Gravitational Wave Cosmology : Constraints on the Hubble Constant using Dark-Sirens

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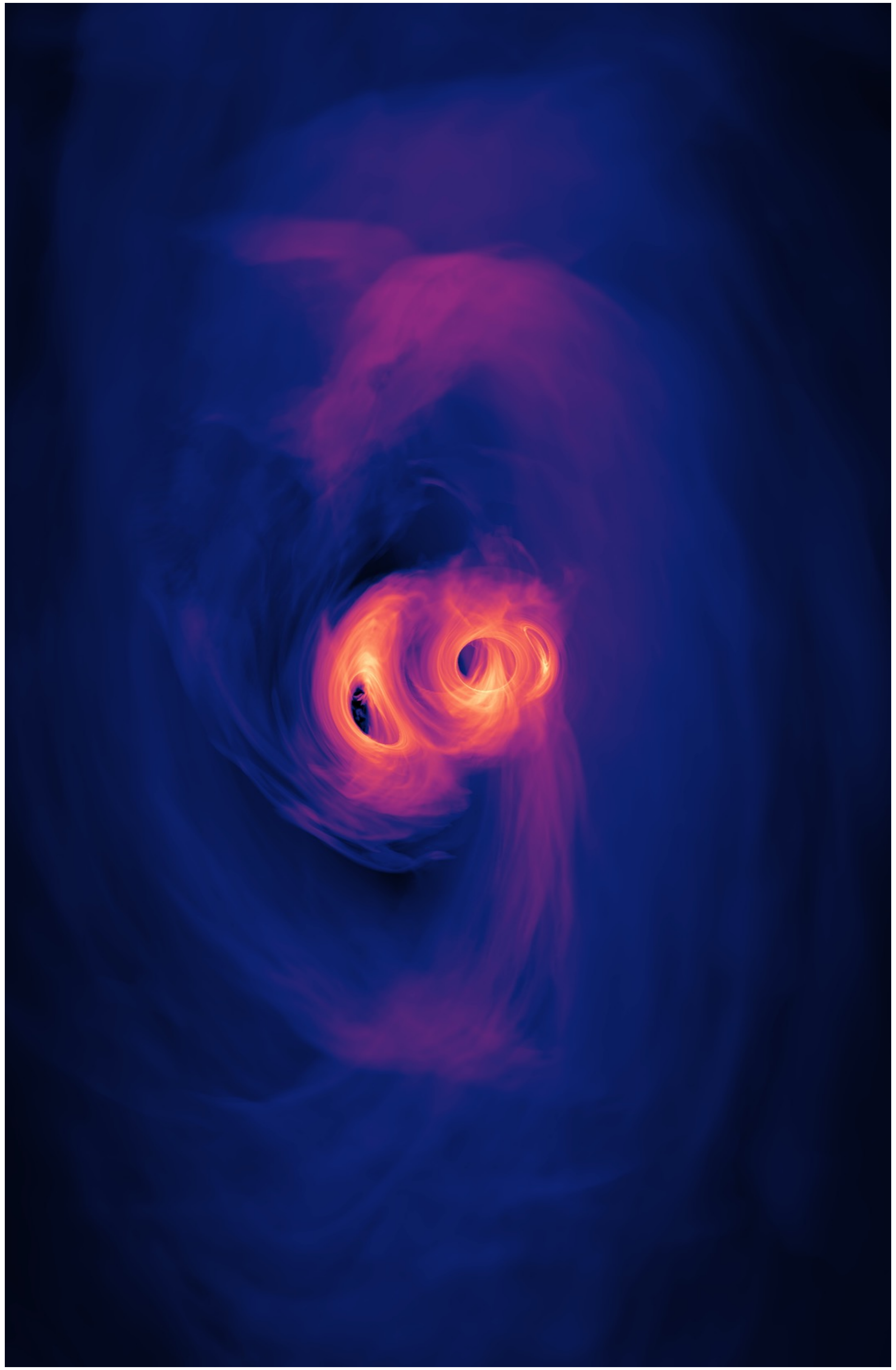
The LIGO-Virgo-KAGRA scientific collaboration

Outline

- Introduction
  - Gravitational wave
  - Cosmology
- Dark sirens
  - Cosmology with GWs
  - IcaroGW
  - Black hole spins
- Results



An introduction for beginners

$$\frac{N_{exp}}{N} = \int p_{det}(m_{1,2,s}^{new}, z_{new}, \chi_{eff}^{new}, \chi_p^{new}, H_0^{new})\pi(m_{1,2,s}^{new}, z_{new}, \chi_{eff}^{new}, \chi_p^{new}|\Lambda^{new})dm_{1,2,s}^{new}dz_{new}d\chi_{eff,p}^{new}$$

$$\frac{N_{exp}}{N} = \frac{1}{N_{sim}}\sum_i^{N_{det}} \frac{\pi_{new}(m_{1,2,s}^{new}|\Lambda^{new})\pi_{new}(z^{new}|\Lambda^{new})}{\frac{dz^{inj}}{dd_L}\frac{dd^L}{dz_{new}}(1+z^{inj})^{-2}(1+z^{new})^2 p_{inj}(m_{1,2,s}^{inj}(m_{1,2,s}^{new}, z^{new}), z^{new}, H_0^{new}, H_0^{inj}))}$$

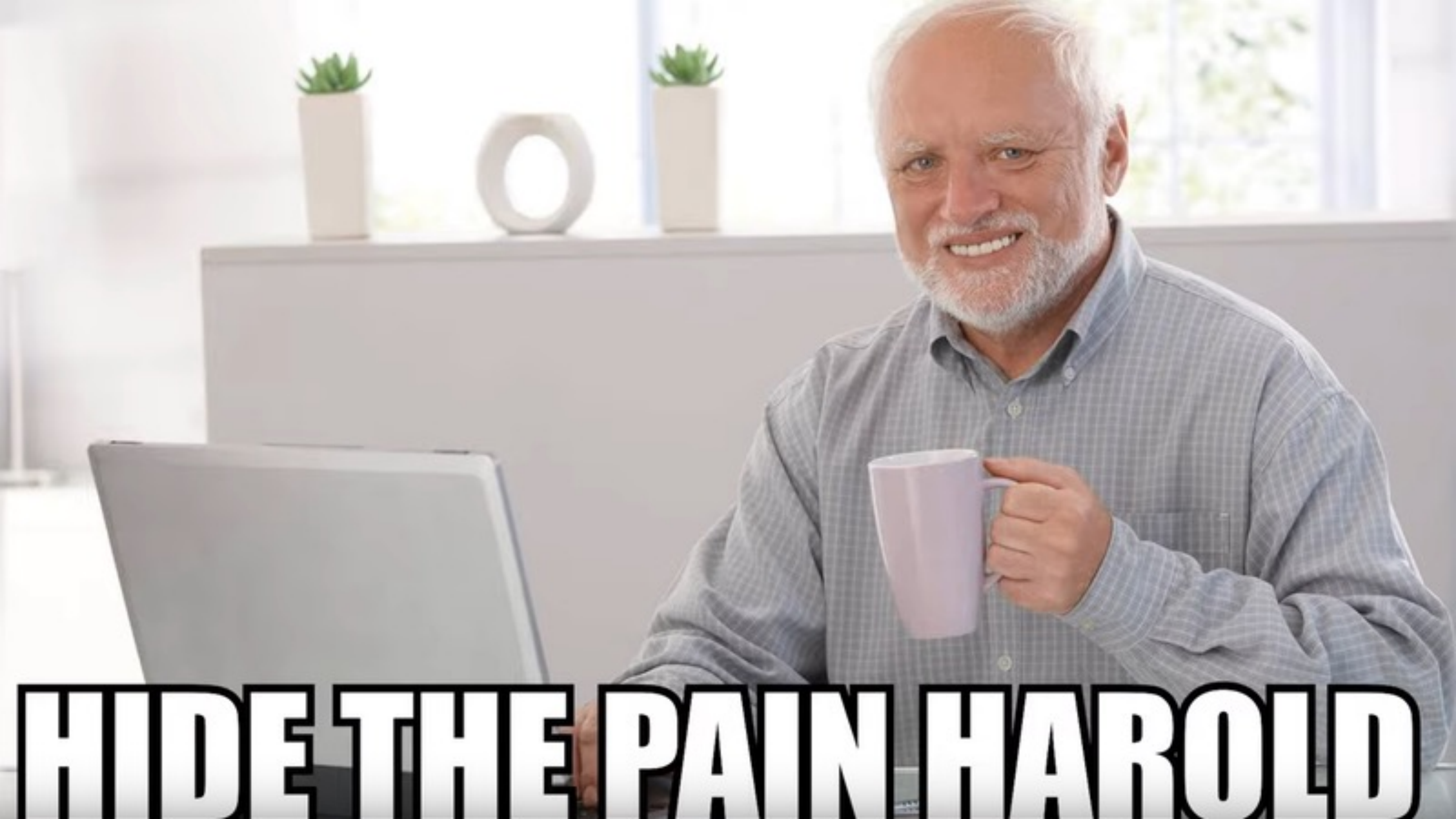
$$p(x_i|\Lambda) = \frac{1}{N_s}\sum_i^{N_s} \frac{\pi(m_{1,s}^i, m_{2,s}^i|\Lambda)\pi(z^i|\Lambda)\pi(\chi_{eff}^i, \chi_p^i|\Lambda)}{d_L^2(1+z)^2\frac{\partial d_L}{\partial z}\left|\frac{\partial(\chi_1, \chi_2, cos(\theta_1), cos(\theta_2), \phi_1, \phi_2)}{\partial(\chi_{eff}, \chi_p, cos(\theta_1), cos(\theta_2), \phi_1, \phi_2)}\right|} \times \frac{\pi_{new}(\chi_{eff}^{new}, \chi_p^{new}|\Lambda^{new})}{p_{new}(\chi_{eff}^{new})p_{new}(\chi_p^{new})}$$

$$p(x_i|\Lambda) = \int \frac{p(\theta_i|x_i, \Lambda)p(x_i)}{\pi_0(\theta|\Lambda)}\pi(\theta|\Lambda)d\theta$$

$$\Pi_{pop}(\chi_{eff}|\Lambda_{\chi_{eff}}, m_1, z) = \mathcal{N}(\chi_{eff}; \mu_\chi, \sigma_\chi)$$

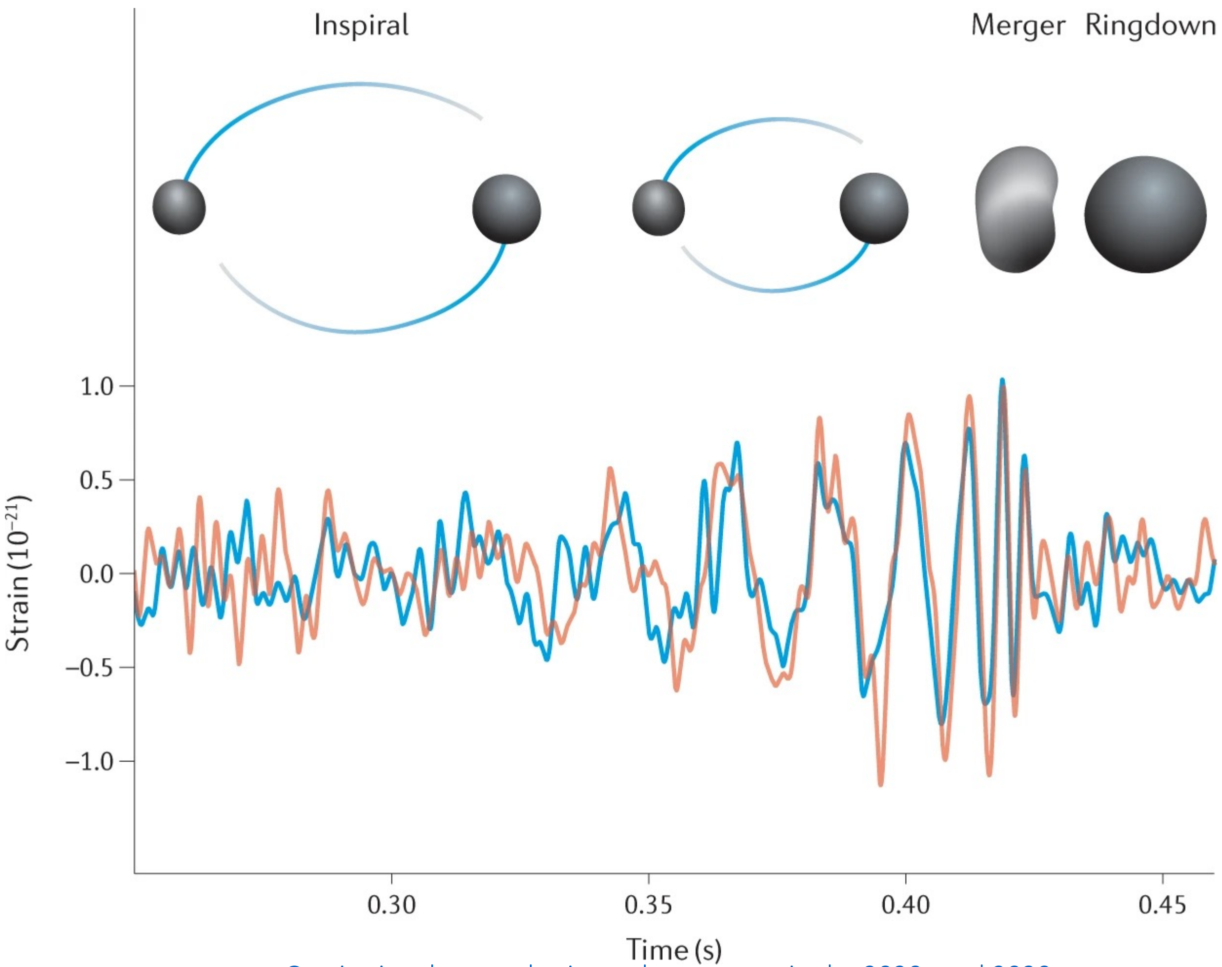
$$\mu_\chi(z, m_1) = \mu_0 + \delta\mu_z(z-0.5) + \delta\mu_{m_1}(\frac{m_1}{10M_{sol}} - 1)$$

$$log\sigma_\chi(z, m_1) = log\sigma_0 + \delta log\sigma_z(z-0.5) + \delta log\sigma_{m_1}(\frac{m_1}{10M_{sol}} - 1)$$


HIDE THE PAIN HAROLD

Let’s start again folks!

Gravitational merger



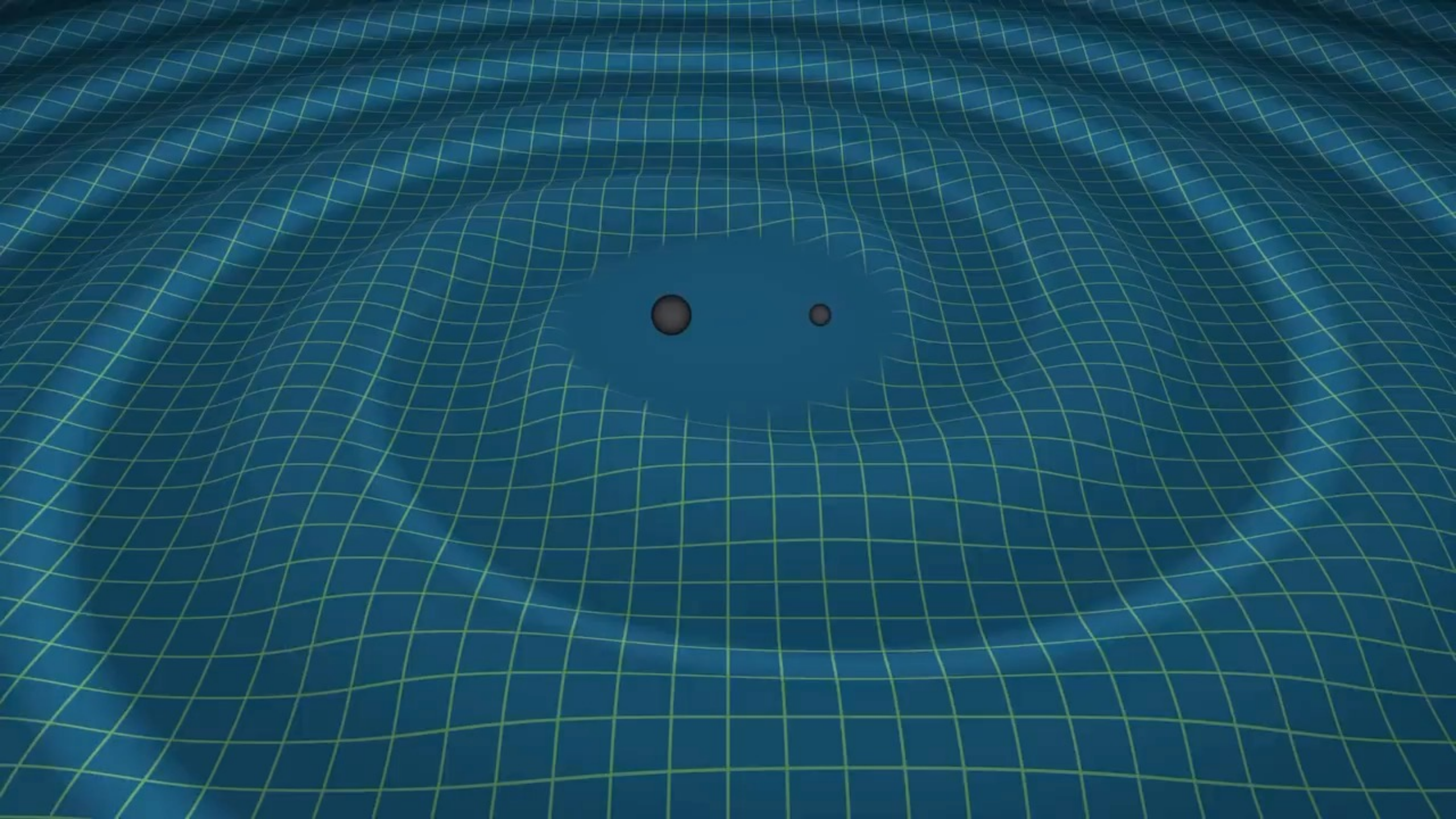
[Gravitational-wave physics and astronomy in the 2020s and 2030s]

- Main source of GWs are compact binary mergers
- Merger has 3 phases
- Several properties about the source can be deduced directly from the GW signal :

Luminosity distance d_L

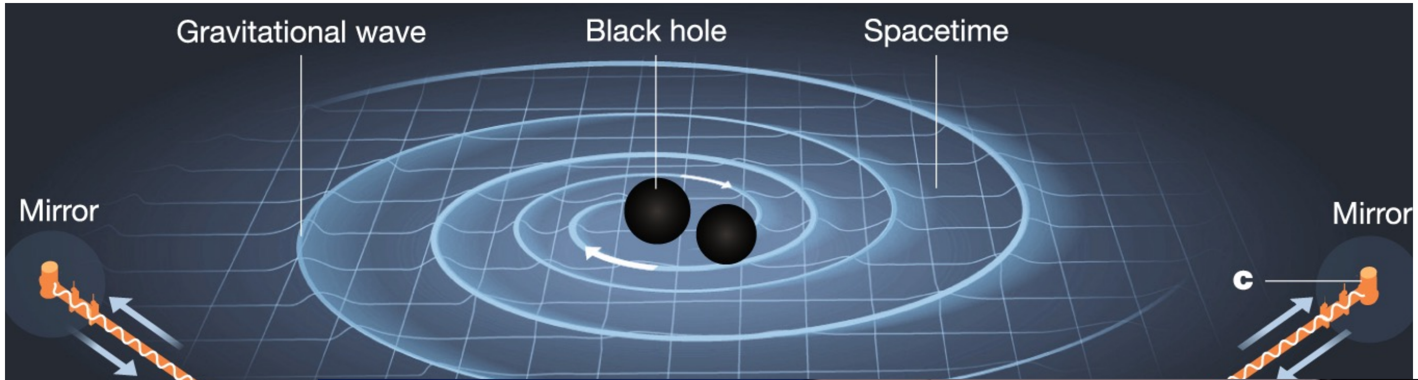
Spins of black holes χ

Primary masses of black holes m_1, m_2



Detection of GWs

Detection of GWs



Gws propagates across the Universe at $v = c$ and go through the detector on Earth

↓

The arms will be contracted/extended due to the waves

↓

Change in the light pattern

↓

GW detection

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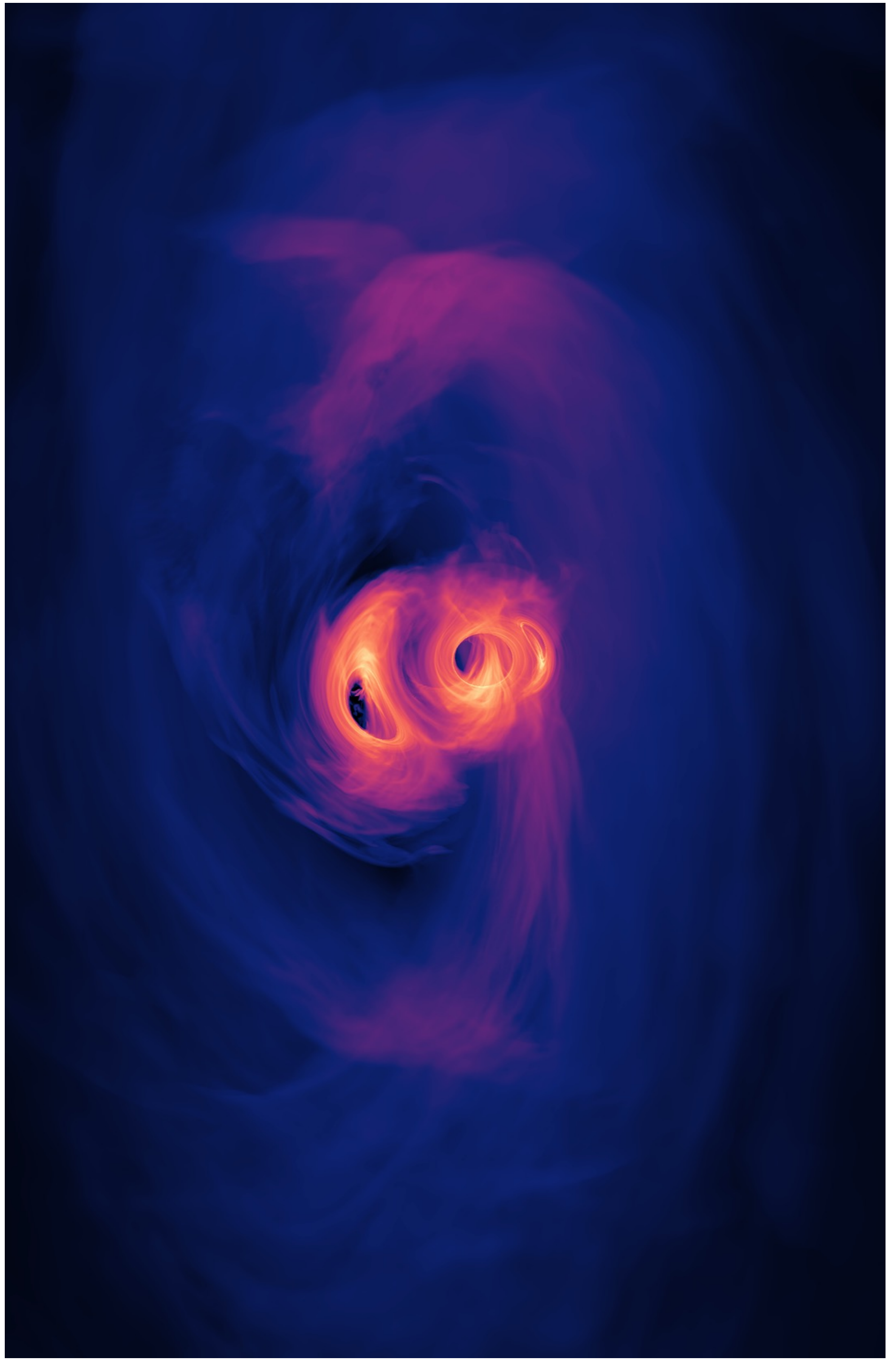
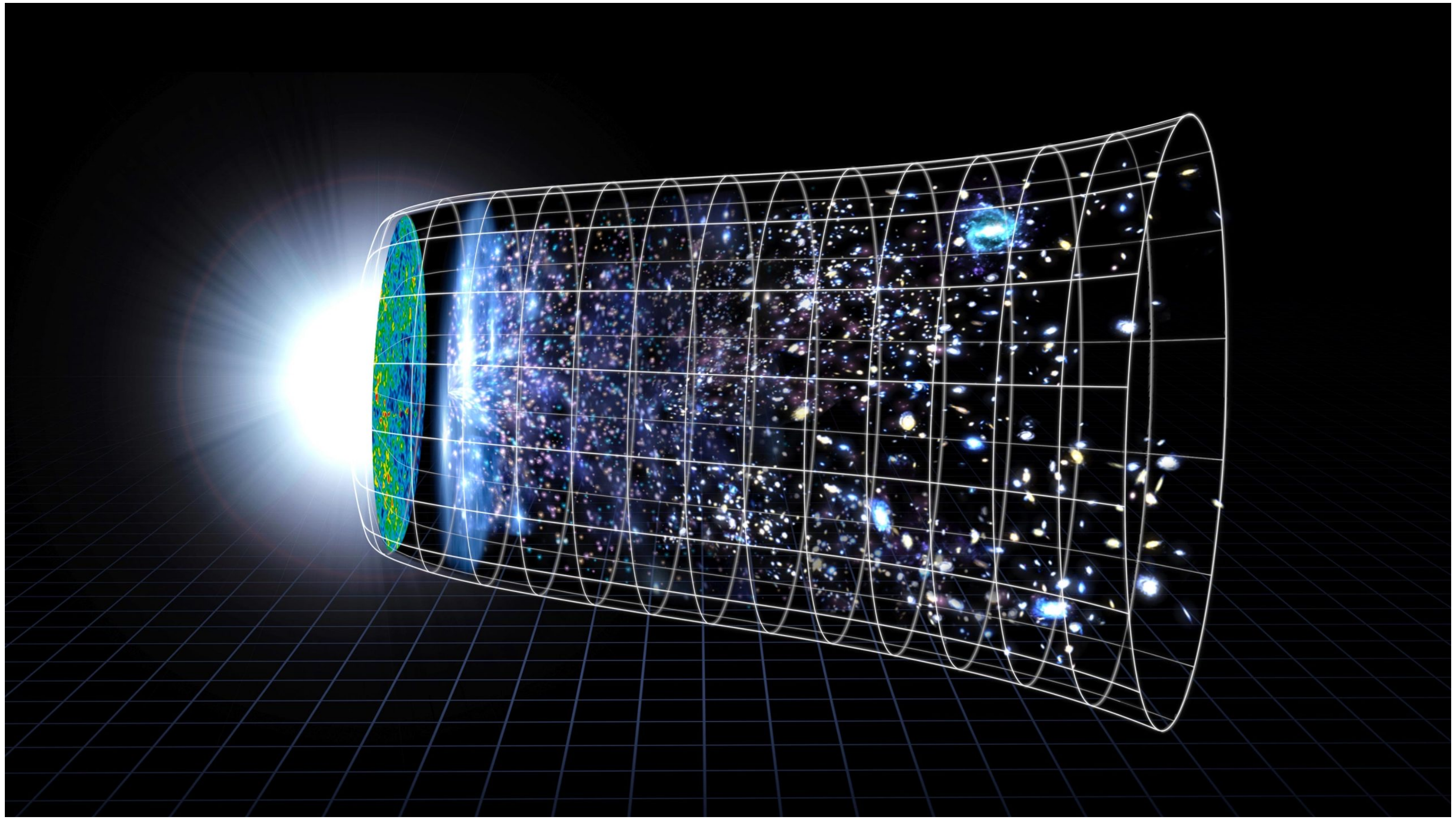


Hubble constant

H_0 = Expansion rate of the universe today
in $km.s^{-1}.Mpc^{-1}$

$\Leftrightarrow$

How fast our universe is expanding



How can we observe the universe expansion ?

Mogette cluster

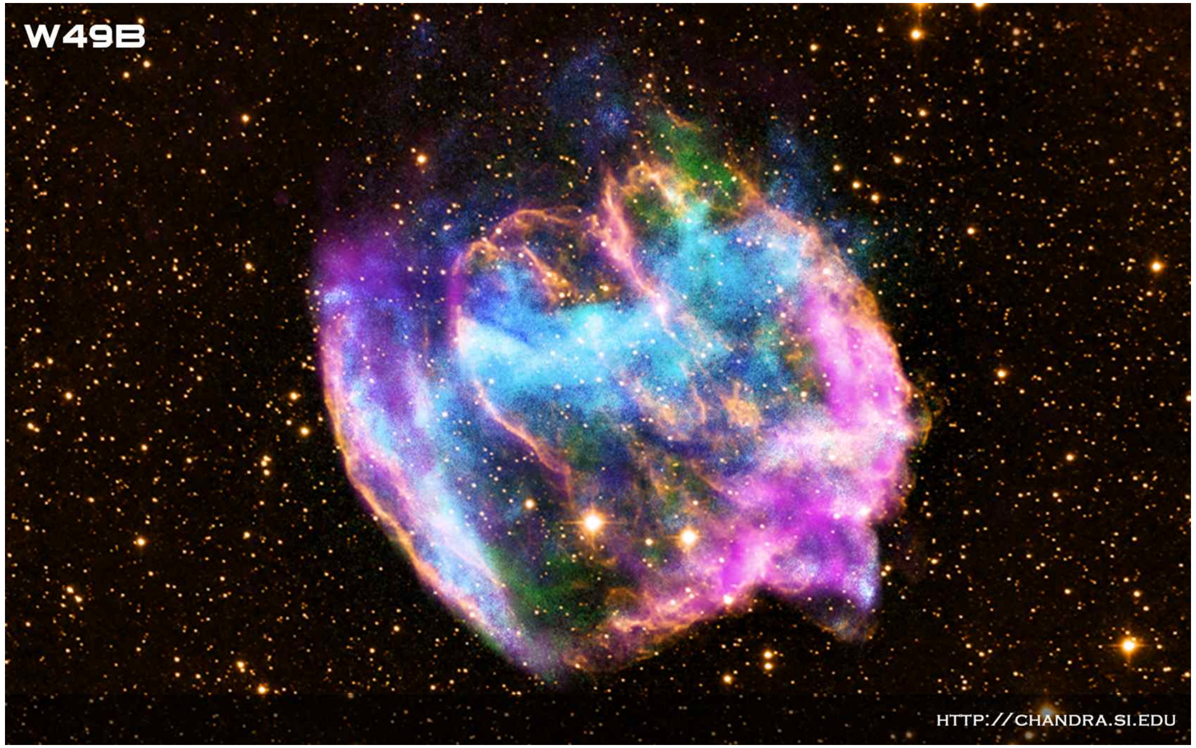
Hubble tension

H_0 = Expansion rate of the universe today
in $km.s^{-1}.Mpc^{-1}$

❖ Two measurements late & early universe :

Supernovae Ia

$H_0 = 74.03 \pm 1.42$



Hubble tension

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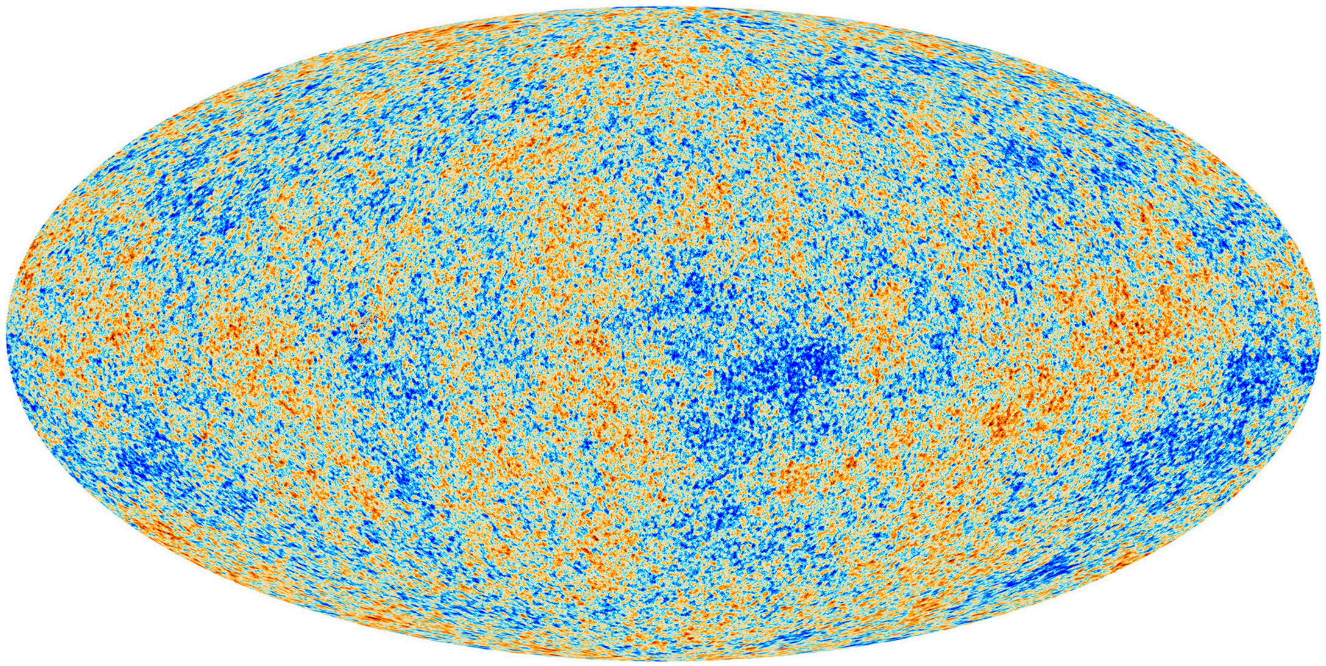
Cosmic microwave background

$H_0 = 67.4 \pm 0.5$



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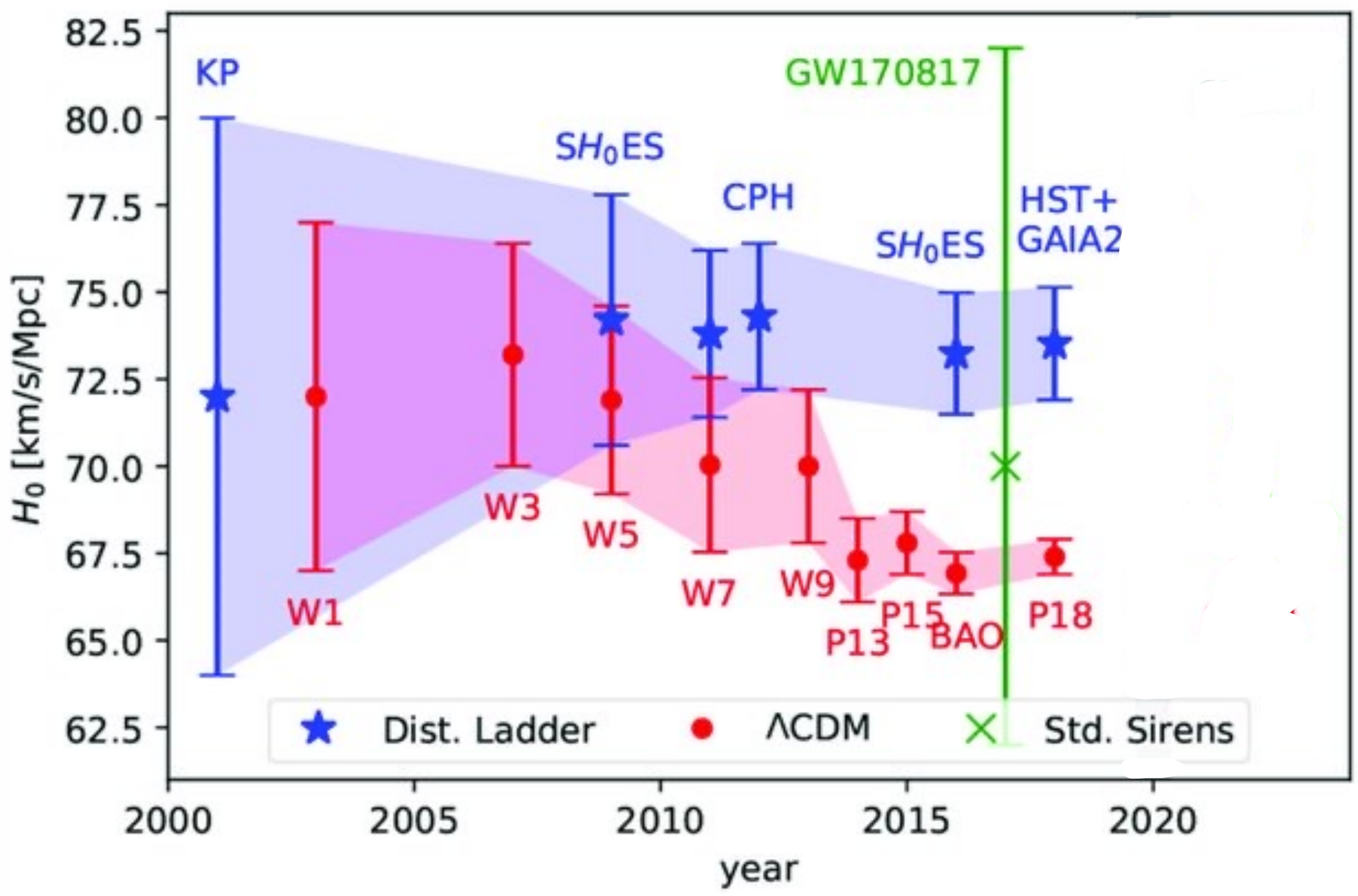


Dark Energy in light of multi-messenger gravitational-wave astronomy

Hubble tension

H_0 = Expansion rate of the universe today
in $km.s^{-1}.Mpc^{-1}$

❖ Two measurements late & early universe :

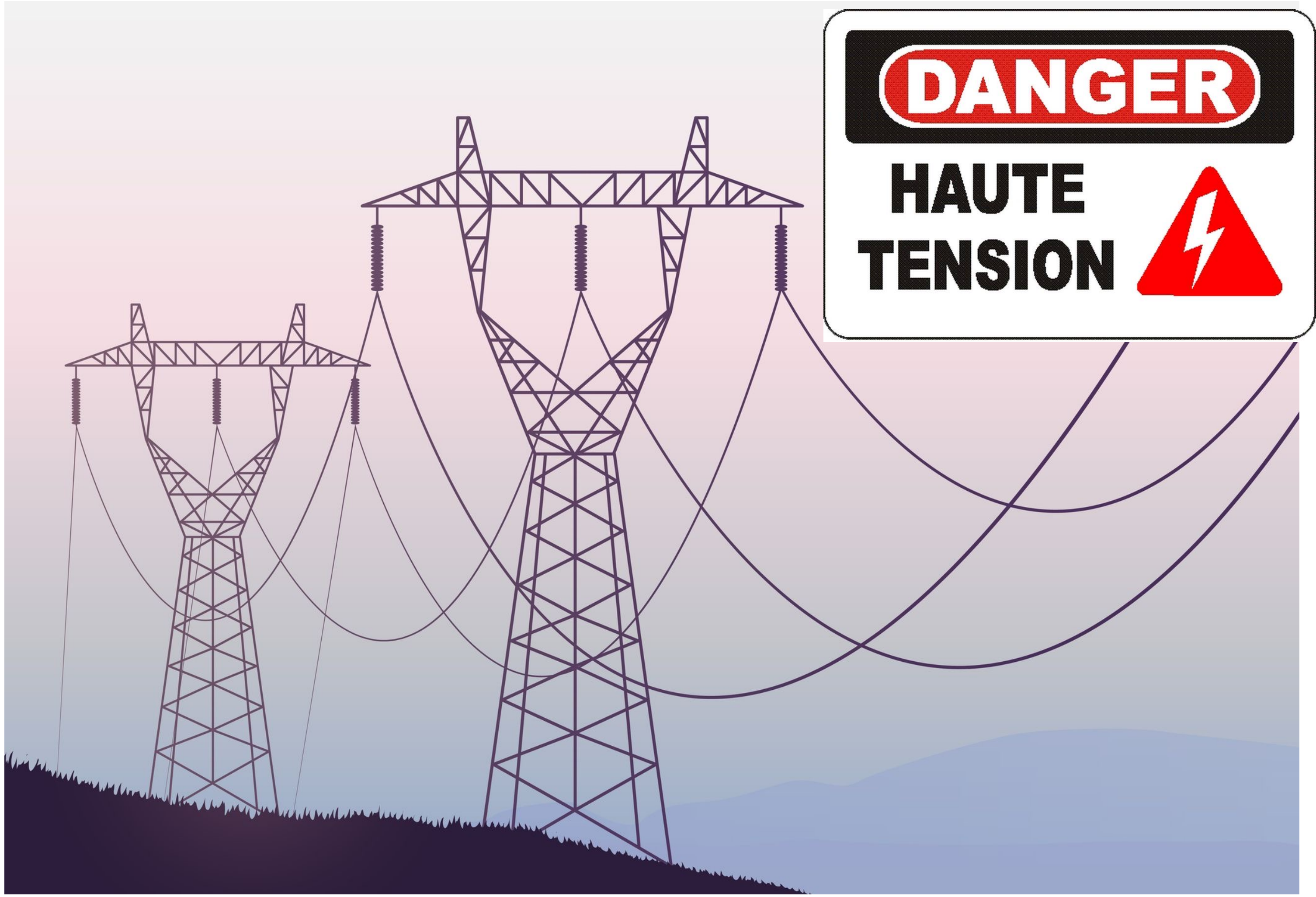
Supernovae Ia

$H_0 = 74.03 \pm 1.42$

Cosmic microwave background

$H_0 = 67.4 \pm 0.5$

⟶ 4.4σ tension



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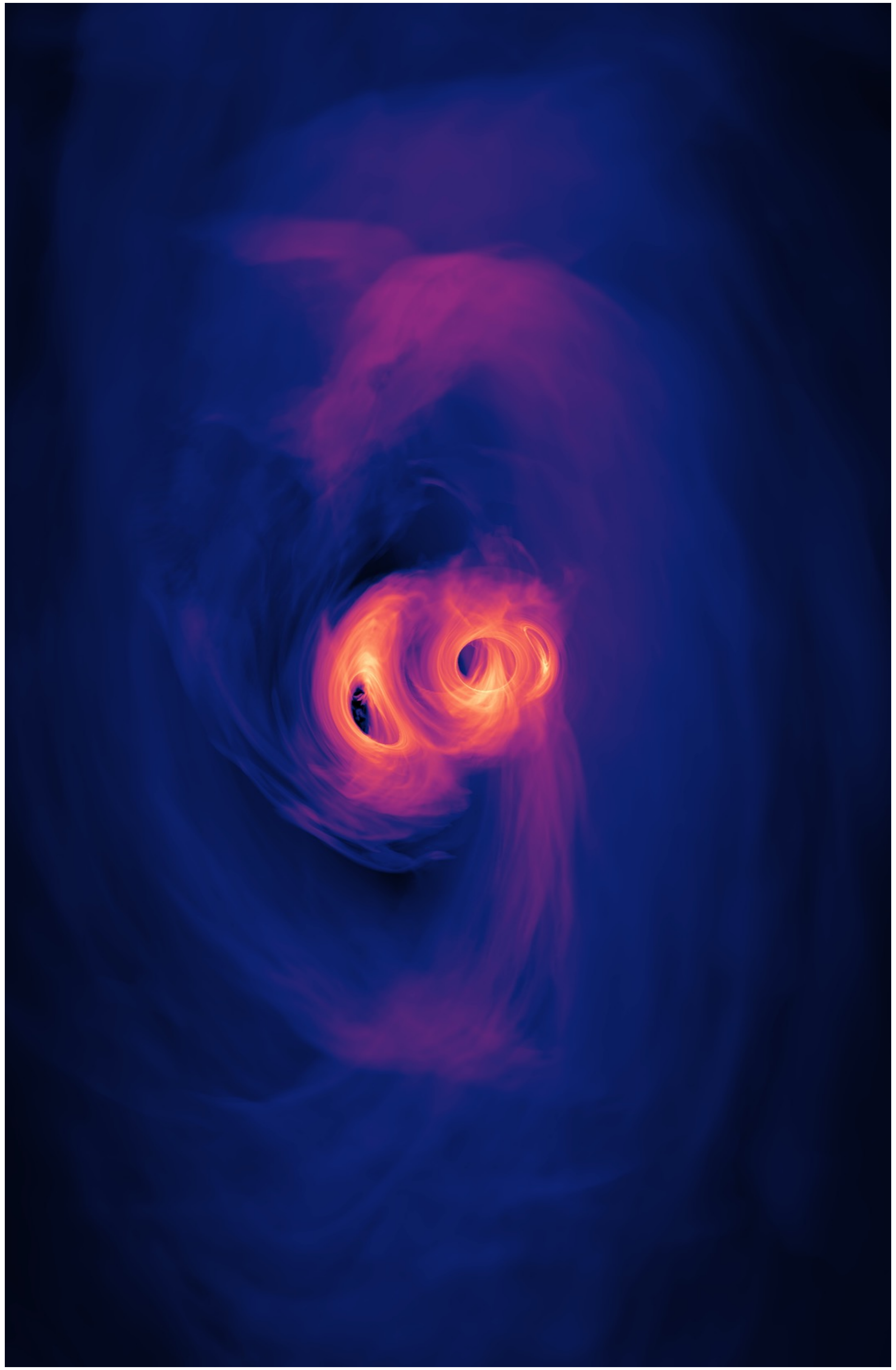


How do we measure H_0 ?

But how do we measure it using GWs ?

- The redshift and the luminosity distance of a source are related through the cosmology

Redshift : Need to be found through other methods

$$d_L \simeq \frac{c}{H_0} z$$

(low redshift approximation)

Luminosity distance : Directly measurable from the amplitude of the GW signal

Break the degeneracy between d_L and z

GW170817 : golden event

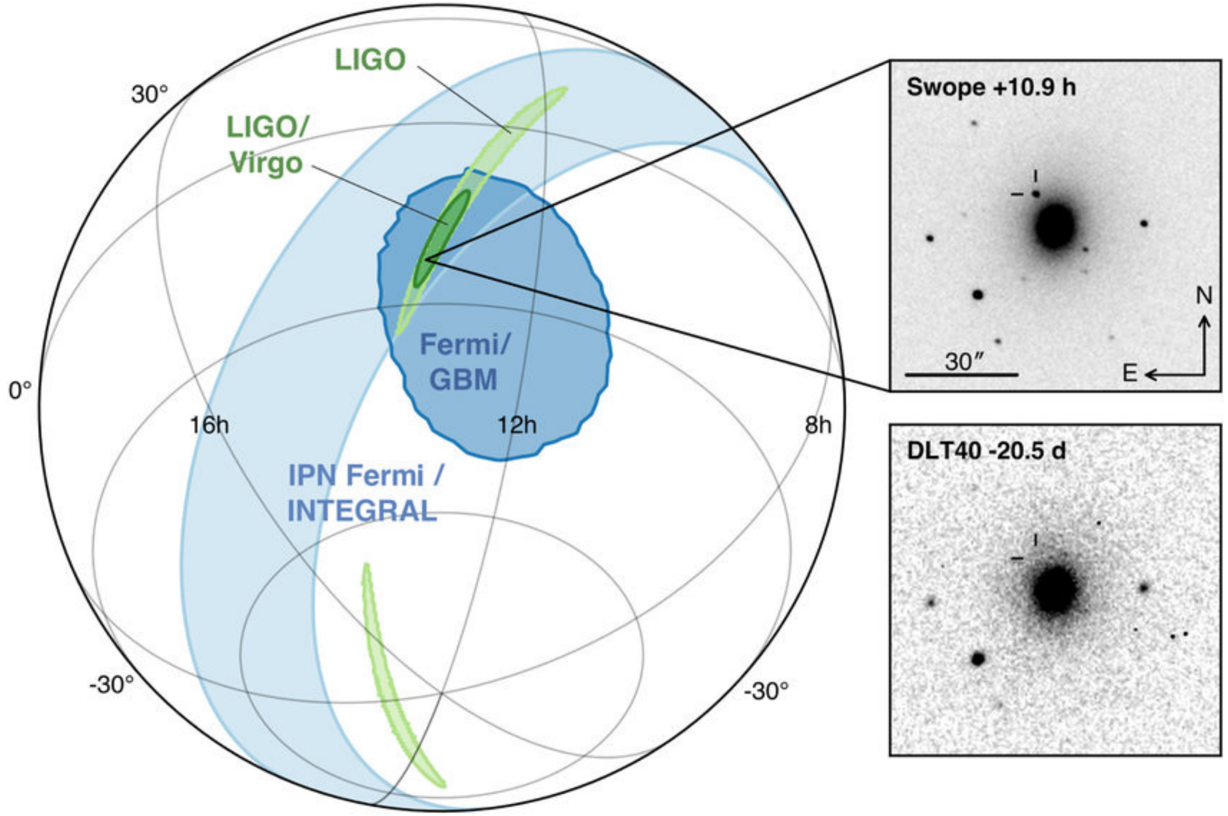
GW170817 is a **binary neutron stars** event

Two type of signals were emitted :

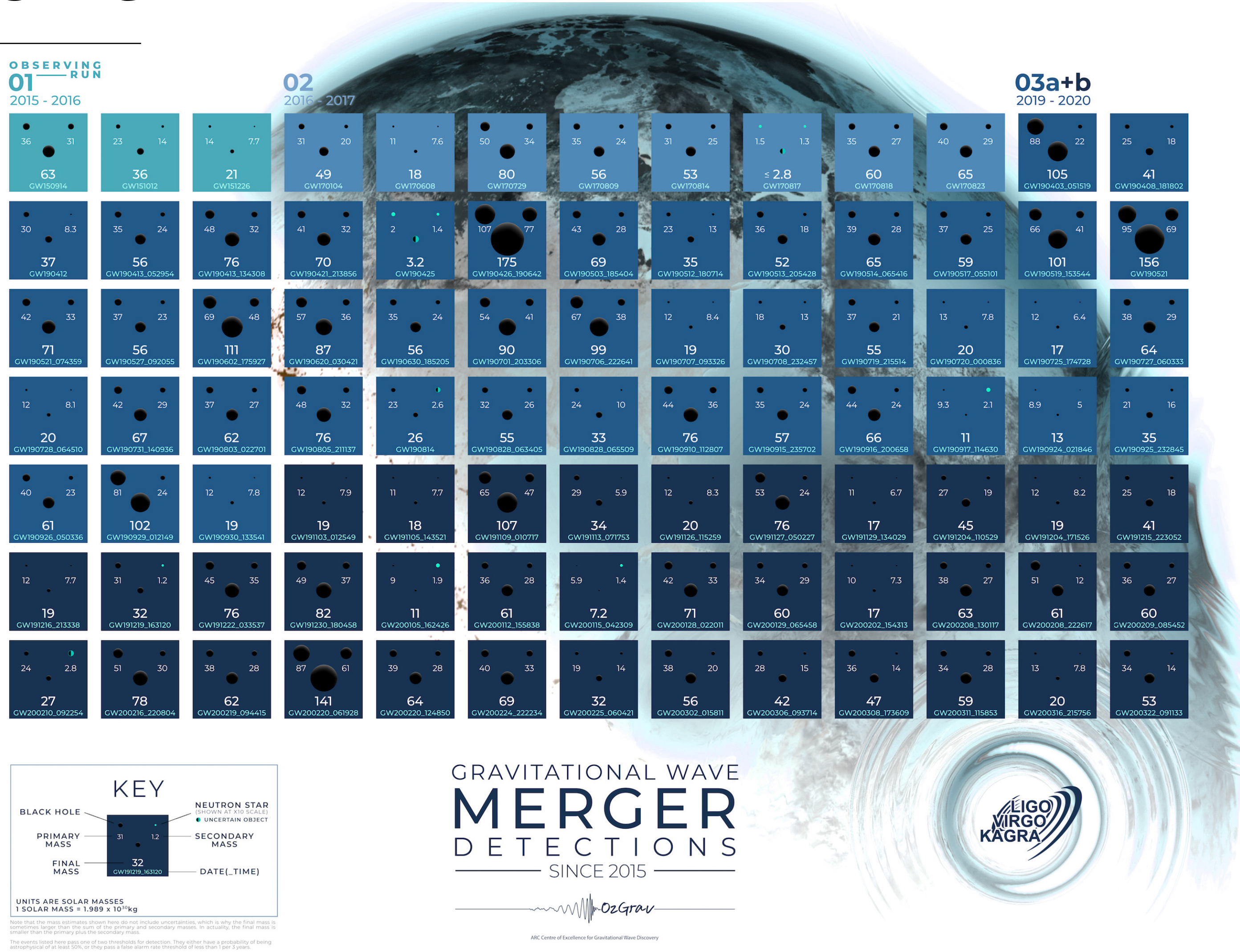
GWs	Gamma ray burst
↓	↓
Luminosity distance	Redshift

$$H_0 = 70.0^{+12.0}_{-8.0} \;\; km.s^{-1}.Mpc^{-1}$$

But still in agreement with the early and late universe values...



GWs state of the art



Since 2015, LIGO and Virgo had 3 observation runs

- 90 Compact binary mergers

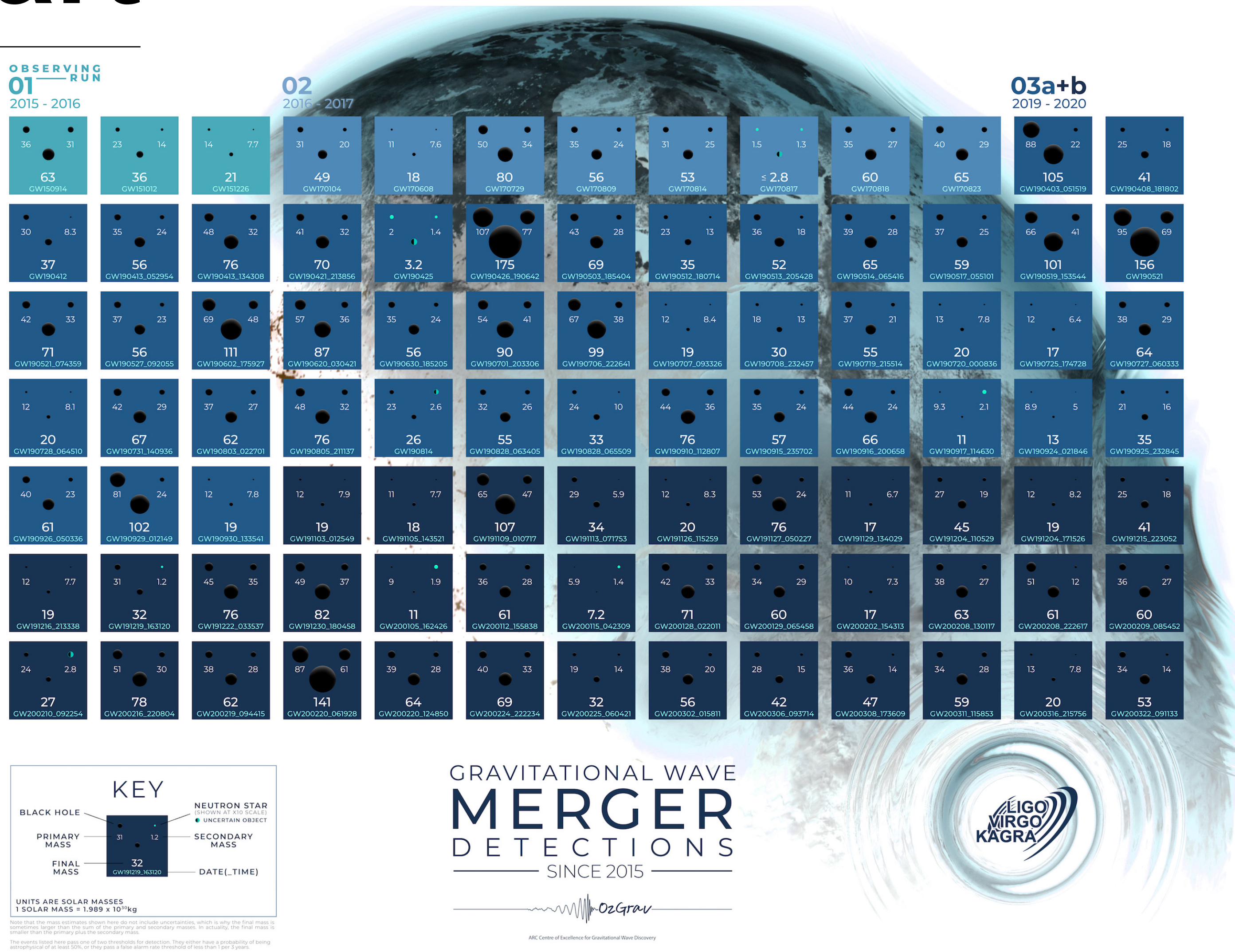
- 89 of them are Dark-Sirens
  (no E.M. counterpart)

GWs state of the art

Need alternative methods to derive the redshift of GWs sources

Population inference using binary black-holes

Galaxy catalog inference



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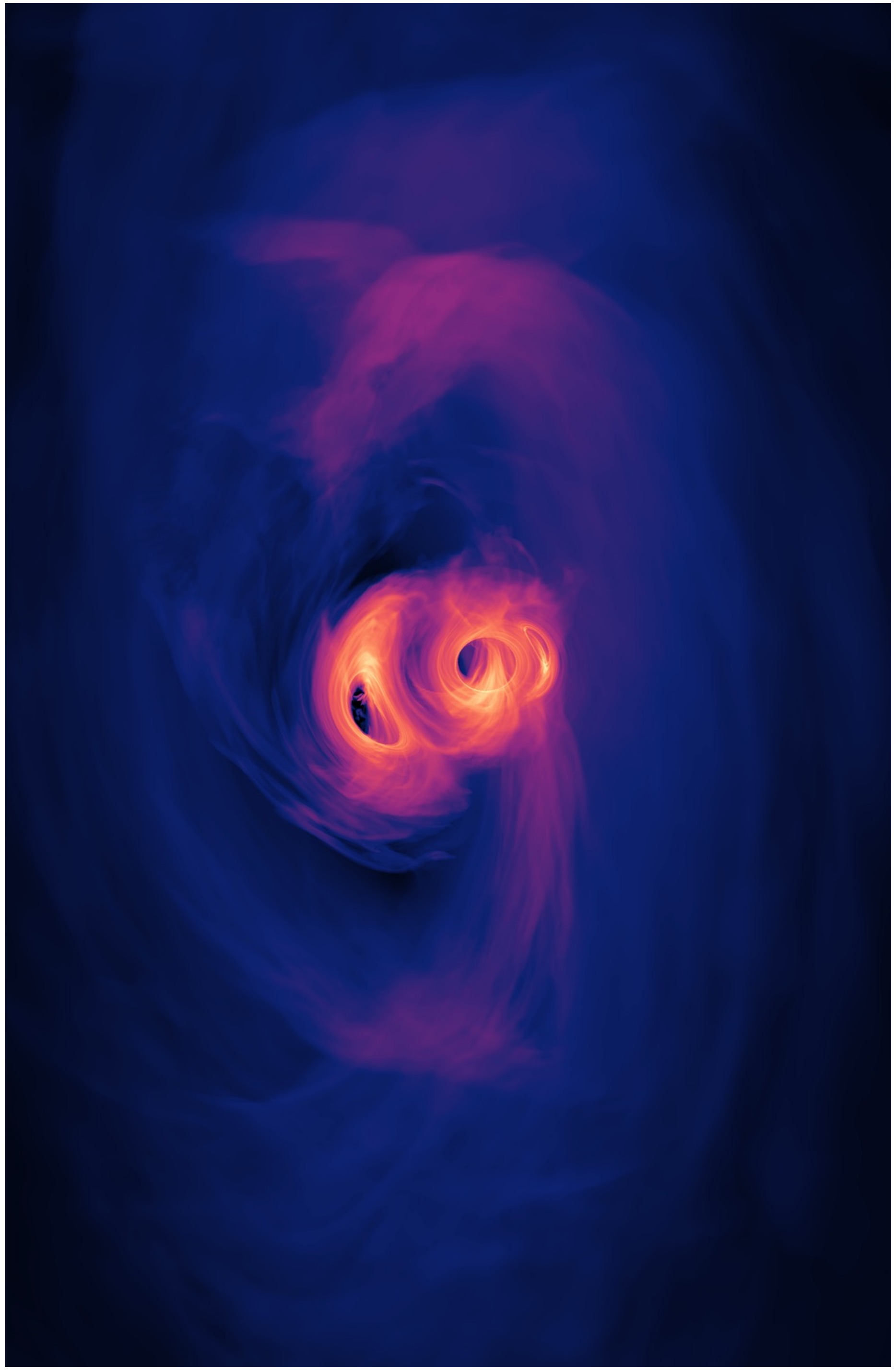
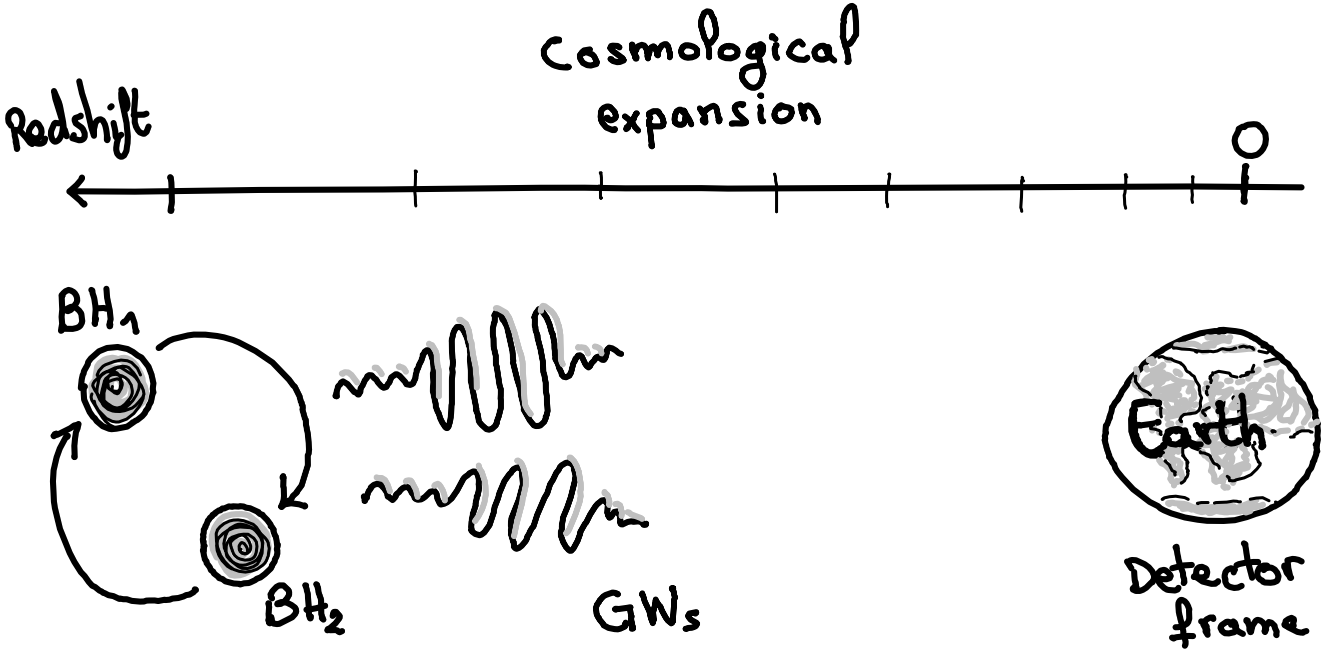


Population method

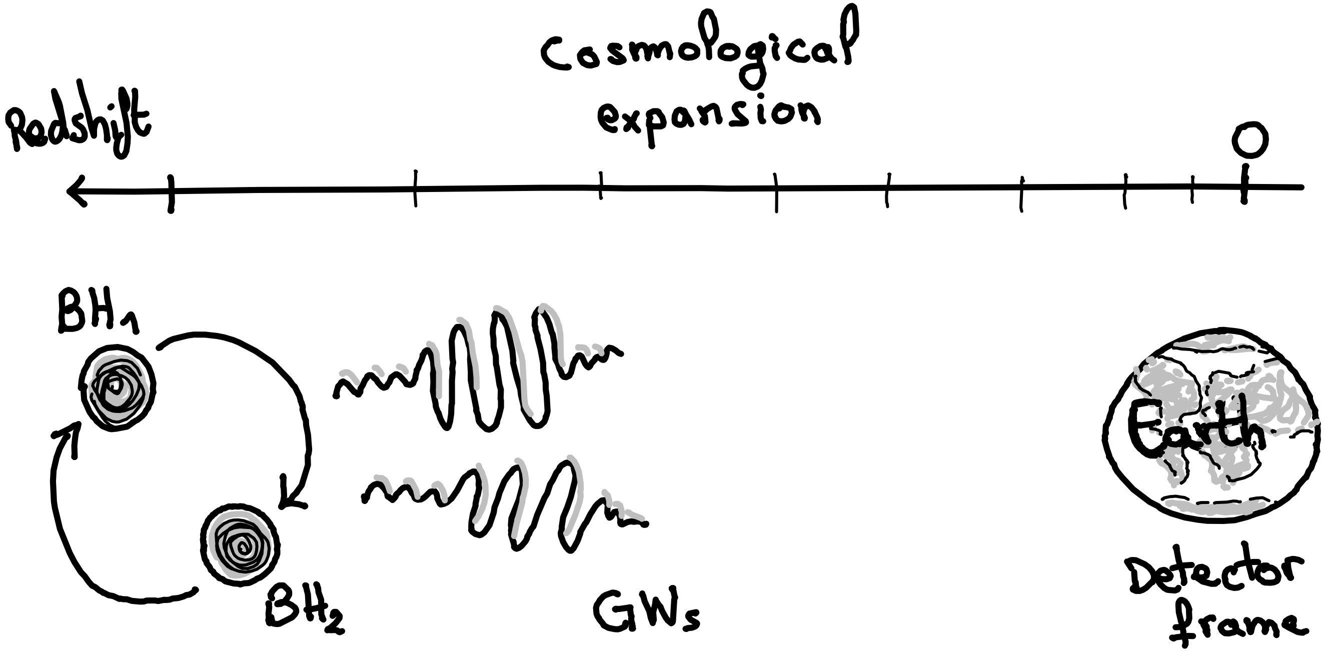
❖ GWs are redshifted due to cosmological expansion



❖ Degeneracy between the detected and the source mass of BBH : $m_i^d = (1+z)m_i^s$

Population method

❖ GWs are redshifted due to cosmological expansion



Need to break this degeneracy in order to measure z

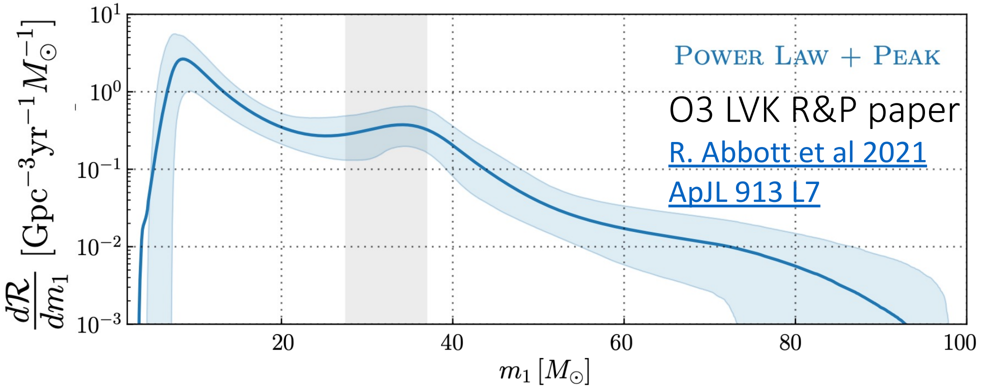
❖ Degeneracy between the detected and the source mass of BBH : $m_i^d = (1+z)m_i^s$

Population method

❖ By fixing the cosmology :

⟶ Deduce the true mass distribution of BBH

$$m_i^d = (1+z)m_i^s$$



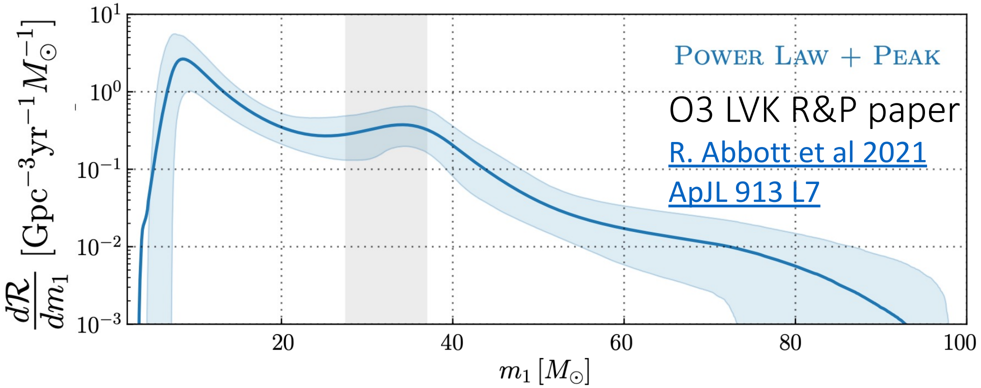
O3 LVK R&P paper
R. Abbott et al 2021
ApJL 913 L7

Population method

❖ By fixing the cosmology :

→ Deduce the true mass distribution of BBH

$$m_i^d = (1+z)m_i^s$$



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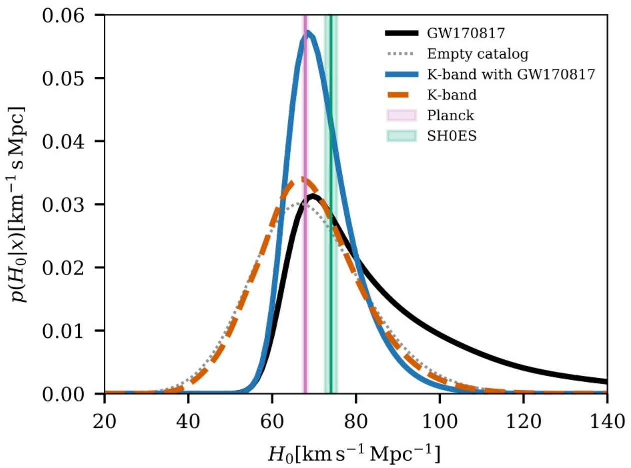
❖ By fixing the population :

→ Deduce the cosmology

$$m_i^d = (1+z)m_i^s$$



O3 LVK Cosmology paper
arXiv:2111.03604


FUSIOOOOOOOOOOOOOOOOOOON
Deduce the true mass distribution
Deduce the cosmology

GWs data

Fake GWs data

Population models

IcaroGW

Hierarchical Bayesian Inference

El classico mogette en sauce

GWs data

Fake GWs data

Population models

↓

IcaroGW

Hierachical bayesian inference

↓

Population of black holes

$\{m_1, m_2, d_L, ...\}$

Cosmology

$\{H_0, \Omega_m, \omega_0\}$

Basic principle behind IcaroGW

❖ Infer jointly the population and cosmological parameters via hierarchical Bayesian inference :

$$p(\mathrm{cosmology}|\mathrm{GWs}) \propto \prod_{i}^{N_{\mathrm{obs}}} \frac{\int p(\mathrm{GW_i}|\theta,\Lambda)\pi(\theta|\Lambda)d\theta}{N_{\mathrm{eff}}}$$

Probability of esti… …ertain cosmology
given so… …ata.

Our final result

Basic principle behind IcaroGW

❖ Infer jointly the population and cosmological parameters via hierarchical Bayesian inference :

$$p(\mathrm{cosmology}|\mathrm{GWs}) \propto \prod_{i}^{N_{\mathrm{obs}}} \frac{\int p(\mathrm{GW_i}|\theta, \Lambda)\pi(\theta|\Lambda)d\theta}{N_{\mathrm{eff}}}$$

Probability of detecting GWs given our models

Basic principle behind IcaroGW

- Infer jointly the population and cosmological parameters via hierarchical Bayesian inference :

$$p(\text{cosmology}|\text{GWs}) \propto \prod_{i}^{N_{\text{obs}}} \frac{\int p(\text{GW}_\text{i}|\theta,\Lambda)\pi(\theta|\Lambda)d\theta}{N_{\text{eff}}}$$

Prior knowledge on the models (masses, spins, redshift, etc)

Basic principle behind IcaroGW

- Infer jointly the population and cosmological parameters via hierarchical Bayesian inference :

$$p(\mathrm{cosmology}|\mathrm{GWs}) \propto \prod_{i}^{N_{\mathrm{obs}}} \frac{\int p(\mathrm{GW_i}|\theta, \Lambda)\pi(\theta|\Lambda)d\theta}{N_{\mathrm{eff}}}$$

↓

Expected number of detection (efficiency)

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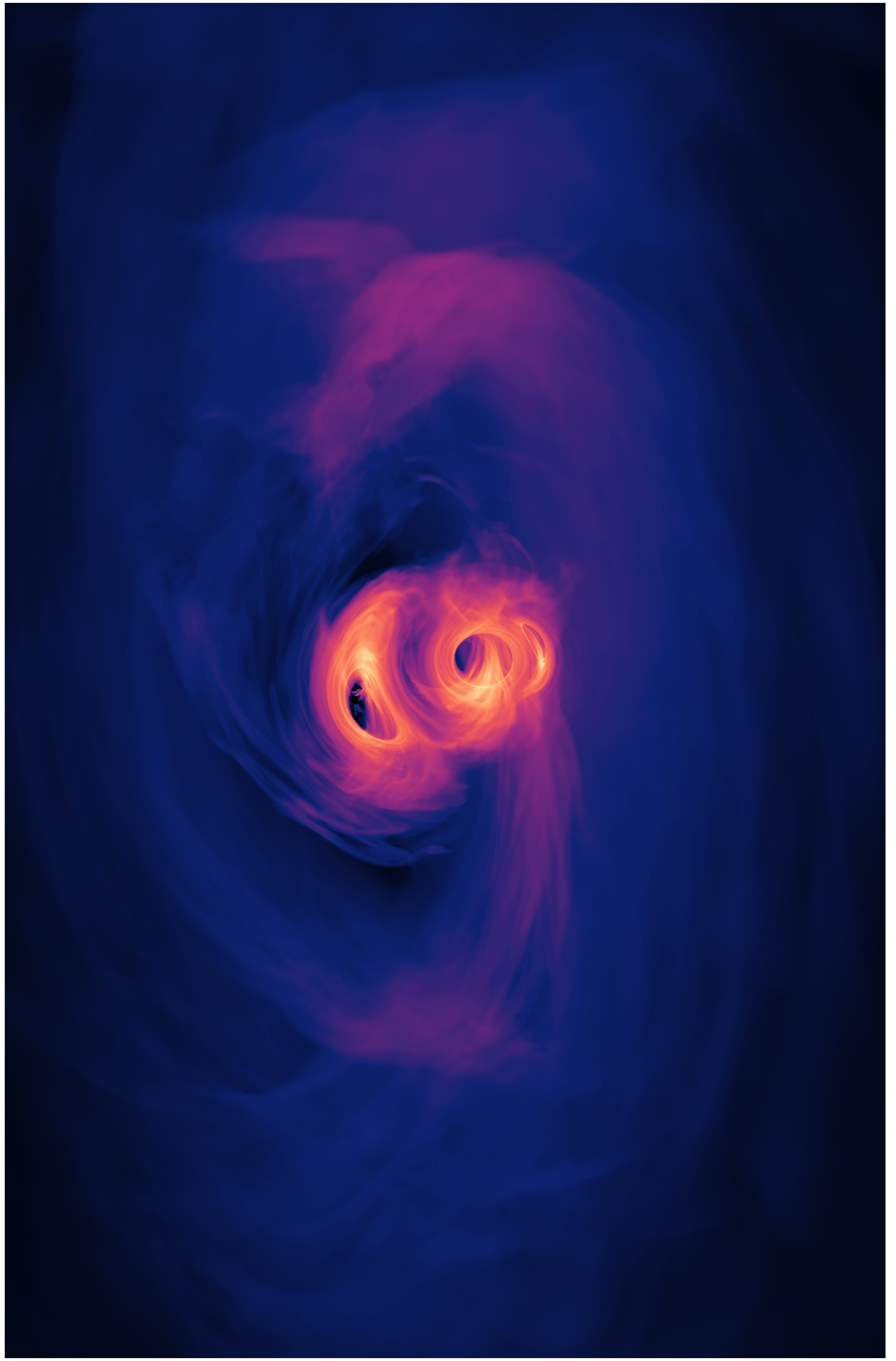


IcaroGW and spins of black holes

- **Aim** : Add BBH spin distributions to generalize the analysis :
  - Development of two spin models for BBH

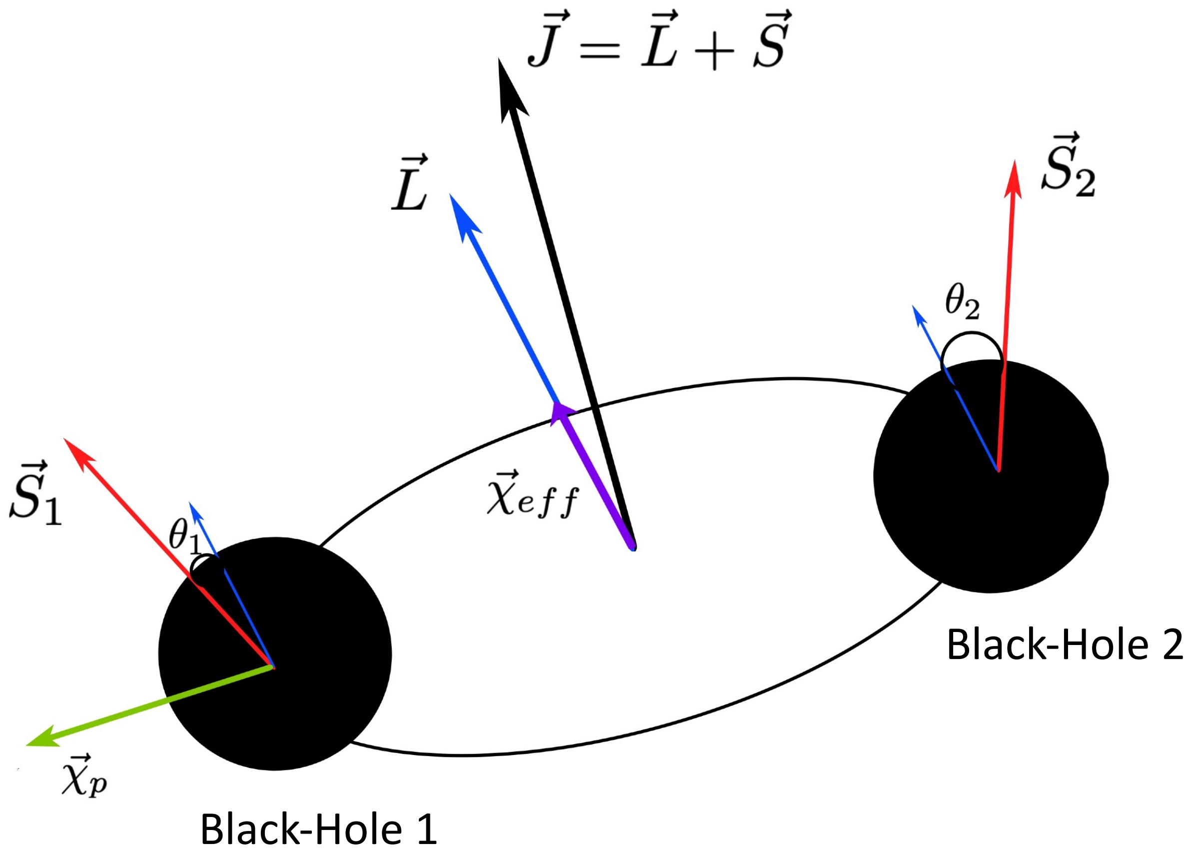
    Default spin model : First introduced by LSC collaboration arXiv:1811.12940

    Gaussian spin model : First introduced by Schmidt et al. arXiv:1408.1810

Motivations

- Generalize the analysis and be ready for spins measurement in the future
- Spins could correlate with the Hubble constant H_0
- BBH formation channels might prefer specific spin configurations

Spin models of BBH



Default spin model

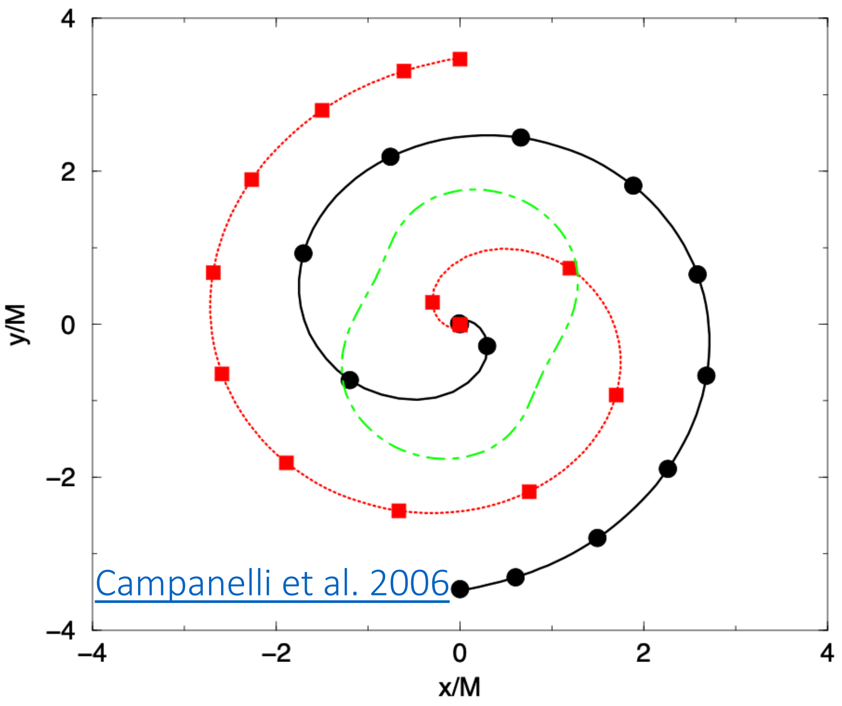
- ❖ Multiple formation channels
- ❖ Covers all 6 degrees of freedom

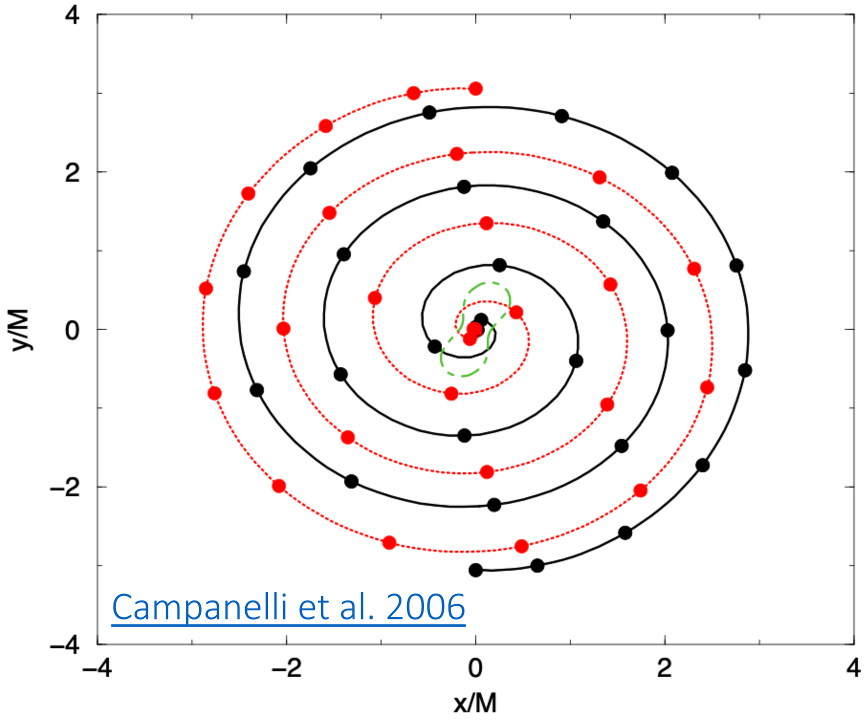
Gaussian spin model

- ❖ Sensitive to precession effects
- ❖ Sensitive to spin asymmetries

Impact of the spins

Number of orbits depends on the spin orientation of each BHs





→ Direct impact on the amplitude of the GWs

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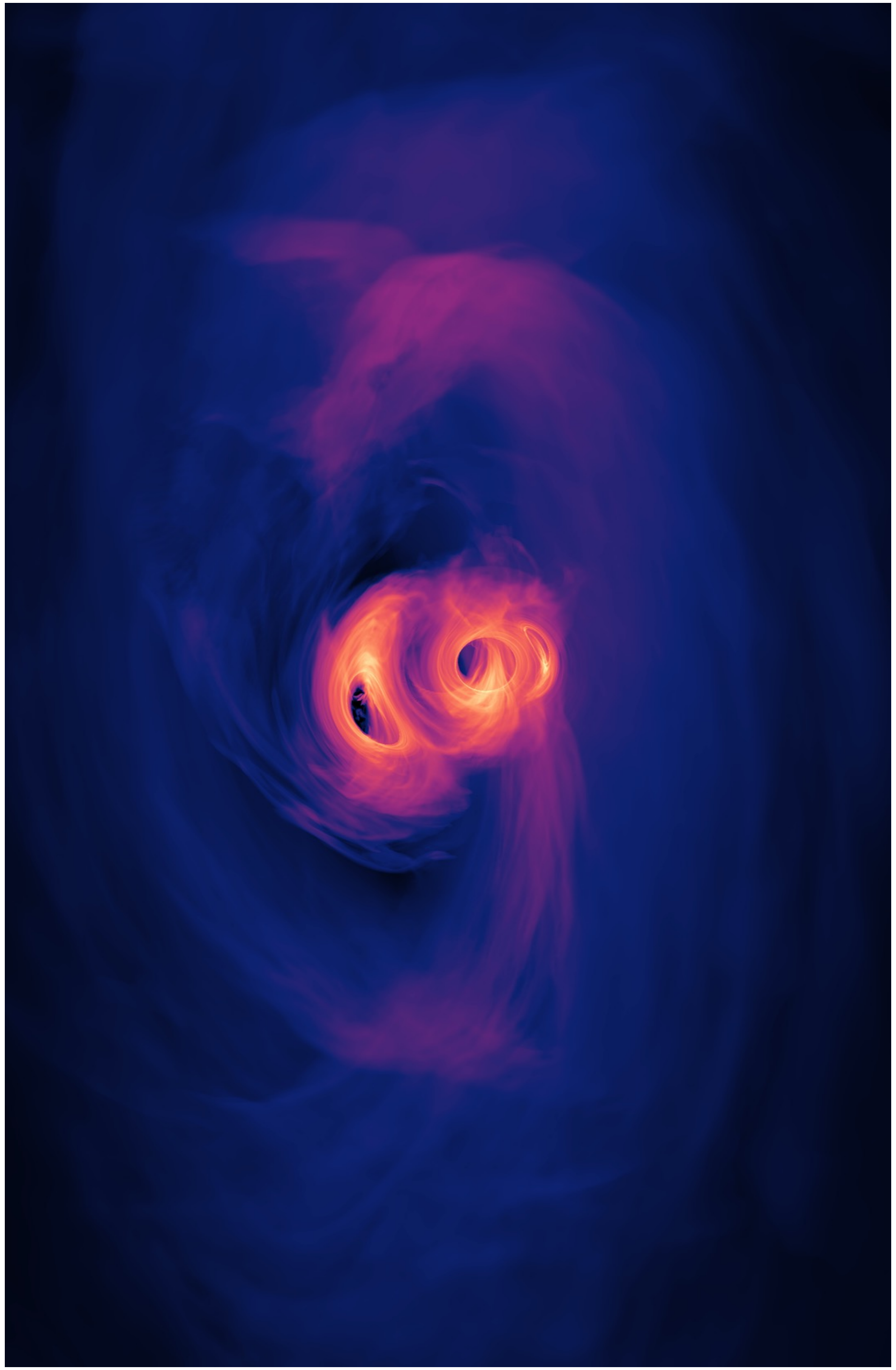


Set-Up of the analysis

- First time analysis that jointly estimates the cosmology and spins parameters of black-holes !

- These results have been obtain using 42 BBHs events from the O3 run of LVK, with IFAR > 4

- We tested both spin models

Default model

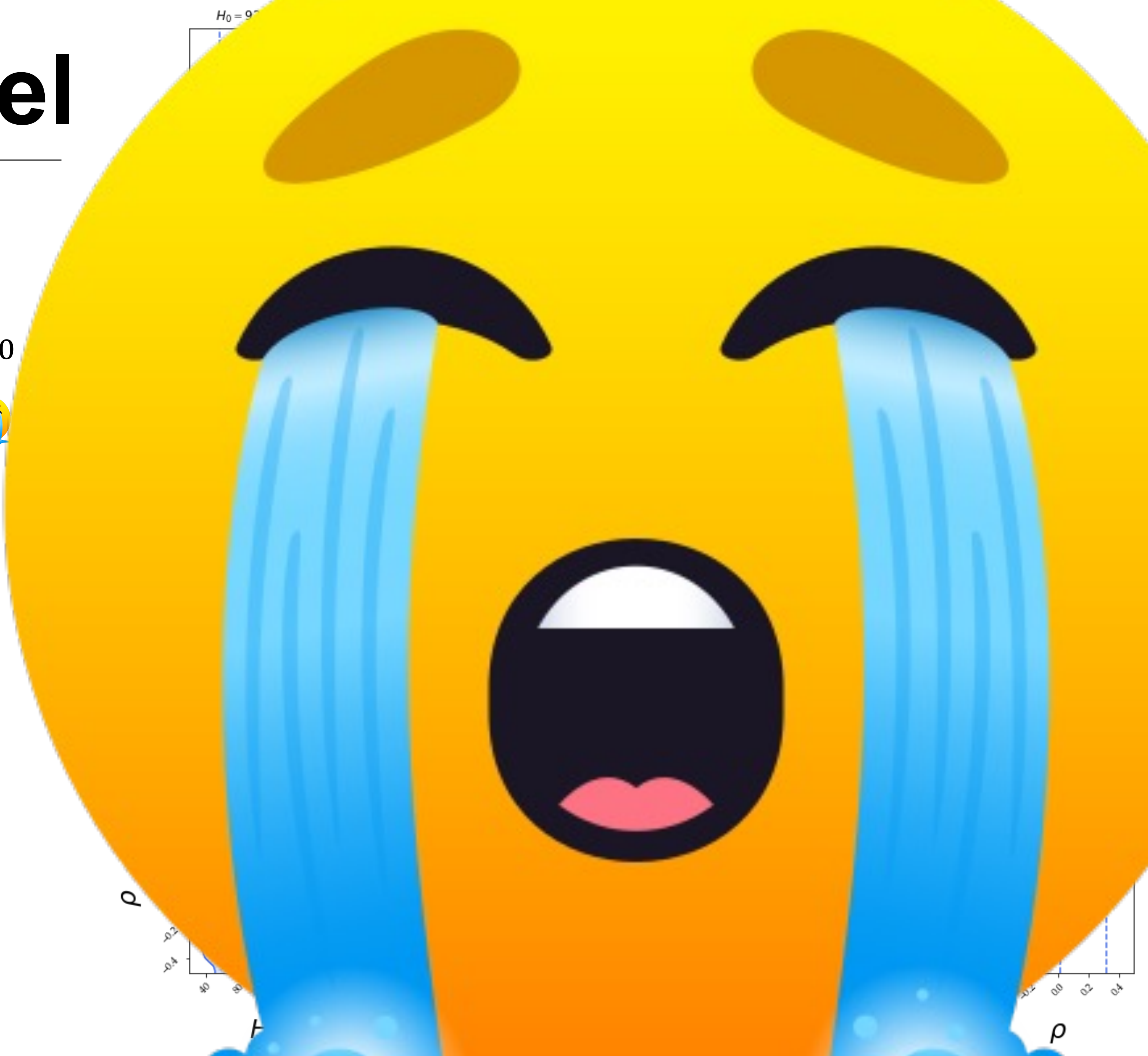
- ❖ No spin correlation with H_0 so far !
- ❖ Preference for a slowly spinning black-hole population
- ❖ Mixed population between isolated and dynamical mergers

Gaussian model

❖ Again : no spin correlation with H_0

❖ Aligned spins are favoured

❖ Slowly precessing black-holes

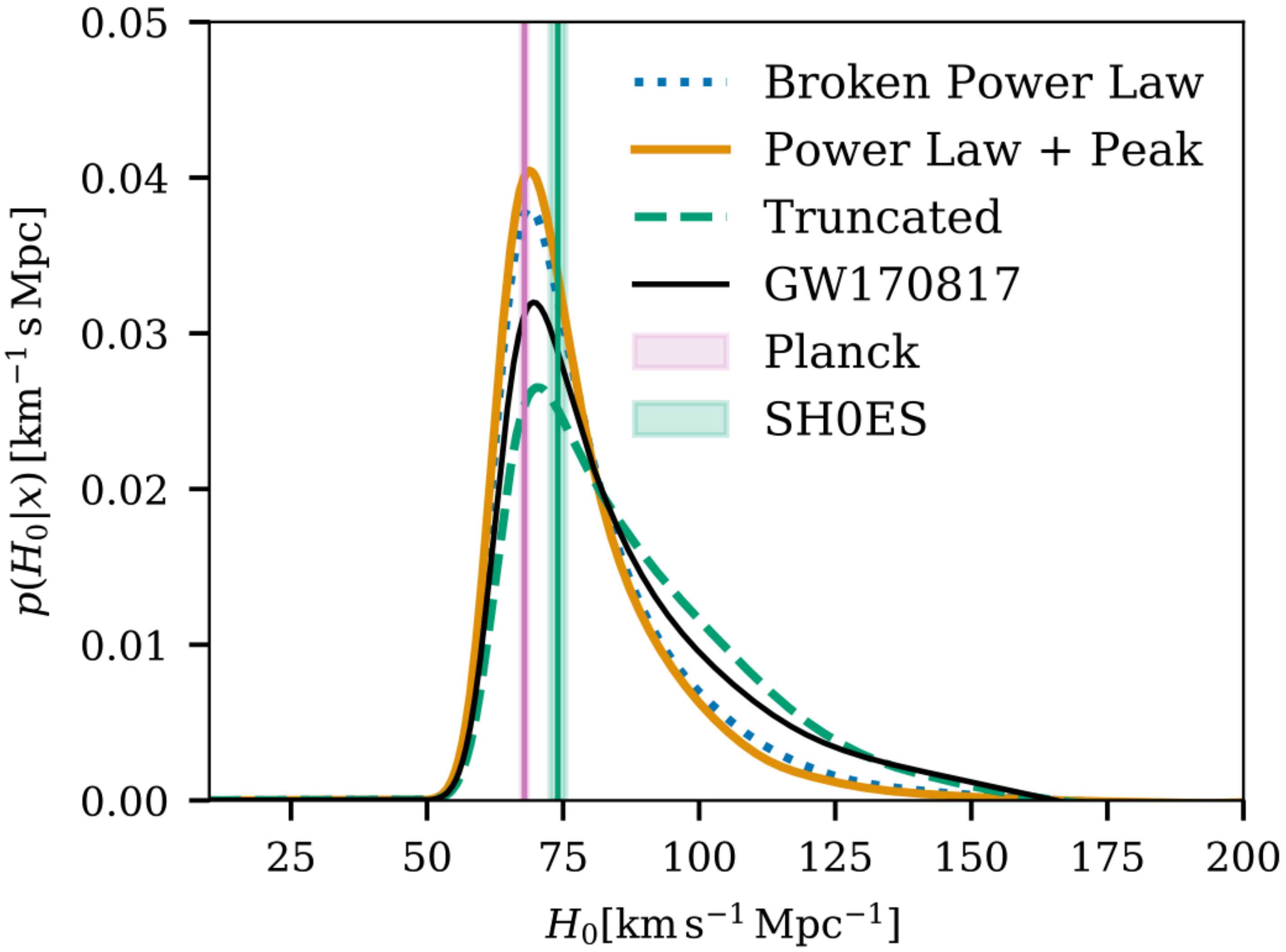


Best measure so far

Constraints on the cosmic expansion history from GWTC–3

Combined with GW1708187 :

$$H_0 = 68^{+8}_{-6} \;\; km.s^{-1}.Mpc^{-1}$$



Why is IcaroGW crazy af ? 🥵🥵🥵🥵

- Able to estimate population parameters of any given model (masses, rates, spins, ...) !
- Able to put constraints on cosmological parameters !
- The more events we use, the better the constraints will be !
- Code is very modular, and easy to use for any other analysis that needs parameter estimation !

Perspectives

- Next run of LVK starting in 2023 (hundreds of GWs events hopefully !)
- IcaroGW with spins in being officially reviewed
- IcaroGW will be used as one of the two cosmology codes in the LVK collaboration for the next observation run
- Constraints on cosmology and population parameters will get better and better with higher statistic

VERY SEXY FUTURE FOR GWs COSMOLOGY

On-going work

- Existence sub-population of BHHs
- Merger rate dependency on z

Big question : What is the impact of sub population BBHs on H_0 inference ?

Thanks

BACK UP

Galaxy catalog results

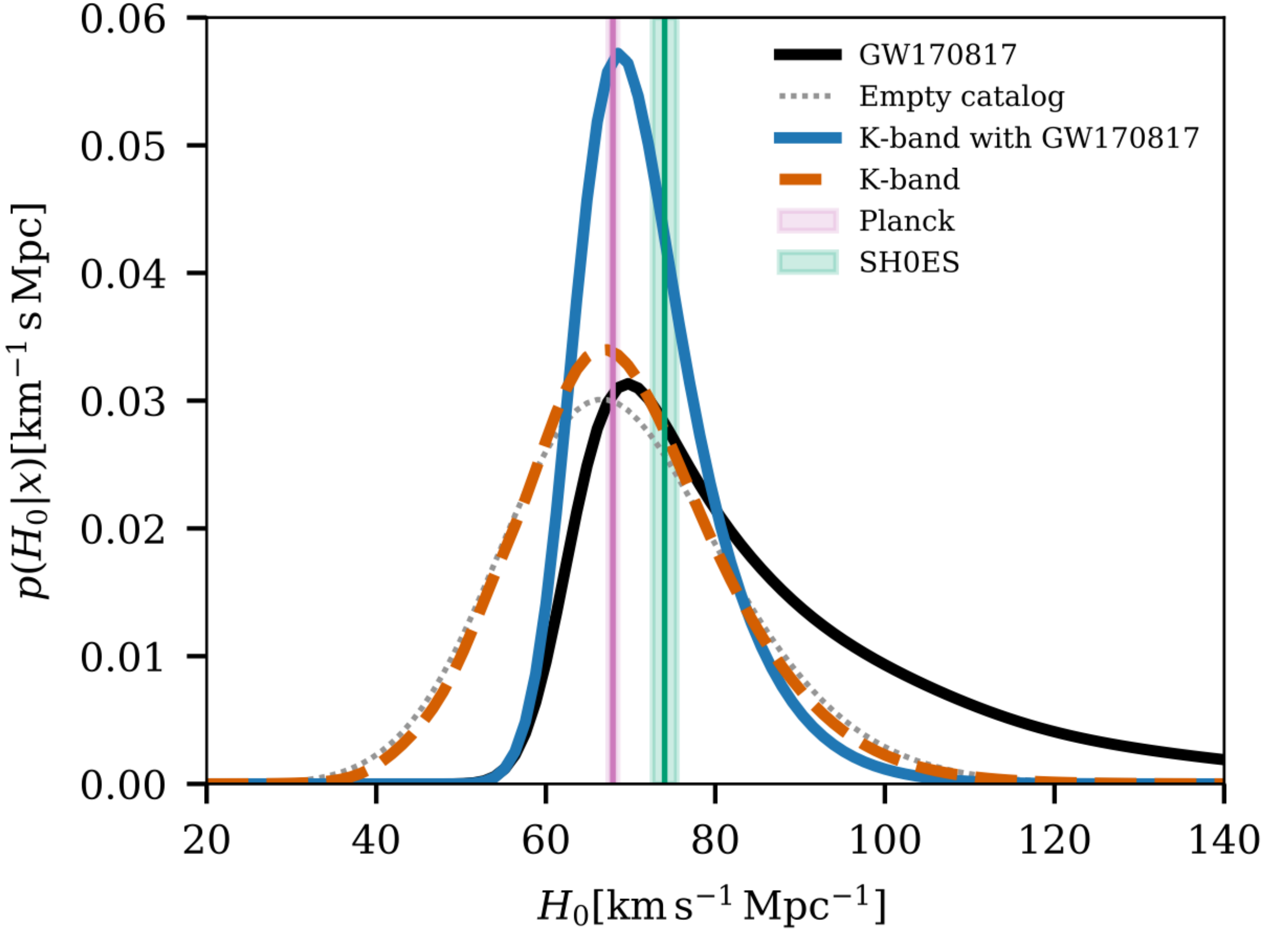
From the galaxy catalog only :

$$H_0 = 67^{+13}_{-12} \ \ km.s^{-1}.Mpc^{-1}$$

Combined with GW1708187 :

$$H_0 = 68^{+8}_{-6} \ \ km.s^{-1}.Mpc^{-1}$$

Constraints on the cosmic expansion history from GWTC–3

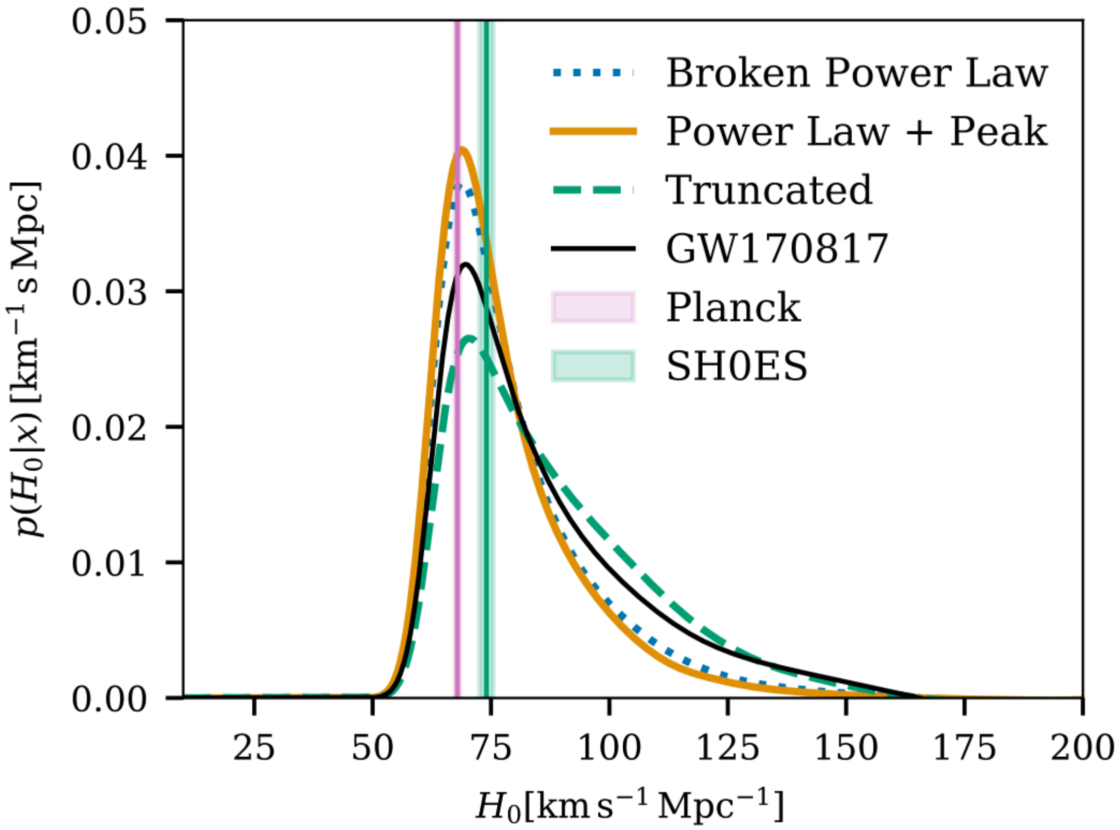


Back up

$$d_L(z) = \frac{1+z}{H_0} c \int_0^z \frac{dz}{(\Omega_m(1+z)^3 + \Omega_\Lambda)}$$

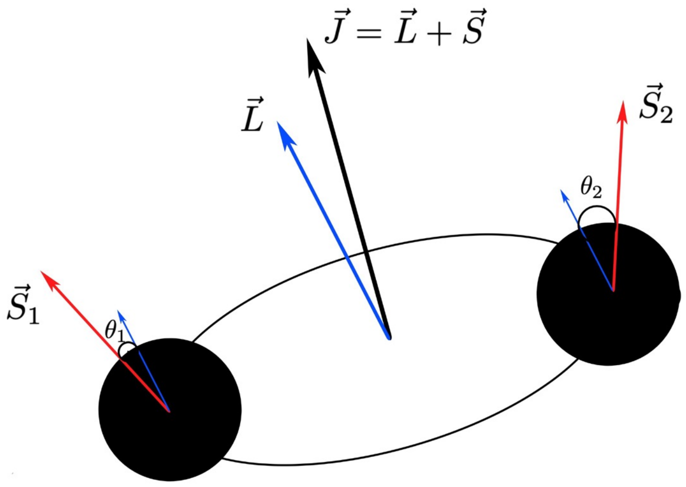
and

$$z = \frac{m^d}{m^s} - 1$$



Spin models of BBH

Default spin model



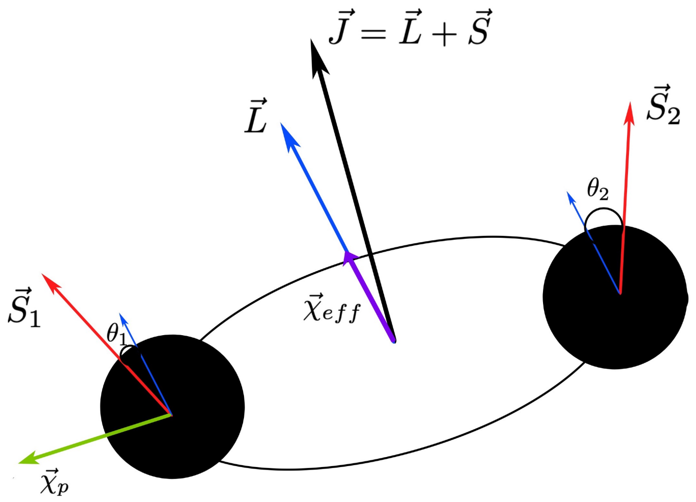
$$\chi_i = |\frac{\vec{S}_i}{m_i^2}|$$

Dimensionless spin magnitudes & tilt angle :

$$\pi(\chi_{1,2}|\alpha_\chi, \beta_\chi) = Beta(\alpha_\chi, \beta_\chi)$$

$$\pi(cos\theta|\zeta, \sigma_t) = \zeta G_t(cos\theta|\sigma_t) + (1-\zeta)\mathcal{F}(cos\theta)$$

Gaussian spin model



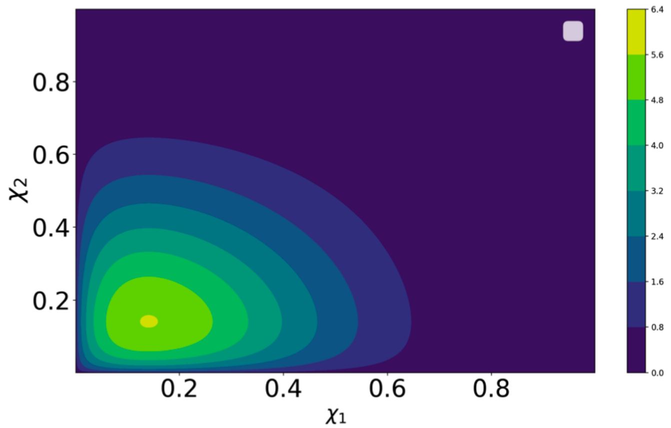
Effective & precession spin :

$$\pi(\chi_{eff}, \chi_p|\mu_{eff}, \sigma_{eff}, \mu_p, \sigma_p) = G^{2D}_{[-1,1],[0,1]}(\chi_{eff}, \chi_p|\mu, \Sigma)$$

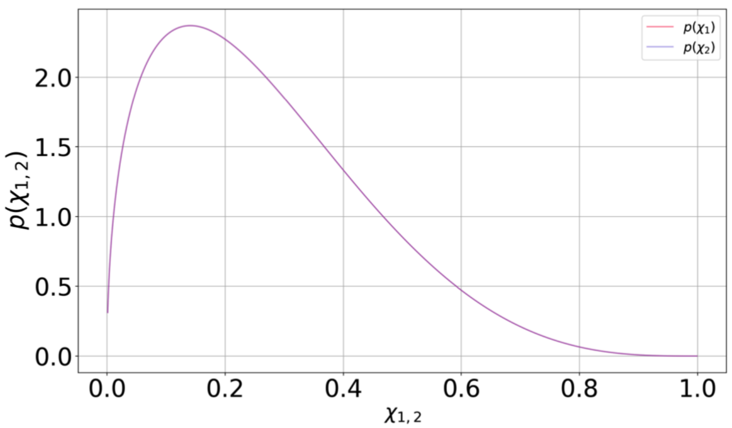
$$\vec{\mu} = (\mu_{eff}, \mu_p)$$

$$\Sigma = \begin{pmatrix} \sigma_{eff}^2 & \rho\sigma_{eff}\sigma_p \\ \rho\sigma_{eff}\sigma_p & \sigma_p^2 \end{pmatrix}$$

Spin distributions : Default

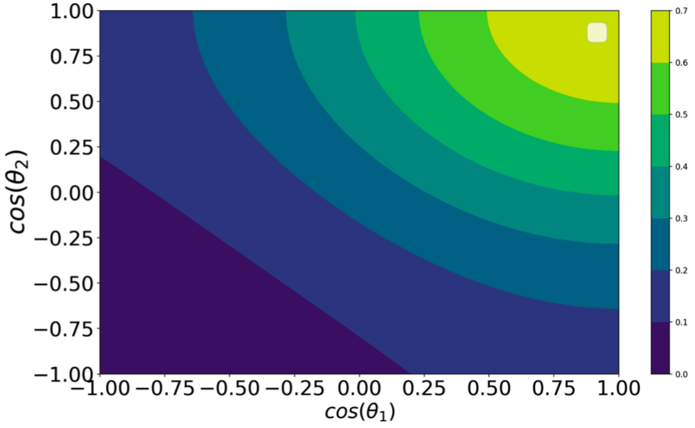


(a)

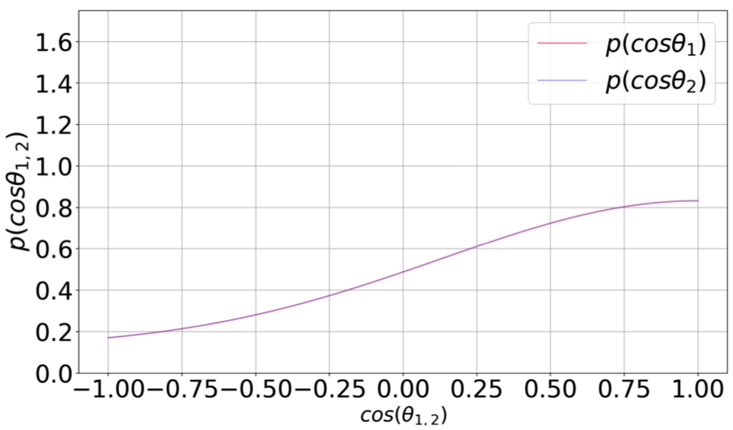


(b)

$$\{\alpha = 2.6, \beta = 6.6, \zeta = 0.76, \sigma_t = 0.87\}$$



(c)



(d)

(a) and (c) :
Joint 2D probability density functions

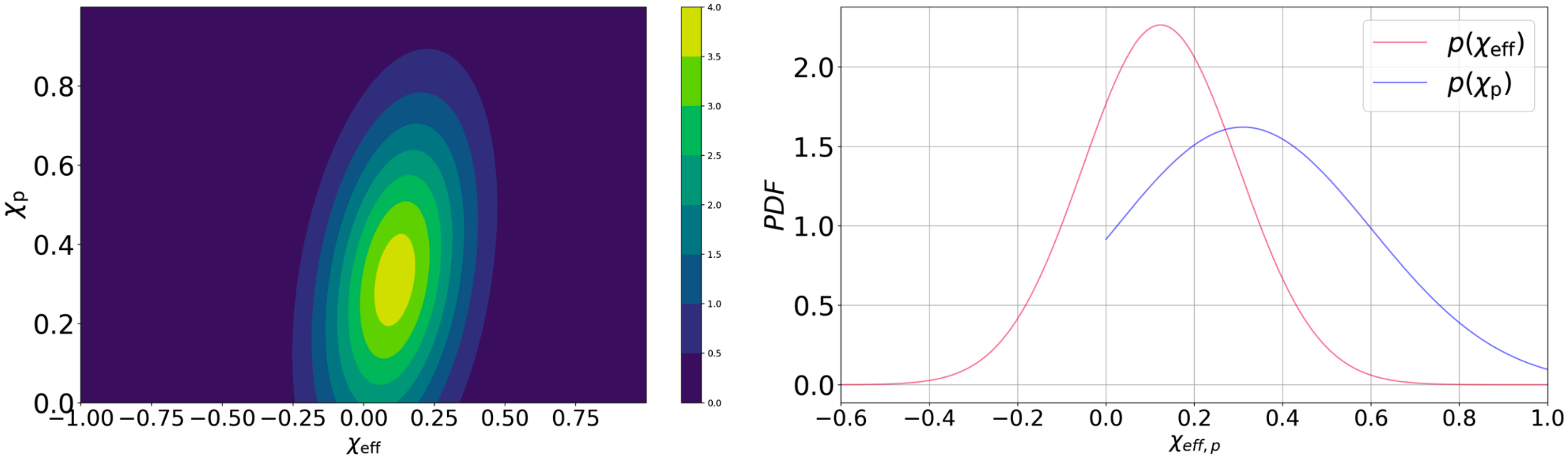
(b) and (d) :
Marginalized probability density functions

Described black-holes :

- Slowly rotating
- Mostly aligned with $\vec{L}$

Spin distributions : Gaussian

$$\{\mu_{eff} = 0.11, \sigma_{eff} = -.18, \mu_p = 0.31, \sigma_p = 0.29, \rho = 0.32\}$$



(a) (b)

- (a) : Joint 2D probability density functions
- (b) : Marginalized probability density functions

Described black-holes :

- Slight asymmetry toward aligned spins
- Slowly precessing systems