Laboratoire de Physique Théorique-Orsay
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Lepton Flavour Violation from Effective Operators

GDR Neutrinos à l'X
École Polytechnique, 29-30 April 2010

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The fate of lepton flavour numbers

☞ Neutrinos change flavours after propagating a finite distance

- Atmospheric Experiments: $\nu_\mu \to \nu_\tau$, $\bar{\nu}_\mu \to \bar{\nu}_\tau$
- Solar Experiments: $\nu_e \to \nu_{\mu,\tau}$
- Reactor Experiments: $\bar{\nu}_e \to \bar{\nu}_{\text{other}}$
- Accelerator Experiments: $\nu_\mu \to \nu_{\text{other}}$

☞ INDISPUTABLE: Neutral Lepton FLAVOUR Number is thus VIOLATED
Charged Lepton Flavour Number should also be violated in SM extensions

➞ **ν-SM**

☞ ν-SM will allow for many new phenomena

- LFV: $\ell_i \to \ell_j\ \ell_k\ \ell_l$, $\ell_i \to \ell_j \gamma$, ...
- Contributions to $g-2$
- Lepton EDMs
- Collider searches for new, heavy states ...

Determination of ν-SM model: requires combinations of different observables

From Neutrino Oscillations to ν-SM

☞ (1) Neutrino Oscillations ➔ Neutrino are massive and mix

Oscillations	$m_\nu > \sqrt{\Delta m^2_{\rm atm}} \sim 5 \times 10^{-2}$ eV
Cosmology	$\sum_i m_{\nu_i} <\sim 1$ eV
Tritium	$m_{\nu_e} \lesssim 2.2$ eV

☞ (2) A possibly related observation: baryon asymmetry of the Universe η_B

Primordial abundances of light elements + CMB Anisotropies

$\propto \eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10}$

Need a dynamical mechanism to generate BAU

☞ AIM: find models that are simultaneously consistent with (1) & (2)

➔ BAU from Leptogenesis ➔ complex Yukawa couplings (➔ EDMs)

▶ Favourite option: new physics at high scale M

▶ Heavy fields and their dynamics manifest in the low energy effective theory (SM) via higher dimensional operators $\delta\mathcal{L}^{d=5}$, $\delta\mathcal{L}^{d=6}$, ...

$m_\nu \neq 0 \Rightarrow$ New Physics

Standard Model

- ν_L and no ν_R $\Longrightarrow$ No Dirac mass term: $\mathcal{L}_{m_D} = m_D\left(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L\right)$

- No Higgs triplet $\Longrightarrow$ No Majorana mass term: $\mathcal{L}_{m_M} = \frac{1}{2} M \bar{\nu_L^c} \nu_L + h.c.$

- Lepton symmetry is accidental $\Longrightarrow$ Broken by non-renormalisable operators dim$=$ 5, 6 ...

SM $\equiv$ Effective theory of a larger one valid at a scale Λ

↓

$$\delta\mathcal{L}^{d=5} = c^{d=5}\mathcal{O}^{d=5}, \quad \mathcal{O}^{d=5} = \frac{1}{\Lambda}\left\{\left(\phi\ell\right)^{\mathrm{T}}\left(\phi\ell\right) + h.c.\right\} \xrightarrow{\langle\phi\rangle = v} m_\nu \sim v^2/\Lambda$$

$m_\nu \sim \sqrt{\Delta m^2_{\mathrm{atm}}} \sim \sqrt{2 \times 10^{-3}\mathrm{eV}^2} \Rightarrow \Lambda \sim 10^{15}$ GeV (Remarkably near Λ_{GUT} !)

$m_\nu \neq 0$: Beyond the Standard Model

Typically 3 possible ways to generate $m_\nu \neq 0$: can be embedded in SUSY, ExtraDims, GUT...

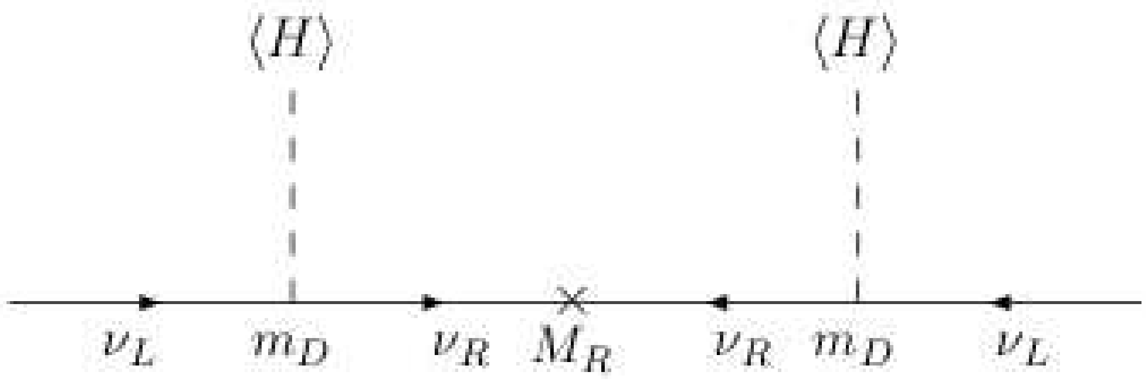
- Seesaw mechanism
  1. type I with RH neutrino exchange
  2. type II with scalar triplet exchange
  3. type III with fermionic triplet exchange
- Radiative corrections ➛ MSSM or various extensions $+\not{R}_p$, Zee model, $\cdots$
- Extra dimensions ➛ alternative to the seesaw

Seesaw Type I, SM + ν_R

$$\mathcal{L} = \mathcal{L}_{SM} + \lambda^{\nu}_{Jk} \bar{L}_k \nu_{R_J} H - \frac{1}{2} \bar{\nu}_{R_J} M_{R_J} \nu^c_{R_J} + \lambda_\alpha H^c \bar{e}_{R_\alpha} \ell_\alpha, \quad m_D = \lambda^\nu \, v$$

Majorana Eigenstates (3×3)

$$\begin{cases} \nu = L + L^c = \nu^c & \rightarrow \quad \tilde{m}_L \sim -m_D \frac{1}{M_R} m_D^T \\ N = R + R^c = N^c & \rightarrow \quad \tilde{M}_R \sim M_R \end{cases}$$



$$\begin{cases} m_D \sim 200 \text{ GeV} \\ M_R \sim 10^{15} \text{GeV} \end{cases} \rightarrow m_\nu \propto \sqrt{\Delta m^2_{\text{atm}}} \sim (10^{-2} \; - \; 10^{-1}) \text{ eV} \quad \leftarrow \quad \lambda^\nu \sim \mathcal{O}(1)$$

$$\lambda^\nu \sim h_e \quad \leftarrow \quad M_R \sim \text{few} \;\; \text{TeV}$$

Effective approach

- Neutrino masses require new heavy fields (or extremely tiny Y_ν)
- Effects at low energy: **effective theorie approach**

  Effective operators obtained when expanding the heavy field propagators in $\frac{1}{M}$

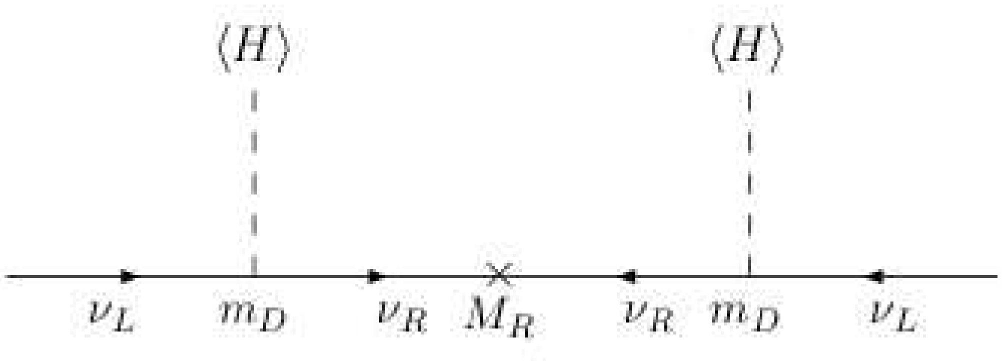
  ☞ heavy fermion: $\frac{1}{\not{D}-M} \sim -\frac{1}{M} - \frac{1}{M}\not{D}\frac{1}{M} + \ldots$

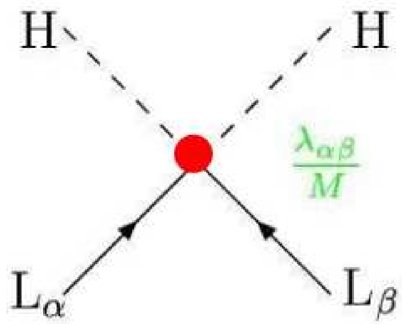
  ☞ heavy scalar : $\frac{1}{D^2-M^2} \sim -\frac{1}{M^2} - \frac{D^2}{M^4} + \ \ldots$

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + c^{d=5}\mathcal{O}^{d=5} + c^{d=6}\mathcal{O}^{d=6} + \cdots$$

Dimension 5

$$\delta\mathcal{L}^{d=5} = \frac{1}{2}\, c^{d=5}_{\alpha\beta}\, \left(\overline{\ell^c_{L\alpha}}\tilde{\phi}^*\right)\left(\tilde{\phi}^\dagger\, \ell_{L\beta}\right) + \text{h.c.}\,,$$





$$m_\nu = \frac{v^2 Y^\dagger Y}{M_R}$$

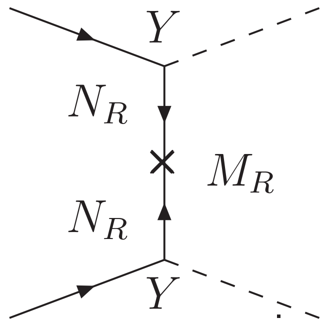
$$m_\nu = c^{d=5} v^2, \quad c^{d=5} = \frac{Y^\dagger Y}{M_R}$$

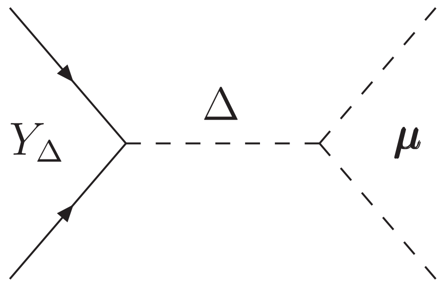
- ☞ $Y \sim 1$ ➔ $M_R \sim M_{\rm GUT}$
- ☞ $Y \sim 10^{-6}$ ➔ $M_R \sim \text{TeV}$

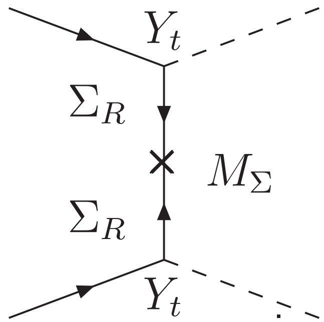
$d = 5$ Operator violates lepton number L ➔ Majorana neutrino mas

- $\mathcal{O}^{d=5}$ **is common to all models of Majorana neutrinos**

3 ways to generate tree level $\mathcal{O}^{d=5}$: Seesaw I, II, III







type I (fermionic singlet)

$$\boldsymbol{m_\nu} = -\frac{1}{2}v^2\, Y_N^T \frac{1}{M_N} Y_N$$

Minkowski, Gell-Man,

Ramond, Slansky

Yanagida, Glashow

Mohapatra, Senjanovic

type II (scalar triplet)

$$\boldsymbol{m_\nu} = -2v^2 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$$

Magg, Wetterich,

Nussinov

Mohapatra, Senjanovic

Schechter, Valle

Ma, Sarkar

type III (fermionic triplet)

$$\boldsymbol{m_\nu} = -\frac{v^2}{2}\, Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$$

Ma, Hambye et al.

Bajc, Senjanovic, Lin

A.A., Biggio, Bonnet, Gavela,

Notari, Strumia, Papucci, Dorsner

Fileviez-Perez, Foot, Lew...

Distinguishing among the 3 types of seesaw

- The $\mathcal{O}^{d=5}$ operator is the same for all extensions of SM responsible for Majorana neutrino masses and mixing

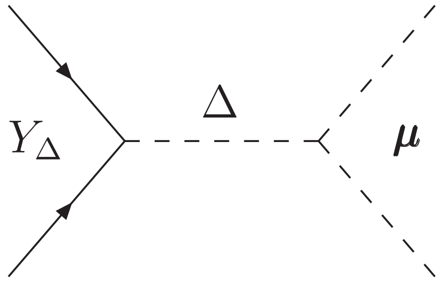
- To distinguish the several seesaw mechanisms, either
  - ☞ produce the heavy states
  - ☞ or use $\mathcal{O}^{d=6}$ operators (and study their phenomenological impact)

Dimension 6 operators

Model	Effective Lagrangian $\mathcal{L}_{eff} = c_i \mathcal{O}_i$		
	$c^{d=5}$	$c_i^{d=6}$	$\mathcal{O}_i^{d=6}$
Fermionic Singlet	$Y_N^T \frac{1}{M_N} Y_N$	$\left(Y_N^\dagger \frac{1}{M_N^\dagger} \frac{1}{M_N} Y_N\right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \widetilde{\phi}\right) i\partial\!\!\!/ \left(\widetilde{\phi}^\dagger \ell_{L\beta}\right)$ LFV
Scalar Triplet	$4 Y_\Delta \frac{\mu_\Delta}{M_\Delta^2}$	$\frac{1}{M_\Delta^2} Y_{\Delta\alpha\beta} Y^\dagger_{\Delta\gamma\delta}$	$\left(\overline{\widetilde{\ell_{L\alpha}}} \vec{\tau} \ell_{L\beta}\right)\left(\overline{\ell_{L\gamma}} \vec{\tau} \widetilde{\ell_{L\delta}}\right)$ LFV
		$\frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	$\left(\phi^\dagger \vec{\tau} \widetilde{\phi}\right)\left(\overleftarrow{D_\mu} \overrightarrow{D^\mu}\right)\left(\widetilde{\phi}^\dagger \vec{\tau} \phi\right)$ Higgs-Gauge
		$-2\left(\lambda_3 + \lambda_5\right) \frac{\lvert\mu_\Delta\rvert^2}{M_\Delta^4}$	$\left(\phi^\dagger \phi\right)^3$ Higgs
Fermionic Triplet	$Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma$	$\left(Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger} \frac{1}{M_\Sigma} Y_\Sigma\right)_{\alpha\beta}$	$\left(\overline{\ell_{L\alpha}} \vec{\tau} \widetilde{\phi}\right) iD\!\!\!\!/ \left(\widetilde{\phi}^\dagger \vec{\tau} \ell_{L\beta}\right)$ LFV

Fermions: if $Y \sim \mathcal{O}(1)$, $c^{d=6} \sim (c^{d=5})^2$ and the smallness m_ν would preclude observable effects from $\mathcal{O}_i^{d=6}$. Not the case for scalars!

Scalar triplet (type II)



$$\Delta = \begin{pmatrix} \Delta^{++} \\ \Delta^{+} \\ \Delta^{0} \end{pmatrix} \sim (1,3,2) \qquad L_\Delta = -2$$

Yukawa couplings:

$$Y_{\Delta ij} \overline{(l_L)}^c_{ia} (l_L)_{jb} (i\tau_2\tau_\alpha)_{ab} \Delta^\alpha + h.c.$$

Scalar coupling:

$$\mu\ \phi^t_a \phi_b (i\tau_2\tau_\alpha) \left(\Delta^\dagger\right)^\alpha + h.c.$$

$$-M^2_\Delta \Delta^\dagger\Delta - \frac{1}{2}\lambda_2 \left(\Delta^\dagger\Delta\right)^2$$

$$-\lambda_3 \left(\phi^\dagger\phi\right)\left(\Delta^\dagger\Delta\right) + \ldots$$

d=5 Operator (Mass) $\quad m_\nu = v^2\, Y_\Delta\, \frac{\mu}{M^2_\Delta}$ ➞ 2 different scales μ, M_Δ

possible to have $Y_\Delta \sim \mathcal{O}(1)$ $\quad M_\Delta \sim 1$ **TeV** ($\mu \sim 100$ **eV**)

Low energy effects of dimension 6 operators:

$$\frac{1}{2M_\Delta^2} Y_{\Delta ij} Y^\dagger_{\Delta kl} \left(\overline{l_{Li}} \gamma^\mu l_{Lk}\right) \left(\overline{l_{Lj}} \gamma_\mu l_{Ll}\right)$$ → LFV, $g-2$, EDMs

constraints not suppressed by μ

$$\left.\begin{array}{l} -2\frac{\mu^2}{M_\Delta^4} \partial_\mu \left(\phi^\dagger \phi\right) \partial^\mu \left(\phi^\dagger \phi\right) \\ 2\lambda_3 \frac{\mu^2}{M_\Delta^4} \left(\phi^\dagger \phi\right)^3 \\ 4\frac{\mu^2}{M_\Delta^4} \left[\phi^\dagger D_\mu \phi\right]^\dagger \left[\phi^\dagger D_\mu \phi\right] \end{array}\right\} \rightarrow$$

EW precision data,
couplings to gauge bosons

$$-2\frac{\mu^2}{M_\Delta^4} \left(\phi^\dagger \phi\right) \left\{Y_e \bar{l} e_R \phi + Y_d \bar{q} d \phi - Y_u \bar{q} i \tau_2 u \phi + h.c.\right\} \rightarrow$$ top physics...

Direct LFV: $d = 5$ operator supressed by small scale μ;
$d = 6$ operator not supressed

Disentangling seesaws

❑ Scalar seesaw (Type II): Lepton Flavour Violation

$\mu \to eee$, $\tau \to \ell\ell\ell$, $\mu \to e\gamma$, $\tau \to e\gamma$

➡ bounds on various combinations of $\frac{Y_\Delta}{M_\Delta}$

❑ Fermionic seesaw (Type I and III): non-unitarity

$$|NN^\dagger - 1|_{\alpha\beta} = |\epsilon^\Sigma| = \frac{v^2}{2}\,|c^{d=6}|_{\alpha\beta} = \frac{v^2}{2}\,|Y_\Sigma^\dagger \frac{1}{M_\Sigma^\dagger}\frac{1}{M_\Sigma} Y_\Sigma|_{\alpha\beta}$$

- Type I: also Lepton Flavour Violation
  $\mu \to e\gamma$, $\tau \to e\gamma$, W, Z (semi) leptonic decays
- Type III: same as Type I + tree level Lepton Flavour Violation
  ($\mu \to eee$, $\tau \to \ell\ell\ell$) at tree level

Bounds on Type II seesaw

$\mu \to eee$ and $\tau \to 3l$

$$\Gamma(\mu^- \to e^+ e^- e^-) = \frac{m_\mu^5}{192\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\mu e}}|^2 |Y_{\Delta_{ee}}|^2$$

which gives

$$\mathrm{BR}(\mu^- \to e^+ e^- e^-) \simeq \frac{\Gamma(\mu^- \to e^+ e^- e^-)}{\Gamma(\mu^- \to e^- \nu_\mu \overline{\nu}_e)} = \frac{1}{M_\Delta^4 G_F^2} |Y_{\Delta_{e\mu}}|^2 |Y_{\Delta_{ee}}|^2$$

For τ decays:

$$\Gamma(\tau^- \to l_i^+ l_j^- l_j^-) = \frac{m_\tau^5}{192\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\tau i}}|^2 |Y_{\Delta_{jj}}|^2 \quad (\text{for any } i \text{ and } j)$$

$$\Gamma(\tau^- \to l_i^+ l_j^- l_k^-) = \frac{m_\tau^5}{96\pi^3} \frac{1}{M_\Delta^4} |Y_{\Delta_{\tau i}}|^2 |Y_{\Delta_{jk}}|^2 \quad (\text{for any } i, j, k \text{ with } j \neq k)$$

Using experimental bounds on BRs constrain the Yukawa couplings

Process	Constraint on	Bound $\left(\times\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2\right)$
$\mu^- \to e^+e^-e^-$	$\lvert Y_{\Delta\,\mu e}\rvert\lvert Y_{\Delta\,ee}\rvert$	$< 1.2\times 10^{-5}$
$\tau^- \to e^+e^-e^-$	$\lvert Y_{\Delta\,\tau e}\rvert\lvert Y_{\Delta\,ee}\rvert$	$< 1.3\times 10^{-2}$
$\tau^- \to \mu^+\mu^-\mu^-$	$\lvert Y_{\Delta\,\tau\mu}\rvert\lvert Y_{\Delta\,\mu\mu}\rvert$	$< 1.2\times 10^{-2}$
$\tau^- \to \mu^+e^-e^-$	$\lvert Y_{\Delta\,\tau\mu}\rvert\lvert Y_{\Delta\,ee}\rvert$	$< 9.3\times 10^{-3}$
$\tau^- \to e^+\mu^-\mu^-$	$\lvert Y_{\Delta\,\tau e}\rvert\lvert Y_{\Delta\,\mu\mu}\rvert$	$< 1.0\times 10^{-2}$
$\tau^- \to \mu^+\mu^-e^-$	$\lvert Y_{\Delta\,\tau\mu}\rvert\lvert Y_{\Delta\,\mu e}\rvert$	$< 1.8\times 10^{-2}$
$\tau^- \to e^+e^-\mu^-$	$\lvert Y_{\Delta\,\tau e}\rvert\lvert Y_{\Delta\,\mu e}\rvert$	$< 1.7\times 10^{-2}$

Non-observation of these LFV transitions ➞ bounds on M_Δ

Most stringent decay: $\mu \to eee$ ➞ $M_\Delta \geq 294$ TeV, for $Y_\Delta \sim \mathcal{O}(1)$

EW Prec. Measurements + EDMs ... further constrain Yukawa couplings

$\mu \to e\gamma$ and $\tau \to l\gamma$

Radiative processes due to exchange of Δ^{++} and Δ^{+} between charged leptons

$$\mathrm{BR}(l_1 \to l_2\gamma) = \frac{\alpha}{48\pi}\frac{25}{16}\frac{\left|\sum_l Y_{\Delta\, l l_1}{}^{\dagger} Y_{\Delta\, l_2 l}\right|^2}{G_F^2 M_\Delta^4}\mathrm{BR}(l_1 \to e\nu_1\bar{\nu}_e)$$

Process	Constraint on	Bound $\left(\times\left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2\right)$
$\mu \to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y_{\Delta}{}^{\dagger}_{l\mu} Y_{\Delta\, el}\rvert$	$< 4.7 \times 10^{-3}$
$\tau \to e\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y_{\Delta}{}^{\dagger}_{l\tau} Y_{\Delta\, el}\rvert$	< 1.05
$\tau \to \mu\gamma$	$\lvert\Sigma_{l=e,\mu,\tau} Y_{\Delta}{}^{\dagger}_{l\tau} Y_{\Delta\, \mu l}\rvert$	$< 8.4 \times 10^{-1}$

Bounds on combinations of Y_Δ

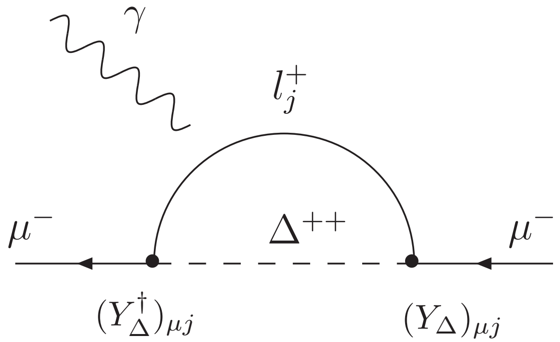
Combined bounds		
Process	Yukawa	Bound $\left(\times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)^2\right)$
$\mu \to e\gamma$	$\left\| {Y_\Delta}^\dagger_{\mu\mu} {Y_\Delta}_{\mu e} + {Y_\Delta}^\dagger_{\tau\mu} {Y_\Delta}_{\tau e} \right\|$	$< 4.7 \times 10^{-3}$
$\tau \to e\gamma$	$\left\| {Y_\Delta}^\dagger_{\tau\tau} {Y_\Delta}_{\tau e} \right\|$	< 1.05
$\tau \to \mu\gamma$	$\left\| {Y_\Delta}^\dagger_{\tau\tau} {Y_\Delta}_{\tau\mu} \right\|$	$< 8.4 \times 10^{-1}$

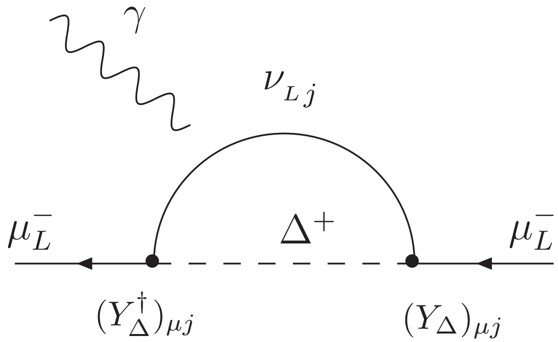
Other bounds on Y_Δ

- Stringent bounds from M_W ➛ $|{Y_\Delta}_{\mu e}|^2 < 7.3 \times 10^{-2} \times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)$
- Additional bounds from Bhabha scattering, **muon anomalous magnetic moment**, ...

Muon Anomalous Magnetic Moment, $(g-2)_\mu$

Shift in the Anomalous Magnetic Moment induced by the exchange of $\Delta^{\pm\pm}$ and $\Delta^{\pm}$





$\delta(a_\mu)\big\|_{\Delta^{\pm\pm}} = -\frac{m_\mu^2}{3\pi M_\Delta^2} \sum_{j=e,\mu,\tau} \left\|Y_{\Delta_{\mu j}}\right\|^2$	$\delta(a_\mu)\big\|_{\Delta^{\pm}} = -\frac{m_\mu^2}{24\pi M_\Delta^2} \sum_{j=e,\mu,\tau} \left\|Y_{\Delta_{\mu j}}\right\|^2$

Scalar triplet contribution: opposite sign with respect to the observed deviation

For instance $\delta(a_\mu) < 20 \times 10^{-10}$ ➞ $\sum_{j=e,\mu,\tau} \left|Y_{\Delta_{\mu j}}\right|^2 < 1.9 \times (M_\Delta / 1 \text{ TeV})^2$

Fermionic singlet and triplet contributions: C. Biggio, 0806.2558; W. Chao, 0806.0889

always too small to account for observed deviation

Scalar triplet: summary

- $Y_\Delta \lesssim 10^{-1} \times \left(\frac{M_\Delta}{1\,\mathrm{TeV}}\right)$ or stronger

- Observation of $\mu \to e\gamma$ at MEG (sensitivity of 10^{-13}?)
  - for $Y_\Delta \sim \mathcal{O}(1)$ ➞ $15\ \mathrm{TeV} < M_\Delta < 50\ \mathrm{TeV}$
  - for $Y_\Delta \sim \mathcal{O}(10^{-2})$ ➞ $0.15\ \mathrm{TeV} < M_\Delta < 0.50\ \mathrm{TeV}$

- If M_Δ turns out to be as low as $\mathcal{O}(\mathrm{TeV})$, possibility of clean signals in hadronic accelerators (Tevatron, LHC)
  - Production of Δ^{++} and Δ^{--}, decaying into pairs of same-sign leptons ➞ striking signals, free from SM backgrounds
  - If Δ observed, must verify whether a scalar-mediated seesaw is at work ➞ observe in addition at least three LFV processes (to measure and disentangle the individual $Y_{\Delta ij}$ couplings)

Type III: bounds on Yukawa couplings

- Lepton Flavour Violation at tree level $\mu \to eee$, $\tau \to 3l$
- Lepton Flavour Violation at loop level $\mu \to e\gamma$, $\tau \to e\gamma$
- Z decays ($Z \to e\mu$, $Z \to e\tau$, $Z \to \mu\tau$)
- W decays, Universality tests, Invisible Z width...
- $(g-2)_\mu$ and lepton EDMs

➜ For $M \sim$ 1 TeV, $|Y| < 10^{-2}$

$\ell \to \ell'\gamma$ versus $\ell \to 3\ell'$ **Generic results:**

☞ In type I seesaw (fermionic singlet): $\Gamma(\ell \to \gamma\ell') >> \Gamma(\ell \to 3\ell')$
☞ In type III seesaw (fermionic triplet): $\Gamma(\ell \to \gamma\ell') << \Gamma(\ell \to 3\ell')$

➜ a measurement of a $\Gamma(\ell \to 3\ell')$ in the next generation of expt. would rule out TeV Fermionic triplet seesaw as its origin

Conclusions

- 3 different seesaw mechanisms induce **different dimension 6 operators**
  ➡ Thus **different Lepton Flavour phenomenology**
- Combination of different observables allows to **discriminate among models** of Majorana ν
- Rich phenomenology associated to TeV seesaw: LFV, $(g-2)_\mu$ and lepton EDMs ...

Detailed analysis - see
JHEP 0712:061, 2007; Phys.Rev.D78:033007, 2008