Dark Matter Caustics

Pierre Sikivie (U of Florida)

Seminar at CEA de Saclay
June 13, 2022

The dark matter (WIMPs, axions, sterile neutrinos, ...) is thought to be cold and collisionless

If this is so,

- its velocity distribution at any physical location is a sum of discrete flows

- dark matter caustics, surfaces where the physical space density quasi-diverges, exist in galactic halos.

James R. Ipser

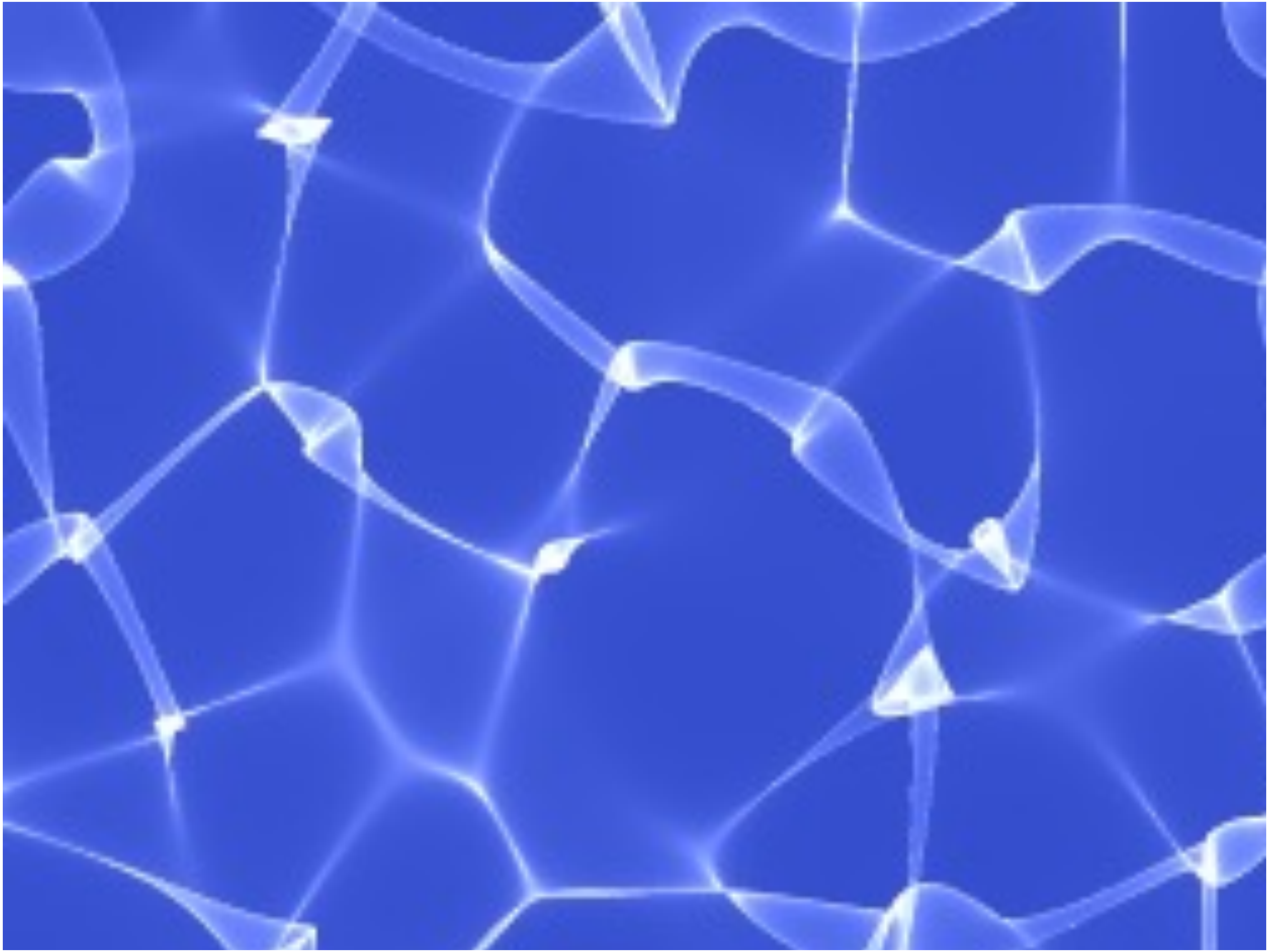
Galactic halos live in phase-space

ordinary fluid

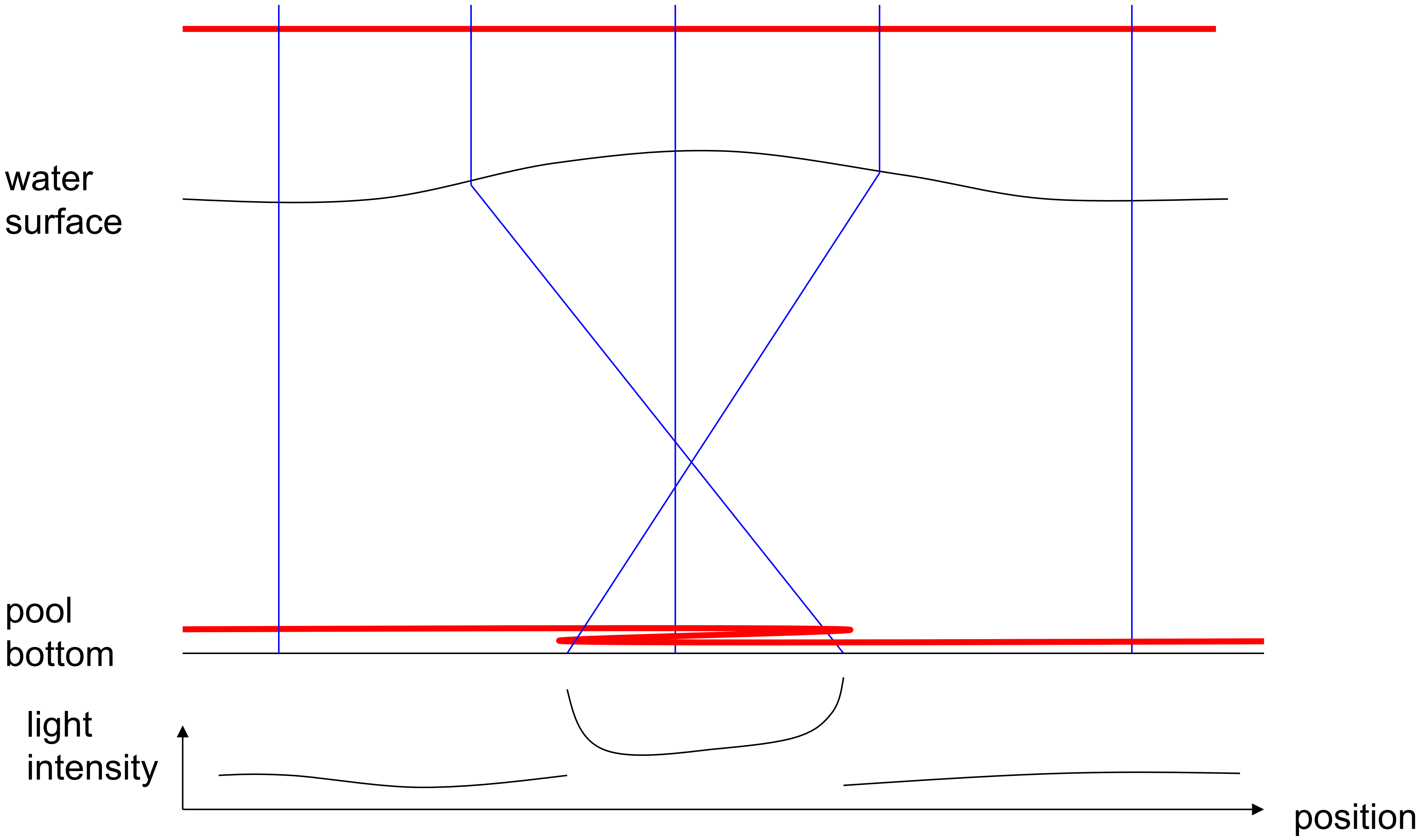
$$d(\vec{r};t) \qquad \vec{\mathrm{v}}(\vec{r};t)$$

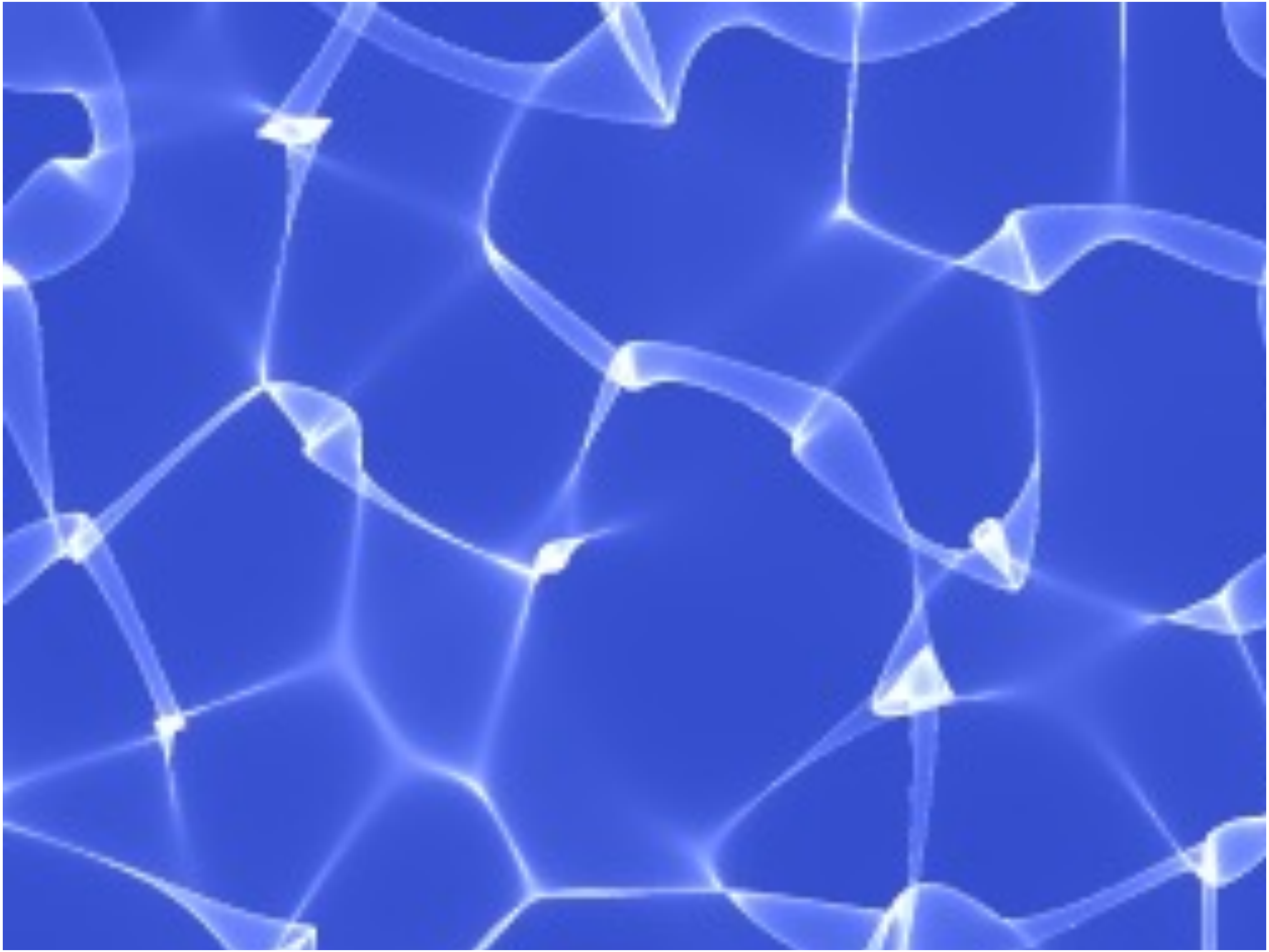
dark matter (collisionless) fluid

$$f(\vec{r},\vec{\mathrm{v}};t)$$



Caustics of light at the bottom of a swimming pool on a sunny breezy day





Rene Thom

(1923-2002)

Caustics are common in the propagation of light
(rainbows, the twinkling of stars, shimmering
of the sea, gravitational lenses …)

because

1. light is collisionless

2. in many cases, the flow of light
   has small velocity dispersion

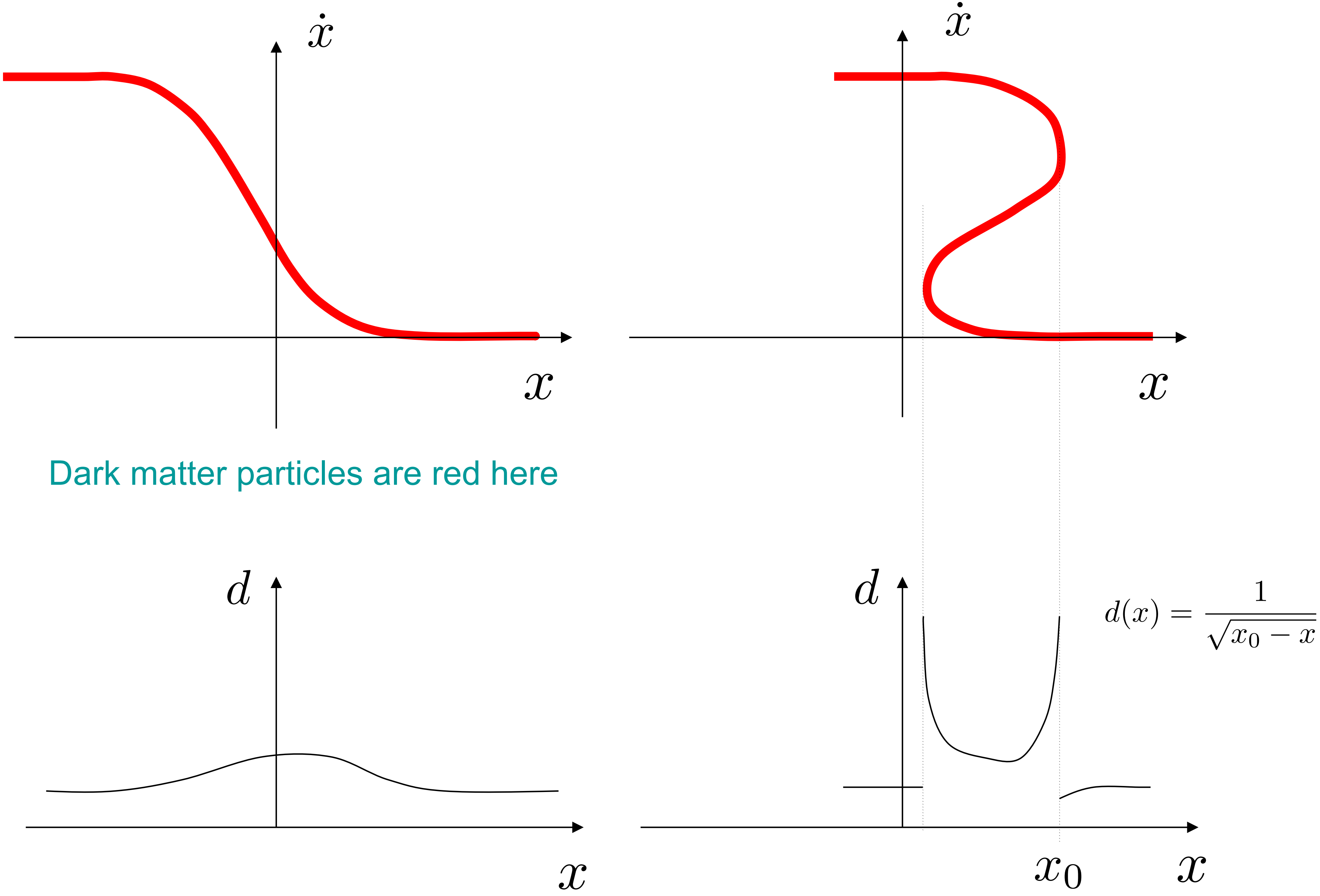
Caustics are rare in the propagation of ordinary matter because of collisions.

On the other hand, cold dark matter

1. is collisionless
2. has negligible primordial velocity dispersion

hence will form caustics.

Caustics in 2-dimensional phase-space

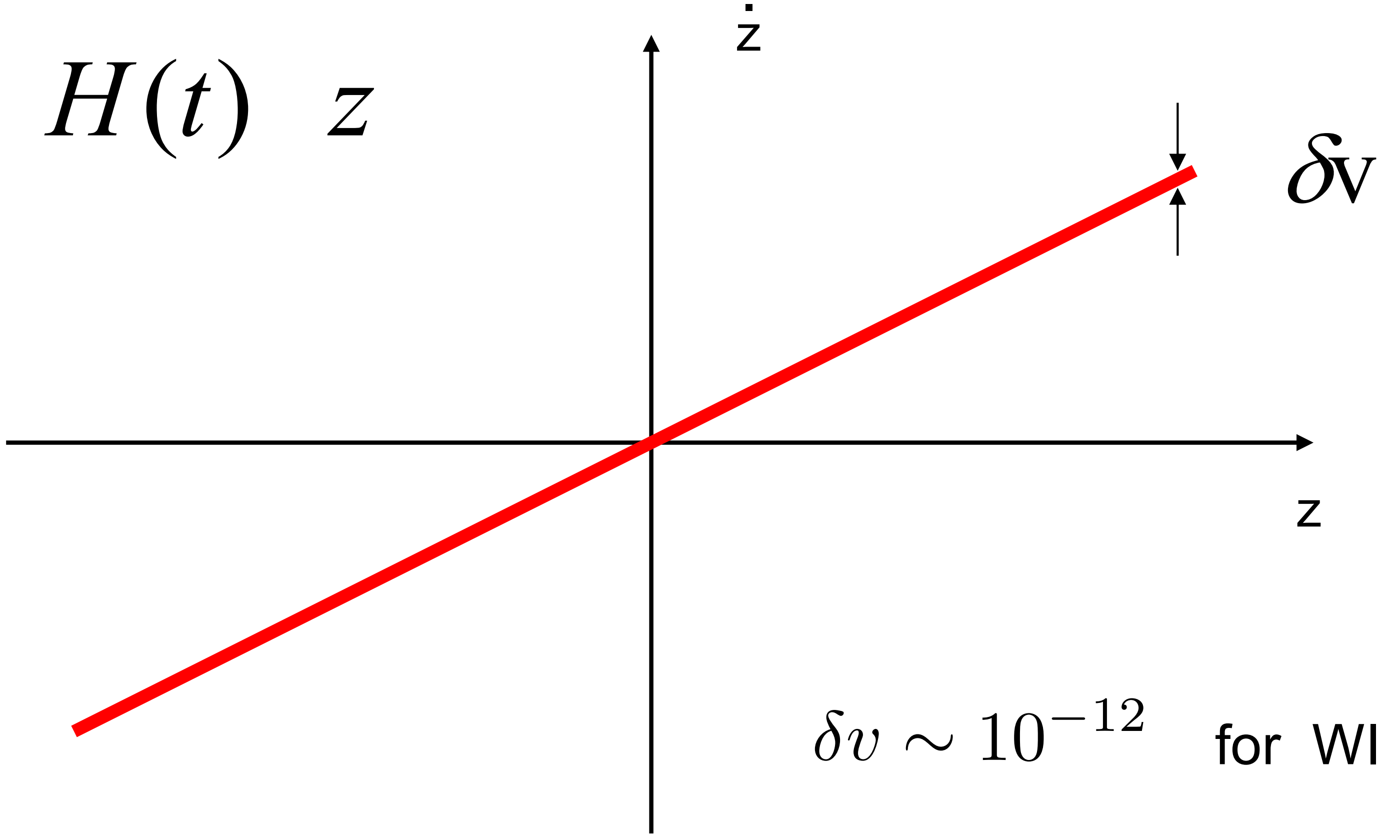


Dark matter particles are red here

$$d(x) = \frac{1}{\sqrt{x_0 - x}}$$

Phase space distribution of CDM in a homogeneous universe

$$\dot{z} = H(t) \; z$$



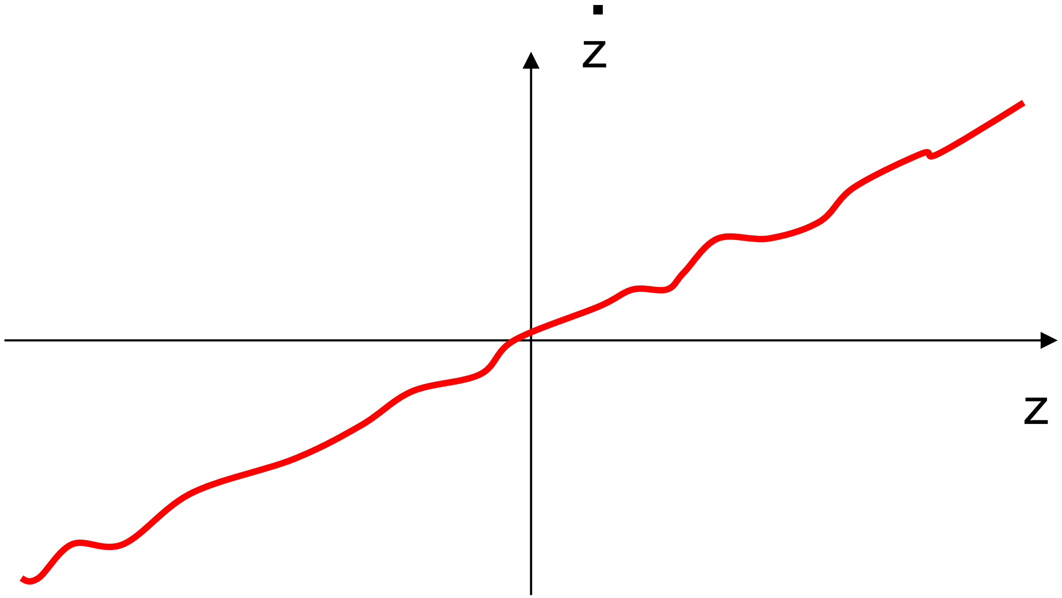
$\delta v \sim 10^{-12}$ for WIMPs

$\delta v \sim 10^{-8}$ for ~keV neutrinos

$\delta v \sim 0$ for axions

The cold dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase-space

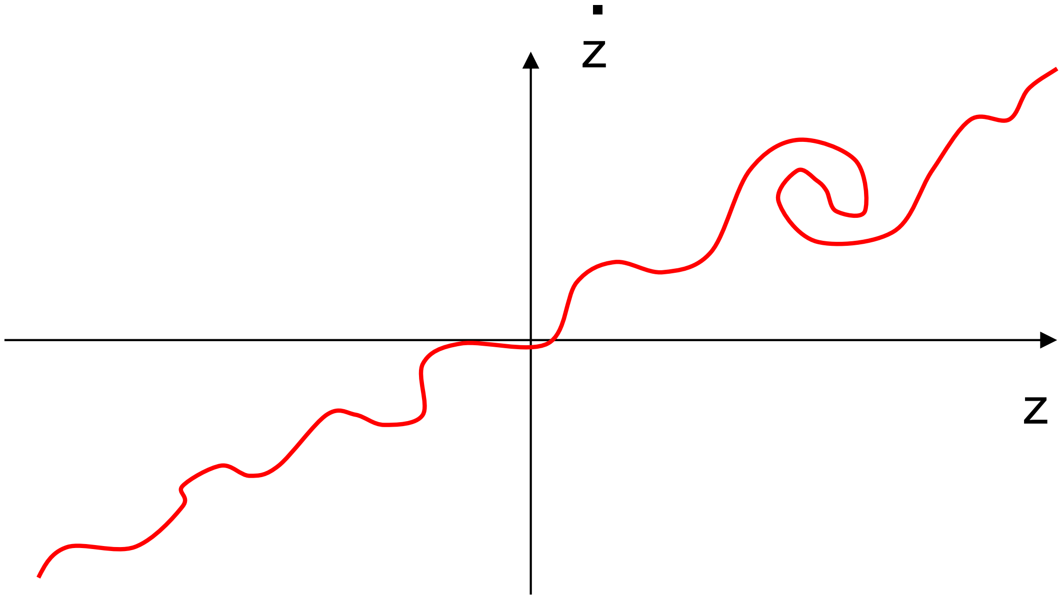
the physical density is the projection of the phase-space sheet onto position space



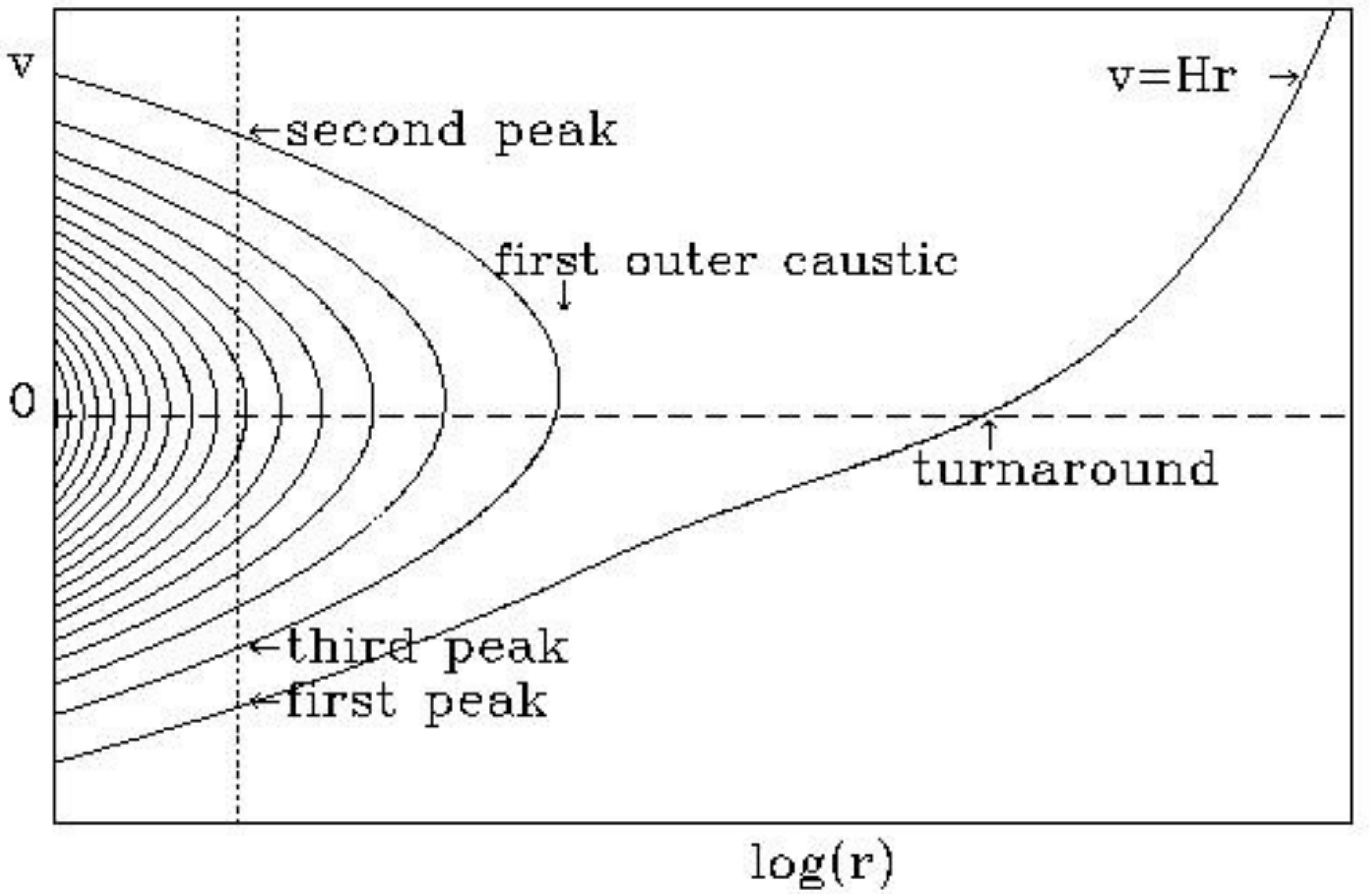
$$\vec{v}(\vec{r},t) = H(t)\vec{r} + \Delta\vec{v}(\vec{r},t)$$

The cold dark matter particles lie on a 3-dimensional sheet in 6-dimensional phase space

the physical density is the projection of the phase space sheet onto position space



Phase space structure of spherically symmetric halos



Implications:

1. At every point in physical space, the distribution of velocities is discrete, each velocity corresponding to a particular flow at that location.

2. At some locations in physical space, where the number of flows changes, there is a caustic, i.e. a surface where the density of dark matter is very high.

- the number of flows at our location in the Milky Way halo is of order 100

- small subhalos from hierarchical structure formation produce an effective velocity dispersion

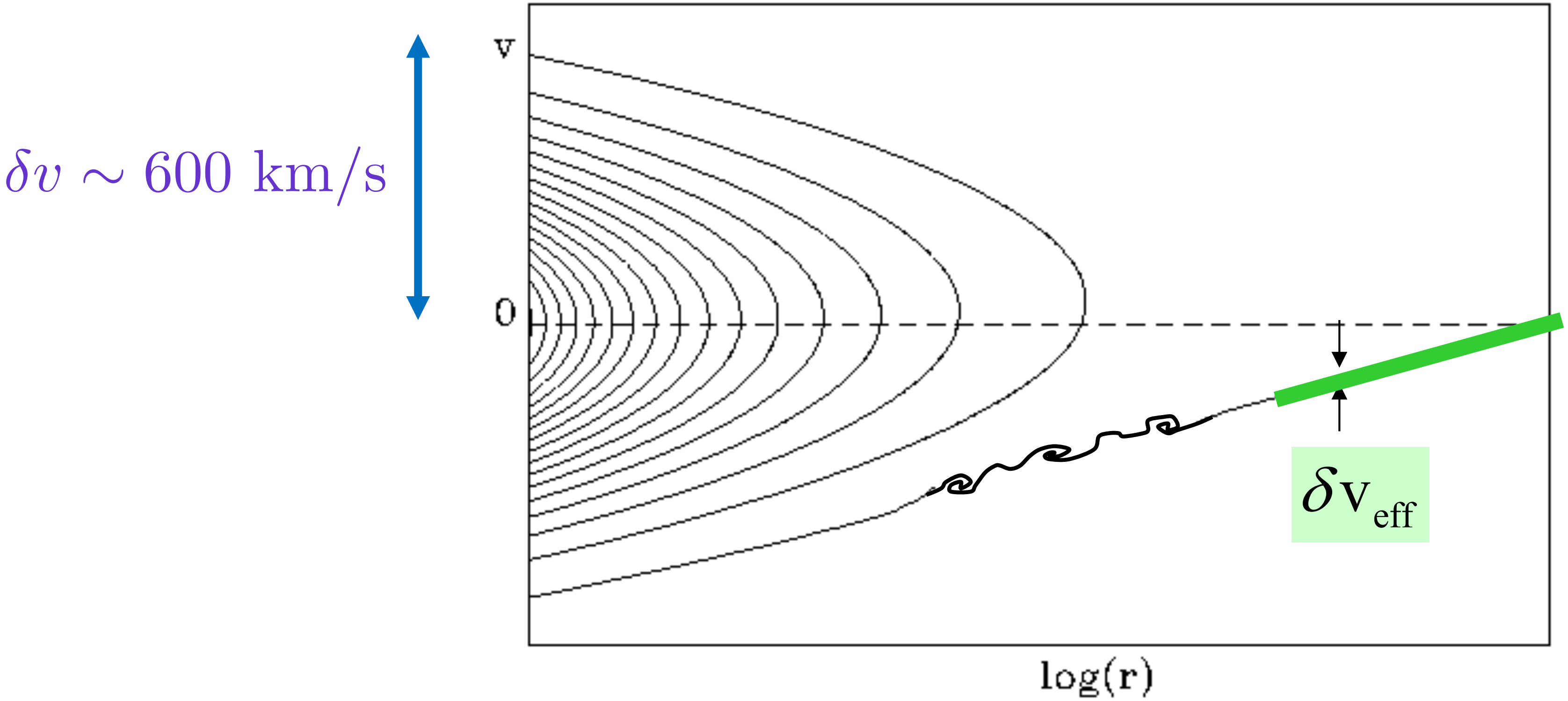
$$\delta v_{\rm eff} \lesssim 10 \ {\rm km/s}$$

  but do not destroy the sheet structure in phase space

- the known inhomogeneities in the distribution of matter are insufficient to diffuse the flows by gravitational scattering

- present N-body simulations do not have enough particles to resolve the expected flows and caustics

Hierarchical clustering introduces effective velocity dispersion



$$\delta v_{\mathrm{eff}} \lesssim 10\ \mathrm{km/s}$$

- the number of flows at our location in the Milky Way halo is of order 100

- small subhalos from hierarchical structure formation produce an effective velocity dispersion

$$\delta v_{\rm eff} \lesssim 10 \ {\rm km/s}$$

but do not destroy the sheet structure in phase space

- the known inhomogeneities in the distribution of matter are insufficient to diffuse the flows by gravitational scattering

- present N-body simulations do not have enough particles to resolve the expected flows and caustics

Gravitational scattering by a class of objects of mass M and density n results in diffusion over a cone of opening angle $\Delta\theta$:

J. Ipser and PS, 1992

$$(\Delta\theta)^2 = 7\times 10^{-7}\left(\frac{10^{-3}c}{v}\right)^3\left(\frac{M}{M_\odot}\right)^2\left(\frac{n}{\mathrm{pc}^{-3}}\right)\left(\frac{t}{10^{10}\ \mathrm{year}}\right)$$

Main contributors: molecular clouds, globular clusters

$$(\Delta\theta)^2 \simeq 10^{-6}\times(\text{number of throughfalls})$$

- the number of flows at our location in the Milky Way halo is of order 100

- small subhalos from hierarchical structure formation produce an effective velocity dispersion

  $$\delta v_{\rm eff} \lesssim 10 \ {\rm km/s}$$

  but do not destroy the sheet structure in phase space

- the known inhomogeneities in the distribution of matter are insufficient to diffuse the flows by gravitational scattering

- present N-body simulations do not have enough particles to resolve the expected flows and caustics

$N \sim 10^8$ $\quad N^{\frac{1}{6}} \sim 21$ $\quad M \sim 10^5 M_\odot$

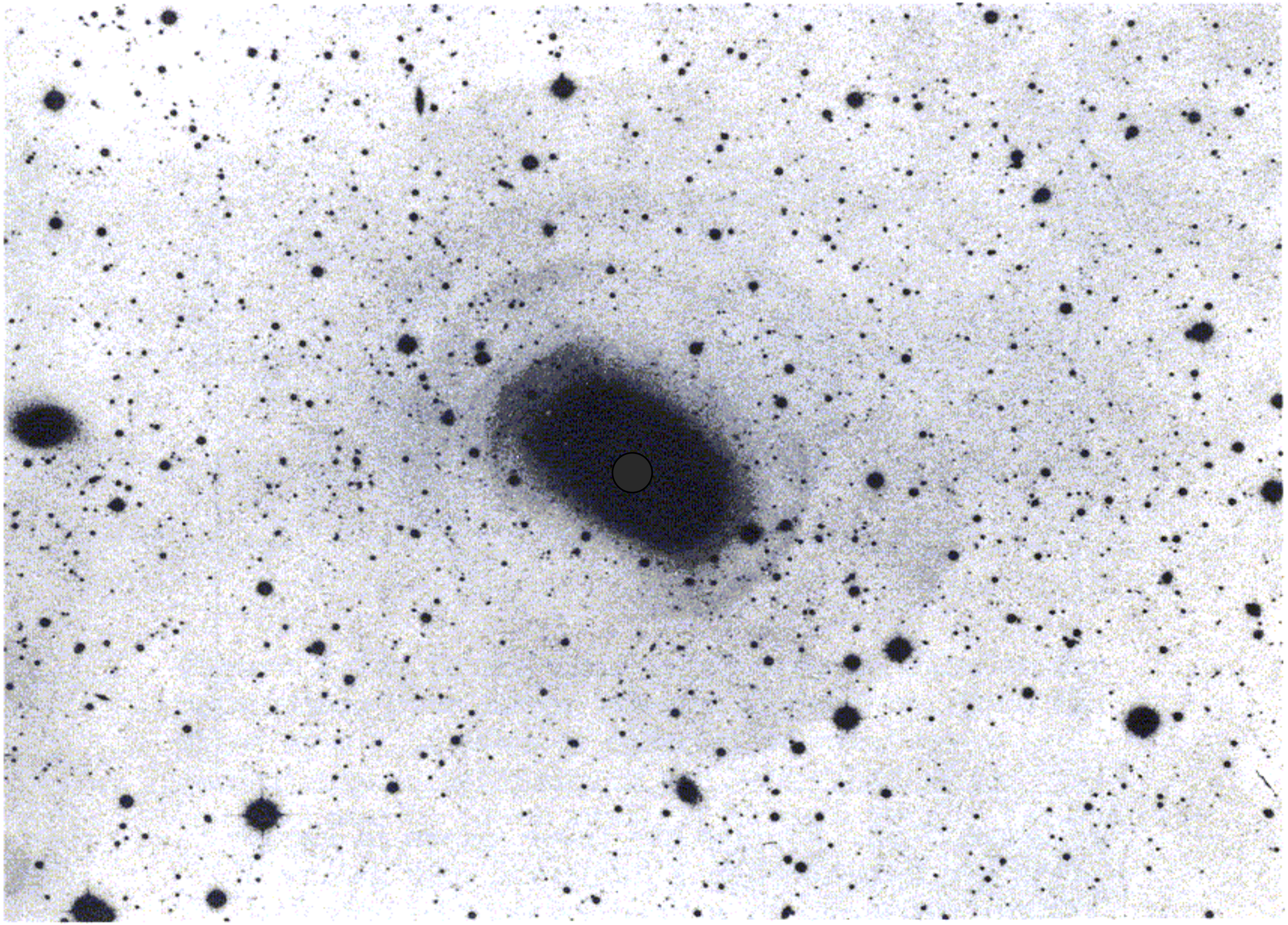


Figure 7-22. The giant elliptical galaxy NGC 3923 is surrounded by faint ripples of brightness. Courtesy of D. F. Malin and the Anglo-Australian Telescope Board.

(from Binney and Tremaine's book)

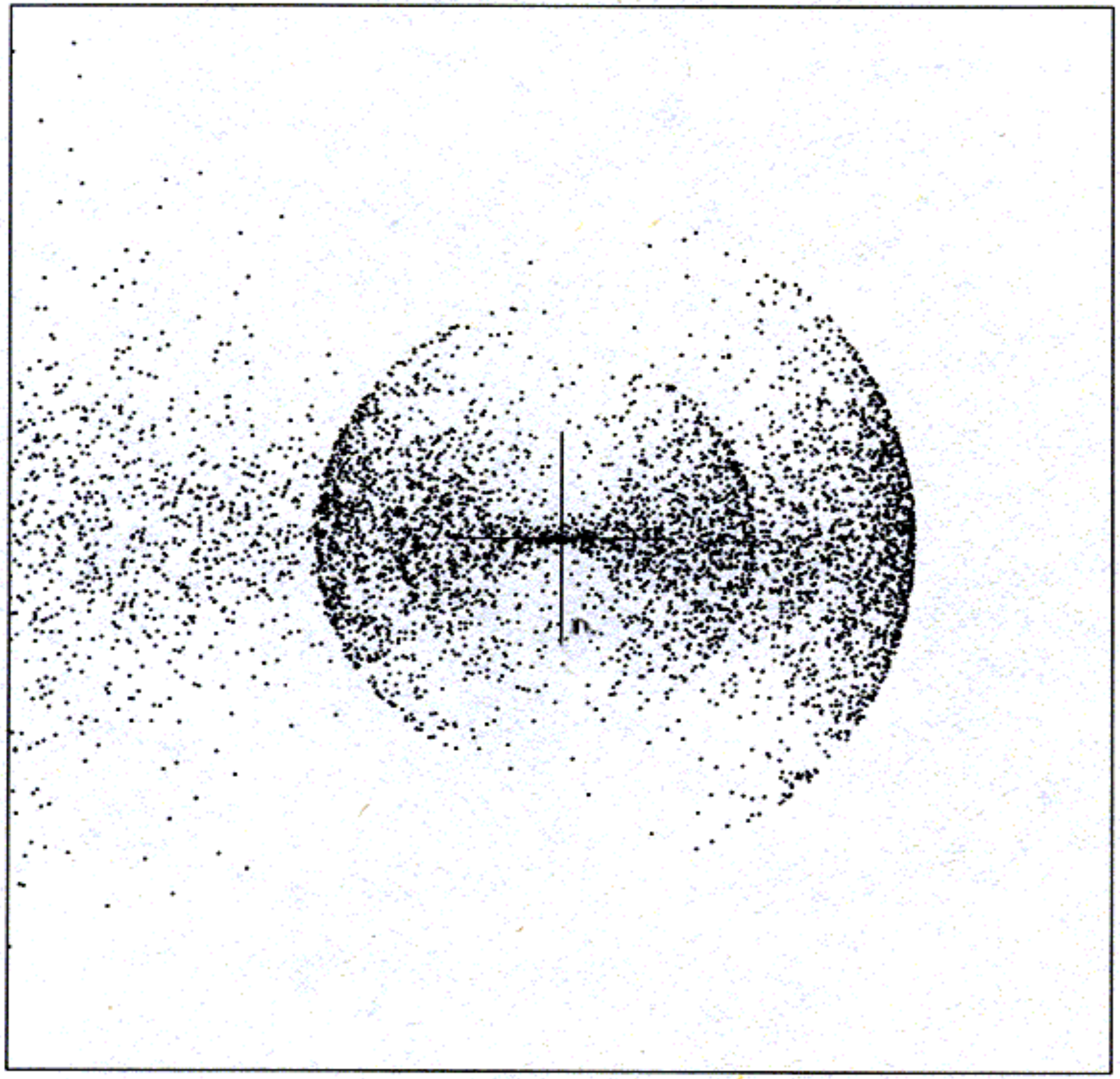


Figure 7-23. Ripples like those shown in Figure 7-22 are formed when a numerical disk galaxy is tidally disrupted by a fixed galaxy-like potential. (See Hernquist & Quinn 1987.)

The flow of cold collisionless particles from all directions in and out of a region necessarily forms an inner caustic (Arvind Natarajan and PS, 2005).

Galactic halos have inner caustics as well as outer caustics.

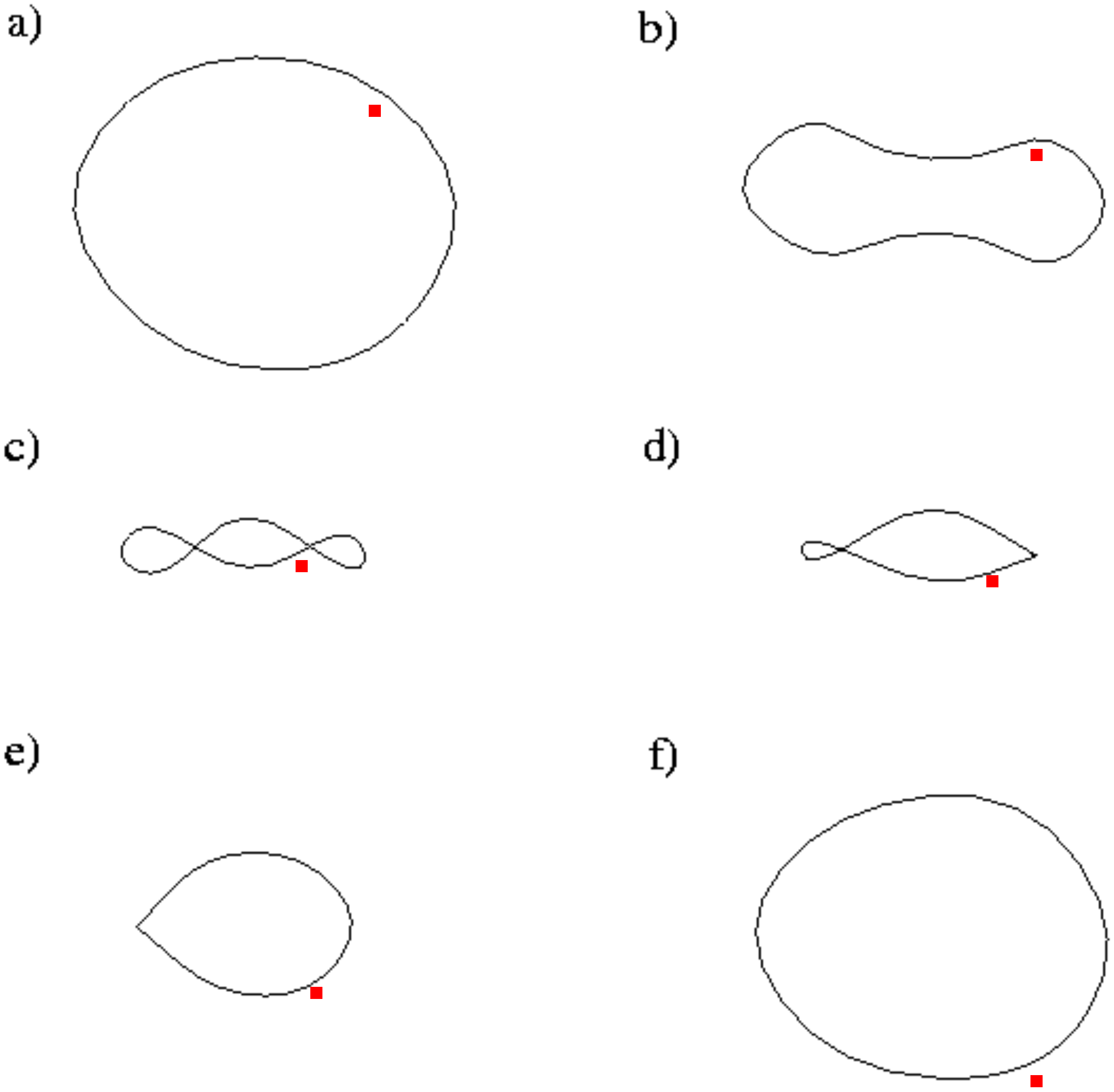
If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

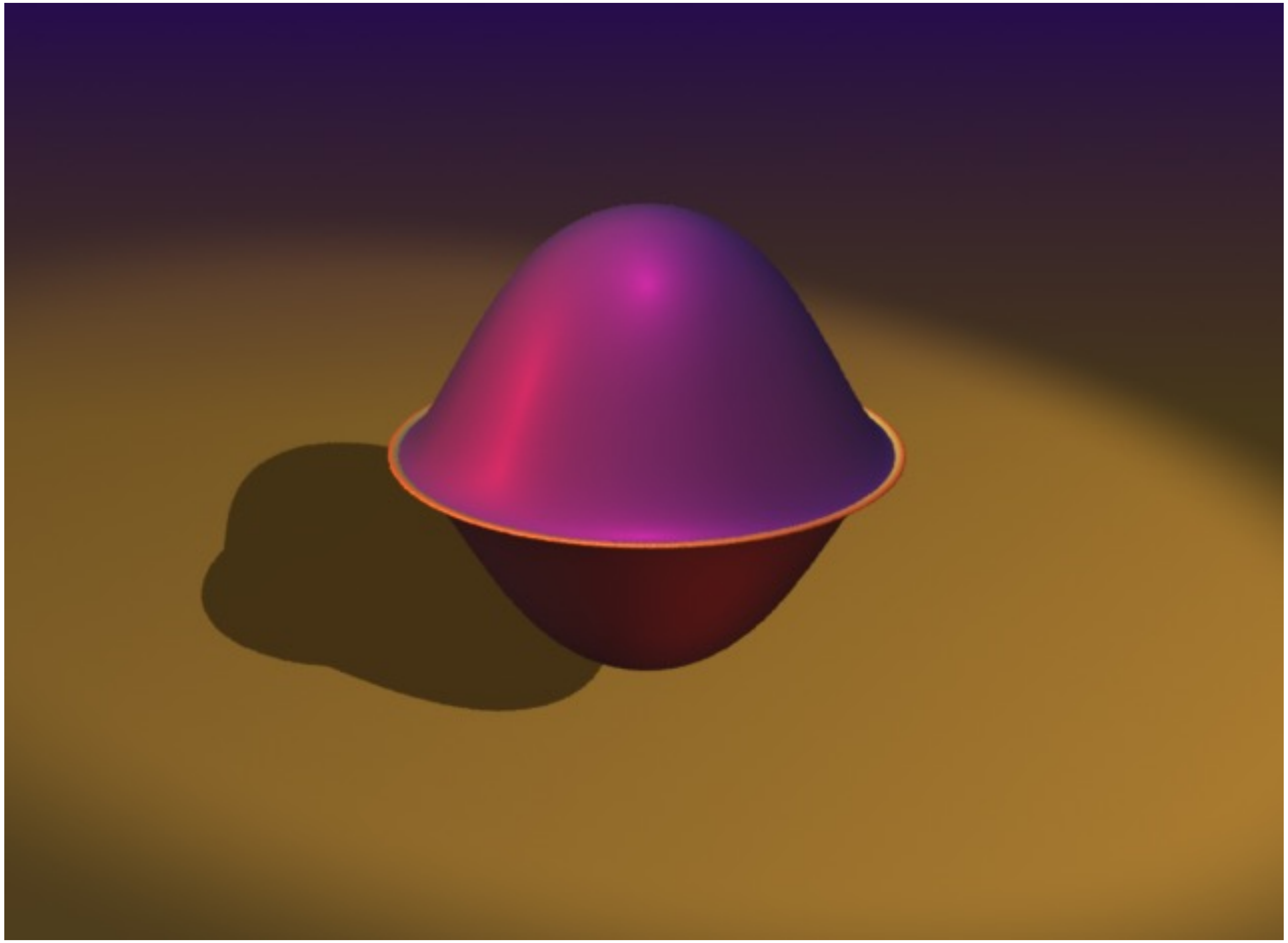
A shell of particles, part of a continuous flow.

The shell has net angular momentum.

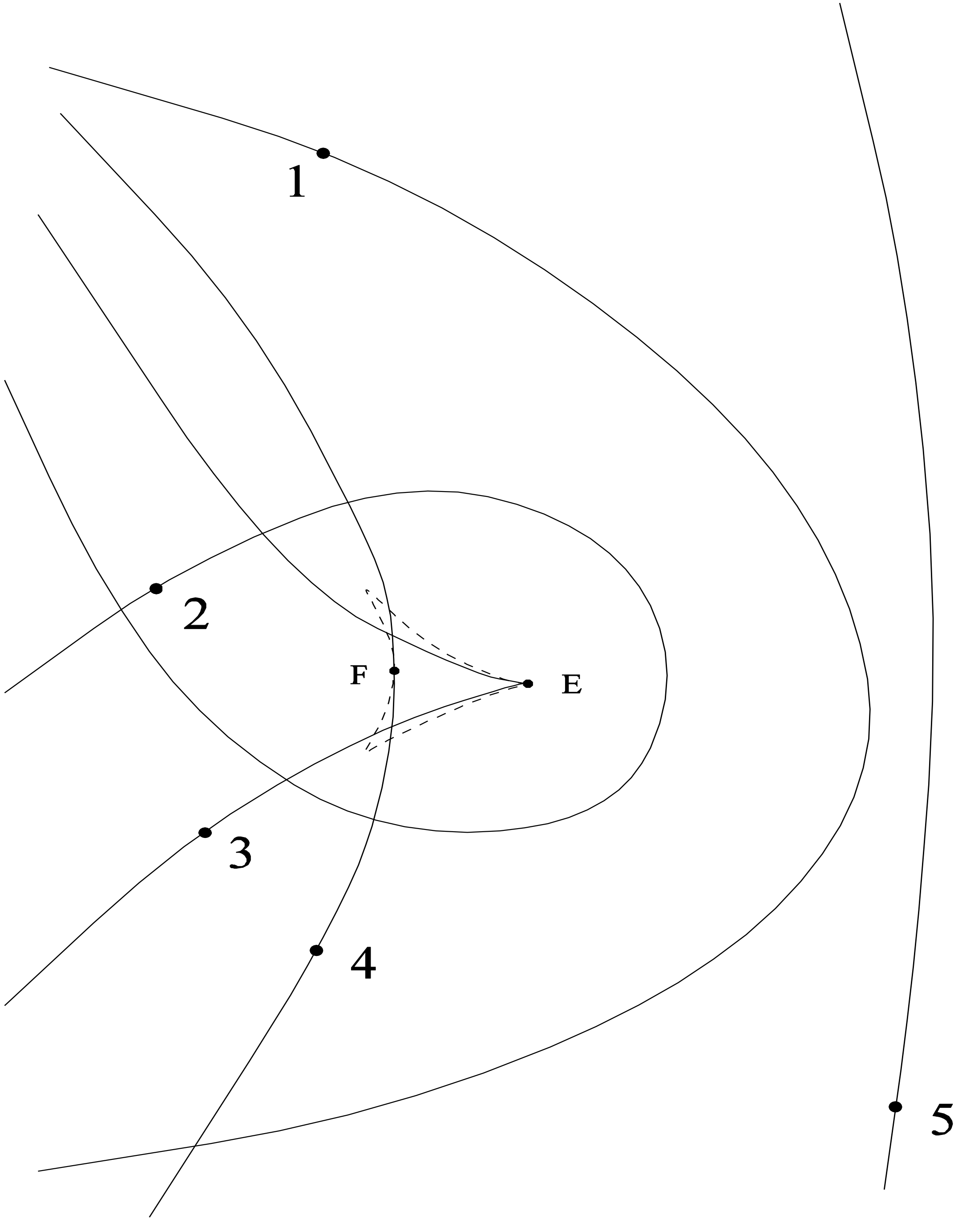
As the shell falls in and out of the galaxy, it turns itself inside out.



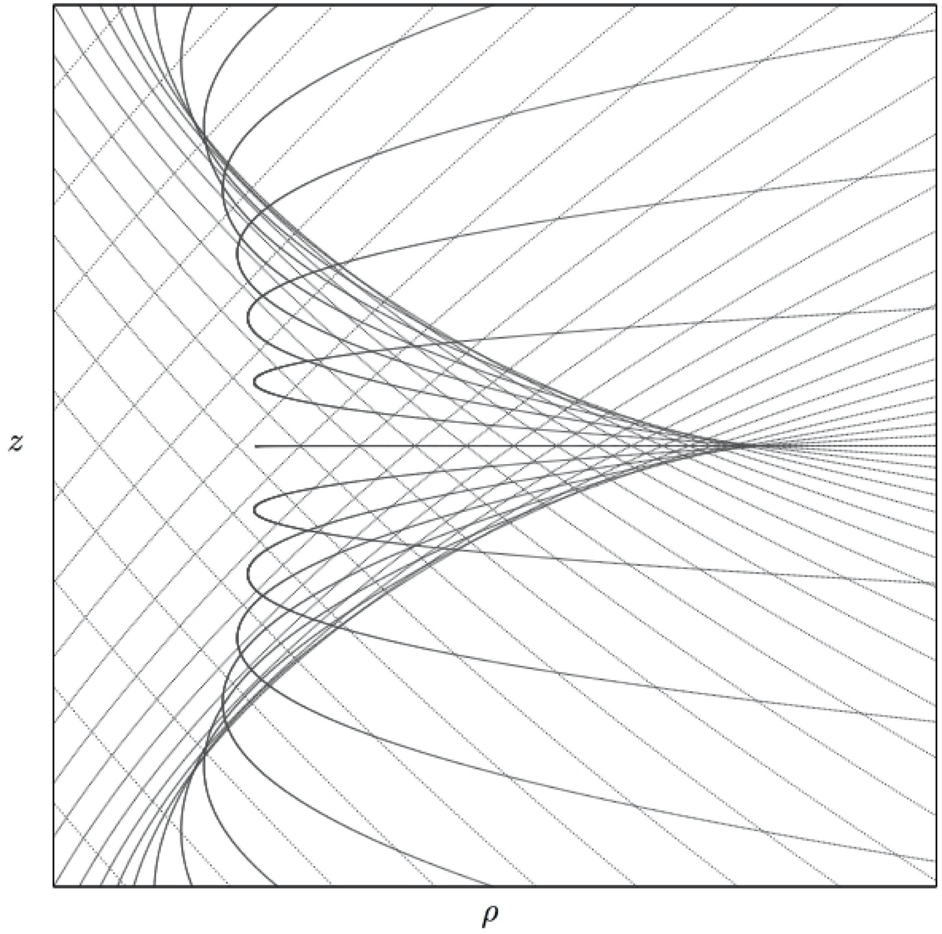
Sphere turning inside out



A caustic forms where the particles with the most angular momentum are at their closest approach to the galactic center.



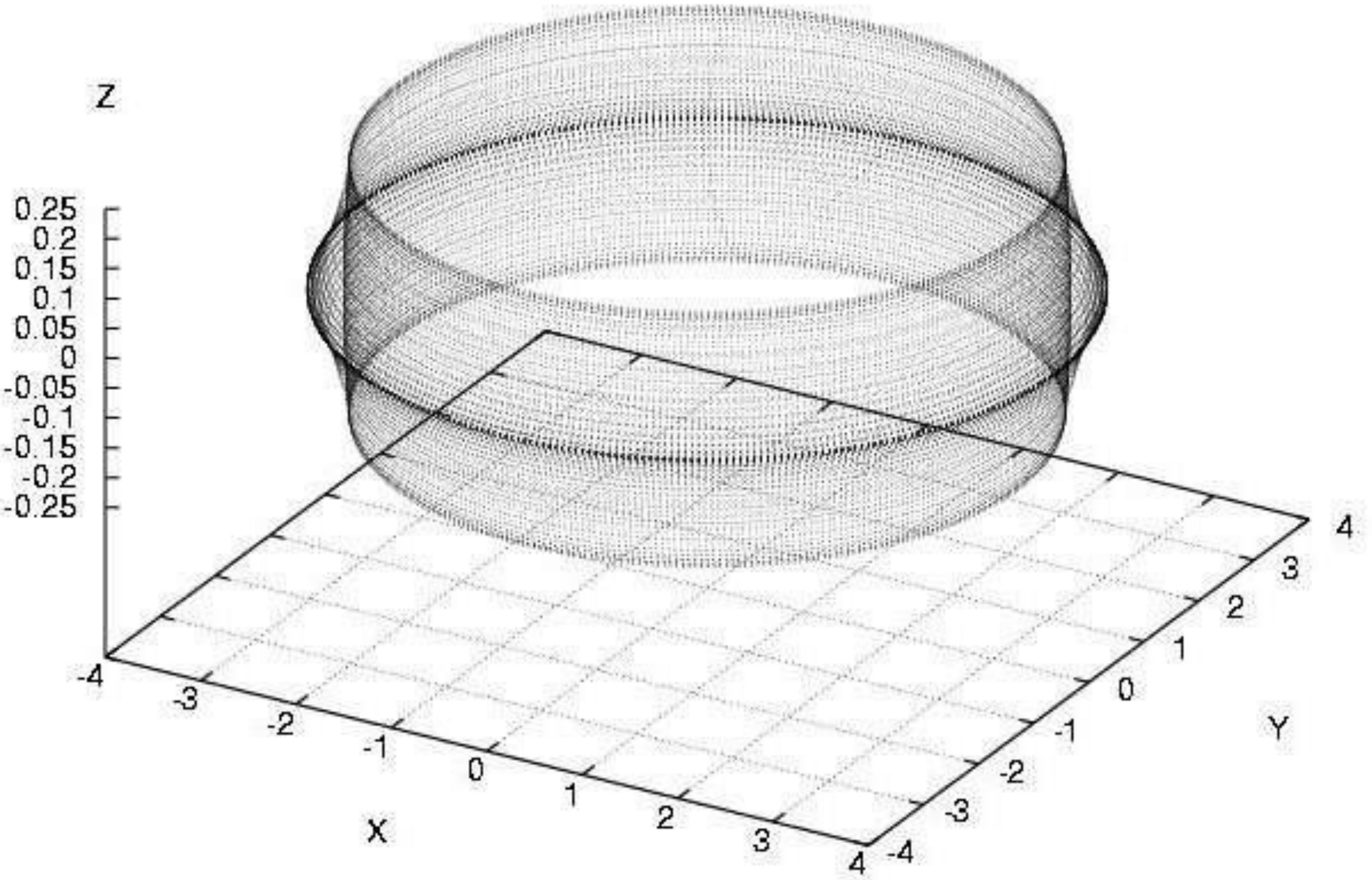
The caustic ring cross-section

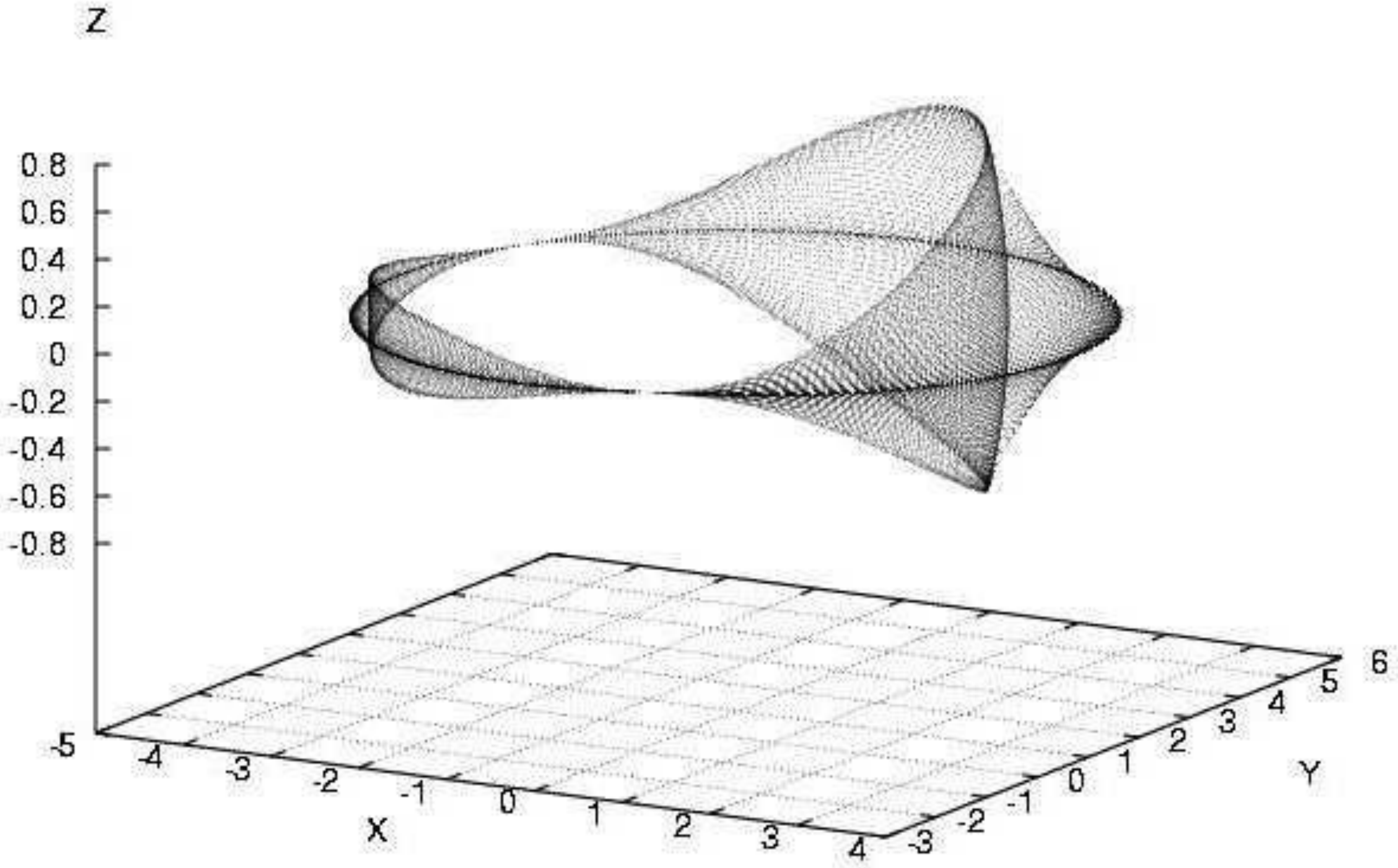


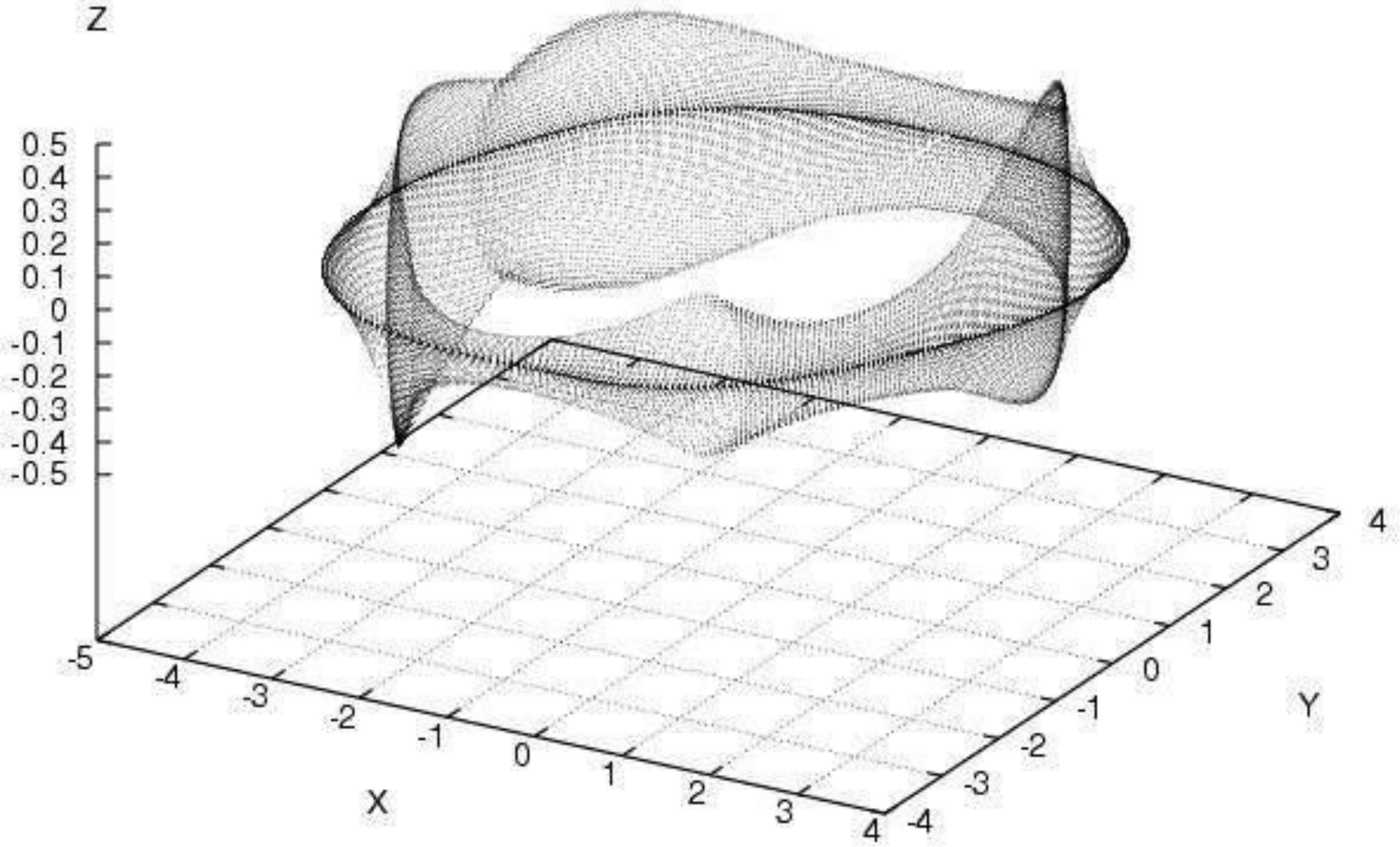
D_{-4}

an elliptic umbilic catastrophe

simulations by Arvind Natarajan




Z
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-5
-4
-3
-2
-1
0
1
2
3
4
X
-3
-2
-1
0
1
2
3
4
5
6
Y


Z
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-5
-4
-3
-2
-1
0
1
2
3
4
X
4
3
2
1
0
-1
-2
-3
-4
Y

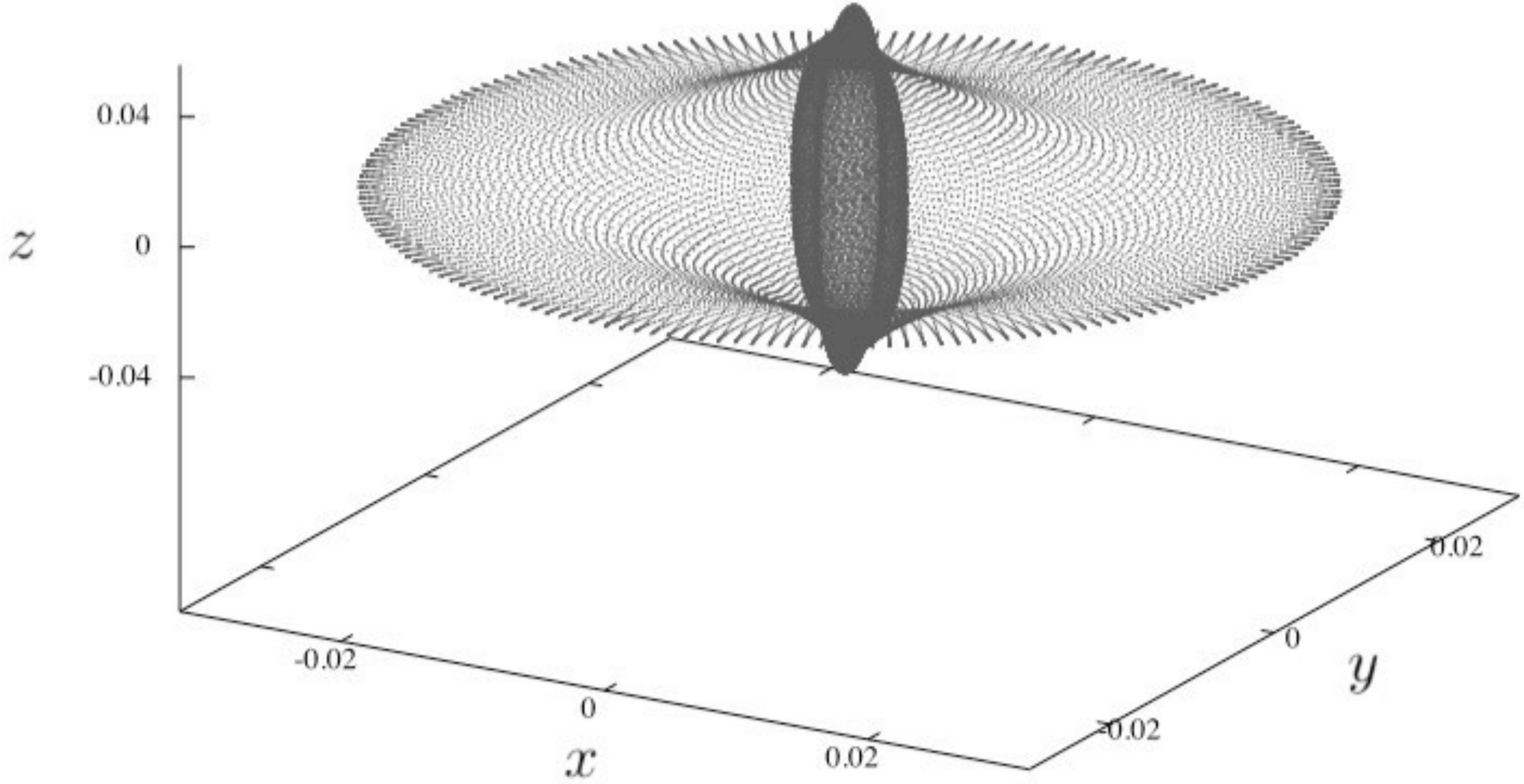
The flow of cold collisionless particles from all directions in and out of a region necessarily forms an inner caustic (Arvind Natarajan and PS, 2005).

Galactic halos have inner caustics as well as outer caustics.

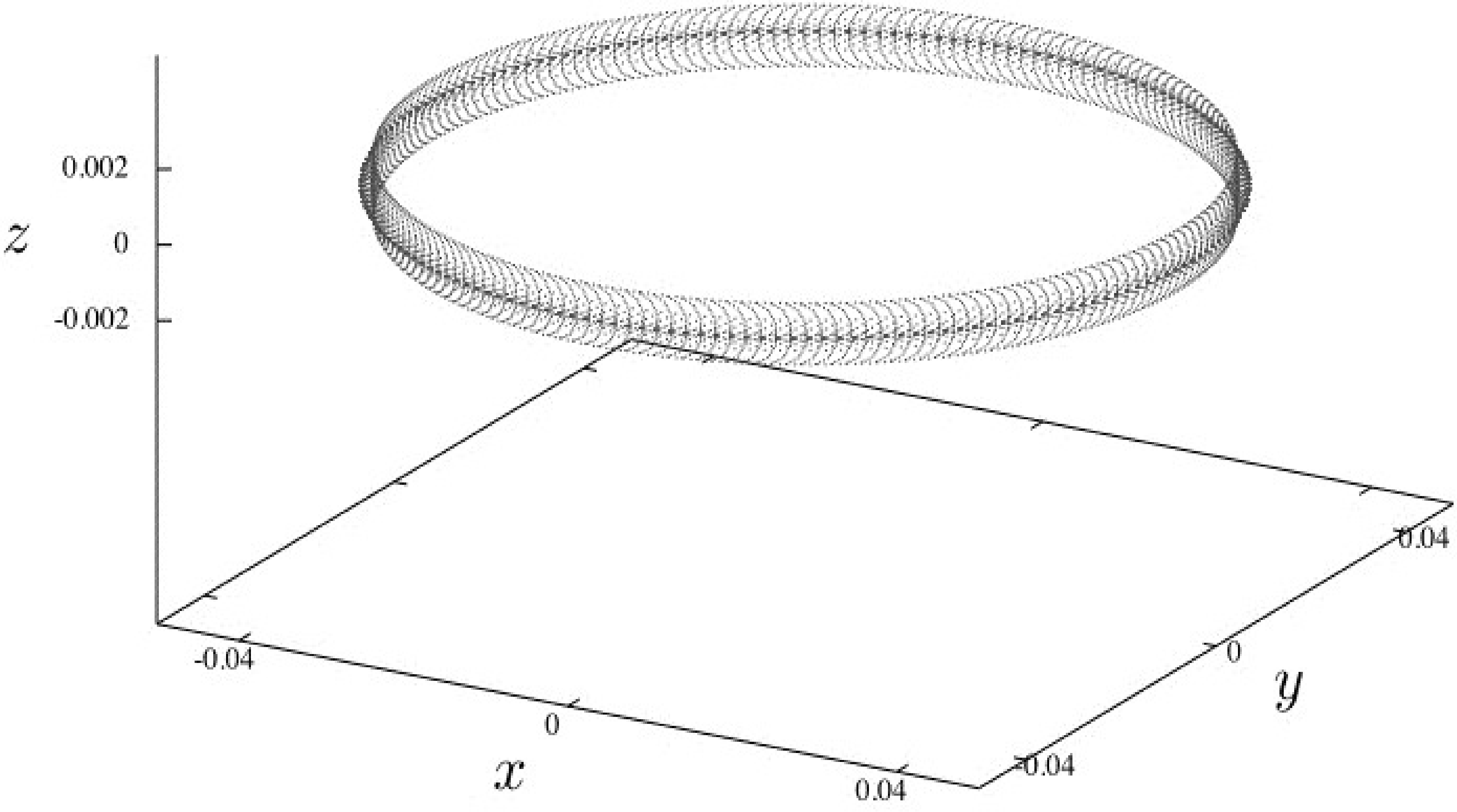
If the initial velocity field is dominated by net overall rotation, the inner caustic is a ‘tricusp ring’.

If the initial velocity field is irrotational, the inner caustic has a ‘tent-like’ structure.

in case of irrotational flow



in case of rotational flow



On the basis of the self-similar infall model (Filmore and Goldreich, Bertschinger) with angular momentum (Tkachev, Wang + PS), the caustic rings were predicted to be

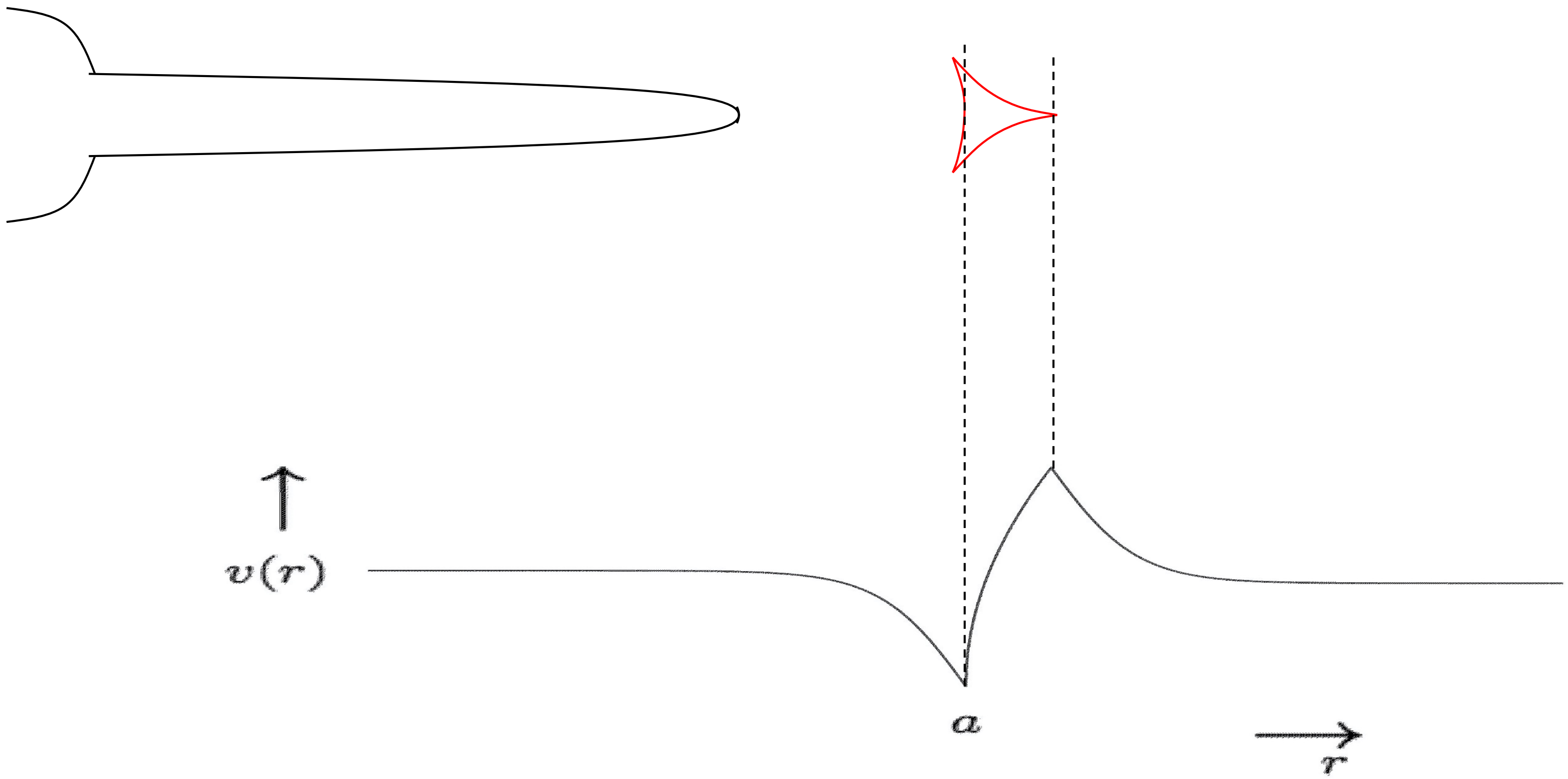
in the galactic plane

with radii $(n = 1, 2, 3 \ldots)$

$$a_n = \frac{40\text{kpc}}{n} \left(\frac{v_{\text{rot}}}{220\text{km/s}} \right) \left(\frac{j_{\max}}{0.26} \right)$$

$j_{\max} \cong 0.26$ was expected for the Milky Way halo from the effect of angular momentum on the inner rotation curve.

Effect of a caustic ring of dark matter upon the galactic rotation curve



Rotation curve of Andromeda Galaxy

from L. Chemin, C. Carignan & T. Foster, arXiv: 0909.3846

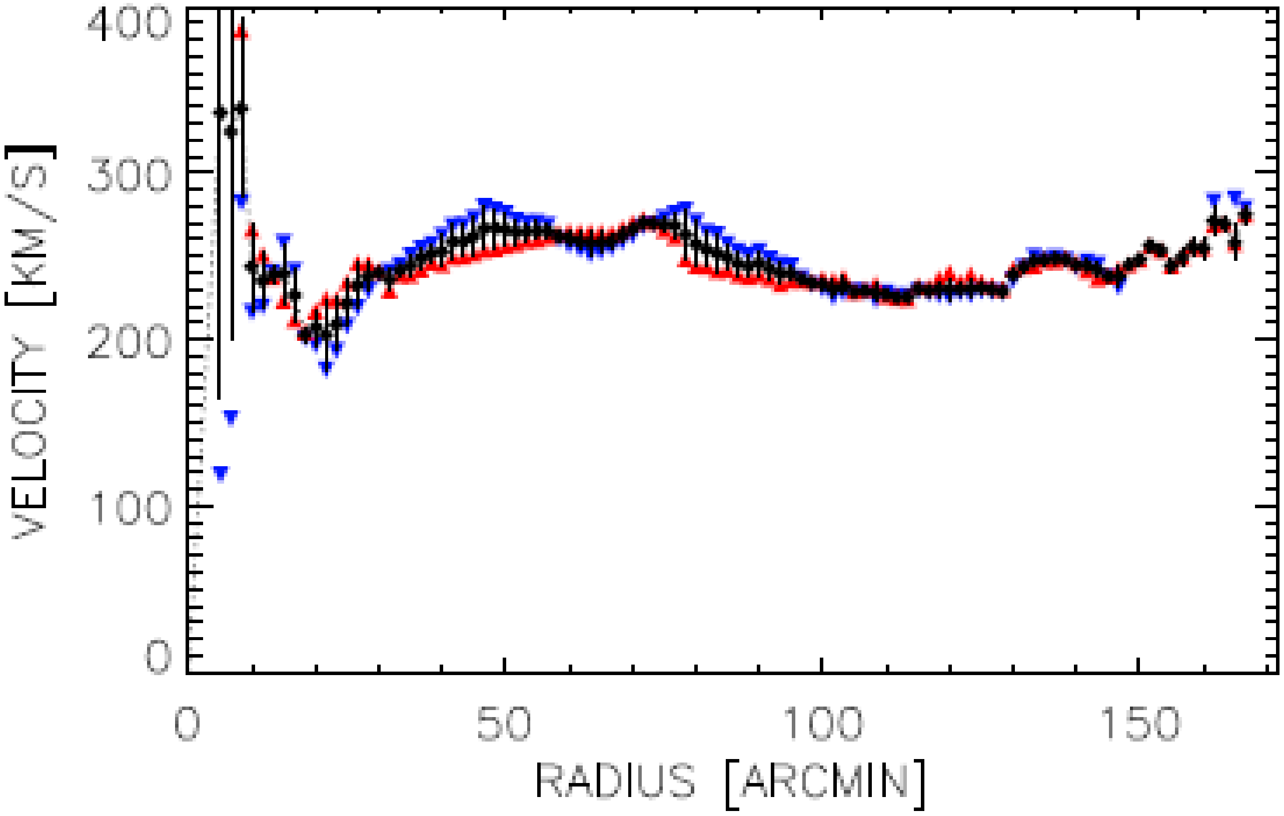
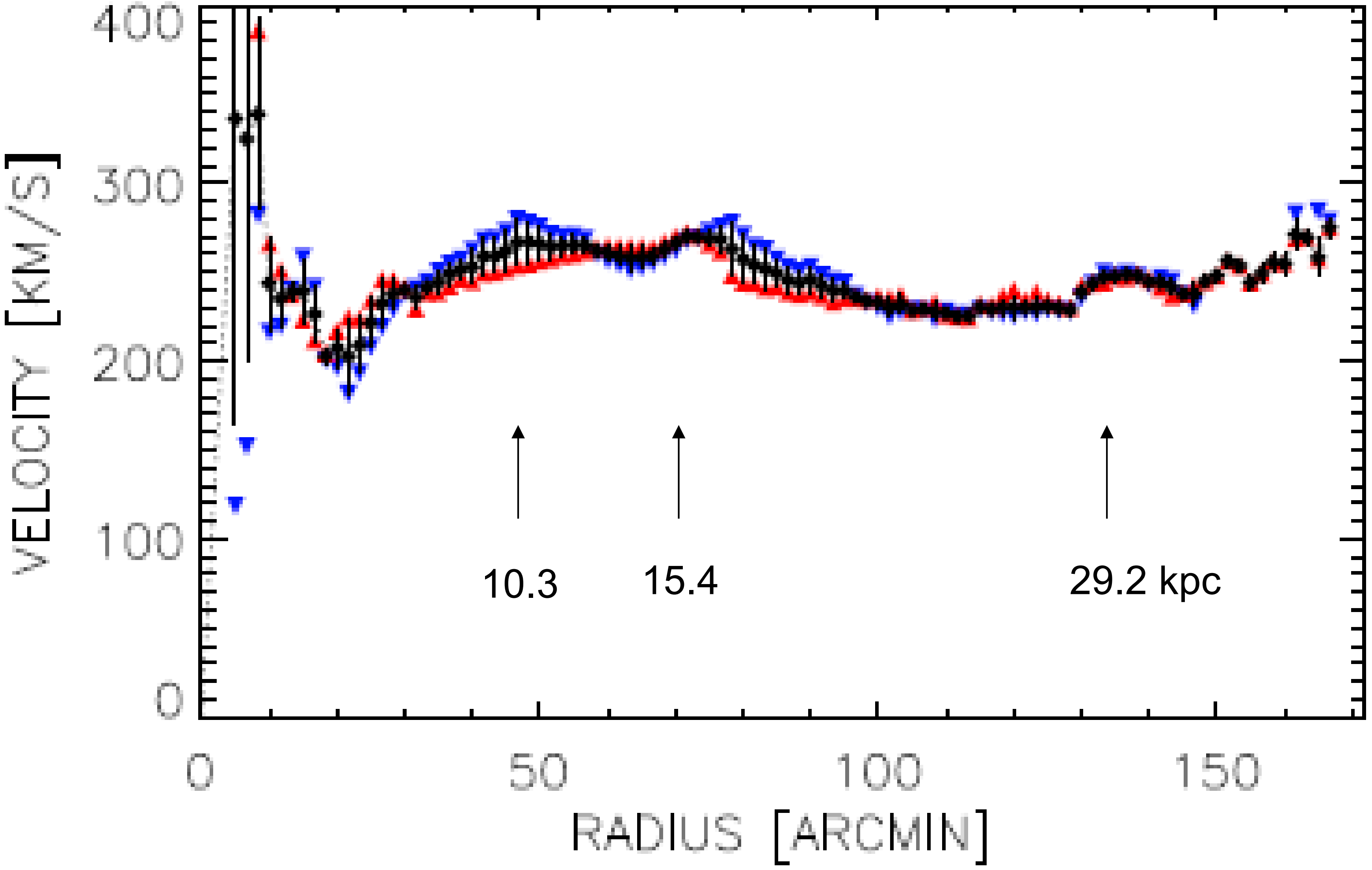


Fig. 10.— HI rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



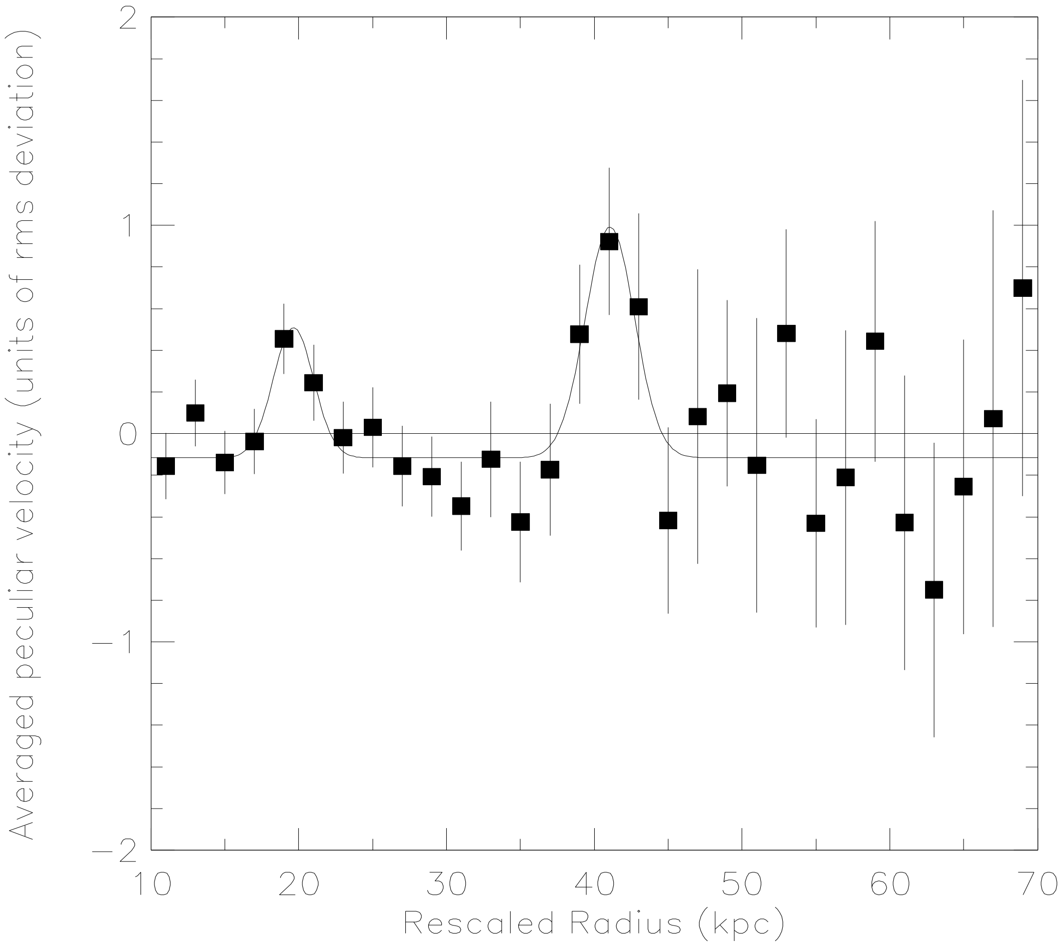
10 arcmin = 2.2 kpc

S. Chakrabarty, Y. Han, A. Gonzalez
& PS, arXiv:2007.10509

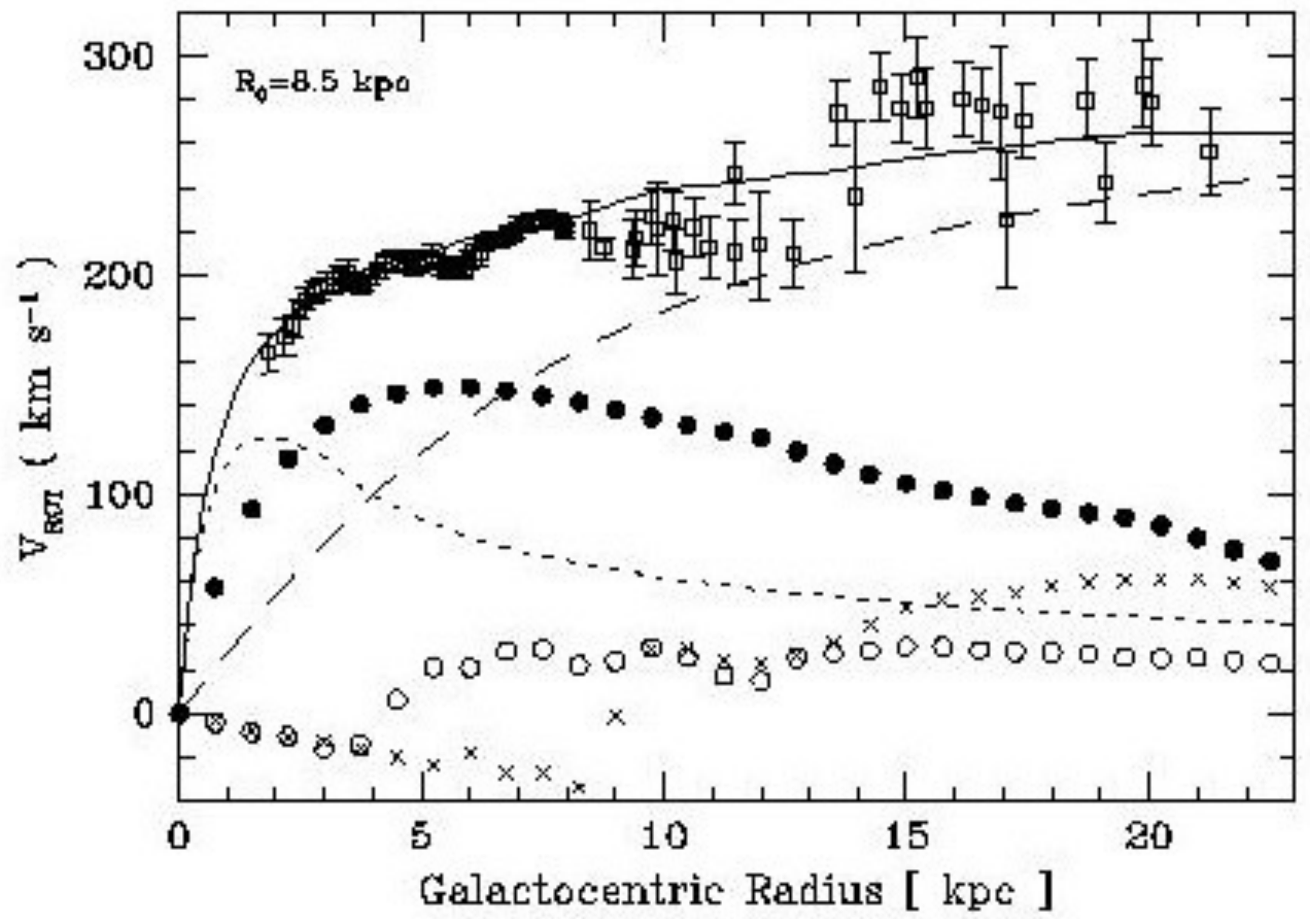
Composite rotation curve

(W. Kinney and PS, astro-ph/9906049)

- combining data on 32 well measured extended external rotation curves
- scaled to our own galaxy

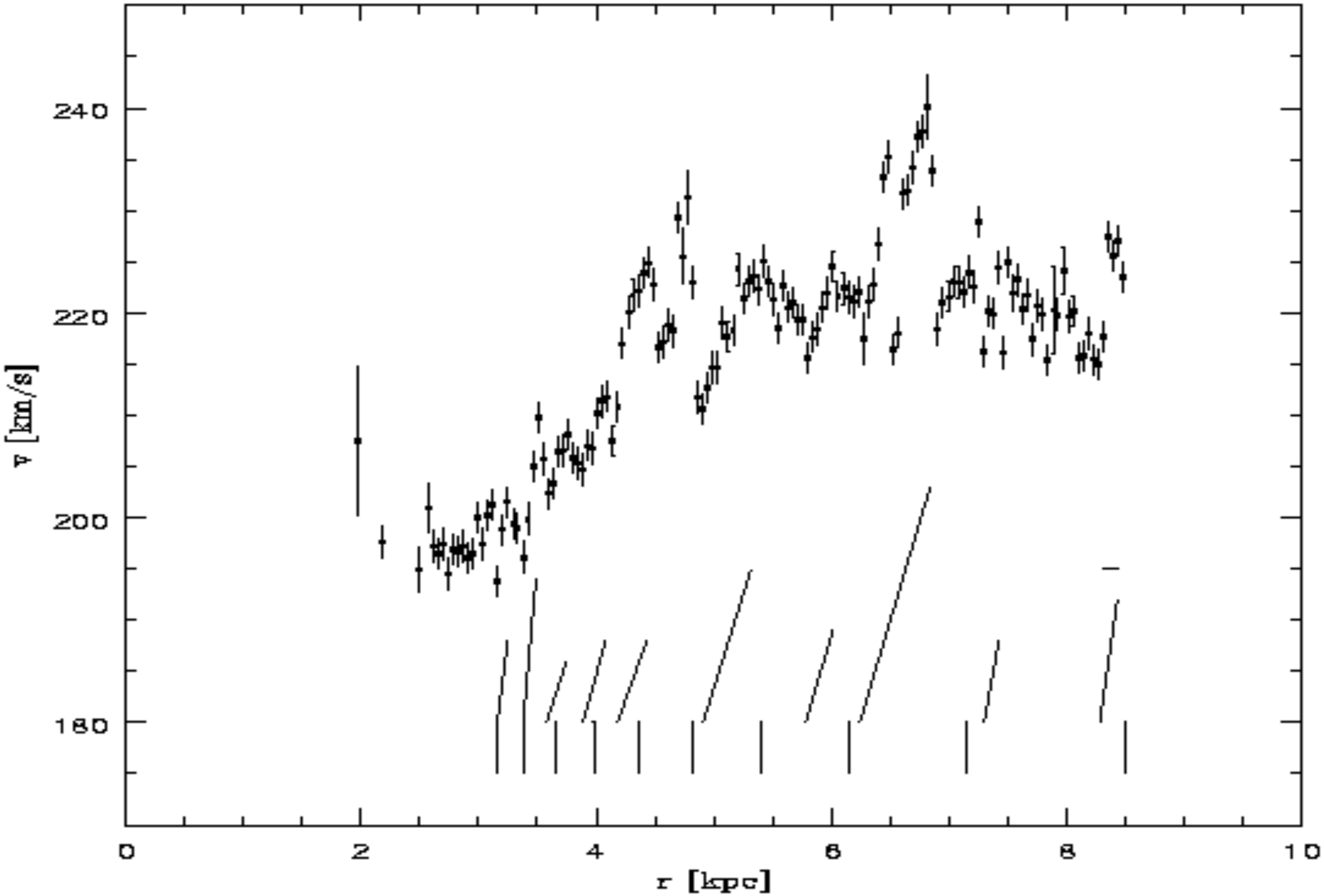


Outer Galactic rotation curve

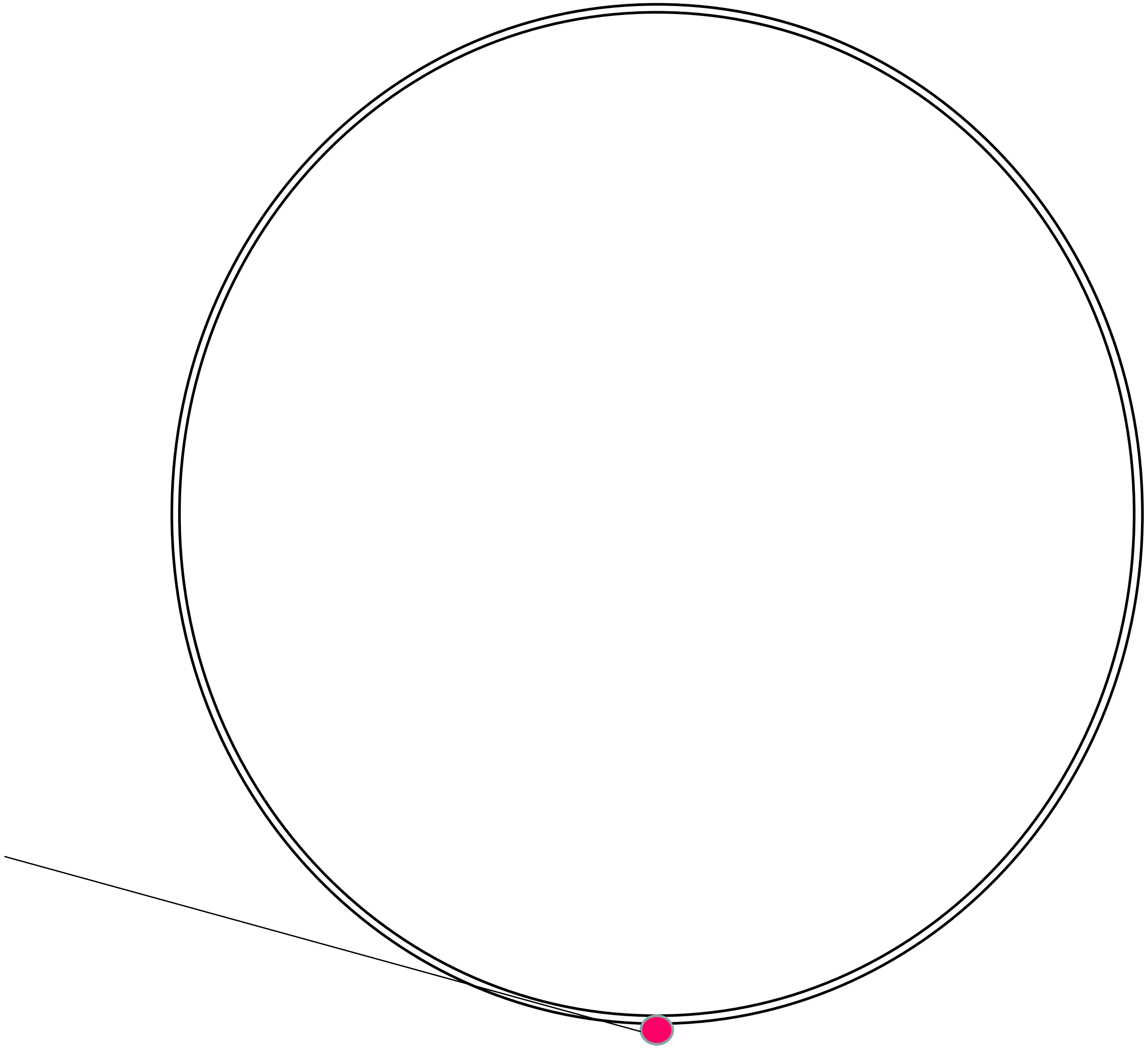


R.P. Olling and M.R. Merrifield, MNRAS 311 (2000) 361

Inner Galactic rotation curve



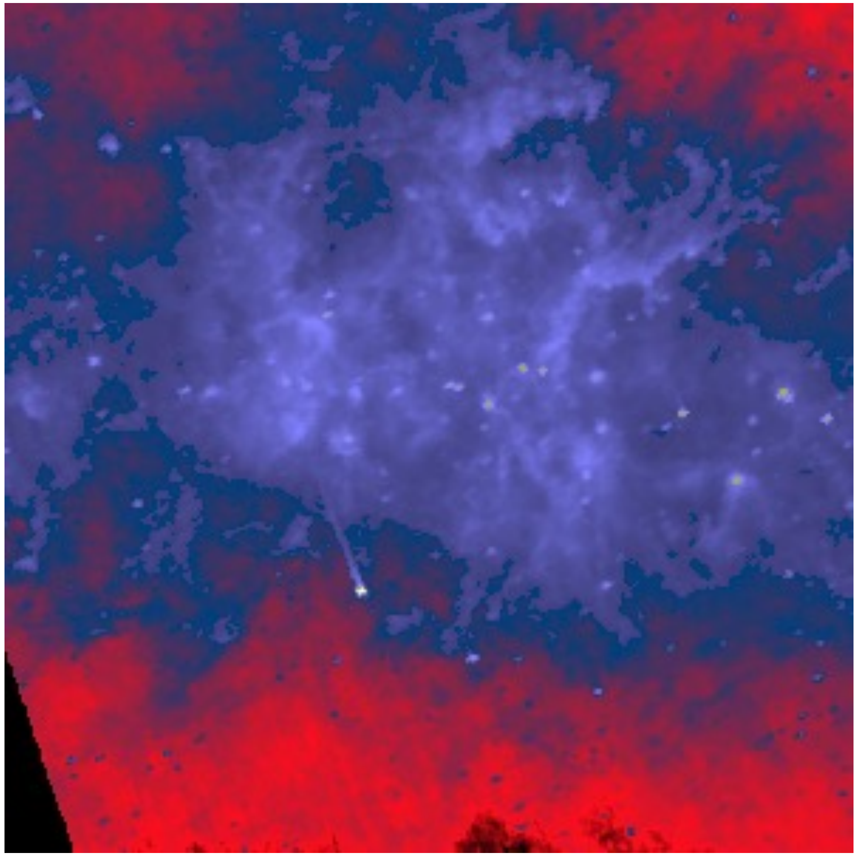
from Massachusetts-Stony Brook North Galactic Pane CO Survey (Clemens, 1985)



Sun

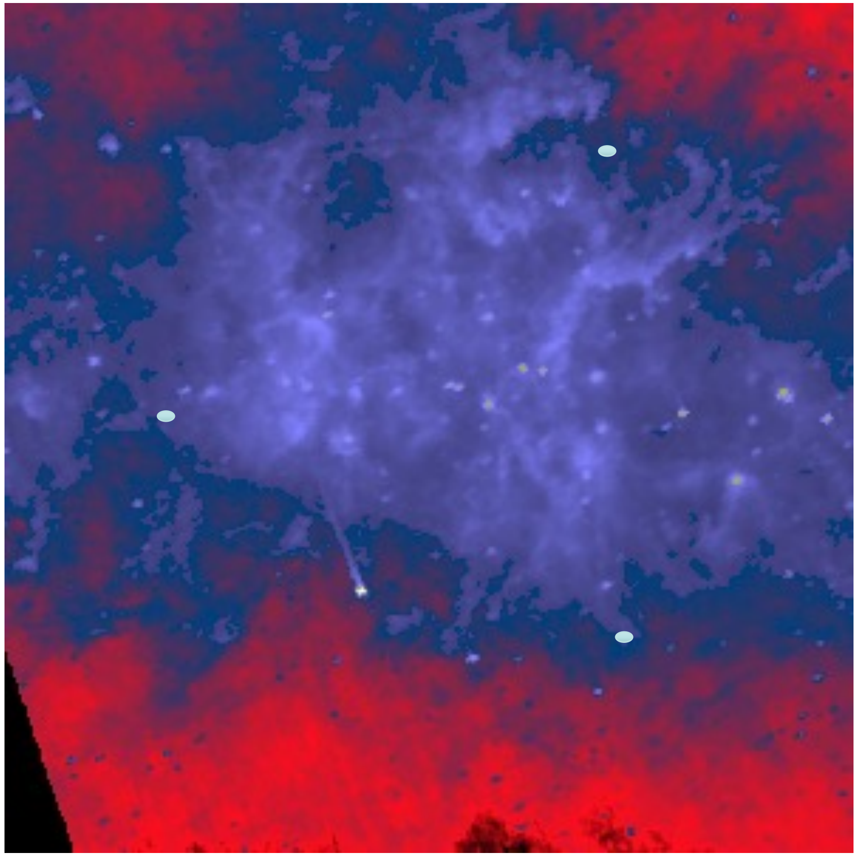
IRAS **12 μm**

$(l, b) = (80^o, 0^o)$ $10^o \times 10^o$



IRAS $\mathbf{12}\ \mu\mathrm{m}$

$(\mathrm{l}, \mathrm{b}) = (80^{\mathrm{o}}, 0^{\mathrm{o}})$ $10^{\mathrm{o}} \times 10^{\mathrm{o}}$

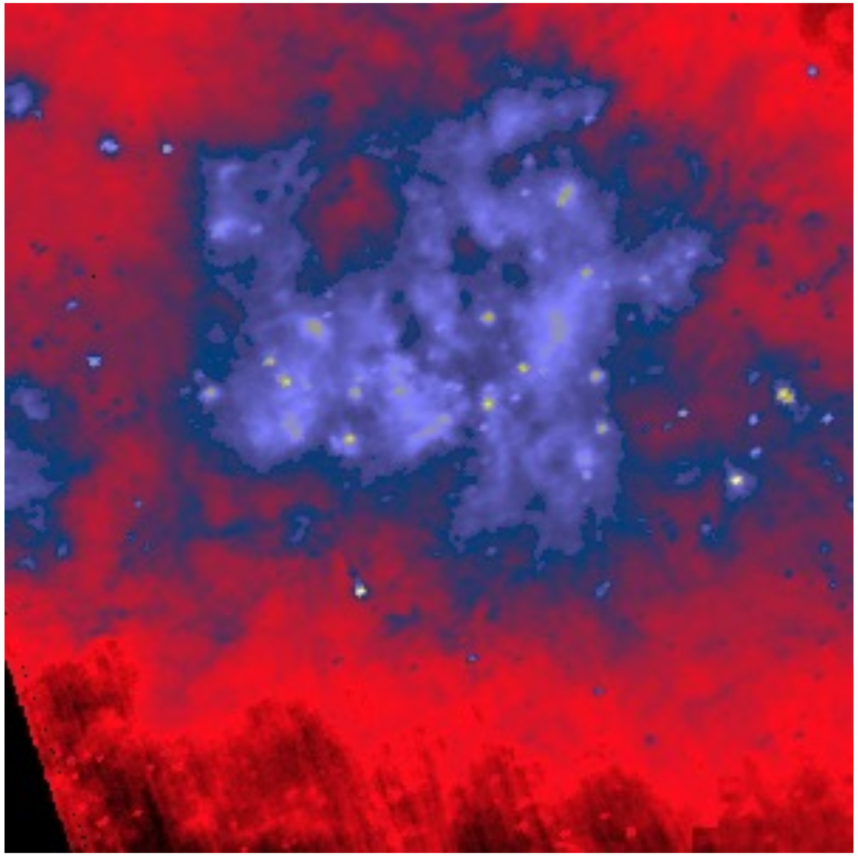


IRAS

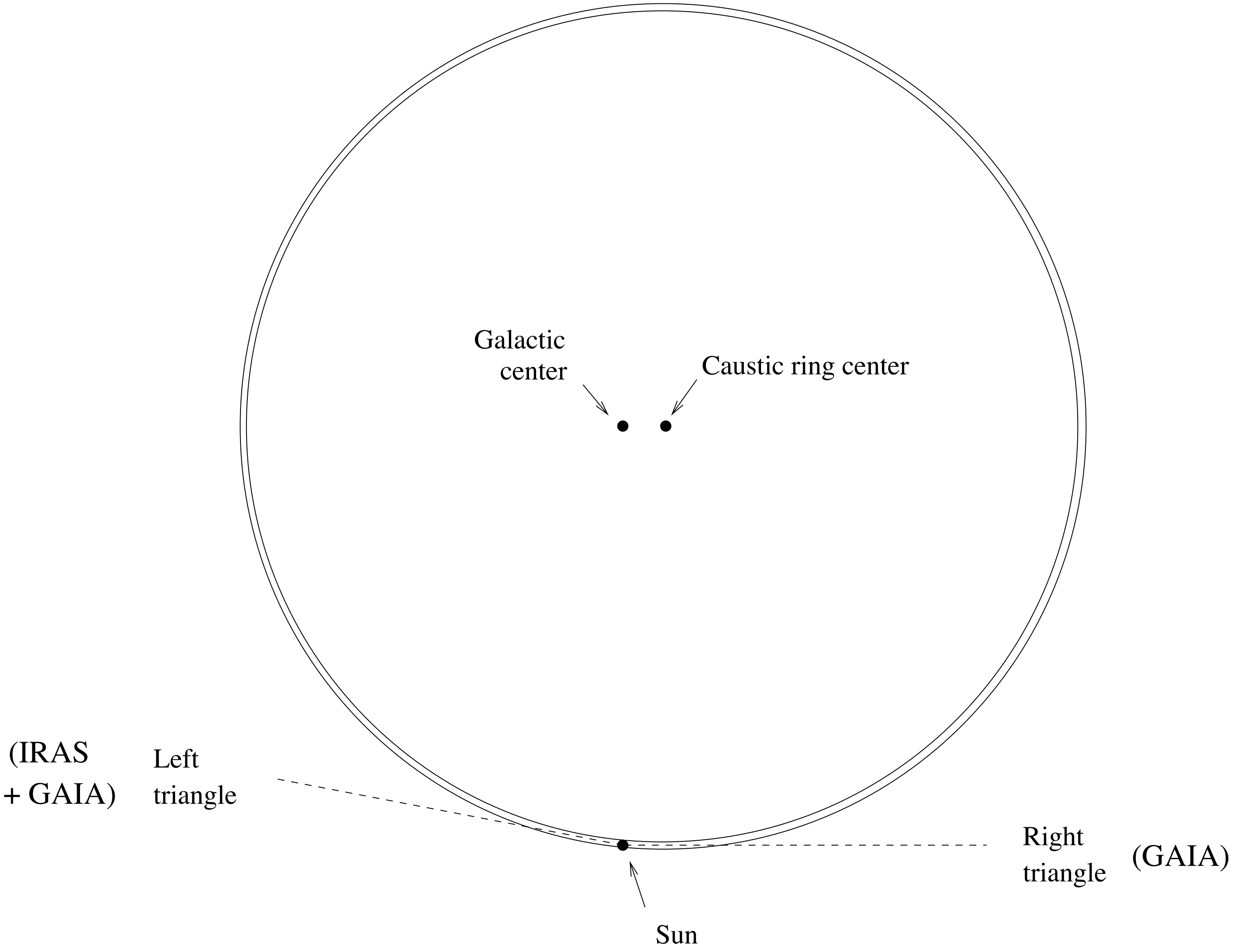
$\mathbf{25}\ \mu\mathrm{m}$

$(\mathrm{l}, \mathrm{b}) = (80^{\mathrm{o}}, 0^{\mathrm{o}})$

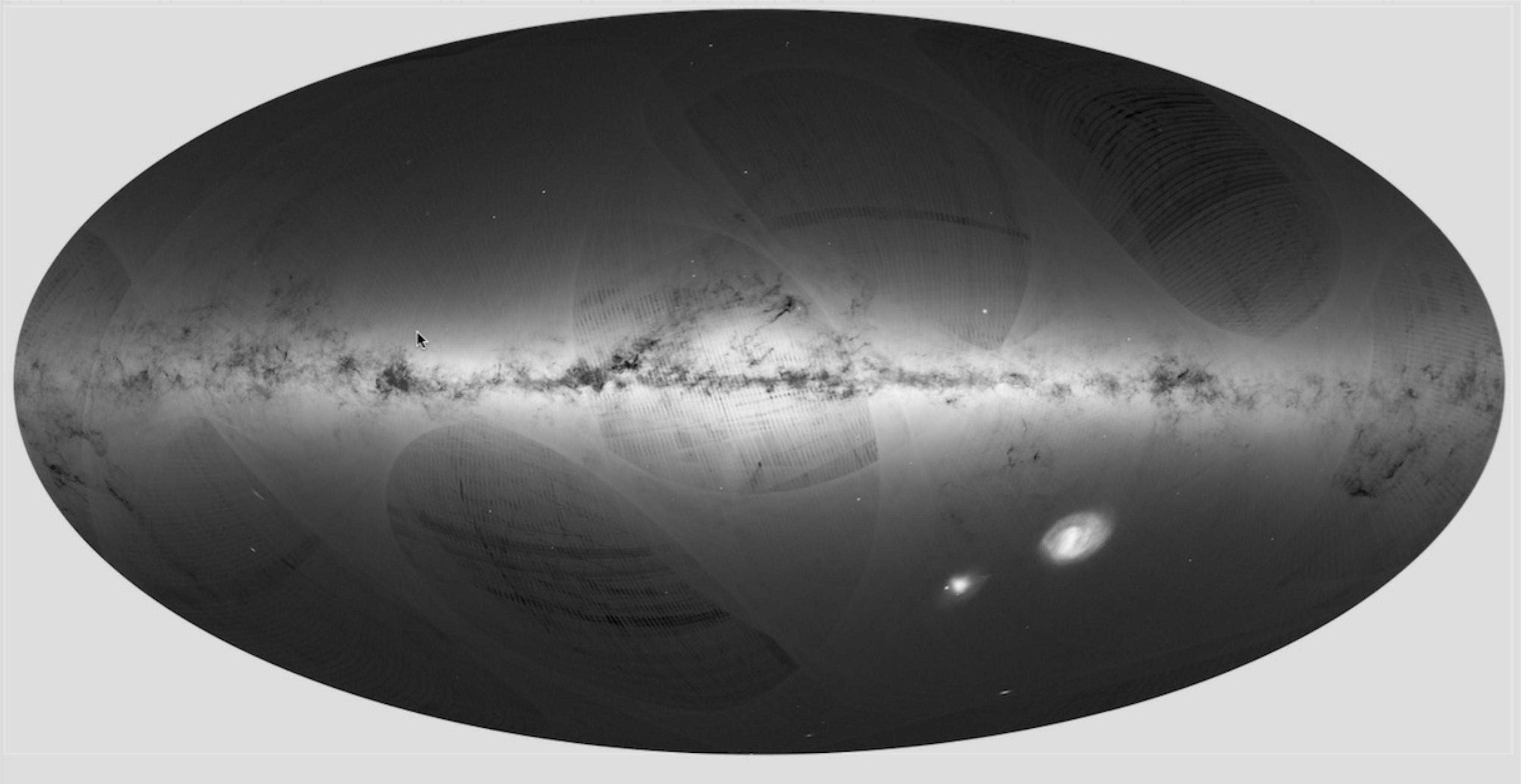
$10^{\mathrm{o}} \times 10^{\mathrm{o}}$

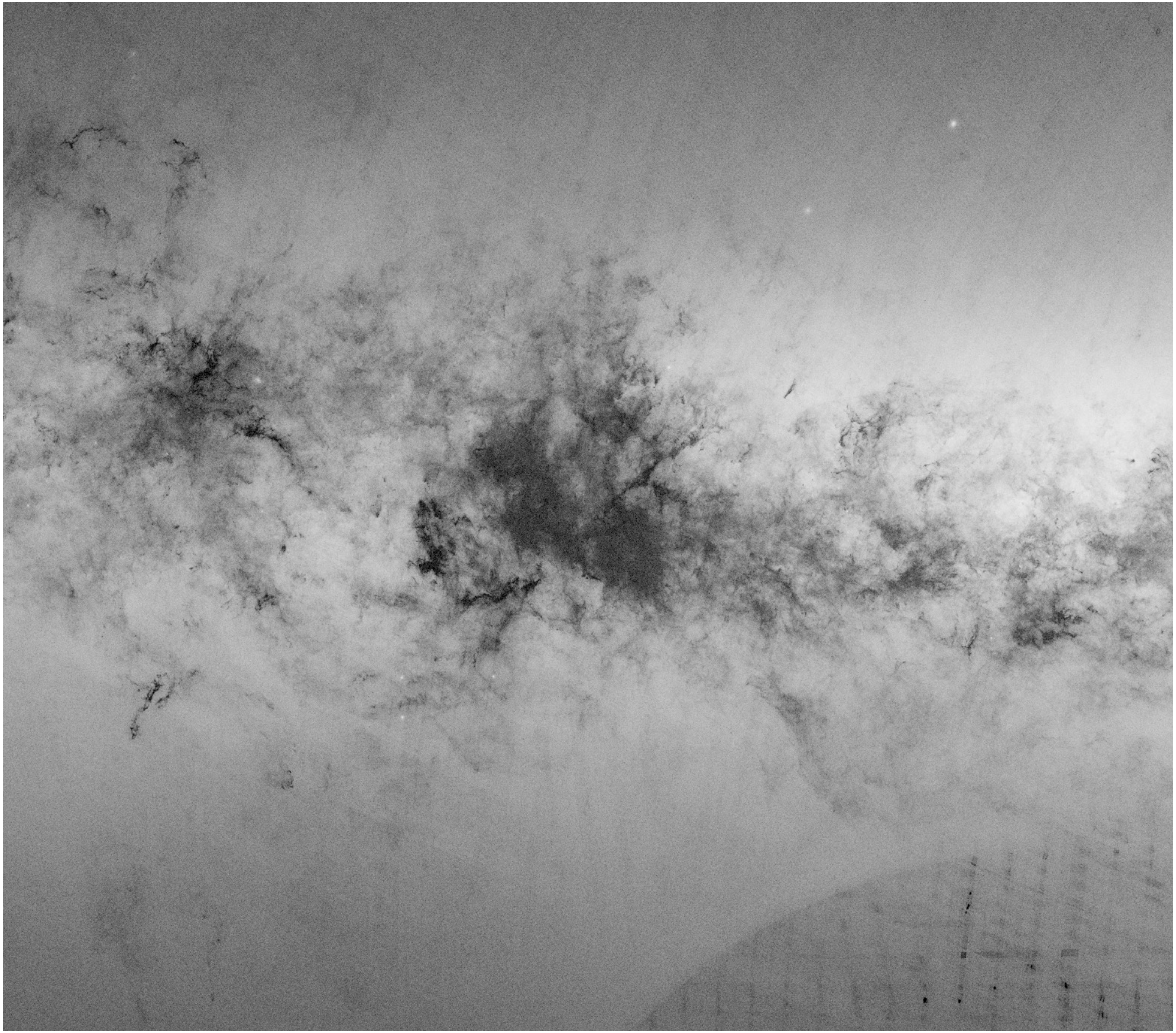


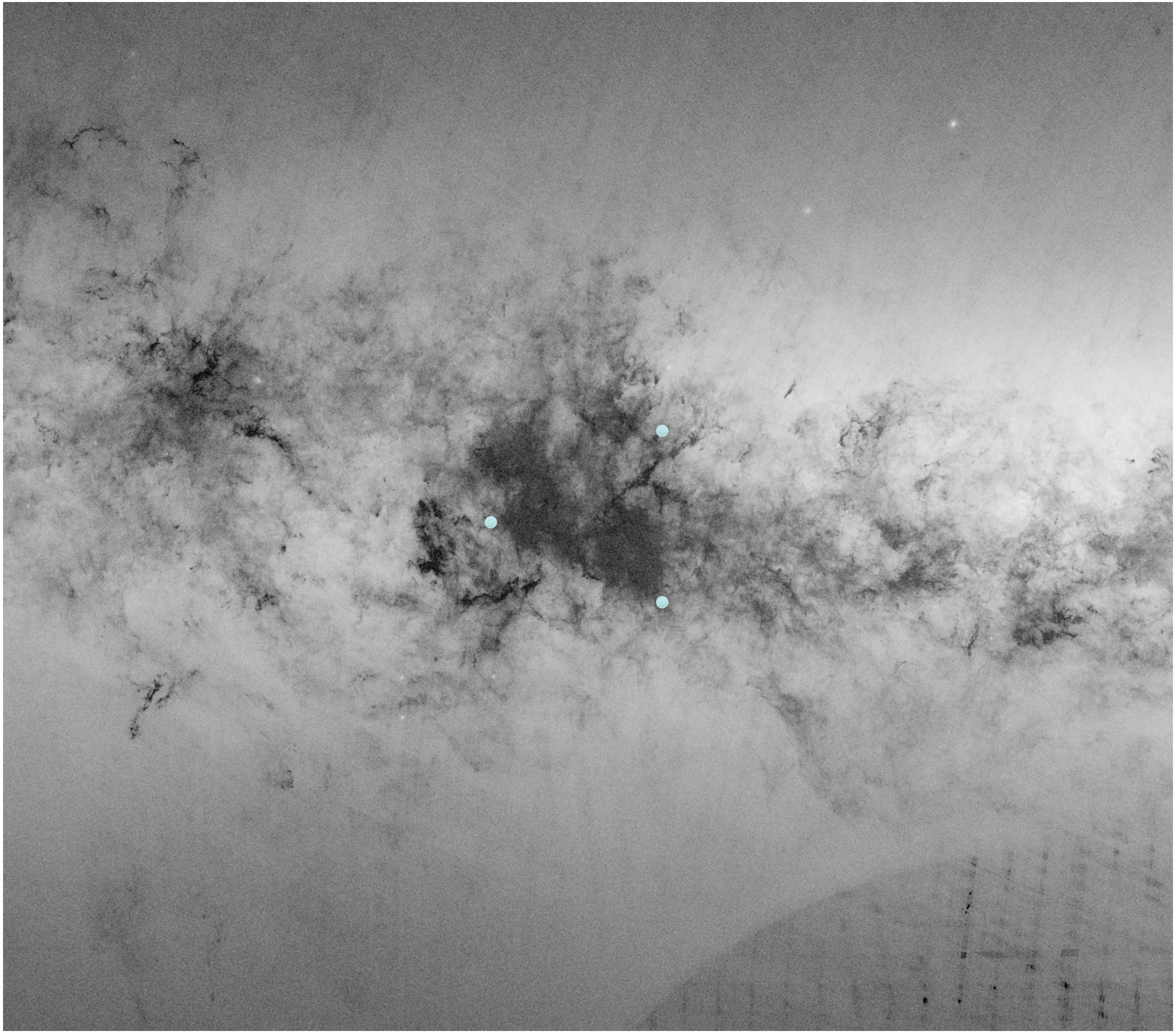
S. Chakrabarty, Y. Han, A. Gonzalez and PS, arXiv: 2007.10509



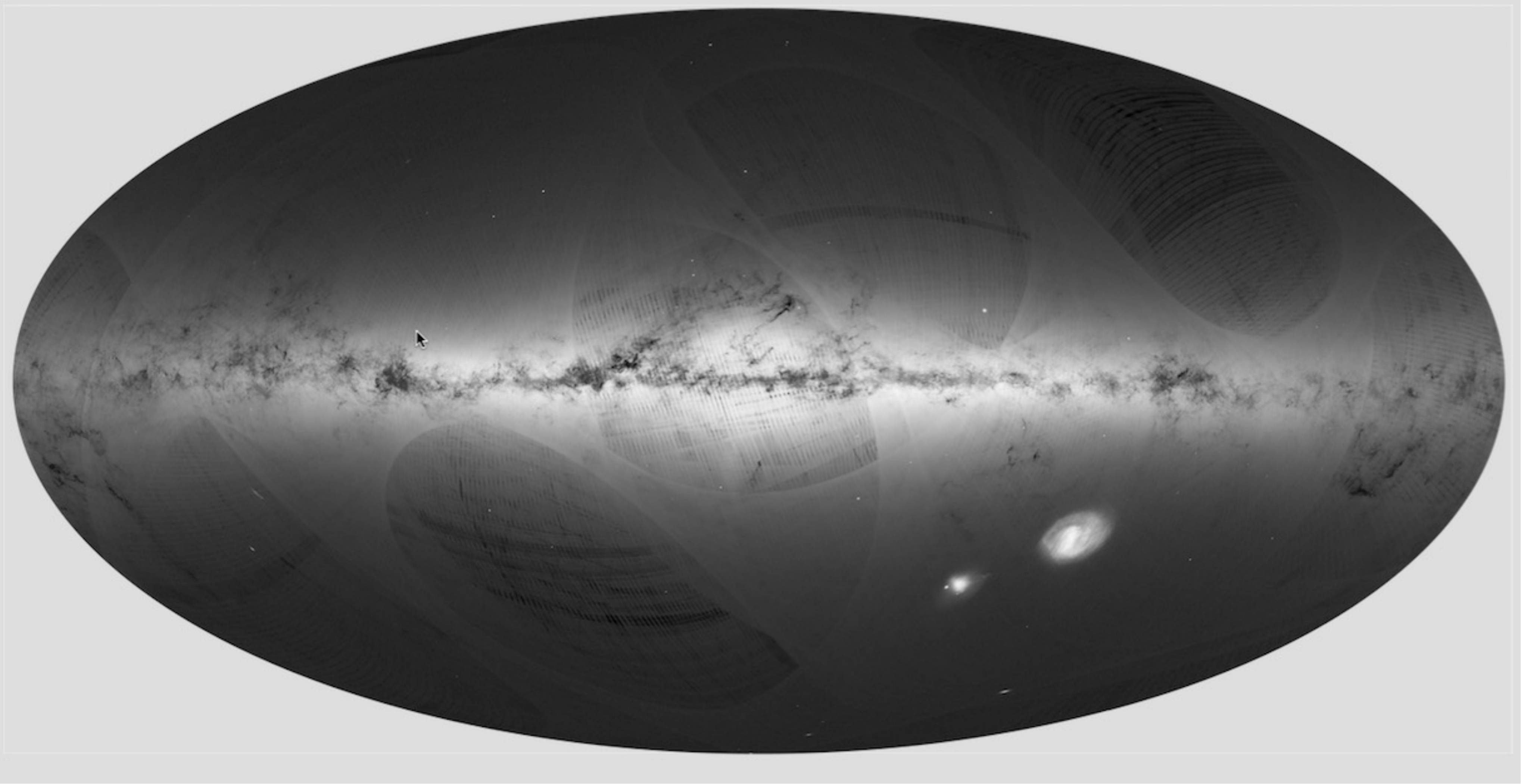
GAIA sky map

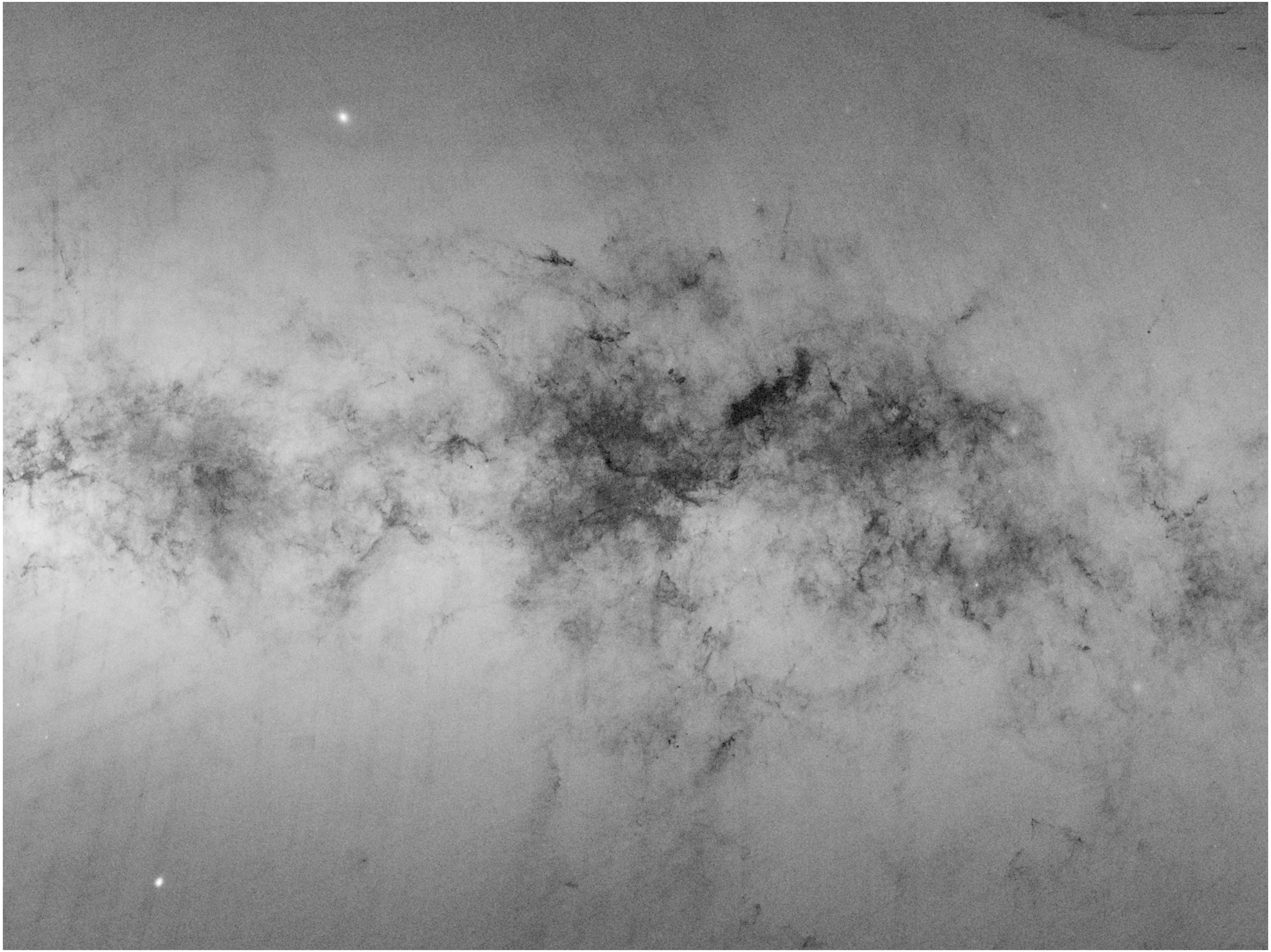


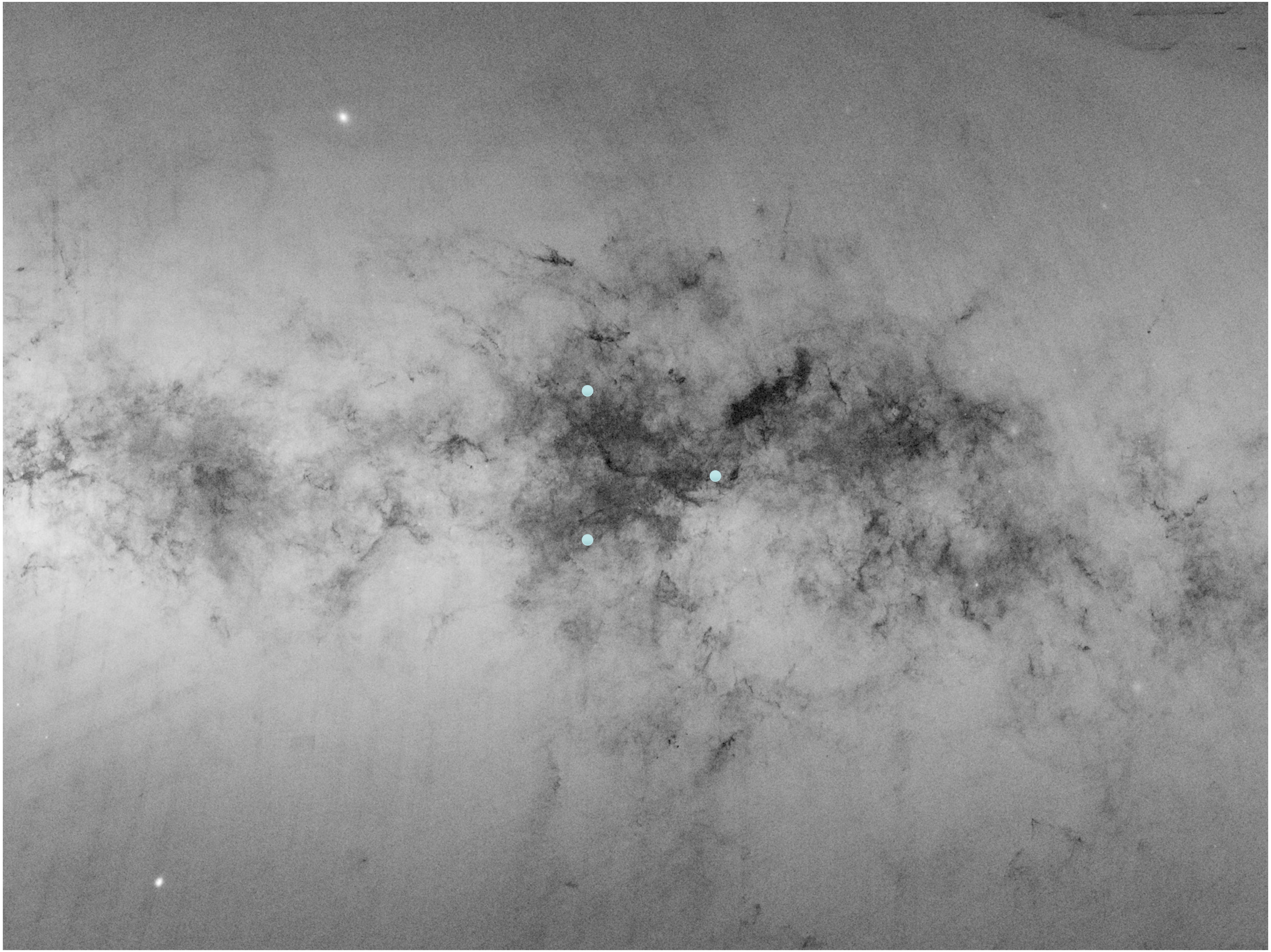


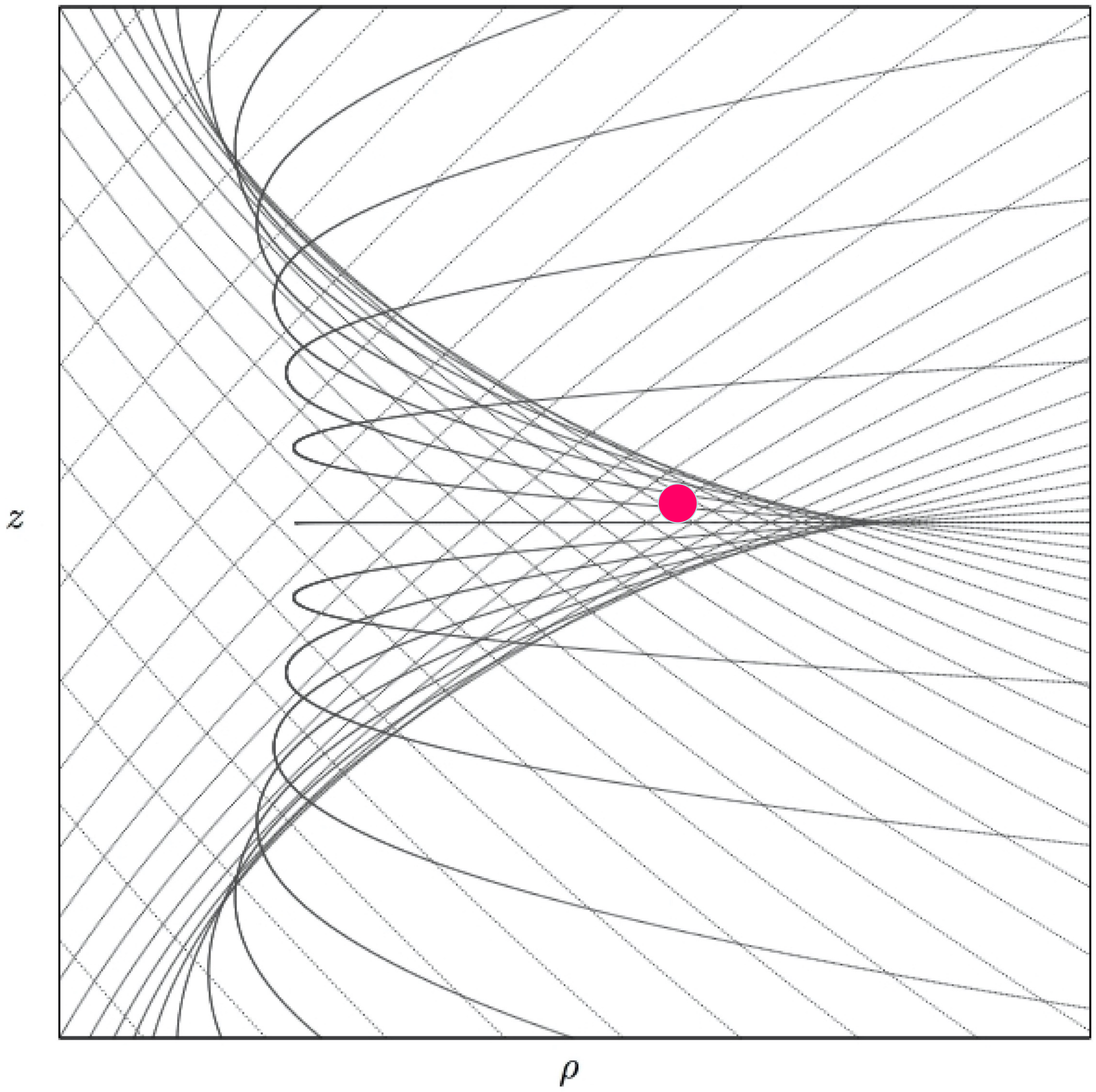


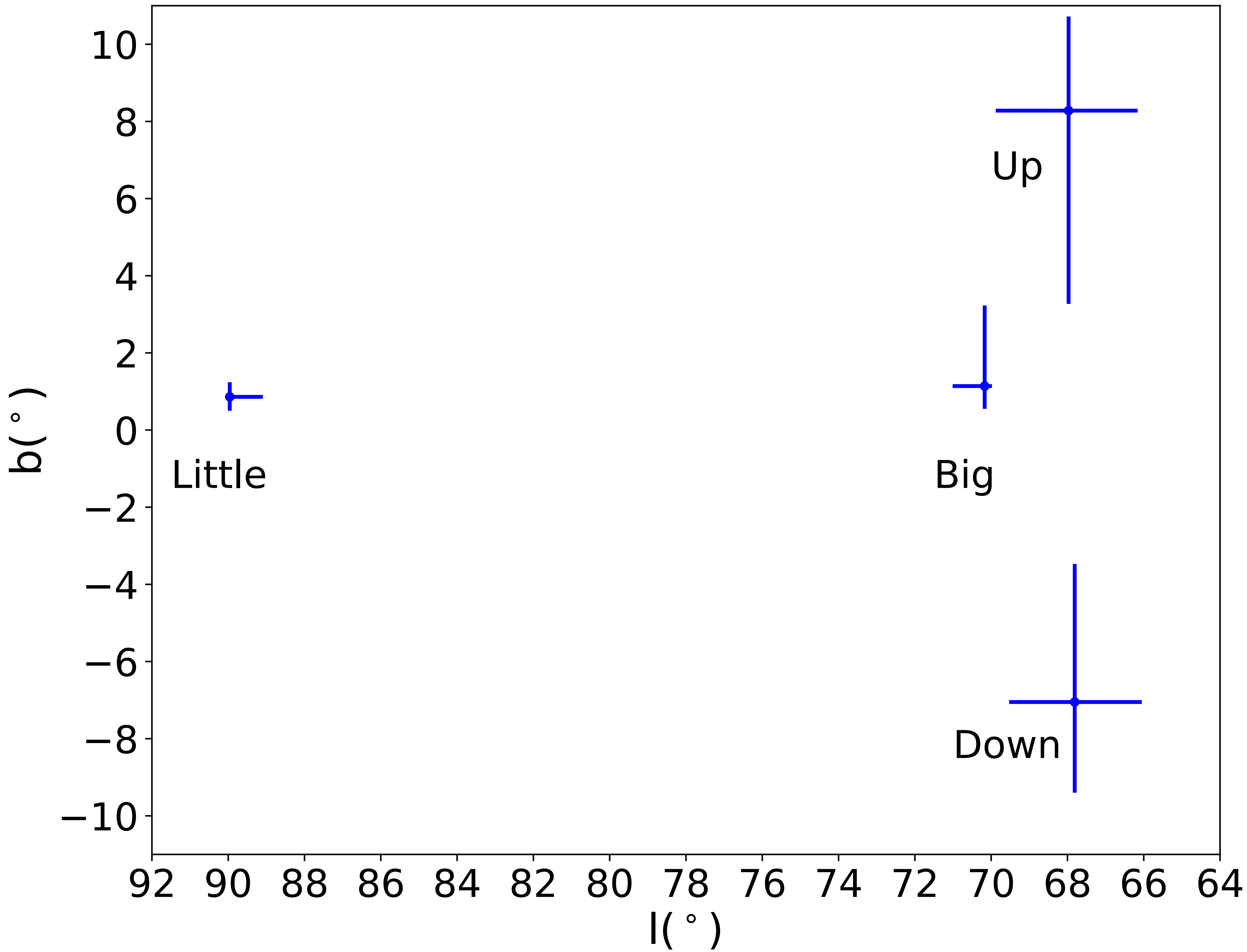
GAIA sky map








z
ρ



Up
Big
Little
Down
b(°)
l(°)
10
8
6
4
2
0
−2
−4
−6
−8
−10
92
90
88
86
84
82
80
78
76
74
72
70
68
66
64

Conclusions

Discrete flows and caustics are a robust prediction of cold dark matter cosmology.

There is observational evidence for caustic rings in galactic halos. It implies that the dark matter falls in with a rotational velocity field.