MCMC parameter estimation methods for LISA massive black holes

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Outline

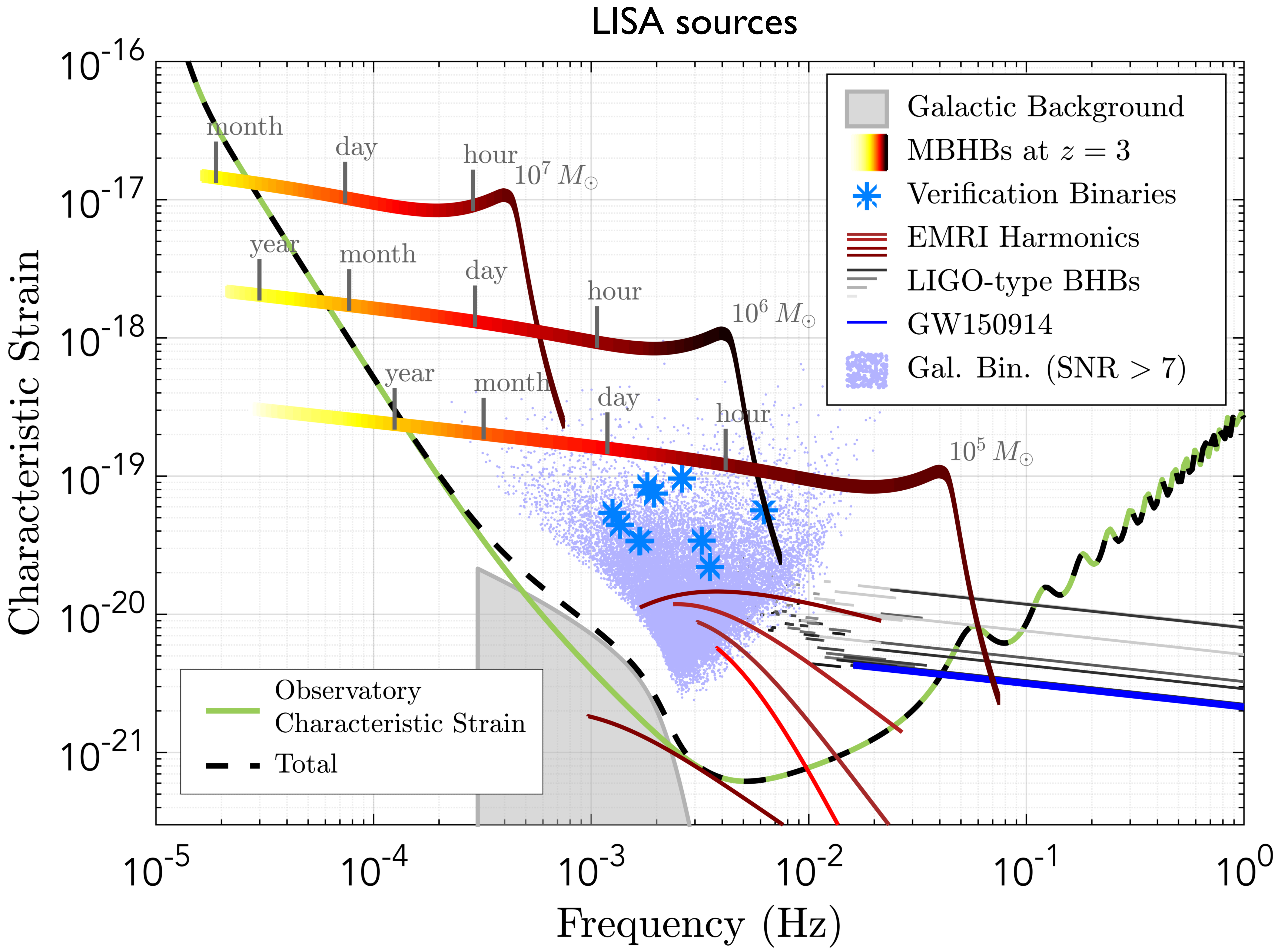
- **MBHB signals in LISA**
- Parameter space degeneracies
- Tools for Bayesian parameter estimation
- LISA Data Challenge: Sangria
- MBHB results for Sangria

The LISA sources and MBHBs

Massive black hole binaries

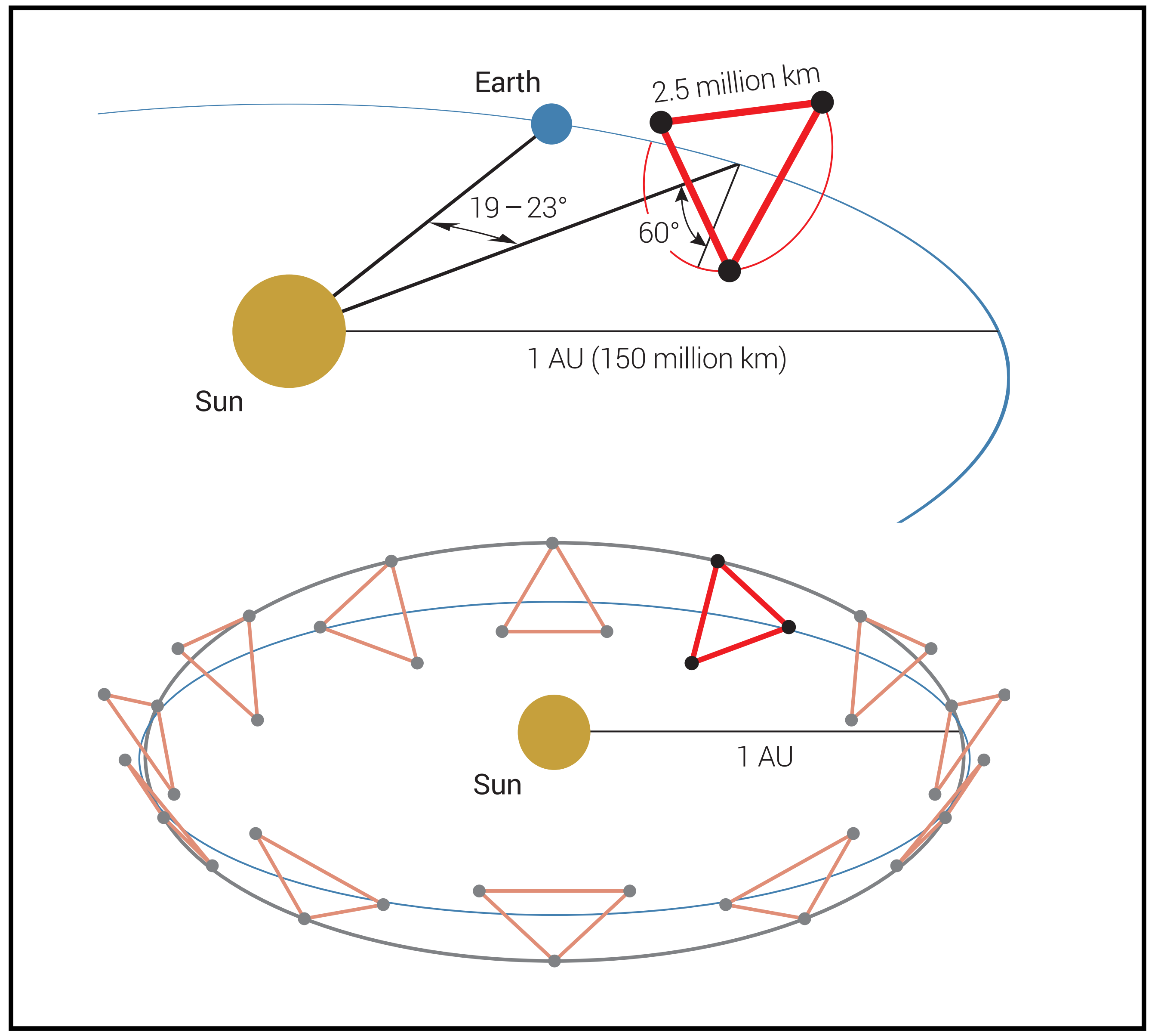
- **Primary sources** for LISA: very loud signals, signal dominated regime
- Precision GW science: details matter, waveform systematics important
- Primary candidates for EM counterparts, crucial for astrophysics and cosmology
- Primary candidates for TGR (with EMRIs): controlling biases and residuals crucial, need tools for extended waveform models

Not fully explored yet:
- IMR waveforms with precession, eccentricity
- Realistic instrument (gaps&glitches), global fit



LISA instrumental response

LISA orbits



Response

From spacecraft s to spacecraft r through link s: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

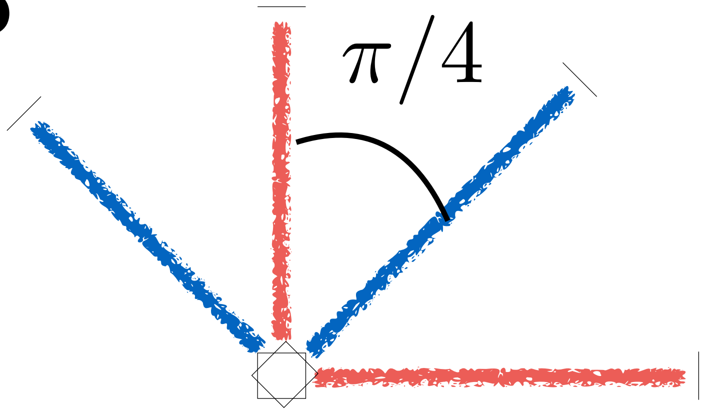
+ Time-delay interferometry (TDI)

Fourier-domain (separation of timescales [Marsat-Baker 2018])

$$\mathcal{T}_{slr} = \frac{i\pi f L}{2} \mathrm{sinc}\left[\pi f L \left(1 - k\cdot n_l\right)\right] \exp\left[i\pi f \left(L + k\cdot(p_r + p_s)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

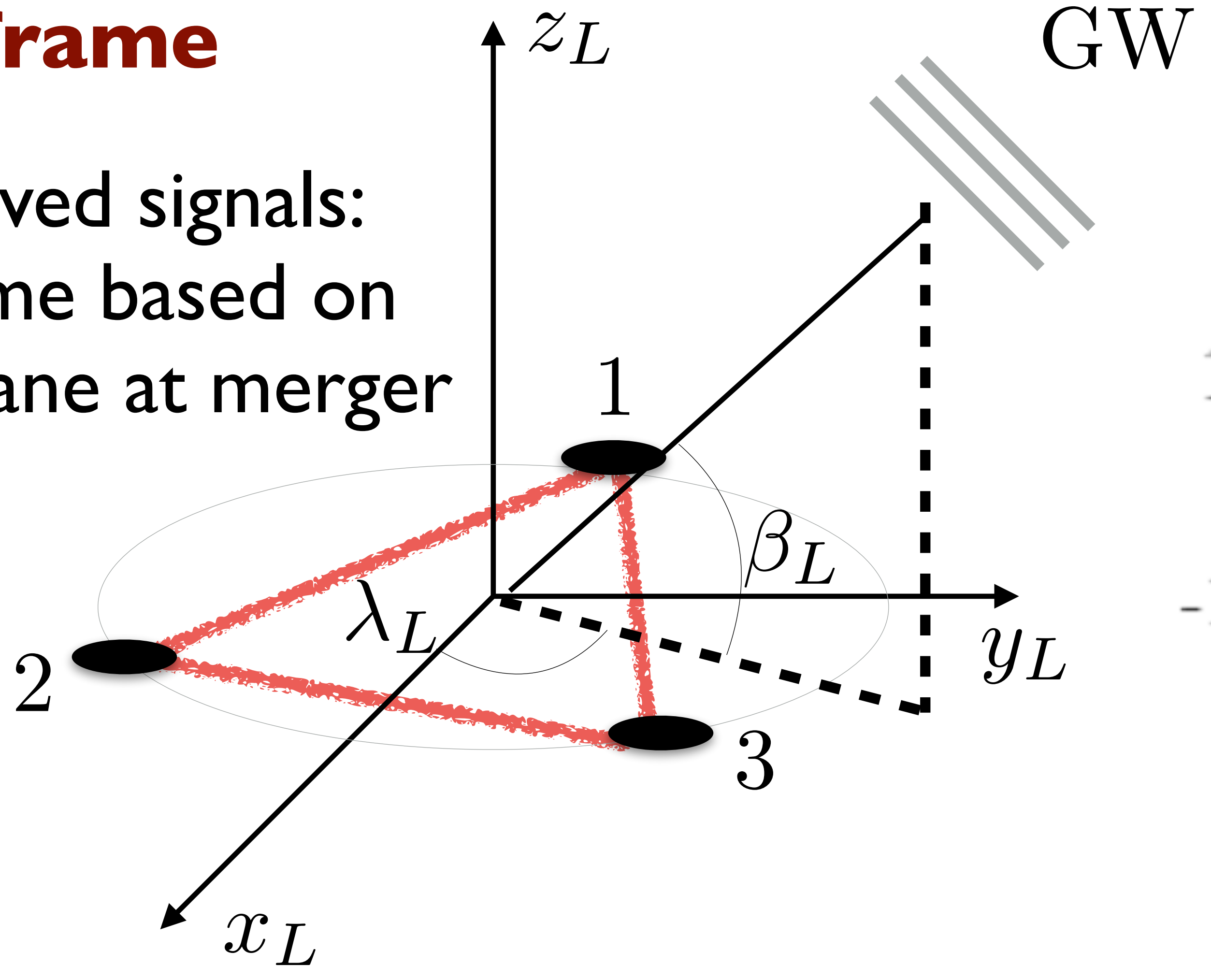
Time and frequency-dependency
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength

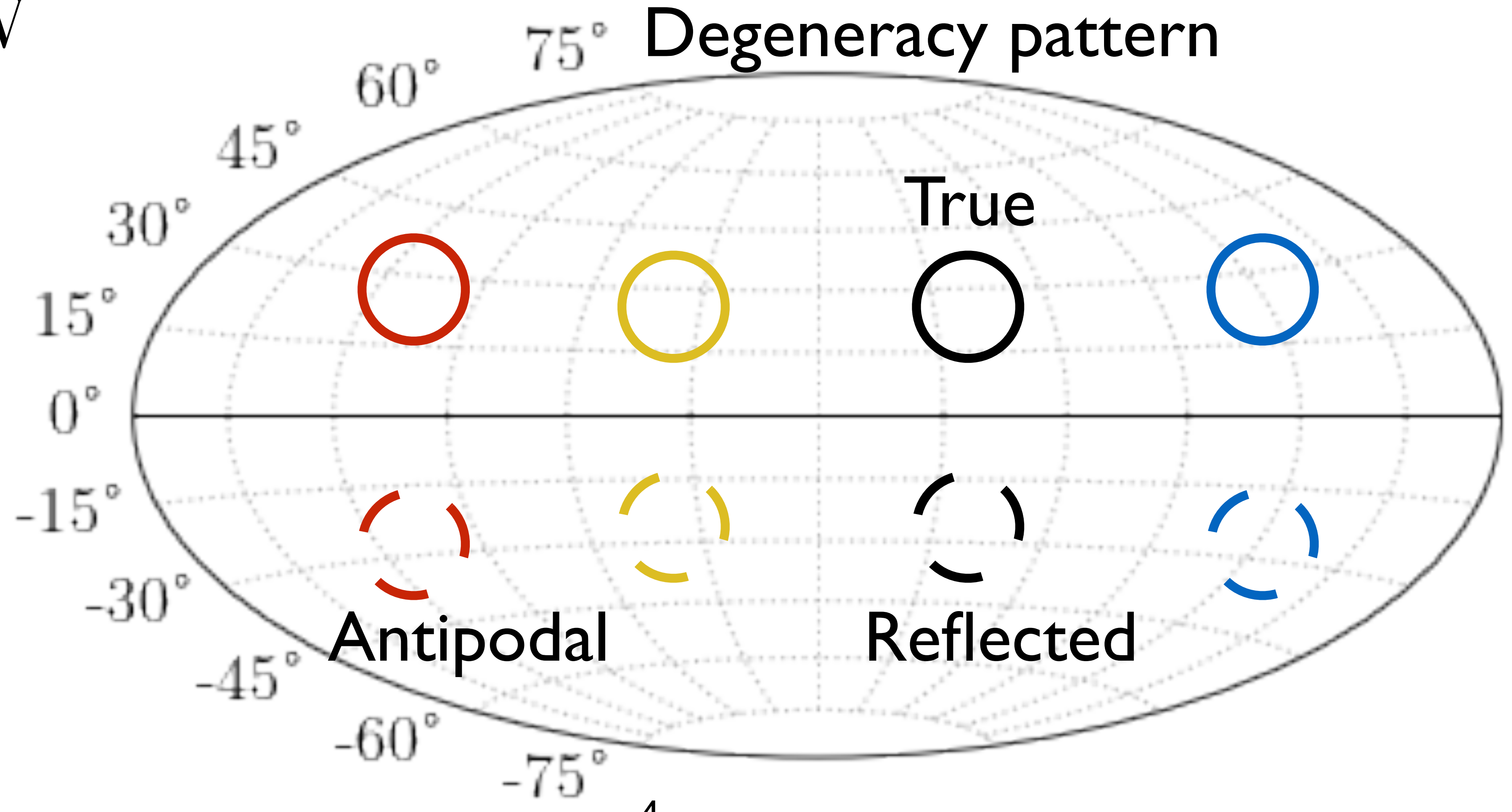
Low-f approximation: **two LIGO-type detectors** in motion [Cutler 1997]
High-f: more complicated



LISA frame

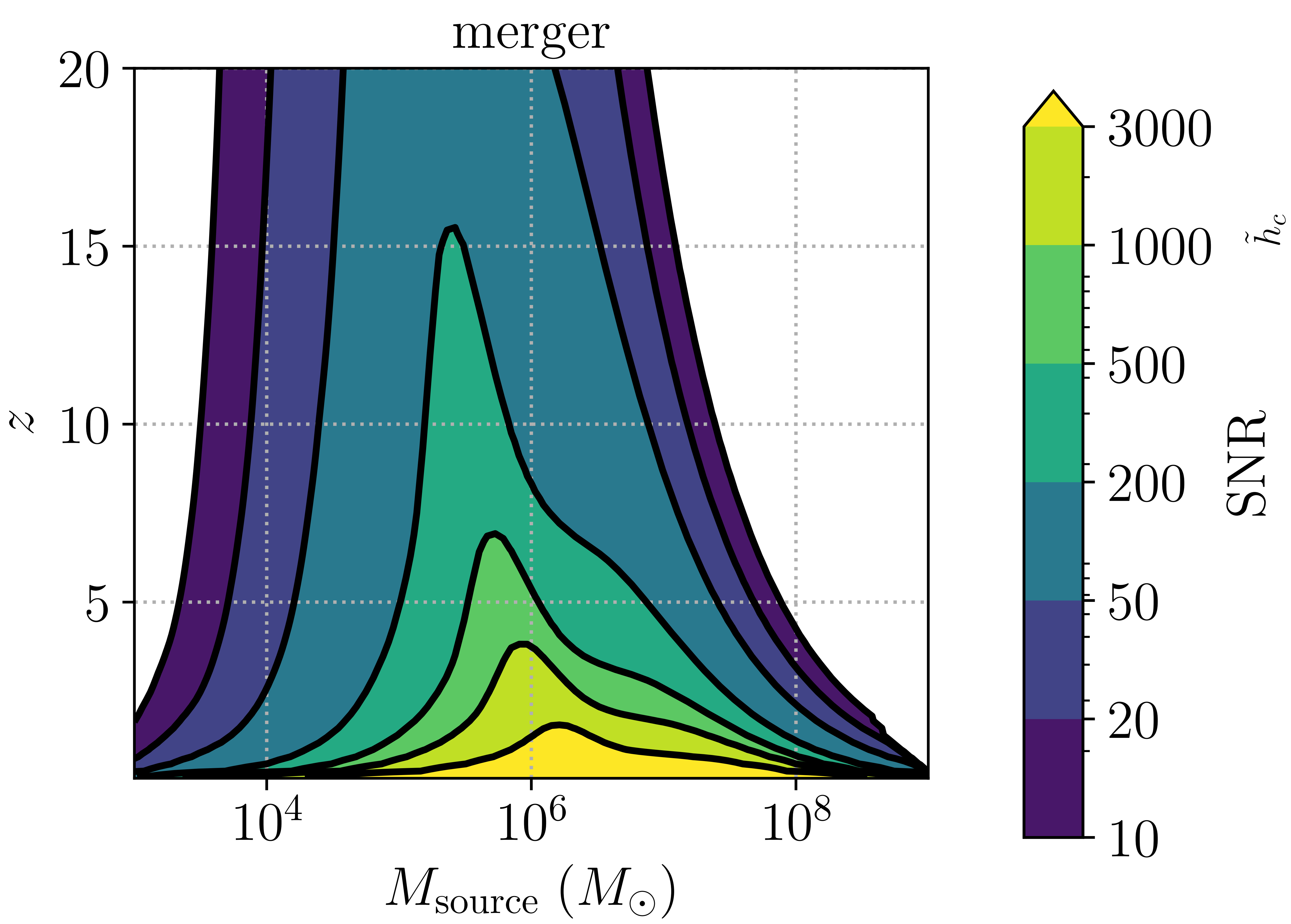
Short-lived signals: use frame based on LISA plane at merger





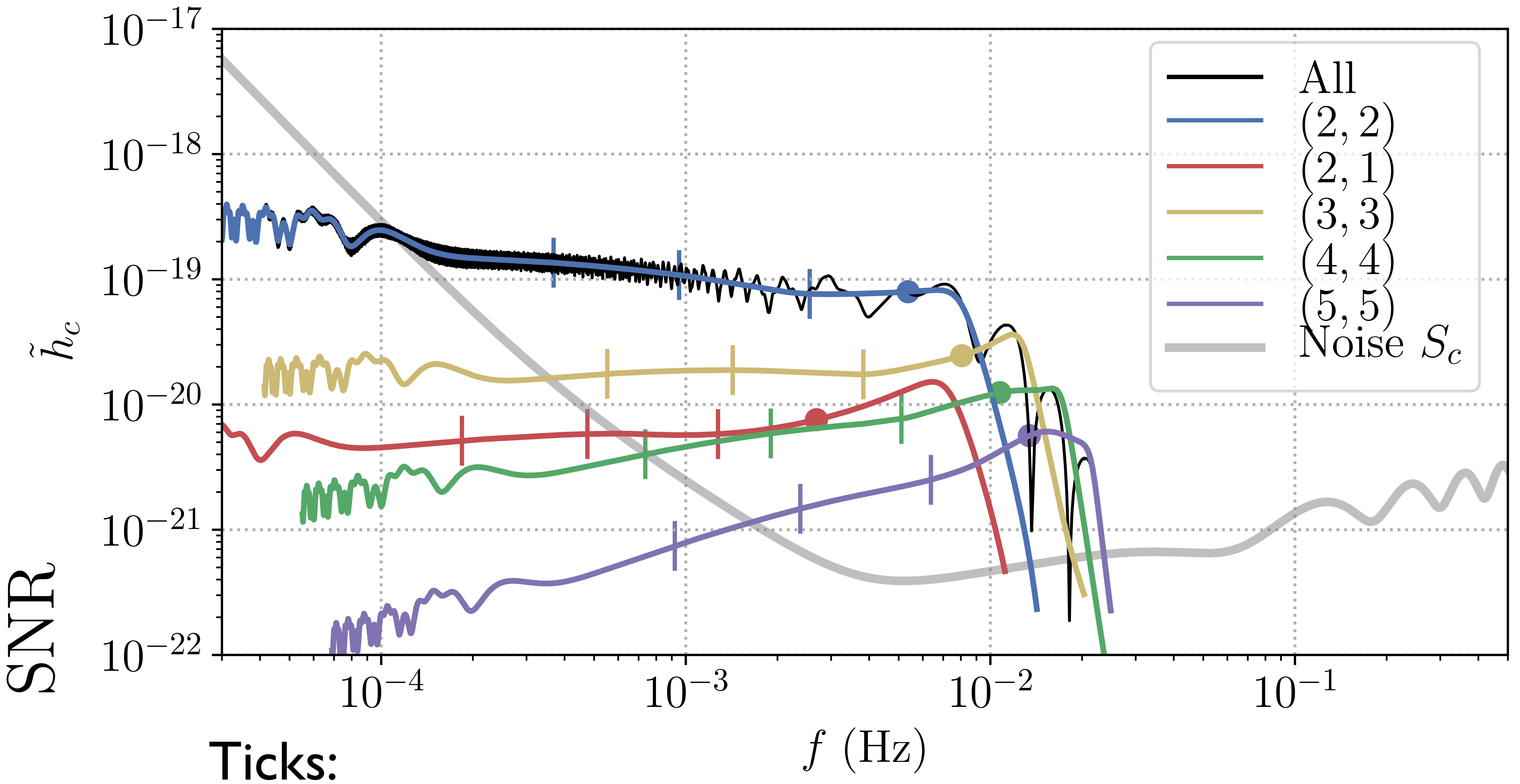
Massive black hole binaries

MBHB SNR contours post-merger



extremely loud signals…

Example MBHB GW signal with higher harmonics



Ticks:

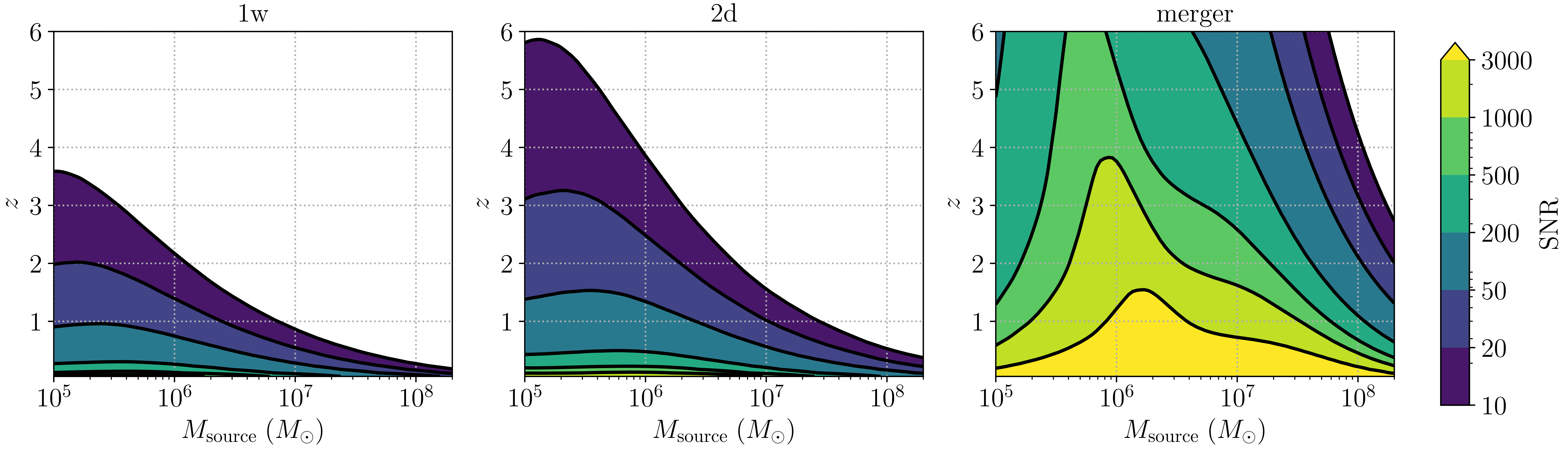
- SNR/64 (40h)
- SNR/16 (2.5h)
- SNR/4 (7min)
- merger

Higher harmonics strong at merger (break degeneracies)

Data analysis simulations still missing precession, eccentricity

MBHB signals are merger-dominated in SNR

MBHB SNR contours pre-merger and post-merger



Most of the SNR accumulates in the last hours before coalescence

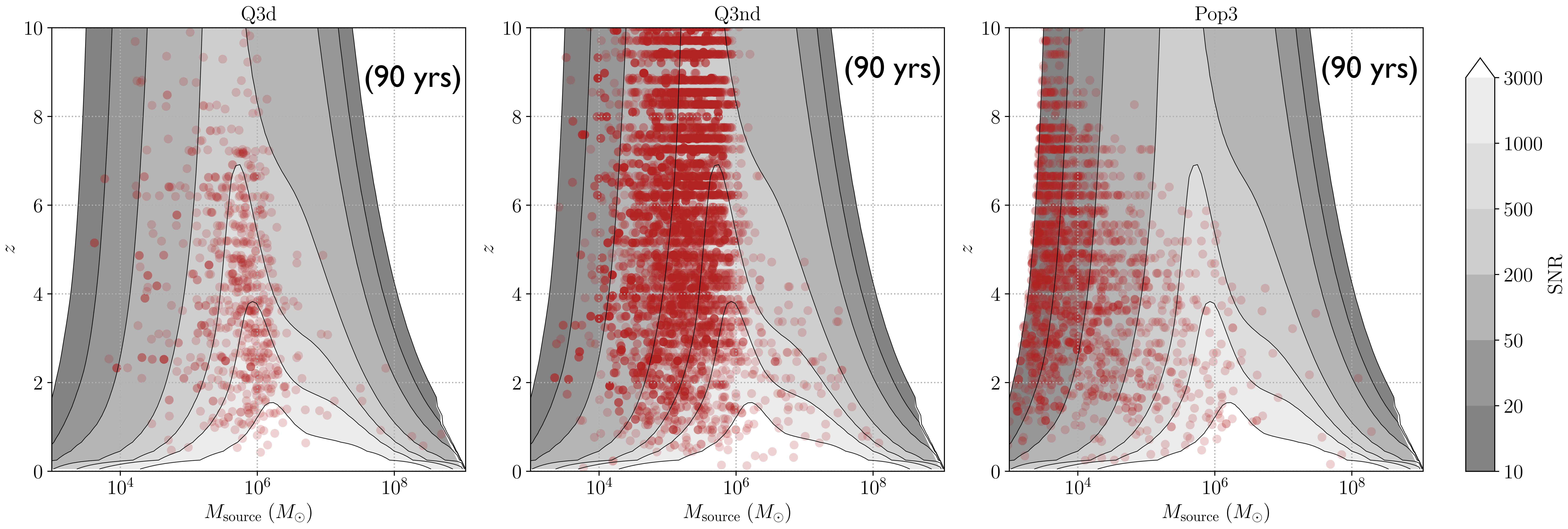
MBHB catalogs

Astrophysical models [Barausse 2012]:

- Heavy seeds - delay (Q3d)
- Heavy seeds - no delay (Q3nd)
- PopIII seeds - delay (Pop3)

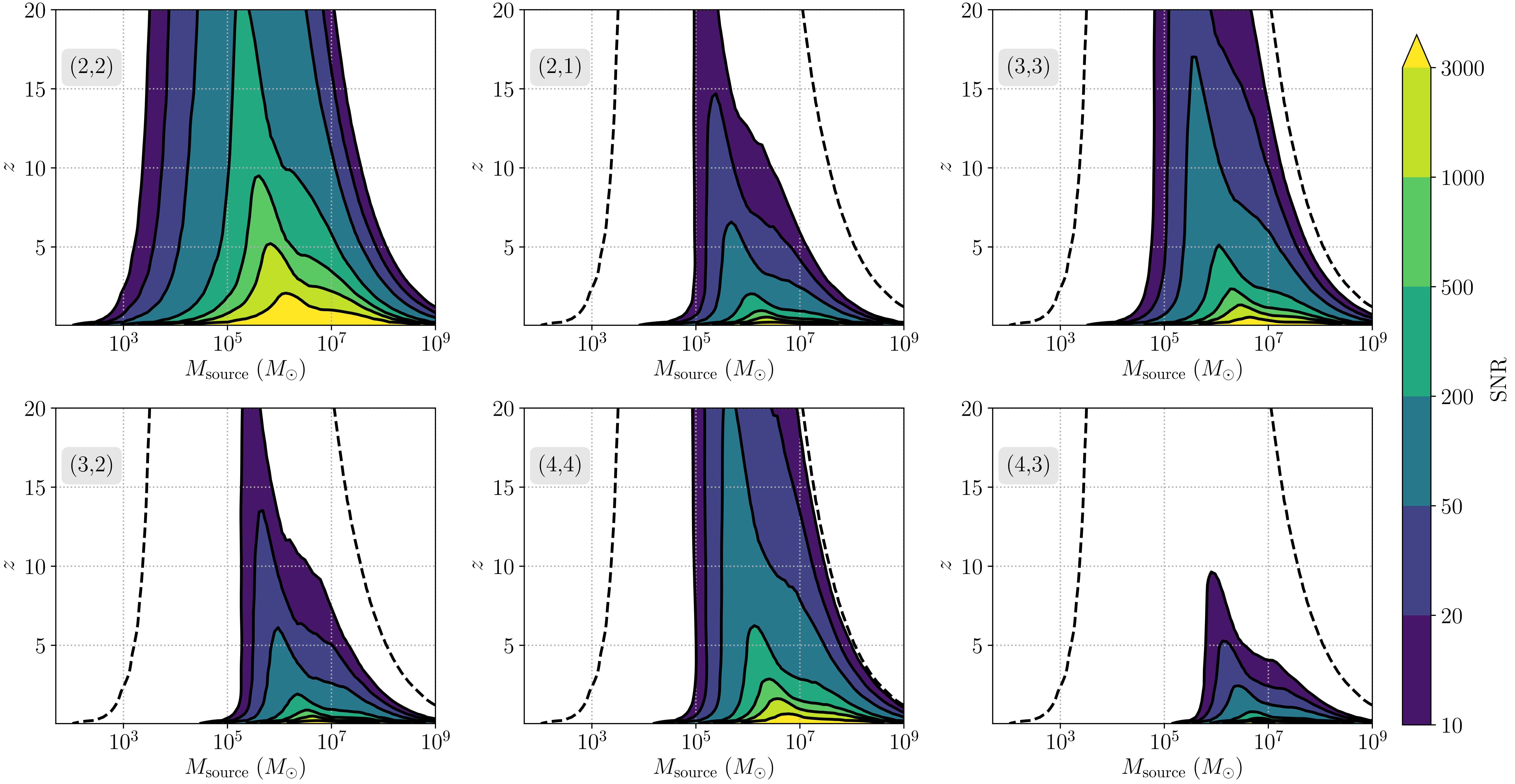
LISA detection rates from 90 yrs simulated:

- Q3d: 30 / 4yrs
- Q3nd: 471 / 4 yrs
- Pop3: 129 / 4yrs



MBHBs: SNR of higher harmonics

PhenomHM, q=5, averaging over spins and orientations



'Mode SNR':

$$\sqrt{(s_{\ell m}|s_{\ell m})}$$

Cross-terms not negligible

$$\sum_{(\ell m)\neq(\ell' m')} (s_{\ell m}|s_{\ell' m'})$$

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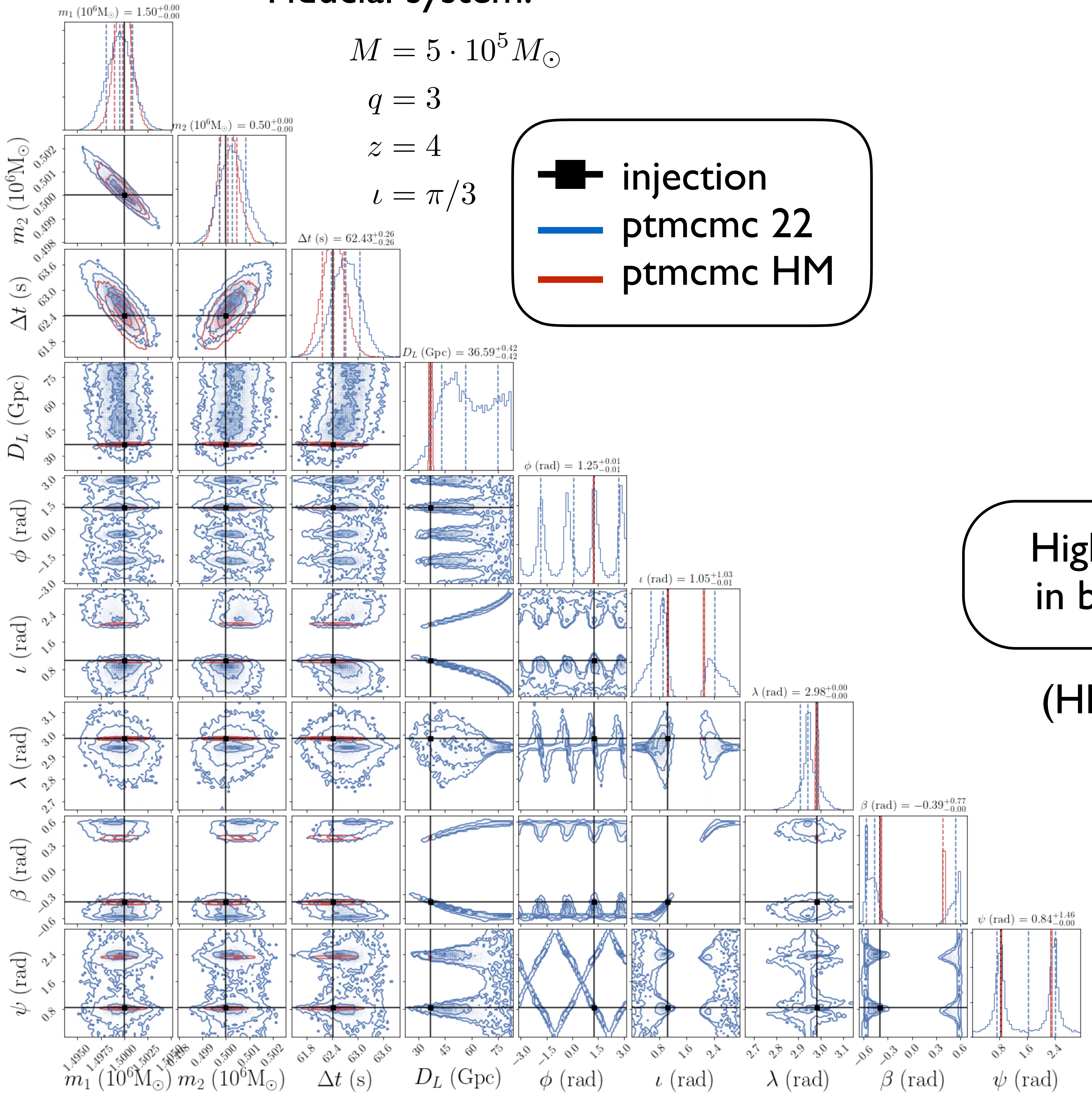
MBHBs: importance of higher modes in parameter estimation

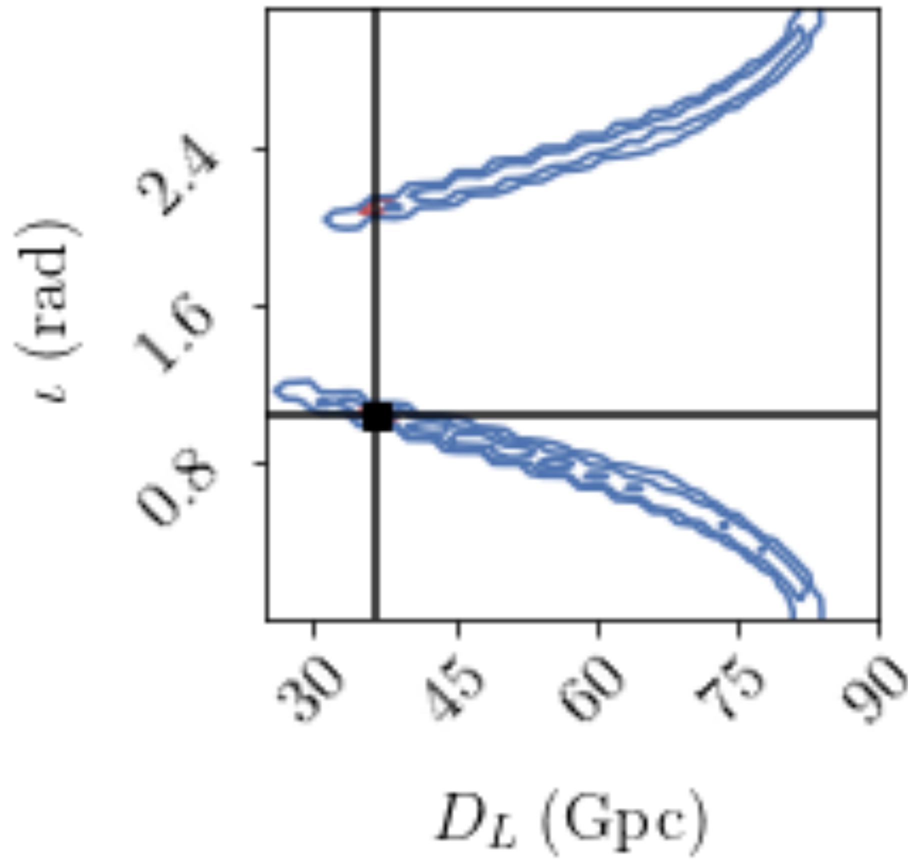
Fiducial system:

$$M = 5 \cdot 10^5 M_\odot$$
$$q = 3$$
$$z = 4$$
$$\iota = \pi/3$$



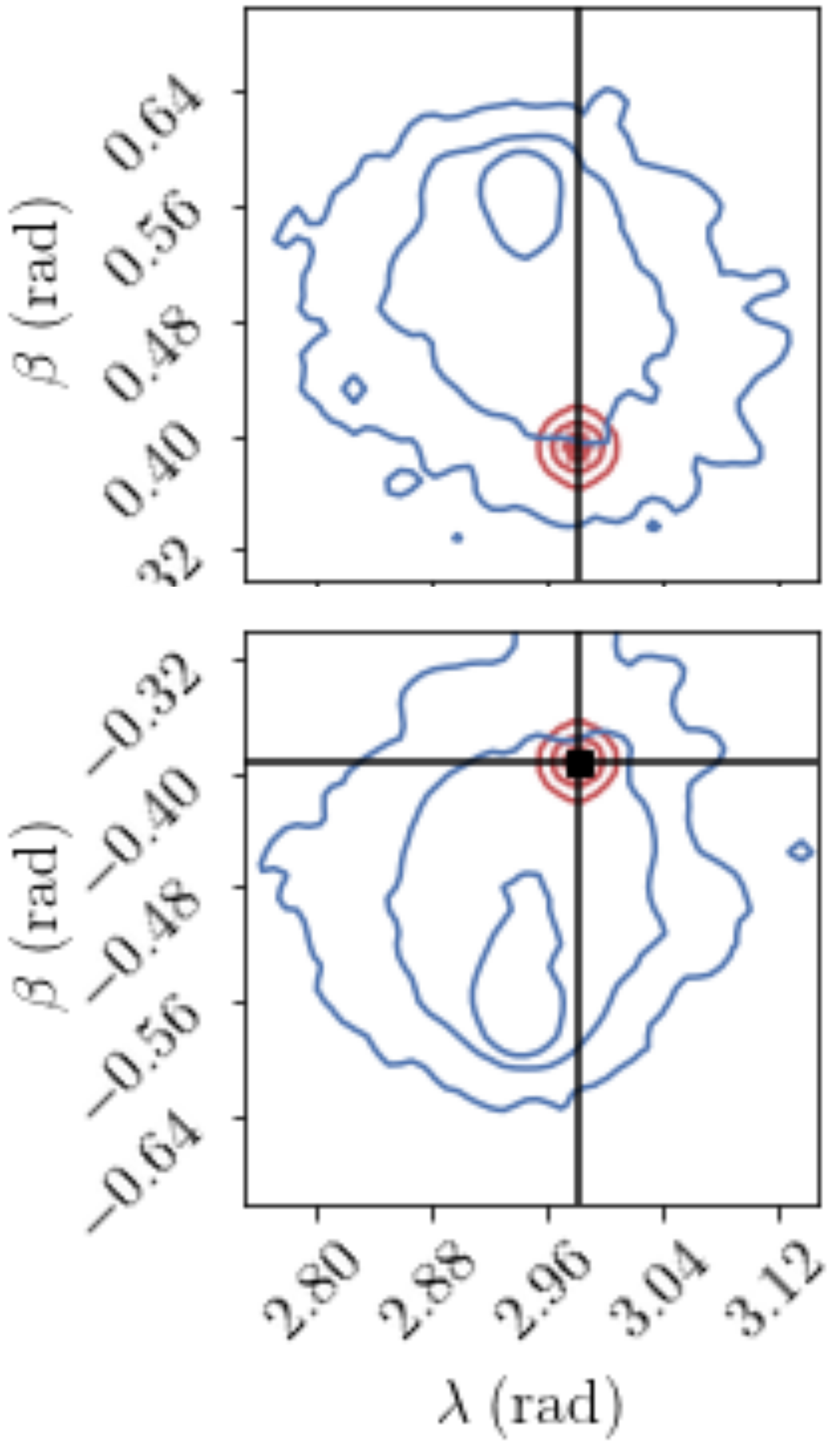
- injection
- ptmcmc 22
- ptmcmc HM

Distance-inclination



[Marsat&al 2020]

Sky position

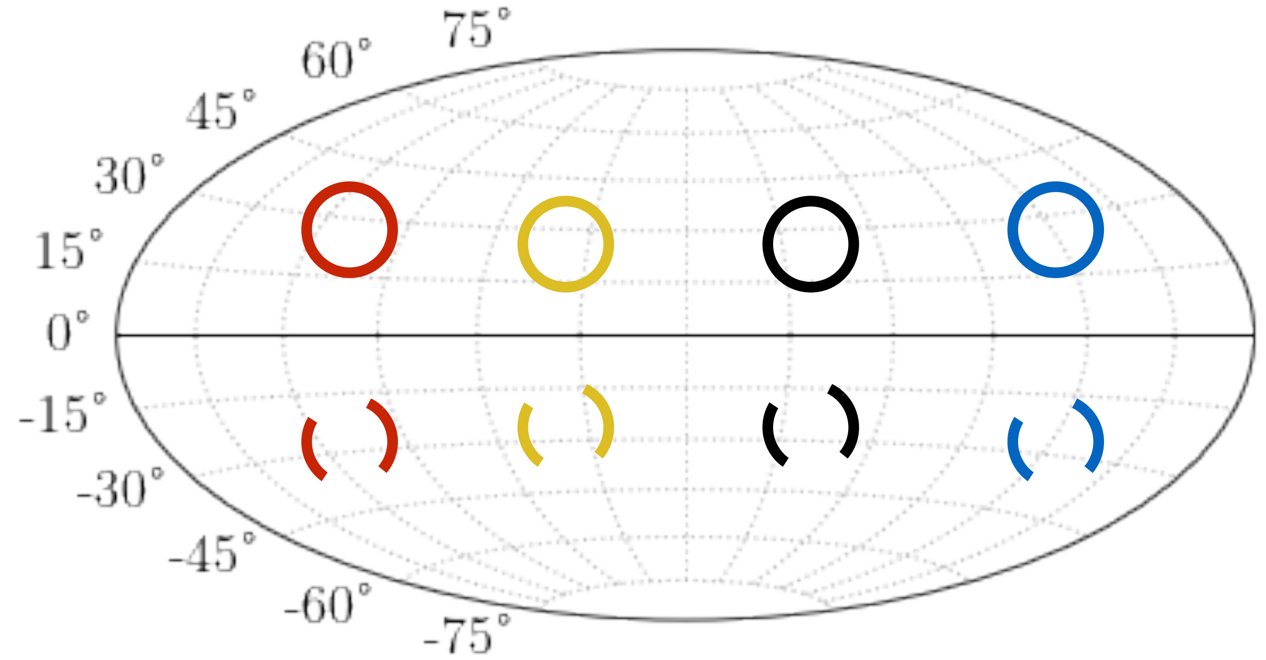


Higher harmonics crucial in breaking degeneracies

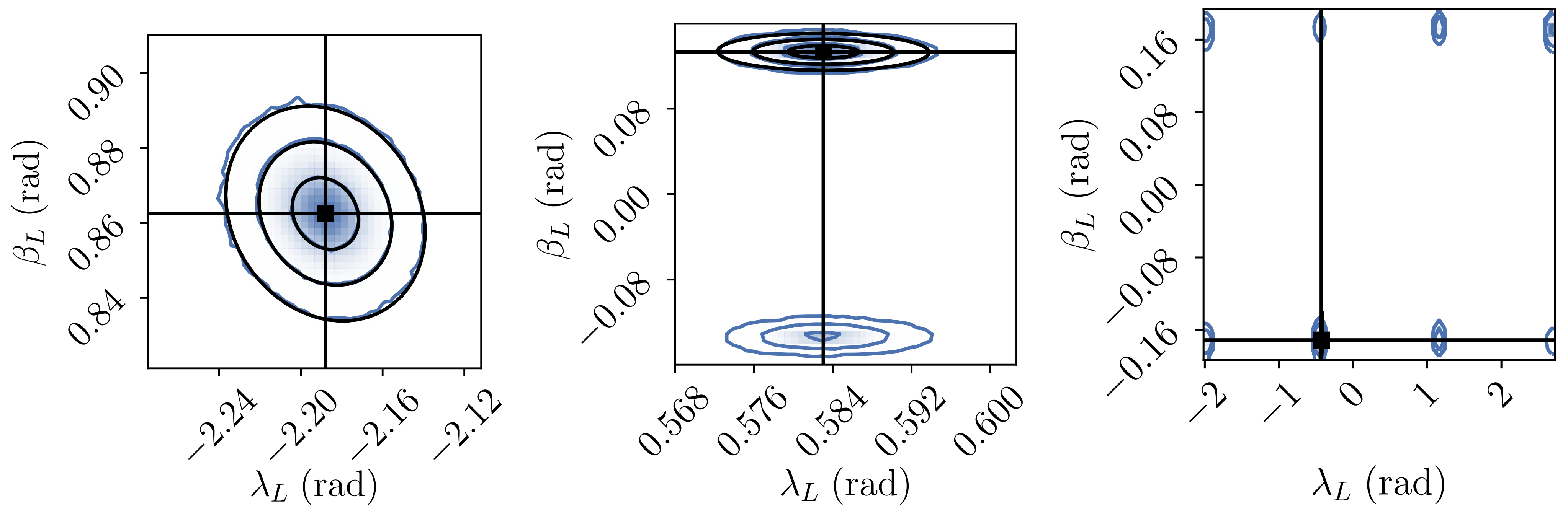
(HM weaker premerger and at low-mass)

Degenerate sky localization possible even with HM

MBHB catalogs: sky multimodality

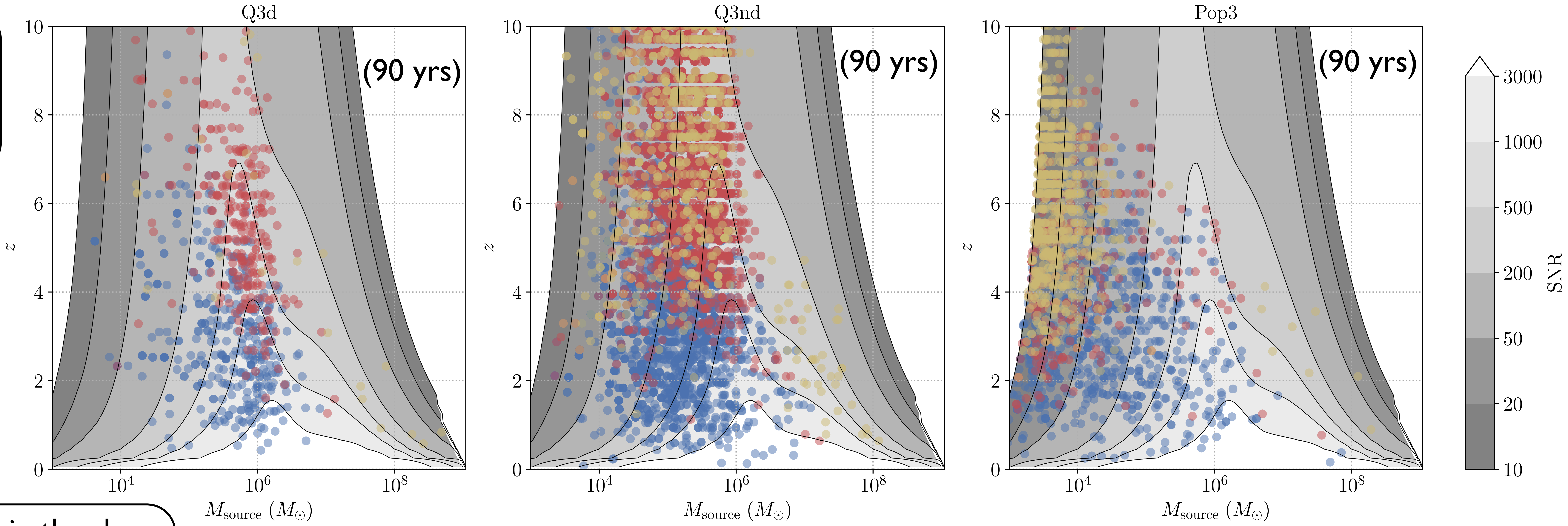


- Bayesian PE required to explore multimodal posteriors
- Simulation of 90yrs catalogs
- Custom proposals for degeneracies



- 1 mode
- 2 modes
- >2 modes

Threshold: 5% probability in the sky mode



Multimodality in the sky present, but rare for counterpart candidates post-merger

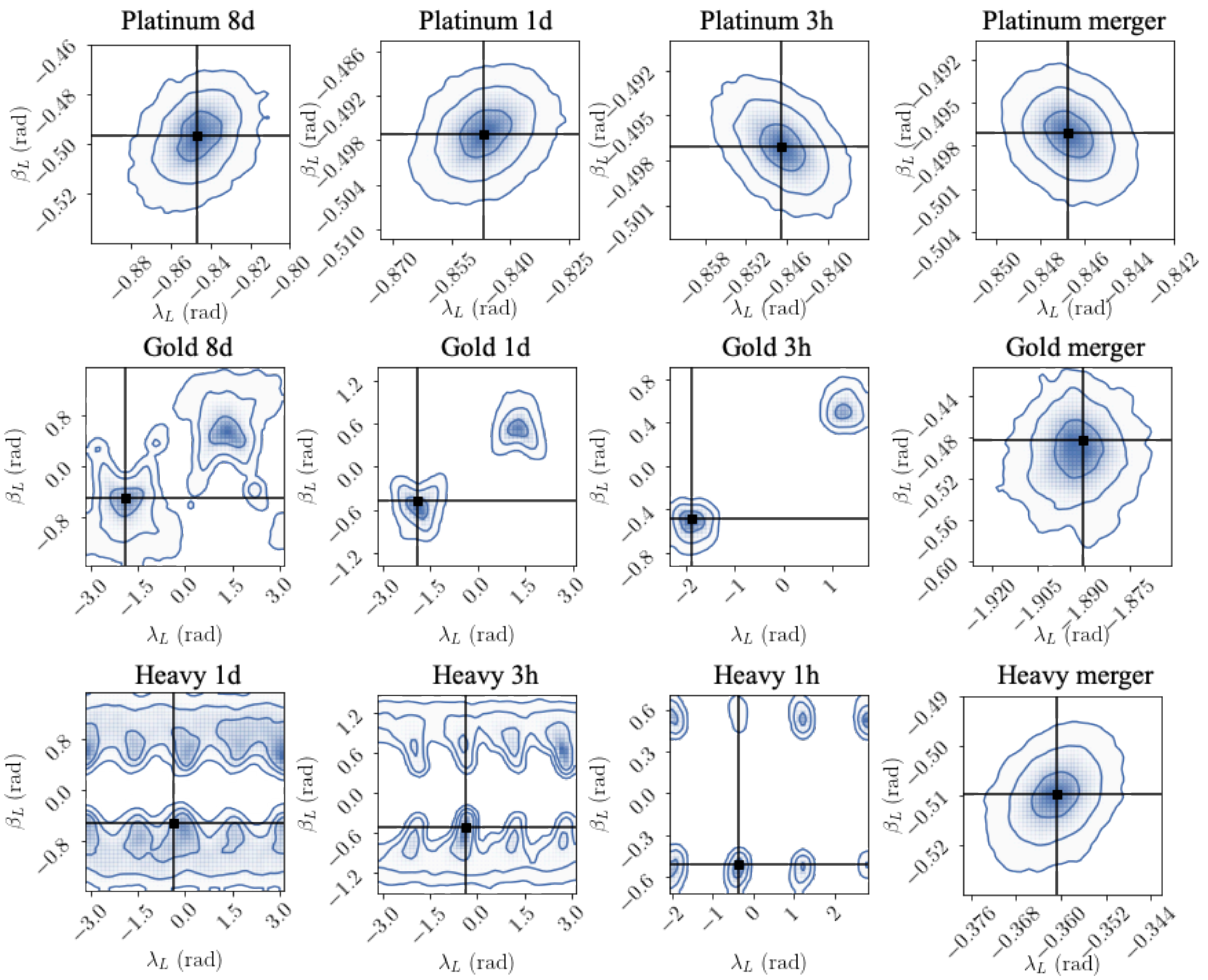
[see also Mangiagli&al 2022, A. Mangiagli's talk]

Localization of ‘golden’ MBHB sources: degeneracies

Bayesian sky localization
cutting at different times

- ‘Gold’: M3e6, z=1
- ‘Heavy’: M1e7, z=1
- ‘Platinum’: M3e5, z=0.3

- Wide range of multimodalities dep. on parameters
- Post-merger localization unimodal for ‘golden’ MBHBs



[Piro&al, in prep]

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Parameter estimation tool: lisabeta

Bayesian analysis

$$(h_1|h_2) = 4\mathrm{Re}\int df\, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$

Posterior: $p(\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$

Likelihood: $\ln\mathcal{L}(d|\theta) = -\sum_{\text{channels}} \frac{1}{2}(h(\theta) - d|h(\theta) - d)$

Data: signal+noise $d = s + n$

Producing samples from the posterior takes millions of evaluations of lnL !

Approximation levels

- Fisher matrix: local Gaussian approx. for lnL for high may work in high SNR limit, but misses degeneracies
- Simplified PE: MCMC initialized from Fisher, 0-noise
- Full simulation with unknown signal, noise (LDC)
- Superposition of sources, unknown noise, noise artifacts…

lisabeta package

- Science prospective
- Prototyping real analysis (LDC)
- Source types: MBHBs, SBHBs for now — GBs soon
- Consortium-available (full members, public soon)

https://gitlab.in2p3.fr/marsat/lisabeta_release

Features

- MBHB waveforms: PhenomD, PhenomHM, aligned spins with HM
- Fast Fourier-domain response
- SNR computations
- Fisher matrices
- MCMC: ensemble sampler with parallel tempering (ptemcee, [Vousden&al 2015])
- Informed proposals to deal with sky degeneracies
- Accelerated likelihoods (few ms)

Accelerating likelihoods: heterodyning

Overview

- Structure of the likelihood

$$\ln \mathcal{L} = -\frac{1}{2}(h-d|h-d) \qquad (a|b) = 4\mathrm{Re}\int df\, \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}$$

$h = Ae^{i\Phi}$ smooth amp/phase $\quad d$ numerical data

- Introduce a reference waveform $\quad \overline{h}(f)$

$\zeta(f) \equiv h(f)/\overline{h}(f)$ now **slowly variable**

in the vicinity of reference parameters

- Separate integrand in slowly and rapidly variable parts

$$(h|d) \sim \int df\, \frac{\overline{h}d^*}{S_n} \times \zeta \qquad (h|h) \sim \int df\, \frac{\overline{h}\overline{h}*}{S_n} \times \zeta\zeta^*$$

- Interpolate and precompute

$\zeta \quad$ interpolated on a coarse, reduced grid

$$(h|d) \sim \sum_i \int_{f_i}^{f_{i+1}} df\, \frac{\overline{h}d^*}{S_n} \times (a_i + b_i f)$$

$$(h|h) \sim \sum_i \int_{f_i}^{f_{i+1}} df\, \frac{\overline{h}\overline{h}*}{S_n} \times (a_i + b_i f + c_i f^2)$$

- Evaluate

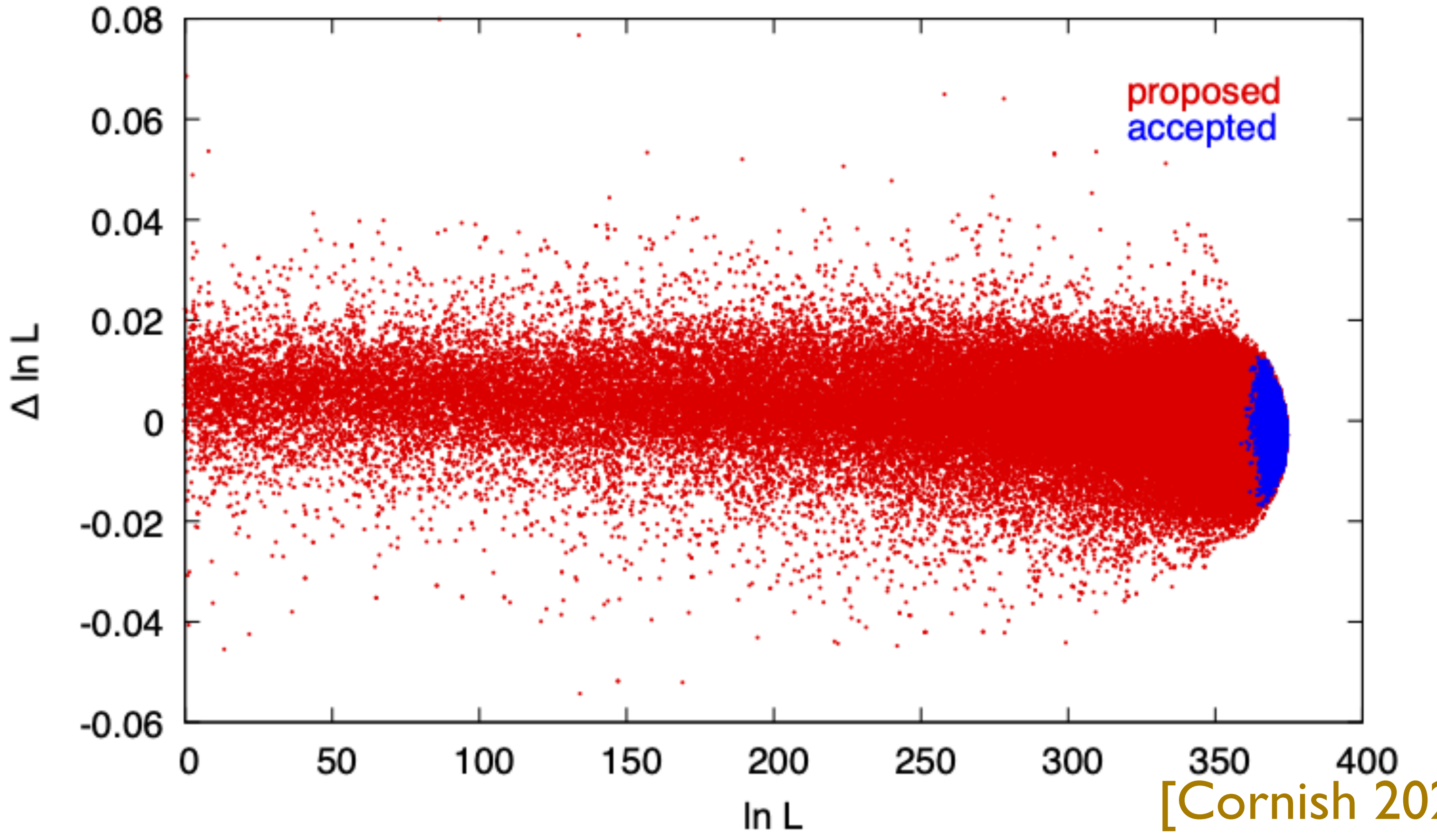
$h \quad$ on coarse grid, then sum weights and coeffs

[Cornish 2010, Cornish 2021]

[Zackay+ 2018] (relative binning)

Usage in practice

- Small reduced grid (N~100): cost <~ ms
- Different interpolation methods (linear, polynomial)
- Requires reference waveform (first guess for signal parameters) — can be updated on the way
- Distinguish burn-in from actual sampling, the latter happens close to the true signal



[Cornish 2021]

Accelerating likelihoods: heterodyning example for MBHB

Decomposing the likelihood:

$$\ln \mathcal{L} = -\frac{1}{2}(s-d|s-d)$$
$$= -\frac{1}{2}(s-s_0|s-s_0) + (s-s_0|d-s_0) - \frac{1}{2}(s_0-d|s_0-d)$$

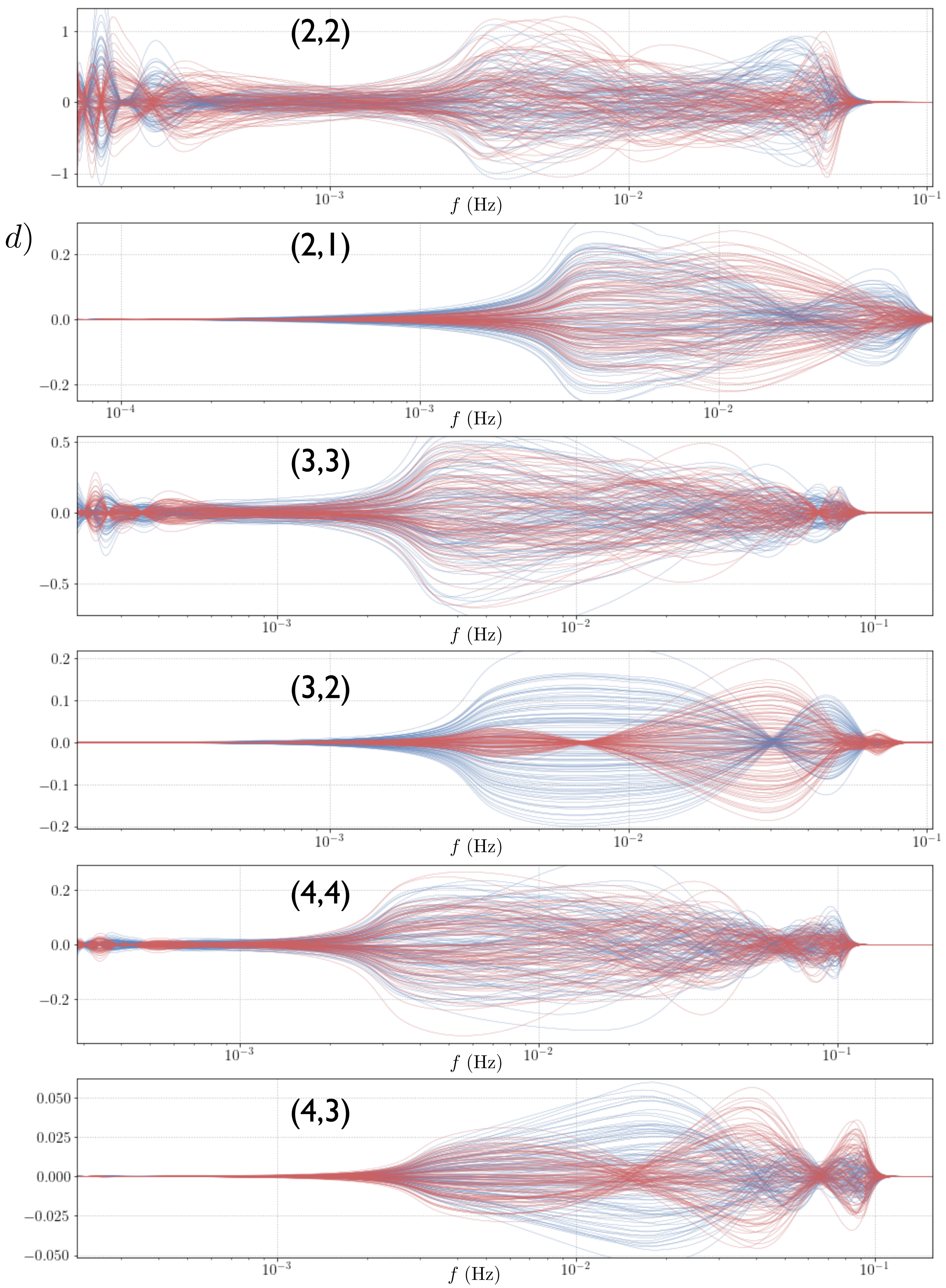
Residuals from reference waveform:

$$s_{\ell m} - s^0_{\ell m} = r_{\ell m} e^{i\Phi^0_{\ell m}}$$

Implementation:

$$(s-s_0|s-s_0) = \sum_{\ell m}\sum_{\ell' m'} (r_{\ell m} r^*_{\ell' m'} | e^{i(\Phi^0_{\ell' m'} - \Phi^0_{\ell m})})$$
$$(s-s_0|d-s_0) = \sum_{\ell m} (r_{\ell m} | e^{-i\Phi^0_{\ell m}}(d-s_0))$$

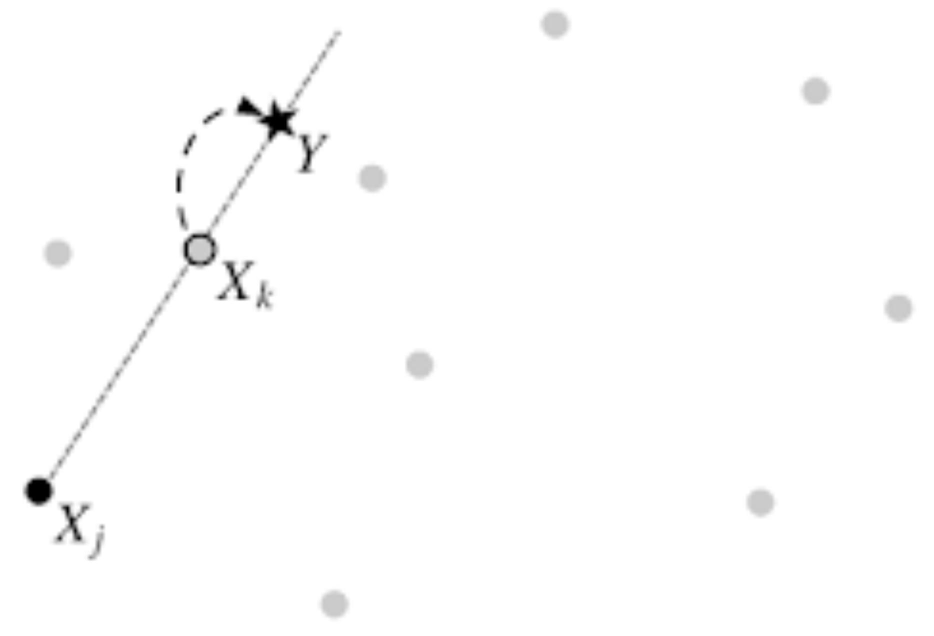
- Fix a sparse frequency grid (~128)
- Linear interpolation of the residuals, mode-by-mode
- Precompute 0-th and 1st polynomial inner products against phase and data terms, with a fine resolution



Dealing with degeneracies

Ensemble sampling

- Evolve a population of walker in parallel
- Self-tuning proposal based on the other walkers



Parallel tempering

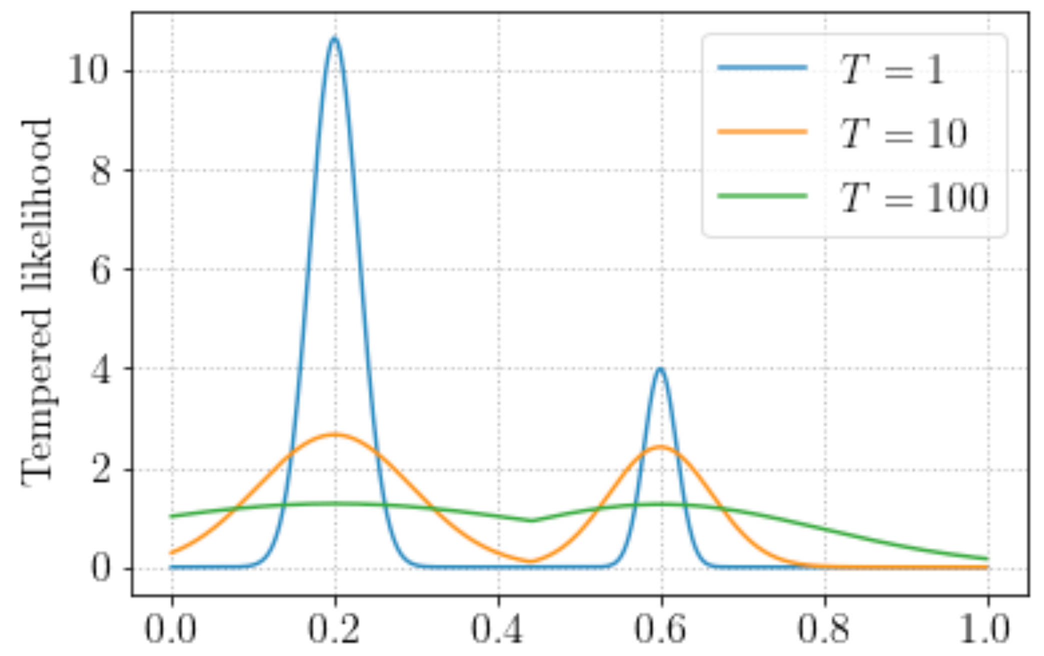
- Introduce parallel chains with temperatures, posterior: $p(\theta)^{\beta_i}$

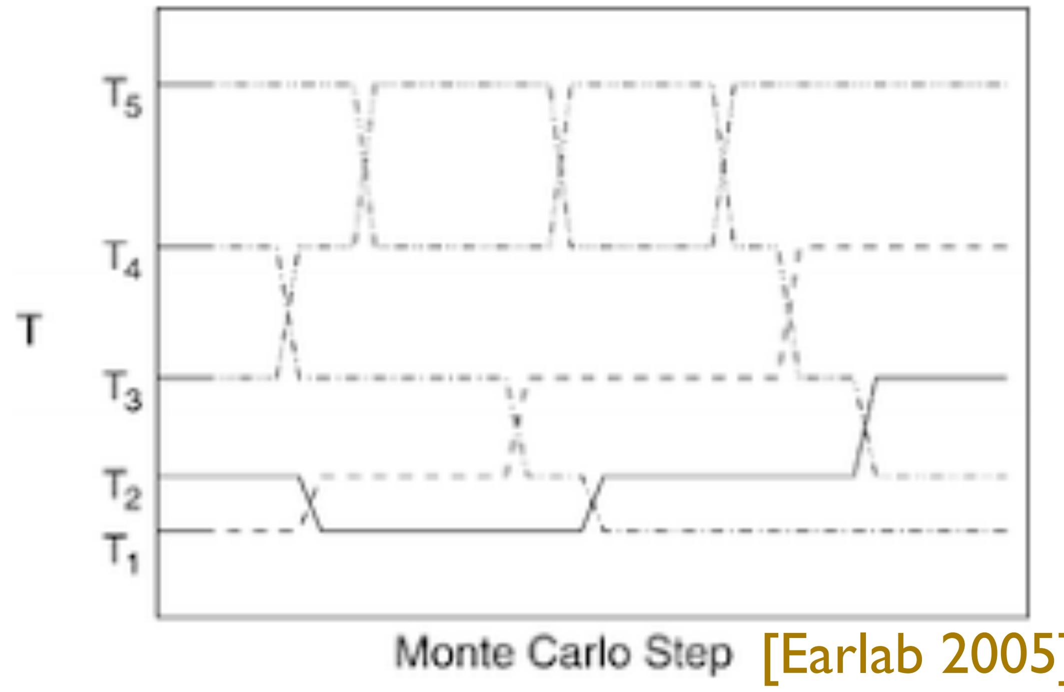
$$\beta_i = 1/T_i$$

- Propose swaps with acceptance:

$$p_{\rm swap} = \min \left[1, \left(\frac{p(\theta_i)}{p(\theta_j)} \right)^{\beta_j - \beta_i} \right]$$

- Crucial for robustness, avoids being stuck in a local maximum



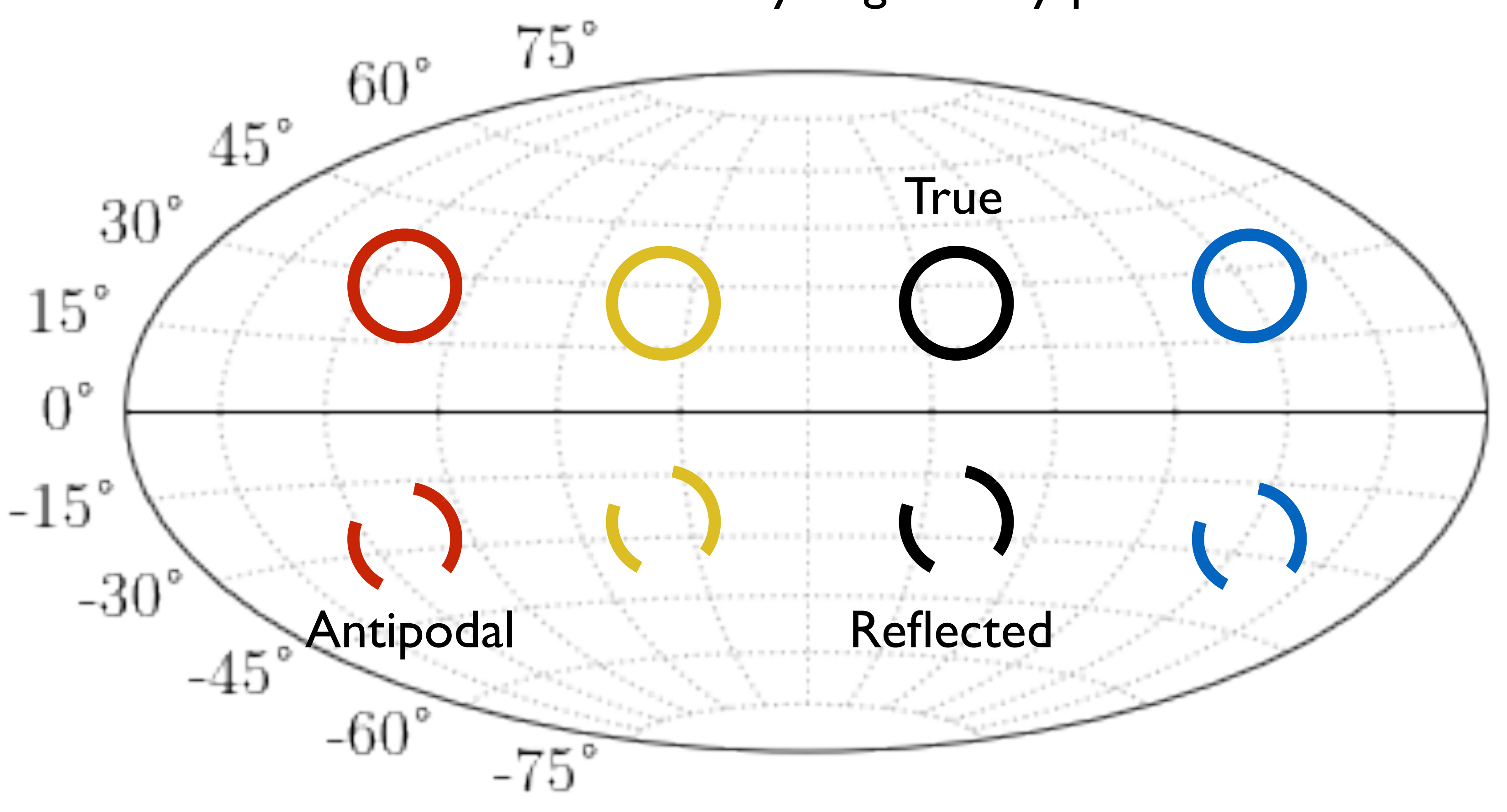


[Earlab 2005]

Tailored jump proposals

- In presence of known degeneracies, include jumps in proposal
- Very efficient for very disconnected multimodal posteriors

LISA MBHB sky degeneracy pattern

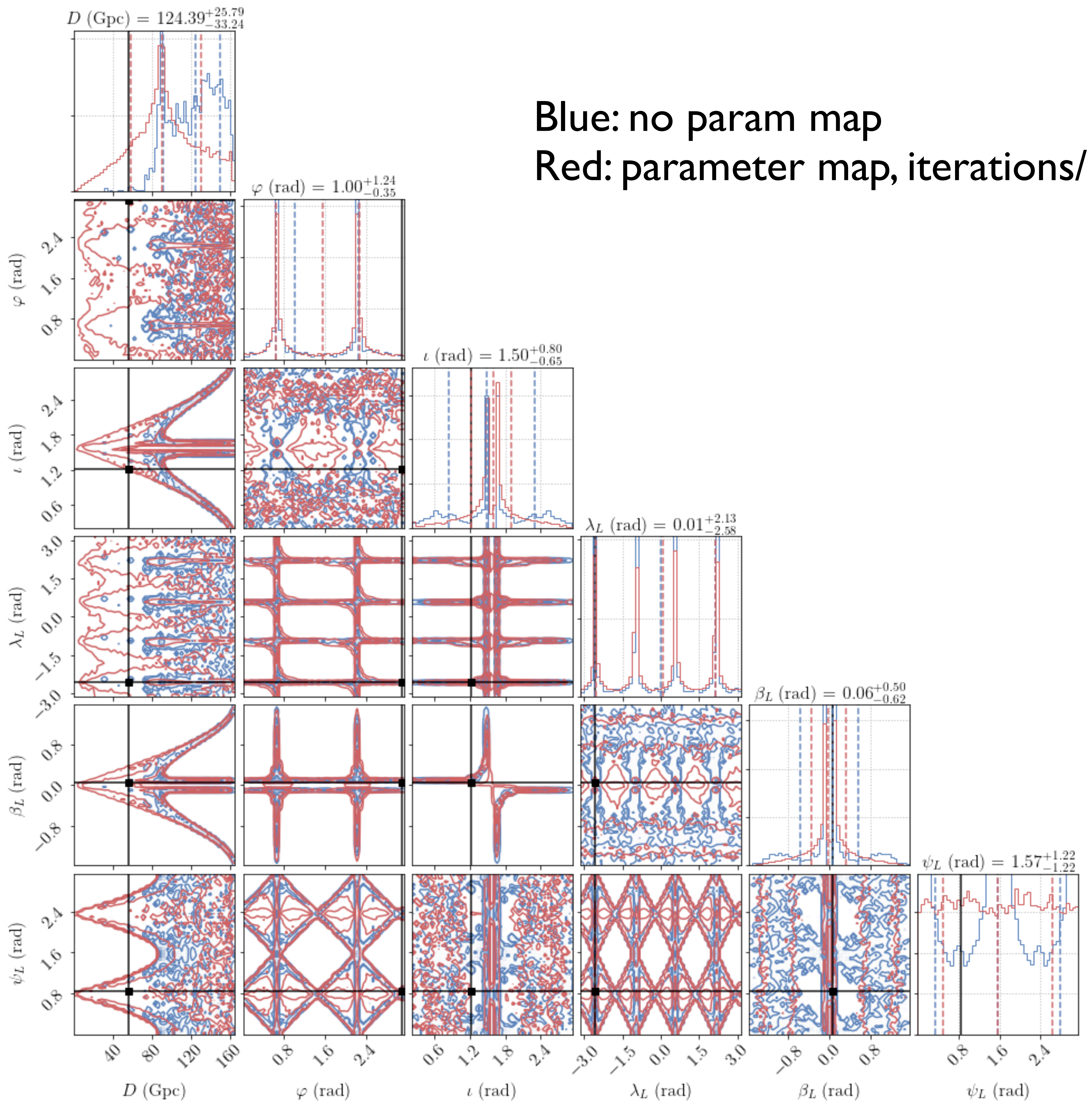


Tailored parameter map

- Transform to new variables (close to observables) in which the posterior is close to a Gaussian
- Easy to implement if transformation and Jacobian analytic

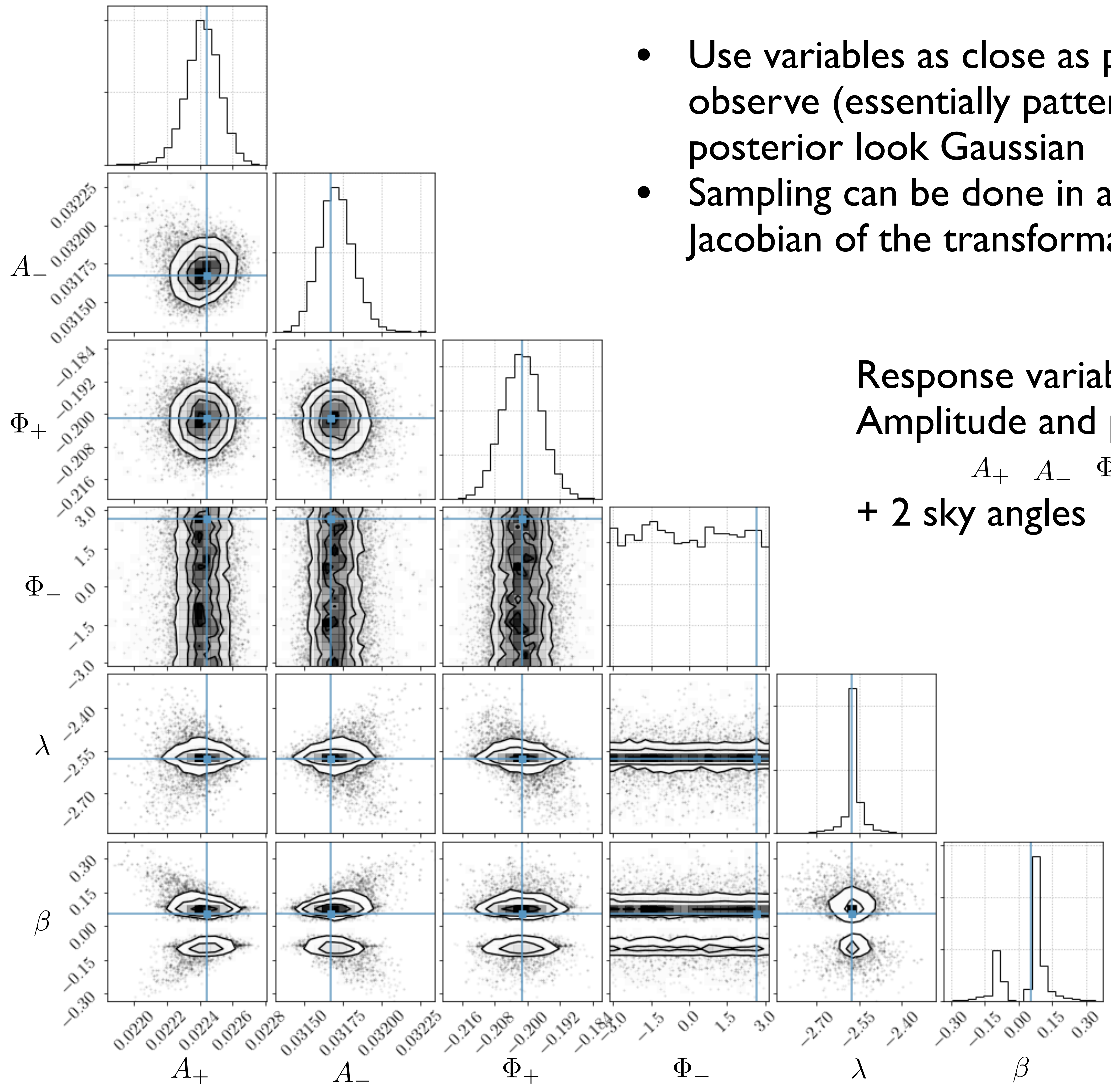
Dealing with degeneracies: maximally degenerate case

Toy problem, completely degenerate extrinsic 22 likelihood without motion and high-f effects



Blue: no param map
Red: parameter map, iterations/10

Dealing with degeneracies: parameter map



- Use variables as close as possible to what we really observe (essentially pattern functions), to make the posterior look Gaussian
- Sampling can be done in any set of parameters, with Jacobian of the transformation analytic here

Response variables: 2 complex pattern functions
Amplitude and phase

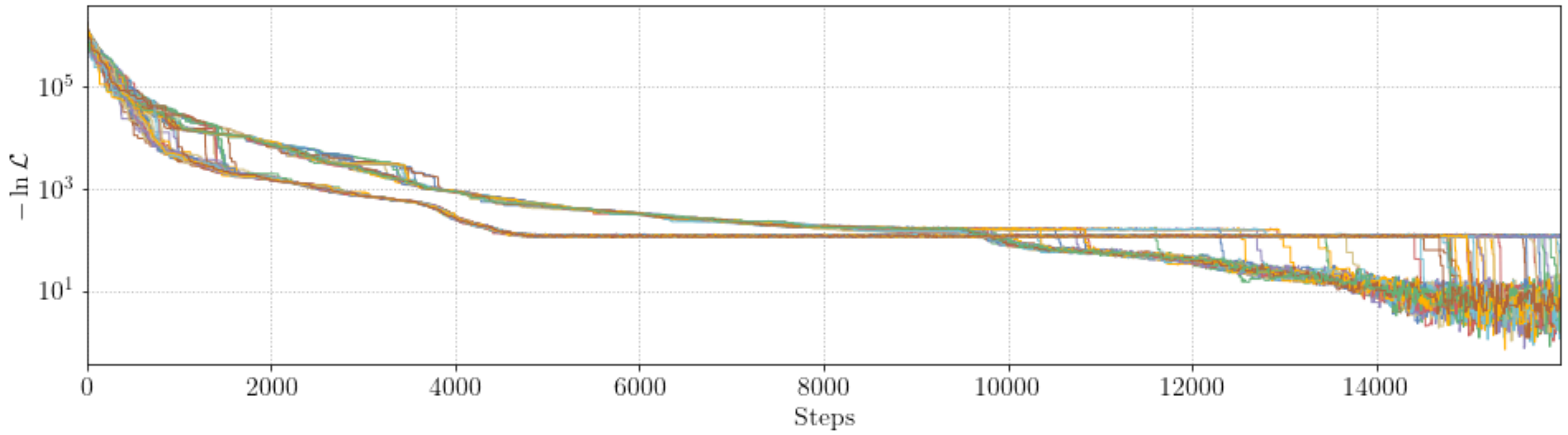
$A_+ \quad A_- \quad \Phi_+ \quad \Phi_-$

+ 2 sky angles $\quad \lambda \quad \beta$

Analytical transformation, no extra cost

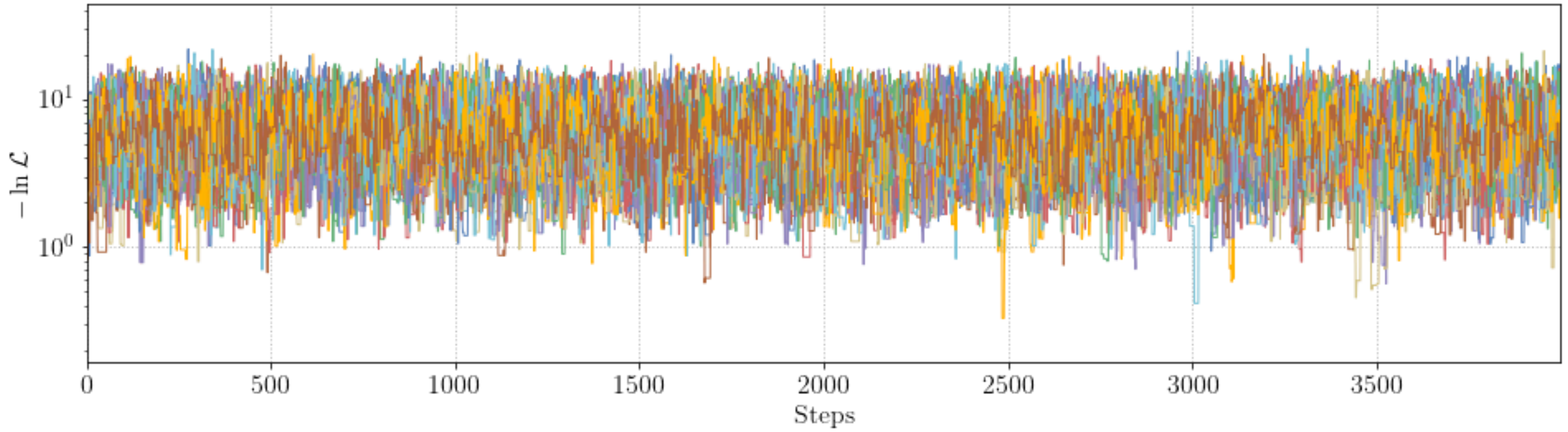
Accelerating PE: burn-in vs sampling

Burn-in (here struggling to find the signal !)



Scale of likelihood with completely wrong signal: $\ln \mathcal{L}_{\rm bad} \sim -{\rm SNR}^2$

Sampling (not moving much in likelihood)



Search and burn-in are different:

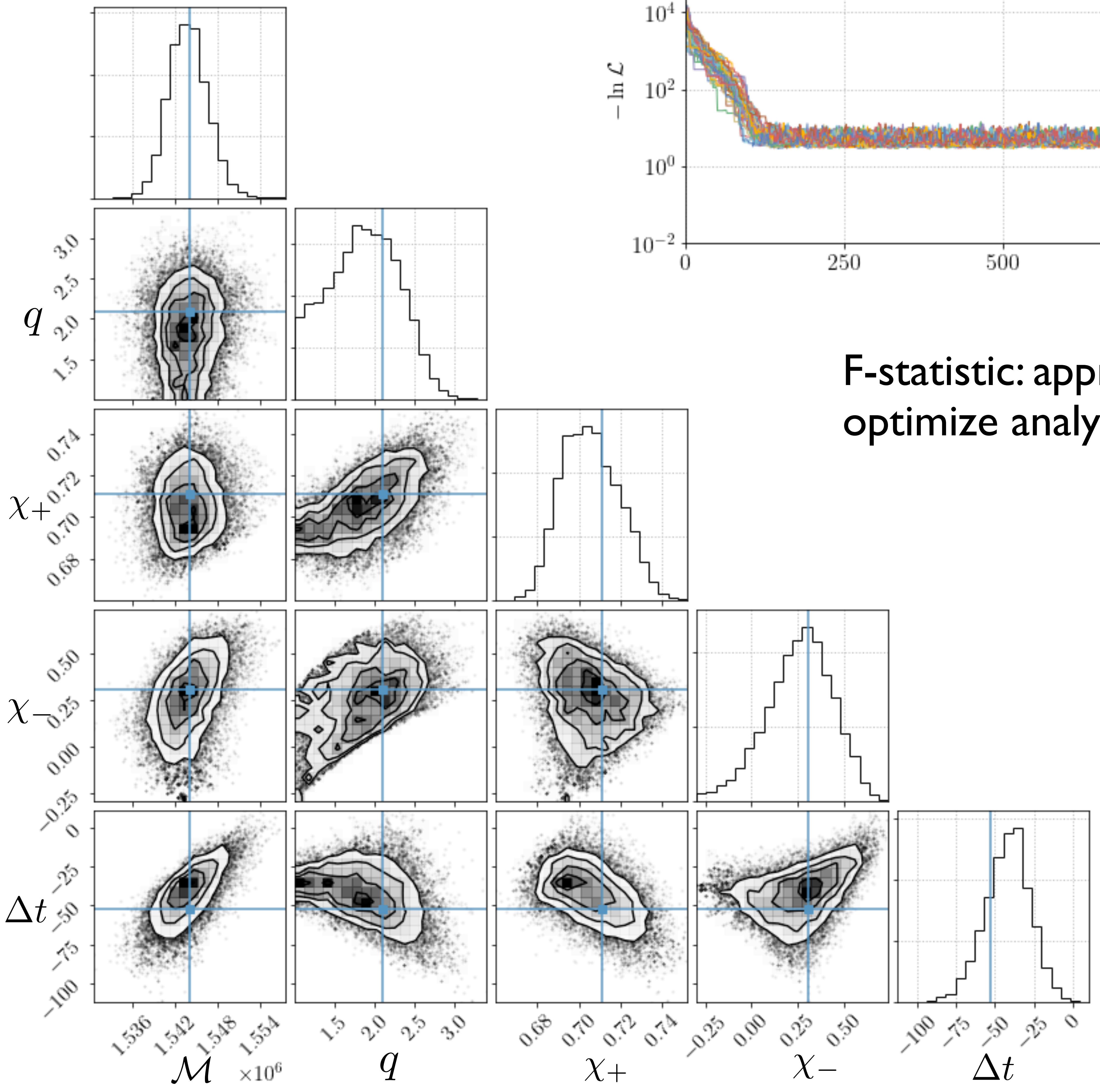
- Sampling algorithm can be inefficient to search for a signal
- First guess of the signal's parameters allows to accelerate likelihoods with heterodyning

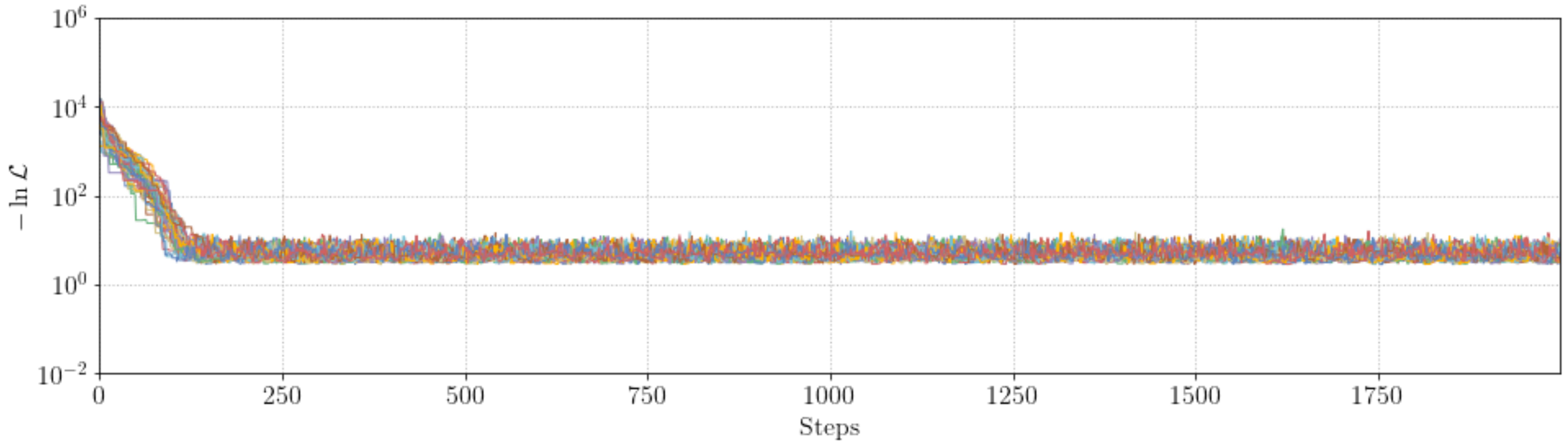
Different use cases for PE:

- Simulating realistic PE: start from prior
- Prospective parameter estimation, only interested in final result: cheat with initialization

Techniques for burn-in (search) or sampling can differ !

MBHB example: I) F-statistic search on small data segments



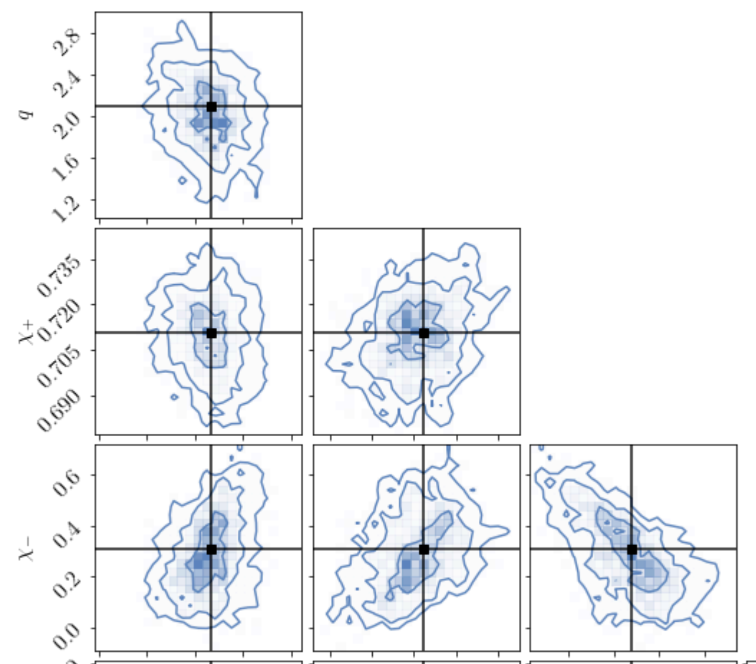


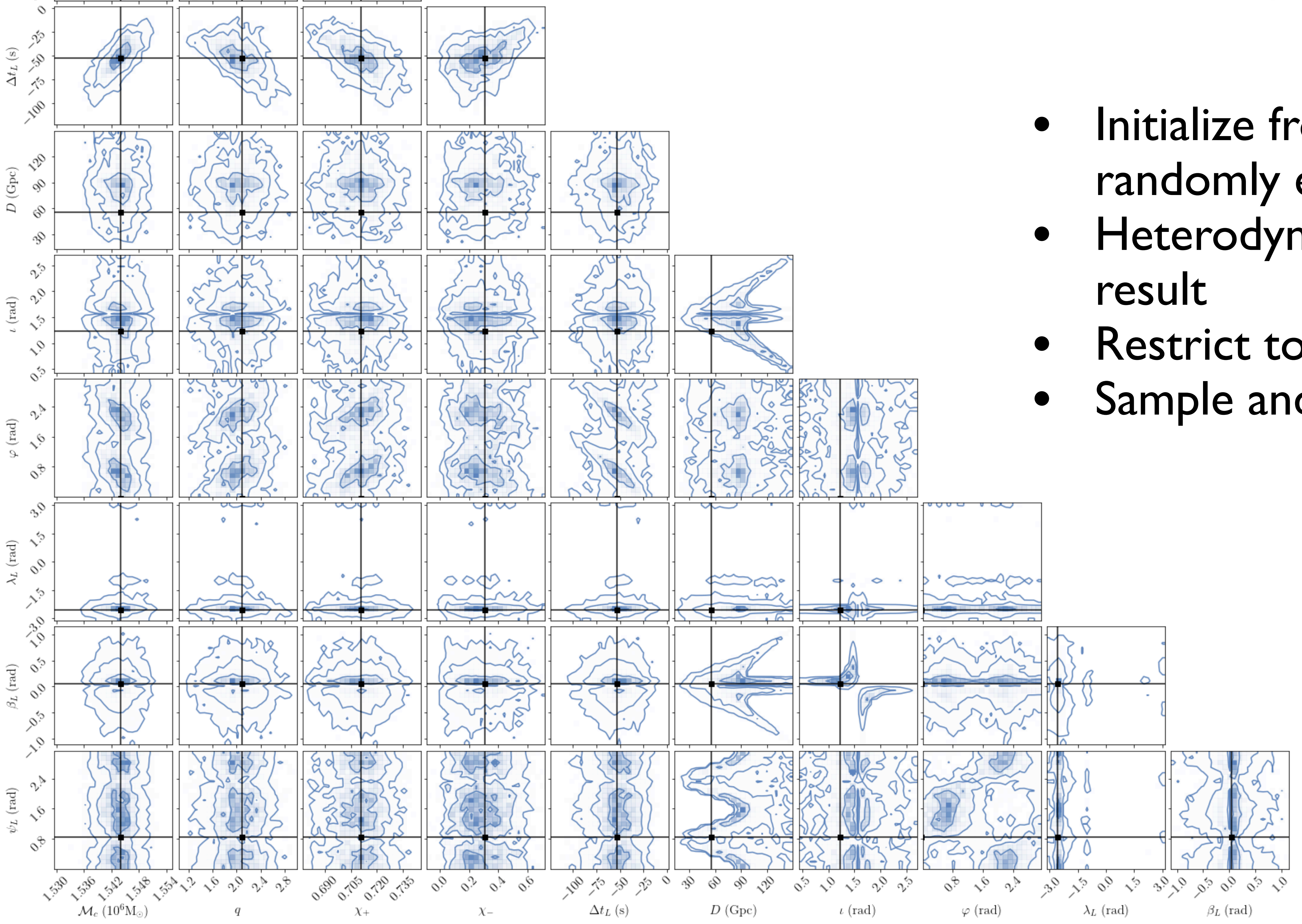
F-statistic: approximate response (low-frequency, no motion, 22 mode), optimize analytically over distance, inclination, phase, polarization, sky

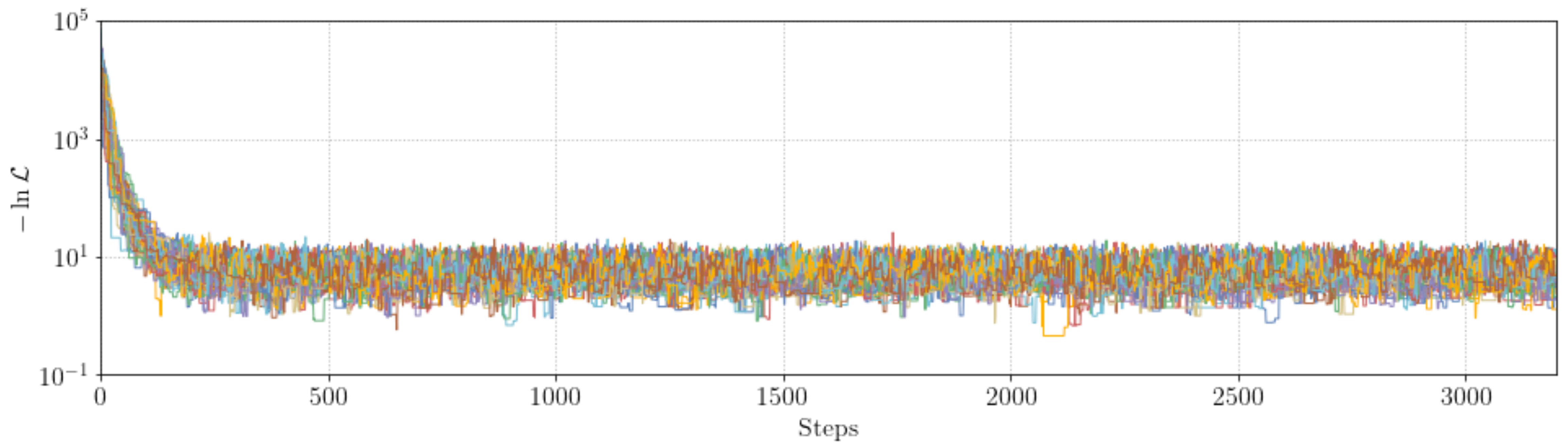
$$\mathcal{F} = \max_\lambda \ln \mathcal{L}(\theta, \lambda)$$

- Select short data segment
- F-statistic as pseudo-likelihood
- Sampling easier for a lower dimensionality
- Get a first guess of intrinsic parameters + time

MBHB example: II) initial PE with low frequencies

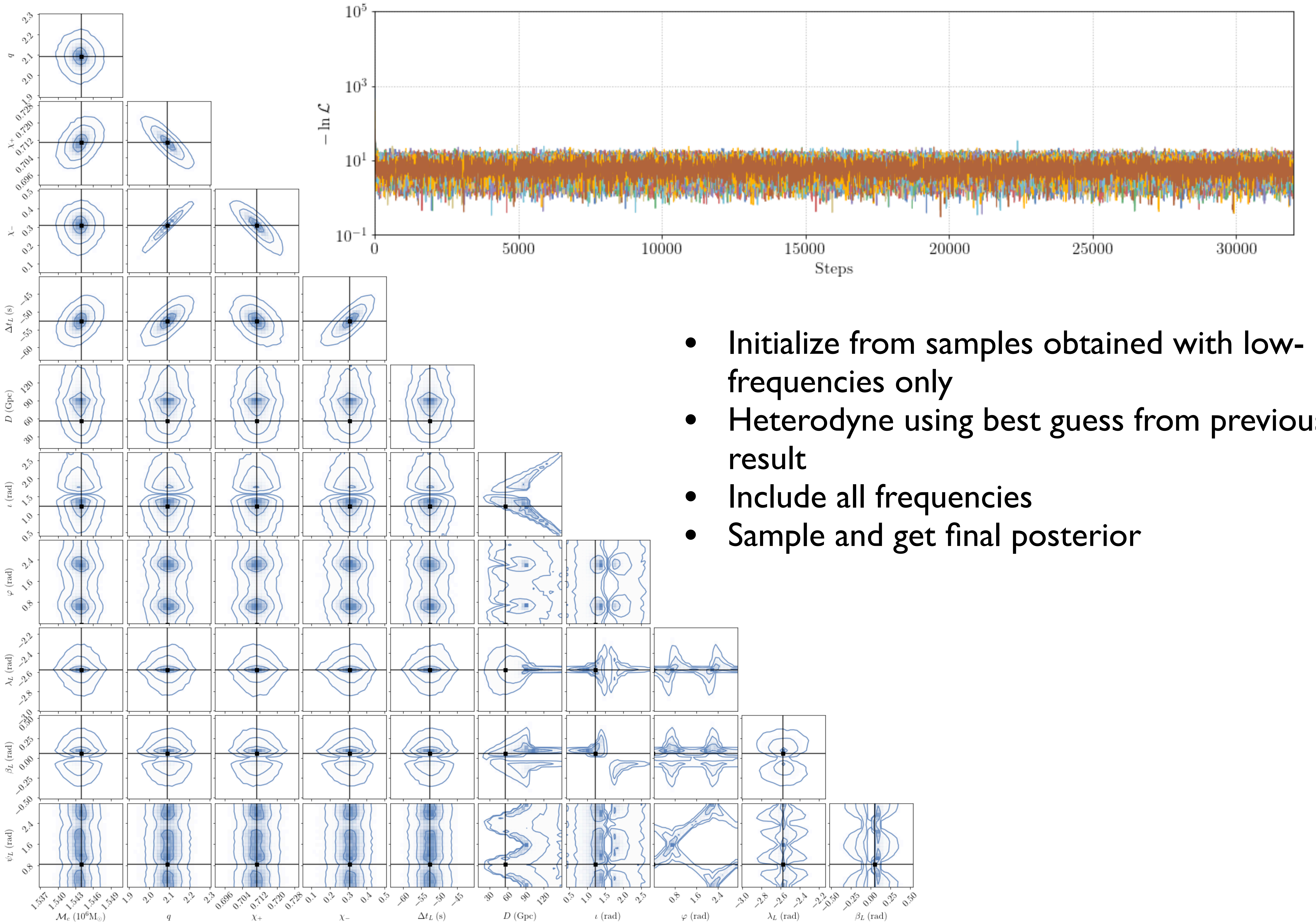






- Initialize from F-statistic samples, draw randomly extrinsic parameters
- Heterodyne using best guess from previous result
- Restrict to low frequencies
- Sample and get a first guess of all params

MBHB example: III) sampling with all frequencies



- Initialize from samples obtained with low-frequencies only
- Heterodyne using best guess from previous result
- Include all frequencies
- Sample and get final posterior

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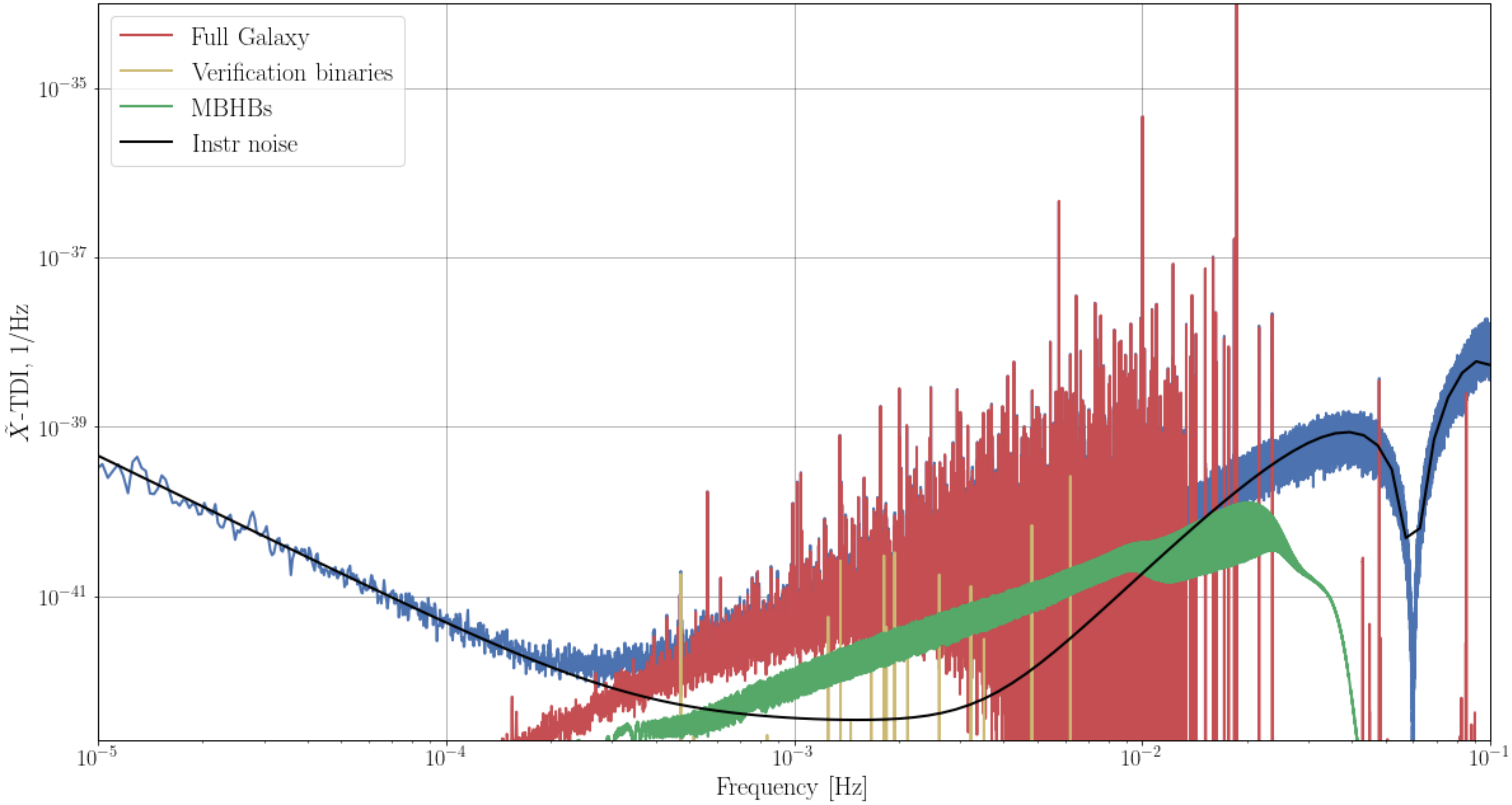
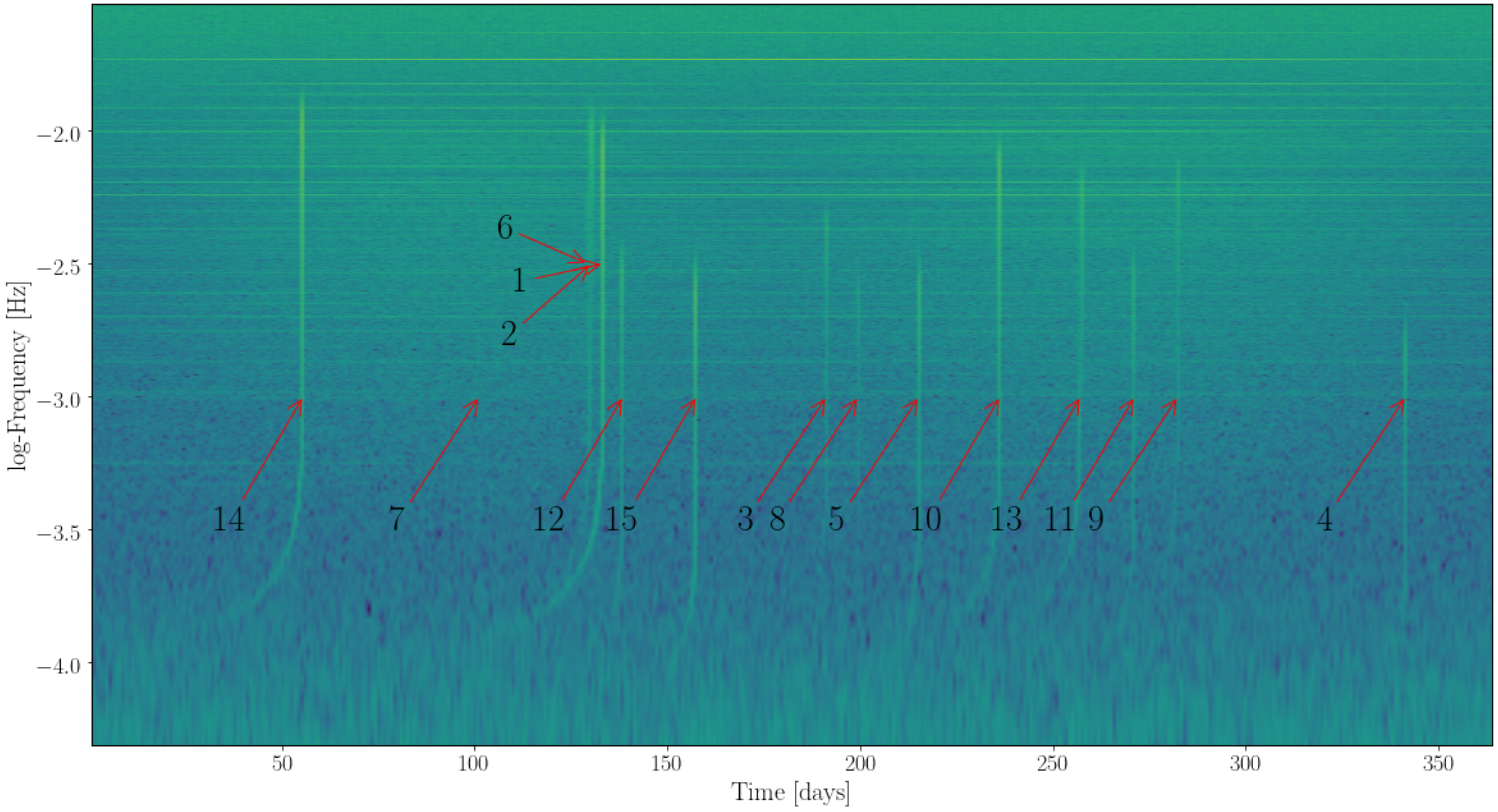
LISA Data Challenge: Sangria



- **MBHBs**: loud and merger-dominated, localized in time but extended in frequency
- **GBs**: continuous signals very local in frequency, both individually resolvable and building up a background

LISA Data Challenge: Sangria



- **MBHBs**: loud and merger-dominated, localized in time but extended in frequency
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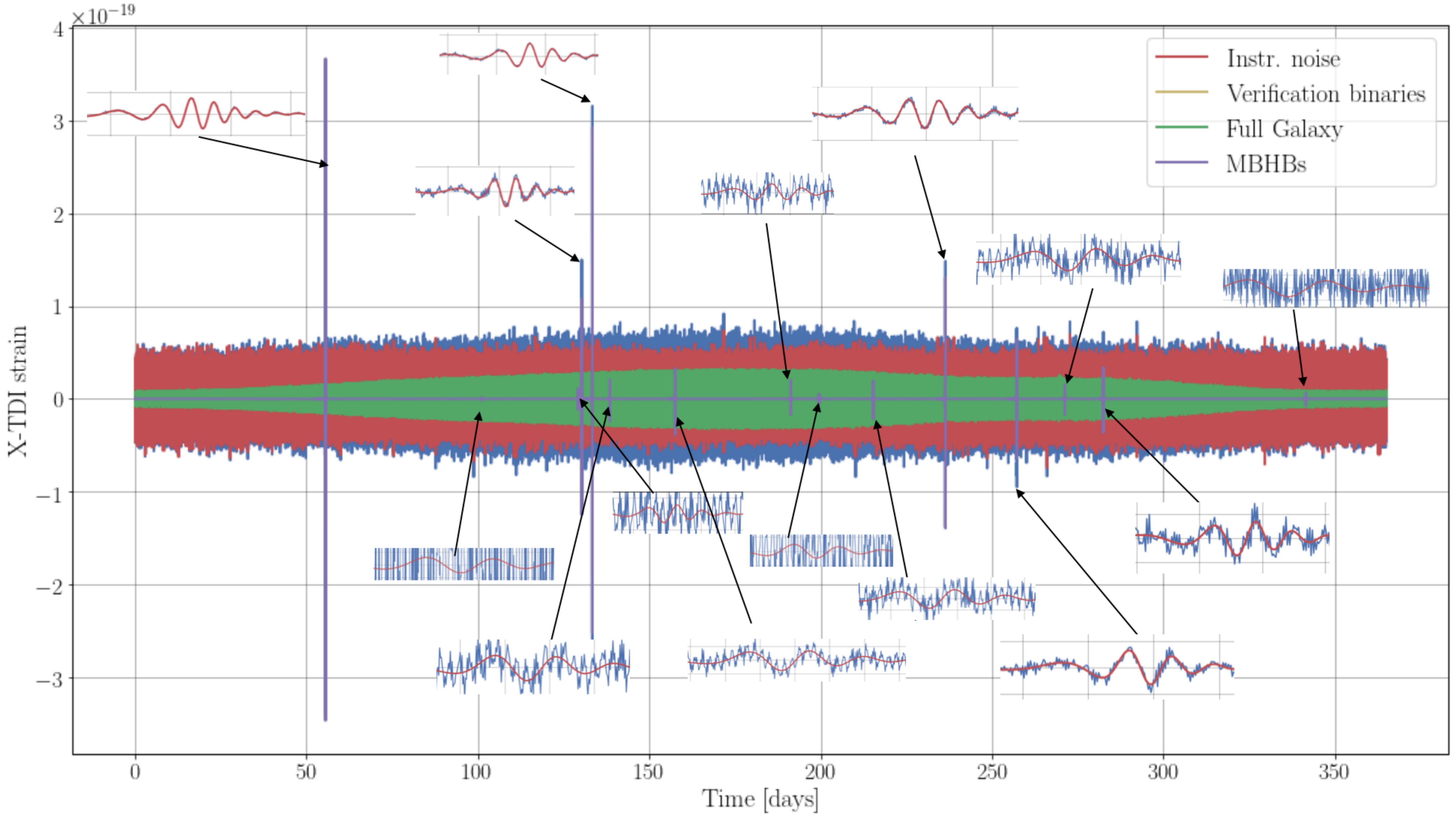
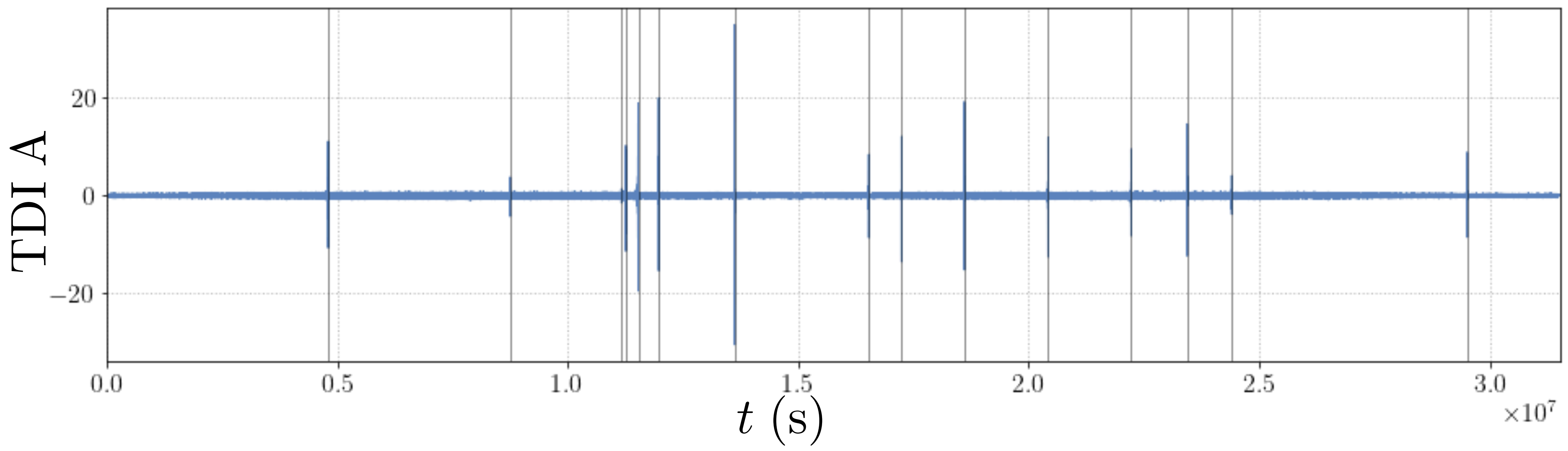
LISA data - band-passed, whitened in time domain

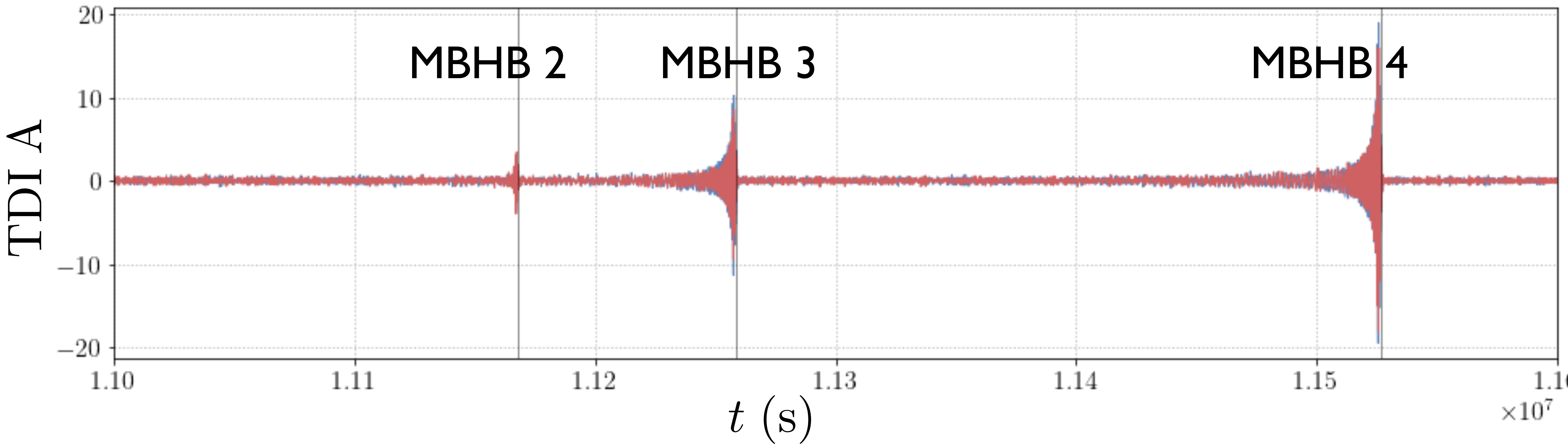


Whitened, band-passed data

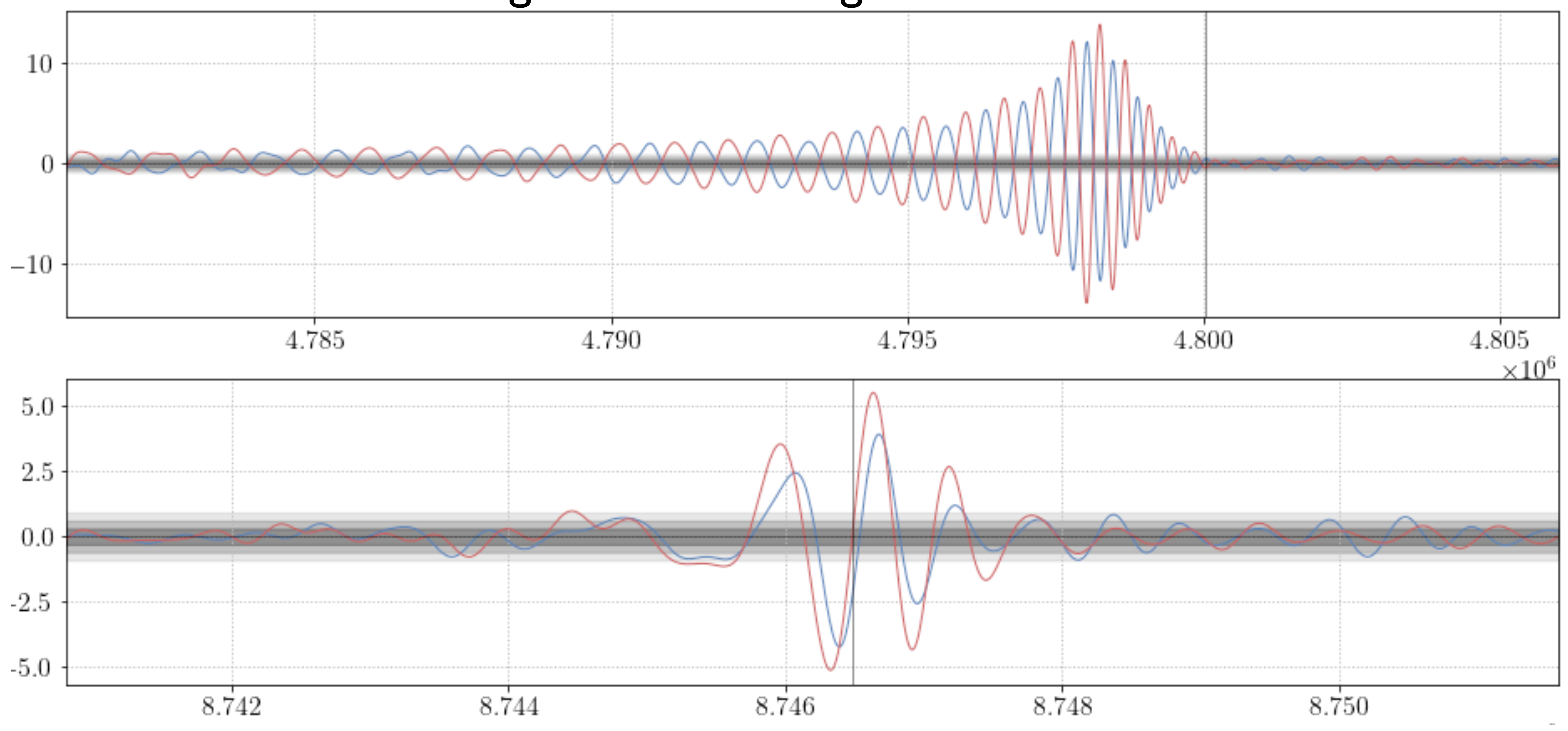


LISA data - band-passed, whitened in time domain

Whitened, band-passed data



sigma-thresholding for detection



Detecting these MBHBs and getting tc is simple
Might be different at low masses

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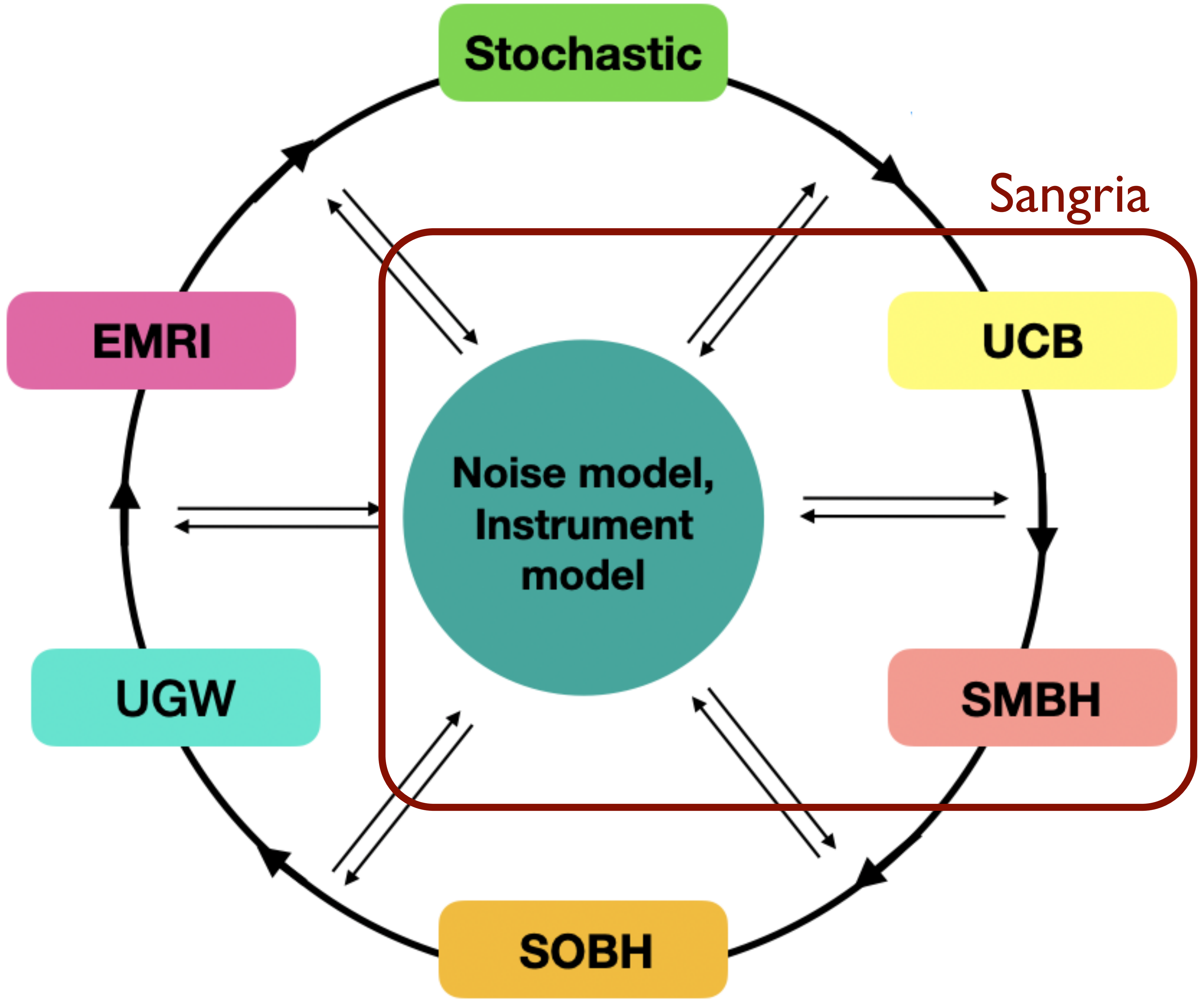
LDC Sangria: first steps of a global fit



[Cornish]

Chicken-and-egg problem

- MBHB analysis with full galaxy / GB analysis with full MBHBs are typically biased
- Some form of signal subtraction seems to be required

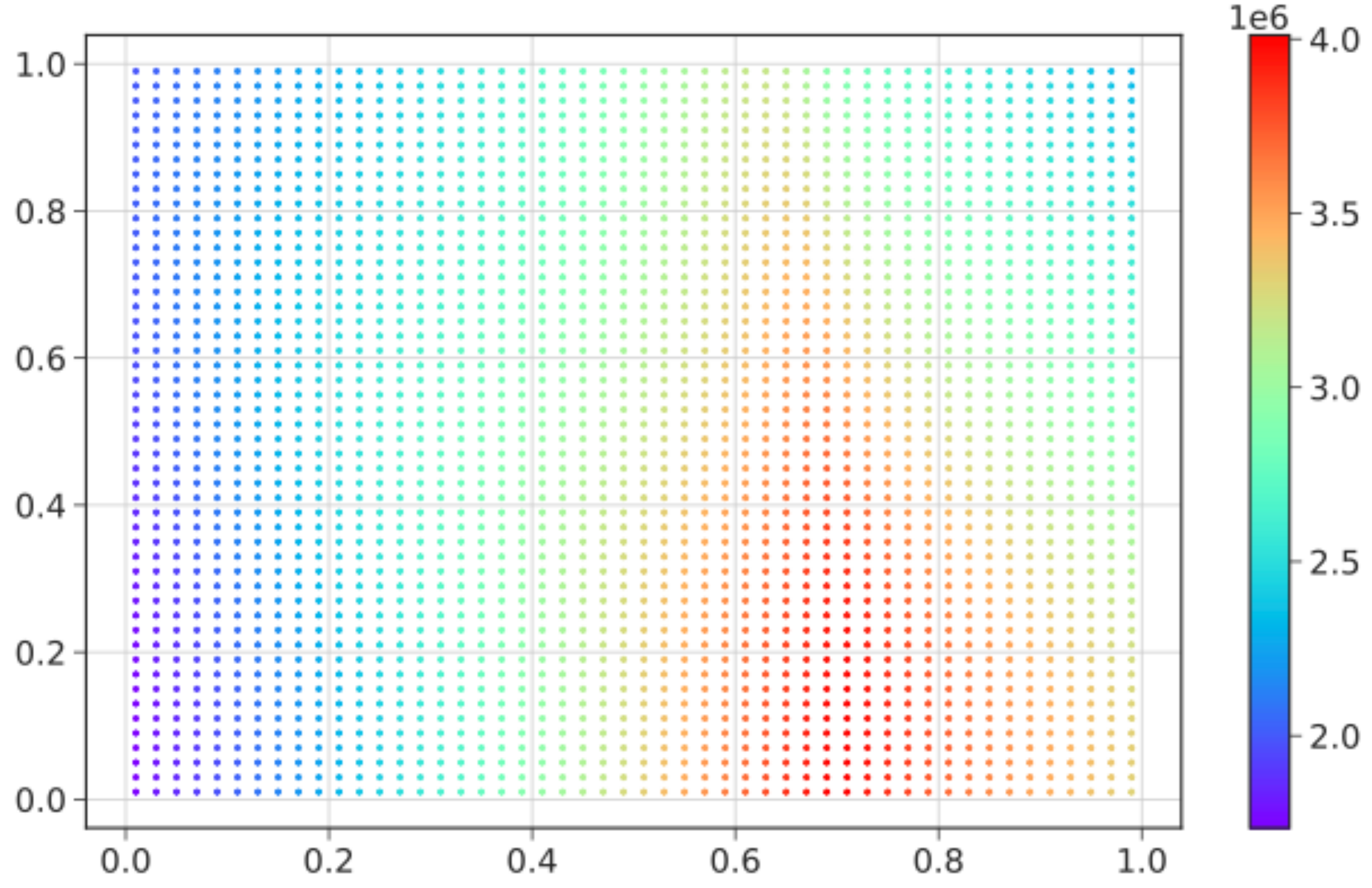
Global fit: a first approach

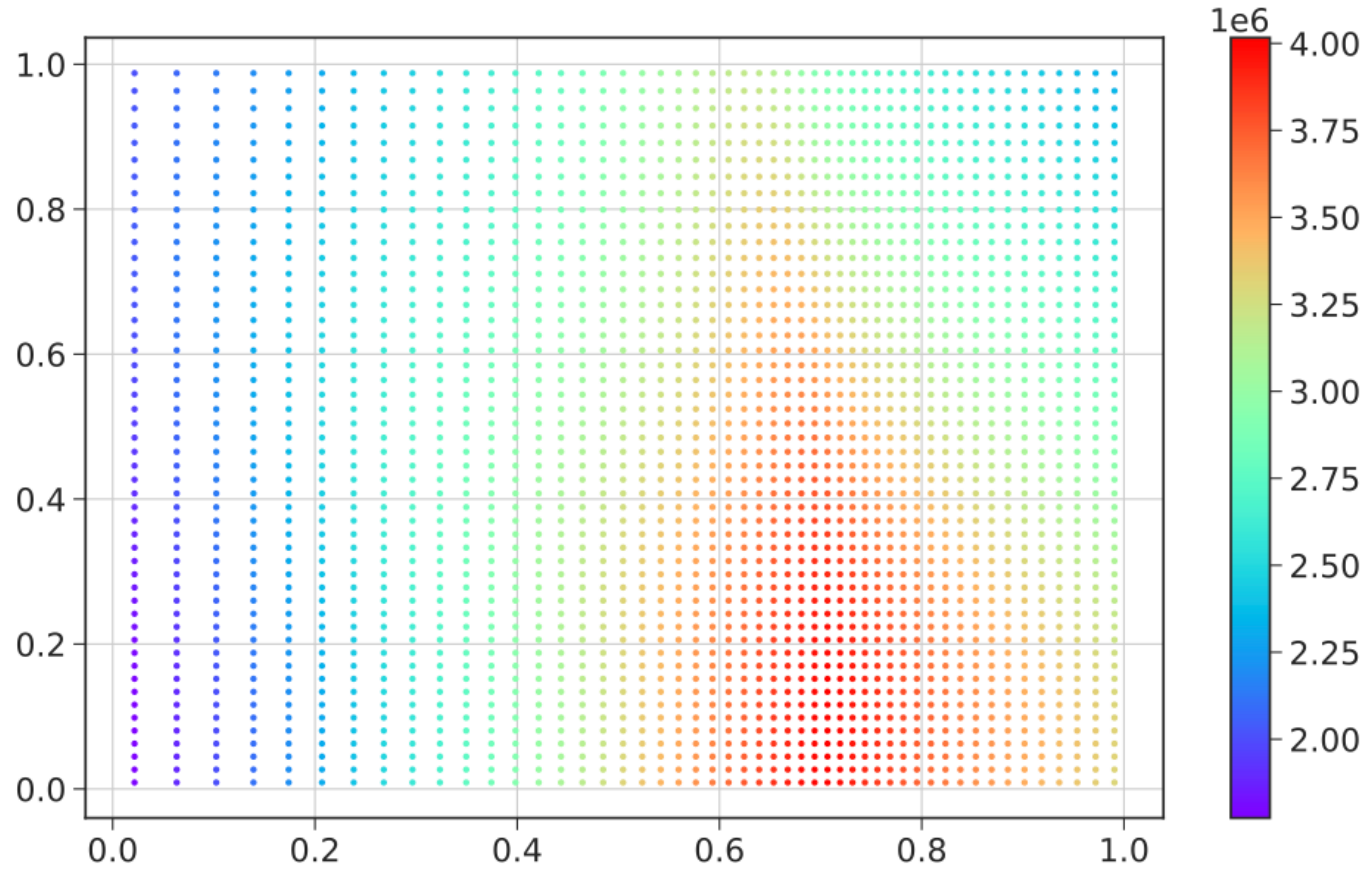
- First detection of MBHB
- Signal subtraction for MBHBs (no PE yet)
- First analysis of GBs, and noise estimation
- First PE (3-stage search/PE) for MBHBs
- Second analysis of GBs and noise ongoing…
- …

No unique approach ! See low-latency analysis of [Cornish 2021]

LDC Sangria: a first subtraction of MBHBs

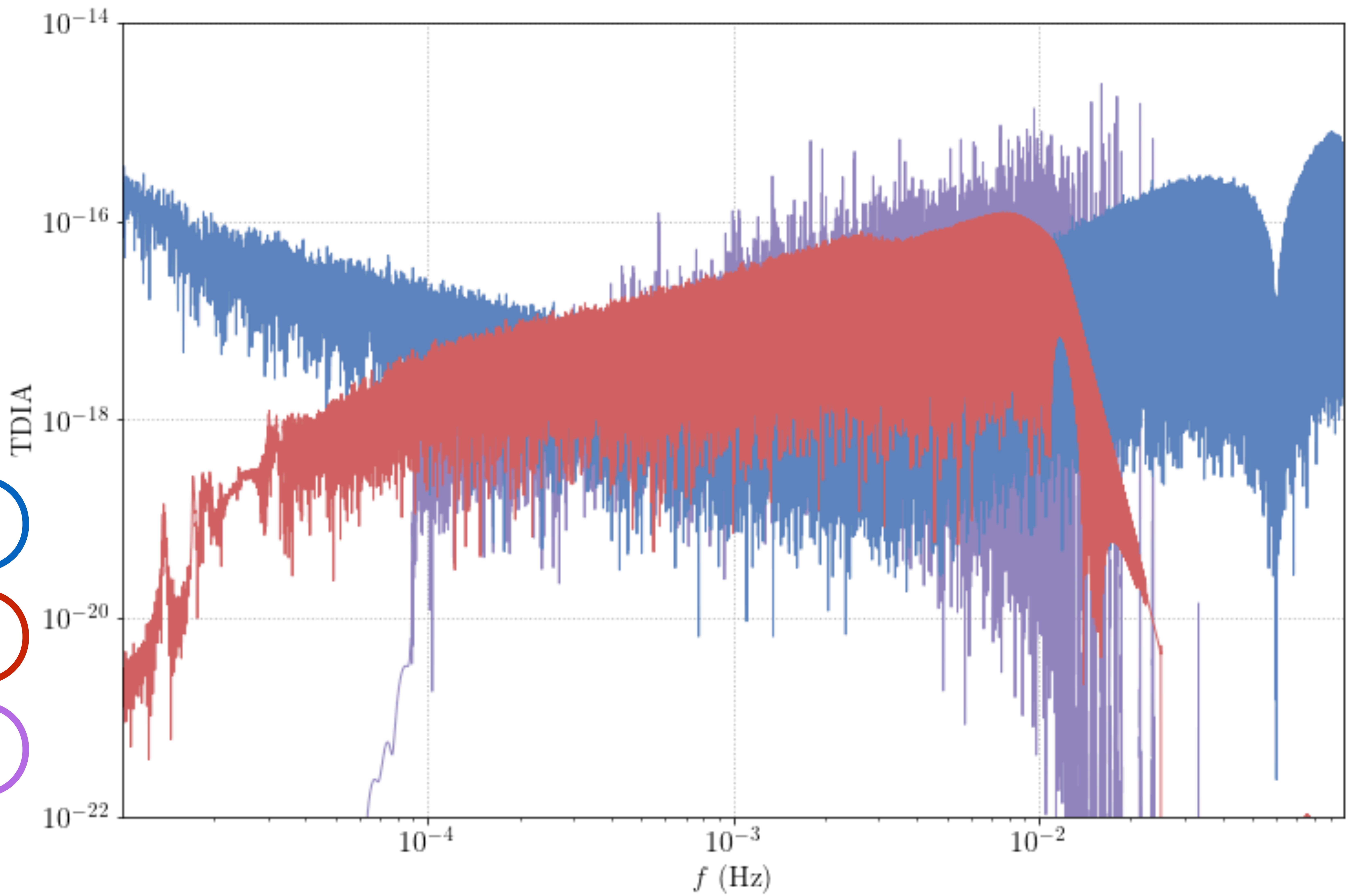
Analysis by [Senwen Deng, Stas Babak]





- Vegas: grid-based method with adaptive mesh refinement [Lepage 79]
- Restrict to low dimensions: masses+primary spin
- Produces best-fit estimate for MBHB signal subtraction

LDC Sangria: first steps of a global fit

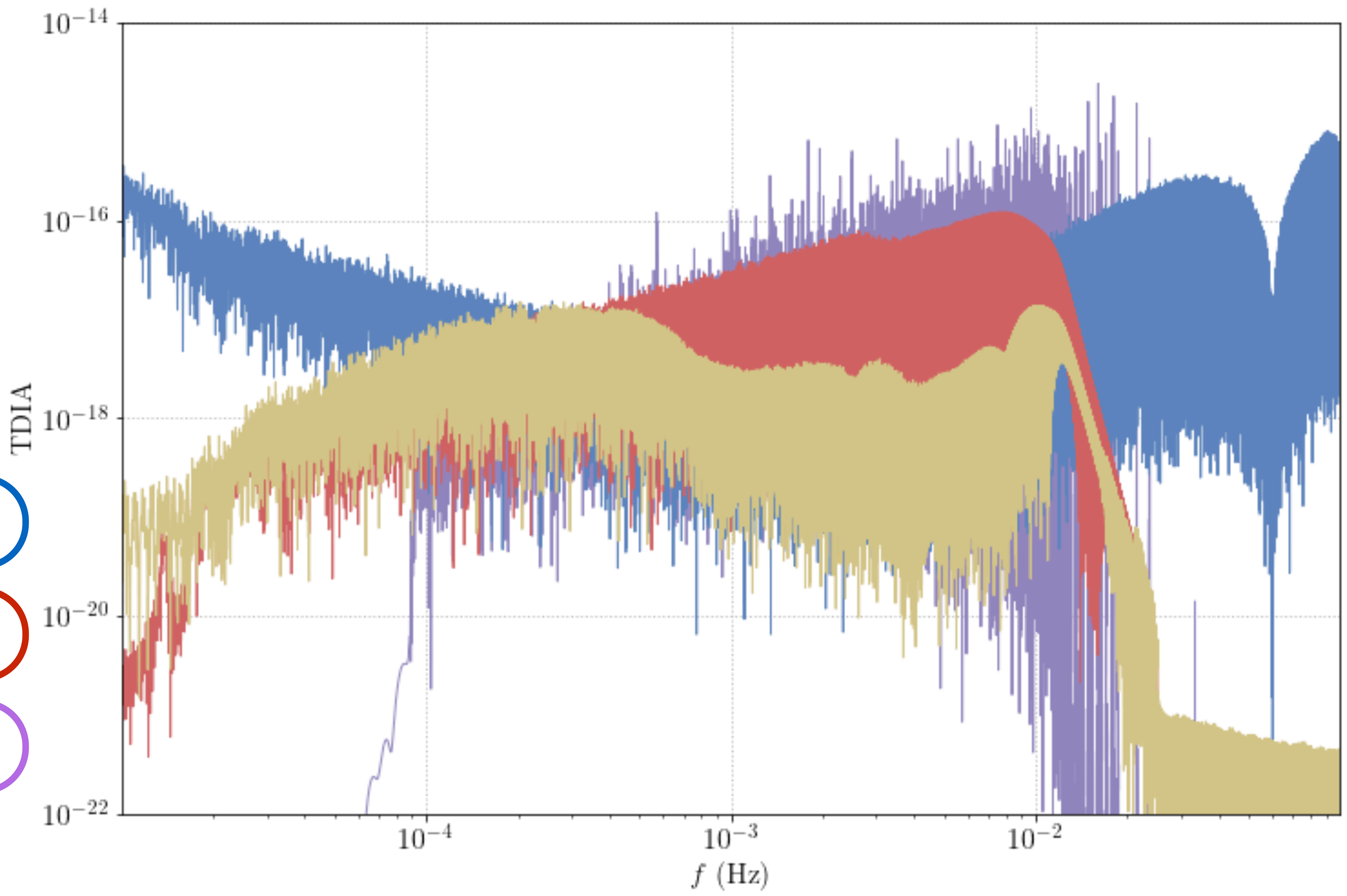


Instr. noise

MBHBs

GBs

LDC Sangria: first steps of a global fit



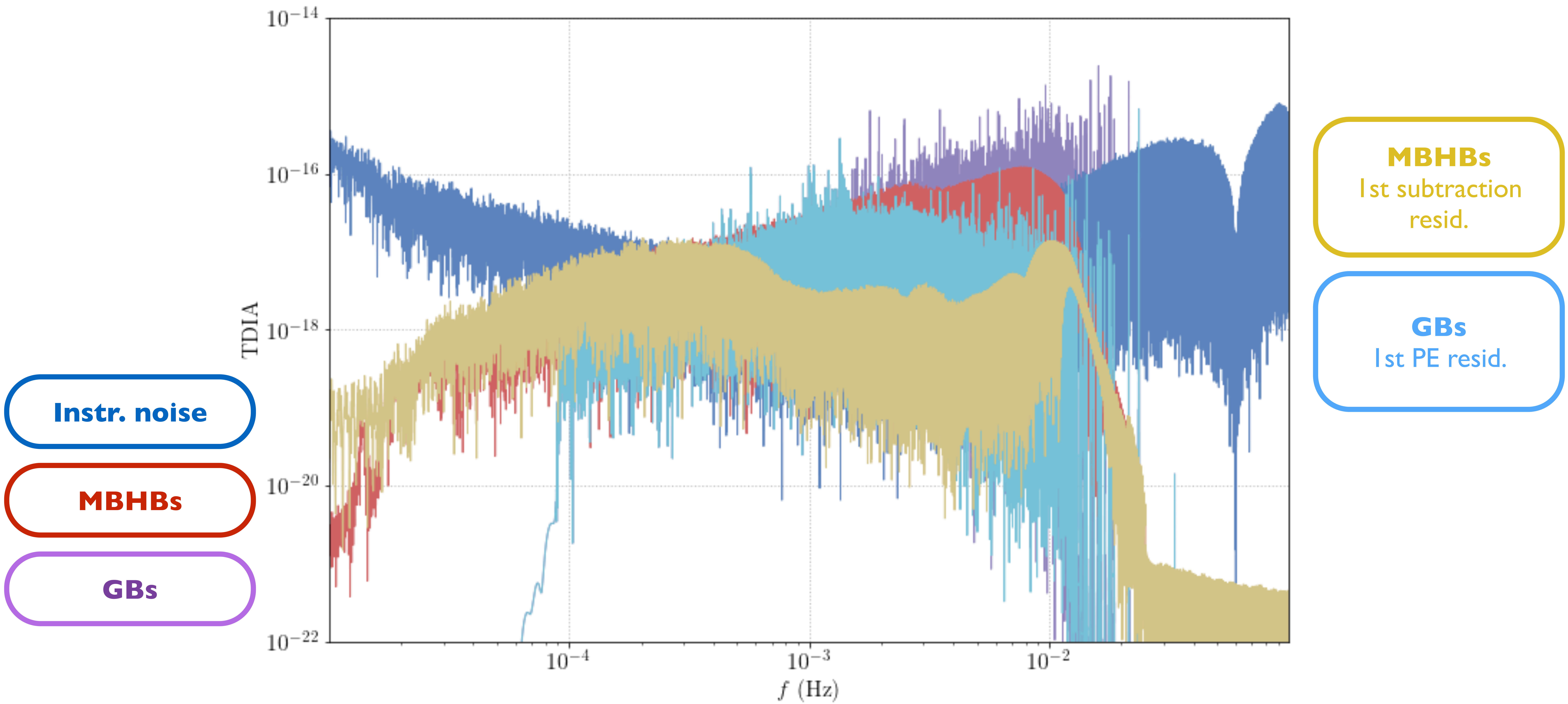
Instr. noise

MBHBs

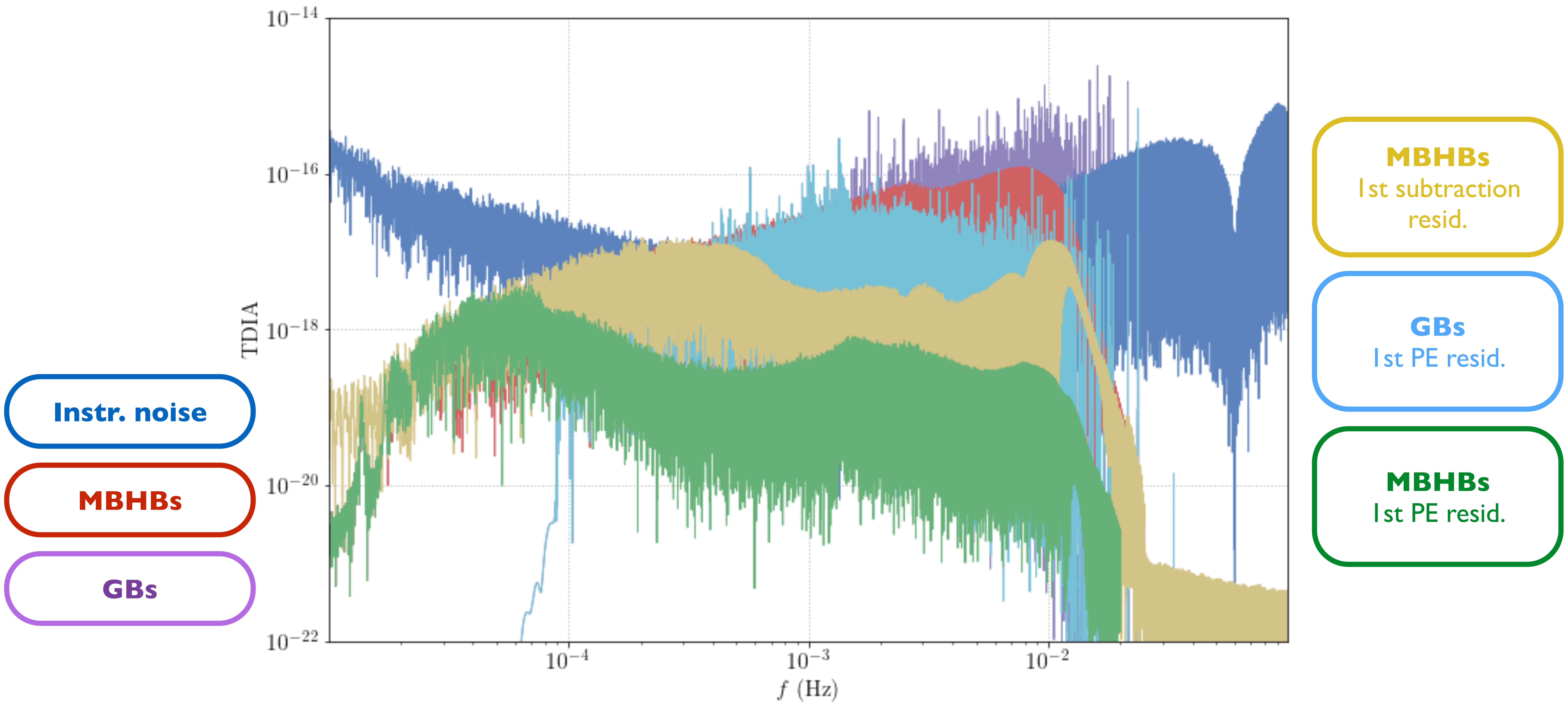
GBs

MBHBs
1st subtraction
resid.

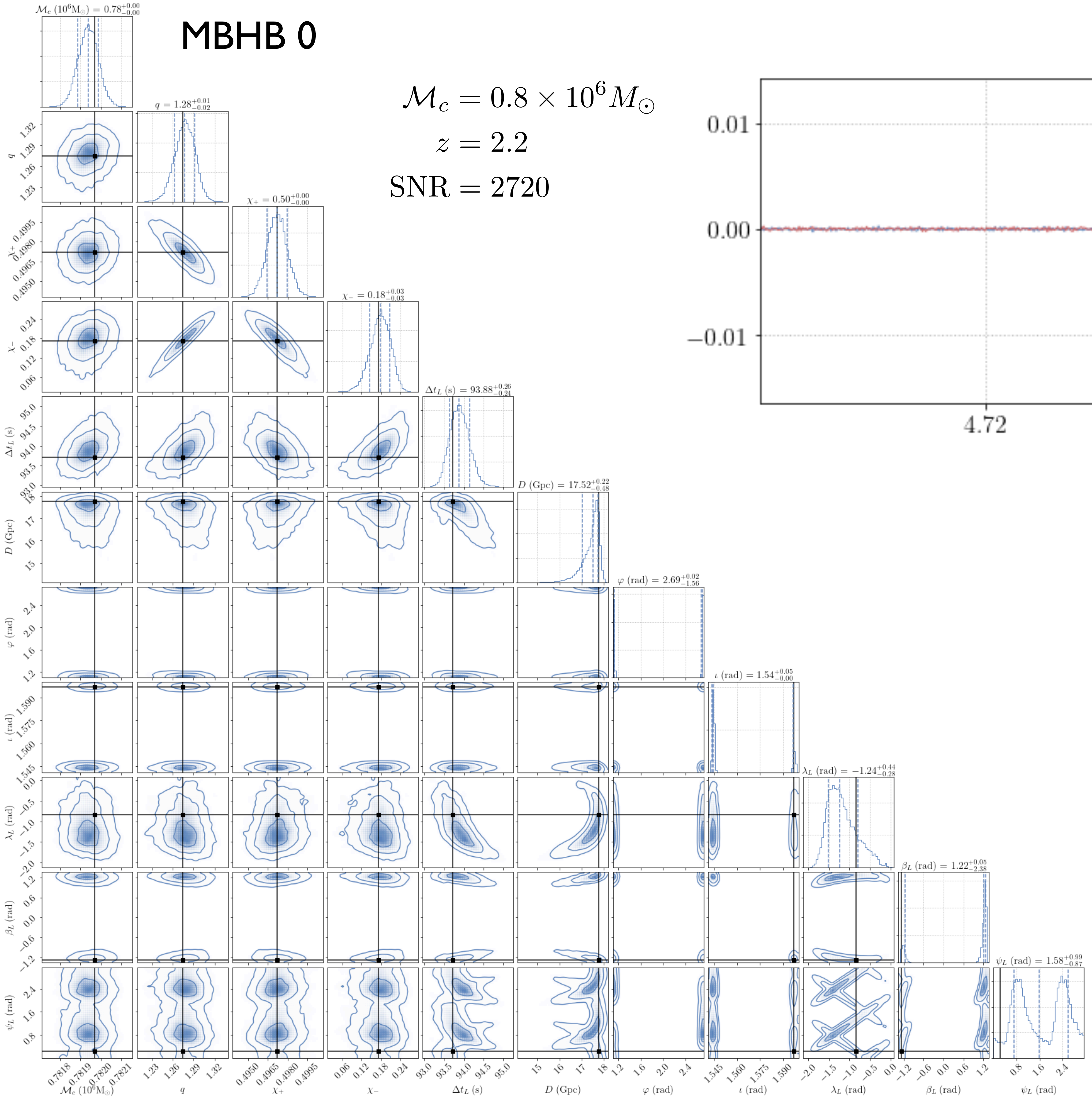
LDC Sangria: first steps of a global fit



LDC Sangria: first steps of a global fit

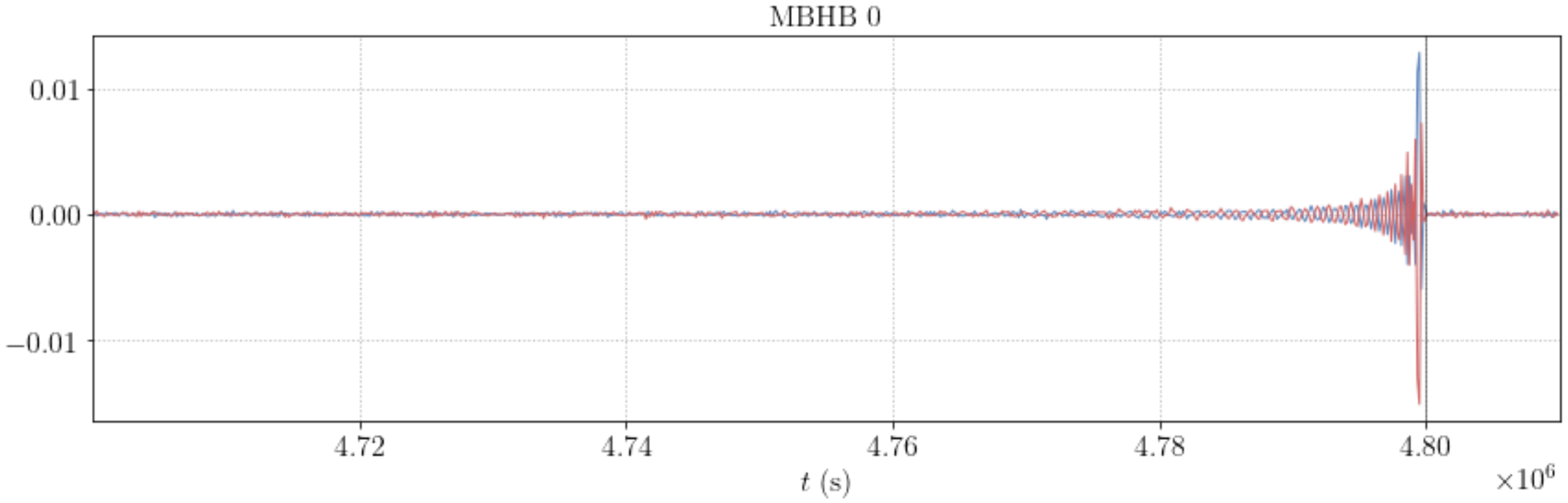


LDC Sangria: individual MBHB posteriors



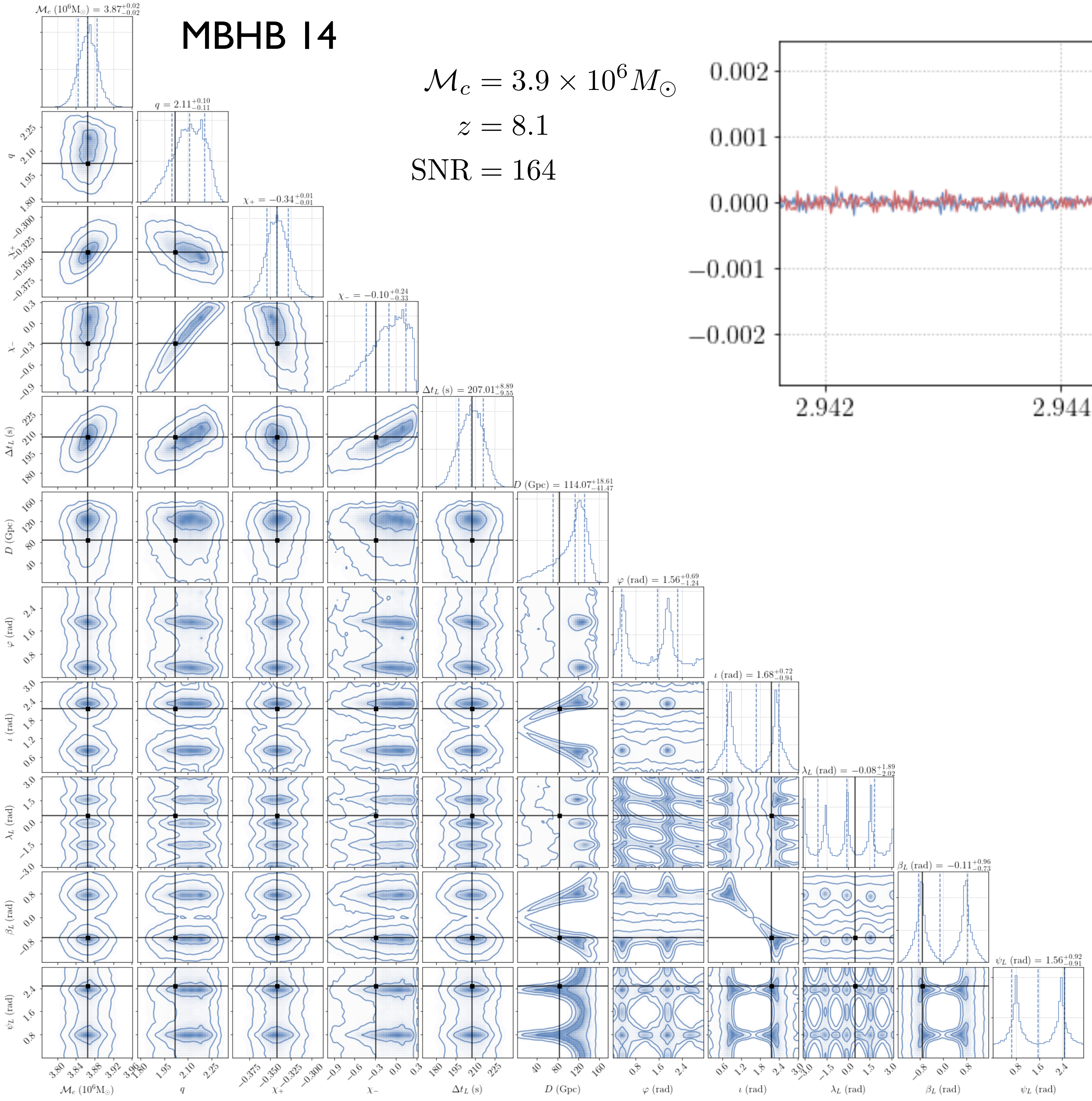
MBHB 0

$$\mathcal{M}_c = 0.8 \times 10^6 M_\odot$$
$$z = 2.2$$
$$\text{SNR} = 2720$$



LDC Sangria: individual MBHB posteriors

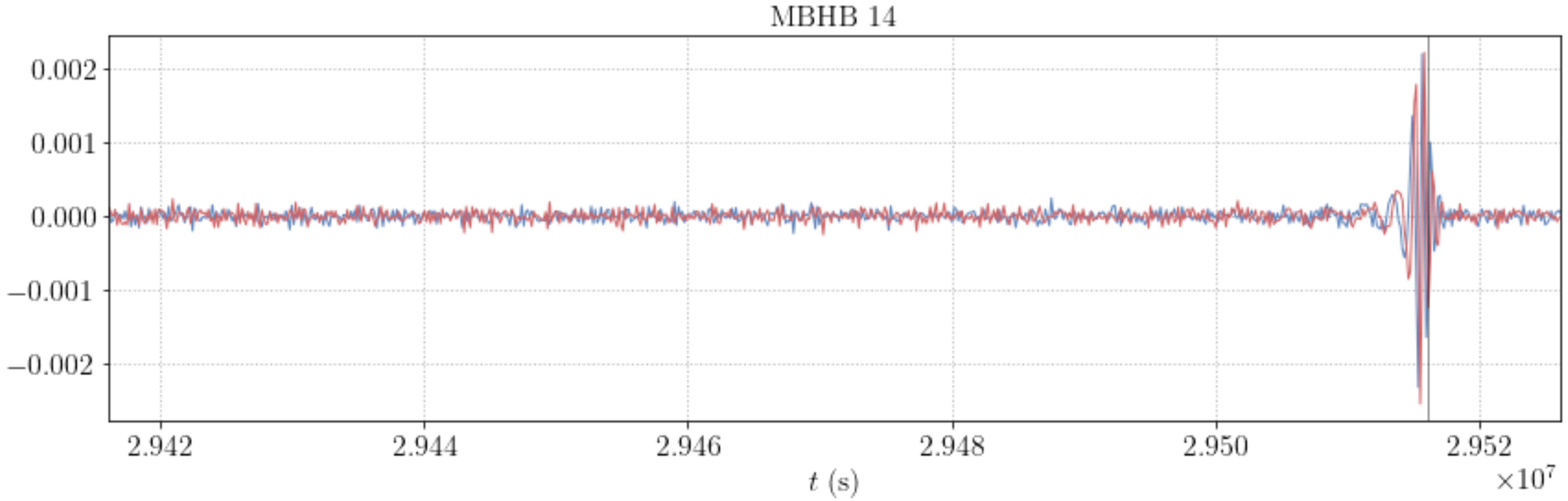
MBHB 14



$$\mathcal{M}_c = 3.9 \times 10^6 M_\odot$$

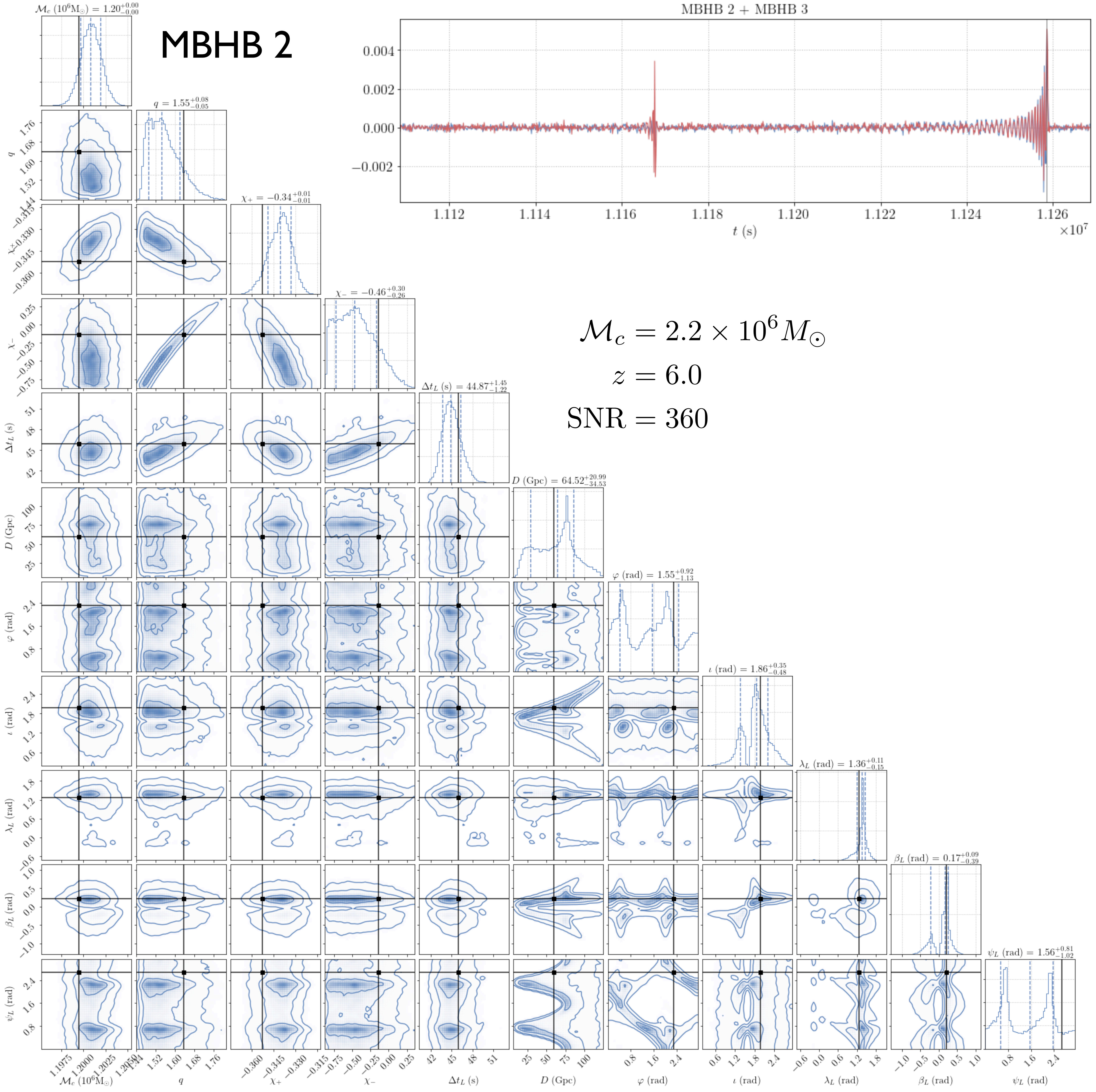
$$z = 8.1$$

$$\mathrm{SNR} = 164$$



LDC Sangria: individual MBHB posteriors

MBHB 2

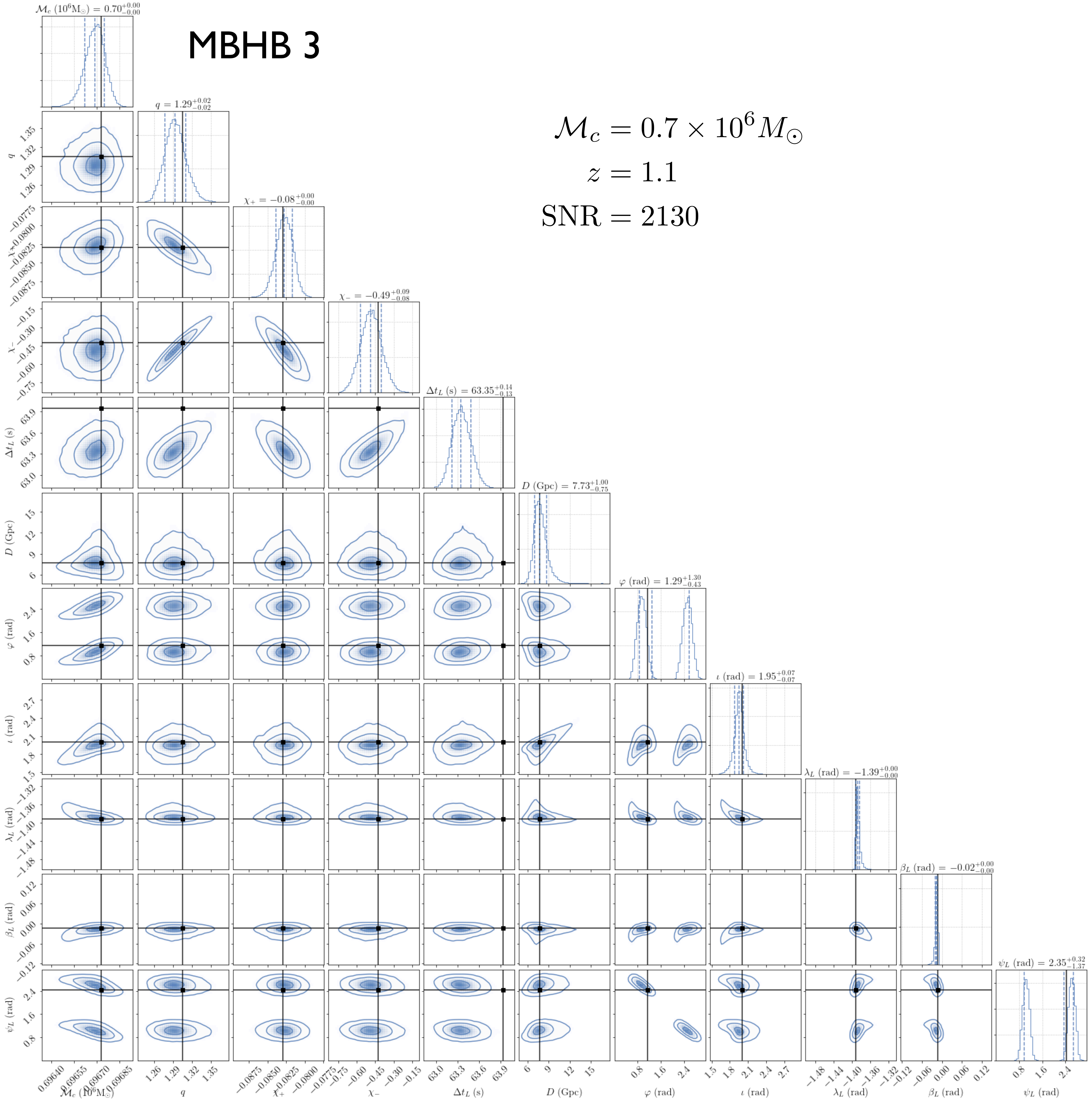


$\mathcal{M}_c = 2.2 \times 10^6 M_\odot$

$z = 6.0$

$\mathrm{SNR} = 360$

MBHB 3



$\mathcal{M}_c = 0.7 \times 10^6 M_\odot$

$z = 1.1$

$\mathrm{SNR} = 2130$

LDC Sangria: work in progress

- Resolve outstanding issues: biases in coalescence time (data generation ?), biases in intrinsic parameters for one source
- 2nd analysis of GBs (ongoing…) and noise
- 2nd analysis of MBHBs
- Confusion problem: do we have to analyze MBHBs jointly if they are correlated ?
- Multiple Gibbs iterations

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Thank you for your attention