_{\mu\nu} U^\dagger_\mathrm{R}$
2^{-+}, $^1D_2$	$\pi_2(1670)$; $K_2(1770)$; $\eta_2(1645), \eta_2(1870)$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j (\mathrm{i}\gamma^5 \overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_\mathrm{R} (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i_\mathrm{L})$	$\Phi_{\mu\nu} \to \mathrm{e}^{-2\mathrm{i}\alpha} U_\mathrm{L} \Phi_{\mu\nu} U^\dagger_\mathrm{R}$
2^{++}, $^3F_2$	$a_2(?)$; $K_2^*(?)$; $f_2(?), f_2'(?)$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j (\overleftrightarrow{D}_\mu \overleftrightarrow{D}_\nu + \cdots) q^i$		
3^{--}, $^3D_3$	$\rho_3(1690)$; $K_3^*(1780)$; $\omega_3(1670), \phi_3(1850)$	$\vdots$	$\vdots$	$\vdots$

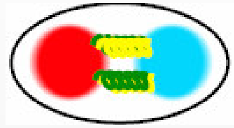
Uniwersytet Jana Kochanowskiego w Kielcach

Table from:

F.G., R. Pisarski, A. Koenigstein Phys.Rev.D 97 (2018) 9, 091901 e-Print: 1709.07454

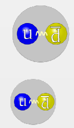
Non-conventional mesons: beyond qq

1) Glueballs 

2) Hybrids 

3) Four-quark states

Compact diquark-antidiquark states 

Molecular states (a type of dynamical generation) 

Companion poles (another type of dynamical generation)

Models for conventional mesons: from J=3 downwards

Strategy

- For a given nonet , write down the corresponding model-Lagrangian respecting flavor (or if possible chiral) symmetry.
- Consider only C, P, invariant terms
- Calculate decays in all possible channels (first at tree-level, in some selected case including finite width or loop effects;
- Fit free parameters to known experimental value;
- Make postdictions and predictions.

Mesons with J=3

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Phenomenology of $J^{PC} = 3^{--}$ tensor mesons

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We study the strong and radiative decays of the antiquark-quark ground state $J^{PC} = 3^{--}$ ($n^{2S+1}L_J = 1^3D_3$) nonet $\{\rho_3(1690), K_3^*(1780), \phi_3(1850), \omega_3(1670)\}$ in the framework of an effective quantum field theory approach, based on the $SU_V(3)$-flavor symmetry. The effective model is fitted to experimental data listed by the Particle Data Group. We predict numerous experimentally unknown decay widths and branching ratios. An overall agreement of theory (fit and predictions) with experimental data confirms the $\bar{q}q$ nature of the states and qualitatively validates the effective approach. Naturally, experimental clarification as well as advanced theoretical description is needed for trustworthy quantitative predictions, which is observed from some of the decay channels. Besides conventional spin-3 mesons, theoretical predictions for ratios of strong and radiative decays of a hypothetical glueball state $G_3(4200)$ with $J^{PC} = 3^{--}$ are also presented.

Decays of J=3-mesons

TABLE III. Effective relativistic interaction terms describing the strong decays of mesons with $J^{PC} = 3^{--}$.

Decay mode	Interaction Lagrangians
$3^{--} \to 0^{-+} + 0^{-+}$	$\mathcal{L}_{w_3pp} = g_{w_3pp}\mathrm{tr}[W_3^{\mu\nu\rho}[P, (\partial_\mu\partial_\nu\partial_\rho P)]_-]$
$3^{--} \to 0^{-+} + 1^{--}$	$\mathcal{L}_{w_3v_1p} = g_{w_3v_1p}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}[W_{3,\mu\alpha\beta}\{(\partial_\nu V_{1,\rho}), (\partial^\alpha\partial^\beta\partial_\sigma P)\}_+]$
$3^{--} \to 0^{-+} + 2^{++}$	$\mathcal{L}_{w_3a_2p} = g_{w_3a_2p}\varepsilon_{\mu\nu\rho\sigma}\mathrm{tr}[W_3{}^\mu{}_{\alpha\beta}[(\partial^\nu A_2^{\rho\alpha}), (\partial^\sigma\partial^\beta P)]_-]$
$3^{--} \to 0^{-+} + 1^{+-}$	$\mathcal{L}_{w_3b_1p} = g_{w_3b_1p}\mathrm{tr}[W_3^{\mu\nu\rho}\{B_{1,\mu}, (\partial_\nu\partial_\rho P)\}_+]$
$3^{--} \to 0^{-+} + 1^{++}$	$\mathcal{L}_{w_3a_1p} = g_{w_3a_1p}\mathrm{tr}[W_3^{\mu\nu\rho}[A_{1,\mu}, (\partial_\nu\partial_\rho P)]_-]$
$3^{--} \to 1^{--} + 1^{--}$	$\mathcal{L}_{w_3v_1v_1} = g_{w_3v_1v_1}\mathrm{tr}[W_3^{\mu\nu\rho}[(\partial_\mu V_{1,\nu}), V_{1,\rho}]_-]$

$$W_3^{\mu\nu\rho} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{3,N}^{\mu\nu\rho}+\rho_3^{0\mu\nu\rho}}{\sqrt{2}} & \rho_3^{+\mu\nu\rho} & K_3^{+\mu\nu\rho} \\ \rho_3^{-\mu\nu\rho} & \frac{\omega_{3,N}^{\mu\nu\rho}-\rho_3^{0\mu\nu\rho}}{\sqrt{2}} & K_3^{0\mu\nu\rho} \\ K_3^{-\mu\nu\rho} & \bar{K}_3^{0\mu\nu\rho} & \omega_{3,S}^{\mu\nu\rho} \end{pmatrix}$$

$$P = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

$$V_1^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{1,N}^\mu+\rho_1^{0\mu}}{\sqrt{2}} & \rho_1^{+\mu} & K_1^{*+\mu} \\ \rho_1^{-\mu} & \frac{\omega_{1,N}^\mu-\rho_1^{0\mu}}{\sqrt{2}} & K_1^{*0\mu} \\ K_1^{*-\mu} & \bar{K}_1^{*0\mu} & \omega_{1,S}^\mu \end{pmatrix}$$

TABLE IV. Decay amplitudes for different decay modes.

Decay mode	$\frac{1}{7}\lvert\mathcal{M}\rvert^2$
$3^{--} \to 0^{-+} + 0^{-+}$	$g_{w_3pp}^2 \frac{2}{35}\lvert\vec{k}_{p^{(1)},p^{(2)}}\rvert^6$
$3^{--} \to 0^{-+} + 1^{--}$	$g_{w_3v_1p}^2 \frac{8}{105}\lvert\vec{k}_{v_1,p}\rvert^6 m_{w_3}^2$
$3^{--} \to 0^{-+} + 2^{++}$	$g_{w_3a_2p}^2 \frac{2}{105}\lvert\vec{k}_{a_2,p}\rvert^4 \frac{m_{w_3}^2}{m_{a_2}^2}(2\lvert\vec{k}_{a_2,p}\rvert^2 + 7m_{a_2}^2)$
$3^{--} \to 0^{-+} + 1^{+-}$	$g_{w_3b_1p}^2 \frac{2}{105}\lvert\vec{k}_{b_1,p}\rvert^4(7 + 3\frac{\lvert\vec{k}_{b_1,p}\rvert^2}{m_{b_1}^2})$
$3^{--} \to 0^{-+} + 1^{++}$	$g_{w_3a_1p}^2 \frac{2}{105}\lvert\vec{k}_{a_1,p}\rvert^4(7 + 3\frac{\lvert\vec{k}_{a_1,p}\rvert^2}{m_{a_1}^2})$
$3^{--} \to 1^{--} + 1^{--}$	$g_{w_3v_1v_1}^2 \frac{1}{105}(m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2)^{-1}\lvert\vec{k}_{v_1^{(1)},v_2^{(2)}}\rvert^2[6\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^4 + 35m_{v_1^{(1)}}^2 m_{v_1^{(2)}}^2 + 14\lvert\vec{k}_{v_1^{(1)},v_1^{(2)}}\rvert^2(m_{v_1^{(1)}}^2 + m_{v_1^{(2)}}^2)]$

Results (post- and predictions)

TABLE V. Decays of $J^{PC} = 3^{--}$ mesons into two pseudoscalars. Experimental data is taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \pi\pi$	32.7 ± 2.3	38.0 ± 3.2
$\rho_3(1690) \to \bar{K}K$	4.0 ± 0.3	2.54 ± 0.45
$K_3^*(1780) \to \pi\bar{K}$	18.5 ± 1.3	29.9 ± 4.3
$K_3^*(1780) \to \bar{K}\eta$	7.4 ± 0.5	48 ± 22
$K_3^*(1780) \to \bar{K}\eta'(958)$	0.021 ± 0.001	
$\omega_3(1670) \to \bar{K}K$	3.0 ± 0.2	
$\phi_3(1850) \to \bar{K}K$	18.8 ± 1.3	Seen

TABLE VI. Decays of $J^{PC} = 3^{--}$ mesons into a pseudoscalar-vector pair. Experimental data taken from Ref. [1].

Decay process	Theory Γ/MeV	Experiment Γ/MeV
$\rho_3(1690) \to \rho(770)\eta$	3.8 ± 0.8	Seen
$\rho_3(1690) \to \bar{K}^*(892)K$	3.4 ± 0.7	
$\rho_3(1690) \to \omega(782)\pi$	35.8 ± 7.4	25.8 ± 9.8
$\rho_3(1690) \to \phi(1020)\pi$	0.036 ± 0.007	
$K_3^*(1780) \to \rho(770)K$	16.8 ± 3.5	49.3 ± 15.7
$K_3^*(1780) \to \bar{K}^*(892)\pi$	27.2 ± 5.6	31.8 ± 9.0
$K_3^*(1780) \to \bar{K}^*(892)\eta$	0.09 ± 0.02	
$K_3^*(1780) \to \omega(782)\bar{K}$	4.3 ± 0.9	
$K_3^*(1780) \to \phi(1020)\bar{K}$	1.2 ± 0.3	
$\omega_3(1670) \to \rho(770)\pi$	97 ± 20	Seen
$\omega_3(1670) \to \bar{K}^*(892)K$	2.9 ± 0.6	
$\omega_3(1670) \to \omega(782)\eta$	2.8 ± 0.6	
$\omega_3(1670) \to \phi(1020)\eta$	$(7.6 \pm 1.6) \times 10^{-6}$	
$\phi_3(1850) \to \rho(770)\pi$	1.1 ± 0.2	
$\phi_3(1850) \to \bar{K}^*(892)K$	35.5 ± 7.3	Seen
$\phi_3(1850) \to \omega(782)\eta$	0.015 ± 0.003	
$\phi_3(1850) \to \omega(782)\eta'(958)$	0.003 ± 0.001	
$\phi_3(1850) \to \phi(1020)\eta$	3.8 ± 0.8	

TABLE VII. Theoretical predictions for the radiative decays $W_3 \to \gamma P$.

Decay process	Theory Γ/keV
$\rho_3^{\pm/0}(1690) \to \gamma\pi^{\pm/0}$	69 ± 14
$\rho_3^0(1690) \to \gamma\eta$	157 ± 32
$\rho_3^0(1690) \to \gamma\eta'(958)$	20 ± 4
$K_3^{\pm}(1780) \to \gamma K^{\pm}$	58 ± 12
$K_3^0(1780) \to \gamma K^0$	231 ± 48
$\omega_3(1670) \to \gamma\pi^0$	560 ± 120
$\omega_3(1670) \to \gamma\eta$	19 ± 4
$\omega_3(1670) \to \gamma\eta'(958)$	1.4 ± 0.3
$\phi_3(1850) \to \gamma\pi^0$	4 ± 1
$\phi_3(1850) \to \gamma\eta$	129 ± 26
$\phi_3(1850) \to \gamma\eta'(958)$	35 ± 7

Isoscalar mixing is small

$$\begin{pmatrix} \omega_3(1670) \\ \phi_3(1850) \end{pmatrix} = \begin{pmatrix} \cos\beta_{w_3} & \sin\beta_{w_3} \\ -\sin\beta_{w_3} & \cos\beta_{w_3} \end{pmatrix} \begin{pmatrix} \omega_{3,N} \\ \omega_{3,S} \end{pmatrix}$$

$$\beta_{w_3} = 3.5^\circ$$

Tensor and (axial-)tensors

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Based on:

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From well-known tensor mesons to yet unknown axial-tensor mesons

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While the ground-state tensor ($J^{PC} = 2^{++}$) mesons $a_2(1320)$, $K_2^*(1430)$, $f_2(1270)$, and $f_2'(1525)$ are well known experimentally and form an almost ideal nonet of quark-antiquark states, their chiral partners, the ground-states axial-tensor ($J^{PC} = 2^{--}$) mesons are poorly settled: only the kaonic member $K_2(1820)$ of the nonet has been experimentally found, whereas the isovector state ρ_2 and two isoscalar states ω_2 and ϕ_2 are still missing. Here, we study masses, strong, and radiative decays of tensor and axial-tensor mesons within a chiral model that links them: the established tensor mesons are used to test the model and to determine its parameters, and subsequently various predictions for their chiral partners, the axial-tensor mesons, are obtained. The results are compared to current lattice QCD outcomes as well as to other theoretical approaches and show that the ground-state axial-tensor mesons are expected to be quite broad, the vector-pseudoscalar mode being the most prominent decay mode followed by the tensor-pseudoscalar one. Nonetheless, their experimental finding seems to be possible in ongoing and/or future experiments.

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Building the Lagrangian

$2^{++},\ {}^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\mu + \cdots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_L^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q_L^i)$	$L_{\mu\nu} \to U_L L_{\mu\nu} U_L^\dagger$
$2^{--},\ {}^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \cdots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_R^j(\gamma_\mu \overleftrightarrow{D}_\nu + \cdots)q_R^i)$	$R_{\mu\nu} \to U_R R_{\mu\nu} U_R^\dagger$

$$\mathcal{L}_{g_2^{\text{ten}}} = \frac{g_2^{\text{ten}}}{2}\left(\text{Tr}\left[\mathbf{L}_{\mu\nu}\{L^\mu, L^\nu\}\right] + \text{Tr}\left[\mathbf{R}_{\mu\nu}\{R^\mu, R^\nu\}\right]\right)$$

$$2^{++} \longrightarrow 0^{-+} + 0^{-+}\ ;$$
$$2^{--} \longrightarrow 0^{-+} + 1^{--}\ .$$

Also in this case: small isoscalar mixing angle

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\beta_T & \sin\beta_T \\ -\sin\beta_T & \cos\beta_T \end{pmatrix} \begin{pmatrix} f_{2,N} \\ f_{2,S} \end{pmatrix}$$

$$\beta_T = (3.16 \pm 0.81)^\circ$$

Postdictions (left) predictions (right)

Decay process (in model)	eLSM (MeV)	PDG (MeV)
$a_2(1320) \longrightarrow \bar{K}\,K$	4.06 ± 0.14	$7.0^{+2.0}_{-1.5} \leftrightarrow (4.9 \pm 0.8)\%$
$a_2(1320) \longrightarrow \pi\,\eta$	25.37 ± 0.87	$18.5 \pm 3.0 \leftrightarrow (14.5 \pm 1.2)\%$
$a_2(1320) \longrightarrow \pi\,\eta'(958)$	1.01 ± 0.03	$0.58 \pm 0.10 \leftrightarrow (0.55 \pm 0.09)\%$
$K_2^*(1430) \longrightarrow \pi\,\bar{K}$	44.82 ± 1.54	$49.9 \pm 1.9 \leftrightarrow (49.9 \pm 0.6)\%$
$f_2(1270) \longrightarrow \bar{K}\,K$	3.54 ± 0.29	$8.5 \pm 0.8 \leftrightarrow (4.6^{+0.5}_{-0.4})\%$
$f_2(1270) \longrightarrow \pi\,\pi$	168.82 ± 3.89	$157.2^{+4.0}_{-1.1} \leftrightarrow (84.2^{+2.9}_{-0.9})\%$
$f_2(1270) \longrightarrow \eta\,\eta$	0.67 ± 0.03	$0.75 \pm 0.14 \leftrightarrow (0.4 \pm 0.08)\%$
$f_2'(1525) \longrightarrow \bar{K}\,K$	23.72 ± 0.60	$75 \pm 4 \leftrightarrow (87.6 \pm 2.2)\%$
$f_2'(1525) \longrightarrow \pi\,\pi$	0.67 ± 0.14	$0.71 \pm 0.14 \leftrightarrow (0.83 \pm 0.16)\%$
$f_2'(1525) \longrightarrow \eta\,\eta$	1.81 ± 0.05	$9.9 \pm 1.9 \leftrightarrow (11.6 \pm 2.2)\%$

Decay process (in model)	eLSM (MeV)	PDG-2020 (MeV)
$a_2(1320) \longrightarrow \rho(770)\,\pi$	71.0 ± 2.6	$73.61 \pm 3.35 \leftrightarrow (70.1 \pm 2.7)\%$
$K_2^*(1430) \longrightarrow \bar{K}^*(892)\,\pi$	27.9 ± 1.0	$26.92 \pm 2.14 \leftrightarrow (24.7 \pm 1.6)\%$
$K_2^*(1430) \longrightarrow \rho(770)\,K$	10.3 ± 0.4	$9.48 \pm 0.97 \leftrightarrow (8.7 \pm 0.8)\%$
$K_2^*(1430) \longrightarrow \omega(782)\,\bar{K}$	3.5 ± 0.1	$3.16 \pm 0.88 \leftrightarrow (2.9 \pm 0.8)\%$
$f_2'(1525) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	19.89 ± 0.73	

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow \rho(770)\,\eta$	$\approx 99 \pm 50$
$\rho_2(?) \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\rho_2(?) \longrightarrow \omega(782)\,\pi$	$\approx 419 \pm 210$
$\rho_2(?) \longrightarrow \phi(1020)\,\pi$	≈ 0.8
$K_{2,A} \longrightarrow \rho(770)\,K$	$\approx 195 \pm 98$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\pi$	$\approx 316 \pm 158$
$K_{2,A} \longrightarrow \bar{K}^*(892)\,\eta$	≈ 0.01
$K_{2,A} \longrightarrow \omega(782)\,\bar{K}$	$\approx 51 \pm 26$
$K_{2,A} \longrightarrow \phi(1020)\,\bar{K}$	$\approx 50 \pm 25$
$\omega_{2,N} \longrightarrow \rho(770)\,\pi$	$\approx 1314 \pm 657$
$\omega_{2,N} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 85 \pm 43$
$\omega_{2,N} \longrightarrow \omega(782)\,\eta$	$\approx 93 \pm 47$
$\omega_{2,N} \longrightarrow \phi(1020)\,\eta$	≈ 0.06
$\omega_{2,S} \longrightarrow \bar{K}^*(892)\,K + \text{c.c.}$	$\approx 510 \pm 255$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta$	$\approx 1.0 \pm 0.5$
$\omega_{2,S} \longrightarrow \omega(782)\,\eta'(958)$	≈ 0.3
$\omega_{2,S} \longrightarrow \phi(1020)\,\eta$	$\approx 101 \pm 51$

Decay process (in model)	eLSM (MeV)
$\rho_2(?) \longrightarrow a_2(1320)\,\pi$	≈ 88
$K_{2,A} \longrightarrow K_2^*(1430)\,\pi$	≈ 49
$K_{2,A} \longrightarrow a_2(1320)\,K$	≈ 84
$K_{2,A} \longrightarrow f_2(1270)\,K$	≈ 4
$\omega_{2,S} \longrightarrow K_2^*(1430)\,K + \text{c.c.}$	≈ 15

Pseudotensor mesons

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

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Regular Article – Theoretical Physics

Phenomenology of pseudotensor mesons and the pseudotensor glueball

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Abstract. We study the decays of the pseudotensor mesons $(\pi_2(1670), K_2(1770), \eta_2(1645), \eta_2(1870))$ interpreted as the ground-state nonet of $1^1D_2\ \bar{q}q$ states using interaction Lagrangians which couple them to pseudoscalar, vector, and tensor mesons. While the decays of $\pi_2(1670)$ and $K_2(1770)$ can be well described, the decays of the isoscalar states $\eta_2(1645)$ and $\eta_2(1870)$ can be brought in agreement with the present experimental data only if the mixing angle between nonstrange and strange states is surprisingly large (about -42°, similar to the mixing in the pseudoscalar sector, in which the chiral anomaly is active). Such a large mixing angle is however at odd with all other conventional quark-antiquark nonets: if confirmed, a deeper study of its origin will be needed in the future. Moreover, the $\bar{q}q$ assignment of pseudotensor states predicts that the ratio $[\eta_2(1870) \to a_2(1320)\,\pi]/[\eta_2(1870) \to f_2(1270)\,\eta]$ is about 23.5. This value is in agreement with Barberis *et al.*, (20.4 ± 6.6), but disagrees with the recent reanalysis of Anisovich *et al.*, (1.7 ± 0.4). Future experimental studies are necessary to understand this puzzle. If Anisovich's value is confirmed, a simple nonet of pseudoscalar mesons cannot be able to describe data (different assignments and/or additional states, such as an hybrid state, will be needed). In the end, we also evaluate the decays of a pseudoscalar glueball into the aforementioned conventional $\bar{q}q$ states: a sizable decay into $K_2^*(1430)\,K$ and $a_2(1230)\,\pi$ together with a vanishing decay into pseudoscalar-vector pairs (such as $\rho(770)\,\pi$ and $K^*(892)\,K$) are expected. This information can be helpful in future studies of glueballs at the ongoing BESIII and at the future PANDA experiments.

ArXiv: 1608.08777

Other considerations on pseudotensor mesons

Our model

- couples pseudotensor mesons to pseudoscalar, vector and tensor mesons.
- reproduces present experimental data for $\pi_2(1670)$ and $K_2(1770)$.
- identifies $\eta_2(1870)$ and $\eta_2(1645)$ with the $\bar{q}q$ pseudotensor meson nonet, if non-strange-strange mixing is large.
- predicts a large non-strange-strange mixing angle $\beta_{pt} \approx -40°$ in the isoscalar sector.
- contributes to the discussion on conflicting experimental results for the branching ratios of $\eta_2(1870)$.

Results for I = 1 and I = 1/2

Decay process	Theory (MeV)	Experiment (MeV)
$\pi_2(1670) \to \rho(770)\,\pi$	80.6 ± 10.8	80.6 ± 10.8
$\pi_2(1670) \to f_2(1270)\,\pi$	146.4 ± 9.7	146.4 ± 9.7
$\pi_2(1670) \to \bar{K}^*(892)\,K + c.c.$	11.7 ± 1.6	10.9 ± 3.7
$\pi_2(1670) \to \bar{K}_2^*(1430)\,K + c.c.$	0	
$\pi_2(1670) \to f_2'(1525)\,\pi$	0.1 ± 0.1	
$\pi_2(1670) \to a_2(1320)\,\pi$	0	not seen
$\pi_2(1670) \to a_2(1320)\,\eta$	0	
$\pi_2(1670) \to a_2(1320)\,\eta'(958)$	0	
$K_2(1770) \to \rho(770)\,K$	22.2 ± 3.0	
$K_2(1770) \to \bar{K}^*(892)\,\pi$	25.5 ± 3.4	seen
$K_2(1770) \to \bar{K}^*(892)\,\eta$	10.5 ± 1.4	
$K_2(1770) \to \bar{K}^*(892)\,\eta'(958)$	0	
$K_2(1770) \to \omega(782)\,K$	8.3 ± 1.1	seen
$K_2(1770) \to \phi(1020)\,K$	4.2 ± 0.6	seen
$K_2(1770) \to a_2(1320)\,K$	0	
$K_2(1770) \to \bar{K}_2^*(1430)\,\pi$	84.5 ± 5.6	dominant
$K_2(1770) \to \bar{K}_2^*(1430)\,\eta$	0	
$K_2(1770) \to \bar{K}_2^*(1430)\,\eta'(958)$	0	
$K_2(1770) \to f_2(1270)\,K$	5.8 ± 0.4	seen
$K_2(1770) \to f_2'(1525)\,K$	0	

Table 4: Decays of $I = 1$ and $I = 1/2$ pseudotensor states. The first two entries were used to determine the coupling constants of the model, see Eq. (3.2). The total decay widths are $\Gamma^{tot}_{\pi_2(1670)} = (260 \pm 9)$ MeV and $\Gamma^{tot}_{K_2(1770)} = (186 \pm 14)$ MeV.

ArXiv: 1608.08777

Results in the isoscalar (large isoscalar mixing!)

Decay process	Theory (MeV) ($\beta_{pt} = -42°$)	Experiment (MeV)
$\eta_2(1645) \to \bar{K}^*(892)\, K + c.c.$	24.7	seen
$\eta_2(1645) \to a_2(1320)\, \pi$	186.5	
$\eta_2(1645) \to \bar{K}_2^*(1430)\, K + c.c.$	0	
$\eta_2(1645) \to f_2(1270)\, \eta$	0	not seen
$\eta_2(1645) \to f_2(1270)\, \eta'(958)$	0	
$\eta_2(1645) \to f_2'(1525)\, \eta$	0	
$\eta_2(1645) \to f_2'(1525)\, \eta'(958)$	0	
$\eta_2(1870) \to \bar{K}^*(892)\, K + c.c.$	3.3	
$\eta_2(1870) \to a_2(1320)\, \pi$	221.0	
$\eta_2(1870) \to \bar{K}_2^*(1430)\, K + c.c.$	0	
$\eta_2(1870) \to f_2(1270)\, \eta$	9.4	
$\eta_2(1870) \to f_2(1270)\, \eta'(958)$	0	
$\eta_2(1870) \to f_2'(1525)\, \eta$	0	
$\eta_2(1870) \to f_2'(1525)\, \eta'(958)$	0	

Table 6: Decays of $I = 0$ pseudotensor states. The total decay widths are $\Gamma^{tot}_{\eta_2(1645)} = (181 \pm 11)$ MeV and $\Gamma^{tot}_{\eta_2(1870)} = (225 \pm 14)$ MeV.

ArXiv: 1608.08777

For a recent re-analysis with decay widhts partial-wave :
V. Shastry, E. Trotti, F.G., Phys. Rev.D 105 (2022) 5, 054022 • e-Print: 2107.13501

Considerations

If new experimental data confirms our results,

- we have good candidates for a ground-state pseudotensor meson nonet.
- the large mixing angle $\beta_{pt} \approx -40^{\circ}$ would be a mystery which deserves a detailed study.
- the current phenomenological study should be redone, including higher order corrections.

If new experimental data is at odd with our results,

- an understanding of the lowlying pseudotensor states as a standard quark-antiquark nonet would be hard.
- $\eta_2(1870)$ could be wrongly assigned as a $\bar{q}q$-state.
- possible further mixings with (hybrid) states could be included in the model.

Large mixing angle: where does it come from?

PHYSICAL REVIEW D **97,** 091901(R) (2018)

Rapid Communications

How the axial anomaly controls flavor mixing among mesons

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$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\beta_{pt} & \sin\beta_{pt} \\ -\sin\beta_{pt} & \cos\beta_{pt} \end{pmatrix} \begin{pmatrix} \eta_{2,N} \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} \equiv \bar{s}s \end{pmatrix}$$

$$\beta_{pt} = -42^\circ$$

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42^\circ$$

(Excited) vector mesons

$n^{2S+1}L_J$	J^{PC}	I=1 $u\bar{d}$, $d\bar{u}$ $\frac{d\bar{d}-u\bar{u}}{\sqrt{2}}$	I=1/2 $u\bar{s}$, $d\bar{s}$ $s\bar{d}$, $s\bar{u}$	I=0 $\approx \frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	I=0 $\approx s\bar{s}$	Meson names	Chiral Partners
1^1S_0	0^{-+}	π	K	$\eta(547)$	$\eta'(958)$	Pseudoscalar	$J=0$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^\star(1430)$	$f_0(1370)$	$f_0(1500)/f_0(1710)$	Scalar	
1^3S_1	1^{--}	$\rho(770)$	$K^\star(892)$	$\omega(782)$	$\phi(1020)$	Vector	$J=1$
1^3P_1	1^{++}	$a_1(1260)$	K_{1A}	$f_1(1285)$	$f_1'(1420)$	Axial-vector	
1^1P_1	1^{+-}	$b_1(1235)$	K_{1B}	$h_1(1170)$	$h_1(1415)$	Pseudovector	$J=1^\star$
1^3D_1	1^{--}	$\rho(1700)$	$K^\star(1680)$	$\omega(1650)$	$\phi(???)$	Excited-vector	
1^3P_2	2^{++}	$a_2(1320)$	$K_2^\star(1430)$	$f_2(1270)$	$f_2'(1525)$	Tensor	$J=2$
1^3D_2	2^{--}	$\rho_2(???)$	$K_2(1820)$	$\omega_2(???)$	$\phi_2(???)$	Axial-tensor	
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	Pseudotensor	
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^\star(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$J=3$ - Tensor	

Citation: R.L. Workman *et al.* (Particle Data Group), to be published (2022)

$\phi(2170)$

$I^G(J^{PC}) = 0^-(1^{--})$

See the review on "Spectroscopy of Light Meson Resonances."

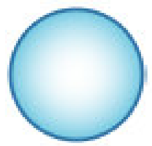
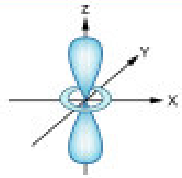
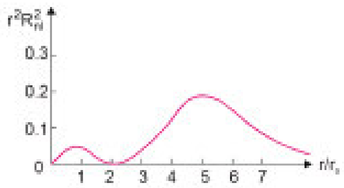
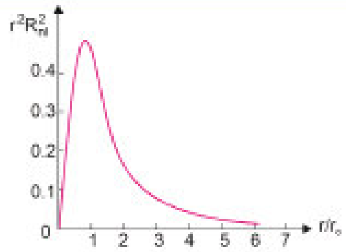
$\phi(2170)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
2162 ± 7	**OUR AVERAGE**	Error includes scale factor of 1.1.		

$\phi(2170)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*C*
100 $^{+31}_{-23}$	**OUR AVERAGE**	Error includes scale factor of 2.5.		

Excited vector mesons: properties

Type of excitation	Radially excited vector mesons	Angular momentum excited vector mesons
Quantum numbers	n $^{2S+1}L_J = 2^3S_1$	n $^{2S+1}L_J = 1^3D_1$
Notation	V_E	V_D
S	1 $\uparrow\uparrow$	1 $\uparrow\uparrow$
n	2	1
L	0	2
orbital		
Radial function		
Associated states	$\rho(1450)$, $K^*(1410)$, $\phi(1680)$, $\omega(1420)$	$\rho(1700)$, $K^*(1680)$, ϕ_P, $\omega(1650)$
Decay types	$V_E \to PP$ $V_E \to VP$ $V_E \to \gamma P$	$V_D \to PP$ $V_D \to VP$ $V_D \to \gamma P$

Radially excited vector mesons: some results

TABLE X. Decays widths of (predominantly) orbitally excited vector mesons into a pseudoscalar meson and a ground-state vector meson ($V_D \to VP$).

Decay process $V_D \to VP$	Theory (MeV)	Experiment (MeV)
$\rho(1700) \to \omega\pi$	140 ± 59	Seen (see text)
$\rho(1700) \to K^*(892)K$	56 ± 23	83 ± 66 MeV (see text)
$\rho(1700) \to \rho\eta$	41 ± 17	68 ± 42 MeV (see text)
$\rho(1700) \to \rho\eta'$	≈ 0	Not listed in PDG
$K^*(1680) \to K\rho$	64 ± 27	101 ± 35 by PDG
$K^*(1680) \to K\phi$	13 ± 6	Not listed in PDG
$K^*(1680) \to K\omega$	21 ± 9	Not listed in PDG
$K^*(1680) \to K^*(892)\pi$	81 ± 34	96 ± 33 by PDG
$K^*(1680) \to K^*(892)\eta$	0.5 ± 0.2	Not listed in PDG
$K^*(1680) \to K^*(892)\eta'$	≈ 0	Not listed in PDG
$\omega(1650) \to \rho\pi$	370 ± 156	~ 205, 154 ± 44, ~ 273, 120 ± 18 (see text)
$\omega(1650) \to K^*(892)K$	42 ± 18	Not listed in PDG
$\omega(1650) \to \omega(782)\eta$	32 ± 13	~ 100, 56 ± 30 (see text)
$\omega(1650) \to \omega(782)\eta'$	≈ 0	Not listed in PDG
$\phi(1930) \to K\bar{K}^*$	260 ± 109	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta$	67 ± 28	Resonance not yet known
$\phi(1930) \to \phi(1020)\eta'$	≈ 0	Resonance not yet known

Prediction for ϕ(1930)

Can one find this state?

TABLE XII. Summary table for the putative state $\phi(1930)$.

Meson $\phi(1930)$	
Quark composition	$\approx s\bar{s}$
Old spectroscopy notation	(Predom.) $n^{2S+1}L_J = 1^3D_1$
n	(Predom.) 1
S	(Predom.) 1↑↑
L	(Predom.) 2
J^{PC}	1^{--}
Mass	$\approx 1930 \pm 40$ MeV
Decays	
Decay channel	Decay width (MeV)
$\phi(1930) \to \bar{K}K$	104 ± 28
$\phi(1930) \to K\bar{K}^*$	260 ± 109
$\phi(1930) \to \Phi(1020)\eta$	67 ± 28
$\phi(1930) \to \Phi(1020)\eta'$	≈ 0
$\phi(1930) \to \gamma\eta$	0.19 ± 0.12
$\phi(1930) \to \gamma\eta'$	0.13 ± 0.08

arXiv: 1708.02593; **it does not fit with ϕ(2170)**

Toward a nonet of hybrid state/PDG

$\pi_1(1600)$

$I^G(J^{PC}) = 1^-(1^{-+})$

See the review on "Spectroscopy of Light Meson Resonances" and a note in PDG 06, Journal of Physics **G33** 1 (2006).

$\pi_1(1600)$ T-Matrix Pole $\sqrt{s}$

$\pi_1(1600)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1661^{+15}_{-11} OUR AVERAGE	Error includes scale factor of 1.2.			

$\pi_1(1600)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
240± 50 OUR AVERAGE	Error includes scale factor of 1.7. See the ideogram below.			

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$\eta\pi$	
Γ_7	$f_1(1285)\pi$	seen

$\pi_1(1400)$

$I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1400)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE	Error includes scale factor of 1.8. See the ideogram below.				

$\pi_1(1400)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

$\pi_1(1400)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen

A unique I=1 hybrid state

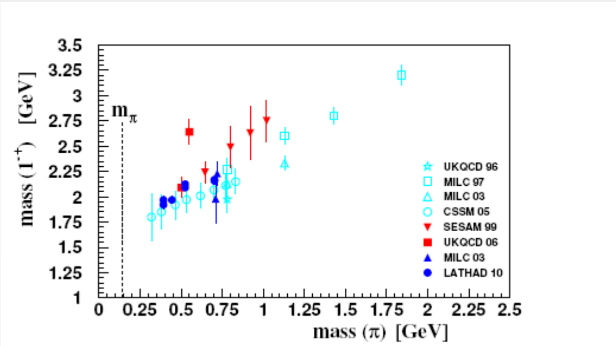
PHYSICAL REVIEW LETTERS **122**, 042002 (2019)

Determination of the Pole Position of the Lightest Hybrid Meson Candidate

A. Rodas,[1,*] A. Pilloni,[2,3,†] M. Albaladejo,[2,4] C. Fernández-Ramírez,[5] A. Jackura,[6,7] V. Mathieu,[2] M. Mikhasenko,[8] J. Nys,[9] V. Pauk,[10] B. Ketzer,[8] and A. P. Szczepaniak[2,6,7]

Mapping states with explicit gluonic degrees of freedom in the light sector is a challenge, and has led to controversies in the past. In particular, the experiments have reported two different hybrid candidates with spin-exotic signature, $\pi_1(1400)$ and $\pi_1(1600)$, which couple separately to $\eta\pi$ and $\eta'\pi$. This picture is not compatible with recent Lattice QCD estimates for hybrid states, nor with most phenomenological models. We consider the recent partial wave analysis of the $\eta^{(\prime)}\pi$ system by the COMPASS Collaboration. We fit the extracted intensities and phases with a coupled-channel amplitude that enforces the unitarity and analyticity of the S matrix. We provide a robust extraction of a single exotic π_1 resonant pole, with mass and width $1564 \pm 24 \pm 86$ and $492 \pm 54 \pm 102$ MeV, which couples to both $\eta^{(\prime)}\pi$ channels. We find no evidence for a second exotic state. We also provide the resonance parameters of the $a_2(1320)$ and $a_2'(1700)$.

π1(1600) and π1(1400) are the same state
(in agreement with various models and lattice QCD)



C. Meyer and E. Swanson,
Hybrid Mesons,
Prog. Part. Nucl. Phys. 82 (2015) 21
[arXiv:1502.07276 [hep-ph]].

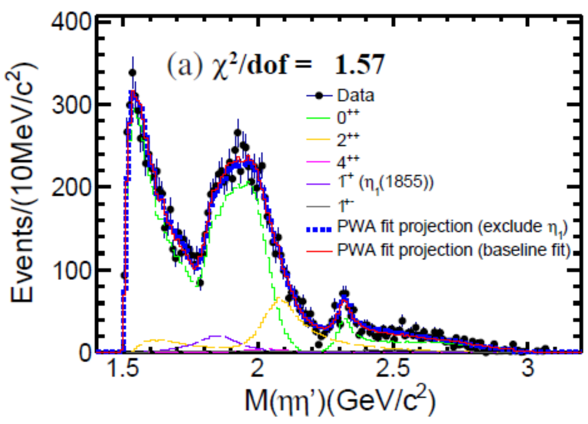
New experimental finding: η(1855)

Observation of an isoscalar resonance with exotic $J^{PC} = 1^{-+}$ quantum numbers in $J/\psi \to \gamma\eta\eta'$

M. Ablikim[1], M. N. Achasov[10,b], P. Adlarson[68], S. Ahmed[14], M. Albrecht[4], R. Aliberti[28], A. Amoroso[67A,67C], M. R. An[32],

Using a sample of $(10.09\pm0.04)\times10^9$ J/ψ events collected with the BESIII detector operating at the BEPCII storage ring, a partial wave analysis of the decay $J/\psi \to \gamma\eta\eta'$ is performed. The first observation of an isoscalar state with exotic quantum numbers $J^{PC} = 1^{-+}$, denoted as $\eta_1(1855)$, is reported in the process $J/\psi \to \gamma\eta_1(1855)$ with $\eta_1(1855) \to \eta\eta'$. Its mass and width are measured to be $(1855\pm9^{+6}_{-1})$ MeV/c^2 and $(188\pm18^{+3}_{-8})$ MeV, respectively, where the first uncertainties are statistical and the second are systematic, and its statistical significance is estimated to be larger than 19σ.

arXiv: 2022.0062



A nonet of hybrid states?

Physics Letters B 834 (2022) 137478

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ELSEVIER

The phenomenology of the exotic hybrid nonet with $\pi_1(1600)$ and $\eta_1(1855)$

Vanamali Shastry [a,*], Christian S. Fischer [b,c], Francesco Giacosa [a,d]

arXiv:2203.04327

Beides π1(1600) and η1(1855), we expect also: K1(1750) and η1(1660). The last two not yet seen.

	M (MeV)	Γ (MeV)
K_1^{hyb}	1761	312 ± 97 170 ± 65
η_1^L	1661	81 ± 15 83 ± 16
η_1^H	1855	259 ± 92 157 ± 68

Table 7
The partial widths and branching ratios of various decay channels and the total width for the hybrid kaon $K_1^{hyb}(1750)$. We have assumed the mass of the state to be 1761 MeV [44].

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{K_1(1270)\pi}$	125 ± 42	48 ± 25	$\Gamma_{\rho K}$	2.18 ± 0.56	2.19 ± 0.57
$\Gamma_{K_1(1400)\pi}$	103 ± 45	98 ± 43	$\Gamma_{\omega K}$	0.82 ± 0.21	0.82 ± 0.21
$\Gamma_{h_1(1170)K}$	1.53 ± 0.28	1.37 ± 0.24	$\Gamma_{\phi K}$	0.49 ± 0.12	0.49 ± 0.13
$\Gamma_{\eta K}$	0.29 ± 0.07	0.29 ± 0.07	$\Gamma_{K^*\pi}$	0.67 ± 0.17	0.67 ± 0.17
$\Gamma_{\eta' K}$	2.77 ± 0.62	2.81 ± 0.62	$\Gamma_{K^*\eta}$	0.30 ± 0.08	0.30 ± 0.08
$\Gamma_{\rho K^*}$	0.045 ± 0.016	0.047 ± 0.016	$\Gamma_{\omega K^*}$	0.011 ± 0.004	0.012 ± 0.004
$\Gamma_{a_1 K}$	11.0 ± 2.32	11.3 ± 2.35	$\Gamma_{b_1 K}$	64 ± 14	3.11 ± 2.88
			Γ_{tot}	312 ± 97	170 ± 65

Table 6
The partial widths and branching ratios of various decay channels and the total width of the η_1^L (left) and the $\eta_1(1855)$ (right) for $\theta_h = 15°$. This corresponds to the "Scenario-2" discussed in the text.

Channel	Width (MeV)		Channel	Width (MeV)	
	Set-1	Set-2		Set-1	Set-2
$\Gamma_{a_1\pi}$	80 ± 15	82 ± 16	$\Gamma_{K_1(1270)K}$	253 ± 92	151 ± 67
Γ_{K^*K}	0.29 ± 0.075	0.29 ± 0.075	Γ_{K^*K}	1.45 ± 0.37	1.46 ± 0.38
$\Gamma_{\eta'\eta}$	0.41 ± 0.09	0.41 ± 0.09	$\Gamma_{\eta'\eta}$	2.28 ± 0.51	2.31 ± 0.51
$\Gamma_{K_1(1270)K}$	0	0	$\Gamma_{a_1\pi}$	0	0
$\Gamma_{\rho\rho}$	0.081 ± 0.028	0.082 ± 0.029	$\Gamma_{\rho\rho}$	0	0
$\Gamma_{K^*K^*}$	0	0	$\Gamma_{K^*K^*}$	0.075 ± 0.027	0.077 ± 0.028
$\Gamma_{\omega\phi}$	0	0	$\Gamma_{\omega\phi}$	$\sim 10^{-4}$	$\sim 10^{-4}$
$\Gamma_{f_1\eta}$	0	0	$\Gamma_{f_1\eta}$	2.15 ± 0.56	2.21 ± 0.57
Γ_{tot}	81 ± 15	83 ± 16	Γ_{tot}	259 ± 92	157 ± 68

Uniwersytet Jana Kochanowskiego w Kielcach

Dulcis in fundo: scalar sector

Eur. Phys. J. C (2022) 82:487
https://doi.org/10.1140/epjc/s10052-022-10403-z

THE EUROPEAN PHYSICAL JOURNAL C

Check for updates

Regular Article - Theoretical Physics

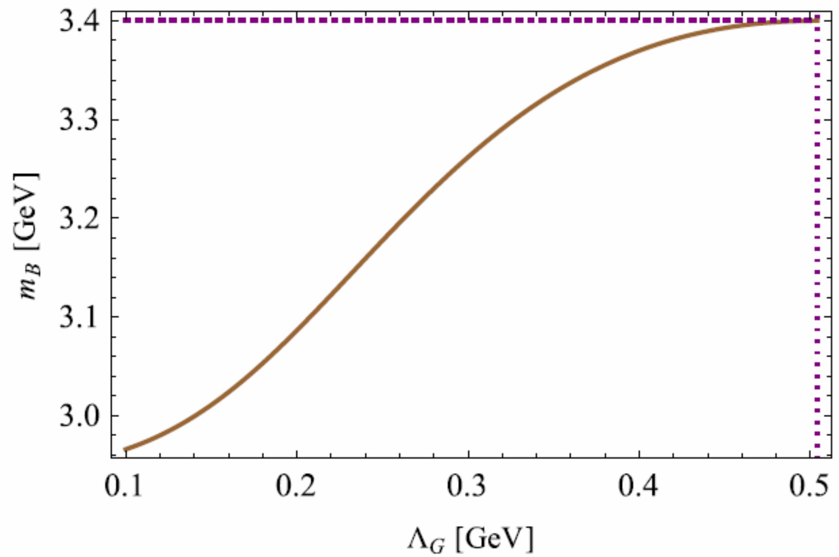
Glueball–glueball scattering and the glueballonium

Francesco Giacosa[1,2]**, Alessandro Pilloni**[3,4]**, Enrico Trotti**[1,a]

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}(\partial_\mu G)^2 - V(G),$$

with

$$V(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$



Conclusions and outlook

Many nonets fit well in the quark-antiquark picture, but…

- axial-tensor mesons basically unknown;
- pseudotensor mesons, is there a large isoscalar mixing?
- vector mesons: which is the orbitally excited ϕ meson?

Unconventional mesons:

- hybrid mesons: a new nonet?

Outlook:

- tensor glueball (ongoing)

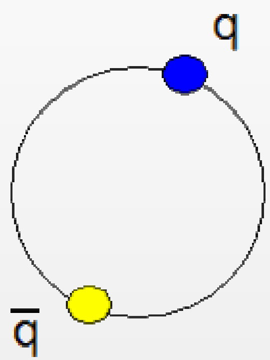
Thanks

Back-up slides

Conventional mesons

Quark: u,d,s,… R,G,B

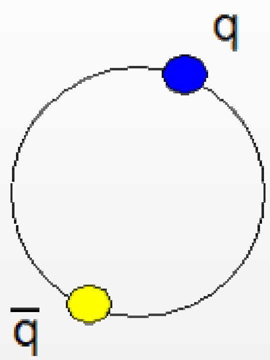
Quark-antiquark bound states: conventional mesons



$$\left| color \right\rangle = \sqrt{1/3}\,(\overline{R}R + \overline{B}B + \overline{G}G)$$

Conventional mesons/2

Surely, with quark-antiquark states we can understand a lot of QCD, but definitely not everything.

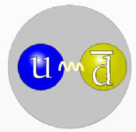


$$P = -(-1)^{L} \qquad C = (-1)^{L+S}$$

L , S → J = L+ S J^{PC}

$$L = S = 0 \rightarrow J^{PC} = 0^{-+} \quad \text{pseudoscalar mesons}$$

$$\left|\pi^{+}\right\rangle = \left|u\bar{d}\right\rangle \left|space : L = 0\right\rangle \left|spin : S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$



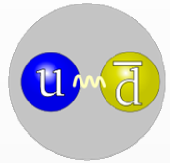
$$\left|K^{+}\right\rangle = \left|u\bar{s}\right\rangle \left|space : L = 0\right\rangle \left|spin : S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

...

$$\left|D^{0}\right\rangle = \left|u\bar{c}\right\rangle \left|space : L = 0\right\rangle \left|spin : S = 0\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

...

$L=0\ ,\ S=1 \rightarrow J^{PC}=1^{--}$ vector mesons



$$\left|\rho^{+}\right\rangle = \left|u\bar{d}\right\rangle \left|space:L=0\right\rangle \left|spin:S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|K^{*}(892)^{+}\right\rangle = \left|u\bar{s}\right\rangle \left|space:L=0\right\rangle \left|spin:S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|D^{*0}\right\rangle = \left|u\bar{c}\right\rangle \left|space:L=0\right\rangle \left|spin:S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

...

$$\left|j/\Psi\right\rangle = \left|c\bar{c}\right\rangle \left|space:L=0\right\rangle \left|spin:S=1\right\rangle \left|\bar{R}R+\bar{B}B+\bar{G}G\right\rangle$$

$$L = S = 1 \rightarrow J^{PC} = 0^{++}$$ scalar mesons

$$\left|\sigma\right\rangle = \left|u\bar{u} + d\bar{d}\right\rangle \left|space : L = 1\right\rangle \left|spin : S = 1\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

corresponds to the resonance $f_0(1370)$.

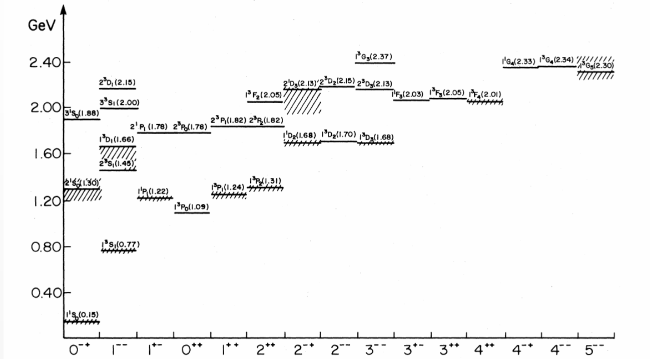
...

...

$$\left|\chi_{c0}(1S)\right\rangle = \left|c\bar{c}\right\rangle \left|space : L = 1\right\rangle \left|spin : S = 1\right\rangle \left|\bar{R}R + \bar{B}B + \bar{G}G\right\rangle$$

Quark model(s) and their QFT extensions

Mesons in a Relativized Quark Model with Chromodynamics
S. Godfrey, N. Isgur
Phys.Rev. D32 (1985) **189-231**



QCD phenomenology based on a chiral effective Lagrangian
T. Hatsuda, T. Kunihiro
Phys.Rept. **247** (1994) 221-367

NJL: quark-based model with
chiral symmetry and SSB
chiral condensate
Effective quark mass
Mesons as quarkonia (pion: ok)

The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states
R. Alkofer, L. von Smekal
Phys.Rept. **353** (2001) 281

Baryons as relativistic three-quark bound states
G. Eichmann et al.
Progr. Part. Nucl. Phys. **91** (2016) 1

DS:
quarks and gluons propagators
from QCD
Condensates
Effective quark and gluon masses
Spectra of mesons as quarkonia
(pion: ok) and baryons as qqq states

Quark-antiquark currents

Meson	$n^{2S+1}L_J$	J^{PC}	S	L	Hermitian quark current operators
pseudoscalar	1^1S_0	0^{-+}	0	0	$P_{ij} = \bar{q}_j \, i\gamma^5 \, q_i$
vector	1^3S_1	1^{--}	1		$V^\mu_{ij} = \bar{q}_j \, \gamma^\mu \, q_i$
pseudovector	1^1P_1	1^{+-}	0	1	$P^\mu_{ij} = \bar{q}_j \, \gamma^5 \overleftrightarrow{\partial}^\mu \, q_i$
scalar	1^3P_0	0^{++}	1		$S_{ij} = \bar{q}_j \, q_i$
axial vector	1^3P_1	1^{++}	1		$A^\mu_{ij} = \bar{q}_j \, \gamma^5\gamma^\mu \, q_i$
tensor	1^3P_2	2^{++}	1		$X^{\mu\nu}_{ij} = \bar{q}_j \, i \left[\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3} \tilde{G}^{\mu\nu} \overleftrightarrow{\not{\partial}}\right] q_i$
pseudotensor	1^1D_2	2^{-+}	0	2	$T^{\mu\nu}_{ij} = \bar{q}_j \, i \left[\gamma^5 \overleftrightarrow{\partial}^\mu \overleftrightarrow{\partial}^\nu - \frac{2}{3} \tilde{G}^{\mu\nu} \overleftrightarrow{\partial}_\alpha \overleftrightarrow{\partial}^\alpha\right] q_i$
excited vector	1^3D_1	1^{--}	1		$S^\mu_{ij} = \bar{q}_j \, \overleftrightarrow{\partial}^\mu \, q_i$
axial tensor	1^3D_2	2^{--}	1		$B^{\mu\nu}_{ij} = \bar{q}_j \, i \left[\gamma^5\gamma^\mu \overleftrightarrow{\partial}^\nu + \gamma^5\gamma^\nu \overleftrightarrow{\partial}^\mu - \frac{2}{3} \tilde{G}^{\mu\nu} \gamma^5 \overleftrightarrow{\not{\partial}}\right] q_i$
spin-3 tensor	1^3D_3	3^{--}	1		$\ldots$

The eLSM: a chiral model of QCD

PHYSICAL REVIEW D **87,** 014011 (2013)

Meson vacuum phenomenology in a three-flavor linear sigma model with (axial-)vector mesons

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(Received 7 August 2012; published 8 January 2013)

PHYSICAL REVIEW D **90,** 114005 (2014)

Is $f_0(1710)$ a glueball?

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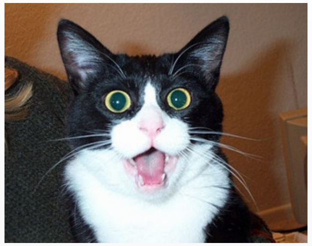
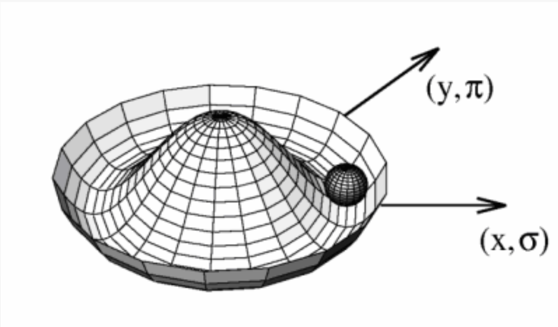
(Received 26 August 2014; published 2 December 2014)

Model of QCD – eLSM with scalar Glueball





$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu G)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda^2}\left(G^4 \ln\left|\frac{G}{\Lambda}\right| - \frac{G^4}{4}\right) + \mathrm{Tr}\left[(D^\mu\Phi)^\dagger(D_\mu\Phi)\right] \\
& - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\Phi^\dagger\Phi\right] - \lambda_1(\mathrm{Tr}\left[\Phi^\dagger\Phi\right])^2 - \lambda_2 \mathrm{Tr}\left[(\Phi^\dagger\Phi)^2\right] \\
& + \left(\frac{G}{G_0}\right)^2 \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)\left((L^\mu)^2 + (R^\mu)^2\right)\right] \\
& - \frac{1}{4}\mathrm{Tr}\left[(L^{\mu\nu})^2 + (R^{\mu\nu})^2\right] + \mathrm{Tr}\left[H\left(\Phi^\dagger + \Phi\right)\right] \\
& + c_1[\det(\Phi) - \det(\Phi^\dagger)]^2 + \frac{h_1}{2}\mathrm{Tr}[\Phi^\dagger\Phi]\mathrm{Tr}[L_\mu L^\mu + R_\mu R^\mu] \\
& + h_2\mathrm{Tr}[\Phi^\dagger L_\mu L^\mu \Phi + \Phi R_\mu R^\mu \Phi^\dagger] + 2h_3\mathrm{Tr}[\Phi R_\mu \Phi^\dagger L^\mu]
\end{aligned}
$$

$$
\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix}
\frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star +} + iK^+ \\
a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\
K_0^{\star -} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S
\end{pmatrix}
$$

$$
L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix}
\frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star +} \pm K_1^+ \\
\rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\
K^{\star -} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S}
\end{pmatrix}
$$

S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011**)
D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

Results of the eLSM (11 parameters, 21 exp. quantities)

Uniwersytet Jana Kochanowskiego w Kielcach

Error from PDG or 5% of exp. Scalar-isoscalar sector not included.

$$\chi^2_{red} = 1.2$$

Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

arXiv:1208.0585

Pseudotensor: Lagrangians and decays

Pseudotensor mesons: {π2(1670), K2(1770),η2(1645),η2(1870)}
Lagrangians based on flavour symmetry

$$\mathcal{L}_{TVP} = c_{TVP}\,\mathrm{Tr}\{T_{\mu\nu}\,[V^{\mu},(\partial^{\nu}P)]_{-}\}\,,$$

$$P=\begin{pmatrix}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+\\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0\\ K^- & \bar{K}^0 & \eta_S\end{pmatrix},\quad V^{\mu}=\begin{pmatrix}\frac{\omega_N^{\mu}+\rho^{0\mu}}{\sqrt{2}} & \rho^{+\mu} & K^{*+\mu}\\ \rho^{-\mu} & \frac{\omega_N^{\mu}-\rho^{0\mu}}{\sqrt{2}} & K^{*0\mu}\\ K^{*-\mu} & \bar{K}^{*0\mu} & \omega_S^{\mu}\end{pmatrix},$$

$$T^{\mu\nu}=\begin{pmatrix}\frac{\eta_{2,N}^{\mu\nu}+\pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu}\\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu}-\pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu}\\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu}\end{pmatrix}.$$

$$\mathcal{L}_{TXP} = c_{TXP}\,\mathrm{Tr}(T_{\mu\nu}\,\{X^{\mu\nu},P\}_{+})$$

$$P=\begin{pmatrix}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+\\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0\\ K^- & \bar{K}^0 & \eta_S\end{pmatrix},\quad X^{\mu\nu}=\begin{pmatrix}\frac{f_{2,N}^{\mu\nu}+a_2^{0\mu\nu}}{\sqrt{2}} & a_2^{+\mu\nu} & K_2^{*+\mu\nu}\\ a_2^{-\mu\nu} & \frac{f_{2,N}^{\mu\nu}-a_2^{0\mu\nu}}{\sqrt{2}} & K_2^{*0\mu\nu}\\ K_2^{*-\mu\nu} & \bar{K}_2^{*0\mu\nu} & f_{2,S}^{\mu\nu}\end{pmatrix},$$

$$T^{\mu\nu}=\begin{pmatrix}\frac{\eta_{2,N}^{\mu\nu}+\pi_2^{0\mu\nu}}{\sqrt{2}} & \pi_2^{+\mu\nu} & K_2^{+\mu\nu}\\ \pi_2^{-\mu\nu} & \frac{\eta_{2,N}^{\mu\nu}-\pi_2^{0\mu\nu}}{\sqrt{2}} & K_2^{0\mu\nu}\\ K_2^{-\mu\nu} & \bar{K}_2^{0\mu\nu} & \eta_{2,S}^{\mu\nu}\end{pmatrix}.$$

Tree-level decay widths:

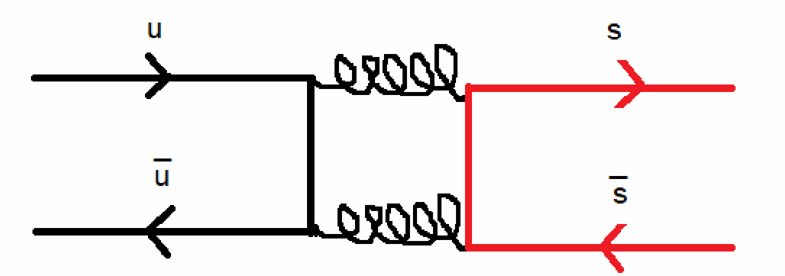
$$\Gamma^{tl}_{T\to VP} = \frac{k_f}{8\pi\, m_T}\,\frac{g^2_{TVP}}{15}\left(2\,\frac{k_f^4}{m_V^2}+5\,k_f^2\right)\Theta(m_T-m_V-m_P)\,,$$

and

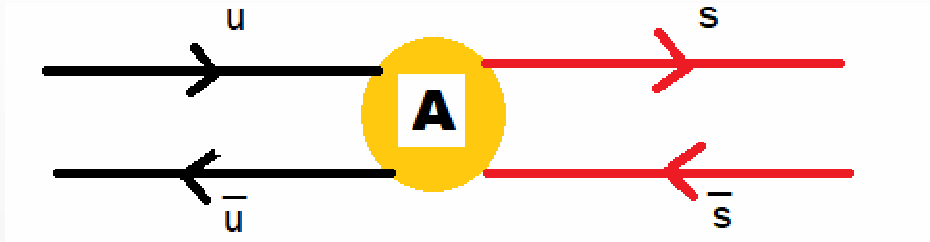
$$\Gamma^{tl}_{T\to XP} = \frac{k_f}{8\pi\, m_T}\,\frac{g^2_{TXP}}{45}\left(4\,\frac{k_f^4}{m_X^4}+30\,\frac{k_f^2}{m_X^2}+45\right)\Theta(m_T-m_X-m_P)\,.$$

Large mixing angle: where does it come from?

Such a mixing is suppressed...



But this can be large



- For pseudoscalar mesons: η(547) and η'(958). Θmix = -42° Large mixing caused by the axal anomaly.
- For vector mesons: ω(782) and φ(1020). Θmix = -3° Very small mixing.
- For tensor mesons: f2(1270) and f'2(1525). Θmix = 3° Also very small mixing. Why?
- Pseudotensor mesons: also large, but confirmation is needed.

Details in: **1709.07454**

Excited vectors: Lagrangians

The Lagrangian of the model is:

$$\mathcal{L} = \mathcal{L}_{1,E} + \mathcal{L}_{1,D} + \mathcal{L}_{2,E} + \mathcal{L}_{2,D},$$

where:

$$\mathcal{L}_{1,E} = ia_E Tr[\partial^\mu P, V_{E,\mu}]P \qquad \mathcal{L}_{1,D} = ia_D Tr[\partial^\mu P, V_{D,\mu}]P$$

$$\mathcal{L}_{2,E} = b_E Tr[\tilde{V}_E^{\mu\nu}\{V_{\mu\nu}, P\}] \qquad \mathcal{L}_{2,D} = b_D Tr[\tilde{V}_D^{\mu\nu}\{V_{\mu\nu}, P\}]$$

$a_E, a_D, b_E, b_D -$ coupling constants of the different decay types.

- $R \to \gamma P$ through „vector meson dominance"

$$V_{\mu\nu} \to V_{\mu\nu} + \frac{e_0}{g_\rho} Q F_{\mu\nu}$$

$F_{\mu\nu} -$ field strength tensor for photons
$e_0 = \sqrt{4\pi\alpha} \qquad \alpha \approx 1/137 \qquad g_\rho \approx 5.5 \pm 0.5 \qquad Q = diag(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$

Strong and radiative decay widths

TYPE OF DECAY

- $R \to PP$

$$\Gamma_{R\to PP} = S\frac{|\vec{k}|^3}{6\pi m_R^2}[\frac{a_i}{2}\lambda_{RPP}]^2$$

- $R \to VP, R \to \gamma P$

$$\Gamma_{R\to VP} = S\frac{|\vec{k}|^3}{12\pi}[\frac{b_i}{2}\lambda_{RVP}]^2$$

EXAMPLES

- $K^*(1410) \to K\eta$

$$\Gamma_{K^*(1410)\to K\eta} = \frac{|\vec{k}|^3}{6\pi m^2_{K^*(1410)}}[\frac{a_E}{2}\frac{1}{2}(cos\theta_p - \sqrt{2}sin\theta_p)]^2$$

- $\phi(1680) \to \phi(1020)\eta$

$$\Gamma_{\phi(1680)\to\phi(1020)\eta} = \frac{|\vec{k}|^3}{12\pi}[\frac{b_E}{2}\frac{sin\theta_p}{\sqrt{2}}]^2$$

where:

$|\vec{k}| = \frac{\sqrt{m_R^2+(m_a^2-m_b^2)^2-2(m_a^2+m_b^2)m_R^2}}{2m_R}$;

m_R− mass of the decaying resonance;

a_i, b_i− coupling constants ($i = E, D$);

m_a, m_b− masses of decay products;

S− symmetry factor;

Matrices of fields

$$P = \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \qquad V^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega^\mu+\rho^{\mu 0}}{\sqrt{2}} & \rho^{\mu +} & K_i^{\mu\star +} \\ \rho^{\mu -} & \frac{\omega^\mu-\rho^{\mu 0}}{\sqrt{2}} & K^{\mu\star 0} \\ K^{\mu\star -} & \bar{K}^{\mu\star 0} & \phi^\mu \end{pmatrix}$$

$$V_E^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_E^\mu+\rho_E^{\mu 0}}{\sqrt{2}} & \rho_E^{\mu +} & K_E^{\mu\star +} \\ \rho_E^{\mu -} & \frac{\omega_E^\mu-\rho_E^{\mu 0}}{\sqrt{2}} & K_E^{\mu\star 0} \\ K_E^{\mu\star -} & \bar{K}_E^{\mu\star 0} & \phi_E^\mu \end{pmatrix} \qquad V_D^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_D^\mu+\rho_D^{\mu 0}}{\sqrt{2}} & \rho_D^{\mu +} & K_D^{\mu\star +} \\ \rho_D^{\mu -} & \frac{\omega_D^\mu-\rho_D^{\mu 0}}{\sqrt{2}} & K_D^{\mu\star 0} \\ K_D^{\mu\star -} & \bar{K}_D^{\mu\star 0} & \phi_D^\mu \end{pmatrix}$$

- $P = \{\pi, K, \eta, \eta'\}$
- $V = \{\rho(770), K^*(892), \phi(1020), \omega(782)\}$
- $V_E = \{\rho(1450), K^*(1410), \phi(1680), \omega(1420)\}$
- $V_D = \{\rho(1700), K^*(1680), \phi_P, \omega(1650)\}$

Which mass for the missing state?

TABLE I. Mass differences between the members of the two nonets of excited vector mesons.

V_E	$\rho(1450)$	$K^*(1410)$	$\omega(1420)$	$\phi(1680)$
V_D	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$	$\phi(???)$
Difference	250 MeV	270 MeV	230 MeV	?

Hence, we can estimate the mass of $\phi(???)$ as

$$m_{\phi(???)} \simeq (m_{\phi(1680)} + 250 \pm 20)\ \text{MeV} = 1930 \pm 20\ \text{MeV}.$$

From now on we shall call this hypothetical state

$$\phi(???) \equiv \phi(1930).$$