Neutrino-nucleus cross sections

Marco Martini

IPSA ÉCOLE D'INGÉNIEURS DE L'AIR, DE L'ESPACE ET DE LA MOBILITÉ DURABLE

SORBONNE UNIVERSITÉ

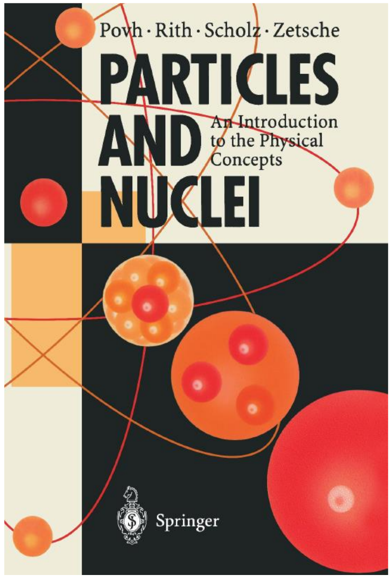
LPNHE PARIS

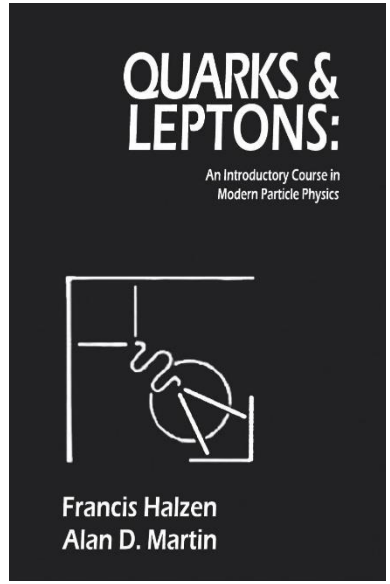
Neutrino-nucleus cross sections -- Plan

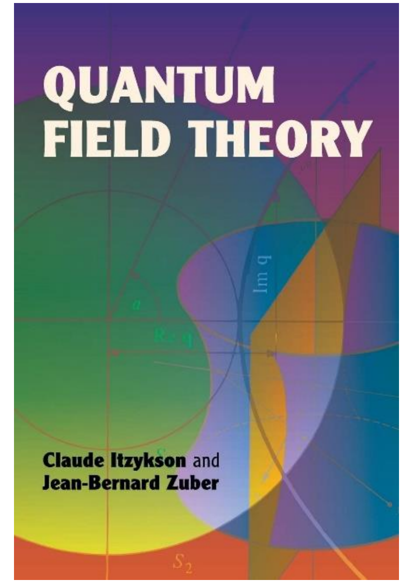
- First lecture: generalities (theory and experiment)

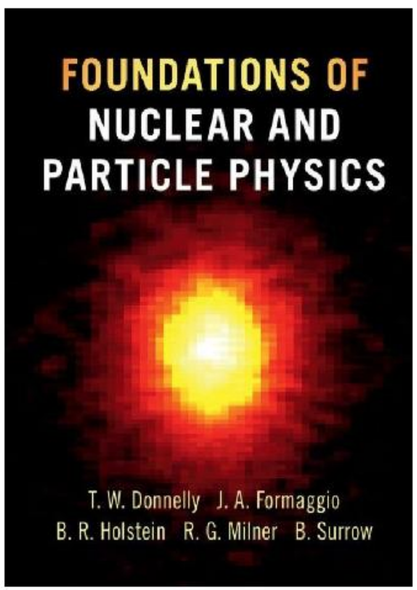
- Second lecture: results and perspectives

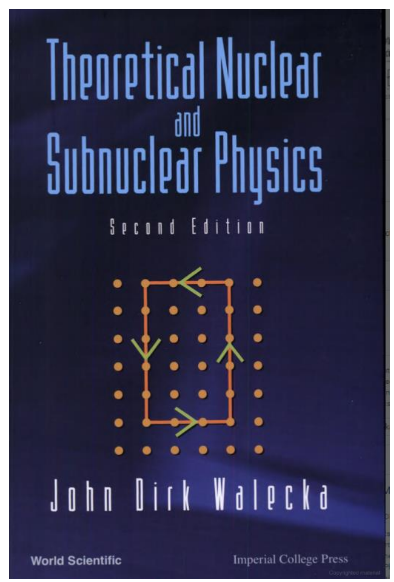
Some Books

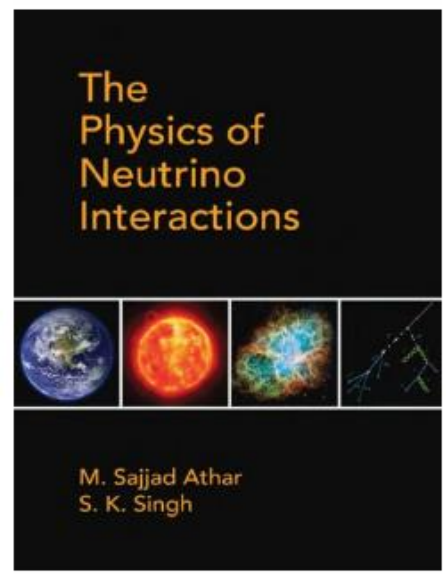

Povh · Rith · Scholz · Zetsche
PARTICLES AND NUCLEI
An Introduction to the Physical Concepts
Springer


QUARKS & LEPTONS:
An Introductory Course in Modern Particle Physics
Francis Halzen
Alan D. Martin


QUANTUM FIELD THEORY
Claude Itzykson and
Jean-Bernard Zuber


FOUNDATIONS OF NUCLEAR AND PARTICLE PHYSICS
T. W. Donnelly J. A. Formaggio
B. R. Holstein R. G. Milner B. Surrow


Theoretical Nuclear and Subnuclear Physics
Second Edition
John Dirk Walecka
World Scientific
Imperial College Press


The Physics of Neutrino Interactions
M. Sajjad Athar
S. K. Singh

Some Review papers

1305.7513.pdf (arxiv.org)

REVIEWS OF MODERN PHYSICS, VOLUME 84, JULY–SEPTEMBER 2012

From eV to EeV: Neutrino cross sections across energy scales

J. A. Formaggio*

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Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

1706.03621.pdf (arxiv.org)

Progress in Particle and Nuclear Physics 100 (2018) 1–68

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

NuSTEC[1] White Paper: Status and challenges of neutrino–nucleus scattering

L. Alvarez-Ruso[a], M. Sajjad Athar[b], M.B. Barbaro[c], D. Cherdack[d], M.E. Christy[e], P. Coloma[f], T.W. Donnelly[g], S. Dytman[h], A. de Gouvêa[i], R.J. Hill[j,f], P. Huber[k], N. Jachowicz[l], T. Katori[m], A.S. Kronfeld[f], K. Mahn[n], M. Martini[o], J.G. Morfín[f,*], J. Nieves[a], G.N. Perdue[f], R. Petti[p], D.G. Richards[q], F. Sánchez[r], T. Sato[s,t], J.T. Sobczyk[u], G.P. Zeller[f]

1611.07770.pdf (arxiv.org)

IOP Publishing — Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **45** (2018) 013001 (98pp) https://doi.org/10.1088/1361-6471/aa8bf7

Topical Review

Neutrino–nucleus cross sections for oscillation experiments

Teppei Katori[1,4,5] **and Marco Martini**[2,3,4,5]

[1] School of Physics and Astronomy, Queen Mary University of London, London, United Kingdom

[2] ESNT, CEA, IRFU, Service de Physique Nucléaire, Université de Paris-Saclay, F-91191 Gif-sur-Yvette, France

[3] Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium

2206.13792.pdf (arxiv.org)

Neutrinos and their interactions with matter

M. Sajjad Athar[a,*], A. Fatima[a], S. K. Singh[a]

[a] *Department of Physics, Aligarh Muslim University, Aligarh - 202002, India*

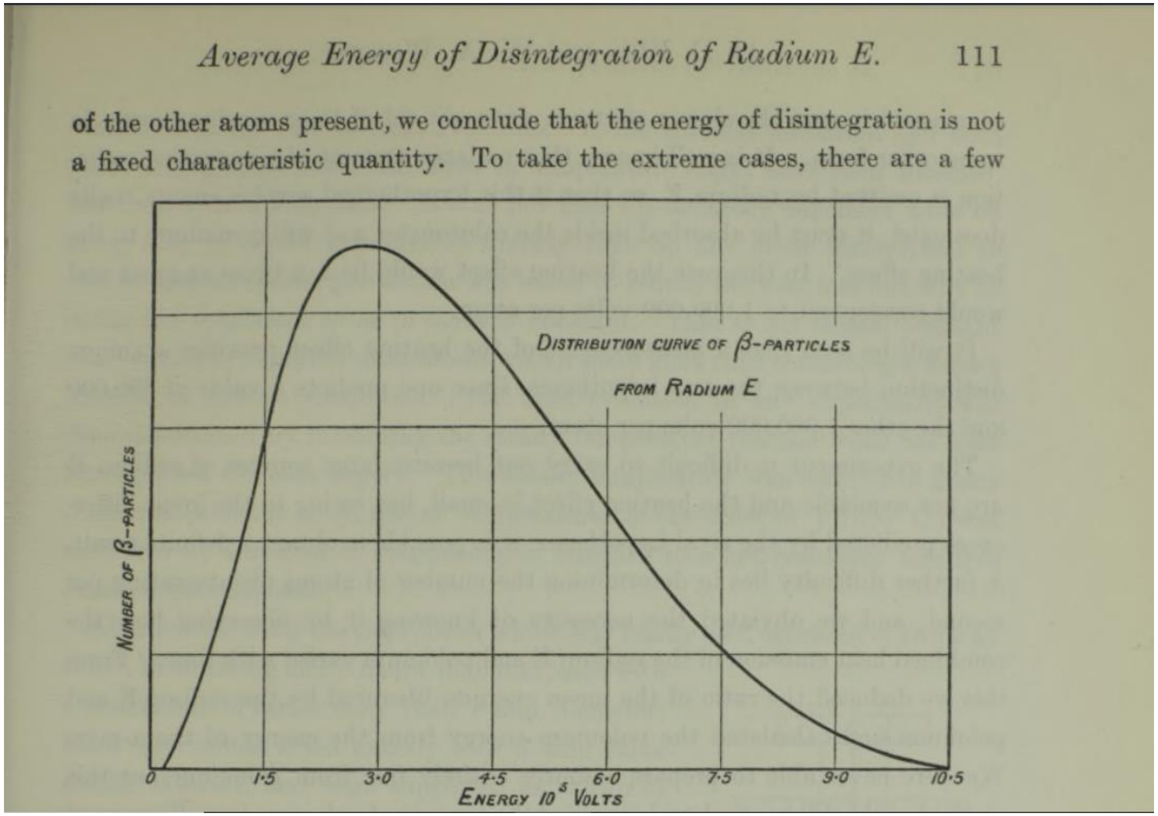
Appetizer:

Neutrino (interactions) (pre)history

Nuclear Beta Decay

Electron energy spectrum in Nuclear Beta Decay is continuous (*J. Chadwick 1914*)

C. D. Ellis and W. A. Wooster (1927)


Average Energy of Disintegration of Radium E. 111

of the other atoms present, we conclude that the energy of disintegration is not a fixed characteristic quantity. To take the extreme cases, there are a few

Historical papers (in2p3.fr)

$$^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + e^-$$

Two-body final state

$\Downarrow$

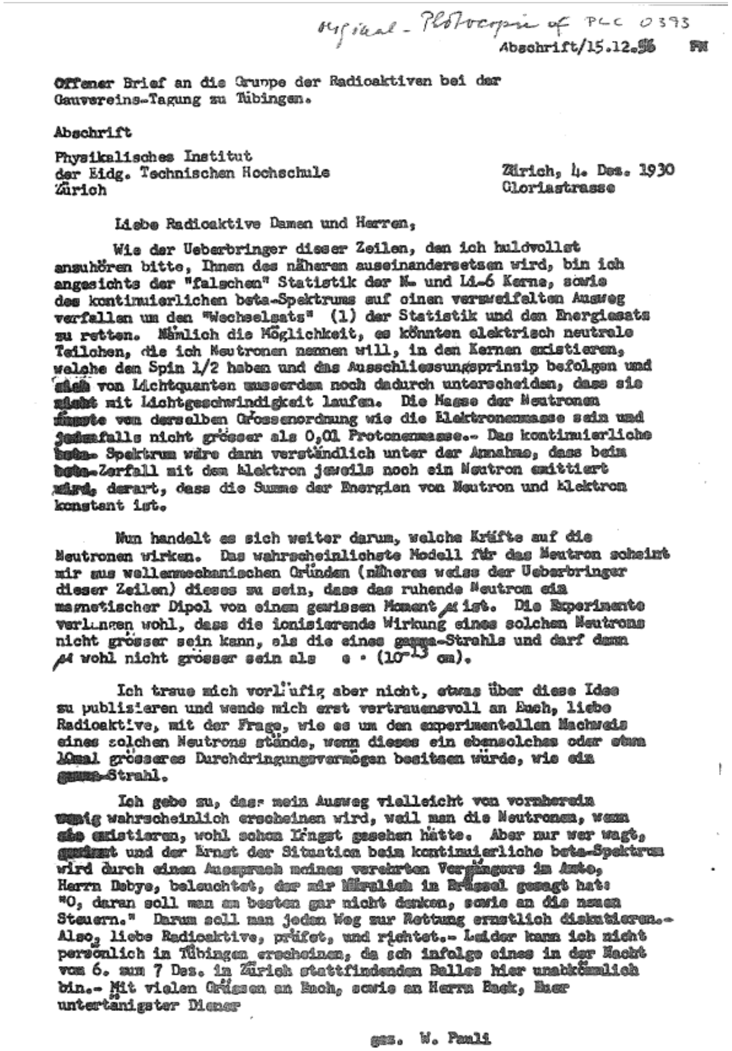
Electron should have a unique energy value

Puzzle

Niels Bohr  questioned the strict validity of energy conservation in subatomic processes

4 December 1930: Neutrino birth – W. Pauli letter

$$^{210}_{83}Bi \rightarrow ^{210}_{84}Po + e^{-} + "neutron"$$



original - Photocopie of PLC 0393

Abschrift/15.12.56 FN

Offener Brief an die Gruppe der Radioaktiven bei der Gauvereins-Tagung zu Tübingen.

Abschrift

Physikalisches Institut
der Eidg. Technischen Hochschule
Zürich

Zürich, 4. Dez. 1930
Gloriastrasse

Liebe Radioaktive Damen und Herren,

Wie der Ueberbringer dieser Zeilen, den ich huldvollst anzuhören bitte, Ihnen des näheren auseinandersetzen wird, bin ich angesichts der "falschen" Statistik der N- und Li-6 Kerne, sowie des kontinuierlichen beta-Spektrums auf einen verzweifelten Ausweg verfallen um den "Wechselsatz" (1) der Statistik und den Energiesatz zu retten. Nämlich die Möglichkeit, es könnten elektrisch neutrale Teilchen, die ich Neutronen nennen will, in den Kernen existieren, welche den Spin 1/2 haben und das Ausschliessungsprinzip befolgen und sich von Lichtquanten ausserdem noch dadurch unterscheiden, dass sie nicht mit Lichtgeschwindigkeit laufen. Die Masse der Neutronen müsste von derselben Grossenordnung wie die Elektronenmasse sein und jedenfalls nicht grösser als 0,01 Protonenmasse.- Das kontinuierliche beta- Spektrum wäre dann verständlich unter der Annahme, dass beim beta-Zerfall mit dem Elektron jeweils noch ein Neutron emittiert wird, derart, dass die Summe der Energien von Neutron und Elektron konstant ist.

Nun handelt es sich weiter darum, welche Kräfte auf die Neutronen wirken. Das wahrscheinlichste Modell für das Neutron scheint mir aus wellenmechanischen Gründen (näheres weiss der Ueberbringer dieser Zeilen) dieses zu sein, dass das ruhende Neutron ein magnetischer Dipol von einem gewissen Moment μ ist. Die Experimente verlangen wohl, dass die ionisierende Wirkung eines solchen Neutrons nicht grösser sein kann, als die eines gamma-Strahls und darf dann μ wohl nicht grösser sein als $e \cdot (10^{-13}$ cm).

Ich traue mich vorläufig aber nicht, etwas über diese Idee zu publizieren und wende mich erst vertrauensvoll an Euch, liebe Radioaktive, mit der Frage, wie es um den experimentellen Nachweis eines solchen Neutrons stände, wenn dieses ein ebensolches oder etwa 10mal grösseres Durchdringungsvermögen besitzen würde, wie ein gamma-Strahl.

Ich gebe zu, dass mein Ausweg vielleicht von vornherein wenig wahrscheinlich erscheinen wird, weil man die Neutronen, wenn sie existieren, wohl schon längst gesehen hätte. Aber nur wer wagt, gewinnt und der Ernst der Situation beim kontinuierliche beta-Spektrum wird durch einen Ausspruch meines verehrten Vorgängers im Amte, Herrn Debye, beleuchtet, der mir kürzlich in Brüssel gesagt hat: "O, daran soll man am besten gar nicht denken, sowie an die neuen Steuern." Darum soll man jeden Weg zur Rettung ernstlich diskutieren.- Also, liebe Radioaktive, prüfet, und richtet.- Leider kann ich nicht persönlich in Tübingen erscheinen, da ich infolge eines in der Nacht vom 6. zum 7 Dez. in Zürich stattfindenden Balles hier unabkömmlich bin.- Mit vielen Grüssen an Euch, sowie an Herrn Back, Euer untertänigster Diener

gez. W. Pauli

Dear Radioactive Ladies and Gentlemen!

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, because of the "wrong" statistics of the N- and Li-6 nuclei and the continuous beta spectrum, **I have hit upon a desperate remedy to save the "exchange theorem" (1) of statistics and the law of conservation of energy.** Namely, the **possibility that *in the nuclei* there could exist electrically neutral particles, which I will call neutrons** [now neutrinos]**, that have spin 1/2 and obey the exclusion principle**…

The continuous beta spectrum would then make sense with the assumption that in beta decay, in addition to the electron, a neutron is emitted such that the sum of the energies of neutron and electron is constant…

But so far, I do not dare to publish anything about this idea, and trustfully turn first to you, dear radioactive ones, with the question of how likely it is to find experimental evidence for such a neutron…

I admit that my remedy may seem almost improbable because one probably would have seen those neutrons, if they exist, for a long time. But nothing ventured, nothing gained…

Thus, dear radioactive ones, scrutinize and judge.

Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7.

Pauli Archives - CERN Document Server
meitner_0393.pdf (cern.ch)

Neutrino invention | symmetry magazine

Neutrino naming and the Fermi theory of Weak Interactions

To avoid confusion with the neutron, discovered in 1932 by J. Chadwick, E. Fermi in his theory of weak interactions (1933-34) called this particle neutrino, meaning little neutron in Italian. This term was coined by E. Amaldi, one of the young members of Fermi's group (« I Ragazzi di Via Panisperna »).

An **attempt** to a β rays theory

$$(Z,A) \rightarrow (Z+1,A) + e^{-} + \bar{\nu}_e$$

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

E. Fermi, Ricerca Scientifica 4 (1933) 491

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

Theory of the emission of β rays by radioactive substances, based on the hypothesis that the **electrons emitted by nuclei do not exist before the disintegration** but are formed, together with a **neutrino**, in a way which is analogous to the formation of a quantum of light which accompany a quantum jump of an atom. Comparison of theory with experience

TENTATIVO DI UNA TEORIA DEI RAGGI β

Nota (1) di ENRICO FERMI

Sunto. - *Si propone una teoria quantitativa dell'emissione dei raggi β in cui si ammette l'esistenza del « neutrino » e si tratta l'emissione degli elettroni e dei neutrini da un nucleo all'atto della disintegrazione β con un procedimento simile a quello seguito nella teoria dell' irradiazione per descrivere l'emissione di un quanto di luce da un atomo eccitato. Vengono dedotte delle formule per la vita media e per la forma dello spettro continuo dei raggi β, e le si confrontano coi dati sperimentali.*

E. Fermi, Nuovo Cimento 11 (1934) 1

A quantitative theory of β-rays emission is proposed in which **the existence of the ≪neutrino≫ is admitted**; electrons and neutrinos emission from a nucleus at a β decay is treated with a procedure similar to the one followed for radiation theory to describe a light quantum emission by an excited atom. Formulae are derived for the mean life and for the distribution of the β-rays continuum spectrum, which are compared with experimental data.

FB.dvi (uniroma3.it)

Rejected from *Nature* for being 'too removed from reality'

8 Scientific Papers That Were Rejected Before Going on to Win a Nobel Prize (sciencealert.com)

Neutrino interaction and neutrino detection

$$\nu_e + (Z, A) \rightarrow (Z+1, A) + e^-$$ $$\nu_e + n \rightarrow p + e^-$$

$$\bar{\nu}_e + (Z, A) \rightarrow (Z-1, A) + e^+$$ $$\bar{\nu}_e + p \rightarrow n + e^+$$

The "Neutrino", Nature 133 (1934) 532

The possibility of creating neutrinos necessarily implies the existence of annihilation processes. The most interesting amongst them would be the following: a neutrino hits a nucleus and a positive or negative electron is created while the neutrino disappears and the charge of the nucleus changes by 1.

The cross section σ for such processes for a neutrino of given energy may be estimated from the lifetime t of β-radiating nuclei giving neutrinos of the same energy. (This estimate is in accord with Fermi's model but is more general.) Dimensionally, the connexion will be

$$\sigma = A/t$$

where A has the dimension cm.^{2} sec. The longest length and time which can possibly be involved are $\hbar/mc$ and $\hbar/mc^2$. Therefore

$$\sigma < \frac{\hbar^3}{m^3c^4t}$$

For an energy of $2 \cdot 3 \times 10^6$ volts, t is 3 minutes and therefore $\sigma < 10^{-44}$ cm.^{2} (corresponding to a penetrating power of 10^{16} km. in solid matter). It is therefore absolutely impossible to observe processes of this kind with the neutrinos created in nuclear transformations.

With increasing energy, σ increases (in Fermi's model[3] for large energies as $(E/mc^2)^2$) but even if one assumes a very steep increase, it seems highly improbable that, even for cosmic ray energies, σ becomes large enough to allow the process to be observed.

If, therefore, the neutrino has no interaction with other particles besides the processes of creation and annihilation mentioned—and it is not necessary to assume interaction in order to explain the function of the neutrino in nuclear transformations—one can conclude that there is no practically possible way of observing the neutrino.

!!

H. BETHE. 
R. PEIERLS.

```
hbarc = 197.3269804 #MeV fm
me = 0.51099895 # MeV
c = 299792458 #ms-1
t = 3*60 #s
hbarc**3*(10**-13)**3/(me**3*c*10**2*t) #cm2

1.0671026026257229e-44
```

Penetrating power: 10^16 km ≈ 10^3 light-year

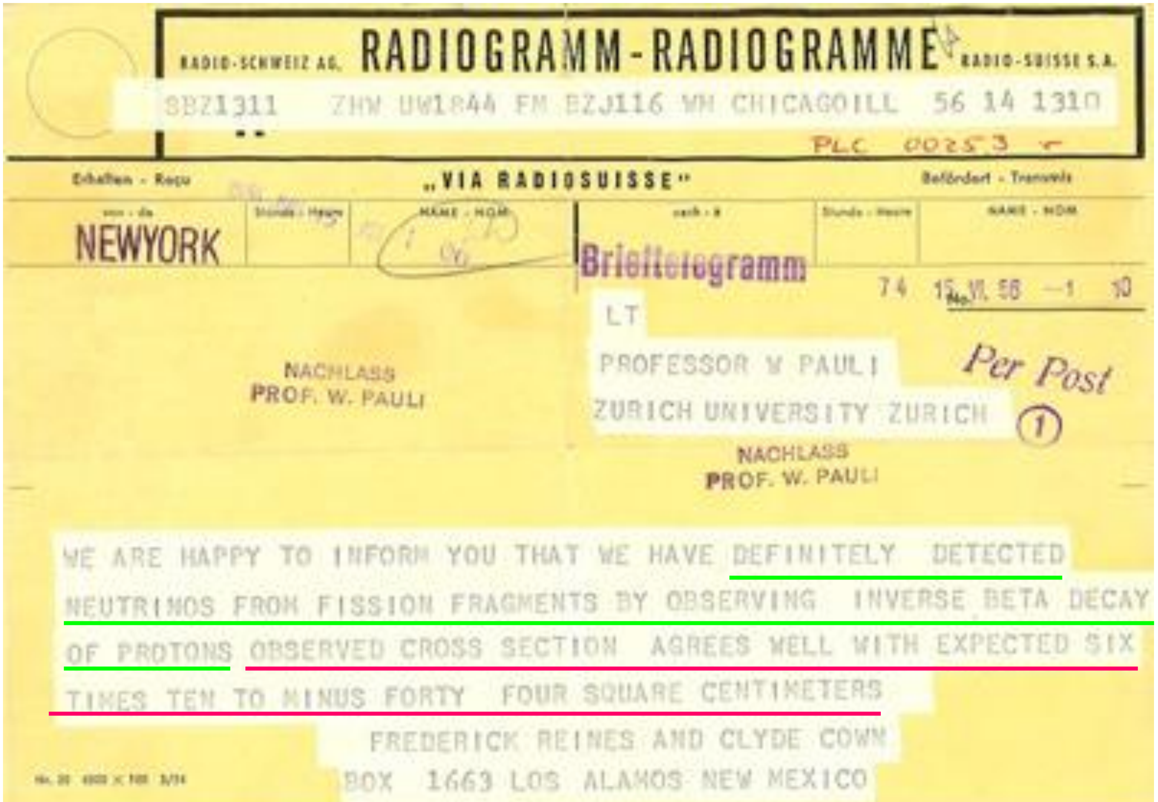
σ ~ 10^−44 cm2 ↔ Probability ~ 10^ − 18 to interact in a solid detector of 1m thickness

↔ Probability ~ 10^−11 to interact inside the Earth along a trajectory passing through its center

For many years no one thought about how to detect the neutrinos

1956: C. Cowan and F. Reines detect (anti)neutrinos

- Antineutrinos were produced by the Savannah River nuclear reactor
- A typical reactor emits $\sim 2 \times 10^{20}\ \bar{\nu}_e/s$ per each GW of the thermal energy power ; $E_{\bar{\nu}} \sim MeV$



RADIO-SCHWEIZ AG. RADIOGRAMM - RADIOGRAMME RADIO-SUISSE S.A.

SBZ1311 ZHW UW1844 FM BZJ116 WH CHICAGOILL 56 14 1310

„VIA RADIOSUISSE"

NEWYORK

Brieftelegramm

LT

PROFESSOR W PAULI
ZURICH UNIVERSITY ZURICH

Per Post

NACHLASS PROF. W. PAULI

WE ARE HAPPY TO INFORM YOU THAT WE HAVE DEFINITELY DETECTED NEUTRINOS FROM FISSION FRAGMENTS BY OBSERVING INVERSE BETA DECAY OF PROTONS OBSERVED CROSS SECTION AGREES WELL WITH EXPECTED SIX TIMES TEN TO MINUS FORTY FOUR SQUARE CENTIMETERS

FREDERICK REINES AND CLYDE COWN
BOX 1663 LOS ALAMOS NEW MEXICO

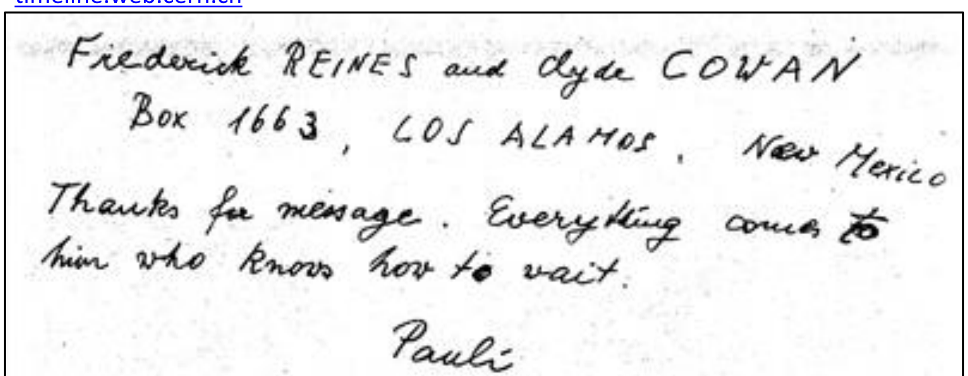
$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$\sigma = 6 \times 10^{-44}\,\text{cm}^2$$

Experimental discovery (in2p3.fr)

Neutrinos detected at last! | timeline.web.cern.ch

Frederick Reines - Nobel Lecture (nobelprize.org)



Frederick REINES and Clyde COWAN
Box 1663, LOS ALAMOS, New Mexico

Thanks for message. Everything comes to him who knows how to wait.

Pauli

conclude that there is no practically possible way of observing the neutrino.

Nature 133 (1934) 532 H. Bethe. R. Peierls.

Reines: I confronted Bethe, with this pronouncement some 20 years later and with his characteristic good humor he said, **"Well you shouldn't believe everything you read in the papers."**

Example of (simple) evaluation of the cross section

$$\bar{\nu}_e + p \rightarrow n + e^+$$

P. Vogel and J. F. Beacom, Phys. Rev. D 60, 053003 (1999)

$$\sigma_{tot}^{(0)} = \sigma_0 (f^2 + 3g^2) E_e^{(0)} p_e^{(0)}$$

$$= 0.0952 \left(\frac{E_e^{(0)} p_e^{(0)}}{1 \ \ \mathrm{MeV}^2} \right) \times 10^{-42} \ \ \mathrm{cm}^2$$

$$\sigma_0 = \frac{G_F^2 \cos^2 \theta_C}{\pi} (1 + \Delta_{inner}^R)$$

```
import numpy as np

import matplotlib.pyplot as plt
%matplotlib inline
```

```
GF = 1.16637*10**(-5) #GeV-2
cthc = 0.9746
hbarc = 197.3269804 #MeV fm
f = 1
g = 1.266
```

```
s0=hbarc**2*(10**(-13))**2*GF**2*(1/10**6)**2*cthc**2/np.pi*(1+0.024)
fact=s0*(f**2+3*g**2)
fact
```

9.525661986246197e-44

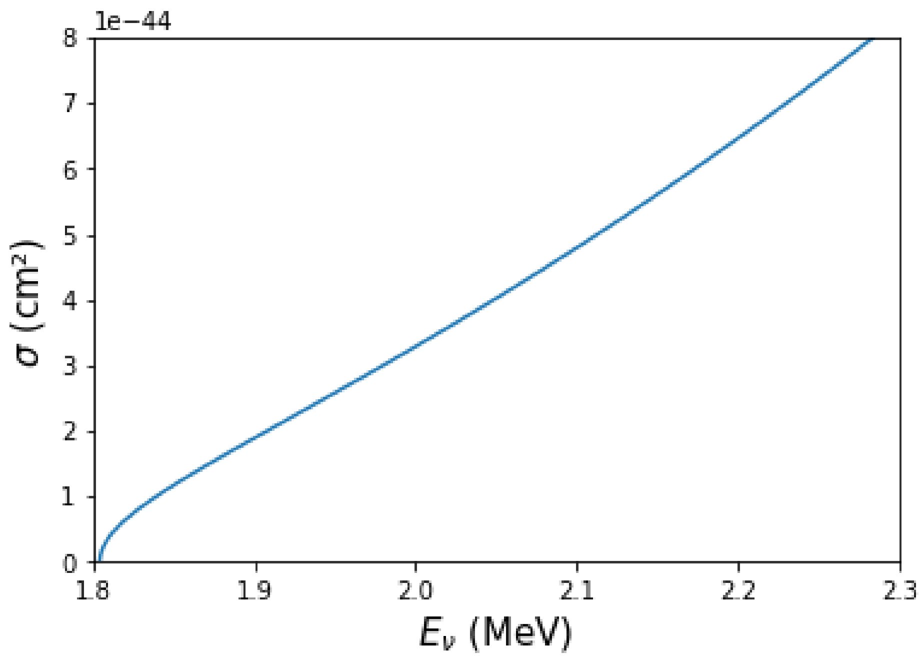
```
me = 0.51099895 # MeV
Mp = 938.27208816 # MeV
Mn = 939.56542052 # MeV

D = Mn-Mp

Enu = np.linspace(D+me, 2.3, 10000)

Ee = Enu-D
pe = np.sqrt(Ee**2-me**2)

plt.plot(Enu,fact*Ee*pe)
plt.ylim(0,8*10**-44)
plt.xlim(1.8,2.3)
plt.xlabel(r'$E_\nu$ (MeV)',fontsize=15)
plt.ylabel(r'$\sigma$ (cm²)', fontsize = 15);
```



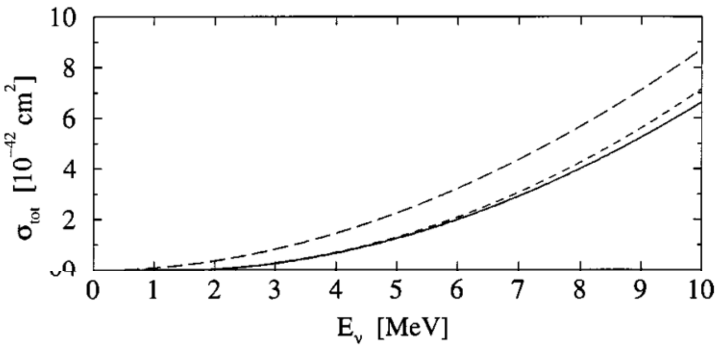
Neutrino-nucleon (quasielastic) cross section at different ν energies

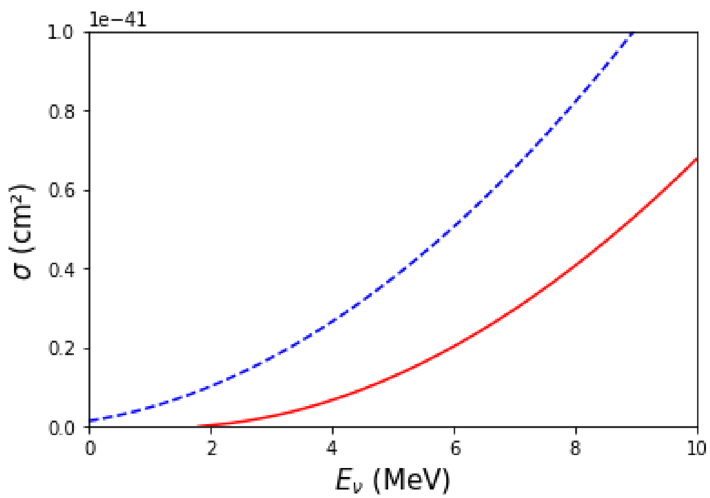
$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$\nu_e + n \rightarrow p + e^-$$

Reactor

P. Vogel and J. F. Beacom, Phys. Rev. D 60, 053003 (1999)

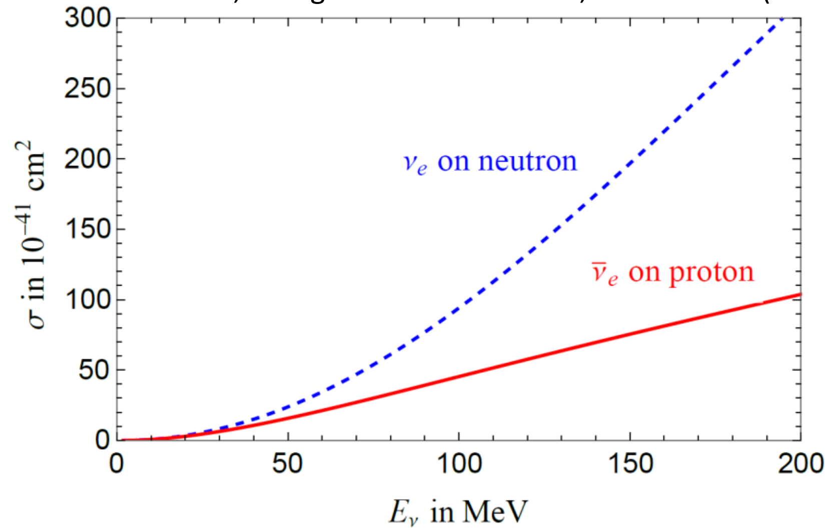




Supernova

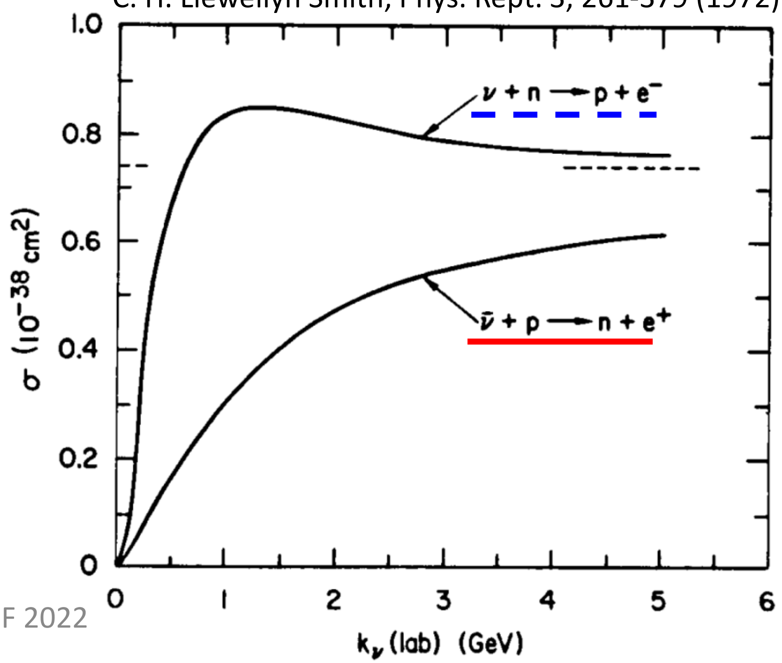
A. Strumia and F. Vissani, Phys. Lett. B 564, 42-54 (2003)

G. Ricciardi, N. Vignaroli and F. Vissani, JHEP 08 212 (2022)



Accelerator

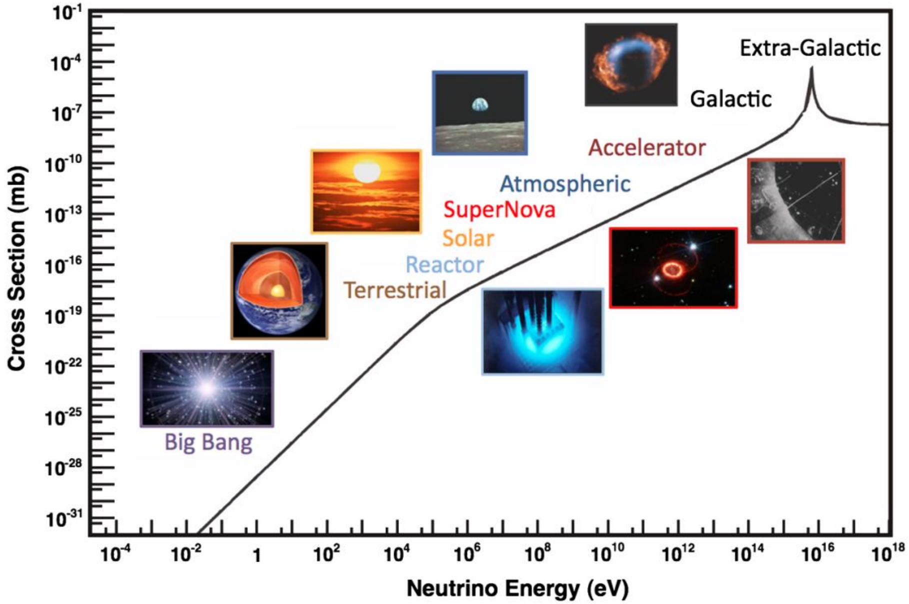
C. H. Llewellyn Smith, Phys. Rept. 3, 261-379 (1972)



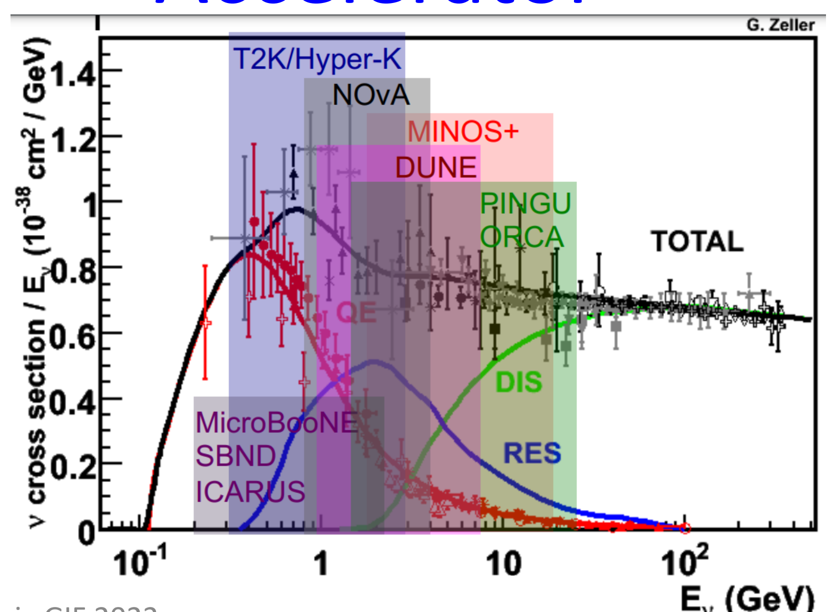
REVIEWS OF MODERN PHYSICS, VOLUME 84, JULY–SEPTEMBER 2012

From eV to EeV: Neutrino cross sections across energy scales

J. A. Formaggio, G. P. Zeller



Accelerator



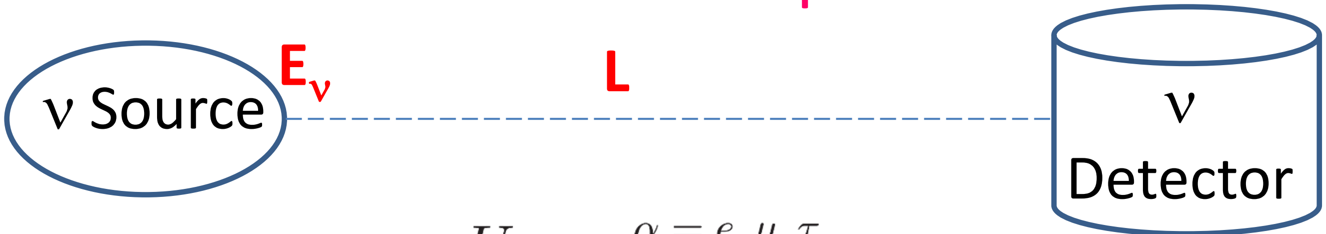
Neutrino-nucleus cross sections

(for ν oscillation experiments ↔ at accelerator energies)

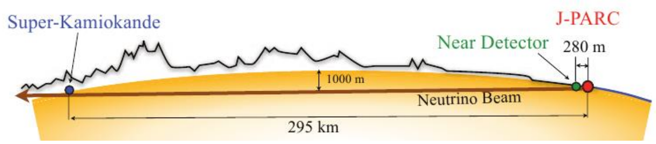
Generalities

Neutrino oscillation experiments

$\nu_\alpha \rightarrow \nu_\beta$



$$\nu_\alpha = U_{\alpha i}\nu_i \quad \begin{array}{l}\alpha = e, \mu, \tau \\ i = 1, 2, 3\end{array}$$



Reconstructed ν energy — True ν energy

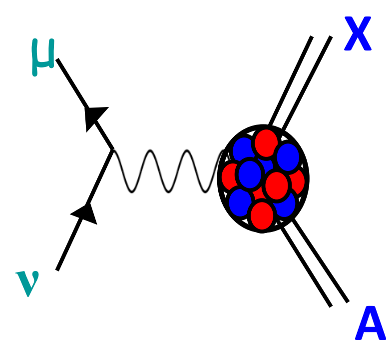
$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \rightarrow \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Number of detected events | ν flux | ν oscillation probability | ν cross section | Detector efficiency | Migration matrix

Modern accelerator-based neutrino oscillation experiments:

- The neutrino energy is reconstructed from the final states

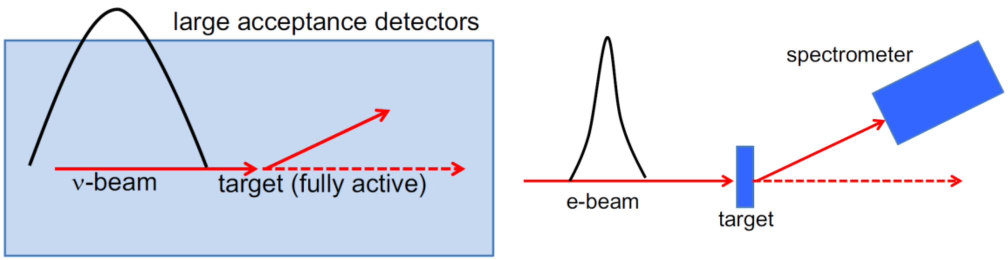
- Nuclear targets (C, O, Ar, Fe...)

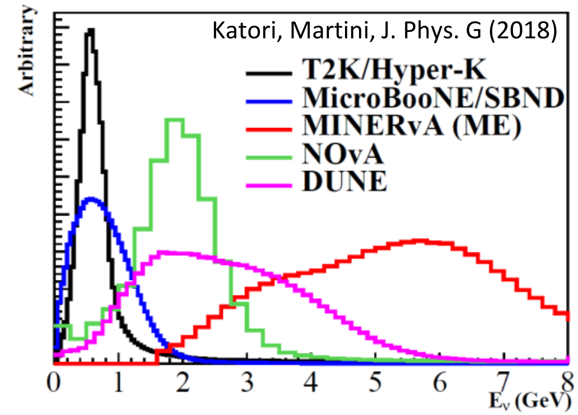


→ the knowledge of the neutrino-nucleus cross section is crucial

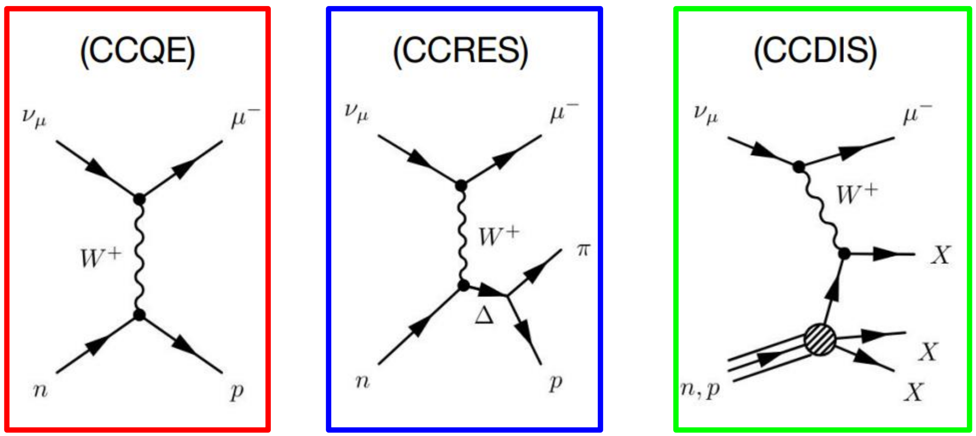
Some crucial points of the accelerator-based ν experiment

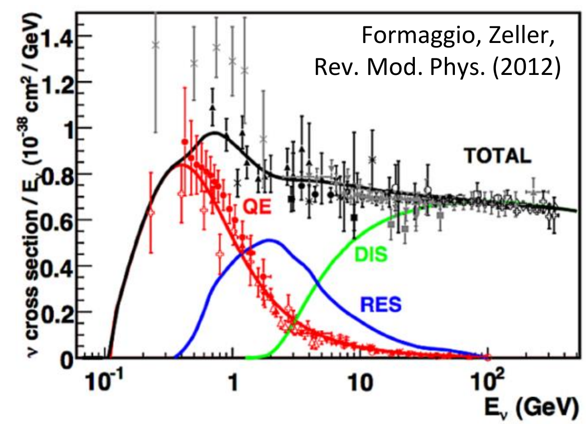
- Neutrino beams are not monochromatic
  (at difference with respect to electron beams)



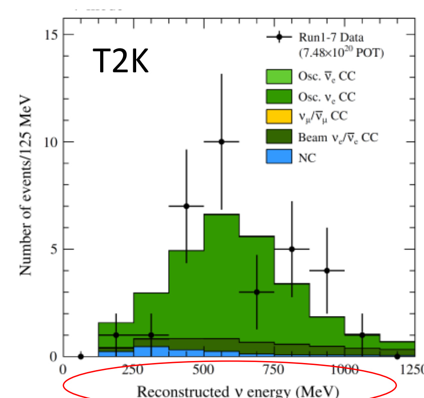


- Different reaction mechanisms contribute

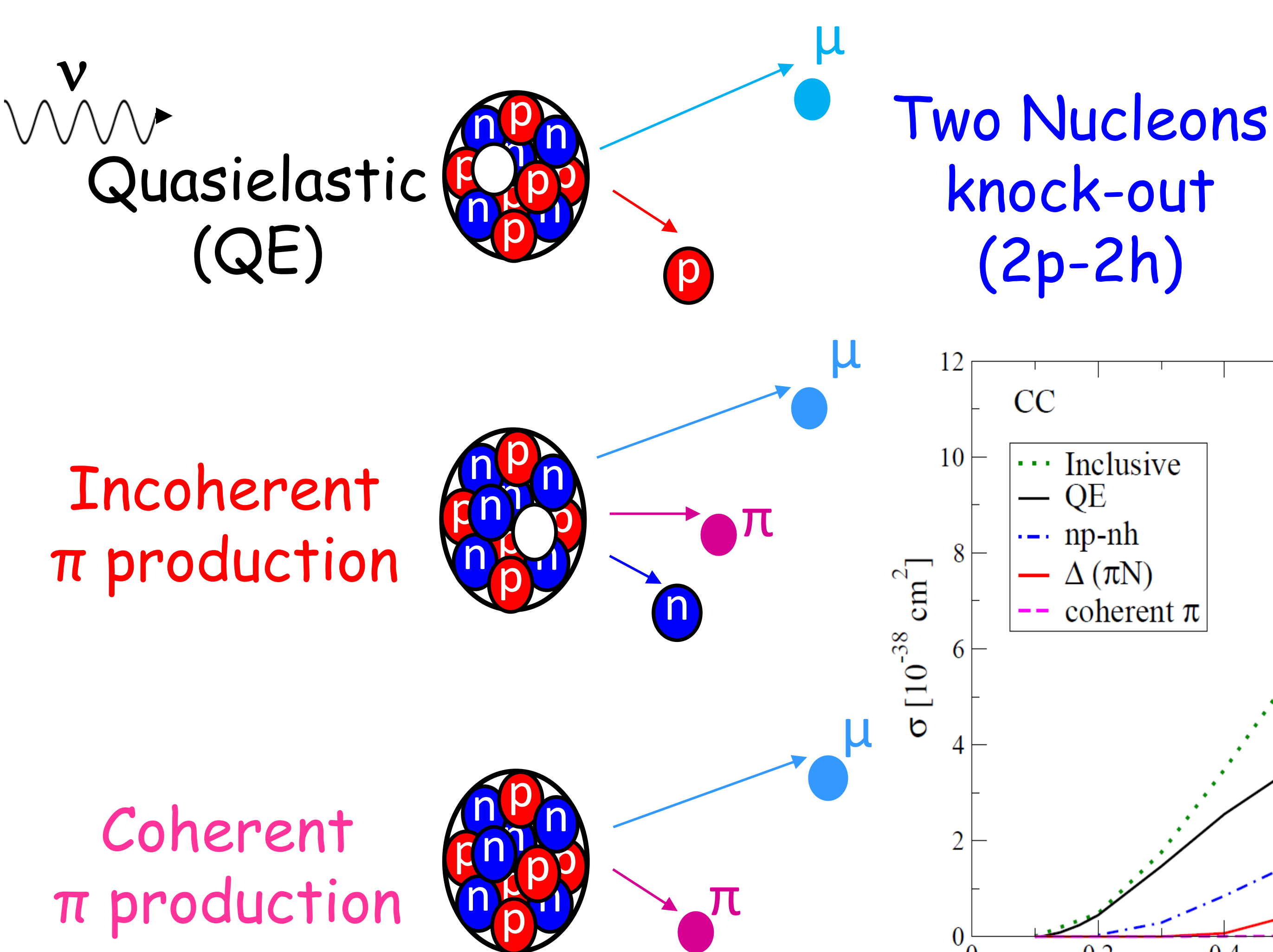


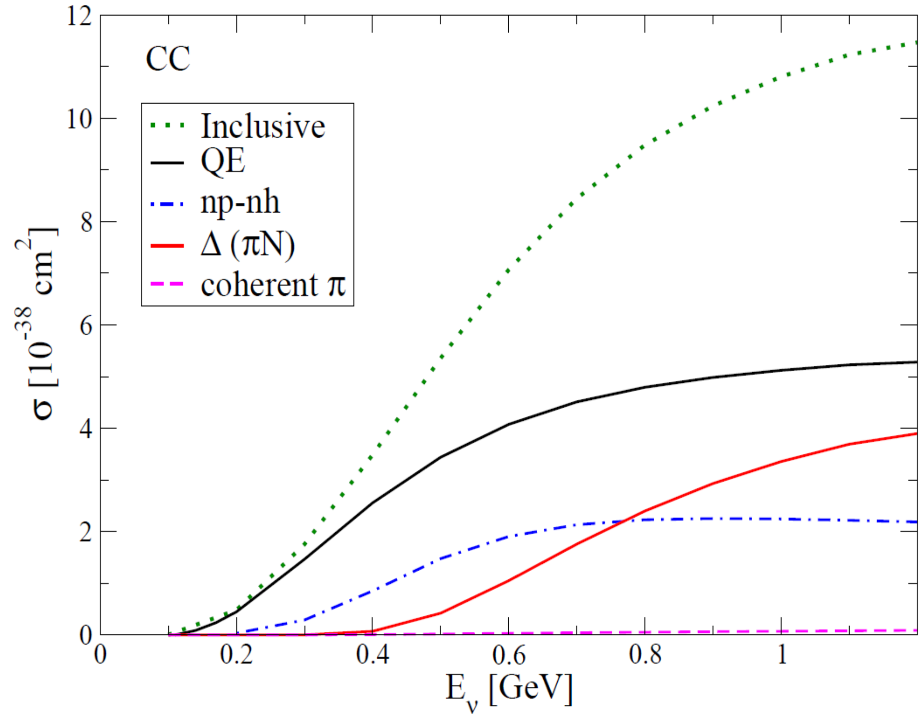


- The neutrino energy is reconstructed
  from the final states of the reaction
  (often from CCQE events)



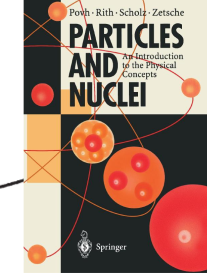
In these lectures: Neutrino - nucleus interaction @ E_ν~ O(1 GeV)



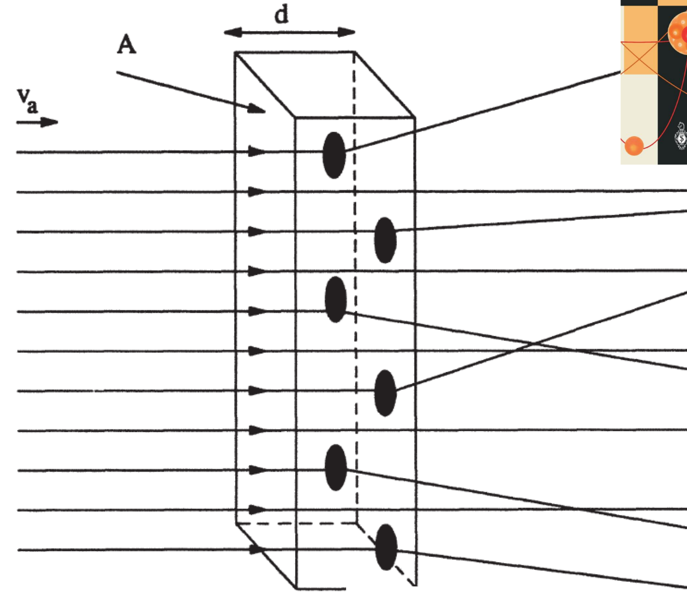


Different processes are entangled

Cross Section generalities - textbook definitions



- **Definition**: The Cross Section is a measure for the probability of a process to happen
- **Dimensions**: Area

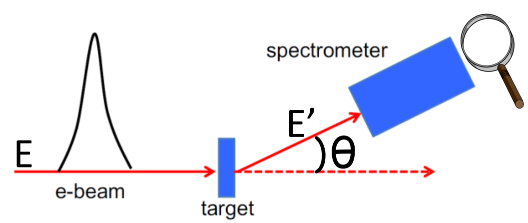


$$a + b \rightarrow a' + b^*$$

$$\Phi_a = \frac{\dot{N}_a}{A} = n_a \cdot v_a \qquad N_b = n_b\,Ad$$

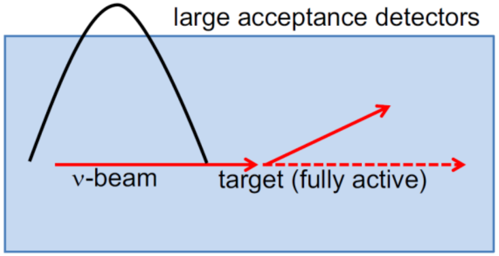
$$\sigma_b = \frac{\dot{N}}{\Phi_a \cdot N_b}$$

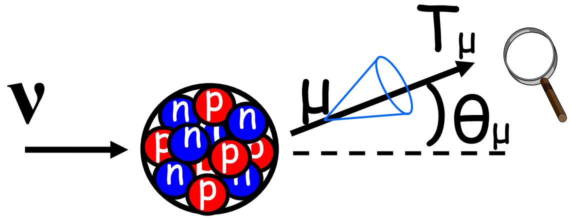
$$= \frac{\text{number of reactions per unit time}}{\text{beam particles per unit time per unit area} \times \text{scattering centres}}$$

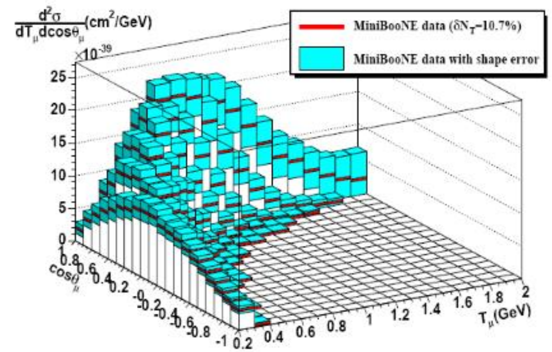


$$\sigma_{tot}(E) = \int_0^{E'_{max}} \int_{4\pi} \frac{d^2\sigma(E, E', \theta)}{d\Omega\, dE'}\, d\Omega\, dE'$$

Neutrino flux integrated double differential cross sections







Flux-integrated differential cross section is where theorists and experimentalists meet for ν interaction

Theory

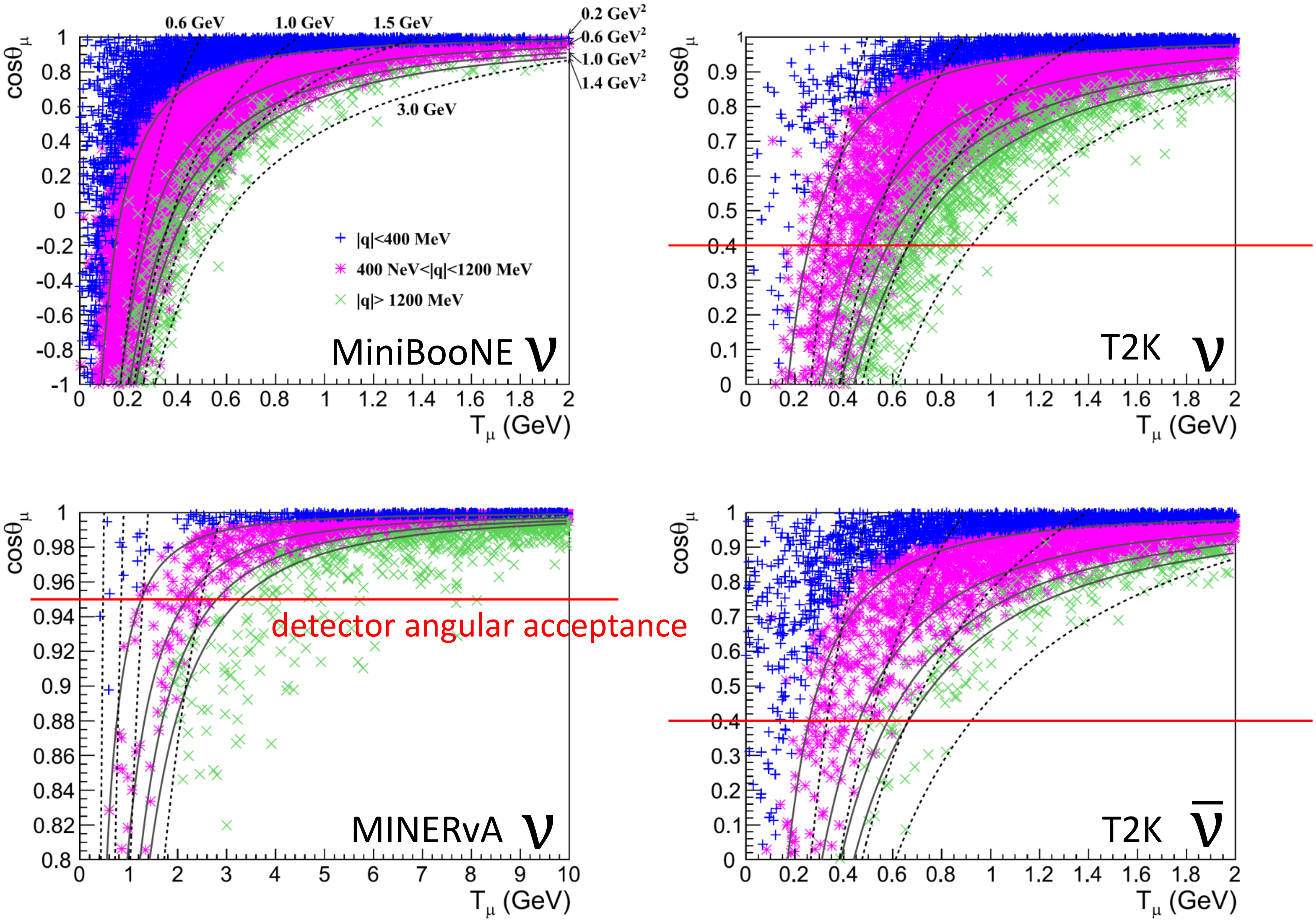
$$\frac{d^2\sigma}{dT_l \ d\cos\theta} = \frac{1}{\int \Phi(E_\nu) \ dE_\nu} \int dE_\nu \left[\frac{d^2\sigma}{d\omega \ d\cos\theta} \right]_{\omega = E_\nu - E_l} \Phi(E_\nu)$$

Experimental Definition

$$\left(\frac{d^2\sigma}{dT_l \cos\theta} \right)_i = \frac{\sum_j U_{ij} (d_j - b_j)}{\Phi \cdot T \cdot \epsilon_i \cdot (\Delta T_l, \Delta\cos\theta)_i}$$

- Unfolding matrix to remove detector effects
- Number of observed events
- Background contribution
- Total integrated flux
- Number of target nucleons in the Fiducial Volume
- Efficiency
- Bin widths

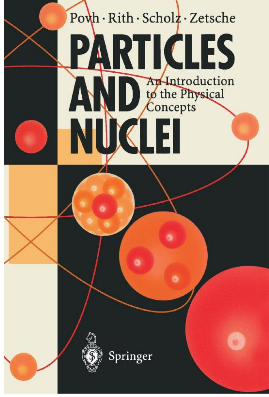
($\cos\theta_\mu$, T_μ) distributions of neutrino flux integrated CCQE generated events



Katori, Martini, J. Phys. G (2018)

Cross Section generalities - Theory

Quantum Mechanics (Time-dependent perturbation theory)



Reaction rate per target particle and per beam particle:

Fermi's golden rule

$$W = \frac{2\pi}{\hbar} |\mathcal{M}_{fi}|^2 \cdot \varrho(E')$$

Transition matrix element

$$\mathcal{M}_{fi} = \langle \psi_f | \mathcal{H}_{\text{int}} | \psi_i \rangle$$

Density of final states

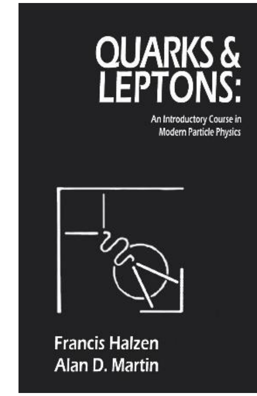
$$\varrho(E') = \frac{\mathrm{d}n(E')}{\mathrm{d}E'} = \frac{V \cdot 4\pi p'^2}{v' \cdot (2\pi\hbar)^3}$$

$$W = \frac{\dot{N}(E)}{N_\mathrm{b} \cdot N_\mathrm{a}} = \frac{\Phi_\mathrm{a} \cdot N_\mathrm{b} \cdot \sigma}{N_\mathrm{b} \cdot N_\mathrm{a}} = \frac{\sigma \cdot v_\mathrm{a}}{V}$$

(see equations slide 18)

$$\sigma = \frac{2\pi}{\hbar \cdot v_\mathrm{a}} |\mathcal{M}_{fi}|^2 \cdot \varrho(E') \cdot V$$

Lorentz invariant general expression of differential Cross Section



$$d\sigma = \frac{|M|^2}{Flux} dQ$$

Scattering of 2 particles leading to N outgoing particles

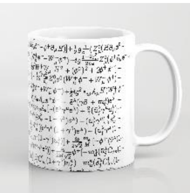
$$p_1 + p_2 \rightarrow p_{f1} + p_{f2} + \cdots + p_{fN}$$

$$\mathrm{d}\sigma = \frac{(2\pi)^4}{4[(p_1 \cdot p_2)^2 - m_1^2 m_2^2]^{1/2}} \delta^4\left(\sum_f p_f - \sum_i p_i\right)\left(\prod_f \frac{\mathrm{d}^4 p_f}{(2\pi)^3}\,\delta(p_f^2 - m_f^2)\right)|\bar{\mathcal{M}}|^2$$

Invariant flux

Lorentz invariant phase space factor *dQ*

Invariant squared amplitude averaged and summed over initial and final states

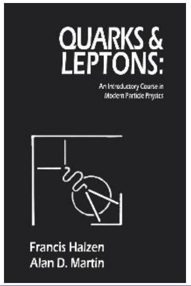


Standard Model of electroweak interaction

Electroweak interaction Lagrangian

$$\mathcal{L}_{\text{int}} = -eJ^{\mu}_{\text{EM}}A_{\mu} - \frac{g}{2\sqrt{2}}\left(J^{\mu}_{\text{CC}}W_{\mu}{}^{\dagger} + \text{h. c.}\right) - \frac{g}{2\cos\theta_W}J^{\mu}_{\text{NC}}Z_{\mu}$$

Feynman Rules



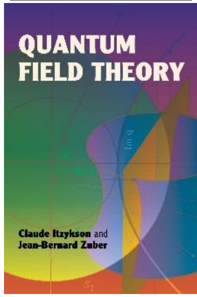
Gauge Bosons Propagators

- Photon (γ) $\quad \frac{\mathrm{i}}{q^2}(-g^{\mu\nu})$

γ , W , Z

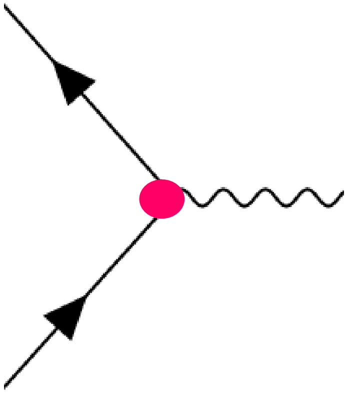
- Massive vector bosons (W,Z)

$$\frac{\mathrm{i}}{q^2 - M_V^2}\left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{M_V^2}\right) \xrightarrow{|q^2| \ll M_V^2} \frac{\mathrm{i}\,g^{\mu\nu}}{M_V^2}$$



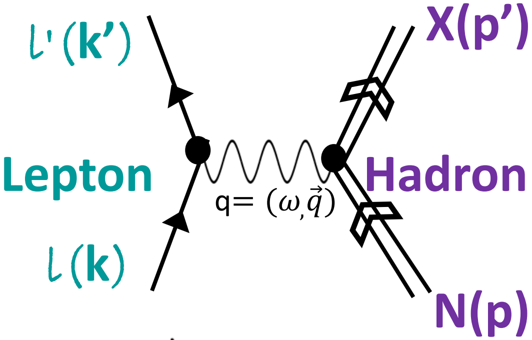
Fermion Vertices

- Electromagnetic $\quad -\mathrm{i}eQ_f\gamma_{\mu}$
  - $Q_{e-} = -1, Q_{up} = 2/3, \ldots$



- Weak (W exchange) $\quad -\mathrm{i}\frac{g}{\sqrt{2}}c\,\gamma_{\mu}\frac{1-\gamma^5}{2}$
  - c = 1 for leptons
  - c = Cabibbo-mixing matrix element for quarks

Electroweak transition matrix elements



Electromagnetic transition $\ell^- N \rightarrow \ell^- X$

$$-i\mathcal{M} = -(ie)^2 \underbrace{\bar{u}(k')\gamma_\mu u(k)}_{\text{e.m. lepton current}} \frac{-ig^{\mu\nu}}{q^2} \underbrace{\left\langle X(p'_f) \left| J_\nu(0) \right| N(p) \right\rangle}_{\text{hadronic current (Vector)}}$$

Charged current transition $\nu N \rightarrow \ell^- X$

$$-i\mathcal{M} = \left(\frac{-ig}{2\sqrt{2}}\right)^2 \cos\theta_C \underbrace{\bar{u}(k')\gamma_\mu(1-\gamma^5)u(k)}_{\text{weak lepton current}} \frac{ig^{\mu\nu}}{M_W^2} \underbrace{\left\langle X(p'_f) \left| J_\nu(0) \right| N(p) \right\rangle}_{\text{hadronic current (Vector-Axial)}}$$

θ_C
Cabibbo angle

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

Fermi coupling constant

Invariant squared amplitude (and Cross Section) in terms of Leptonic and Hadronic tensors

$$|\bar{\mathcal{M}}|^2 = C^2_{\mathrm{EM,CC,NC}} L_{\mu\nu} W^{\mu\nu}$$

Hadronic tensor ($W^{\mu\nu}$)

Leptonic tensor ($L_{\mu\nu}$)

$$C_{\mathrm{EM}} = 4\pi\alpha/q^2 \quad C_{\mathrm{CC}} = G_F \cos\theta_C/\sqrt{2} \quad C_{\mathrm{NC}} = G_F/\sqrt{2}$$

($4\pi\alpha = e^2$)

$$\alpha = 1/137 \quad G_F = 1.16637 \cdot 10^{-5}\,\mathrm{GeV}^{-2} \quad \cos\theta_C = 0.9746$$

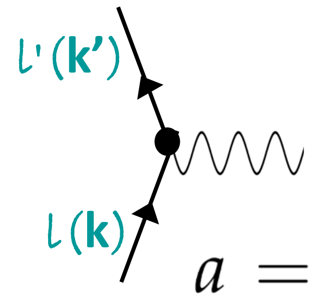
$$\mathrm{d}\sigma \propto L_{\mu\nu} W^{\mu\nu}$$

A universal structure, valid for any lepton and hadron and maintained at different energy scales

The leptonic tensor

$$L_{\mu\nu} = \frac{1+|a|}{2}\sum_{s_i}\sum_{s_f} j_\mu{}^\dagger j_\nu = \frac{1+|a|}{2}\sum_{s_i}\sum_{s_f} \bar{u}(k)\tilde{l}_\mu u(k')\bar{u}(k')l_\nu u(k)$$
$$= \frac{1+|a|}{2}\mathrm{Tr}\left[(\not{k}+m_\ell)\tilde{l}_\mu(\not{k}'+m_{\ell'})l_\nu\right]$$

Leptonic component of the electroweak current



$$j_\mu = \bar{u}(k')l_\mu u(k)$$

$$\not{p} = \gamma_\mu p^\mu$$

$$l_\mu = \gamma_\mu(1-a\gamma^5) \quad \tilde{l}_\mu = \gamma_0 l_\mu^\dagger \gamma_0$$

$a = 0$ EM

$a = 1\ (-1)$ CC and NC

Electron scattering

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}(k\,k' - m_e^2)$$

$$g_{\mu\nu} = (+,-,-,-)$$
$$\varepsilon_{0123} = +1$$

Neutrino scattering

$$L_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k\cdot k' \mp i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta$$

ν (upper sign), $\bar{\nu}$ (lower sign)

p.s. In literature L is defined with different multiplicative and normalization factors

The hadronic tensor

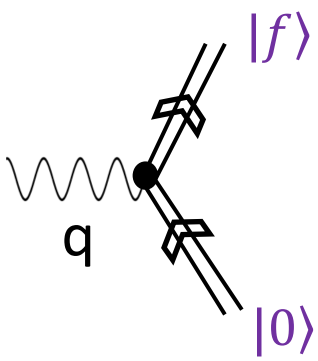
The hadronic tensor contains all the information on the target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\delta^{(4)}(p_0 + q - p_f)$$

$|0\rangle$ hadronic initial state

$|f\rangle$ hadronic final state

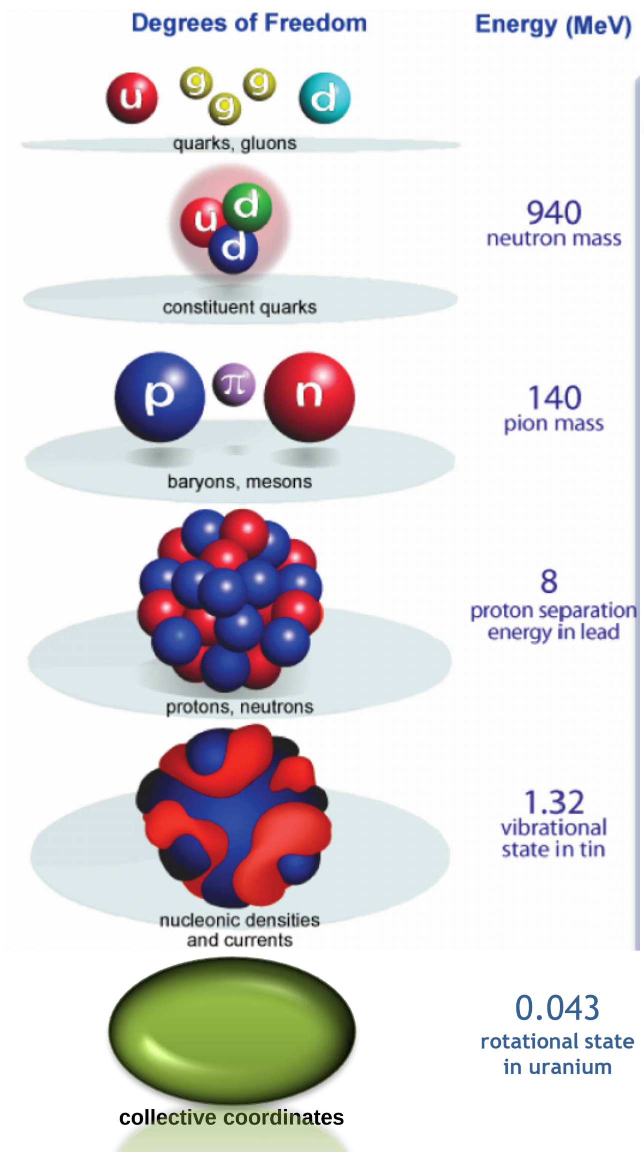
J^{ν} Hadronic component of the electroweak current

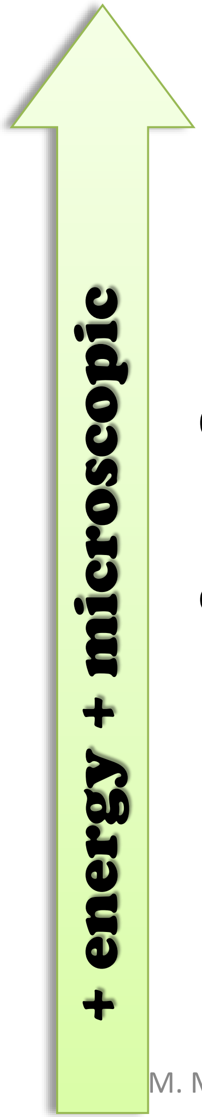


A general expression

- valid for different degrees of freedom (quark, nucleon, nucleon resonances, nucleus)
- valid for different currents (electromagnetic, weak; one-body, two-body,…)

Parenthesis: Nuclear physics and strong interaction

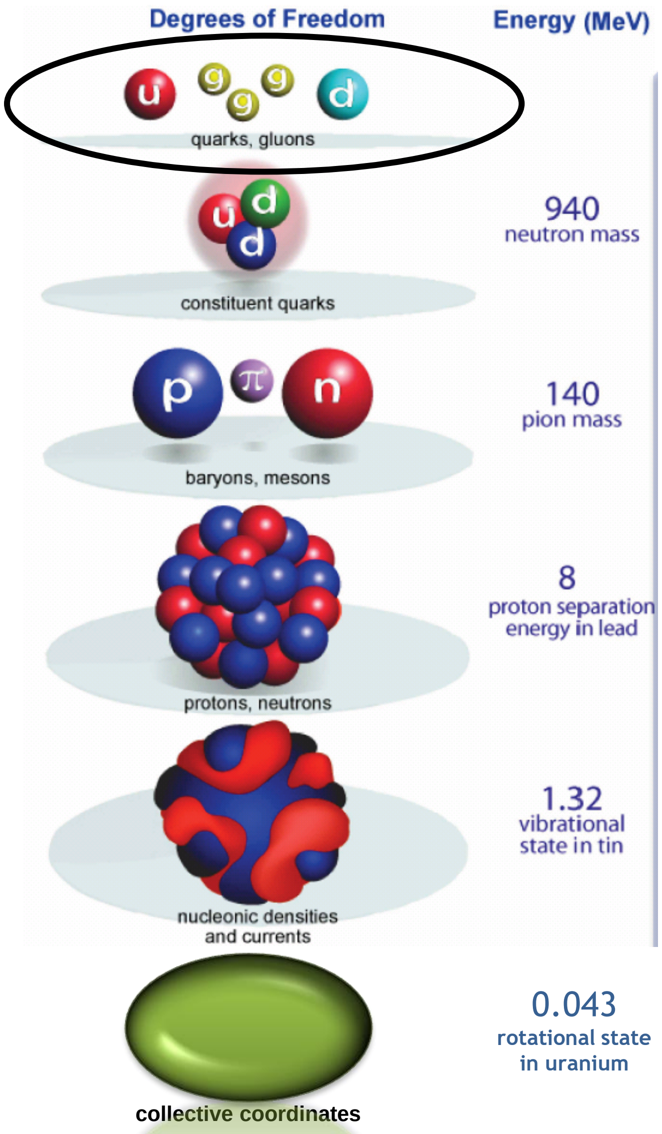


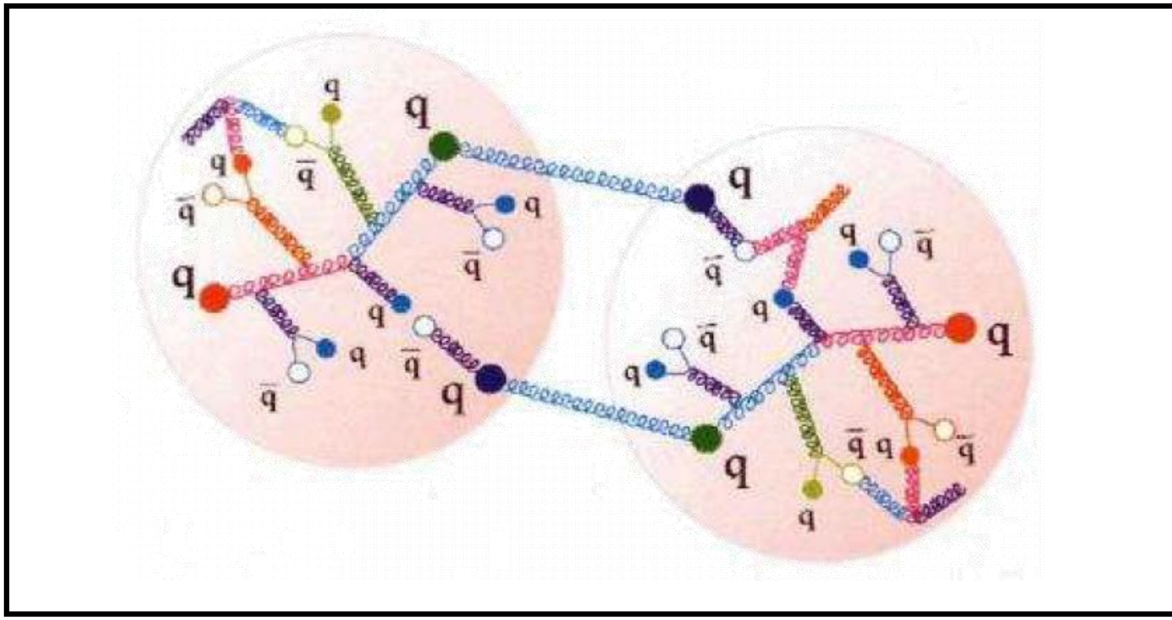


QCD is non-perturbative at low energies

Currently there is no knowledge on how to use it directly in a system as complex as the atomic nucleus

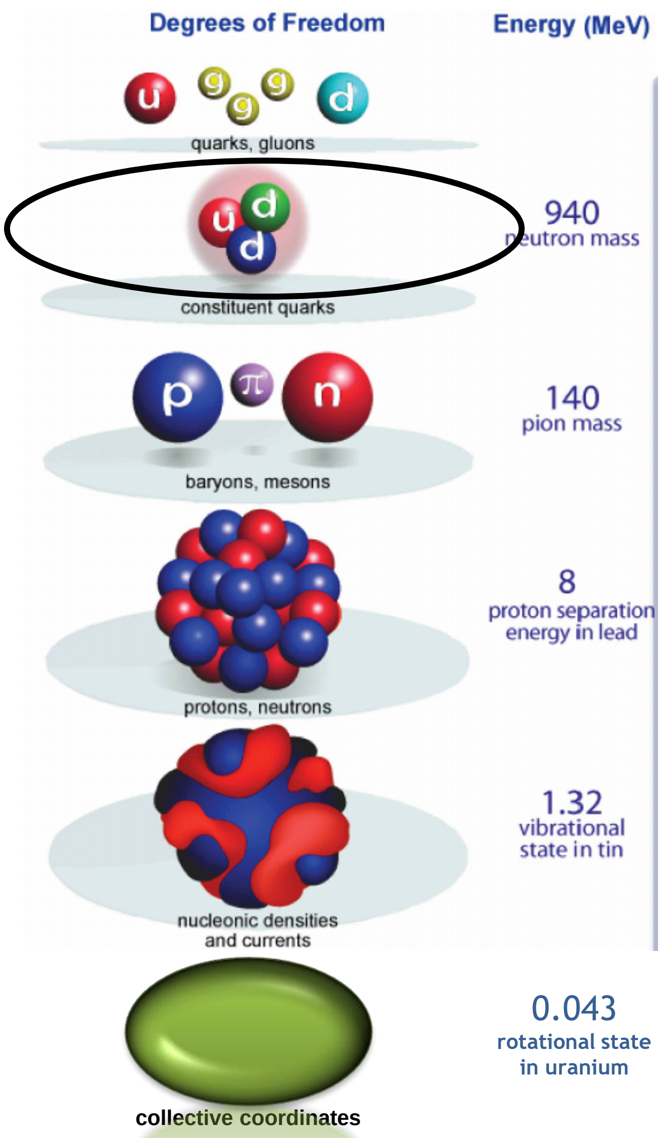
Choice of appropriated Degrees of Freedom

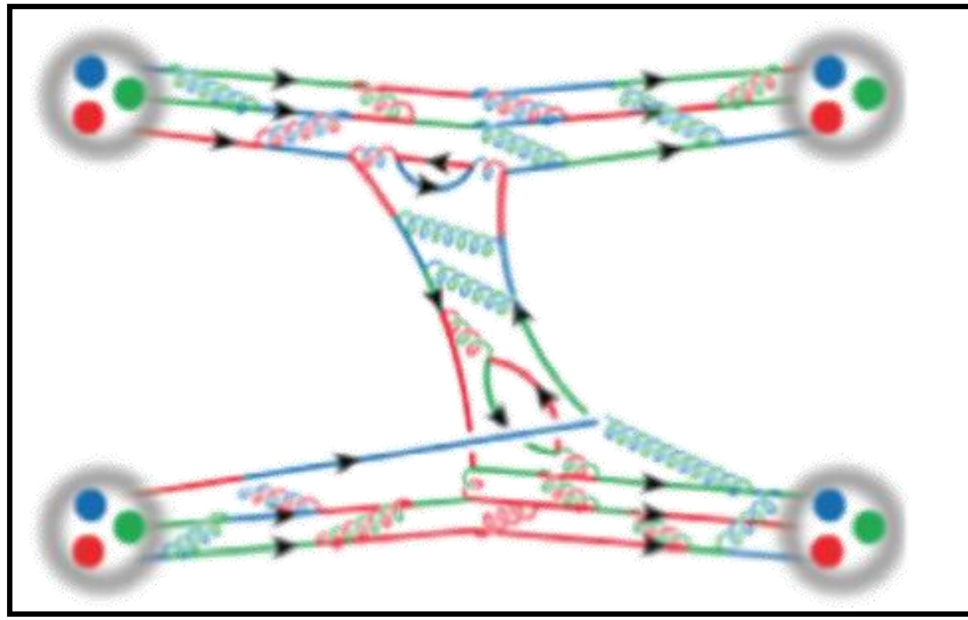




- DoF = quarks and gluons
- QCD

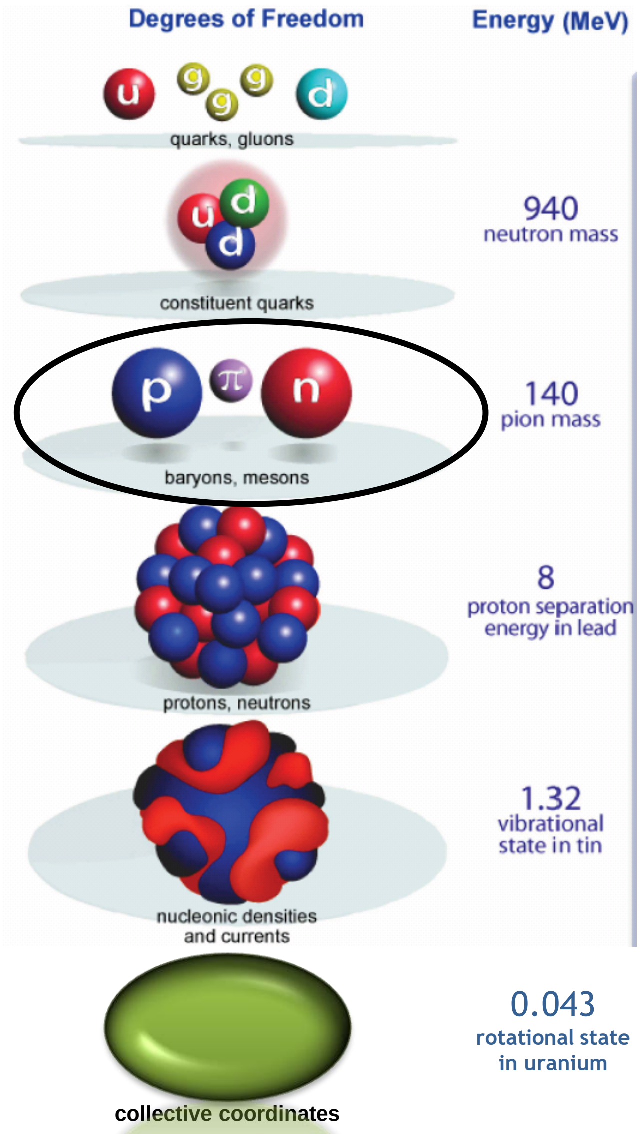
Choice of appropriated Degrees of Freedom

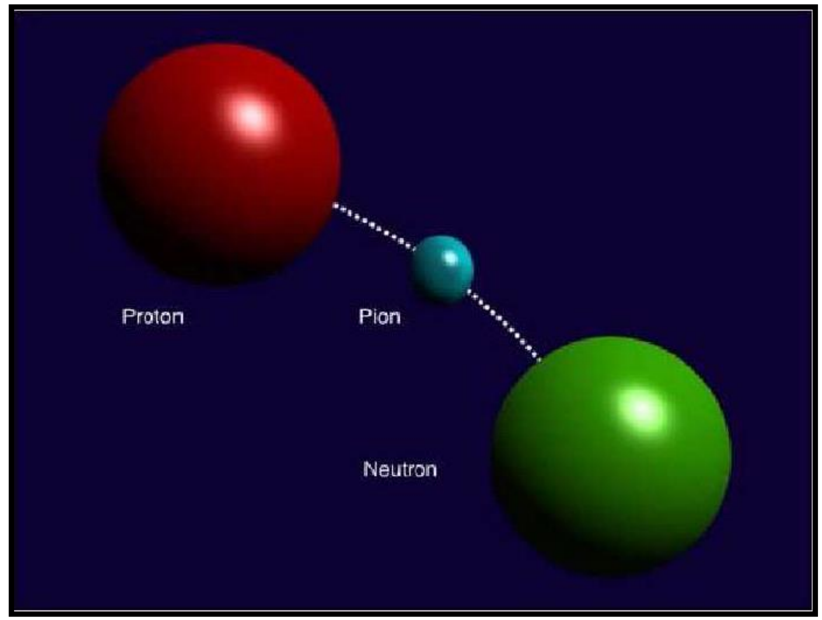




- DoF = valence quarks and gluons
- Low energy QCD, Effective Theories

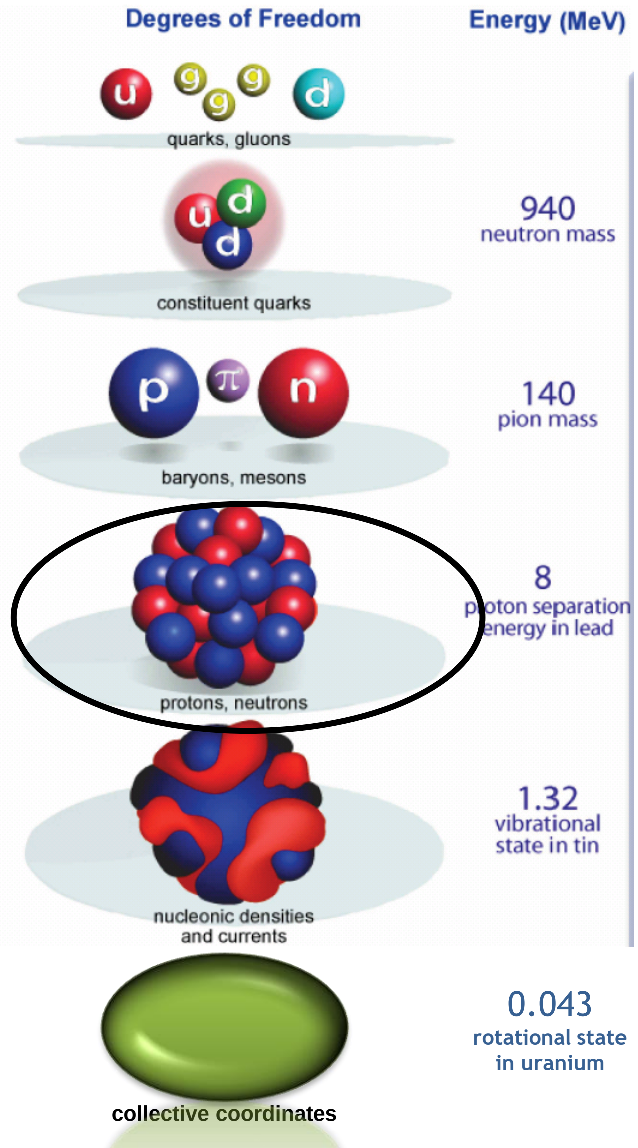
Choice of appropriated Degrees of Freedom

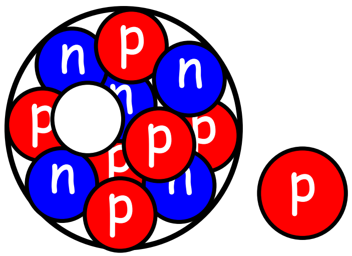




- DoF = baryons and mesons
- Effective Field Theories

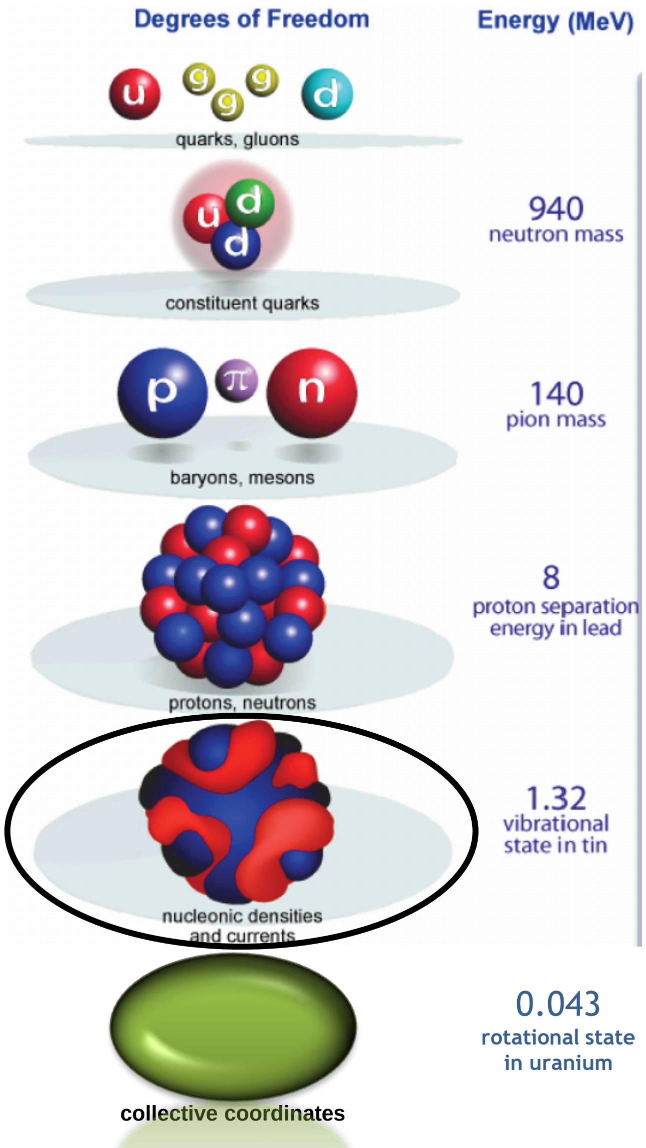
Choice of appropriated Degrees of Freedom

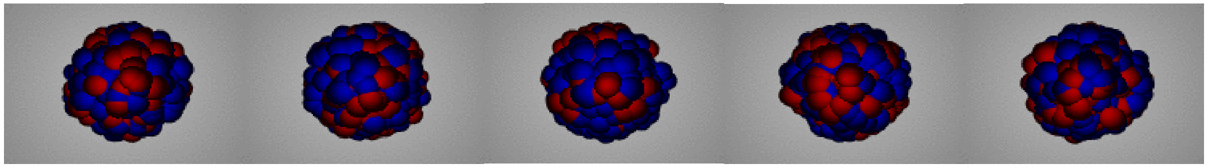




- DoF = Nucleons
- Nuclear Many Body Physics

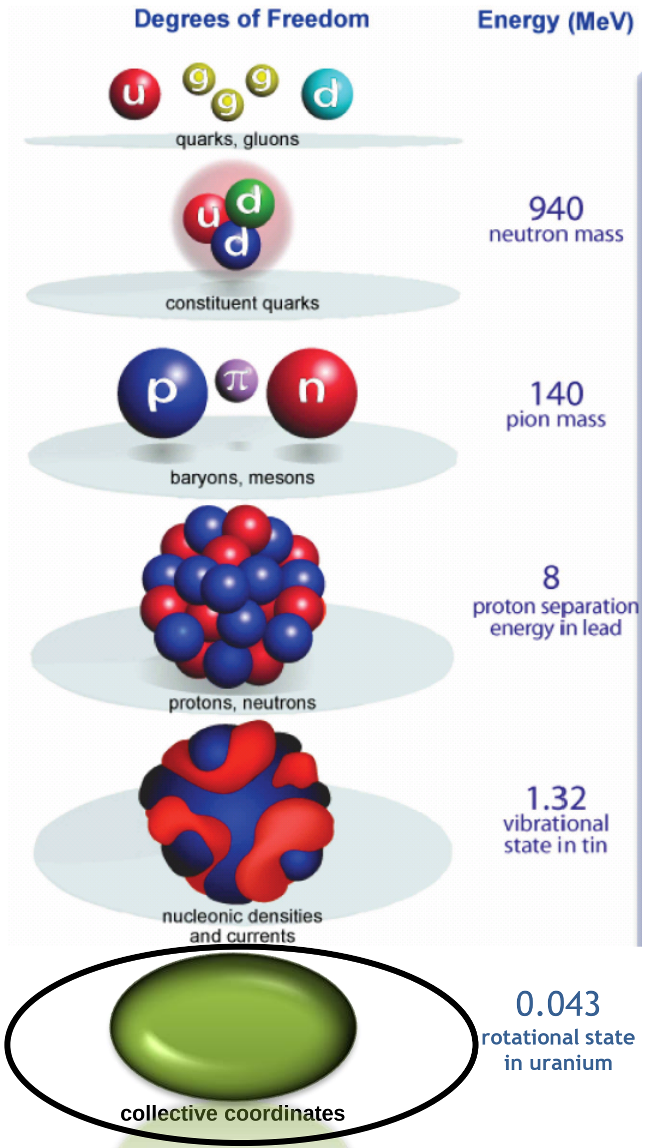
Choice of appropriated Degrees of Freedom

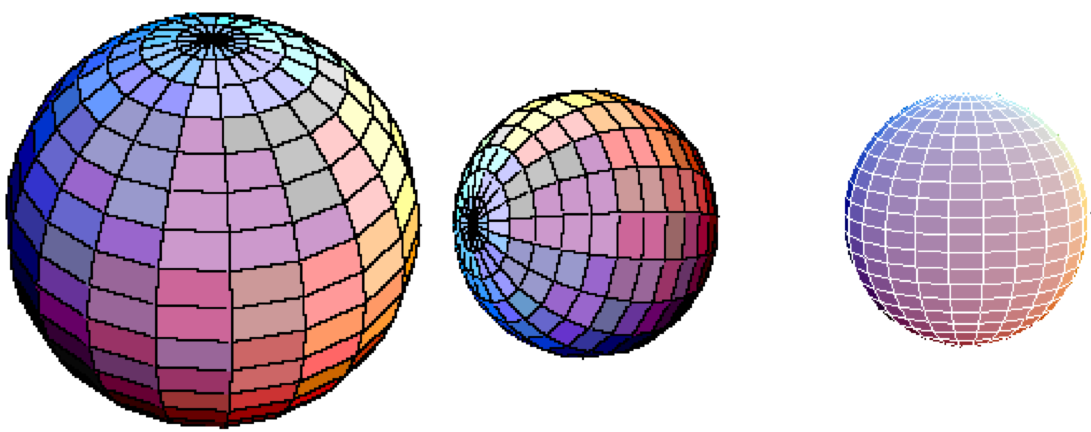




- DoF = nucleonic densities and currents
- Nuclear Many Body Physics

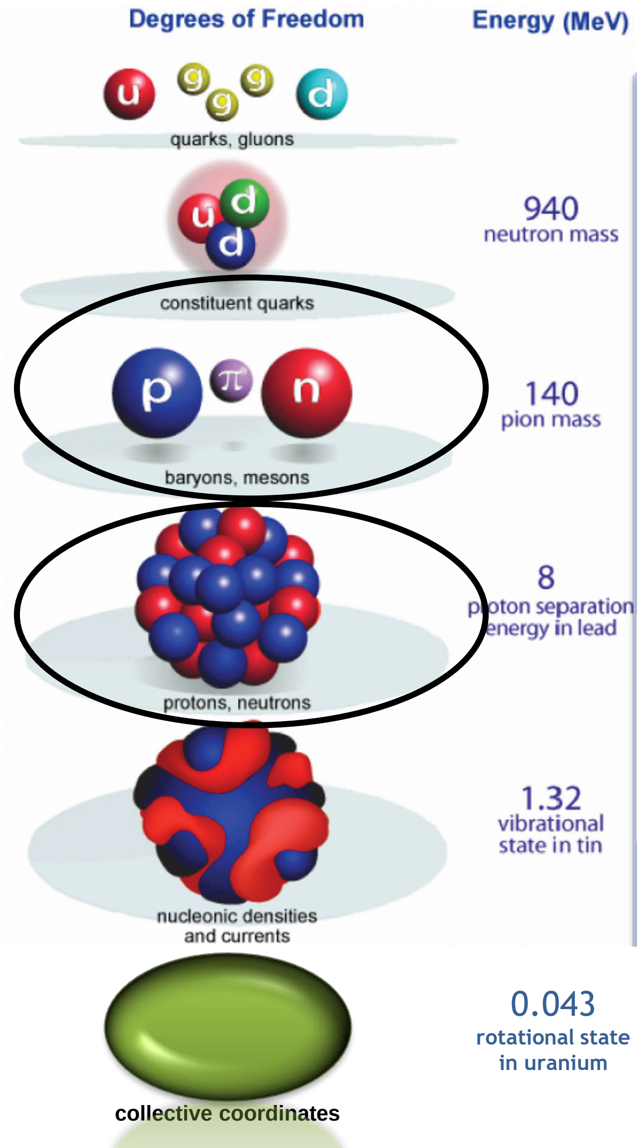
Choice of appropriated Degrees of Freedom

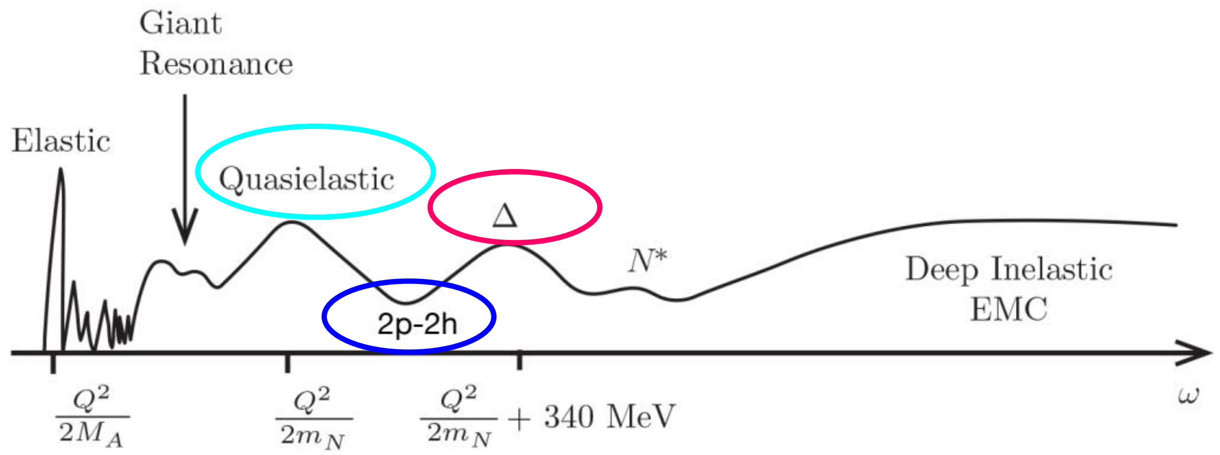


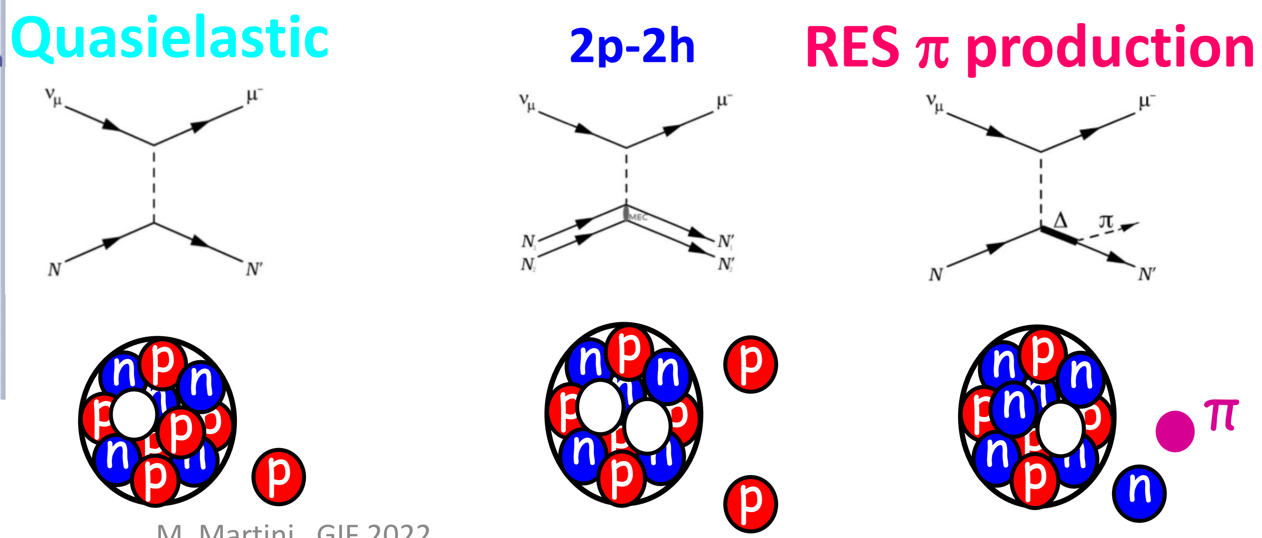


- DOF = collective coordinates
- Macroscopic models

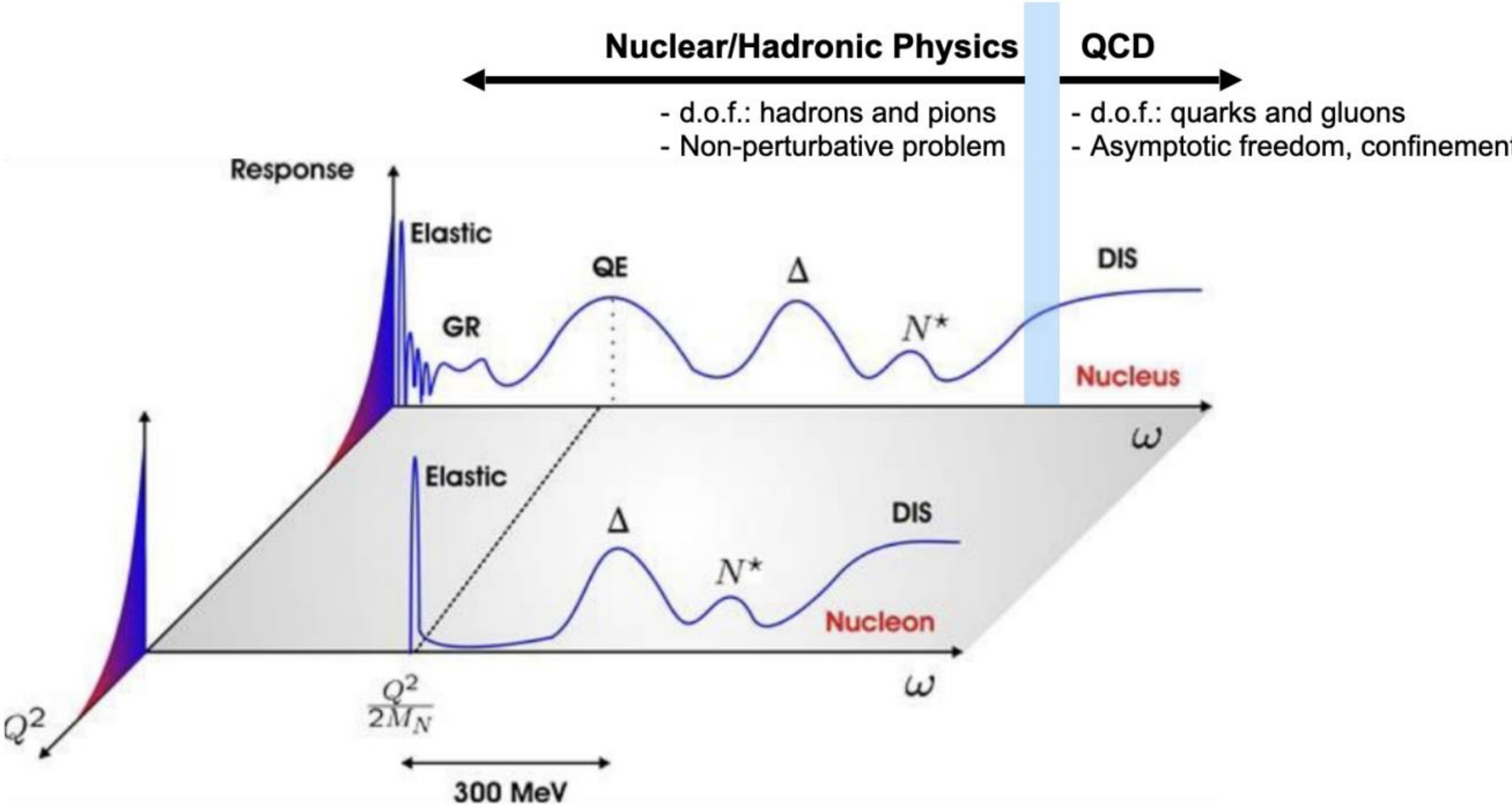
In the following:







Choice of appropriated Degrees of Freedom



The hadronic tensor

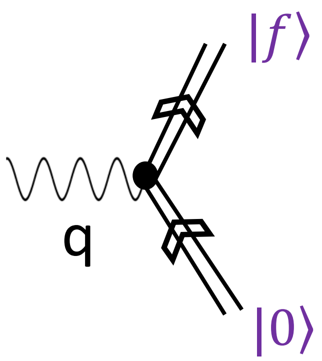
The hadronic tensor contains all the information on the target response

$$W^{\mu\nu} = \sum_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\delta^{(4)}(p_0 + q - p_f)$$

$|0\rangle$ hadronic initial state

$|f\rangle$ hadronic final state

J^{ν} Hadronic component of the electroweak current



A general expression

- valid for different degrees of freedom (quark, nucleon, nucleon resonances, nucleus)
- valid for different currents (electromagnetic, weak; one-body, two-body,…)

Let's start by considering the single nucleon electroweak current

The single nucleon electroweak current

Electromagnetic current - Electron scattering

$$J^{\mu}_{s's}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)i\sigma^{\mu\nu}\frac{q_{\nu}}{2m_N}\right]u_s(\mathbf{p})$$

$$Q^2 = -q^2 \qquad \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu},\gamma^{\nu}]$$

Weak current – CC neutrino scattering

$$\boxed{J^{\mu} = V^{\mu} - A^{\mu}}$$

Vector – Axial

Vector $$V^{\mu}_{s's}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[2F_1^V\gamma^{\mu} + 2F_2^V i\sigma^{\mu\nu}\frac{q_{\nu}}{2m_N}\right]u_s(\mathbf{p})$$

Conserved Vector Current (CVC) $q_{\alpha}V^{\alpha} = 0$ and isospin symmetry $\Rightarrow$ $F_i^V = F_i^p - F_i^n$

Axial $$A^{\mu}_{s's}(\mathbf{p}',\mathbf{p}) = \overline{u}_{s'}(\mathbf{p}')\left[G_A\gamma^{\mu}\gamma_5 + G_P\frac{q^{\mu}}{2m_N}\gamma_5\right]u_s(\mathbf{p})$$

Partially Conserved Axial Current (PCAC) and pion-pole dominance $\Rightarrow$ $G_P = \frac{4m_N^2}{m_{\pi}^2 + Q^2}G_A$

$$q_{\alpha}A^{\alpha} = i(m_u + m_d)\bar{q}_u\gamma_5 q_d \to 0$$

The nucleon form factors

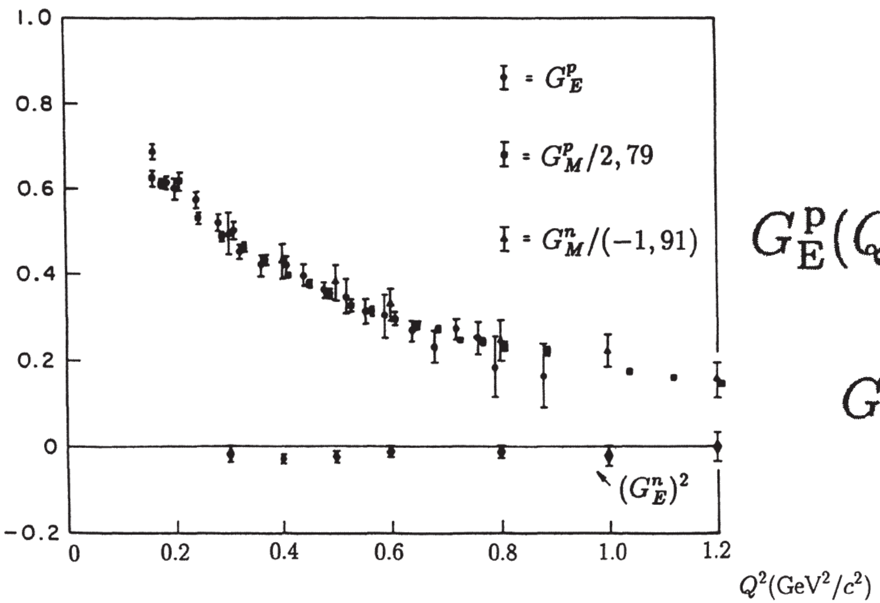
The form factors are corrections to "point-like coupling''
They reflect the fact that the nucleon has an internal structure and a finite size

F_1 and F_2 can be written as a combination of the Electric and Magnetic form factors G_E and G_M

$$F_1^{p,n} = \left[G_E^{p,n} + \frac{Q^2}{4m_N^2} G_M^{p,n}\right]\left[1 + \frac{Q^2}{4m_N^2}\right]^{-1} \qquad F_2^{p,n} = \left[G_M^{p,n} - G_E^{p,n}\right]\left[1 + \underbrace{\frac{Q^2}{4m_N^2}}_{\tau}\right]^{-1}$$

Electron-nucleon cross section

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) = \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right)_{\mathrm{Mott}} \cdot \left[\frac{G_{\mathrm{E}}^2(Q^2) + \tau G_{\mathrm{M}}^2(Q^2)}{1+\tau} + 2\tau G_{\mathrm{M}}^2(Q^2)\tan^2\frac{\theta}{2}\right]$$



Global **dipole**-like behavior

$$G_{\mathrm{E}}^{\mathrm{p}}(Q^2) = \frac{G_{\mathrm{M}}^{\mathrm{p}}(Q^2)}{2.79} = \frac{G_{\mathrm{M}}^{\mathrm{n}}(Q^2)}{-1.91} = G^{\,\mathrm{dipole}}(Q^2)$$

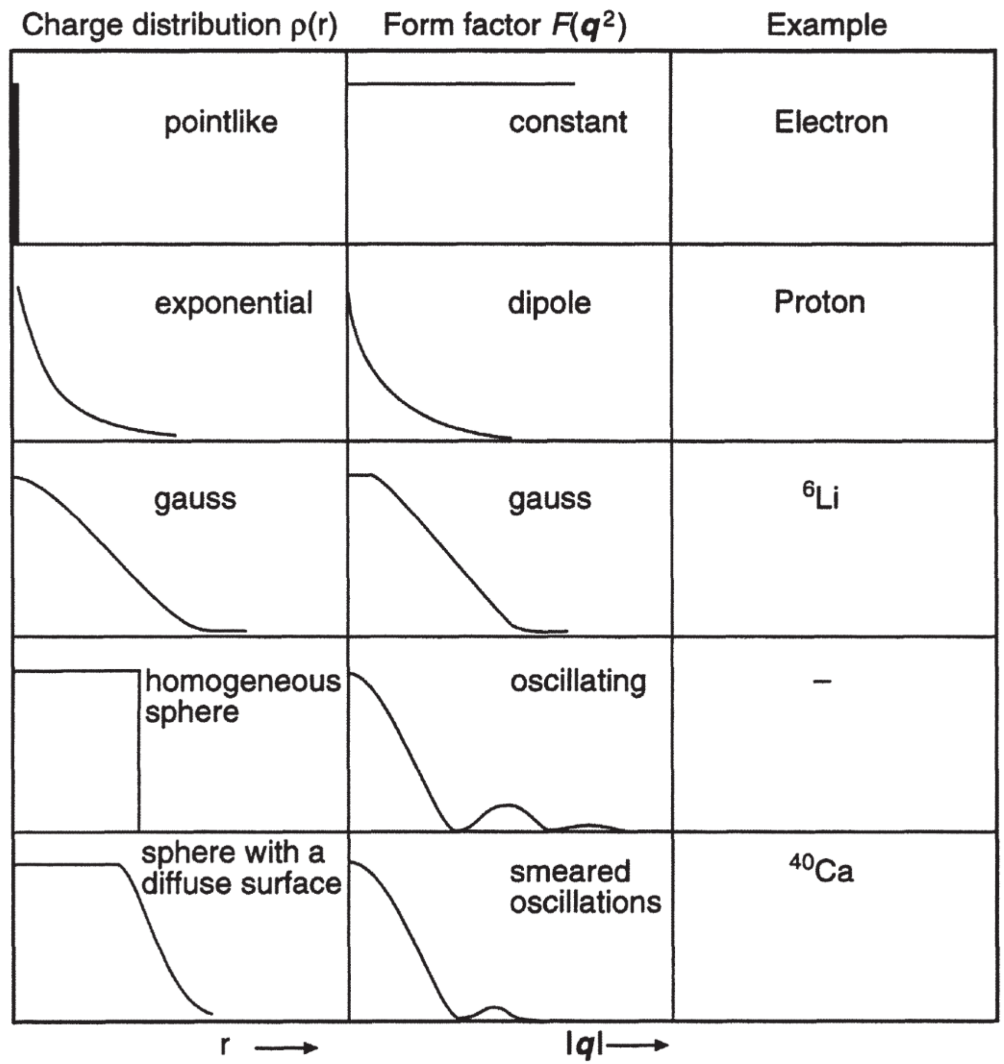
$$G^{\,\mathrm{dipole}}(Q^2) = \left(1 + \frac{Q^2}{0.71\,(\mathrm{GeV}/c)^2}\right)^{-2}$$

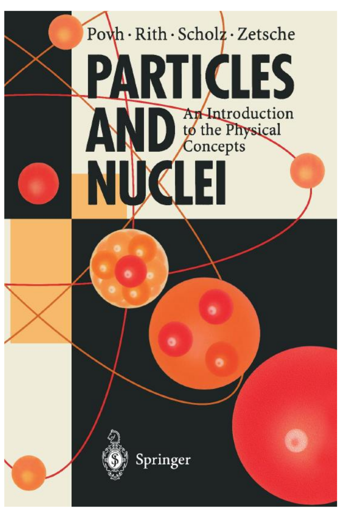
Form factor and spatial distribution

$$\left(\frac{d\sigma}{d\Omega}\right)_{\rm exp} = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \cdot |F(\mathbf{q}^2)|^2$$

$$F(\mathbf{q}^2) = \int d^3\mathbf{r}\ \rho(\mathbf{r}) \cdot e^{i\mathbf{q}\cdot\mathbf{r}}$$

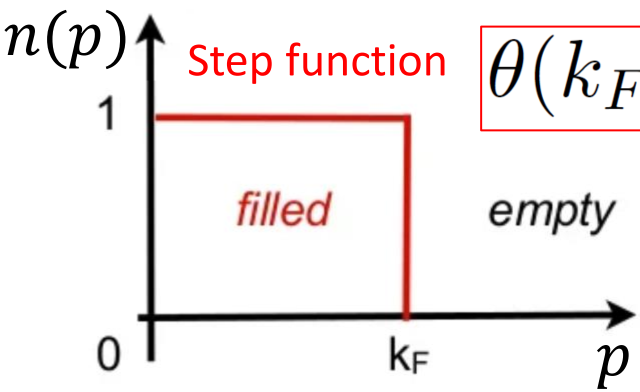
Form factors and spatial distributions
(charge, magnetization)
are Fourier pairs





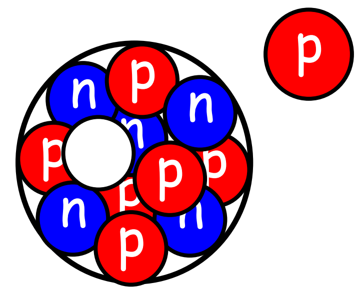
An example of hadronic tensor for the nucleus excitations: Quasielastic (1p-1h) excitation in the Relativistic Fermi Gas (RFG)

- Relativistic Fermi Gas: Nucleus as ensemble of non interacting fermions (nucleons)
- In the RFG ground state all the momenta **p** with |**p**| < k_F (Fermi momentum) are filled



$$\theta(k_F - p)$$

EW current approximated by 1-body operator, which can produce only 1particle--1hole (1p1h) excitations



RFG 1p-1h (QE) hadronic tensor

J.E. Amaro et al. J.Phys.G 47 (2020) 12, 124001

$$W^{\mu\nu}(q,\omega) = \sum_{\mathbf{p}} \sum_{s,s'} \delta(E' - E - \omega) \frac{m_N^2}{EE'} J^{\mu *}_{s's}(\mathbf{p}', \mathbf{p}) J^{\nu}_{s's}(\mathbf{p}', \mathbf{p}) \theta(k_F - p) \theta(p' - k_F)$$

$$W^{\mu\nu}(q,\omega) = \frac{V}{(2\pi)^3} \int d^3p\, \delta(E' - E - \omega) \frac{m_N^2}{EE'} 2 w^{\mu\nu}_{s.n.}(\mathbf{p}', \mathbf{p}) \theta(k_F - p) \theta(p' - k_F)$$

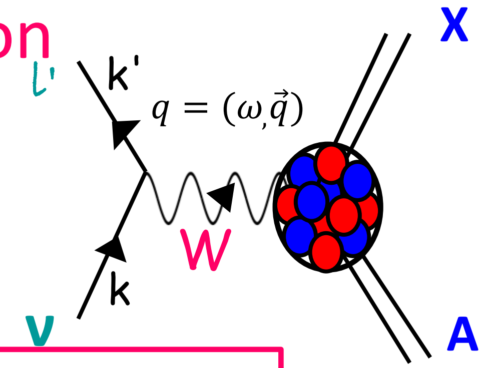
Single-nucleon hadronic tensor

$$w^{\mu\nu}_{s.n.}(\mathbf{p}', \mathbf{p}) = \frac{1}{2} \sum_{ss'} J^{\mu *}_{s's}(\mathbf{p}', \mathbf{p}) J^{\nu}_{s's}(\mathbf{p}', \mathbf{p})$$

see slide 38 for the expressions of J^{μ}

Charged current neutrino-nucleus cross section

$$\nu_l\ (\bar{\nu}_l) + A \longrightarrow l^-\ (l^+) + X$$



Lab frame

$$\frac{d^2\sigma}{d\Omega_{k'}d\omega} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu} W^{\mu\nu}(\mathbf{q}, \omega)$$

$d\Omega_{k'}$ differential solid angle in the direction specified by the charged-lepton momentum **k'**

$k \equiv (E_\nu, \mathbf{k})\ \ k' \equiv (E'_l, \mathbf{k}')$ initial and final lepton 4-momenta

$q = k - k' \equiv (\omega, \mathbf{q})$ four-momentum transfer

$\omega = E_\nu - E'_l$ energy transfer

The charged current cross section is a linear combination of five contributions

$$\frac{d^2\sigma}{d\Omega_{k'}d\omega} = \sigma_0 \left[L_{00}W^{00} + L_{33}W^{33} + (L_{03} + L_{30})W^{03} + (L_{11} + L_{22})W^{11} \pm (L_{12} - L_{21})W^{12} \right]$$

- The notation {00; 03; 33; 11; 12} is often replaced by {00; 0z; zz; xx; xy} or {CC;CL; LL;T; T'} where the letters C, L and T stand for Coulomb, Longitudinal and Transverse respectively
- The explicit expression of the lepton coefficients L (which depend only on lepton kinematics) and of the components of the hadronic tensor W can be found in many books and articles. For example:

Walecka, J. D. (1995), "Theoretical nuclear and subnuclear physics", Oxford Stud. Nucl. Phys., 16

O' Connell et al. PRC 6 719–733 (1972); Nieves et al. PRC 70 055503 (2004); Amaro et al. PRC C 71 065501 (2005); Martini et al. PRC 80 065501 (2009); Shen et al. PRC 86 035503 (2012)

A simplified expressions particularly useful for illustration

- Final lepton mass contributions ignored (m_l=0)
- Obtained by keeping only the leading terms for the hadronic tensor in the development of the hadronic current in p/M_N

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2}G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

$$\left. + 2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

Explicitly appear:

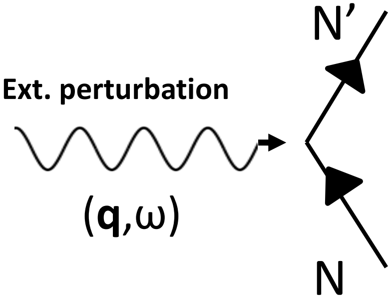
1. The different **kinematic variables** (related to the leptonic tensor)
2. The nucleon Electric, Magnetic, and Axial **form factors** (↔ nucleon properties)
3. The **nuclear response functions** (↔ nuclear dynamics)

Nuclear response functions R(q,ω):

$$R_\alpha^{PP'}(\mathbf{q},\omega) = \sum_n \langle n|\sum_{j=1}^{A} O_\alpha^P(j)\, e^{i\,\mathbf{q}\cdot\mathbf{x}_j}|0\rangle\langle n|\sum_{k=1}^{A} O_\alpha^{P'}(k)\, e^{i\,\mathbf{q}\cdot\mathbf{x}_k}|0\rangle^*\,\delta(\omega-E_n+E_0),$$

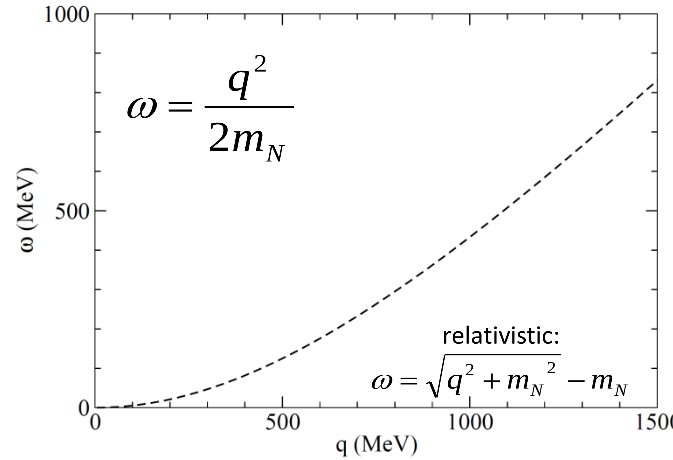
Isovector R_τ	Isospin Spin-Longitudinal $R_{\sigma\tau(L)}$	Isospin Spin-Transverse $R_{\sigma\tau(T)}$
$O_\alpha^N(j)=\tau_j^\pm$	$(\boldsymbol{\sigma}_j\cdot\widehat{q})\,\tau_j^\pm$	$(\boldsymbol{\sigma}_j\times\widehat{q})^i\,\tau_j^\pm$

Free (or bare) nuclear response function



- **Free nucleon at rest:**

  Response functions $\propto \delta(\omega - q^2/2m_N)$



- **Nucleon inside the nucleus:**

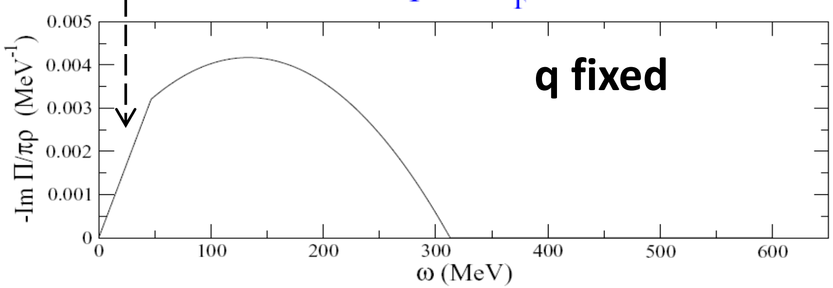
Nucleon-Nucleon interaction switched off ↔ Nucleons respond individually

Fermi Gas Quasielastic Response

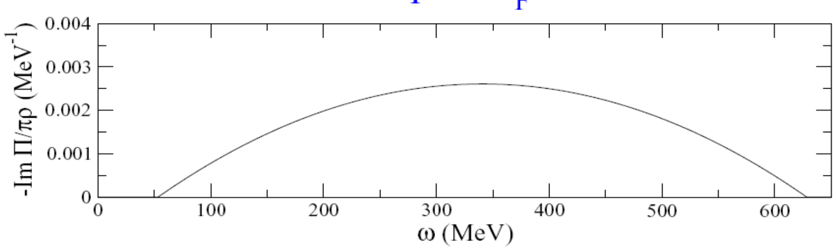
- **Fermi motion** spreads δ distribution
- **Pauli blocking** cuts part of the low q and ω response

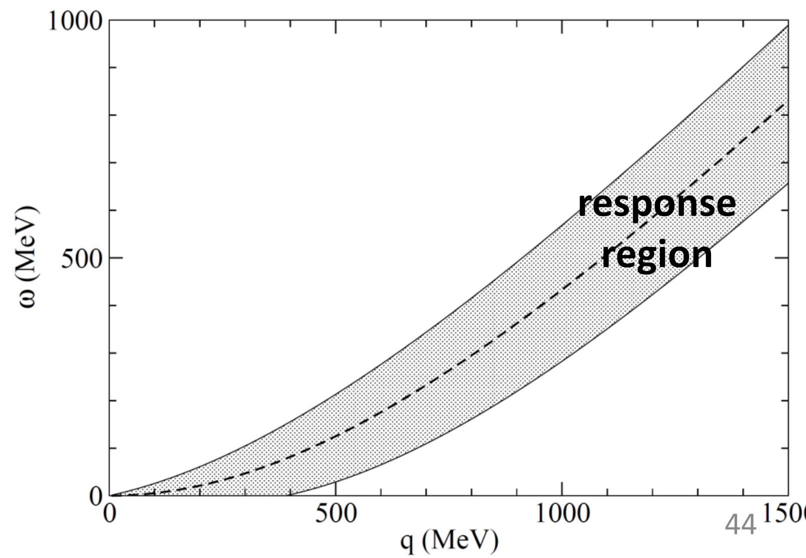
$$\frac{q^2}{2m_N} - \frac{qk_F}{m_N} \le \omega \le \frac{q^2}{2m_N} + \frac{qk_F}{m_N}$$

$q < 2\,k_F$



$q > 2\,k_F$

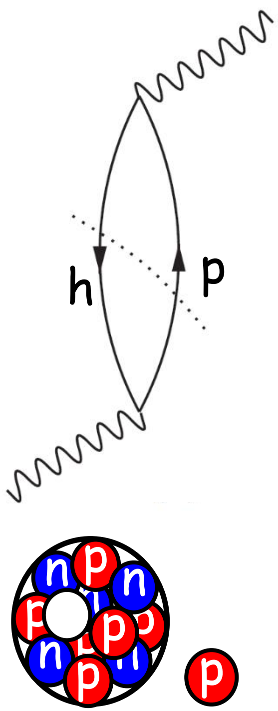




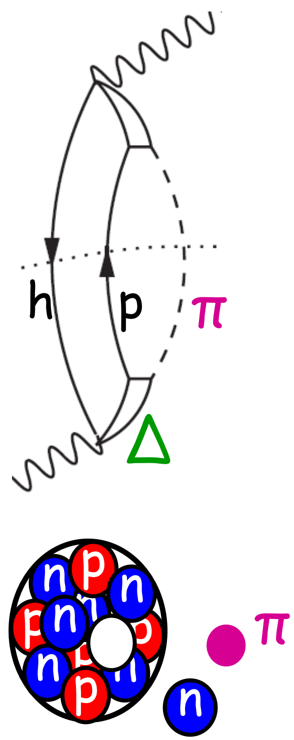
Nuclear Responses for different excitations

$$R_\alpha = \sum_{n \neq 0} |\langle n|\hat{O}_{(\alpha)}|0\rangle|^2 \; \delta[\omega - (E_n - E_0)]$$

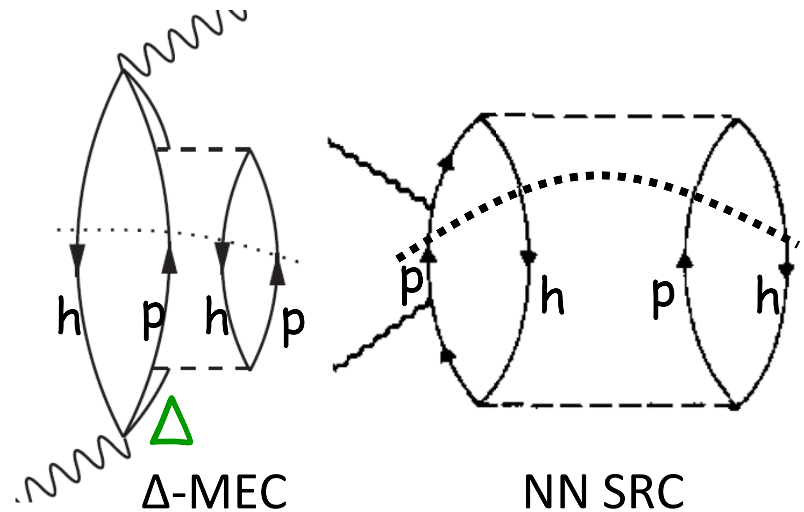
1p-1h
Quasielastic



1p-1h
(Δ→πN) 1π production

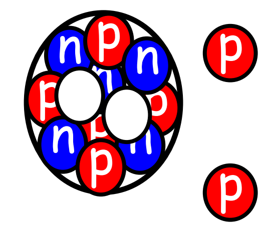


2p-2h:
two examples

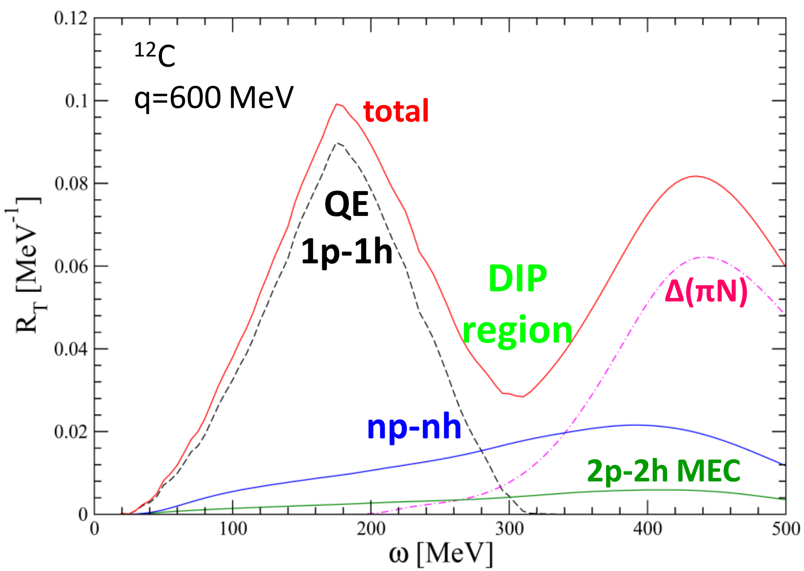


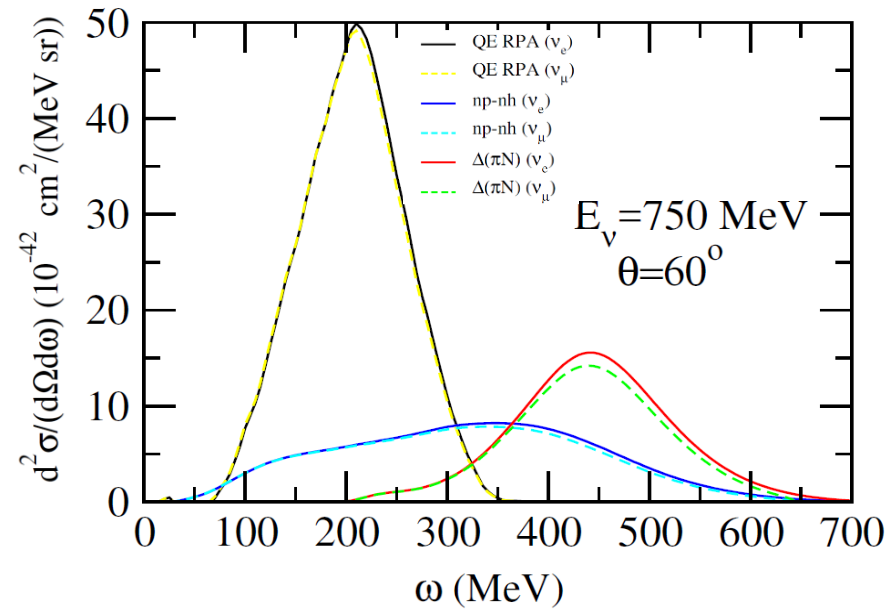
Δ-MEC

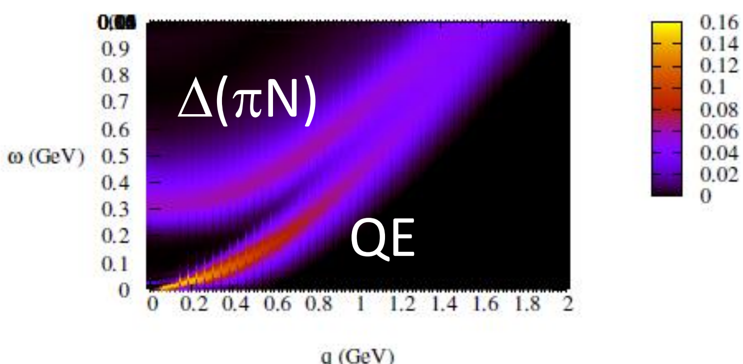
NN SRC

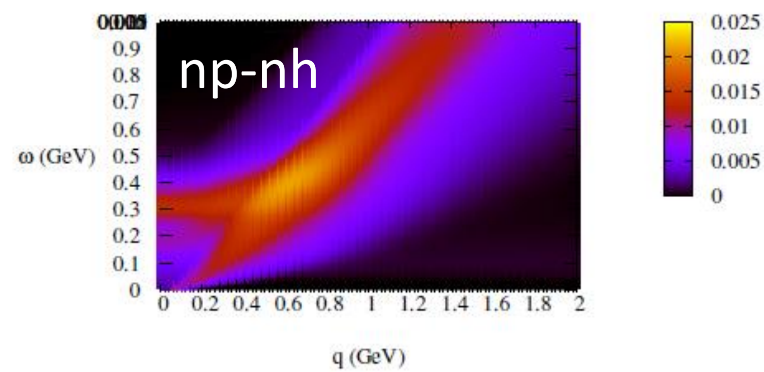


Nuclear responses and neutrino cross sections at fixed kinematics









QE peak:

$$\omega = \sqrt{\mathbf{q}^2 + M_N^2} - M_N = \frac{Q^2}{2M_N} = \frac{\mathbf{q}^2 - \omega^2}{2M_N}$$

Δ peak:

$$\omega = \sqrt{\mathbf{q}^2 + M_\Delta^2} - M_N = \frac{Q^2}{2M_N} + \frac{M_\Delta^2 - M_N^2}{2M_N}$$

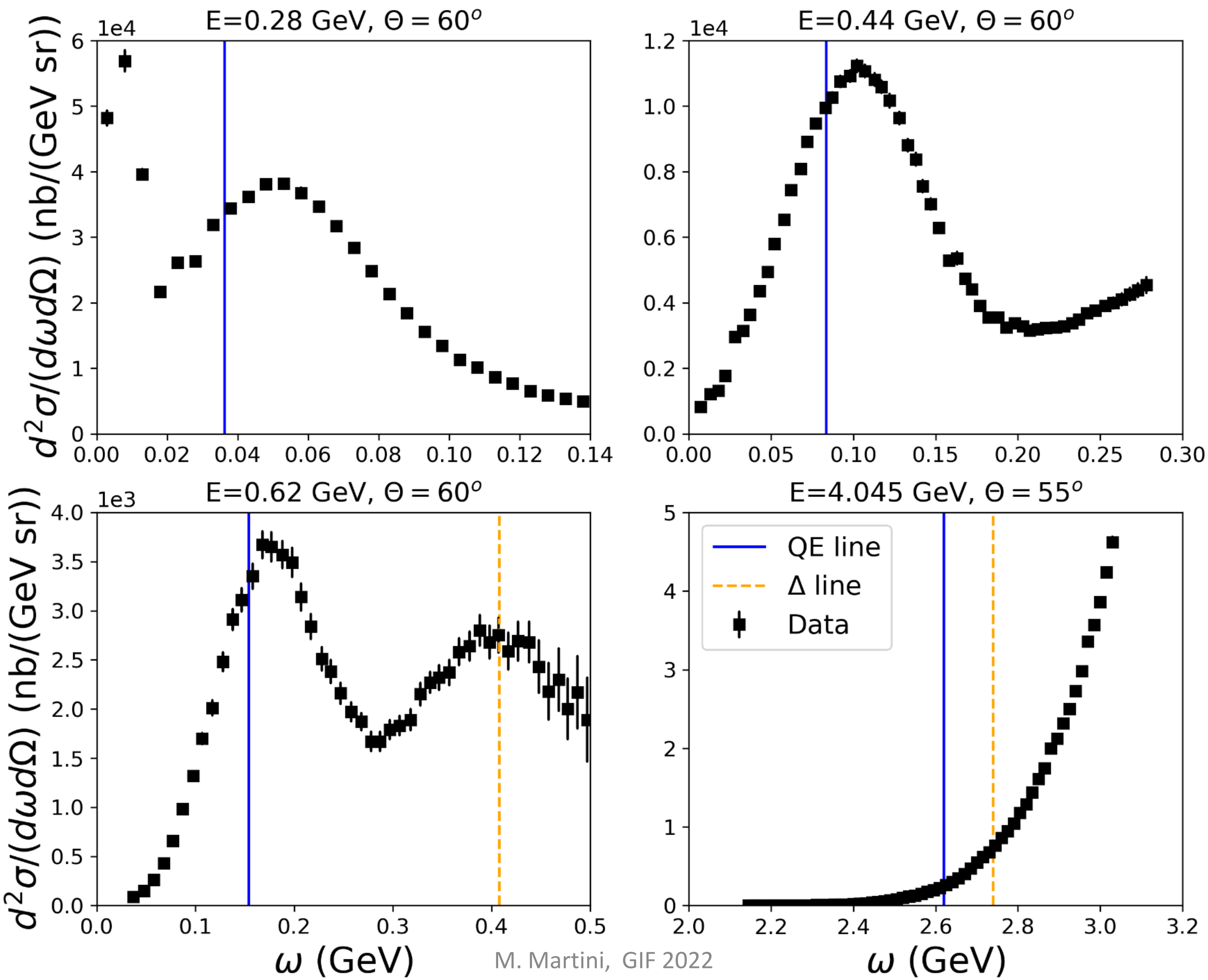
np-nh excitations fill the DIP region

np-nh enlarges the region of response to the whole (ω,q) plane

Examples of electron scattering cross section on ^{12}C

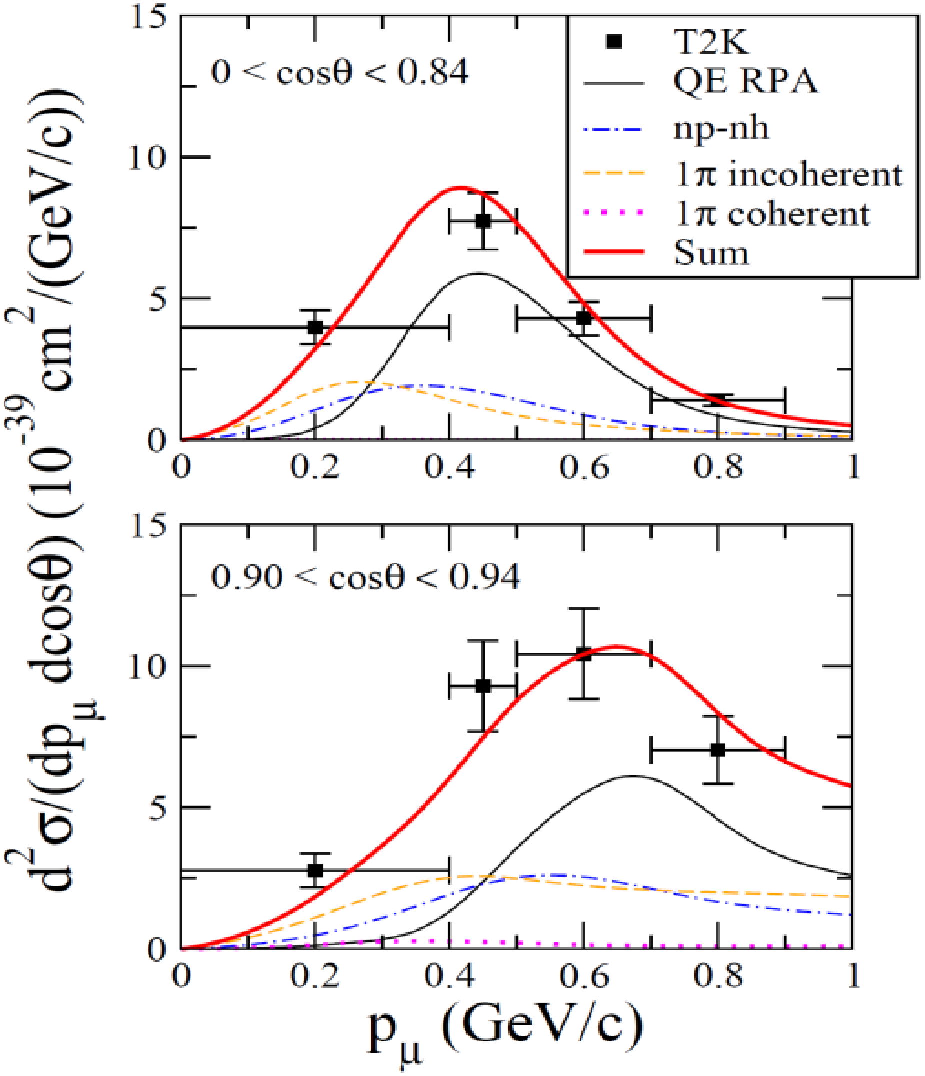
Remind: monochromatic beam

$$\omega_{QE} = \frac{E^2(1-\cos\theta)}{M_N + E(1-\cos\theta)} \qquad \omega_\Delta = \frac{M_N \Delta M + E^2(1-\cos\theta)}{M_N + E(1-\cos\theta)},$$

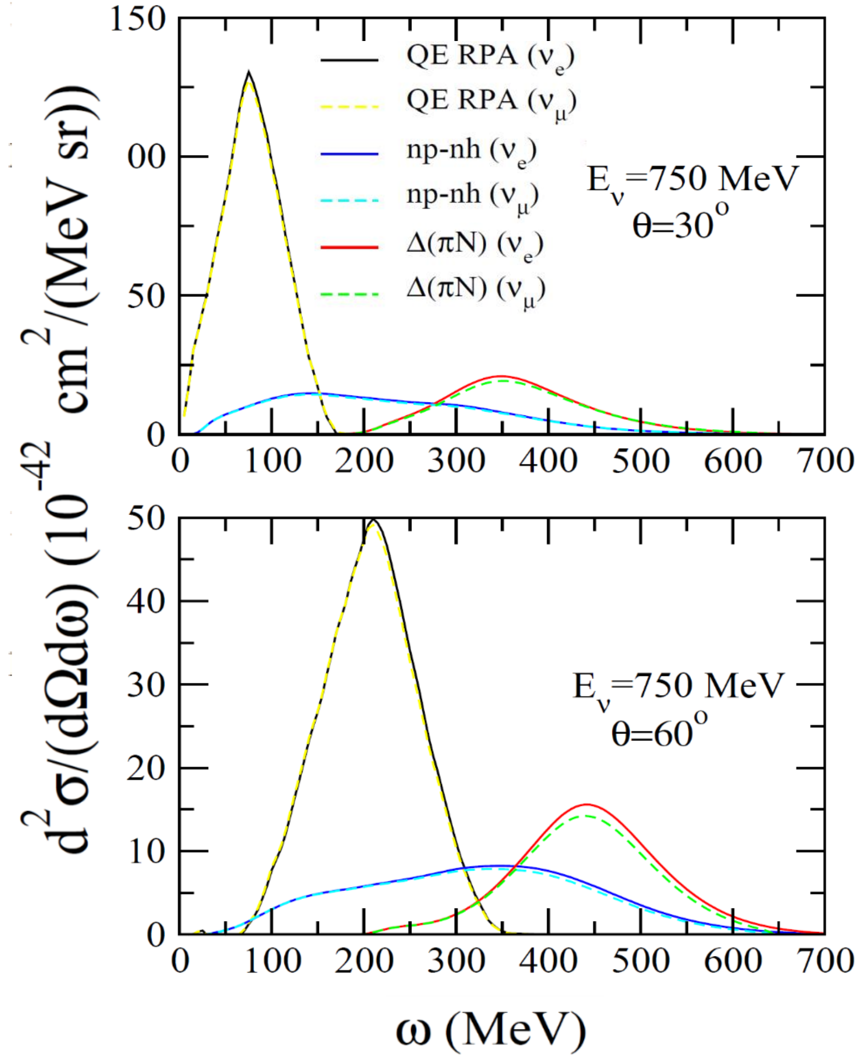


Remark: flux-integrated .vs. monochromatic beam cross sections

Neutrino flux-integrated d2σ



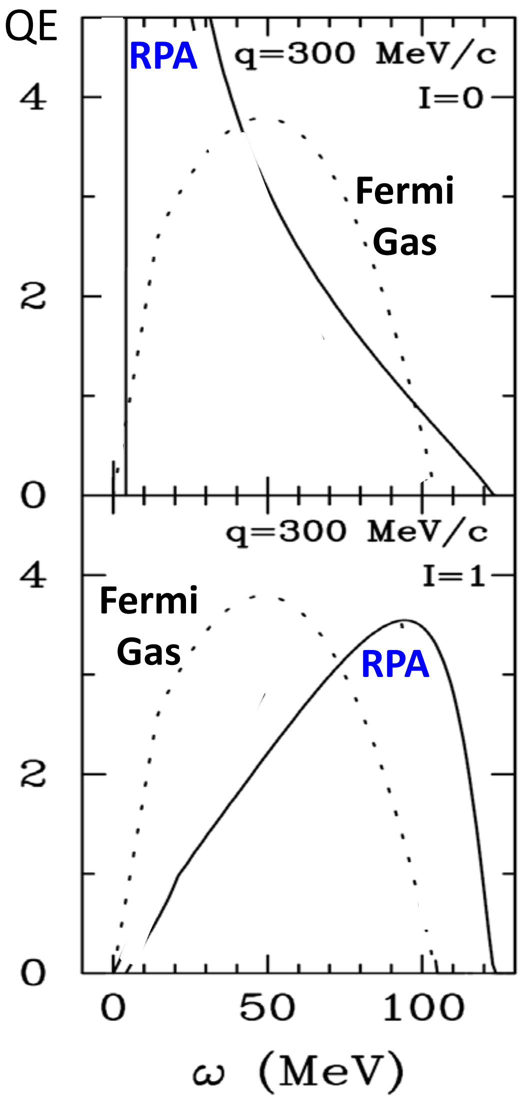
Monochromatic d2σ

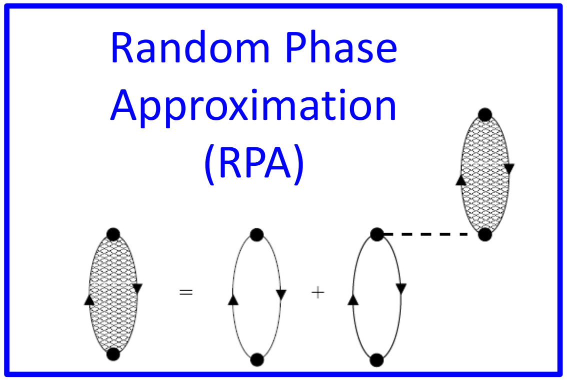


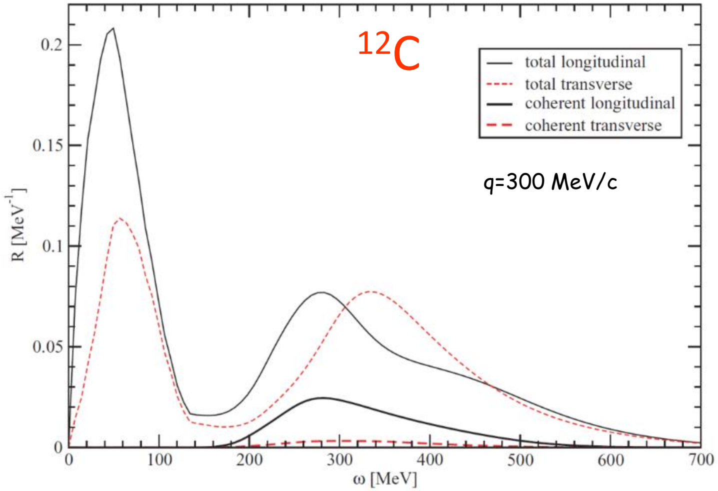
In the flux-integrated cross sections the different channels are entangled

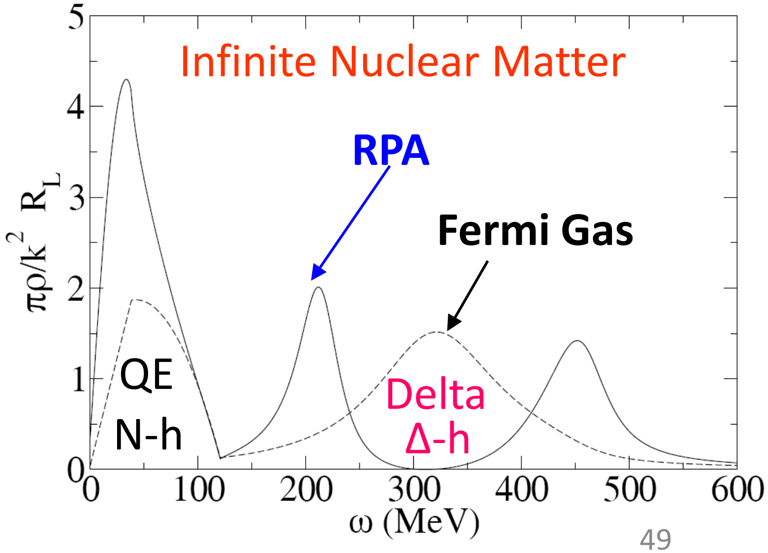
Switching on the nucleon- nucleon interaction

- External force acting on one nucleon is transmitted to the neighbors by the interaction – **Long Range Correlations**
- The nuclear response becomes collective
- Shift of the peak with respect to Fermi Gas, decrease, increase depending on the channels of excitation









Neutrino scattering - Effects of the RPA in the genuine quasielastic channel

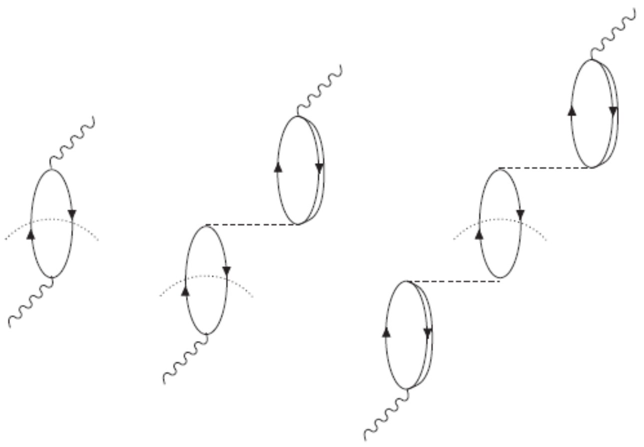
QE totally dominated by isospin spin-transverse response $R_{\sigma\tau(T)}$

RPA reduction

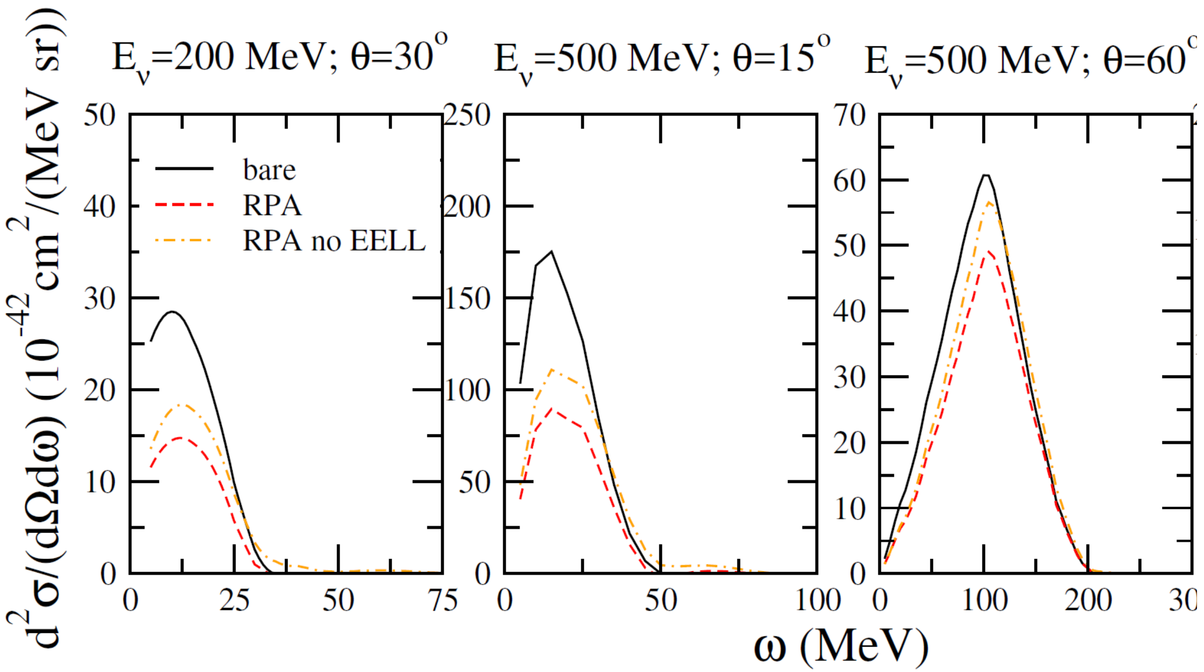
- expected from the repulsive character of p-h interaction in T channel
- also due to interference term $R^{N\Delta} < 0$

(Lorentz-Lorenz or Ericson-Ericson effect *[M.Ericson, T. Ericson, Ann. Phys. 36, 323 (1966)]*)

Lowest order contribution to QE:

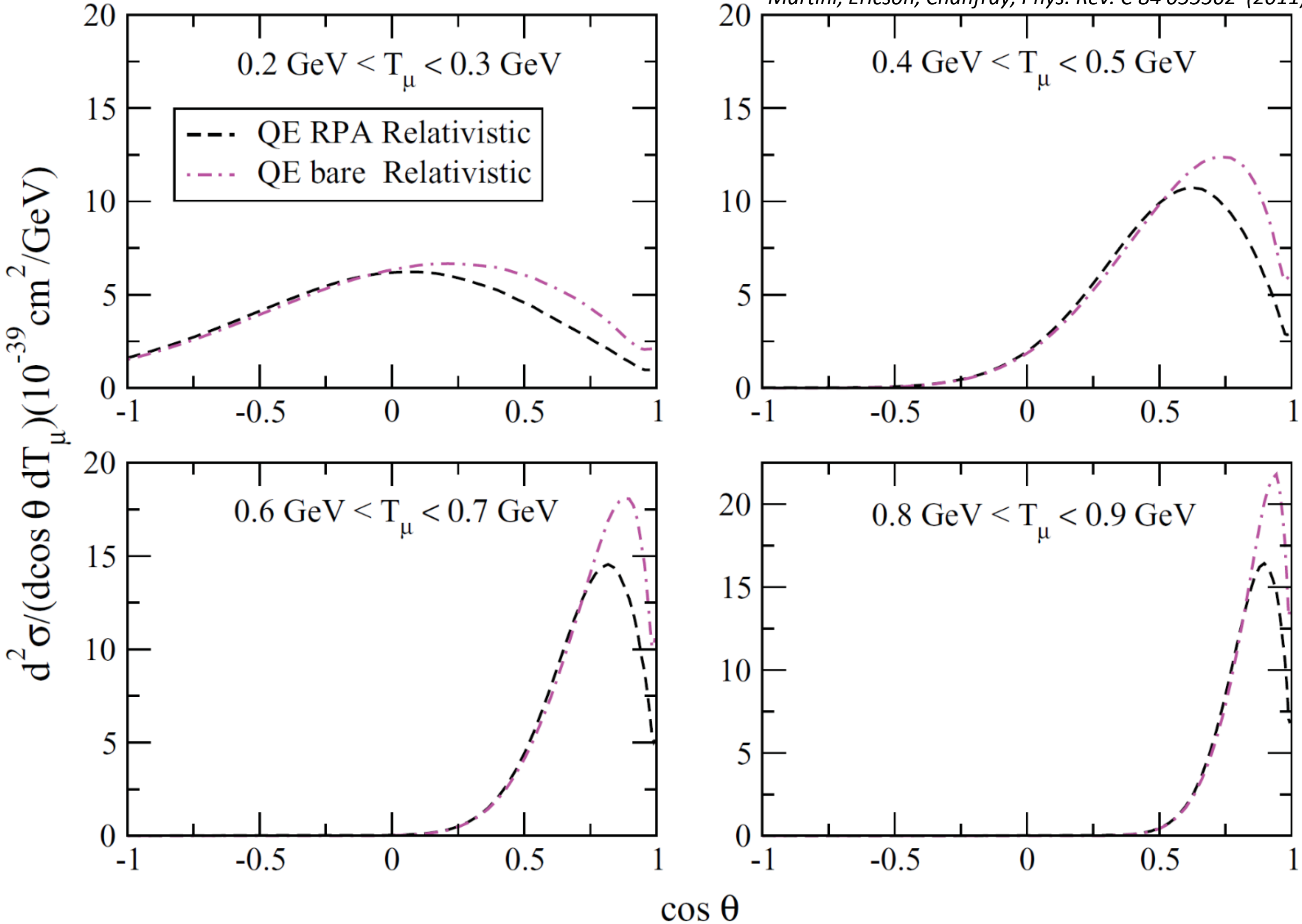


R^{NN}_{QE} $R^{N\Delta}_{QE}$ $R^{\Delta\Delta}_{QE}$



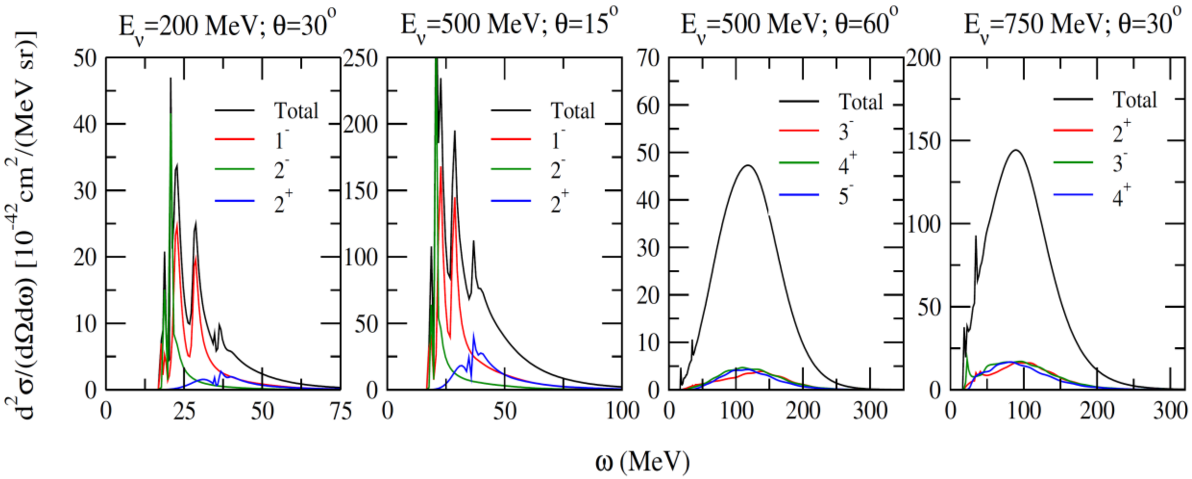
Bare *vs* RPA for MiniBooNE flux integrated $d^2\sigma$ (genuine QE)

Martini, Ericson, Chanfray, Phys. Rev. C 84 055502 (2011)



RPA produces a quenching and some shift towards larger angles

The Hartree Fock + Continuum RPA for giant resonances and QE

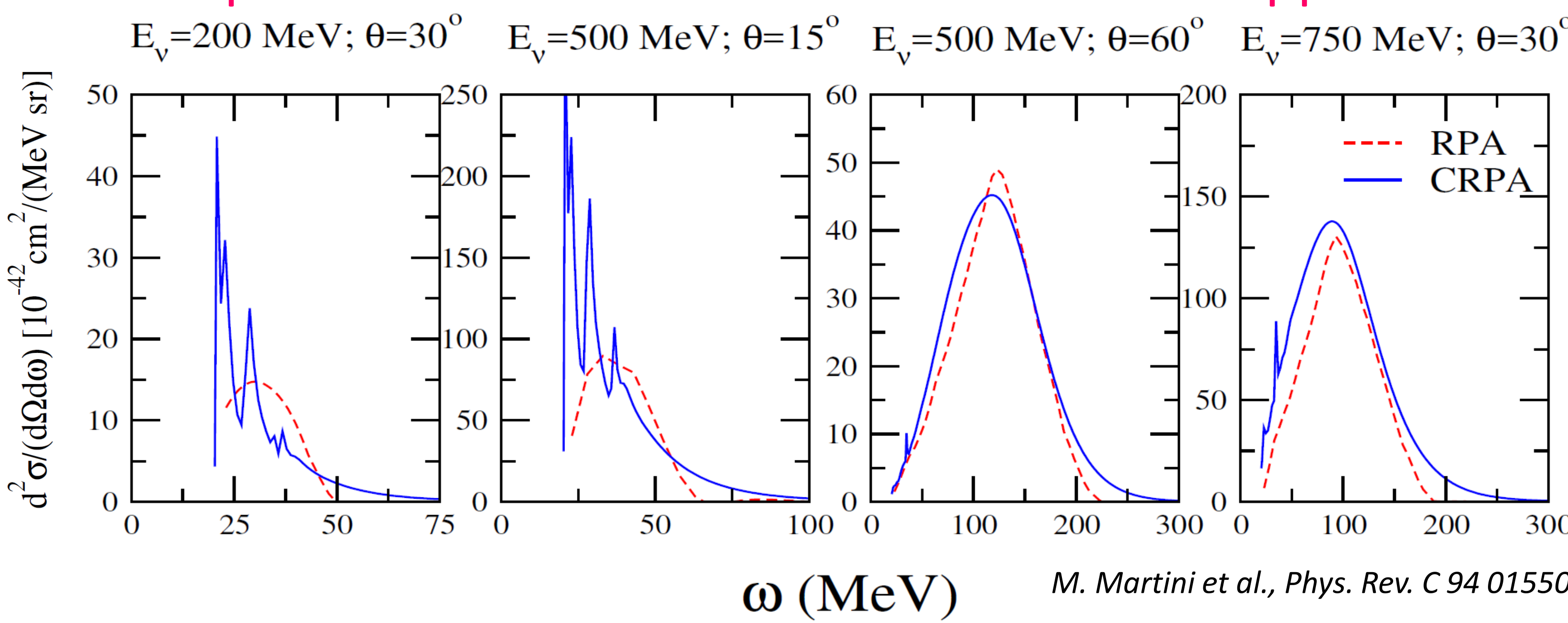


HF+CRPA (Ghent)

Pandey et al. Phys.Rev. C94 054609 (2016)

- Shell effects and giant resonances
- Different multipolar excitations

Comparison between LFG+RPA and HF+CRPA approaches



M. Martini et al., Phys. Rev. C 94 015501 (2016)

- The two approaches are essentially in agreement
- In the low energy part the LFG+RPA results represent the average of the HF+CRPA ones

Several models to calculate the responses and the ν cross sections

- Local Fermi Gas + Random Phase Approximation

Lyon *M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)*

Valencia *J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, Phys. Rev. C 83 045501 (2011)*

- Hartree-Fock + (Continuum) Random Phase Approximation

Ghent *V. Pandey, N. Jachowicz, T. Van Cuyck, J. Ryckebusch, M. Martini, Phys. Rev. C 92 024606 (2015)*

Other groups focused on giant resonances and below *Kolbe et al. ; Volpe et al.; Co' et al.; …*

- SuSAv2 superscaling/relativistic mean field

Granada, Madrid, MIT, Sevilla, Torino

G.D. Megias, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, I. Ruiz Simo, PRD 94 093004 (2016)

- Spectral function approach

Roma *N. Rocco, C. Barbieri, O. Benhar, A. De Pace, A. Lovato, Phys. Rev. C 99 025502 (2019)*

- Relativistic Green's function

Pavia *A. Meucci, C. Giusti, F. D. Pacati, Nucl.Phys.A 739 277-290 (2004)*

- Green's function Monte Carlo ("ab initio")

Argonne, Los Alamos *A. Lovato, J. Carlson, S. Gandolfi, N. Rocco, R. Schiavilla, PRX 10 031068 (2020)*

- GiBUU transport theory

Giessen *O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A.B. Larionov, T. Leitner, J. Weil, U. Mosel, Phys.Rept. 512 1-124 (2012)*

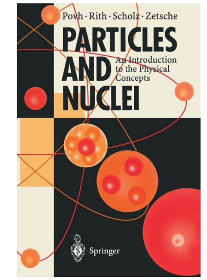
p.s. only one representative reference for each approach (not necessarily the founding paper)

For discussions and comparisons of different models see for example:

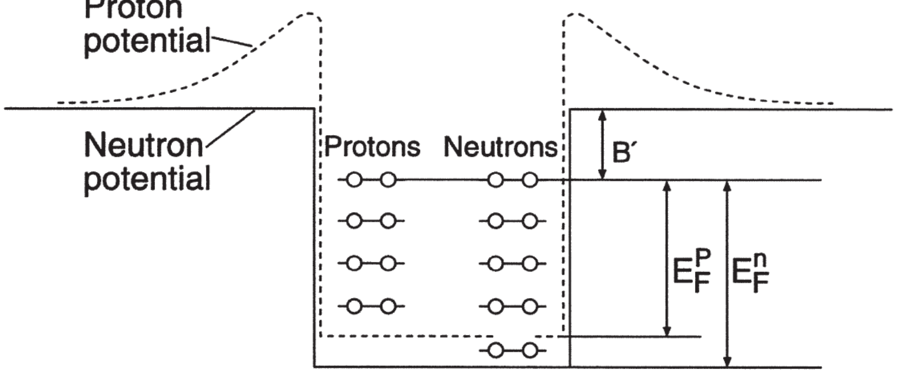
- *G.T. Garvey, D.A. Harris, H.A. Tanaka, R. Tayloe, G.P. Zeller, Phys.Rept. 580 (2015) 1-45*
- *T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001*
- *M. Sajjad Athar, A. Fatima, S. K. Singh arxiv. 2206.13792*

Simple nuclear models in introductory books

Independent particles models



Fermi Gas



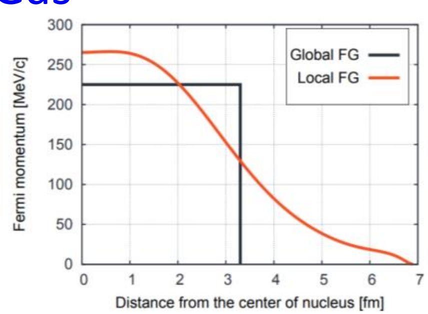
- Protons and neutrons move freely within the nuclear volume V
- Retained only statistical correlations (Pauli principle)
- In the nuclear ground state, the lowest states are all occupied up to a maximal momentum called Fermi momentum k_F

$$\frac{Z}{V} = \rho_p = \frac{(k_F^p)^3}{3\pi^2} \quad \frac{N}{V} = \rho_n = \frac{(k_F^n)^3}{3\pi^2} \quad \frac{A}{V} = \rho = \frac{2k_F^3}{3\pi^2}$$

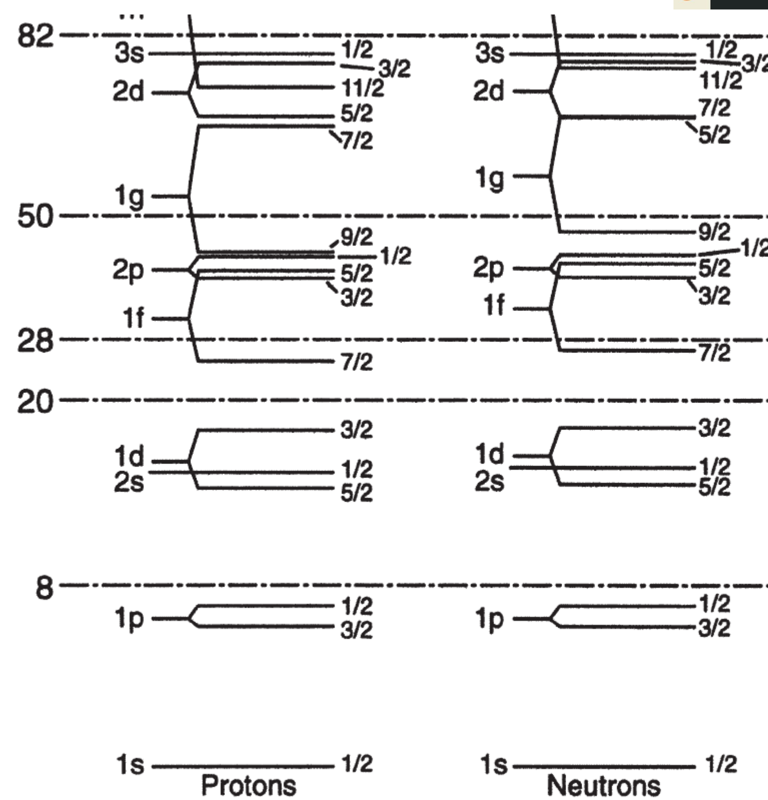
Local Fermi Gas

Local Density approximation

$$k_F(r) = [3/2 \quad \pi^2 \rho(r)]^{1/3}$$

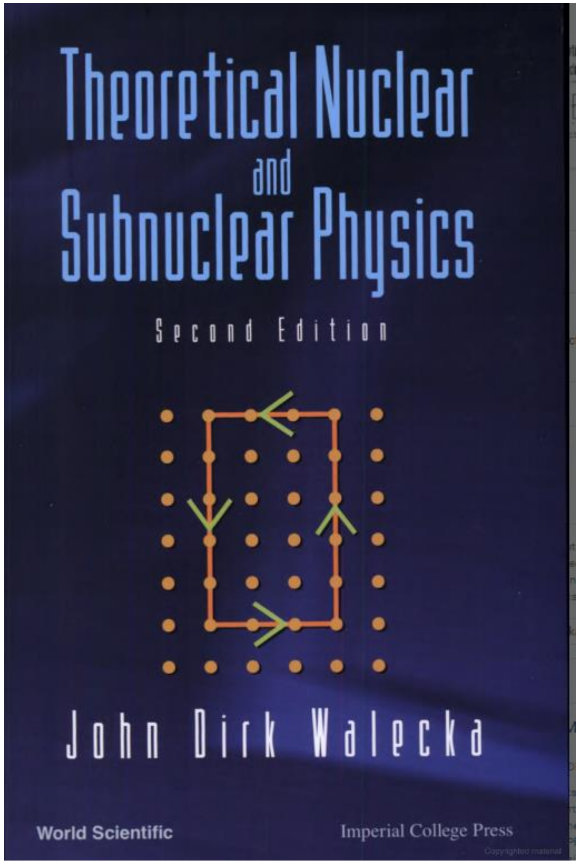


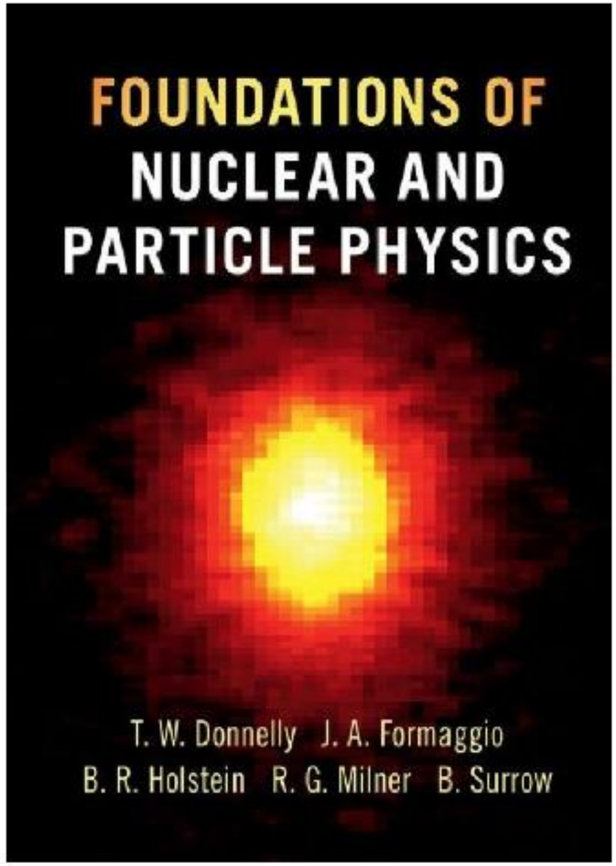
Shell Model

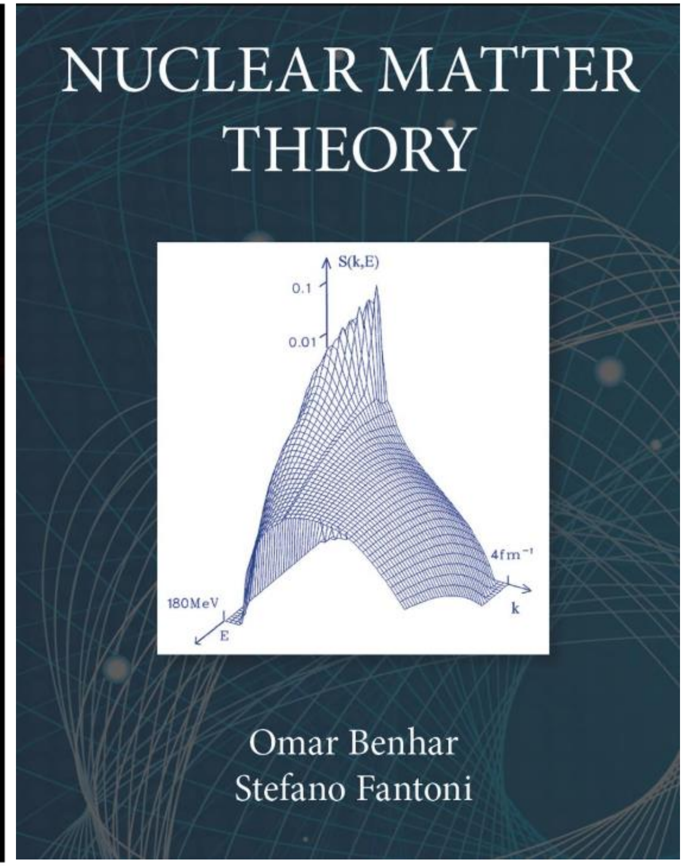


- The nucleons move inside a mean field potential produced by the other nucleons
- Discrete energy levels arise which are filled up according to the Pauli principle

More sophisticated models in advanced books







- Hartree-Fock
- RPA
- Relativistic Mean Field
- Quantum Hadrodynamics

- Hartree-Fock
- Scaling
- Spectral function

- Spectral function
- Green's function methods
- Monte Carlo methods
- Variational methods (CBF, FHNC)
- Relativistic Mean Field

Monte Carlo Event Generators

Monte Carlo event generators connects theoretical models to experimental measurements

Main Event Generators for neutrino interactions:

NEUT

L. Alvarez-Ruso et al., EPJ Spec. Top. 230, 4449 (2021)

O. Buss et al., Phys.Rept. 512 1-124 (2012)

Y. Hayato and L. Pickering, EPJ Spec. Top. 230, 4469 (2021)

T. Golan et al., NPB 229–232, 499 (2012)

PHYSICAL REVIEW D **105,** 092004 (2022)

Comparisons and challenges of modern neutrino-scattering experiments

M. Buizza Avanzini,[1] M. Betancourt,[2] D. Cherdack,[3] M. Del Tutto,[2,4] S. Dytman,[5] A. P. Furmanski,[6,7] S. Gardiner,[2] Y. Hayato,[8] L. Koch,[9] K. Mahn,[10] A. Mastbaum,[11] B. Messerly,[5,7] C. Riccio,[12,13] D. Ruterbories,[14] J. Sobczyk,[15] C. Wilkinson,[16] and C. Wret[14]

Main models implemented for the quasielastic (and 2p-2h):

- Relativistic global and local Fermi Gas
- RPA
- Spectral Function
- SuperScaling (SuSAv2)

We have already rapidly illustrated the Fermi Gas and the RPA.
In the following the SuperScaling and the spectral function will be briefly sketched
2p-2h will be discussed in the second lecture

SuperScaling

- The basic idea of the approach [J.E. Amaro et al., PRC71 (2005) 015501] is to exploit electron scattering in order to predict the neutrino scattering cross section based on the "superscaling" properties of inclusive electron scattering data, extensively analysed in the 90s [Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990); Donnelly and Sick, PRL82; PRC60 (1999)]

- Extract a **SuperScaling function** from electron scattering inclusive data

$$f(q,\omega;k_F) = k_F \times \frac{\left[d^2\sigma/d\omega d\Omega\right]^{(e,e')}_{exp}}{\overline{\sigma}_{eN}}$$

- Plot it as function of a **Scaling variable** which is a combination of q and ω

$$\psi \equiv \psi(q,\omega;k_F)$$

- **SuperScaling** is realized if:

$$\psi(q,\omega;k_F) \longrightarrow f(\psi)$$

I) f is independent of the kinematics (q) for a given nucleus (scaling of firs kind)
II) f is independent of the nucleus (k_F) for given kinematics (scaling of second kind)

The SuperScaling function f is a universal function encoding the nuclear dynamics.
It can be extracted from electron scattering experiment or calculated within a model.

- Final step: Use the SuperScaling function to predict the **neutrino cross sections**

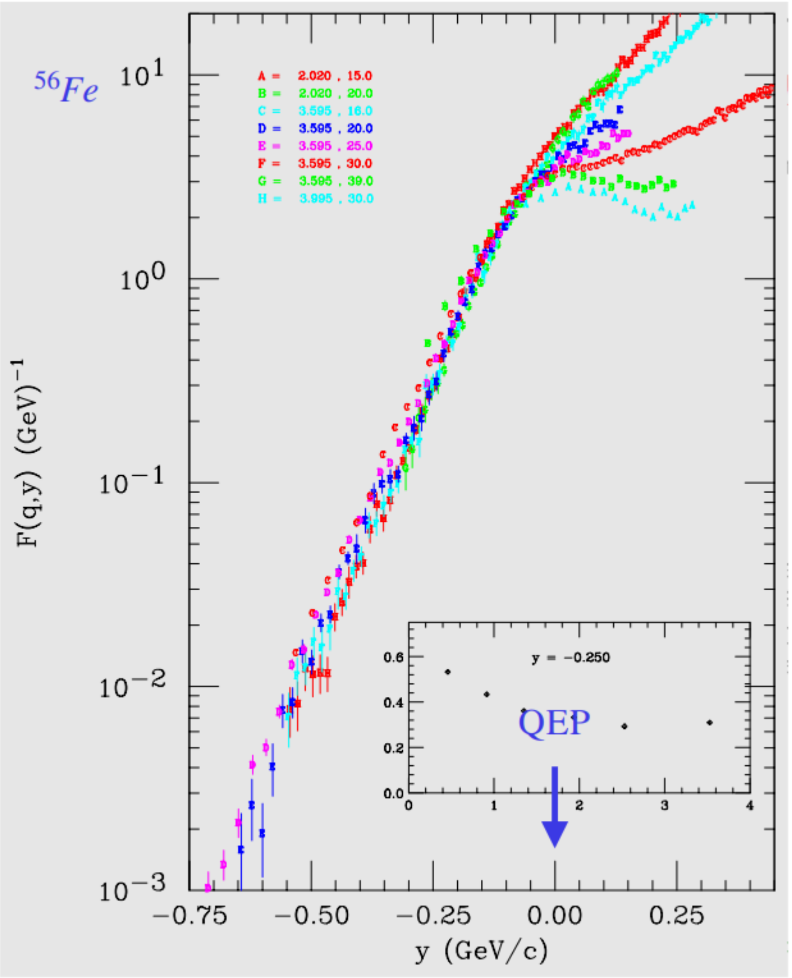
$$\left[d^2\sigma/d\omega d\Omega\right]^{(\nu,l)} = \frac{1}{k_F}\overline{\sigma}_{\nu N}\ f(\psi)$$

SuperScaling of inclusive electron scattering data

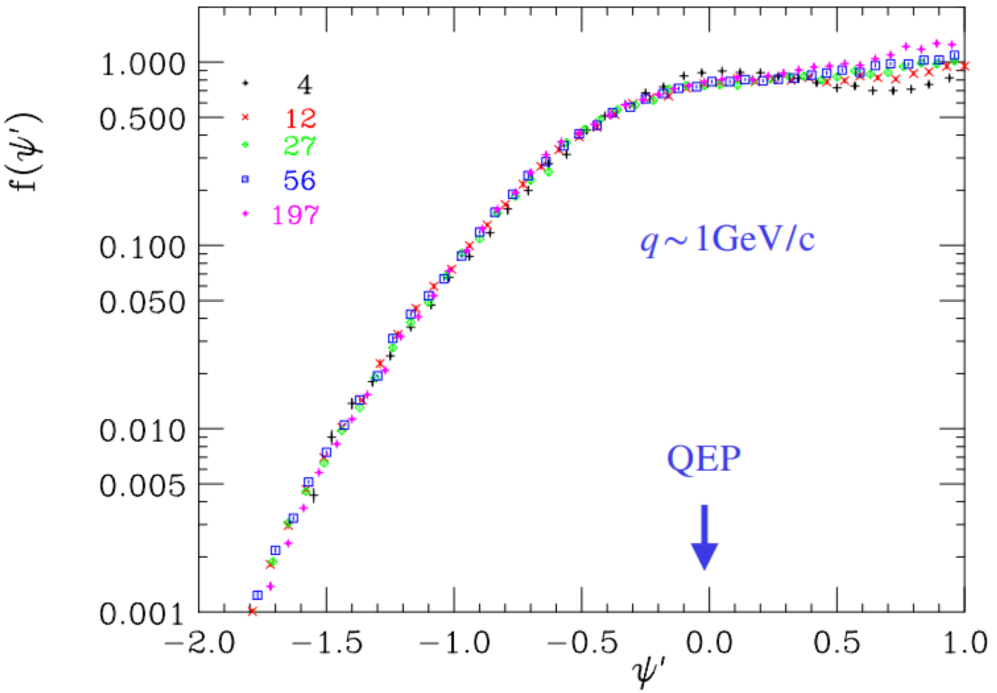
- How well and at which kinematics does SuperScaling work?

  Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990); Donnelly and Sick, PRL82; PRC60 (1999)

I kind: fixed target, varying kinematics



II kind: fixed kinematics, varying target



- SuperScaling is well realized below the Quasi Elastic Peak
- Scaling violations occur beyond the Quasi Elastic Peak

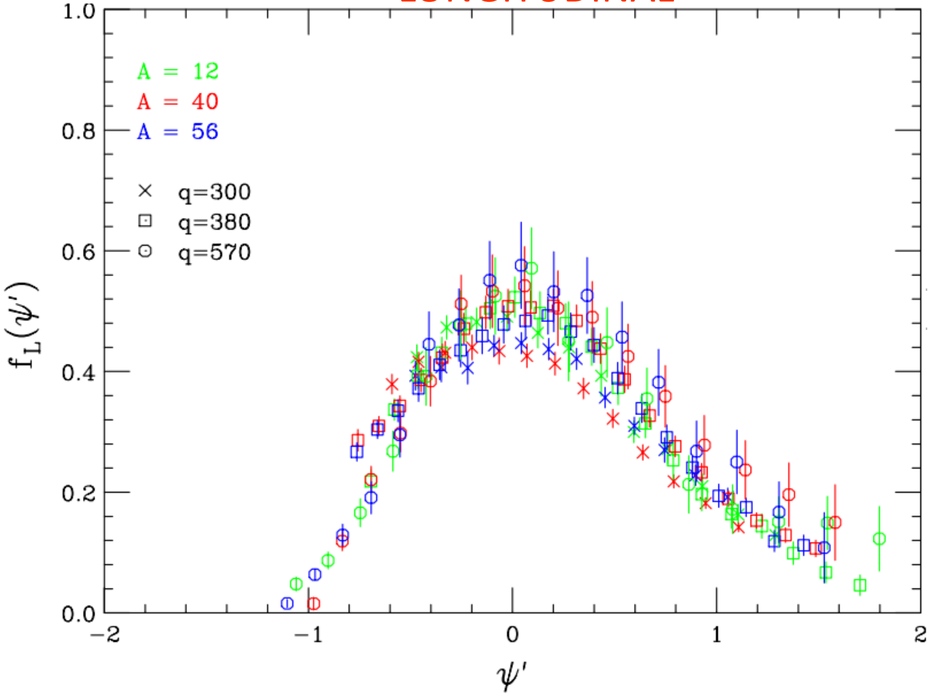
Scaling violations: longitudinal and transverse Superscaling

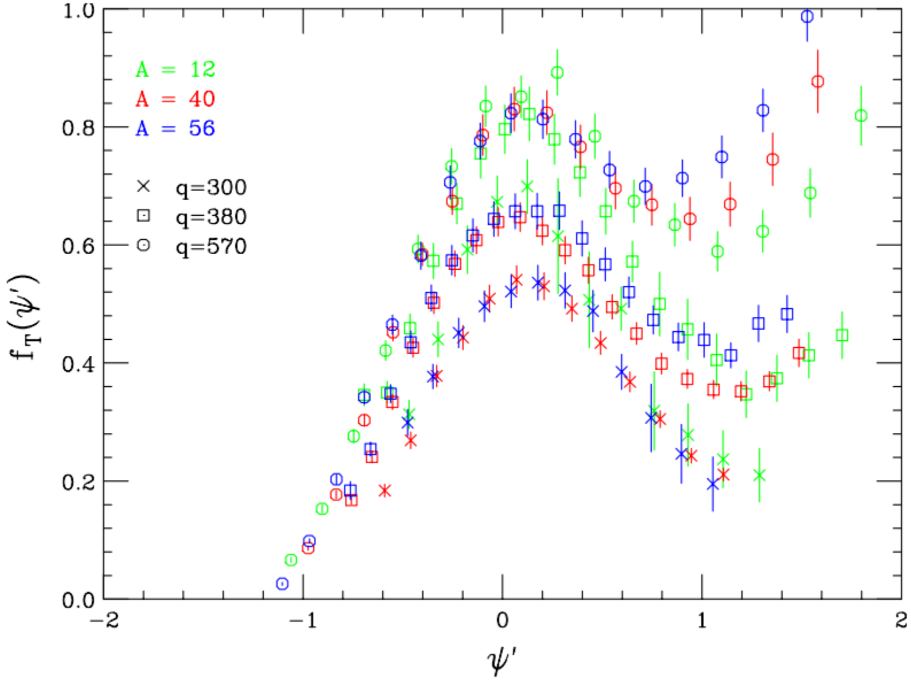
$$\frac{d^2\sigma}{d\theta\, d\omega} = \sigma_M \left\{ \frac{(\omega^2 - q^2)^2}{q^4} R_L(\omega, q) + \left[\tan^2\left(\frac{\theta}{2}\right) - \frac{\omega^2 - q^2}{2q^2} \right] R_T(\omega, q) \right\}$$

LONGITUDINAL

Donnelly et al. PRC 60 '99

TRANSVERSE





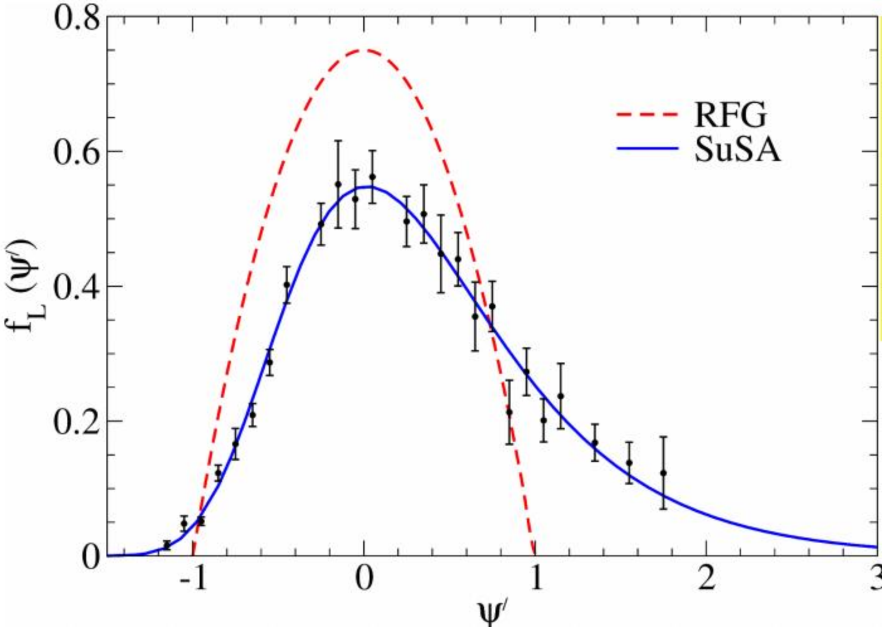
$$f_L(\psi) = k_F \frac{R_L(q,\omega)}{G_L^{e,e'}(q,\omega)}$$

$$f_T(\psi) = k_F \frac{R_T(q,\omega)}{G_T^{e,e'}(q,\omega)}$$

- The **longitudinal** response **scales**
- **Scaling violations** are mainly **transverse** (2p-2h, Δ resonance and other inelastic processes)

The SuSA and SuSAv2 models in the quasielastic region

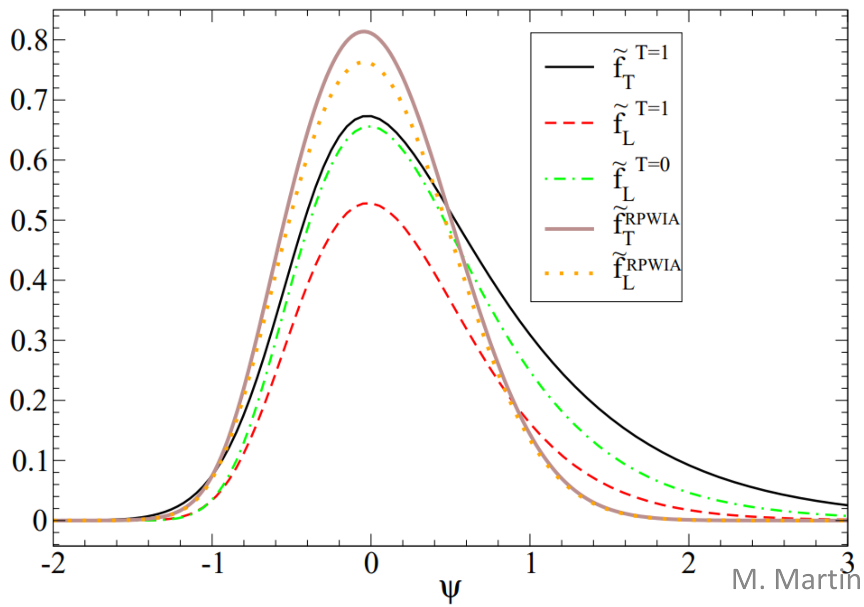
The scaling function(s) are used to describe simultaneously electron and neutrino scattering



SuSA model - phenomenological

J.E. Amaro et al., PRC71 (2005) 015501

- One scaling function extracted from longitudinal inclusive (e,e') data



SuSAv2 model - microscopic

R. Gonzalez-Jimenez et al., PRC90 (2014) 035501

- Based on Relativistic Mean Field calculation
- A set of scaling functions in L,T and isospin channels

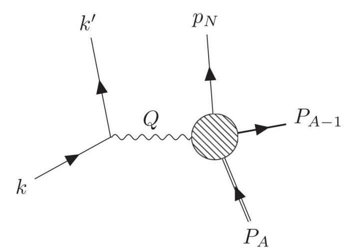
The Spectral Function

- The spectral function $S(E_m, \mathbf{p}_m)$ represents the joint probability of removing a nucleon of given momentum $\mathbf{p}_m$ from the nuclear ground state A leaving the residual nucleus A-1 in a state characterized by missing energy E_m

Missing Energy $E_m = \omega - T_N - T_{A-1}$

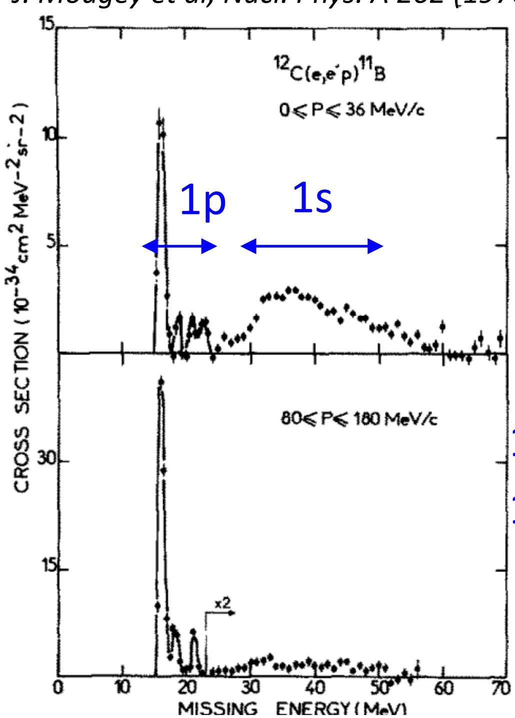
Missing momentum $\boldsymbol{p}_m = \boldsymbol{q} - \boldsymbol{p}_N = \boldsymbol{p}_{A-1}$ recoil momentum

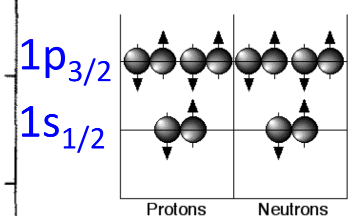
p.s. Often in literature the sign is opposite : $\boldsymbol{p}_m = \boldsymbol{p}_N - \boldsymbol{q} = -\boldsymbol{p}_{A-1}$



$$\frac{d\sigma}{de' d\Omega_e dT' d\Omega_p} = K \left(\frac{d\sigma}{d\Omega_e}\right)_{ep} S(E, \boldsymbol{P})$$

J. Mougey et al, Nucl. Phys. A 262 (1976)





- This approach has been largely used in the electron scattering experiments where the energy and the momentum transferred to the nucleus (ω,q) are measured. In particular it has been used in the **(e,e'p) exclusive experiments** where $\mathbf{p}_m$ and E_m can be selected by fixing the outgoing nucleon kinematics
- Assuming that the interaction occurs on a single nucleon and that the energy and momentum of the outgoing nucleon are not modified by FSI (Plane Wave Impulse Approximation), $\mathbf{p}_m$ and E_m are the impulse and kinetic energy of the struck nucleon inside the nucleus

470 J. MOUGEY *et al.*

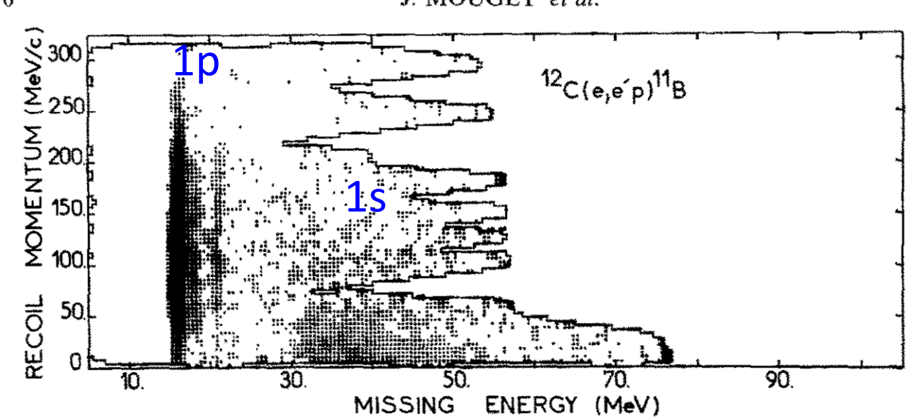
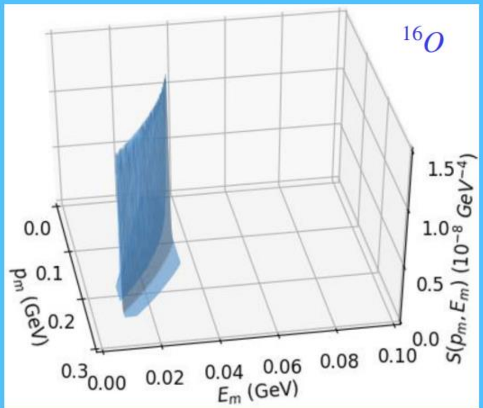


Fig. 8. Cross section $\sigma(E, P)$ for ^{12}C, showing the 1p shell at 17 MeV and the 1s shell around 38 MeV.

Different ^{16}O Theoretical Spectral Functions

RFG

$$S_{RFG}(p_m, E_m) = \theta(p_F - p_m)\,\delta\left(E_m - \sqrt{p_m^2 + m_N^2}\right)$$



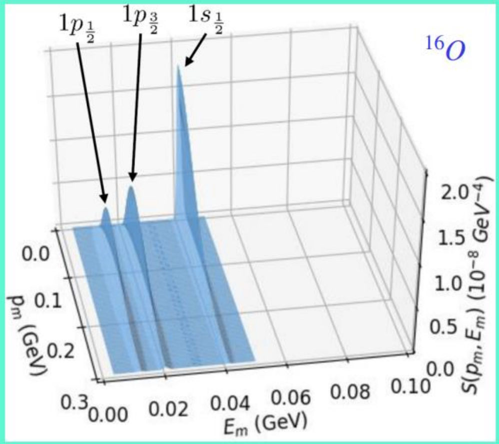
Independent-particle models

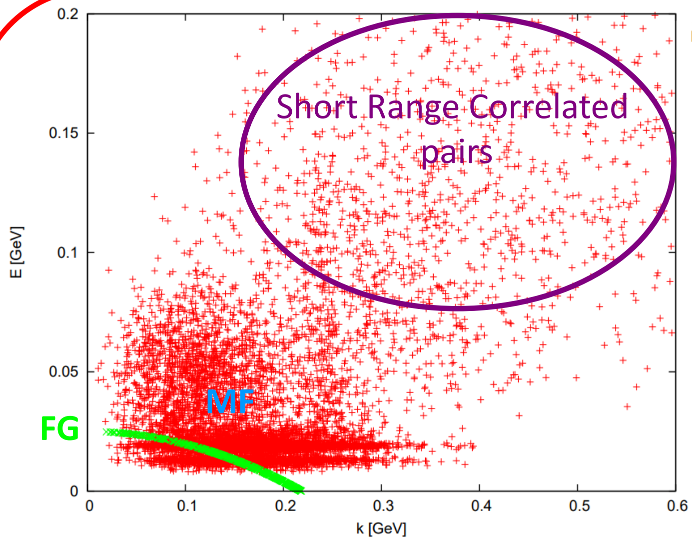
J. M. Franco Patino et al, PRC 102 064626 (2020)

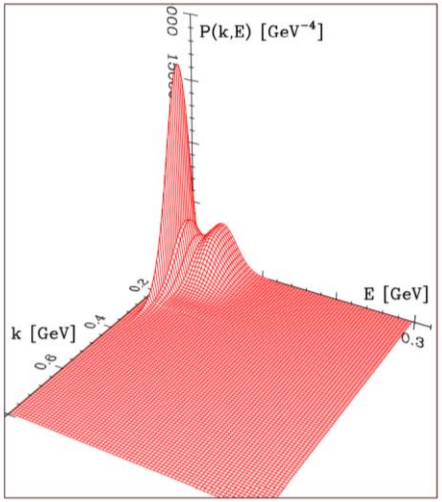
Figures from M. B. Barbaro talk @NUFACT 2021

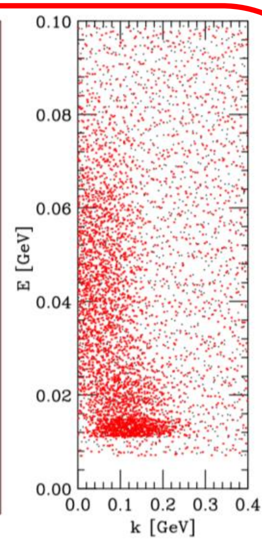
IPSM/RMF

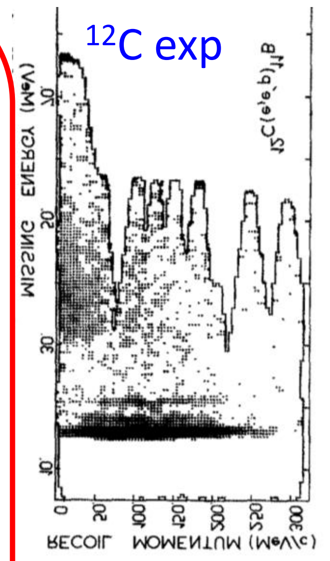
$$S_{IPSM}(p_m, E_m) = \sum_{nlj} (2j+1)\, n_{nlj}(p_m)\, \delta(E_m - E_{nlj})$$











Correlated Basis Functions

$$S_{CBF}(p_m, E_m) = S_{MF}(p_m, E_m) + S_{corr}(p_m, E_m)$$

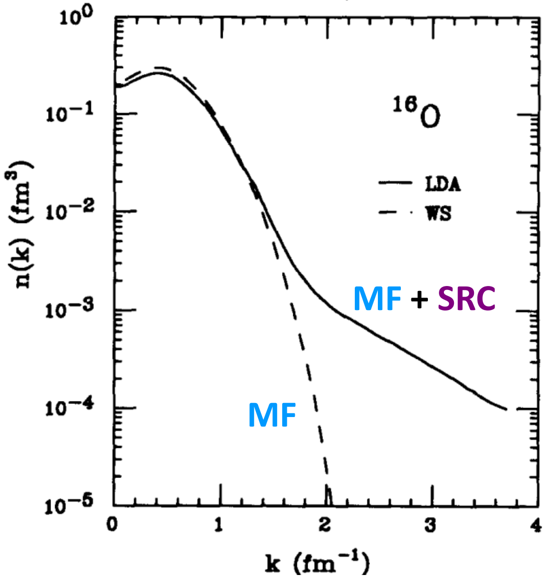
O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989)

Figures from N. Rocco talk @ESNT-CEA workshop 2016

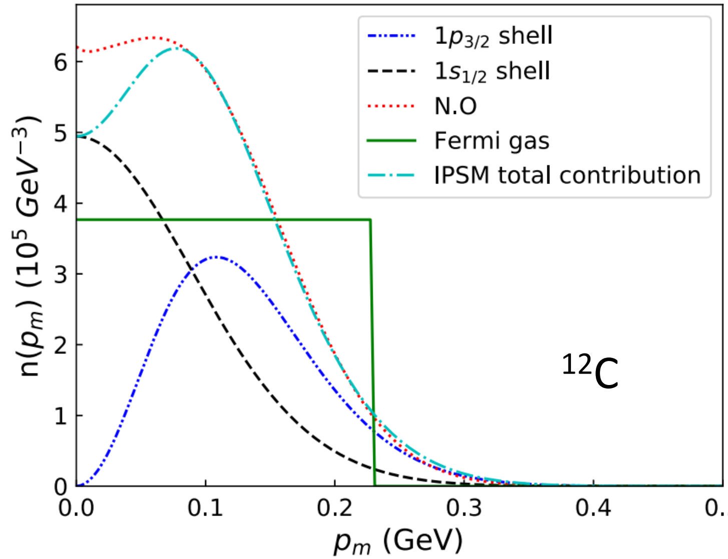
Nucleon momentum distribution

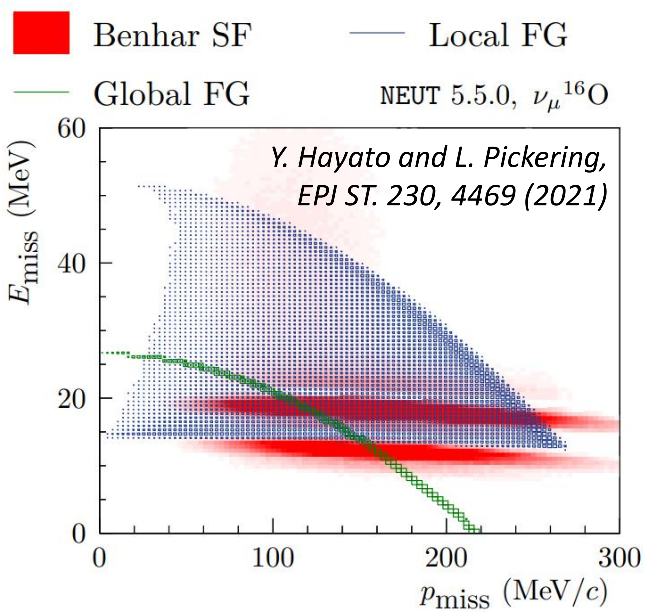
$$n(p_m) = \int_0^\infty dE_m S(p_m, E_m)$$

O. Benhar et al. Nucl.Phys. A 579 (1994)

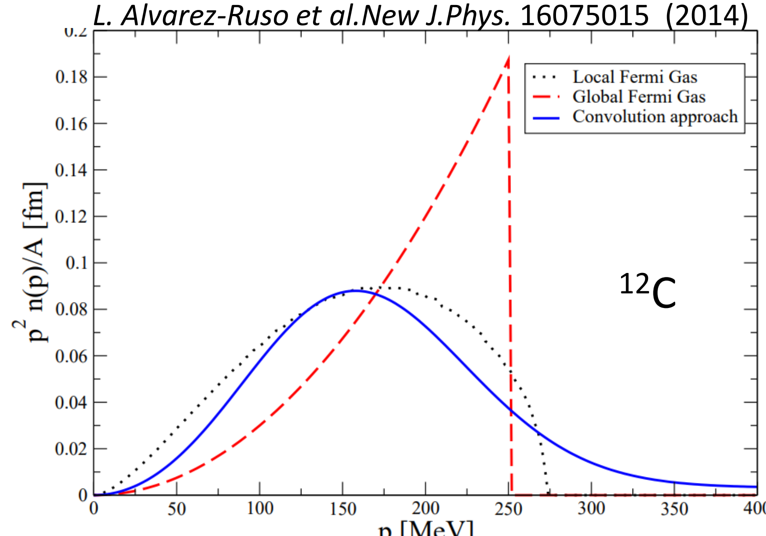


J. M. Franco Patino et al, PRC 102 064626 (2020)



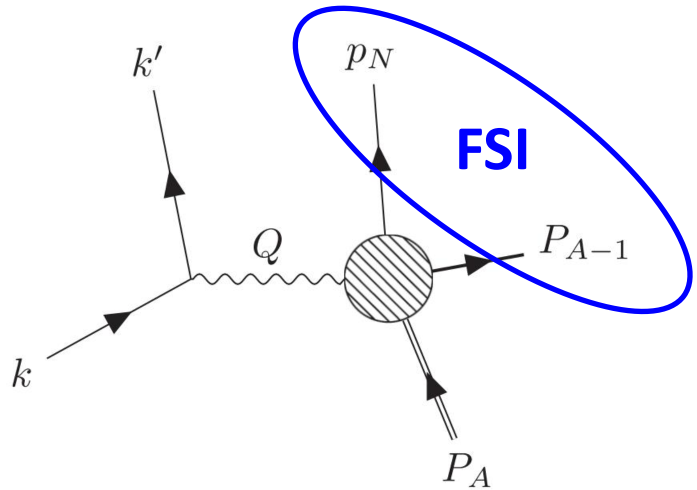


L. Alvarez-Ruso et al.New J.Phys. 16075015 (2014)



Final State Interactions

FSI between the knocked-out particle(s) and the residual nucleus



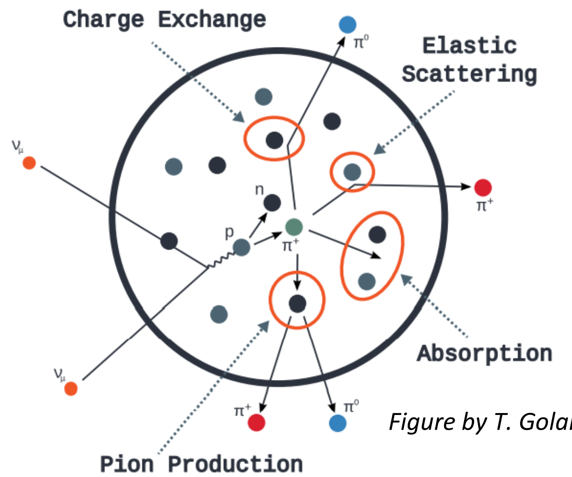


Figure by T. Golan

- FSI describe the propagation of the particles produced at the interaction vertex through the nucleus
- FSI include both elastic and inelastic reactions: elastic scattering with energy change, charge exchange, production of new particles, absorption
- Different interaction vertices can lead to the same final state due to FSI
- The inclusion of FSI effects is extremely important for the description of semi-inclusive data
- Monte Carlo event generators includes different models of intra-nuclear cascades: particles are assumed to be classical and move along a straight line
- FSI between the knocked-out nucleon and the residual nucleus can be theoretically treated using different approaches: Optical Potential, RMF, Energy-Dependent RMF
- Some recent references: *R. Gonzalez-Jimenez et al., PRC 101, 015503 (2020) ; J. Isaacson et al., PRC 103 015502 (2021); A. Nikolakopoulos et al. PRC 105, 054603 (2022); A. Ershova et al., PRD 106 032009 (2022)*

Neutrino-nucleus cross sections

Second Lecture

Results and Perspectives

Charged current neutrino-nucleus cross section (remind)

Lab frame

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \frac{G_F^2 \cos^2\theta_C}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu} W^{\mu\nu}(\mathbf{q},\omega)$$

The charged current inclusive cross section is a linear combination of five contributions

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \sigma_0 \left[L_{00} W^{00} + L_{33} W^{33} + (L_{03}+L_{30}) W^{03} + (L_{11}+L_{22}) W^{11} \pm (L_{12}-L_{21}) W^{12} \right]$$

A simplified expressions particularly useful for illustration

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c}{\pi} |\mathbf{k}'| E_l' \cos^2\frac{\theta}{2} \left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega) \right.$$
$$\left. + 2\left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2\right) R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\,\frac{E_\nu + E_l'}{M_N} \tan^2\frac{\theta}{2}\, G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega) \right]$$

Explicitly appear:

1. The different **kinematic variables** (related to the leptonic tensor)
2. The nucleon Electric, Magnetic, and Axial **form factors** (↔ nucleon properties)
3. The **nuclear response functions** (↔ nuclear dynamics)

The Form Factors

Standard dipole parameterization

Vector (see slide 39)

$$Q^2 = q^2 - \omega^2$$

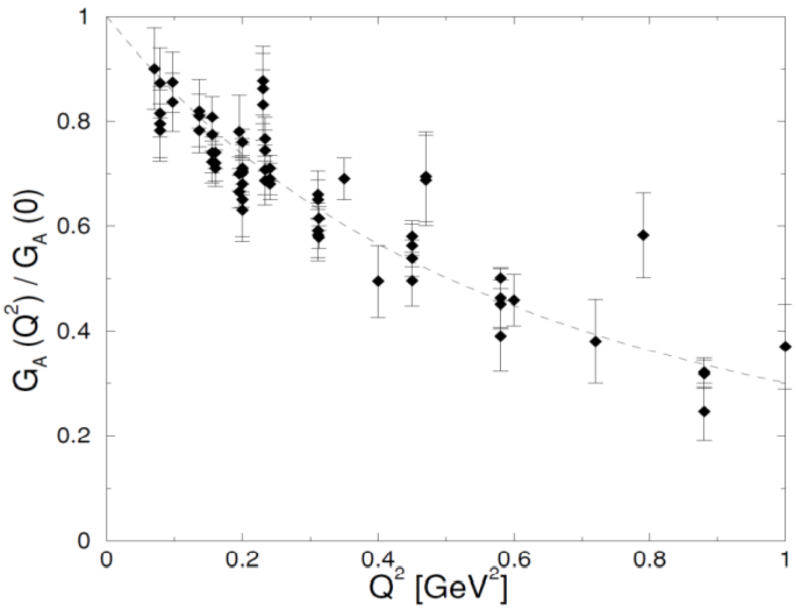
$$G_E(Q^2) = G_M(Q^2)/(\mu_p - \mu_n) = (1 + Q^2 / M_V^2)^{-2}$$

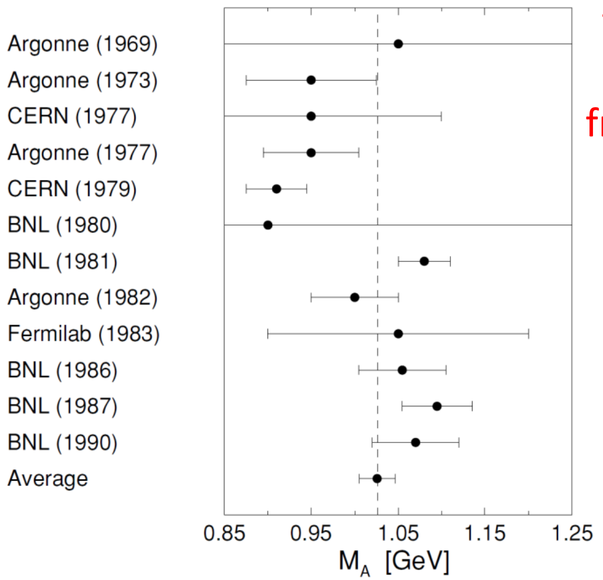
Axial

$$G_A(Q^2) = g_A (1 + Q^2 / M_A^2)^{-2}$$

$g_A = 1.26$ from neutron β decay

$$M_A = (1.026 \pm 0.021)\ GeV/c^2$$





from ν-deuterium CCQE
and
from π electroproduction

V. . Bernard, J.Phys. G28 (2002) R1-R35

CCQE, CCQE-like and CC0π

MiniBooNE CC Quasielastic cross section on Carbon and the M_A puzzle

First Measurement of Muon Neutrino Charged Current Quasielastic (CCQE) Double Differential Cross Section

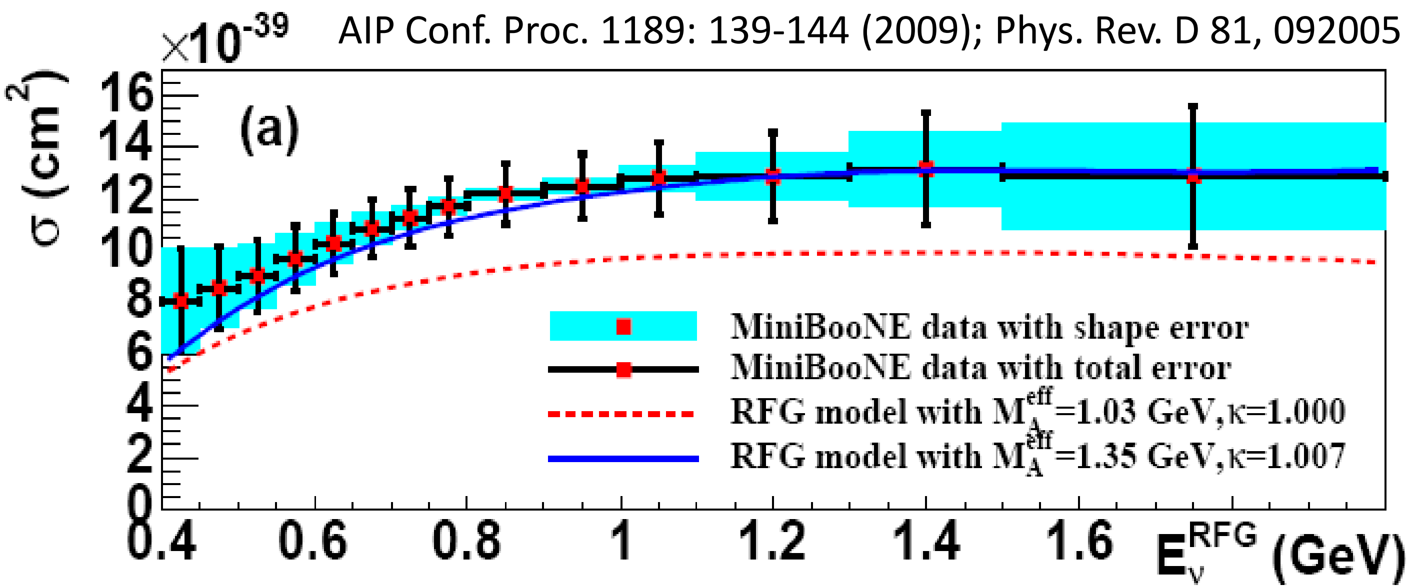
Cite as: AIP Conference Proceedings **1189**, 139 (2009); https://doi.org/10.1063/1.3274144
Published Online: 02 December 2009

Teppei Katori and MiniBooNE collaboration

PHYSICAL REVIEW D **81**, 092005 (2010)

First measurement of the muon neutrino charged current quasielastic double differential cross section

AIP Conf. Proc. 1189: 139-144 (2009); Phys. Rev. D 81, 092005 (2010)



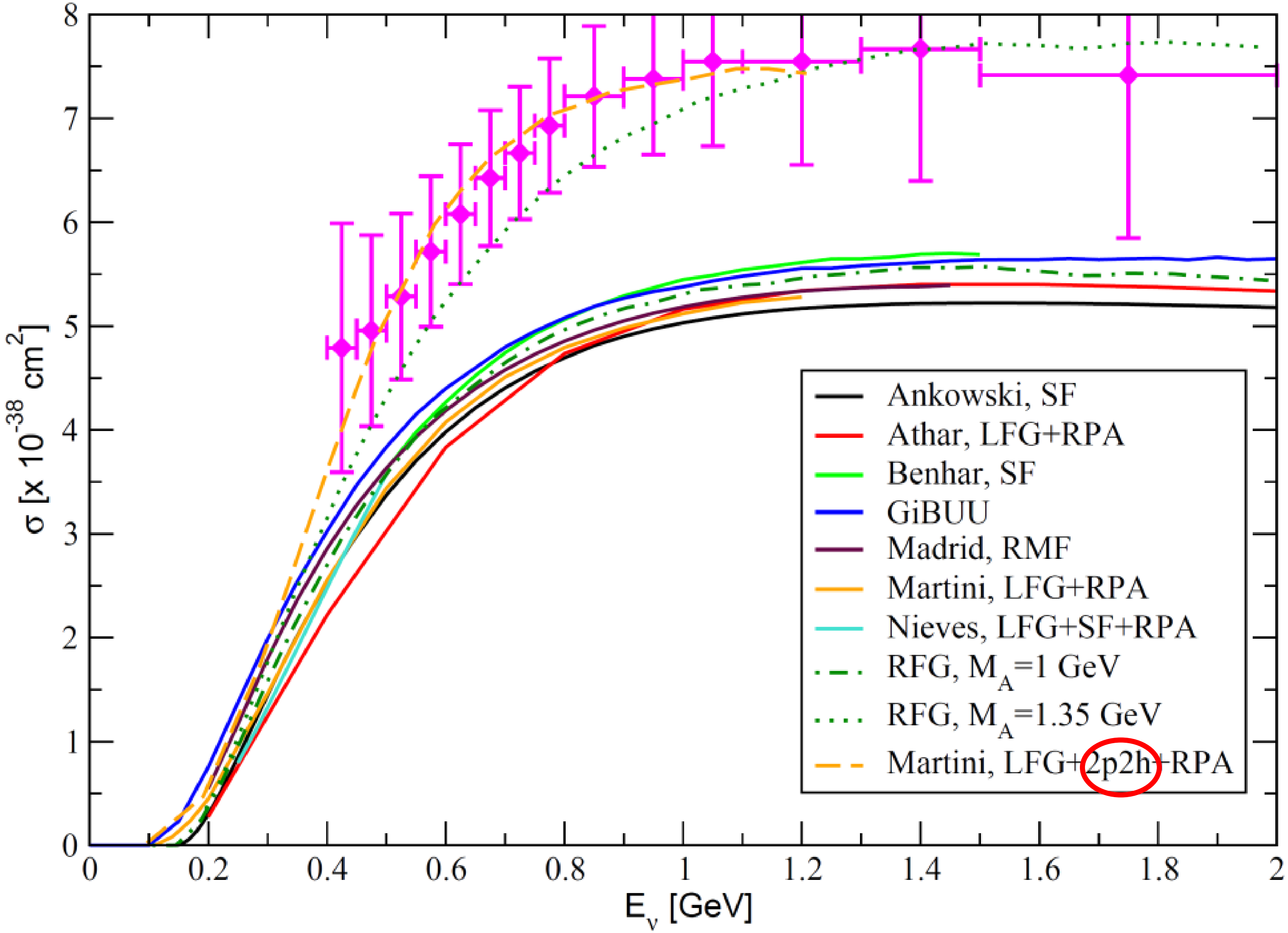
Comparison with a prediction based on **RFG** using **M_A=1.03 GeV** (standard value) reveals a discrepancy

In the Relativistic Fermi Gas (RFG) model an axial mass of **1.35 GeV** is needed to account for data

puzzle??

Comparison of different theoretical models for Quasielastic

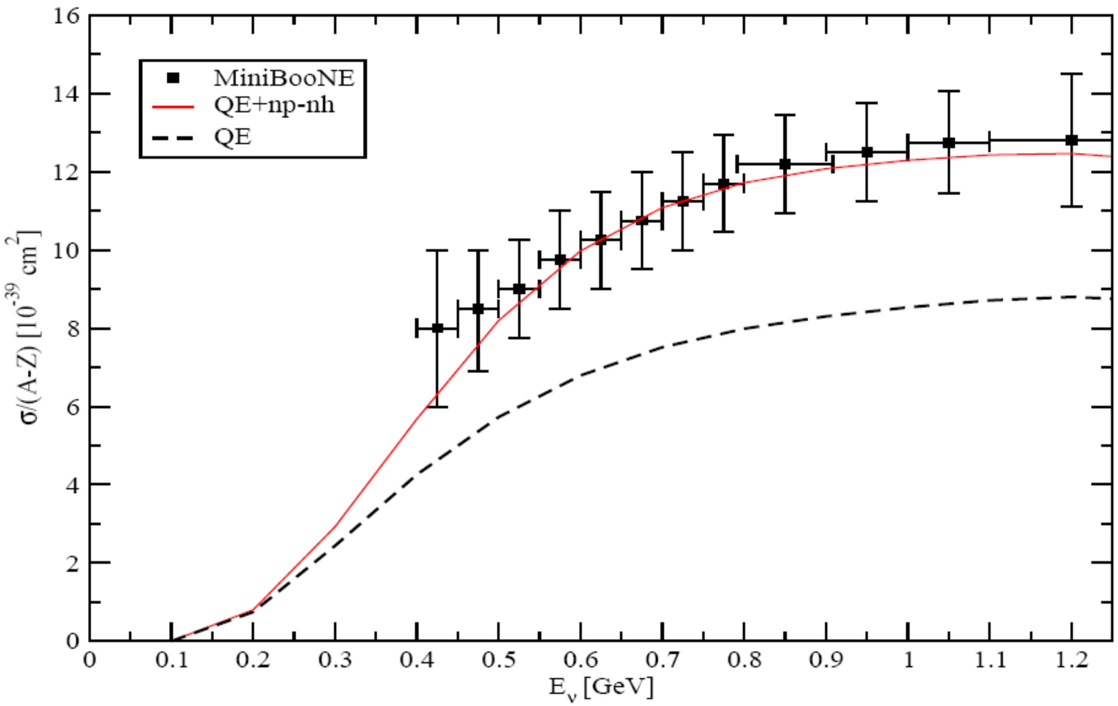
L. Alvarez-Ruso , arXiv:1012.3871 (Neutrino 2010)

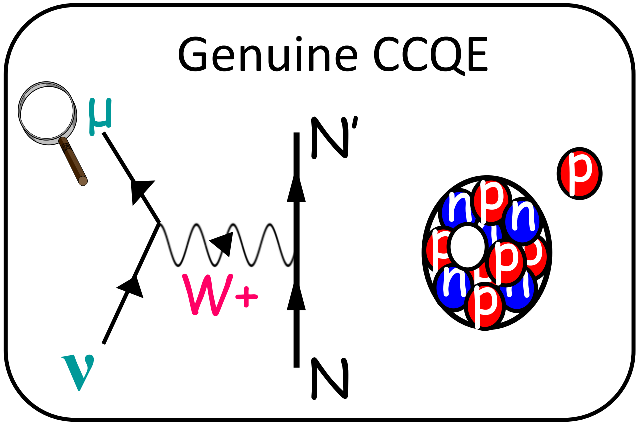


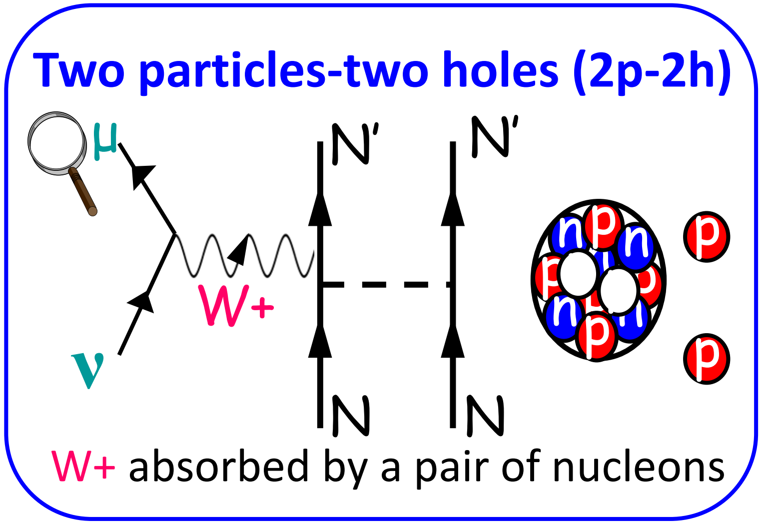
puzzle??

An explanation of this puzzle

Inclusion of the multinucleon emission channel
(np-nh = 2p-2h + 3p-3h)







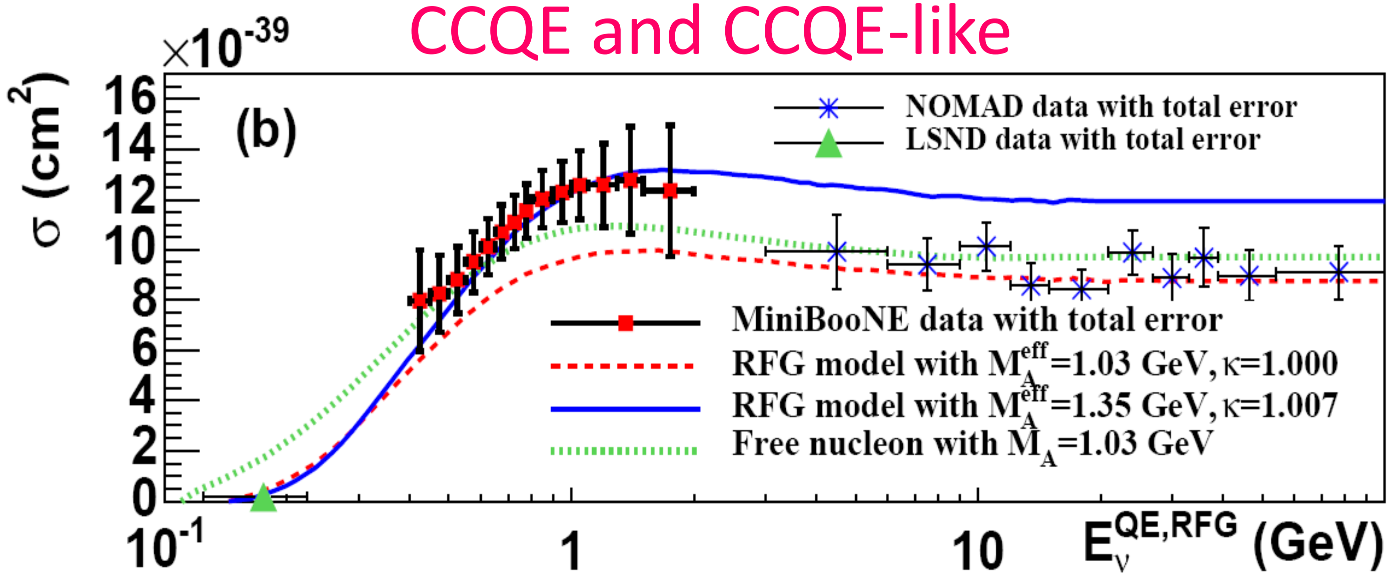
CCQE-like = Genuine CCQE + np-nh

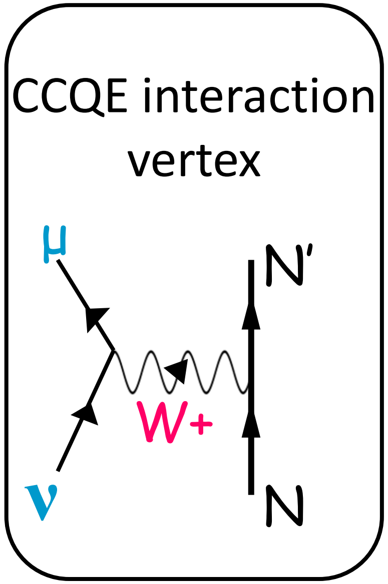
M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)

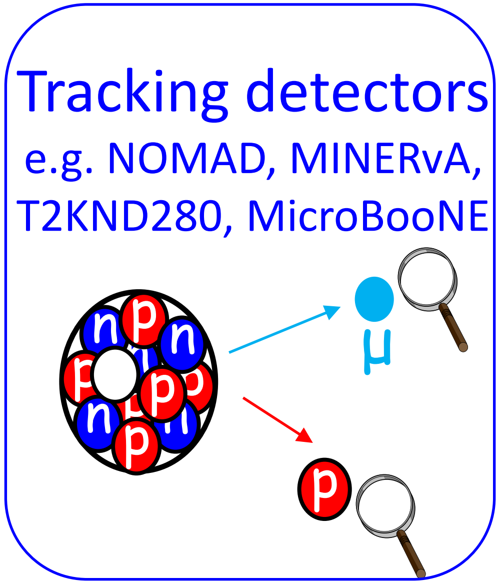
Agreement with MiniBooNE without increasing M_A

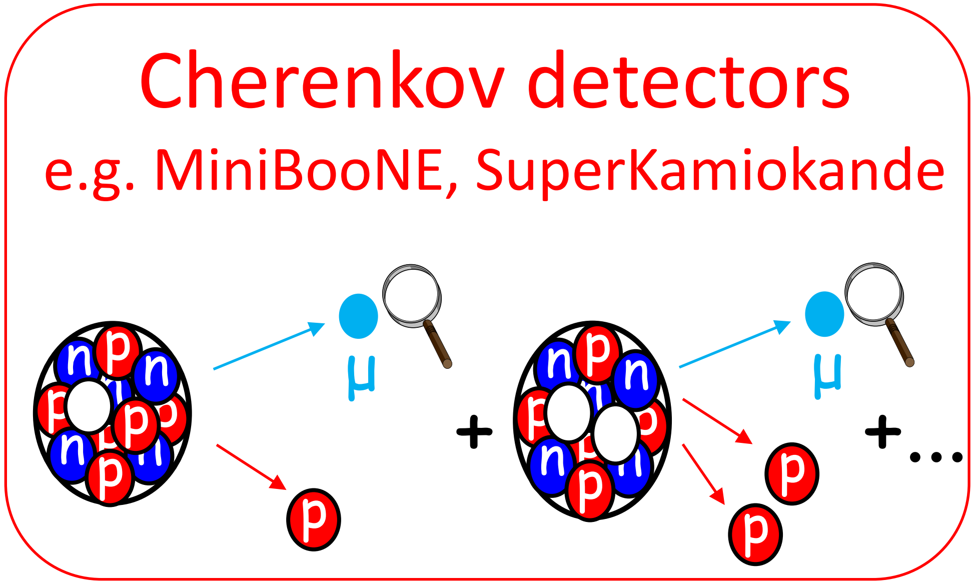
⇨ MiniBooNE measured CCQE-like, not genuine CCQE

CCQE and CCQE-like









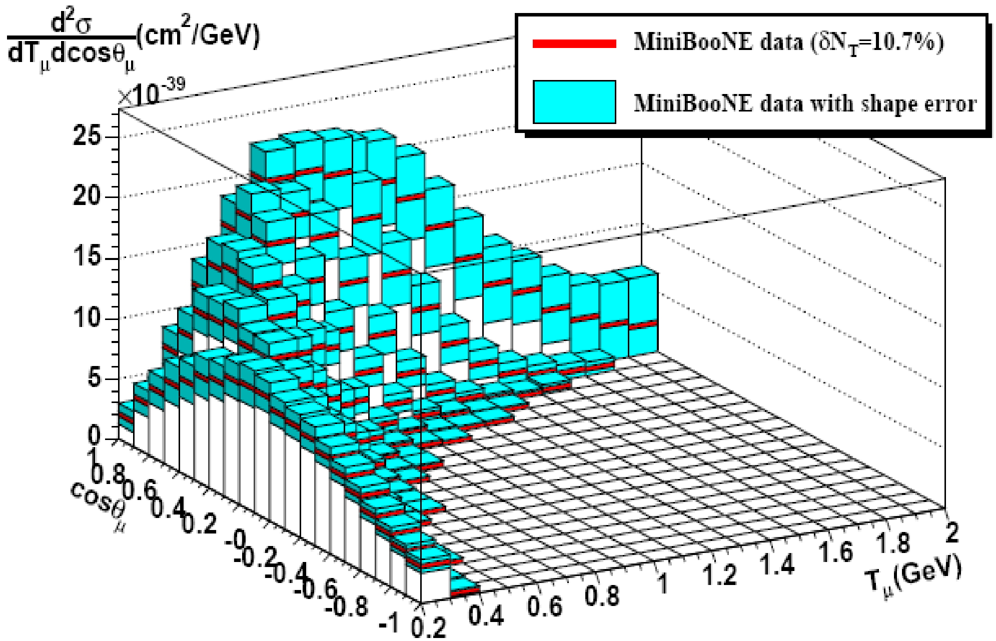
- Cherenkov detectors measure “CCQE-like” which includes np-nh contributions
- After MiniBooNE: CCQE-like = CCQE + np-nh
- Very recently [e.g. MicroBooNE PRL 125, 201803, 2020] “CCQE-like” has been used with another meaning
- After MiniBooNE it has become more popular to present the data in terms of final state particles

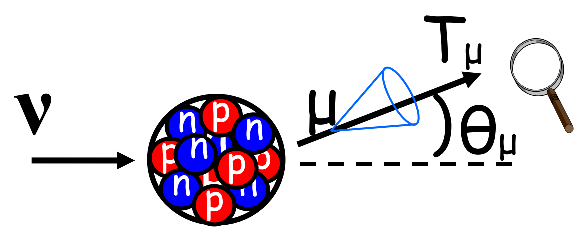
Flux-integrated double differential cross section

$$\left(\frac{d^2\sigma}{dT_l \cos\theta}\right)_i = \frac{\sum_j U_{ij}(d_j - b_j)}{\Phi \cdot T \cdot \epsilon_i \cdot (\Delta T_l,\, \Delta\cos\theta)_i} \qquad \text{(see slide 19)}$$

PHYSICAL REVIEW D **81,** 092005 (2010)

First measurement of the muon neutrino charged current quasielastic double differential cross section



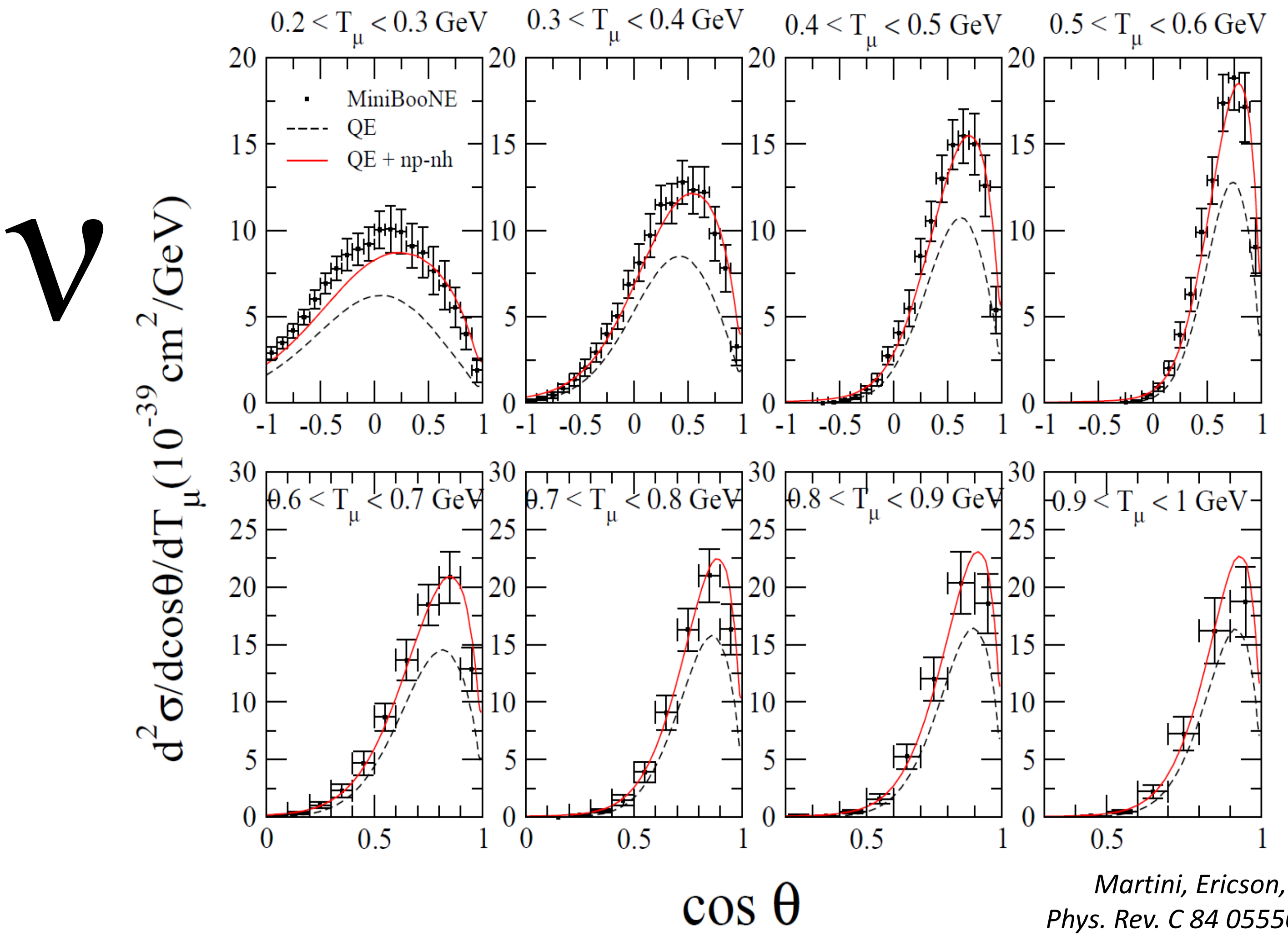


- Function of two measured variables
- Less model dependent than $\sigma(E_\nu)$: free from the neutrino energy reconstruction problem (see later)
- Flux dependent

Flux-integrated differential cross section is where theorists and experimentalists meet for ν interaction

MiniBooNE CCQE-like flux-integrated double differential cross section

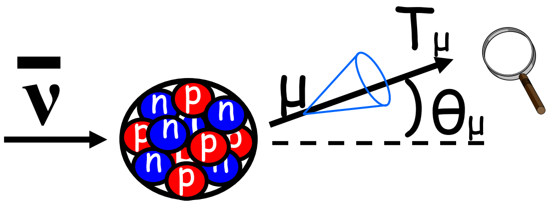
$$\frac{d^2\sigma}{dT_l\ d\cos\theta} = \frac{1}{\int \Phi(E_\nu)\ dE_\nu} \int dE_\nu \left[\frac{d^2\sigma}{d\omega\ d\cos\theta}\right]_{\omega=E_\nu-E_l} \Phi(E_\nu)$$

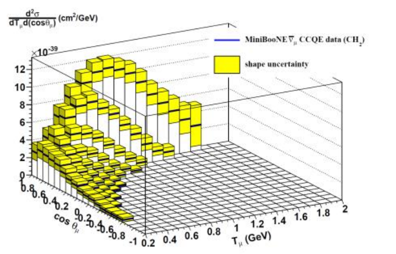


Martini, Ericson, Chanfray,
Phys. Rev. C 84 055502 (2011)

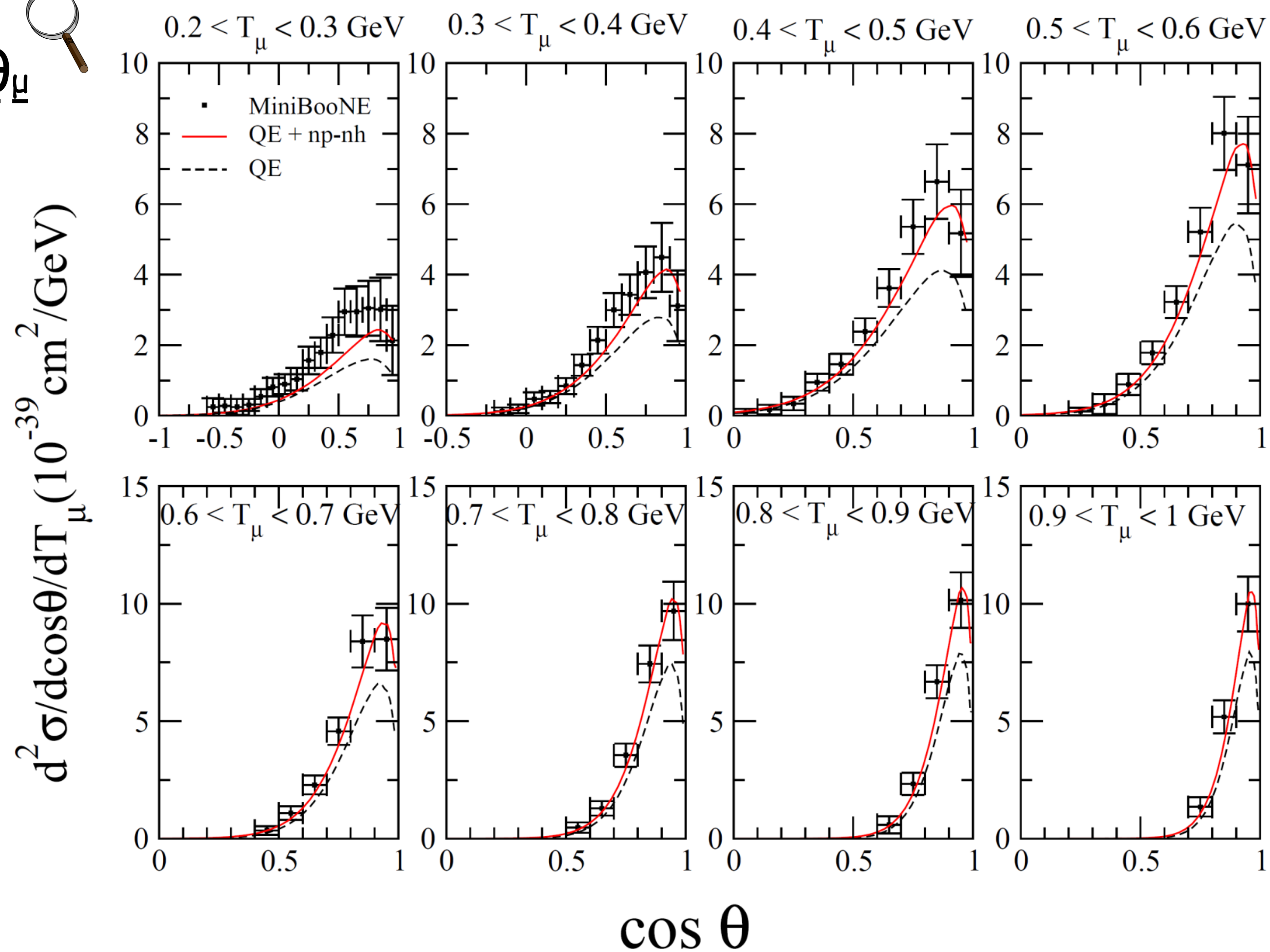
- Good agreement with data once multinucleon contributions are included
- Similar conclusions obtained by different theoretical calculations (see later)

MiniBooNE CCQE-like flux-integrated double differential cross section





MiniBooNE, Phys. Rev. D 88 032001 (2013)



Martini, Ericson, Phys. Rev. C 87 065501 (2013)

Similar conclusion also for the MiniBooNE CCQE-like antineutrino cross sections

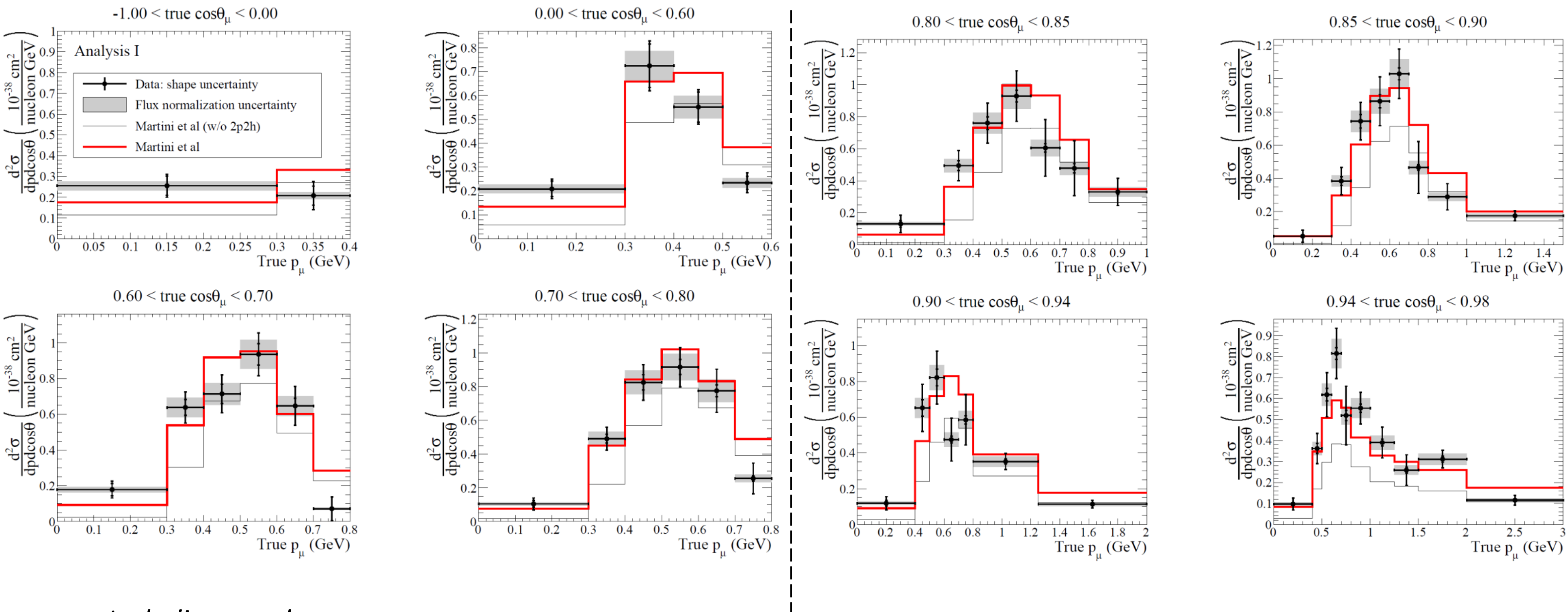
The **CC0π** measurement

After MiniBooNE, it has become more popular to present the data in terms of final state particles

CC0π = CCQE-like without subtraction of π absorption background (CC0π ≥ CCQE-like)

PHYSICAL REVIEW D **93,** 112012 (2016)

Measurement of double-differential muon neutrino charged-current interactions on C_8H_8 without pions in the final state using the T2K off-axis beam



Including np-nh

Without np-nh

Better agreement including np-nh

The **CC0π** measurement

After MiniBooNE, it has become more popular to present the data in terms of final state particles

CC0π = CCQE-like without subtraction of π absorption background

PHYSICAL REVIEW D **93,** 112012 (2016)

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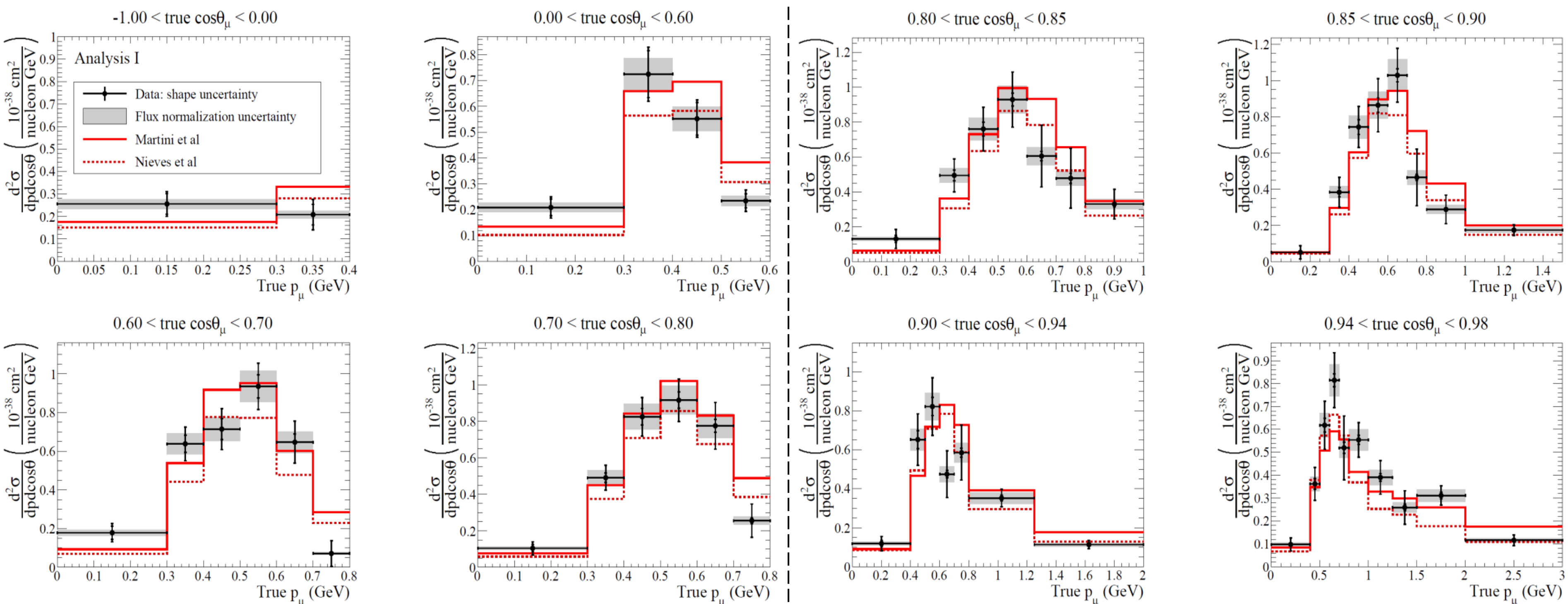


Martini et al.

Nieves et al.

- Two theoretical models including np-nh are compatible with data
- Differences between models' predictions

The T2K **CC0π** data and the Monte Carlo predictions

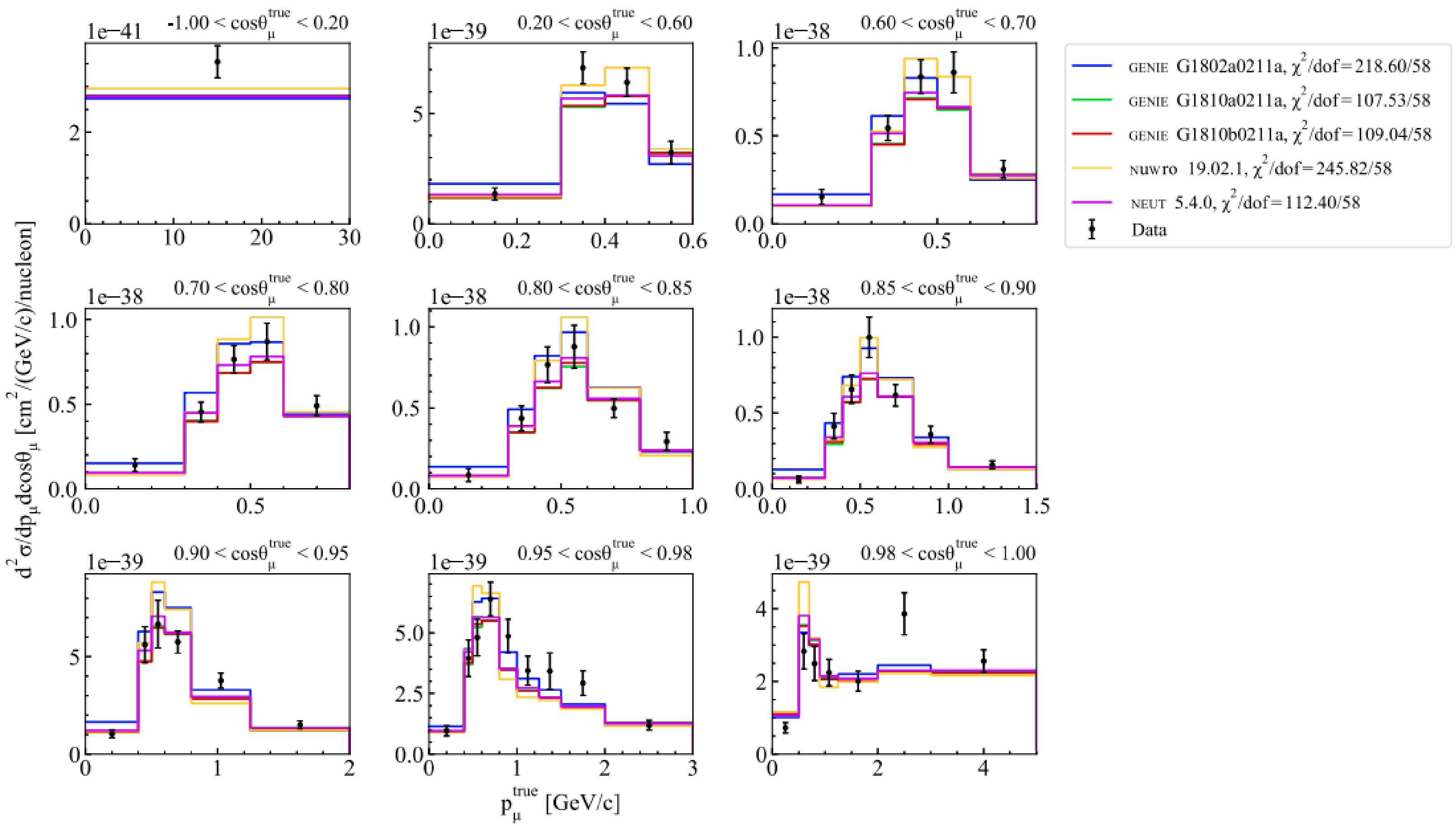


FIG. 10. Measured T2K ν_μ CC-0π double-differential cross sections on hydrocarbon in bins of true muon kinematics. The results are compared to GENIE v3 G18_02a (blue), G18_10a (green), and G18_10b (red), NuWro 19.02.1 (orange), and NEUT 5.4.0 (violet). The last bin in momentum is not displayed for readability.

The multinucleon emission channel (or np-nh, or 2p-2h)

- A lot of interest in these last 13 years (starting from the explanation of MiniBooNE CCQE-like)

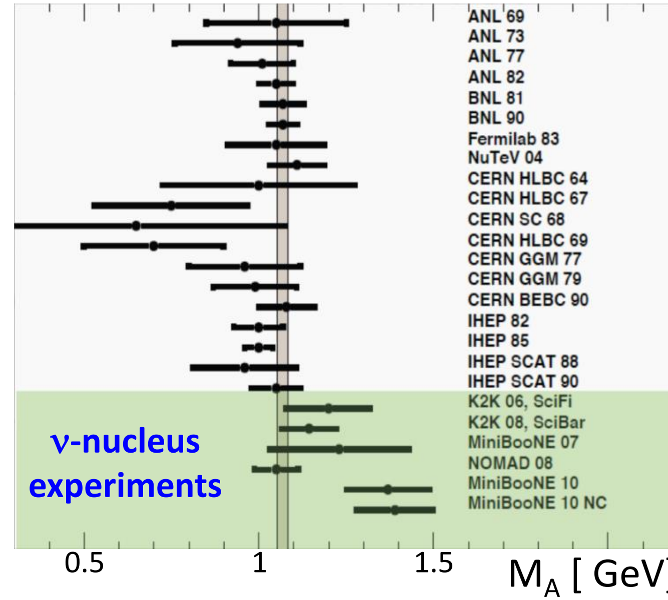
- Explanation of the axial mass puzzle

- Before MiniBooNE it was not included in the generators used for the analyses of ν cross sections and oscillations experiments

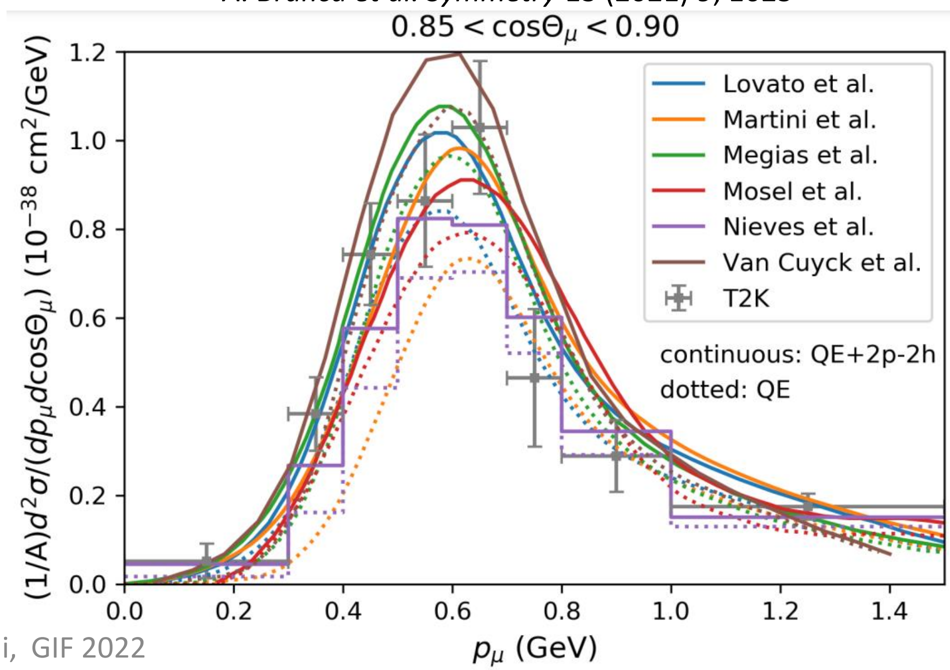
- The effort to include this np-nh channel in several Monte Carlo is still in progress

- Several theoretical calculations agree on its crucial role but there are differences on the results obtained for this channel

- One of the most important source of the cross section uncertainties (systematic errors in oscillation experiments)



A. Branca et al. Symmetry 13 (2021) 9, 1625

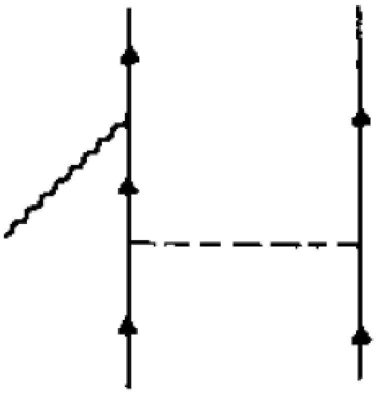


Some theoretical details on 2p-2h

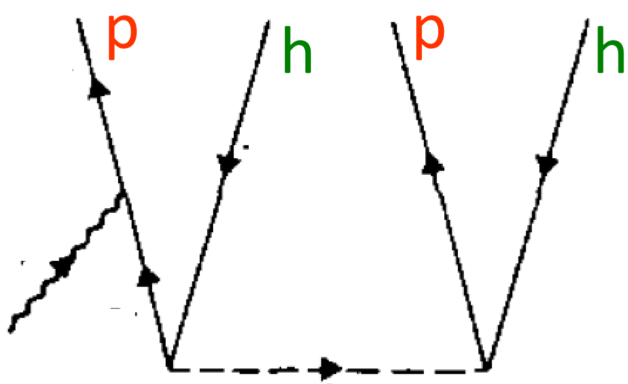
Two particle-two hole sector (2p-2h)

Three equivalent representations of the same process

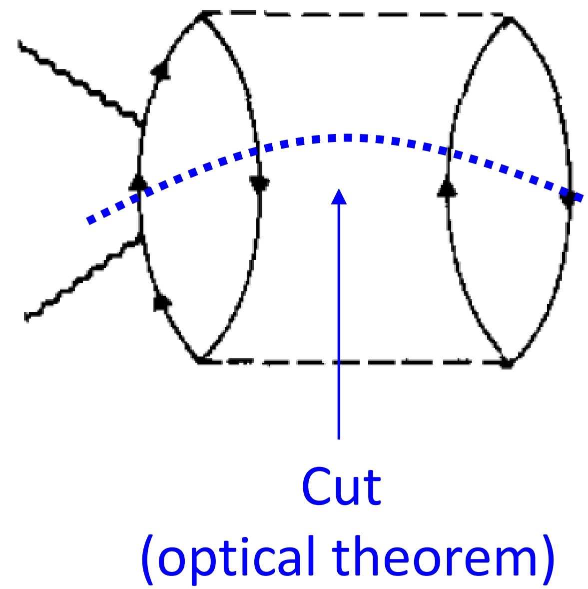
2 body current



2p-2h matrix element



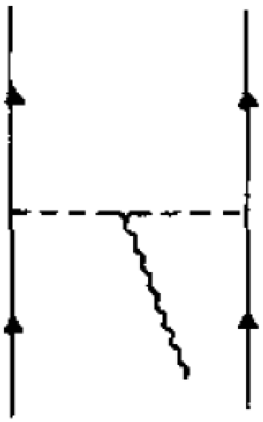
2p-2h response



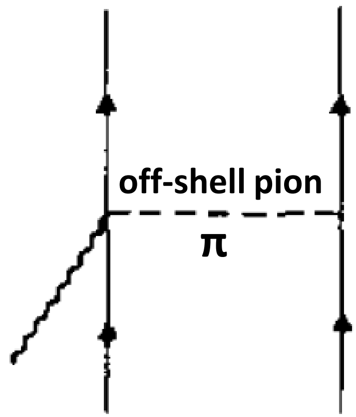
Final state: two particles-two holes

Diagrams for 2 body currents

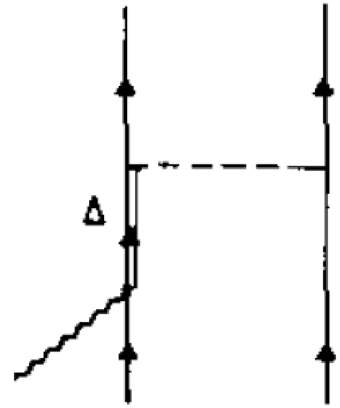
Meson Exchange Currents (MEC) J^{MEC}



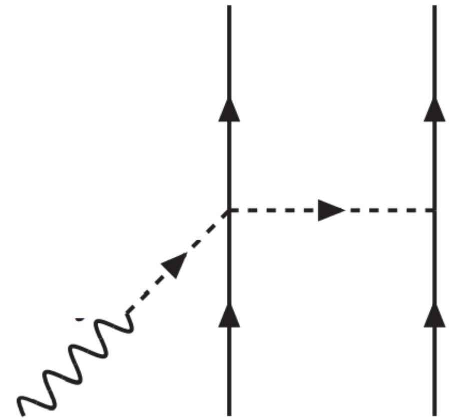
Pion in flight



Seagull or Contact

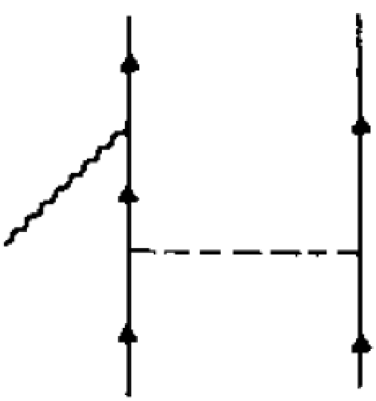


Delta



Pion pole (purely axial)

Nucleon-Nucleon Correlations (SRC) J^{corr}



- An additional two-body current to be included in the framework of independent particle models such as LFG or Hartree-Fock.
- Absent in the approaches which start from the description of the nucleus in terms of correlated wave functions (such as CBF spectral function or GFMC) since the matrix elements of the one body current already includes this contribution.
- There is a risk of a double counting of SRC in the Monte Carlo if different contributions to the neutrino cross sections are taken from different models.

Some two-body currents

Electromagnetic

- Seagull or contact:

$$j_{\rm s}^{\mu}(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_1,\mathbf{p}_2) = \frac{f^2}{m_\pi^2}\,\mathrm{i}\epsilon_{3ab}\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1)\frac{F_1^{\rm V}}{K_1^2-m_\pi^2}\,\overline{u}(\mathbf{p}_2')\tau_b\gamma_5\gamma^\mu u(\mathbf{p}_2) + (1\leftrightarrow 2)\,.$$

- Pion-in-flight:

$$j_{\rm p}^{\mu}(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_1,\mathbf{p}_2) = \frac{f^2}{m_\pi^2}\,\mathrm{i}\epsilon_{3ab}\frac{F_\pi(K_1-K_2)^\mu}{(K_1^2-m_\pi^2)(K_2^2-m_\pi^2)}\,\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1)\overline{u}(\mathbf{p}_2')\tau_b\gamma_5 \not{K}_2 u(\mathbf{p}_2)\,.$$

- Correlation:

$$j_{\rm cor}^{\mu}(\mathbf{p}_1',\mathbf{p}_2',\mathbf{p}_1,\mathbf{p}_2) = \frac{f^2}{m_\pi^2}\,\overline{u}(\mathbf{p}_1')\tau_a\gamma_5 \not{K}_1 u(\mathbf{p}_1)\frac{1}{K_1^2-m_\pi^2}\overline{u}(\mathbf{p}_2')[\tau_a\gamma_5 \not{K}_1 S_{\rm F}(P_2+Q)\Gamma^\mu(Q)$$

$$+\Gamma^\mu(Q)S_{\rm F}(P_2'-Q)\tau_a\gamma_5\not{K}_1]u(\mathbf{p}_2) + (1\leftrightarrow 2)\,.$$

Amaro et al. Phys.Rev.C 82 044601 (2010)

Weak

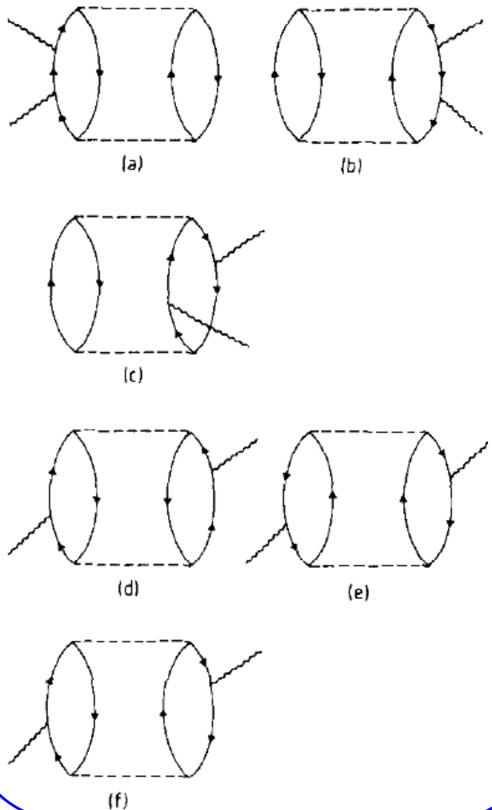
- CC Seagull

$$j_s^\mu(\mathbf{p}_1',\mathbf{p}_2',\mathbf{h}_1,\mathbf{h}_2) = (\tau_0\otimes\tau_{+1}-\tau_{+1}\otimes\tau_0)\frac{f}{m_\pi}\frac{1}{\sqrt{2}f_\pi}\overline{u}(\mathbf{p}_1')\gamma_5\ \not{K}_1 u(\mathbf{h}_1)\ \frac{\overline{u}(\mathbf{p}_2')\left[g_A F_1^V(Q^2)\gamma_5\gamma^\mu + F_\rho(K_2^2)\gamma^\mu\right]u(\mathbf{h}_2)}{K_1^2-m_\pi^2} - (1\leftrightarrow 2)$$

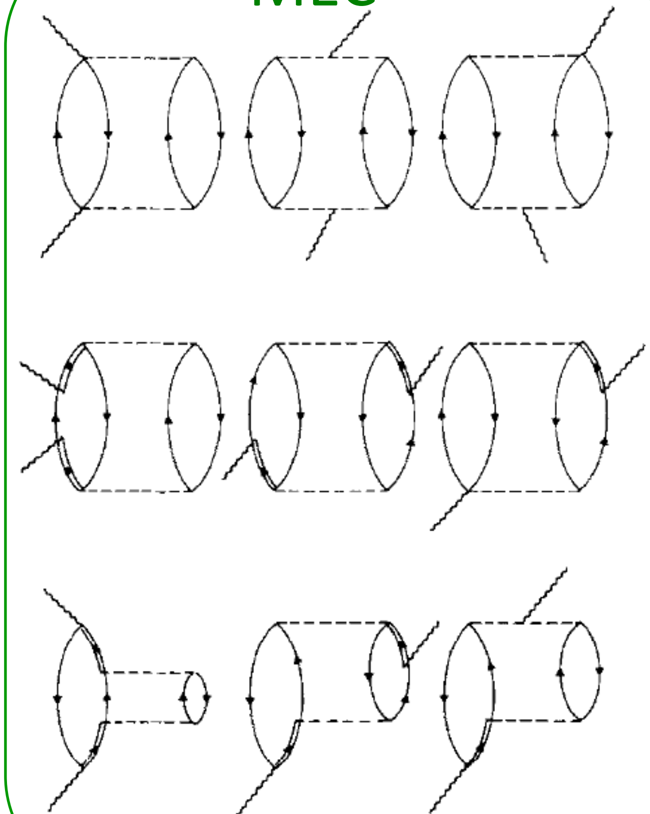
Ruiz-Simo et al. Phys.Rev.D 90 033012 (2014); J.Phys.G 44 065105 (2017)

Some diagrams for 2p-2h responses

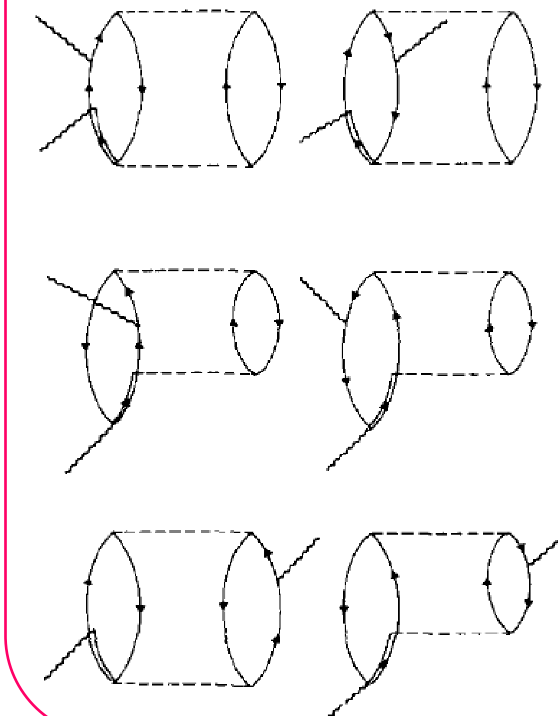
NN correlations



MEC

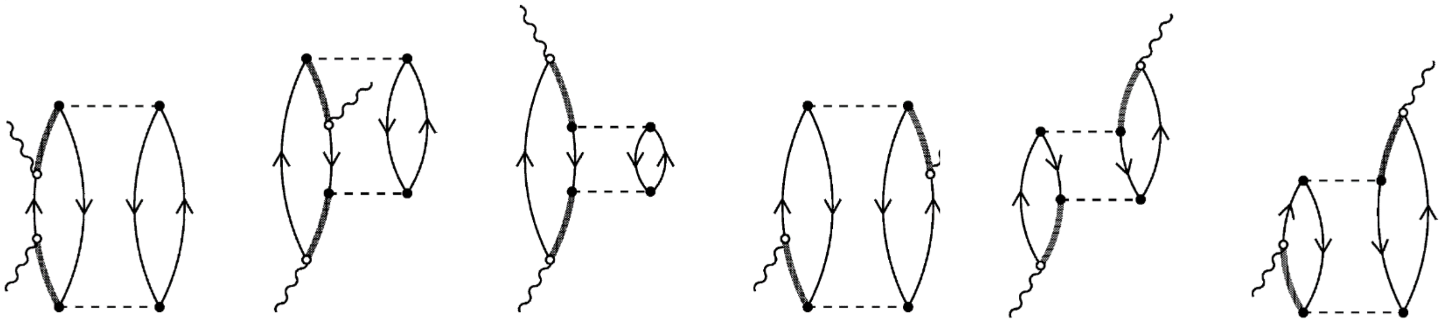


NN correlation-MEC interference

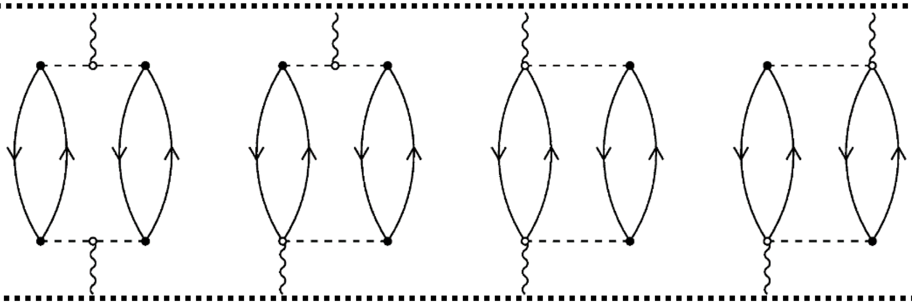


Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

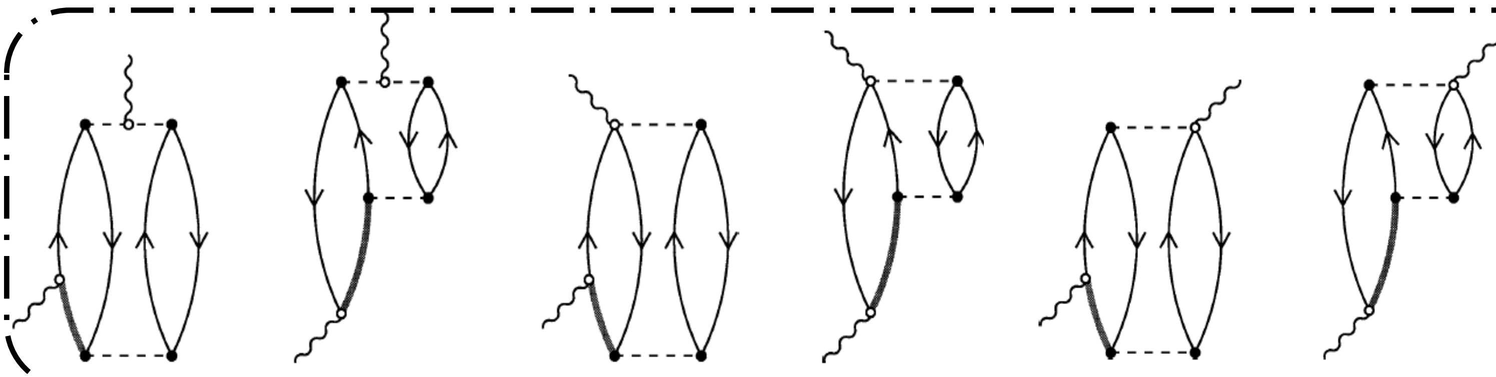
MEC contributions



Δ-MEC



Pionic

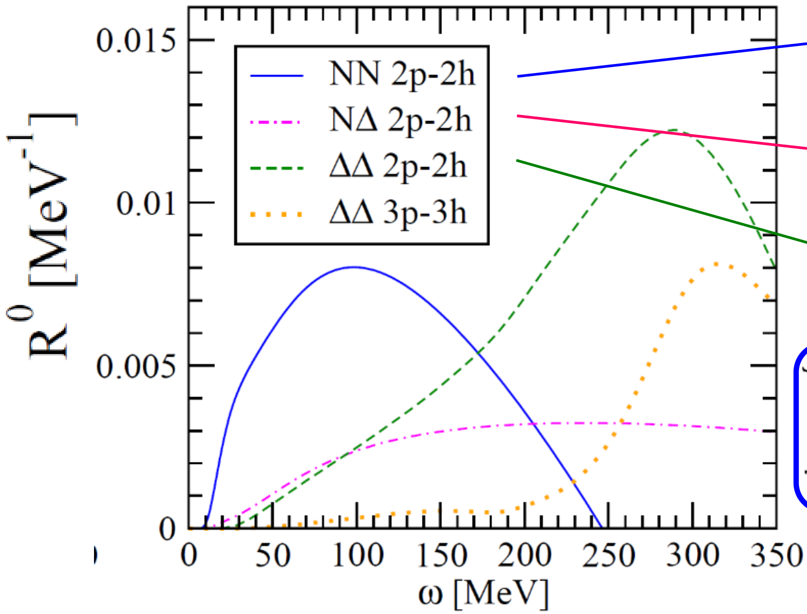


π-Δ interference

De Pace, Nardi, Alberico, Donnelly, Molinari, NPA741 (2004)

Separation of np-nh contributions in the nuclear responses

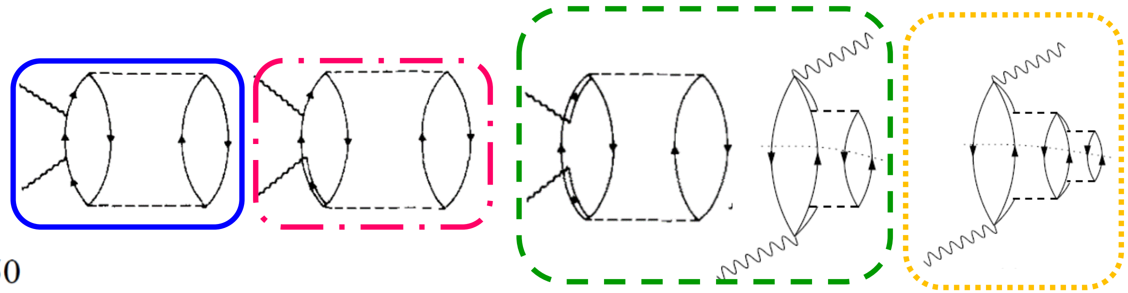
M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



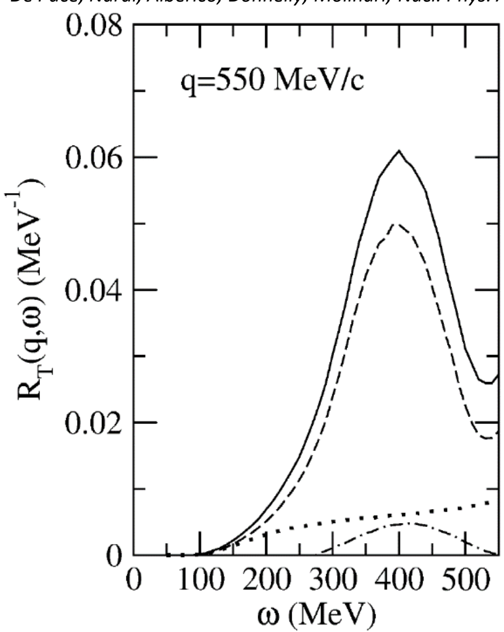
also called NN SRC; part of 1-body current contribution in correlated nuclear wave functions approaches, like SF or GFMC

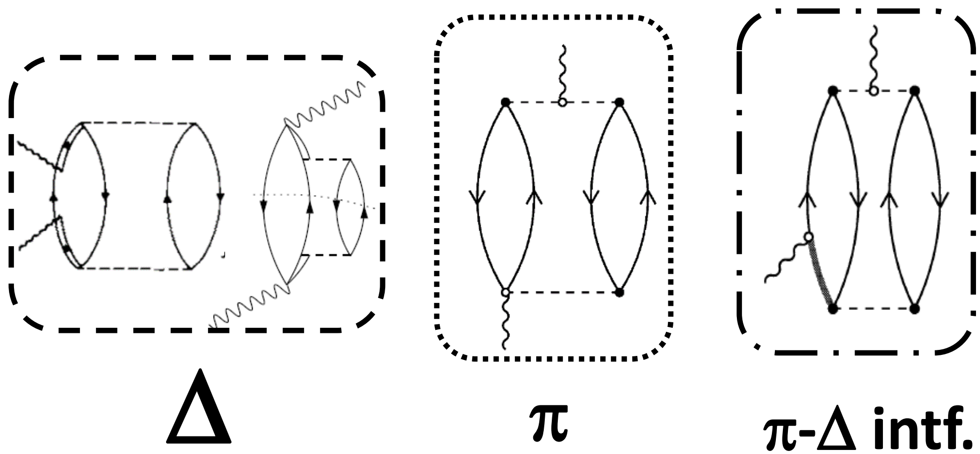
NΔ interference, also called NN correlation-ΔMEC interference or 1-body—2-body interference

Δ mediated MEC



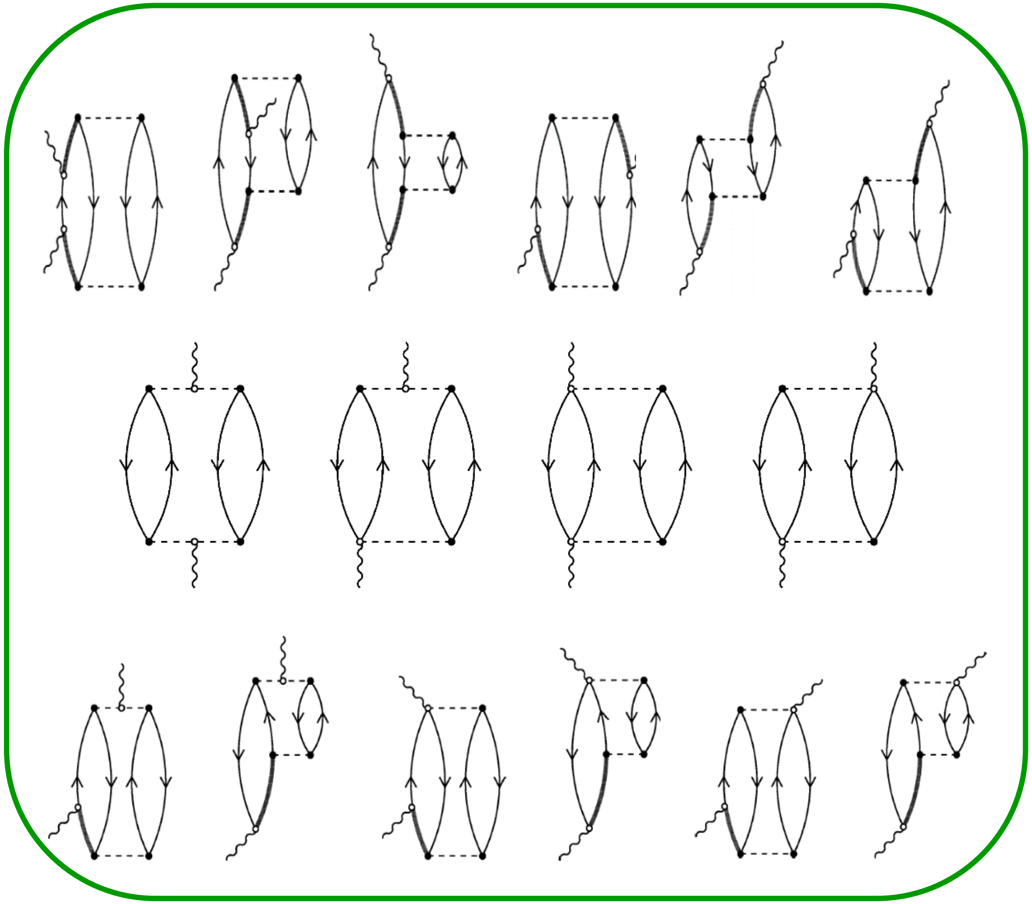
De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)



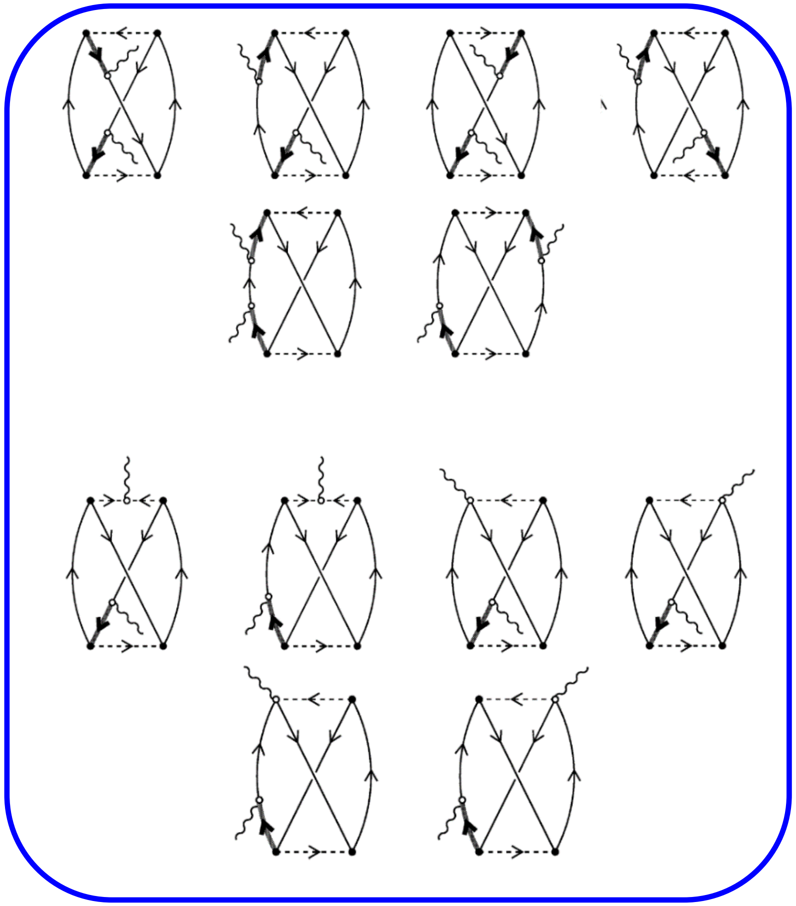


Direct and exchange MEC contributions

Direct



Exchange



Fully relativistic calculation of *De Pace, Nardi, Alberico, Donnelly, Molinari, NPA741 (2004):*

3000 direct terms

More than **100 000** exchange terms

Main difficulties in the np-nh sector

$$W^{\mu\nu}(\mathbf{q},\omega) = W^{\mu\nu}_{1p1h}(\mathbf{q},\omega) + W^{\mu\nu}_{2p2h}(\mathbf{q},\omega) + \cdots$$

$$W^{\mu\nu}_{2p-2h}(\mathbf{q},\omega) = \frac{V}{(2\pi)^9}\int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1E_2E'_1E'_2}\theta(p'_2-k_F)\theta(p'_1-k_F)\theta(k_F-h_1)\theta(k_F-h_2)$$
$$\langle 0|J^\mu|\mathbf{h}_1\mathbf{h}_2\mathbf{p}'_1\mathbf{p}'_2\rangle\langle\mathbf{h}_1\mathbf{h}_2\mathbf{p}'_1\mathbf{p}'_2|J^\nu|0\rangle\delta(E'_1+E'_2-E_1-E_2-\omega)\delta(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{h}_1-\mathbf{h}_2-\mathbf{q})$$

matrix elements

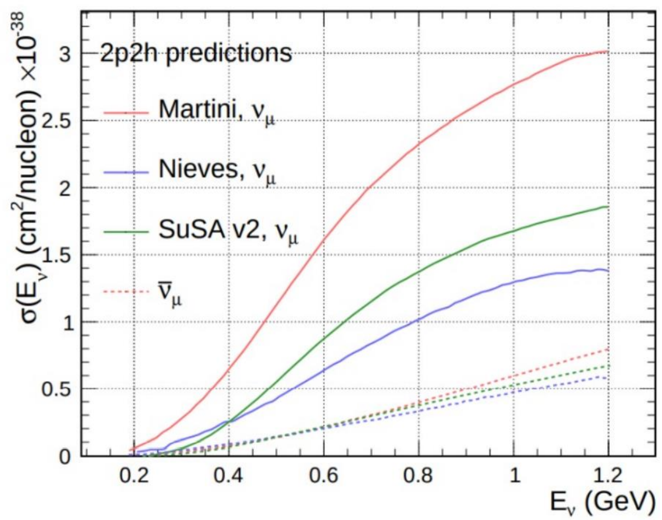
- 7-dimensional integrals $\int d^3h_1 d^3h_2\; d\theta'_1$ of thousands of terms
- Huge number of diagrams and terms
- Divergences (angular distribution; NN correlations contributions)
- Calculations for all the kinematics compatible with the experimental neutrino flux

Computing very demanding

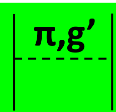
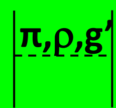
Hence different approximations by different groups:

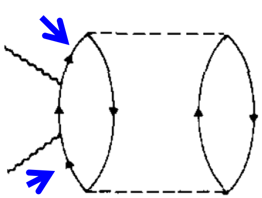
- choice of subset of diagrams and terms;
- different prescriptions to regularize the divergences;
- reduce the dimension of the integrals (7D --> 2D if non relativistic; 7D -->1D if $h_1 = h_2 = 0$)

$\Rightarrow$ Different final results by different groups



Different approximations for the 2p-2h calculations

Approach	Vector	Axial	NN correlations	MEC	NN-MEC interference	Relativistic	
RPA Lyon Martini et al.	Yes	Yes		Yes (Only Δ MEC)	Yes	Some ingredients	No
RPA Valencia Nieves et al.	Yes	Yes		Yes	Yes	Approximations in the WNNπ vertex	No
SuSAv2	Yes	Yes	Already in Superscaling function (1p-1h part)	Yes	No	Fully Relativistic	Yes



$$(p_0 - E_{\mathbf{p}} + i\epsilon)^{-2}$$

- Divergences in NN correlations, prescriptions:

-nucleon propagator only off the mass shell (*Alberico et al. Ann. Phys. 1984*)

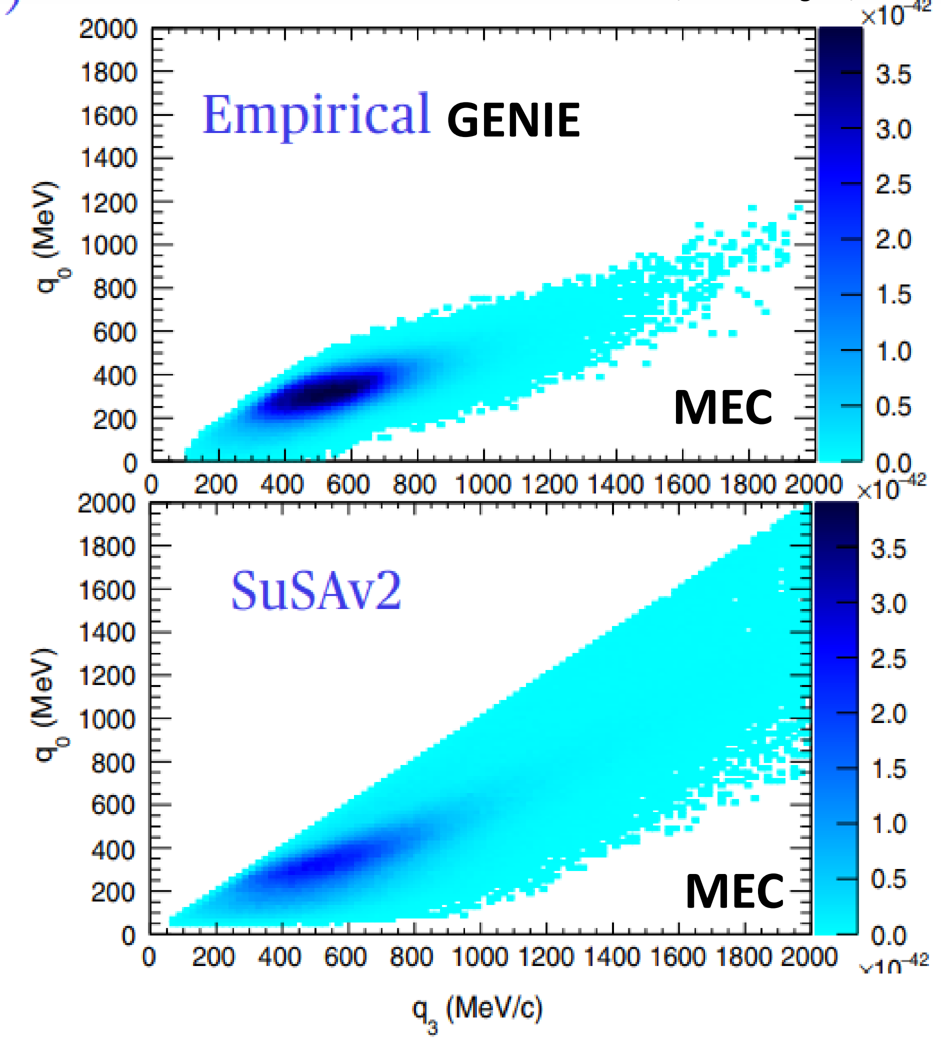
-kinematical constraints + nucleon self energy in the medium (*Nieves et al PRC 83*)

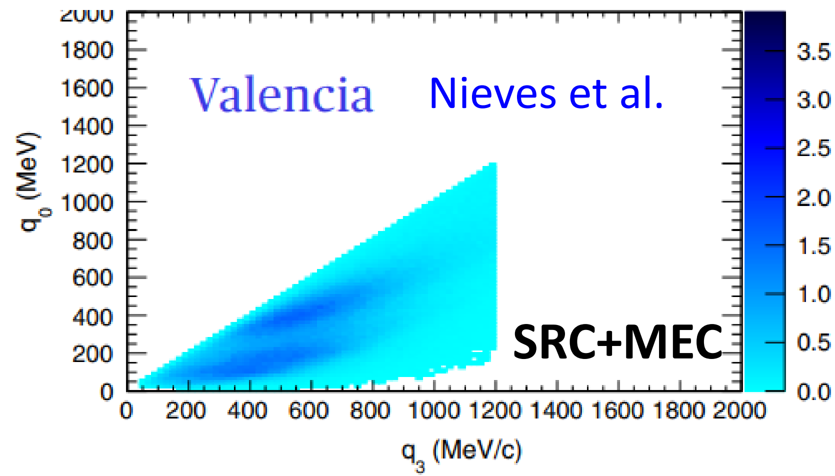
- regularization parameter taking into account the finite size of the nucleus to be fitted to data (*Amaro et al. PRC 82 044601 2010*)

T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001

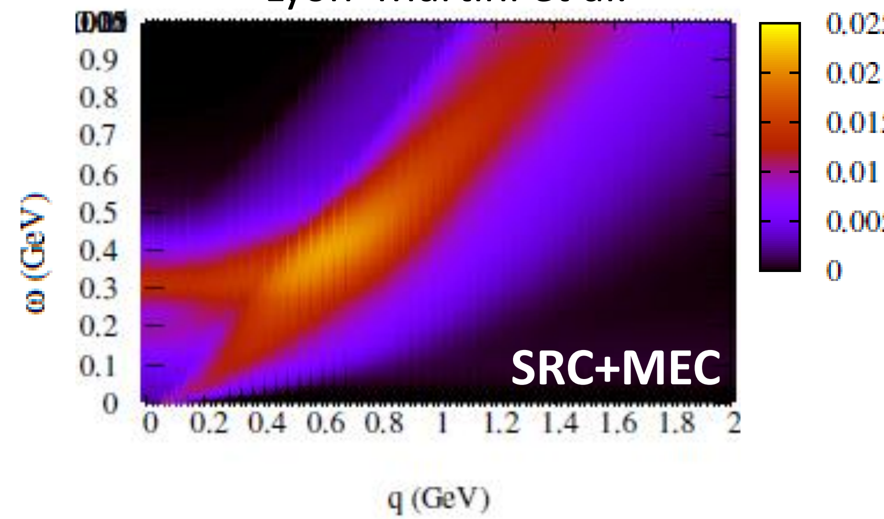
Example of different results for 2p-2h in the (q,ω) or (q_0,q_3) plane

S. Dolan, G.D. Megias, S. Bolognesi, Phys.Rev.D 101 033003 (2020)





Lyon Martini et al.



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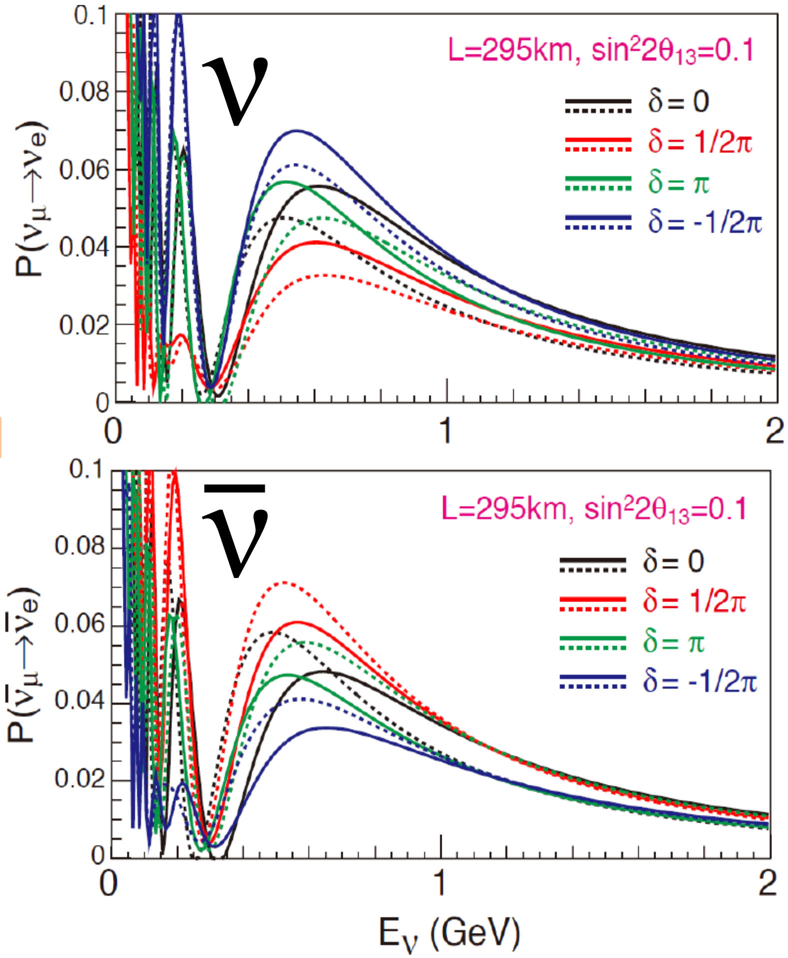
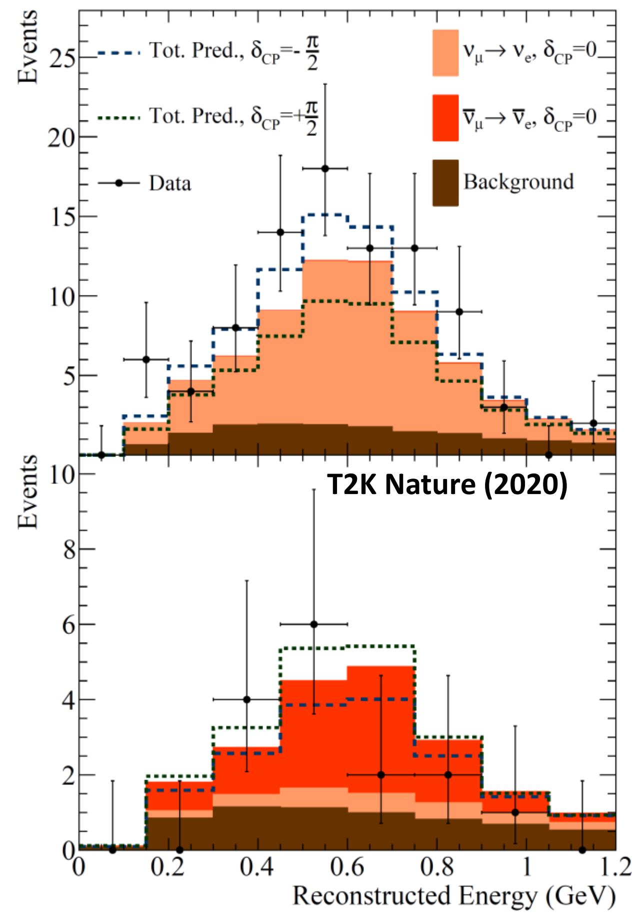
ν .vs. $\bar{\nu}$ and ν_μ .vs. ν_e

What about ν vs $\overline{\nu}$ interaction? And ν_μ vs ν_e?

$$P(\nu_\mu \to \nu_e) \overset{?}{\neq} P(\bar{\nu}_\mu \to \bar{\nu}_e)$$





$$\nu_\mu \to \nu_e$$

$$\bar{\nu}_\mu \to \bar{\nu}_e$$

A precise and simultaneous knowledge of the four cross sections is important in connection to the oscillation experiments aiming at the search for CP violation in the lepton sector.

Neutrino *vs* Antineutrino interactions

The ν and anti ν cross sections differ by the sign of the V-A interference term

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2R_\tau(\mathbf{q},\omega)+\frac{\omega^2}{\mathbf{q}^2}G_A^2R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

$$\left.+2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega)\pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_AG_MR_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

Vector-Axial interference

$$\begin{cases} + & (\nu) \\ - & (\bar{\nu}) \end{cases}$$

Vector-Axial interference:

basic asymmetry from weak interaction theory

different sign in the Leptonic tensor

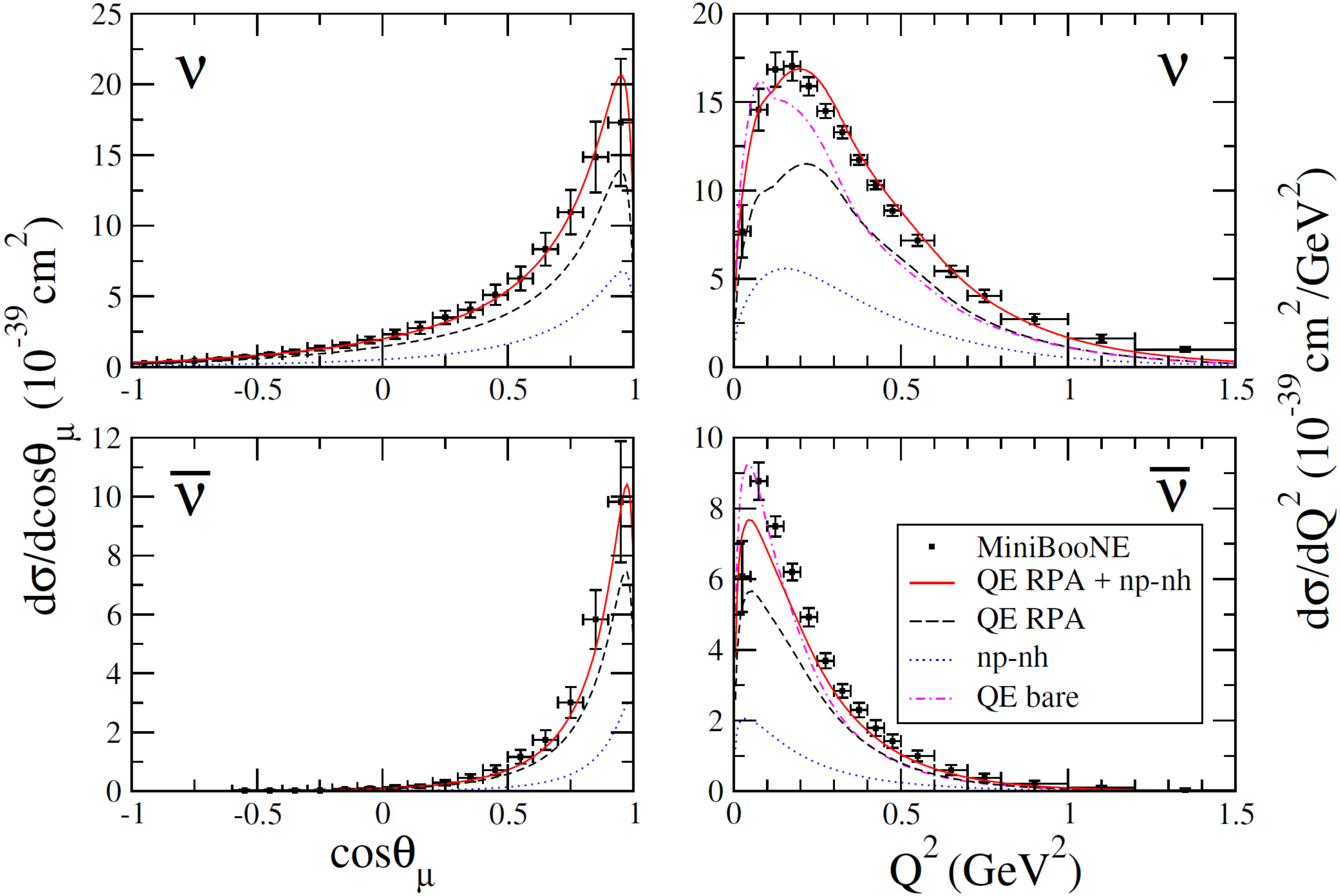
$$L_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}k\cdot k' \mp i\varepsilon_{\mu\nu\alpha\beta}k^\alpha k'^\beta$$

(upper sign: ν; lower sign: $\bar{\nu}$)

⇒ Even neglecting nuclear effects, the absolute value and the kinematic behavior of neutrino and antineutrino cross sections are different

dσ/dcosθ　　　Q^2 distribution

T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001



- Antineutrino cross section falls more rapidly than the neutrino one
- Antineutrino Q^2 distribution peaks at smaller Q^2 values than the neutrino one

Neutrino *vs* Antineutrino interactions and nuclear effects

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2\cos^2\theta_c}{\pi}|\mathbf{k}'|E_l'\cos^2\frac{\theta}{2}\left[\frac{(\mathbf{q}^2-\omega^2)^2}{\mathbf{q}^4}G_E^2 R_\tau(\mathbf{q},\omega) + \frac{\omega^2}{\mathbf{q}^2}G_A^2 R_{\sigma\tau(L)}(\mathbf{q},\omega)\right.$$

$$\left. + 2\left(\tan^2\frac{\theta}{2}+\frac{\mathbf{q}^2-\omega^2}{2\mathbf{q}^2}\right)\left(G_M^2\frac{\mathbf{q}^2}{4M_N^2}+G_A^2\right)R_{\sigma\tau(T)}(\mathbf{q},\omega) \pm 2\frac{E_\nu+E_l'}{M_N}\tan^2\frac{\theta}{2}G_A G_M R_{\sigma\tau(T)}(\mathbf{q},\omega)\right]$$

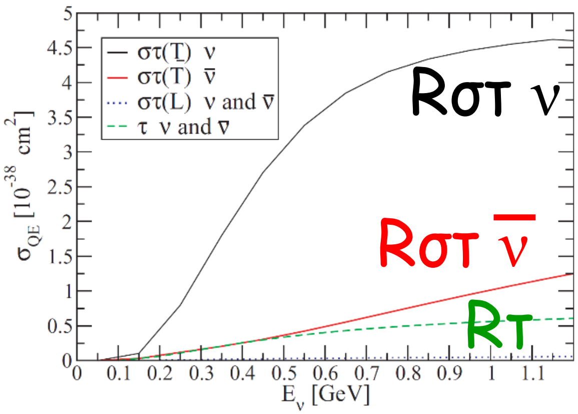
Vector-Axial interference

The ν and anti ν interactions differ by the sign of the V-A interference term

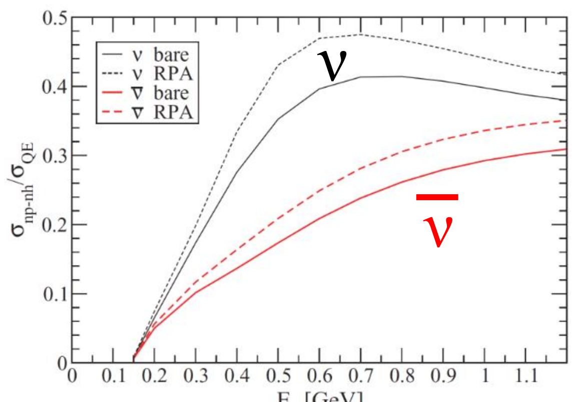
→the relative weight of the different nuclear responses is different for neutrinos and antineutrinos

→the relative role of np-nh contributions is different for neutrinos and antineutrinos

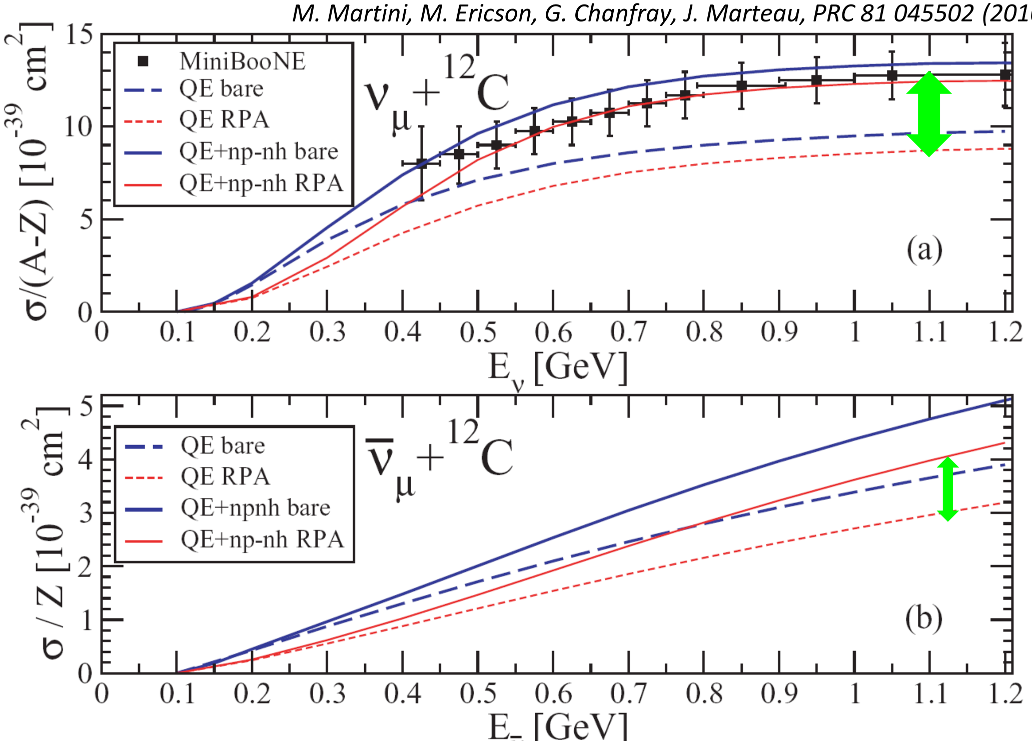
QE



$\frac{np-nh}{QE}$



M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 81 045502 (2010)



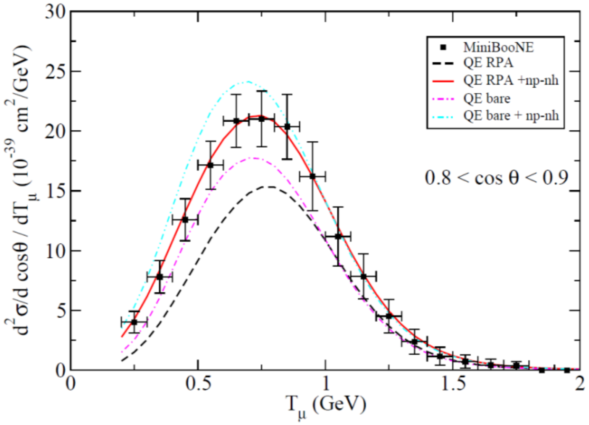
Nuclear effects generate an asymmetry unrelated to CP violation

ν

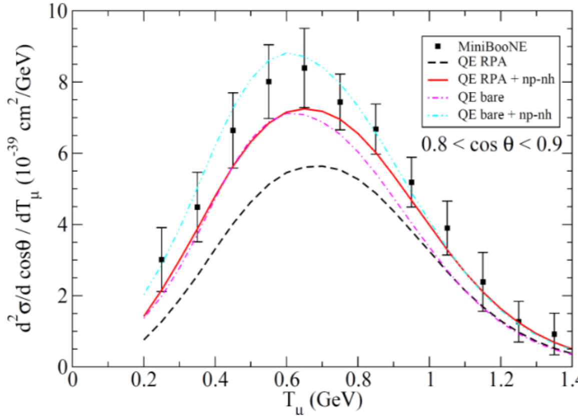
The relative role of np-nh for neutrinos and antineutrinos is different in different approaches

$\bar{\nu}$

Lyon RPA
Martini et al.

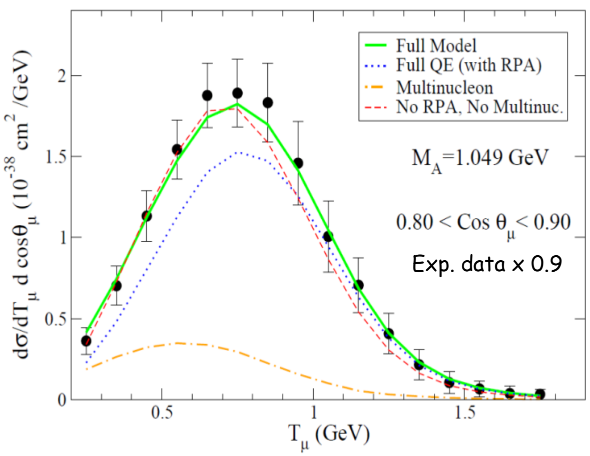


PRC 84 (2011)

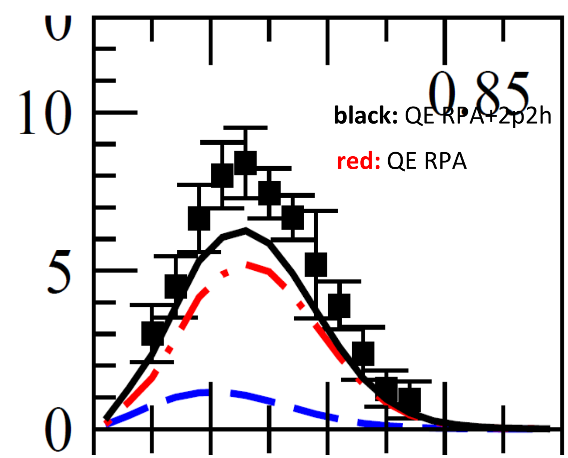


PRC 87 (2013)

Valencia RPA
Nieves et al.

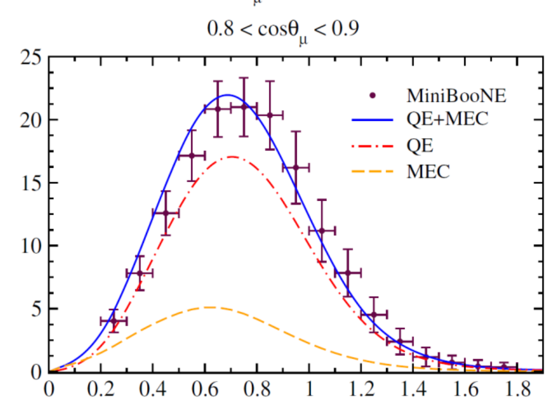


PLB 707 (2012)

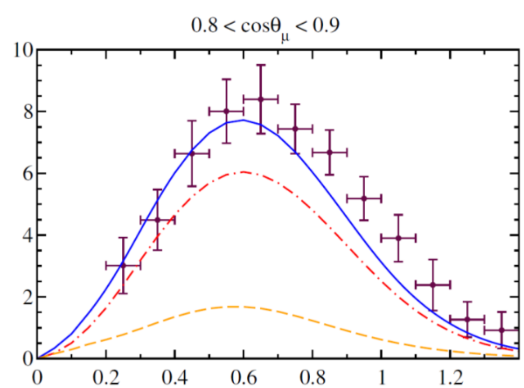


PLB 721 (2013)

SuSAv2



PRD 94 (2016)



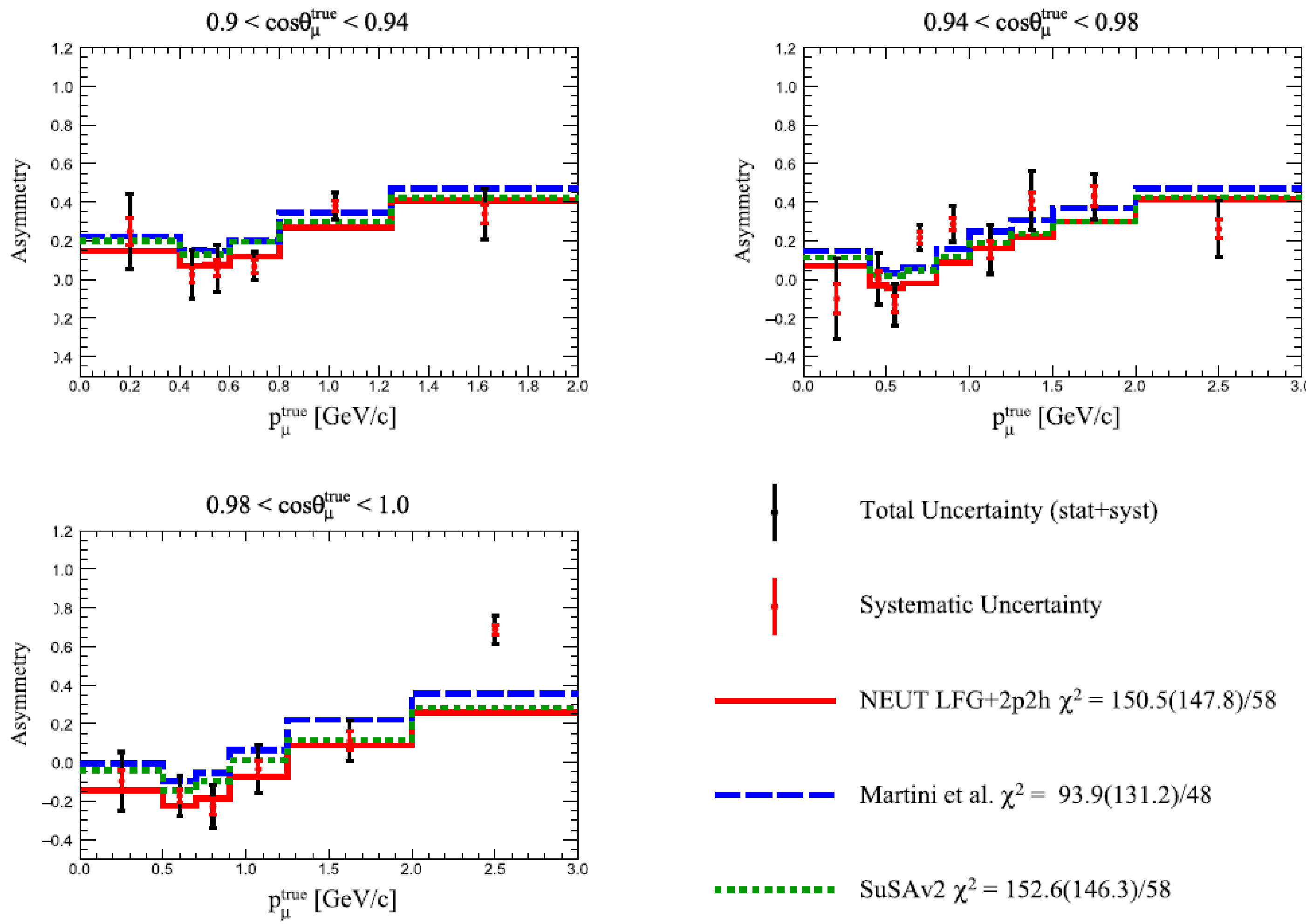
PRD 94 (2016)

T. Katori, M. Martini, J.Phys.G 45 (2018) 1, 013001

PHYSICAL REVIEW D **101,** 112001 (2020)

First combined measurement of the muon neutrino and antineutrino charged-current cross section without pions in the final state at T2K

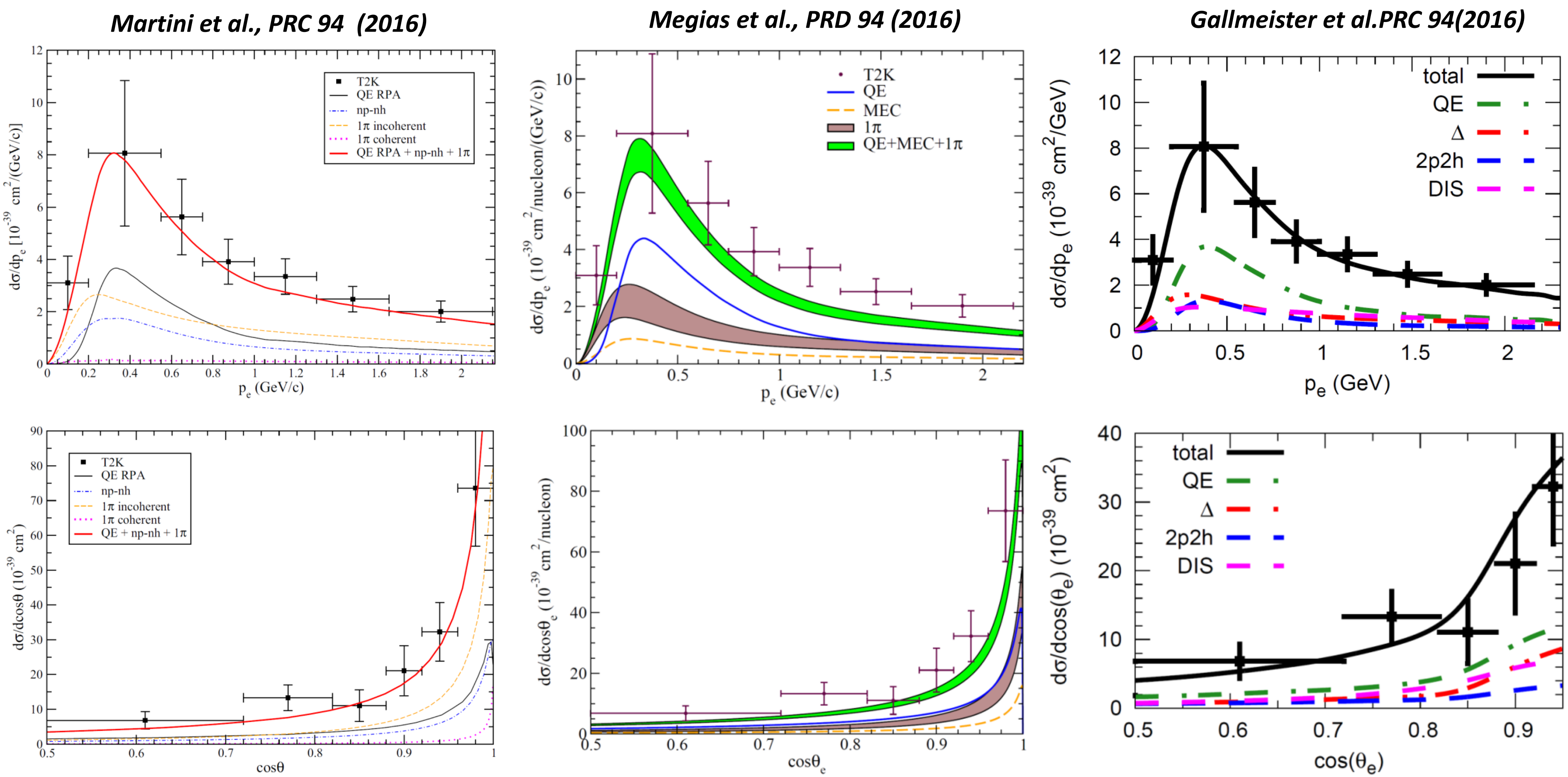
$$\frac{\nu-\bar{\nu}}{\nu+\bar{\nu}}$$



ν_e cross sections

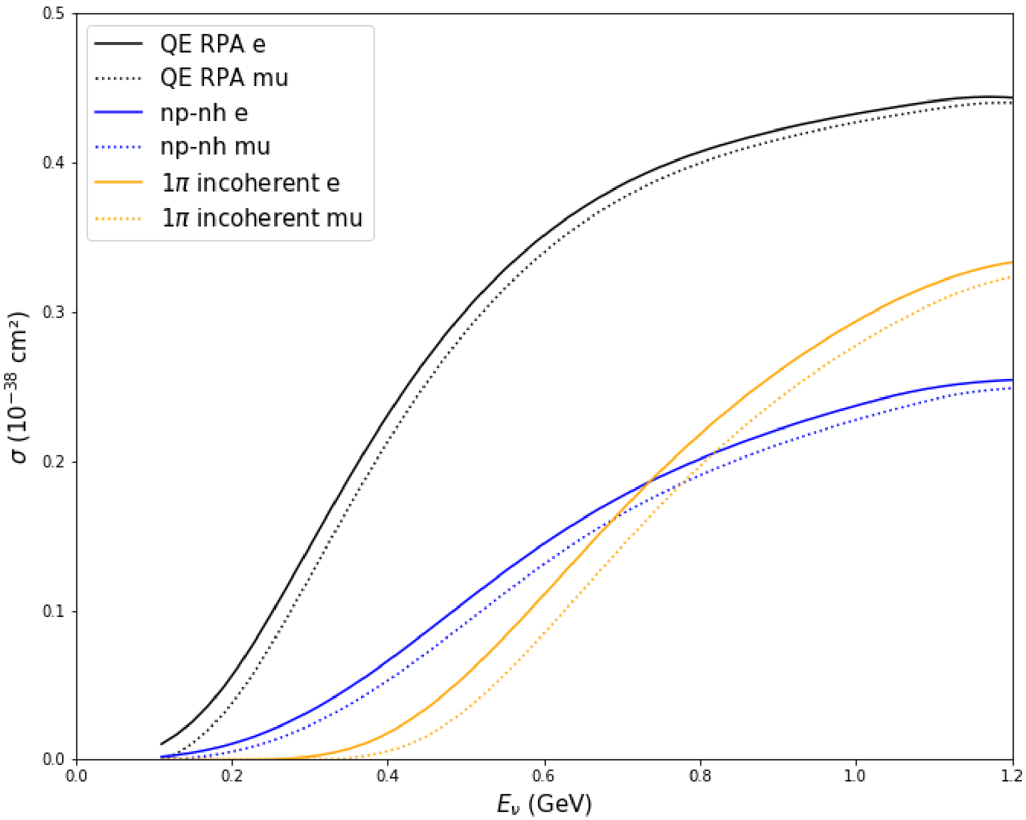
- There are few published results on ν_e cross sections. This is essentially due the relatively small component of ν_e fluxes with respect to the ν_μ ones hence to small statistics.
- The ν_e experimental published results essentially concern inclusive cross sections

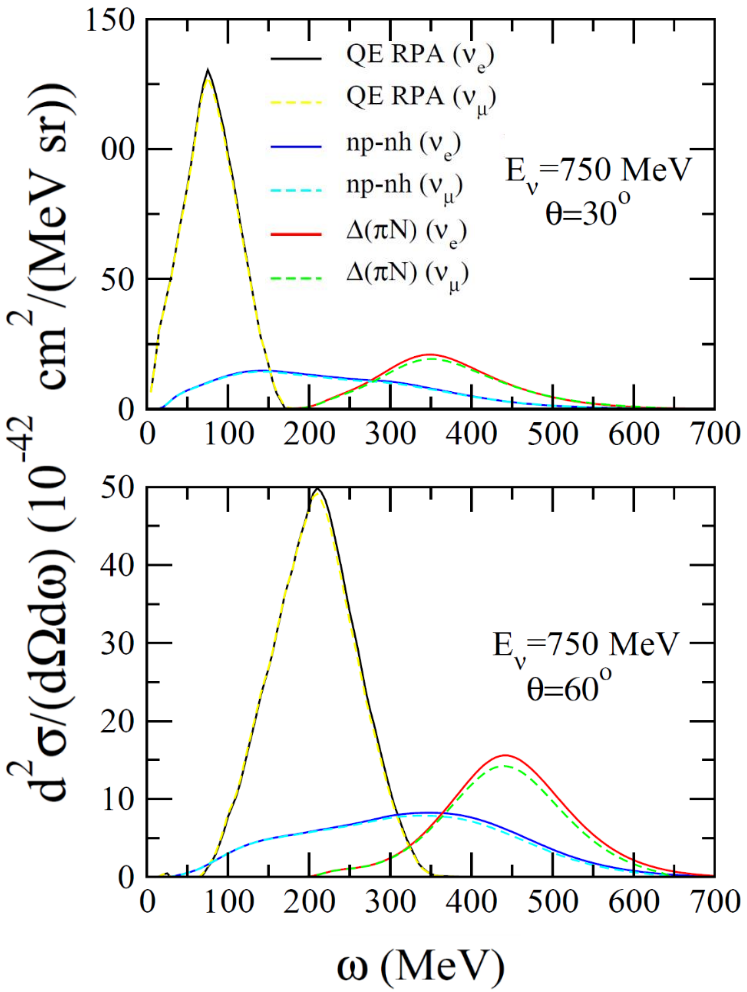
T2K flux-integrated ν_e CC inclusive differential cross sections on carbon



- Theoretical results agree with data
- Similarity of the theoretical results for the inclusive dσ

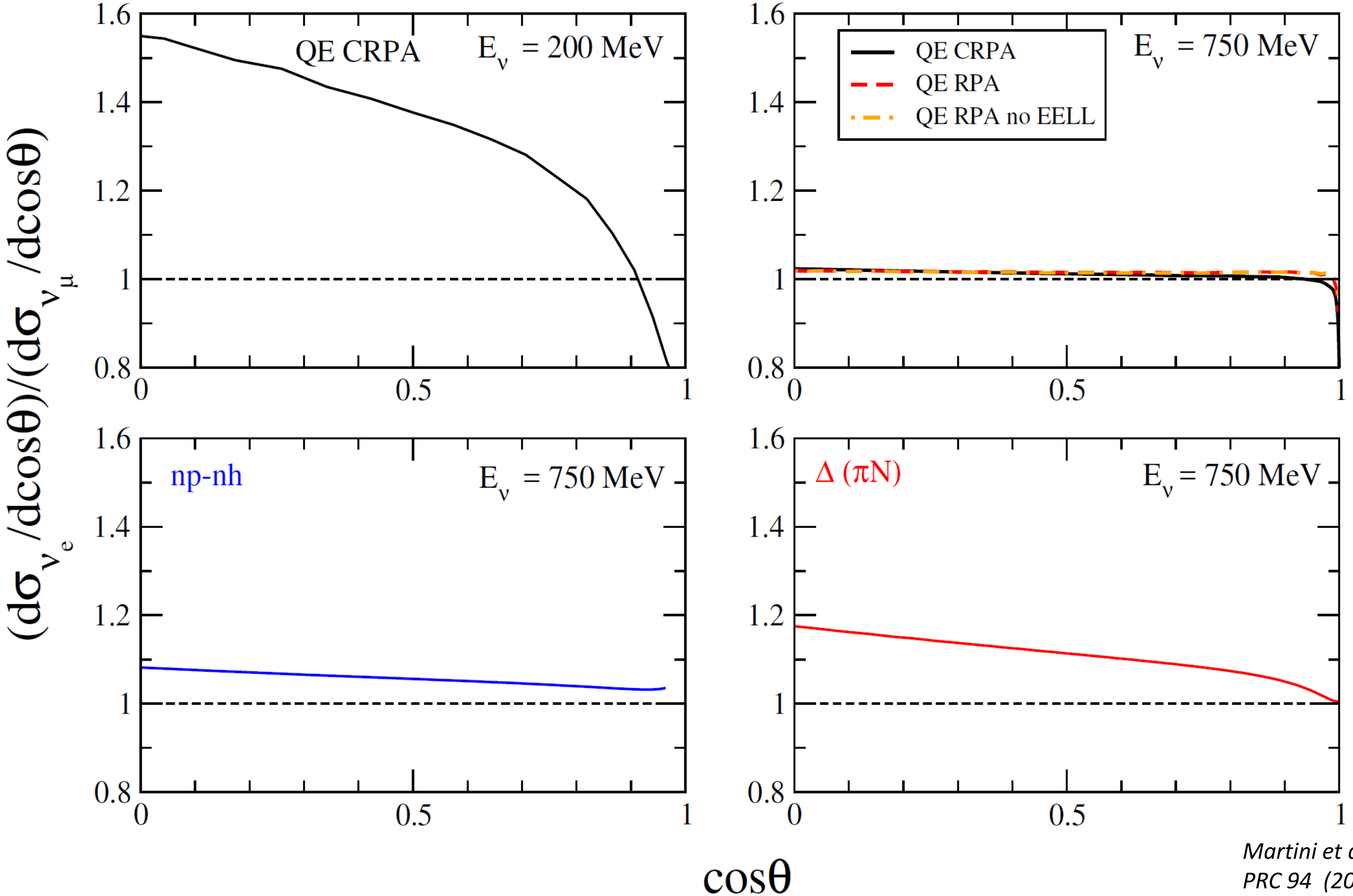
ν_e and ν_μ total and double differential cross sections





Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones

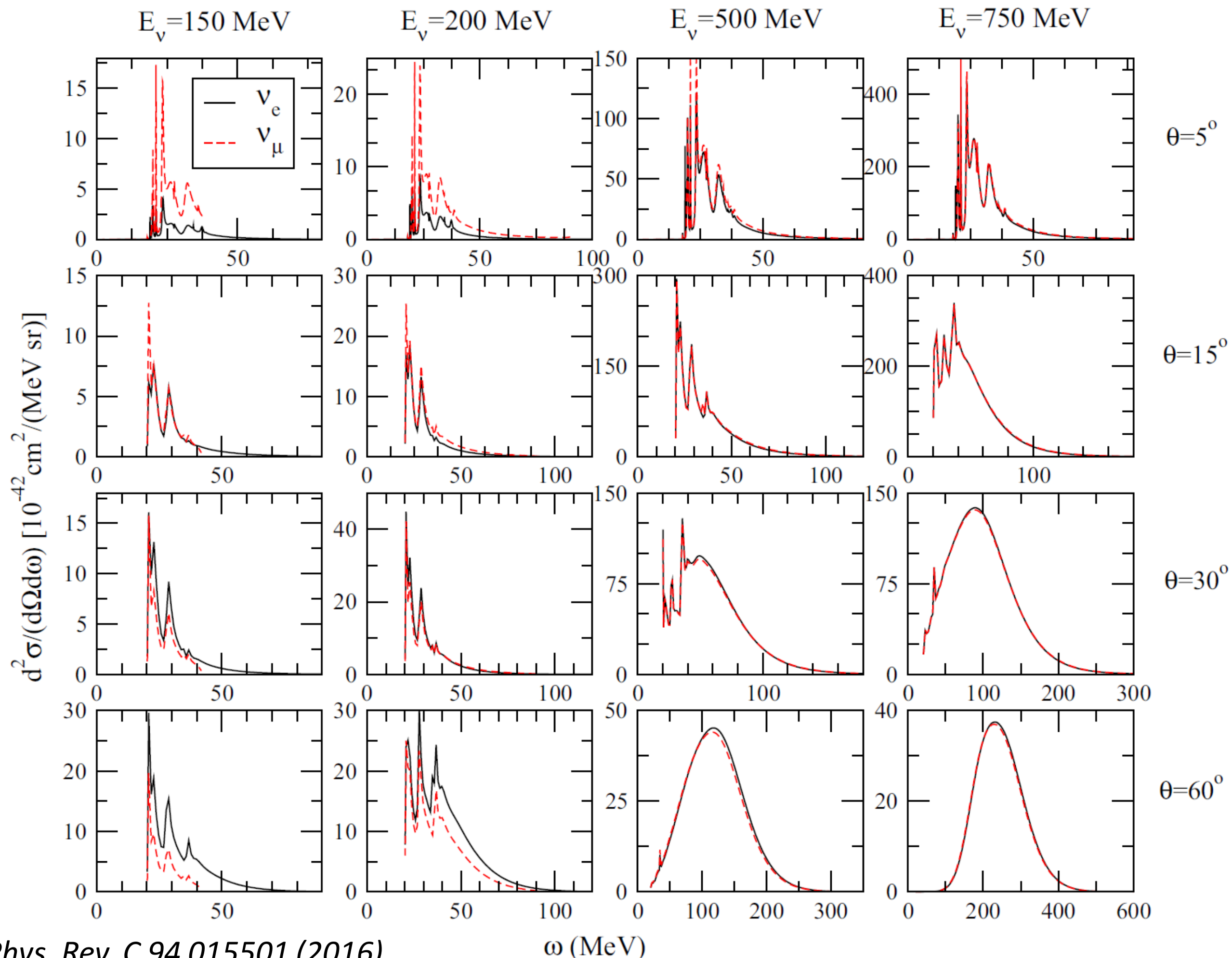
Ratio ν_e/ν_μ for $d\sigma/d\cos\theta$ in different channels



Martini et al., PRC 94 (2016)

Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones.
However for forward scattering angles this hierarchy is opposite in the QE channel.

A theoretical study (HF+CRPA Ghent) of the ν_μ and ν_e $d^2\sigma$



M. Martini et al., Phys. Rev. C 94 015501 (2016)

Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones. However for forward scattering angles this hierarchy is opposite.

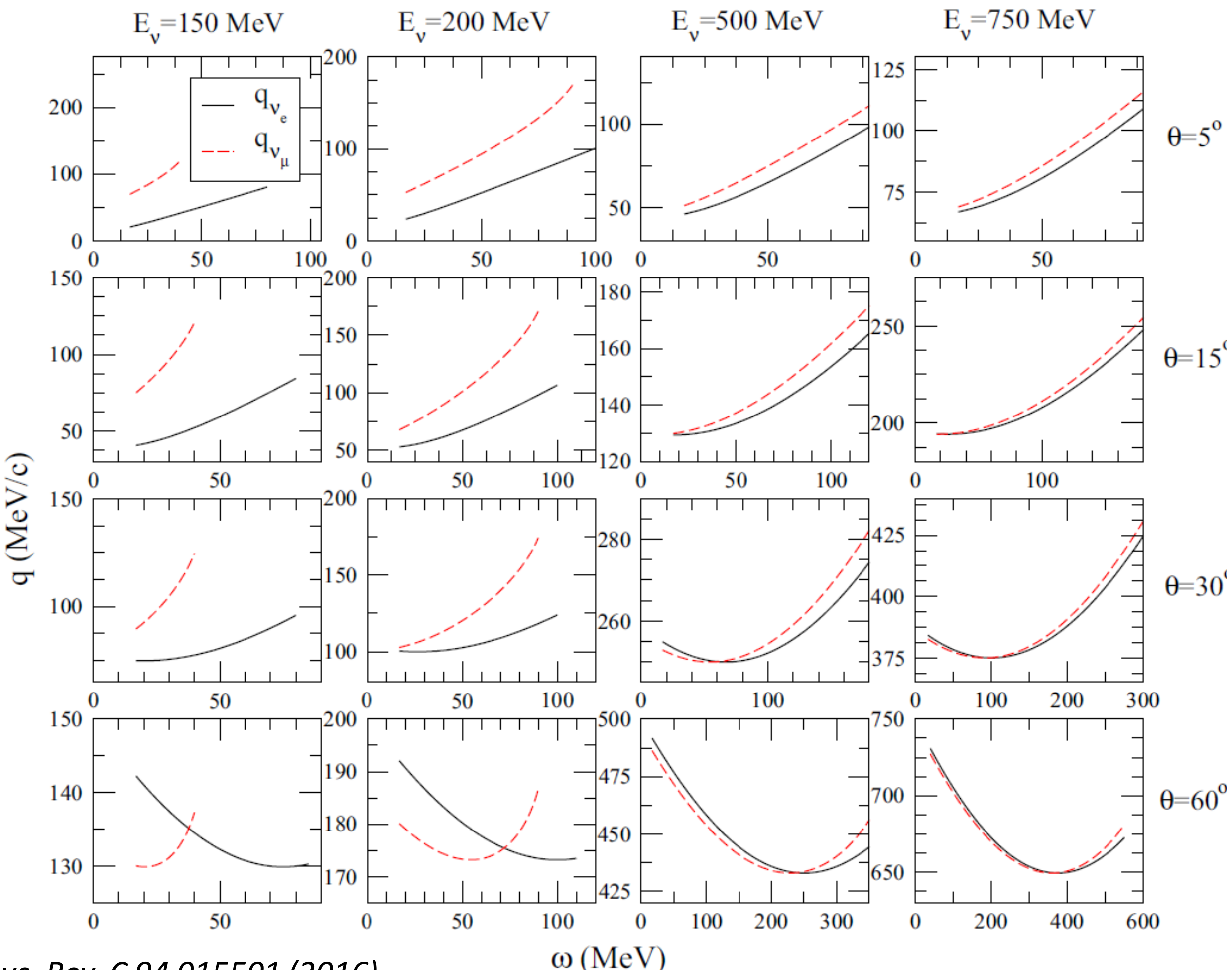
The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton.

But the mass affects the three momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

Further studies: A Nikolakopoulos et al. , PRL 123, 052501 (2019); R. González-Jiménez, PRC, 100, 045501 (2019)

Momentum transfer q versus transferred energy ω for ν_μ and ν_e $d^2\sigma$

Kinematical conditions of the previous slide



M. Martini et al., Phys. Rev. C 94 015501 (2016)

$$q^2 = E_\nu^2 + p_l^2 - 2E_\nu p_l \cos\theta \qquad p_l^2 = E_l^2 - m_l^2 = (E_\nu - \omega)^2 - m_l^2$$

The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton.
But the mass affects the three-momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

Neutrino energy reconstruction

Energy reconstruction in neutrino oscillation experiments

Reconstructed ν energy

True ν energy

$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \to \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Number of detected events

ν flux | ν oscillation probability | ν cross section | Detector efficiency | Migration matrix

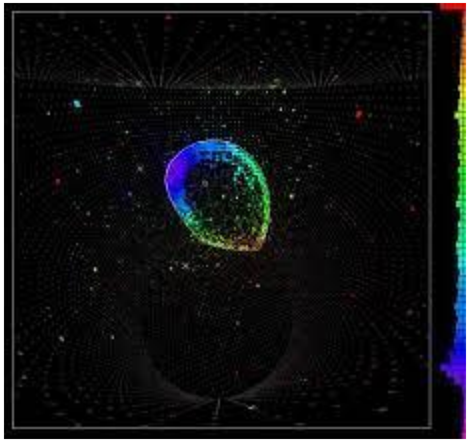
Two methods for ν energy reconstruction

Tracking detectors

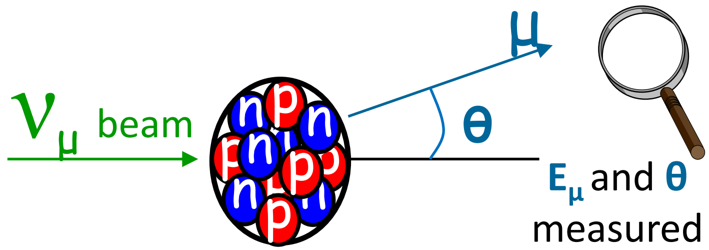
- Use all the detected particles
- Calorimetric method

Cherenkov detectors

- Use only lepton
- Quasielastic-based method



Quasielastic-based neutrino energy reconstruction



$\nu_\mu\, n \rightarrow \mu^-\, p$ CCQE

Reconstructed neutrino energy

$$\overline{E_\nu} = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2\left(m_n - E_b - E_\mu + p_\mu \cos\theta_\mu\right)}$$

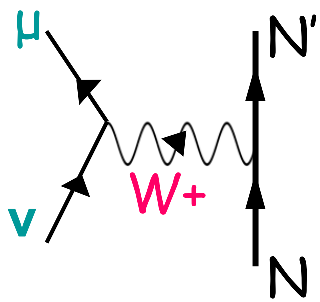
via two-body kinematics

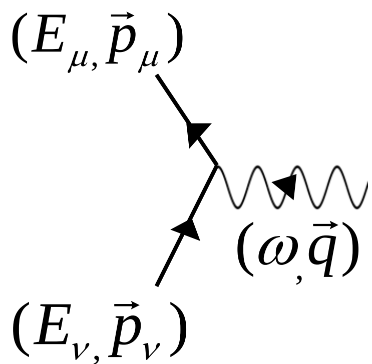
$\overline{E_\nu} = E_\nu$

is exact only for CCQE with free nucleon

reconstructed neutrino energy $\overline{E_\nu} \overset{?}{\longleftrightarrow} E_\nu$ true neutrino energy

QE Scattering with free nucleon at rest: two-body kinematics





$$\omega = E_\nu - E_\mu$$

$$q^2 = E_\nu^2 + p_\mu^2 - 2E_\nu p_\mu \cos\theta$$

$$q^2 - \omega^2 = 4(E_\mu + \omega)E_\mu \sin^2\frac{\theta}{2} - m_\mu^2 + 2(E_\mu + \omega)(E_\mu - p_\mu)\cos\theta$$

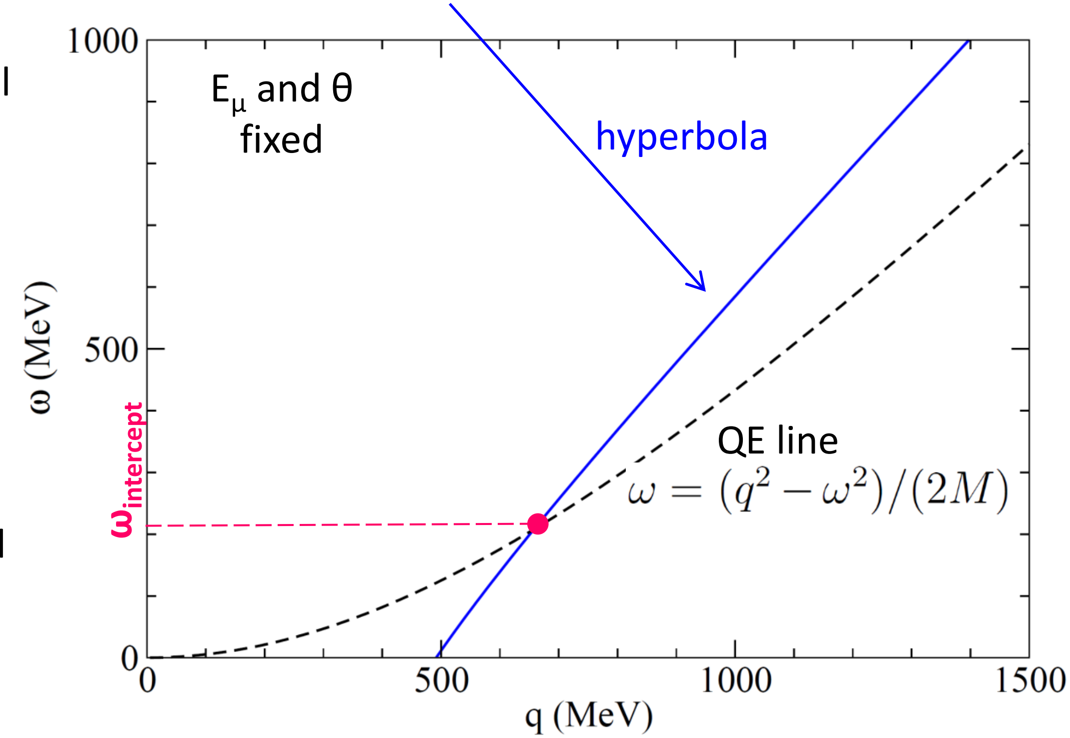
The nuclear response function is proportional to the delta distribution

$$\delta\left[\omega - \left(\sqrt{q^2 + M^2} - M\right)\right]$$

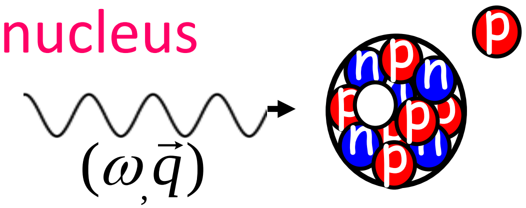
The intercept of the **hyperbola** with the **QE line** fixes the possible ω and q values for given E_μ and θ.

Hence the neutrino energy is determined

$$E_\nu = E_\mu + \omega_{\text{intercept}}$$



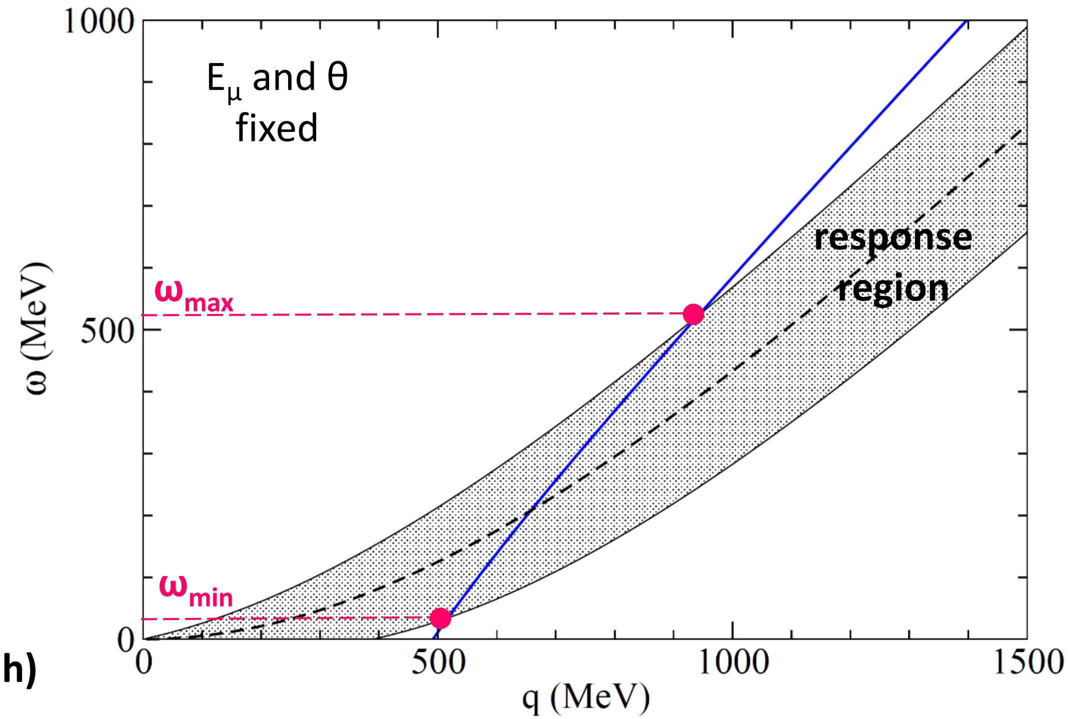
QE Scattering with nucleon inside the nucleus



The intercept of the **hyperbola** with the **response region** gives several possible ω

Broadening of the neutrino energy

$$E_\nu = E_\mu + (\omega_{\min} \le \omega \le \omega_{\max})$$



Nuclear effects in genuine CCQE (1p-1h)

Fermi motion spreads δ distribution

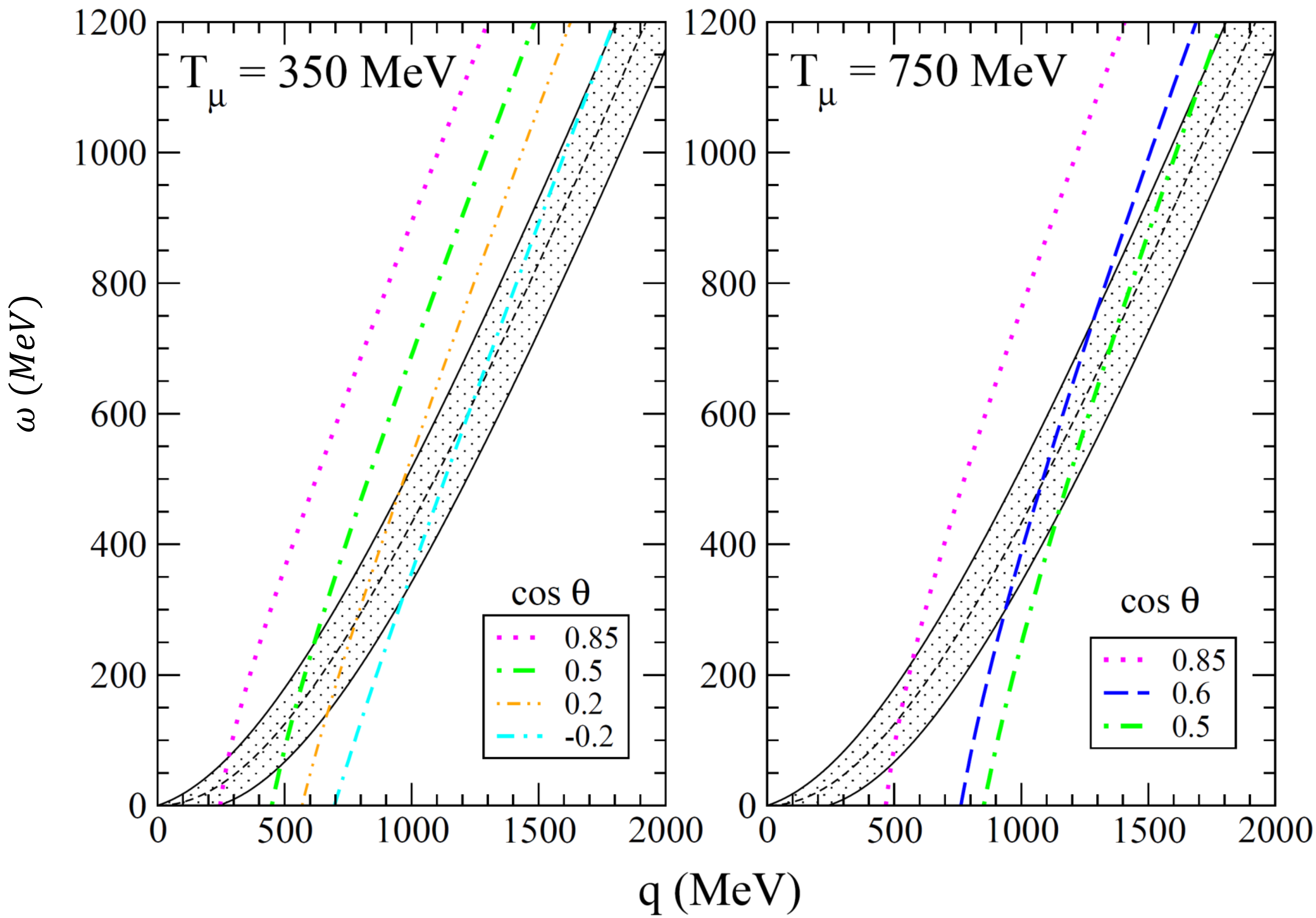
Pauli blocking cuts part of the nuclear response

Binding energy E_B

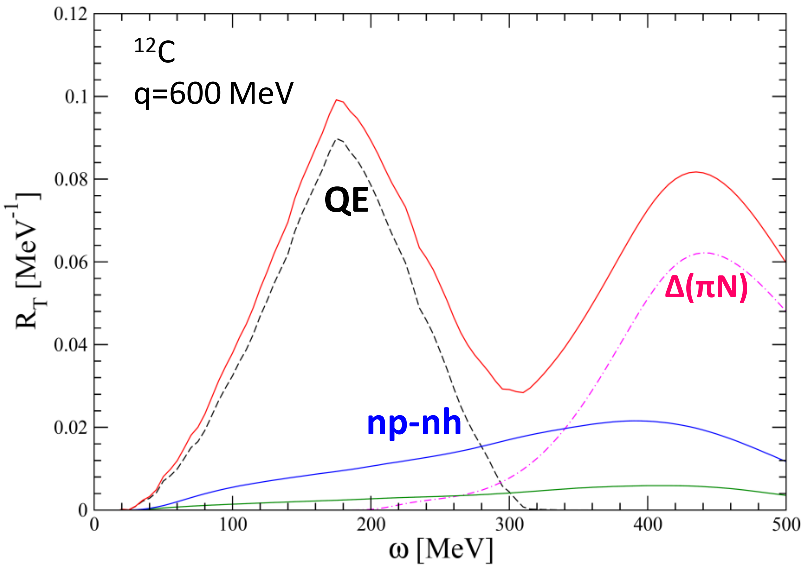
Long Range Correlations (RPA collective effects)

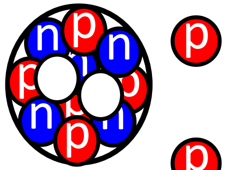
Final State interactions (FSI)

P.S. QE Response region and hyperbolas for several Tμ and θ



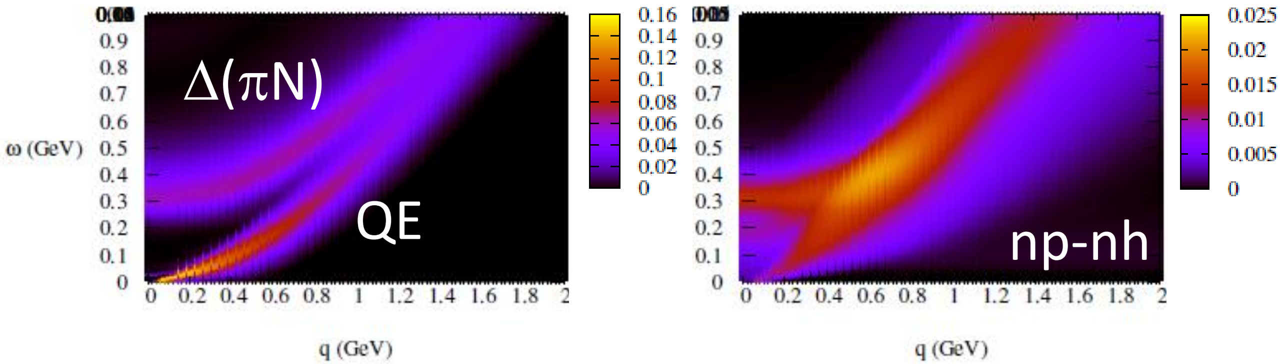
Multinucleon emission



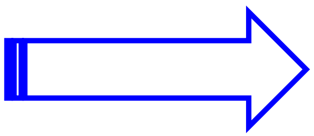


np-nh creates a high energy tail above the QE peak

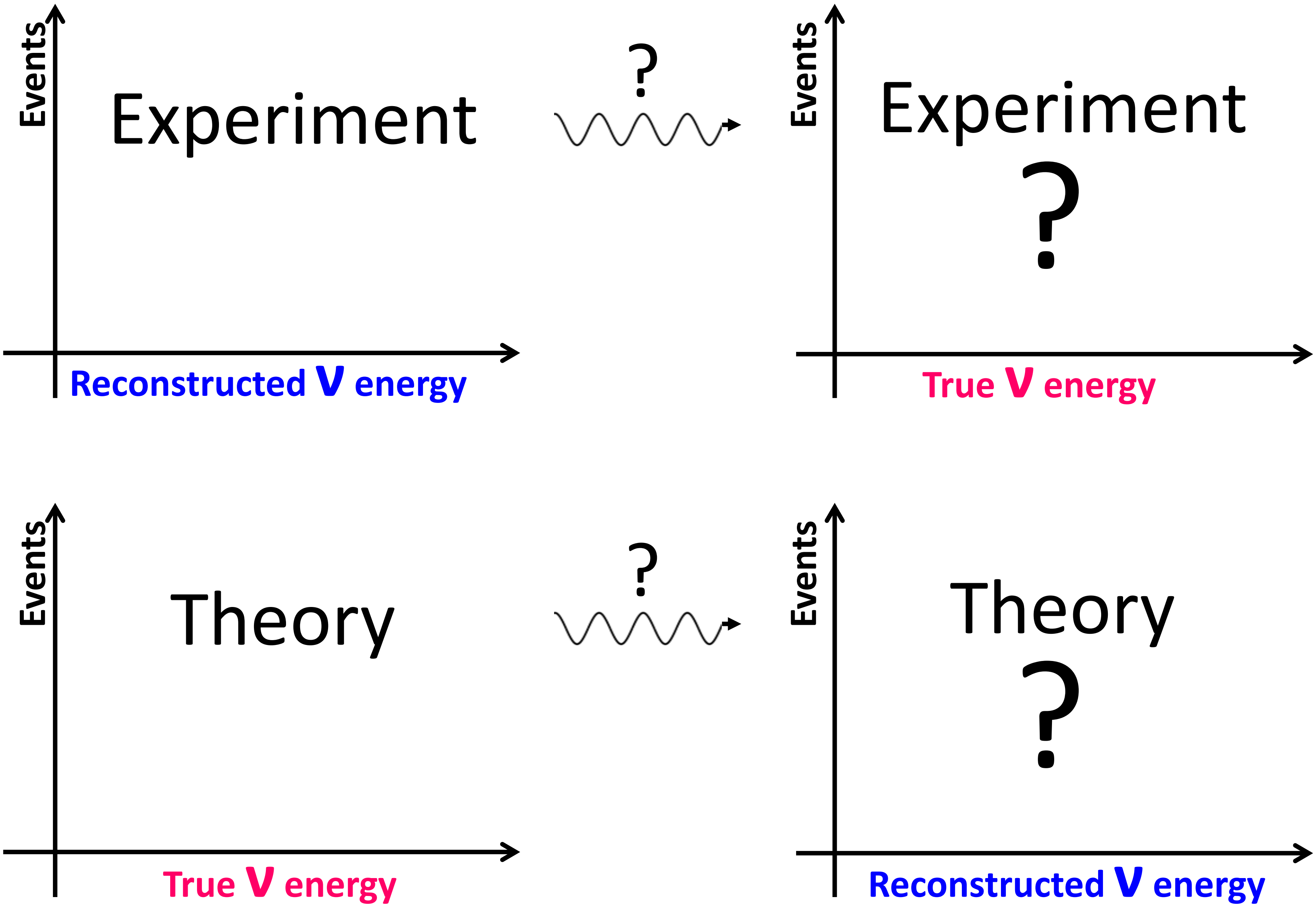
T. Katori, M. Martini, J. Phys. G 45 1, 013001 (2018)



np-nh enlarges the region of response to the whole (ω,q) plane



no reason to fulfill the QE relation for Eν reconstruction

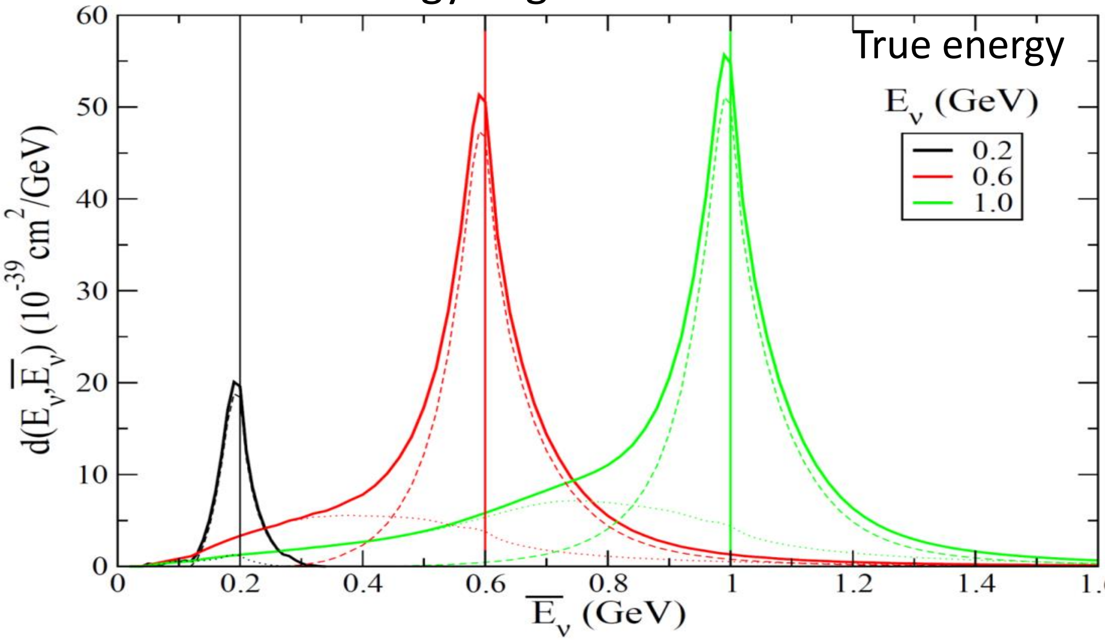

Events
Experiment
Reconstructed ν energy
?
Events
Experiment
?
True ν energy
Events
Theory
True ν energy
?
Events
Theory
?
Reconstructed ν energy

From true neutrino energy to reconstructed neutrino energy

$$D_{rec}(\overline{E_\nu}) = \int dE_\nu \Phi(E_\nu) \underbrace{\int_{E_l^{min}}^{E_l^{max}} dE_l \frac{ME_l - m_l^2/2}{\overline{E}_\nu^2 P_l} \left[\frac{d^2\sigma}{d\omega \; d\cos\theta}\right]_{\omega=E_\nu - E_l, \; \cos\theta = \cos\theta(E_l, \overline{E_\nu})}}_{\nu \text{ energy migration matrix}}$$

The quantity $D_{rec}(\overline{E_\nu})$ corresponds to the product $\sigma(E_\nu)\Phi(E_\nu)$ but in terms of reconstructed neutrino energy

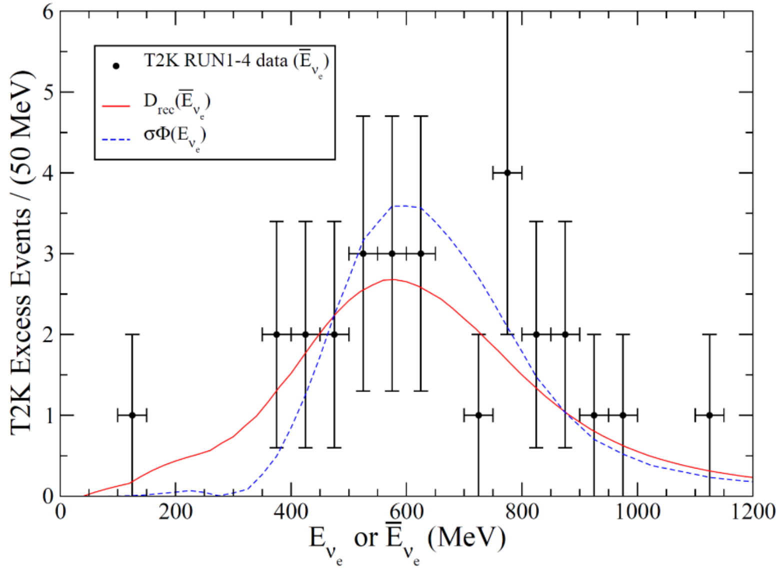
M. Martini, M. Ericson, G. Chanfray
- Phys. Rev. D 85 093012 (2012)
- Phys. Rev. D 87 013009 (2013)



- Distributions not symmetrical around Eν
- Crucial role of np-nh: low energy tail

QE-based neutrino energy reconstruction and neutrino oscillations

ν_e appearance T2K

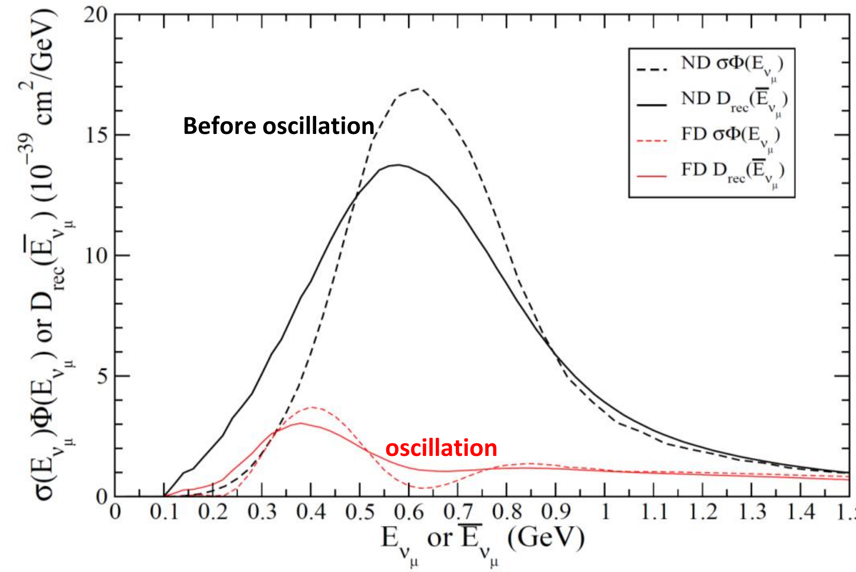


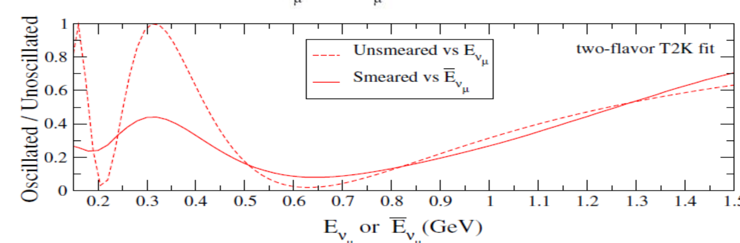
The reconstruction correction tends to make events leak outside the high flux region, especially towards the low energy side

M. Martini, M. Ericson, G. Chanfray
Phys. Rev. D 85 093012 (2012); Phys. Rev. D 87 013009 (2013)

Similar results in:
- Nieves, Sanchez, Simo, Vicente Vacas PRD 85 113008 (2012)
- Lalakulich, Mosel, Gallmeister, PRC 86 054606 (2012)

ν_μ disappearance T2K





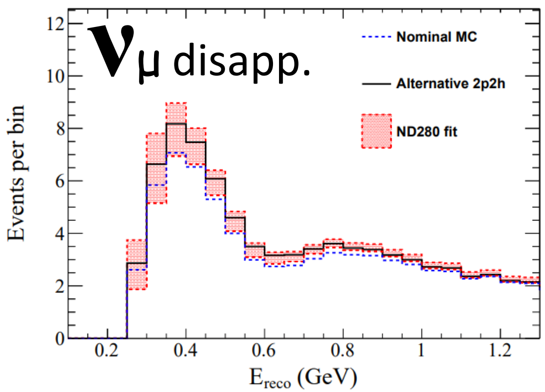
After reconstruction:

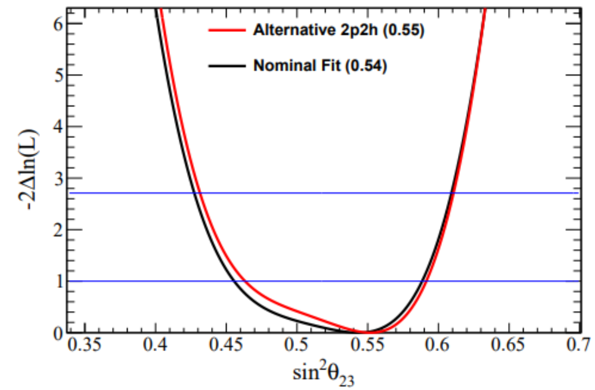
- Near Detector:
  clear low energy enhancement
- Far Detector:
  low energy tail and
  the middle hole is largely filled

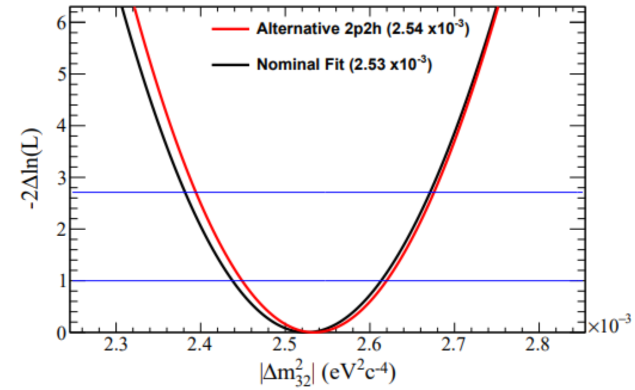
Neutrino energy reconstruction and neutrino oscillation analysis are affected by np-nh

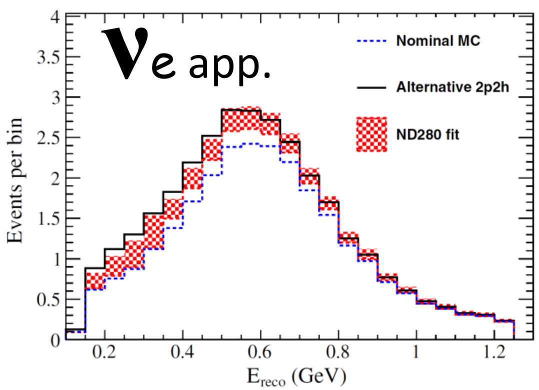
Impact of 2p-2h modeling on T2K oscillation analysis

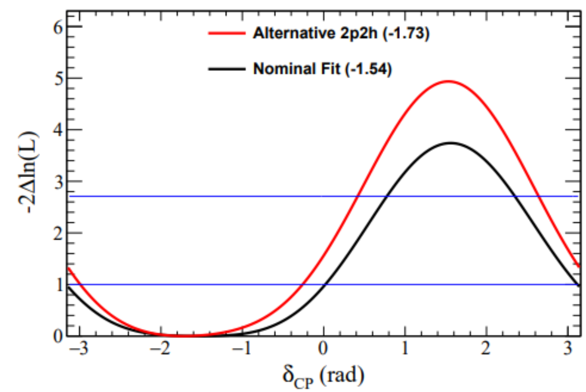
T2K Phys.Rev.D 96 (2017) 9, 092006

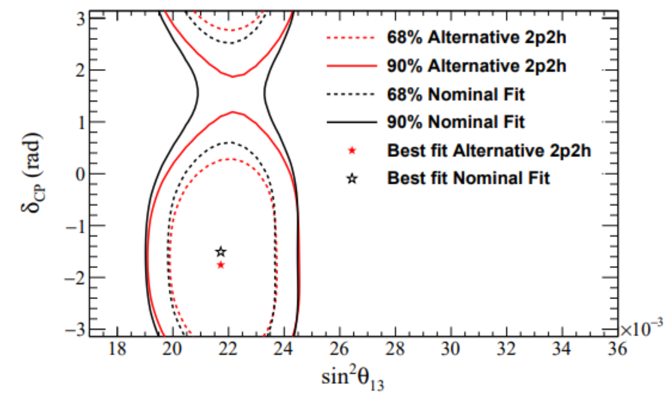








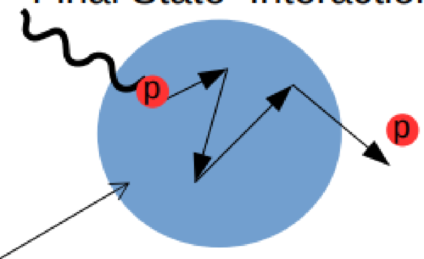




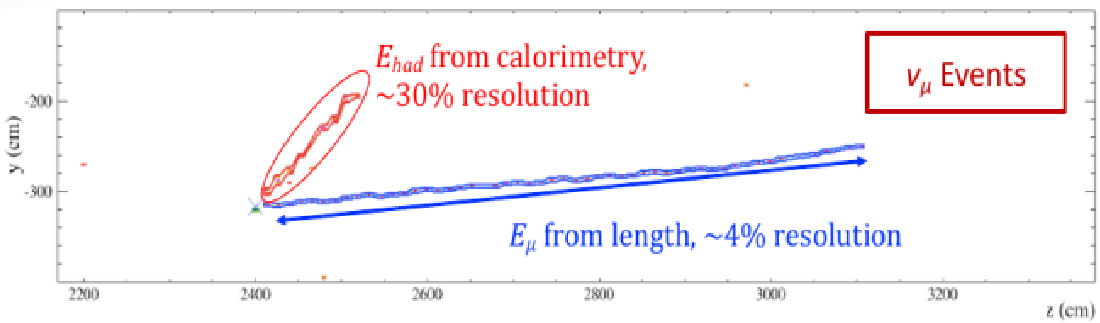
See S. Bolognesi and S. Lavignac lectures

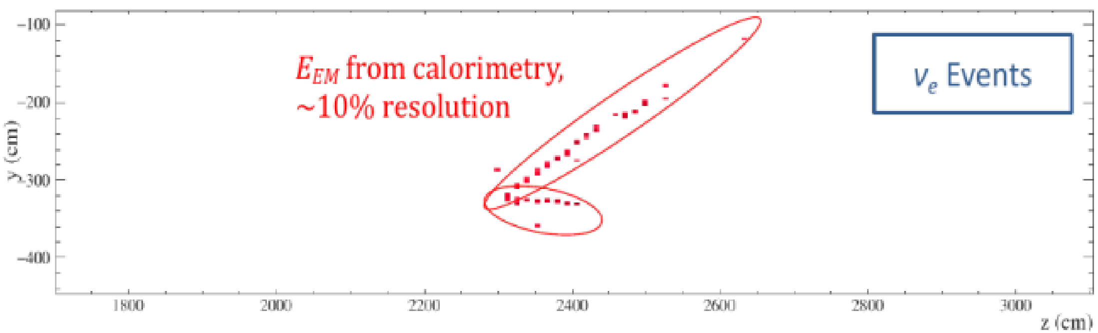
Eν reconstruction NOvA

Final State Interactions

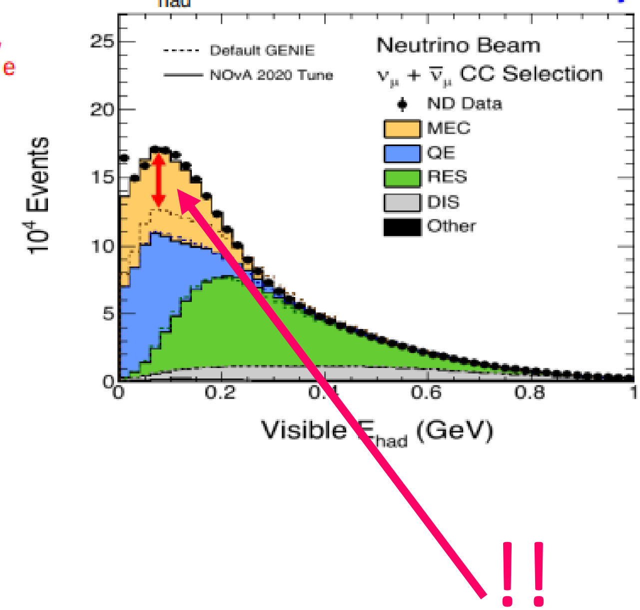


- **Eν reconstructed with hadronic deposits:**
  - important difference ν – $\bar{\nu}$: proton vs neutron (~undetected)
  - proton/pion energy smeared by Final State Interactions
- Different reconstruction and energy resolution for ν_μ and ν_e



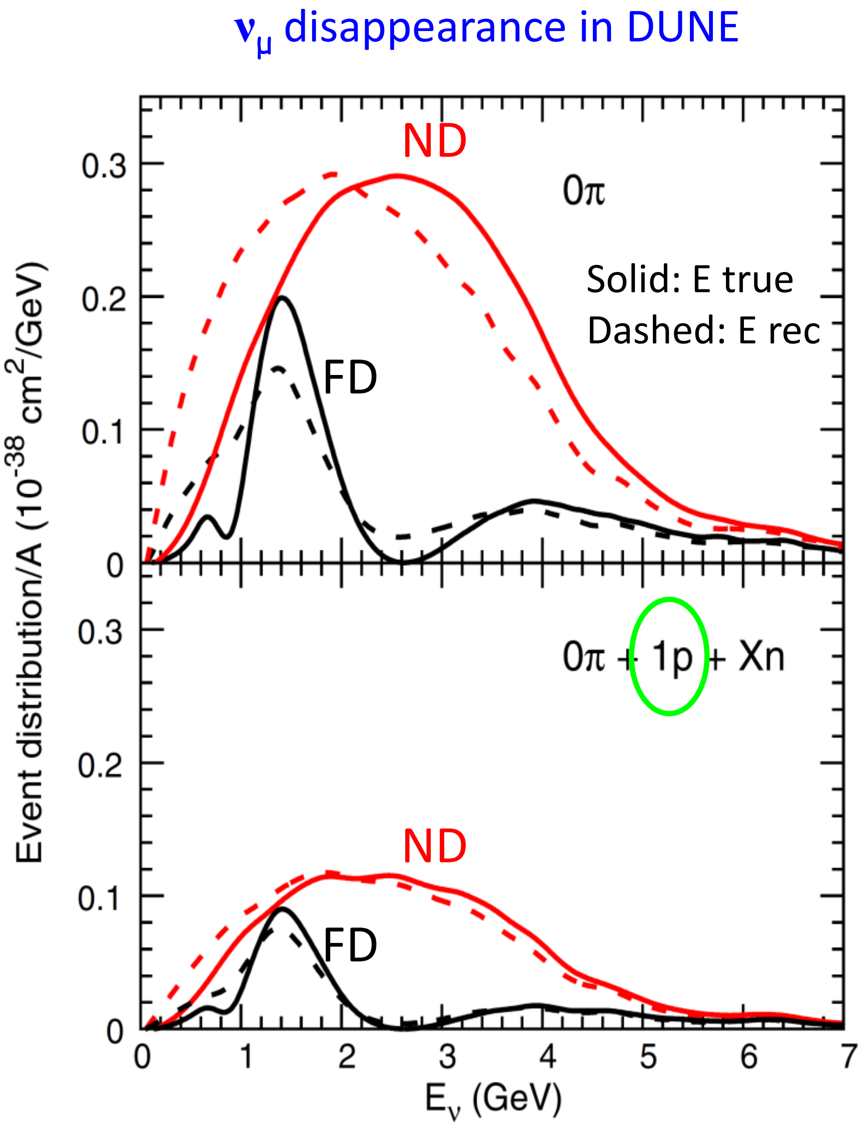


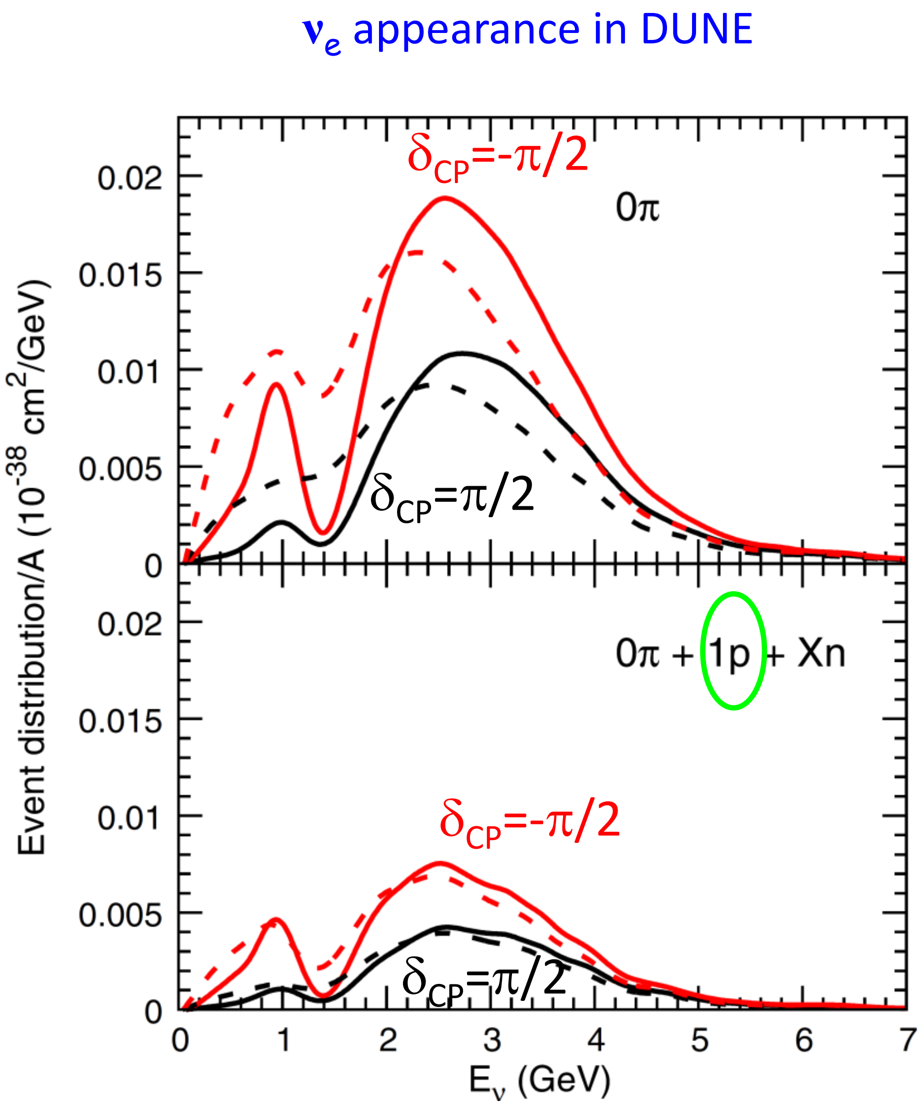
Important to tune model predictions for E_{had}



S. Bolognesi slide 119

QE-based E_ν reconstruction using proton information

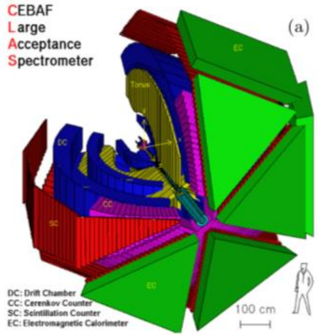




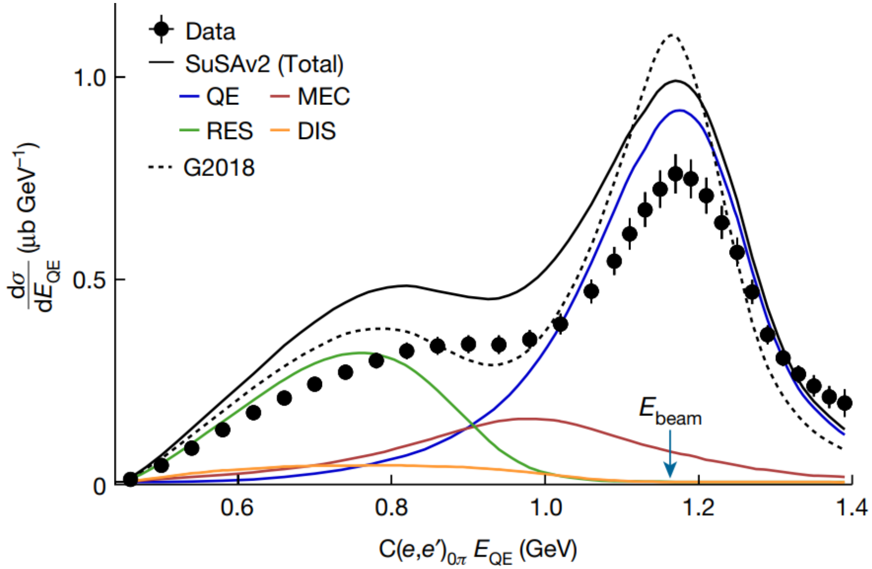
Major improvement in 0π **+ 1p** + Xn sample, events down by only factor 3

Mosel et al. Phys. Rev. Lett. 112 151802 (2014)

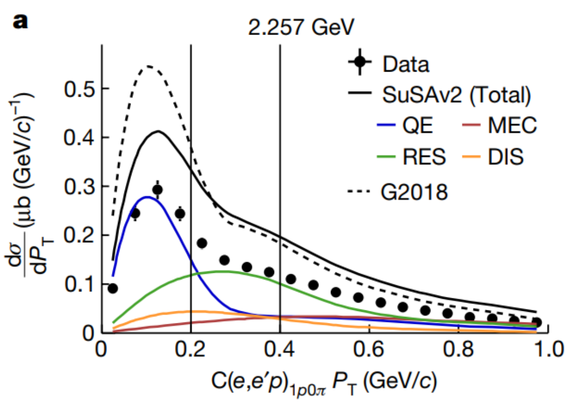
Electron-beam energy reconstruction for ν oscillation measurements

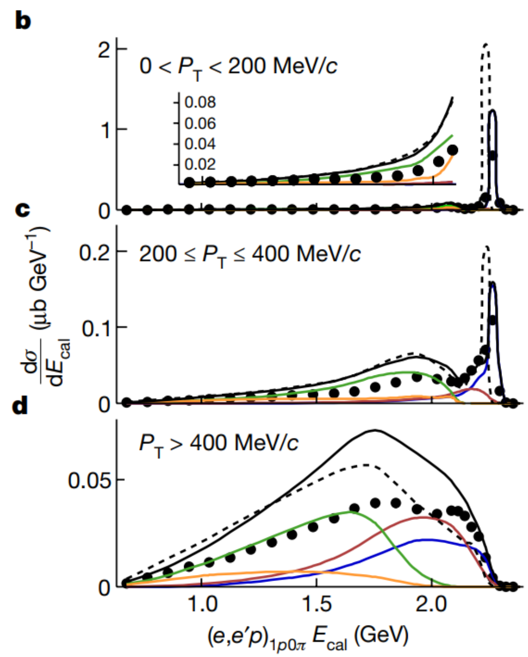


Nature 599 (2021) 7886, 565-570



QE-based (e,e')



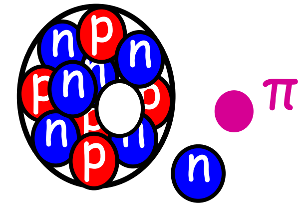


Calorimetric -based (e,e'p)

$$\mathbf{P}_T = \mathbf{P}_T^{e'} + \mathbf{P}_T^{p}$$

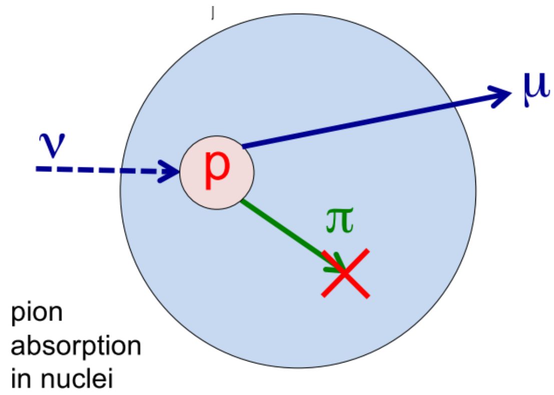
1π production

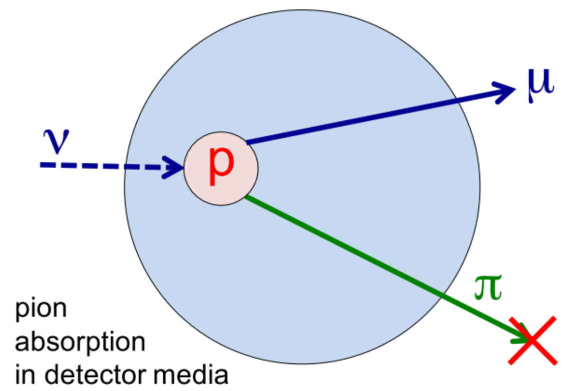
The one pion production channel



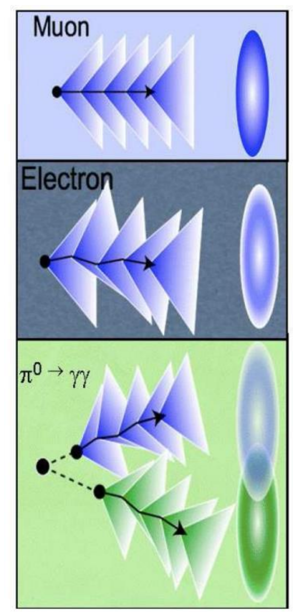
Important for two reasons:

- CC1π can mimic CCQE if the pion is not detected

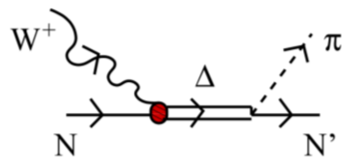




- NC1π^0 can mimic in Cherenkov detectors electron-like signal in $\nu_\mu \to \nu_e$ oscillation search



The 1π production via Δ(1232) resonance excitation and decay



At energies of our interest, it is the dominant mechanism of the reaction $WN \to N'\pi$

E. Hernandez et al. Phys. Rev. D 76, 033005 (2007)

$W^+n \to \Delta^+$ **Hadron matrix element** $\langle \Delta^+; p_\Delta = p + q\,|j^\mu_{cc+}(0)|n;p\rangle = \bar{u}_\alpha(\vec{p}_\Delta)\Gamma^{\alpha\mu}(p,q)\,u(\vec{p})\cos\theta_C$

Electroweak vertex

$$\Gamma^{\alpha\mu}(p,q) = \left[\frac{C_3^V}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_5^V}{M^2}(g^{\alpha\mu}q\cdot p - q^\alpha p^\mu) + C_6^V g^{\mu\alpha}\right]\gamma_5$$
$$+ \left[\frac{C_3^A}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\alpha\mu}q\cdot p_\Delta - q^\alpha p_\Delta^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M^2}q^\mu q^\alpha\right], \quad p_\Delta = p + q$$

Vector form factors $C^V_{3,4,5,6}$ can be extracted from single-pion electro-production data

Axial form factors $C^A_{3,4,5,6}$ $C_5^A(Q^2) = \dfrac{C_5^A(0)}{(1+Q^2/M_{A\Delta}^2)^2}$ $C_6^A \underset{\text{PCAC}}{=} M^2/(m_\pi^2 + Q^2)\cdot C_5^A$ $C_4^A \underset{\text{Adler}}{=} -1/4\cdot C_5^A$ C_3^A Small, usually neglected

Δ propagator

$$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}$$

Spin 3/2 projection operator

$$P^{\mu\nu}(p_\Delta) = -(\not{p}_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu\gamma^\nu - p_\Delta^\nu\gamma^\mu}{M_\Delta}\right]$$

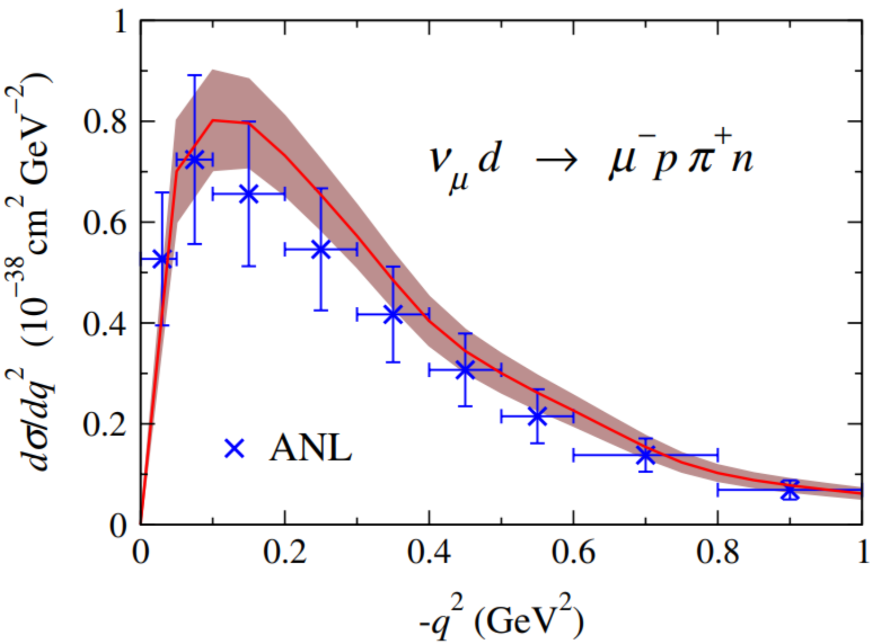
NΔπ coupling

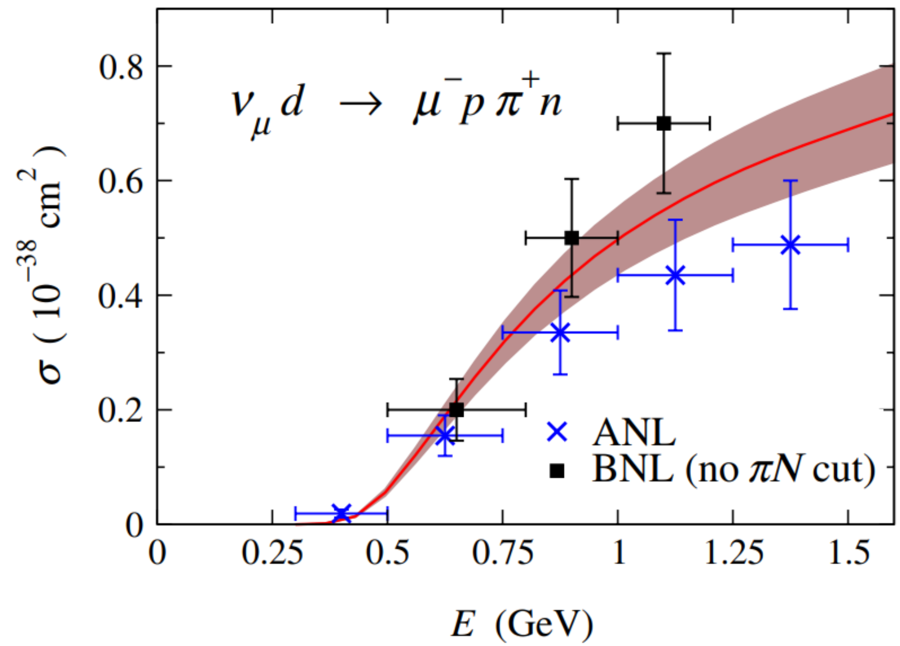
$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi}\bar{\Psi}_\mu \vec{T}^\dagger(\partial^\mu\vec{\phi})\Psi + \text{h.c.}$$

1π production in neutrino deuteron scattering

- "Old" deuteron bubble-chamber experiments (Argonne ANL and Brookhaven BNL)
  [Campbell J et al 1973 Phys. Rev. Lett. 30 335; Radecky G et al 1982 Phys. Rev. D 25 1161; Kitagaki T et al 1986 Phys. Rev. D 34 2554]
- Both ANL and BNL data suffer from a large flux-normalization error

E. Hernandez et al. Phys. Rev. D 87, 113009 (2013)





$$C_5^A(Q^2) = \frac{C_5^A(0)}{(1 + Q^2/M_{A\Delta}^2)^2}$$

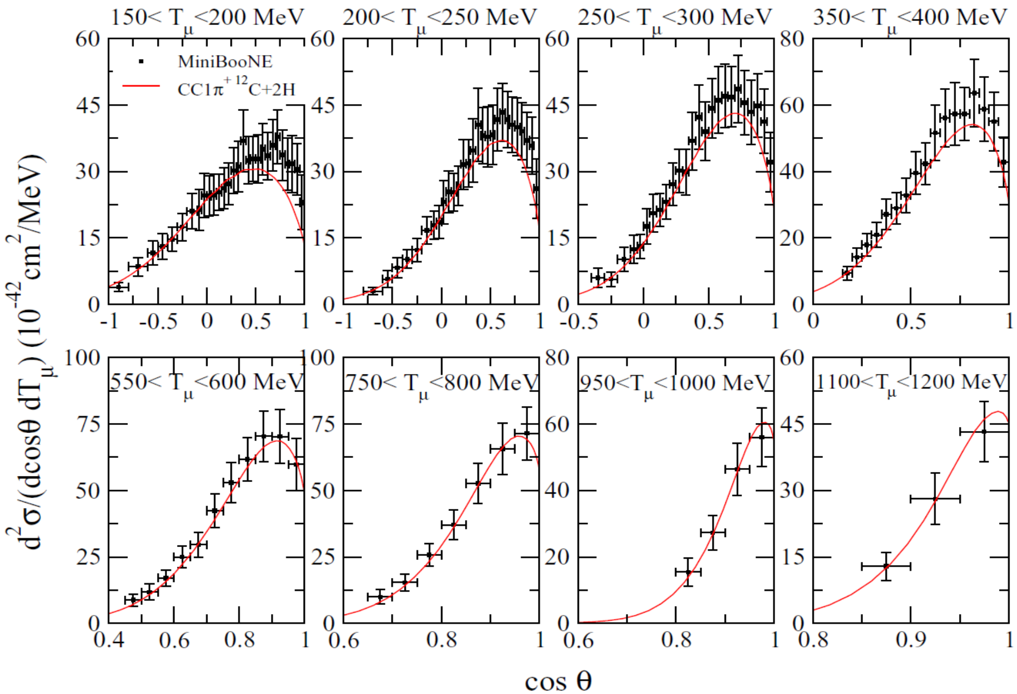
There is also a strong desire to repeat bubble-chamber experiments to better determine the C_5^A form factor

Nowadays due to the tighter safety regulations of modern experiments, hydrogen or deuteron bubble-chamber experiments are not easily approved, especially underground, where most of neutrino beams are located.

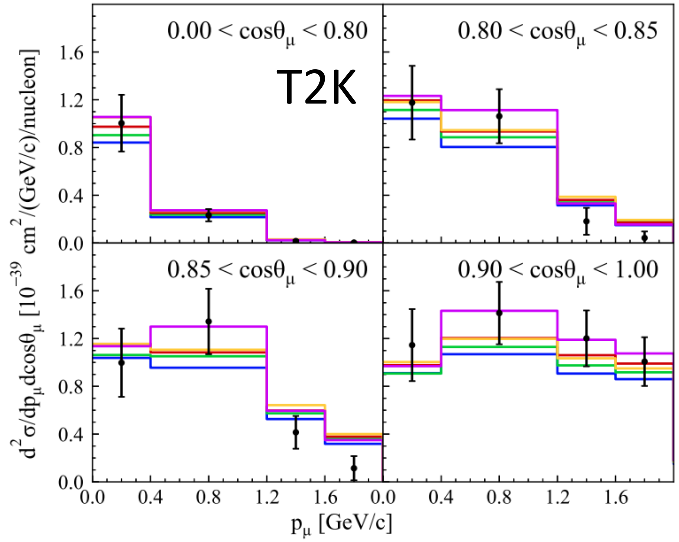
CC1π+ flux-integrated differential cross sections on carbon

Results in terms of muon variables

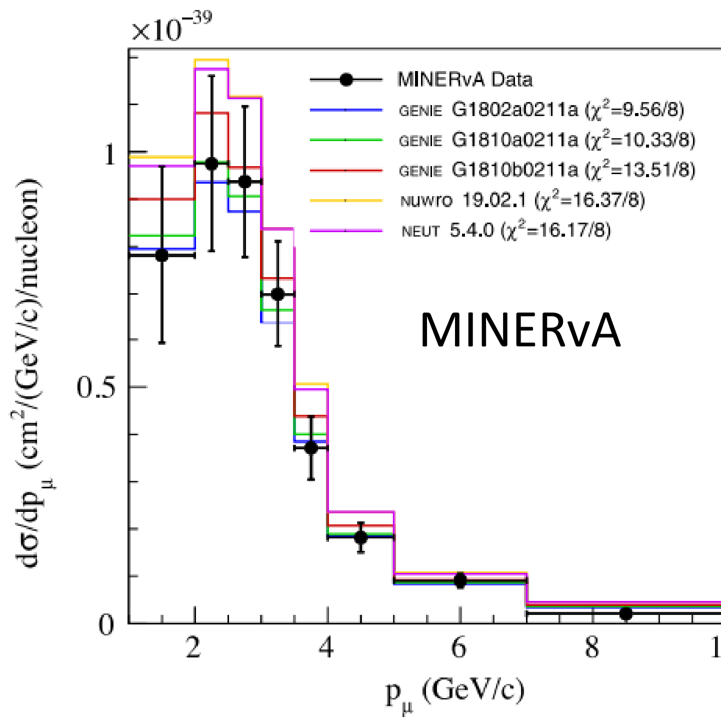
MiniBooNE



M. Martini, M. Ericson, Phys. Rev. C 90 025501 (2014)



M. Buizza Avanzini et al. PRD 105, 092004 (2022)

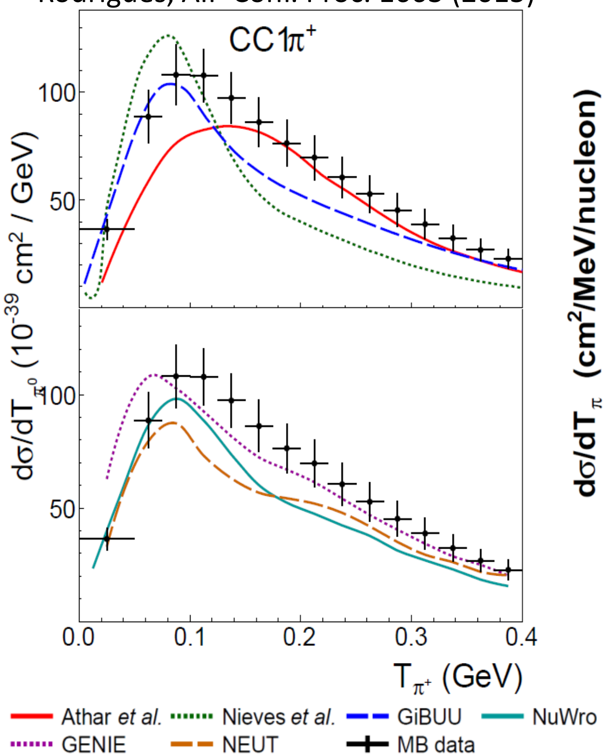


Reasonable agreement between models and data

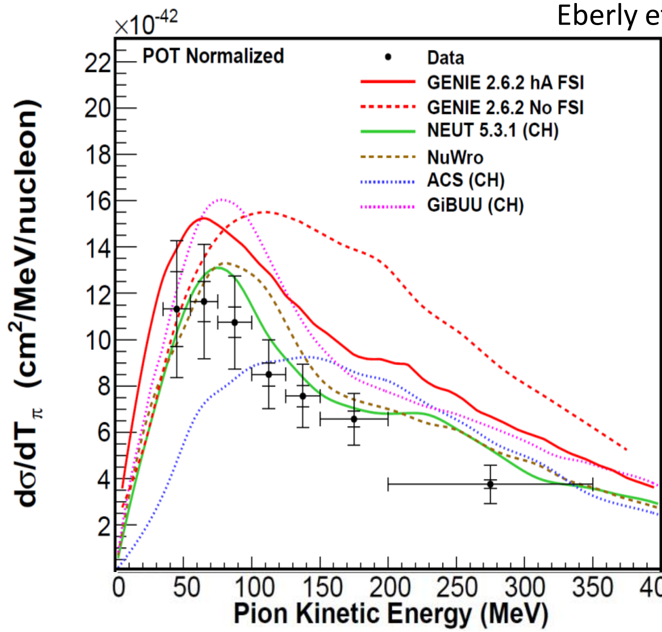
CC1π results in terms of pion variables

MiniBooNE

Rodrigues, AIP Conf. Proc. 1663 (2015)

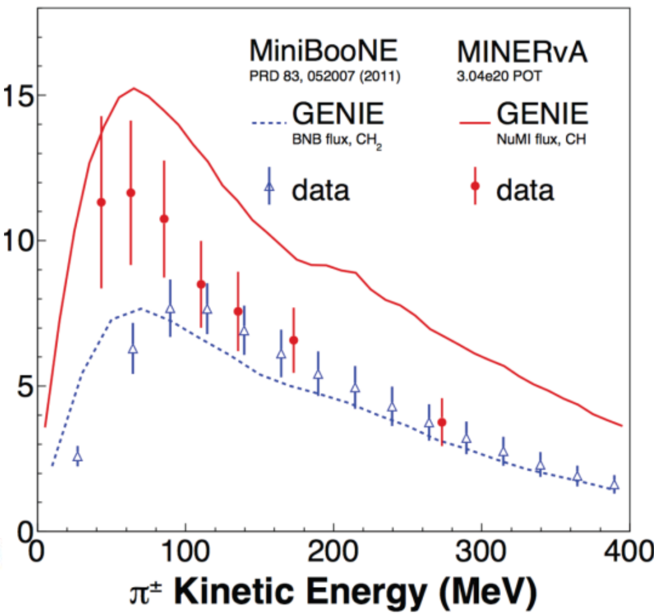


MINERvA

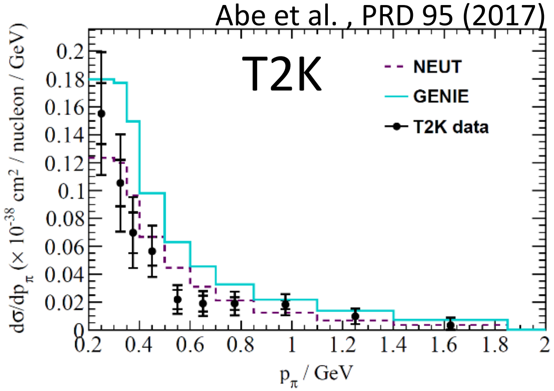


MiniBooNE - MINERvA

Eberly et al. , PRD 92 (2015)



Abe et al. , PRD 95 (2017)



Historically many tensions

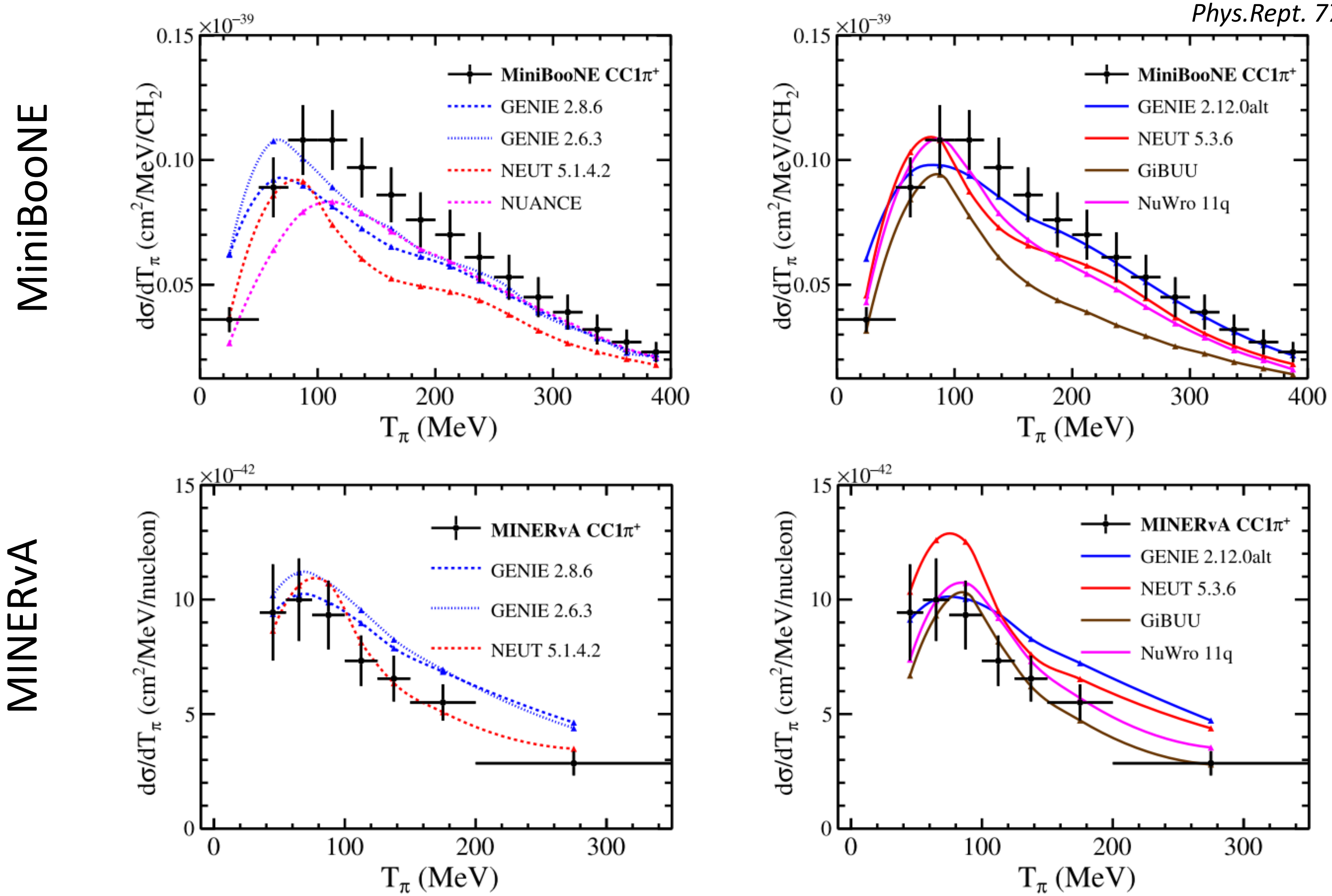
- models .vs. data ??
- models .vs. models??
- data .vs. data (through models)??

the 1π puzzle

Pion puzzle – Tension 2016 Workshop

Old | **New (after Tension)**

M. Betancourt et al.
Phys.Rept. 773-774 (2018) 1-28



- Same models, correct signal definition, proper flux averaging
- Updated flux prediction from MINERvA

Better normalization agreement but shape discrepancies remain

Pion puzzle – Tuning GENIE with MINERvA data (2019)

P. Stowell et al. PRD 100 (2019) 7, 072005

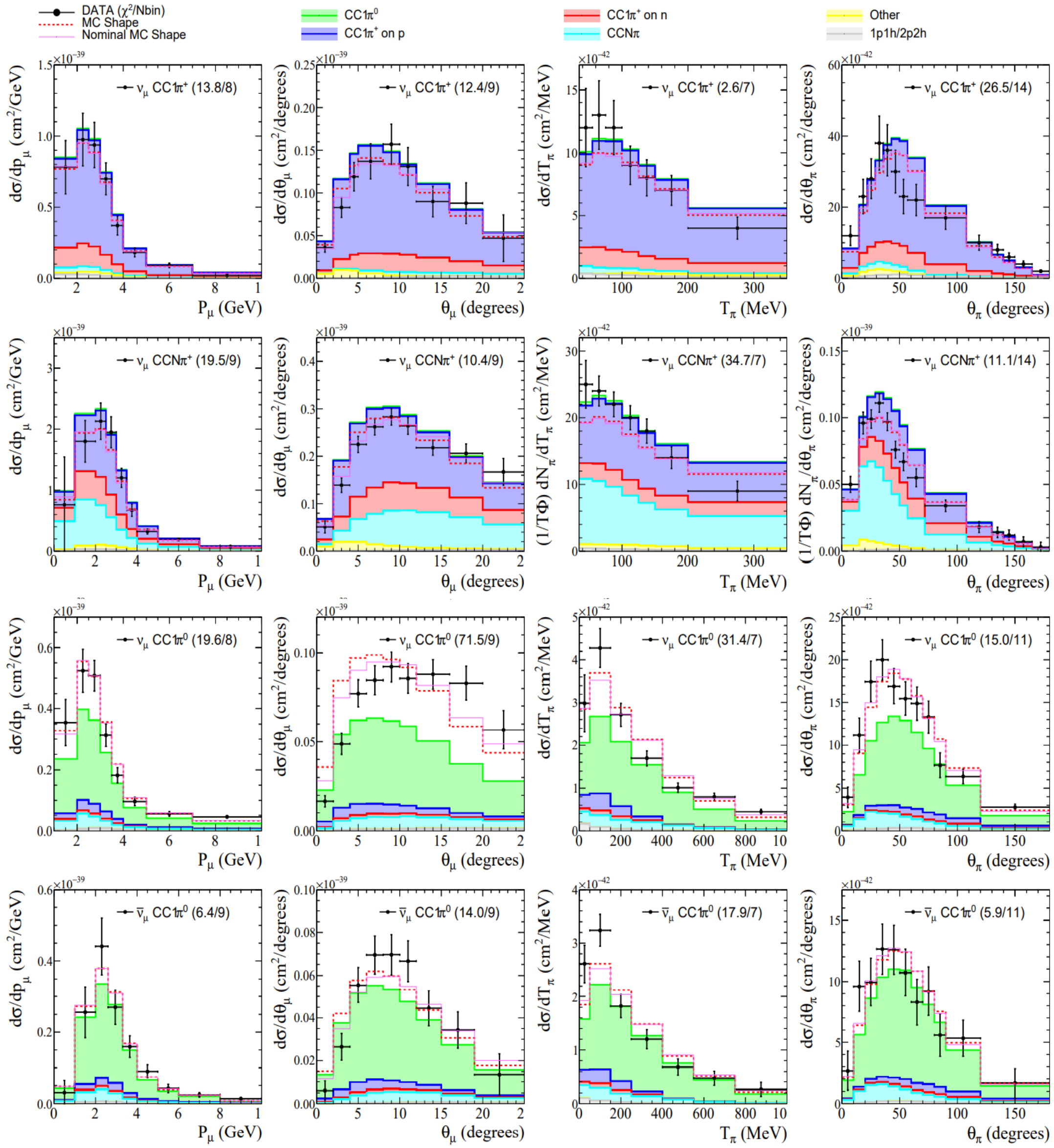
Tune Monte Carlo to simultaneously fit 4 datasets

$\nu_\mu CC1\pi^+$

$\nu_\mu CCN\pi^+$

$\nu_\mu CC1\pi^0$

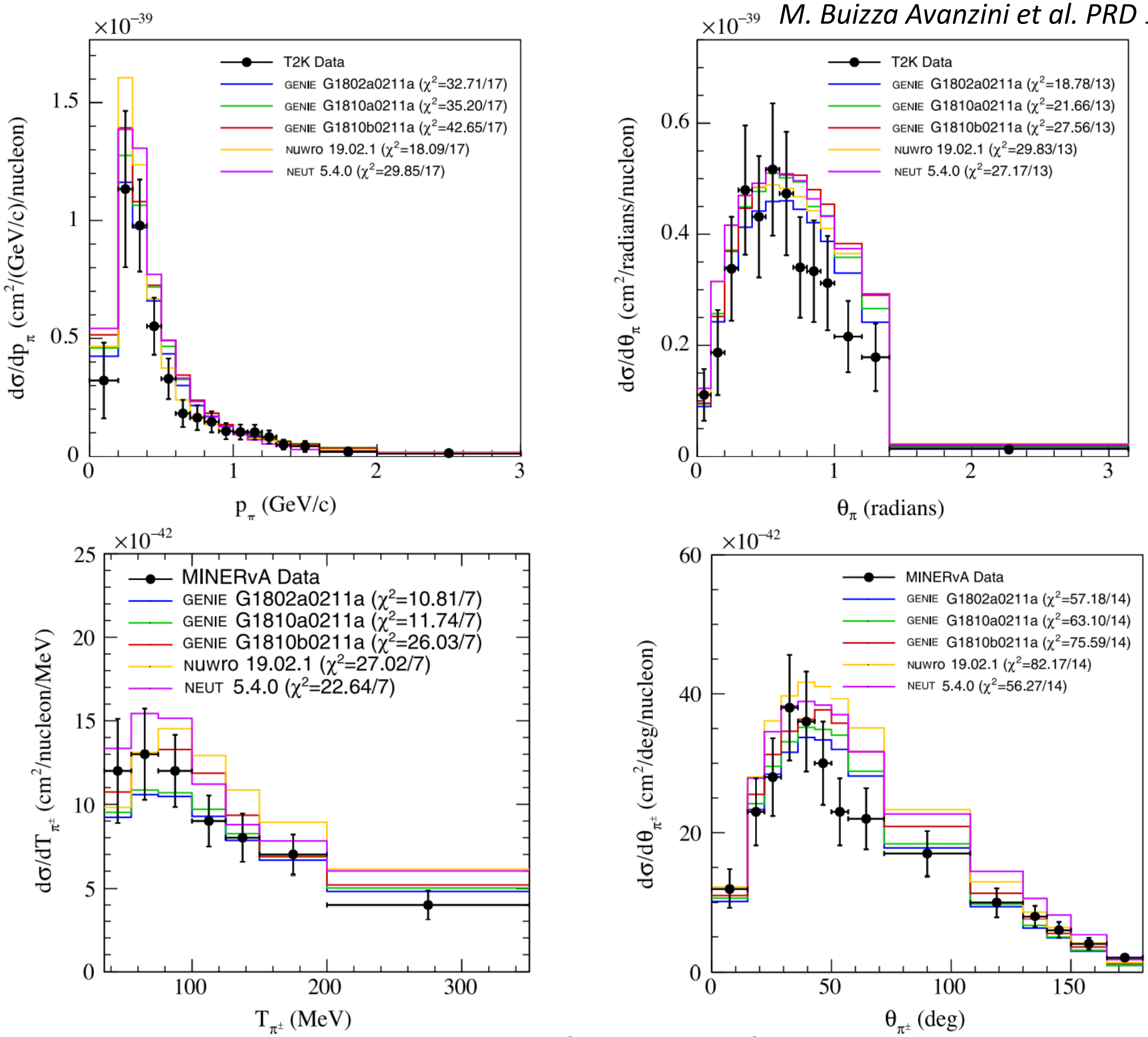
$\overline{\nu_\mu} CC1\pi^0$



The tuning improves the model, but tensions remain

Pion puzzle – T2K and MINERvA data .vs. Monte Carlo (2022)

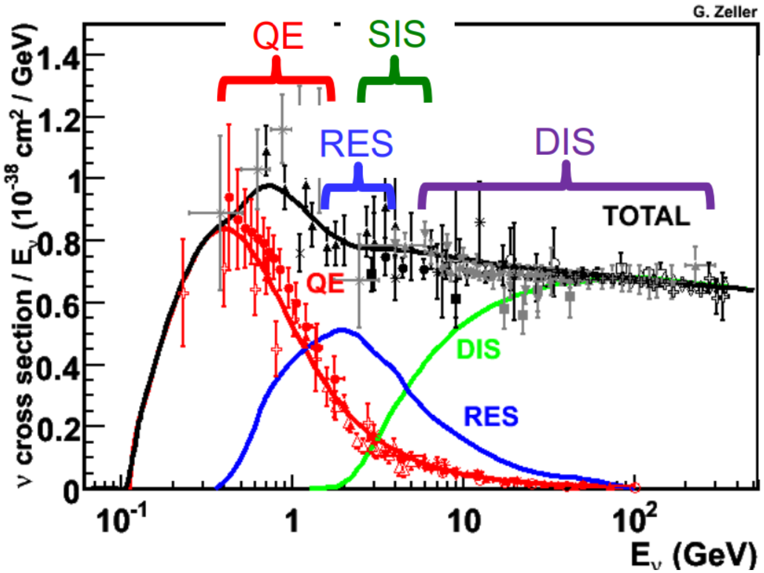
M. Buizza Avanzini et al. PRD 105, 092004 (2022)

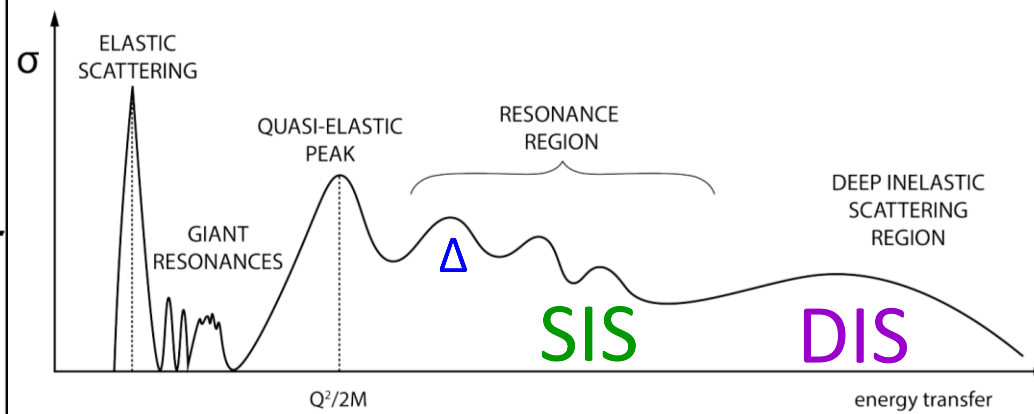


Tensions remain

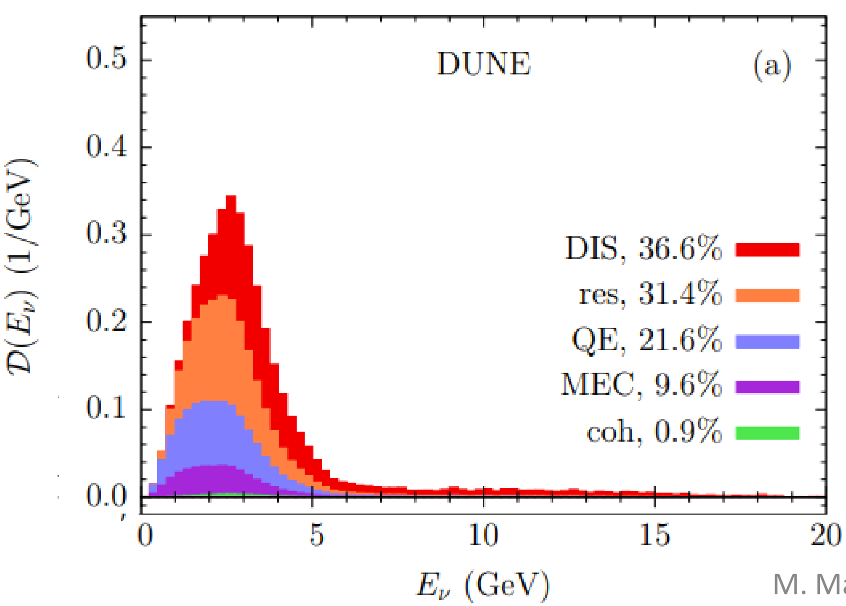
- The generators used to extract the cross section is often the one with the best description of the data
- Experimental collaborations have more advanced analyses in progress
- These studies are Δ dominated interactions
- None of the common event generators include nuclear medium effects for the Δ

Beyond Δ resonance





A. Ankowski



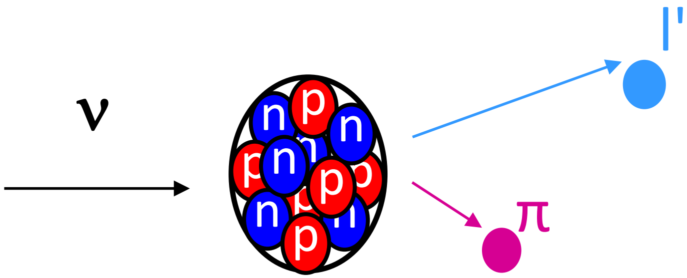
- The complications of pion data analyses lay not only on the modeling of primary production and pion FSI but also on the fact that all hadronic processes related to shallow inelastic scattering (SIS) and DIS regions must be modeled correctly
- SIS and DIS have been minimally studied both experimentally and theoretically with neutrino scattering
- Another major challenge important in particular for DUNE

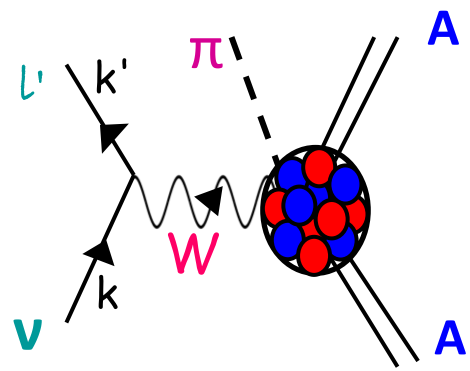
T. Katori, M. Martini, J.Phys.G 45 1, 013001 (2018)
L. Alvarez-Ruso et al. Prog. Part. Nucl. Phys. 100, 1–68 (2018)
M. Sajjad Athar, J. G. Morfín, J.Phys. G 48, 034001 (2021)

The coherent 1π production

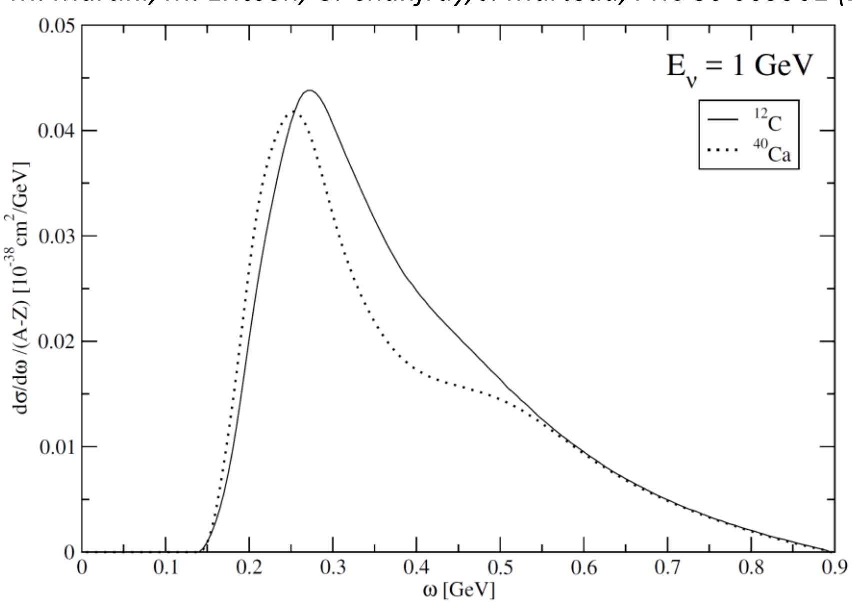
Production of 1 pion with the nucleus remaining in its ground state

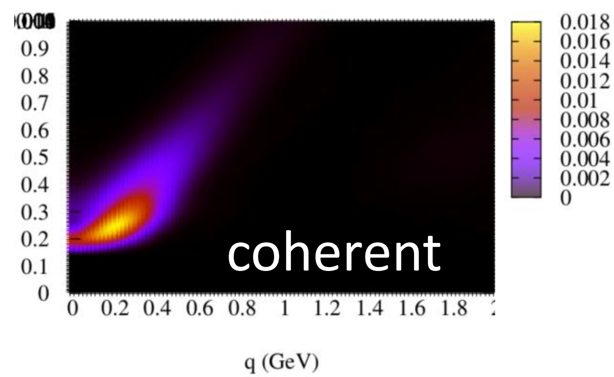
Relatively rare interaction channel, but can mimic oscillation signals

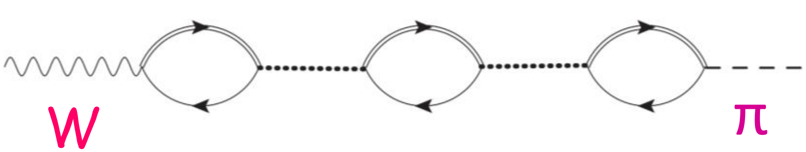




M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)







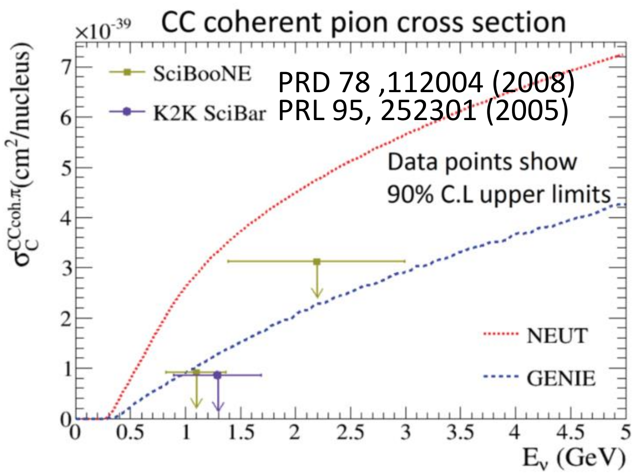
Cross sections reshaped by nuclear collective effects

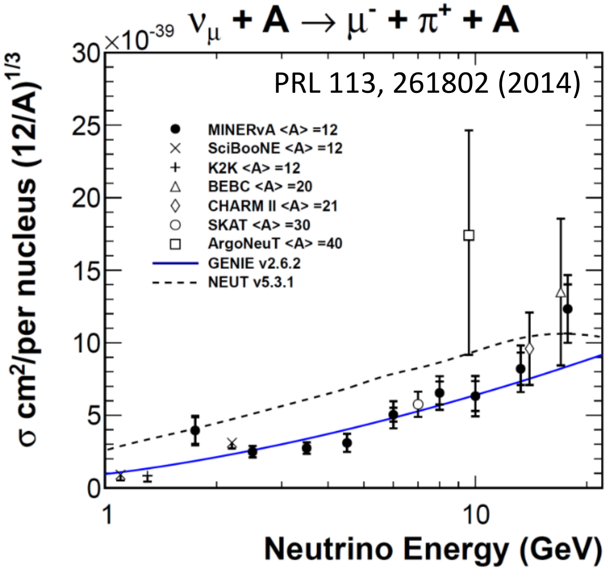
Coherent 1π production experimental results

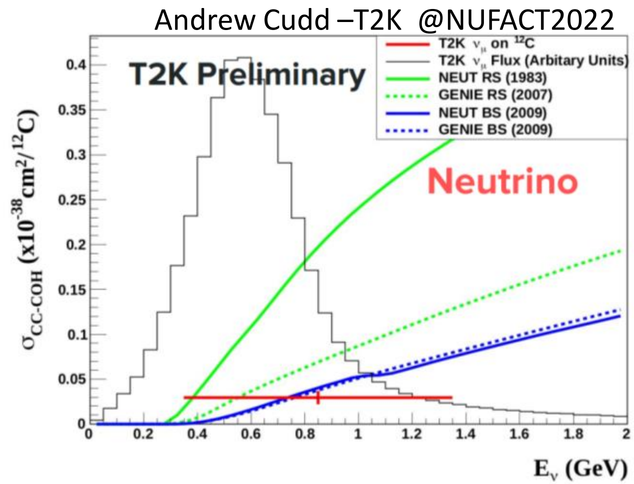
K2K and SciBooNE did not observe coherent π^+ production at neutrino energies ~1GeV

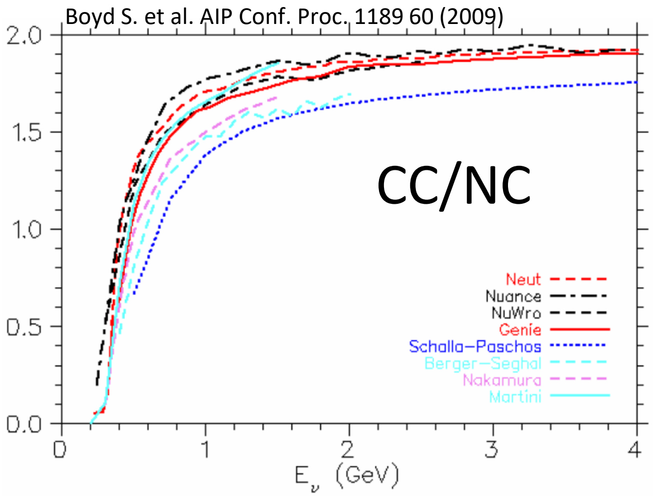
MINERνA and ArgoNeut see evidence for CC coherent pion production

Preliminary T2K cross section measurement: coherent π+ production at neutrino energies ~1GeV









Coherent puzzle at $E_\nu \sim 1$ GeV

Theoretical models:

$$\frac{\pi^+ \text{ coh. CC}}{\pi^0 \text{ coh. NC}} = 1.5 \sim 2$$

SciBooNE:

$$\frac{\pi^+ \text{ coh. CC}}{\pi^0 \text{ coh. NC}} = 0.14^{+0.30}_{-0.28}$$

Kurimoto et al, PRD 81 (2010)

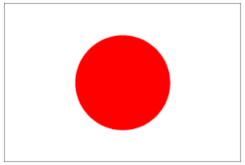
Recent hot topics and perspectives

- Different nuclear targets, in particular Argon
- Semi-inclusive processes (proton detection)
- Single Transverse Kinematics Imbalance

Nuclear targets of present and future LBL oscillation experiments

Present

Future



Carbon: T2K(ND) and NOvA

Oxygen (water): T2K (SuperK) and Hyper-K

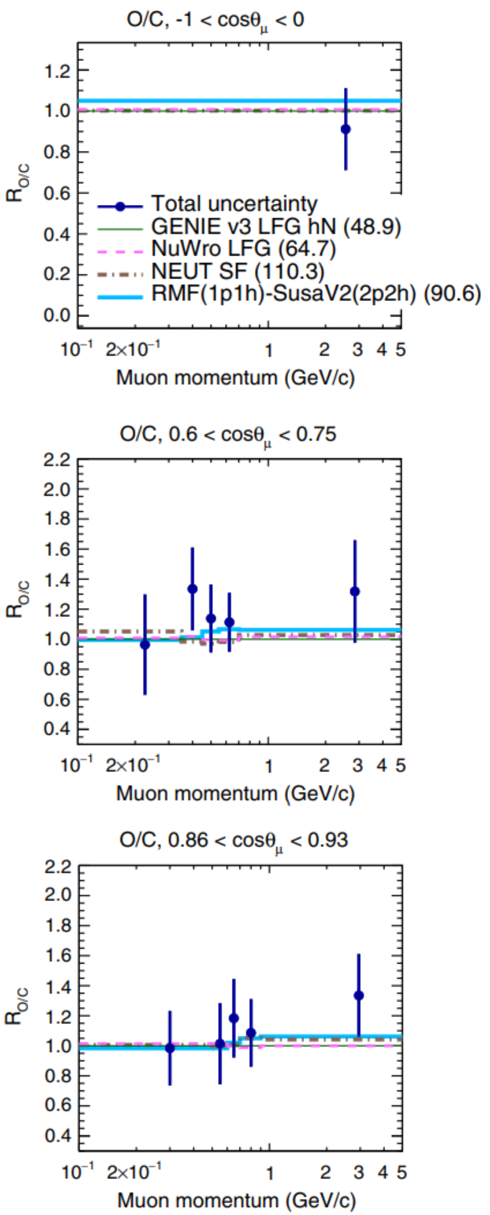
Argon: DUNE

In the last 15 years many cross sections measurements and theoretical studies have been performed for Carbon (^{12}C). Less for Oxygen (^{16}O) and Argon (^{40}Ar)

T2K CC0π d²σ cross sections on oxygen and carbon

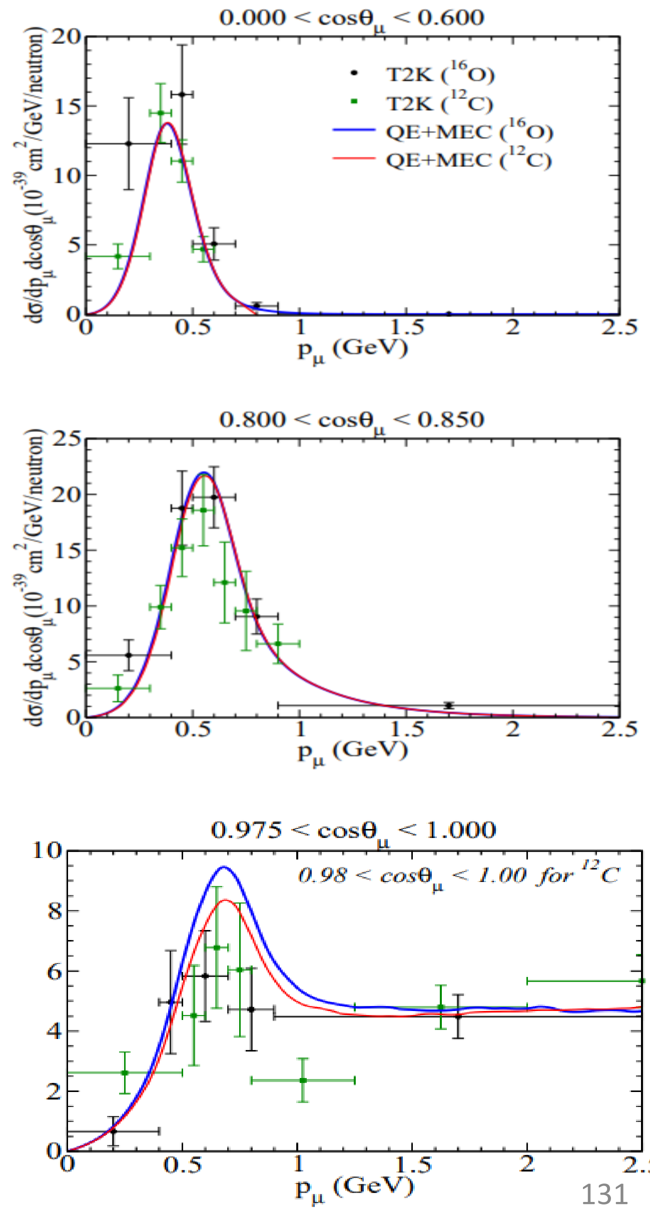
Ratio $^{16}O/^{12}C$ per nucleon

T2K PRD 101 (2020)

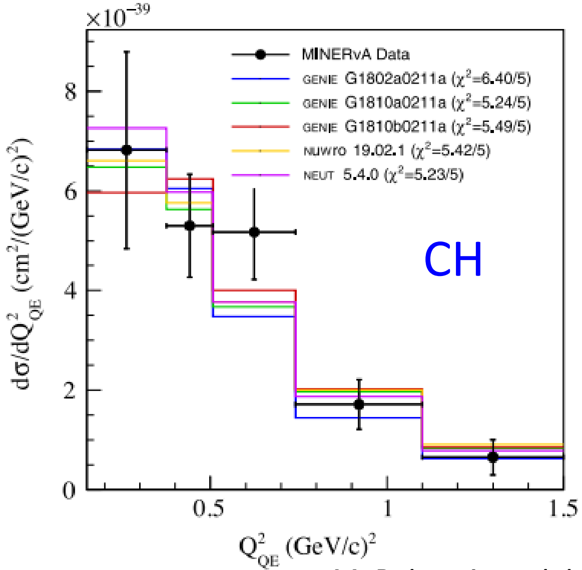


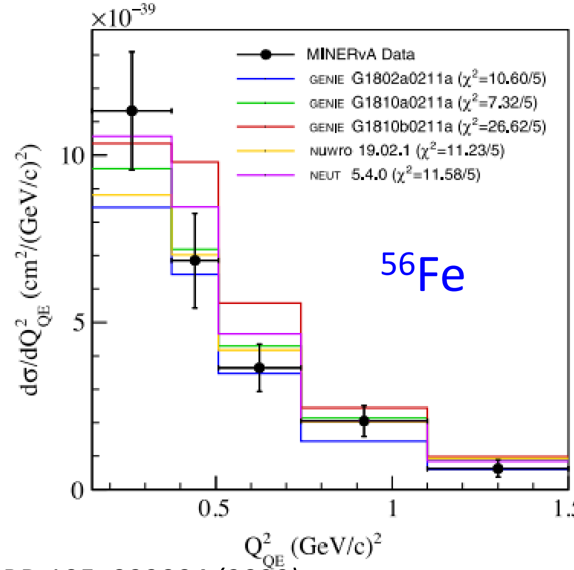
SuSAv2+MEC

Megias et al., JPG46 (2019)

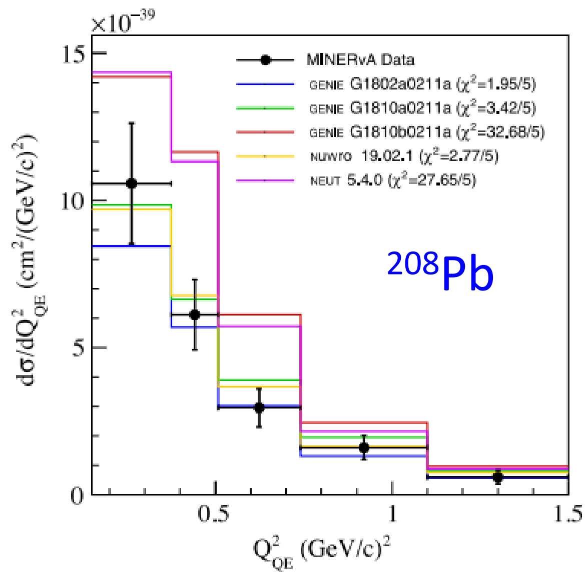


MINERvA CC0π1p(at least) Q^2distributions for carbon, iron, lead





M. Buizza Avanzini et al. PRD 105, 092004 (2022)

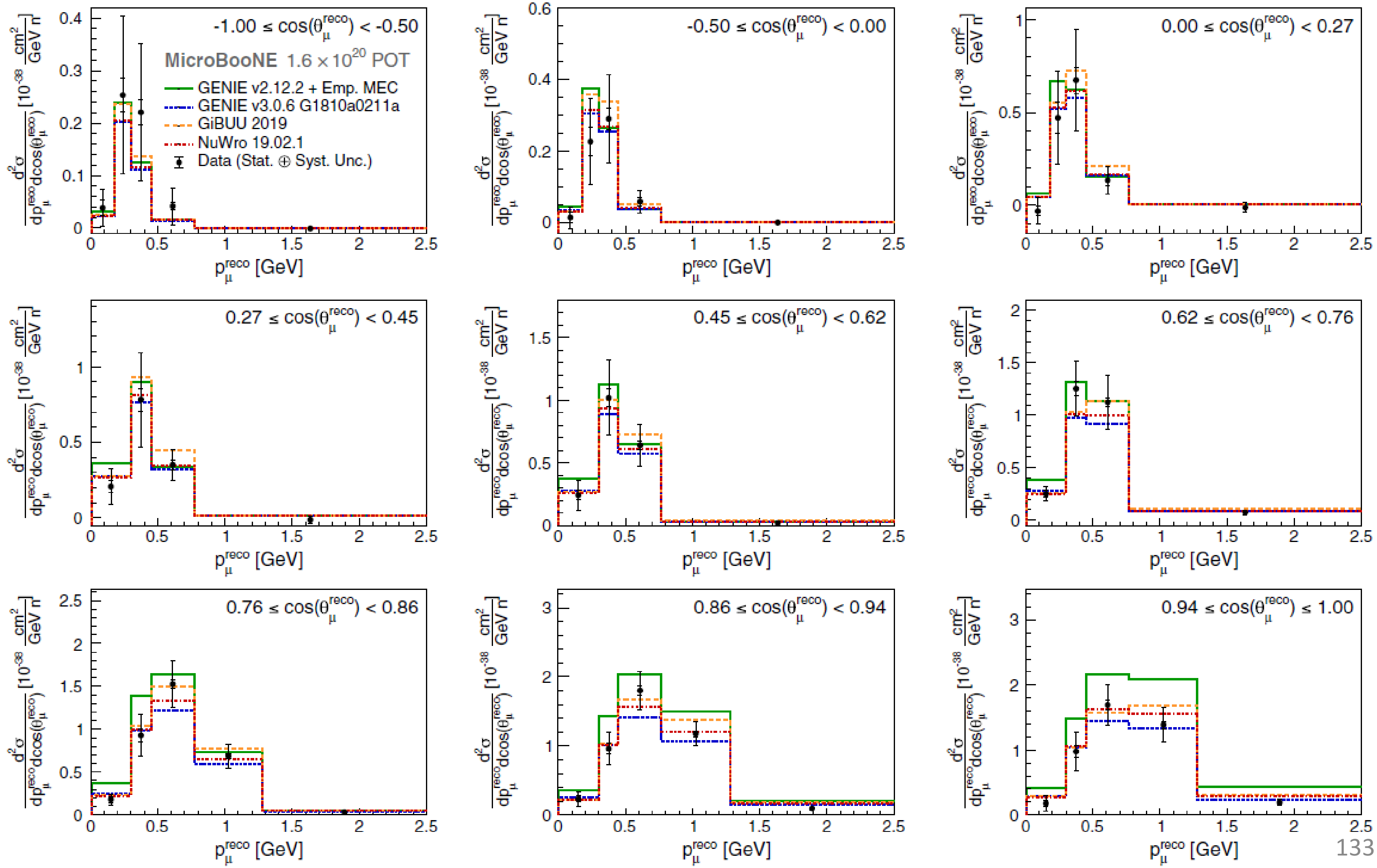


- The spread of distributions predicted by generators increases from carbon to lead
- Most significant deviations are at low Q^2 where nuclear effects are more important

First MicroBooNE measurement on Argon: inclusive $d^2\sigma/dp_\mu d\cos\theta_\mu$

- CC Inclusive: only the charged lepton is detected. All reaction mechanisms contribute
- Inclusive measurements are less affected by background subtraction with respect to exclusive ones
- Inclusive measurements accumulate more rapidly enough statistics of events

PHYSICAL REVIEW LETTERS **123,** 131801 (2019)

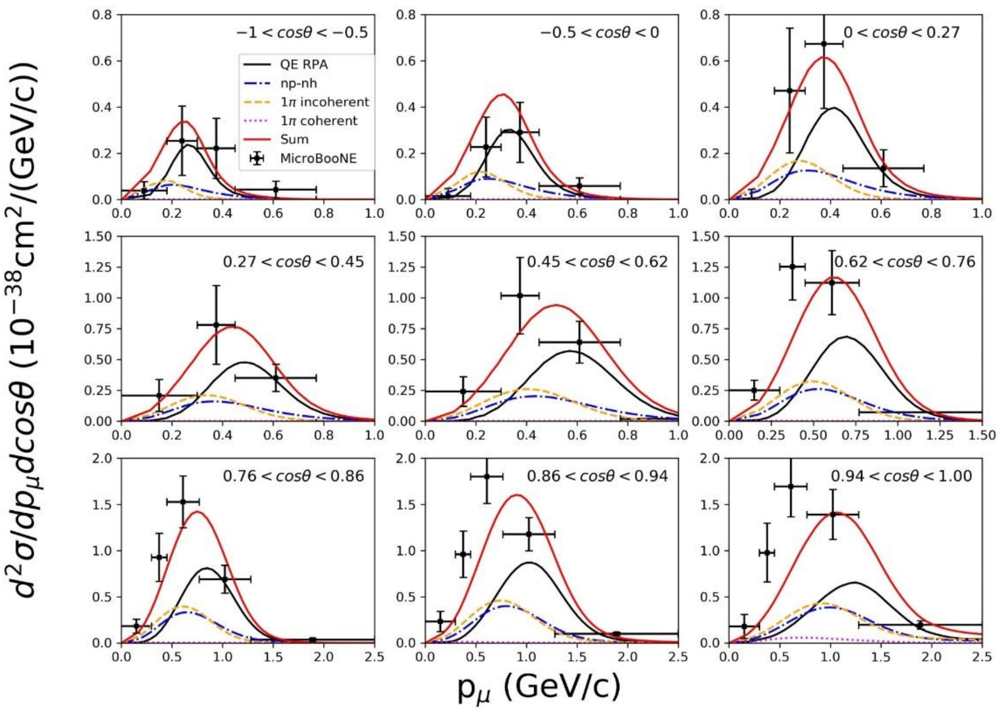


RPA and SuSAv2 calculations of MicroBooNE inclusive $d^2\sigma$ on agon

RPA

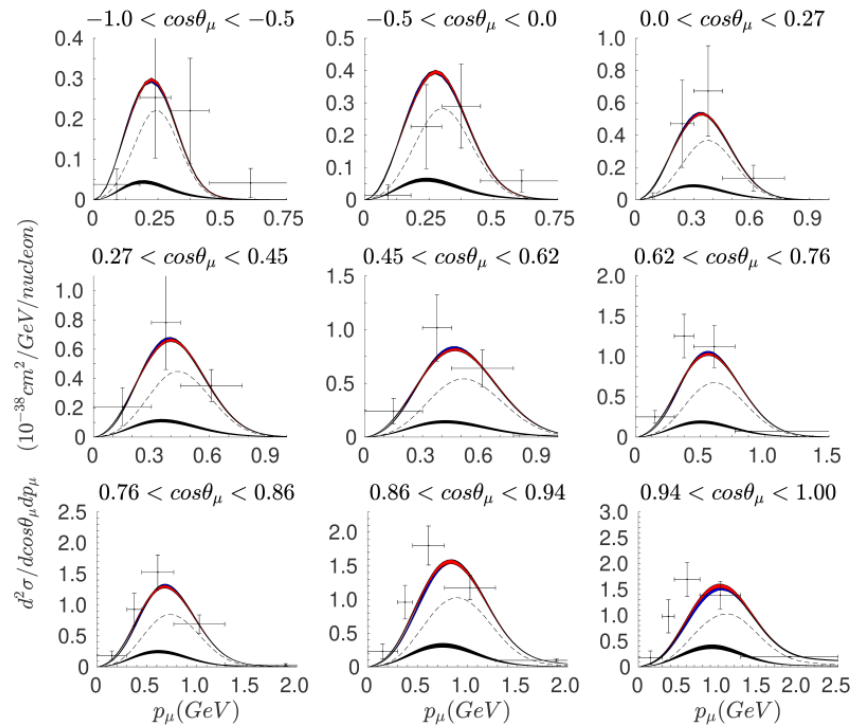
Total = QE + np-nh + 1π inc.+ 1π coh.

M. Martini, M. Ericson, G. Chanfray, PRC 106 (2022)



SuSAv2

Gonzalez-Rosa et al. PRD 105 (2022)



Results also with SuSA
Barbaro et al. Universe 7 (2021)

- Reasonable overall agreement, though not as good as in the ^{12}C T2K inclusive case (see next slide)
- At backward angles the predictions of the different models are slightly shifted to lower values of p_μ, whereas the reverse occurs at forward angles

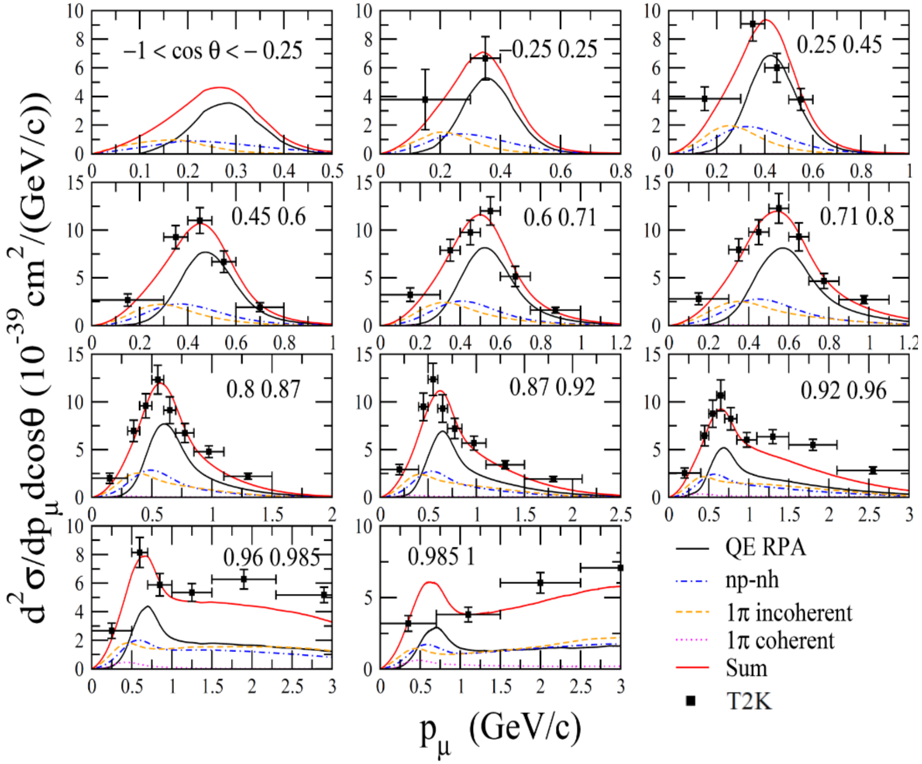
RPA and Monte Carlos calculations of T2K inclusive $d^2\sigma$ on carbon

PHYSICAL REVIEW D **98**, 012004 (2018)

Measurement of inclusive double-differential ν_μ charged-current cross section with improved acceptance in the T2K off-axis near detector

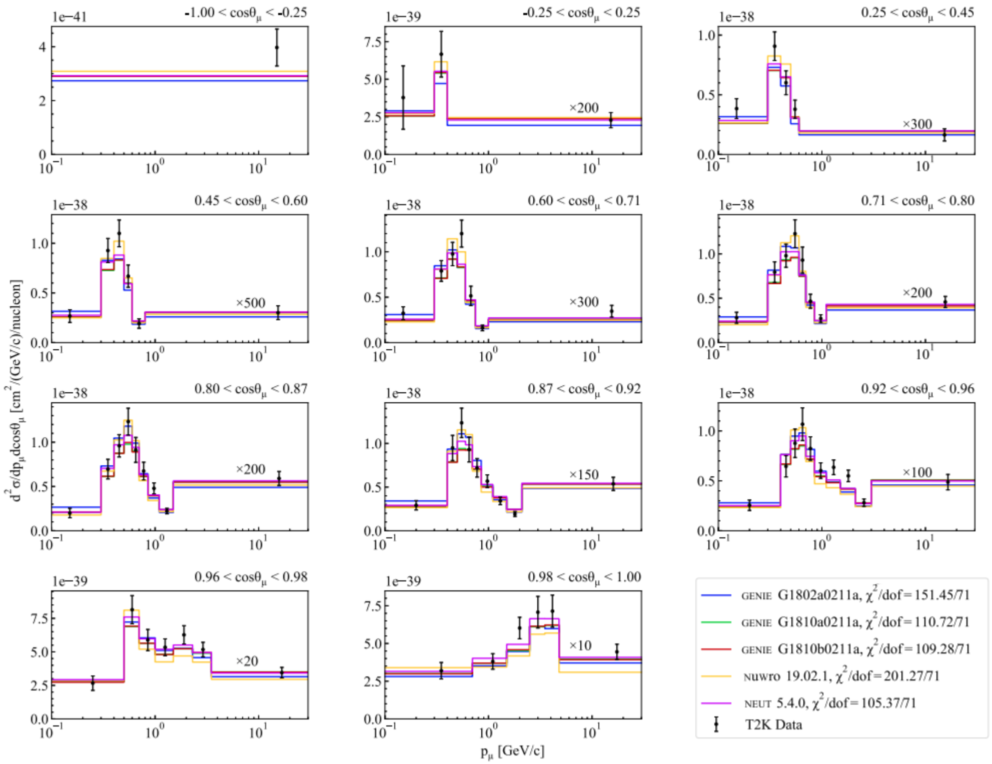
RPA

M. Martini, M. Ericson, G. Chanfray, PRC 106, 015503 (2022)



Monte Carlo

M. Buizza Avanzini et al. PRD 105, 092004 (2022)

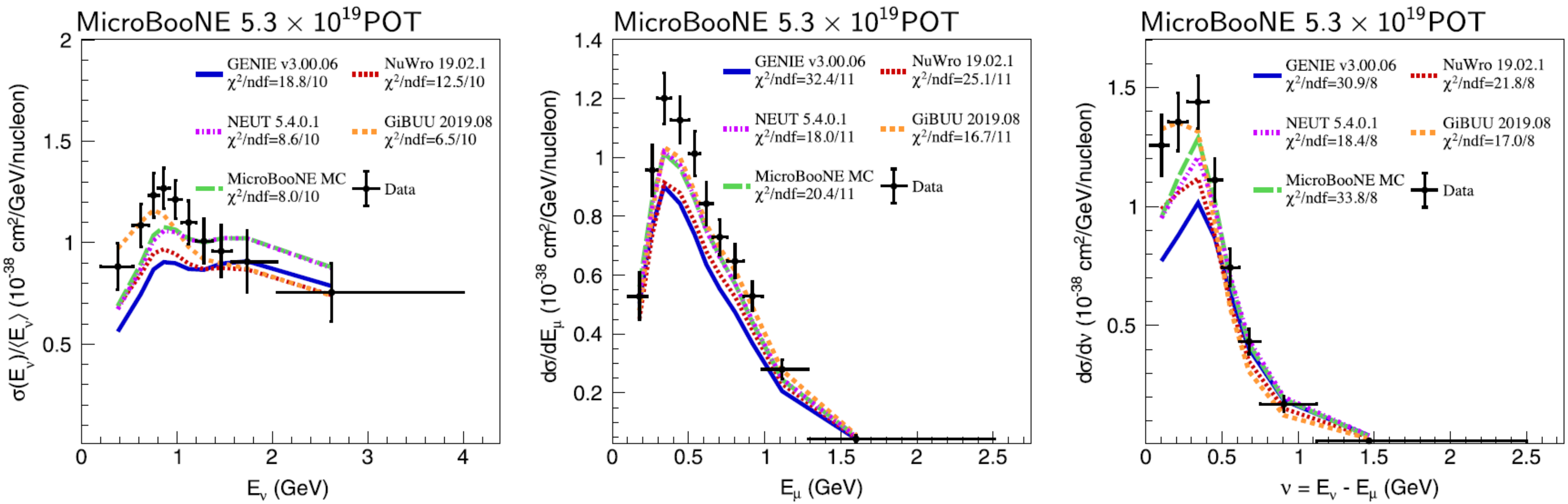


Remarkable agreement

Recent energy-dependent inclusive MicroBooNE cross sections on argon

PHYSICAL REVIEW LETTERS **128,** 151801 (2022)

First Measurement of Energy-Dependent Inclusive Muon Neutrino Charged-Current Cross Sections on Argon with the MicroBooNE Detector



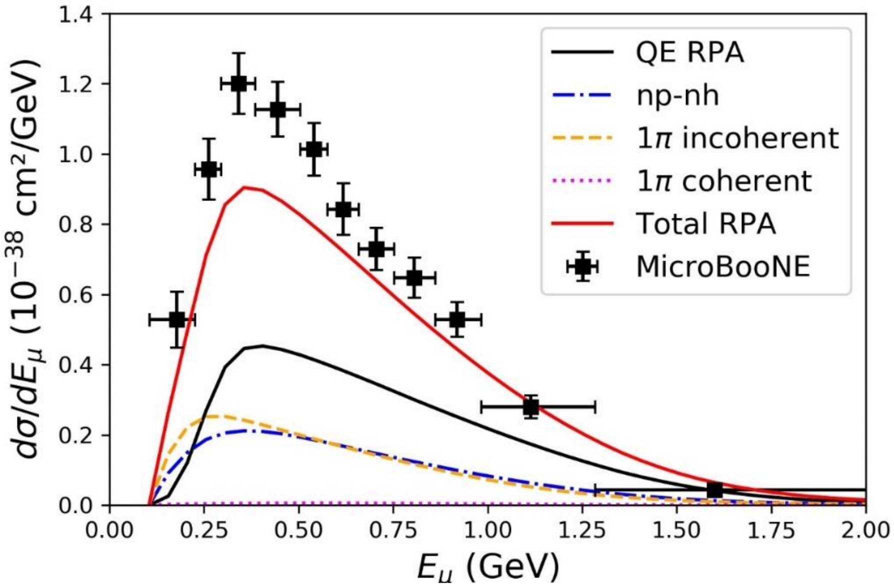
Experimental results presented for the first time as a function of true neutrino energy E_ν and transferred energy (ν or ω)

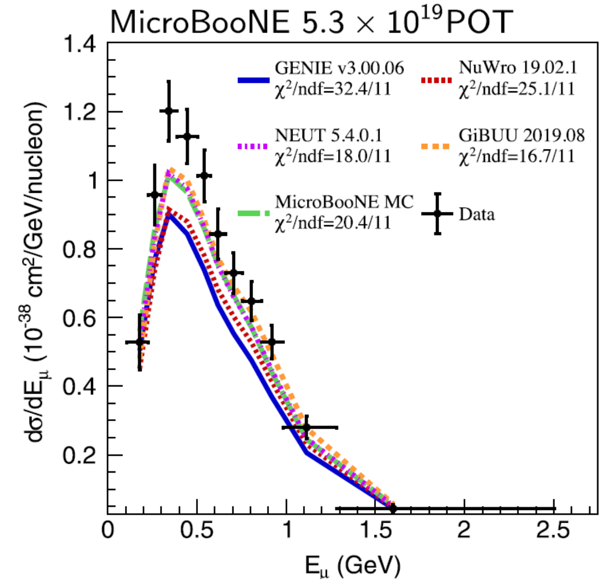
This has been made possible by a new procedure (based on the comparison between the data and the Monte Carlo predictions constrained on the lepton kinematics) allowing the mapping between the true E_ν and ω on one hand, and the reconstructed neutrino energy E_ν^{rec} and hadronic energy E_{had}^{rec} on the other hand

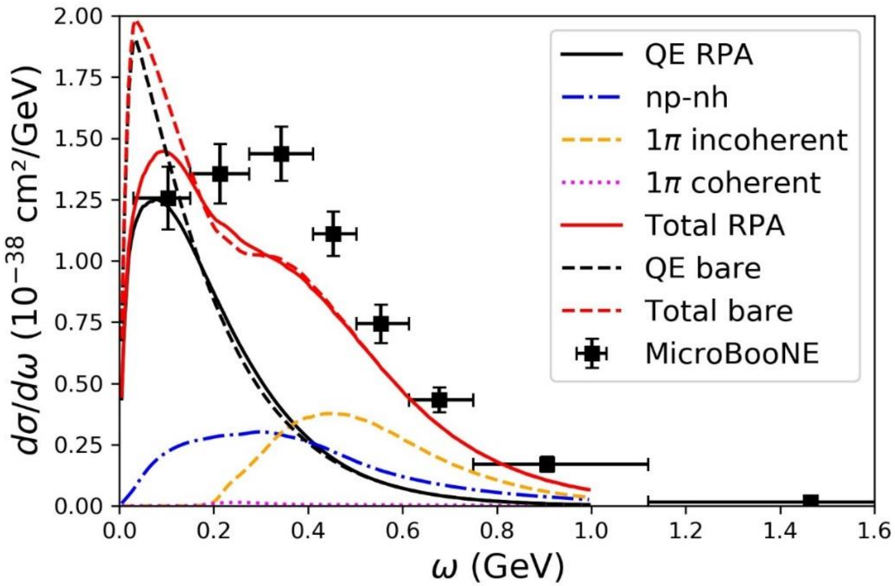
MicroBooNE flux-averaged inclusive $d\sigma/dE_\mu$ and $d\sigma/d\omega$ on argon

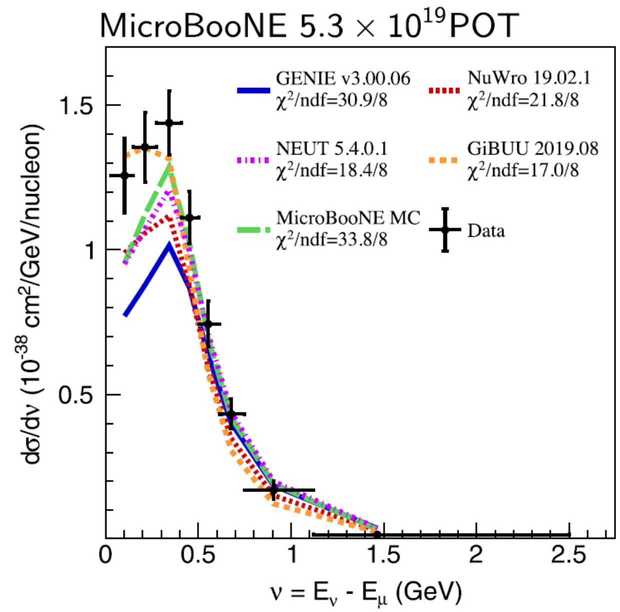
M. Martini, M. Ericson, G. Chanfray, Phys. Rev. C 106, 015503 (2022)

PRL 128, 151801 (2022)







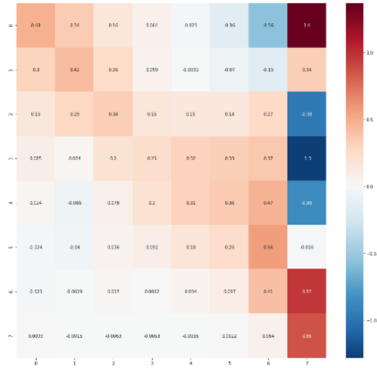


$d\sigma/d\omega$ allows a better separation of the different channels

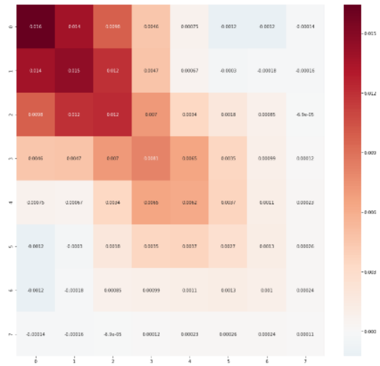
Quantitative analysis of MicroBooNE inclusive dσ/dω on argon

MicroBooNE shared additional smearing and covariant matrices for quantitative analysis

Additional Smearing Matrix



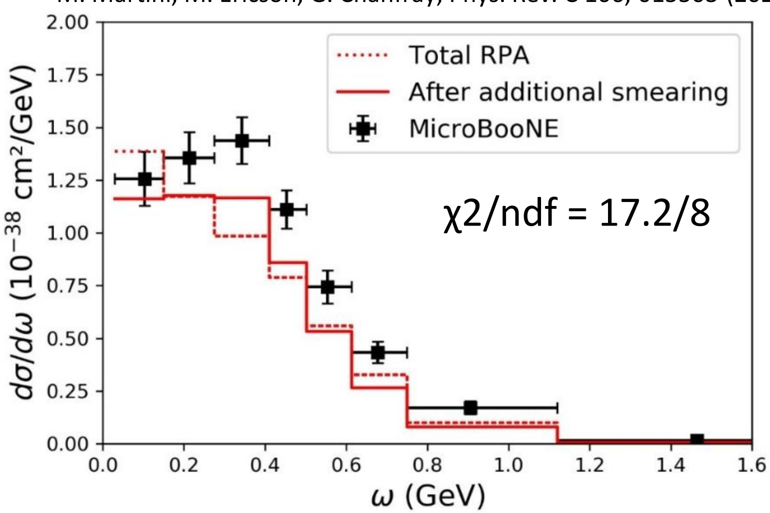
Covariant Matrix



$$\sigma_{smeared} = M_{add\_smr} \times \sigma_{model}$$

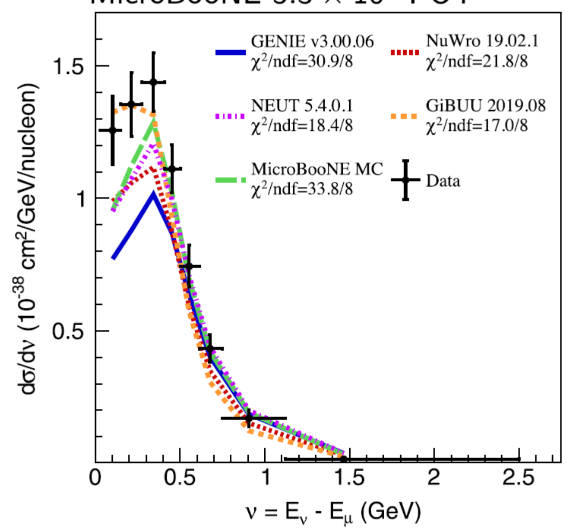
$$\chi^2 = (M - P)^T \times Cov^{-1}_{full}(M, P) \times (M - P)$$

M. Martini, M. Ericson, G. Chanfray, Phys. Rev. C 106, 015503 (2022)



PRL 128, 151801 (2022)

MicroBooNE 5.3 × 10^{19}POT



- RPA χ2/ndf=17.2/8. Comparable with the one of GiBUU and better than all the Monte Carlo predictions
- A possible reason is that GENIEv3, MicroBooNE MC, NEUT and NuWro implement np-nh contribution deduced by Nieves et al. model. This contribution is smaller than the one of other evaluations (GiBUU, Martini et al,…)

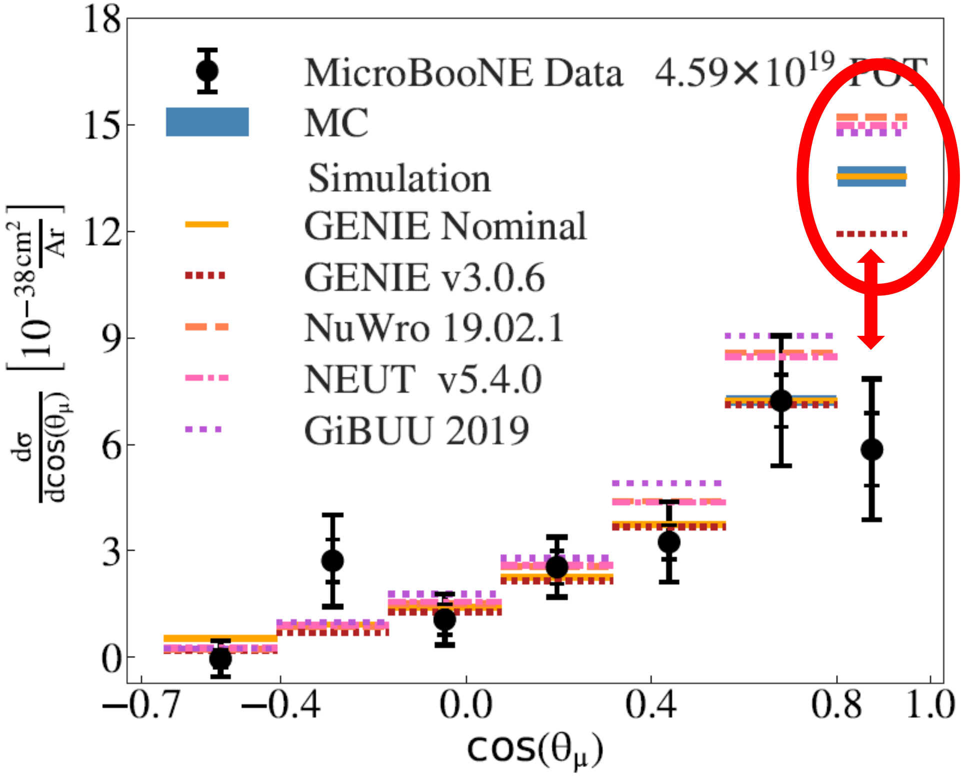
MicroBooNE semi-inclusive CC0π1p on argon

PHYSICAL REVIEW LETTERS **125,** 201803 (2020)

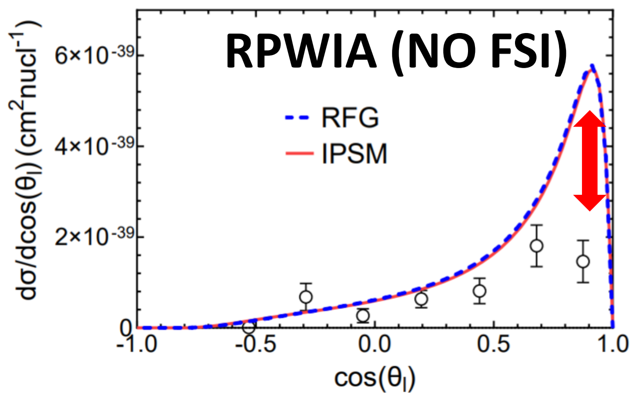
First Measurement of Differential Charged Current Quasielasticlike ν_μ-Argon Scattering Cross Sections with the MicroBooNE Detector

?! CCQE-like with another meaning than in the past

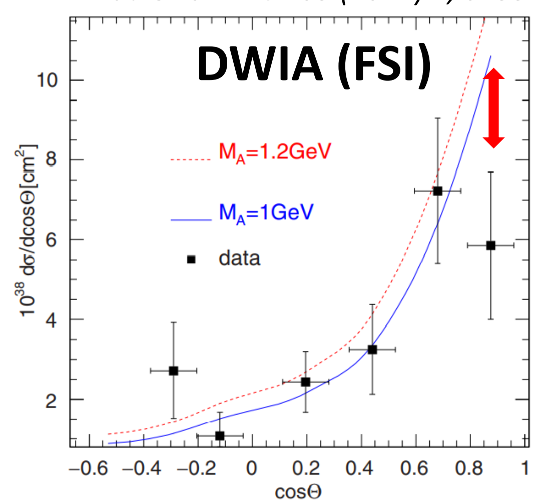
MicroBooNE PRL 125(2020)



J. M. Franco-Patino et al. PRD 104 (2021) 7, 073008



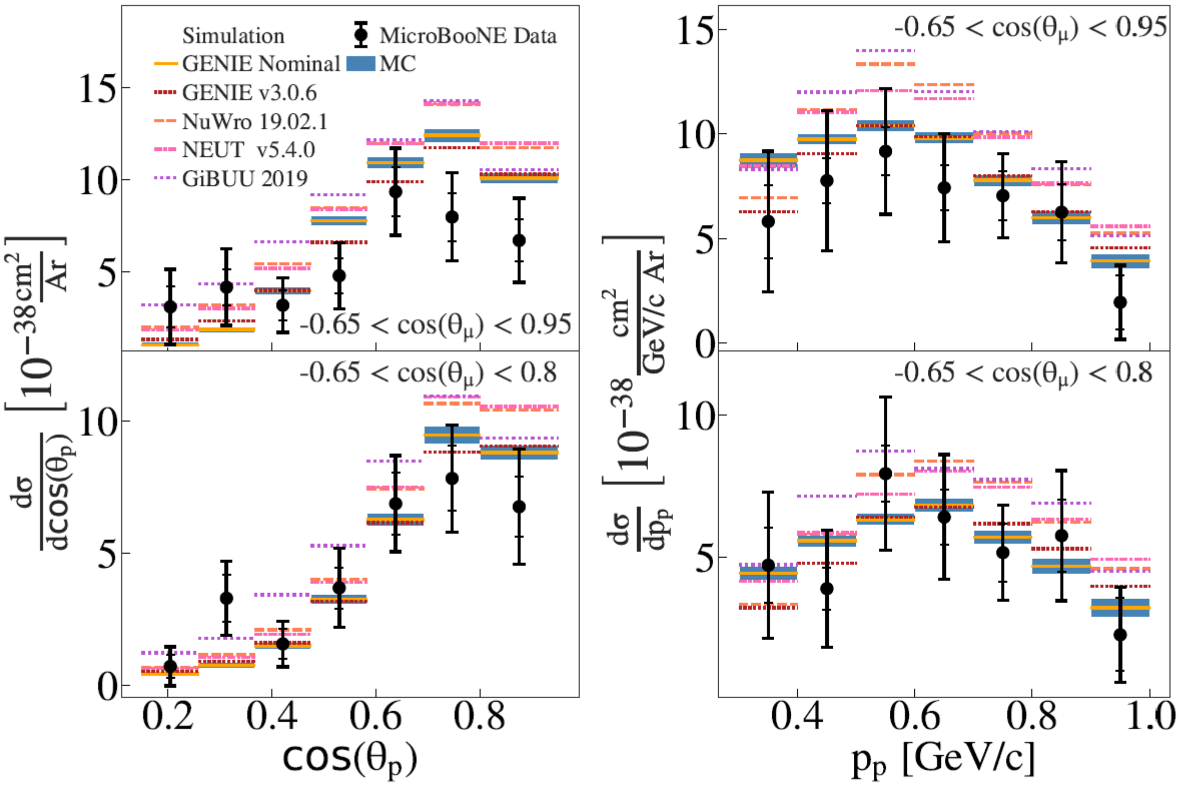
A.V. Butkevich PRC 105 (2022) 2, 025501



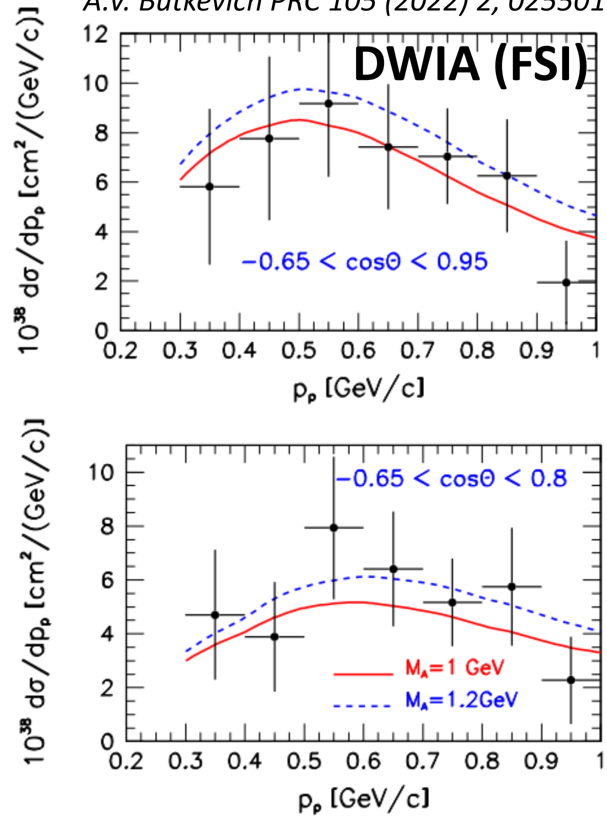
Overestimation in the muon forward direction

MicroBooNE semi-inclusive CC0π1p on argon versus proton variables

MicroBooNE PRL 125(2020)



A.V. Butkevich PRC 105 (2022) 2, 025501

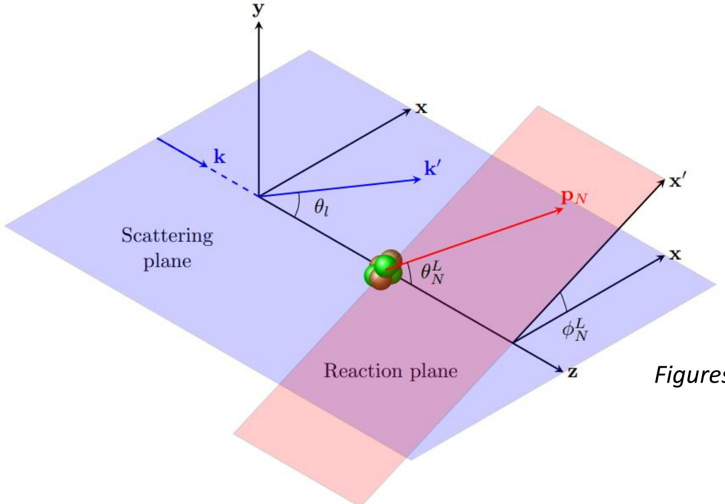


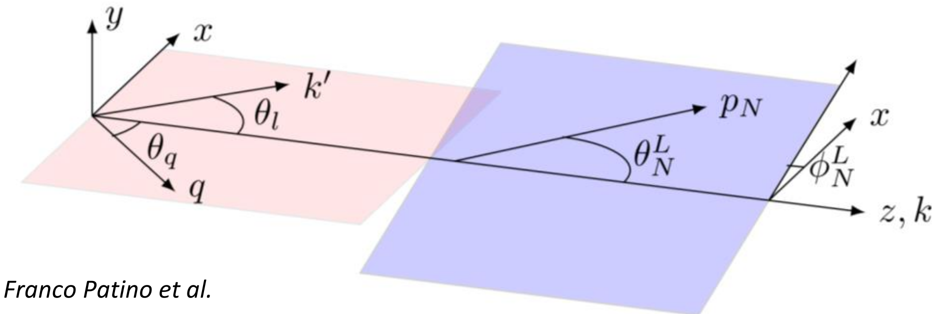
- Poor Monte Carlo – data agreement
- Spread of Monte Carlo predictions

How good are the approximations (use inclusive models, factorization) of the present MC?

The semi-inclusive neutrino cross section

There is a rapidly increasing interest on semi-inclusive cross sections





Figures by J. M. Franco Patino et al.

M. B. Barbaro talk @NUFACT 2021

$$\mathcal{F}_\chi^2 = L_{\mu\nu} W^{\mu\nu}$$
$$= V_{CC}R^{CC} + 2V_{CL}R^{CL} + V_{LL}R^{LL} + V_T R^T + V_{TT}R^{TT} + V_{TC}R^{TC} + V_{TL}R^{TL} + \chi\left(V_{T'}R^{T'} + V_{TC'}R^{TC'} + V_{TL'}R^{TL'}\right)$$

The $(\nu_\mu, \mu p)$ cross section is decomposed in **10 independent response functions** of **5 variables** $(\omega, q, \mathbf{p_N})$.
More complex structure than in the **inclusive** (ν_μ, μ) case: **5 new responses**, which vanish after integration over the final nucleon variables

$R^{TT,TC,TL,TC',TL'} \propto \cos(\phi), \cos(2\phi)$ ϕ outgoing nucleon azimuthal angle

Semi-inclusive —> Inclusive (but not viceversa!)

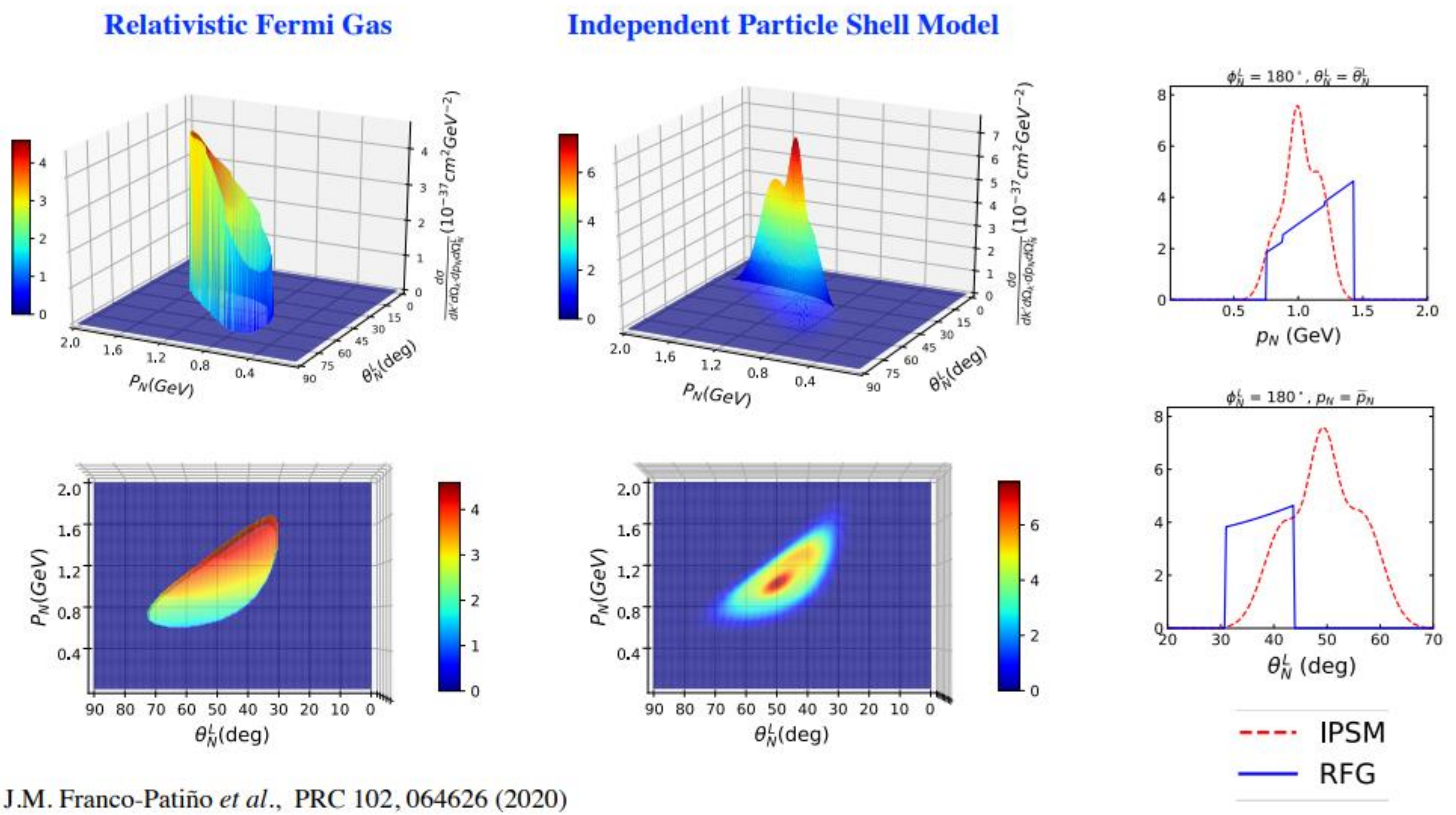
Theoretical situation:
- few models and papers for genuine CCQE [*J. M. Franco Patino et al, PRC 102 (2020); PRD 104 (2021), 2207.02086;* A. V. Butkevich *PRC 105 (2022)]*
- one (incomplete due to the absence of Δ-MEC) model for 2p-2h [T. Van Cuyck et al. PRC 94 (2016); PRC 95 (2017)]

Semi-inclusive cross section: impact of different initial state modeling

M. Barbaro talk @IPSA 2022

6-differential semi-inclusive cross section $(\nu_\mu, \mu p)$ nucleon knock-out

$^{40}Ar(\nu_\mu, \mu^- p)^{39}Cl$ DUNE flux $k' = 1.5\,\text{GeV}, \theta_\mu = 30^0, \phi_N^L = \pi$



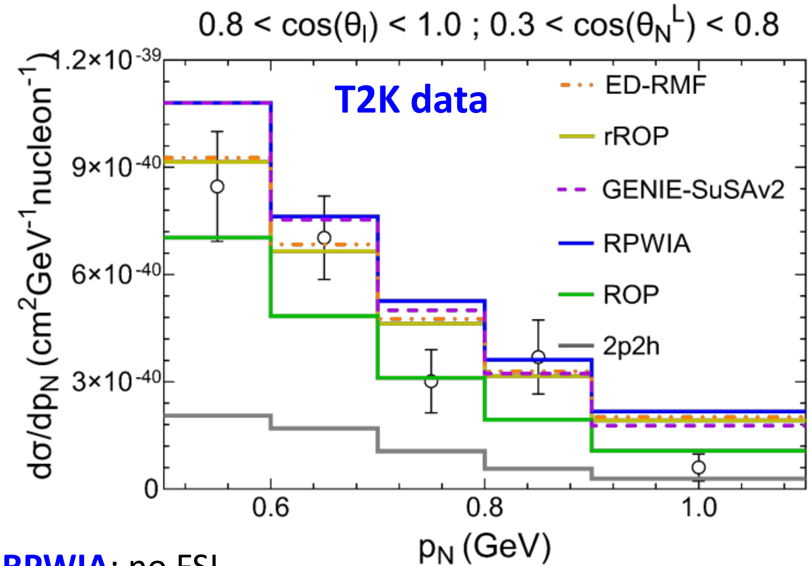
J.M. Franco-Patiño *et al.*, PRC 102, 064626 (2020)

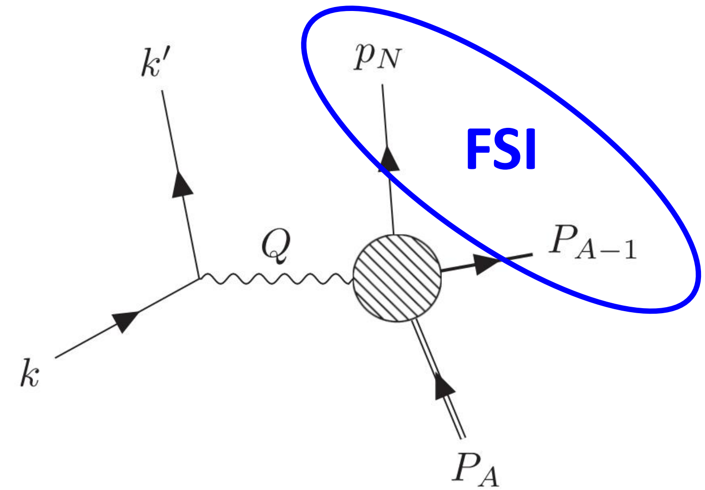
Relativistic Plane Wave Impulse Approximation (no FSI included)

Striking differences in the cross section due to initial state physics described by different spectral functions.
The precise knowledge of the SF is crucial for a reliable modelling of semi-inclusive reactions.

Semi-inclusive CC0π cross section on carbon: role of proton FSI

Adapted from M. Barbaro talk @IPSA 2022

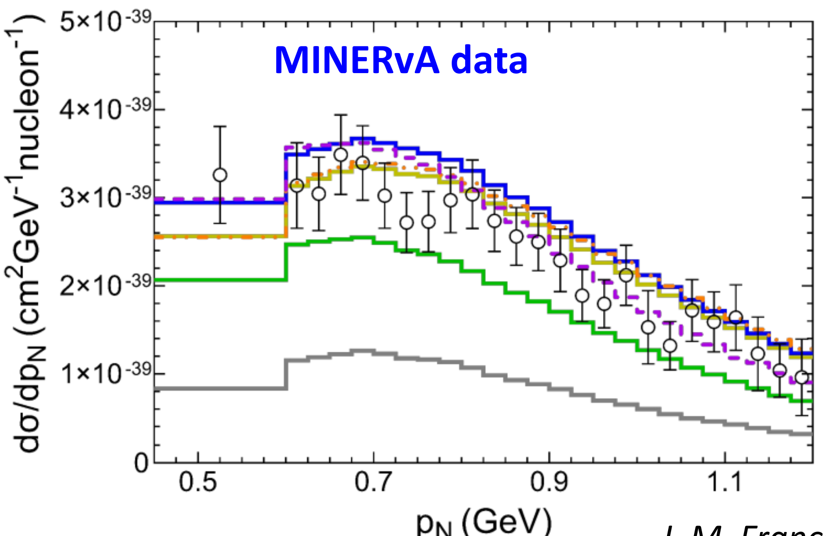




RPWIA: no FSI

GENIE-SuSAv2: include FSI but from inclusive model (factorization)

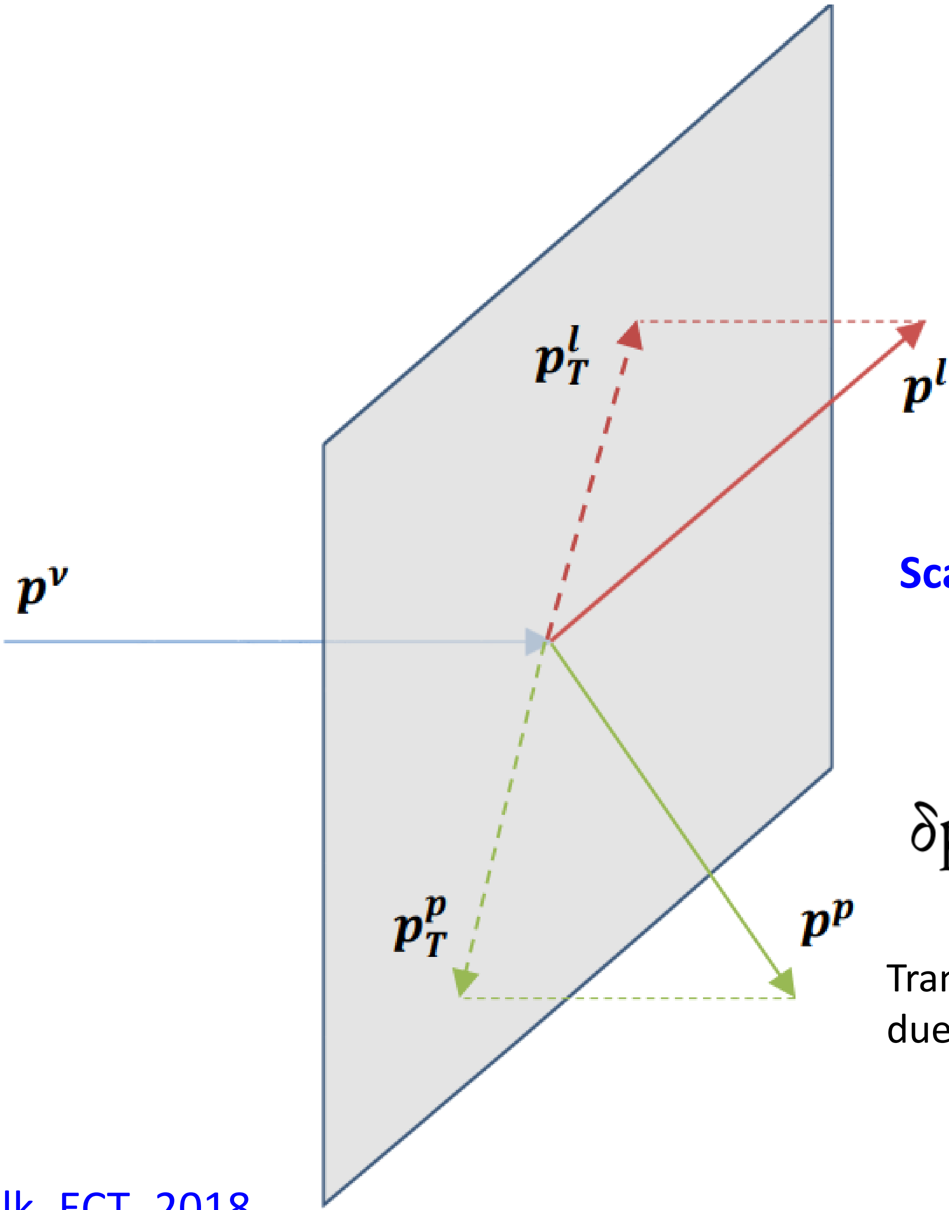
ED-RMF, **rROP**, **ROP**: different theoretical approaches for FSI

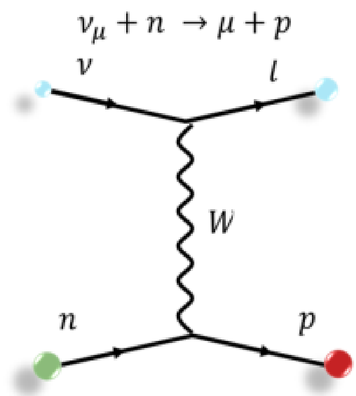


- FSI improve the agreement with data respect to the RPWIA prediction
- Ambiguity in the way of implementing FSI, which the data error bars are not sufficient to resolve
- 2p2h give non-negligible contribution

J. M. Franco-Patino et al. , arXiv 2207.02086

Single Transverse Kinematic Variables





No nuclear Effects

Scattering on a free nucleon at rest

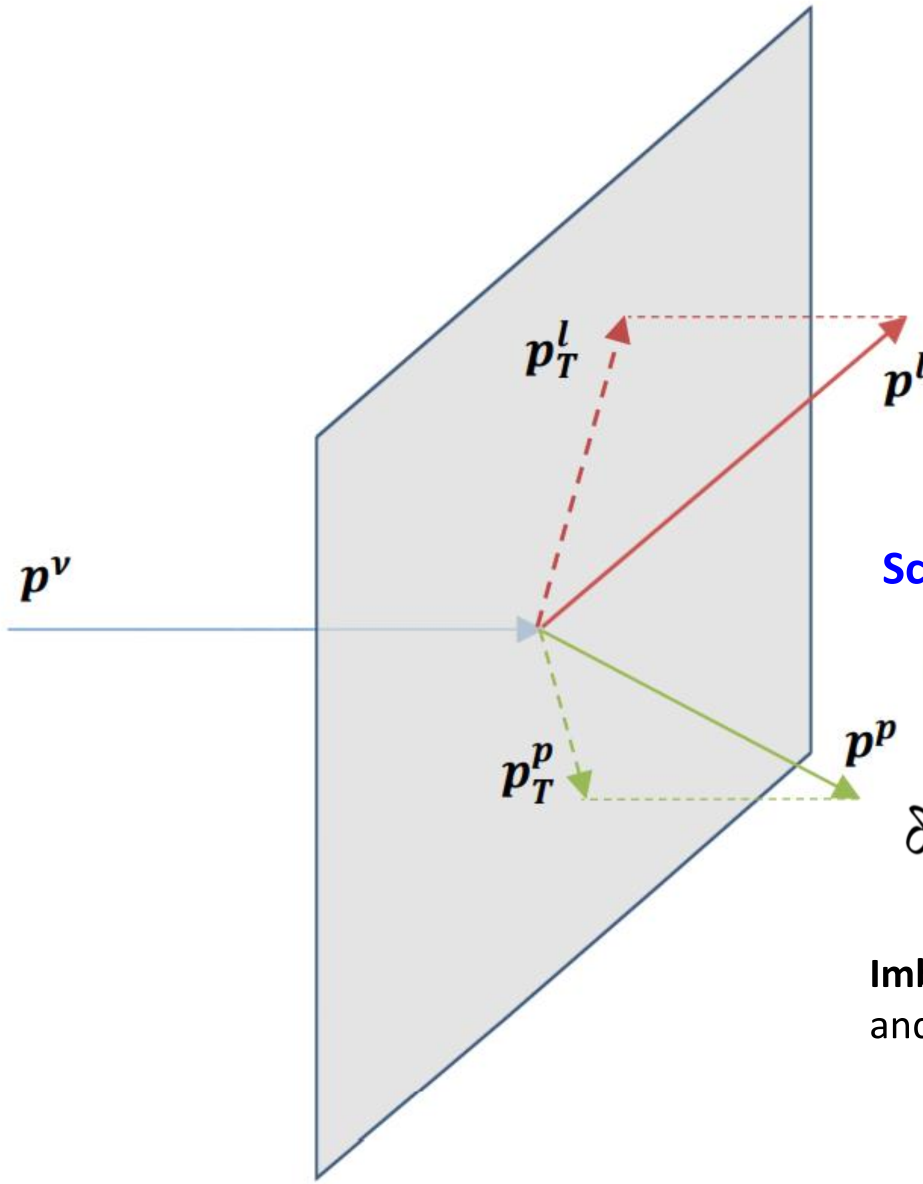
$$p^l_T = -p^p_T$$

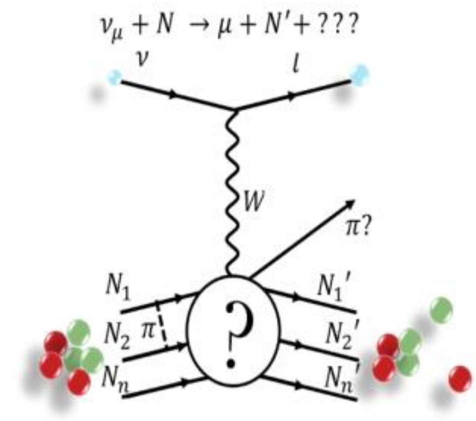
$$\delta p_T = |\, \mathbf{p_T}^\mu + \mathbf{p_T}^p \,| = 0$$

Transverse projections equal and opposite due to momentum conservation

S_Dolan_Talk_ECT_2018

Single Transverse Kinematic imbalance (STKI)





With Nuclear Effects

Scattering on nucleus

$$\boldsymbol{p_T^l} \neq -\boldsymbol{p_T^p}$$

$$\delta p_T = |\, \mathbf{p_T}^{\mu} + \mathbf{p_T}^{p} \,| > 0$$

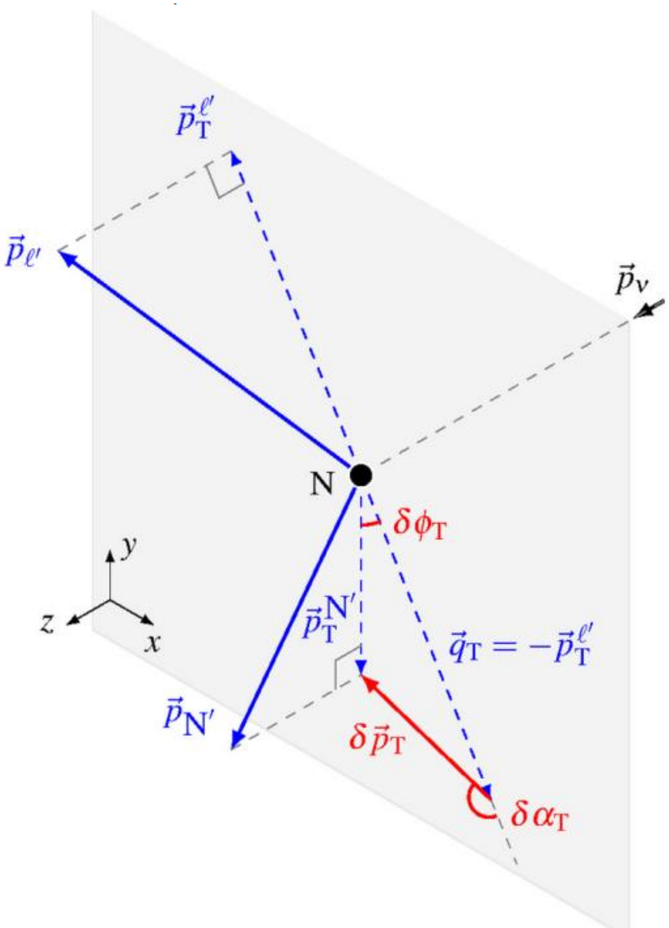
Imbalance due to initial nucleon motion and other nuclear effects

S_Dolan_Talk_ECT_2018

PHYSICAL REVIEW C **94**, 015503 (2016)

Measurement of nuclear effects in neutrino interactions with minimal dependence on neutrino energy

X.-G. Lu,[1,*] L. Pickering,[2] S. Dolan,[1] G. Barr,[1] D. Coplowe,[1] Y. Uchida,[2] D. Wark,[1,3] M. O. Wascko,[2] A. Weber,[1,3] and T. Yuan[4]



Single Transverse Variables (STV)

$$\delta\vec{p}_T \equiv \vec{p}_T^{\,\ell'} + \vec{p}_T^{\,N'}$$

$$\delta\alpha_T \equiv \arccos\frac{-\vec{p}_T^{\,\ell'}\cdot\delta\vec{p}_T}{p_T^{\ell'}\,\delta p_T}$$

$$\delta\phi_T \equiv \arccos\frac{-\vec{p}_T^{\,\ell'}\cdot\vec{p}_T^{\,N'}}{p_T^{\ell'}\,p_T^{N'}},$$

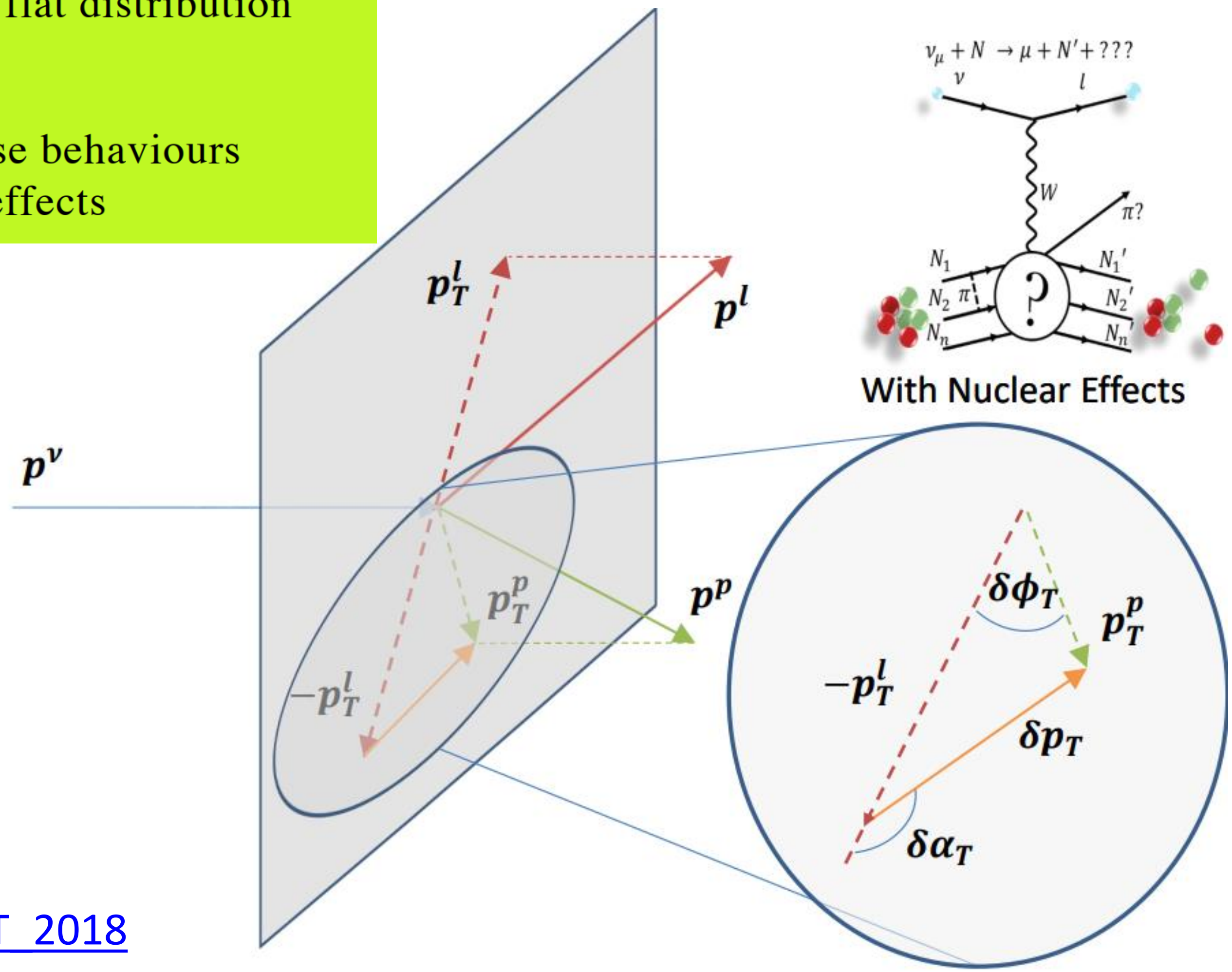
Single Transverse Kinematic imbalance (STKI) – 3 variables (STV)

On a free nucleon at rest $\overrightarrow{p}_T^{l'} = -\overrightarrow{p}_T^{N'}$:

$\delta p_T = \delta\phi_T = 0$ —> peaked distribution

$\delta\alpha_T$ undefined —> flat distribution

Deviations from these behaviours "measure" nuclear effects



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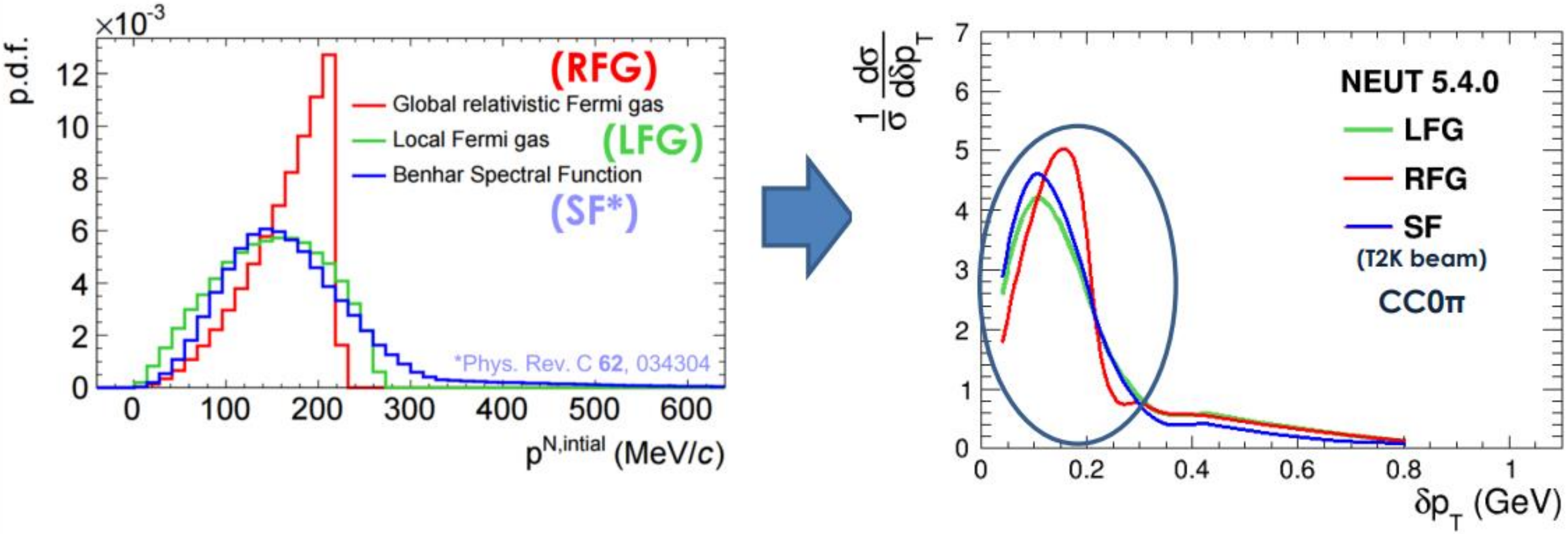
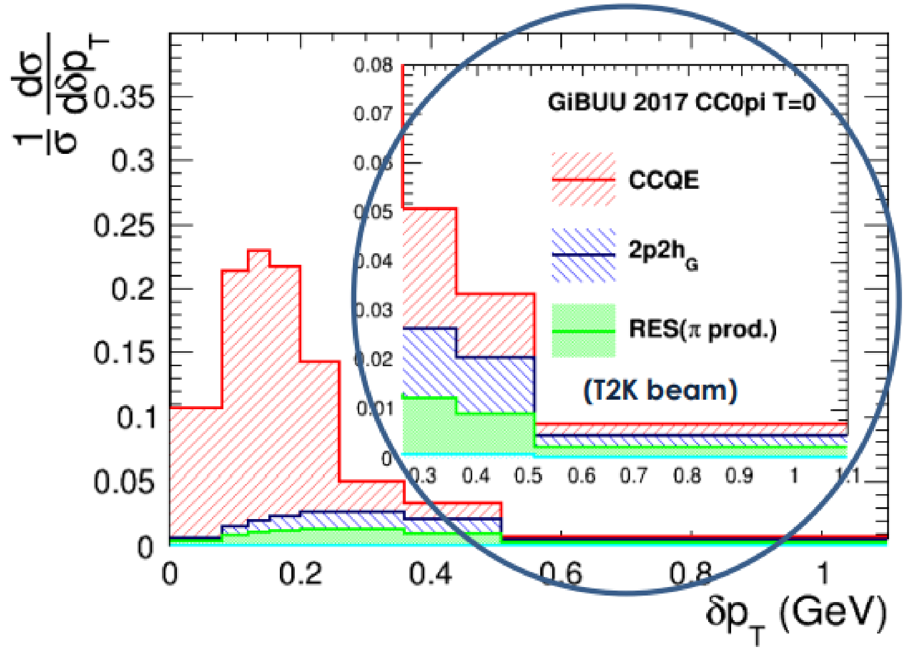
STV model discrimination - δp_T

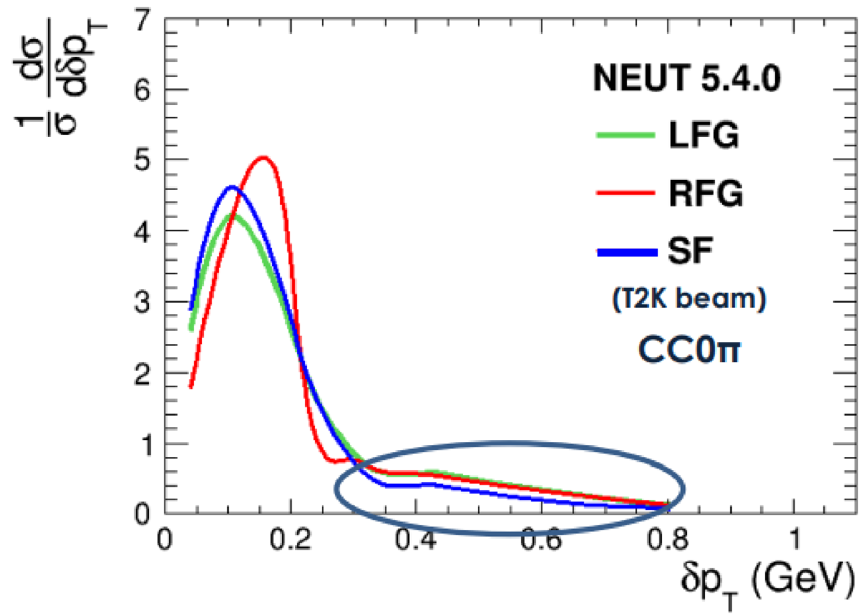


- In the absence of other nuclear effects, δp_T is the transverse projection of the Fermi motion.
- Since this motion is isotropic, $\delta p_T \rightarrow$ Fermi motion

S_Dolan_Talk_ECT_2018

STV model discrimination - δp_T





- In the absence of other nuclear effects, δp_T is the transverse projection of the Fermi motion.
- Since this motion is isotropic, $\delta p_T \rightarrow$ Fermi motion
- Cross section beyond the Fermi momentum must come from physics beyond RFG → 2p2h, FSI, SRCs …

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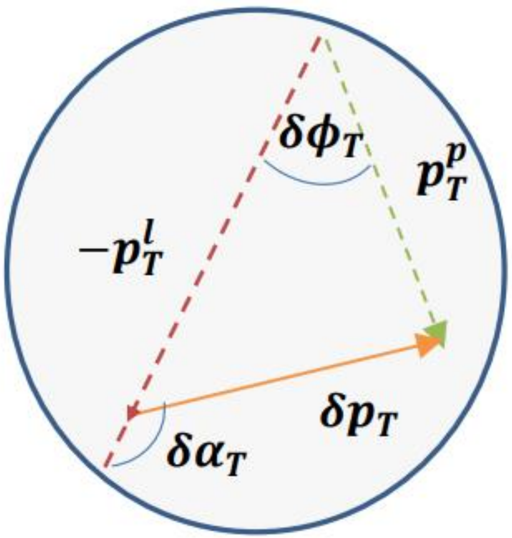
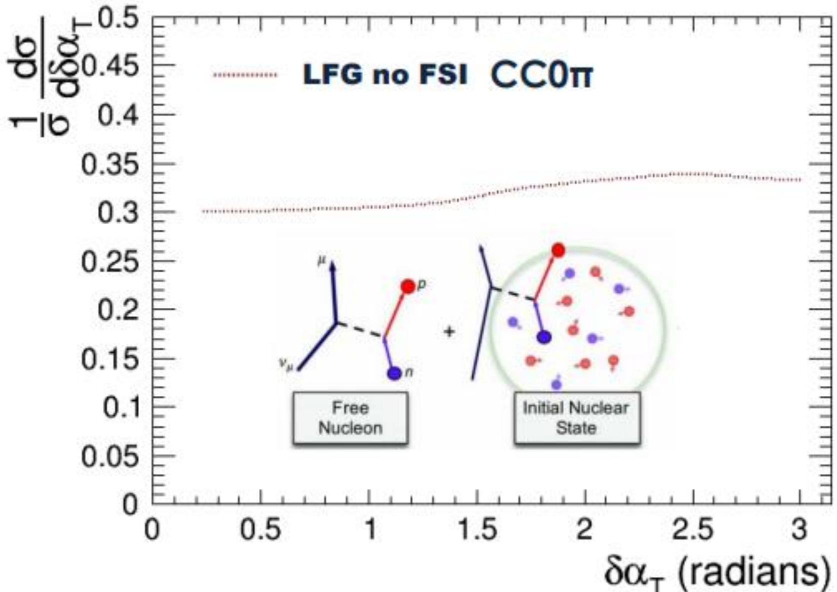
STV model discrimination - $\delta\alpha_T$



Consider imbalance from only Fermi motion



Fermi motion is isotropic so no preferred $\delta\alpha_T$ direction

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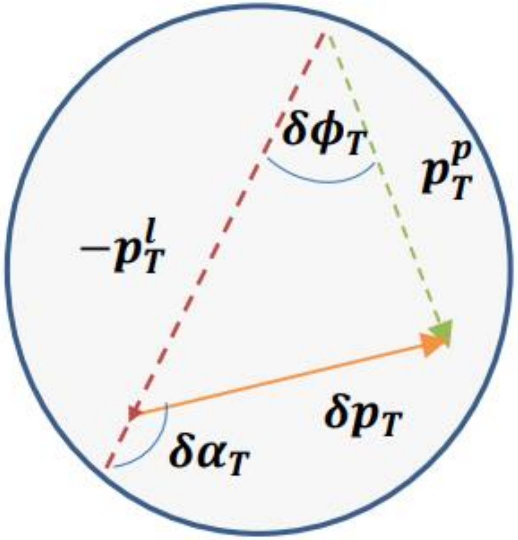
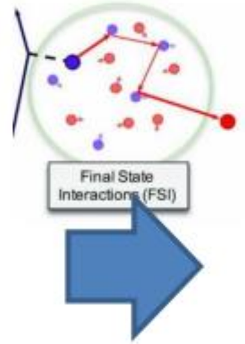
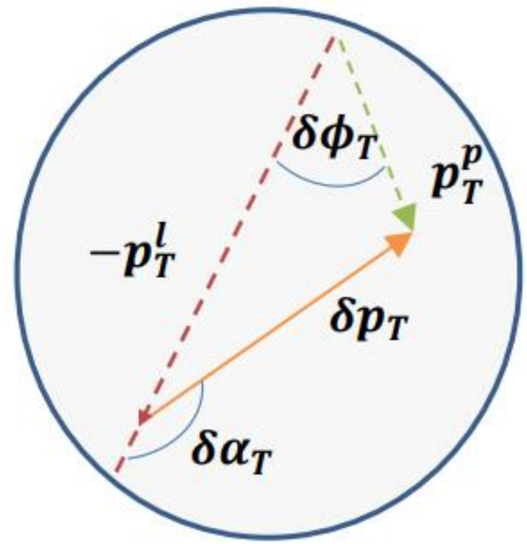
STV model discrimination - $\delta\alpha_T$

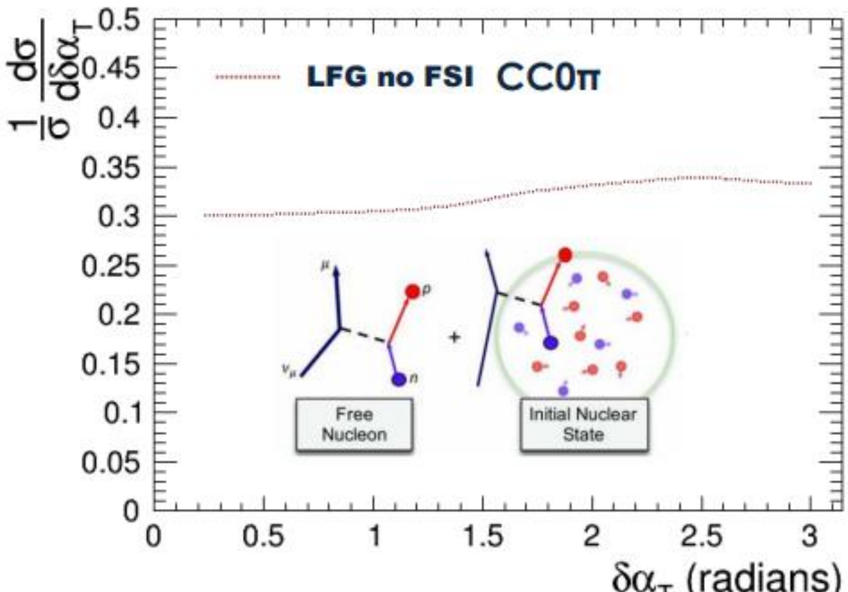


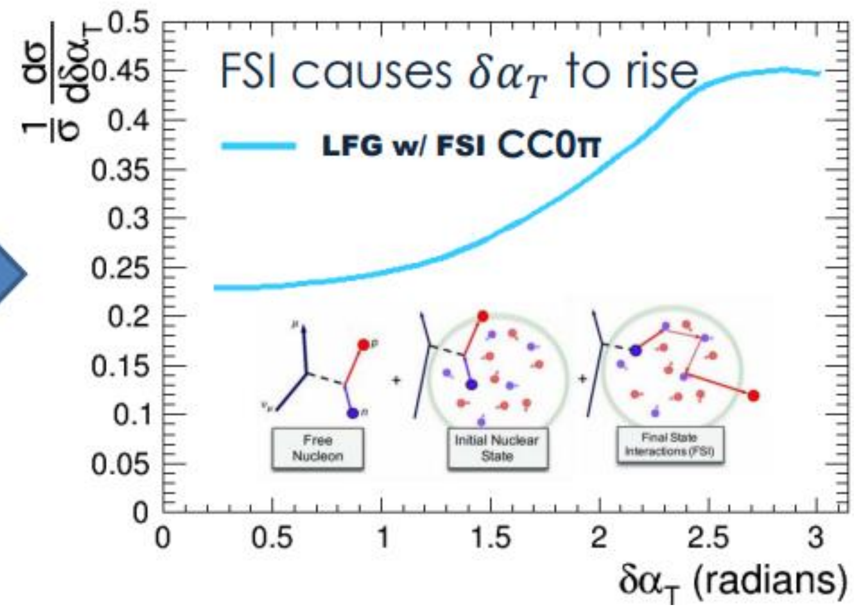


Deceleration of proton from FSI



= larger $\delta\alpha_T$





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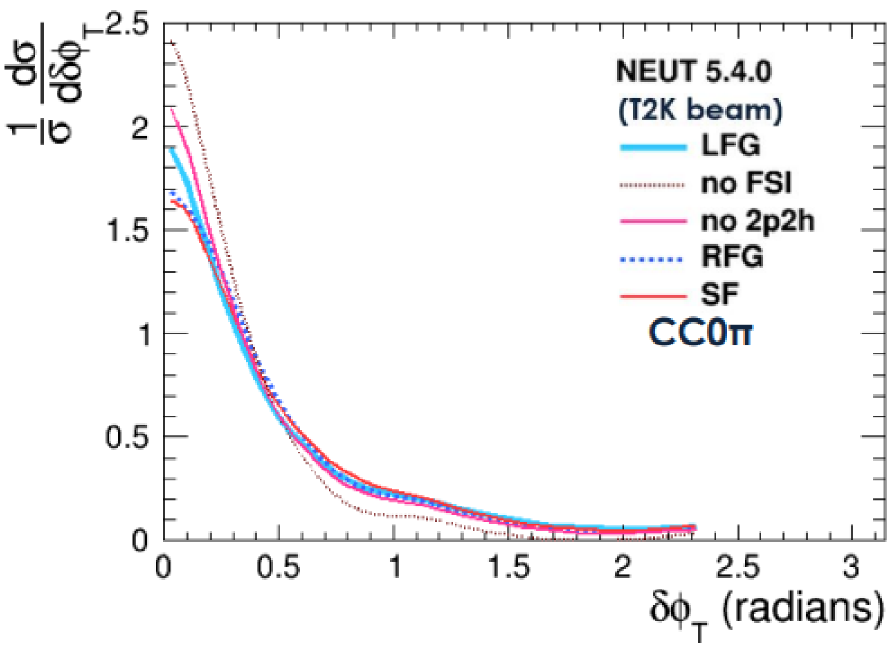
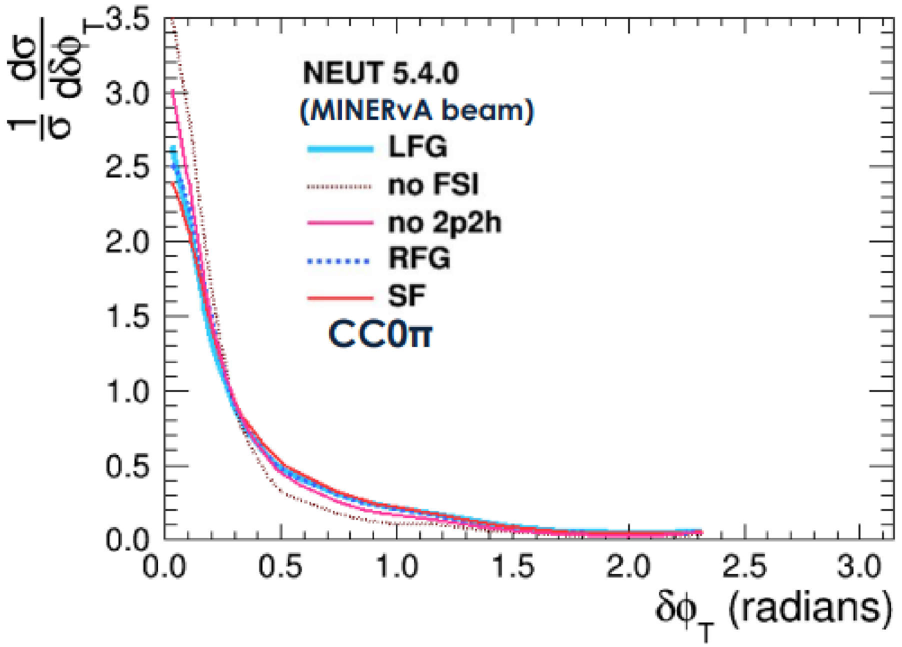
STV model discrimination - $\delta\phi_T$





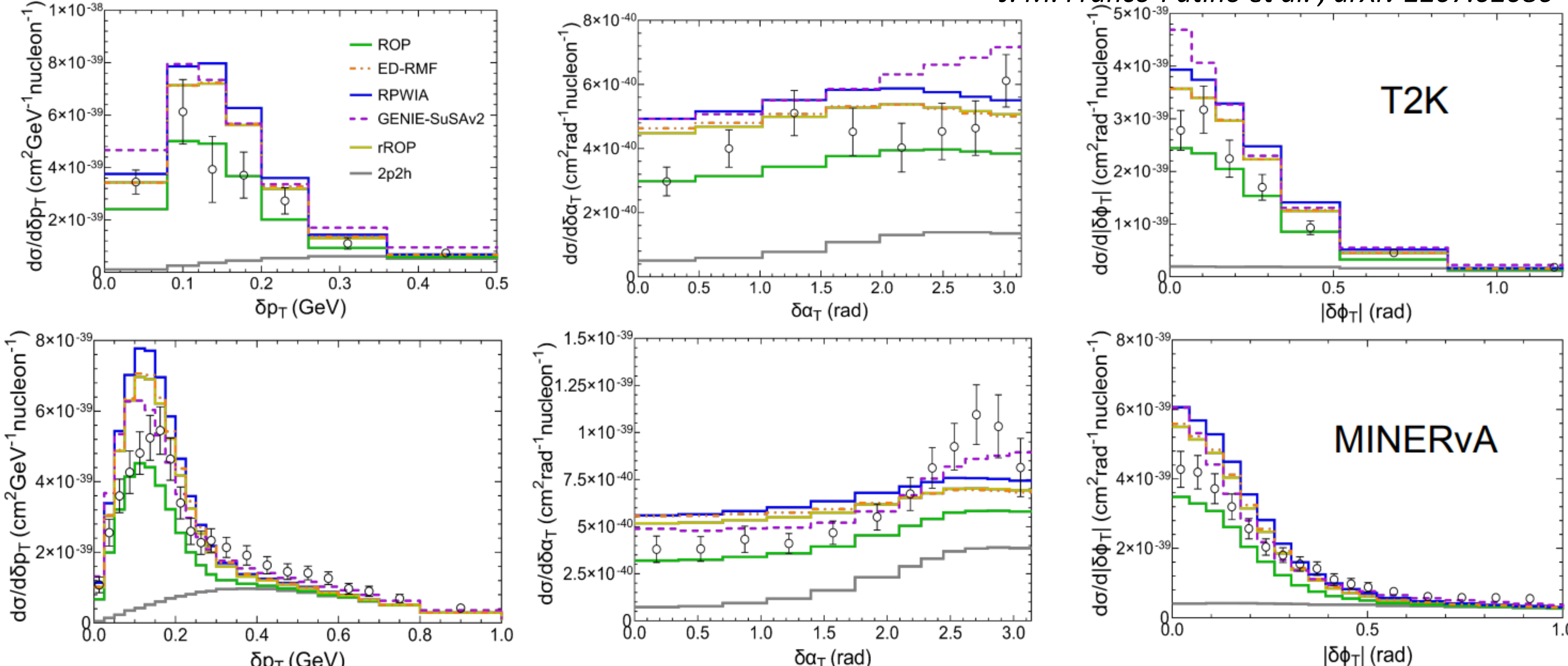
- A more general measure of transverse imbalance, more 2p2h and FSI give a larger contribution in the tail
- Not quite as powerful as δp_T and $\delta\alpha_T$
- But only requires outgoing particle angles and not their momentum → **much better detector resolution on $\delta\phi_T$**

S_Dolan_Talk_ECT_2018

Semi-inclusive CC0π dσ on carbon versus STKI Variables: discrimination of FSI modeling

A very recent theoretical study

J. M. Franco-Patino et al. , arXiv 2207.02086



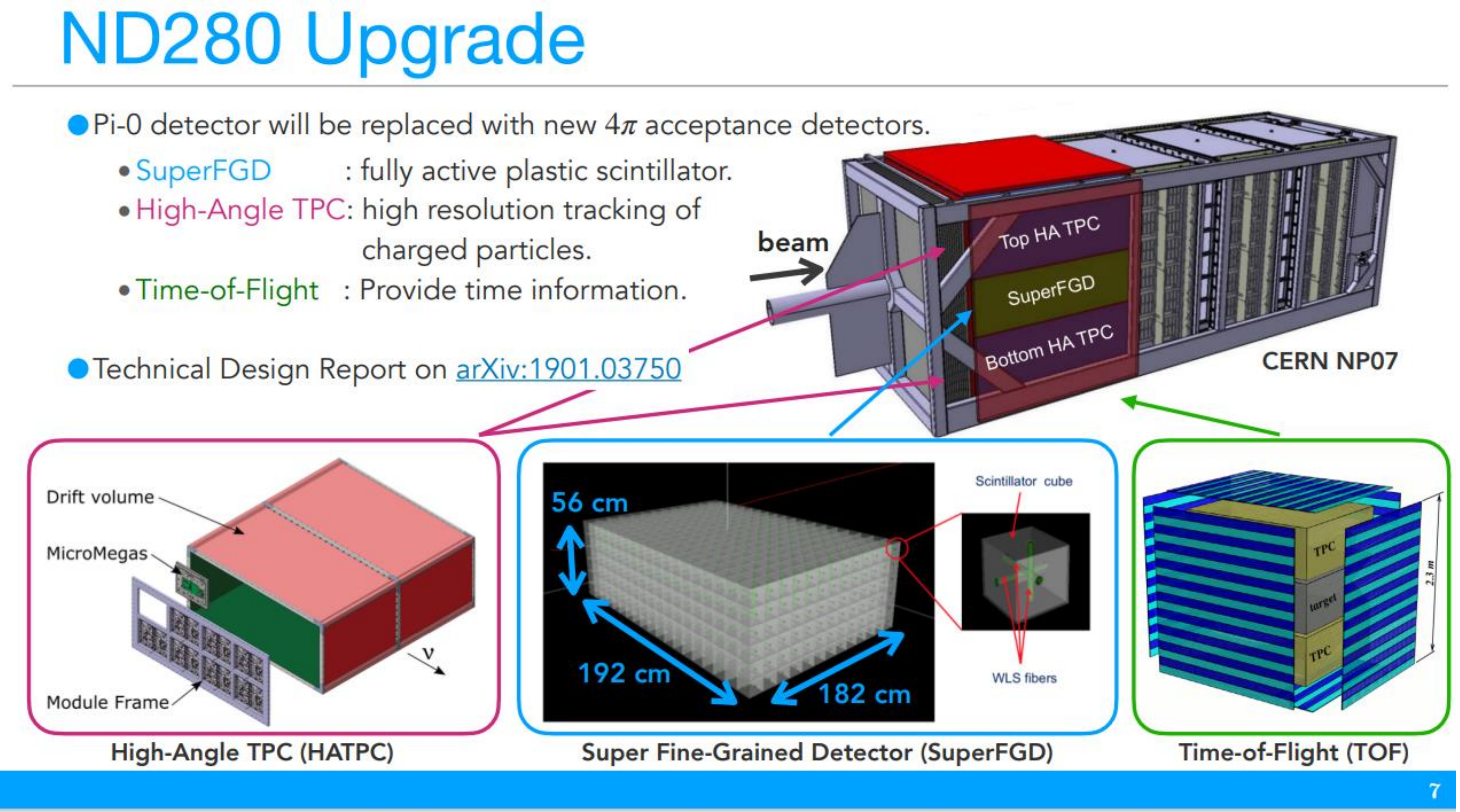
RPWIA: no FSI

GENIE-SuSAv2: include FSI but from inclusive model (factorization)

ED-RMF, **rROP**, **ROP**: different theoretical approaches for FSI

- FSI improve the agreement with data respect to the RPWIA prediction
- STKI Variables helps to discriminate between different FSI models: data (at least T2K) seem to prefer **ROP**
- 2p2h give non-negligible contribution

The T2K ND280 Upgrade



[A_Eguchi_T2KND280upgrade_NuFACT2022 (fnal.gov)]

see also S. Bolognesi lecture

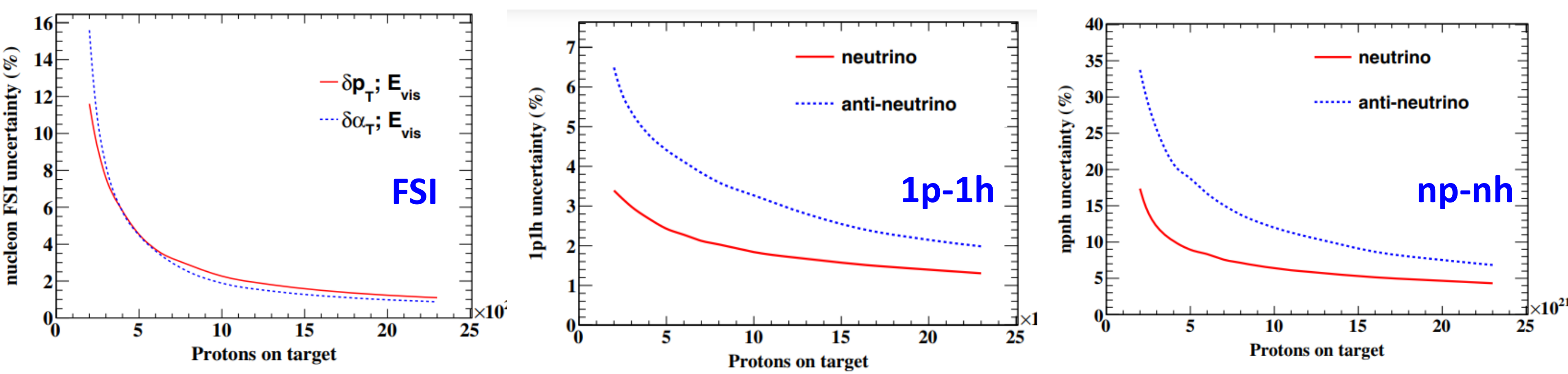
The T2K ND280 Upgrade – Physics sensitivity studies

- More mass, more data, better acceptance
- Improved reconstruction at high and backward lepton angles
- Better reconstruction of outgoing nucleons

The Upgrade opens the door to new multi-dimensional analyses (e.g. δp_T in bins of $\delta\alpha_T$)

PHYSICAL REVIEW D **105,** 032010 (2022)

Sensitivity of the upgraded T2K Near Detector to constrain neutrino and antineutrino interactions with no mesons in the final state by exploiting nucleon-lepton correlations

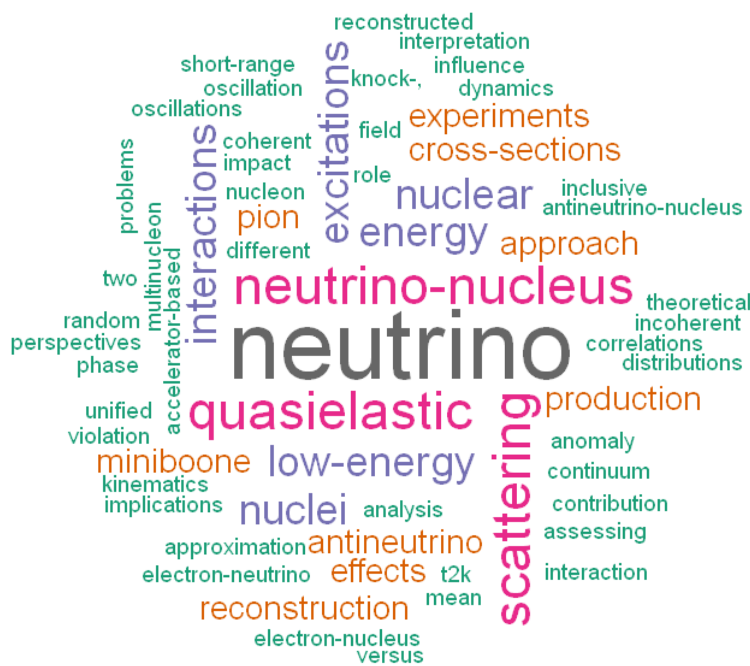


Significant decrease of the nuclear effects uncertainties

Summary and conclusions

Neutrino-nucleus cross sections: exciting, surprising, “incontournable”

In the precision era of neutrino physics new intriguing results, like CP violation, necessary passes through a precise knowledge of neutrino-nucleus cross sections



BACKUP

Electron-nucleon scattering

$$\ell^-(k) + N(p) \to \ell^-(k') + N(p')$$

Scattering on a point-like spinless target

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4E_k^2 \sin^2\frac{\theta}{2}} \frac{E_{k'}}{E_k} \cos^2\frac{\theta}{2}$$

Scattering on a point-like spin ½ target

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 - \frac{q^2}{2M^2} \tan^2\frac{\theta}{2}\right]$$

Scattering on spin ½ particle with an internal structure (protons and neutrons): electric and magnetic form factors

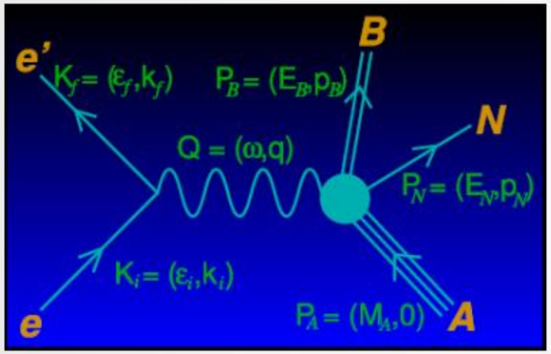
$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[\frac{G_E^2 - \frac{q^2}{4M^2} G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2} G_M^2 \tan^2\frac{\theta}{2}\right]$$

Relativistic scattering: notations and formalism

Four-vectors $A^\mu = (A_0, \vec{A})$

Bjorken&Drell conventions $g^{00} = 1,\, g^{kk} = -1\ (k = 1, 2, 3)$

First Born approximation: one virtual boson exchange



$Q^2 = \omega^2 - q^2 < 0$ space-like virtual boson

$Q^\mu = K^\mu - K'^\mu = P_f^\mu - P_i^\mu$ 4-momentum conservation

$P^2 = M^2$ on-shell condition

$\partial\!\!\!/ \equiv \gamma_\mu \partial^\mu$ $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\,\mathbb{1}$

$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$ $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$

Dirac equation and Dirac spinors:

$(i\,\partial\!\!\!/ - M)\,\Psi = 0$ free particles

$(i\,\partial\!\!\!/ - e\,A\!\!\!/ - M)\,\Psi = 0$ in presence of e.m. field

$\Psi_{\mathbf{p}}^{(+)}(\mathbf{x}, t) = \sqrt{\frac{M}{EV}}\, u(\mathbf{p}, s) e^{-iP_\mu X^\mu}$ positive energy

$\Psi_{\mathbf{p}}^{(-)}(\mathbf{x}, t) = \sqrt{\frac{M}{EV}}\, v(\mathbf{p}, s) e^{iP_\mu X^\mu}$ negative energy

$(P\!\!\!/ - M)\, u(\mathbf{p}, s) = 0$

$(P\!\!\!/ + M)\, v(\mathbf{p}, s) = 0$

$$u(\mathbf{p}, s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\chi_s \end{pmatrix}, \quad v(\mathbf{p}, s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M}\xi_s \\ \xi_s \end{pmatrix}$$

Dirac spinology

Normalization condition and Dirac adjoint operators: [$\overline{u} \equiv u^\dagger \gamma^0$ & $\overline{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0$]

$$\overline{u}(\mathbf{p}, s) u(\mathbf{p}, s) = 1, \quad \overline{v}(\mathbf{p}, s) v(\mathbf{p}, s) = -1$$

Projection operators:

Energy $\hat{\Lambda}_\pm(\mathbf{p}) = \left(\frac{\pm \not{P} + M}{2M} \right) = \sum_{\pm s} u(\mathbf{p}, s) \overline{u}(\mathbf{p}, s)$

Spin $\hat{P}(\pm s) = \frac{1}{2} (\mathbb{1} \pm \gamma_5 \not{S})$ *Four-spin* $S^\mu = (s^0, \mathbf{s})$ *satisfies:* $S^2 = S_\mu S^\mu = -1$ *and* $P_\mu S^\mu = 0$

Bilinear covariants and their properties under Lorentz transformations

$\overline{\Psi}\Psi$ (scalar), $\overline{\Psi}\gamma^5\Psi$ (pseudoscalar), $\overline{\Psi}\gamma^\mu\Psi$ (vector), $\overline{\Psi}\gamma^5\gamma^\mu\Psi$ (pseudovector),

Trace theorems:

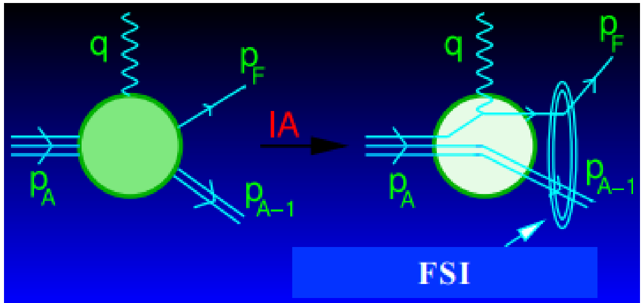
$$\mathrm{Tr}\,(\not{A}\ \not{B}) = 4 A \cdot B, \quad \mathrm{Tr}\,(\gamma^5) = 0, \quad \mathrm{Tr}\,(\gamma^5\ \not{A}\ \not{B}) = 0$$

$$\mathrm{Tr}\,(\not{A}\ \not{B}\ \not{C}\ \not{D}) = 4\,[A \cdot B\ C \cdot D + A \cdot D\ B \cdot C - A \cdot C\ B \cdot D]$$

$$\mathrm{Tr}\,(\gamma^5\ \not{A}\ \not{B}\ \not{C}\ \not{D}) = 4 i \epsilon_{\alpha\beta\gamma\delta} A^\alpha B^\beta C^\gamma D^\delta$$

Realtivistic Mean Field Model

The RMF model is based on the **impulse approximation (IA)**:
scattering off a nucleus = incoherent sum of single nucleon scattering processes.

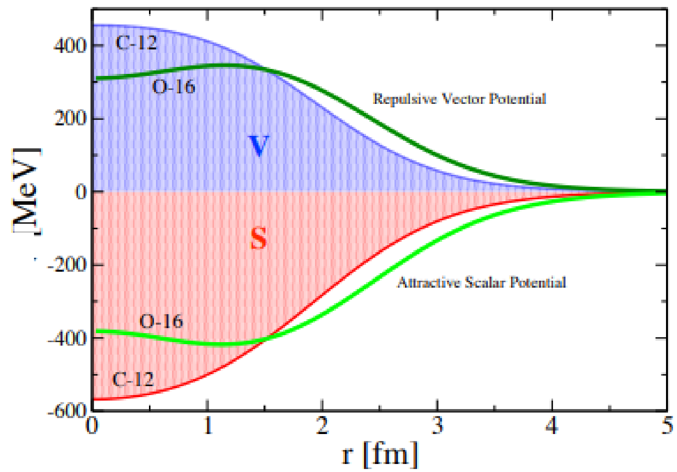


Nuclear Current ⟹ One-body operator

$$J_N^\mu(\omega, \vec{q}) = \int d\vec{p}\, \overline{\Psi}_F(\vec{p}+\vec{q}) \hat{J}_N^\mu \Psi_B(\vec{p})$$

The nucleon wave functions are finite nucleus solutions of the Dirac equation with phenomenological **relativistic scalar and vector potentials** obtained from a Walecka-type Lagrangian fitted to properties of nuclear radii and masses:

$$(i\gamma^\mu \partial_\mu - M - S + V)\psi(\vec{r}, t) = 0$$



Bound wave function

$$\Psi_B = \begin{pmatrix} \phi^{up} \\ \phi^{down} \end{pmatrix} = \begin{pmatrix} \phi^{up} \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{E+M+S-V}\phi^{up} \end{pmatrix} = \alpha u + \beta v$$

Scattered wave function

The ejected nucleon wave function is distorted by **Final State Interactions** (FSI) with the residual nucleus. In the RMF model it is a scattering solution of the same Dirac equation used to describe the bound state.
Orthogonality is preserved: the initial and final nucleon wave functions are eigenstates of the same Hamiltonian.

Scattered Nucleon Description

Regarding the scattered nucleon, we can consider several situations:

- Relativistic Plane-Wave Impulse Approximation (RPWIA): the ejected nucleon is considered a plane-wave (i.e, there are not final state interactions)

- Energy-Dependent Relativistic Mean Field (ED-RMF): W.F. solution of the Dirac equation in the continuum using the same RMF potential that describes the initial state times a phenomenological function that weakens the potentials at high energies

- Relativistic Optical Potential (ROP): The scattered nucleon travels under the influence of a phenomenological relativistic optical potential fitted to reproduce elastic proton scattering data. Keeping only the real part of the OP (rROP) is an effective way to take into account all the channels (elastic and inelastic)

$$\Psi_{s_N}(\mathbf{r},\,\mathbf{p_N}) = 4\pi\sqrt{\frac{E_N+m_N}{2E_N}}\sum_{\kappa,\,m_l,\,m_j} e^{-i\delta^*_\kappa} i^l \langle l\, m_l\, 1/2\, s_N |\, j\, m_j\rangle Y^*_{l\,m_l}(\Omega_N)\psi^{m_j}_\kappa(\mathbf{r})$$

$$\psi^{m_j}_\kappa(\mathbf{r}) = \begin{pmatrix} g_\kappa(r)\phi^{m_j}_\kappa(\Omega_r) \\ i f_\kappa(r)\phi^{m_j}_{-\kappa}(\Omega_r) \end{pmatrix}$$

$$\frac{dg_\kappa}{dr} = -\frac{\kappa}{r}g_\kappa + [E_N + S(r,\,E_N) - V(r,\,E_N)]f_\kappa$$

$$\frac{df_\kappa}{dr} = -\frac{\kappa}{r}f_\kappa + [E_N - S(r,\,E_N) - V(r,\,E_N)]g_\kappa$$

Electromagnetic 2p-2h MEC response

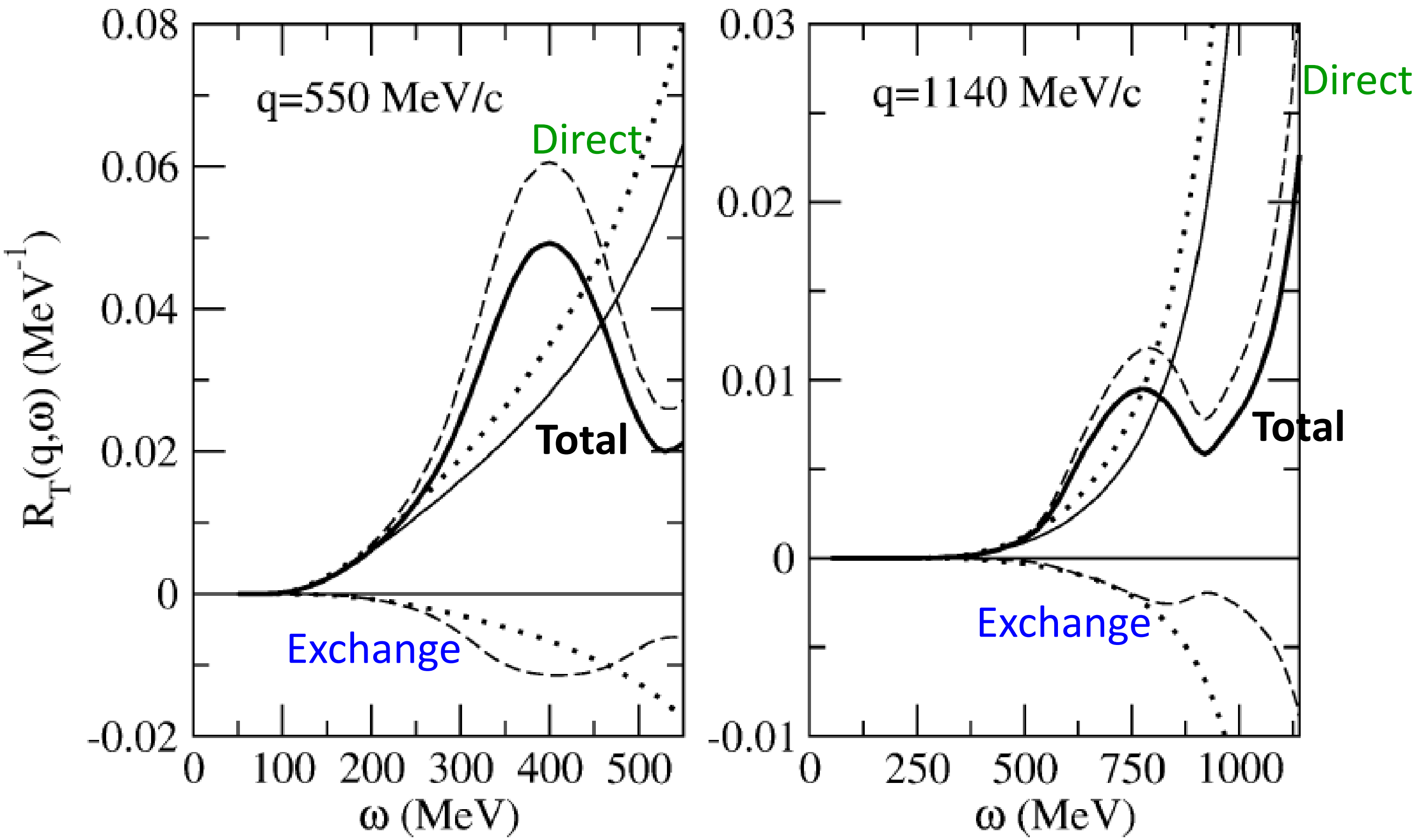


Fig. 12. The transverse response function $R_T(q,\omega)$ at $q = 550$ MeV/c and $q = 1140$ MeV/c including the exchange contributions: non-relativistic direct (positive dotted), non-relativistic exchange (negative dotted), non-relativistic total (light solid), relativistic direct (positive dashed), relativistic exchange (negative dashed) and relativistic total (heavy solid). In all instances $\bar{\epsilon}_2 = 70$ MeV and $k_F = 1.3$ fm^{-1}.

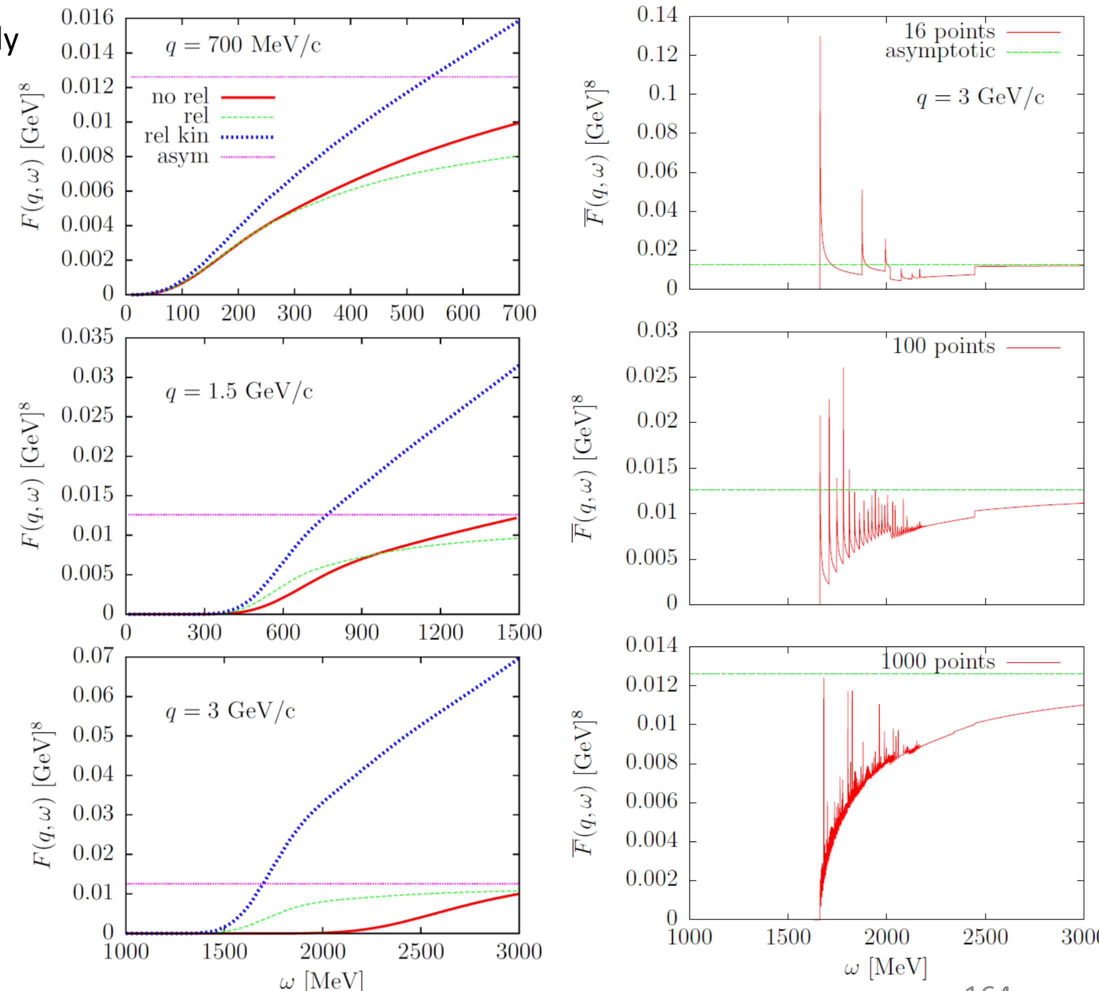
De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)

2p-2h phase space integral

$$F(\omega,q) \equiv \int d^3h_1\, d^3h_2\, d^3p'_1 \frac{m_N^4}{E_1E_2E'_1E'_2}\Theta(p'_1,p'_2,h_1,h_2)\delta(E'_1+E'_2-E_1-E_2-\omega)$$

$$\overline{F}(\omega,q) = \left(\frac{4}{3}\pi k_F^3\right)^2 \int d^3p'_1\, \delta(E'_1+E'_2-\omega-2m_N)\,\Theta(p'_1,p'_2,0,0)\frac{m_N^2}{E'_1E'_2}$$

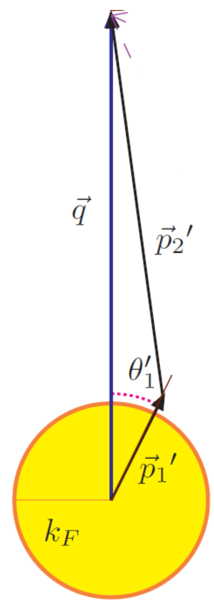
Ruiz Simo, Albertus, Amaro, Barbaro, Caballero, Donnelly
Phys. Rev. D 90 033012 (2014)
Phys. Rev. D 90 053010 (2014)



Angular distribution of ejected nucleons

$$\overline{F}(\omega,q) = \left(\frac{4}{3}\pi k_F^3\right)^2 2\pi \int_0^\pi d\theta_1' \, \Phi(\theta_1')$$

$$\Phi(\theta_1') = \sin\theta_1' \int p_1'^2 dp_1' \delta(E_1 + E_2 + \omega - E_1' - E_2') \times \Theta(p_1', p_2', h_1, h_2) \frac{m_N^4}{E_1 E_2 E_1' E_2'} = \sum_{\alpha=\pm} \frac{m_N^4 \sin\theta_1' p_1'^2 \Theta(p_1', p_2', h_1, h_2)}{E_1 E_2 E_1' E_2' \left| \frac{p_1'}{E_1'} - \frac{\mathbf{p}_2' \cdot \hat{\mathbf{p}}_1'}{E_2'} \right|} \Bigg|_{p_1' = p_1'^{(\alpha)}}$$



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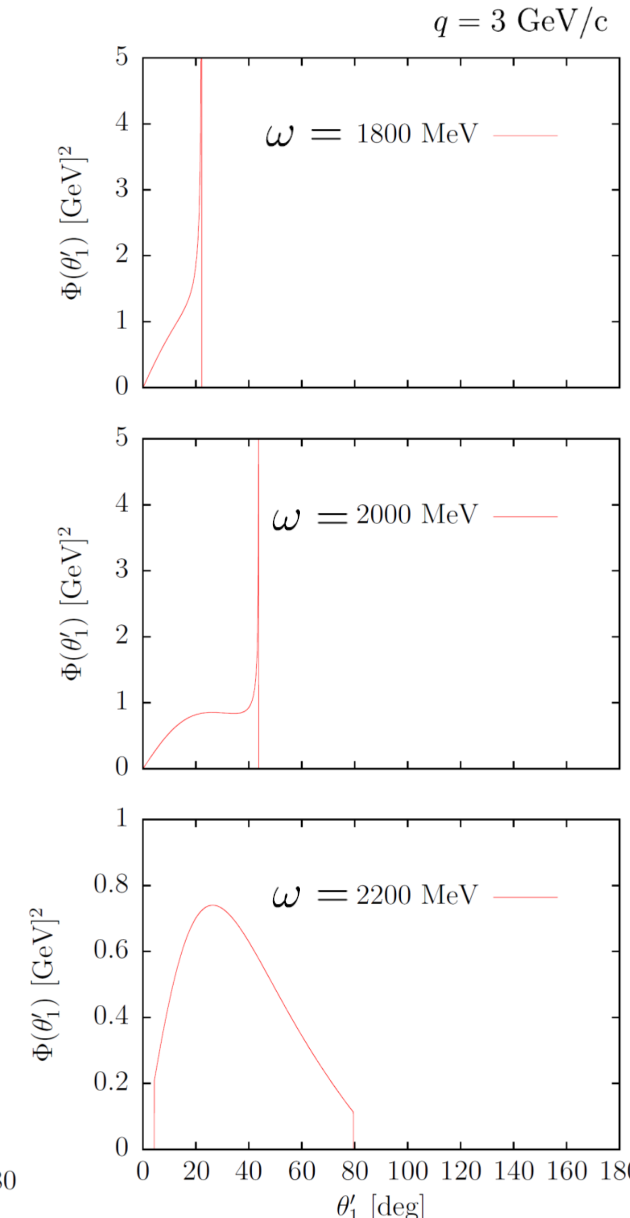
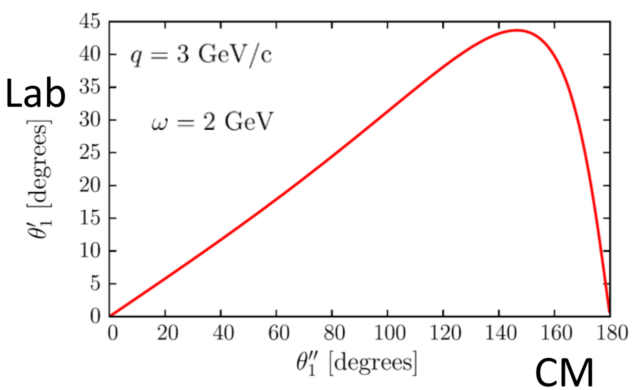
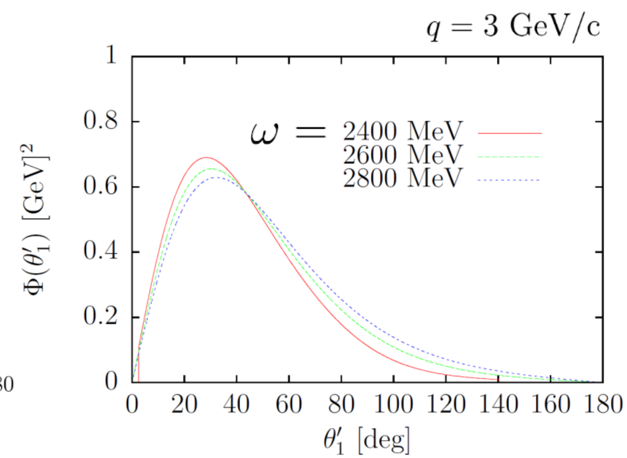
Phys. Rev. D 90 033012 (2014)
Phys. Rev. D 90 053010 (2014)







Difference of ν and antiν cross sections and the VA interference term

$d\sigma \sim d\sigma_L + d\sigma_T \pm d\sigma_{VA}$

$d\sigma_\nu - d\sigma_{\bar{\nu}} \overset{?}{\leftrightarrow} 2d\sigma_{VA}$

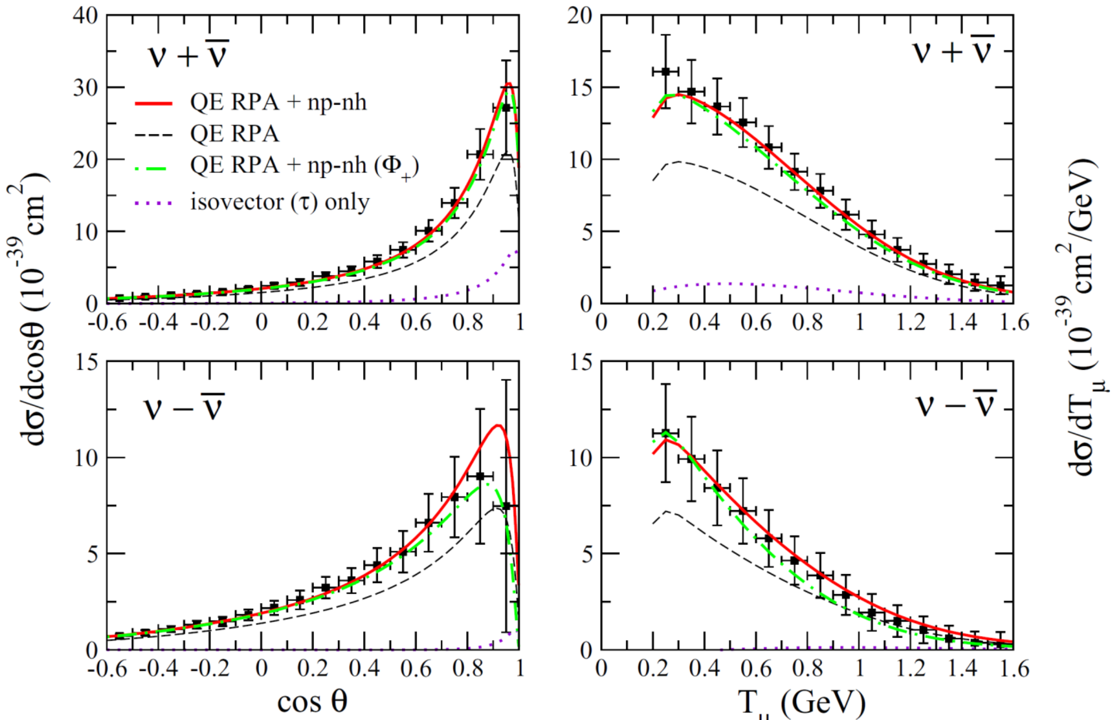
Difference gives only the VA term for identical ν and antiν flux

Problem: flux dependence of dσ $\frac{d^2\sigma}{dE_\mu d\cos\theta} = \int dE_\nu \left[\frac{d^2\sigma}{d\omega d\cos\theta}\right]_{\omega=E_\nu-E_\mu} \Phi(E_\nu)$

We introduce the **mean flux** $\Phi_+ = 1/2[\Phi_\nu + \Phi_{\bar{\nu}}]$

We calculate the sum and the difference using **real** and **mean** MiniBooNE fluxes results

M. Ericson, M. Martini Phys. Rev. C 91 035501 (2015)



The mean flux contribution is dominant

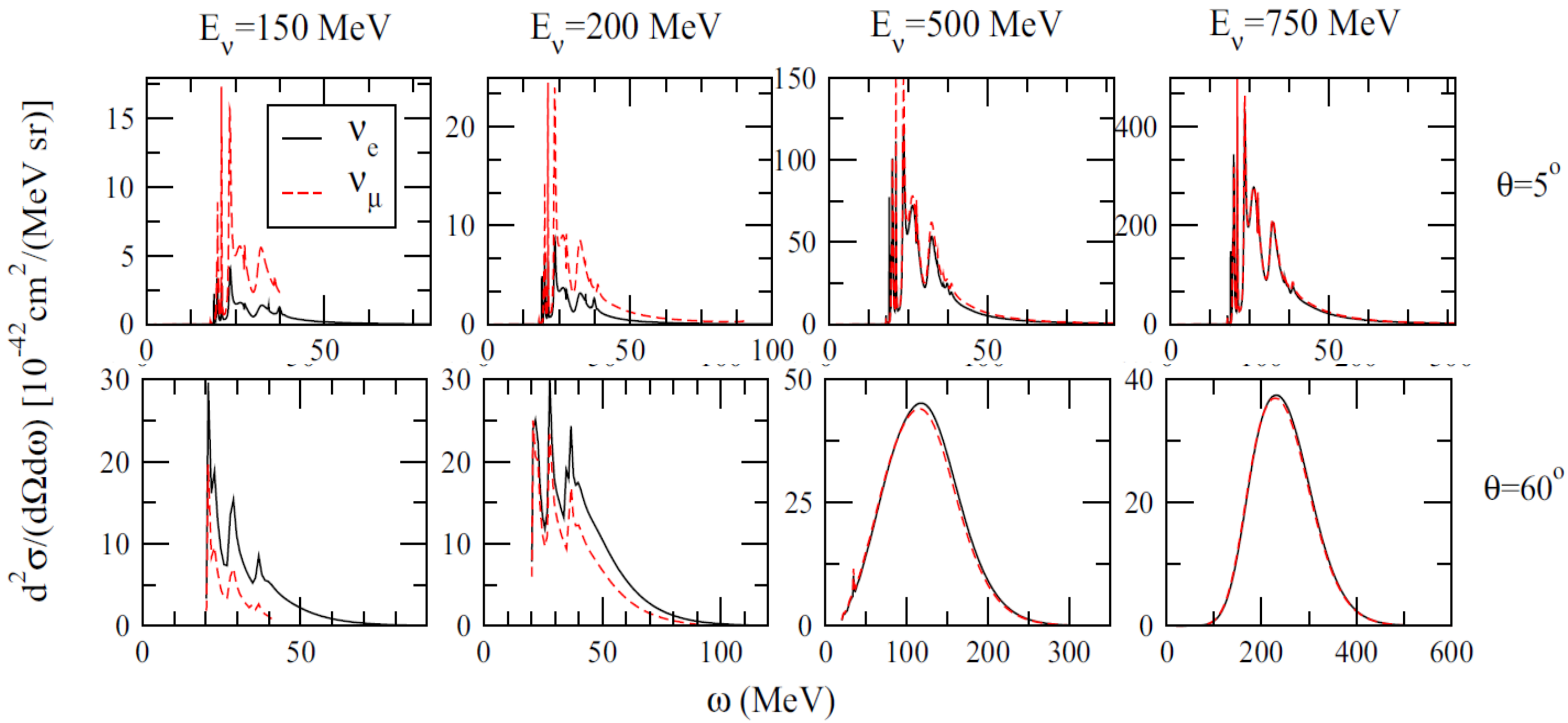
⇓

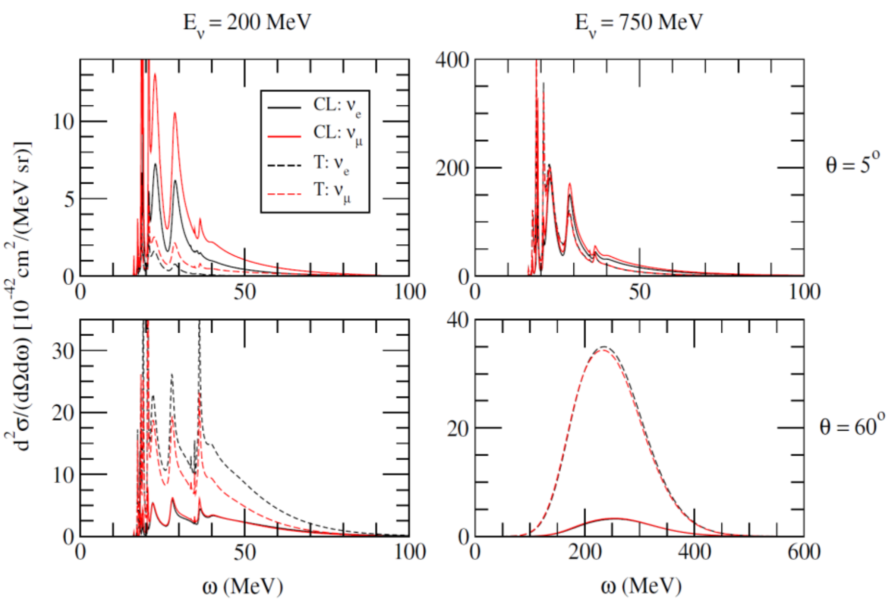
The VA interference term is experimentally accessible in MBdata

⇓

Need for the **multinucleon** component in the **VA interference**

ν_μ and ν_e CCQE $d^2\sigma$ in CRPA

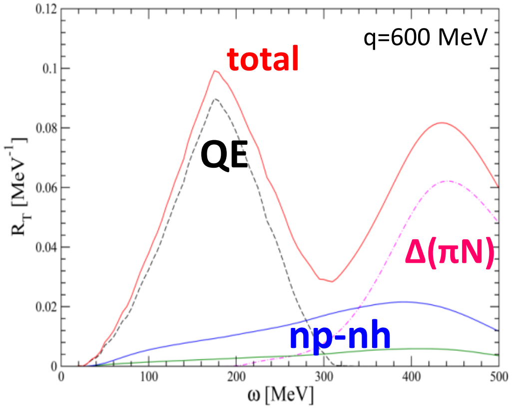


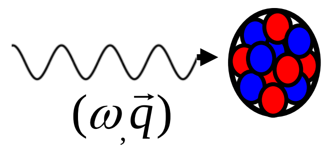


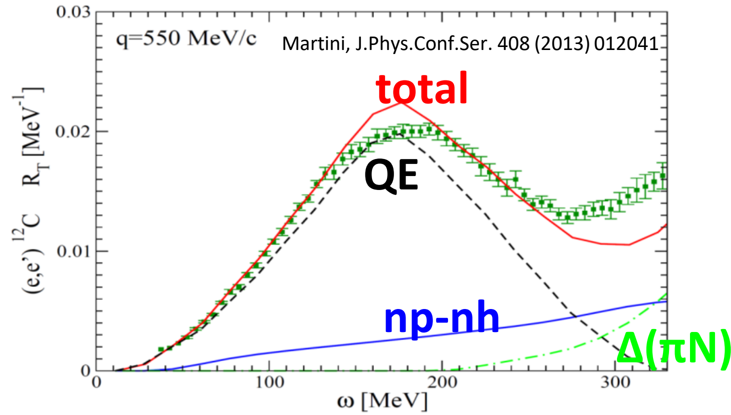
M. Martini et al., Phys. Rev. C 94 015501 (2016)

MINERvA “calorimetric” measurement

Aim: isolate the different contributions (in particular np-nh) in the (ω,q) phase space, as in (e,e’) scattering

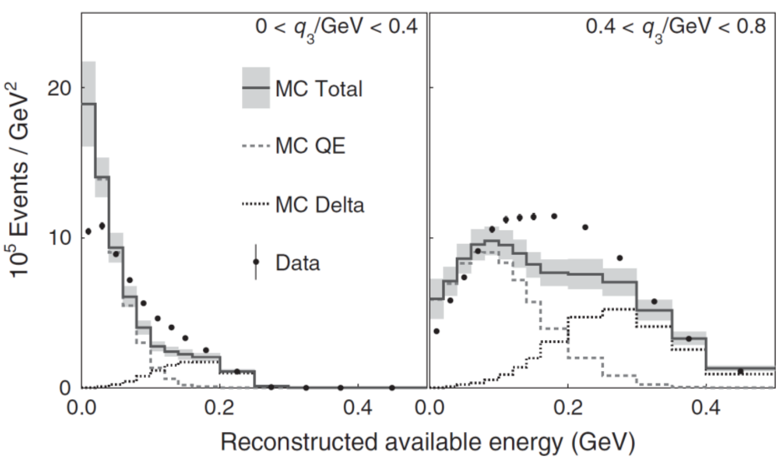


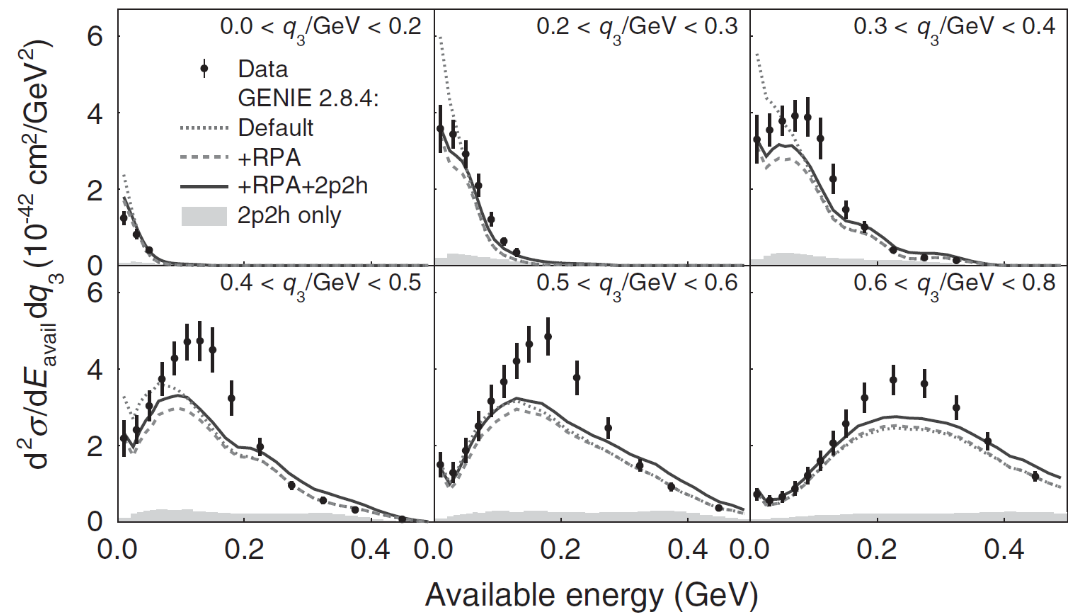




$$E_{avail} = \sum(\text{Proton and } \pi^{\pm} \text{ KE}) + (\text{Total E of other particles except neutrons})$$

P. A. Rodrigues et al., PRL 116, 071802 (2016)



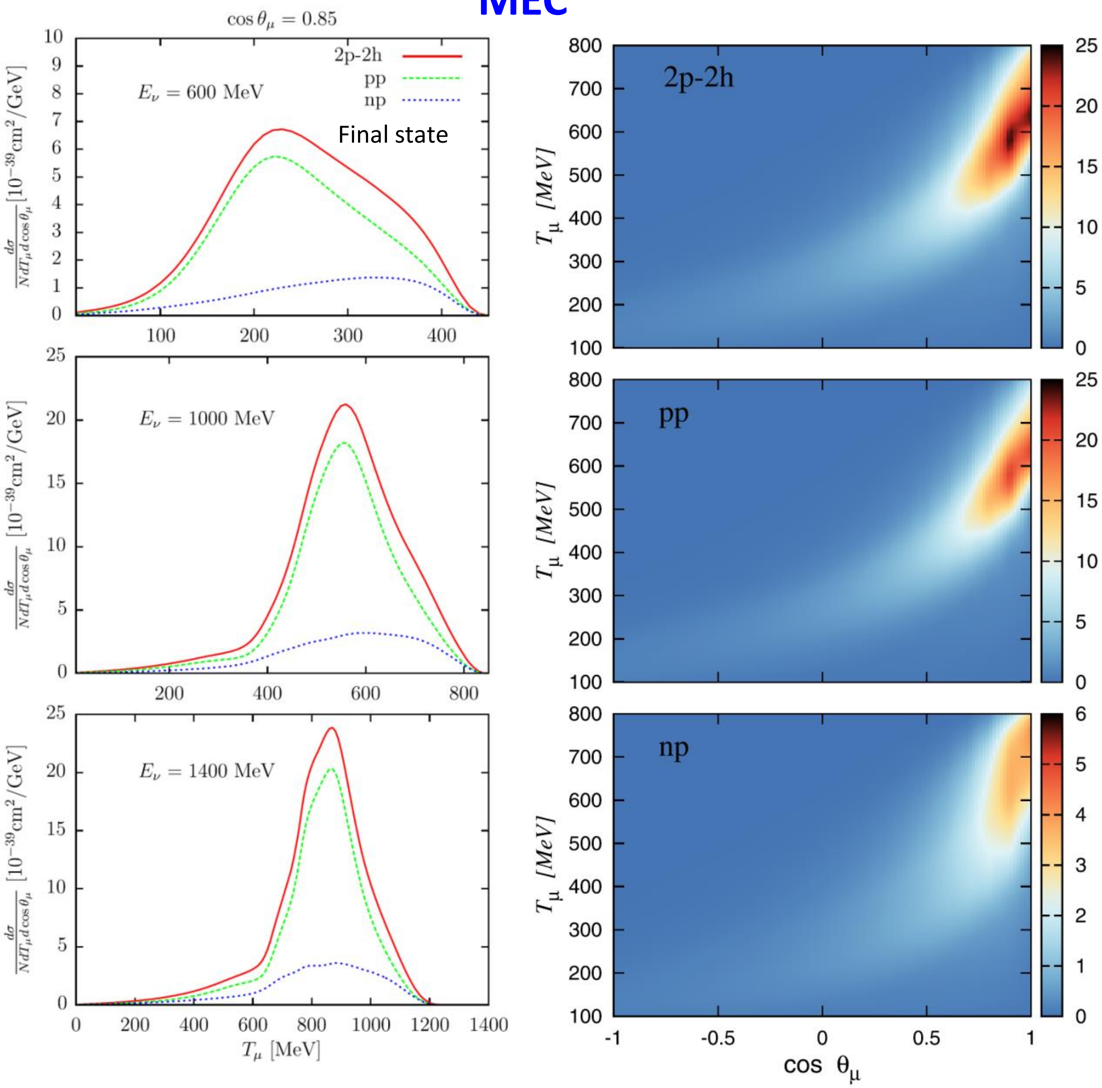


The addition of np-nh excitations via a GENIE implementation of the model of Nieves et al. reduces the discrepancy between simulation and data in the dip region, but more np-nh events would further improve the agreement with data

Theoretical studies on hadron information – Isospin content

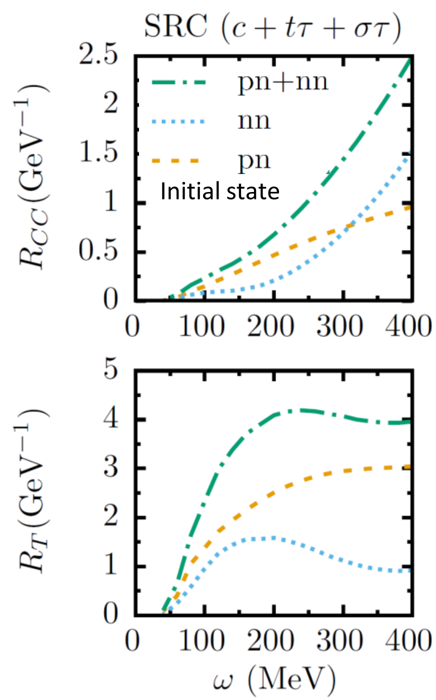
I. Ruiz Simo et al. Phys. Lett.B762, 124 (2016)

MEC



T. Van Cuyck et al. PRC 94, 024611(2016)

NN SRC

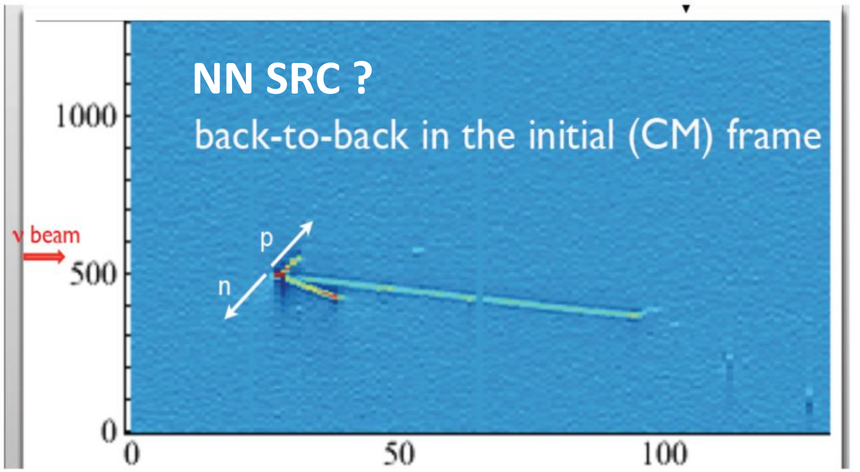


- The **pp** channel final state (**np** in the initial state) dominates in MEC and SRC
- The **pp**/**np** ratio depends on the kinematics

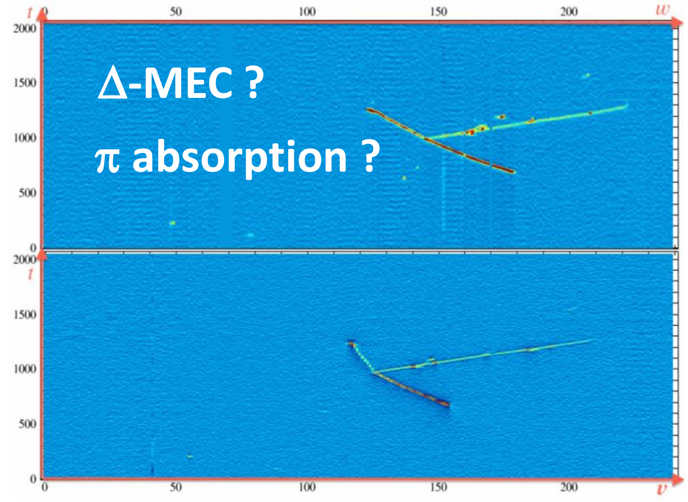
Exclusive processes (ν_μ, μ^-+2p)

ArgoNeut

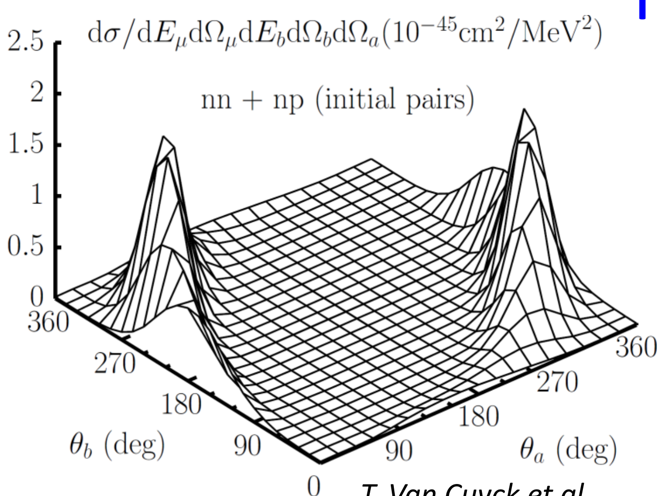
Acciari et al. Phys.Rev. D90 (2014) 012008



Hammer events



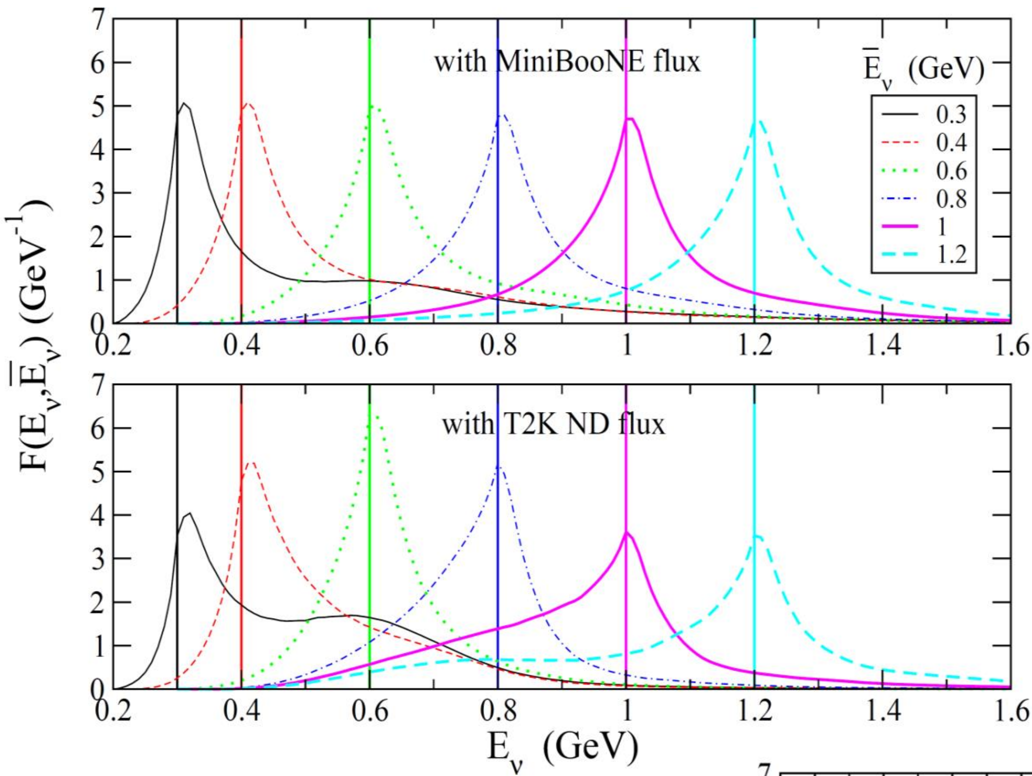
Theoretical studies



T. Van Cuyck et al. PRC 94, 024611(2016)

- Modeling the coincidence reactions is in demand by the experimental community but it is a very challenging task
- Many models used up to now to compare with the neutrino flux-integrated differential cross sections function of the charged lepton variables are not applicable for exclusive studies. More nuclear response functions contribute to the cross section

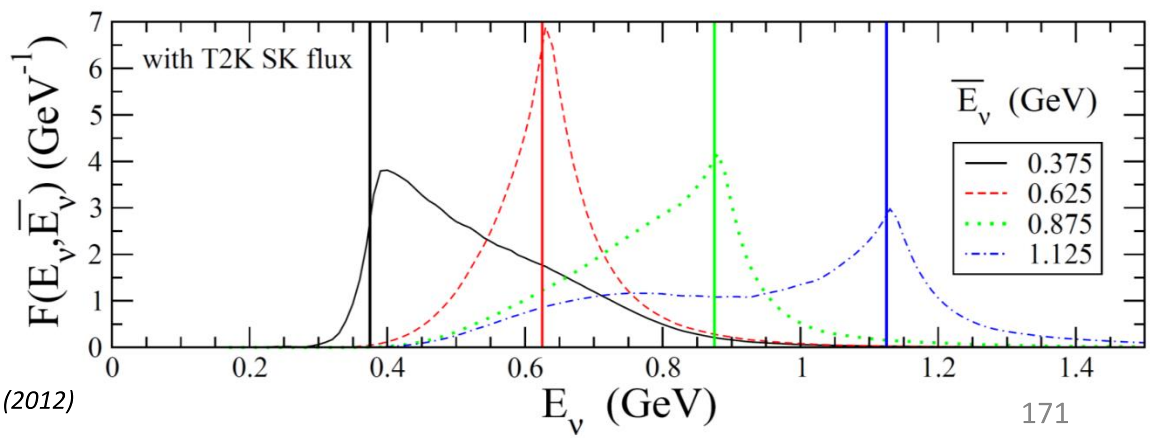
Energy reconstruction: Probability distributions $F(E_\nu, \bar{E}_\nu)$ for several $\bar{E}_\nu$ using three different neutrino fluxes



The distributions are not symmetrical around $\bar{E}_\nu$.

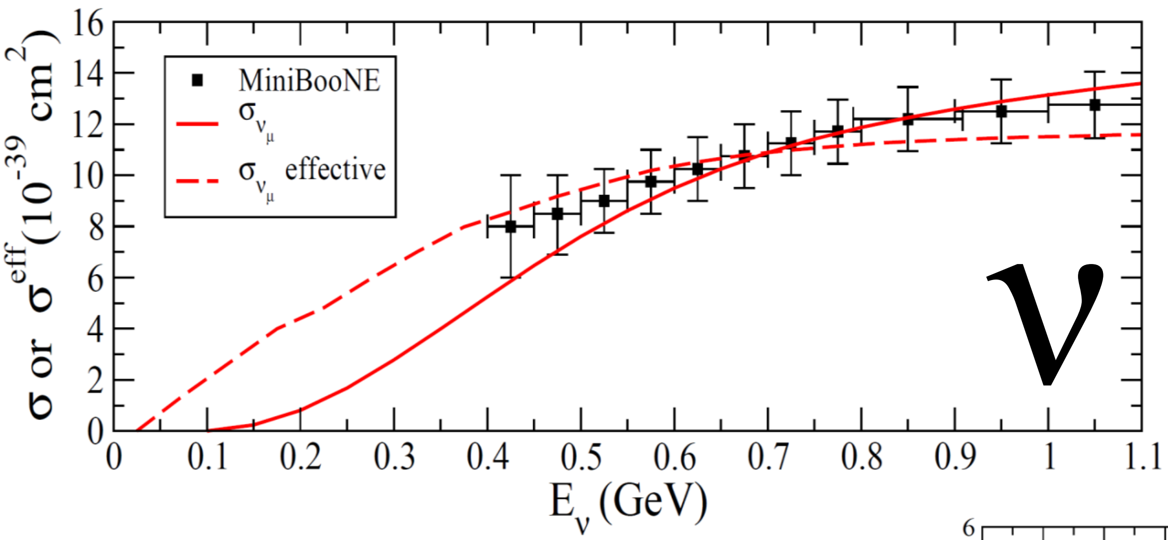
The asymmetry favors higher energies at low $\bar{E}_\nu$ and smaller energies for large $\bar{E}_\nu$.

Crucial role of neutrino flux.

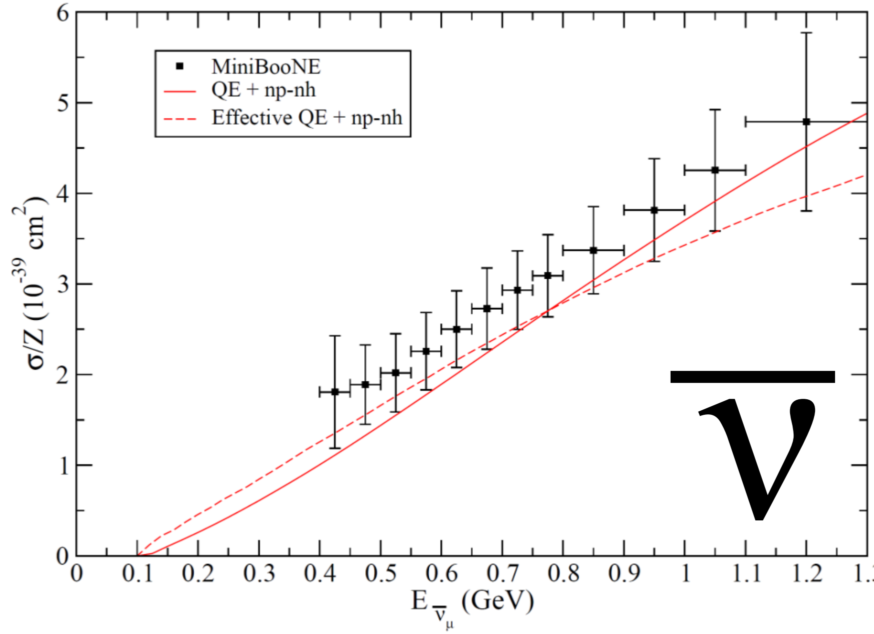


M. Martini, M. Ericson, G. Chanfray, Phys. Rev. D 85 093012 (2012)

CCQE-like cross sections as a function of real (continuous line) and reconstructed (dashed line) neutrino energy



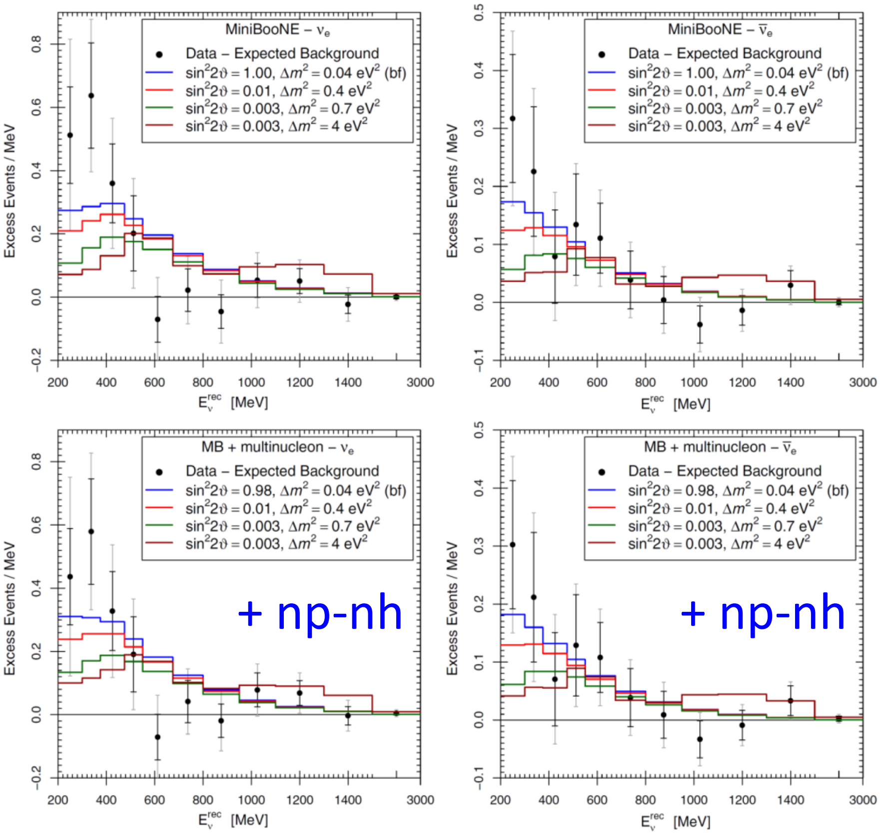
M. Martini, M. Ericson, G. Chanfray, Phys. Rev. D 87 013009 (2013)



Martini, Ericson, Phys. Rev. C 87 065501 (2013)

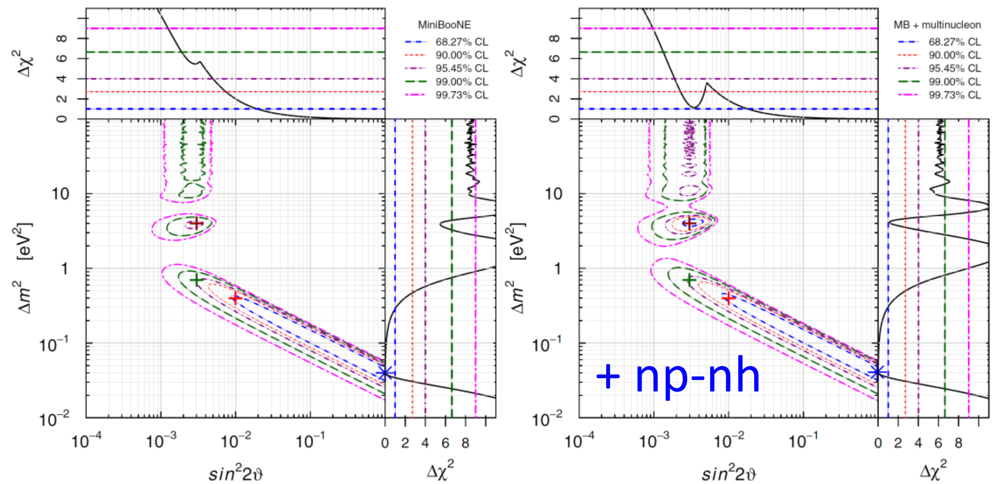
The role of np-nh in the $\nu_\mu \to \nu_e$ MiniBooNE low-energy anomaly

Quantitative analysis

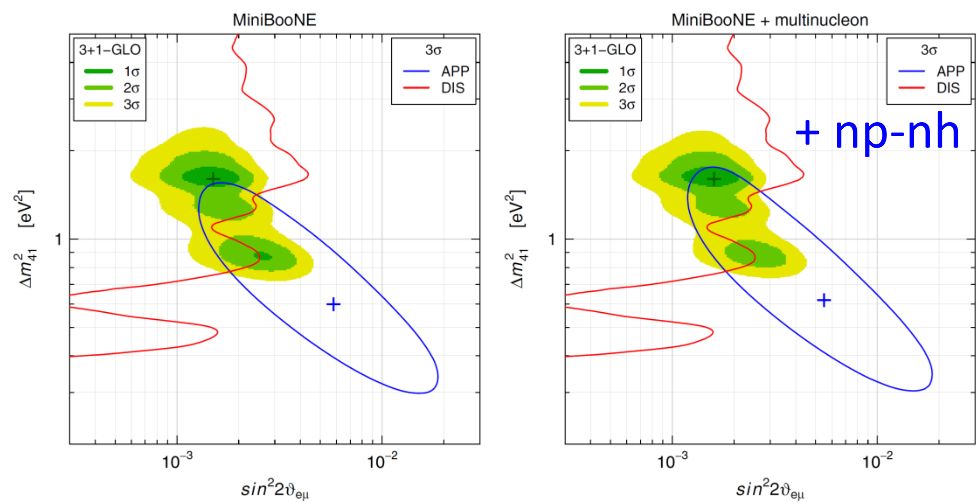


Taking into account np-nh allows a slightly better fit of the MiniBooNE low-energy excess

M.Ericson, M.V.Garzelli, C.Giunti, M.Martini, Phys. Rev. D 93, 073008 (2016)



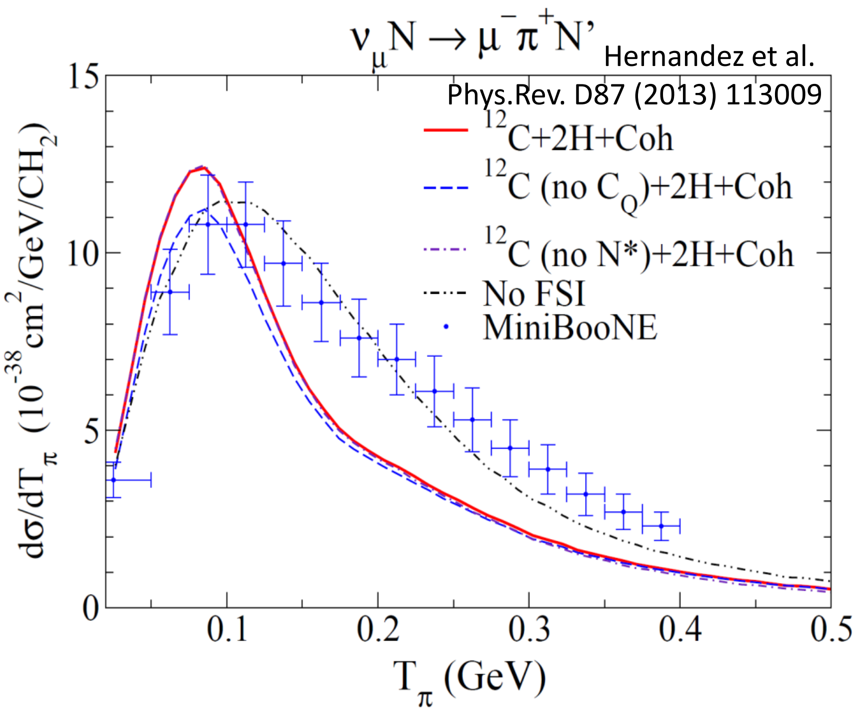
Taking into account np-nh induces a shift of the allowed region towards smaller values of $\sin^2 2\vartheta$ and larger values of Δm^2 in the framework of 2ν oscillations



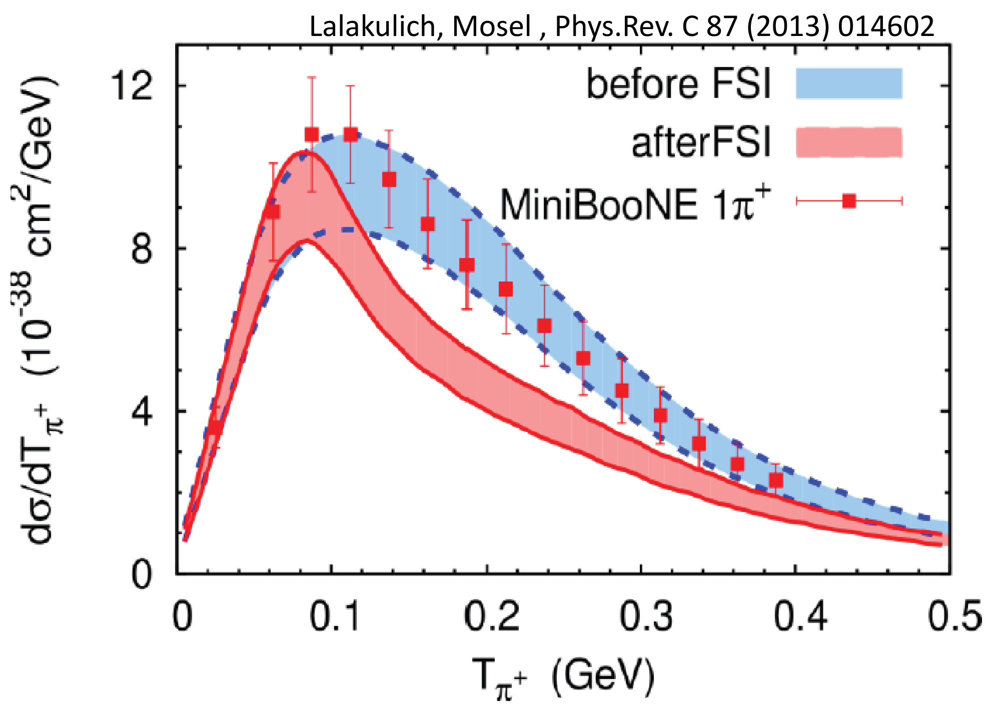
Taking into account np-nh leads to a decrease of the appearance-disappearance tension but not enough to solve the problem in the global fit of short-baseline ν oscillation data

1 Pion production controversy

Best theories (with Δ medium effects and pion rescattering) do not agree with pion KE spectrum



Valencia



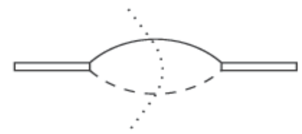
GiBUU

Data prefer calculations with no Final State Interaction for the pion

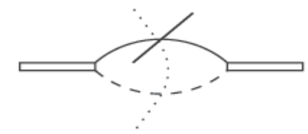
Delta in the nuclear medium

Mass

$$\tilde{M}_\Delta = M_\Delta + 40(MeV)\frac{\rho}{\rho_0}$$



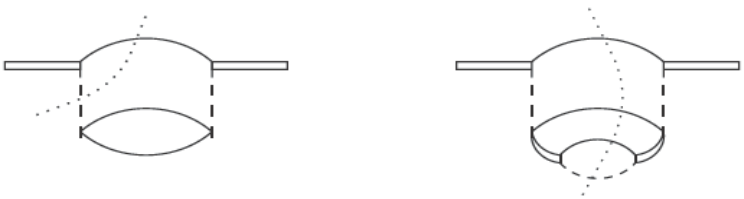
Δ→π N



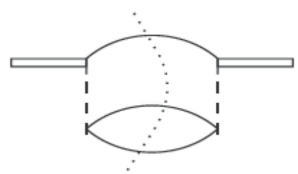
Pauli correction (F_P)

Width

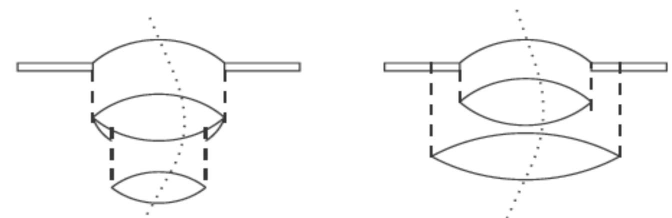
$$\widetilde{\Gamma_\Delta} = \Gamma_\Delta \; F_P - 2\mathrm{Im}(\Sigma_\Delta)$$



Pion distortion (C_Q)



2p-2h



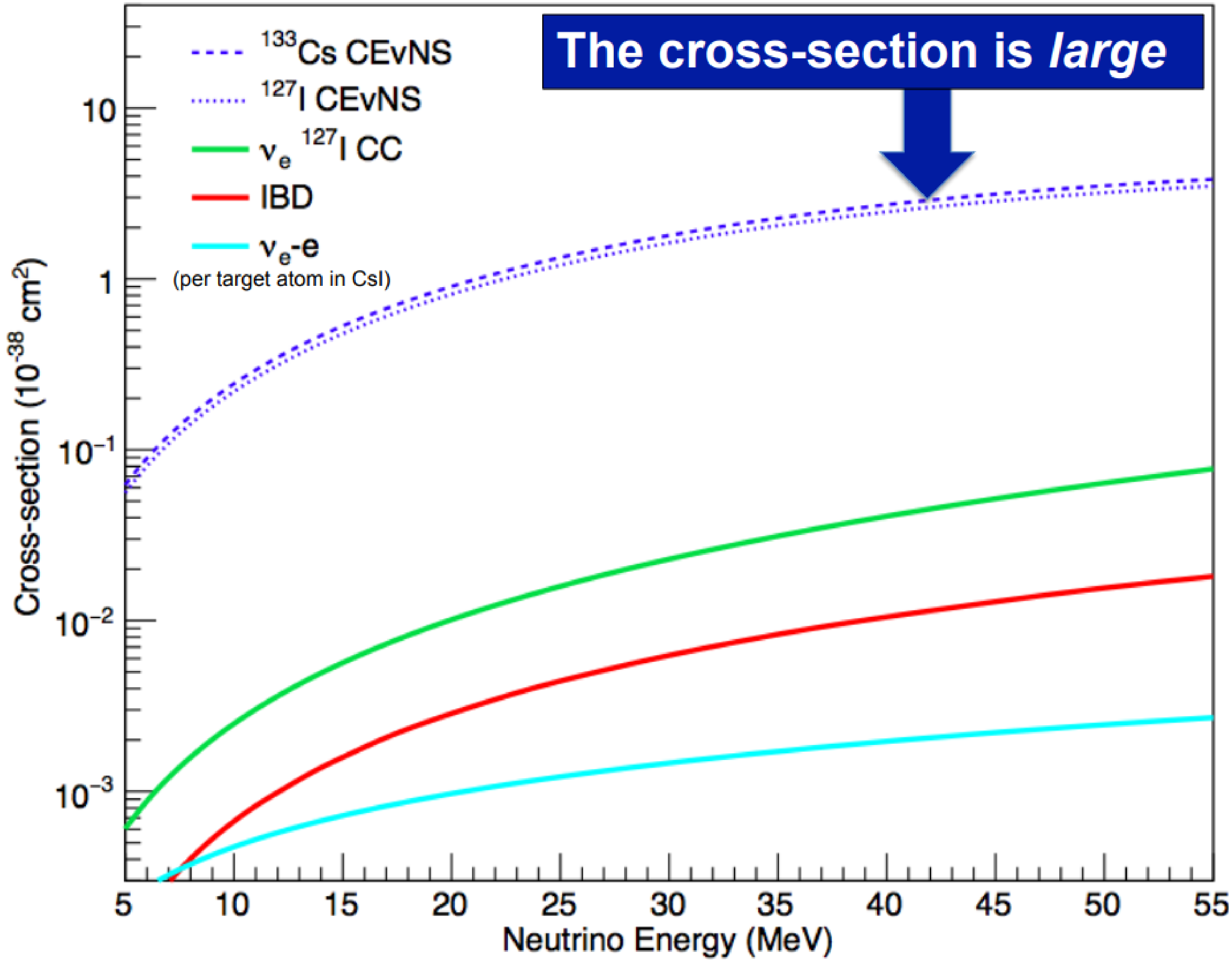
3p-3h

Self energy

$$\mathrm{Im}(\Sigma_\Delta(\omega)) = -\left[C_Q(\frac{\rho}{\rho_0})^\alpha + C_{2p2h}(\frac{\rho}{\rho_0})^\beta + C_{3p3h}(\frac{\rho}{\rho_0})^\gamma\right]$$

E. Oset and L. L. Salcedo, Nucl. Phys. A 468, 631 (1987)

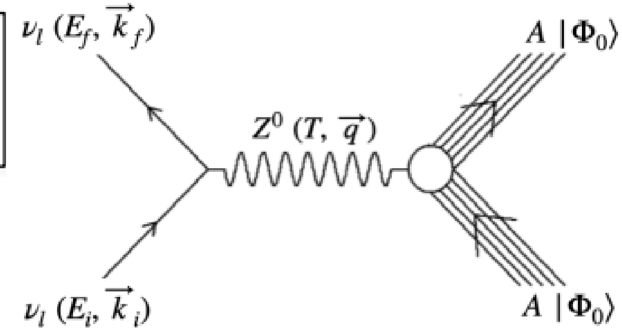
Coherent Elastic neutrino Nucleus Scattering (CEvNS)



CEvNS and Parity Violating Electron Scattering

- **CEvNS cross section**:

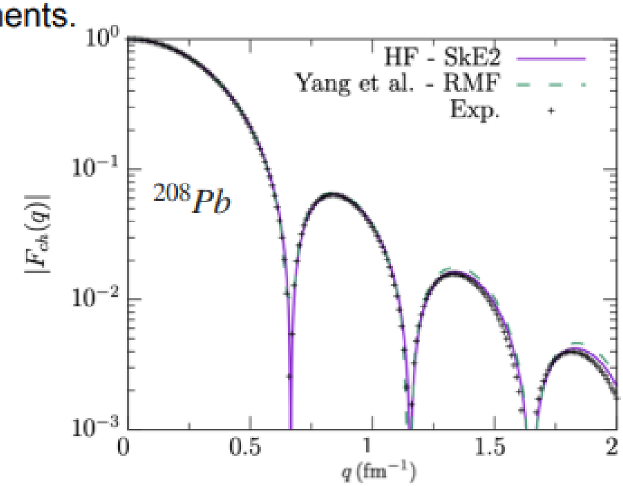
$$\frac{d\sigma}{dT} = \frac{G_F^2}{\pi} M_A \left[1 - \frac{T}{E_i} - \frac{M_A T}{2E_i^2}\right] \frac{Q_W^2}{4} F_W^2(q)$$



$$Q_W\ F_W(q) \approx \langle \Phi_0 | \hat{J}_0(q) | \Phi_0 \rangle$$
$$\approx (1 - 4\sin^2\theta_W)\, Z\, F_p(q) - N\, F_n(q)$$
$$\approx 2\pi \int d^3r\ [(1 - 4\sin^2\theta_W)\rho_p(r) - \rho_n(r)\ ]\, j_0(qr)$$

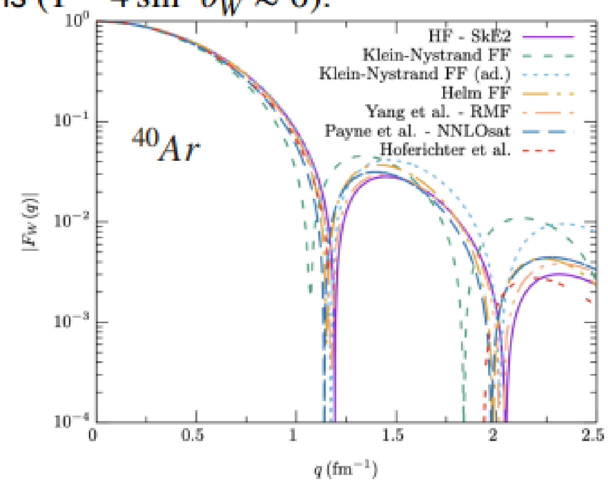
N. Van Dessel, V. Pandey, H. Ray, N. Jachowicz, arXiv:2007.03658 [nucl-th]

Charge density and charge form factor: proton densities and charge form factors are well know through decades of elastic electron scattering experiments.



Data: H. De Vries, et al., Atom. Data Nucl. Data Tabl. 36, 495 (1987)

Neutron densities and neutron form factor: neutron densities and form factors are poorly known. Note that CEvNS is primarily sensitive to neutron density distributions ($1 - 4\sin^2\theta_W \approx 0$).



Fermilab