An Exact Algorithm for the Linear Tape Scheduling Problem

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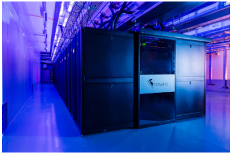
JI – November 2022




AND NOW,
MAGNETIC TAPES!


Insert this side into recorder
Do not touch the tape inside


A
SA
TDK

Tape usage today

$\approx$ 20TB on $\overbrace{1000\text{s} \times 1\text{km}}^{\text{(or 11 football fields)}}$ read at $\underbrace{10\text{m/s}}$ – 100s MB/s

https://commons.wikimedia.org/wiki/File:LTO2-cart-wo-top-shell.jpg
https://commons.wikimedia.org/wiki/File:Usain_Bolt_Rio_100m_final_2016i-cr.jpg

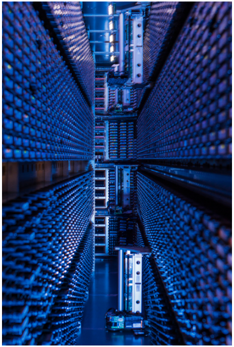
Primordial for HTC (High Throughput Computing)

e.g., CCIN2P3 CERN ECMWF Forecast User Guide (100s PB)

also: media companies, cloud archive...

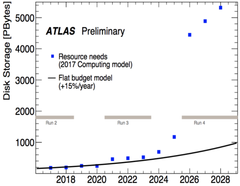
☺ Impressive technology improvements
density: + 30% / year (vs HDD: + 8%)

☹ high latency (mount, load, position $\rightarrow$ few mn)
Adapted for Write Once Read Many



Why not use hard drives?

up to 6-10 times cheaper overall (before 2020)

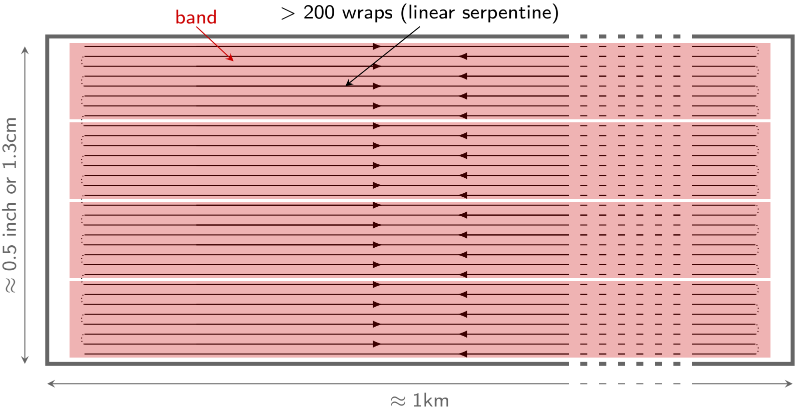


[Xin Zhao, HEPIX 2018]

air gap, power failure, lifetime

energy-efficient

Overview of a tape



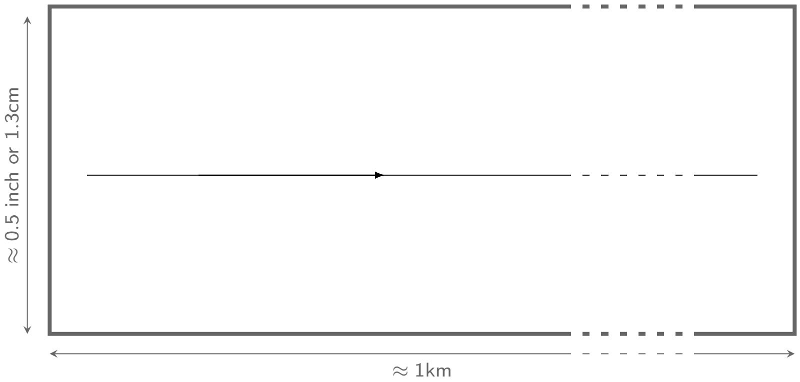
wrap = dozens of tracks read / written simultaneously by parallel heads

Overview of a tape

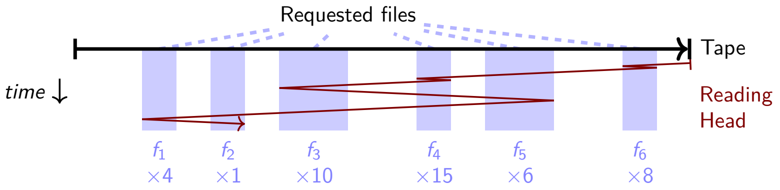
Boustrophedon

wrap = dozens of tracks read / written simultaneously by parallel heads

Overview of our tape model



Linear Tape Scheduling Problem [CardonhaReal'16]



Assumptions:

- files are read left-to-right
- start on the right
- constant speed

Input:

- tape of n_f consecutive files
- n file requests (44 here)
- n_{req} distinct files requested (6)

Objective: average service time

Motivation: lack of fundamental theoretical results, models local files

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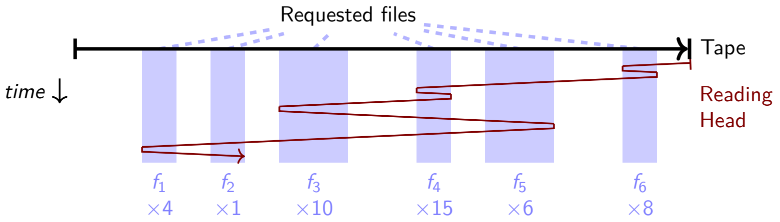

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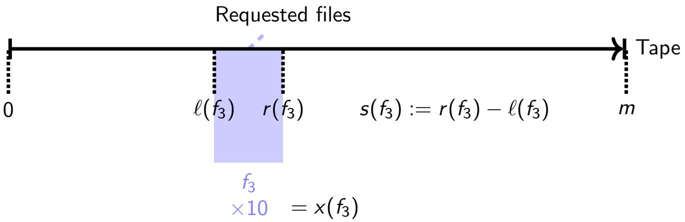

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Related to the Linear Tape Scheduling Problem

Travelling Salesperson Problem (TSP)

- super-famous NP-hard problem
- recent $(1.5 - 10^{-36})$ - approximation [KarlinKG'21]
- ☹ minimizes makespan, trivial on the real line

Minimum Latency Problem / TRP (Repair) - variant

- ☺ minimize average service time $\in P$ on the real line
- delays to repair a node: complexity open

Dial-a-ride variant on the real line

- $\approx$ LTSP but with overlapping files in both directions
  $\longrightarrow$ NP-hard

Tapes except LTSP: 2 specific experimental papers in the 90's

Structural results

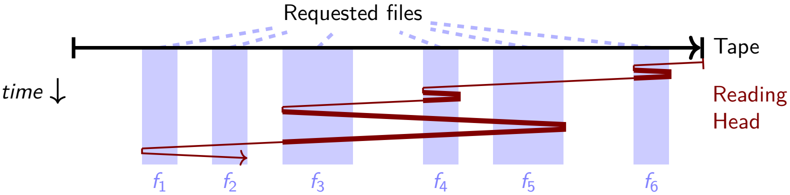
Any optimal solution

- after reaching $\ell(f_1)$, go straight to the rightmost unread request
- can be described by a set of **detours** done before

Definition (Detours)

A solution includes the **detour** (a,b) with $a \leq b$ if:

- the 1st time the head reaches $\ell(a)$, go straight to $r(b)$, back to $\ell(a)$



detours: $[(f_6, f_6), (f_4, f_4), (f_3, f_5)]$

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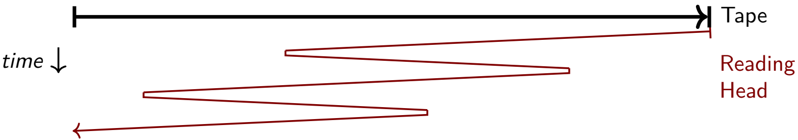

Lemma: detours never partially overlap (*strictly laminar*)

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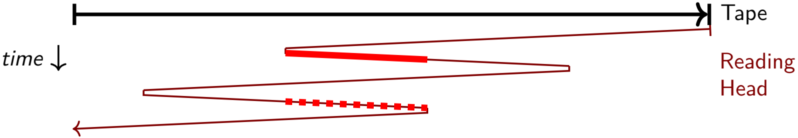

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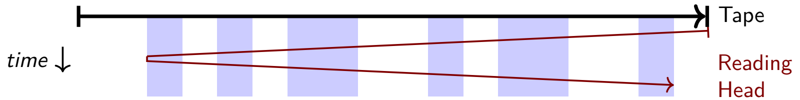
Naive algorithms

NoDetour: go to the leftmost request, then to the rightmost

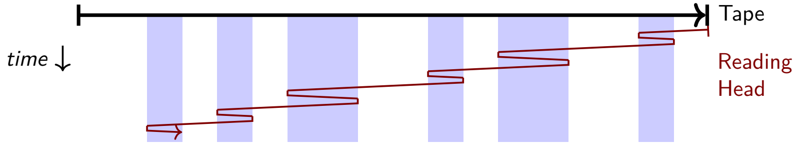
can be arbitrarily bad (place urgent requests on the right)



GS (Greedy Schedule): do all atomic detours, *i.e.*, $\{(f_i, f_i)\}_{\forall i}$

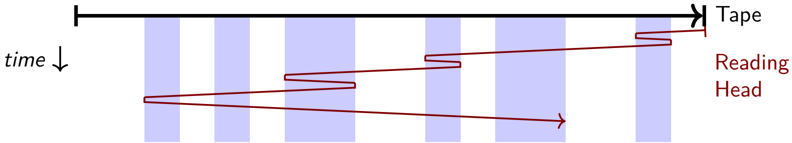
Lemma [CardonhaReal'16] : **GS** is a 3-approximation if $U = 0$

Proof: does $\leq$ 3 times the optimal distance before reading each request

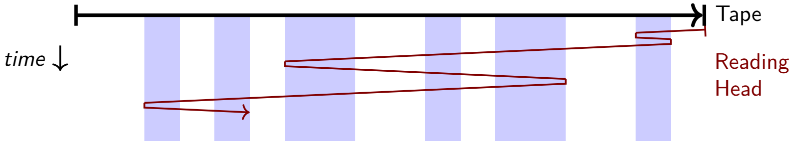


Heuristic improvements [CardonhaCiréReal'18]

FGS (Filtered): remove detrimental atomic detours in $O(n_{req}^2)$



NFGS (Non-atomic): greedily add long detours if currently beneficial. Make one pass from left to right. Complexity in $O(n_{req}^3)$



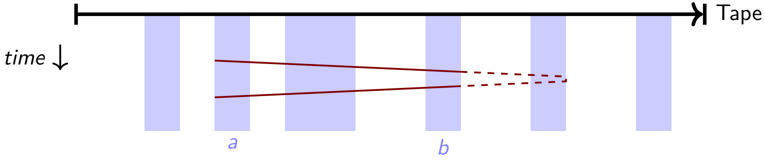
Dynamic Program: overview

Each cell: three parameters $T[a, b, n_{skip}]$

- compute the best strategy from $r(\mathbf{b})$ to $\ell(\mathbf{a})$ assuming:

1. there is a detour $(\mathbf{a}, f)$ for some $f \geq \mathbf{b}$,
2. there is no detour (f_1, f_2) such that $\mathbf{a} < f_1 < \mathbf{b} < f_2$,
3. when reading $r(\mathbf{b})$, exactly n_{skip} requests have been skipped.

$\Rightarrow$ value $\approx$ cost contribution from 'first $r(b)$' to '$r(b)$ after reading a'



Subtleties: $\forall$ request on f, do not count the cost $m \rightarrow \ell(f) \rightarrow r(f)$
if f is read after b, remove one U (counted before)

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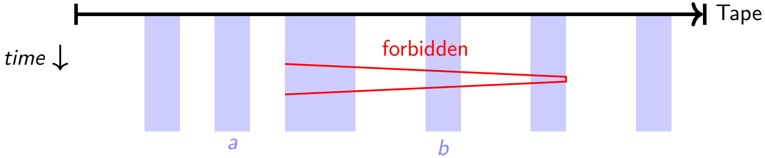

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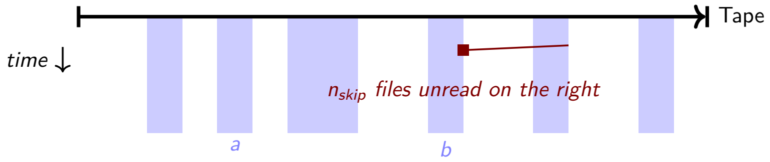

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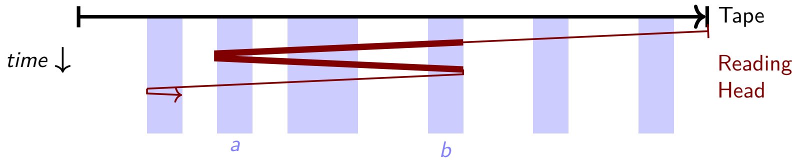

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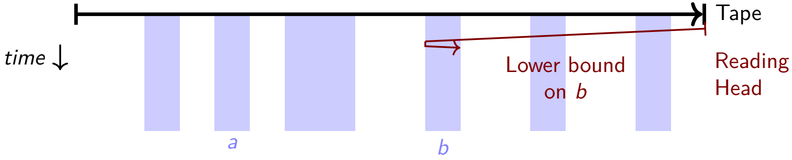

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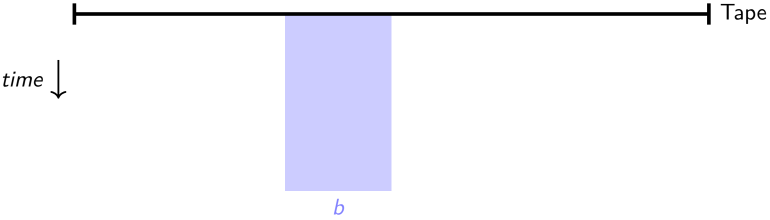
Dynamic program: base case, $a = b$

$a = b \implies$ **a detour starts at** $\ell(b)$

$n_\ell(b) :=$ # file requests strictly on the left of b

$$T[b, b, n_{skip}] = 2 \cdot s(b) \cdot (n_{skip} + n_\ell(b))$$



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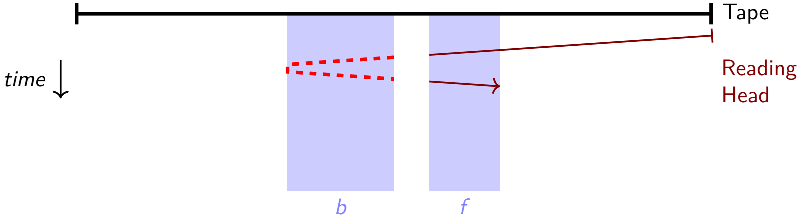

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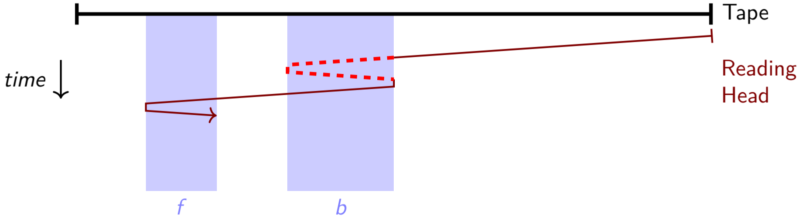

Dynamic program: inductive case 1, skip b

$a < b$ and assume b is skipped

$left(f) :=$ requested file directly on the left of f

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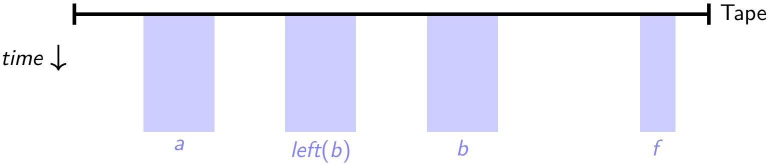

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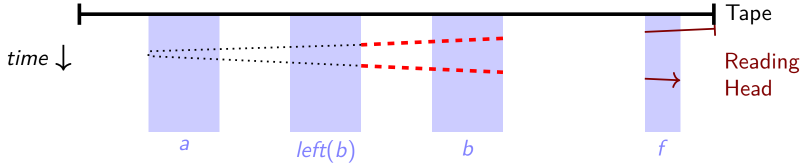

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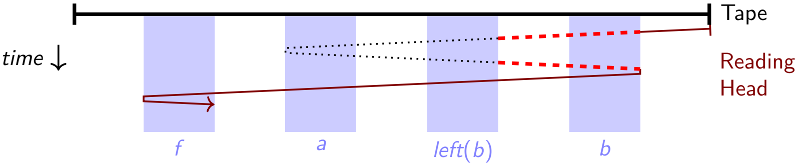

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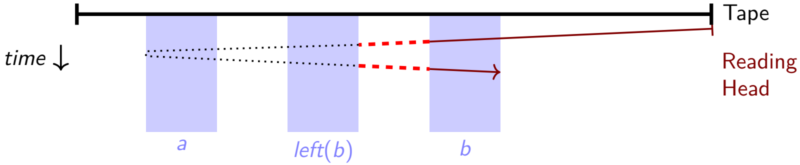

Dynamic program: inductive case 2, intertwined detours

$a < b$, there is a detour (c, b) with $c > a$

$$
\begin{aligned}
detour_c(a, b, n_{skip}) := \; & T[a, left(c), n_{skip}] + T[c, b, n_{skip}] \\
& + 2 \cdot (r(b) - r(left(c))) \cdot (n_{skip} + n_\ell(a)) \\
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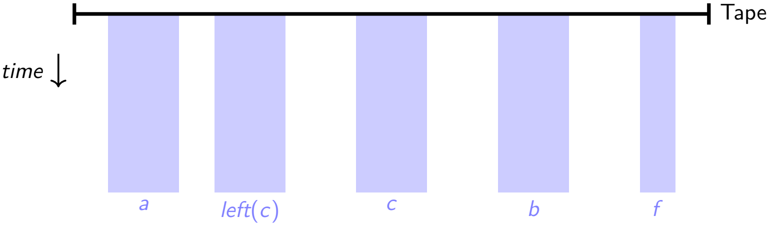

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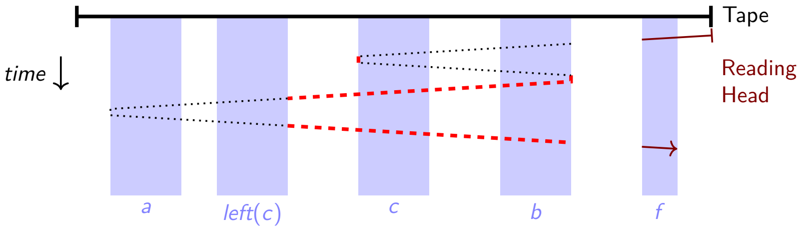

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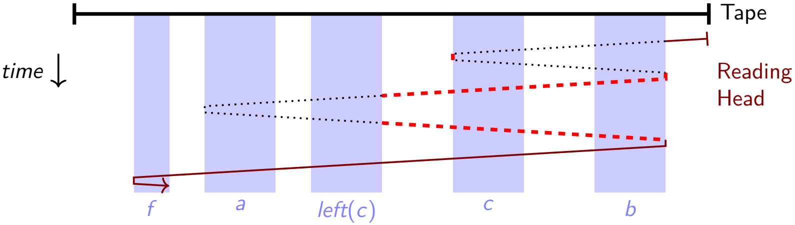

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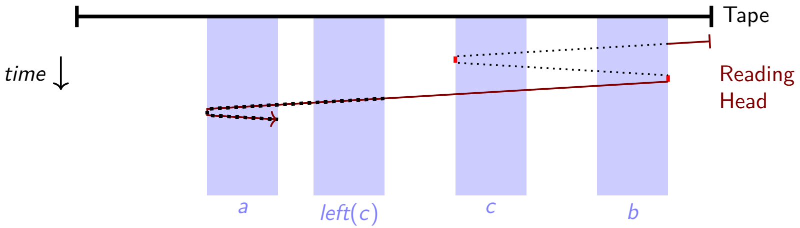

Dynamic program: complete formulation

$$
\begin{aligned}
\mathit{skip}(a, b, n_{skip}) := \; & T[a, \mathit{left}(b), n_{skip} + x(b)] \\
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\end{aligned}
$$

define $F_{a,b}$:= files requested between a and b excluding a

Dynamic program $\mathbf{DP}$ (with $a < b$)

$$
\begin{aligned}
T[b, b, n_{skip}] &= 2 \cdot s(b) \cdot (n_{skip} + n_\ell(b)) \\
T[a, b, n_{skip}] &= \min \left(\mathit{skip}(a, b, n_{skip}) \; ; \; \min_{c \in F_{a,b}} \mathit{detour}_c(a, b, n_{skip}) \right)
\end{aligned}
$$

Theorem

DP *solves LTSP exactly in time* $O(n \cdot n_{req}^3)$.

More dynamic programs

LogDP(λ): **DP** restricted to detours spanning $\lambda \log n_{req}$ requested files

Reduced complexity in $O(\lambda^2 \cdot n_{req} \cdot n \cdot \log^2(n_{req}))$, *tested with* $\lambda \in \{1, 5\}$

SimpleDP: **DP** forbidding intertwined (*i.e.*, overlapping) detours

Similar to **DP** but a is always f_1: no need to call $T[b, c, n_{skip}]$
Reduced complexity in $O(n \cdot n_{req}^2)$

Lemma: for all U, competitive ratio $\in [\frac{5}{3}, 3]$

Note: dependency in n and not $\log n \longrightarrow$ pseudo-polynomial for high-multiplicity instances (harder problem as in scheduling)

Note2: concurrent similar solution from [CardonhaCireReal'21]

Simulations: overview

Dataset: 2 weeks at CC-IN2P3

- 169 tapes, $>$ 3M files
- focus on reading operations
- filtering steps, data processing (e.g., merge reads on aggregates)
- median data: 150 files requested, 3k requests, 50% file size variation

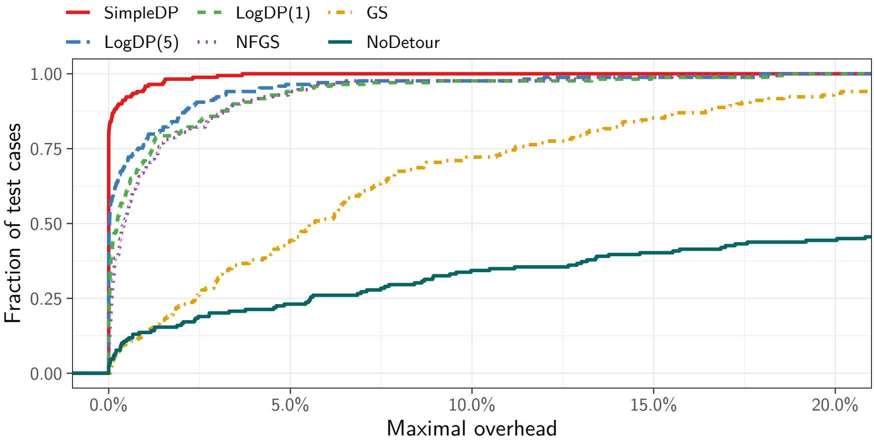
Code + dataset (with statistical descriptions) available online

Experimental methodology

- choose 3 values for U: $\{0, 0.5, 1\}\times$ average file size reading time
- median time performance (seconds, on a compiled Python program):

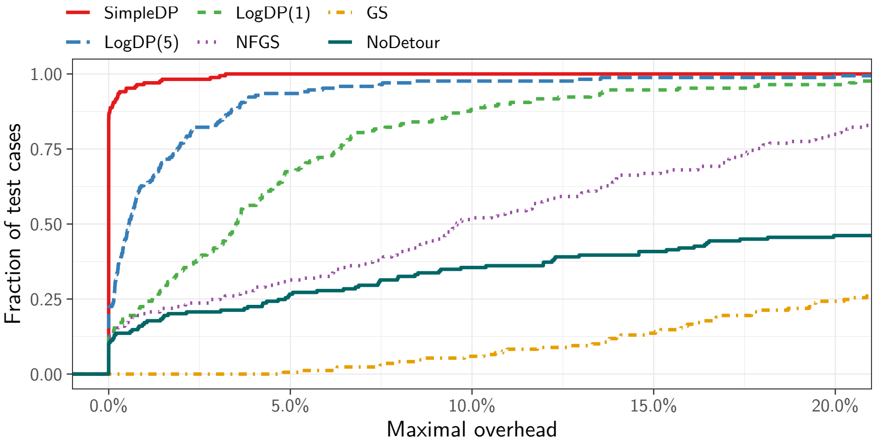
FGS	NFGS	LogDP(1)	SimpleDP	LogDP(5)	DP
< 0.1	1	2	3	7	30

Simulation results, $U = 0$



Performance profile: best is top-left (most instances with low overhead vs OPT)

Simulation results, U = file



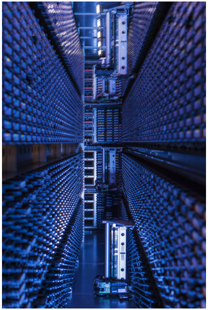
Performance profile: best is top-left (most instances with low overhead vs OPT)

Conclusion

https://commons.wikimedia.org/wiki/File:LT02-cart-wo-top-shell.jpg

General: tapes are past & future

- tapes stay primordial in some fields (€$£¥₩) but neglected by CS research
- fundamental problems are still open



On LTSP

- high-multiplicity variant remains open
- huge gap between theoretically studied models and practical heuristics

Perspectives on other tape-related topics

- multi-tape requests: optimize waiting queues
- optimize tape / disk storage ratio

CS Research

- some engineering problems are unknown to researchers