Theory of neutrino masses and oscillations

Lecture 2

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- neutrino propagation in matter
  - constant matter density – long baseline experiments
  - varying matter density – solar neutrinos
- 3-flavour interpretation of experimental results
- the absolute neutrino mass scale
- the neutrino nature: neutrinoless double beta decay

Ecole de Gif 2022 : La Physique des Neutrinos
LPNHE Paris, 5-9 septembre 2022

Neutrino propagation in matter

The interaction of neutrinos with matter (e-, p, n) affect their propagation $\Longrightarrow$ modified oscillation parameters + a new phenomenon: matter-induced flavour conversion in a medium with varying density

<u>Appropriate description</u>: Schrödinger-like equation

$$i\frac{d}{dt}\,|\nu(t)\rangle \;=\; H\,|\nu(t)\rangle$$

The Hamiltonian H contains a potential term describing the interactions of the neutrinos with the medium and can depend on t

It is convenient to write the Schrödinger equation in the flavour eigenstate basis $\{\,|\nu_\alpha\rangle\,,|\nu_\beta\rangle\,,\cdots\}$, in which $|\nu(t)\rangle = \sum_\beta \nu_\beta(t)\,|\nu_\beta\rangle$:

$$i\frac{d}{dt}\,\nu_\beta(t) \;=\; \sum_\gamma H_{\beta\gamma}\,\nu_\gamma(t)$$

$$\nu_\beta(t) \;=\; \langle\nu_\beta|\nu(t)\rangle$$

$$H_{\beta\gamma} \;=\; \langle\nu_\beta|\,H\,|\nu_\gamma\rangle$$

$\nu_\beta(t)$ is the probability amplitude to find the neutrino in the state $|\nu_\beta\rangle$ at t

if $|\nu(t=0)\rangle = |\nu_\alpha\rangle$, then $P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2$

Vacuum oscillations in the Schrödinger formalism (2-flavour case)

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_0\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad H_0 = U\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}U^\dagger$$

The Hamiltonian in vacuum H0 is diagonalized by the PMNS matrix

One can check that this reproduces the standard oscillation formula (*)

It is customary to subtract a piece proportional to the unit matrix from H0 (which only affects the overall phase of the neutrino state vector $|\nu(t)\rangle$, leaving oscillations unchanged) to bring it to the form:

$$H_0 = \begin{pmatrix}-\frac{\Delta m^2}{2E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta\\ \frac{\Delta m^2}{4E}\sin 2\theta & 0\end{pmatrix}$$

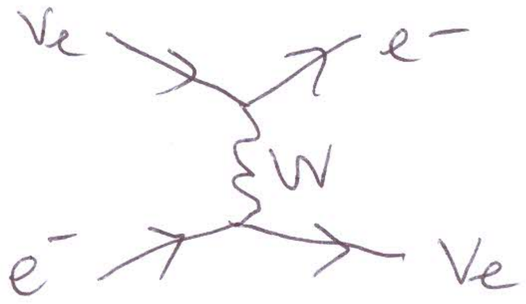
(*) proof: use the fact that $|\nu_\gamma\rangle = \sum_i U^*_{\gamma i}\,|\nu_i\rangle \;\Rightarrow\; \nu_\gamma(t) = \sum_i U_{\gamma i}\,\nu_i(t)$, where $\nu_i(t) \equiv \langle\nu_i|\nu(t)\rangle$ to show that the solution of the Schrödinger equation is $\nu_\gamma(t) = \sum_i U_{\gamma i}\,\nu_i(t)$ with $\nu_i(t) = e^{-iE_i t}\,\nu_i(0) = e^{-iE_i t}\,U^*_{\delta i}\,\nu_\delta(0)$, which reproduces the known oscillation formula

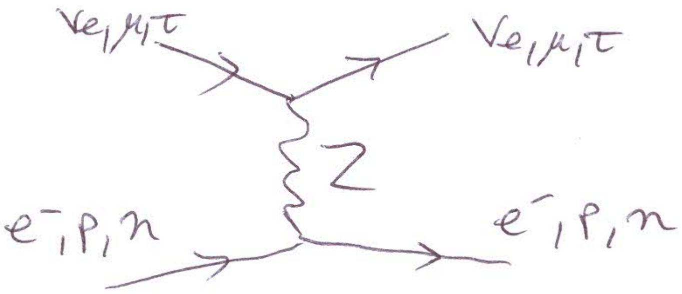
Neutrino Hamiltonian in matter

$$H_0 \;\rightarrow\; H_m = H_0 + V$$

V induced by interactions (anti-)neutrinos / e-, p, n of the medium

Relevant interactions: forward elastic scatterings ($\vec{p}_\nu$ unchanged)





CC – only for ν_e

NC – same for $\nu_{e,\mu,\tau}$ $\Rightarrow$ can be subtracted from Hm

$$V_{\rm CC} = \sqrt{2}\, G_F n_e(x) \qquad\qquad V_{\rm NC} = -\frac{G_F}{\sqrt{2}}\, n_n(x)$$

G_F = Fermi constant

In the flavour eigenstate basis:

$$(H_m)_{\beta\gamma} \;=\; (H_0)_{\beta\gamma} + V_\beta\, \delta_{\beta\gamma} \qquad\qquad V_\beta = V^\beta_{\rm CC} + V^\beta_{\rm NC}$$

For anti-neutrinos, V has the opposite sign: $V \;\rightarrow\; -V$

For a sterile (= insensitive to weak interactions) neutrino: $V_\beta \;=\; 0$

Example with electron neutrinos and another flavour:

$$i\frac{d}{dt}\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} = H_m\begin{pmatrix}\nu_\alpha(t)\\ \nu_\beta(t)\end{pmatrix} \qquad \begin{cases}\alpha = e\\ \beta = \mu, \tau, s\end{cases}$$

$$H_m = \begin{pmatrix} -\frac{\Delta m^2}{2E}\cos 2\theta + \sqrt{2}\,G_F n & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & 0 \end{pmatrix}$$

$$n = \begin{cases} n_e(x) & \text{if } \beta = \mu, \tau \\ n_e(x) - \frac{1}{2}\,n_n(x) & \text{if } \beta = s \end{cases}$$

For anti-neutrinos, $+\sqrt{2}\,G_F n \ \rightarrow\ -\sqrt{2}\,G_F n$

Energy levels in matter and matter eigenstates

In vacuum, the PMNS matrix U relates the flavour eigenstates to the mass eigenstates (= eigenstates of H0):

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U^* \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix} \qquad \begin{matrix} \leftarrow & E_1 = \sqrt{\vec{p}^2 + m_1^2} \\ \leftarrow & E_2 = \sqrt{\vec{p}^2 + m_2^2} \end{matrix}$$

In matter, one defines matter eigenstates = eigenstates of Hm

$$\begin{pmatrix} |\nu_\alpha\rangle \\ |\nu_\beta\rangle \end{pmatrix} = U_m^* \begin{pmatrix} |\nu_1^m\rangle \\ |\nu_2^m\rangle \end{pmatrix} \qquad \left.\begin{matrix} \leftarrow & E_1^m \\ \leftarrow & E_2^m \end{matrix}\right\} \begin{matrix} \text{eigenvalues of } H_m = \\ \text{energy levels in matter} \end{matrix}$$

Um contains the mixing angle in matter that diagonalizes Hm :

$$H_m = U_m \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$$

By analogy with $\nu_\beta(t) = \langle \nu_\beta | \nu(t) \rangle$, one defines $\nu_i^m(t) = \langle \nu_i^m | \nu(t) \rangle$ (amplitude of probability to find the neutrino in the ith matter eigenstate at t)

then
$$\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

Medium with constant matter density

n(x) = n ⇒ Hm, hence the matter eigenstates $|\nu_i^m\rangle$, energy levels E_i^m and mixing matrix Um, do not depend on t

Using $\begin{pmatrix} \nu_\alpha(t) \\ \nu_\beta(t) \end{pmatrix} = U_m \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$, one can rewrite the Schrödinger equation for the probability amplitudes $\nu_i^m(t)$

$$i\frac{d}{dt}\begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m & 0 \\ 0 & E_2^m \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

which is solved by $\nu_i^m(t) = e^{-iE_i^m t}\,\nu_i^m(0)$, giving

$$P(\nu_\alpha \to \nu_\beta) = |\nu_\beta(t)|^2 = |\textstyle\sum_i (U_m)_{\beta i}\,\nu_i^m(t)|^2 = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

→ oscillations in matter with constant density are governed by the same formula as in vacuum, with the replacements

$$\theta \;\to\; \theta_m\,, \qquad \frac{\Delta m^2}{4E} \;\to\; \frac{(E_m^2 - E_m^1)}{2}$$

Oscillation parameters in matter

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta_m \sin^2 \frac{(E_m^2 - E_m^1)t}{2}$$

$$E_2^m - E_1^m = \frac{\Delta m^2}{2E}\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}$$

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$\cos 2\theta_m = \frac{\left(1-\frac{n}{n_{\rm res}}\right)\cos 2\theta}{\sqrt{\left(1-\frac{n}{n_{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta}}$$

$$n_{\rm res} \equiv \frac{\Delta m^2 \cos 2\theta}{2\sqrt{2}\, G_F E}$$

for antineutrinos,

$$n \to -n$$

(n = ne if only active neutrinos)

MSW resonance (Mikheev-Smirnov-Wolfenstein):

$$\sin 2\theta_m = 1 \quad \text{for } n = n_{\rm res}$$

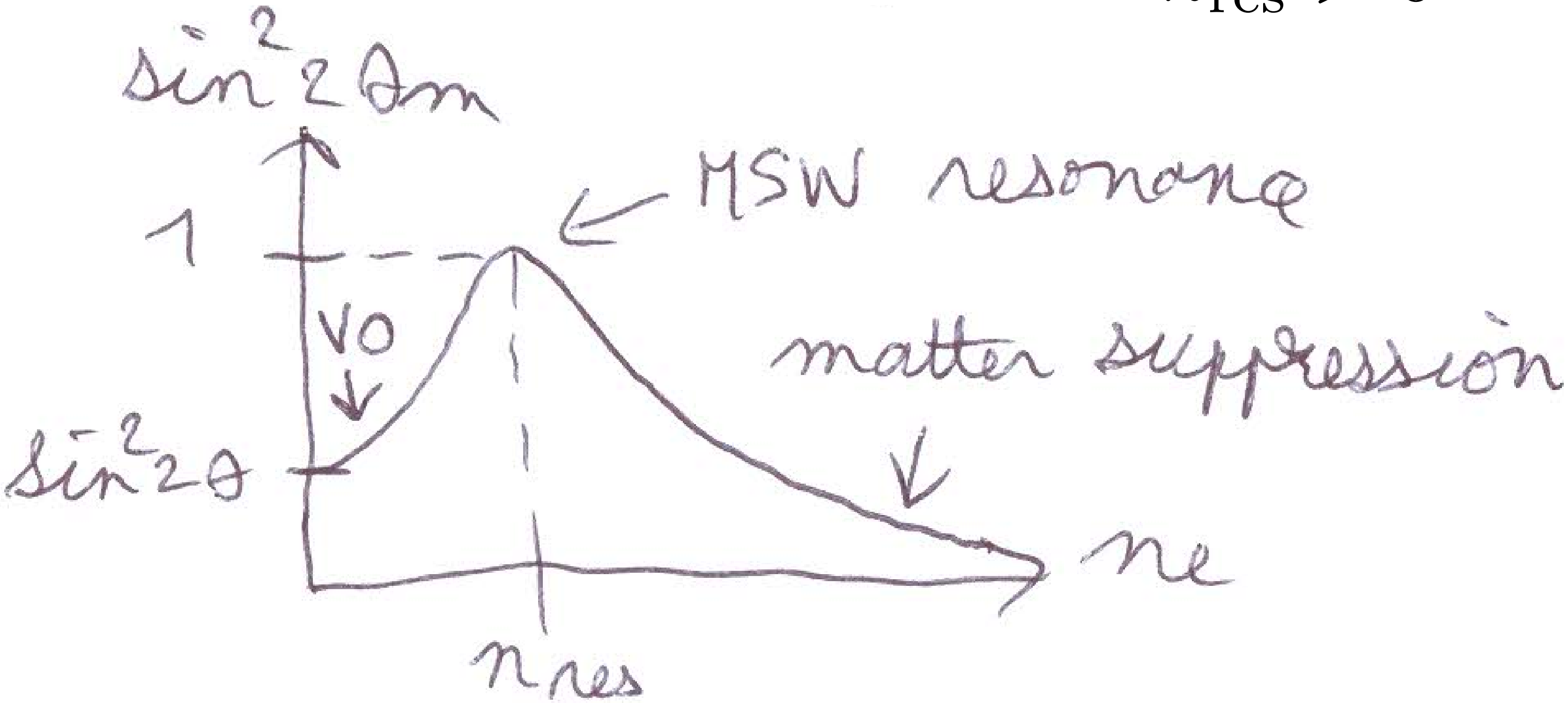
(irrespective of the value of the mixing angle in vacuum θ)

Resonance condition:
$$\begin{cases} \Delta m^2 \cos 2\theta > 0 & \text{for neutrinos} \\ \Delta m^2 \cos 2\theta < 0 & \text{for antineutrinos} \end{cases}$$

When neutrino oscillations are enhanced, antineutrino oscillations are suppressed, and vice versa

Different regimes for oscillations in matter :

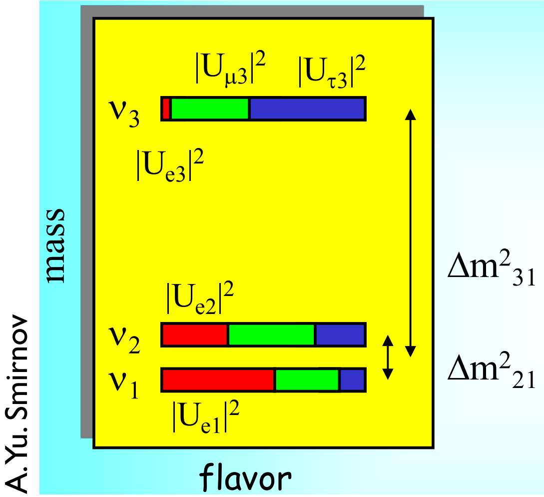
$n_{\rm res} > 0$



- low density ($n \ll n_{\rm res}$) : $\sin 2\theta_m \simeq \sin 2\theta$ $\Rightarrow$ vacuum oscillations
- resonance ($n = n_{\rm res}$) : $\sin 2\theta_m = 1$
- high density ($n \gg n_{\rm res}$) : $\sin 2\theta_m < (\ll) \sin 2\theta$ $\Rightarrow$ oscillations are suppressed by matter effects

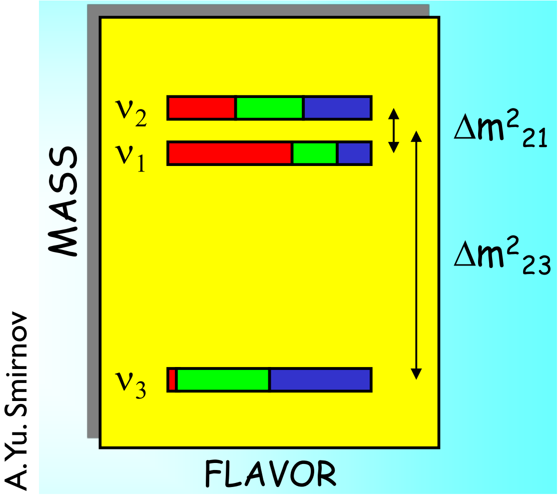
Application: determination of the mass hierarchy in long-baseline experiments

Two mass orderings allowed by experiments:



Normal hierarchy

$$\Delta m^2_{31} > 0$$



Inverted hierarchy

$$\Delta m^2_{31} < 0$$

In vacuum: $$P(\nu_\mu \to \nu_e) \simeq \sin^2\theta_{23}\sin^2 2\theta_{13}\,\sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

For long baselines (> several 100 km), matter effects cannot be neglected

$$n_{\rm res} = \frac{\Delta m^2_{31}\cos 2\theta_{13}}{2\sqrt{2}\,G_F E} \qquad \begin{cases} n_{\rm res} > 0 & \text{for normal hierarchy} \\ n_{\rm res} < 0 & \text{for inverted hierarchy} \end{cases}$$

If nres is close to the Earth crust density, neutrino (antineutrino) oscillations are enhanced for NH (IH), while antineutrino (neutrino) oscillations are suppressed

[may have to disentangle CP violation from matter effect] and outdated but informative plot

$$R = \frac{\overline{\nu}_e \rightarrow \overline{\nu}_\mu}{\nu_e \rightarrow \nu_\mu}$$

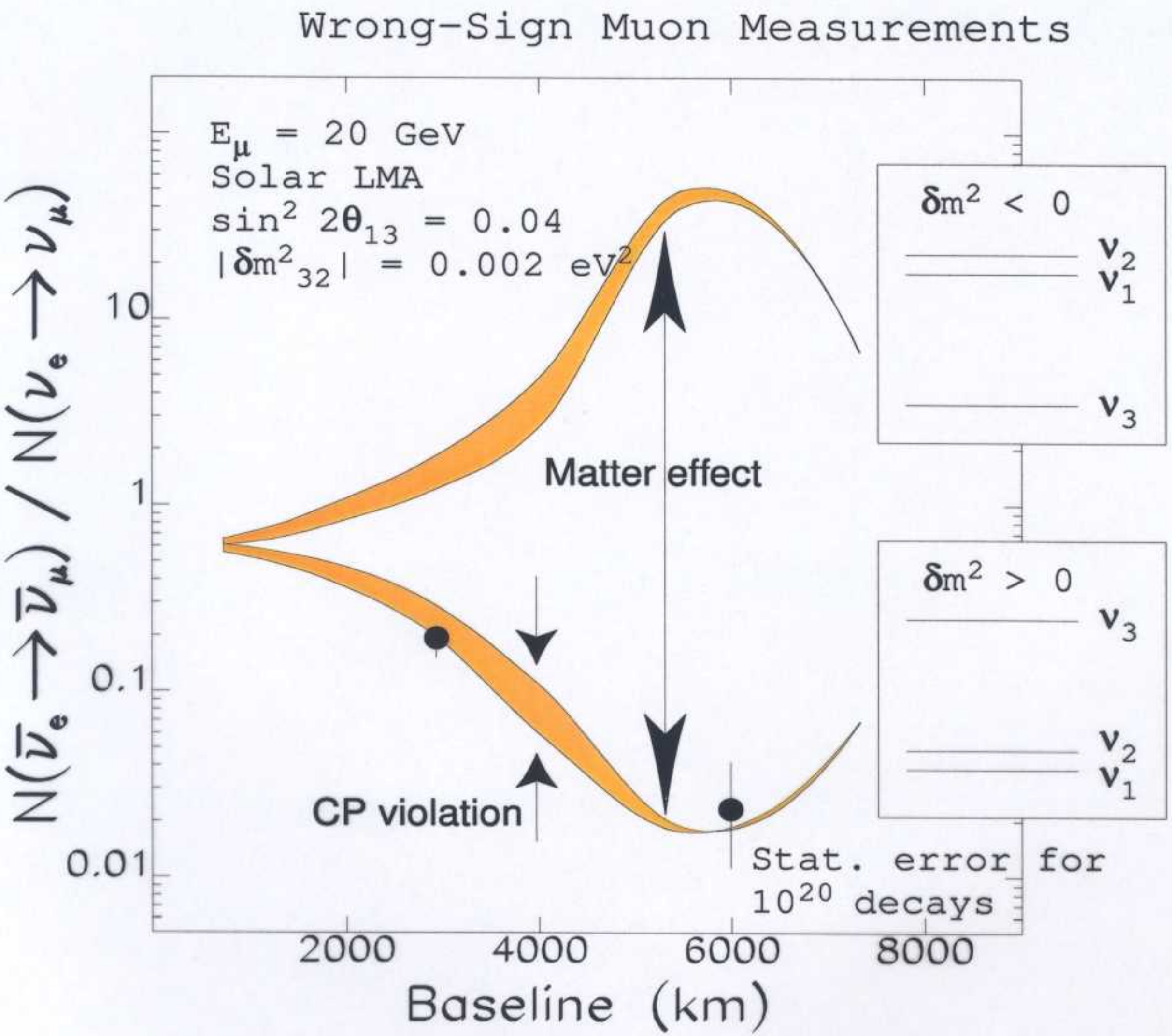


Figure 2: Predicted ratios of $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ to $\nu_e \rightarrow \nu_\mu$ rates at a 20 GeV neutrino factory. The statistical error shown corresponds to 10^{20} muon decays of each sign and a 50 kt detector.

- Un baseline de $L = \mathcal{O}$ (3000 km) est nécessaire/optimale

[Barger, Geer, Raja, Whisnant]

Medium of varying density (e.g. the Sun)

Now the matter eigenstates, energy levels and mixing angle depend on t

→ "instantaneous" matter eigenstates: $|\nu_i^m(t)\rangle \leftarrow E_i^m(t)$

$$H_m = U_m \begin{pmatrix} E_1^m(t) & 0 \\ 0 & E_2^m(t) \end{pmatrix} U_m^\dagger \qquad U_m = \begin{pmatrix} \cos\theta_m(t) & \sin\theta_m(t) \\ -\sin\theta_m(t) & \cos\theta_m(t) \end{pmatrix}$$

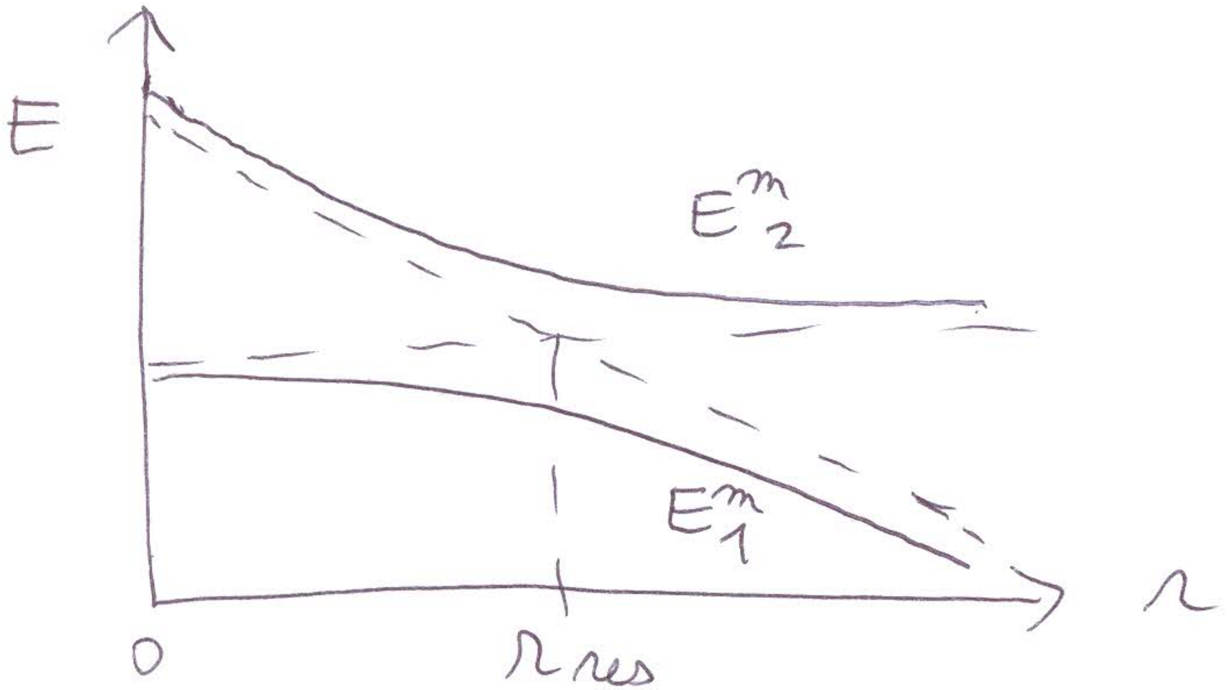
The Schrödinger equation now depends on the time variation of θm:

$$i\frac{d}{dt}\begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix} = \begin{pmatrix} E_1^m(t) & -i\dot{\theta}_m \\ i\dot{\theta}_m & E_2^m(t) \end{pmatrix} \begin{pmatrix} \nu_1^m(t) \\ \nu_2^m(t) \end{pmatrix}$$

In most physical environments (including the Sun), the evolution is <u>adiabatic</u> (the neutrino state has the time to adjust to the variation of density) and one can neglect $\dot{\theta}_m$ in the Schrödinger equation. A neutrino produced in a given matter eigenstate will stay in the same matter eigenstate during its propagation, but its flavour composition will change

→ adiabatic flavour conversion

"Level crossing" in the Sun (case n_e (r=0) >> n_{res})



This is the case for high-energy solar neutrinos (E > 1 MeV)

$$n_e(r=0) \gg n_{\rm res} \;\Rightarrow\; \sin 2\theta_m^0 \simeq 0 \text{ and } \cos 2\theta_m^0 \simeq -1$$

$$\Rightarrow\; \theta_m^0 \simeq \pi/2 \;\Rightarrow\; |\nu_e\rangle \simeq |\nu_2^m(r=0)\rangle$$

⇒ a neutrino produced at the center of the Sun is a quasi pure matter eigenstate and exits the Sun in the eigenstate

$$|\nu_2^m(r=R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\,|\nu_e\rangle + \cos\theta\,|\nu_\beta\rangle \quad (\beta=\mu,\tau)$$

$\Rightarrow$ a high-energy neutrino produced at the center of the Sun is a quasi pure matter eigenstate and exits the Sun in the eigenstate

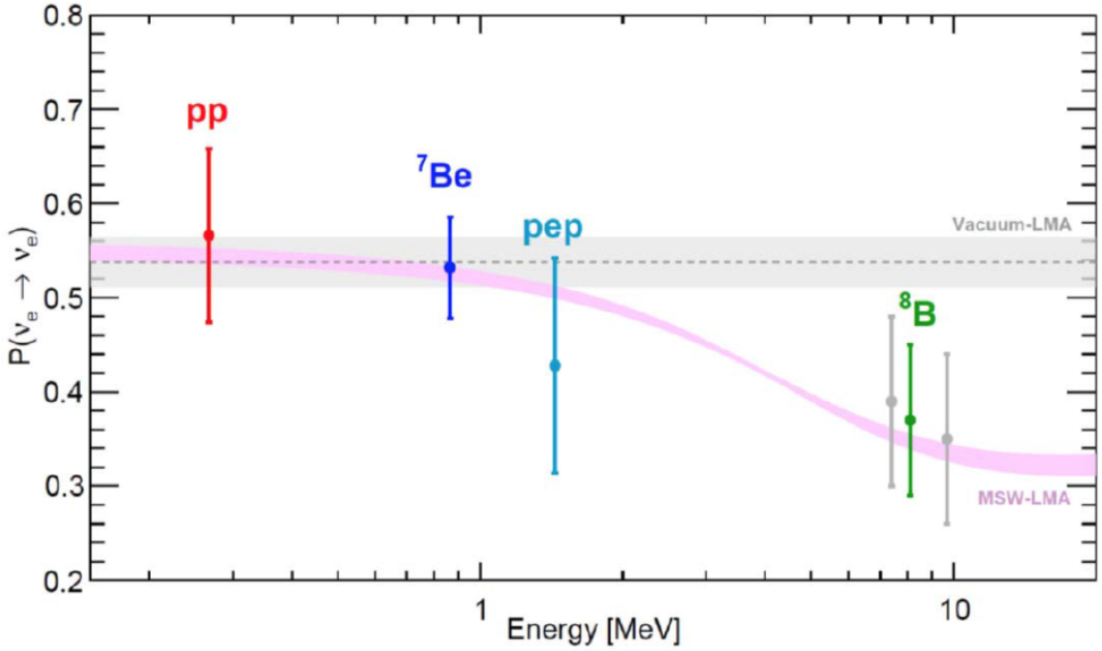
$$|\nu_2^m(r = R_{\rm Sun})\rangle = |\nu_2\rangle = \sin\theta\, |\nu_e\rangle + \cos\theta\, |\nu_\beta\rangle \qquad (\beta = \mu, \tau)$$

and reaches the Earth as a $|\nu_2\rangle$, giving (using the observed value of θ_{12} for θ)

$$P_{ee} \;=\; |\langle\nu_e|\nu_2\rangle|^2 \;=\; \sin^2\theta \;\simeq\; 0.3$$

For low-energy solar neutrinos, the level-crossing condition is not satisfied ($n_e(r=0) \ll n_{\rm res}$) and matter effects are small

$\Rightarrow$ averaged vacuum oscillations: $$P_{ee} \;=\; 1 - \frac{1}{2}\sin^2 2\theta \;\simeq\; 0.58$$



Borexino Phase II results

[talk at TAU2018, arXiv:1810.12967]

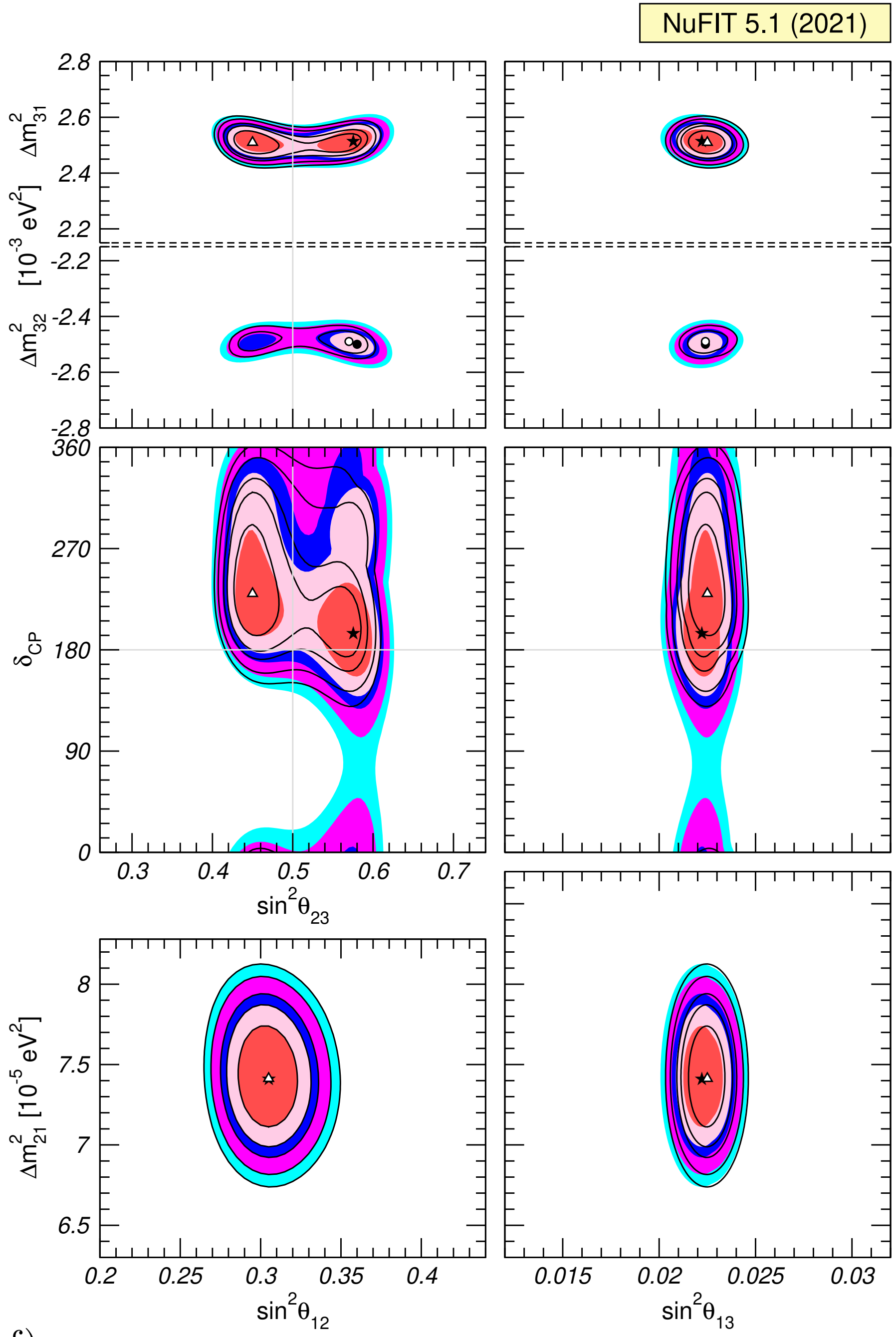
3-flavour interpretation of experimental results

(aka « 3-flavour global fit »)

I. Esteban et al., JHEP 09 (2020) 178
NuFIT 5.1 (2021), www.nu-fit.org

(based on data available in October 2021)

All experimental data (leaving aside a few anomalies) is very well described in the 3-flavour framework, and the determination of oscillation parameters is becoming more and more precise



The different contours correspond to 1σ, 90%, 2σ, 99%, 3σ CL (2 dof).

Allowed ranges for the oscillation parameters (October 2021)

I. Esteban et al., NuFIT 5.1 (2021) , JHEP 09 (2020) 178

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 7.0$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$
	$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \to 35.87$	$33.45^{+0.78}_{-0.75}$	$31.27 \to 35.87$
	$\sin^2\theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.570^{+0.016}_{-0.022}$	$0.410 \to 0.613$
	$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \to 50.9$	$49.0^{+0.9}_{-1.3}$	$39.8 \to 51.6$
	$\sin^2\theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \to 0.02435$	$0.02241^{+0.00074}_{-0.00062}$	$0.02055 \to 0.02457$
	$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \to 8.98$	$8.61^{+0.14}_{-0.12}$	$8.24 \to 9.02$
	$\delta_{\mathrm{CP}}/^\circ$	230^{+36}_{-25}	$144 \to 350$	278^{+22}_{-30}	$194 \to 345$
	$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
	$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \to +2.593$	$-2.490^{+0.026}_{-0.028}$	$-2.574 \to -2.410$

3σ uncertainty around 15% for θ_{12} and Δm^2_{21} , less than 10% for θ_{13} and $\Delta m^2_{3\ell}$

The best known parameters are θ_{13} and Δm^2_{32} (Δm^2_{31} in the case of normal ordering), with 3σ uncertainties below 10%, and θ_{12} and Δm^2_{21} , with 3σ uncertainties around 15%

By contrast, θ_{23} (first [$\theta_{23} < \pi/4$] or second [$\theta_{23} > \pi/4$] octant ?), the mass ordering (or mass hierarchy), and the CP-violating phase δ depend on subleading 3-flavour effects and are poorly known

⇒ not yet statistically significant

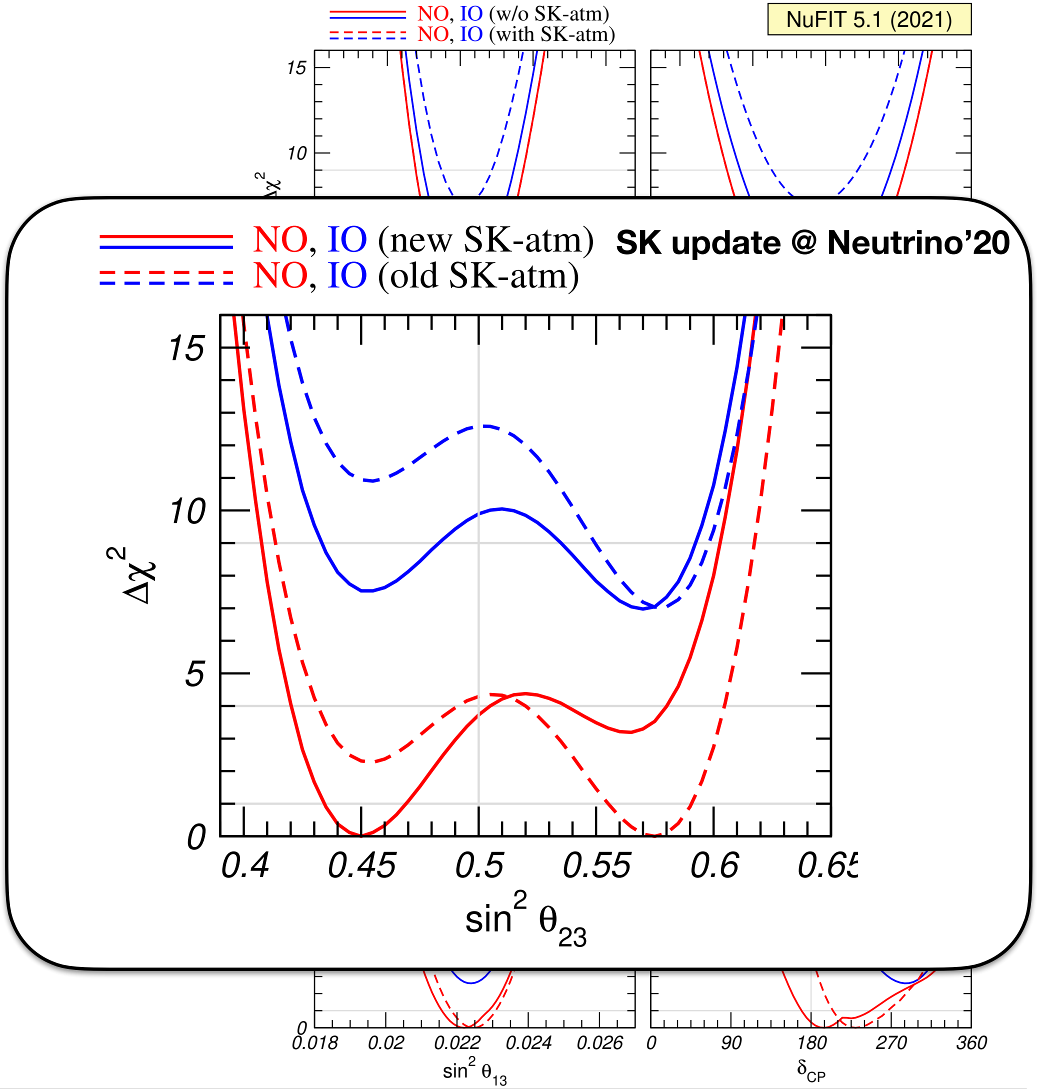
The least known mixing angle

NuFIT 5.1 results www.nu-fit.org

- broad allowed range for θ_{23} (24%)

for quarks: 5.2%

- ambiguity in the octant
  - fragile with respect to atmospheric neutrino analysis and mass ordering

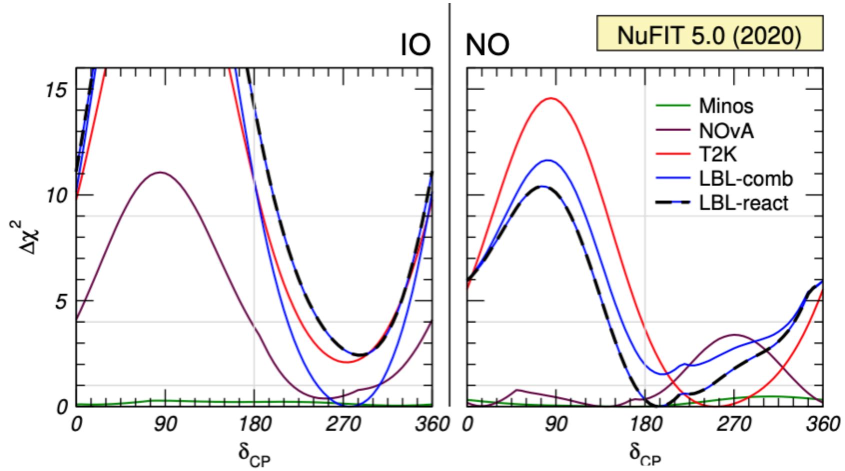


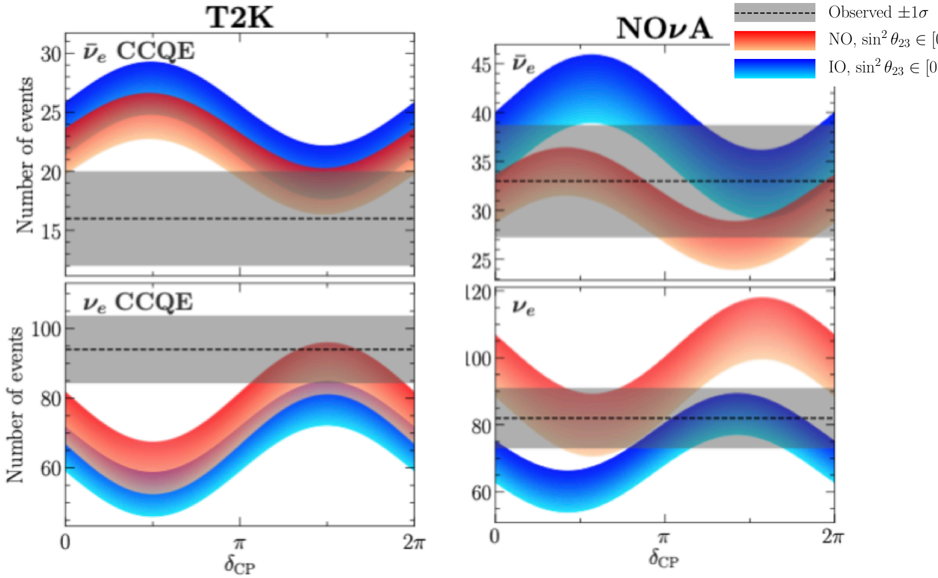
SK update @ Neutrino'20

T. Schwetz, Neutrino 2022

Mass ordering and CP phase

- different tendencies in
  - LBL accelerator data:
    T2K & NOvA better compatible for IO
  - Reactor and LBL data:
    better agreement of $|\Delta m^2_{31}|$ for NO
- overall preference for NO with
  $\Delta\chi^2 = 2.7$ (was 6.2 in 2019)
- CP phase best fit at δ=195° →
  CP conservation allowed at 0.6σ
- for IO: best fit close to δ=270°,
  CP conservation disfavoured at 3σ





T. Schwetz, Neutrino 2022

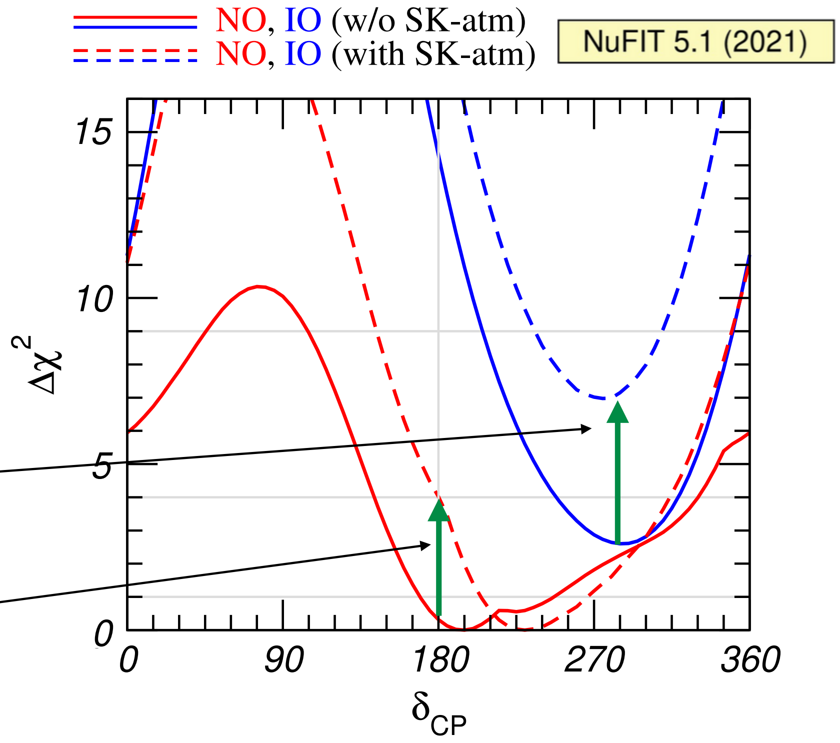
Mass ordering and CP phase: atmospheric neutrinos

- NuFit 5.0 updated with SK I-IV analysis presented @ Neutrino'20

- improved sensitivity to MO:
  χ^2(IO) - χ^2(NO) = 3.2 (atm only)
  pre-Neutrino'20: 4.3

- added to global fit via χ^2 table:
  χ^2(IO) - χ^2(NO) = 2.7 (no SK)
  → 7.1 (w SK) 2.7σ

- CP conservation @ 0.6σ (no SK)
  → 2σ (w SK)
  best fit: $\delta_{CP} \approx 230°$



The absolute neutrino mass scale

Oscillation experiments measure only mass squared differences
→ information on the neutrino mass scale from beta decay or cosmology

Cosmology

Upper bound on sum of neutrino masses from CMB and large structure data
[eV-scale SM neutrinos would be hot dark matter and affect structure formation, leading to fewer small structures than observed ⇒ must be a subdominant DM component]

$$\sum m_\nu < 0.12 \text{ eV} \quad (95\,\%,\ \textit{Planck}\ \text{TT,TE,EE+lowE} \ \text{+lensing+BAO}).$$

[Planck 2018]

[avec + Lyman-a, Palanque-Delabrouille et al. obtiennent < 0.09 eV, 95% CL (JCAP04 (2020) 038)]

Kinematic measurements (beta decay)

The non-vanishing neutrino mass leads to a distortion of the Ee spectrum close to the endpoint

Best bound (KATRIN) : $m_\nu \ < \ 0.8 \text{ eV} \quad (95\% \text{ C.L.})$

[Nature Phys. 18 (2022) 160]

Tritium beta decay

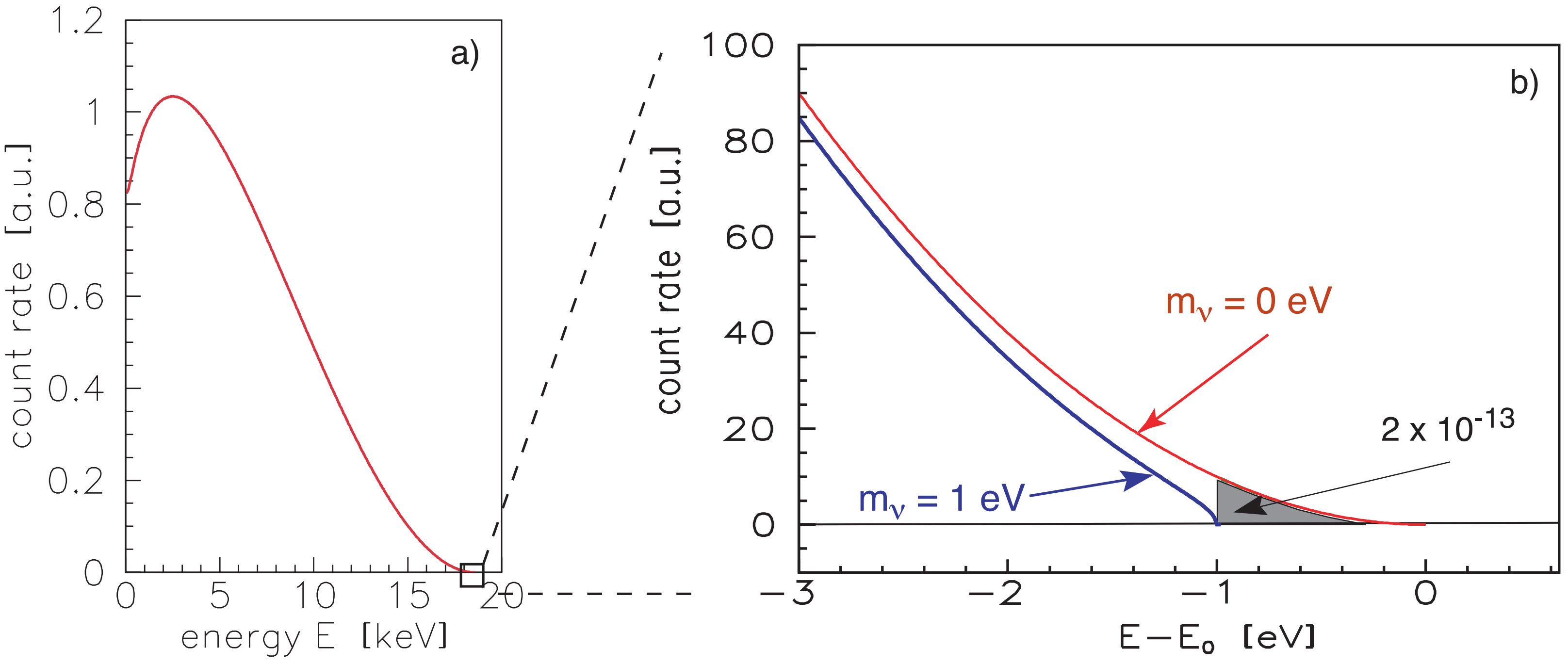
$$^3H \rightarrow \, ^3H_e + e^- + \bar{\nu}_e \qquad\qquad E_0 = m_{^3H} - m_{^3H_e}$$

The electron energy spectrum is given by:

$$\frac{dN}{dE_e} = R(E_e)\sqrt{(E_0 - E_e)^2 - m_\nu^2} \qquad\qquad E_e = E_0 - E_\nu$$

Effect of the non-vanishing neutrino mass: $E_e^{max} = E_0 \rightarrow E_0 - m_\nu$

$\Rightarrow$ distorsion of the Ee spectrum close to the endpoint

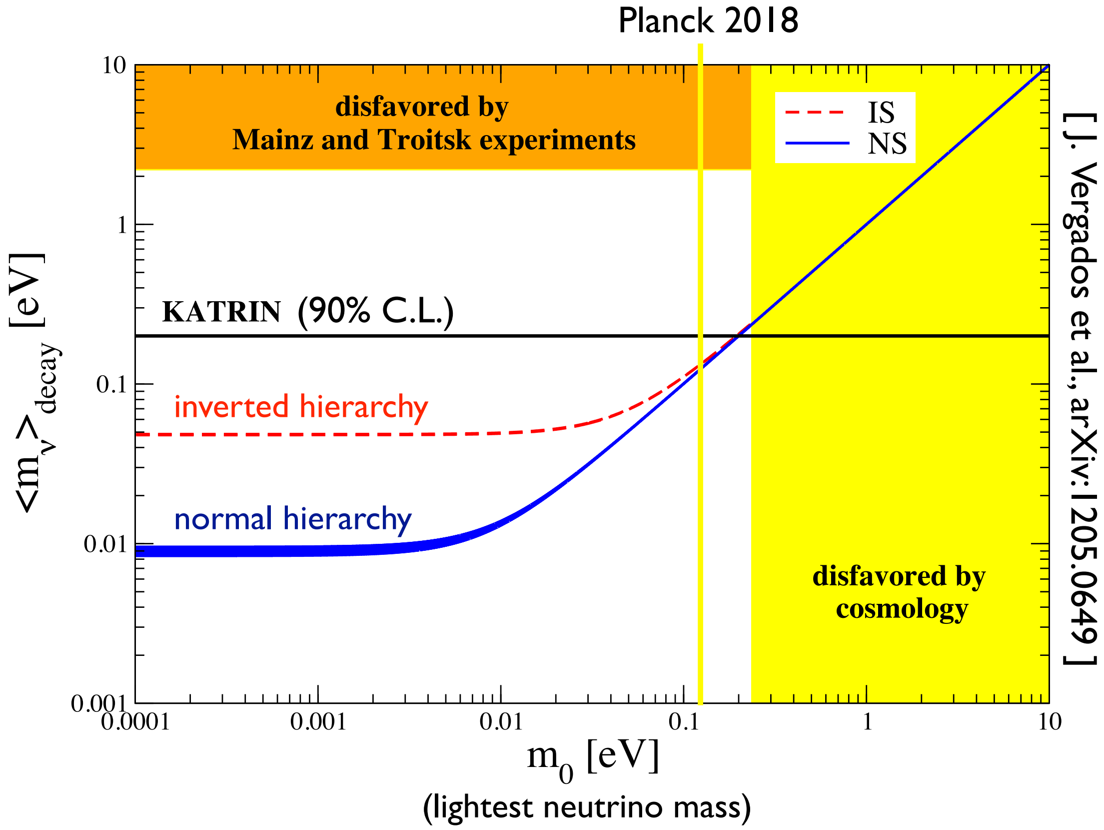


[Katrin Collaboration, hep-ex/0109033]

Present bound (KATRIN) : $m_\nu < 0.8\ \mathrm{eV} \quad (95\%\ \mathrm{C.L.})$

KATRIN will reach a final sensitivity of about 0.3 eV (95% CL)
(5σ discovery potential 0.35 eV)

In pratice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates. However the energy resolution does not allow to resolve them, and what is measured is the effective mass $$m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$



[J. Vergados et al., arXiv:1205.0649]

KATRIN will test only
the degenerate case

Future experiments like Project 8 aim at the 40 meV level

In practice, there is no electron neutrino mass, but 3 (or more) strongly mixed mass eigenstates, and

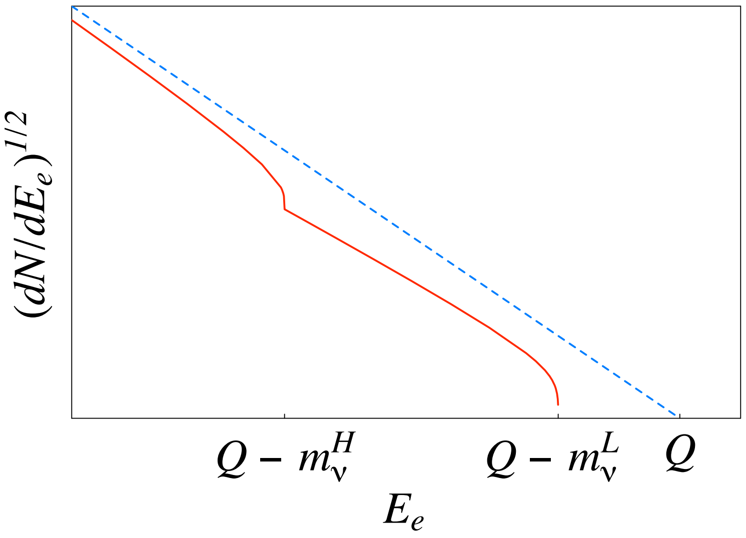
$$\frac{dN}{dE_e} = R(E_e) \sum_i |U_{ei}|^2 \sqrt{(E_0 - E_e)^2 - m_i^2}\, \Theta(E_0 - E_e - m_i)$$

If all mi's are smaller than the energy resolution, this can be rewritten as:

$$\frac{dN}{dE_e} = R(E_e) \sqrt{(E_0 - E_e)^2 - m_\beta^2} \qquad m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$

If there is an eV-scale sterile neutrino (comparable to the energy resolution of KATRIN), its mass may be resolved:

$$\frac{1}{R(E_e)} \frac{dN}{dE_e} = (1 - |U_{e4}|^2) \sqrt{(E_0 - E_e)^2 - m_\beta^2}$$
$$+ |U_{e4}|^2 \sqrt{(E_0 - E_e)^2 - m_4^2}\, \Theta(E_0 - E_e - m_4)$$

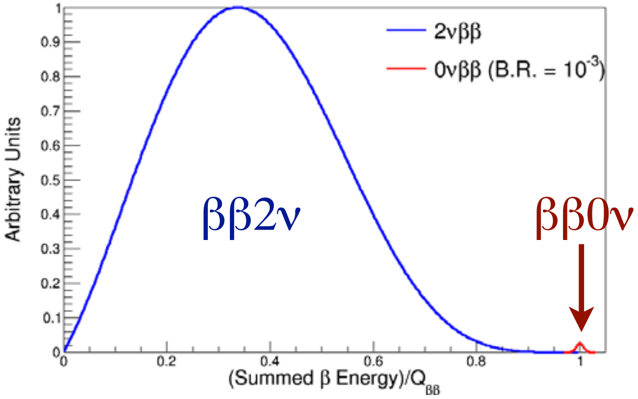


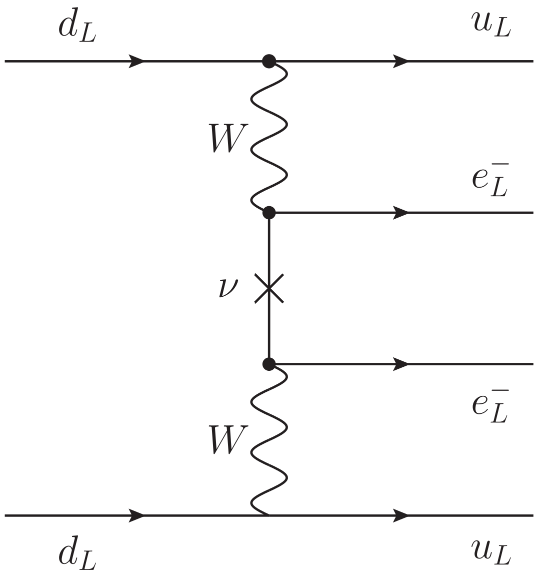
The neutrino nature: neutrinoless double beta decay

$$(A, Z) \;\rightarrow\; (A, Z+2) + e^- + e^-$$

violates lepton number by 2 units

$\Rightarrow$ possible only for Majorana neutrinos





$$Q_{\beta\beta} \equiv M_i - M_f - 2m_e = T_{e_1} + T_{e_2}$$

Half-life:

$$\left[T_{1/2}^{0\nu}\right]^{-1} = \Gamma_{0\nu} = G_{0\nu}(Q_{\beta\beta}, Z)\; |M_{0\nu}|^2\; |m_{\beta\beta}|^2$$

integrated phase-space factor

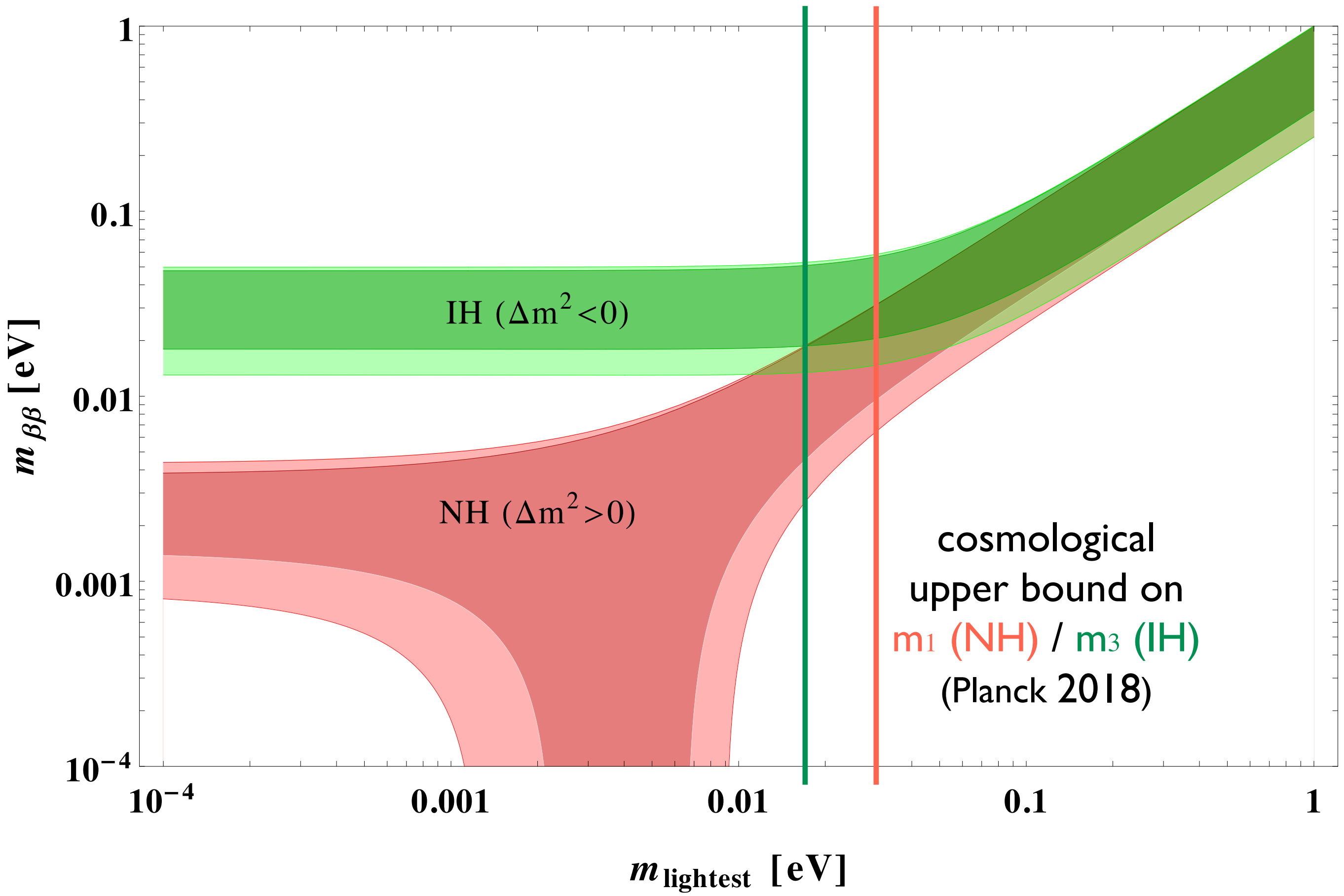
nuclear matrix element (NME)
(large theoretical uncertainty)

Sensitive to the effective mass parameter:

$$m_{\beta\beta} \;\equiv \sum_i m_i U_{ei}^2 = m_1 c_{13}^2 c_{12}^2 e^{2i\alpha_1} + m_2 c_{13}^2 s_{12}^2 e^{2i\alpha_2} + m_3 s_{13}^2$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

[Dell'Oro et al., arXiv:1404.2616]



cosmological
upper bound on
m_1 (NH) / m_3 (IH)
(Planck 2018)

IH

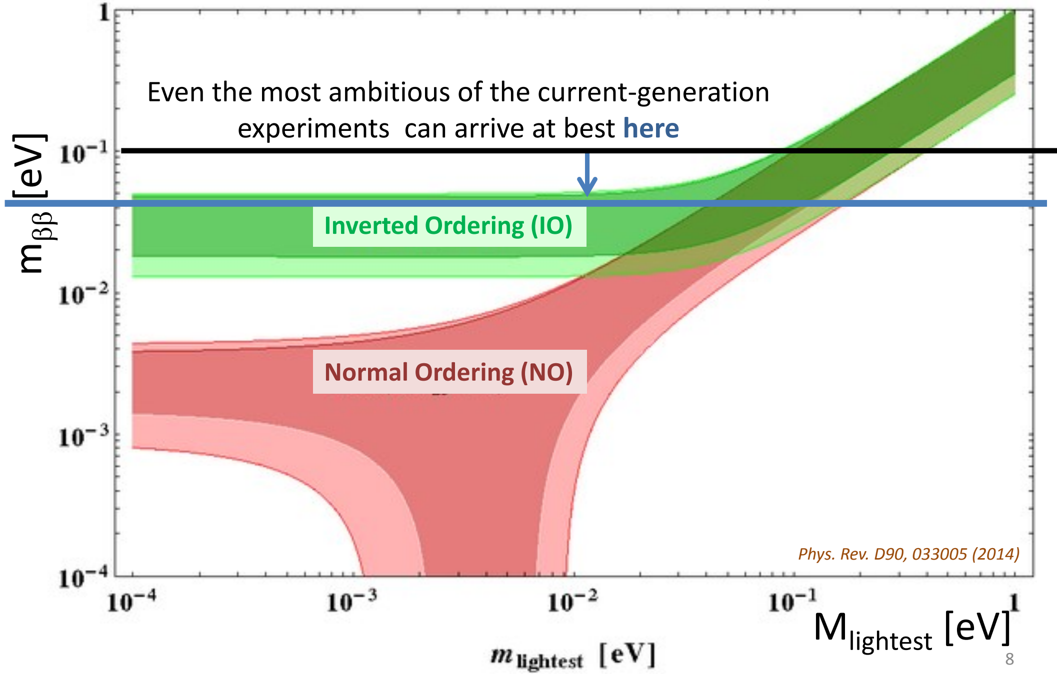
NH

dark shaded areas
= best fit values of
oscillation parameters
(only α_1, α_2 vary)

light shaded areas
= 3σ regions due
to uncertainties on
oscillation parameters
(+ dependence on α_i)

- need to reach 10 meV to exclude IH (lower bound on $m_{\beta\beta}$)
- need to reach few meV to test NH (if no mass degeneracy)
- if unlucky (m_1 ~ 1-10 meV), may not observe ββ0ν even if neutrinos are Majorana (cancellation in $m_{\beta\beta}$ due to α_1, α_2)

A. Giuliani, Neutrino 2018



Even the most ambitious of the current-generation experiments can arrive at best **here**

Phys. Rev. D90, 033005 (2014)

currently here, around 100 meV
(experimental upper bounds
depend on NME calculations
⇒ 2 - 4 uncertainty factor)

around 40 meV

Current best limit (90% C.L.) : KamLAND-Zen (2022)

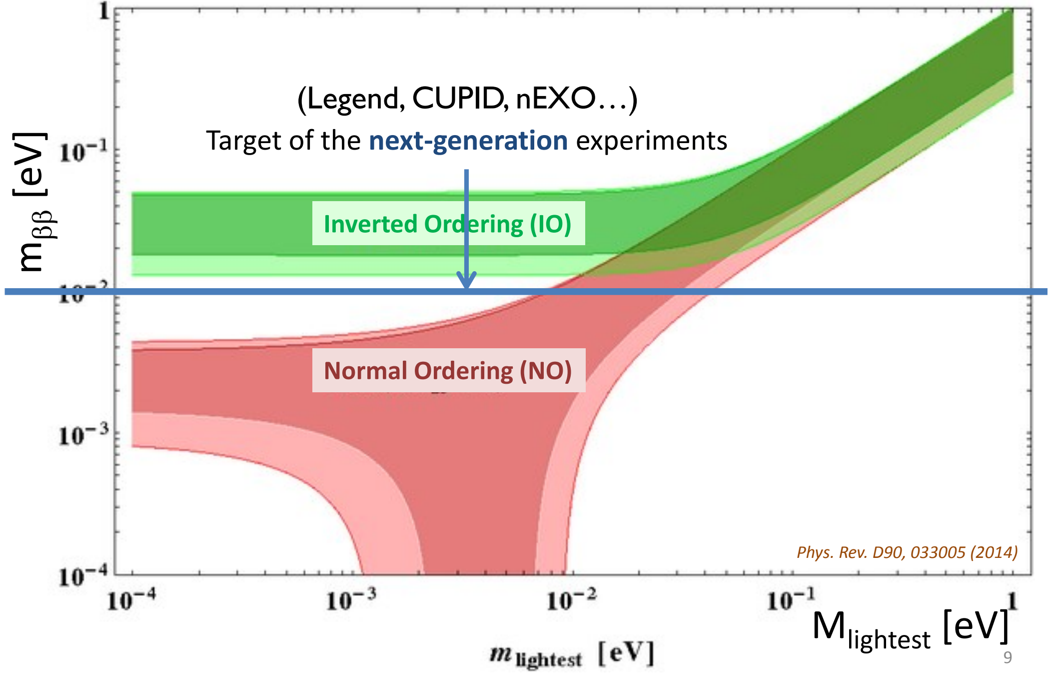
^{136}Xe-loaded liquid scintillator

$$T^{0\nu}_{1/2} \; > \; 2.3 \times 10^{26}\,\text{yr}$$

$$m_{\beta\beta} \; < \; (36 - 156) \text{ meV}$$

(uncertainty from NMEs)

A. Giuliani, Neutrino 2018



(Legend, CUPID, nEXO…)
Target of the **next-generation** experiments

Phys. Rev. D90, 033005 (2014)

around 10 meV