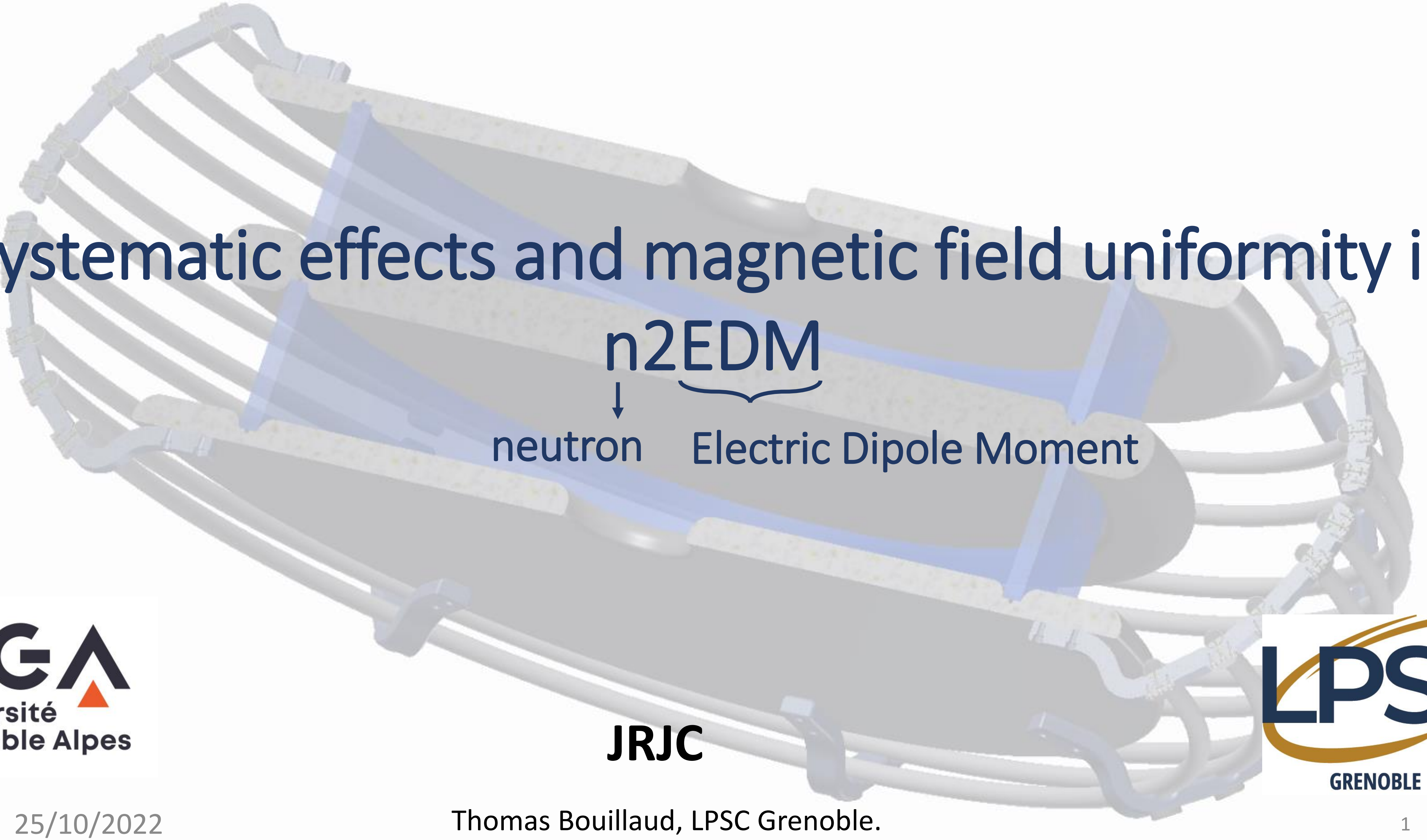


Systematic effects and magnetic field uniformity in n2EDM

n → neutron, EDM → Electric Dipole Moment

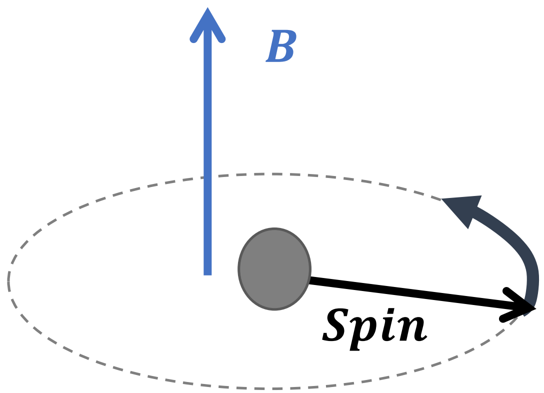
JRJC

Thomas Bouillaud, LPSC Grenoble.

Université Grenoble Alpes

LPSC GRENOBLE | MODANE

What is the electric dipole moment?



Spin ½ particle in a magnetic field

- $H = -\mu\, \boldsymbol{\sigma} \cdot \boldsymbol{B}$
- With $\boldsymbol{B} = B_0 \boldsymbol{u_z}$, precession frequency given by $\hbar 2\pi f = 2\mu\, B_0$
- Neutron in $B_0 = 1\ \mu\mathrm{T}$ → $f \approx 30\ s^{-1}$

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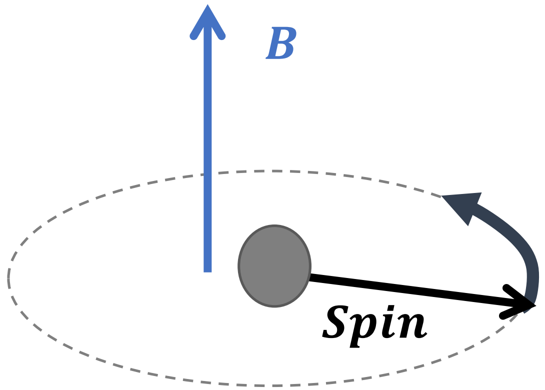

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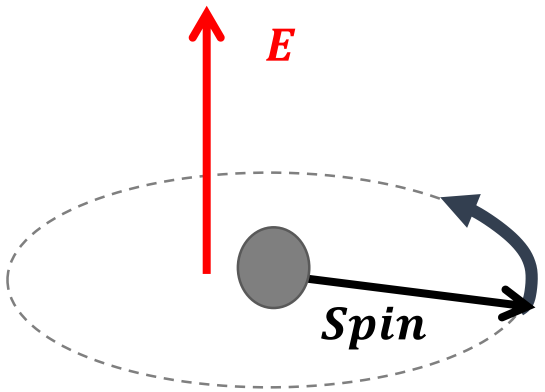

What about a spin ½ particle in an electric field?

- $H = -d\, \boldsymbol{\sigma} \cdot \boldsymbol{E}$
- With $\boldsymbol{E} = E_0 \boldsymbol{u_z}$, precession frequency given by $\hbar 2\pi f = 2d\, E_0$
- Neutron in $E = 1\ k\mathrm{V/cm}$ → $f < 4\ \mathrm{year}^{-1}$ (according to the current nEDM limit)

  ↔ **d_n almost zero**

Why measure the neutron EDM?

Cosmological motivation: explain baryon asymmetry $\eta = \frac{n_B - n_{\bar{B}}}{n_\gamma} \approx 10^{-10}$

Sakharov conditions for baryogenesis:

1. Non-conservation of baryonic number
2. Out-of-equilibrium thermal interactions
3. **Violation of C and CP symmetries**

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3. **Violation of C and CP symmetries**

EDMs are described by couplings that violate CP!

Violates T → violates CP by CPT

Formally: CP violating term (EM field and quark coupling)

$$\mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\ F^{\mu\nu} - \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\ F^{\mu\nu} \quad \xrightarrow{CP} \quad \mathcal{L} = \frac{\mu}{2}\bar{f}\sigma_{\mu\nu}f\ F^{\mu\nu} + \frac{id}{2}\bar{f}\sigma_{\mu\nu}\gamma_5 f\ F^{\mu\nu}$$

$$H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} - d\,\boldsymbol{\sigma}\cdot\boldsymbol{E} \quad \longrightarrow \quad H = -\mu\,\boldsymbol{\sigma}\cdot\boldsymbol{B} + d\,\boldsymbol{\sigma}\cdot\boldsymbol{E}$$

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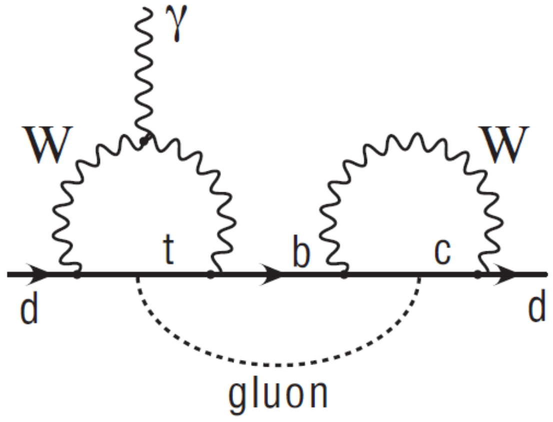
In the Standard Model:

- CKM phase contribution to quark EDMs through at least 3 loops diagrams → very negligible ($d_n \sim 10^{-32} e.\mathrm{cm}$).

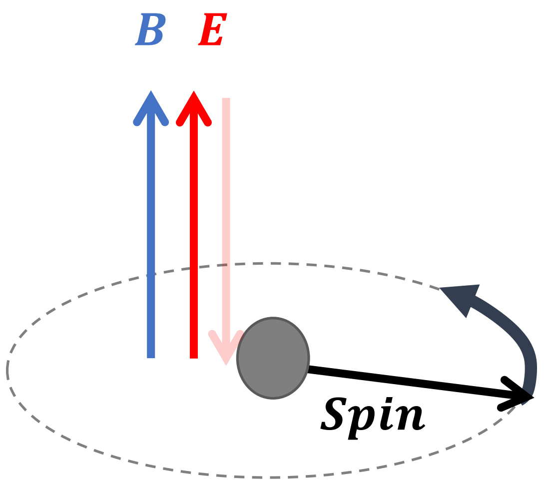


- QCD contribution $\frac{\alpha}{8\pi}\bar{\theta}\,G^{\mu\nu}\widetilde{G_{\mu\nu}}$ should generate huge EDMs ($d_n \sim 10^{-16} e.\mathrm{cm}$). : **current limit $d_n < 10^{-26} e.cm$** $\Rightarrow \bar{\theta} < 10^{-10}$(strong CP problem).

Beyond the SM:

- (ex) modified Higgs-fermion Yukawa coupling $\mathcal{L} = -\frac{y_f}{\sqrt{2}}\left(\kappa_f \bar{f}fh + i\tilde{\kappa}_f\,\bar{f}\gamma_5 fh\right)$ generates EDM at 2 loops.

How do we measure the neutron EDM?



$$2\pi f = \frac{2\mu}{\hbar} B \pm \frac{2d}{\hbar} |E| \quad \Rightarrow \quad f(\uparrow\uparrow) - f(\uparrow\downarrow) = -\frac{2}{\pi\hbar} d\,|E|$$

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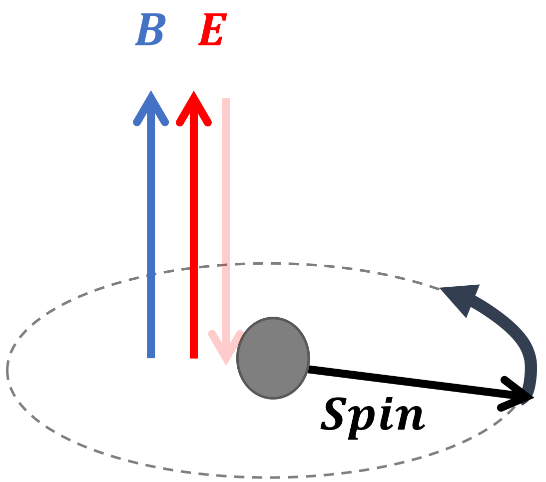

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$f_{\text{larmor}} \approx 30\ s^{-1}$

at $B = 1\mu\text{T}$

If $d_n = 10^{-26} e.\text{cm}$:

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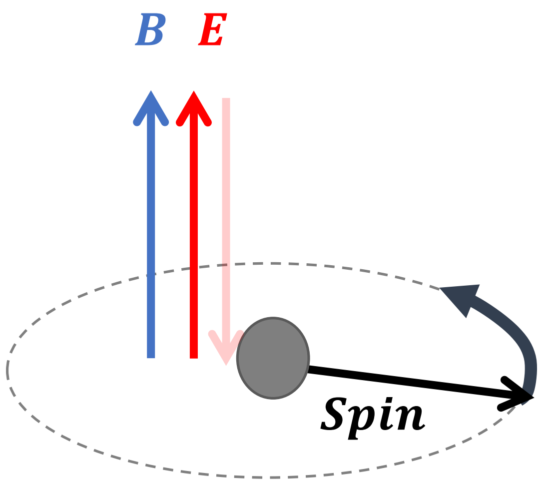

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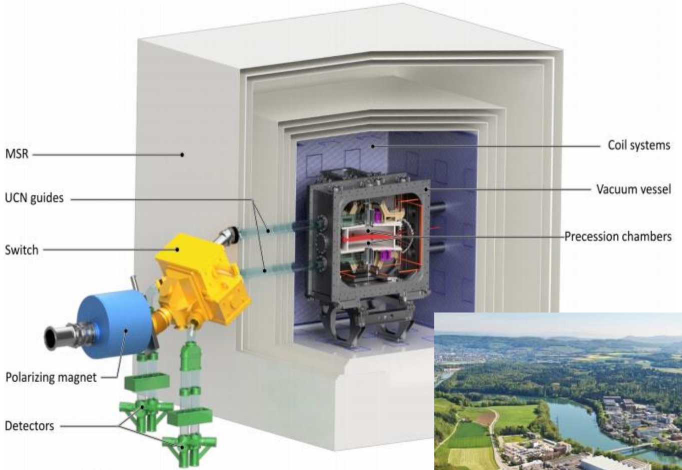

What can we do to detect something that small ?

- Maximize the interaction time → Ultra Cold Neutrons
- Maximize the statistics → Large cell volume, efficient UCN transport
- Control the magnetic field → Hg co-magnetometry, magnetic shielding (MSR, AMS), field mapping

→ Deal with systematics: false EDM, gravitational shift

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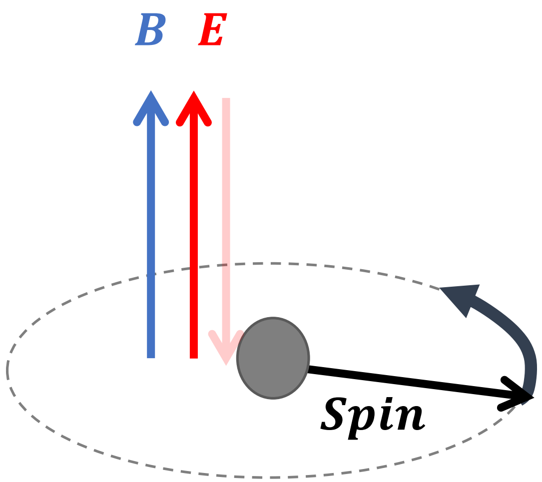

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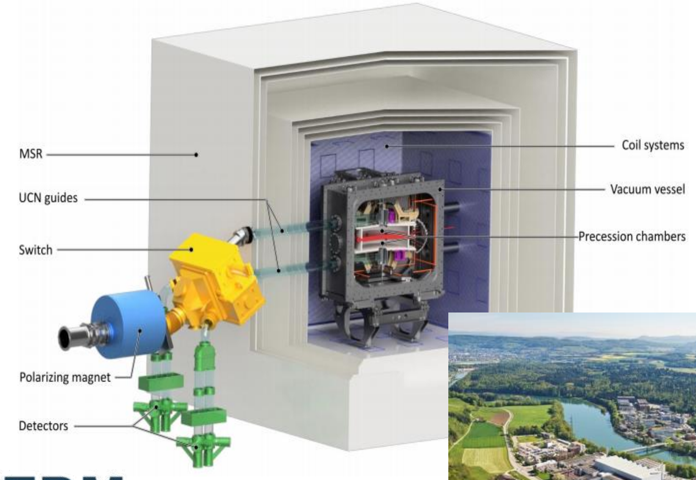

n2EDM

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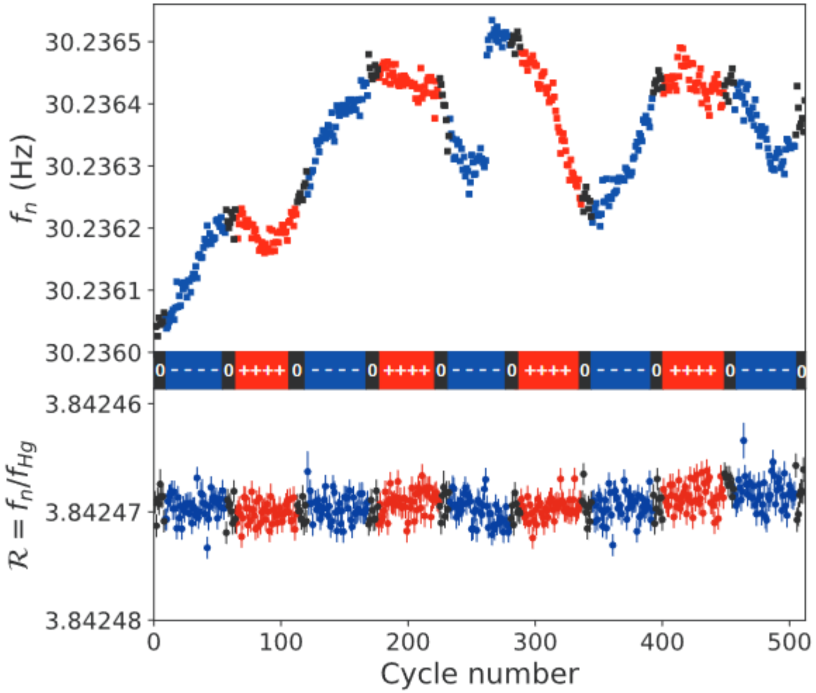
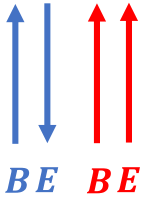
Hg co-magnetometry to compensate magnetic field fluctuations





Problem:
Uncertainty on f dominated by magnetic field fluctuations!

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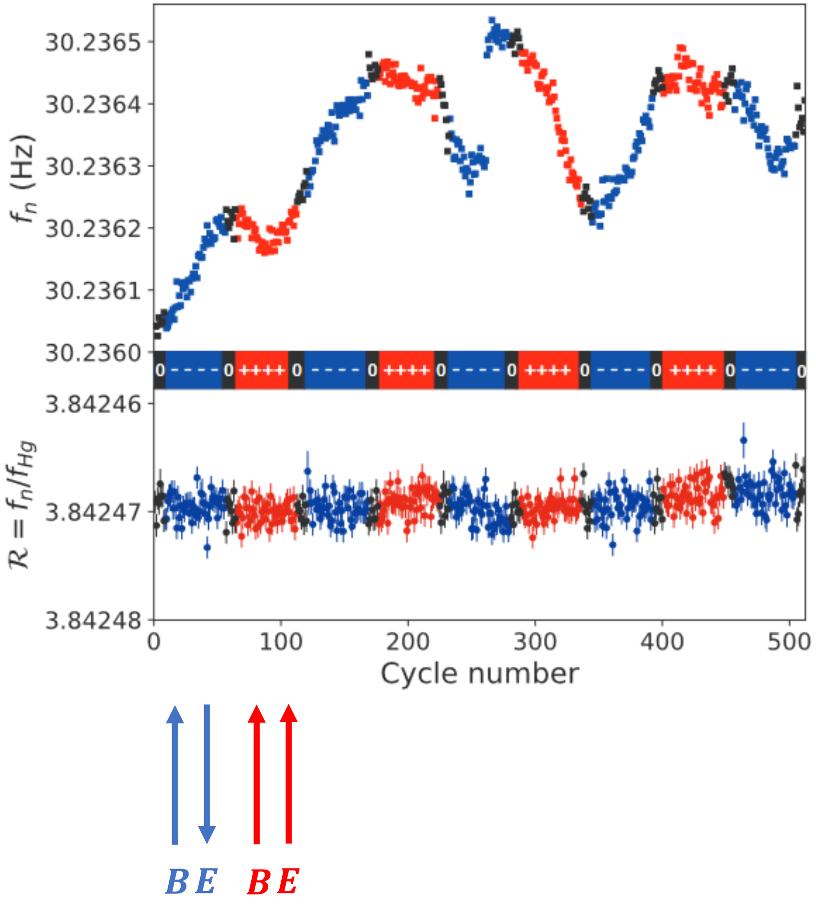

Problem:
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Solution:
Measure instead the ratio of **mercury** and neutron frequencies:

$$\mathcal{R} = \frac{f_n}{f_{Hg}} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \mp \frac{|E|}{\pi\hbar f_{Hg}} d_n$$

Contribution from EDM

$$f_n = \left|\frac{\gamma_n}{2\pi}\right| B_0 \mp \frac{d_n}{\pi\hbar}|E|$$

No contribution from EDM!

$$f_{Hg} = \left|\frac{\gamma_{Hg}}{2\pi}\right| B_0$$

…which is free from the magnetic field fluctuations!

How do we parametrize the magnetic field?

Polynomial field expansion

$$\boldsymbol{B}(\boldsymbol{r}) = \sum_{l=0}^{+\infty} \sum_{m=-l}^{l} G_{lm} \boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$$

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$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$ and $\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{0}$

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Laplace equation in spherical coordinates

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Harmonic modes $\boldsymbol{\Pi}_{\boldsymbol{lm}}(\boldsymbol{r})$ deduced from solutions of Laplace equation

TABLE IV. The basis of harmonic polynomials sorted by degree in cylindrical coordinates.

l	m	Π_ρ	Π_ϕ	Π_z
0	−1	$\sin\phi$	$\cos\phi$	0
0	0	0	0	1
0	1	$\cos\phi$	$-\sin\phi$	0
1	−2	$\rho\sin 2\phi$	$\rho\cos 2\phi$	0
1	−1	$z\sin\phi$	$z\cos\phi$	$\rho\sin\phi$
1	0	$-\frac{1}{2}\rho$	0	z
1	1	$z\cos\phi$	$-z\sin\phi$	$\rho\cos\phi$
1	2	$\rho\cos 2\phi$	$-\rho\sin 2\phi$	0
2	−3	$\rho^2\sin 3\phi$	$\rho^2\cos 3\phi$	0
2	−2	$2\rho z\sin 2\phi$	$2\rho z\cos 2\phi$	$\rho^2\sin 2\phi$
2	−1	$\frac{1}{4}(4z^2-3\rho^2)\sin\phi$	$\frac{1}{4}(4z^2-\rho^2)\cos\phi$	$2\rho z\sin\phi$
2	0	$-\rho z$	0	$-\frac{1}{2}\rho^2+z^2$
2	1	$\frac{1}{4}(4z^2-3\rho^2)\cos\phi$	$\frac{1}{4}(\rho^2-4z^2)\sin\phi$	$2\rho z\cos\phi$
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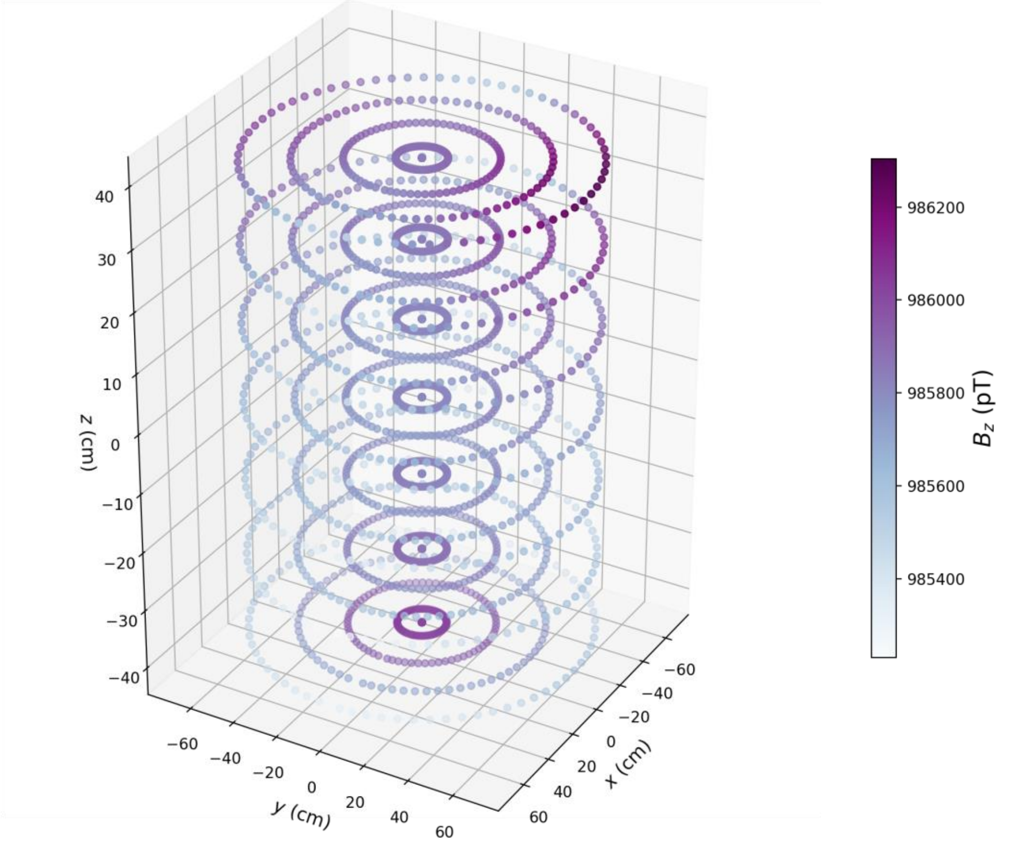
So what do we measure? The *generalized gradients* G_{lm} :

- "Online" with mercury co-magnetometry and cesium magnetometers.
- "Offline" with the mapper.

G_{lm} extraction

1) Do cylindrical map $B_z(\rho, \varphi, z)$



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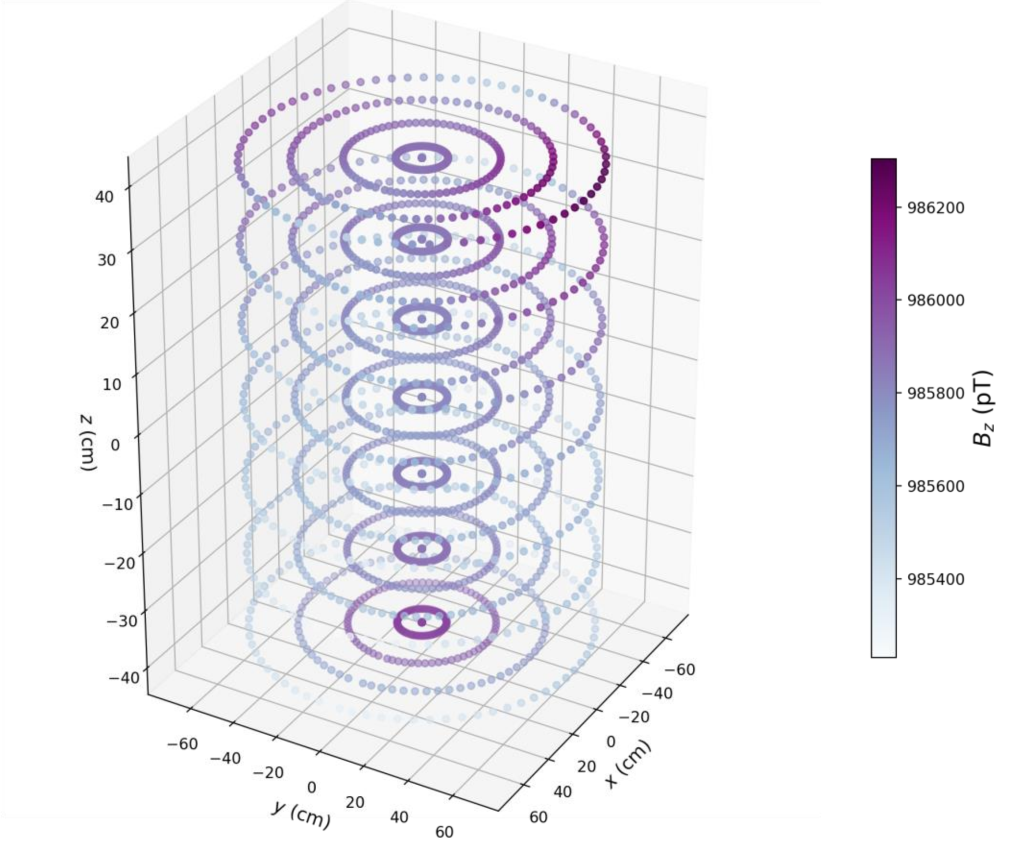
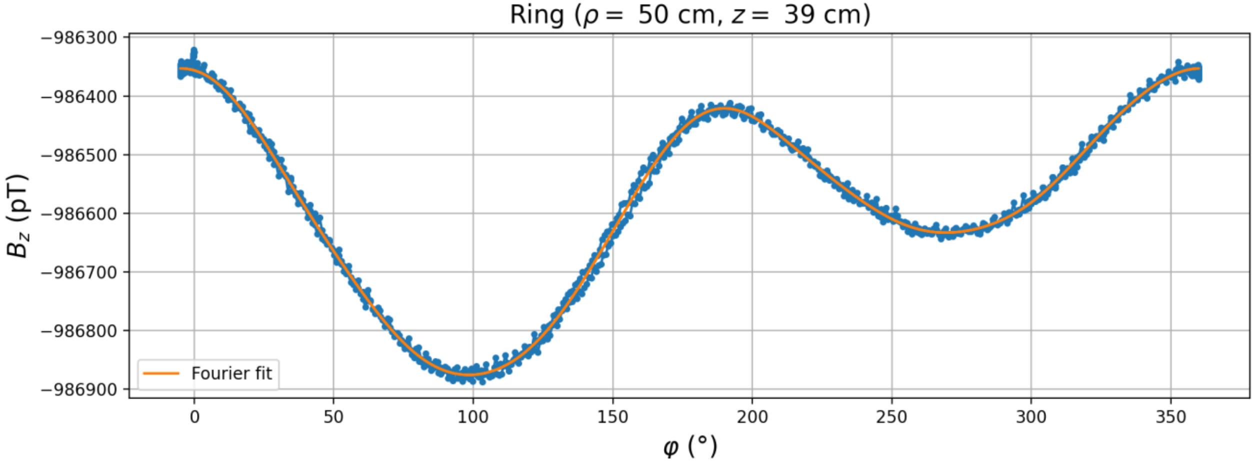


2) Fit rings with fourier series:

$$B_z(\rho, \varphi, z) = \sum_{m \geq 0} a_m^{(z)}(\rho, z)\cos(m\varphi) + b_m^{(z)}(\rho, z)\sin(m\varphi)$$



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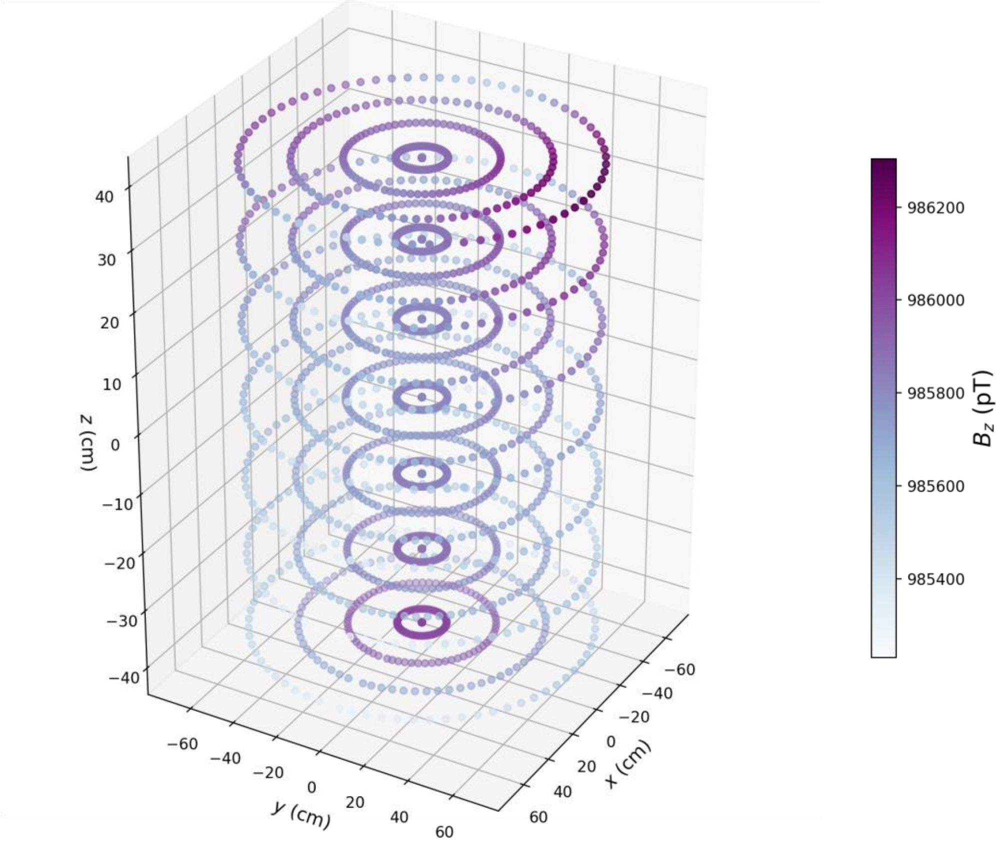

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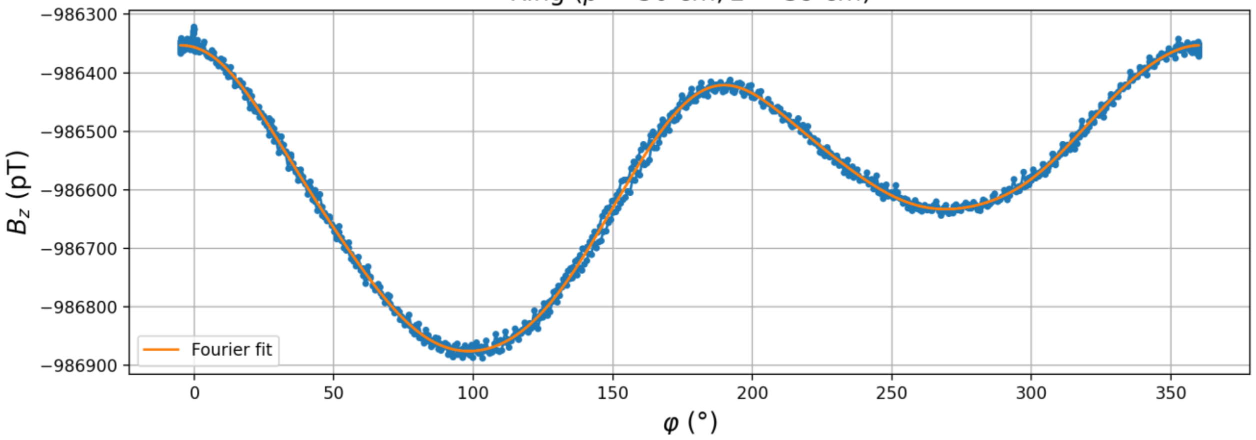

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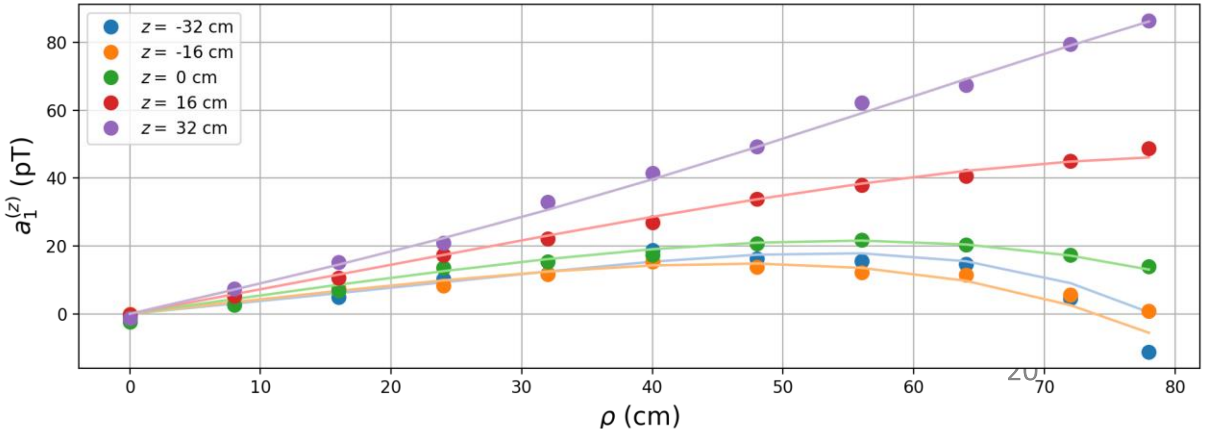

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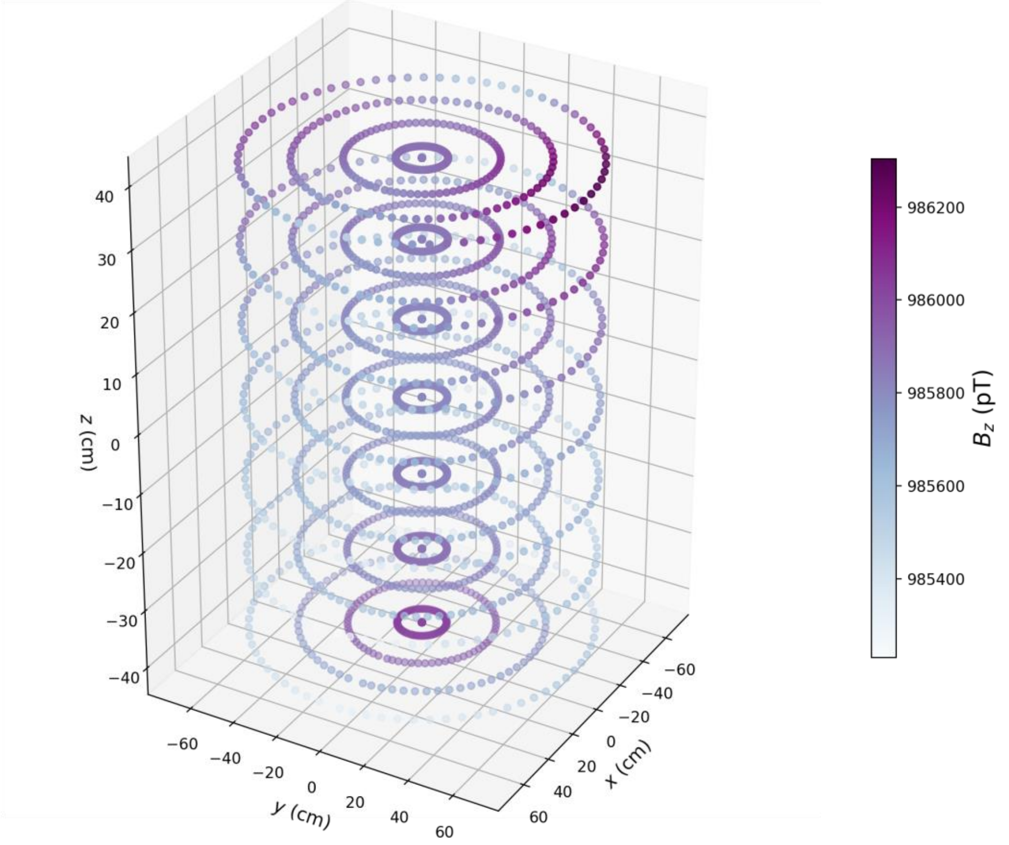

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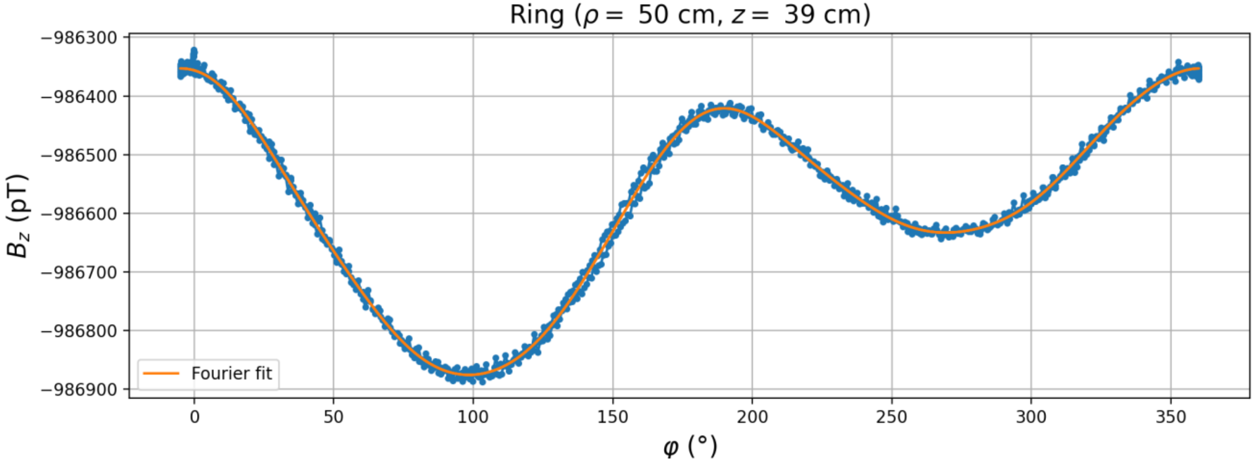

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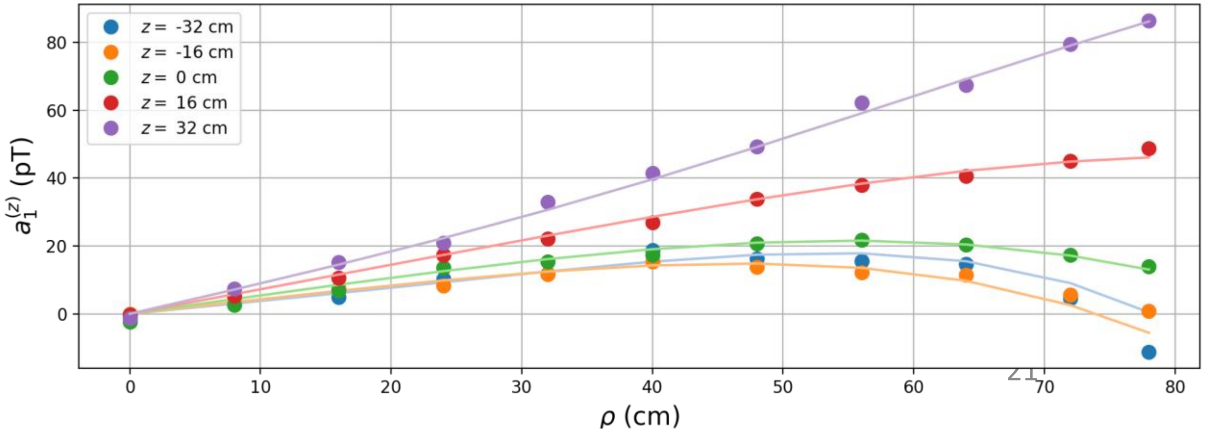
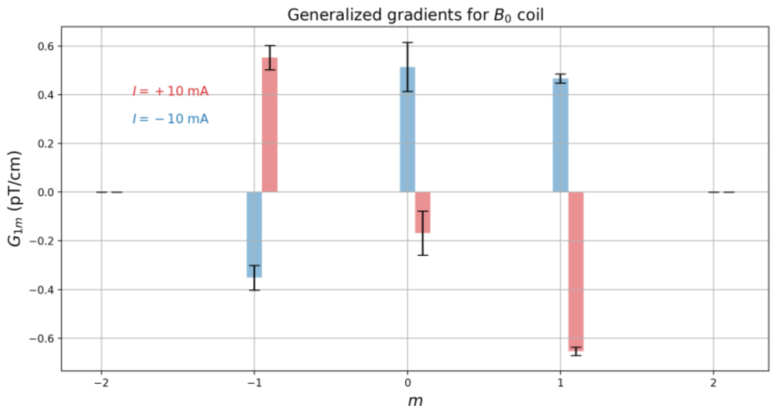
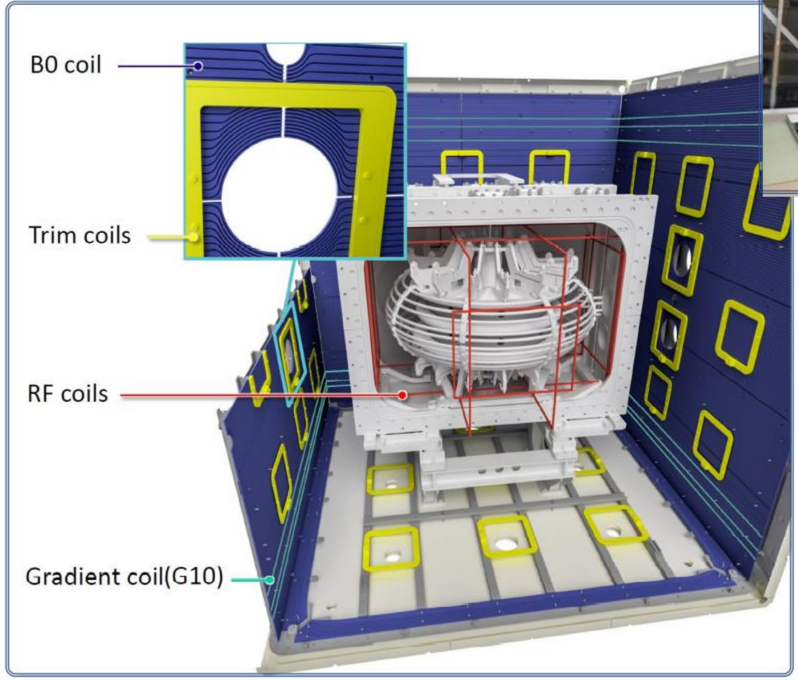


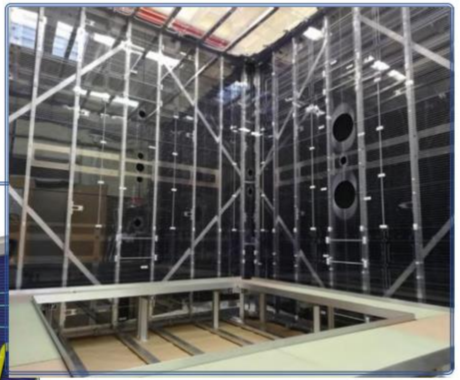
4) Get G_{lm}spectrum



One use of field mapping: moving the B_0 coil

- Produce a very uniform B0 field (1μT)
- Produce specific gradients
- Hold the UCN polarisation
- Neutron spin manipulation





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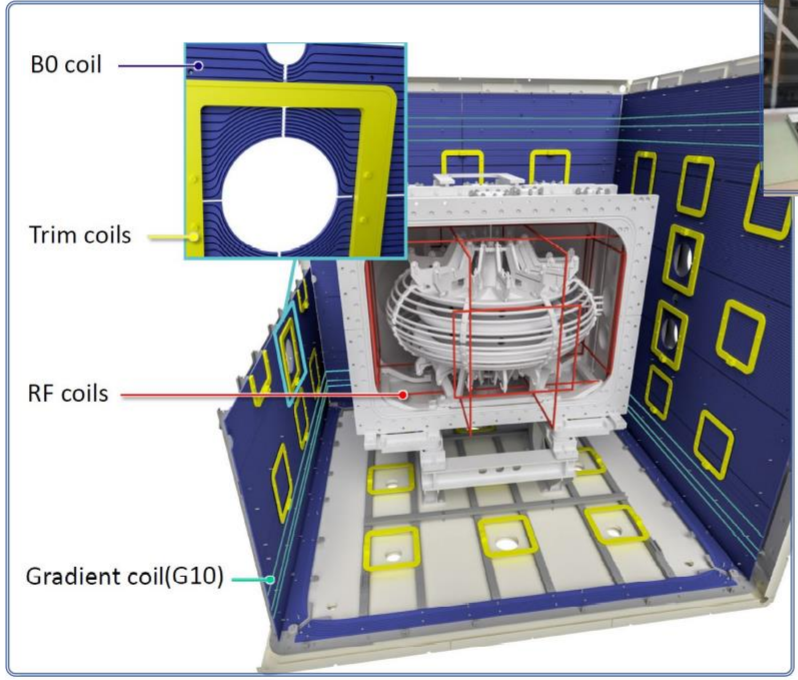

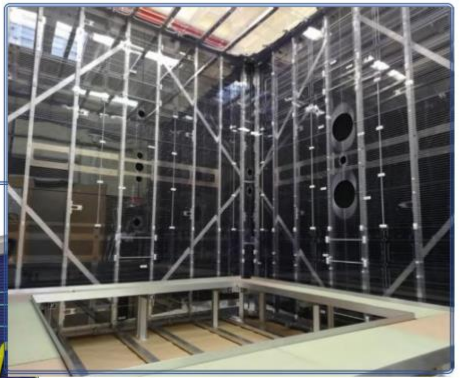

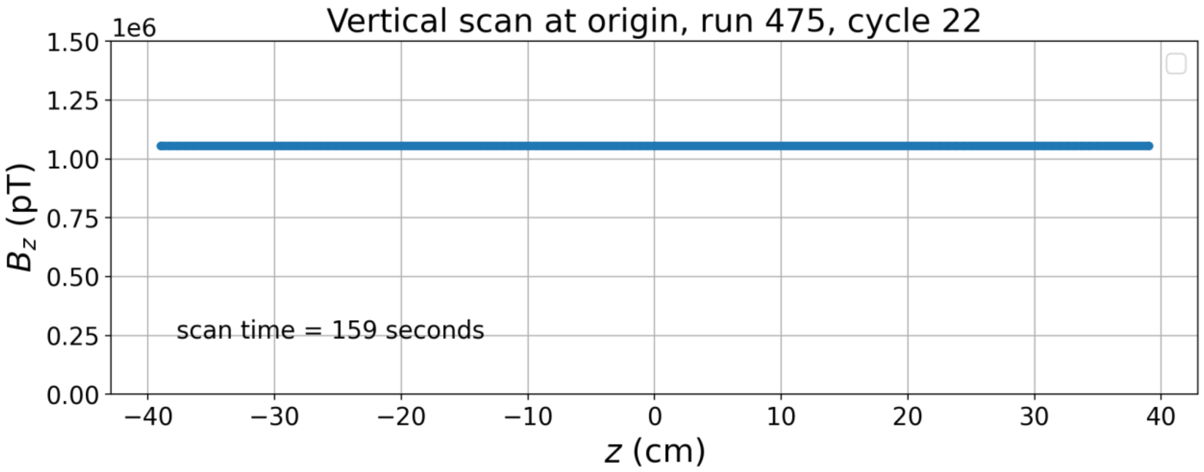

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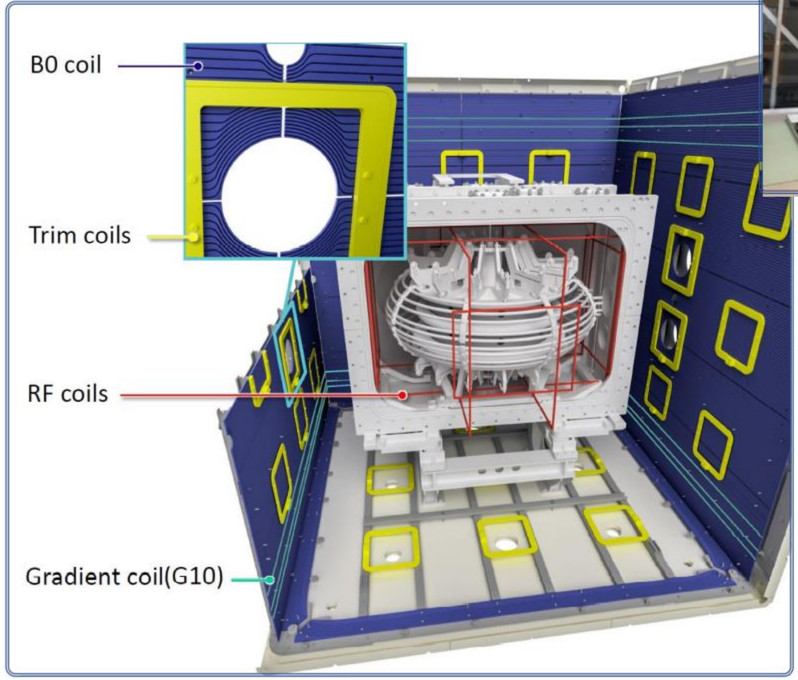

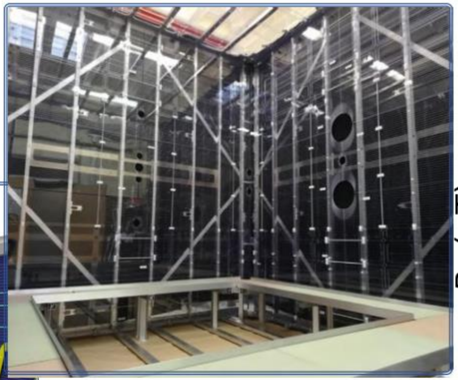

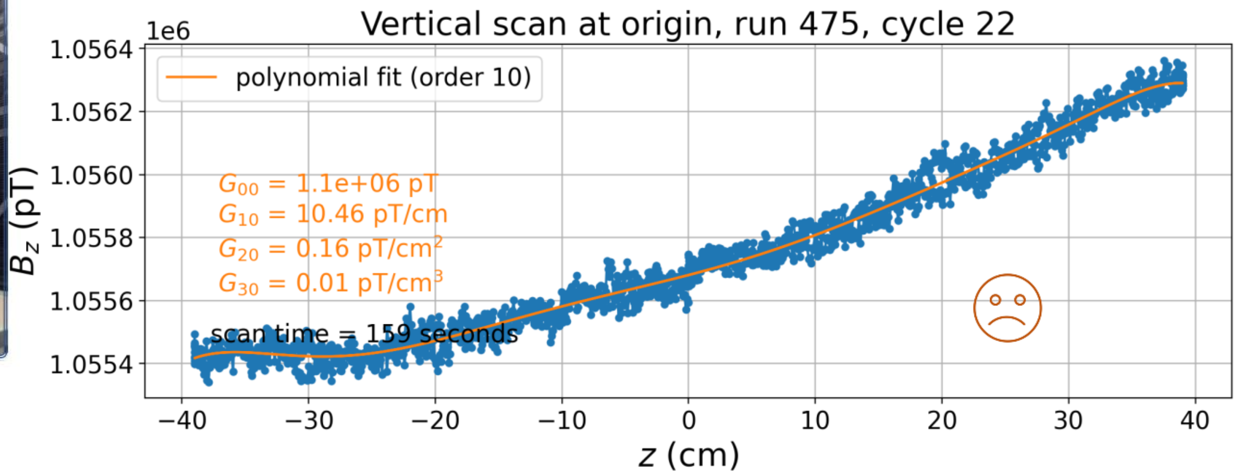

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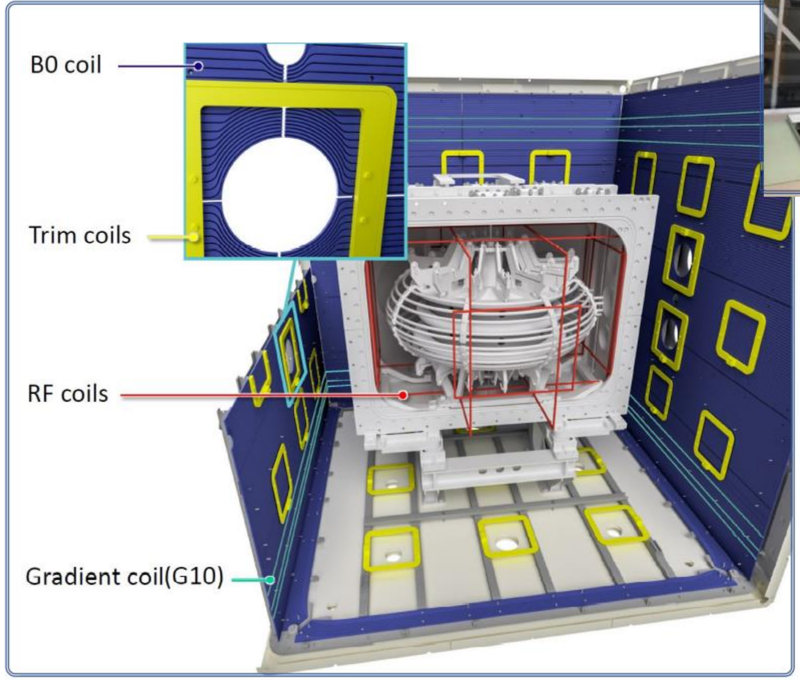
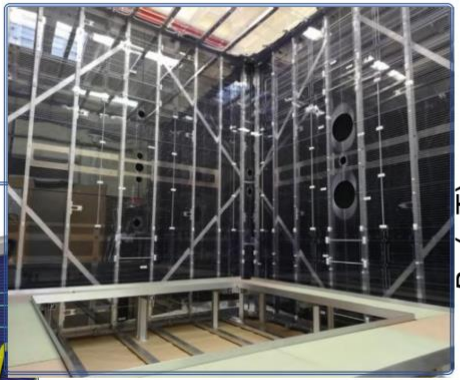
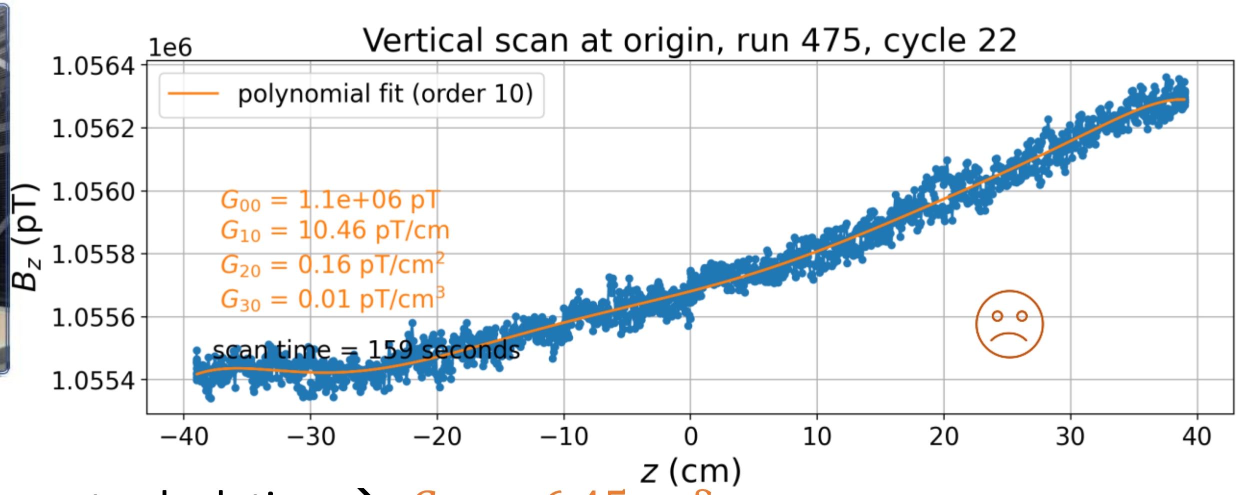
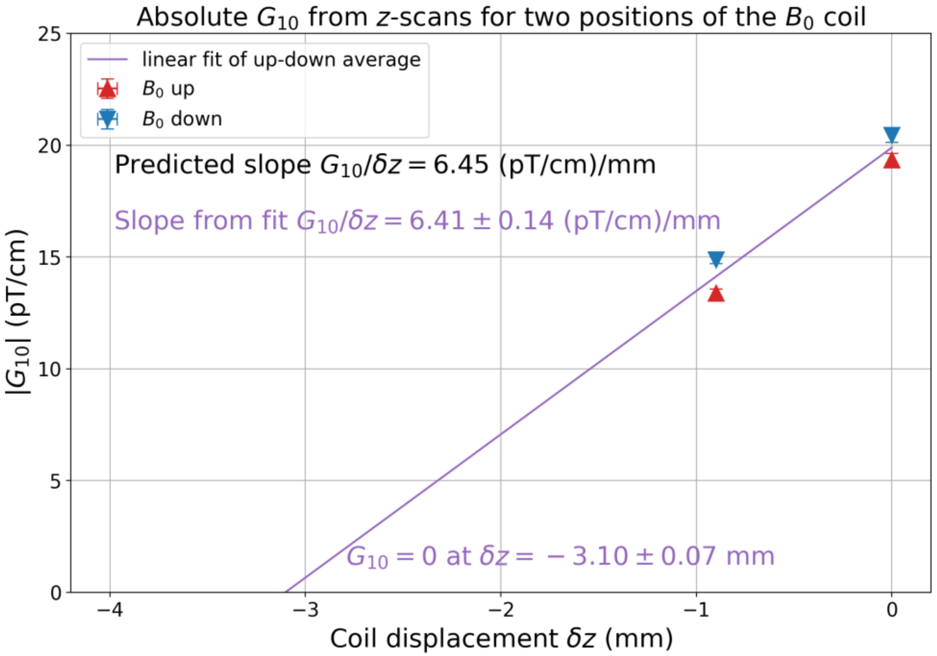






Finite Element calculation → $G_{10} = 6.45 \times \delta_z$



Result: we need to move the coil by 3mm!

An important systematic effect, the “false EDM”

Because the magnetic field is not perfectly uniform and because the mercury atoms and the neutrons do not move at the same velocity, they **do not see the same magnetic field.**
This induces extra terms in the frequency ratio that act like EDMs:

$$\mathcal{R} = \left|\frac{\gamma_n}{\gamma_{Hg}}\right| \mp \frac{2|E|}{\pi\hbar|\gamma_{Hg}B_0|}\left(d_n + d_n^{\text{false}} + d_{n\leftarrow Hg}^{\text{false}} + \cdots\right)$$

False neutron EDM induced by the false mercury EDM

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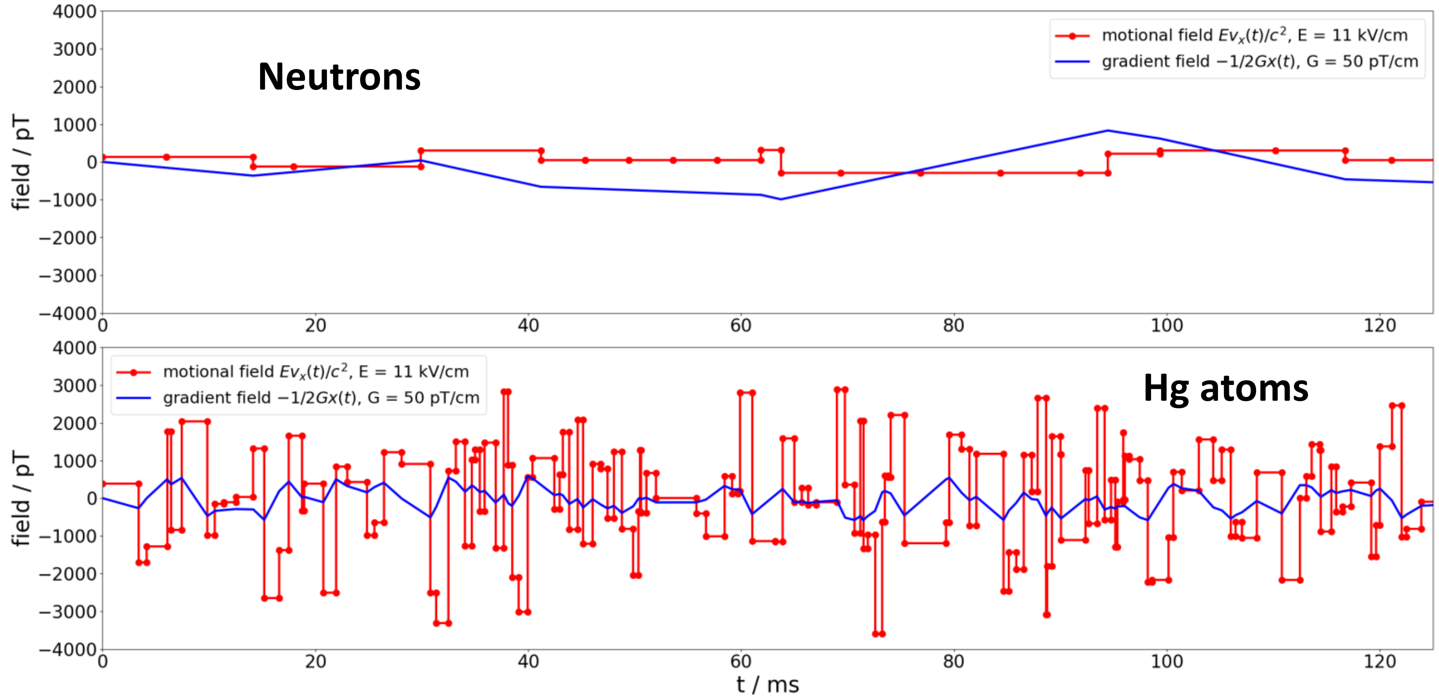
non-uniform field + motional field



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- Larger motional field larger for fast Hg atoms than for slow neutrons.

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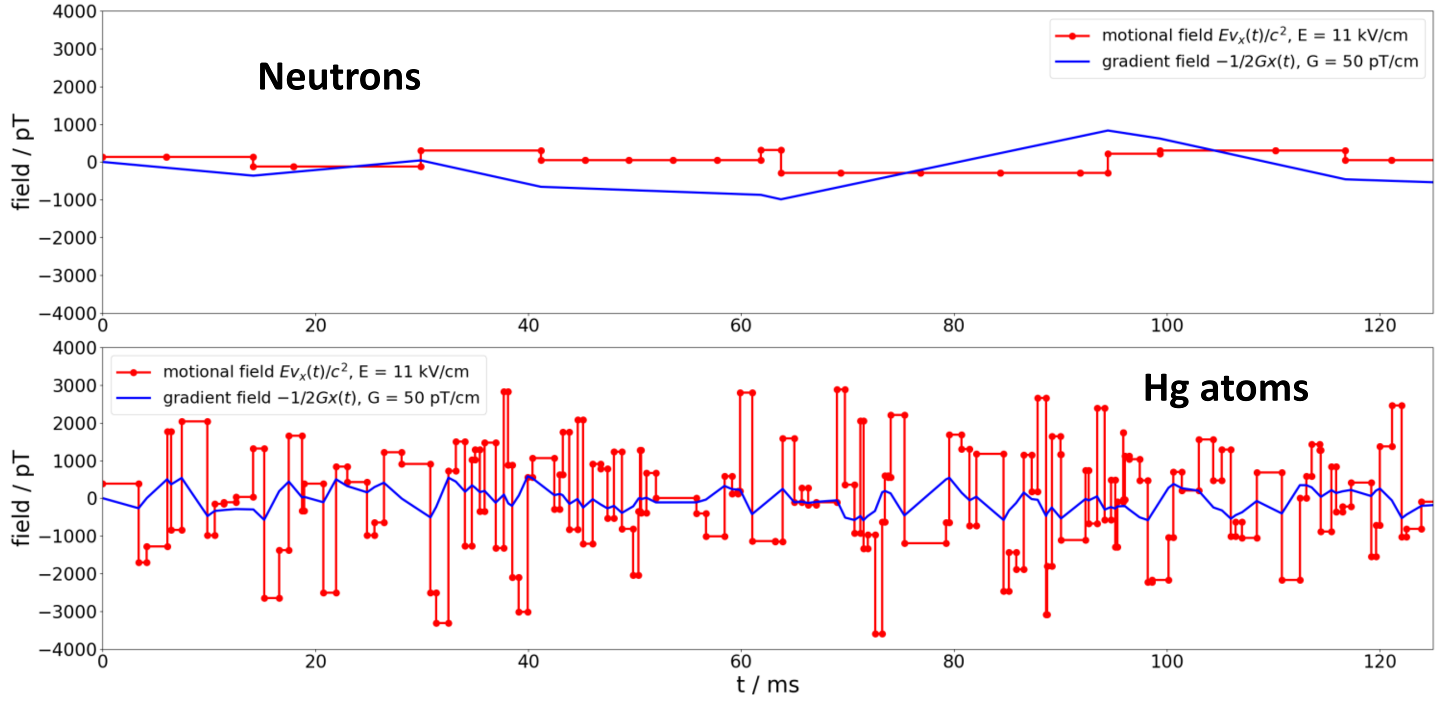
$$\tau_L = \frac{1}{\gamma B_0}$$



$v_n \approx 3\ m.s^{-1} \Rightarrow \tau_{n,c} > \tau_{n,L}$

- Larger motional field larger for fast Hg atoms than for slow neutrons.
- **"Low-frequency regime"** at $B_0 = 1\ \mu T$ for Hg atoms that have shorter correlation time than Larmor time constant.

$v_{\text{Hg}} \approx 150\ m.s^{-1} \Rightarrow \tau_{\text{Hg},c} < \tau_{\text{Hg},L}$

An expression for the false EDM

The **false** EDM is the difference in frequency **shifts** of opposite electric field configurations

$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n|}{4|E|}\left(\delta\omega_{Hg}(-E) - \delta\omega_{Hg}(E)\right)$$

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where the frequency shift is given by spin relaxation theory as a function of the fluctuating transverse magnetic field

$$\delta\omega_{Hg} = \frac{\gamma_{Hg}^2}{2}\int_0^\infty d\tau \, \mathrm{Im}\left[e^{i\omega\tau}\langle b^*(0)b(\tau)\rangle\right]$$

Conclusion: the **combination of a** **non-uniform field** **and** **moving particles** generates a systematic effect

$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar\left|\gamma_n\gamma_{Hg}\right|}{2c^2}\int_0^\infty d\tau \cos(\omega\tau)\frac{d}{d\tau}\left\langle x(\tau)B_x(0) + y(\tau)B_y(0)\right\rangle$$

How do we deal with the false EDM?

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A) Estimate it →

- @ $B_0 = 1\ \mu T$

because $d_{n\leftarrow Hg}^{\text{false}}$ has an analytical expression valid in **low frequency regime:**

$$d_{n\leftarrow Hg}^{\text{false}} = -\frac{\hbar|\gamma_n \gamma_{Hg}|}{2c^2} \langle x B_x + y B_y \rangle$$

$$= \frac{\hbar|\gamma_n \gamma_{Hg}|}{8c^2} R^2 \left(G_{10} - G_{30} \left(\frac{R^2}{2} - \frac{H^2}{4} \right) + \cdots \right)$$

…but need to know the generalized gradients accurately

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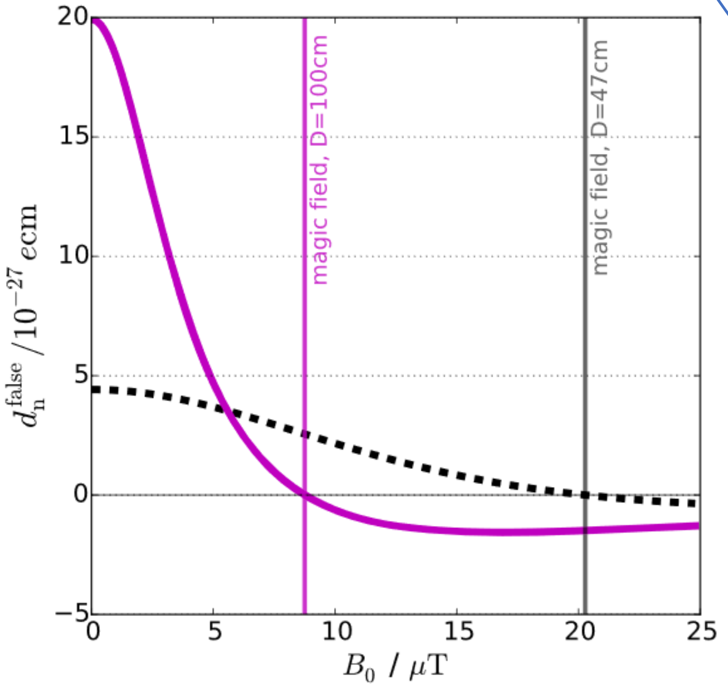
A) Estimate it

OR

B) Suppress it



False EDM produced by a linear gradient field as a function of holding field B_0

- @ $B_0 = 1\ \mu T$

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…but need to know the generalized gradients accurately.

- @ $B_m \approx 10\ \mu T$ **"magic field"**

because $d_{n\leftarrow Hg}^{\text{false}}(B_m) = 0$ for some specific field configuration

…but no analytical expression.

The magic field, take one

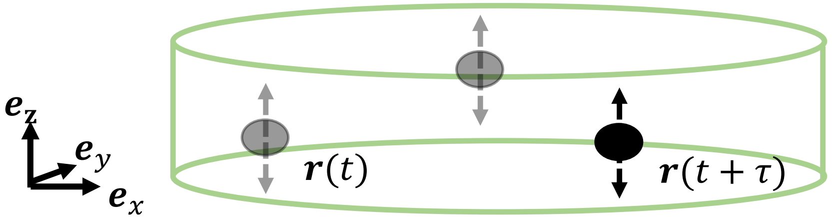
$$d_{n\leftarrow Hg}^{\text{false}} = \frac{\hbar|\gamma_n\gamma_{Hg}|}{2c^2}\int_0^\infty d\tau\cos(\omega\tau)\frac{d}{d\tau}\langle x(\tau)B_x(0)+y(\tau)B_y(0)\rangle$$

1) **Calculate the correlation function with a Monte-Carlo simulation for a given magnetic configuration**

i. Simulate trajectories $\boldsymbol{r}(t) = (x(t), y(t), z(t))$ of Hg atoms

ii. Calculate polynomial pieces $\langle x(\tau)x^i(0)y^j(0)\rangle$ with the **ergodicity property**: average over all particles ⇔ time average of one particle over infinite time

$$\lim_{T\to\infty}\frac{1}{T}\int_0^\infty dt\, x(t)x(t+\tau)$$



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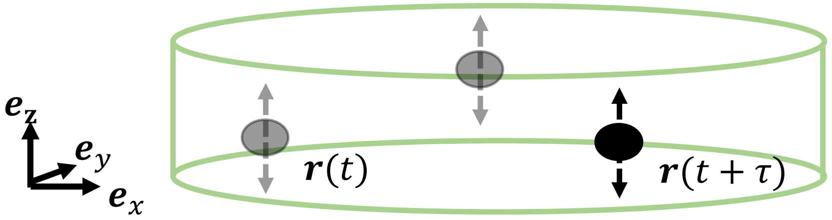
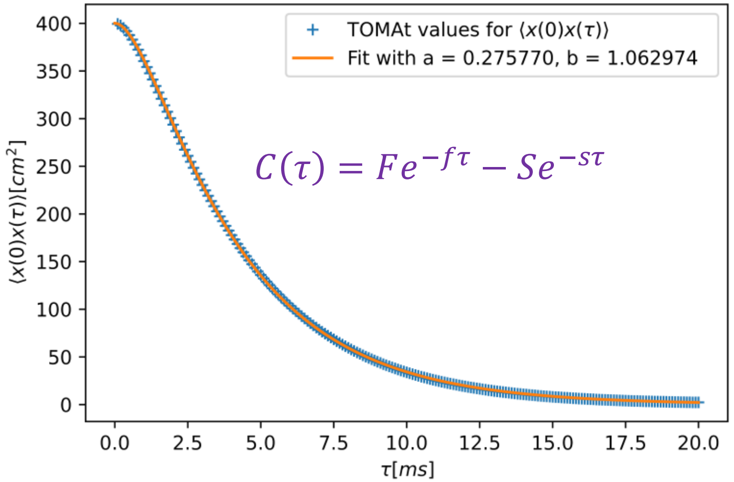


2) Fit the correlation function



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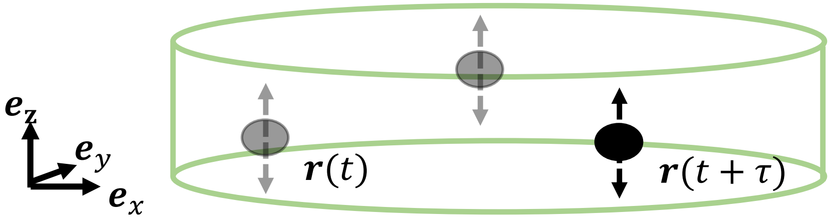

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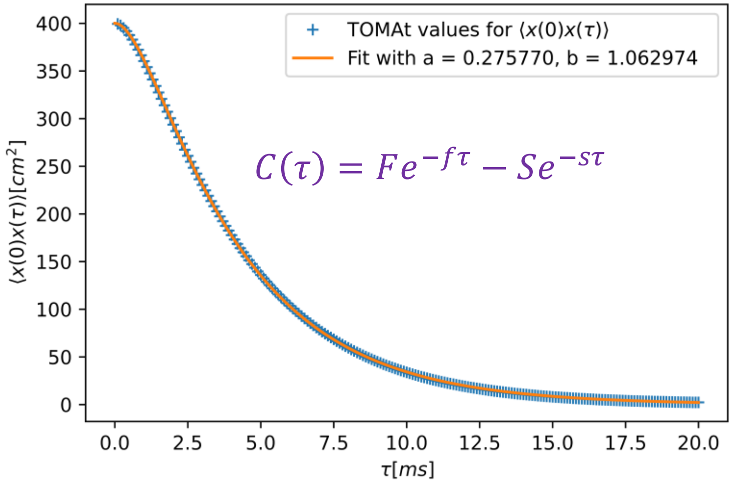
3) Calculate false EDM



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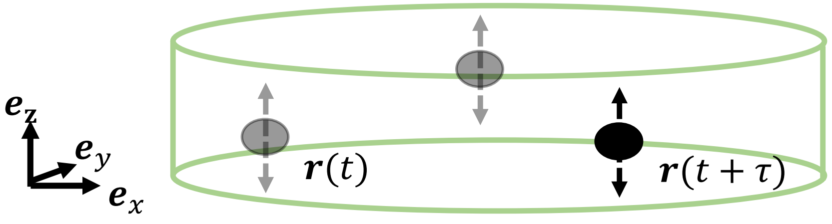

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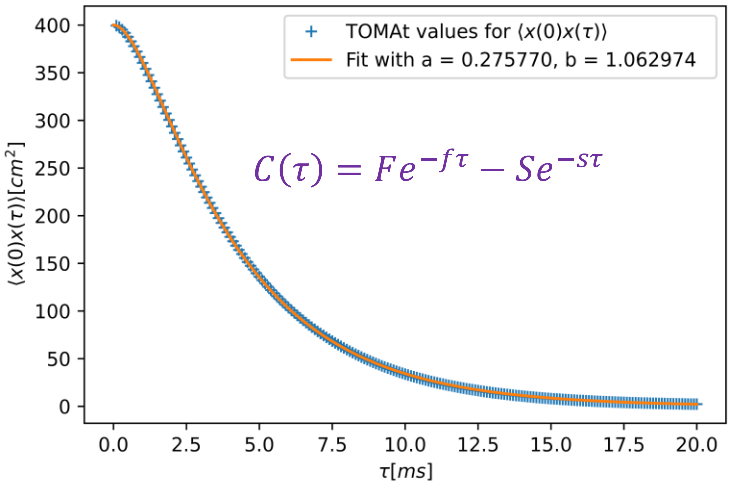
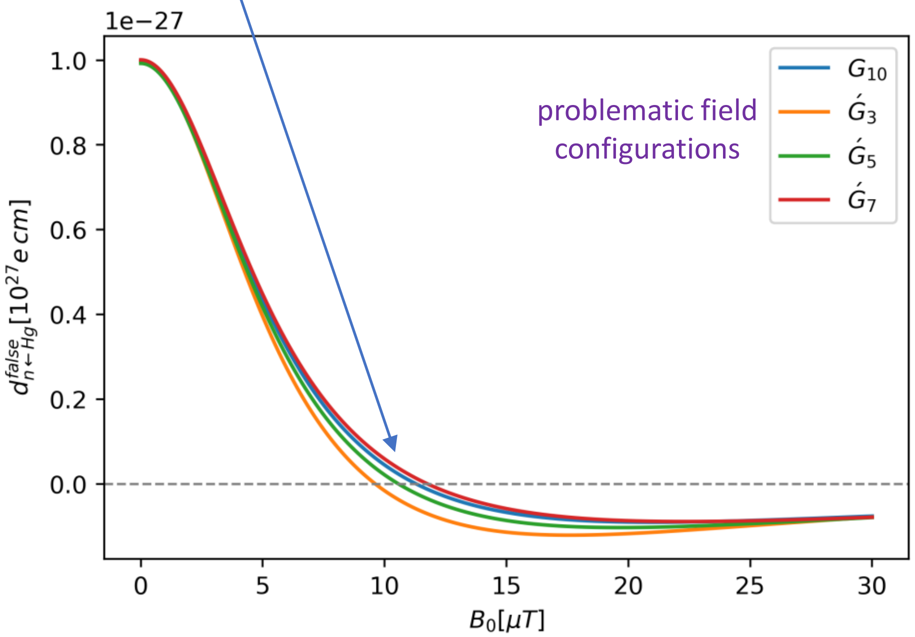


4) Set the holding field to a value that cancels the false EDM generated by this magnetic configuration

Example: the "magic" value that cancels the false EDM generated by

$$\boldsymbol{B}(x,y,z) = G_{10}\begin{pmatrix} -x/2 \\ -y/2 \\ z \end{pmatrix}$$

is $B_{\text{mogette}} = 11.3\ \mu\text{T}$



The magic field, take two

- 1st method is biased by the correlation function fit
- The correlation function of a signal is linked to its Power Spectral Density via the **Wiener-Khinchin theorem**

$$S_{ij}(\omega) = \int_{-\infty}^{+\infty} d\tau \, \langle x_i(0) x_j(\tau) \rangle e^{-i\omega\tau}$$

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For a linear vertical gradient field:

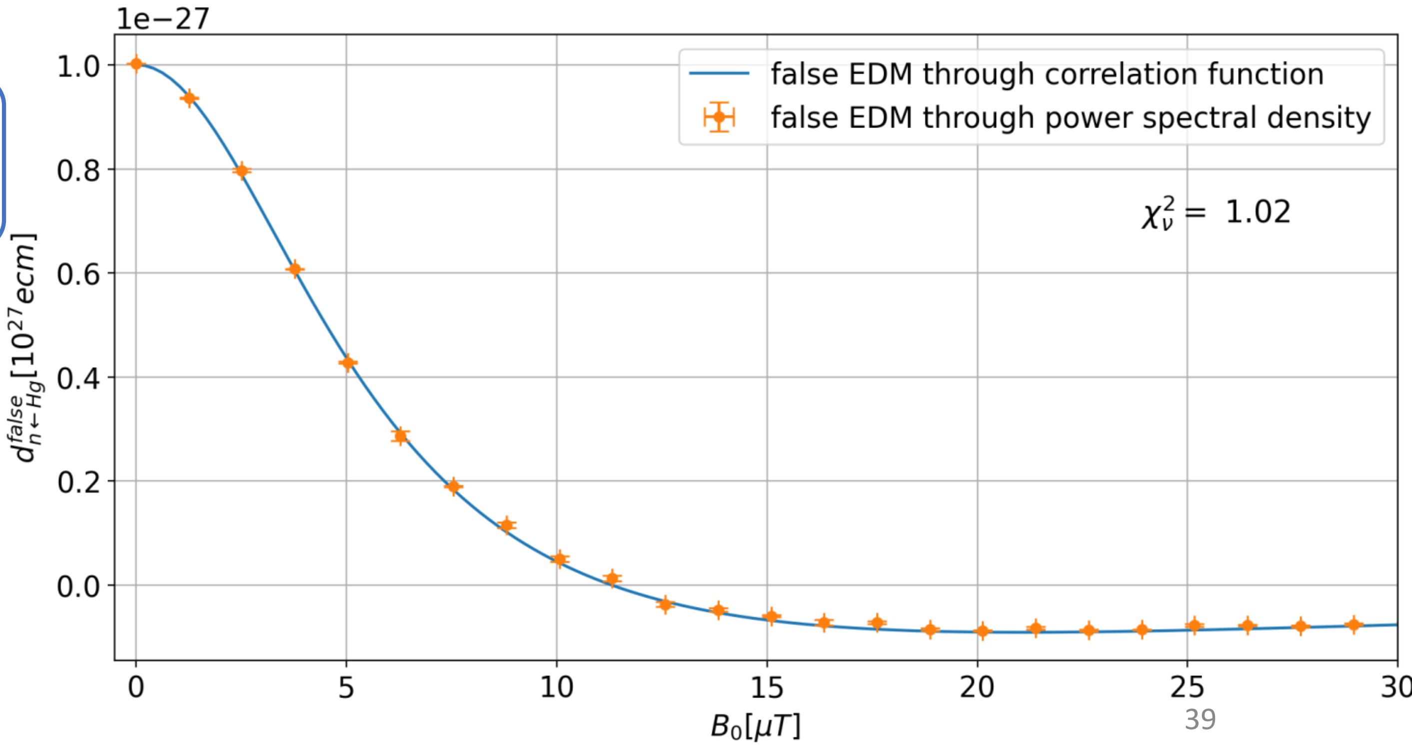
$$d_{n\leftarrow Hg}^{\text{false}}(\omega_0) = -\frac{\hbar |\gamma_n \gamma_{Hg}|}{4\pi c^2} \text{P.V} \int_{-\infty}^{+\infty} d\omega \, \omega \, \frac{S_{xx}(\omega) + S_{yy}(\omega)}{\omega - \omega_0}$$

$$\propto \frac{1}{N_t} \sum_{j,k,l}^{N_t, N_c, N_c} \frac{x_k x_l}{\Delta T_j \omega_0} \sum_{n,m} c_{klmn}(t_{mn}) I_m(\omega_0 t_{mn})$$

(some sum of converging integrals in complex plane)

$$I_m(\omega_0 t) = P.V \int_{-\infty}^{+\infty} d\nu \frac{e^{i\omega_0 t\nu}}{\nu^m(\nu - 1)}$$

$$= i\pi \, \text{sign}(\omega_0 t) \left(e^{i\omega_0 t} - \sum_{k=0}^{m-1} \frac{(i\omega_0 t)^k}{k!} \right)$$



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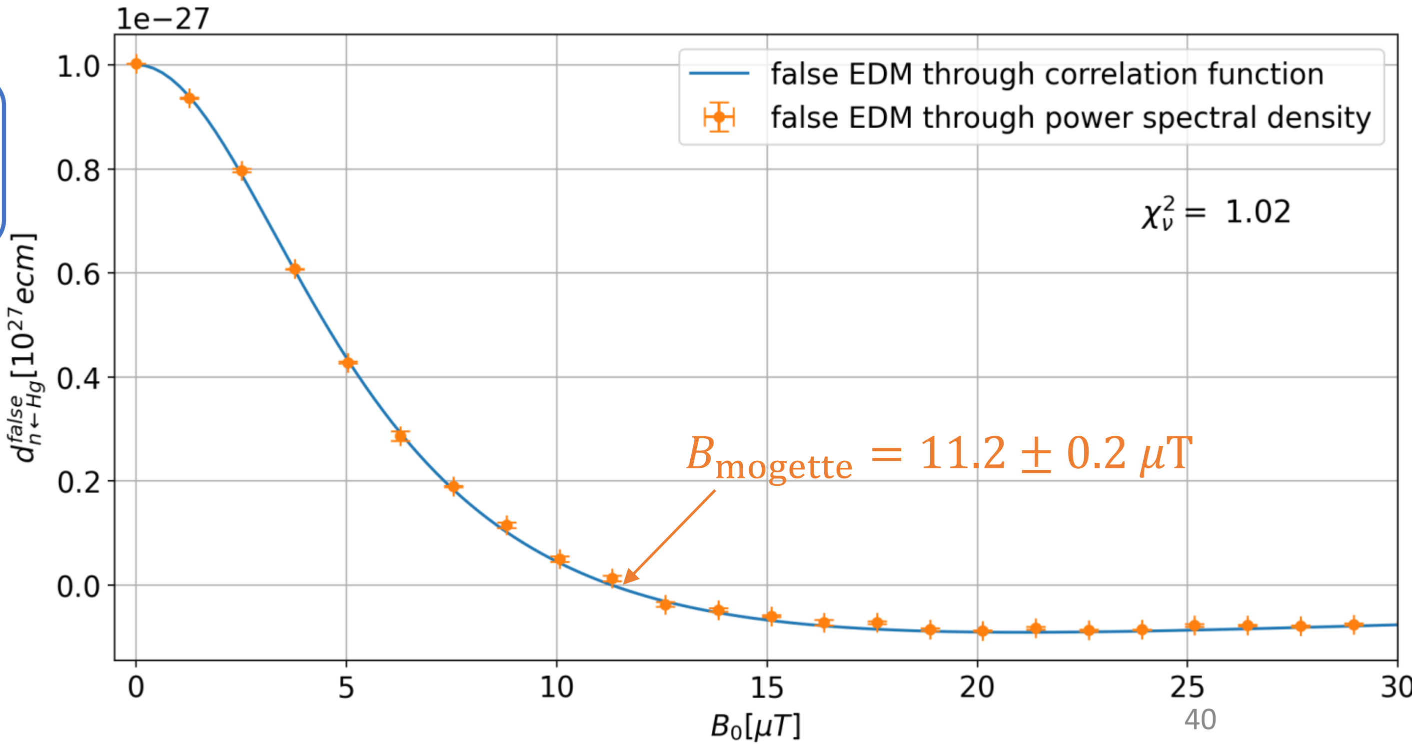


Conclusion

- The combination of non-uniformities and relativistic effects in the magnetic field generate a “false EDM”
- This false EDM can be estimated at low fields or suppressed at specific high fields (“magic fields”)
- Different challenges:
  1. <u>Low fields:</u> require accurate measurement of generalized gradients
  2. <u>Magic fields:</u> require accurate numerical estimation of magic values

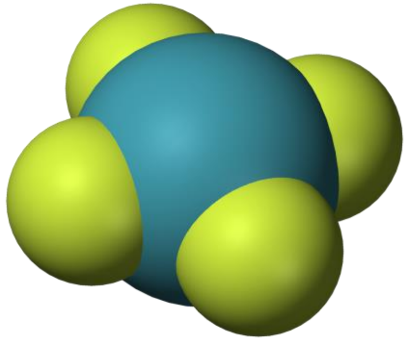
3.1 Symmetries of the B_0 coil

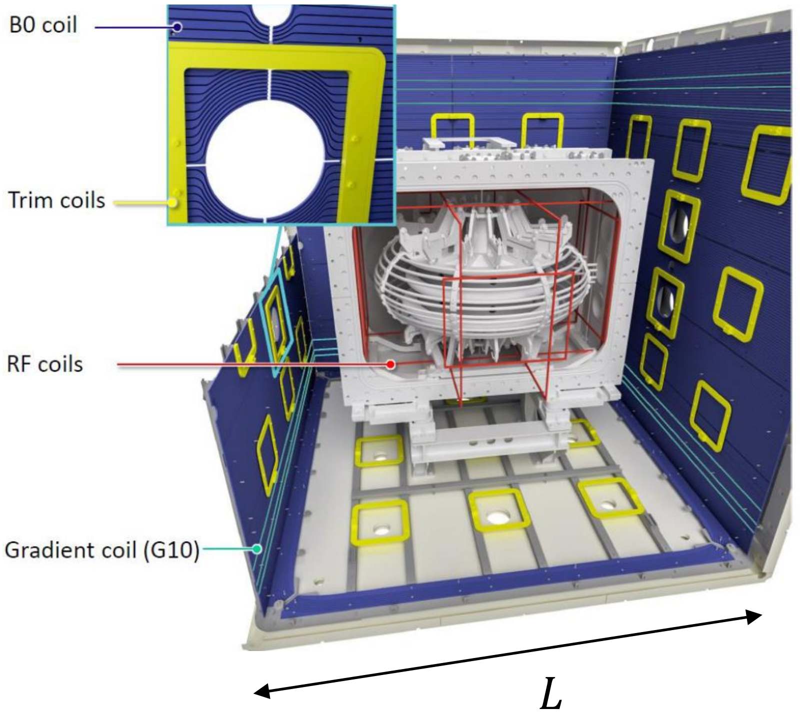
The coil system is described by the vector field

$$I(r) = \begin{cases} (0, -I_0, 0) \text{ if } x = L, |y| < L \\ (I_0, 0, 0) \text{ if } y = L, |x| < L \\ (0, I_0, 0) \text{ if } x = -L, |y| < L \\ (-I_0, 0, 0) \text{ if } y = -L, |x| < L \end{cases}$$

$r = (x, y, z)$

Xenon tetrafluoride





Consider the set of linear transformations

$$D_{4h} = \{\mathbb{I}_3, P, R_z, R_z^2, R_z^{-1}, \sigma_x, \sigma_{xy}, \sigma_y, \sigma_{-xy}, \sigma_z, R_z', R_z^{-1'}, R_x^2, R_y^2, \sigma_{xy}', \sigma_{-xy}'\}$$

$$R_z = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \ldots$$

- This set together with matrix multiplication $(D_{4h}, \times)$ is a group.
- This group acts on the coil system through a **representation** (V_c, ρ_c) of D_{4h} :

$$\rho_c(M)\colon I(r) \to I'(r) = M I(M^{-1} r) \equiv \rho_c(M) I(r)$$

satisfies $\rho_c(MN) = \rho_c(M)\,\rho_c(N), \forall M, N \in D_{4h}$

ρ_c **character** table

irrep	I	P	σ_x	σ_y	σ_z	R_x^2	R_y^2	R_z^2	R_z	R_z^{-1}	σ_{xy}	σ_{-xy}	R_z'	$R_z^{-1\prime}$	σ_{xy}'	σ_{-xy}'
ρ_c	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1

- ρ_c is an irreducible representation.
- All $\rho_c(M)$, $M \in D_{4h}$ are **symmetries** of the coil system because they satisfy:

$$I'(r) = \pm I(r) \Leftrightarrow \rho_c(M) = \pm 1$$

A few facts:

- D_{4h} is a subgroup of the orthogonal group $O(3)$
  $D_{4h} \subset O(3) = \{M \in GL(3, \mathbb{R}) | M^T M = \mathbb{I}_3\}$
- D_{4h} is generated by $\{R_z, \sigma_x, \sigma_z\}$
  $\langle R_z, \sigma_x, \sigma_z \rangle = D_{4h}$

apply σ_x, σ_z, and $\sigma_x \sigma_z$ to $\langle R_z \rangle = \{\mathbb{I}_3, R_z, R_z^2, R_z^{-1}\} \sim \mathbb{Z}_4$

Curie principle → symmetries in the coil system generate symmetries in the magnetic field.

The group D_{4h} acts on the magnetic field $B(r)$ through a **representation** (V_b, ρ_b) of D_{4h} :

$$\rho_b(M): B(r) \to B'(r) = \det(M)\, MB(M^{-1}r)$$

This alone isn't very useful for us, as we describe the magnetic field with the harmonic decomposition $B(r) = \sum_{l=1}^{+\infty} \sum_{m=-l}^{l} G_{lm} \Pi_{lm}(r)$.

→ look for symmetries in the generalized gradients.

Step (1): construct an n-sized vector:

$$g = (G_{0-1}, G_{00}, G_{0-1}, \dots, G_{L,-L-1}, \dots, G_{L,L+1})$$

with $n = (L+1)(L+3)$, and such that

$$B(r) = P(r)g,$$ where $P(r)$ is some $3 \times n$ matrix.

Step (2): find another representation $(\mathbb{R}^n, \rho)$ of D_{4h} which describes the action of D_{4h} on g and satisfies for all $M \in D_{4h}$:

$$B'(r) = \det(M)\, MP(M^{-1}r)g = P(r)\rho(M)g$$

Step (3): decompose ρ into a direct sum of irreducible representations (irreps).

→ Irreps of dimension 1 describe symmetries of the generalized gradients iff their character table matches that of the coil system.

More specifically:

Only need to find irreps for generators R_z, σ_x, σ_z

irrep	m	σ_x	σ_y	σ_z	R_z
ρ_0	0	-1	-1	1	1
ρ_{1l}	$(1,-1)$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$	$(-1)^{l+1}\mathrm{I}_2$	$\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$
ρ_{2l}	2	-1	-1	$(-1)^l$	-1
ρ_{-2l}	-2	1	1	$(-1)^l$	-1
ρ_{3l}	$(3,-3)$	$\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$	$\begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}$	$(-1)^{l+1}\mathrm{I}_2$	$\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$

We can check that all these representations are irreducible using the orthogonality theorem

$$\frac{1}{|G|} = \sum_{g \in G} \chi^{(\mu)}(g)\chi^{(\nu)}(g^{-1}) = \delta^{\mu\nu}$$

→ example for ρ_{1l}: $\sum_{M \in D_{4h}} \mathrm{Tr}[\rho_{1l}(M)] = 16$ ☺

→ Explicit generalized gradients **character table** for $L = 4$.

l	m	I	P	σ_x	σ_y	σ_z	R_x^2	R_y^2	R_z^2	R_z	R_z^{-1}	σ_{xy}	σ_{-xy}	R_z'	$R_z^{-1\prime}$	σ'_{xy}	σ'_{-xy}
0	0	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1
1	−2	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	−1	1	1	1	1
1	0	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	1	1
1	2	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	1	1	−1	−1
2	−2	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
2	0	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1
2	2	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	1	1
3	−4	1	−1	1	1	−1	−1	−1	1	1	−1	1	1	−1	−1	−1	−1
3	−2	1	−1	1	1	−1	−1	−1	1	−1	−1	−1	−1	1	1	1	1
3	0	1	−1	−1	−1	−1	1	1	1	1	1	−1	−1	−1	−1	1	1
3	2	1	−1	−1	−1	−1	1	1	1	−1	−1	1	1	1	1	−1	−1
3	4	1	−1	−1	−1	−1	1	1	1	1	−1	−1	−1	−1	−1	1	1
4	−4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	−2	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1
4	0	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1
4	2	1	1	−1	−1	1	−1	−1	1	−1	−1	1	1	−1	−1	1	1
4	4	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1

Recall: ρ_c **character** table

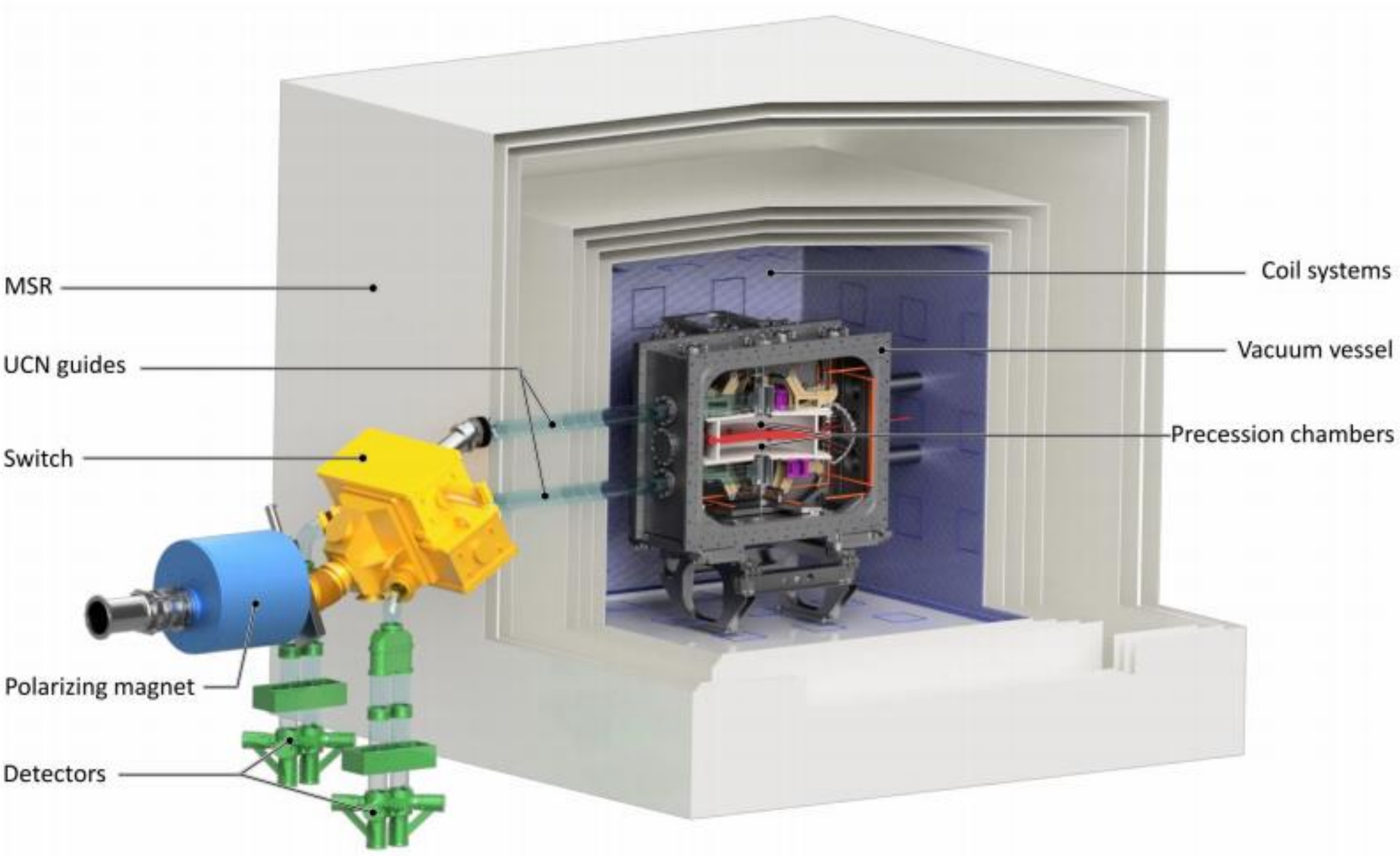
irrep	I	P	σ_x	σ_y	σ_z	R_x^2	R_y^2	R_z^2	R_z	R_z^{-1}	σ_{xy}	σ_{-xy}	R_z'	$R_z^{-1\prime}$	σ'_{xy}	σ'_{-xy}
ρ_c	1	1	−1	−1	1	−1	−1	1	1	1	−1	−1	1	1	−1	−1

Conclusion:

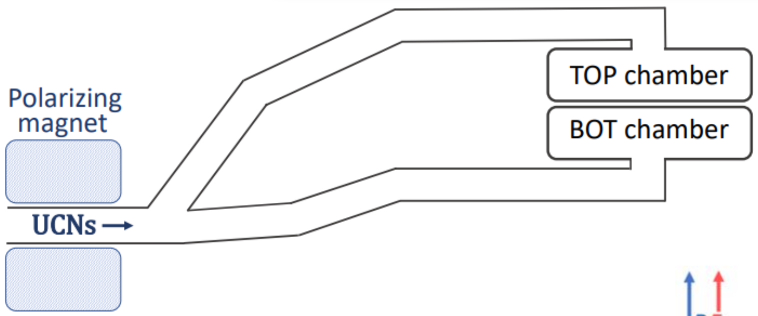
The generalized gradients $G_{00}, G_{20}, \ldots, G_{2n,0}$ are unaffected by transformations that preserve the B_0 coil symmetries.

→ They are said to be *allowed* by the symmetries of the coil.

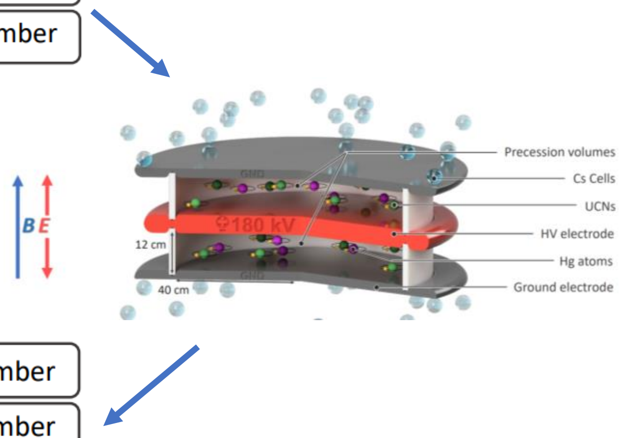
Overview of n2EDM

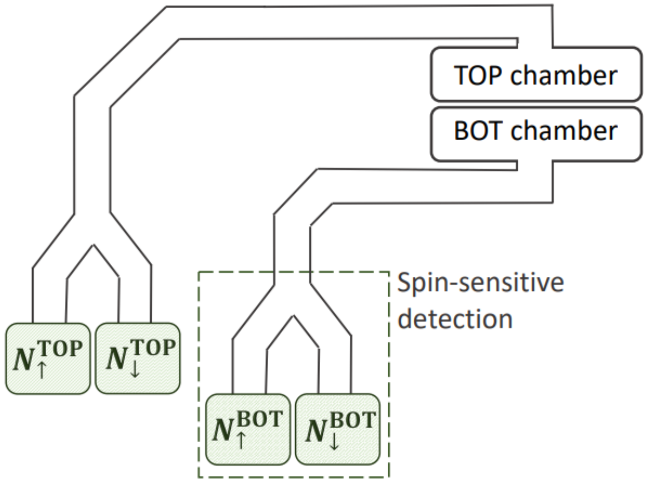


Counting spins with the Ramsey method

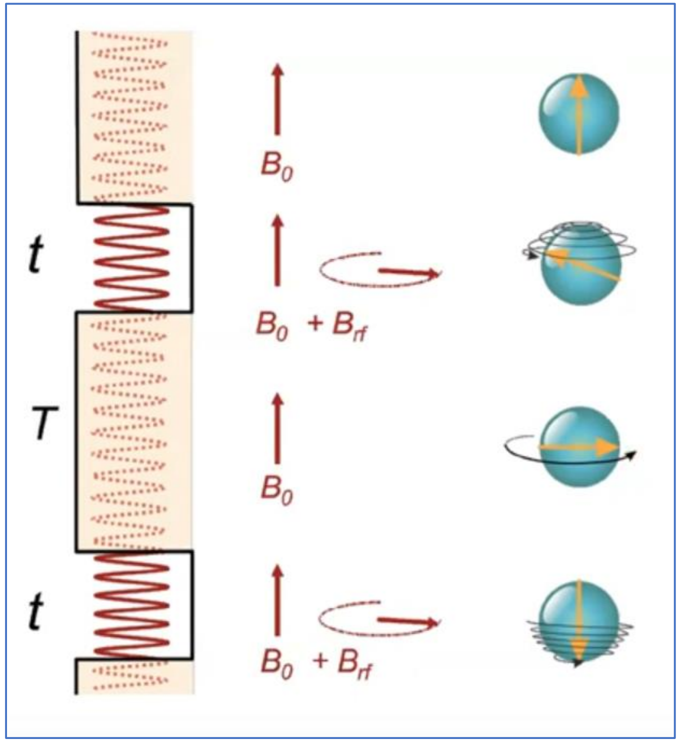


(a) Polarized UCNs fill the precession chambers

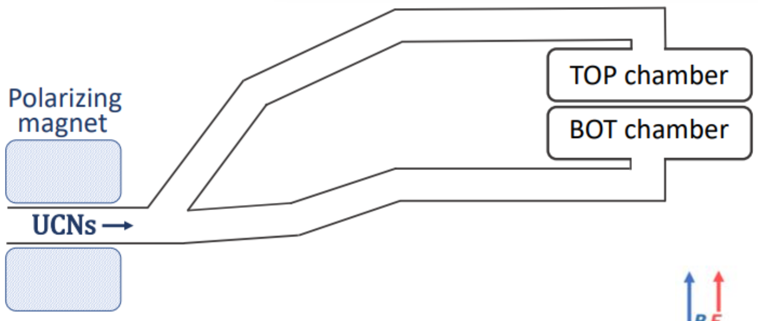




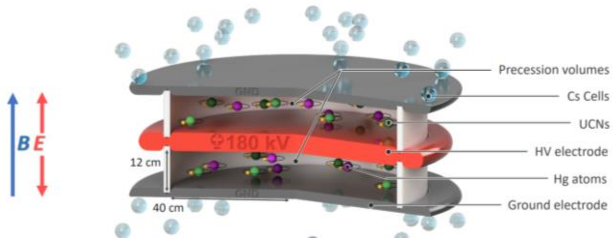
(b) UCNs are guided towards the spin-sensitive detectors.

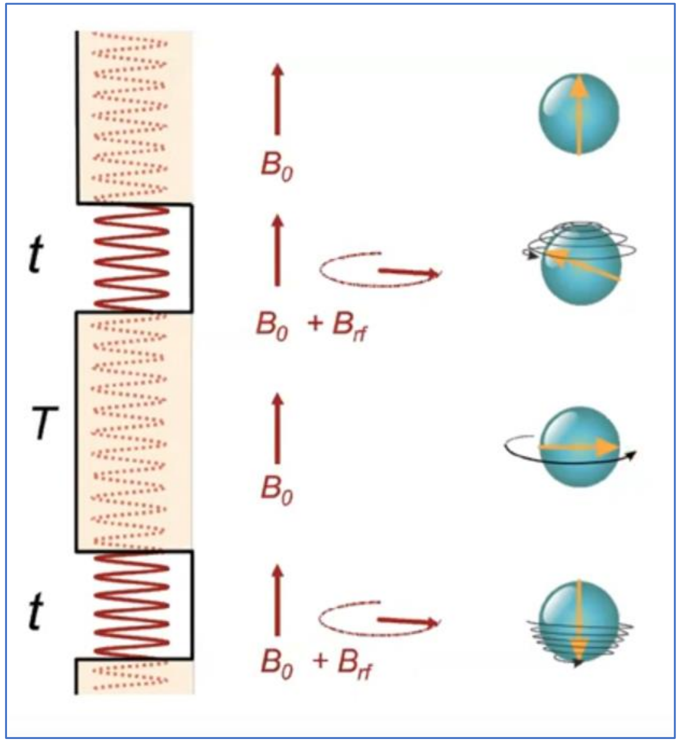


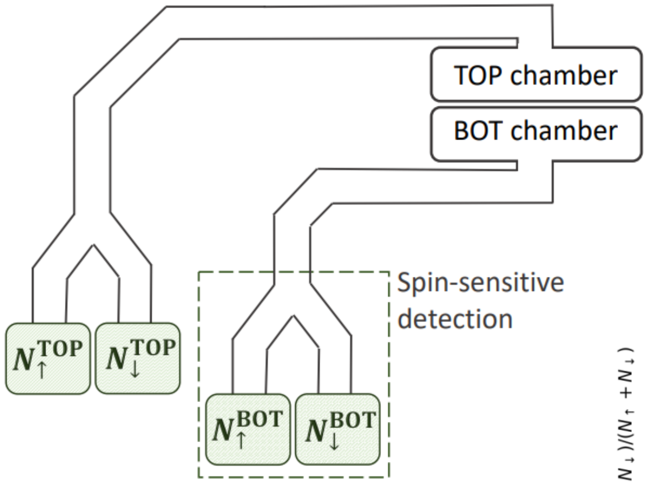
Counting spins with the Ramsey method



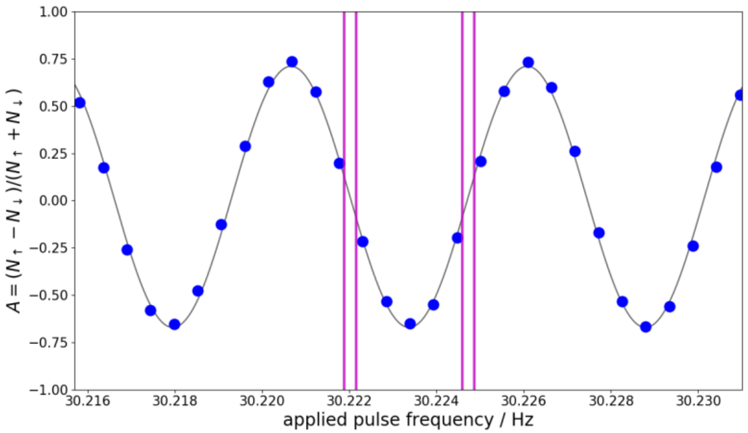
(a) Polarized UCNs fill the precession chambers







(b) UCNs are guided towards the spin-sensitive detectors.



Up-down spin asymmetry A → precession frequency f