Characterizing Anisotropic Stochastic Gravitational Wave Backgrounds and Foregrounds with the Bayesian LISA Pipeline (BLIP)

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with Sharan Banagiri, Jessica Lawrence, Steven Rieck, Malachy Bloom, Joe Romano, and Vuk Mandic

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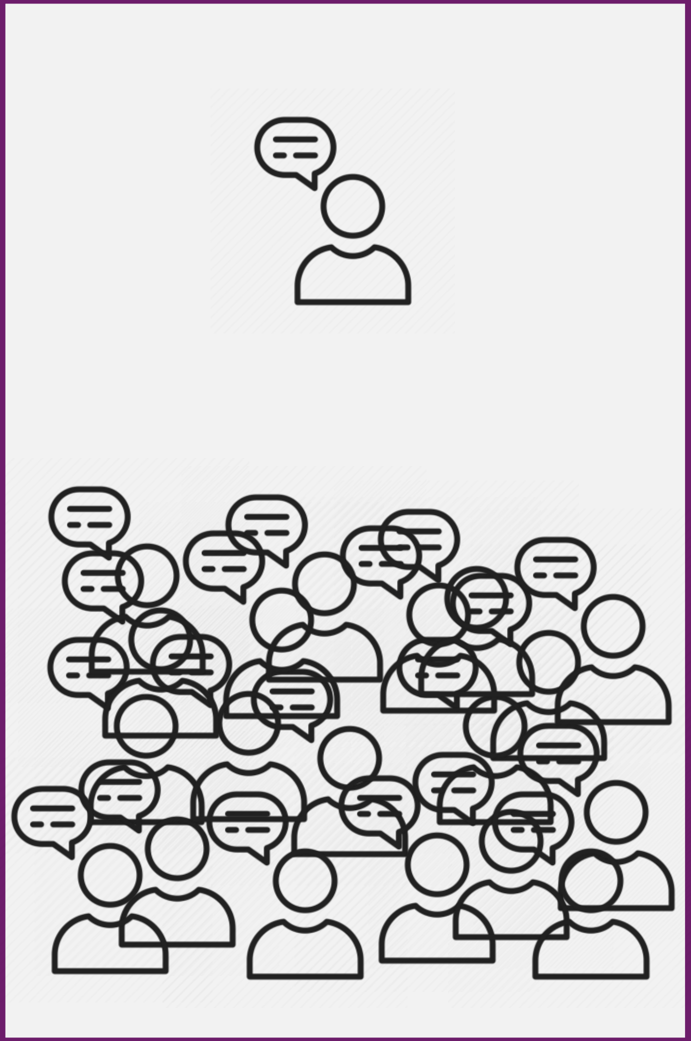
Stochastic Gravitational Wave (GW) Signals:

The stochastic confusion noise arising from the superposition of many **unresolved** gravitational wave sources on the sky [1,2]



Stochastic GW Signals in LISA

Cosmological

Arise from the early universe

- Inflation [3]
- Phase transitions [4]
- Cosmic Strings [5]

Expected to be **isotropic** at scales accessible by LISA

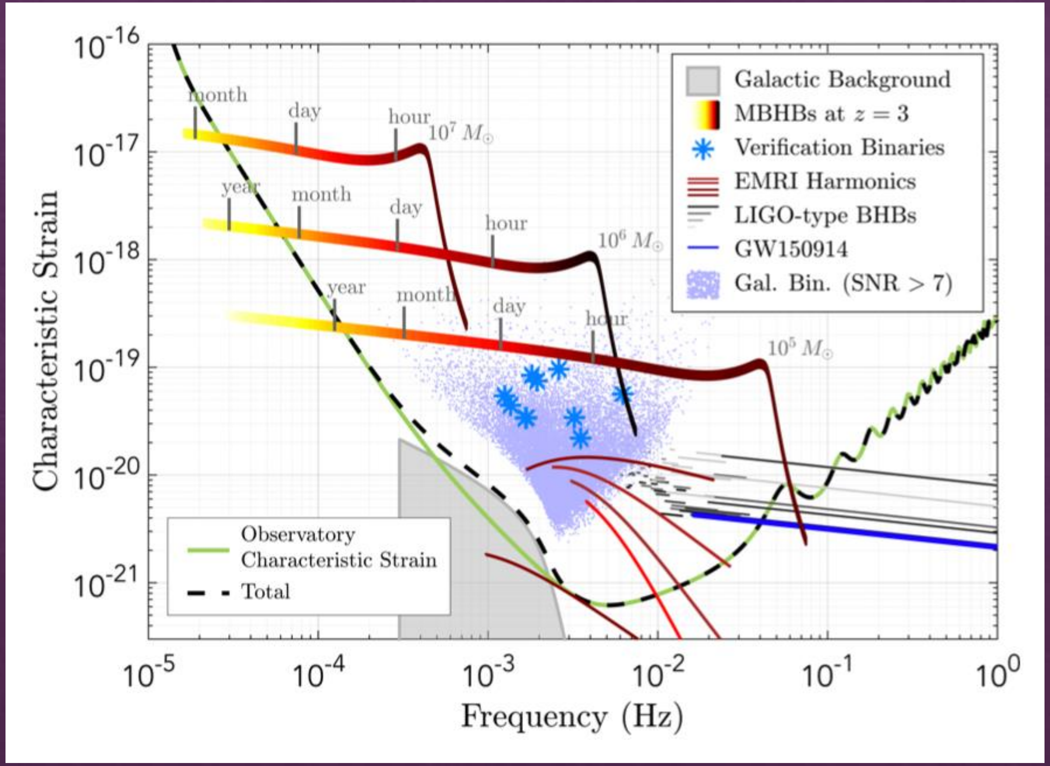
Arise from the superposition of many individual astrophysical events (e.g. unresolved white dwarf binaries[6,7])

Expected to reflect the spatial distributions of their component sources and may be **anisotropic**

Astrophysical

Anisotropic

Stochastic GW Signals in LISA



One scientist's noise is another's signal!

Anisotropic astrophysical signals embed **spatial** and **frequency** information about their source **populations**.

Astrophysical

The Bayesian LISA Pipeline (BLIP)

A Python pipeline for end-to-end simulation and Bayesian analysis of stochastic signals with LISA

- Simulation of time-domain data, Fourier domain analysis
- Flexible, modular GW signal, injection, and likelihood models
- Simultaneous noise and signal characterization

All-sky **isotropic** and **anisotropic** stochastic searches

Sharan Banagiri

Post-doc at Northwestern

github.com/sharanbngr/blip

arxiv.org/abs/2103.00826 [8]

The Bayesian LISA Pipeline (BLIP)

JOURNAL ARTICLE

Mapping the gravitational-wave sky with LISA: a Bayesian spherical harmonic approach

Sharan Banagiri ✉, Alexander Criswell, Tommy Kuan, Vuk Mandic, Joseph D Romano, Stephen R Taylor

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Anisotropic Searches with BLIP

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$$\mathcal{P}(\mathbf{n}) = \frac{1}{\sqrt{4\pi a_{0,0}}} \sum_{\ell,m} a_{\ell,m} Y_{\ell,m}(\mathbf{n}),$$

Standard spherical harmonic expansion of GW power on the sky. But $\mathcal{P}(\boldsymbol{n})$ must be non-negative everywhere on the sky…

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$$\sum_{L,M} a_{L,M} Y_{L,M} = \left(\sum_{\ell,m} b_{\ell,m} Y_{\ell,m}(\mathbf{n})\right)^2$$

We can then infer each $b_{l,m}$ up to some desired $l^b_{\max} = ½ l^a_{\max}$, quickly liaising between our $b_{l,m}$ parameterization and the power on the sky in the $a_{l,m}$s via **Clebsch-Gordon coefficients**

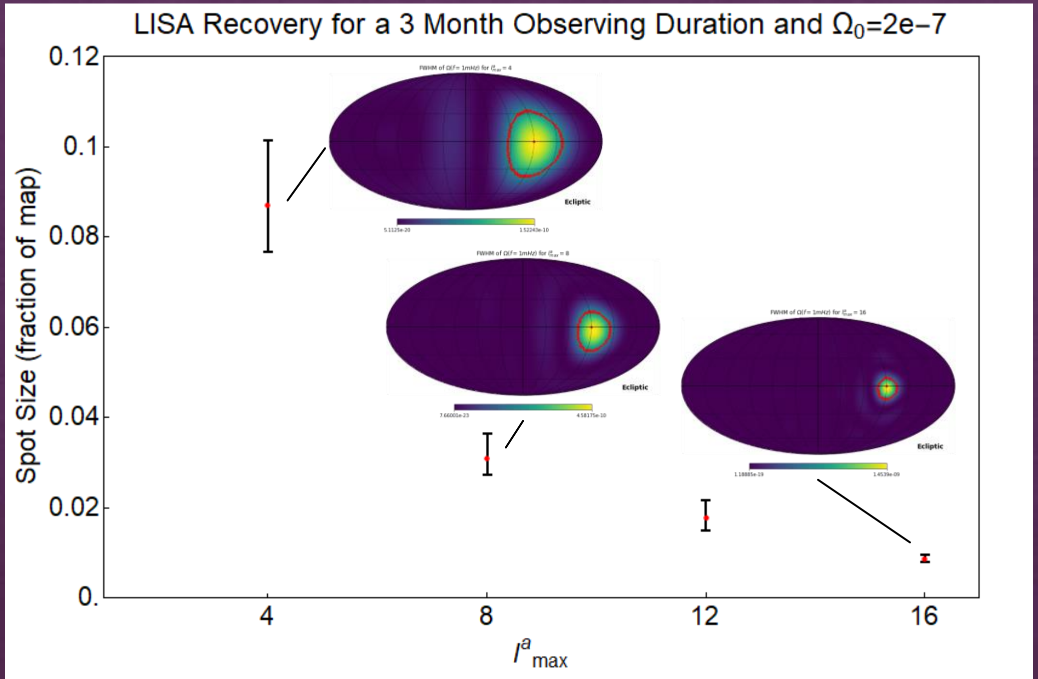
Science with BLIP

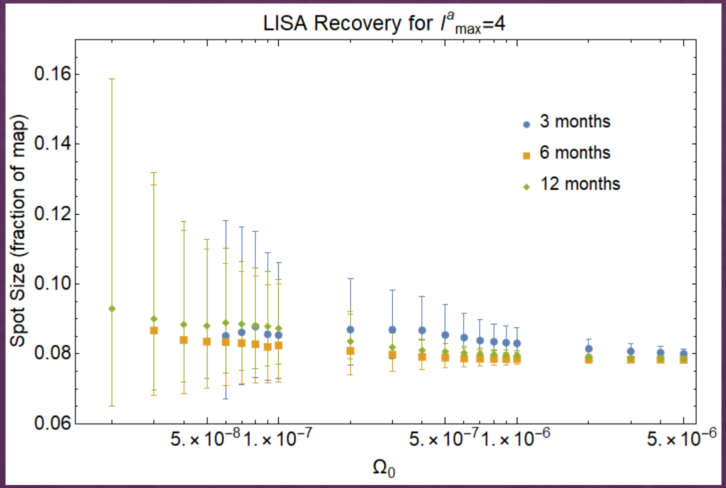
BLIP
is a *tool*.

… so what can we do with it?

1. Explore & characterize BLIP's abilities and limitations.
2. Test our capabilities with known signals in LISA.
3. Investigate detectability of potential new sources!

Angular Resolution of the Anisotropic Search





Point Source Injections

Malachy Bloom

Undergrad at Carleton College

Main takeaway: BLIP's anisotropic search is *currently* computation-limited (primarily), not SNR-limited → need to optimize!

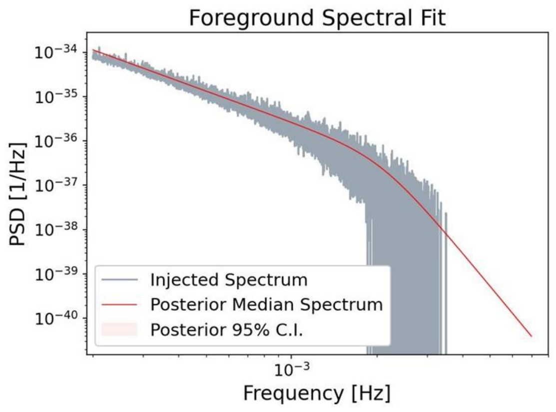
New Capability

Population Injections

BLIP can now take in a GW source population synthesis catalogue and inject/recover the associated spectral and spatial power distribution.

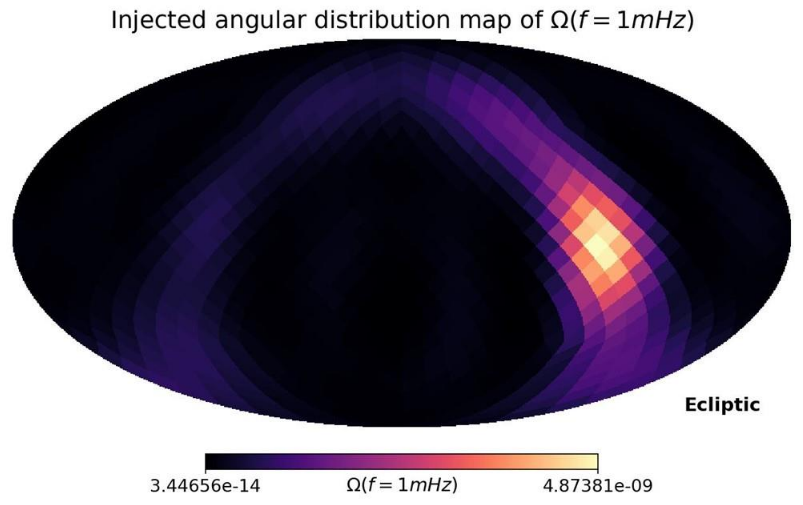
This allows for **realistic** investigations of **population-derived anisotropic stochastic signals.**

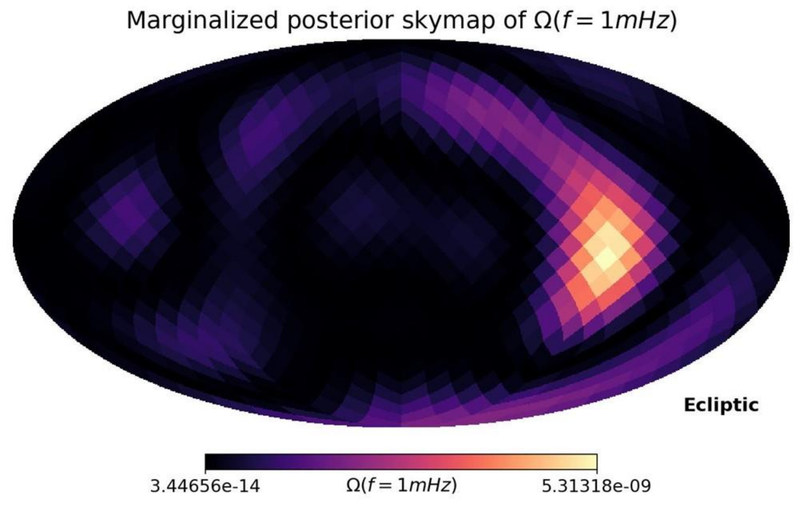
The Milky Way Foreground



Recovery Shown:

- 1 year of data
- Injected pop. synth. catalogue from Korol+2021 [9]
- Broken power law spectral model (3 parameters)
- Acceleration + position noise model (2 parameters)
- $l_{\text{max}}^{a} = 8$ (so $l_{\text{max}}^{b} = 4$; 24 b_{lm} parameters)

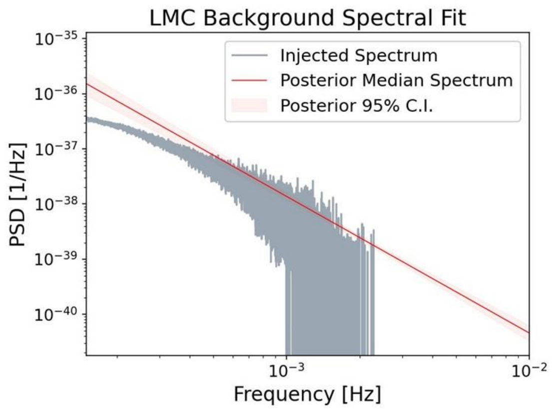




The Large Magellanic Cloud (LMC)!

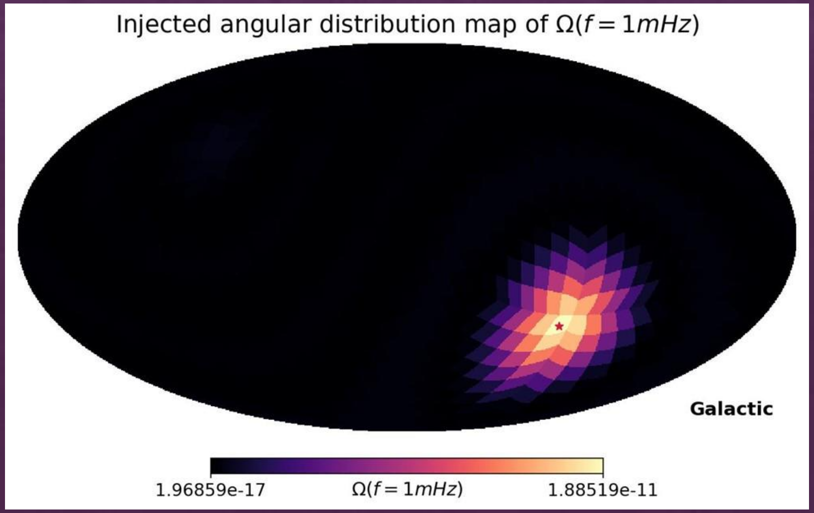
Steven Rieck

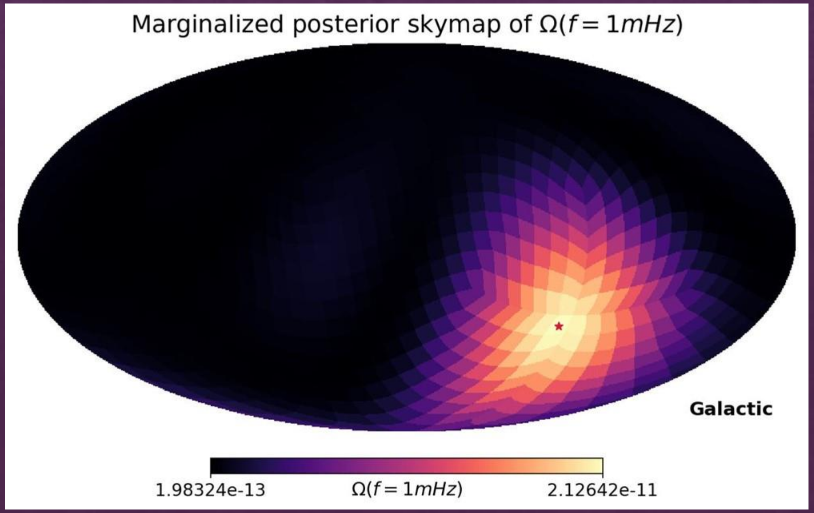
Undergrad at UMN



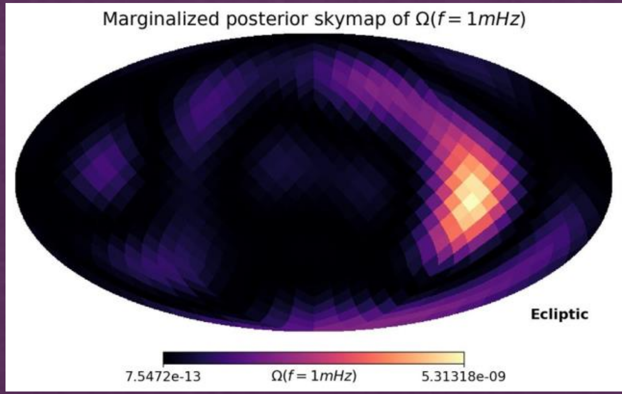
Recovery Shown:

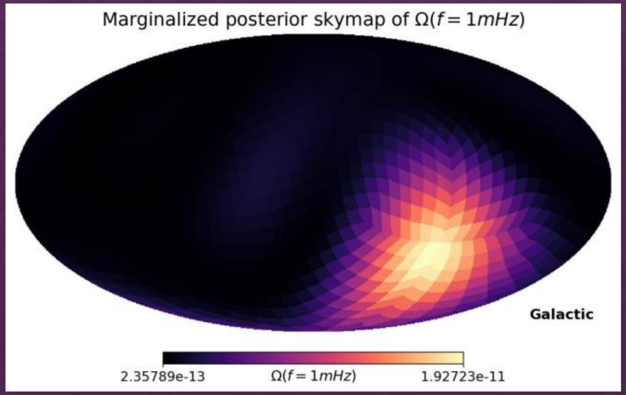
- 2 years of data
- Injected LMC pop. synth. catalogue from Keim+2022 [10]
- Power law spectral model (2 parameters)
- Acceleration + position noise model (2 parameters)
- $l^a_{\mathrm{max}} = 4$ (so $l^b_{\mathrm{max}} = 2$; 8 b_{lm} parameters)





Population studies: what's next?





Hierarchical Parameter Estimation

Stellar evolution

LMC mass

Astrophysics!

Spatial distribution models

& more?

Where do we go from here?

Simultaneous characterization.

1. Simultaneous inference of **isotropic + anisotropic** sources.
2. Simultaneous inference of **multiple anisotropic sources**.
3. Integration of the BLIP stochastic search with **global fit efforts.**

Summary

- End-to-end simulation and recovery of anisotropic stochastic gravitational wave signals with the Bayesian LISA Pipeline (BLIP).
- Ability to inject realistic, **population synthesis catalogue-derived signals**, allowing us to investigate both known and novel stochastic sources in LISA.
- Large numbers of spherical harmonic parameters needed at high l_{max} → **need to optimize sampling** to improve angular resolution.
- Successful characterization of a **realistic galactic foreground** signal.
- The stochastic signal from **unresolved white dwarf binaries in the Large Magellanic Cloud** may be **within reach of LISA**!

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Extra Slide: Clebsch-Gordon Expansion

$$\sum_{L,M} a_{L,M} Y_{L,M} = \left(\sum_{\ell,m} b_{\ell,m} Y_{\ell,m}(\mathbf{n}) \right)^2 .$$

$$\sum_{L,M} a_{L,M} Y_{L,M} = \sum_{\ell,m} \sum_{\ell',m'} b_{\ell,m} b_{\ell',m'} Y_{\ell,m}(\mathbf{n}) Y_{\ell',m'}(\mathbf{n}).$$

$$Y_{\ell,m}(\mathbf{n}) Y_{\ell',m'}(\mathbf{n}) = \sum_{L=L_{\min}}^{L_{\max}} \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi(2L+1)}} \times C^{LM}_{\ell m,\ell' m'} C^{L0}_{\ell 0,\ell' 0} Y_{L,M}(\mathbf{n}).$$

$$\sum_{L,M} a_{L,M} Y_{L,M}(\mathbf{n}) = \sum_{L,M} \left(\sum_{\ell m} \sum_{\ell' m'} b_{\ell,m} b_{\ell',m'} \beta^{\ell m,\ell' m'}_{L,M} \right) \times Y_{L,M}(\mathbf{n}).$$

- $M = m + m'$
- $L_{\min} = \min(|\ell - \ell'|, |m + m'|)$ and $L_{\max} = \ell + \ell'$
- L is an integer

For compactness, let us define $\beta^{L,M}_{\ell m,\ell' m'}$ such that:

$$\beta^{L,M}_{\ell m,\ell' m'} = \sqrt{\frac{(2\ell+1)(2\ell'+1)}{4\pi(2L+1)}} C^{LM}_{\ell m,\ell' m'} C^{L0}_{\ell 0,\ell' 0}, \qquad (3.7)$$

when the selection rules are satisfied, but $\beta^{L,M}_{\ell m,\ell' m'} = 0$ otherwise.

Extra Slide: Spectral Models

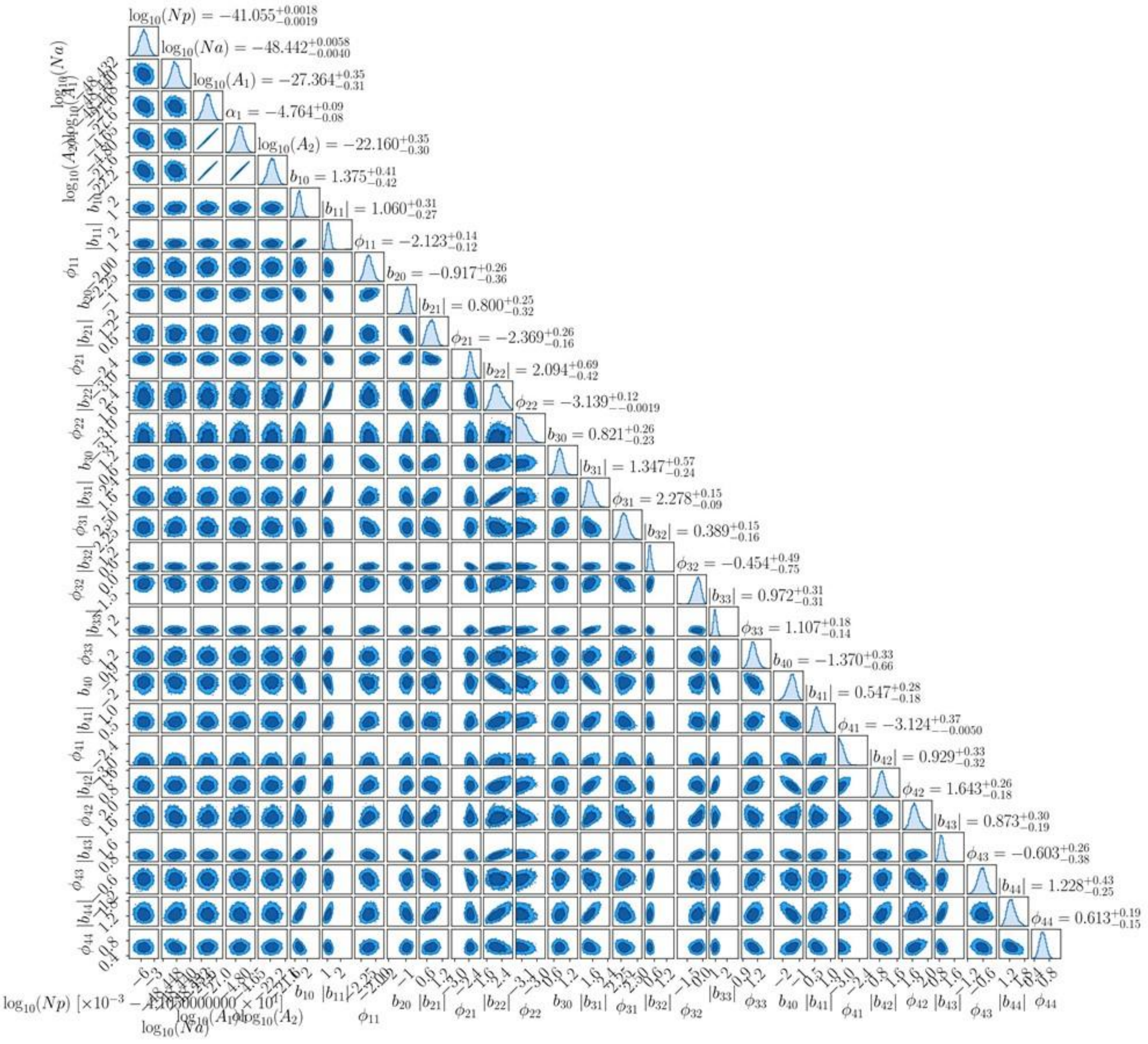
Power law:

$$\Omega(f) = \Omega_{\mathrm{ref}} \left(\frac{f}{f_0}\right)^{\alpha},$$

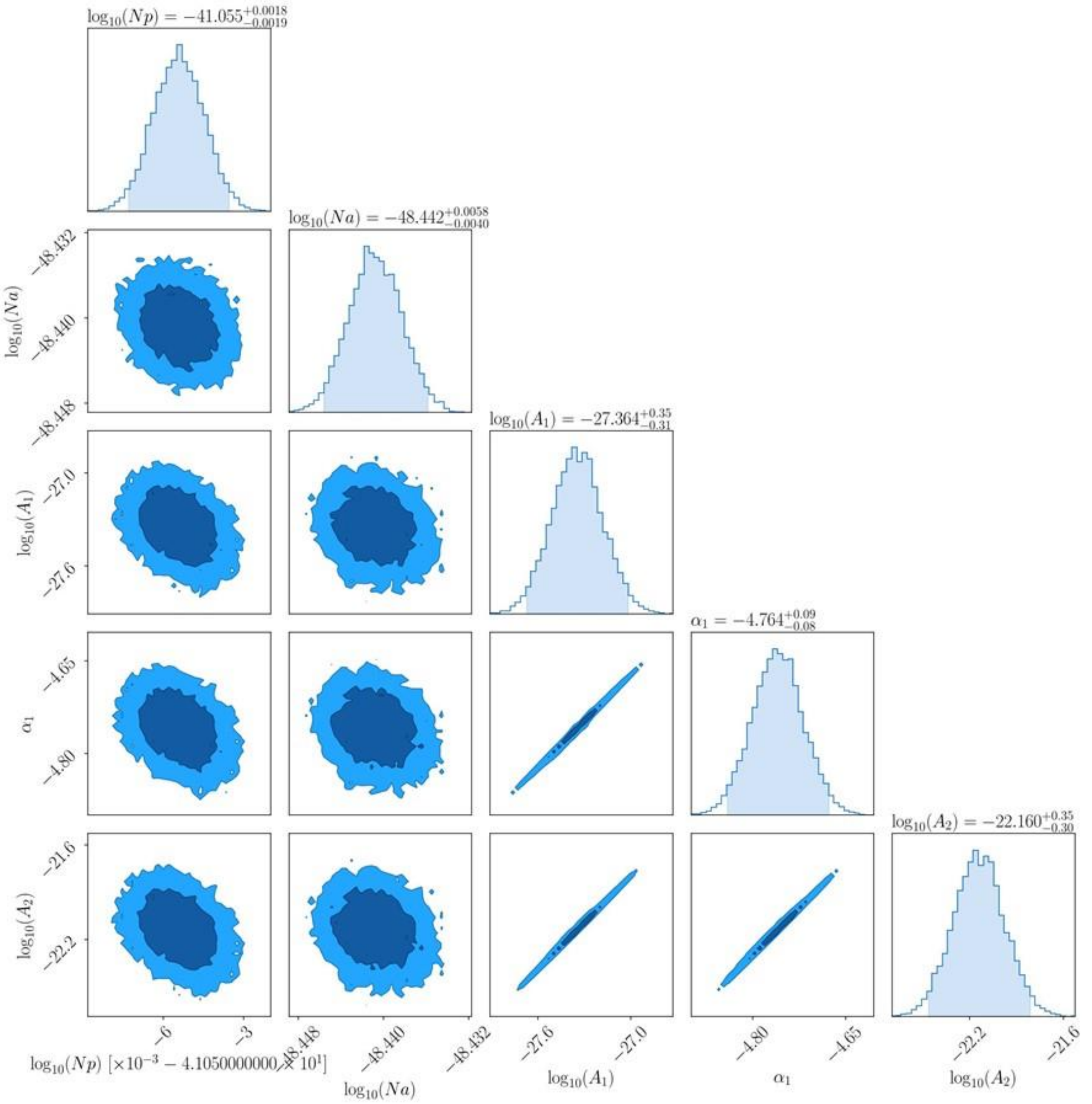
Broken power law from Boileau et al. (2021)

$$\Omega(f) = \frac{A_1\left(\frac{f}{f_{ref}}\right)^{\alpha_1}}{1+A_2\left(\frac{f}{f_{ref}}\right)^{\alpha_2}}$$

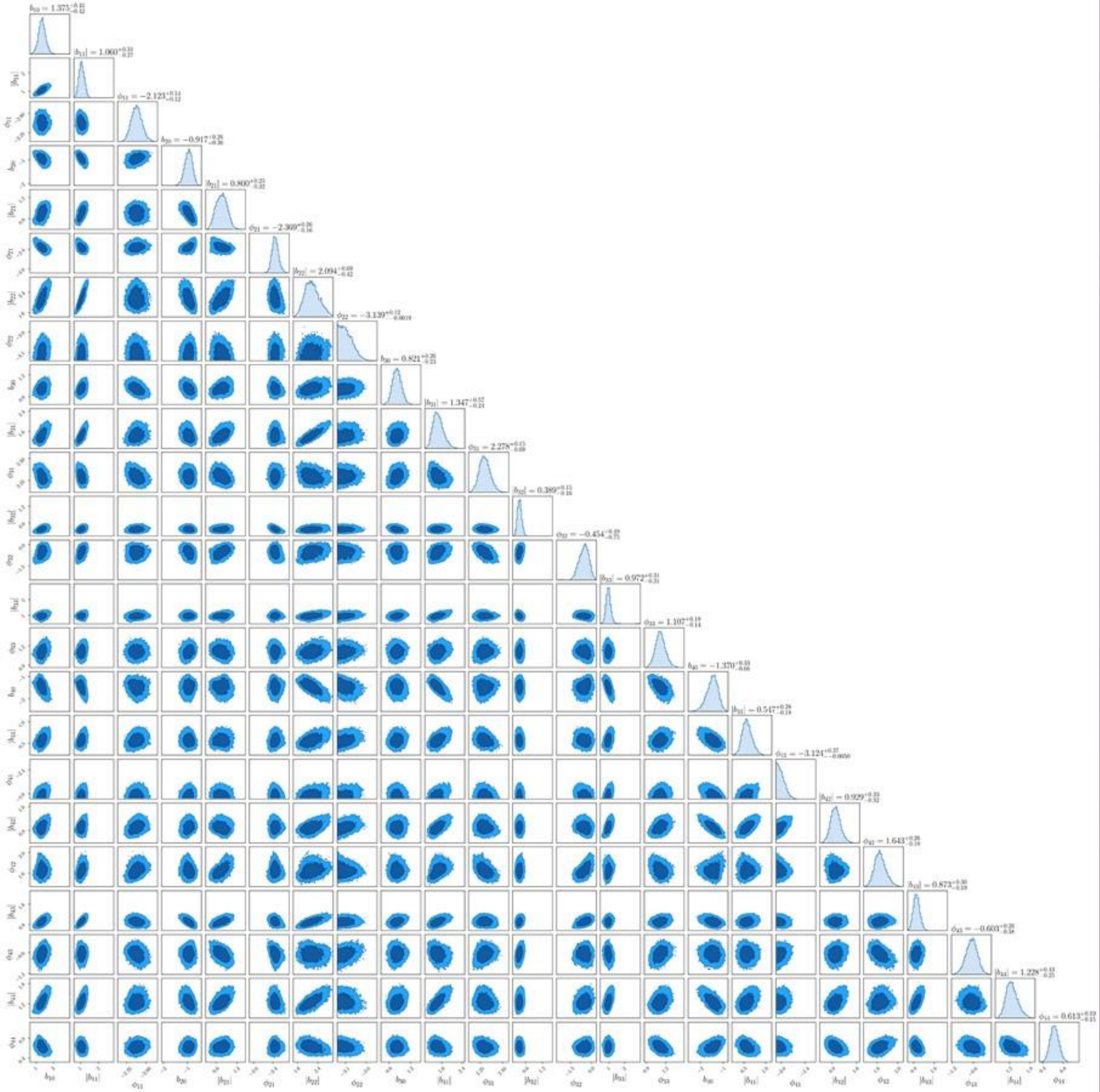
Extra Slide: Foreground Corner Plot (full)



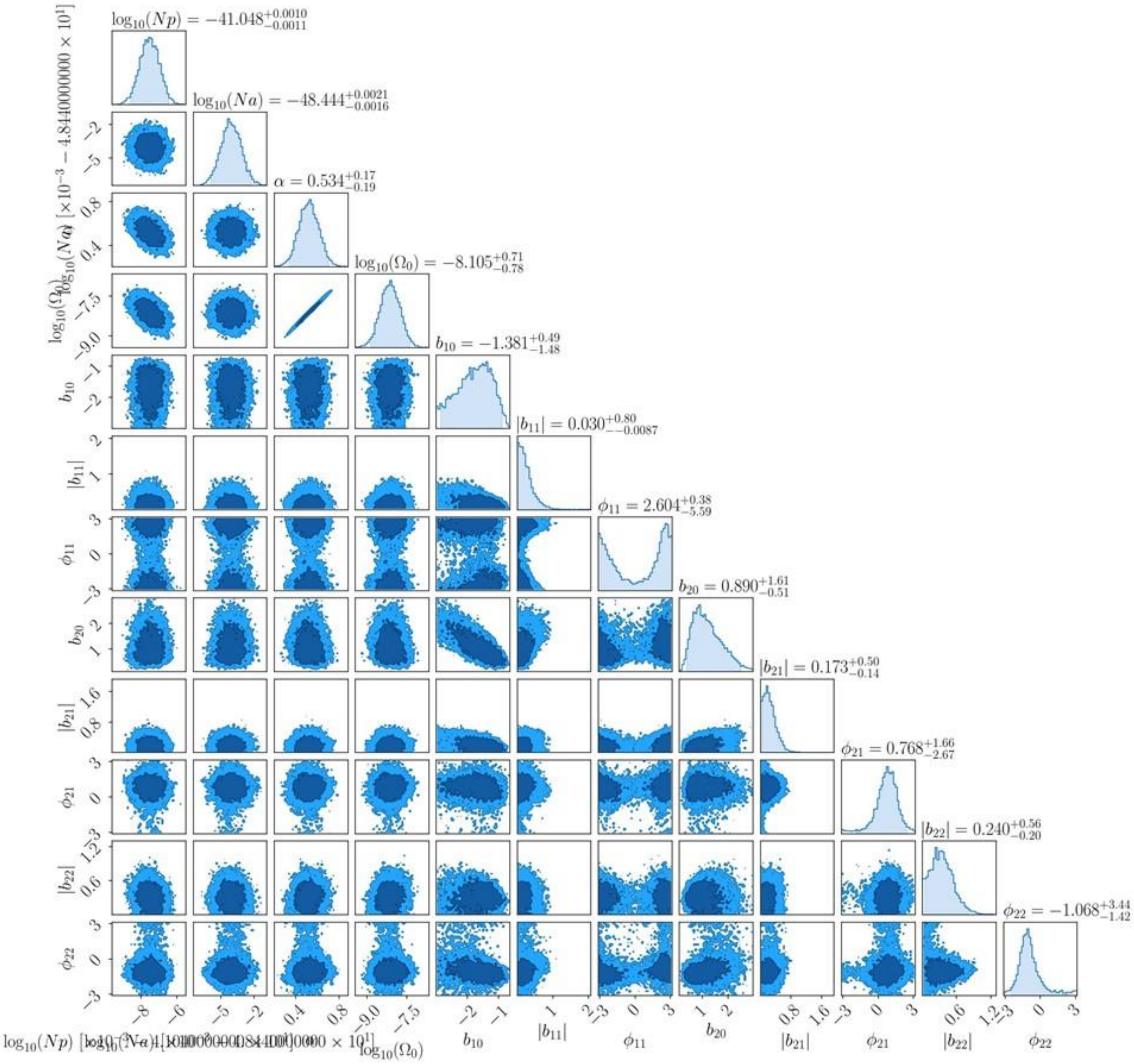
Extra Slide: Foreground Corner Plot (spectral)



Extra Slide: Foreground Corner Plot (spatial)



Extra Slide: LMC Corner Plot (full)



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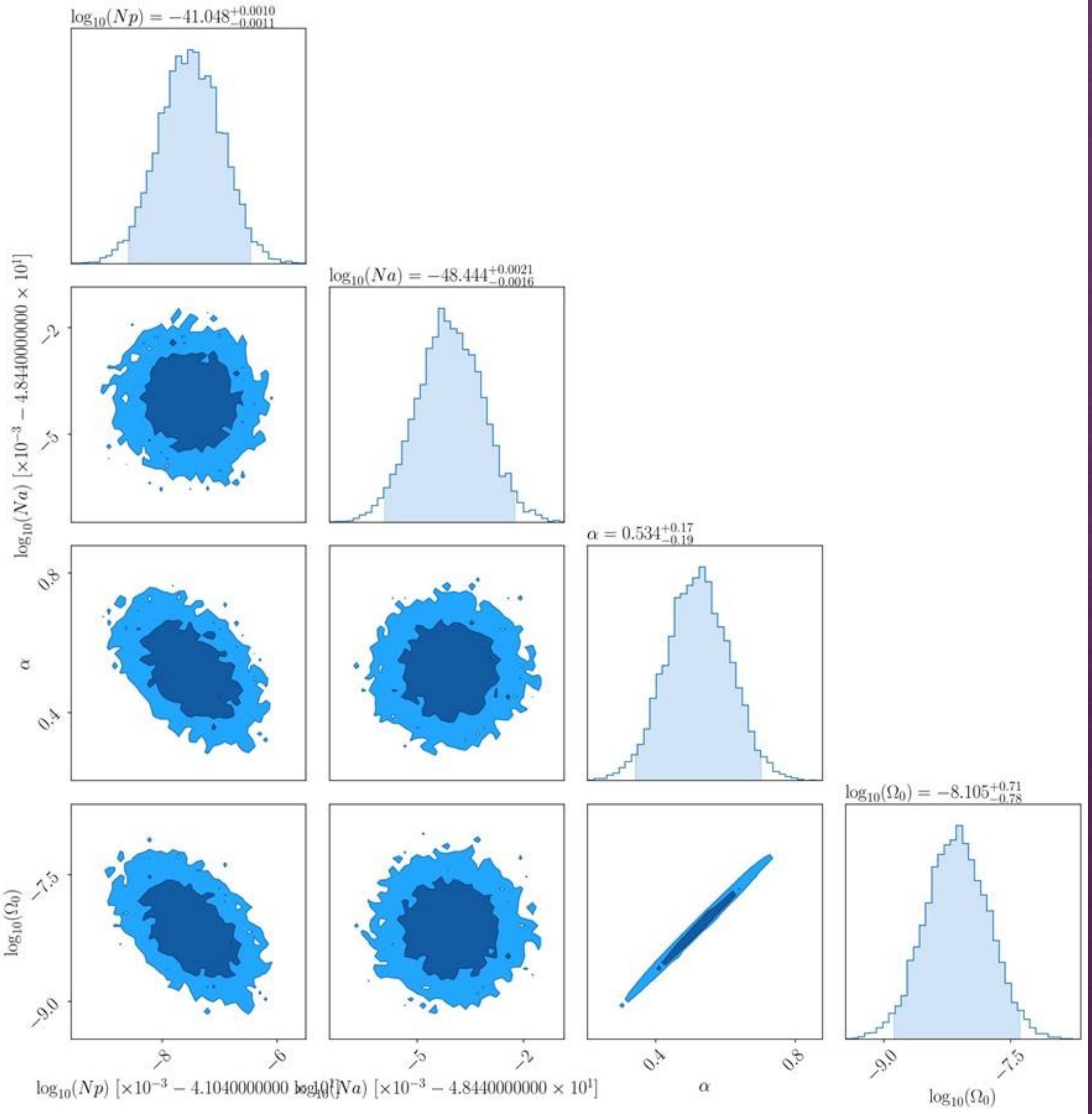

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