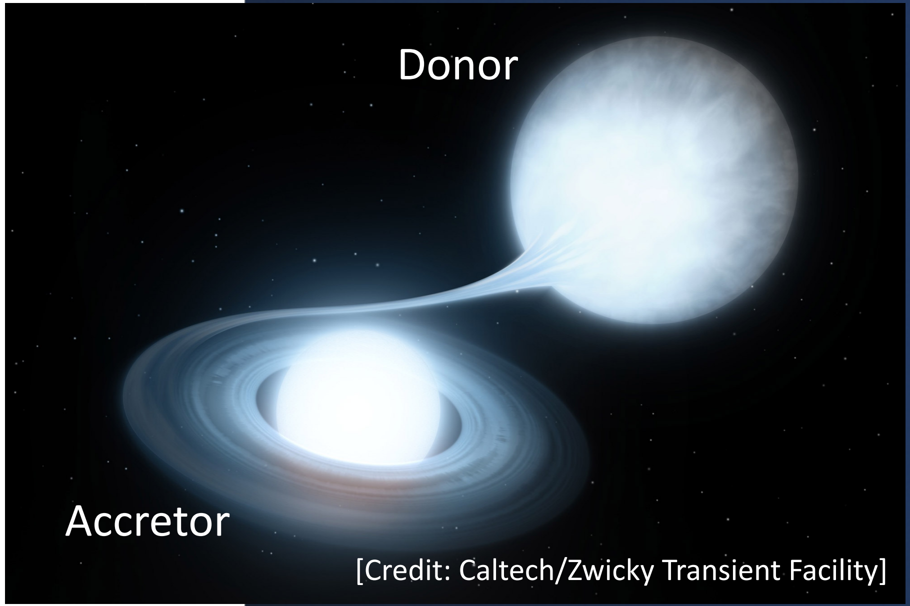
Determining the individual masses of accreting white dwarf binaries

Sophia Yi, with Kent Yagi, Phil Arras, and Vincent Lau
University of Virginia

LIDA Workshop

Objective

- Original objective: investigate how well we will be able to determine the individual masses of accreting binary white dwarfs (BWDs) using LISA's measurements of f_{GW} and $\dot{f}_{GW}$
- Along the way, we have effectively added studies on the measurability of a few other parameters



[Credit: Caltech/Zwicky Transient Facility]

Background

- Sky-averaged gravitational waveforms can be resolved into an amplitude, A, times the cosine of a time-varying phase, $\phi(t)$

Measurable components of A:

- GW frequency, f_{GW}
- Luminosity distance, d
- Chirp mass, $\mathcal{M}_c = \dfrac{(m_d m_a)^{3/5}}{(m_d + m_a)^{1/5}}$

Measurable components of $\phi(t)$:

- GW frequency, f_{GW}
- GW frequency derivative, $\dot{f}_{GW}$

- General concept: the more that parts of the GW waveform depends on WD parameters (mass, radius, etc.), the better we will be able to measure these parameters

Expressing f_{GW} and $\dot{f}_{GW}$

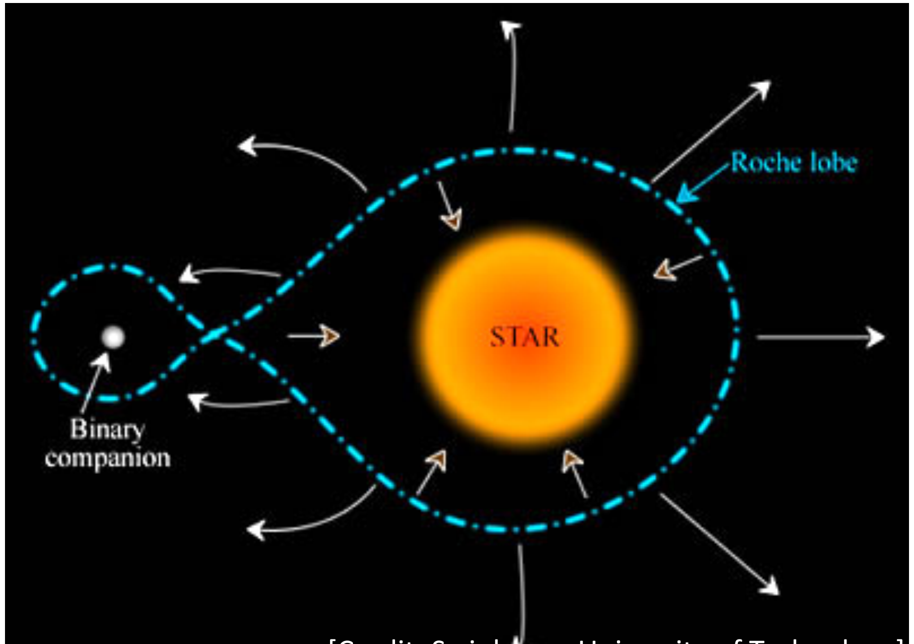
- Roche lobe filling: $r_d \sim a$ (semi-major axis)
- Kepler's third law: $f_{GW} \sim \sqrt{\frac{m_d + m_a}{a^3}}$

$$f_{GW} \sim \left(\frac{m_d + m}{r_d^{\,3}} \right)^{1/2}$$

- Differentiating f_{GW} to get $\dot{f}_{GW}$ introduces a new parameter,

$$\eta_d = \frac{d \ln r_d}{d \ln m_d}$$

$$\dot{f}_{GW} \sim (\mathcal{M}_c)^{5/3} f_{GW}^{\;11/3} \times \frac{-3\eta_d + 1}{\frac{5}{6} + \frac{1}{2}\eta_d - \frac{m_d}{m_a} - r_h^{1/2}\left(1 + \frac{m_d}{m_a}\right)^{1/2}}$$



[Credit: Swinburne University of Technology]

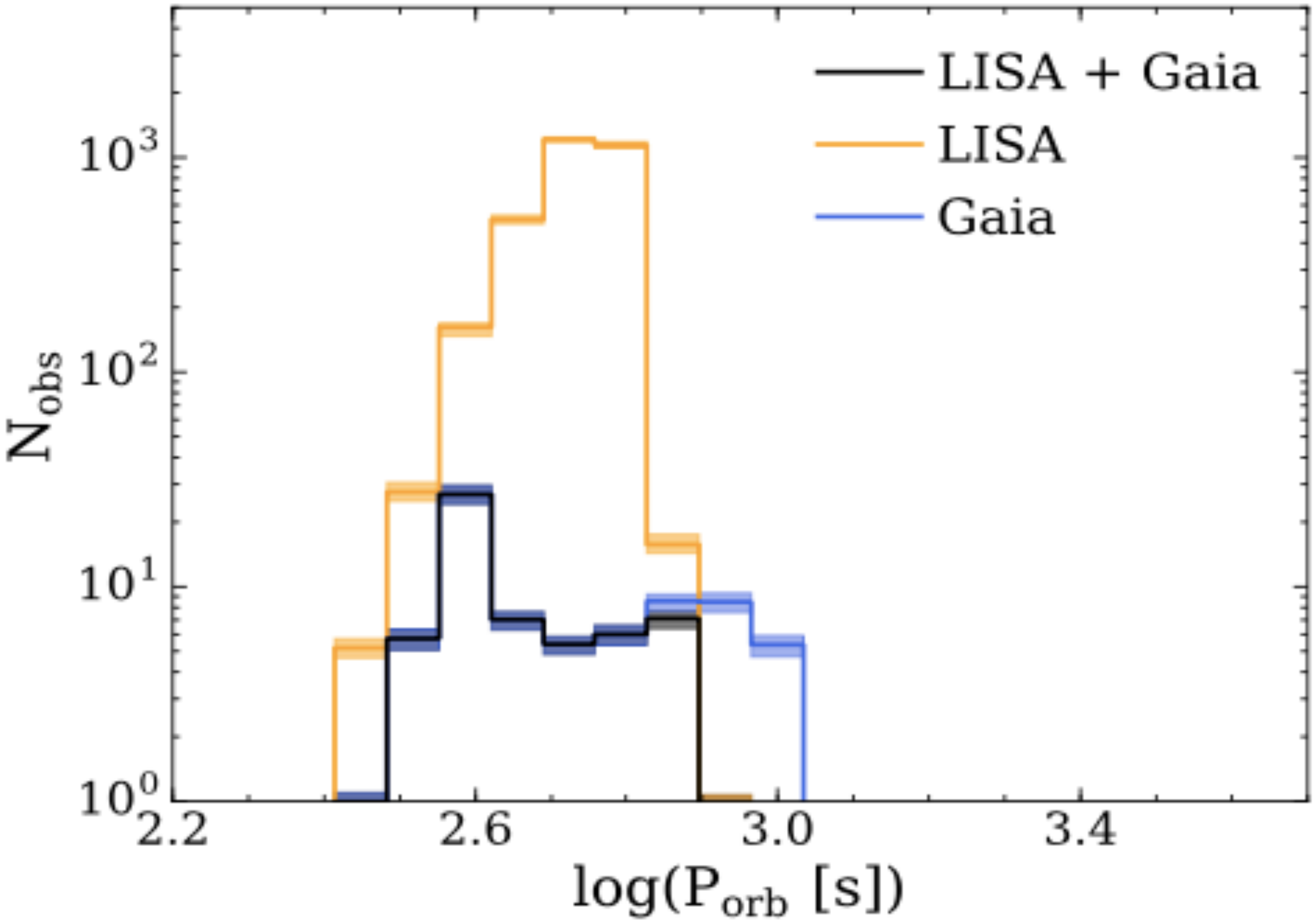
Including d as a parameter?

$$A \sim \frac{\mathcal{M}_c}{d}\left(\mathcal{M}_c\, f(m_d,\ m_a, r_d)\right)^{2/3}$$

$$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d\right)$$

or

$$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d, d\right)$$



[Breivik et al. arXiv:1710.08370]

Parameter estimation technique: Fisher analysis

- Sky-averaged gravitational waveform: $h(t) = A\cos\phi(t)$
- For some parameter set, θ^i, define the Fisher information matrix:

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta^i}\middle|\frac{\partial h}{\partial \theta^j}\right)$$

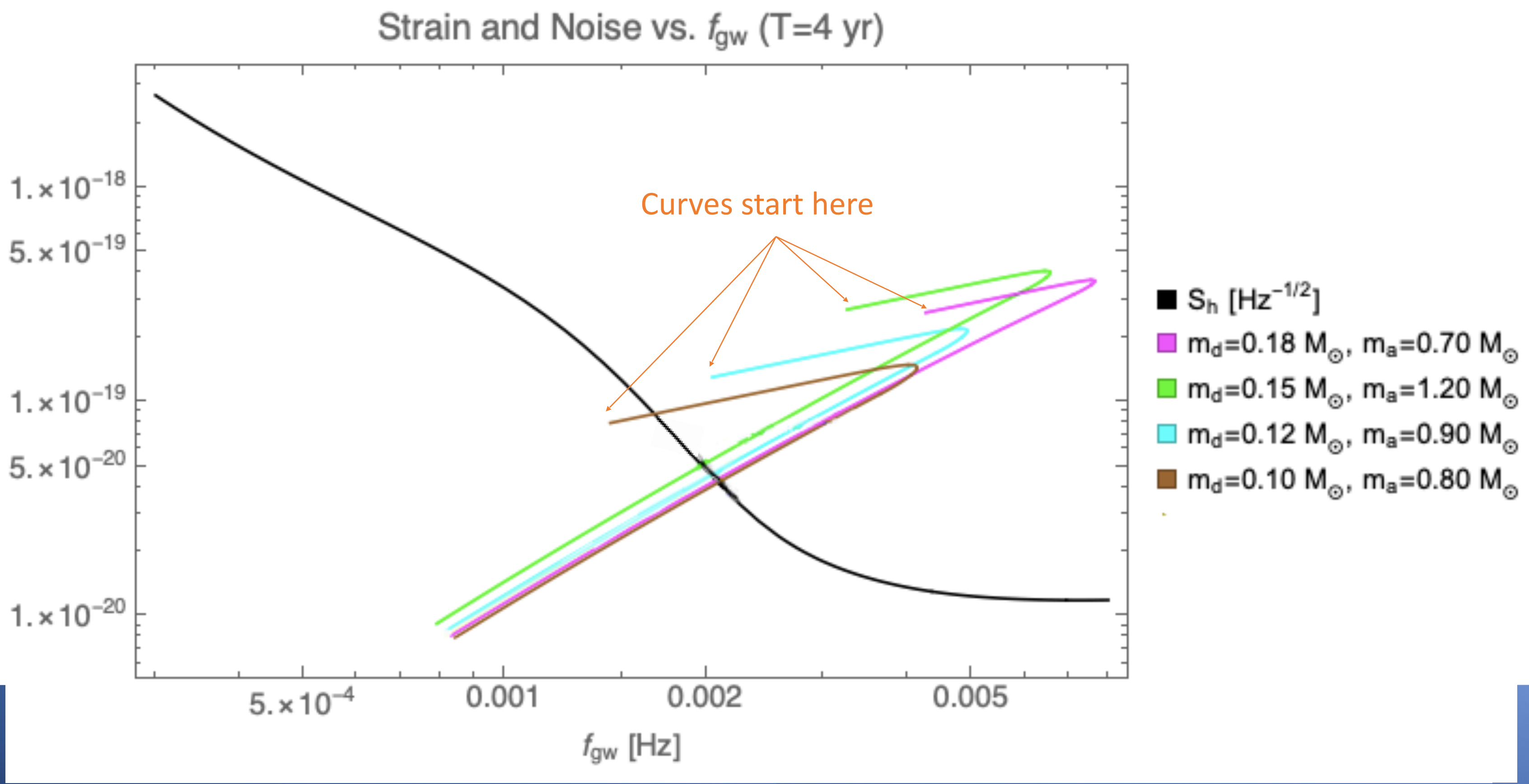
→ 1-σ measurement uncertainty: $\Delta\theta_i = \sqrt{\left(\Gamma^{-1}\right)_{ii}}$

- Separately consider two parameter sets: $\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d\right)$ and

$$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d, d\right)$$

GW strain magnitude vs. LISA's noise

- Strain$\sim\frac{\mathcal{M}_c^{5/3}}{d} f_{GW}^{2/3}$
- Plots assume $d = 10$ kpc
- Good chance of resolving GW strain for systems with larger donor WDs



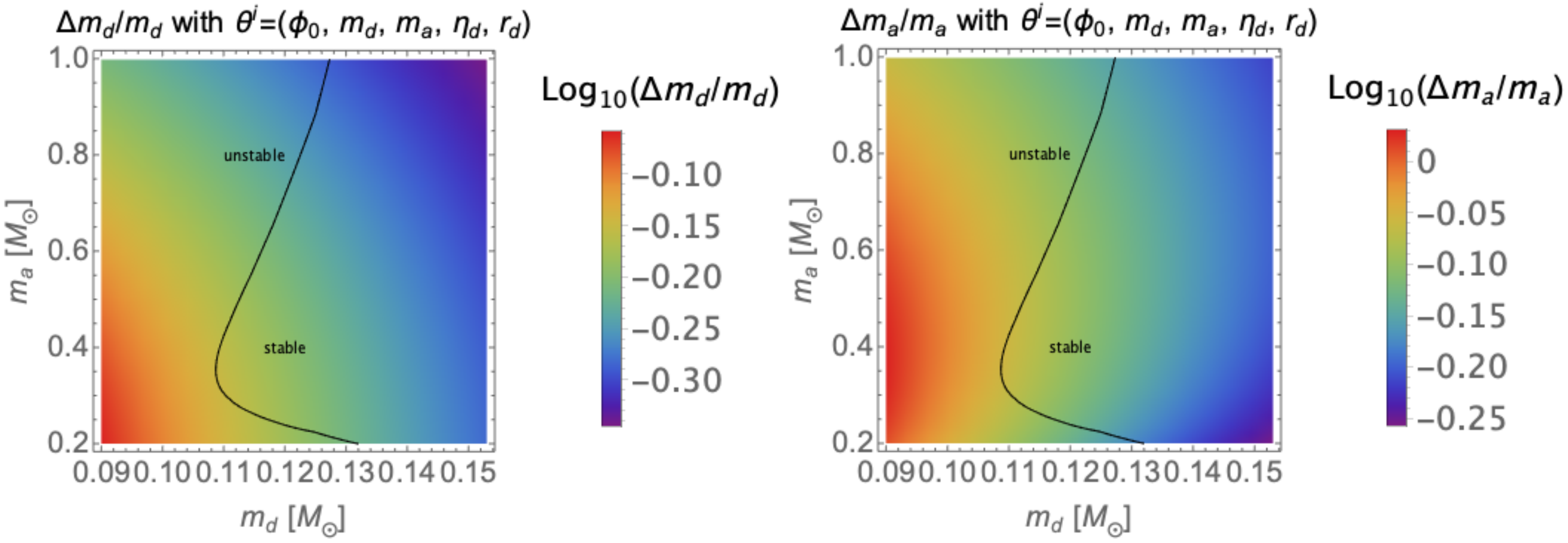
Parameter estimation results:

$$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d \right)$$

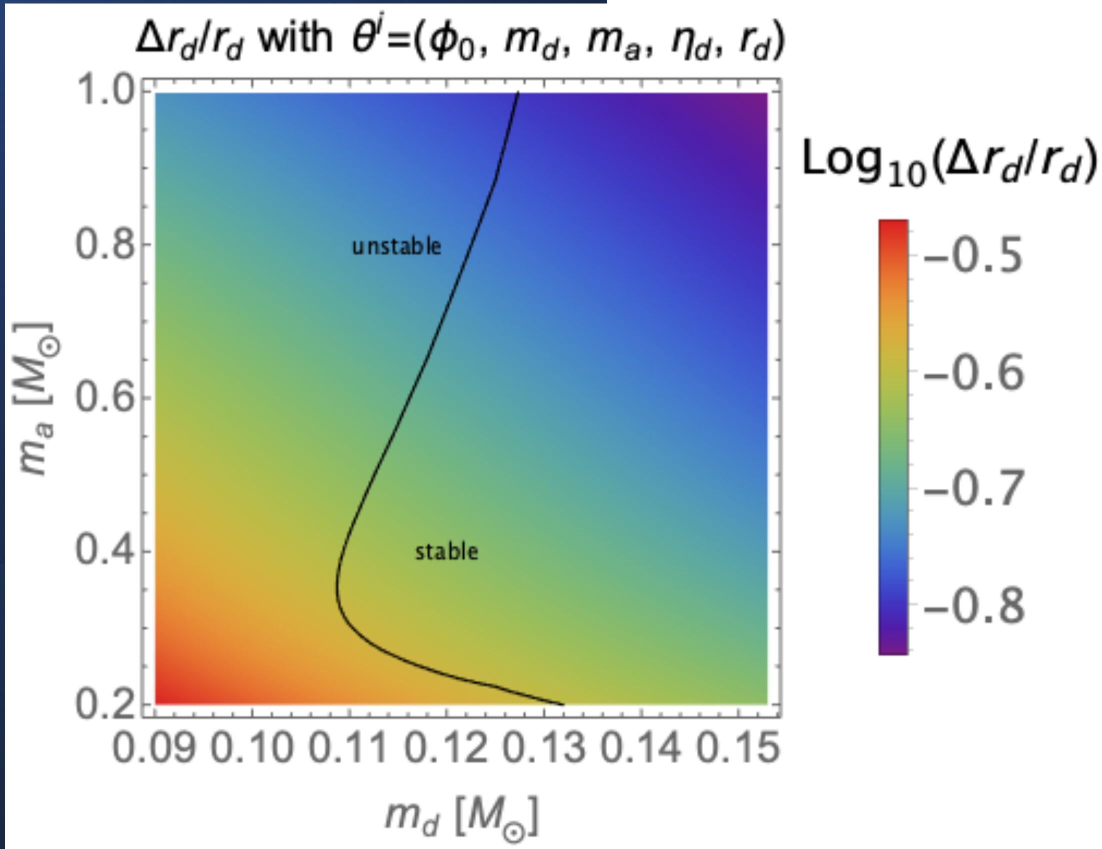
Assumptions

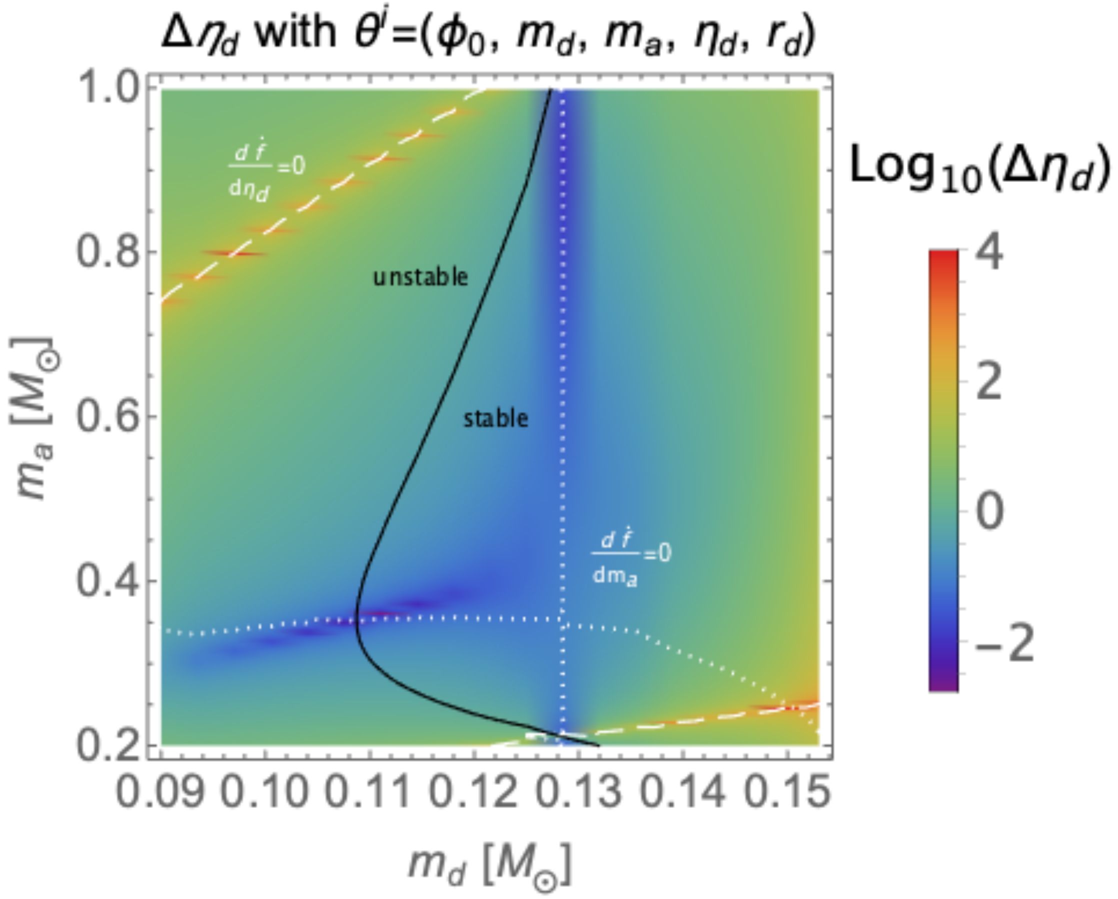
- Luminosity distance is independently measured to be 1 kpc
- LISA's noise curve from Robson et al. [arXiv:1803.01944]
- 4-year observation period
- Gaussian priors: $\sigma_{m_d} = 0.1\ M_\odot,\ \sigma_{m_a} = 1.2\ M_\odot$

Error on m_d, m_a



Error on r_d, η_d





Summary: parameter estimation with $\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d\right)$

- If d is known, we have a fair chance of constraining the individual masses for relatively higher-mass combinations when we impose the priors, $\sigma_{m_d} = 0.1\ M_\odot$ and $\sigma_{m_a} = 1.2\ M_\odot$
- Under these conditions, there is also a fair chance of constraining r_d and η_d

Parameter estimation results:

$$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d, d \right)$$

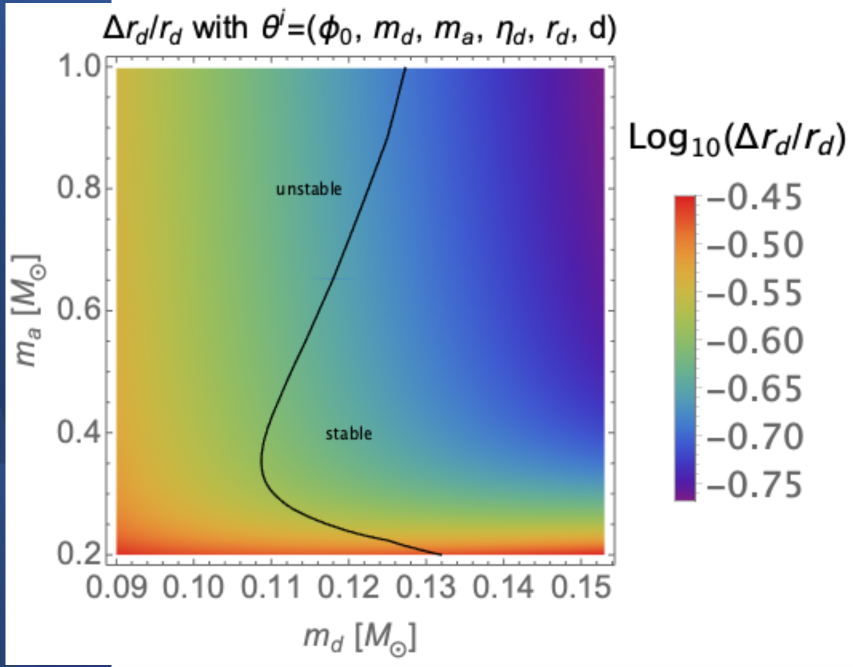
Assumptions

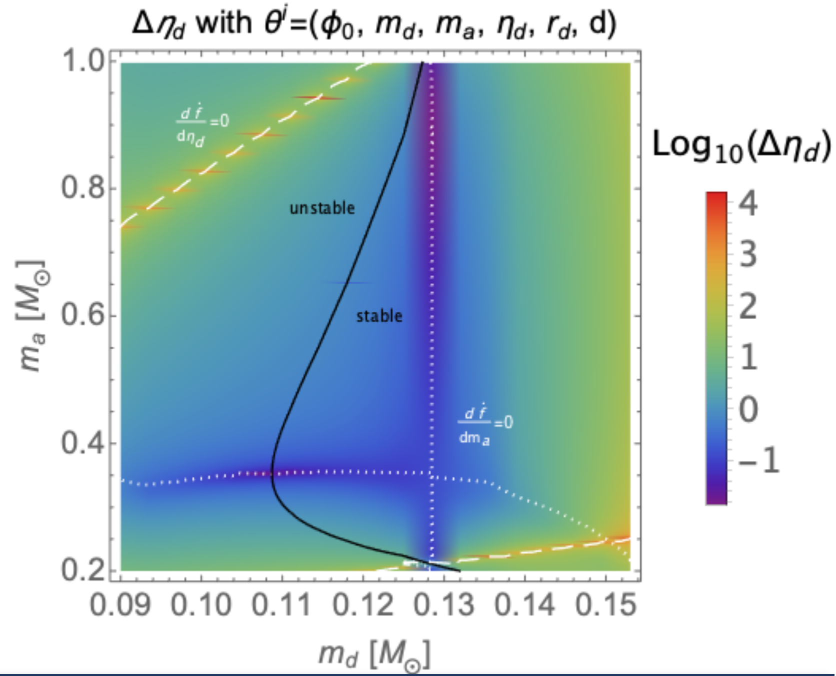
- Now, we make no assumptions for the value of d
- LISA's noise curve from Robson et al. [arXiv:1803.01944]
- 4-year observation period
- Gaussian priors: $\sigma_{m_d} = 0.1\ M_{\odot},\ \sigma_{m_a} = 1.2\ M_{\odot}$

Error on the masses

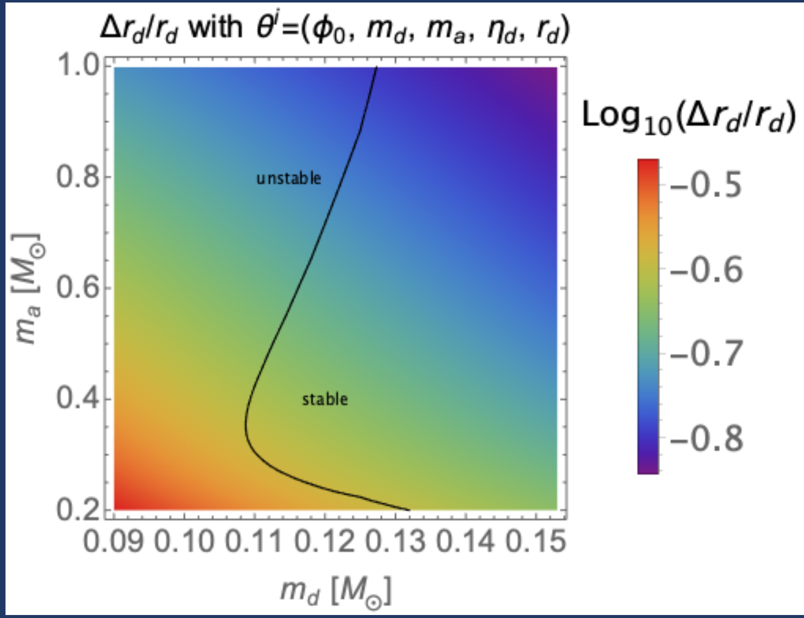
- When d is included as a parameter, the parameter estimation simply returns the priors
- We gain no additional constraints on the individual masses

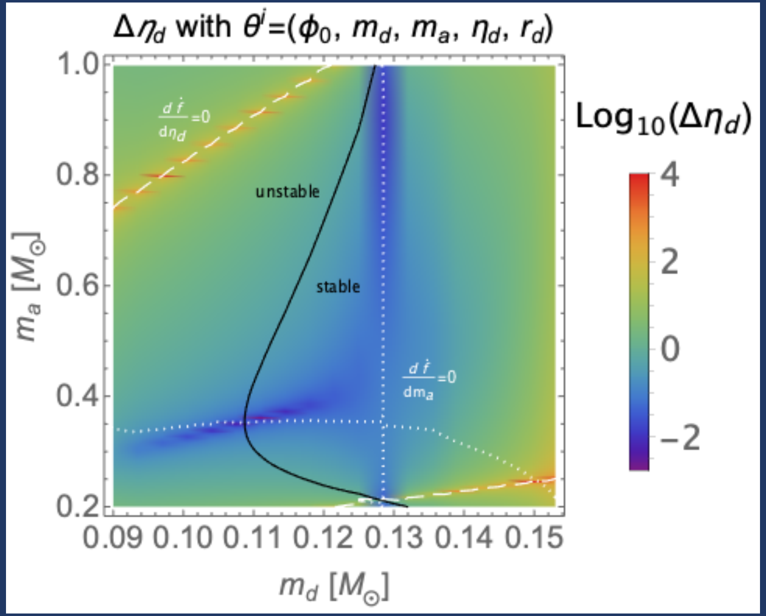
Error on r_d, η_d





Compare with results for the five-parameter set



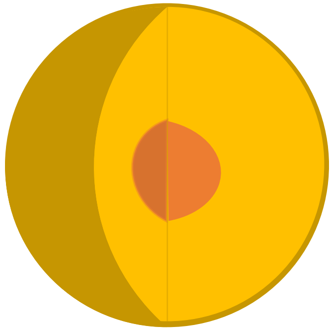


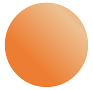
Conclusion and Future Work

- We use a Fisher analysis to estimate the measurability of WD masses and other parameters
  - ➢ If we do not include the luminosity distance as a parameter, we have a fair chance of constraining m_d, m_a, r_d and η_d when we impose priors on the masses
  - ➢ If we do include d as a parameter, the measurability of r_d and η_d does not change significantly, but we can no longer constrain m_d and m_a
- Consider the case of non-conservative mass transfer
  - $\dot{m}_a = -(1 - F)\dot{m}_d, \; F > 0$

Interpreting $\eta_d = \frac{d\ln r_d}{d\ln m_d}$

- η_d defines the response of the donor WD to mass loss
- For the very low-mass WDs we consider ($\lesssim 0.1 M_\odot$), η_d varies depending on the stage of evolution ("early" or "late") as follows:





Early stage

- WD has a thick hydrogen envelope
- Nondegenerate envelope, degenerate core
- Radius decreases as mass decreases $\implies \eta_d > 0$

Time of highest emitted GW strain

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- …emains
- …decreases $\implies \eta_d < 0$

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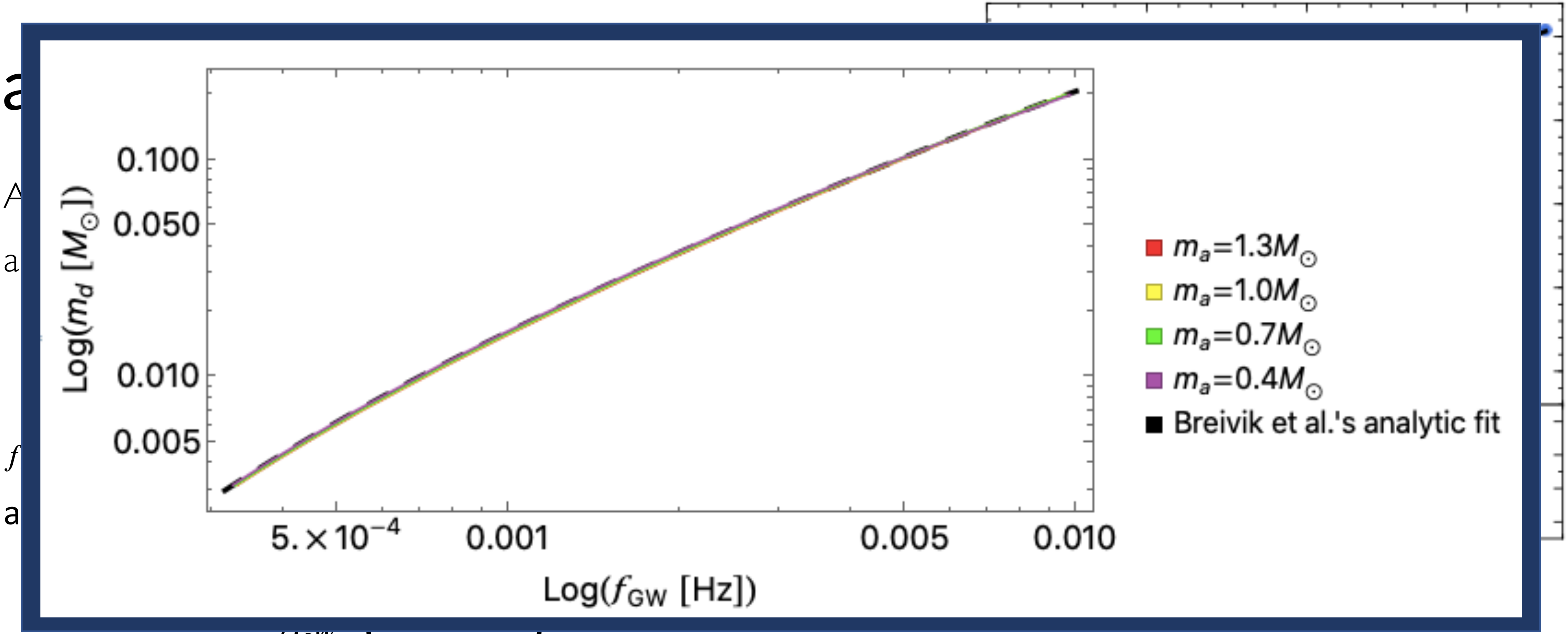
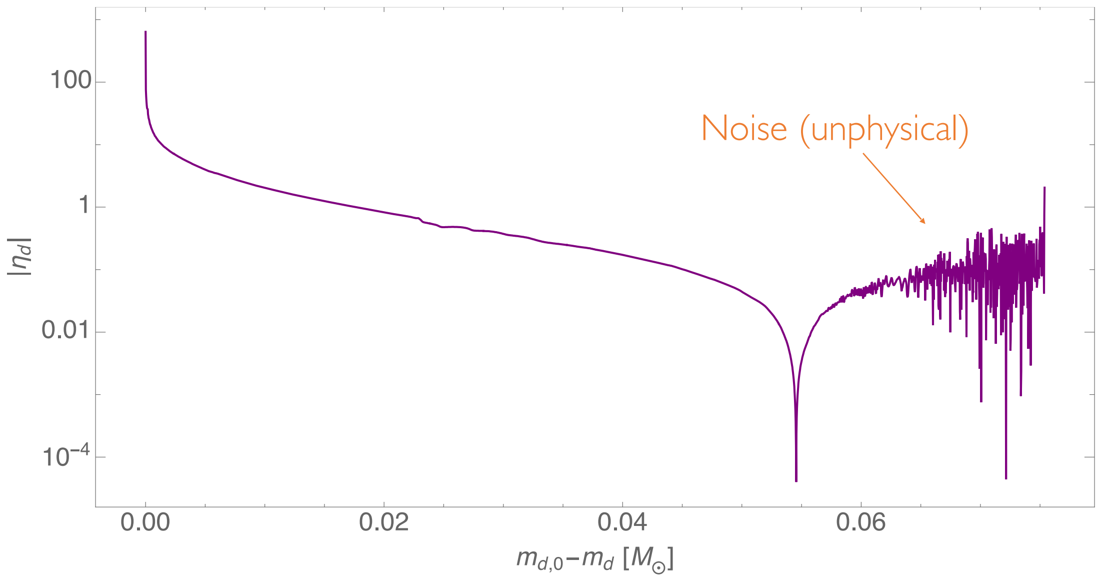
f_{GW} (r_d^3)



[Breivik et al. arXiv:1710.08370]

Early stage of evolution



- No analytic mass-radius formula
- MESA (Modules for Experiments in Stellar Astrophysics) outputs masses and radii of WDs with helium cores and hydrogen envelopes
  - We can interpolate this data for r_d vs. stripped mass
  - However, we experience numerical difficulty in computing η_d vs. stripped mass

Determining the parameter set

Late stage

- Because of the analytic mass-radius formula, both r_d and η_d are completely determined by m_d

$\Longrightarrow$ At most, $\theta^i = \left(\phi_0, m_d, m_a, d\right)$

Early stage

- η_d and r_d are unknown for a given m_d (no analytic mass-radius formula, MESA results are inconclusive)

$\Longrightarrow$ At most,
$\theta^i = \left(\phi_0, m_d, m_a, r_d, \eta_d, d\right)$