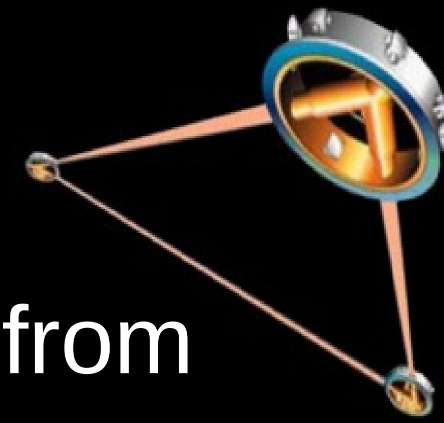

Detecting Gravitational Waves from Cosmic Strings with LISA

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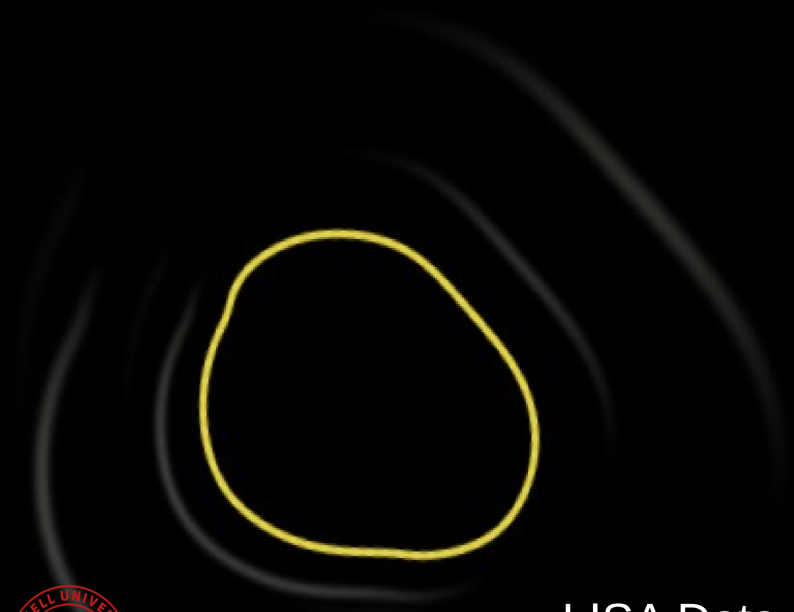


LISA Data Analysis: From Classical Methods to Machine Learning

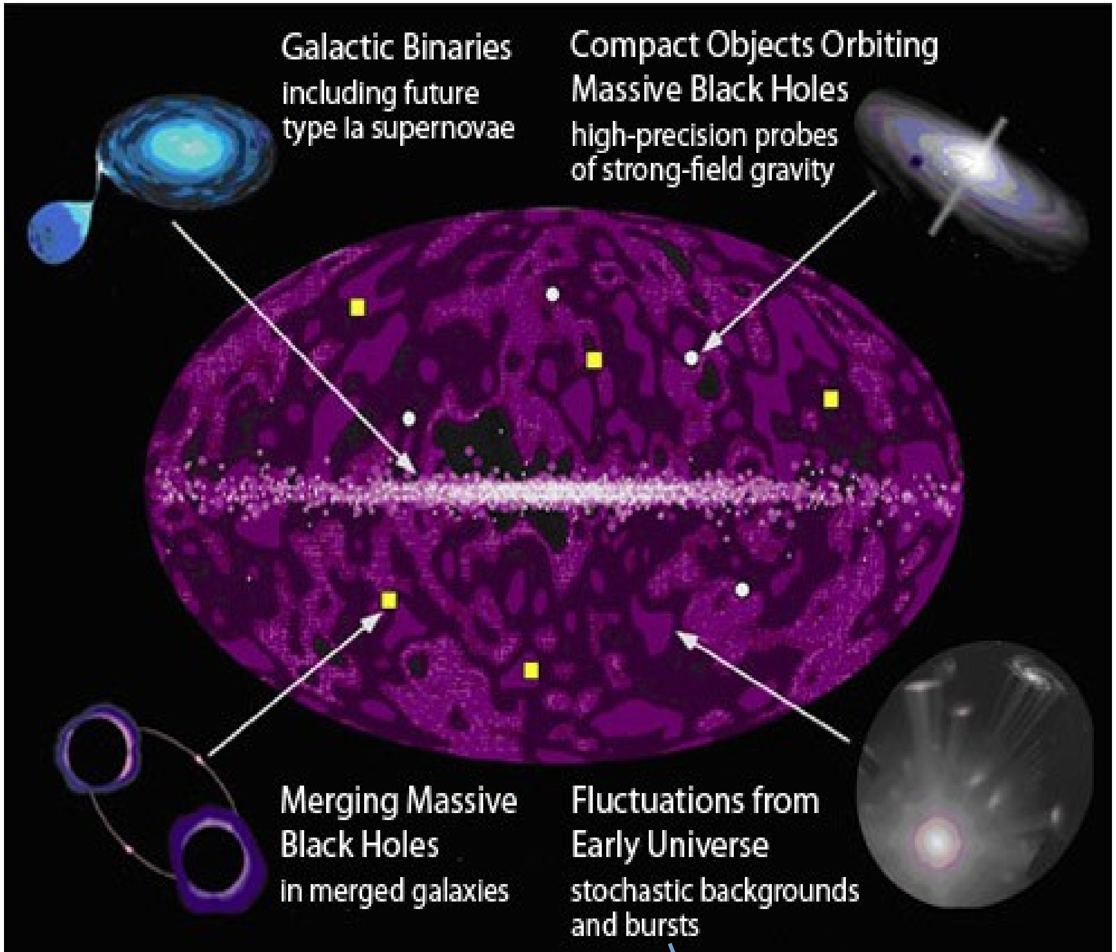
L2IT Toulouse

Nov 24, 2022

Outline

- Motivation
- Cosmic strings and relevant formalisms
- Computation of emission of power – previous and improved approaches
- Results

Motivation



[NASA/ESA]

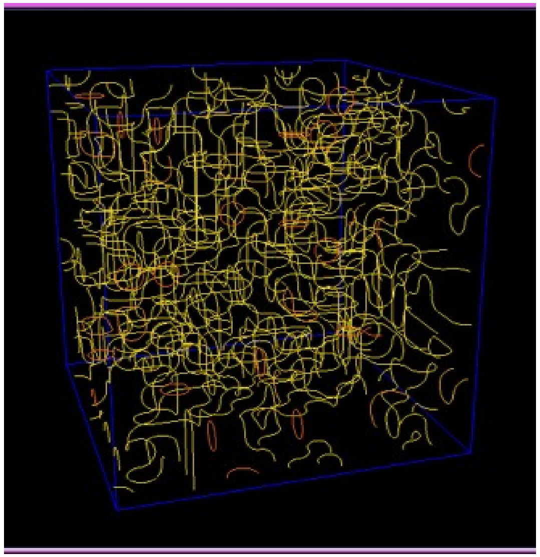
Cosmic strings!

$$\Omega_{\mathrm{gw}}$$

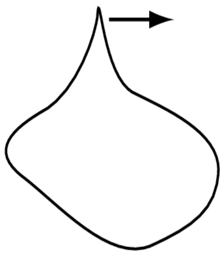
$$n(l,t) \qquad P_{\mathrm{gw}}(f,l)$$

What are cosmic strings?

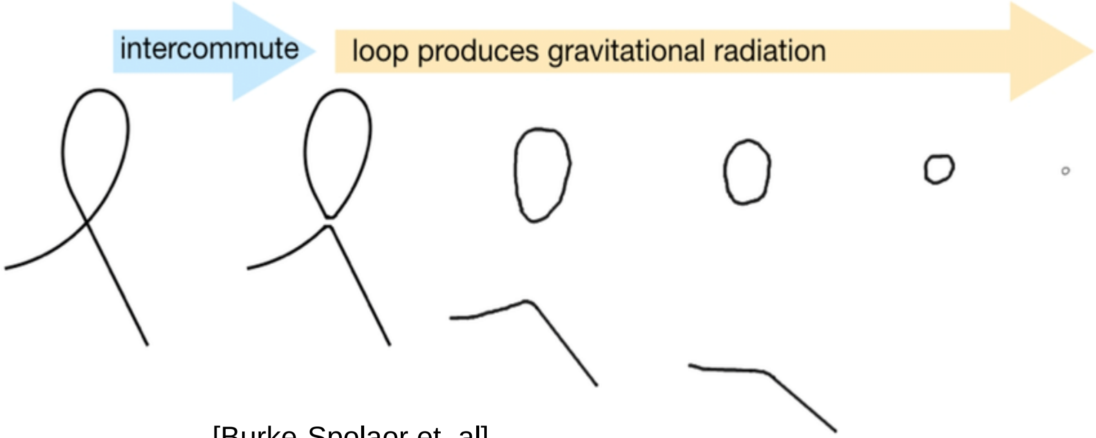
- Old picture: GUT scale topological defects
- New picture: Fundamental strings from superstring theory
- Parameters: String tension μ, length of the string loop l
- Cusps: Regions of the loop moving at the speed of light



[Martins + Shellard]



[Davis, Kibble]



[Burke-Spolaor et. al]

Cosmic string loop dynamics

- Described by Nambu-Goto action

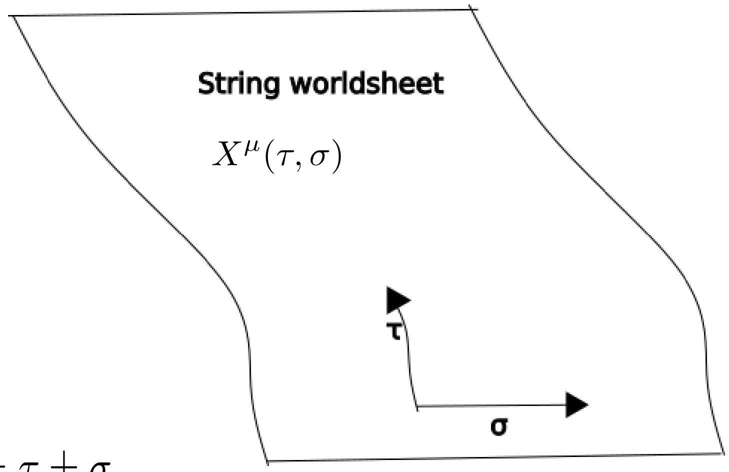
$$S_{NG} = -\mu \int d\tau d\sigma \sqrt{-\gamma}$$

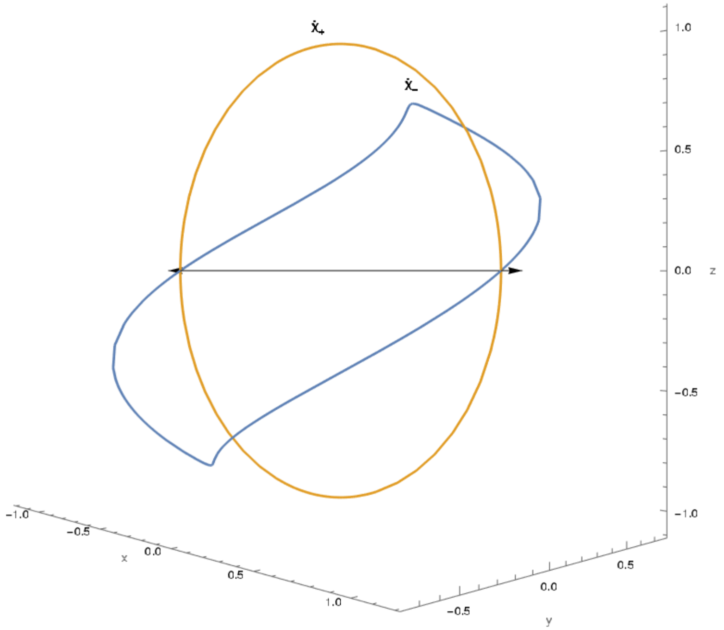
Worldsheet coordinates

$$X^\mu(\tau,\sigma) = \frac{1}{2}\left[X^\mu_-(\sigma_-) + X^\mu_+(\sigma_+)\right] \qquad \sigma_\pm = \tau \pm \sigma$$

- Tangent vectors intersect to form cusps

$$\vec{\dot{X}}_-(\sigma_-^{(c)}) = \vec{\dot{X}}_+(\sigma_+^{(c)})$$





Gravitational radiation from cosmic string loops

- Recipe:

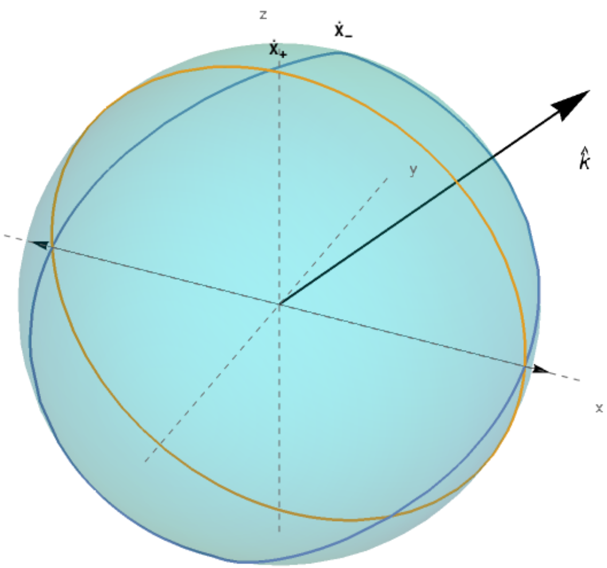
$$T^{\mu\nu} \rightarrow \frac{dP_m}{d\Omega} \rightarrow P_m = \int \frac{dP_m}{d\Omega} d\Omega \rightarrow P = \sum_{m=1}^{\infty} P_m$$

String energy-momentum tensor

Differential power along $\hat{k}$

Power per mode

Total power over all modes



- $T^{\mu\nu} = \frac{\mu l}{2}\left[I_-^{\mu} I_+^{\nu} + I_-^{\nu} I_+^{\mu}\right]$

[Damour + Vilenkin, 2001]

$$I_{\pm}^{\mu} = \frac{1}{l}\int_{-l/2}^{l/2} d\sigma_{\pm}\, \dot{X}_{\pm}^{\mu} e^{-\frac{i}{2} k_m . X_{\pm}}$$

$$k_m^{\mu} = \frac{4\pi m}{l}\left(1, \hat{k}\right)$$

Need to compute this

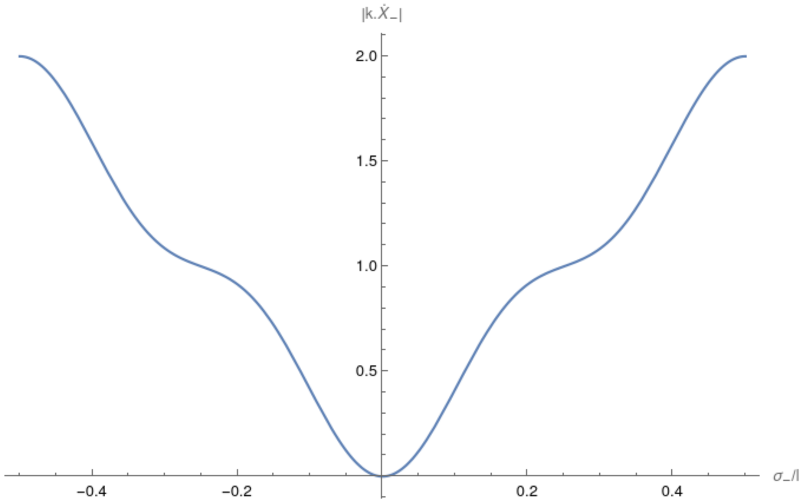
Analytic approximation to $\mathrm{I}_{\pm}^{\mu}$ – previous approaches (single-point)

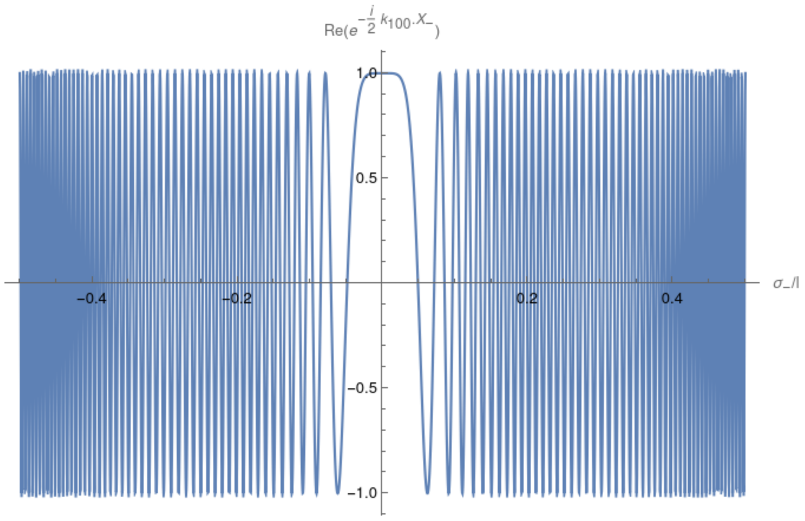
- Highly oscillatory at high modes, cusps dominate emission

  $P_m \propto m^{-4/3}$

- Expand around the cusp → analytic integrals

- Misses out other points on the loop

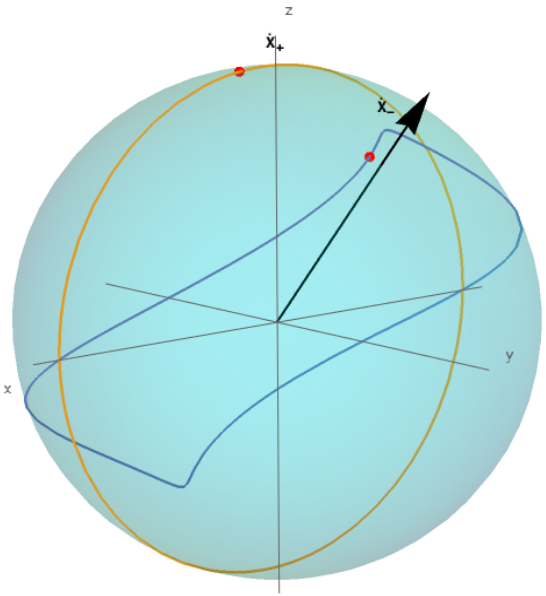


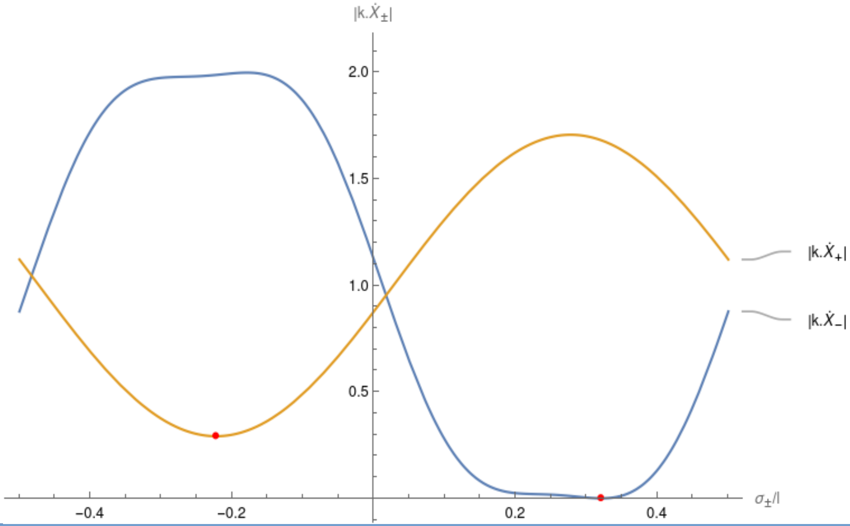


Analytic approximation to $\mathrm{I}^{\mu}_{\pm}$ – improved approach (multi-point)

- Factors in other regions of the loop
- For any $\hat{k}$, find regions of the loop which contribute the most
- Expansion center(s) for each $\hat{k}$:

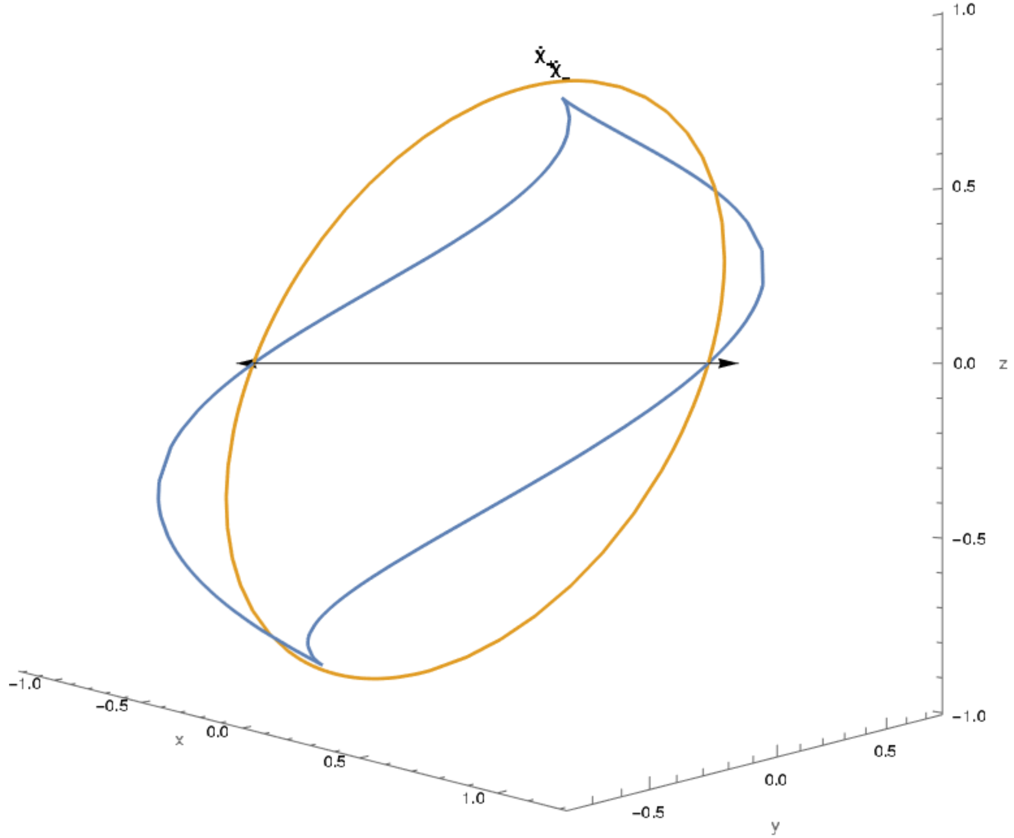
  Minima of $|k.\dot{X}_{\pm}| \rightarrow \sigma^{*}_{\pm}$
- Analytic integrals after Taylor expansion





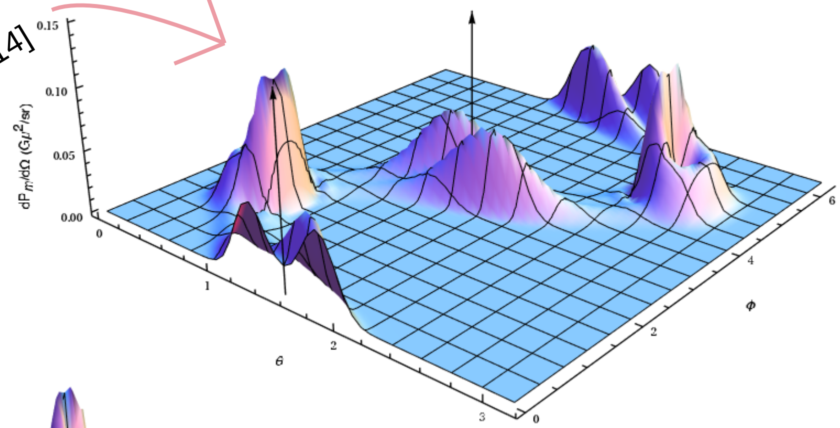
Example calculation of $dP_m/d\Omega$

Loop with two cusps

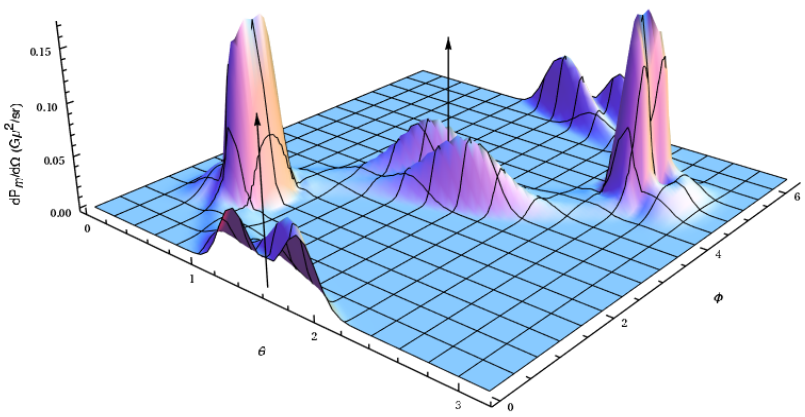


Example calculation of $dP_m/d\Omega$

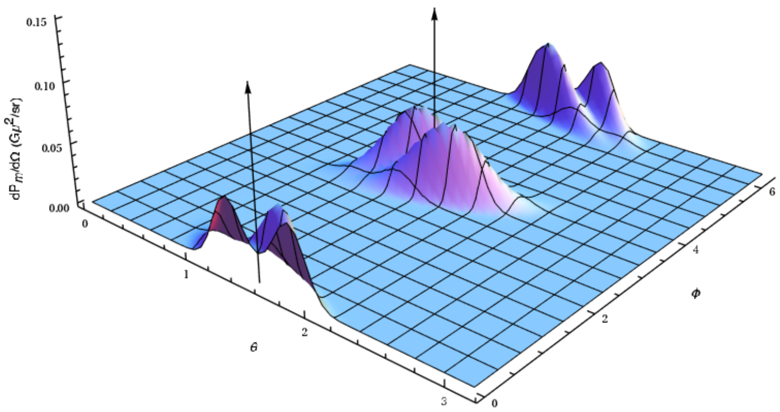
Pseudocusps
[Elghozi et. al., 2014]



Numerical



Multi-point



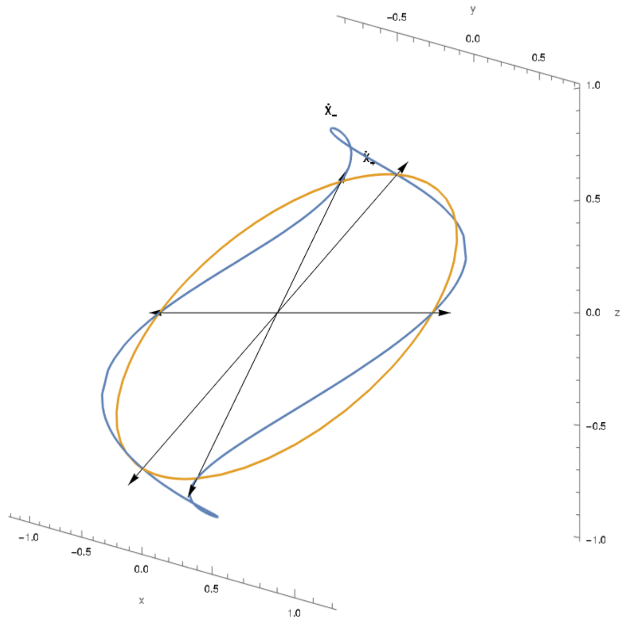
Single-point
[Blanco- Pillado + Olum, 2017]

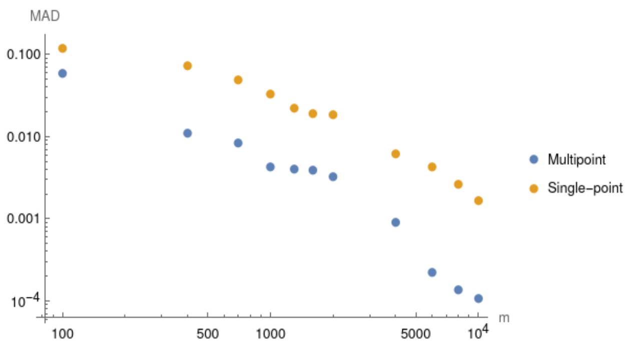
Comparing multi-point vs. single point

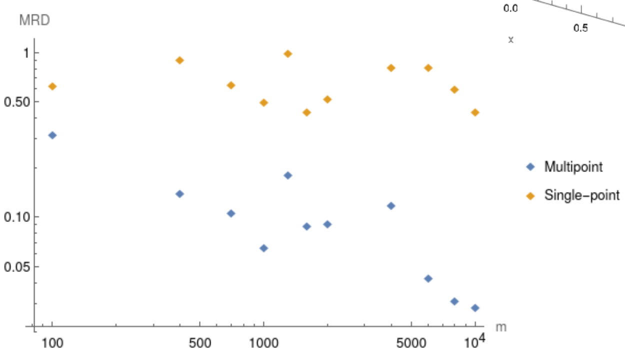
$$\mathrm{MAD} = \max\left|(dP_m/d\Omega)_i - (dP_m/d\Omega)_{\mathrm{numerical}}\right| \qquad i = \text{multipoint, single-point}$$

$$\mathrm{MRD} = \frac{\mathrm{MAD}}{(dP_m/d\Omega)_{max,\mathrm{numerical}}}$$

Example: Loop with 6 cusps and pseudocusps

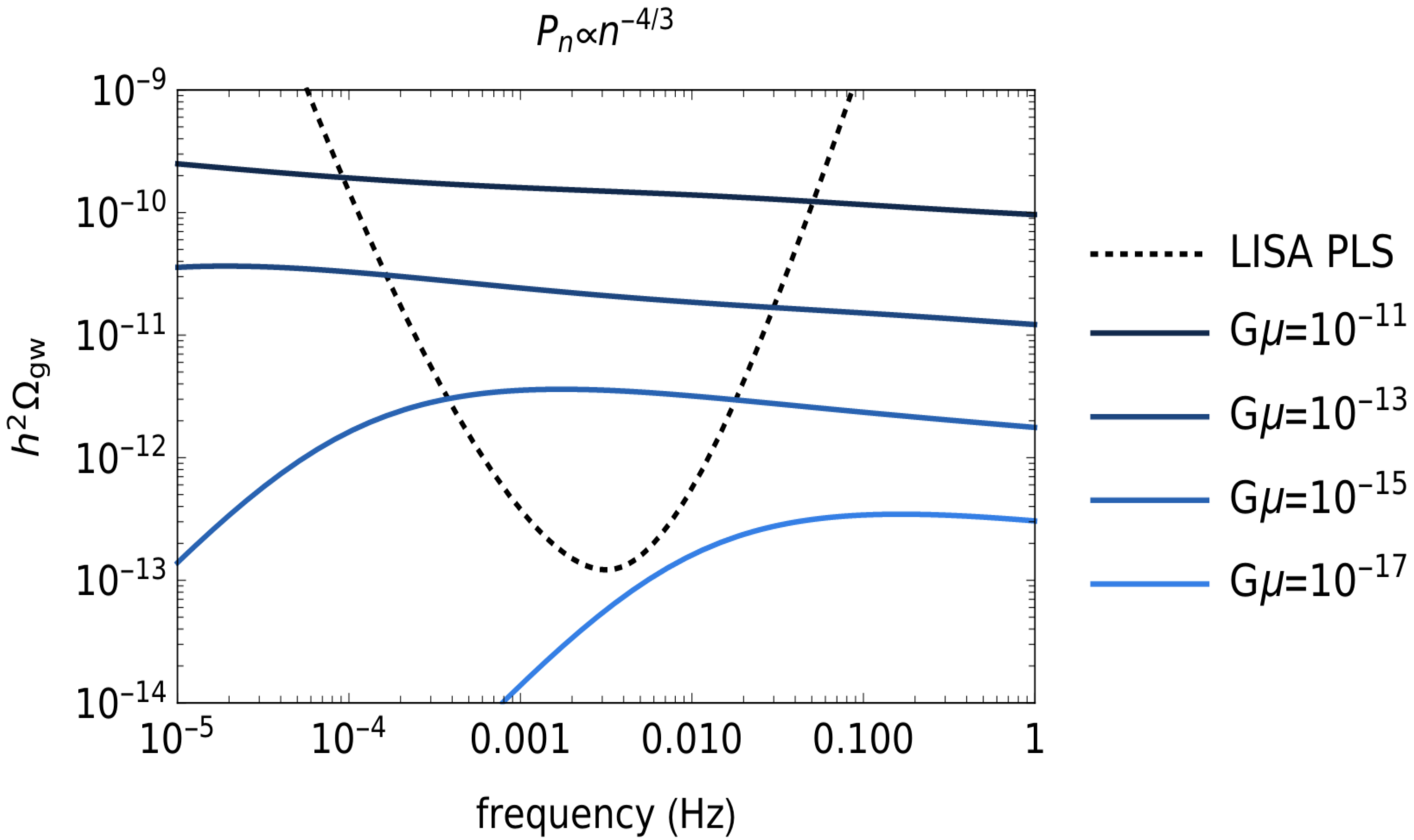






Probing the Gravitational Waves with LISA (Previous models)

- LISA will be sensitive to cosmic strings with tension $G\mu \gtrsim 10^{-17}$



[Auclair et. al., 2019]

Conclusions

- Developed an improved approach for calculating the power spectrum from a cosmic string loop
- Better accuracy, accounts for larger regions of the loop, general formalism applicable to loops without cusps
- Numerical model at low m + Improved analytical approach at high m → complete model of the power spectrum → SGWB calculation

Thank You!

$$f_m = \frac{2}{l} m$$

$$f_1 = \frac{d_H}{l} \times 10^{-18} \mathrm{Hz}$$

$$f_{obs} = \frac{f_m}{1+z}$$

Analytic approximation to $\mathrm{I}_{\pm}^{\mu}$ – improved approach (multi-point)

- Expand phase and prefactor around these points

$$k.X_\pm(\sigma_\pm) = k.\left[X_\pm(\sigma^*_\pm) + \dot{X}_\pm(\sigma^*_\pm)(\sigma_\pm - \sigma^*_\pm) + \frac{1}{2}\ddot{X}_\pm(\sigma^*_\pm)(\sigma_\pm - \sigma^*_\pm)^2 + \frac{1}{6}X^{(3)}_\pm(\sigma^*_\pm)(\sigma_\pm - \sigma^*_\pm)^3\right]$$

$$I_\pm^\mu = \frac{1}{l}\int_{-l/2}^{l/2} d\sigma_\pm\ \dot{X}_\pm^\mu e^{-\frac{i}{2}k_m.X_\pm}$$

$$\int_{-l/2}^{l/2} \to \int_{-\infty}^{\infty}$$

$$\dot{X}^\mu_\pm(\sigma_\pm) = \dot{X}^\mu_\pm(\sigma^*_\pm) + \ddot{X}^\mu_\pm(\sigma^*_\pm)\left(\sigma_\pm - \sigma^*_\pm\right)$$

- Integrals $\int_{-\infty}^{\infty} du\ e^{i(Au^3+Bu^2+Cu+D)}$ and $\int_{-\infty}^{\infty} du\ u\ e^{i(Au^3+Bu^2+Cu+D)}$ can be done analytically