A neoclassical approach to coherent gravitational-wave search

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Coherent search for LISA

- Uses phase comparisons between data *x* & signal template *h*

$$\langle x|h\rangle := 4\Re\sum_f \delta f\,\frac{\tilde{x}^*(f)\tilde{h}(f)}{S_n(f)}$$

- Coherent statistics are also functions on continuous space of templates

$$X(\boldsymbol{\theta}|x) := \frac{\langle x|h(\boldsymbol{\theta})\rangle}{\sqrt{\langle h(\boldsymbol{\theta})|h(\boldsymbol{\theta})\rangle}}$$

- Search involves finding strong maxima & establishing significance (detection)
- Mapping *x* correctly to source parameters $\boldsymbol{\theta}$ is important as well (identification)

Coherent search for LISA

- Needs stochastic rather than grid search
  - Larger signal space than that of CBCs in ground-based observing
  - Stronger correlations between signal & other signals or templates
  - Models can be computationally expensive: MBHs, EMRIs
- Needs sampling rather than optimisation algorithms
  - Coherent statistics are not globally concave or log-concave
  - Monte Carlo sampling of statistic-based probability distributions
  - Thus connects to posterior sampling in Bayesian inference

$$\ln L(\boldsymbol{\theta}|x) := -\frac{1}{2}\langle x - h(\boldsymbol{\theta})|x - h(\boldsymbol{\theta})\rangle \qquad L \propto \exp\left(X\rho\right)$$

Difficulties of coherent search

- We are often looking for a “needle” in a large search area
  - This is especially the case for EMRIs & SOBHs
  - For GBs: Many needles to find, but we want as many as possible
- The needle is not on a hill (i.e. no global gradient leading to it)
  - Global gradient is either zero for X, or SNR-driven for $\ln L$

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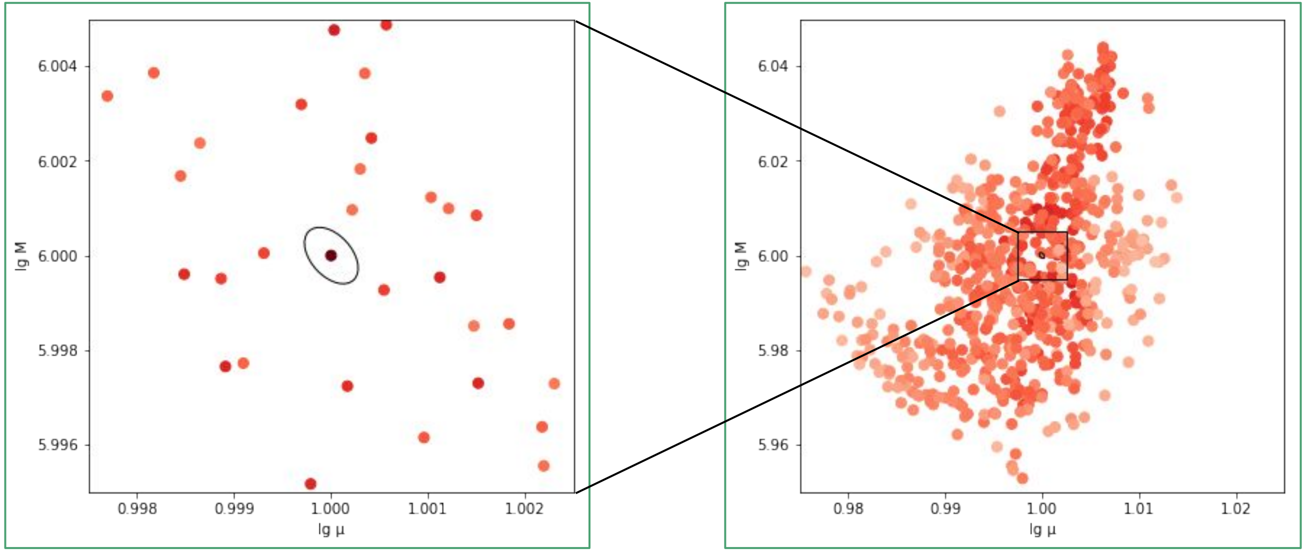
- The needle is not on a hill (i.e. no global gradient leading to it)
  - Global gradient is either zero for X, or SNR-driven for $\ln L$
- The search area is often a haystack, or even a field of haystacks
  - Small variations everywhere from noise–template correlations
  - Larger variations sometimes from signal–template correlations
  - For EMRIs: Severe non-local parameter degeneracy [Chua & Cutler, arXiv:2109.14254]

Difficulties of coherent search

- All of these problems are epitomised by EMRIs in particular
  - The scale of local correlations is much smaller than the global space: Volume ratio < 10^{-20}
  - The secondary maxima can be prevalent & pronounced: Hundreds, with overlaps up to 0.8



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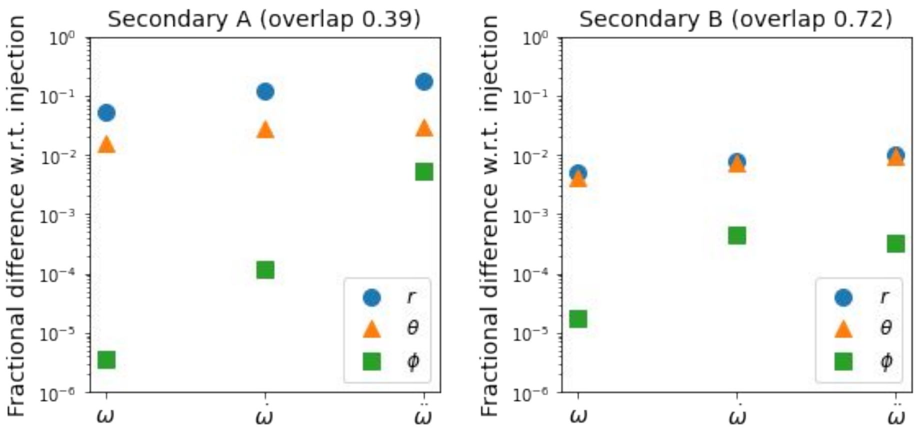
    - The scale of local correlations is much smaller than the global space: Volume ratio < 10^{-20}
    - The secondary maxima can be prevalent & pronounced: Hundreds, with overlaps up to 0.8
- The strongest secondaries have a very generic cause
    - Matched initial frequencies & derivatives of dominant frequency harmonic only



Modified coherent search functions

- Partial maximisation/marginalisation over some degrees of freedom
  - Semi-coherent statistics: Sum of shift-maximised statistics for shorter time segments
  - *F*-statistics: Maximisation of statistics over the coefficients of some basis for the signal space
  - These de-emphasise variations by broadening peaks & coalescing them
  - Still retains variations due to signal & noise correlations ("bigger needle in a haystack")

$$\sum_t \max_{\text{shift}} \langle x | h_t \rangle \qquad \min_{\lambda_i} \left\langle x - \sum_i \lambda_i h_i \middle| x - \sum_i \lambda_i h_i \right\rangle$$

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- Annealing (used by sampling algorithms such as parallel tempering)
    - This de-emphasises variations in the simplest way possible, by a rescaling of the function
    - Still retains strong secondaries relative to primary peak ("needle in a sparser haystack")
- These approaches make sense, but none really cuts to the chase

$$\sum_t \max_{\text{shift}} \langle x | h_t \rangle \qquad \min_{\lambda_i} \left\langle x - \sum_i \lambda_i h_i \middle| x - \sum_i \lambda_i h_i \right\rangle \qquad \frac{1}{T} \ln L$$

A slightly heretical proposal

- Let's stop looking for needles in haystacks — it's suboptimal
- At the same time, there's no way to put the needle on a hill
- What's the next best thing? A needle in an open field
    - Coherent search function with exponential suppression of all unwanted variations
    - Must preserve wanted variations: A strong peak at the signal parameters

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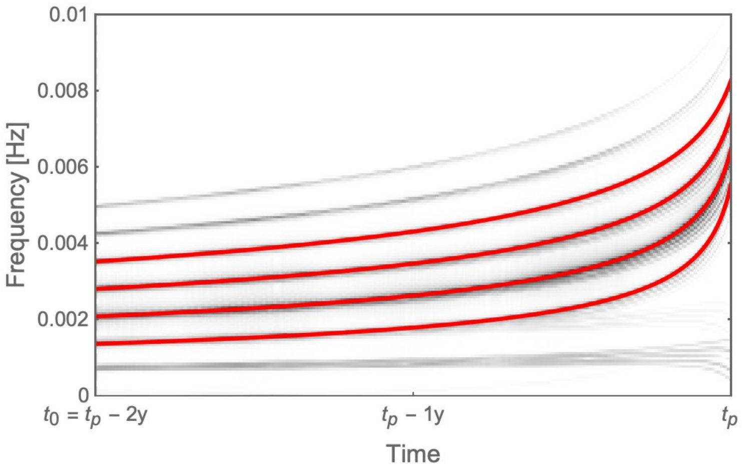
- What’s the next best thing? A needle in an open field
  - Coherent search function with exponential suppression of all unwanted variations
  - Must preserve wanted variations: A strong peak at the signal parameters
- Let’s do even better: We want a simple, natural & well-defined function
  - Analytically tractable, no tuning parameters & fully informed by template model
  - Can characterise its properties as a statistic (for detection)
  - Can characterise its relationship to the Bayesian likelihood (for inference)
  - A one-stop solution to GW signal analysis! At least conceptually...

The one-stop function

- The function is defined for a general mode decomposition

$$h = \sum_m h_m + \epsilon, \quad \langle h_m | h_m \rangle > 1, \quad \langle h_m | h_{m'} \rangle \ll 1, \quad \langle \epsilon | \epsilon \rangle \ll 1$$

- For EMRIs: Frequency harmonics
- For SOBHs: Frequency harmonics
    - Need to extract from current PN models
- For GBs: Time-domain partition
    - Could help with source separation
- For MBHs: Frequency-domain partition
    - Low eccentricity & precession
    - Little power in higher spherical modes



The one-stop function

- It combines information from two standard statistics: X and χ^2

$$f(\boldsymbol{\theta}|x) := X(\boldsymbol{\theta}|x) \exp\left(-\frac{1}{2}\beta(\boldsymbol{\theta})\chi^2(\boldsymbol{\theta}|x)\right)$$

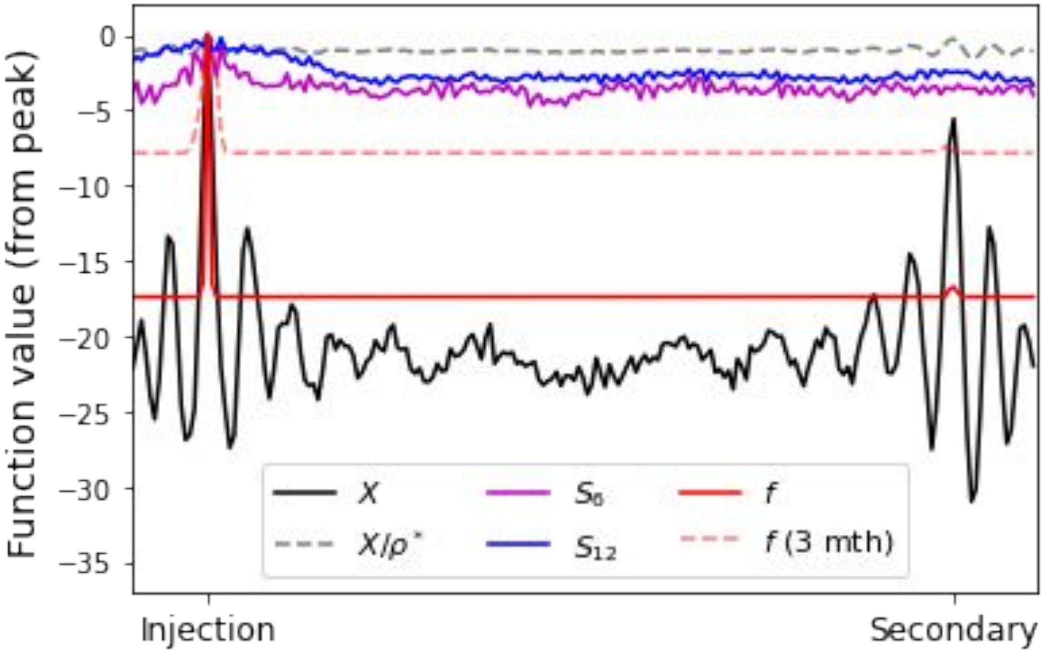
- It is calibrated for EMRIs through the definition of β (not a statistic)

$$\beta(\boldsymbol{\theta}) := \frac{2\ln\left(\alpha(\boldsymbol{\theta})\rho(\boldsymbol{\theta})\right)}{(1-\alpha(\boldsymbol{\theta})^2)\rho(\boldsymbol{\theta})^2}$$

- This definition reduces strongest secondaries to level of noise variations
    - And actual noise variations to effectively zero
- Useful definitions of β for other source types: TBD
    - Needs problem statement first (e.g. source separation for GBs, fast searches for MBHs)

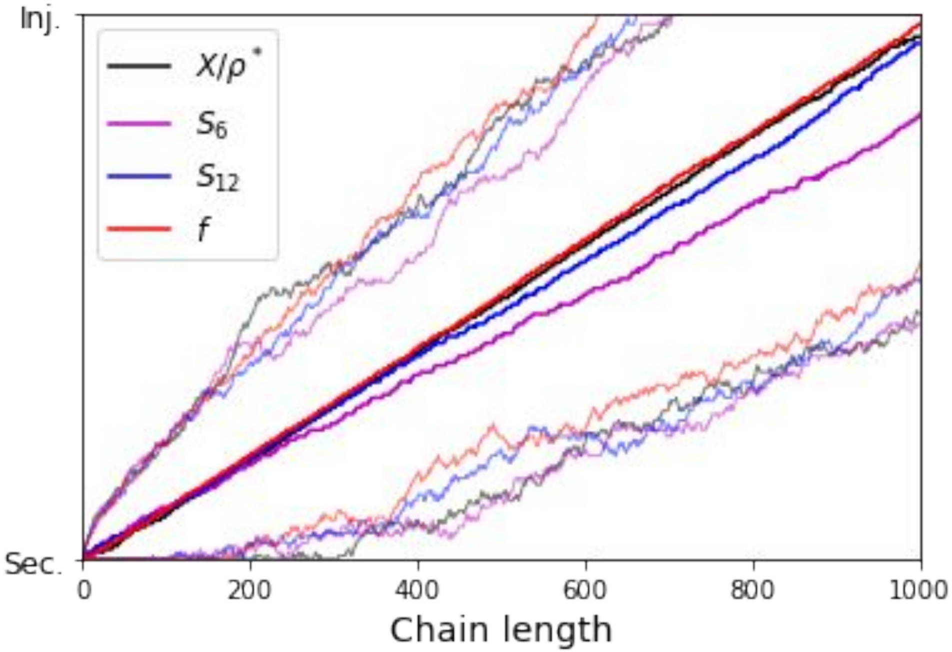
The one-stop function

- The function fulfils its main purpose of exponential suppression



The one-stop function

- The virtually flat baseline of *f* makes it the most intrinsically traversable

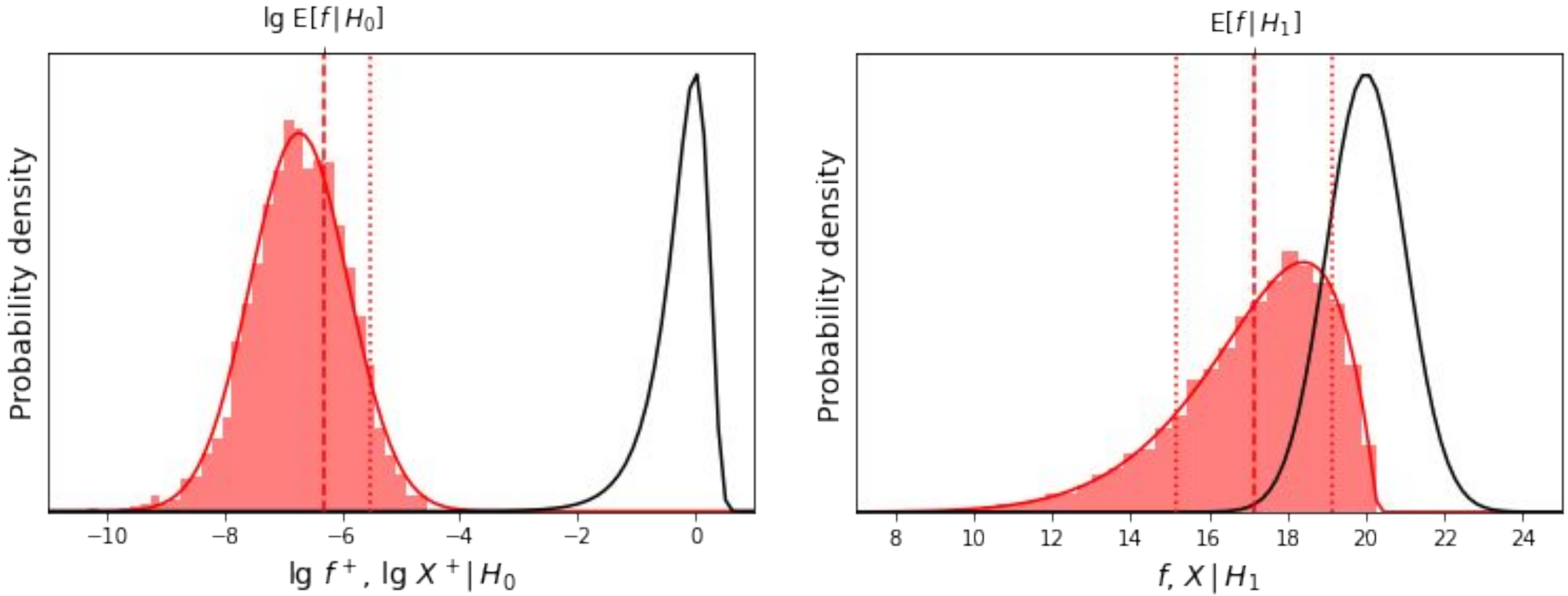


Summary

- Claim: Existing coherent statistics in GW search have suboptimal efficiency
- This is solely due to uncontrolled variations in the search surface
- Proposal: A coherent statistic that exponentially suppresses such variations
- The new function can be calibrated for EMRIs in a natural & well-defined way

The one-stop function

- The function is itself a viable detection statistic



The one-stop function

- It works well for prior localisation or even importance sampling of the posterior
  - The profile of *f* is generally broader than that of ln *L* for SNR > 5