

# Towards a complete L0-L2 pipeline

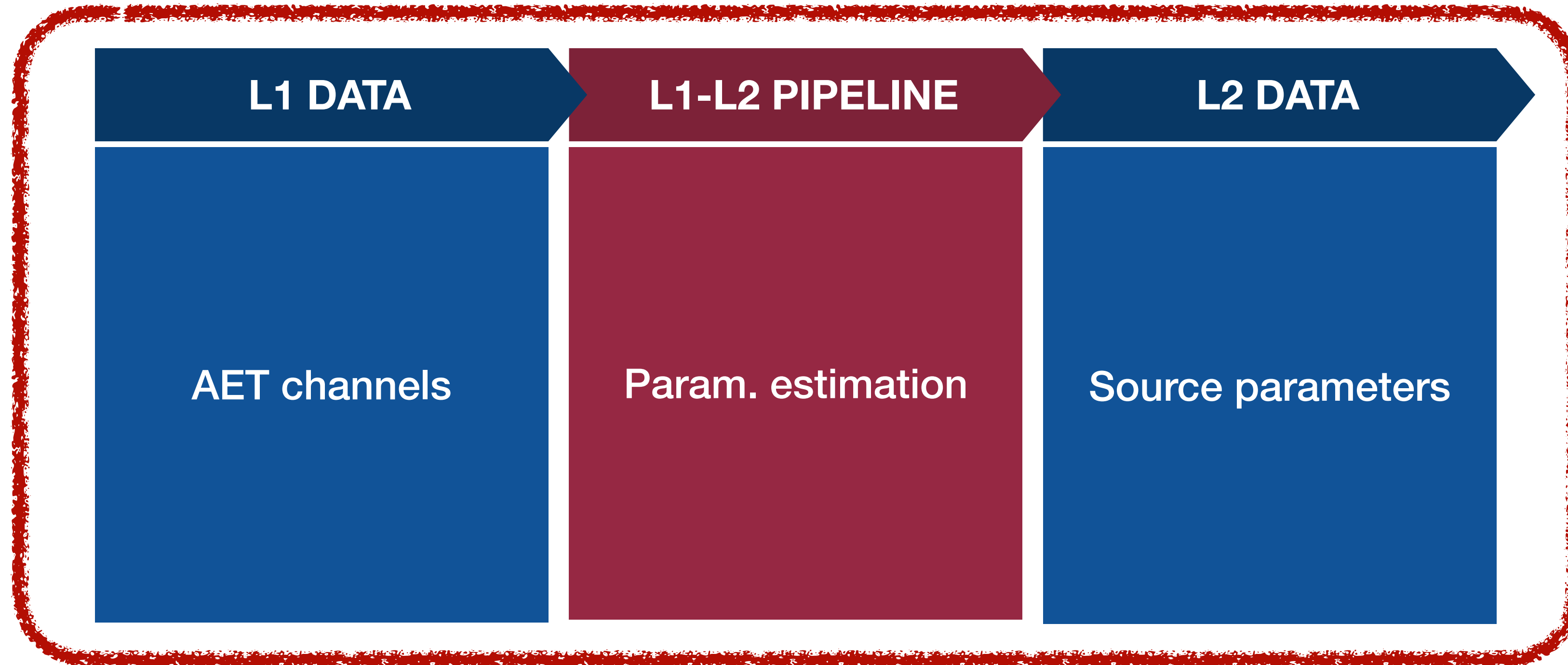
Progress in simulation, processing and analysis

SYRTE  | PSL 

 University  
of Glasgow

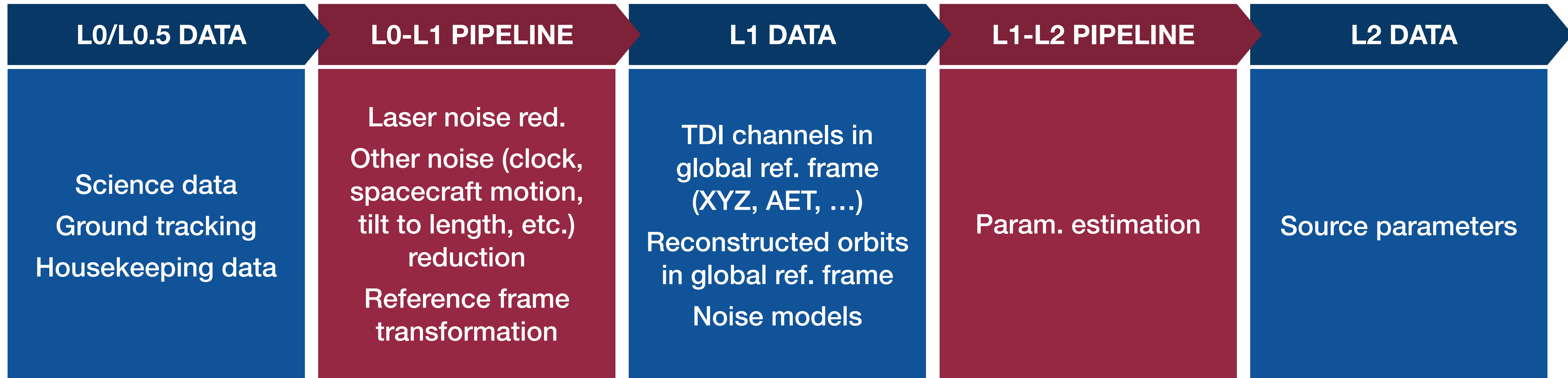
Jean-Baptiste Bayle and Olaf Hartwig – 2022 LIDA (Toulouse)

# LISA Data Analysis Pipeline

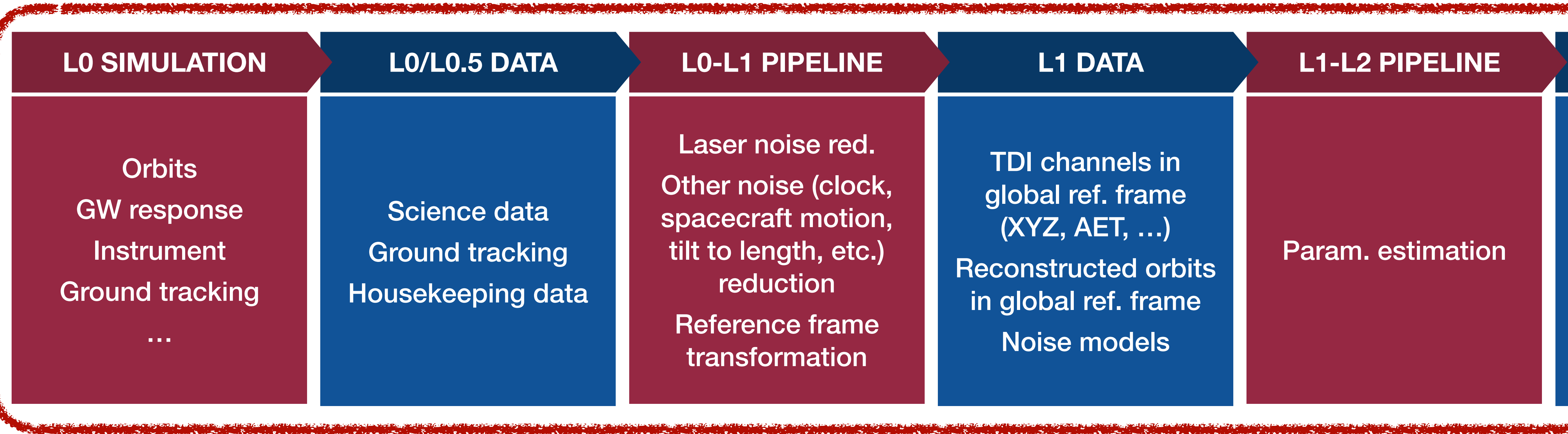


**LDC**

# LISA Data Analysis Pipeline



# LISA Data Analysis Pipeline



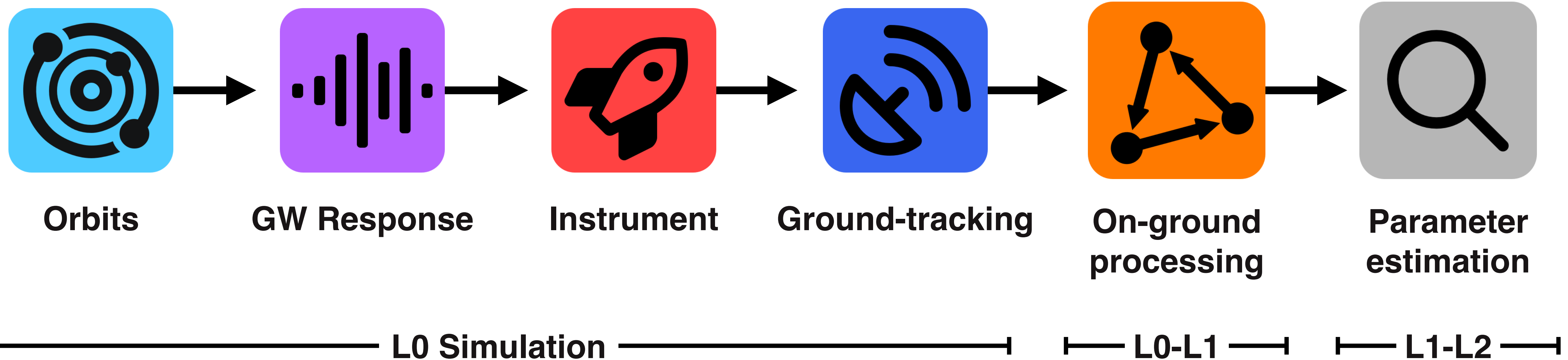
## L0-L2 DEMONSTRATION PIPELINE

# Context

- **Activity requested by ESA & LISA Consortium**
  - FMT Task 4.5 started only a few months ago
  - Preparation for mission adoption over summer 2023
  - Deliverables are a demonstration pipeline and some figures of merits (Before june?)
- **Participants**
  - University of Glasgow (Jean-Baptiste Bayle, Christian Chapman-Bird, Graham Wohan)
  - SYRTE (Olaf Hartwig, Aurelien Hees, Marc Lilley, Peter Wolf)

# Method

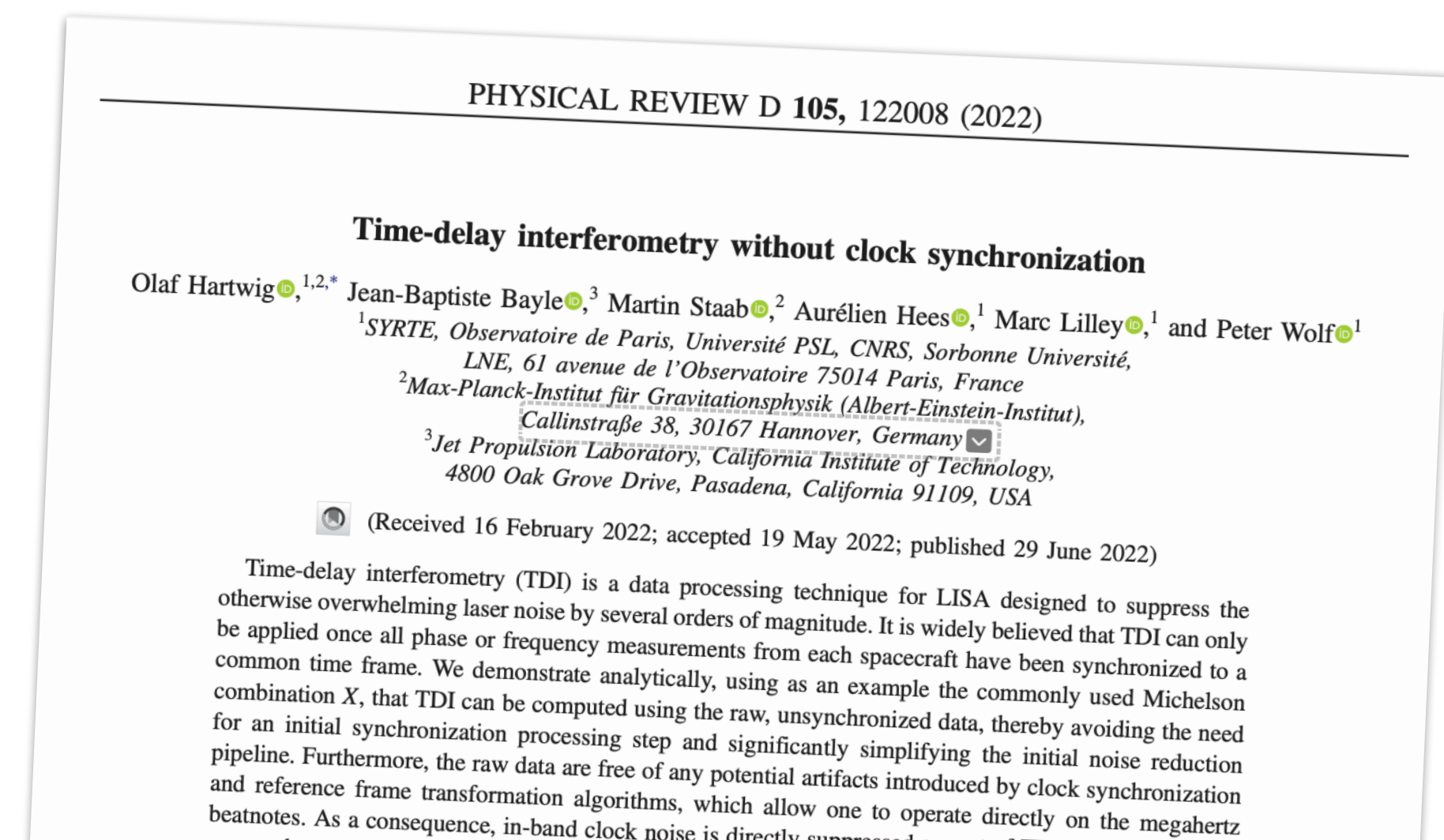
- Build a pipeline with various processing blocks



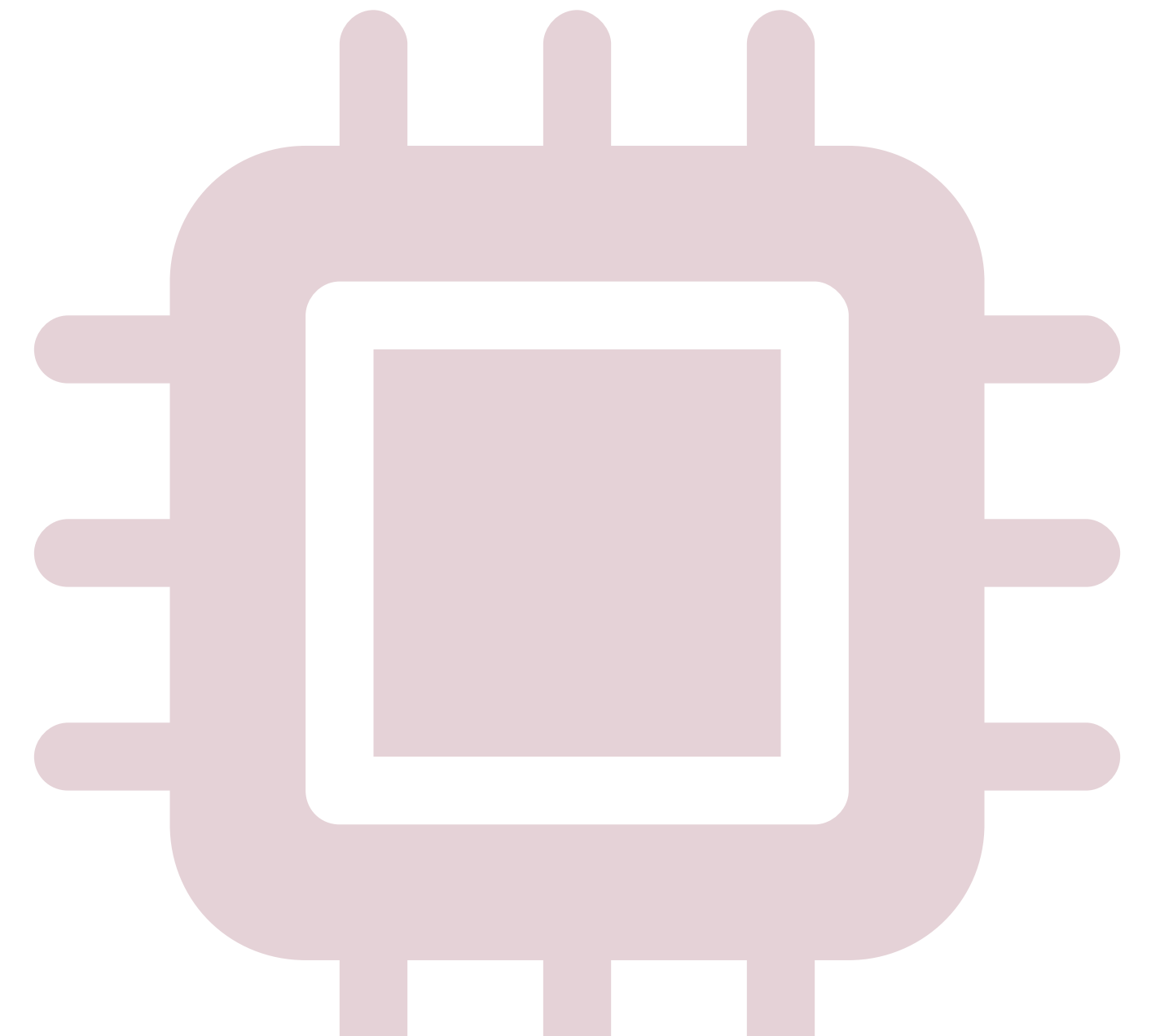
- Assemble the blocks and builds performance metrics

# Method

- Processing blocks exist in multiple versions (various configurations)
  - Start with simplified configurations
  - Increase model faithfulness (and processing accordingly) and assess metric variations
- Short timeframe, so restrict the activity to
  - Rather simple target configuration
    - ▶ Well separated loud Galactic binary sources
    - ▶ Current best simulation model of the instrument
    - ▶ Processing derived from [10.1103/PhysRevD.105.122008](https://arxiv.org/abs/10.1103/PhysRevD.105.122008)
    - ▶ Parameter estimation based on LDC
  - Identify impactful effects (and try to mitigate them?)



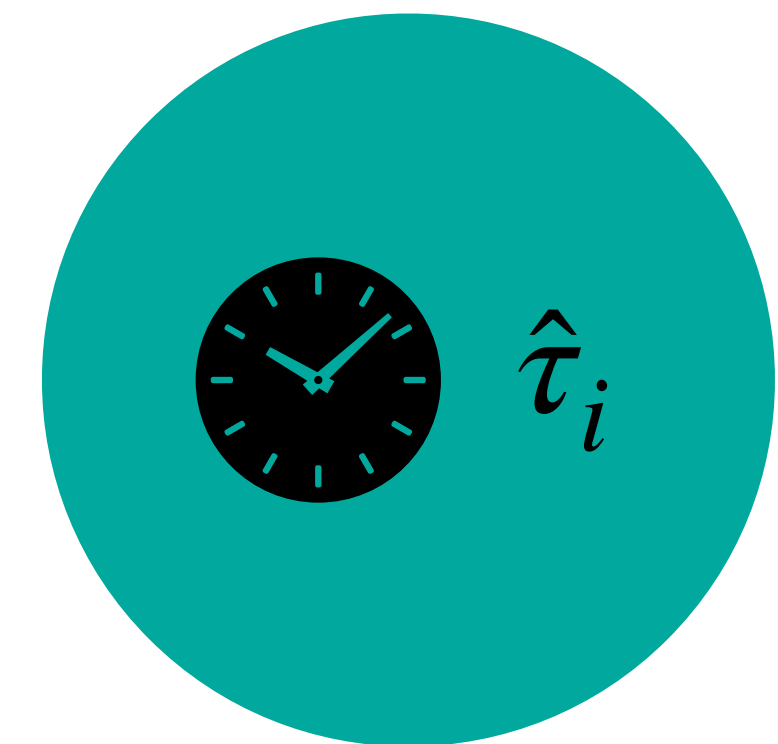
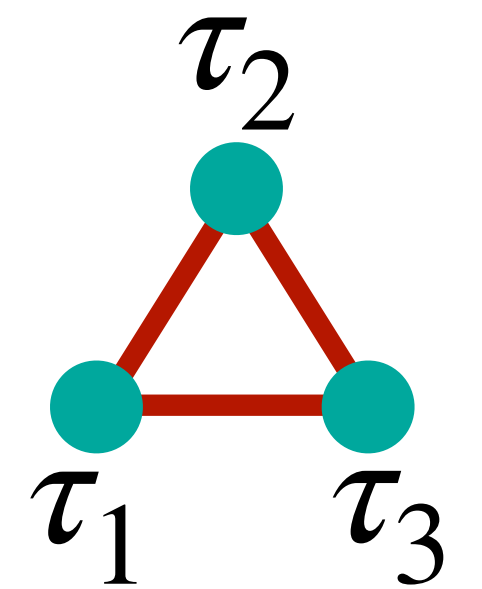
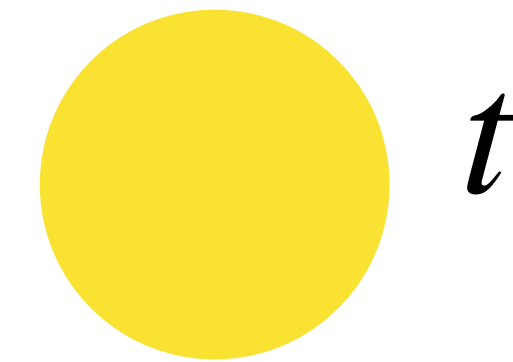
# L0 Simulation





# Timescales

- **Global TCB time  $t$** 
  - Defined as time shown of a perfect clock sitting at Solar system barycenter
  - Global timescale, used for orbits and GW strain
- **One proper time  $\tau_i$  for each spacecraft  $i$  ( $i = 1,2,3$ )**
  - Defined as time shown by a *perfect* clock sitting in spacecraft  $i$
  - Related to  $t$  (and each other) by General Relativity
  - Used to describe physics inside one spacecraft
- **One onboard clock time  $\hat{\tau}_i$  for each spacecraft  $i$  ( $i = 1,2,3$ )**
  - Defined as time shown by the *actual* clock sitting in spacecraft
  - Differs from  $\tau_i$  by instrumental imperfections
  - Only timescales directly accessible by the satellites

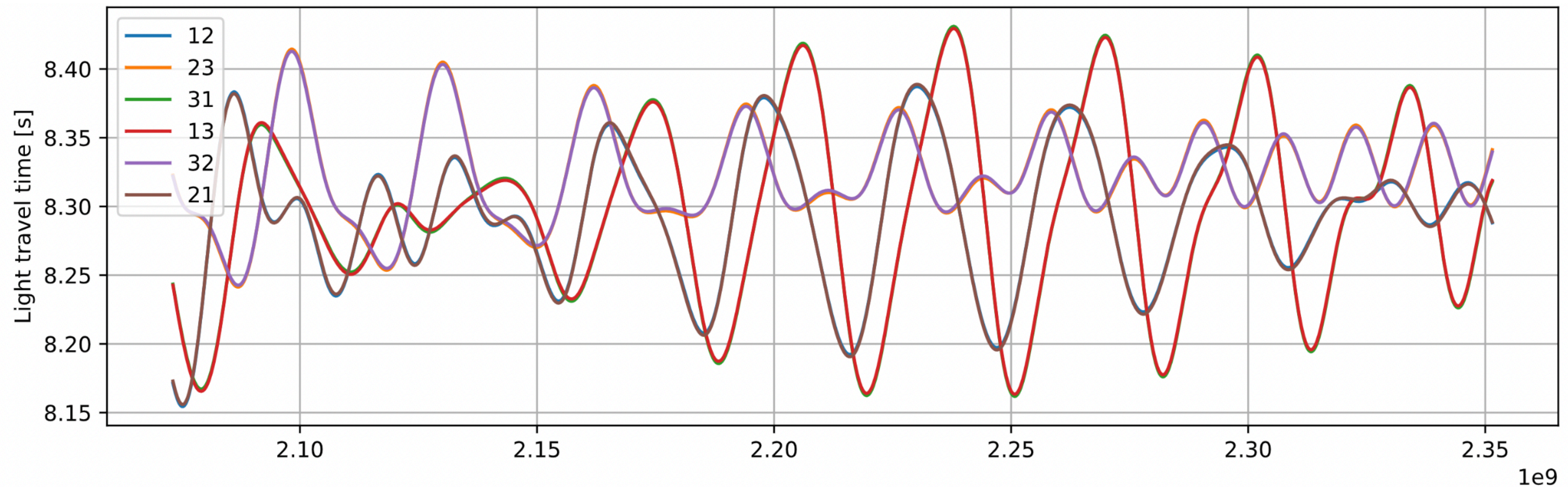


# Orbits

- Use ESA numerically optimized orbits
- Interpolate spacecraft state vectors and compute necessary quantities (e.g., light travel times and relativistic relationships between reference frames) with



Bayle, Jean-Baptiste, Hees, Aurélien, Lilley, Marc, & Le Poncin-Lafitte, Christophe. (2022). LISA Orbits (2.0). Zenodo. <https://doi.org/10.5281/zenodo.6412992>



# GW Response

- Compute frequency shift due to gravitational waves measured on each optical bench (link responses) in the spacecraft proper time frames with



Bayle, Jean-Baptiste, Baghi, Quentin, Renzini, Arianna, & Le Jeune, Maude. (2022). LISA GW Response (1.1). Zenodo. <https://doi.org/10.5281/zenodo.6423436>

- Model fully described in documentation

- Deformation induced on link 12 is  $H_{12}(t) = h_{+}^{\text{SSB}}(t)\xi_{+}(\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{n}}_{12}) + h_{\times}^{\text{SSB}}(t)\xi_{\times}(\hat{\mathbf{u}}, \hat{\mathbf{v}}, \hat{\mathbf{n}}_{12})$
- Reception time  $t_1$  of a photon emitted at  $t_2$  is  $t_1 \approx t_2 + \frac{L_{12}}{c} - \frac{1}{2c} \int_0^{L_{12}} H_{12}(\mathbf{x}(\lambda), t(\lambda)) d\lambda$
- We substitute and differentiate to obtain the frequency shift

$$y_{12}(t_1) \approx \frac{1}{2(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}_{12}(t_1))} \left[ H_{12} \left( t_1 - \frac{L_{12}(t_1)}{c} - \frac{\hat{\mathbf{k}} \cdot \mathbf{x}_2(t_1)}{c} \right) - H_{12} \left( t_1 - \frac{\hat{\mathbf{k}} \cdot \mathbf{x}_1(t_1)}{c} \right) \right]$$

- Resample to proper times  $\hat{\tau}_1(t)$  using orbits

# Instrument Simulation

- Simulation model available in a paper *in prep.* and the LISA Instrument Simulation Model document ([available here](#) for the consortium members)



Bayle, Jean-Baptiste, Hartwig, Olaf, & Staab, Martin. (2022).

LISA Instrument (1.1.1). Zenodo. <https://doi.org/10.5281/zenodo.7071251>

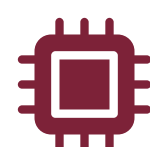
- Instrumental simulation includes



Optics (modulation and propagation of laser beams, main interferometric measurements and auxiliary measurements)



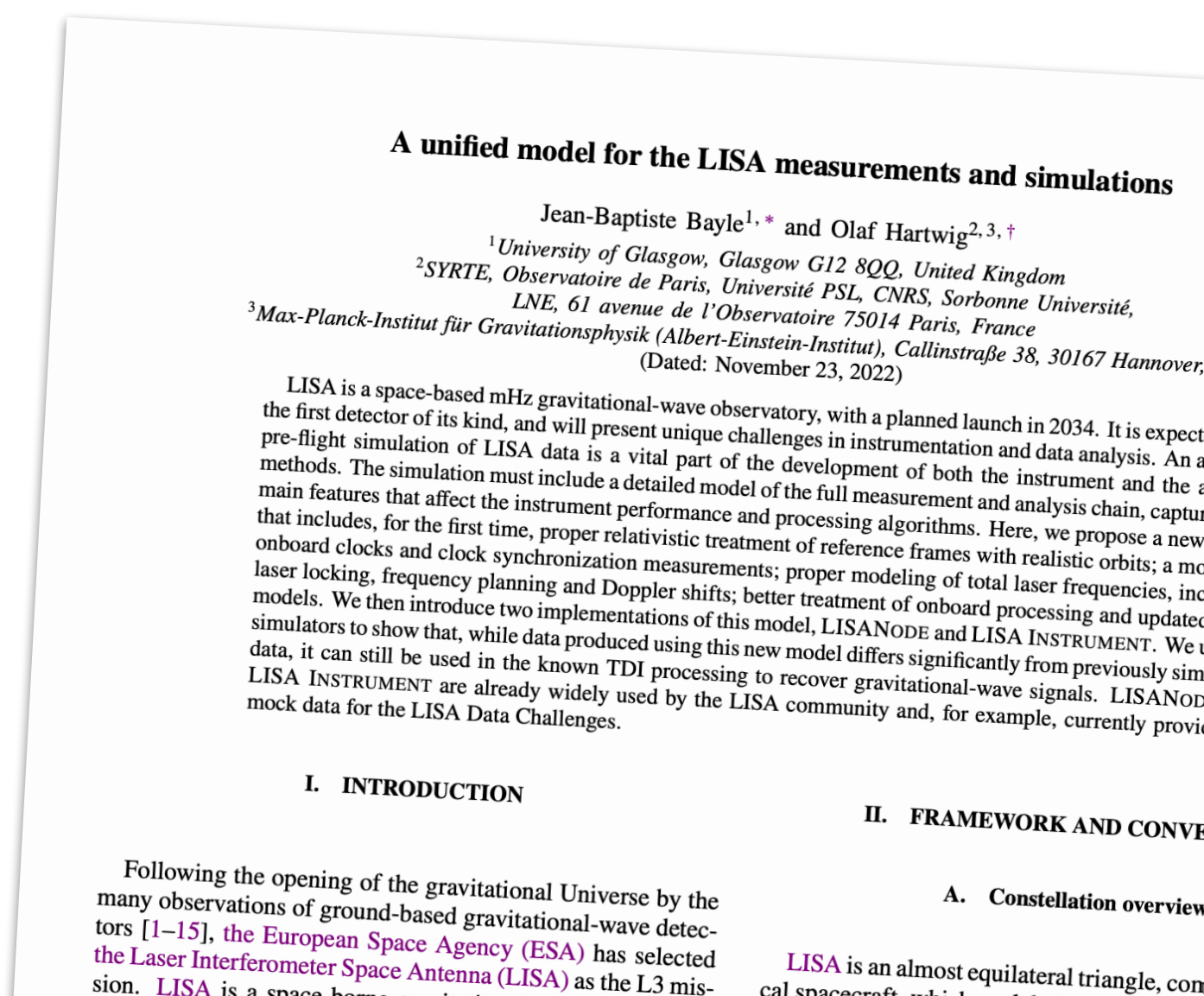
Dynamics (motion of spacecraft and test masses) – currently limited



Onboard processing (digital sampling, filtering and downsampling)

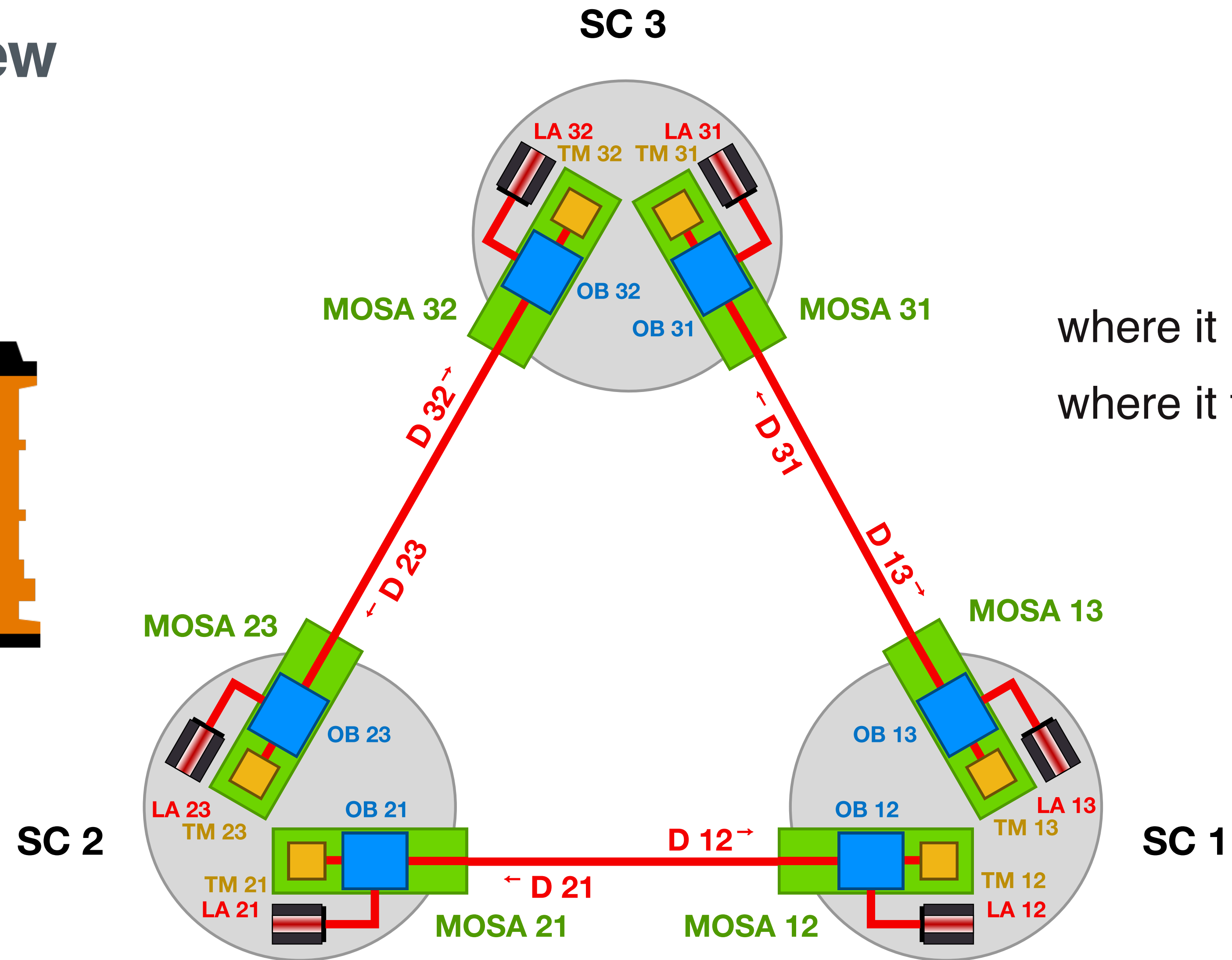
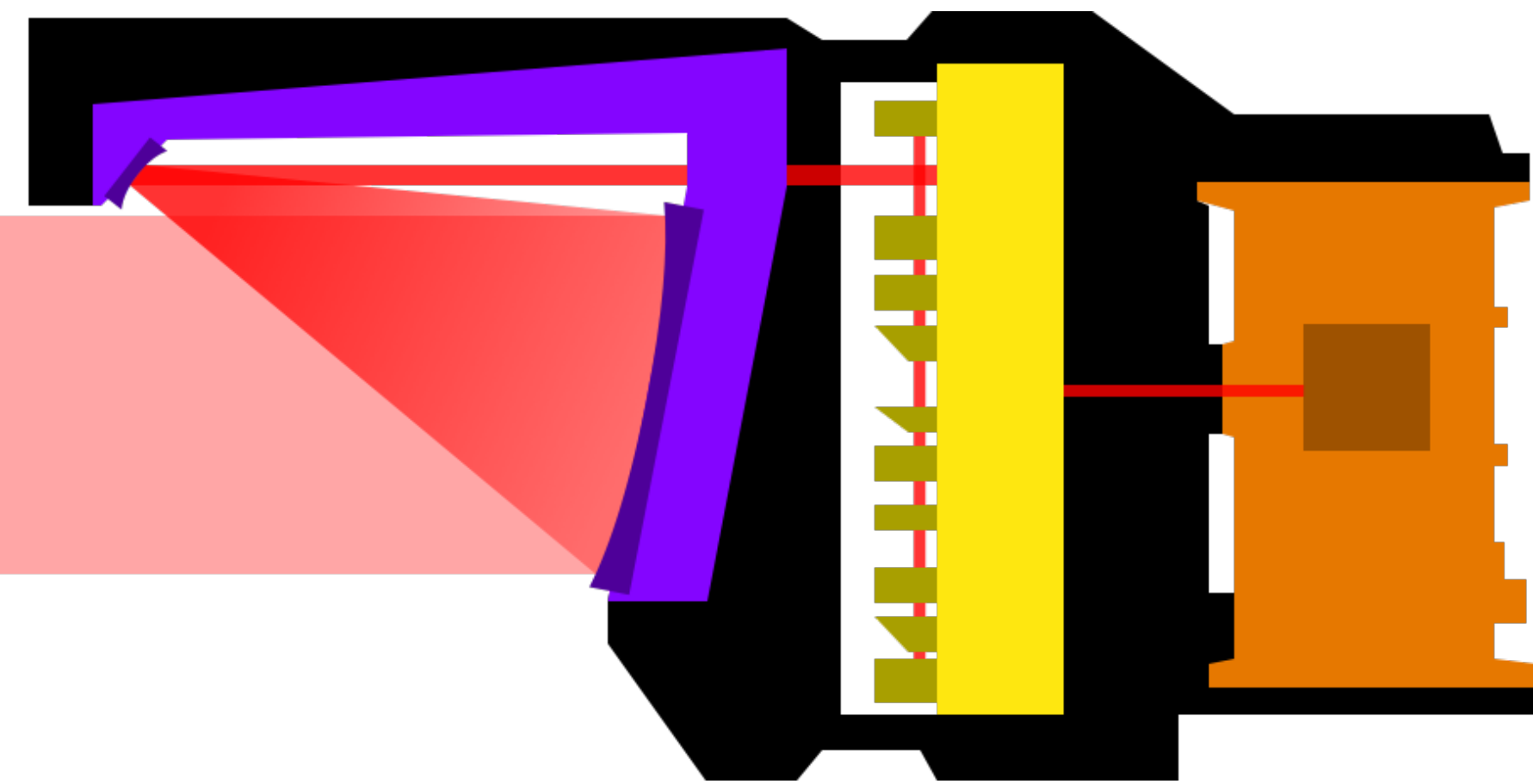
- Two sampling rates

- Measurements telemetered at 4 Hz
- Physics simulated at 16 Hz (inputs upsampled during simulation)



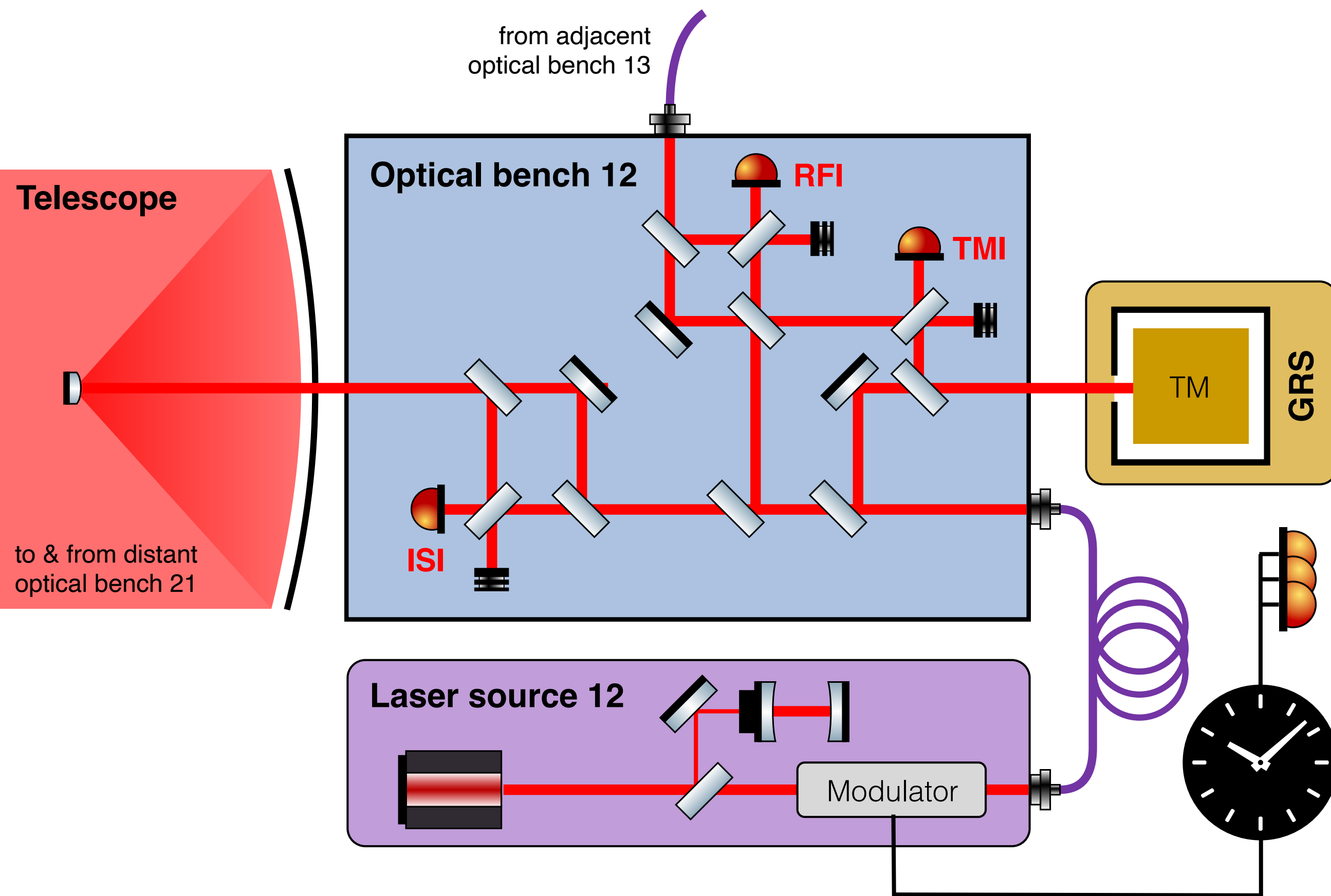
# Instrument Simulation

## Constellation Overview



# Instrument Simulation

## Optical Bench Overview



- 3 main interferometric signals recorded on each optical bench
  - **Inter-spacecraft interferometer (ISI)**
  - **Reference interferometer (RFI)**
  - **Testmass interferometer (TMI)**
- Interferometer data sampled according to onboard clock
- Modulate laser beams using clock signal to correct for sampling errors during L0-L1 processing

# Instrument Simulation

## Laser Beams

- Electromagnetic field  $E(\tau) = E_0(\tau)\cos(2\pi\Phi(\tau))$
- GW signals encoded in the oscillating part of the field
- Simulate total frequencies  $\nu(\tau) = \dot{\Phi}(\tau)/2\pi$  – *not just strain!*

$$\nu(\tau) = \nu_0 + \nu^o(\tau) + \nu^\epsilon(\tau)$$

The diagram illustrates the decomposition of the total instantaneous frequency  $\nu(\tau)$  into three components, each represented by an upward-pointing arrow:

- total instantaneous frequency** (points to  $\nu(\tau)$ )
- central frequency** (points to  $\nu_0$ ): constant 281.6 THz
- frequency offsets** (points to  $\nu^o(\tau)$ ): freq. plan, Dopplers, etc.  $\approx 10$  MHz
- frequency fluctuations** (points to  $\nu^\epsilon(\tau)$ ): Noises, GW, etc.  $\approx 100$  Hz + 100 nHz

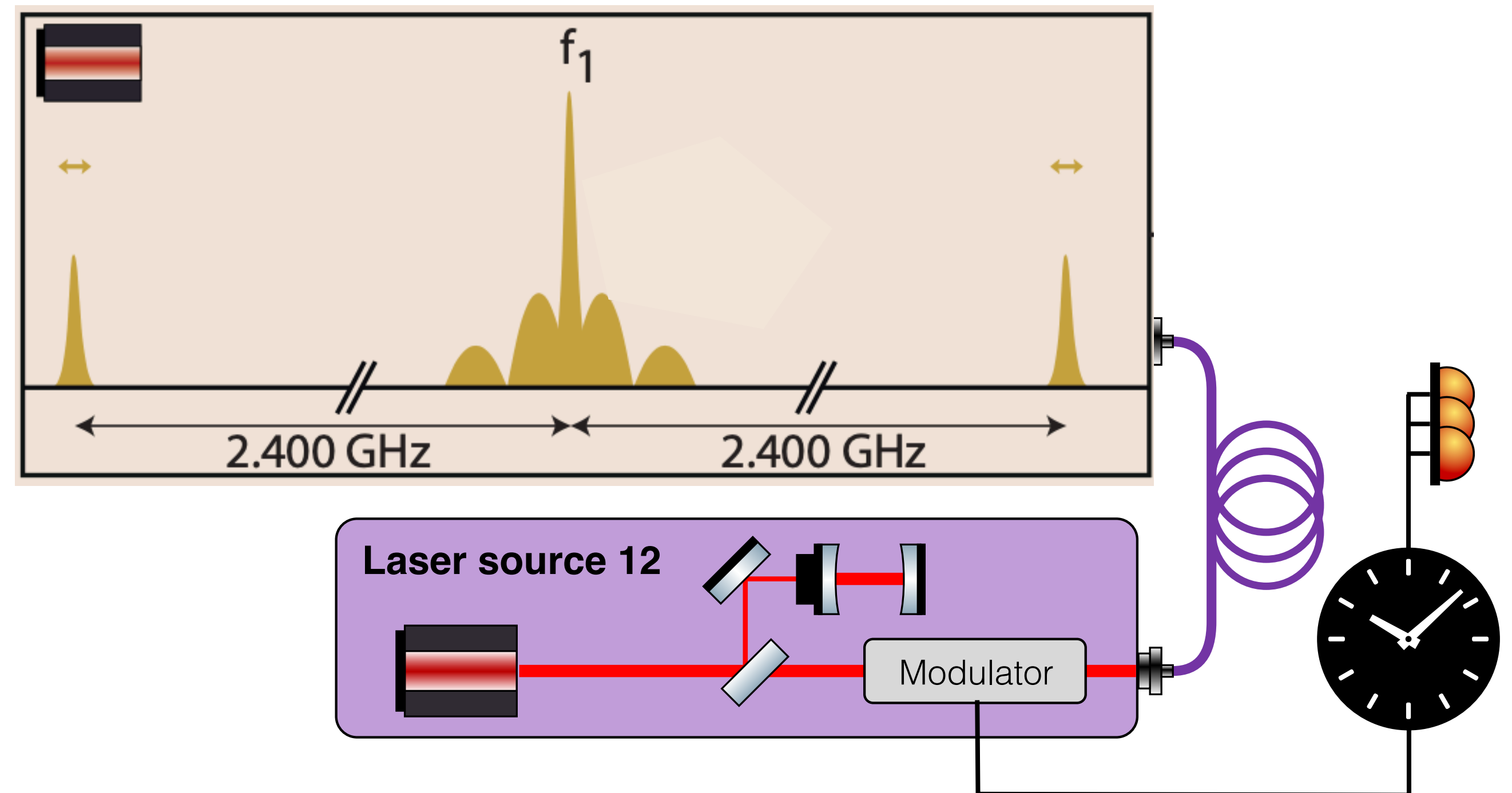
# Instrument Simulation

## Beam Modulation

- Onboard clocks used to sample data, therefore contribute to *phase errors*
- Phase modulation used to measure the in-band part of this clock noise

$$E(\tau) = E_0 e^{j2\pi(\Phi_c(\tau) + m\Phi_m(\tau))}$$

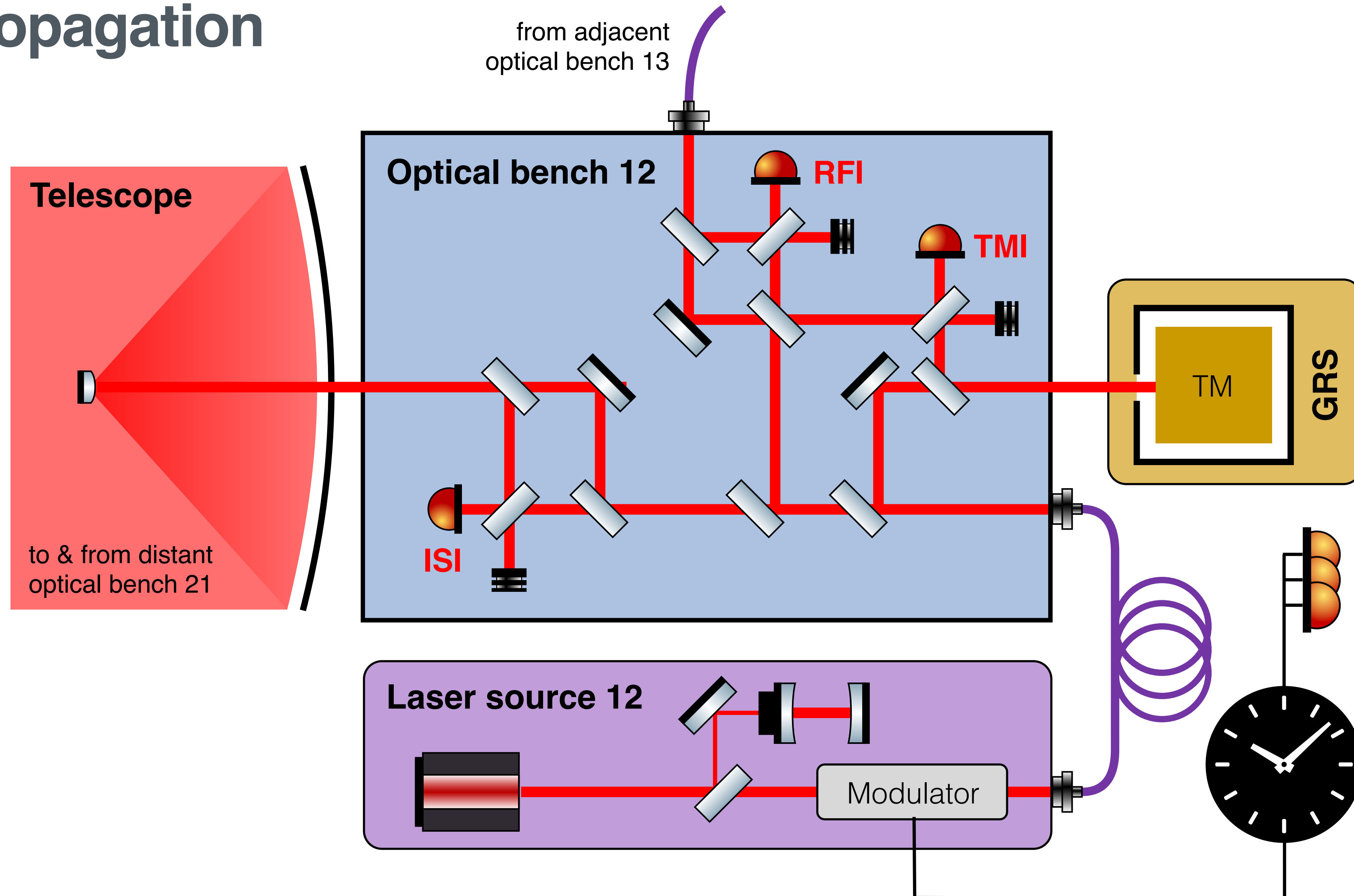
- Modeled as “independent” sideband beams (expansion with Bessel functions)





# Instrument Simulation

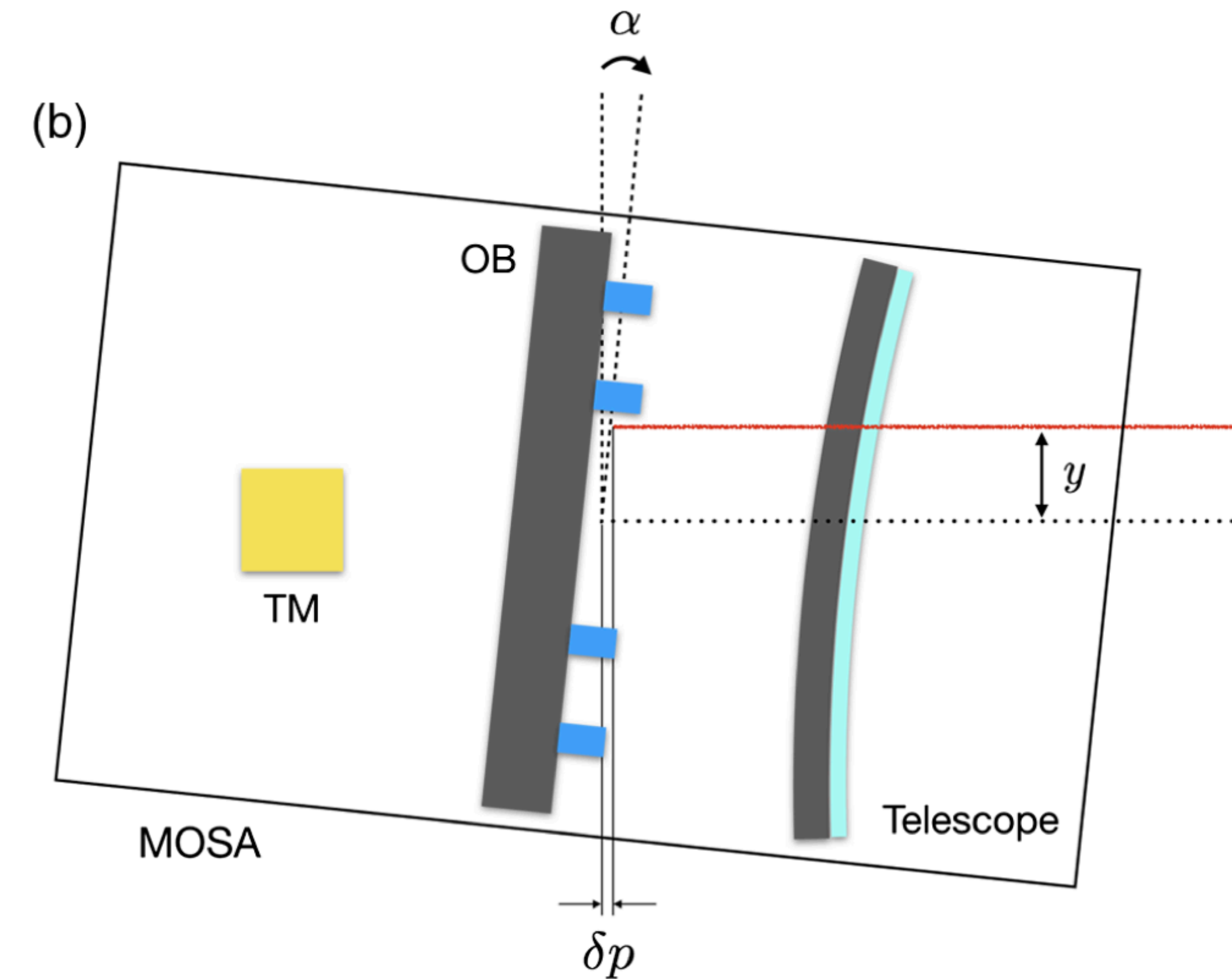
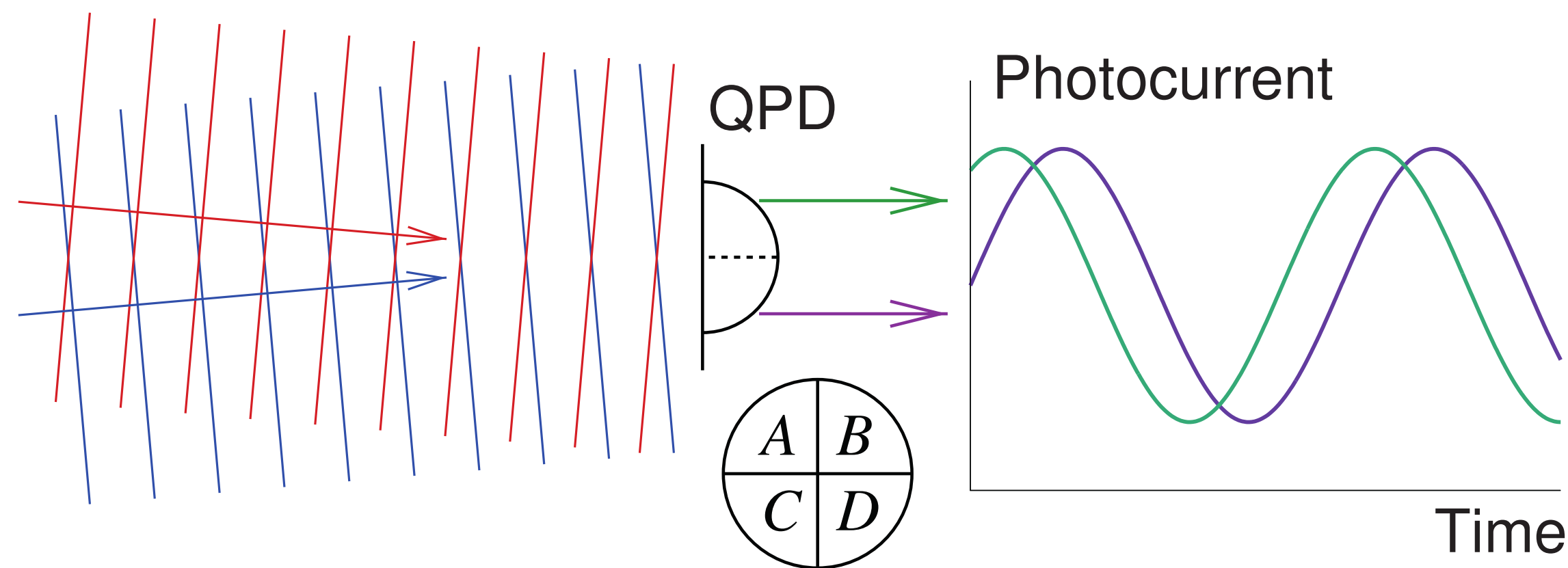
## Beam Propagation



# Instrument Simulation

## Tilt to Length (TTL)

- **Tilt to length couples beam tilts and optical element misalignment to pathlength changes**
- Linear model with a set of 24 coefficients relating tilt angles to pathlength changes
- DWS allows to measure angular tilts of 2 beams by combining outputs of a quadrant photodiode



Paczkowski et al. (2022). 10.1103/PhysRevD.106.04200

# Instrument Simulation

## Inter-spacecraft Propagation

- Phase is a frame invariant quantity, so total phase is equal at reception and emission
- Signals propagated between spacecraft using **proper pseudoranges (PPRs)**, which include **light travel times** and conversion factor between spacecraft proper times
- In addition,
  - GWs cause a tiny ( $\approx 10^{-20}$  s) additional modulation of the PPR
  - Additional Doppler shifts with frequency data

$$\nu_{ij \leftarrow ji}(\tau) = \frac{d}{d\tau} \Phi_{ji}(\tau - d_{ij}(\tau) - H_{ij}(\tau)) = (1 - \dot{d}_{ij}(\tau) - \dot{H}_{ij}(t)) \nu_{ji}(\tau - d_{ij}(\tau))$$

total instantaneous frequency

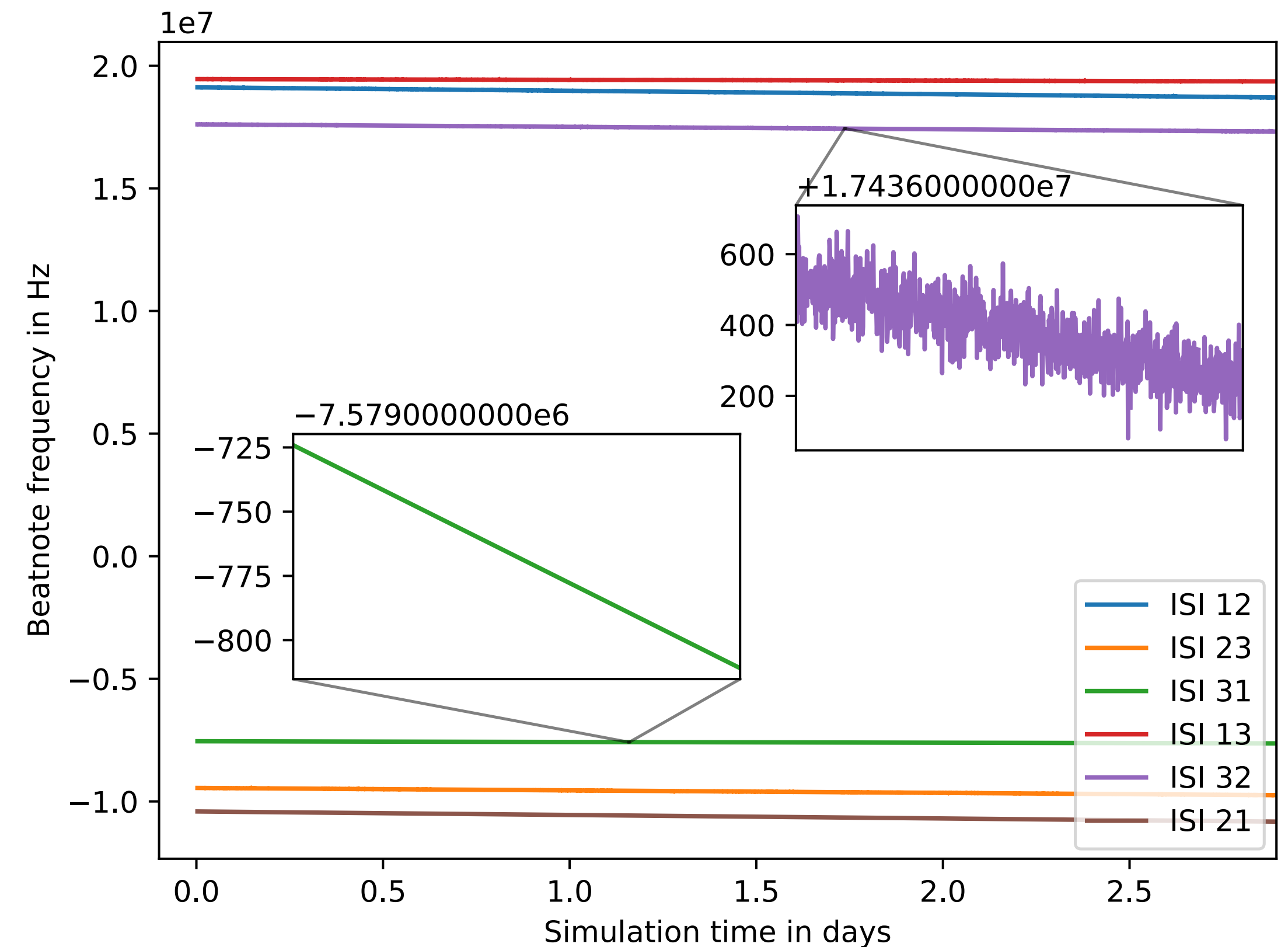
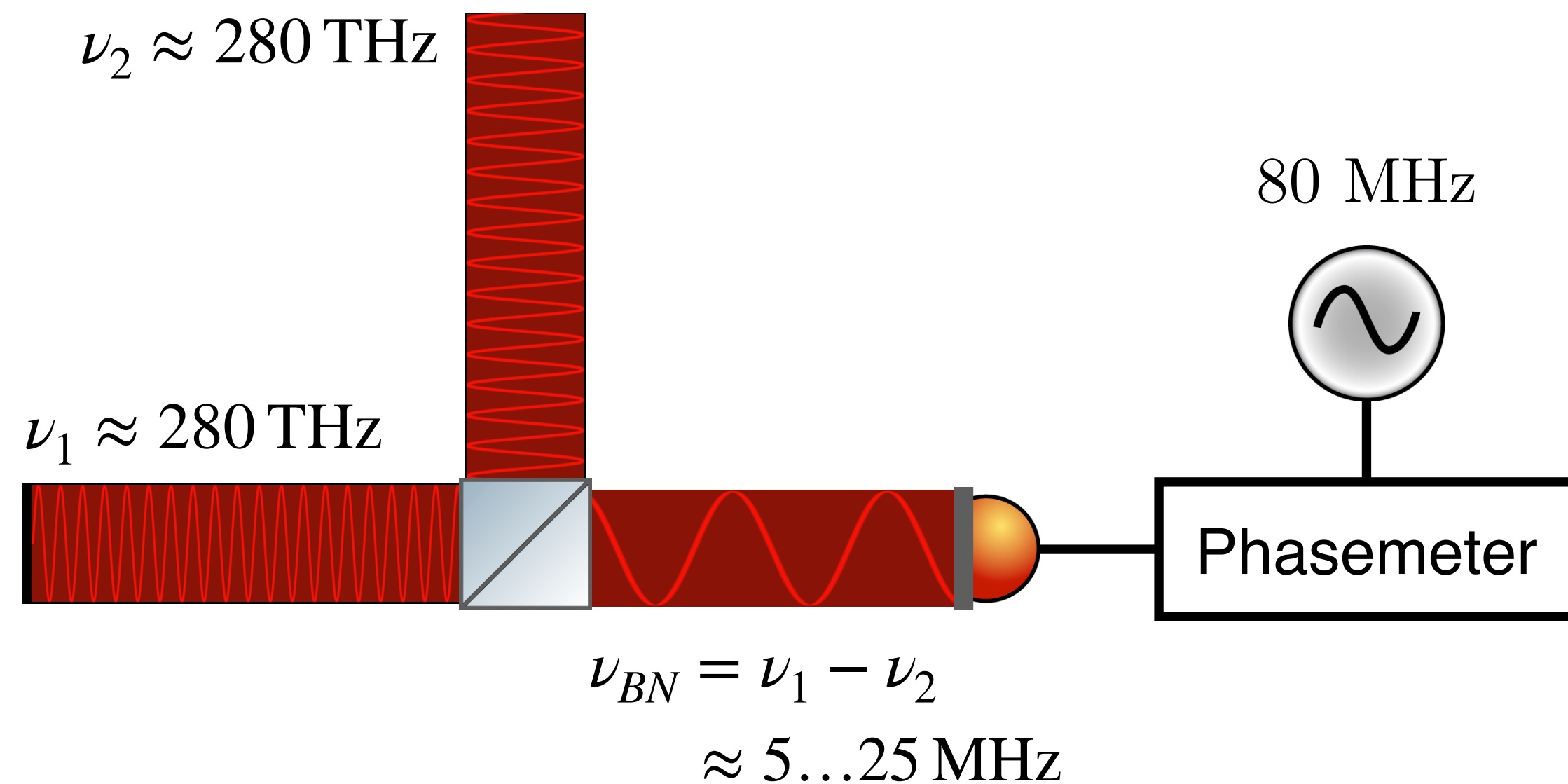
PPRs

Doppler shift

# Instrument Simulation

## Interferometry & Readout

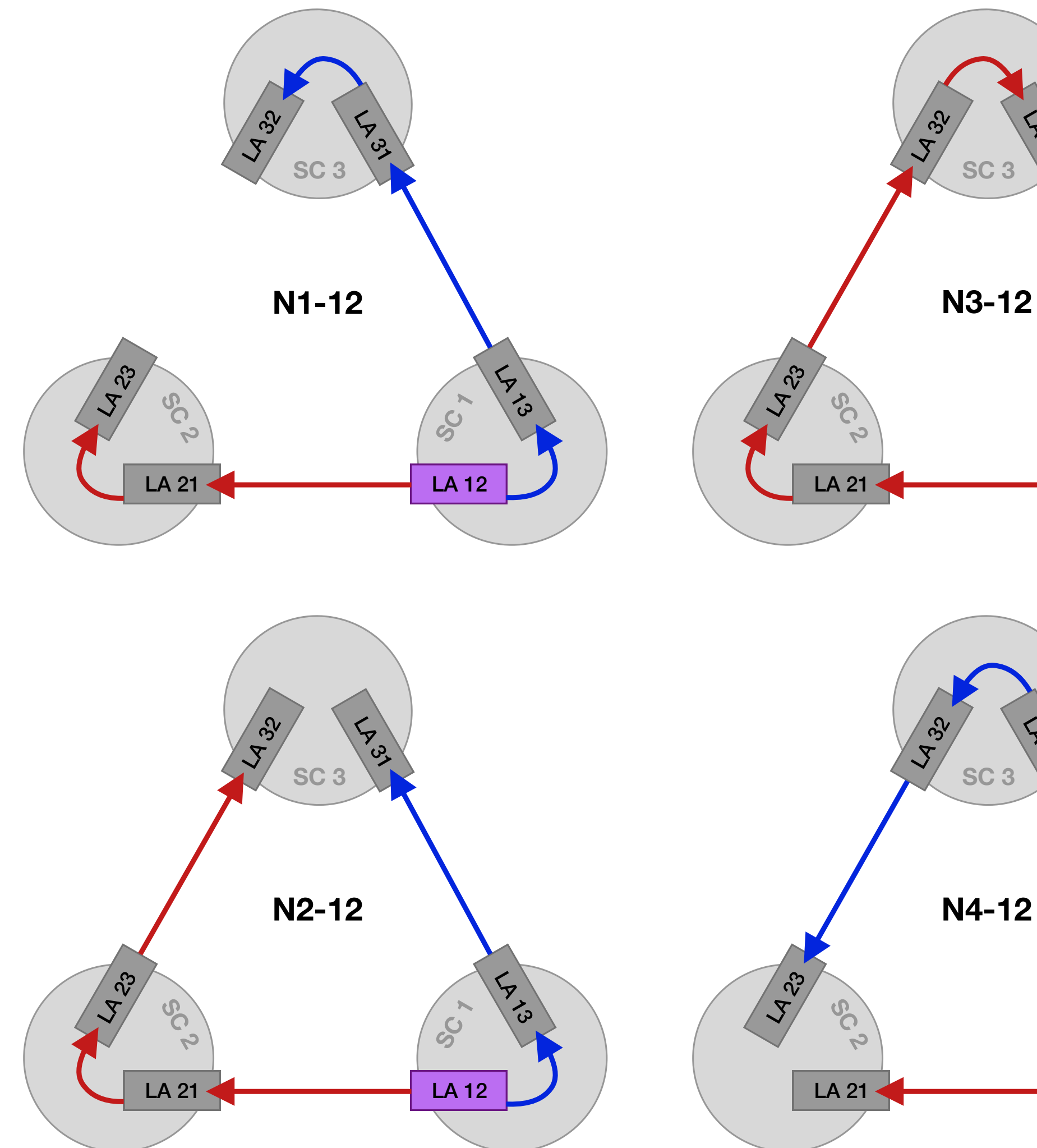
- LISA uses heterodyne interferometry and GW is encoded as  $\mu$ -cycle phase fluctuation in MHz beatnotes (in frequency)
- Beatnotes recorded according to local clock



# Instrument Simulation

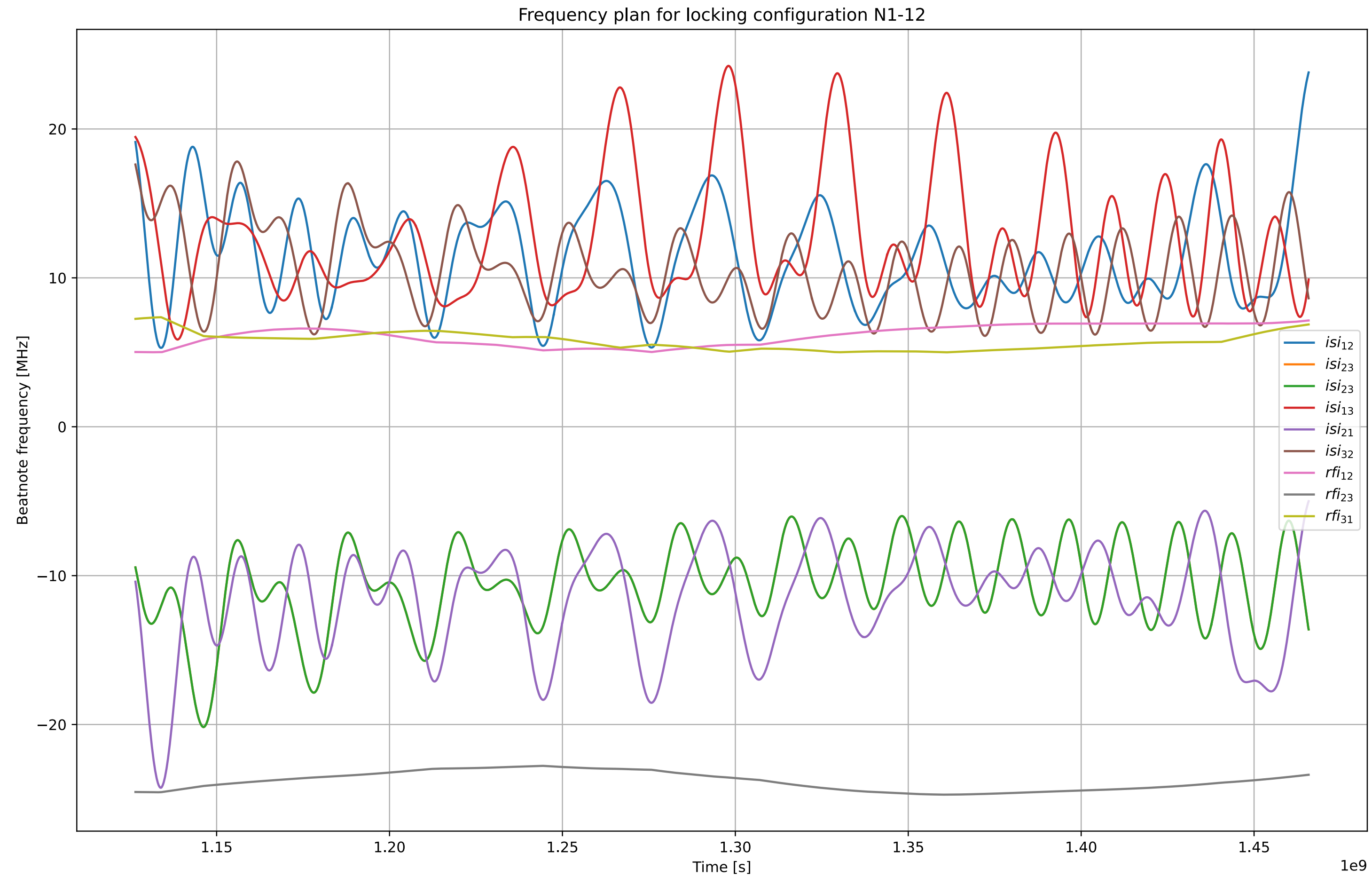
## Laser Locking & Frequency Plan

- All beatnotes should fall into the phasemeter validity frequency range (5 to 25 MHz)
  - Doppler shifts frequencies by 10s of MHz
  - Solution: lock lasers (many configurations possible) with an optimized precomputed frequency plan
- Frequency plan optimized numerically by G. Heinzel
- As a consequence, noises are distributed over different beatnotes



# Instrument Simulation

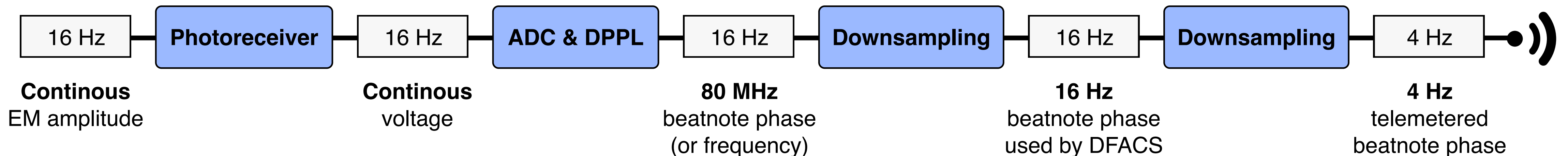
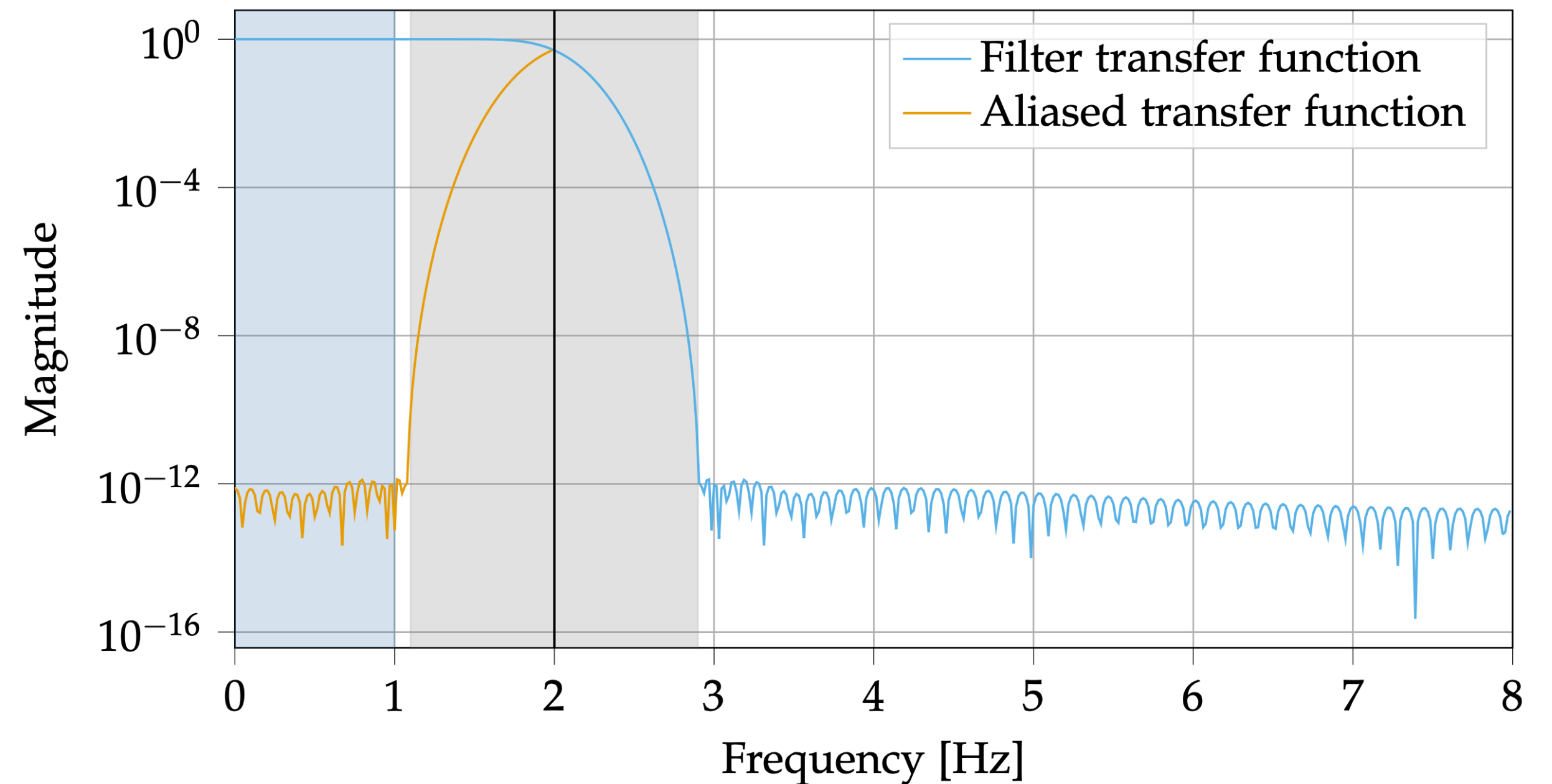
## Laser Locking & Frequency Plan



# Instrument Simulation

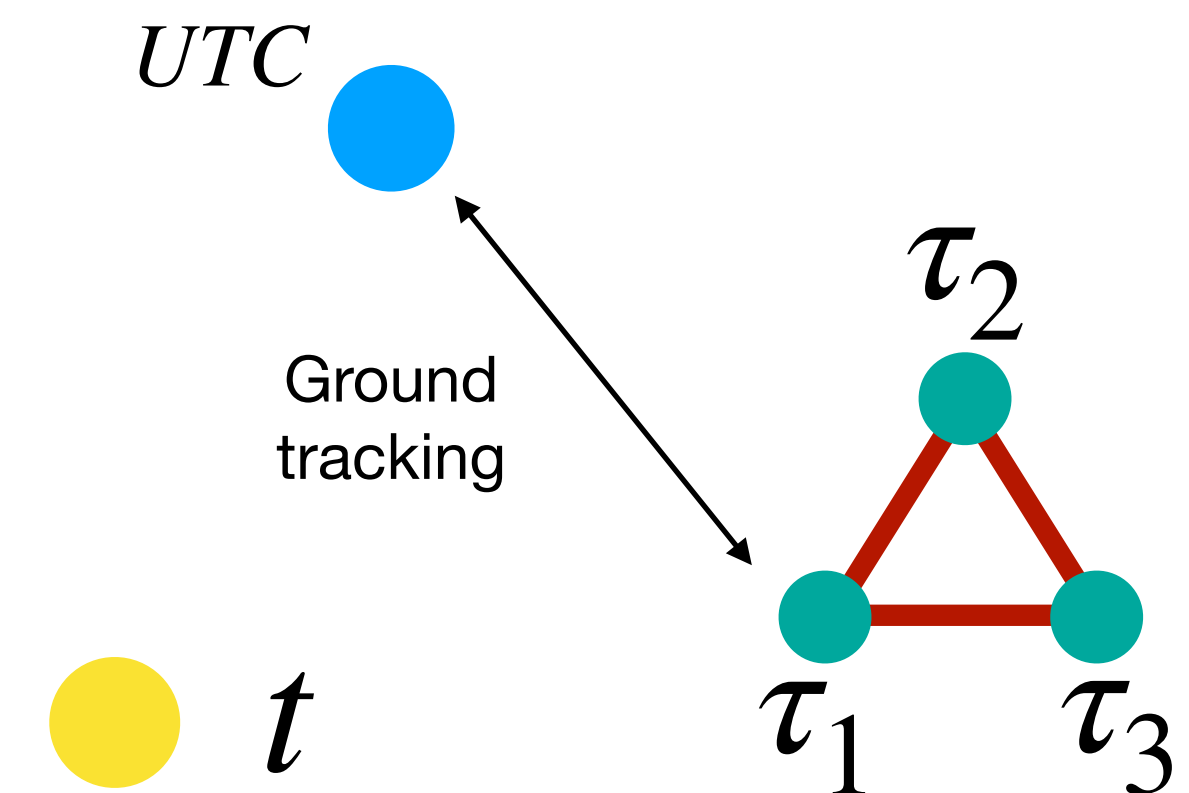
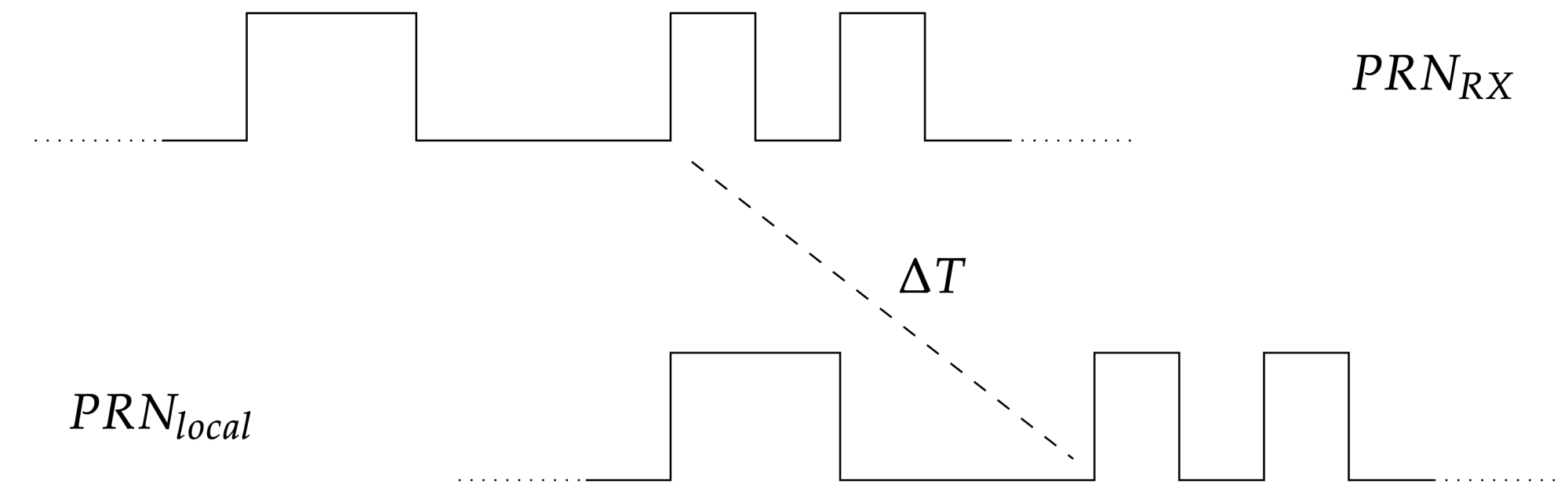
## Onboard Processing

- In reality
  - Phasemeter runs at 80 MHz
  - Filtered and downsampled in several steps to final 4 Hz telemetry
- In simulation
  - Physics and phasemeter at 16 Hz
  - Single filter and decimation step to 4 Hz



# MPR & Ground Tracking

- Measured pseudoranges (MPRs)
  - Correlate signals from two distant clocks
  - Include photon light travel time and transformation between clock times  $\Delta T = \hat{\tau}_i - \hat{\tau}_j$  (and any correlation errors)
- Ground tracking provides estimates of
  - Spacecraft positions with 2-50 km accuracy (direction dependent) and spacecraft velocities with 10 cm/s accuracy
  - Time correlations between clocks and a global time frame (here UTC) better than ms accuracy

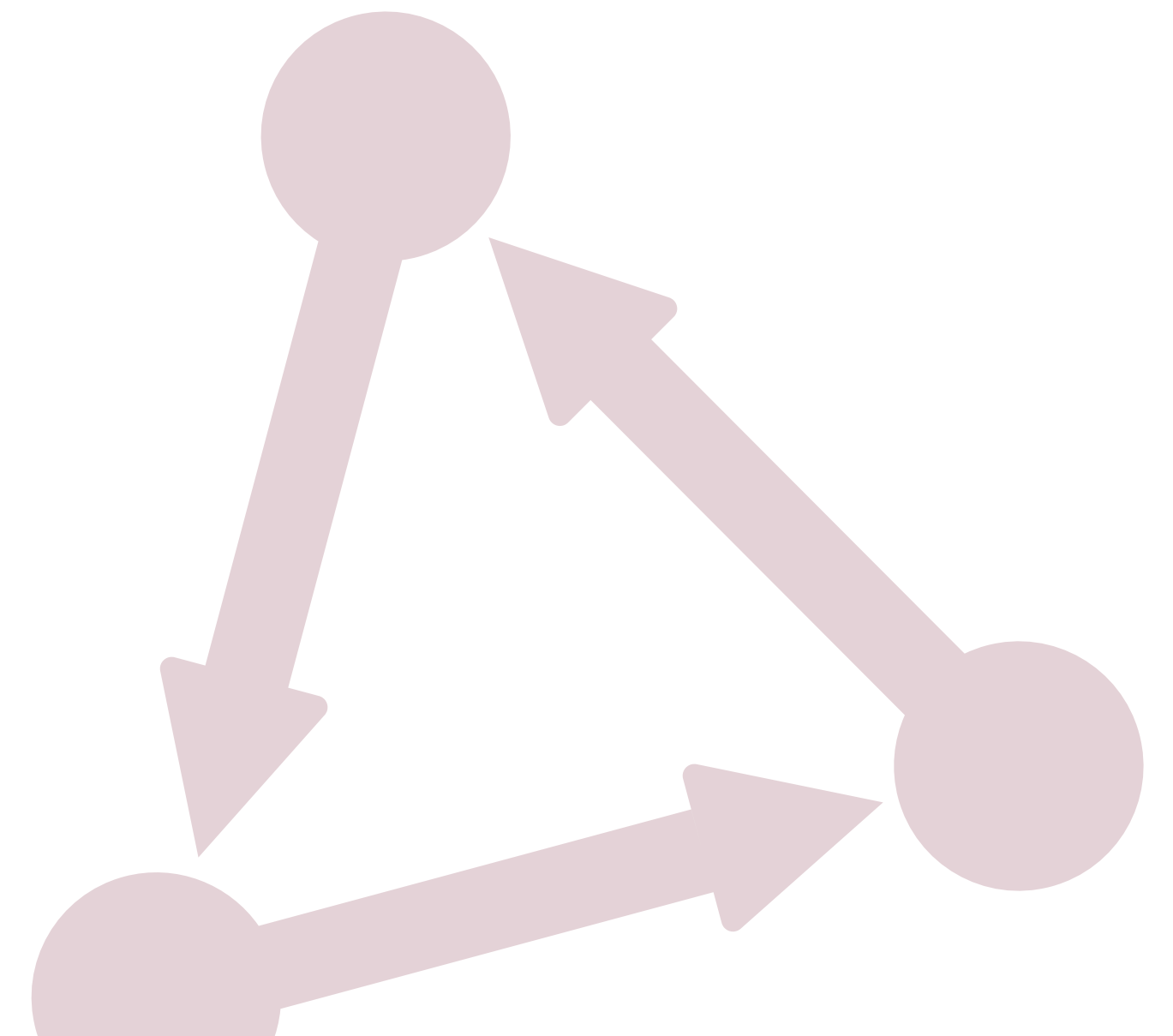




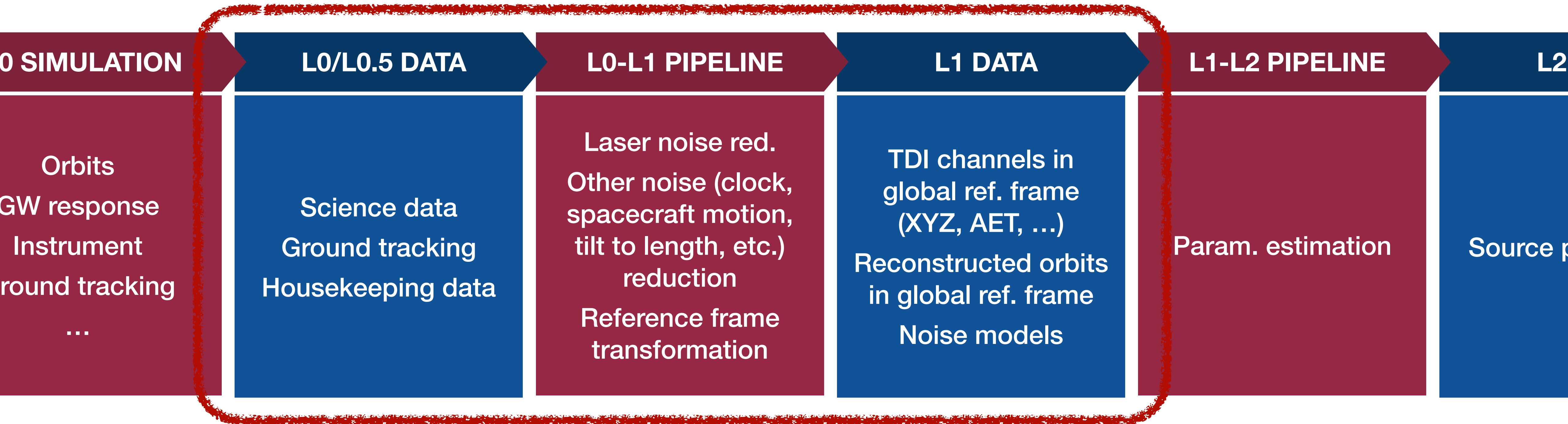
# L0 Data Overview

- Science data
  - 3 interferometer data (carrier and sideband) on the 6 optical benches
  - Measured pseudoranges (MPRs)
  - Differential waveform sensor (DWS) measurements
  - Other quantities not simulated currently (GRS, etc).
- Ground-tracking
  - Reconstructed orbits
  - Time correlations (clock times as a function of a global reference frame)
- Housekeeping and calibration stuff (lots of 'em)

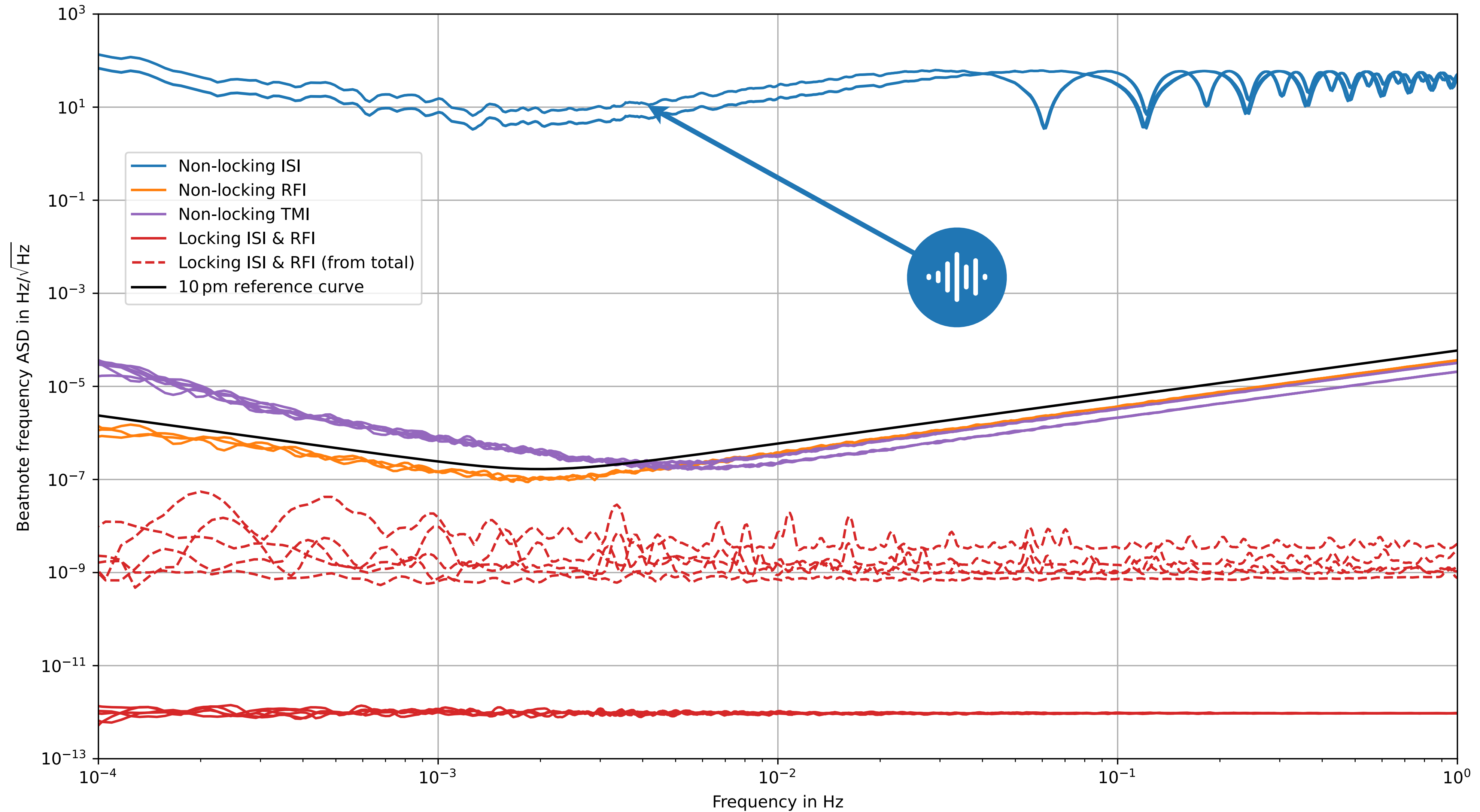
# L0-L1 Processing



# LISA Data Analysis Pipeline



# L0 Data Overview



# One Recipe (Amongst Others)

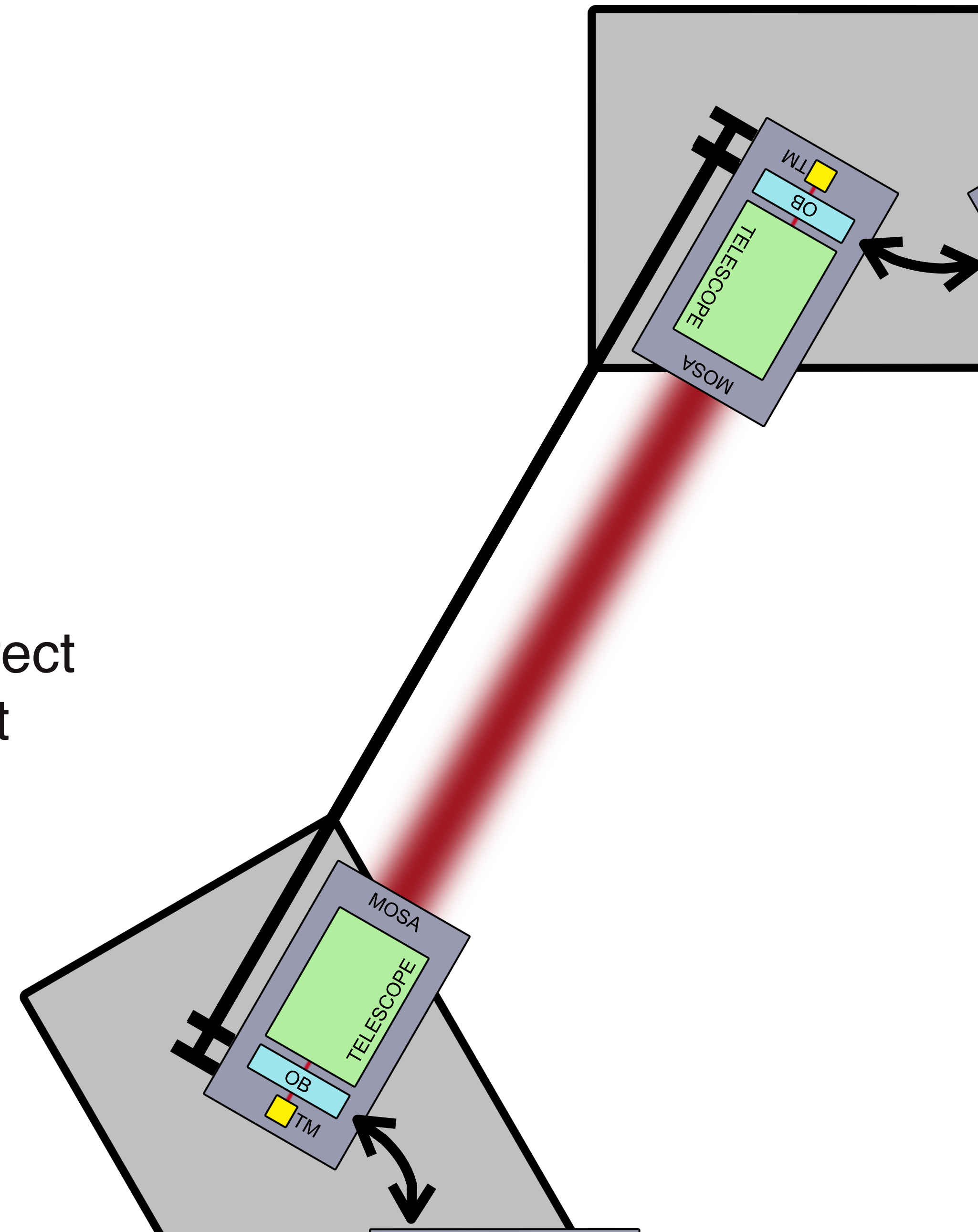
**Here, L0-L1 pipeline for total frequency**

*Phase data will also be available, TBD if we gain anything in PE from using it...*

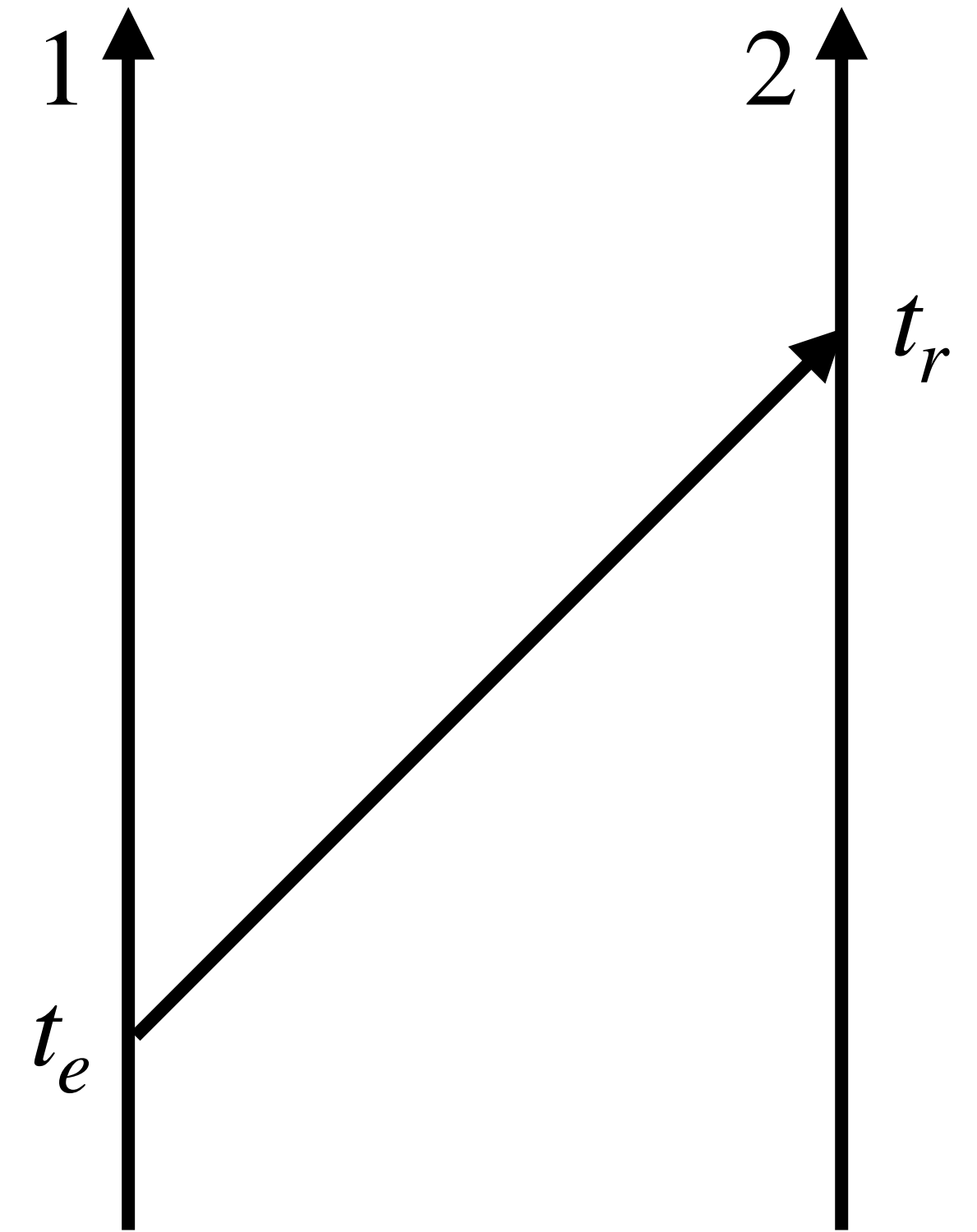
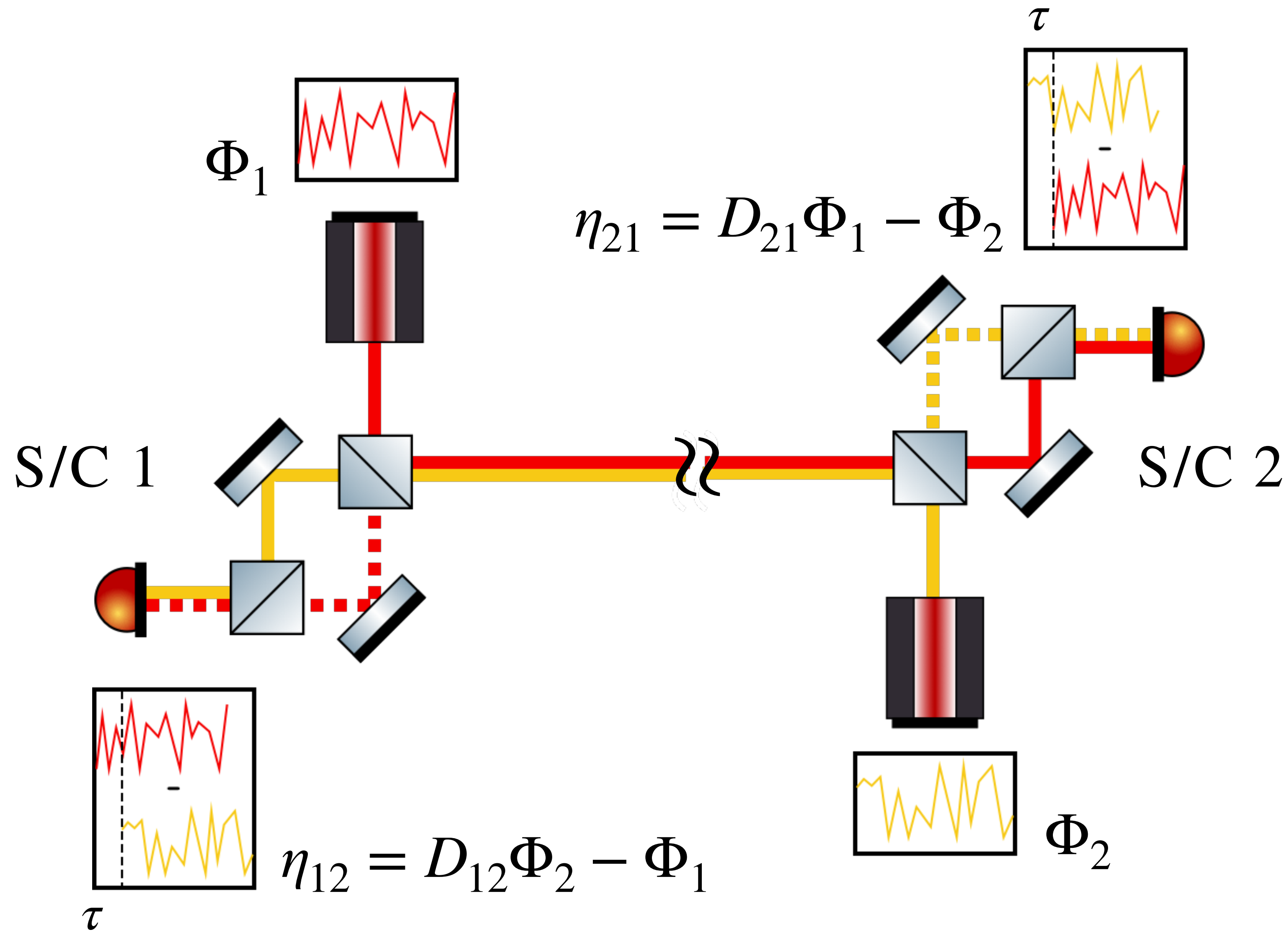
1. Accurate estimation of the pseudoranges
  - Merge ground-tracking, MPRs and sidebands to provide accurate and low-noise estimates
  - Compute light travel times for response function
2. Construct test-mass-to-test-mass measurements to reduce spacecraft motion
3. Reduce tilt-to-length using DWS measurements
4. Correct for laser and clock noise using TDI
5. Synchronize TDI variable to a global reference frame using time correlations
6. Compute AET channels (if actually useful for PE)

# Single-Link Corrections

- **We want to monitor the TM-to-TM measurement**
- 3 Interferometers on each optical bench
  - Inter-spacecraft interferometer (ISI)
  - Test-mass interferometer (TMI)
  - Reference(interferometer (RFI)
- Combined in early processing step to synthesize direct TM-to-TM measurement, with 1 laser per spacecraft
- Then subtract TTL via DWS measurements
  - Coupling coefficients are estimated using dedicated calibration experiments (under investigation)

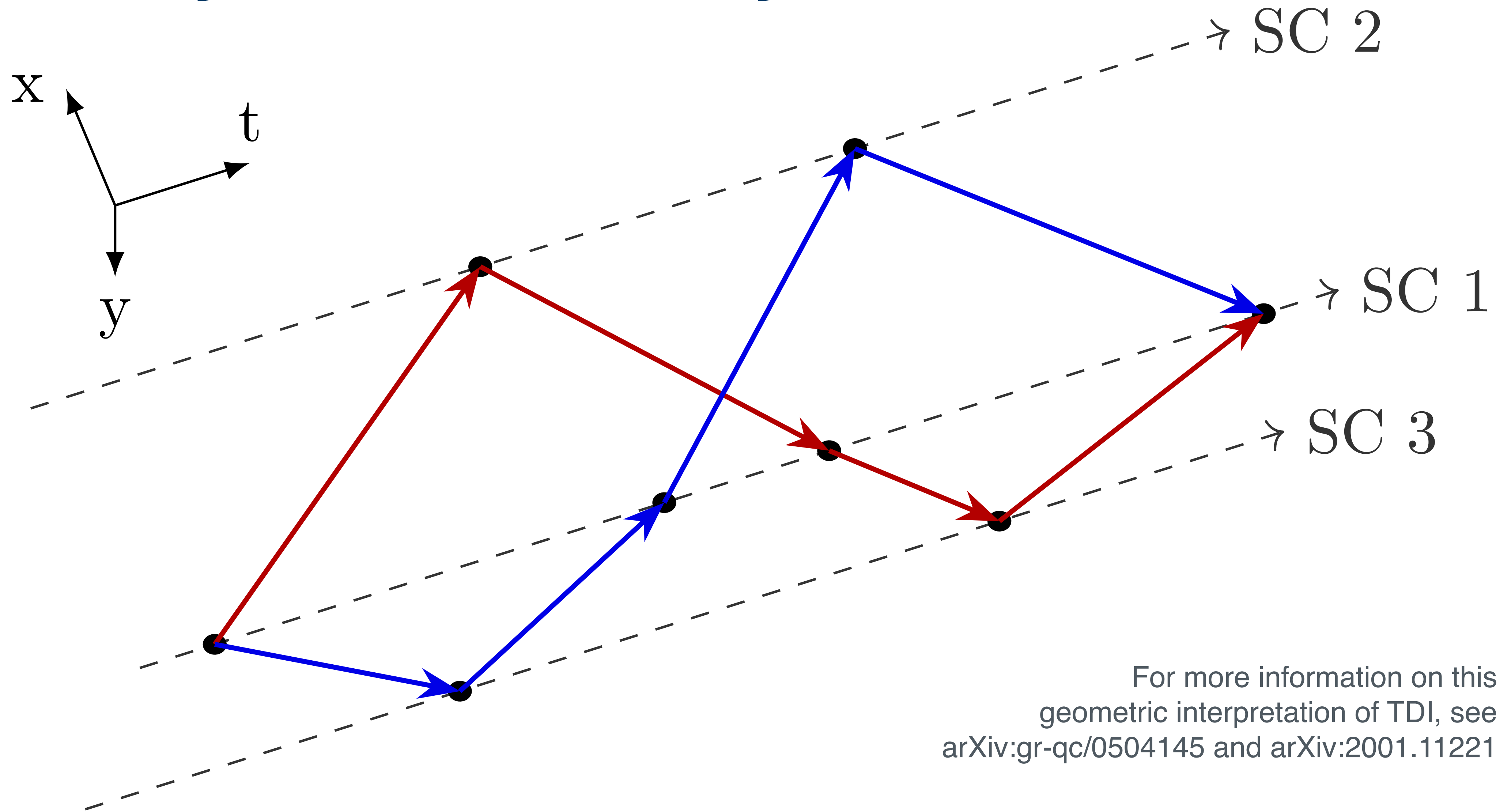


# Simplified LISA Link



$$\begin{aligned}
 \Phi_1(t_e) &\equiv D_{21}\Phi_1(t_r) \\
 &= \Phi_1(t_r - d_{21}(t_r))
 \end{aligned}$$

# Time-Delay Interferometry



For more information on this  
geometric interpretation of TDI, see  
[arXiv:gr-qc/0504145](https://arxiv.org/abs/gr-qc/0504145) and [arXiv:2001.11221](https://arxiv.org/abs/2001.11221)



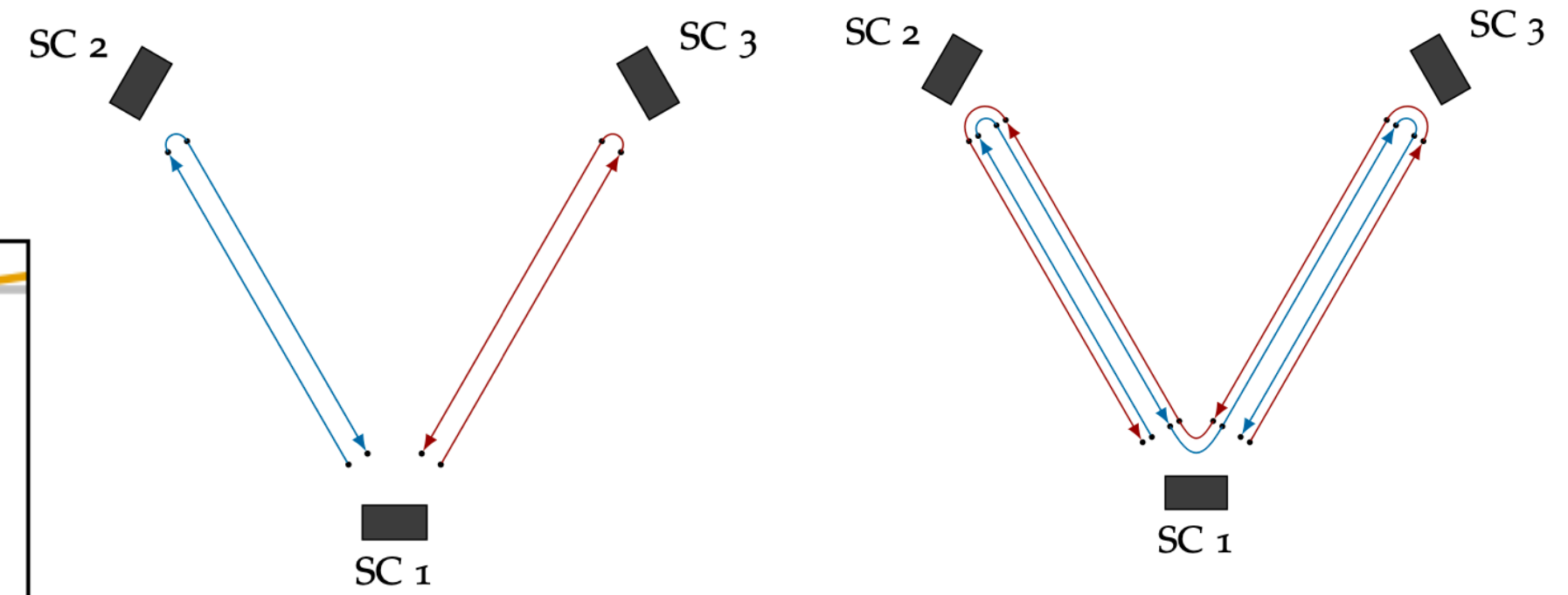
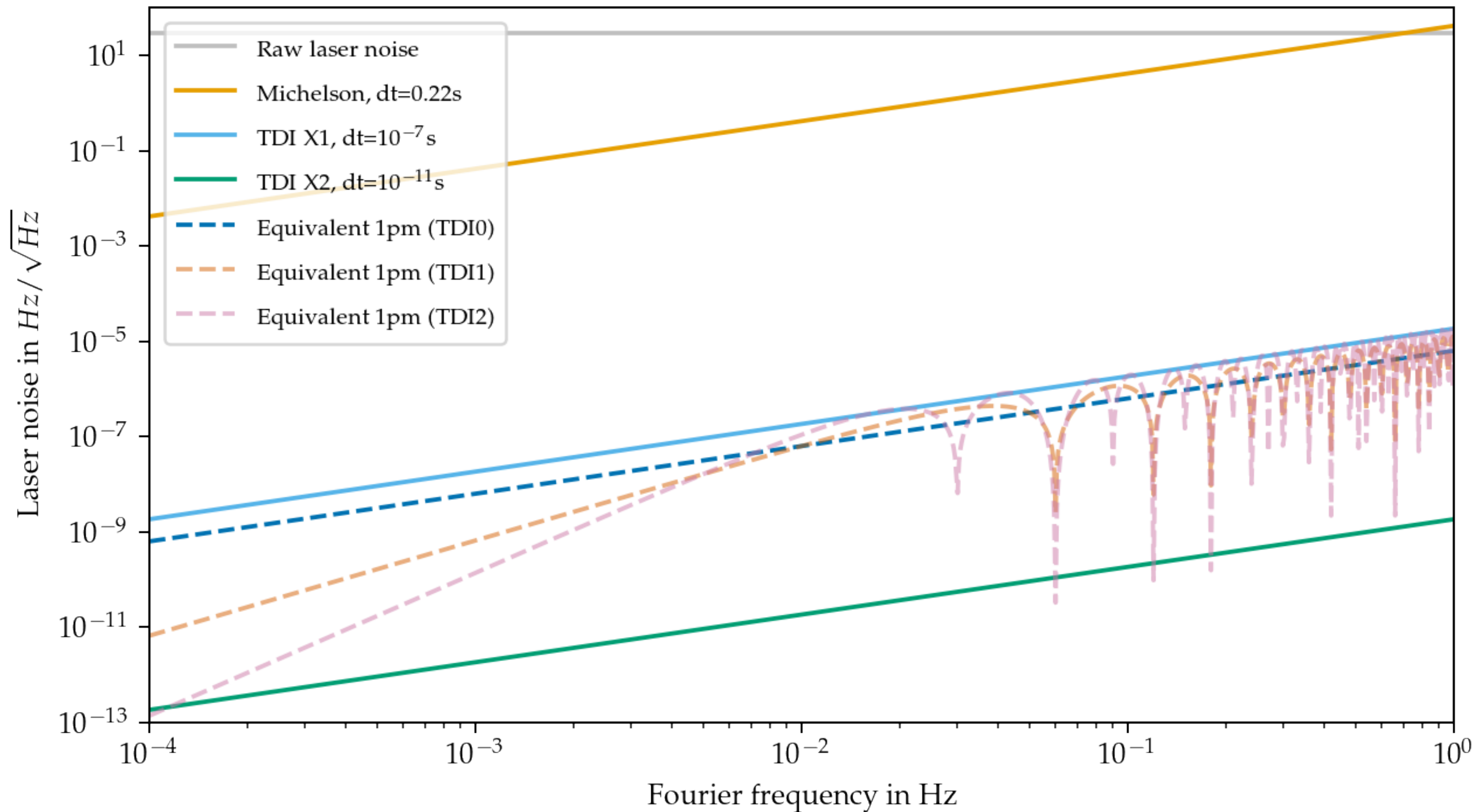
# Laser Noise Residual

... Assuming Realistic Orbits

Average light travel times with ESA orbits:  $d_A \approx 8.3\text{ s}$ ,  $\dot{d}_A \approx 10^{-9}$  and  $\ddot{d}_A \approx 10^{-15}\text{ s}^{-1}$

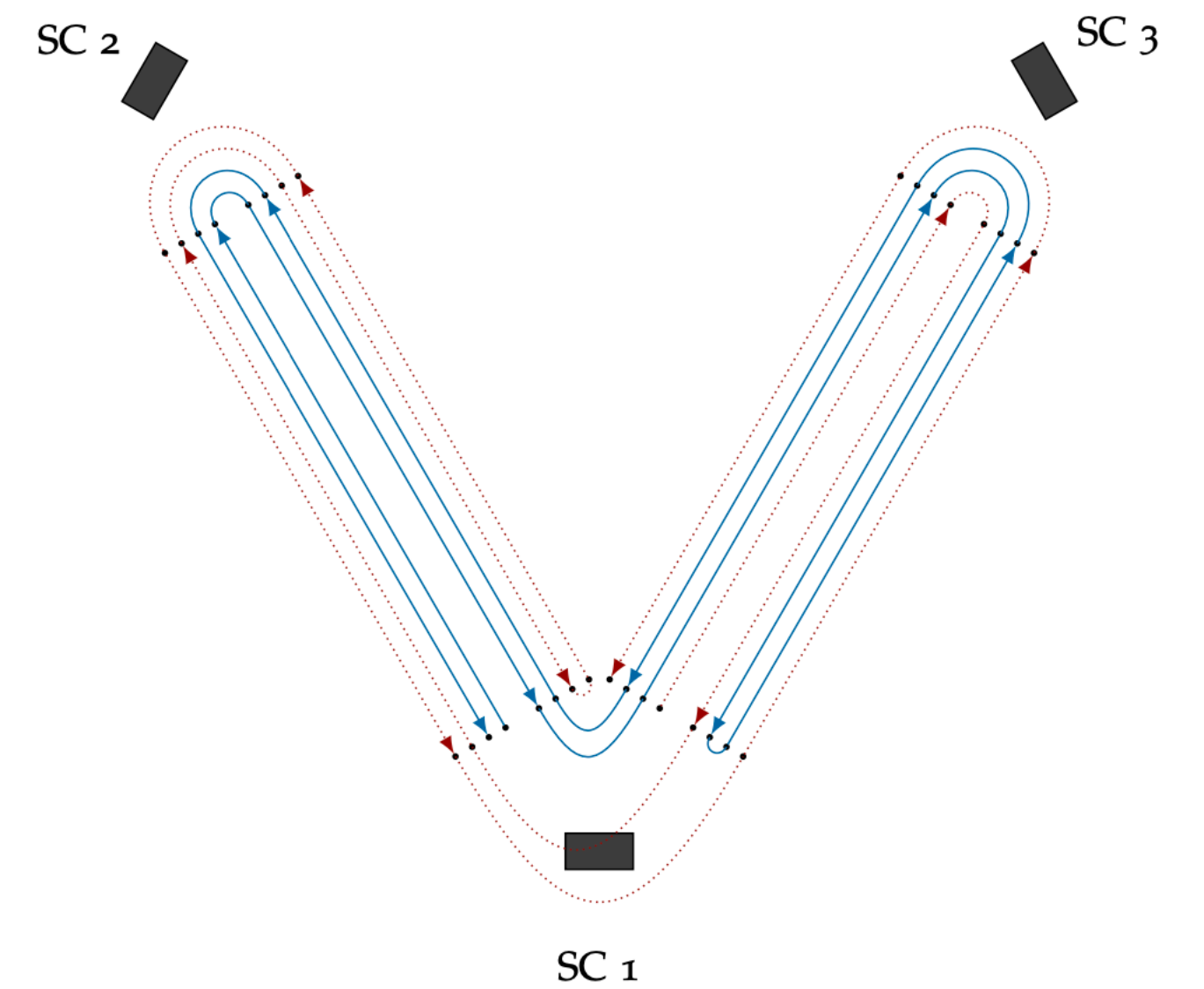
$$S_{\Phi, \text{TDI}} \approx (2\pi f)^2 \delta t^2 S_{\Phi}$$

Residual laser noise



$$\Delta\tau_0 \approx 2(d_{12} - d_{13})$$

$$\Delta\tau_1 \approx 4d(\dot{d}_{31} - \dot{d}_{12})$$



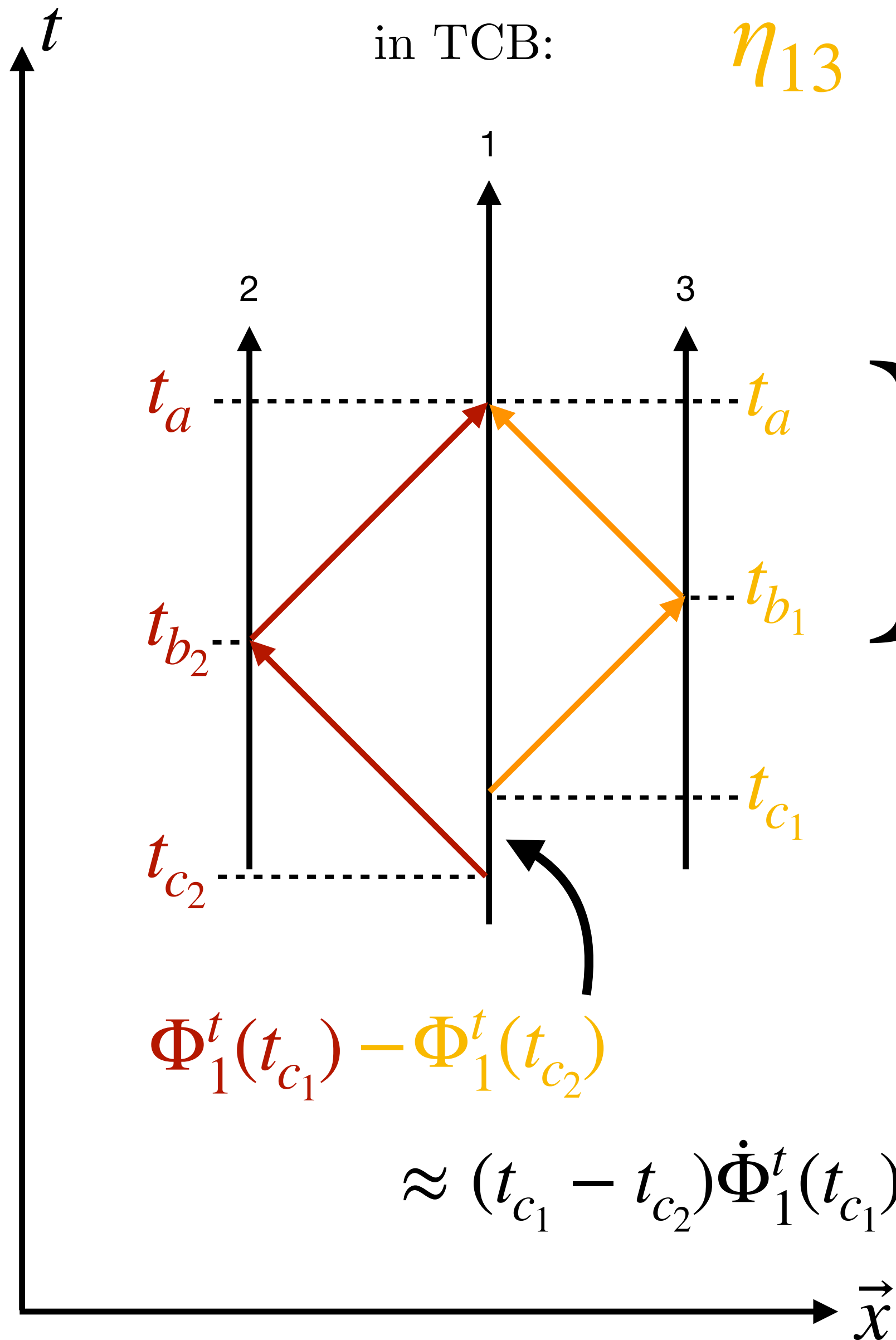
$$\Delta\tau_2 = \approx t[(\dot{d}_{12}^2 - \dot{d}_{31}^2) - 2d(\ddot{d}_{12} - \ddot{d}_{31})]$$

*TDI with desynchronized clocks*

# Geometric TDI with Clock Times

Measurements  
in TCB:

$$\eta_{13} - \eta_{12} + D_{13}\eta_{31} - D_{12}\eta_{21}$$



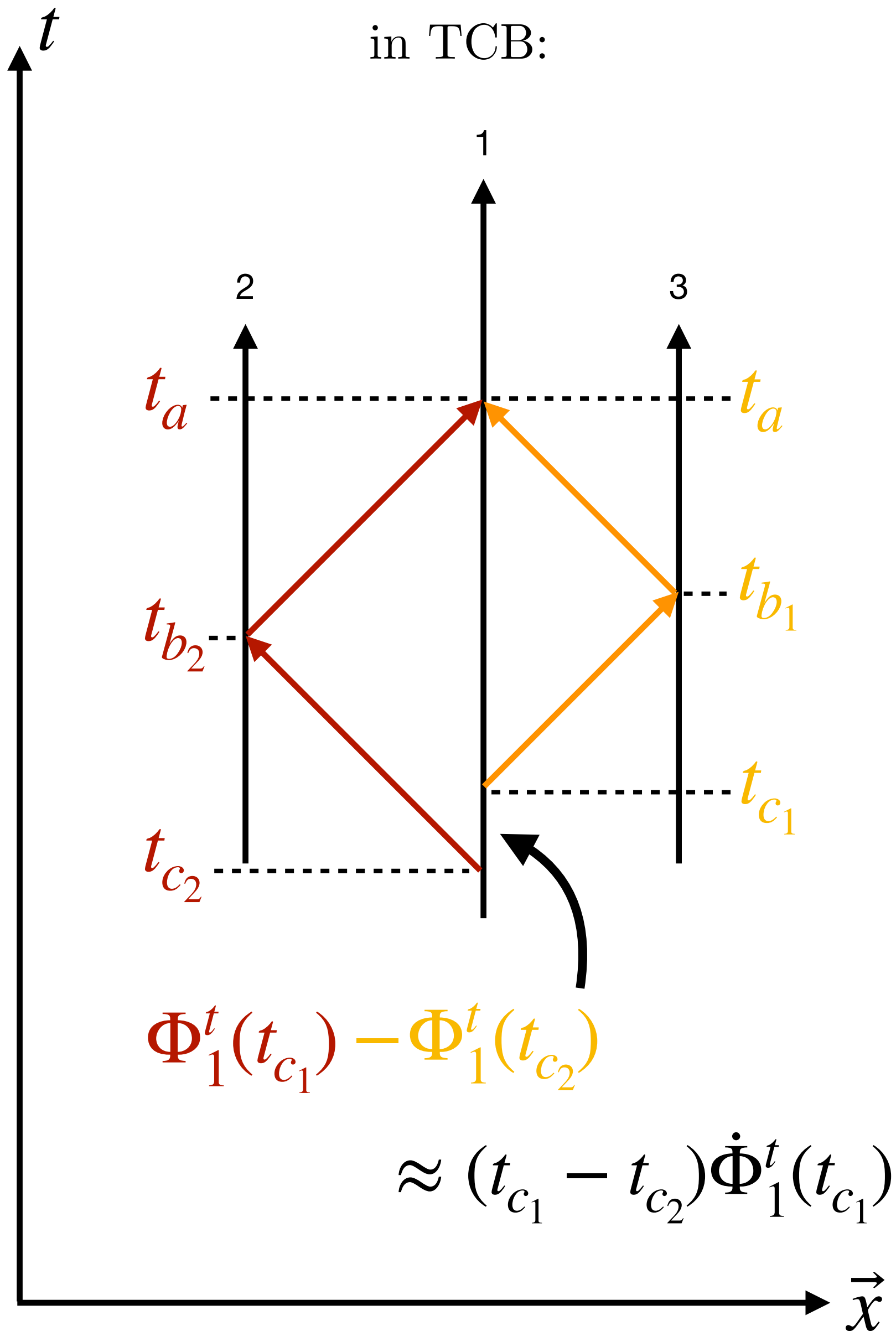
} Light travel time in TCB,  
Computed from S/C position  
estimates

$$\Phi_1^t(t_{c_1}) - \Phi_1^t(t_{c_2})$$

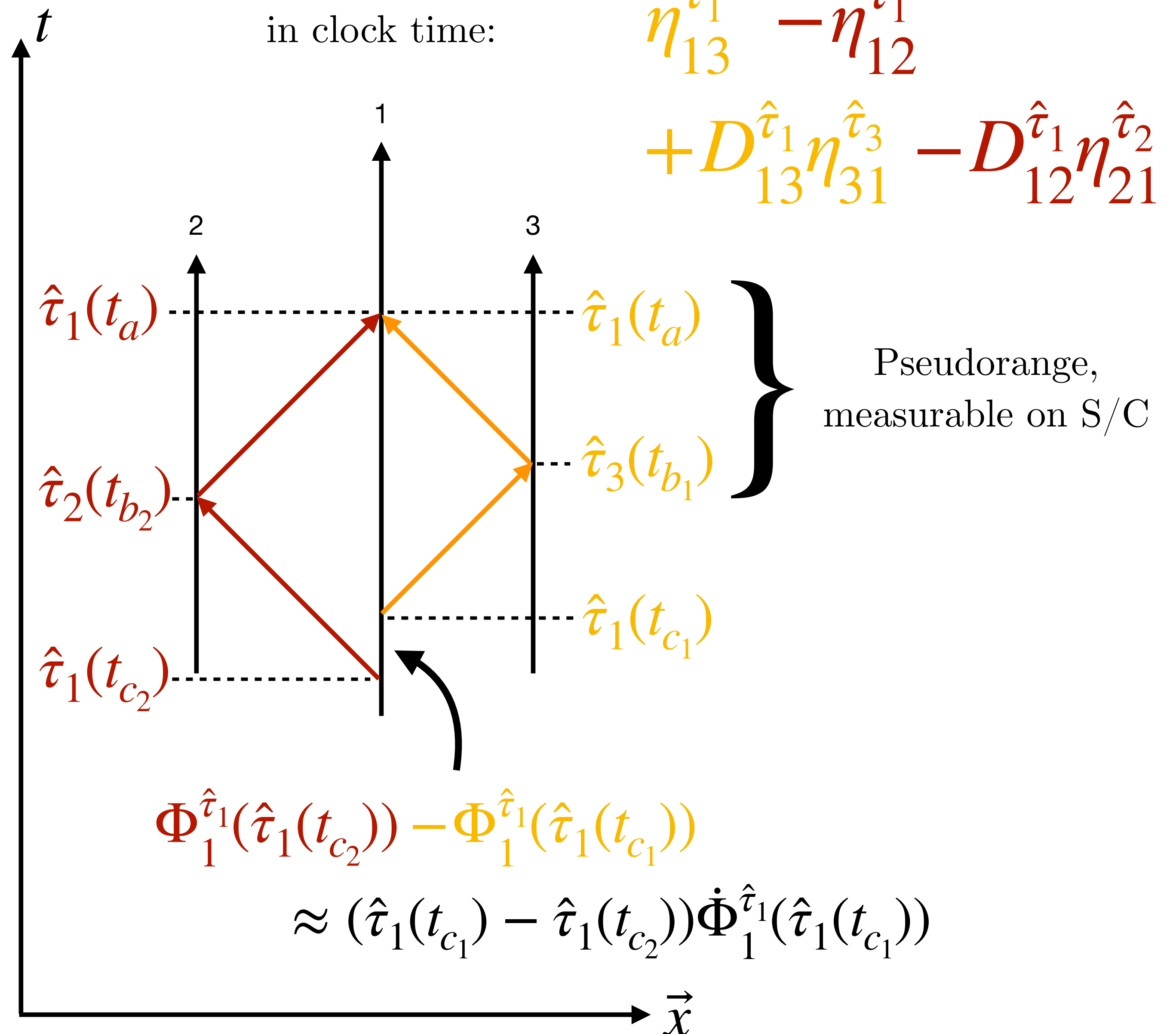
$$\approx (t_{c_1} - t_{c_2})\dot{\Phi}_1^t(t_{c_1})$$

# Geometric TDI with Clock Times

Measurements  
in TCB:



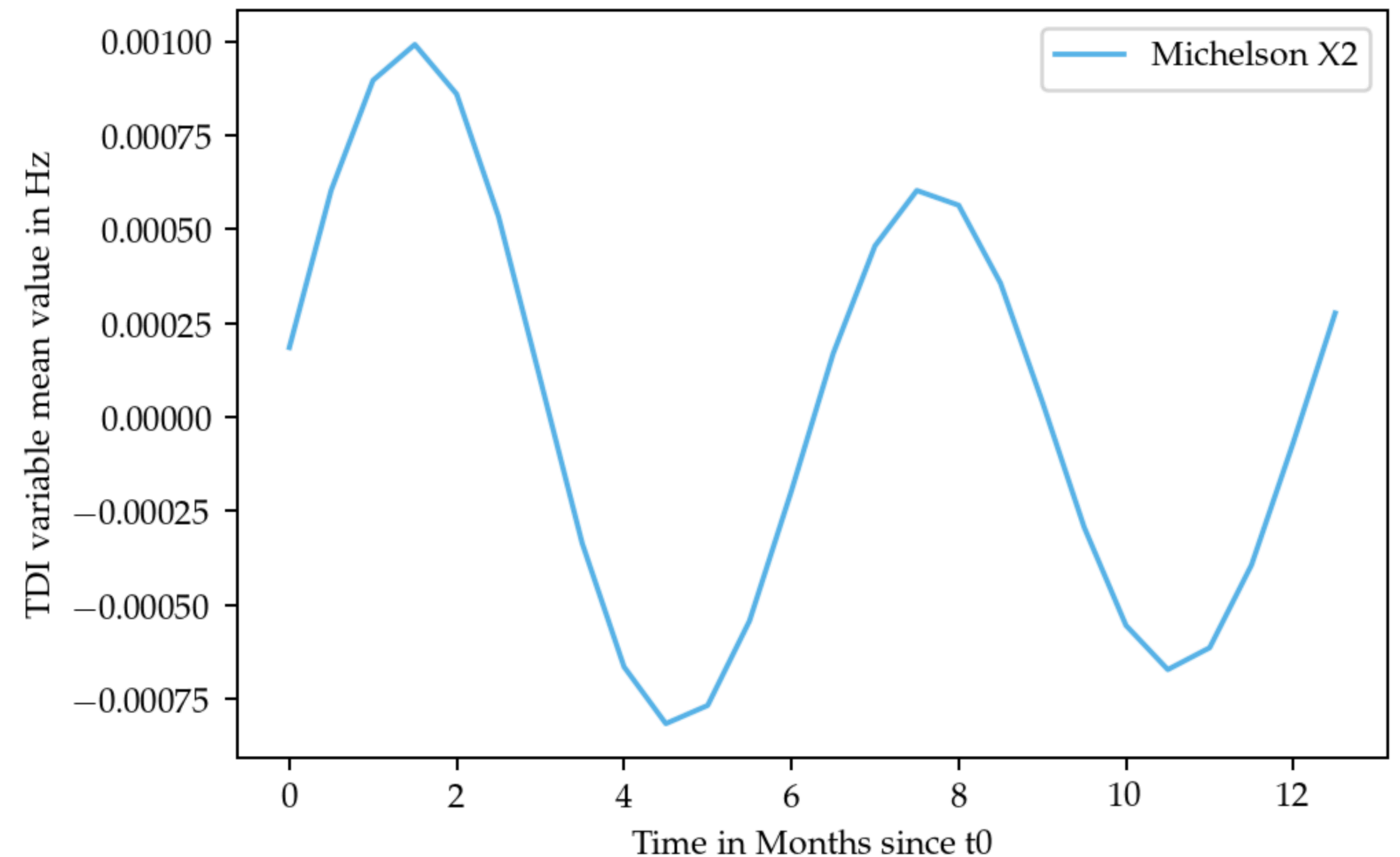
Measurements  
in clock time:



# Clock Noise

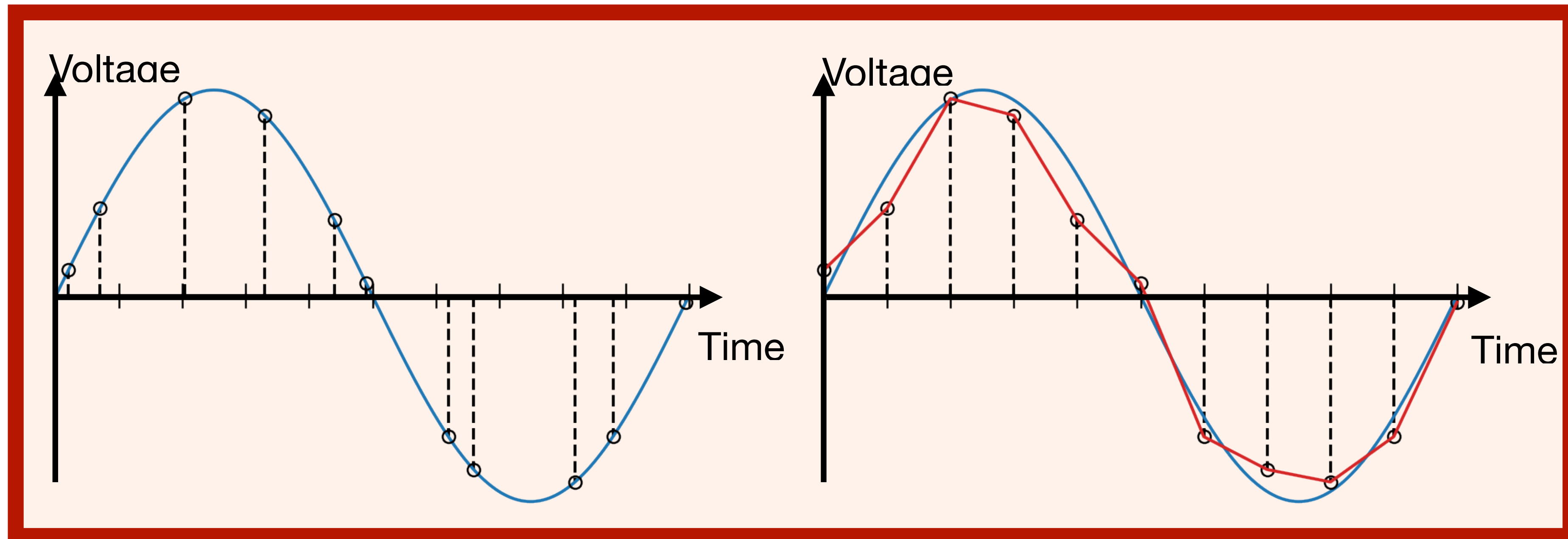
## ... with Perfect Pseudoranges

- We can show that we get  $X^{\hat{\tau}_1}(\tau) = X^t(\tau - \delta\hat{\tau}_1(\tau)) \approx X^t(\tau) - \dot{X}^t(\tau)\delta\hat{\tau}_1(\tau)$
- The mean value of  $\dot{X}^t(\tau)$  varies between  $\pm 1$  mHz, with a period of 2/year
- This is 10 orders of magnitude below the previous coupling to 10 MHz beatnotes!
- Remark: noise-free variables still need to be sync. to TCB
- Remark 2: large drift can be subtracted — should we?



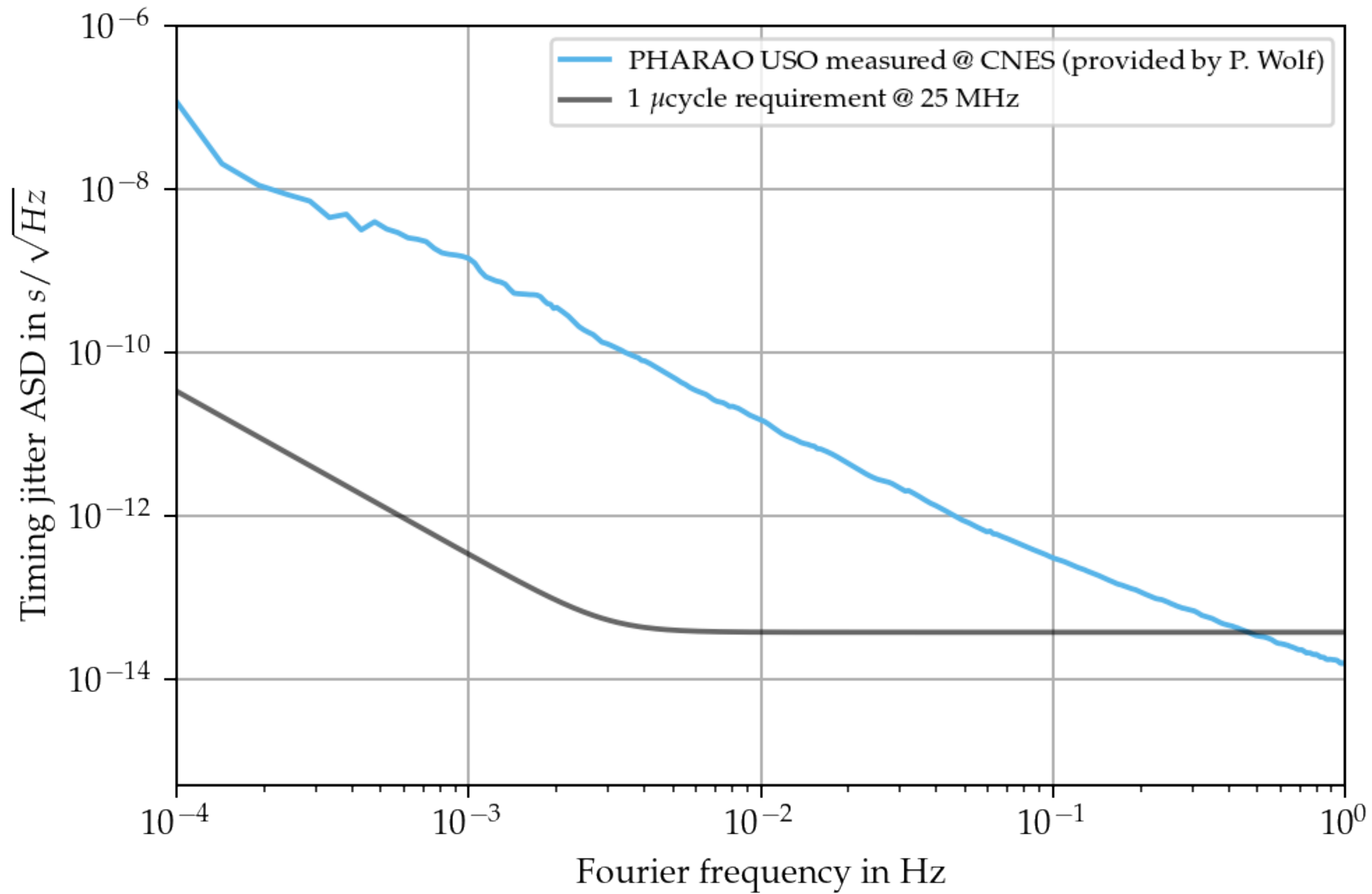
# Clock Noise

## ... With Imperfect Pseudoranges



- 25 MHz beatnotes require 40 fs/sqHz timing precision for  $\mu$ cycle signals: clocks not good enough!
- Correct clock noise alongside laser noise by properly time-shifting each individual sample
- Any time shift applied in TDI inherits same timing requirement, if applied to total phase/frequency

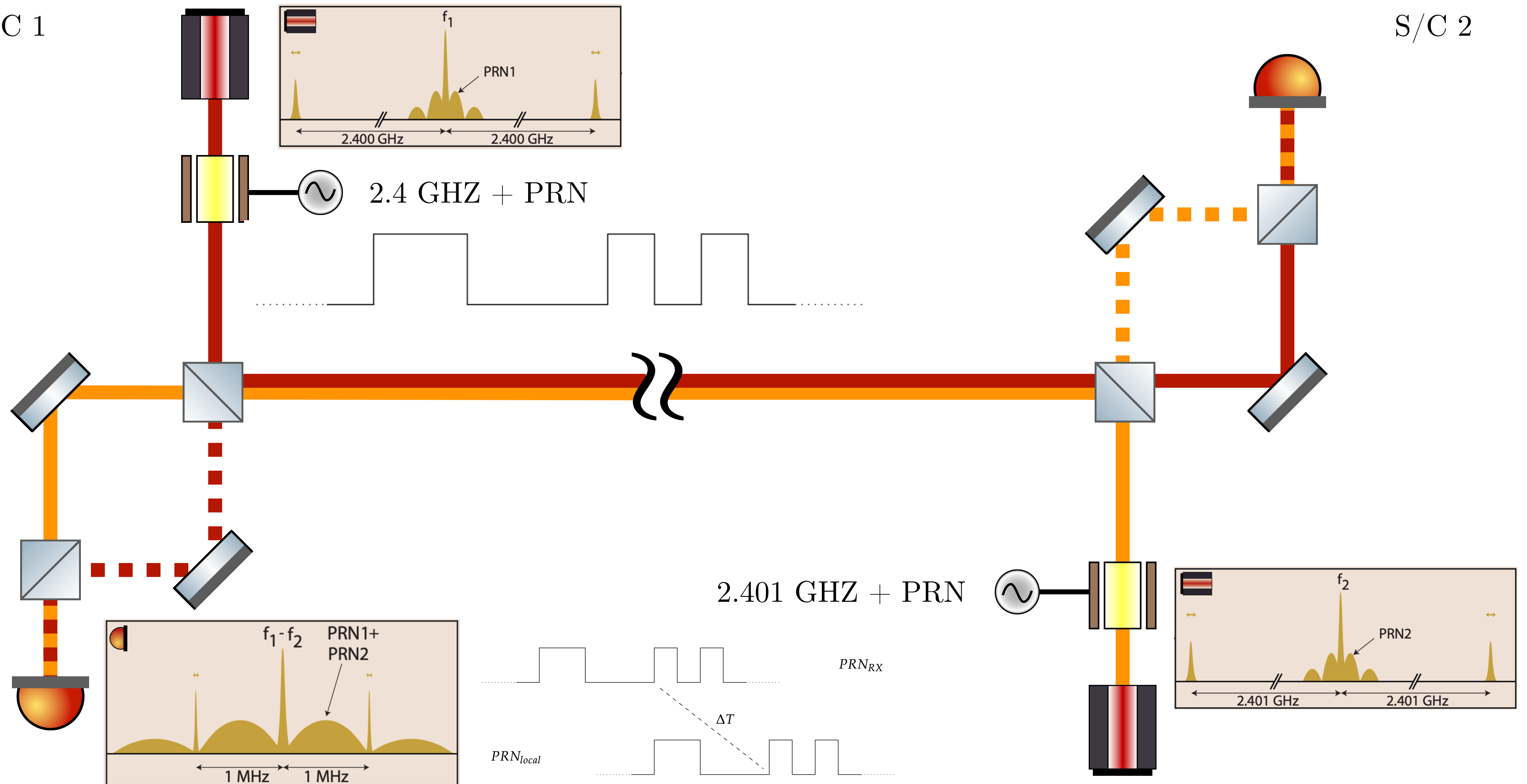
# Clock comparison performance



# Sideband & PRN Modulation

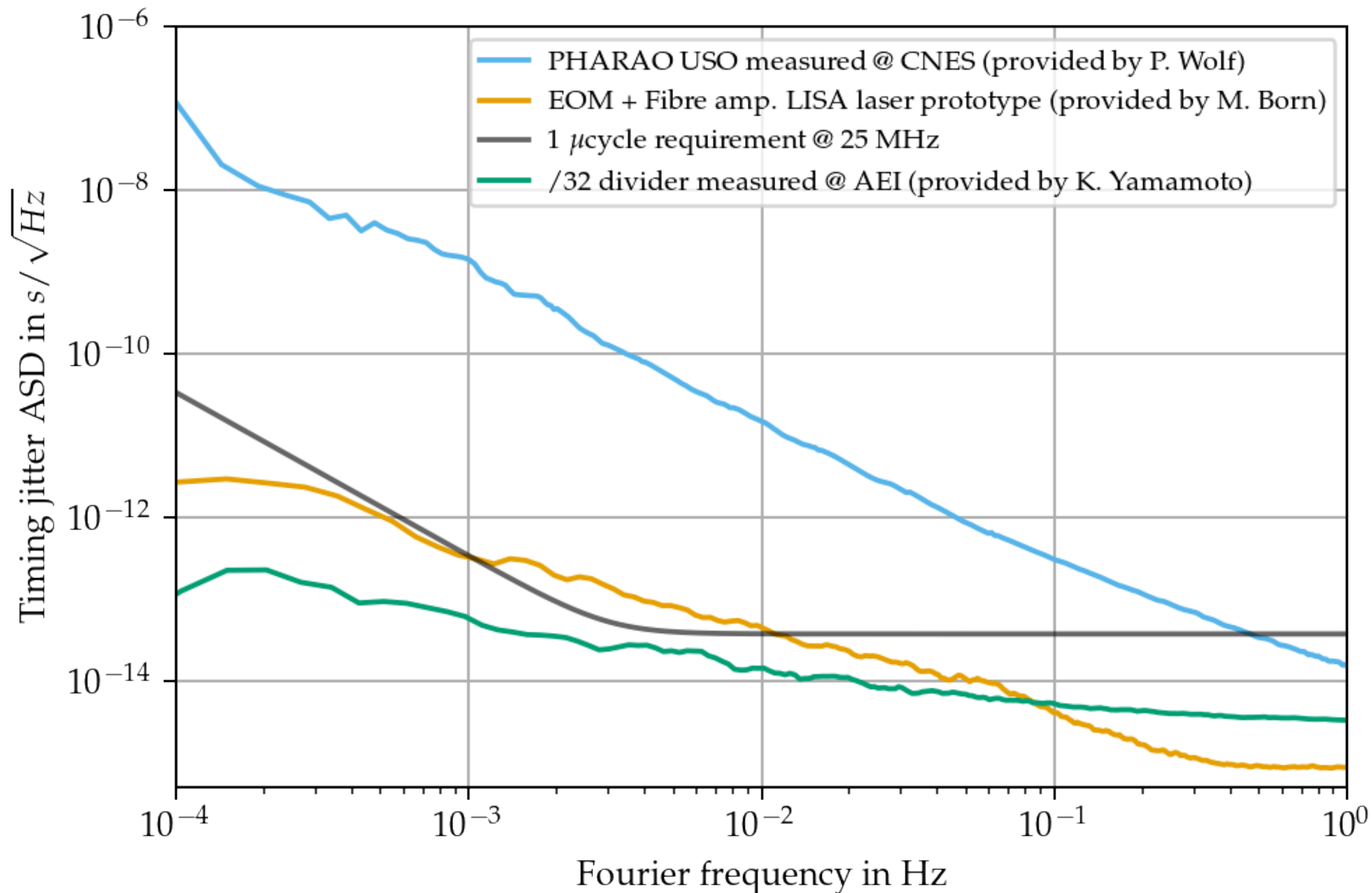
S/C 1

S/C 2





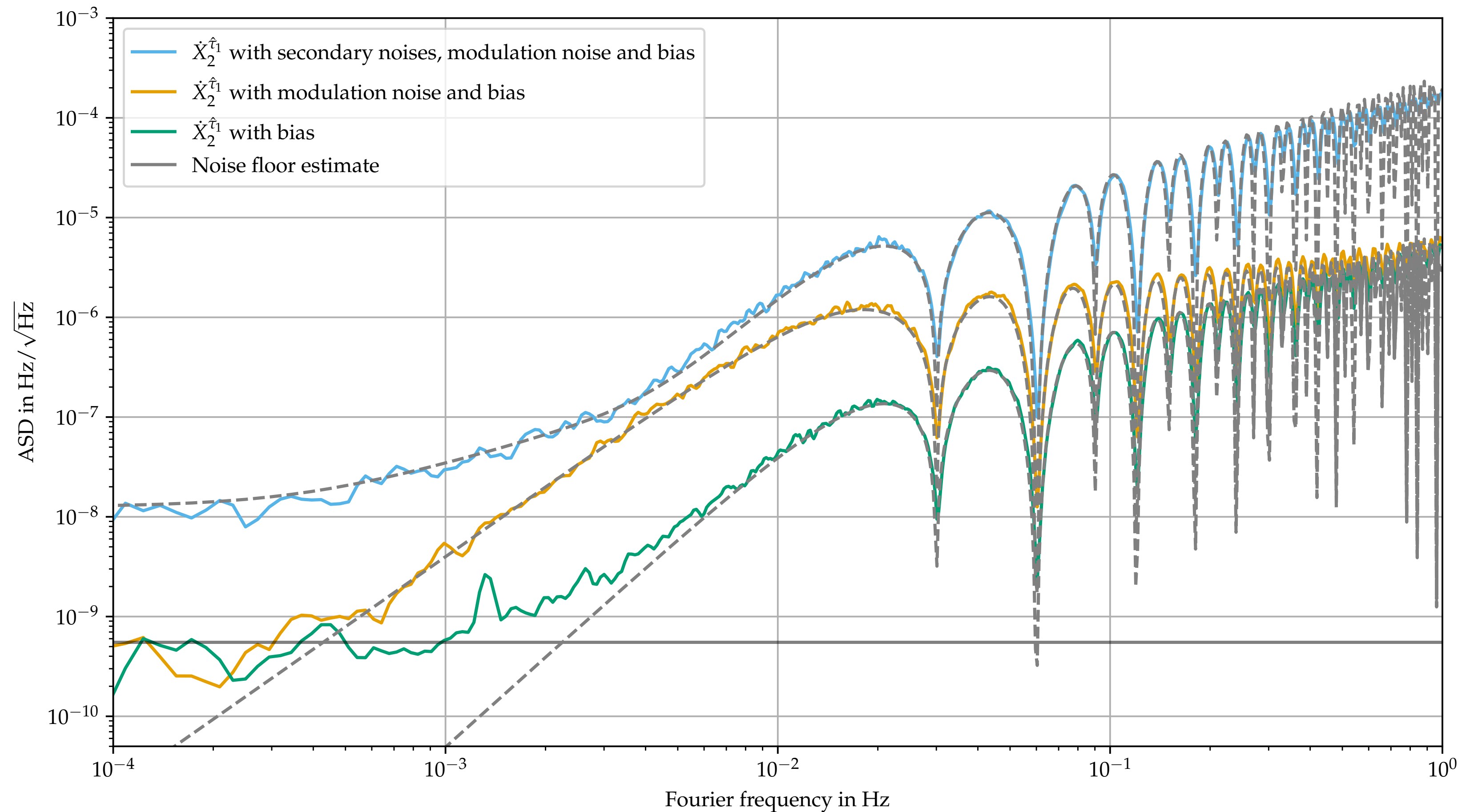
# Clock Compared Performance



- PRN and sidebands allow measurement of pseudorange at  $40 \text{ fs Hz}^{-0.5}$  level
- Absolute value of pseudorange accurate to  $\approx 3 \text{ ns}$  (1 m)
- Clock synchronisation to globale frame accurate to  $\approx 0.1 \text{ ms}$  (30 km)

# TDI Performance

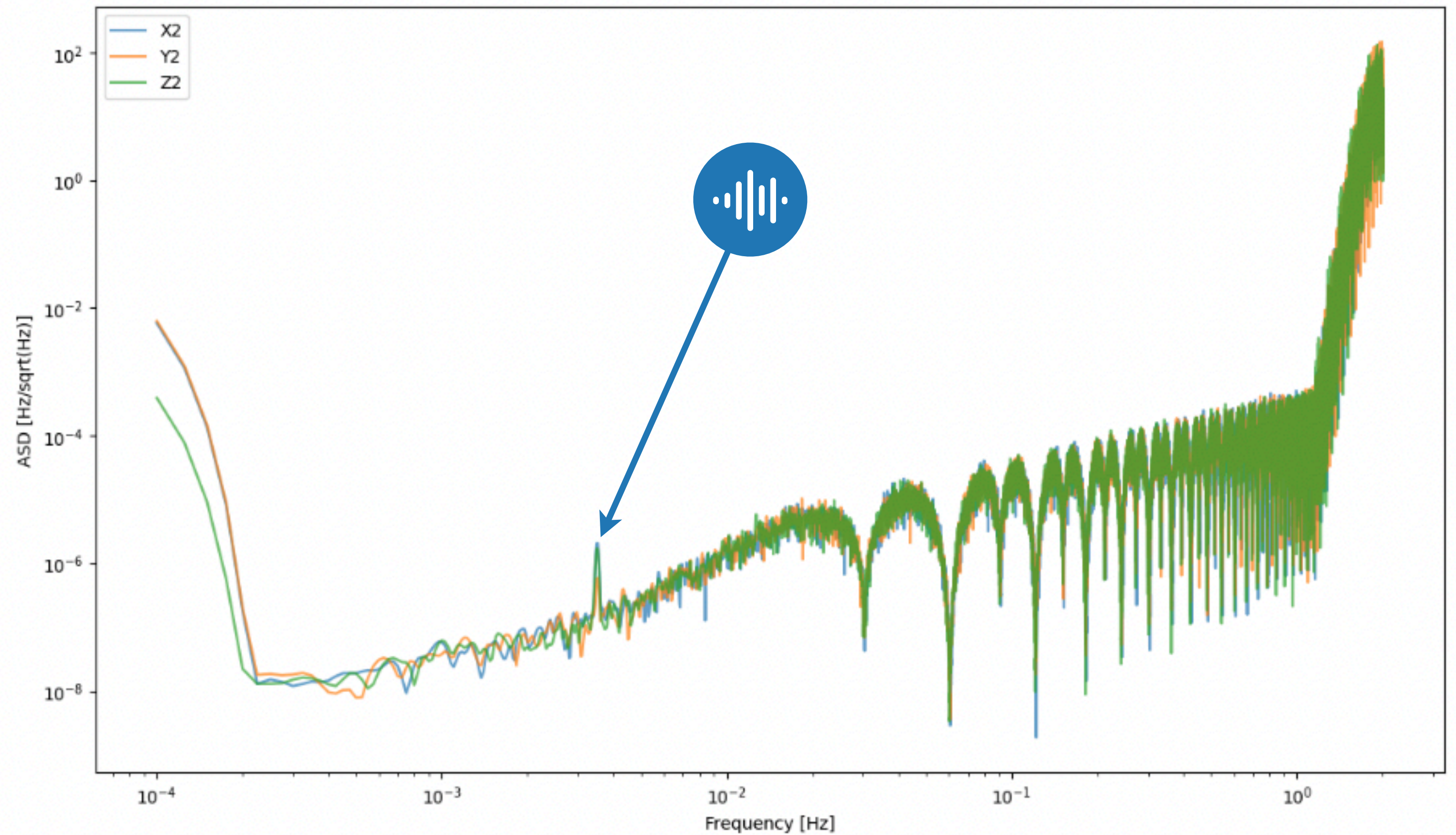
- Perform simulation with
  - Realistic orbits
  - Realistic laser, clock, sideband, PRN noises
  - Ultimately limiting secondary noises
- Performance is unaffected by large clock drifts and offsets
- Noise due to clock correction depends on the beatnote frequency and is nonstationary



Time delay interferometry without clock synchronisation,  
O. Hartwig, J.-B. Bayle, M. Staab, A. Hees, M. Lilley, P. Wolf.  
arXiv:2202.01124

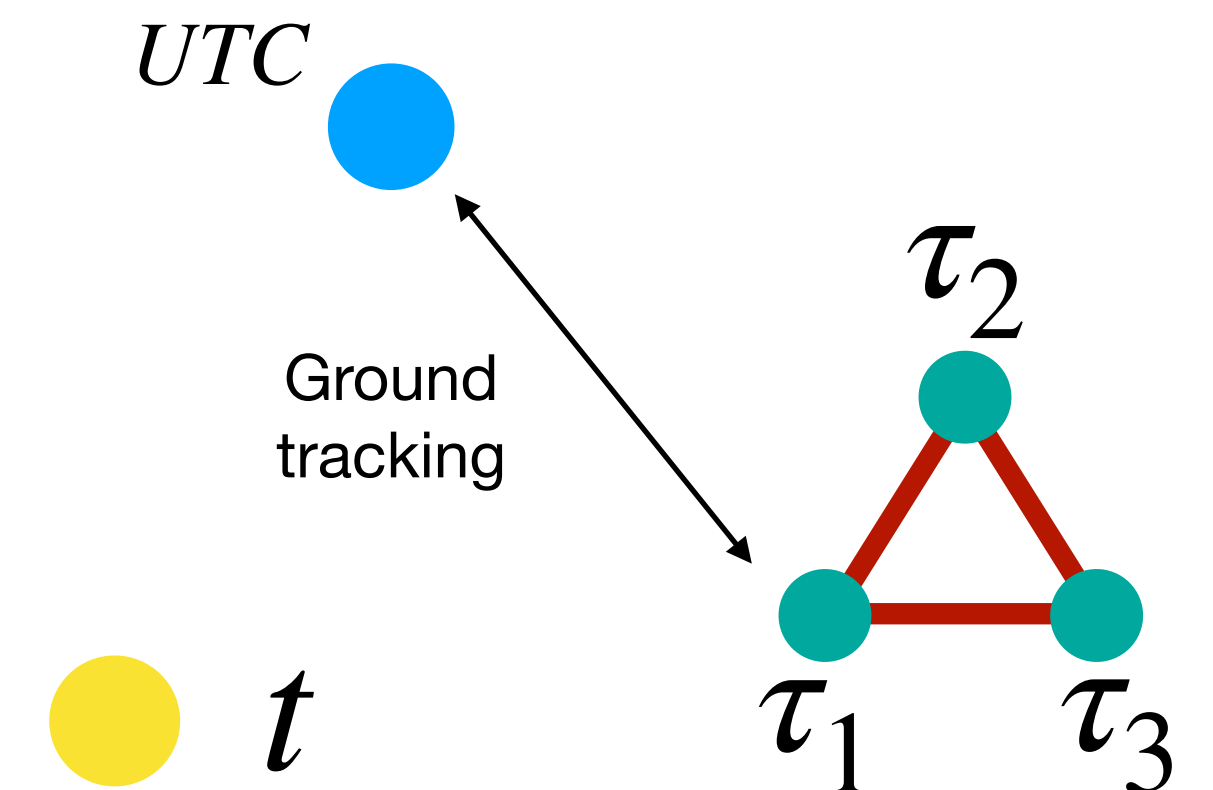
# Current Outputs

- As first step, we consider a single verification binary
- Amplitude is boosted to give 4 year SNR in just 3 days
- Signal is clearly visible in TDI data



# Time Synchronisation & Orbit Calculation

- After noise reduction, still need to synchronize resulting TDI channels to a global time frame at about 1ms accuracy
  - Use directly clock information from ground tracking
- For the parameter estimation, we need estimates of the orbits and light travel times to compute the response function
  - First try: directly feed-forward reconstructed orbits provided by MOC
  - Also compute LTTs directly from MOC orbits (  $\approx 1$  ms accuracy )
- Impact of these procedures is one output of this study



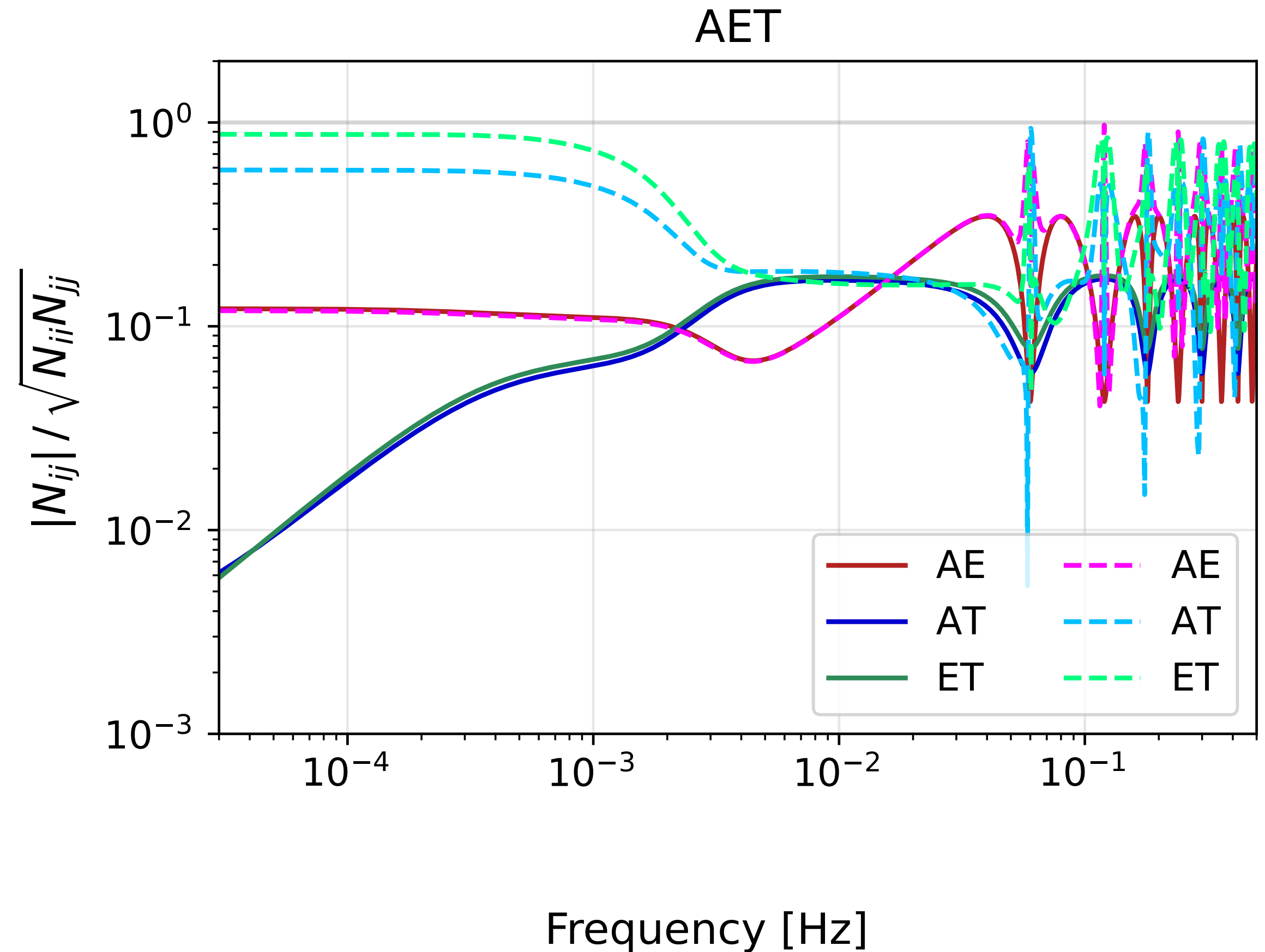
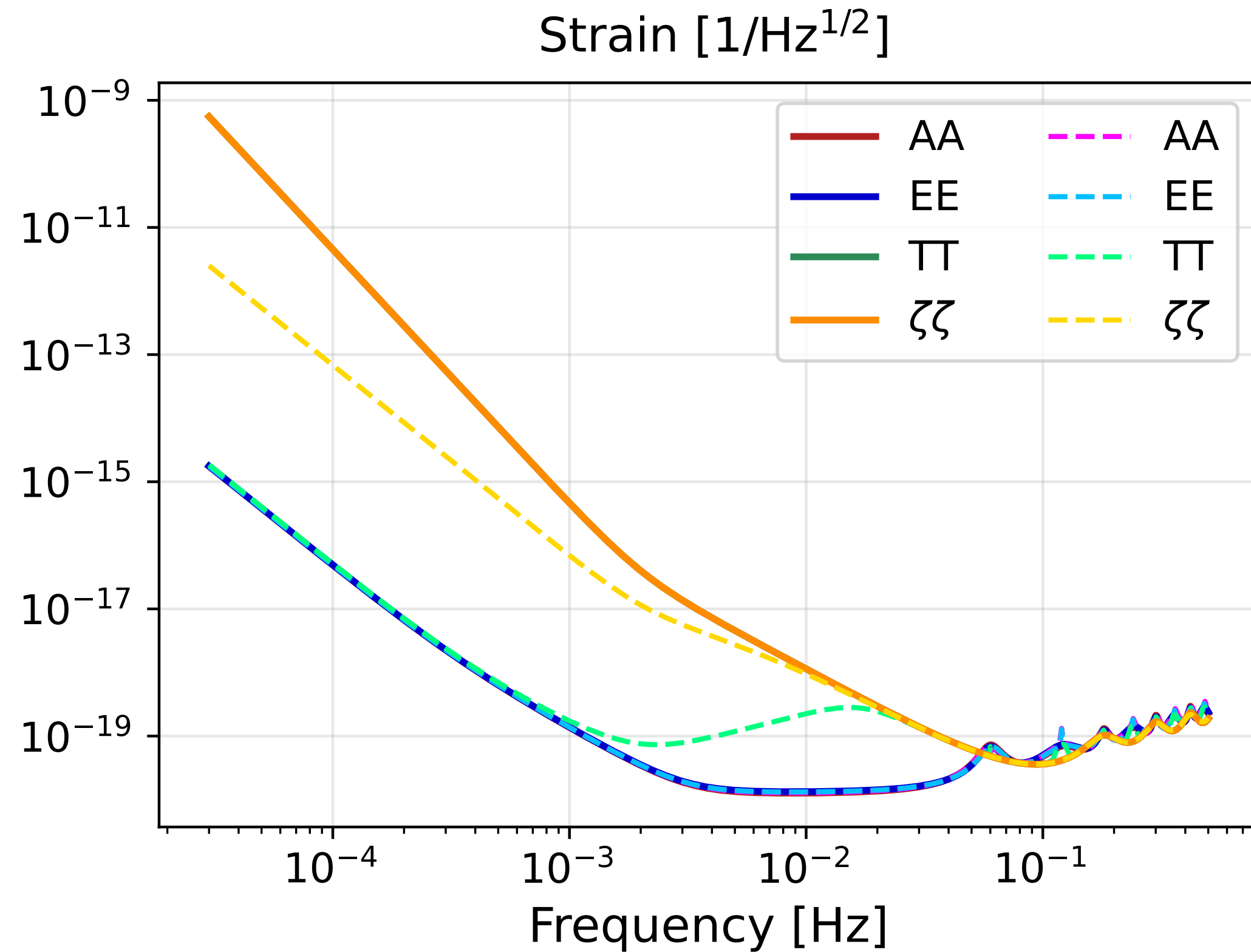
# Orthogonal Channels

- Once synchronized, we can combine multiple channels to construct quasi-orthogonal channels. For example,

$$A = \frac{Z - X}{\sqrt{2}}, \quad E = \frac{X - 2Y + Z}{\sqrt{6}}, \quad T = \frac{X + Y + Z}{\sqrt{3}}$$

- **Warning: these are *not* orthogonal in realistic scenarios!**
  - Arm lengths and individual noise levels not equal
- Impact under investigation...
- Could use other base channels, better orthogonalization, or skip *AET* and use the full covariance matrix

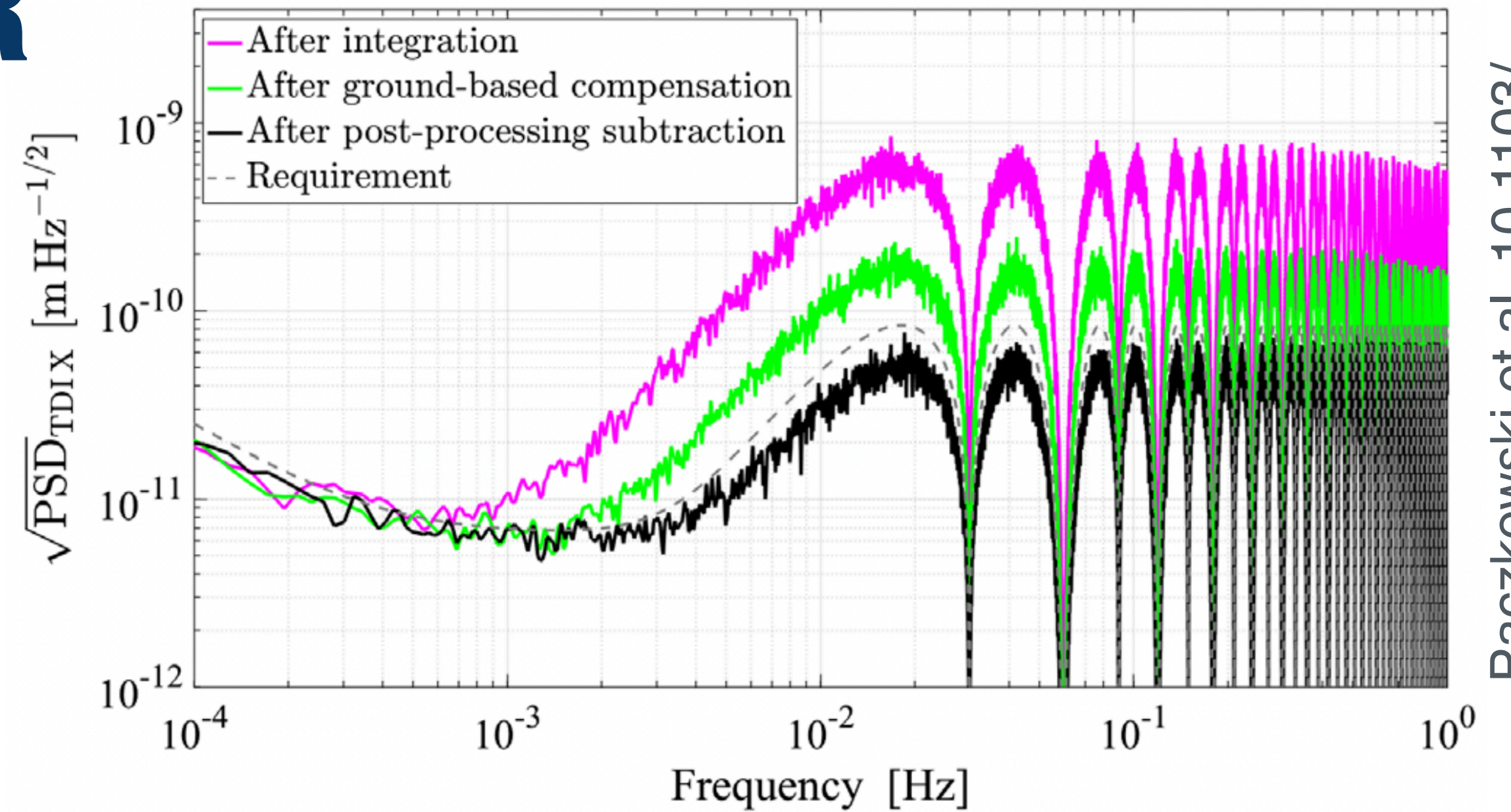
# Analytical Michelson



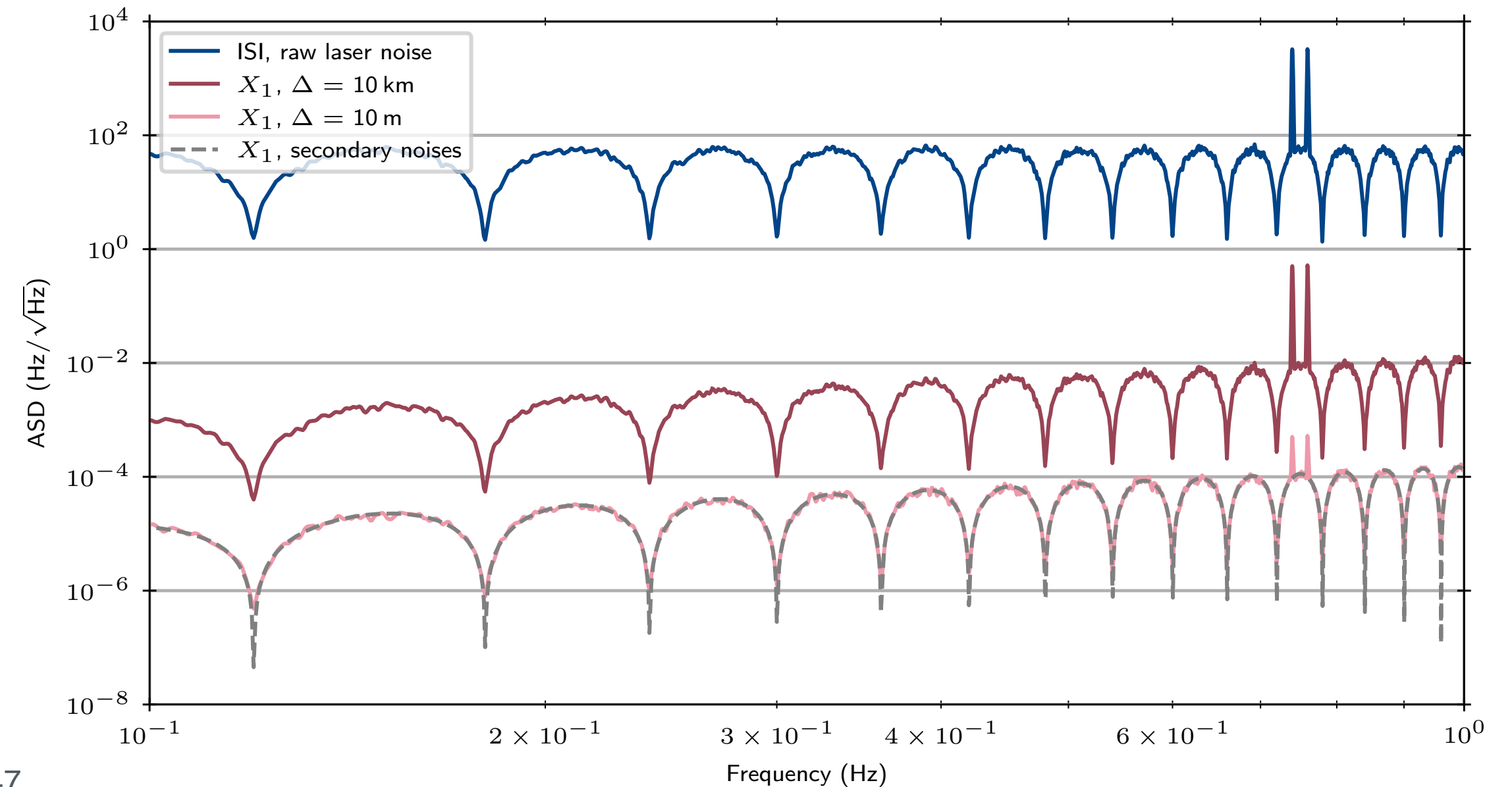
- Assume realistic (but static) arm length mismatches
- Assume noise levels drawn from normal distribution with  $\sigma = 0.2$

# A Word on TTL & TDIR

- Some processing steps envisioned as part of L0-L1 will require to fit some parameters to the data
- TTL coefficients are not known sufficiently well a-priori
  - Fit DWS measurement coupling factors by minimizing the noise
- Pseudorange measurements might contain additional unmodeled biases
  - Fit ranging bias by minimizing noise in TDI combinations



Paczkowski et al. 10.1103/PhysRevD.106.042005



Staab et al., in prep.

# L1-L2 Parameter Estimation

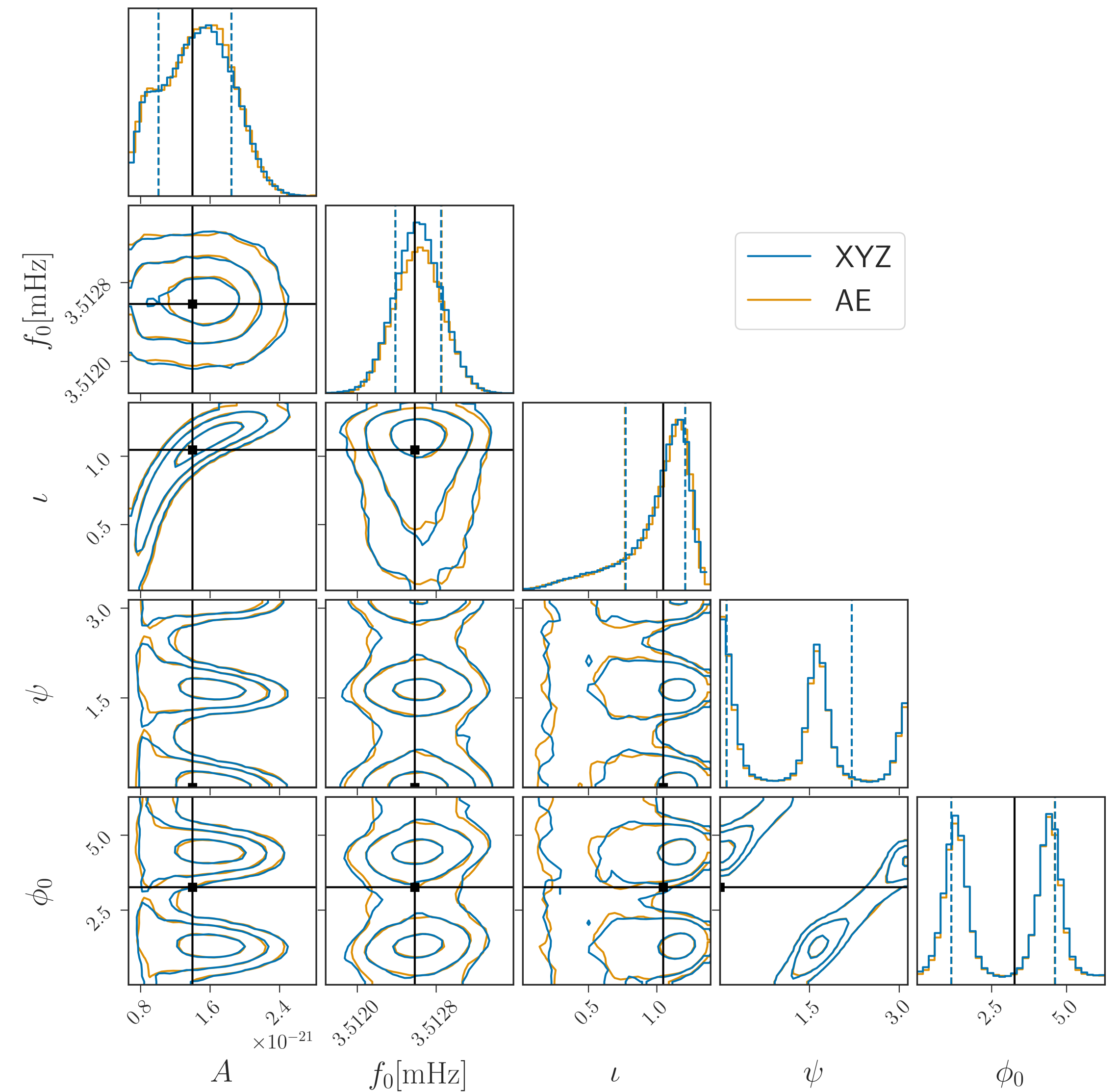
Thanks Christian Chapman-Bird!





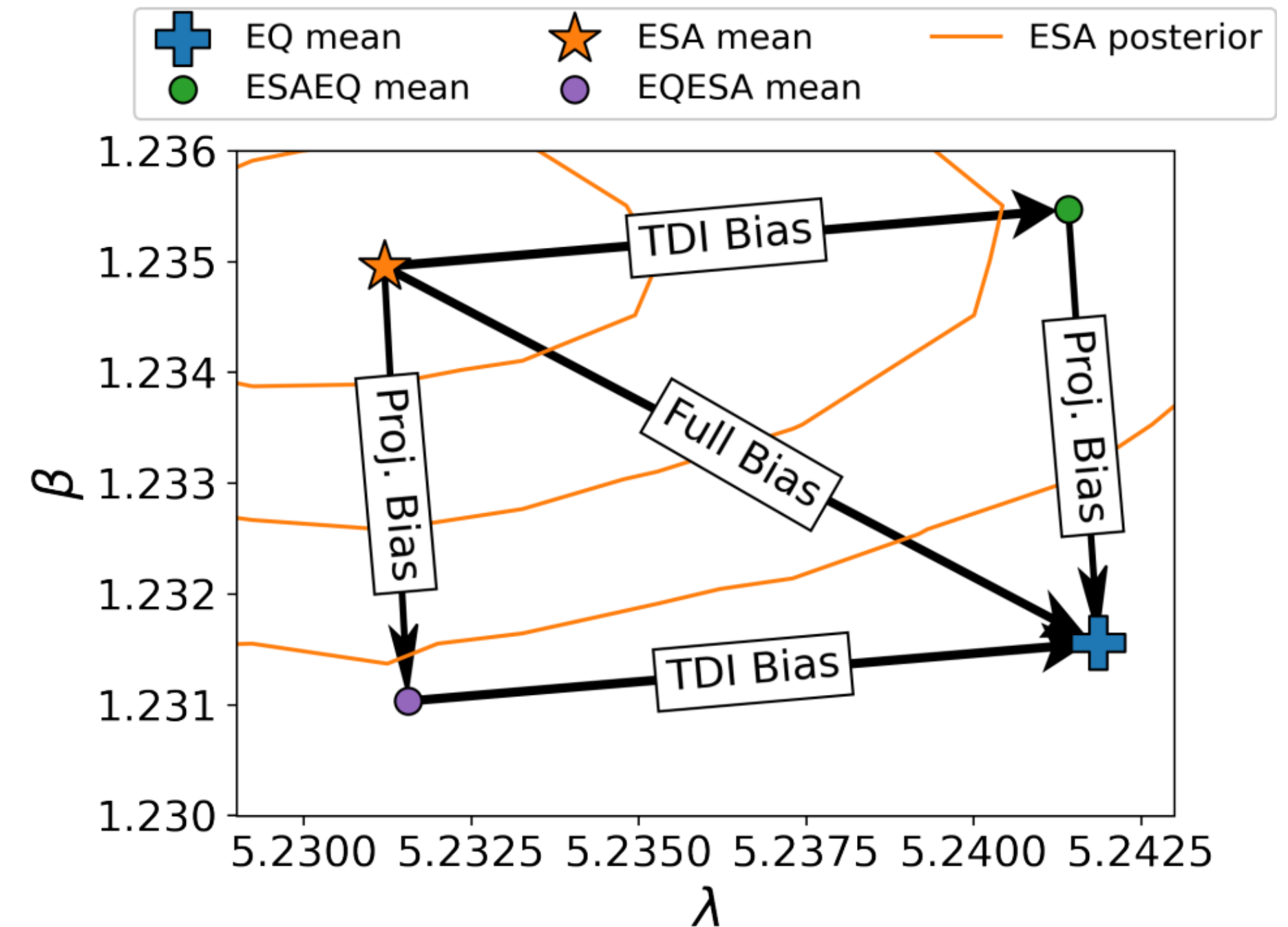
# Current Progress

- Single galactic binary with optimal SNR of 100 for a 4-year dataset
  - For ease of testing, reduce to 3 days of data and rescale amplitude
  - Sky position fixed: "verification binary"
  - Equal-armlength orbits and aforementioned primary/secondary noise sources enabled
- Parameters are recovered well (except phase, likely an error with epochs somewhere!)
- We see good agreement between working in 2nd-generation  $AE(T)$  or  $XYZ$ , but for more realistic orbits we expect biases to emerge if diagonal covariance matrix is assumed



# Next Steps

- Adapt the FastGB waveform model to incorporate
  - More realistic orbits, including unequal and time-varying armlengths
  - Use of orbital information obtained from ground tracking, instead of evolving a dynamical model
  - Eventually, merge these changes with existing GBGPU waveform model
- Extend simulation duration to 4 years and enable further noise sources
  - Verify that sky localisation and frequency derivative inference are performed successfully
- Experiment with various noise sources to probe the resulting impact on parameter estimation
  - Explore under which scenarios parameter estimation will incur biases



# Conclusion & Outlook



# Conclusion & Outlook

- Many analysis methods to extract source parameters under development
- These methods often rely on simplifying assumptions, not necessarily reflecting the full complexity of the LISA data
- We work on checking (some of) these assumptions by building a (more) realistic simulation-processing-analysis pipeline
- We go from simple configurations (close to current LDC) and slowly add realistic features and processing elements to check that they do not break anything
- Activity started by defining the target configuration and run the pipeline with a simple configuration – PE works (mostly) as expected!