Likelihood and parameter estimation acceleration

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Introduction

- Bayesian parameter estimation: $$p(\theta|d) = \frac{p(d|\theta)p(\theta)}{p(d)}$$

- Performance of likelihood function is crucial (millions of evals) - performance of PE algorithm is also crucial, time to converge can be unreasonable

- Standard likelihood for stationary Gaussian noise: $$\ln \mathcal{L} = -\frac{1}{2}(h-d|h-d)$$

$$(a|b) = 4\mathrm{Re}\int df\, \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)} \qquad (a|b) = 4\mathrm{Re}\Delta f \sum_i \frac{\tilde{a}(f_i)\tilde{b}^*(f_i)}{S_n(f_i)} \qquad f_i = i\Delta f$$

- Ingredients: fast waveforms, fast likelihoods, fast PE algorithms

- Accelerating waveforms: Reduced Order Models (ROMs), Surrogates — crucial for NR and EOB

- Acceleration of likelihoods go beyond simply using a fast waveform — important for data interface

Outline

- How long are GW signals ? Ground-based detectors and LISA
- Accelerating the likelihood: multibanding, heterodyning, Reduced Order Quadratures
- Accelerating PE: burn-in vs sampling, marginalization/extremization, fast/slow parameters
- Dealing with degeneracies: example of MBHBs for LISA

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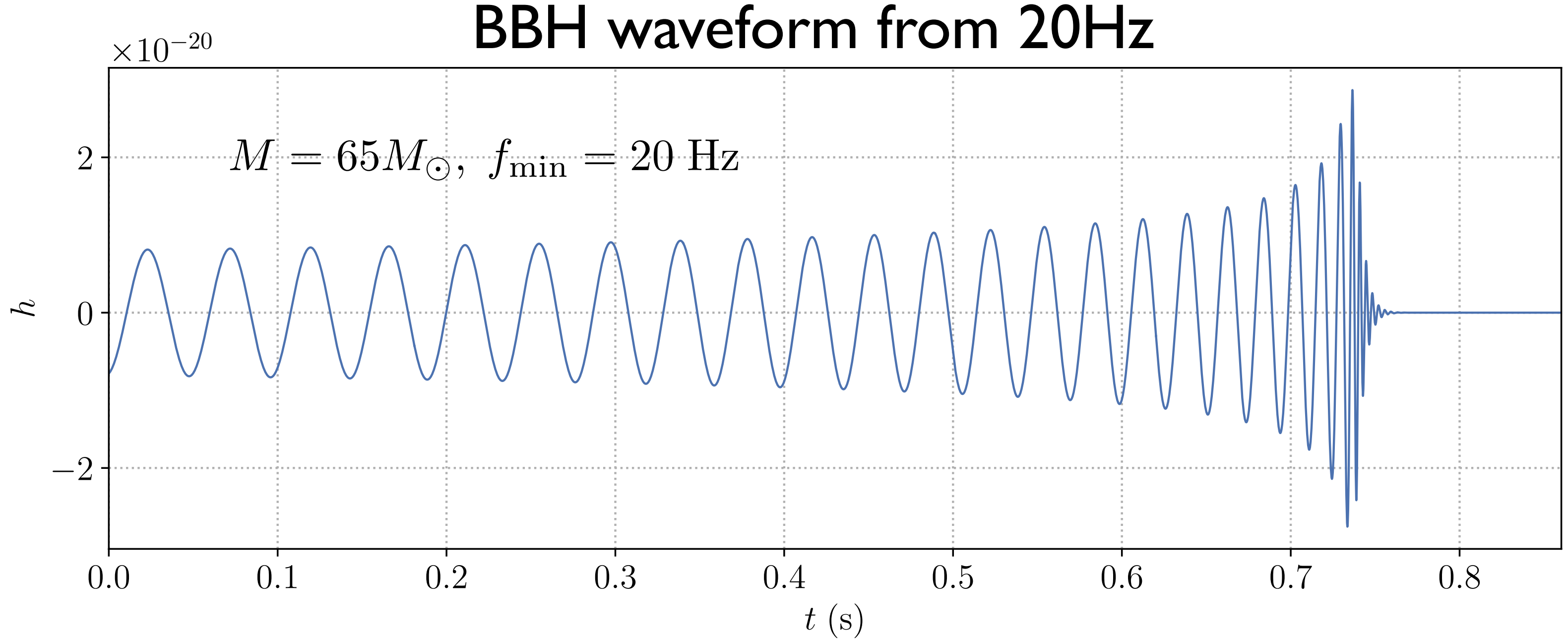
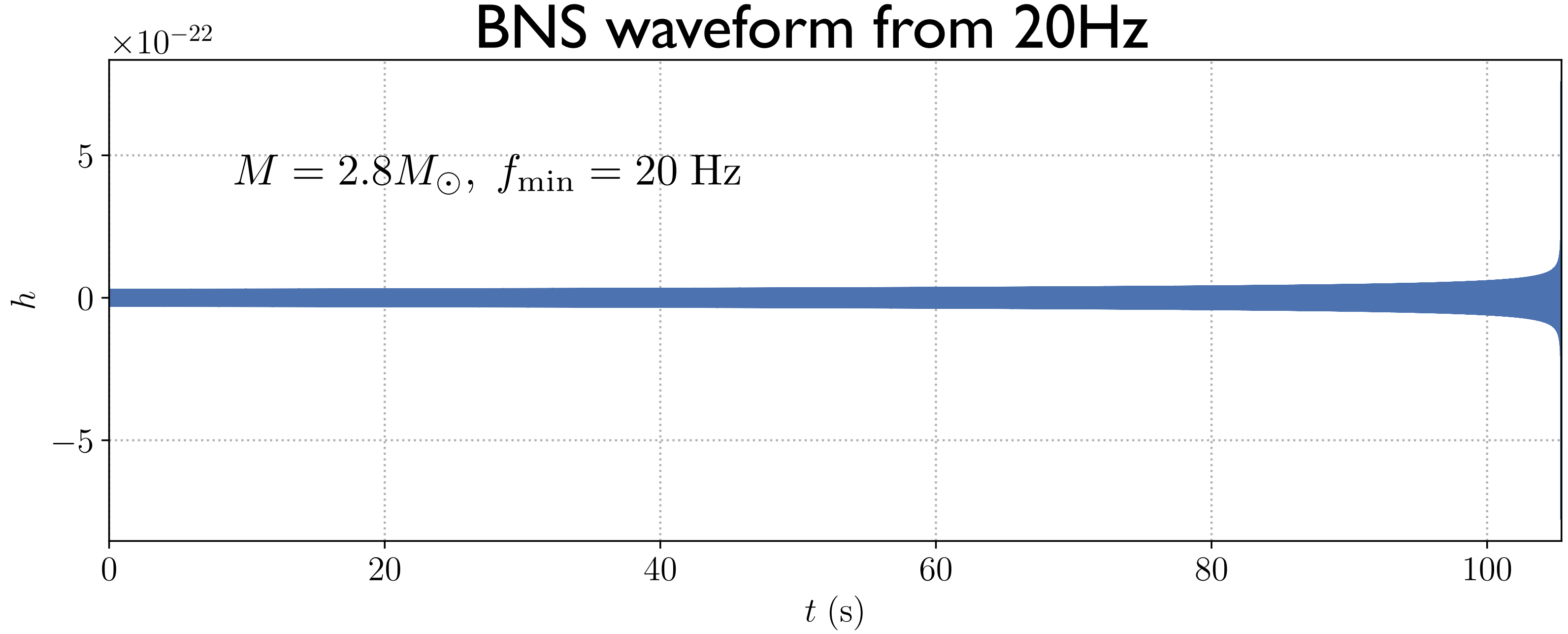
How long are GW signals ?

BBH waveform from 20Hz



BNS waveform from 20Hz



Time-frequency relation

With $M_s = GM/c^3$ the total mass in seconds ($M_\odot = 5 \times 10^{-6} s$):

$$f = \frac{1}{\pi M_s} \left[\frac{256\nu}{5} \frac{\Delta T}{M_s} \right]^{-3/8}$$

$$\Delta T = M_s \frac{5}{256\nu} \left[\pi M_s f \right]^{-8/3}$$

at leading order, for a circularized binary

Nyquist sampling

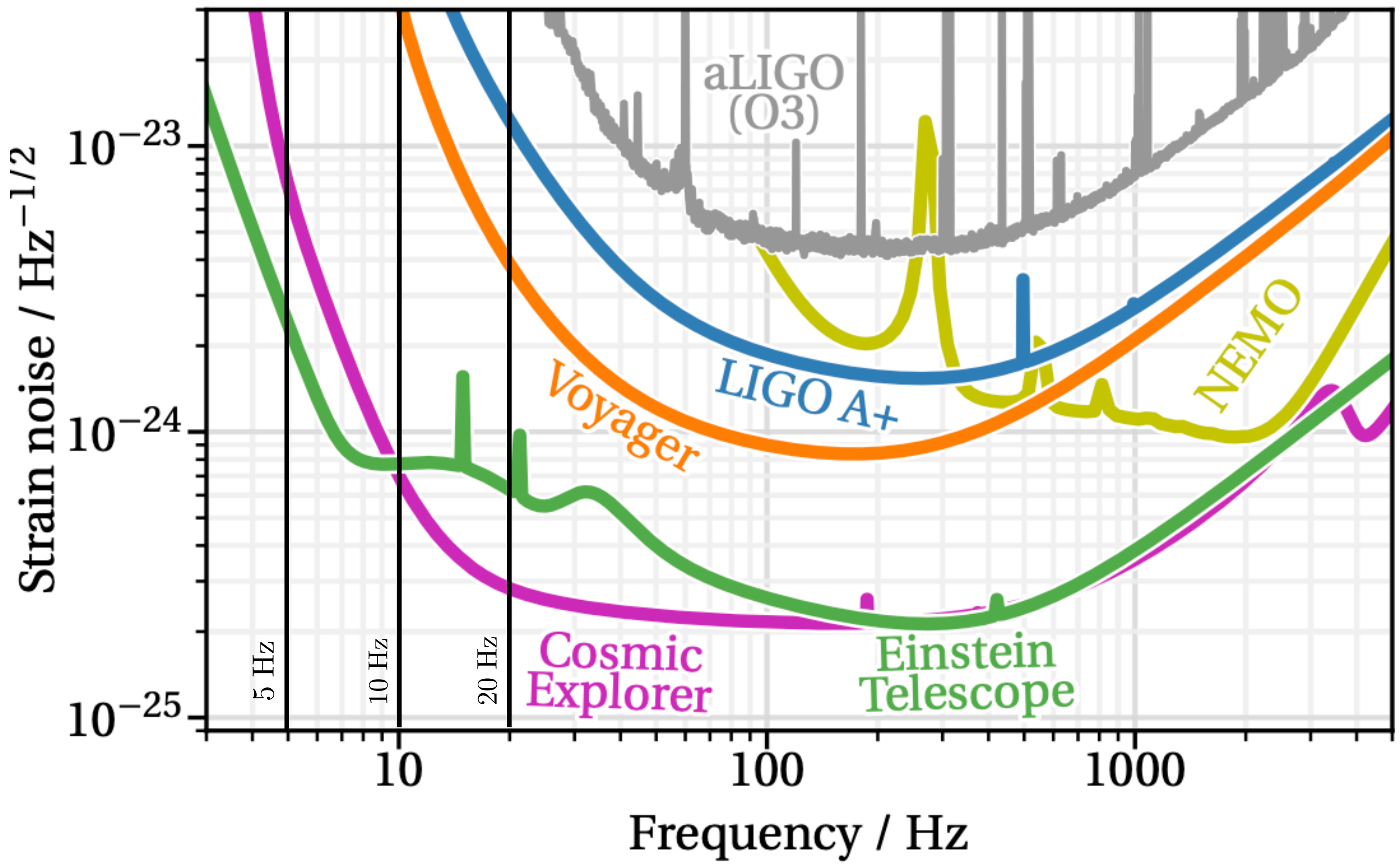
$\Delta t = 1/f_s \qquad f_s = 2 f_{\rm Nyquist}$

$f_{\rm Nyquist}$ highest frequency present in the signal

How long are signals for ground-based detectors ?

Steep dependency of signal duration on M and starting frequency

High sampling rate is overkill for the inspiral…



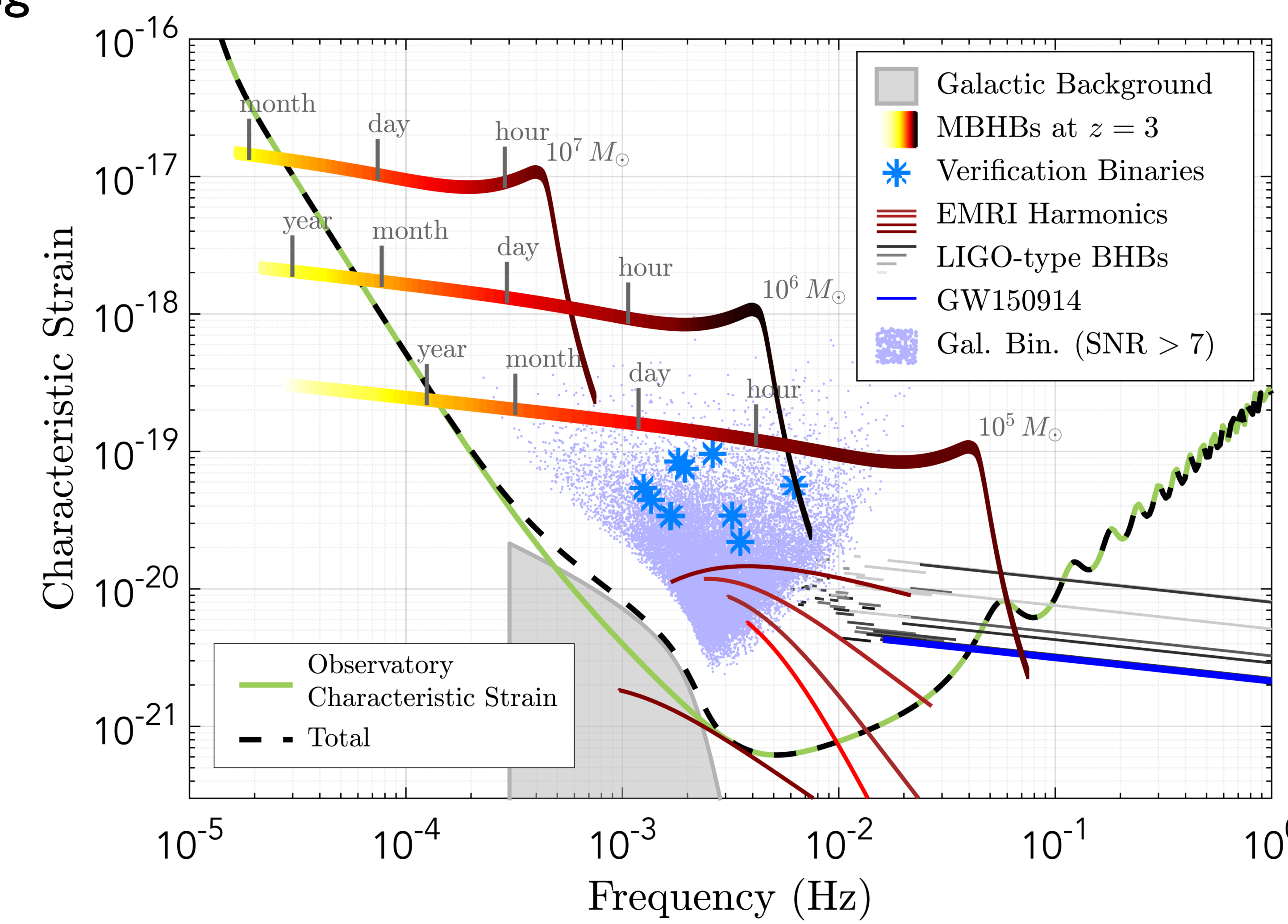
BBH $M = 65 M_\odot$	ΔT	N $f_s = 4096\,\mathrm{Hz}$	$\Delta\phi_{\rm orb}/(2\pi)$
$f_{\min} = 20\,\mathrm{Hz}$	0.8 s	3.5e3	14
$f_{\min} = 10\,\mathrm{Hz}$	5.5 s	2.3e4	44
$f_{\min} = 5\,\mathrm{Hz}$	35 s	1.4e5	140

BNS $M = 2.8 M_\odot$	ΔT	N $f_s = 4096\,\mathrm{Hz}$	$\Delta\phi_{\rm orb}/(2\pi)$
$f_{\min} = 20\,\mathrm{Hz}$	163 s	6.7e5	2600
$f_{\min} = 10\,\mathrm{Hz}$	1030 s	4.2e6	8300
$f_{\min} = 5\,\mathrm{Hz}$	6560 s	2.7e7	26000

How long are LISA signals ?

LISA signals

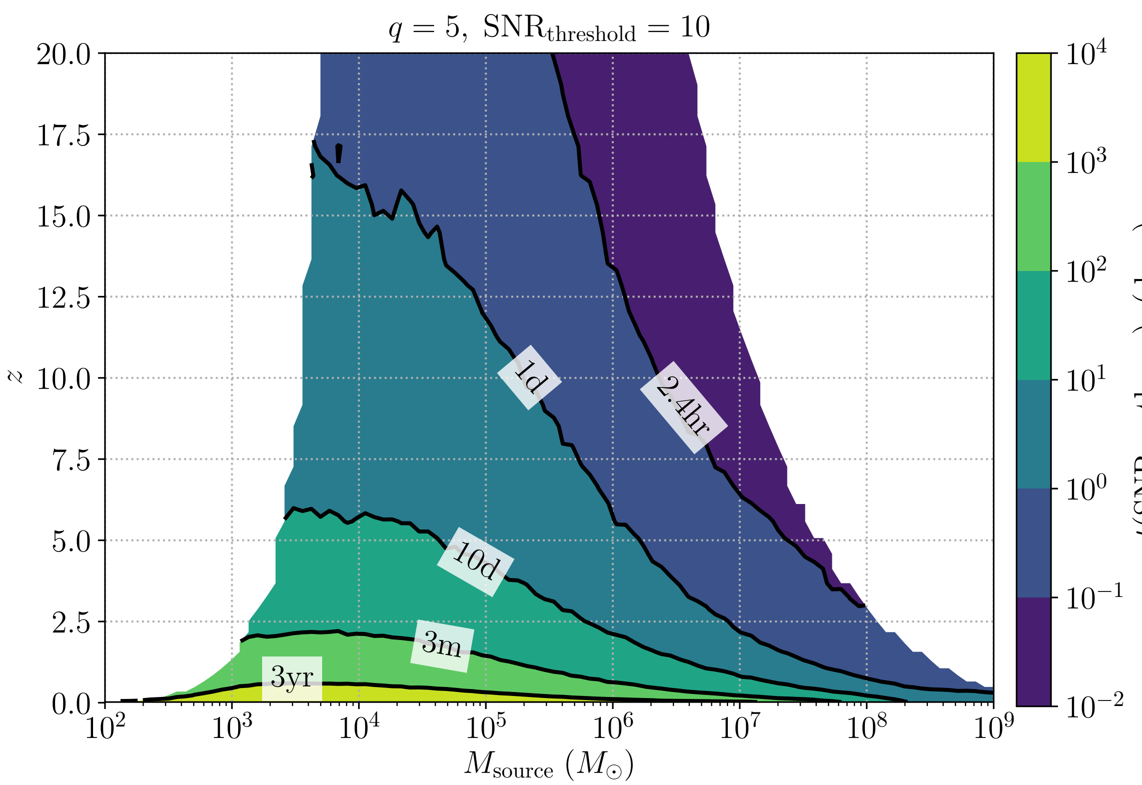
- **MBHBs**: very loud, merger-dominated (mostly short)
- **SBHBs**: early inspiral, some chirping during LISA obs. (multiband ?)
- **GBs**: quasi-monochromatic, superposed
- **EMRIs**: long-lived, many harmonics
- **Stochastic backgrounds**

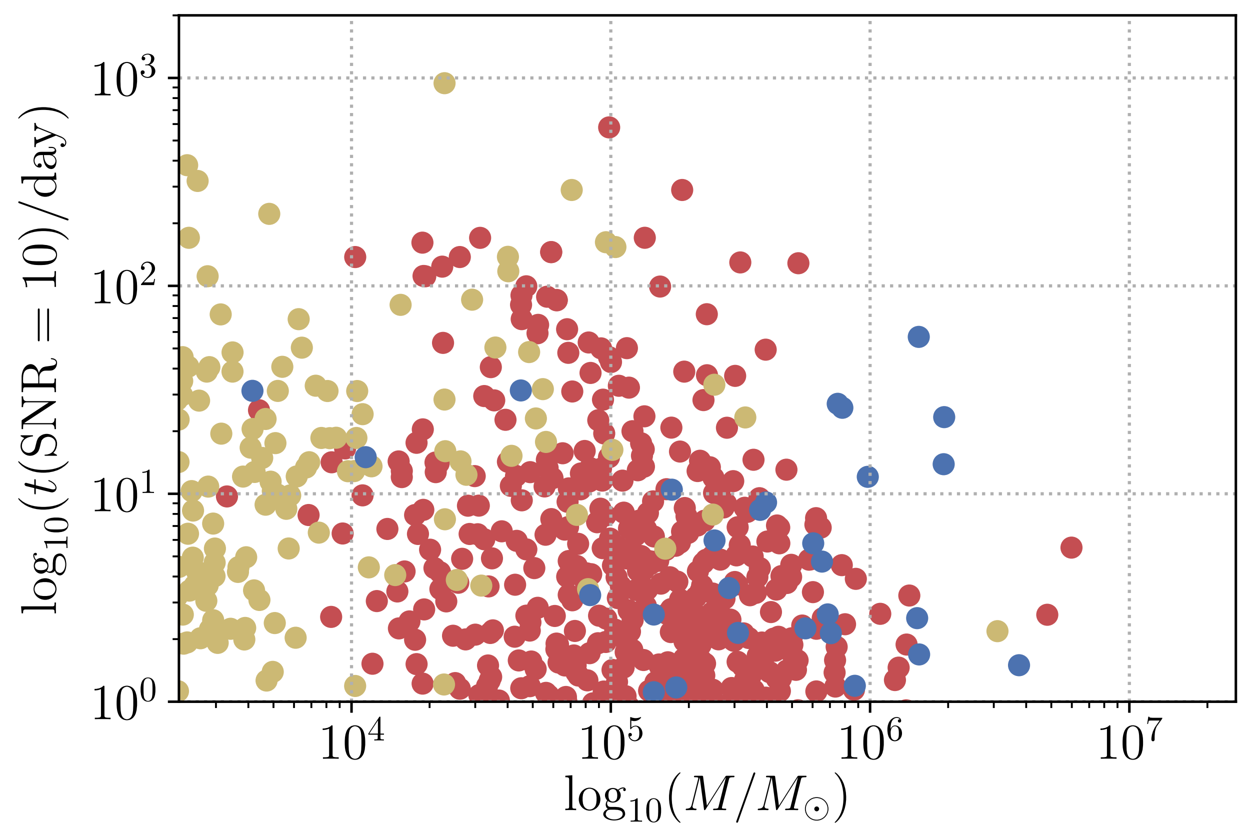


How long are LISA signals ? MBHBs

- How long before merger can we detect the signal ?
- SNR=10 to claim detection

Astrophysical models [Barausse 2012]:
- Heavy seeds - delay
- Heavy seeds - no delay
- PopIII seeds - delay





MBHB detected signals:
Bulk shorter than ~10days
Tail extending to ~3months

How long are LISA signals ? Other signals

SBHB signals

- Can last for years in the LISA band and chirp to exit the band at high frequencies
- Data with Nyquist sampling 0.2Hz for 10yrs: N=6.3e6

GB signals

- Last for the whole mission
- Quasi mono-chromatic, very compact in Fourier domain
- Millions superposed, ~20000 resolvable !

EMRI signals

- Can last for months
- Waveform generation and representation very challenging, complex signal with many harmonics

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Accelerating likelihoods: ‘multibanding’

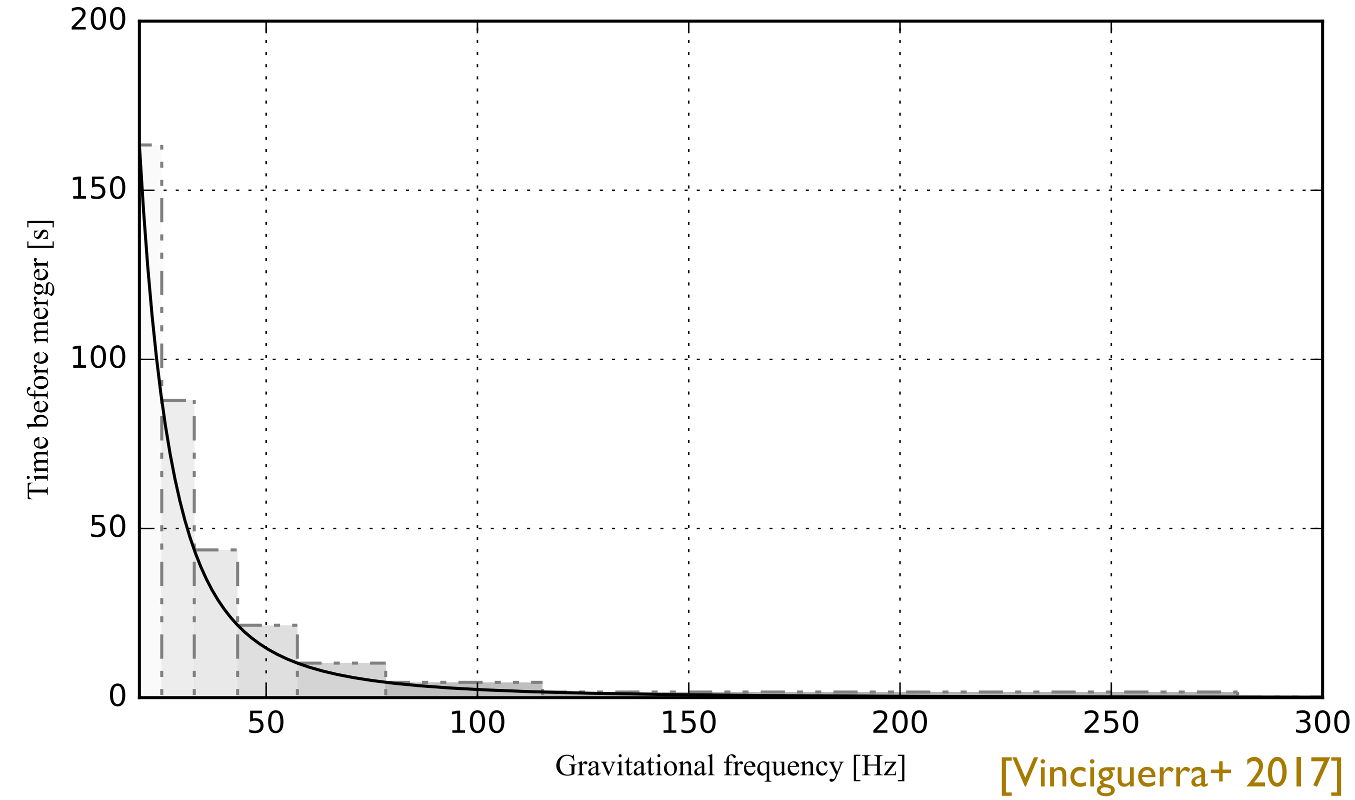
Beyond Nyquist sampling

$$N = f_{\max}/\Delta f = f_{\max} * \Delta T$$

Max frequency set by merger, duration dominated by inspiral

Nyquist sampling is far from optimal for chirping signals

Solution: separate data in signal-adapted bands



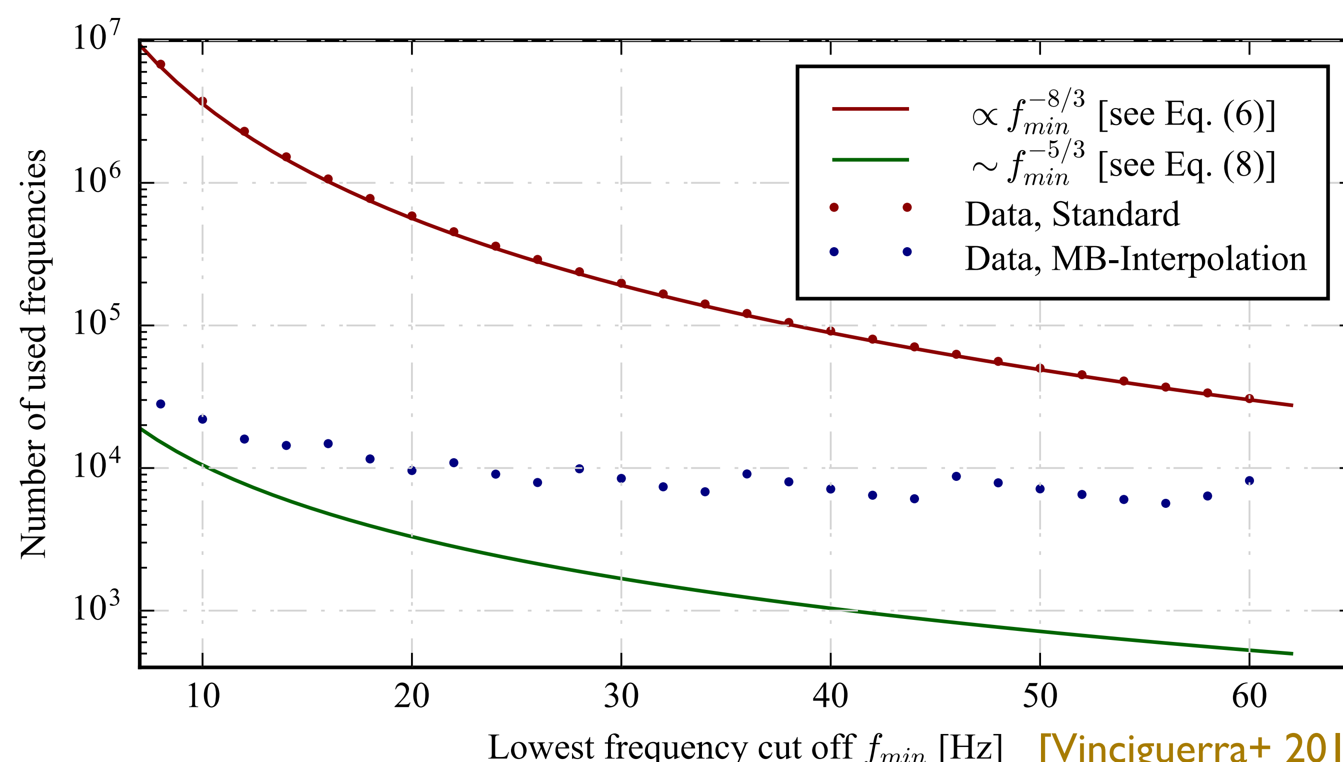
[Vinciguerra+ 2017]

Interpolation acceleration

If on each band, phase is linearly interpolated, replace exp. with mult.

$$e^{i\lambda(j+1)\Delta f} = e^{i\lambda j\Delta f} e^{i\lambda\Delta f}$$

Generalizes to higher-order polynomials



[Vinciguerra+ 2017]

Accelerating likelihoods: quasi-monochromatic heterodyning

GB signals in LISA: quasi-monochromatic

$$\Phi = \Phi_0 + 2\pi \left(f_0 t + \frac{1}{2} \dot{f}_0 t^2 + \dots \right)$$

$$\frac{\dot{f}_0 T}{f_0} \ll 1$$

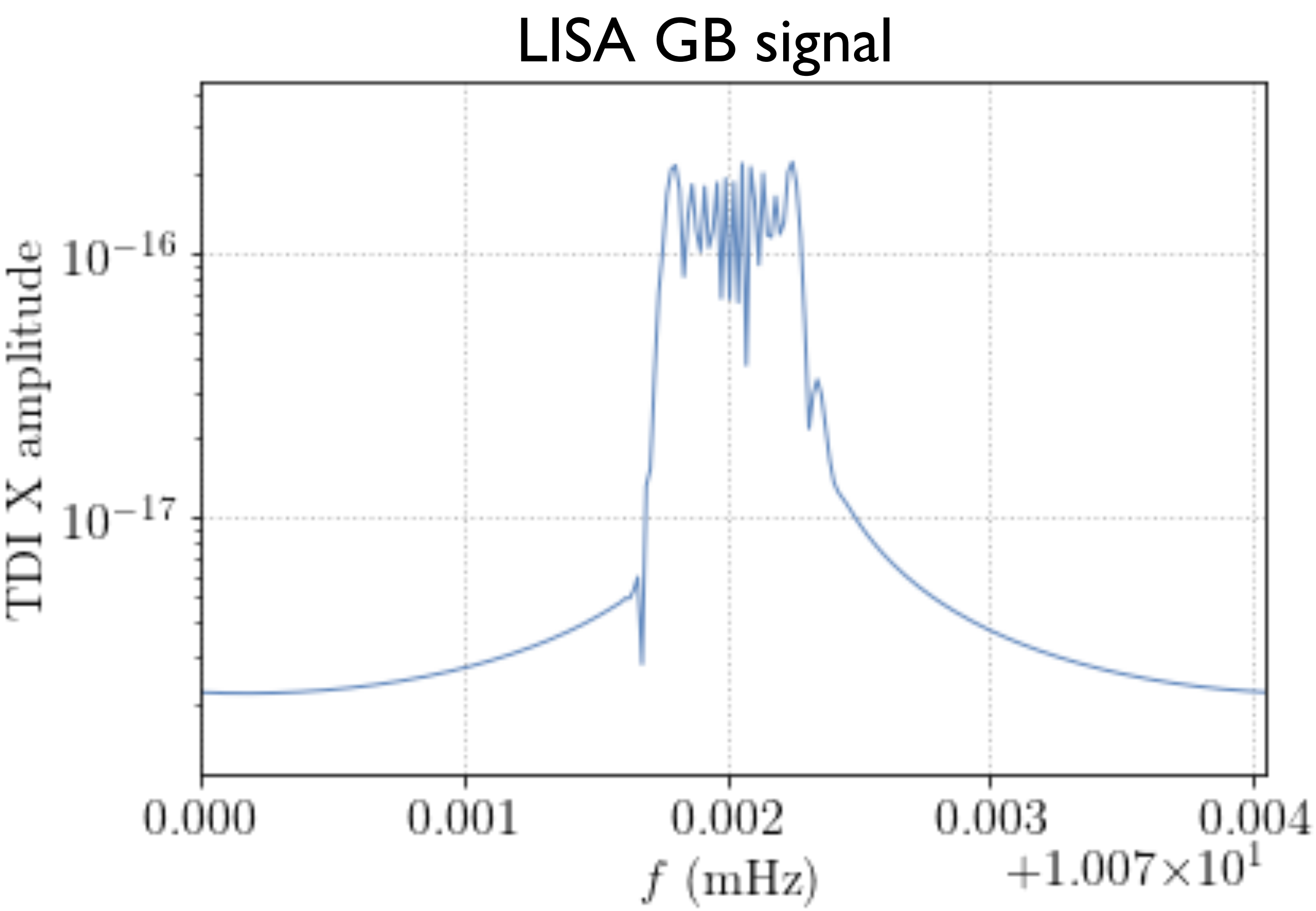
Sidebands: created by chirp and modulation of the LISA response

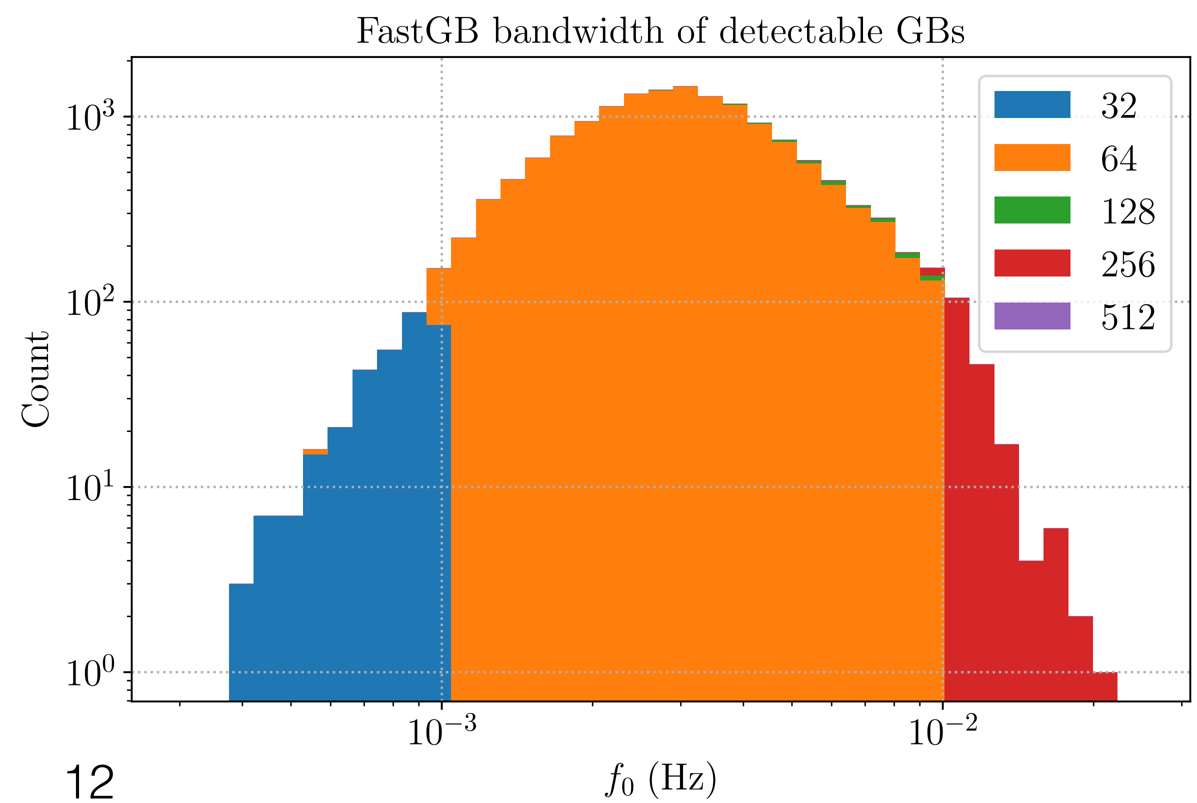
- Idea: factor out the fast-varying part, work with the slow-varying part [Conish-Littenberg 2007]

$$s(t) = \overline{s}(t) \exp[2i\pi f_0 t]$$

- FFT of the slow part, with a very reduced Nyquist frequency (bandwidth: from 32 to 512)

- Go back to original signal with a simple shift in frequency





Accelerating likelihoods: heterodyning

Overview

- Structure of the likelihood

$$\ln \mathcal{L} = -\frac{1}{2}(h-d|h-d) \qquad (a|b) = 4\mathrm{Re}\int df\, \frac{\tilde{a}(f)\tilde{b}^*(f)}{S_n(f)}$$

$h = Ae^{i\Phi}$ smooth amp/phase d numerical data

- Introduce a reference waveform $\overline{h}(f)$

$\zeta(f) \equiv h(f)/\overline{h}(f)$ now **slowly variable**

in the vicinity of reference parameters

- Separate integrand in slowly and rapidly variable parts

$$(h|d) \sim \int df\, \frac{\overline{h}d^*}{S_n} \times \zeta \qquad (h|h) \sim \int df\, \frac{\overline{h}\overline{h}*}{S_n} \times \zeta\zeta^*$$

- Interpolate and precompute

ζ interpolated on a coarse, reduced grid

$$(h|d) \sim \sum_i \int_{f_i}^{f_{i+1}} df\, \frac{\overline{h}d^*}{S_n} \times (a_i + b_i f)$$

$$(h|h) \sim \sum_i \int_{f_i}^{f_{i+1}} df\, \frac{\overline{h}\overline{h}*}{S_n} \times (a_i + b_i f + c_i f^2)$$

- Evaluate

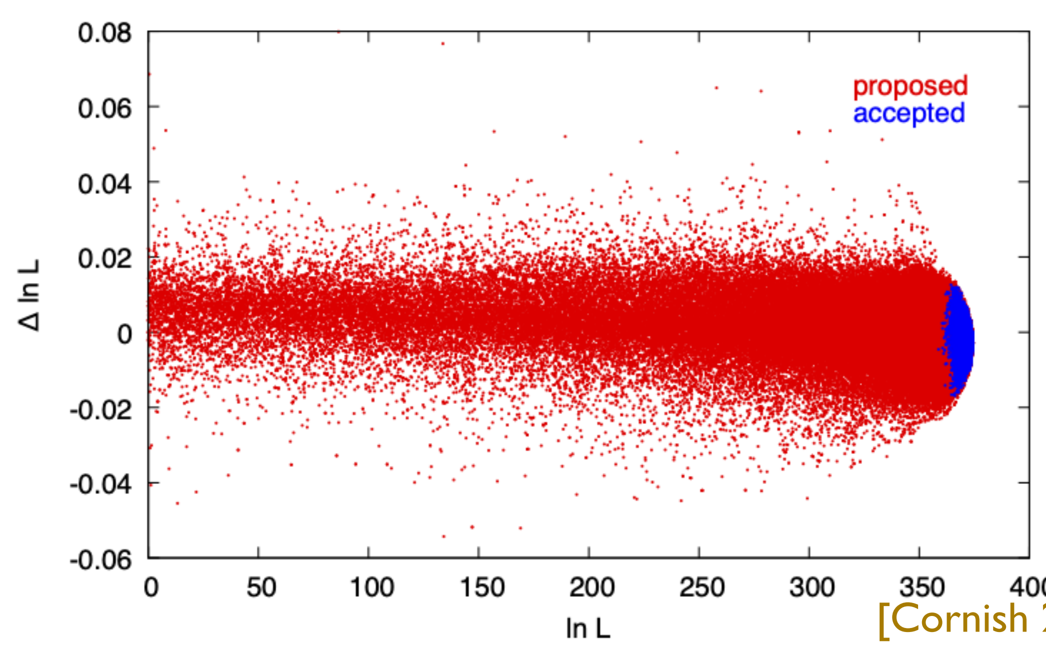
h on coarse grid, then sum weights and coeffs

[Cornish 2010, Cornish 2021]

[Zackay+ 2018] (relative binning)

Usage in practice

- Small reduced grid (N~100)
- Different interpolation methods (linear, polynomial)
- Requires reference waveform (first guess for signal parameters) — can be updated on the way
- Distinguish burn-in from actual sampling, the latter happens close to the true signal



[Cornish 2021]

Accelerating likelihoods: heterodyning example for MBHB

Decomposing the likelihood:

$$\ln \mathcal{L} = -\frac{1}{2}(s-d|s-d)$$
$$= -\frac{1}{2}(s-s_0|s-s_0) + (s-s_0|d-s_0) - \frac{1}{2}(s_0-d|s_0-d)$$

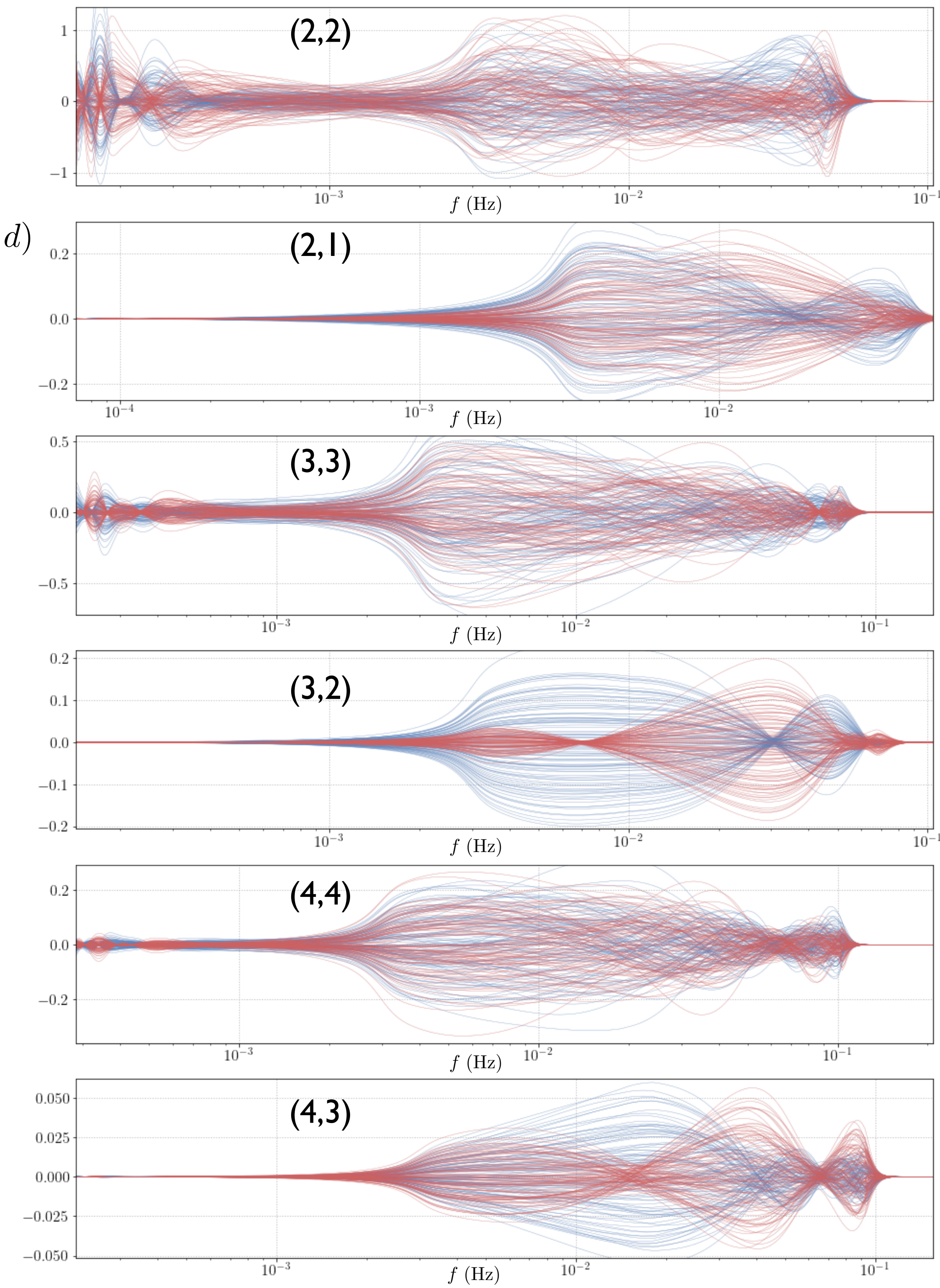
Residuals from reference waveform:

$$s_{\ell m} - s^0_{\ell m} = r_{\ell m} e^{i\Phi^0_{\ell m}}$$

Implementation:

$$(s-s_0|s-s_0) = \sum_{\ell m}\sum_{\ell' m'} (r_{\ell m} r^*_{\ell' m'} | e^{i(\Phi^0_{\ell' m'} - \Phi^0_{\ell m})})$$
$$(s-s_0|d-s_0) = \sum_{\ell m} (r_{\ell m} | e^{-i\Phi^0_{\ell m}}(d-s_0))$$

- Fix a sparse frequency grid (~128)
- Linear interpolation of the residuals, mode-by-mode
- Precompute 0-th and 1st polynomial inner products against phase and data terms, with a fine resolution



Accelerating likelihoods: ROQs

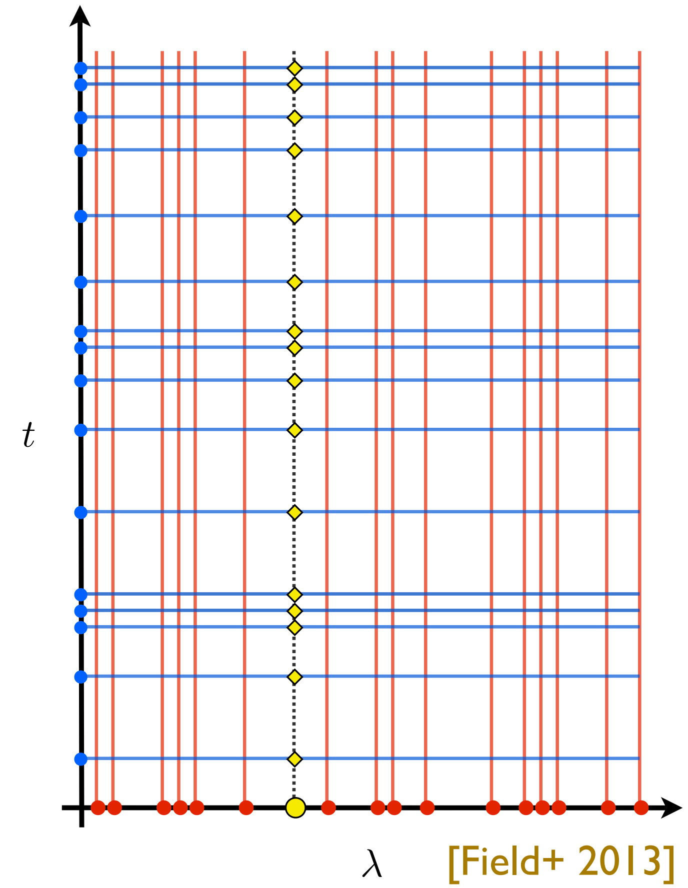
Linear ROM for GW signal

Building a reduced basis and empirical interpolant directly for $h_+, h_\times$ and products $h_A h_B\,(A=+,\times)$:

$$h[\theta](f) = \sum_{j=0}^{p} B_j(f) h[\theta](f_j^{\rm node}) \qquad h_A[\theta]h_B[\theta](f) = \sum_{k=0}^{q} C_k(f) h_A[\theta]h_B[\theta](f_k^{\rm node})$$

p,q size of the reduced basis (linear and quadratic)

+ efficient representation for likelihood
- larger basis for longer signals, challenging to build



[Field+ 2013]

Likelihood evaluation

- Precompute all inner products

$$(B_j|d) \quad (C_k|1)$$

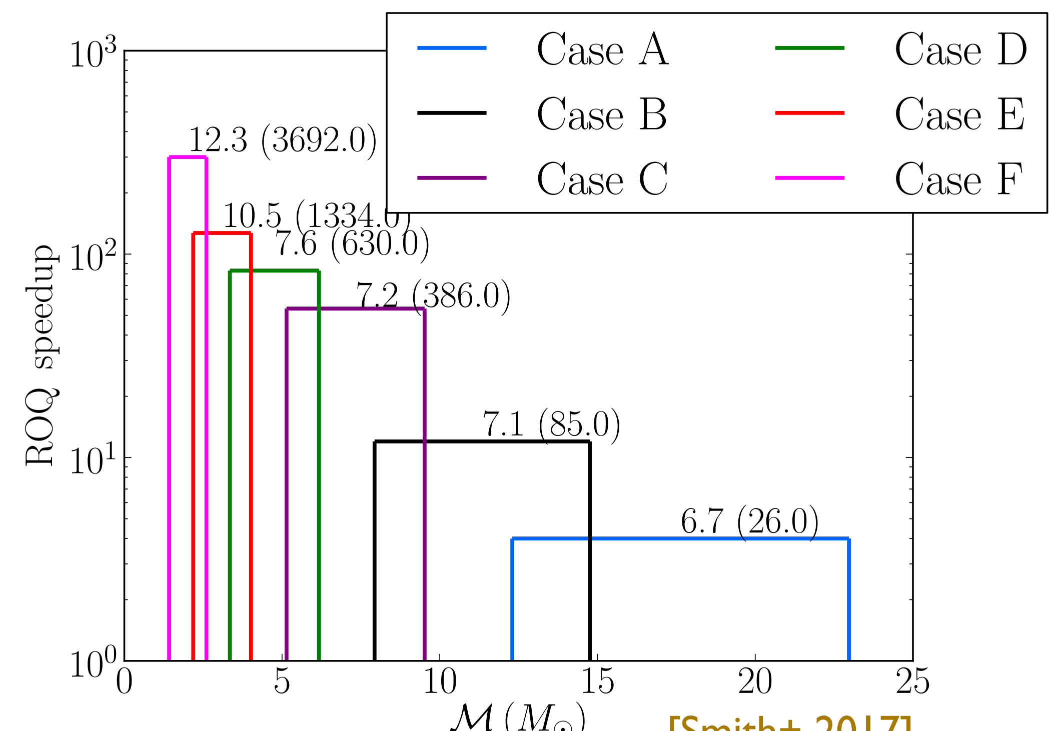
- Evaluate waveform model on interpolant nodes

$$\lambda_j = h[\theta](f_j^{\rm node}) \qquad \mu_k = h_A[\theta]h_B[\theta](f_k^{\rm node})$$

- Use linear/quad structure to compute likelihood

$$\ln\mathcal{L} = -\frac{1}{2}(h-d|h-d)$$

$$\ln\mathcal{L} = \sum_{j=0}^{p}\lambda_j(B_j|d) + \sum_{k=0}^{q}\mu_k(C_k|1) + \text{const}$$



[Smith+ 2017]

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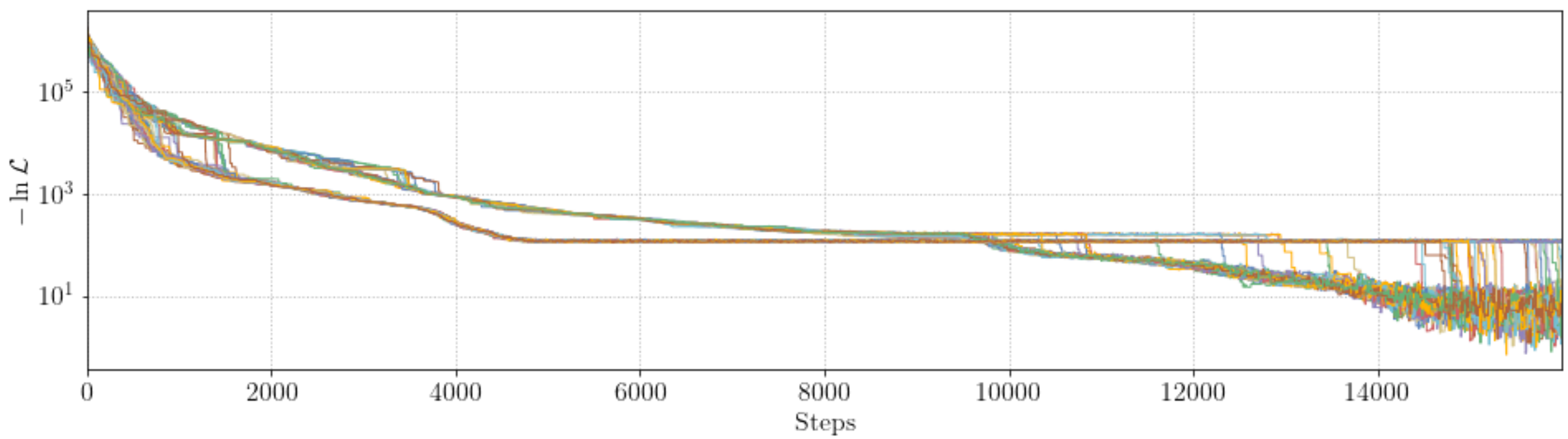
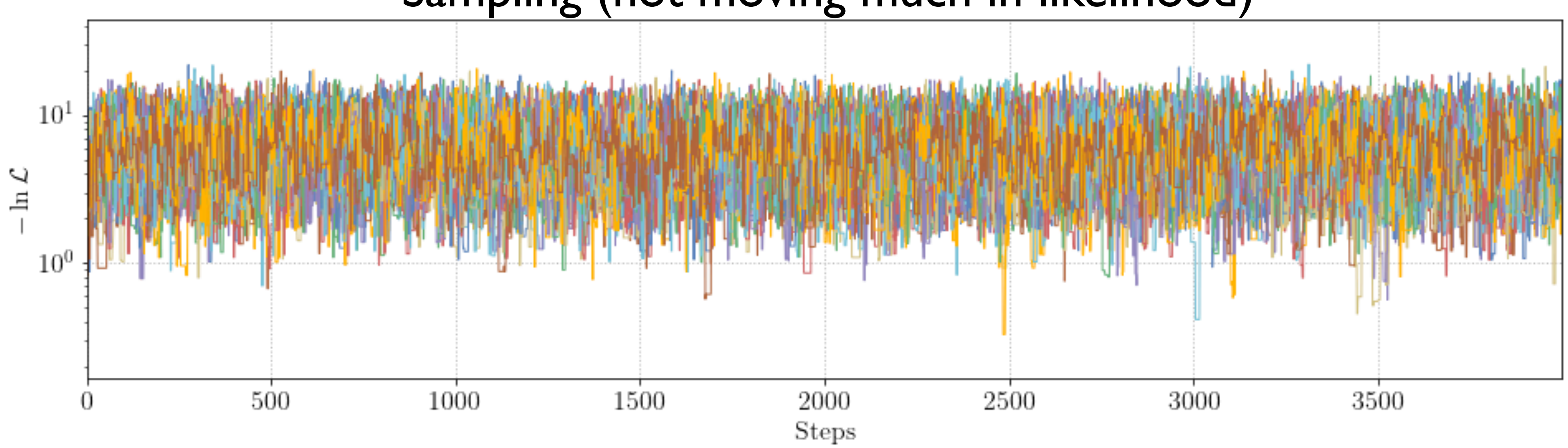
Accelerating PE: burn-in vs sampling

Burn-in (here struggling to find the signal !)



Scale of likelihood with completely wrong signal: $\ln \mathcal{L}_{\rm bad} \sim -{\rm SNR}^2$

Sampling (not moving much in likelihood)



- Simulating realistic PE: start from prior
- Prospective parameter estimation, only interested in final result: cheat with initialization

Techniques for burn-in (search) or sampling can differ !

Accelerating PE: marginalization, optimization

Separate parameters to reduce dimensionality:

θ 'interesting' parameters (intrinsic)

λ parameters to eliminate (extrinsic)

Marginalization

$$p(\theta|d) = \int d\lambda \, p(\theta, \lambda|d)$$

review: [Talbot-Thrane 2018]

- Approximate marginalization on time: IFFT of integrand mimics time shifts
- Phase marginalization for 22-only signals (likelihood becomes modified Bessel function)
- Distance marginalization

Optimization

$$\mathcal{F} = \max_{\lambda} \ln \mathcal{L}(\theta, \lambda)$$

Useful at search stage, for faster burn-in

- F-stat to build a proposal
- Directly search 'sampling' F-stat as a pseudo-likelihood

- F-stat analytical over distance, inclination, phase, polarization
- LISA GB in low-frequency approximation: +sky
- LISA MBHB low-f and short signal approximation: +sky

Accelerating PE: fast and slow parameters

Separate parameters for computational efficiency:

θ 'costly' parameters (intrinsic)
λ 'cheap' parameters (extrinsic)

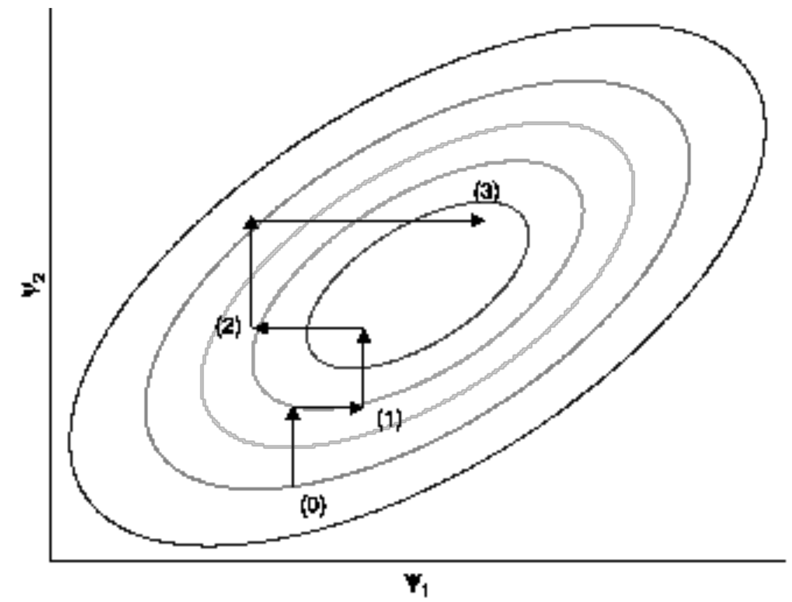
Intrinsic parameters: requires to solve GR (analytical models, numerical relativity)

Extrinsic parameters: geometry and signal propagation, simple and completely universal

Gibbs sampling

Successive sampling steps by blocks

$$\theta_{i+1} \sim p(\theta|\lambda_i)$$
$$\lambda_{i+1} \sim p(\lambda|\theta_{i+1})$$



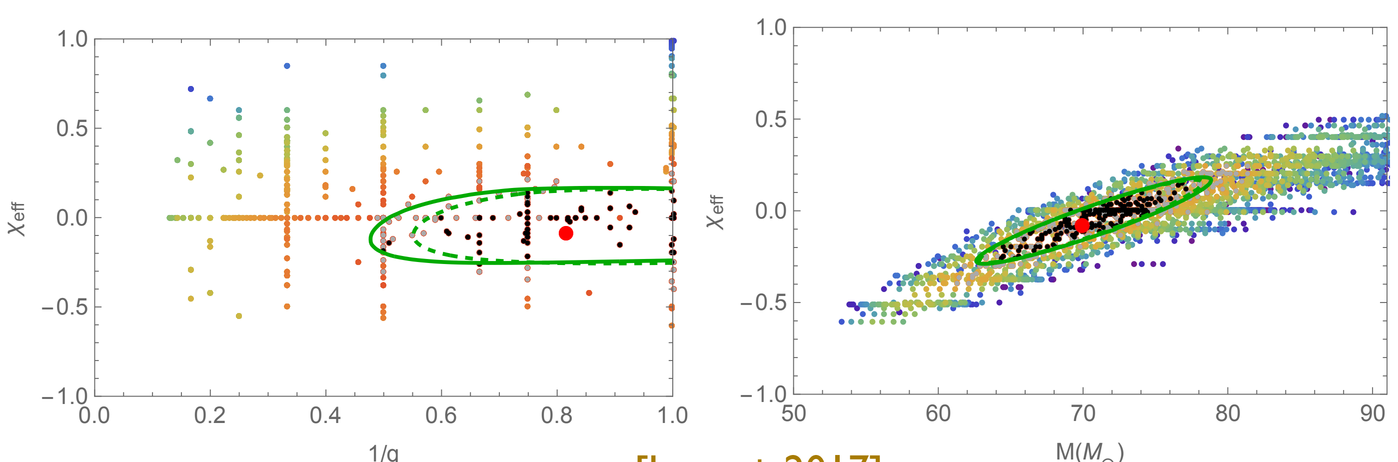
\+ cache intrinsic waveform (waveform modes) and sample efficiently in cheap extrinsic parameters

\- resolving correlations might be costly

Likelihood pre-interpolation

Pre-interpolate intrinsic likelihood (e.g. Gaussian Process Regression)

- LIGO/Virgo: RIFT [Lange+ 2017]
- LISA GB: [Strub+ 2022]

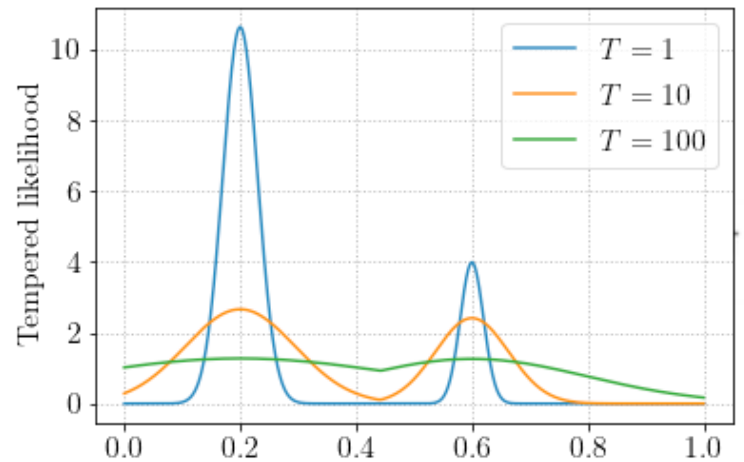


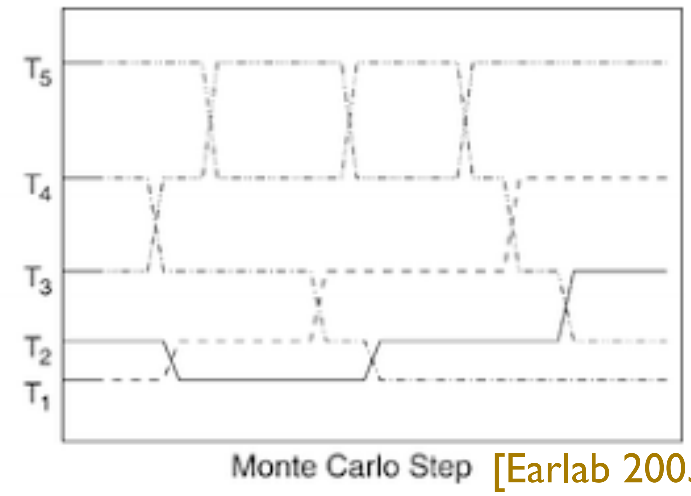
[Lange+ 2017]

Accelerating PE: dealing with degeneracies

Parallel tempering

- Introduce parallel chains with temperatures, posterior: $p(\theta)^{\beta_i}$
$$\beta_i = 1/T_i$$
- Propose swaps with acceptance:
$$p_{\mathrm{swap}} = \min\left[1, \left(\frac{p(\theta_i)}{p(\theta_j)}\right)^{\beta_j - \beta_i}\right]$$





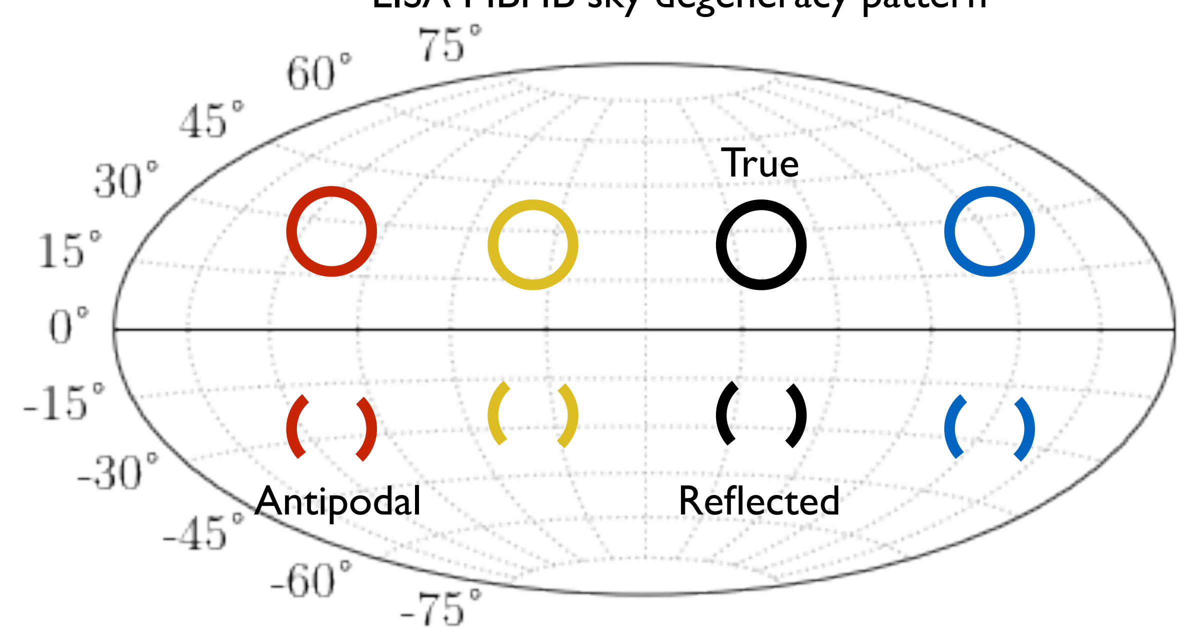
[Earlab 2005]

- Crucial for robustness, avoids being stuck in a local maximum and ensures exploration of the parameter space

Tailored proposals

- In presence of known degeneracies, include jumps in proposal
- Very efficient for very disconnected multimodal posteriors

LISA MBHB sky degeneracy pattern



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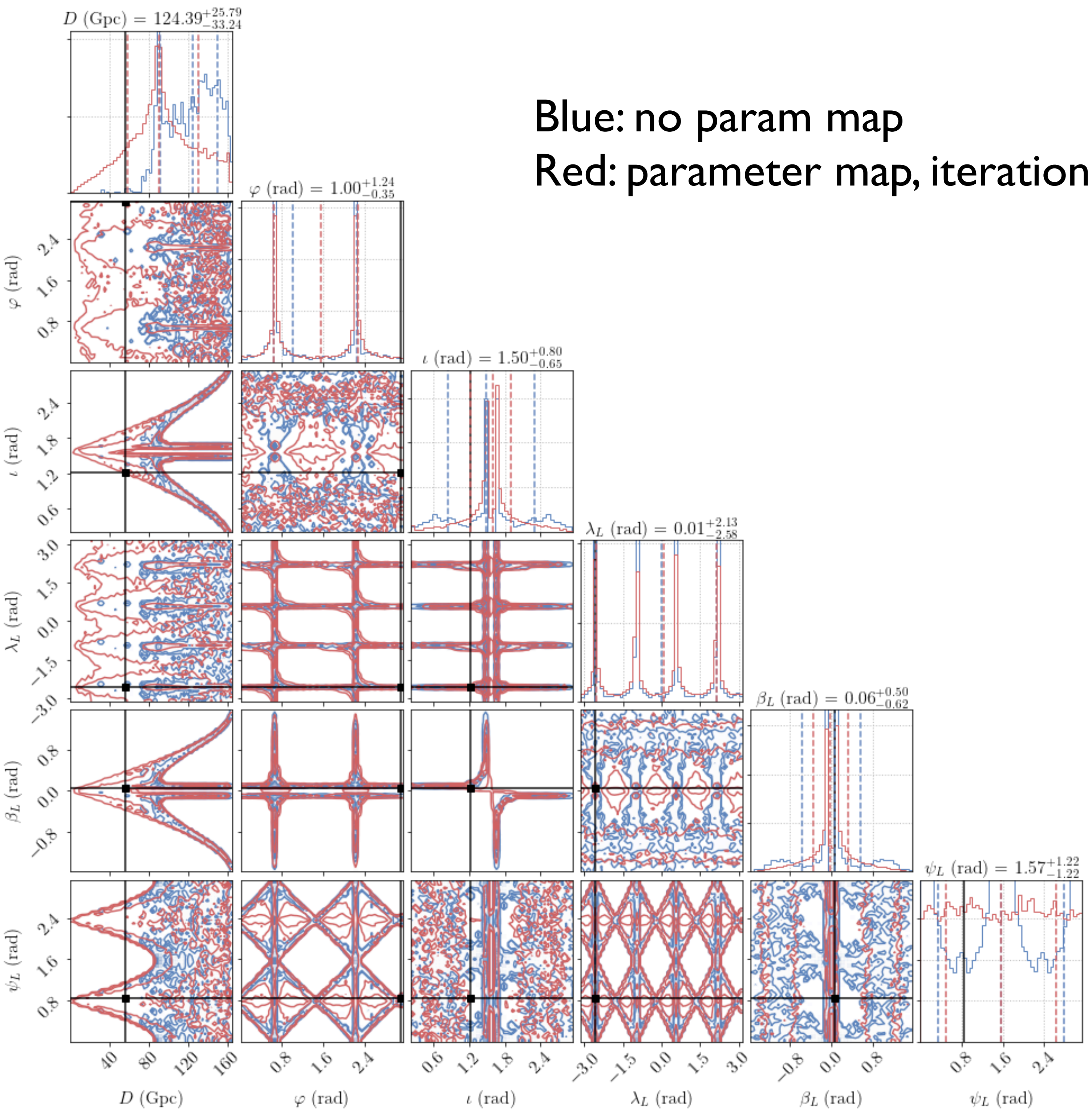
- **Dealing with degeneracies: example of MBHBs for LISA**

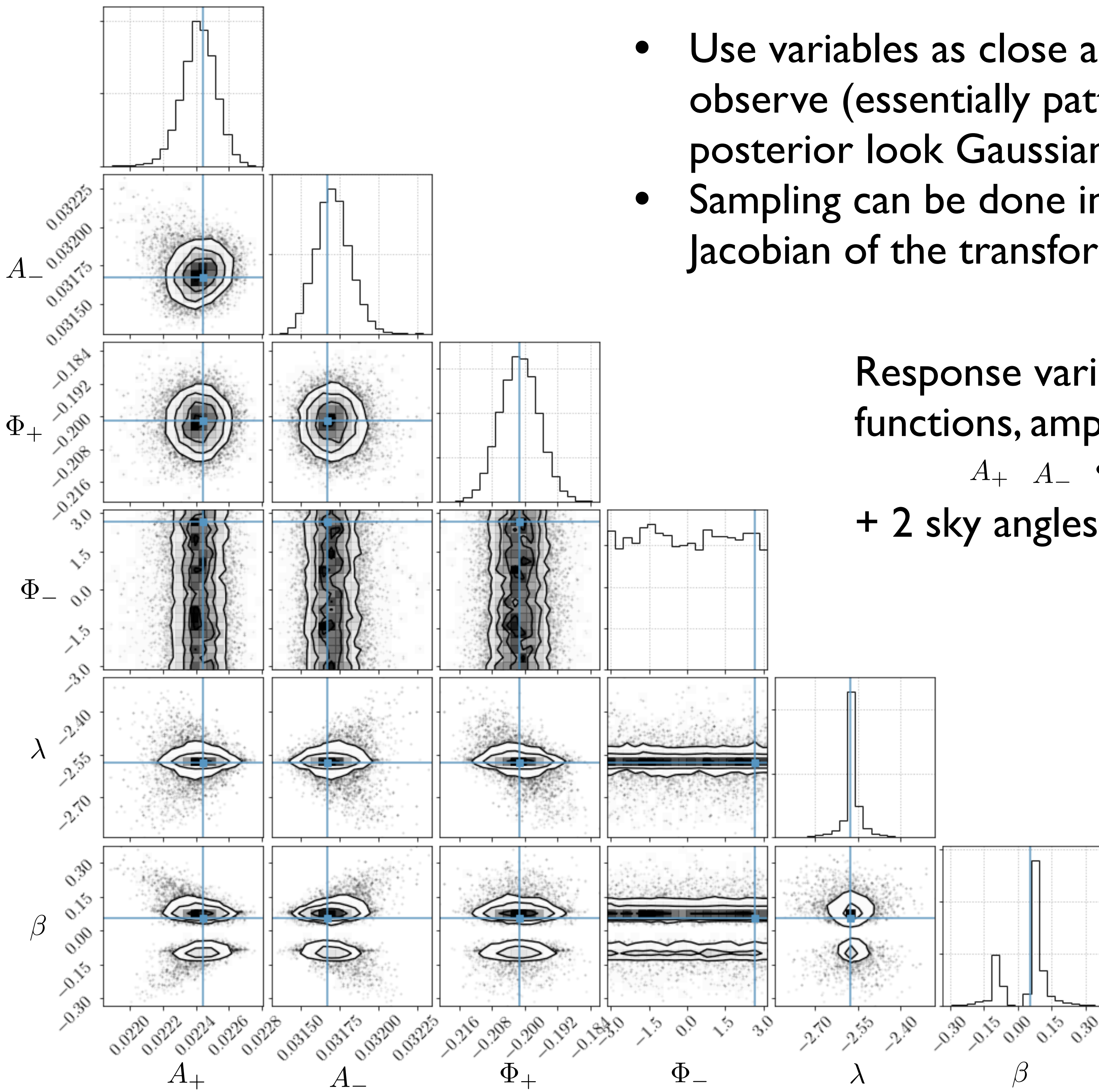
Dealing with degeneracies: MBHB example

Toy problem, completely degenerate extrinsic 22 likelihood without motion and high-f effects

Blue: no param map
Red: parameter map, iterations/10



Dealing with degeneracies: parameter map



- Use variables as close as possible to what we really observe (essentially pattern functions), to make the posterior look Gaussian
- Sampling can be done in any set of parameters, with Jacobian of the transformation analytic here

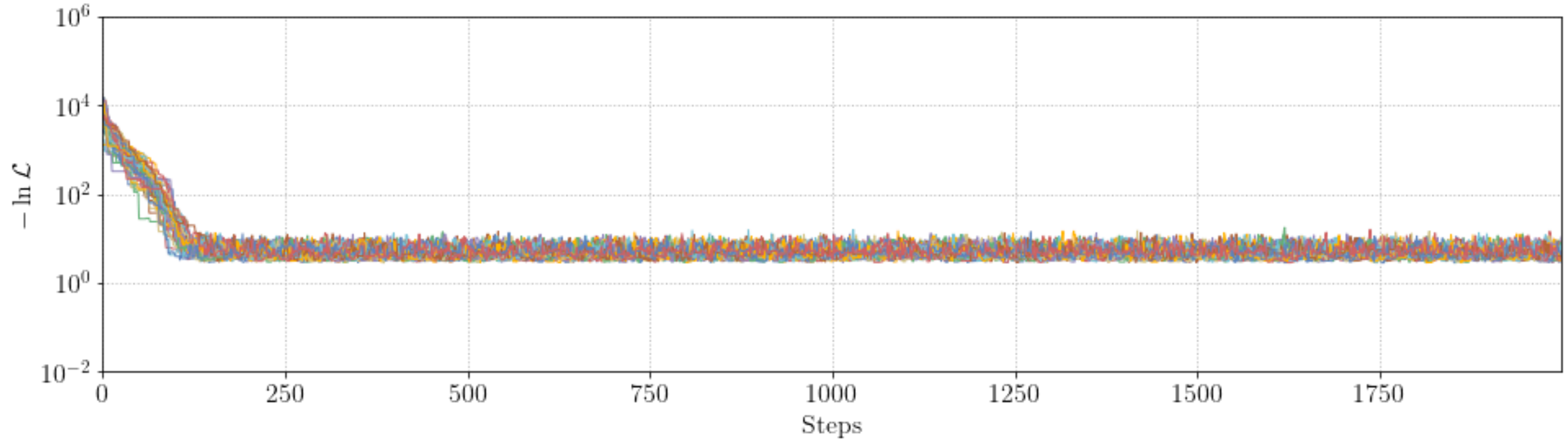
Response variables: 2 complex pattern functions, amplitude and phase

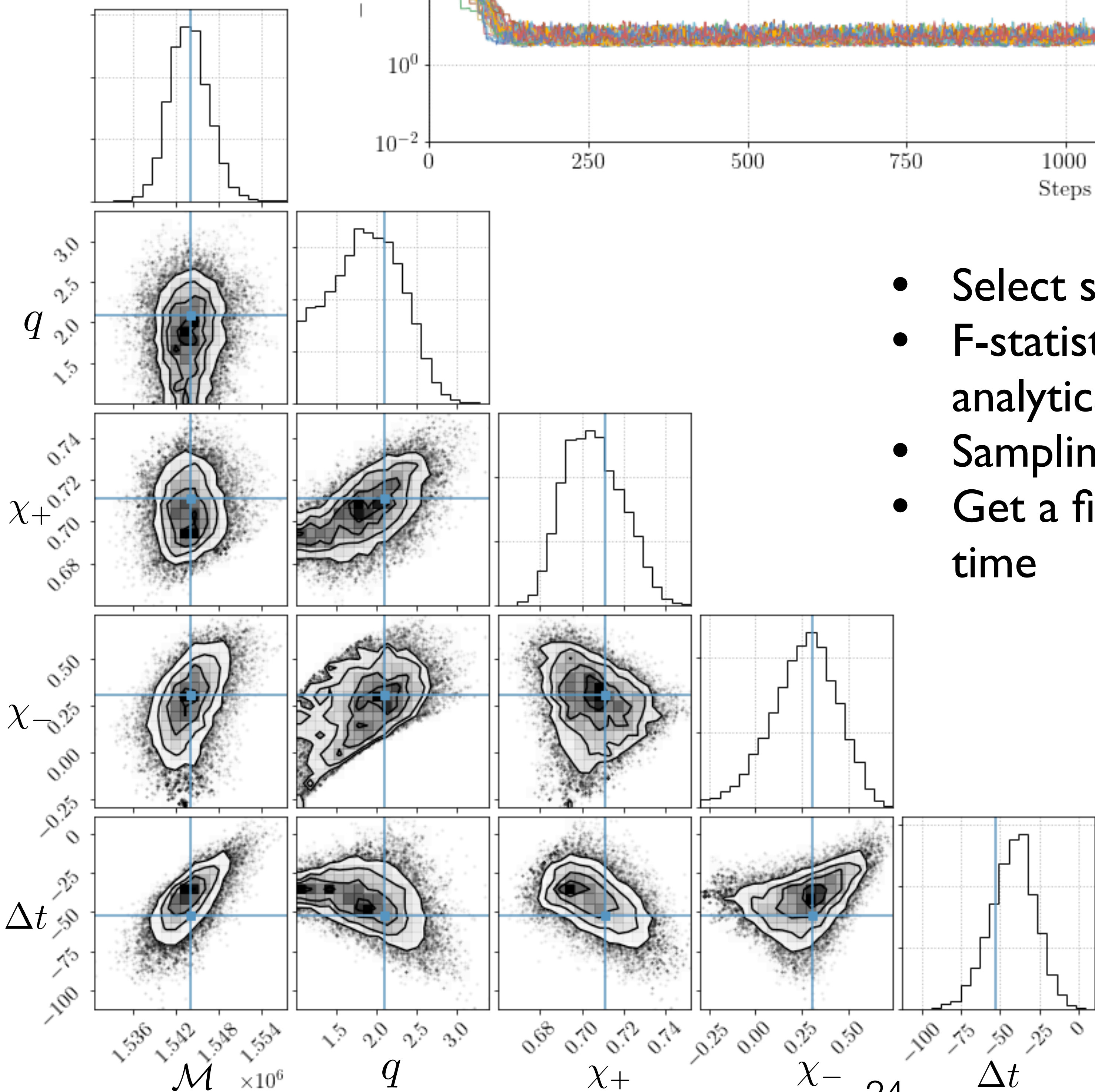
$A_+ \quad A_- \quad \Phi_+ \quad \Phi_-$

+ 2 sky angles $\lambda \quad \beta$

Analytical transformation, no extra cost

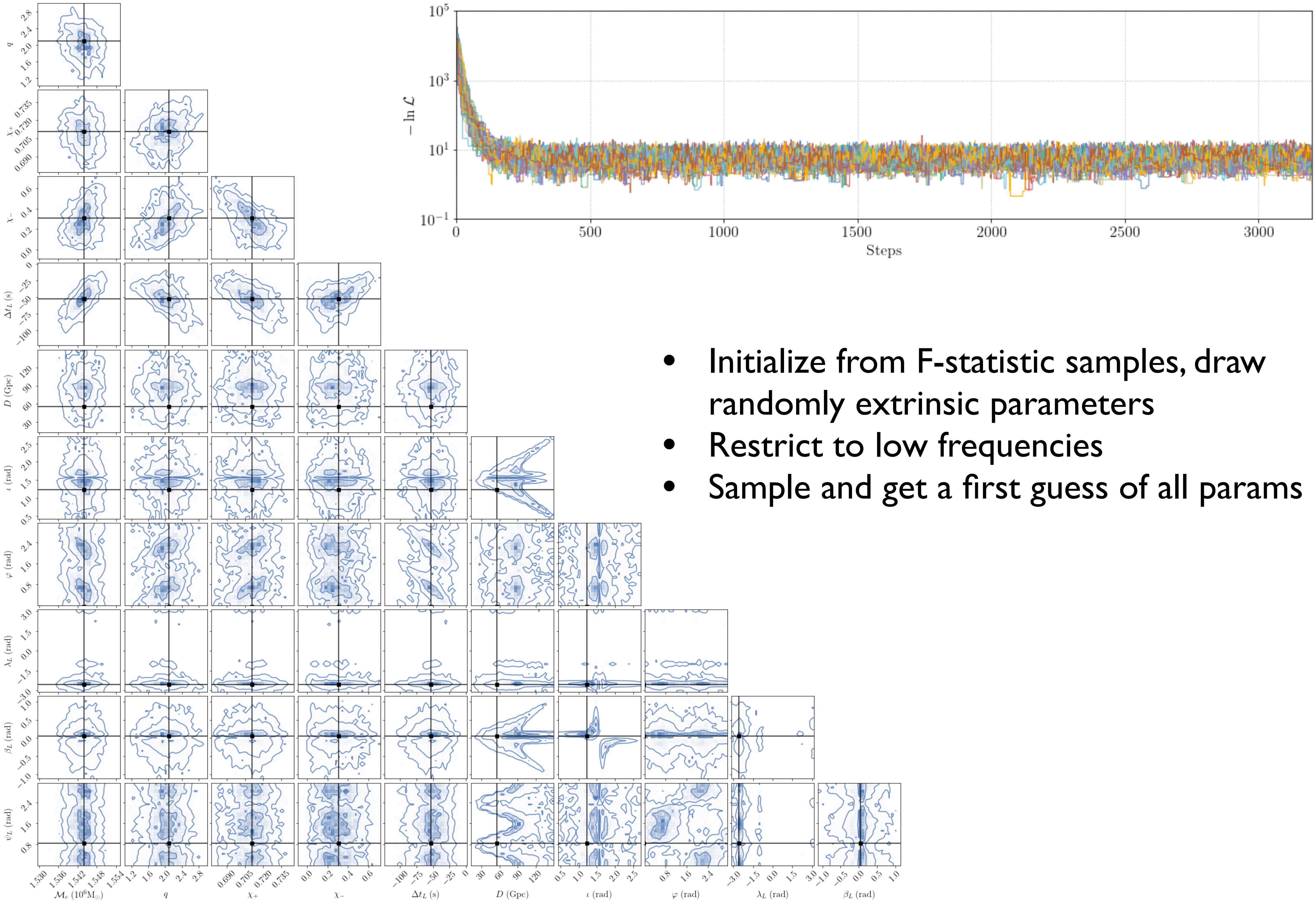
MBHB example: I) F-statistic search on small data segments





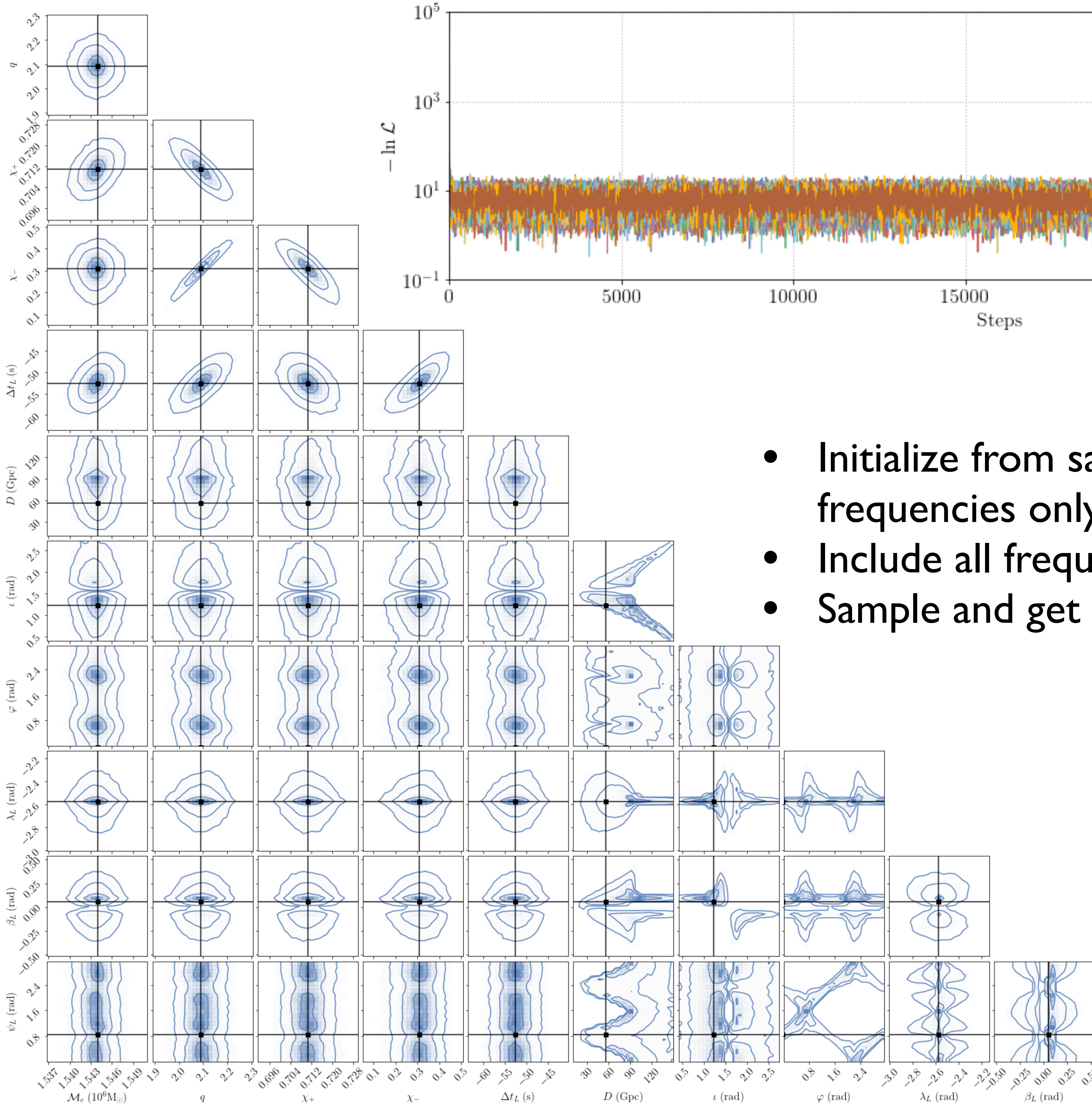
- Select short data segment
- F-statistic: approximate response, optimize analytically over extrinsic parameters
- Sampling easier for a lower dimensionality
- Get a first guess of intrinsic parameters + time

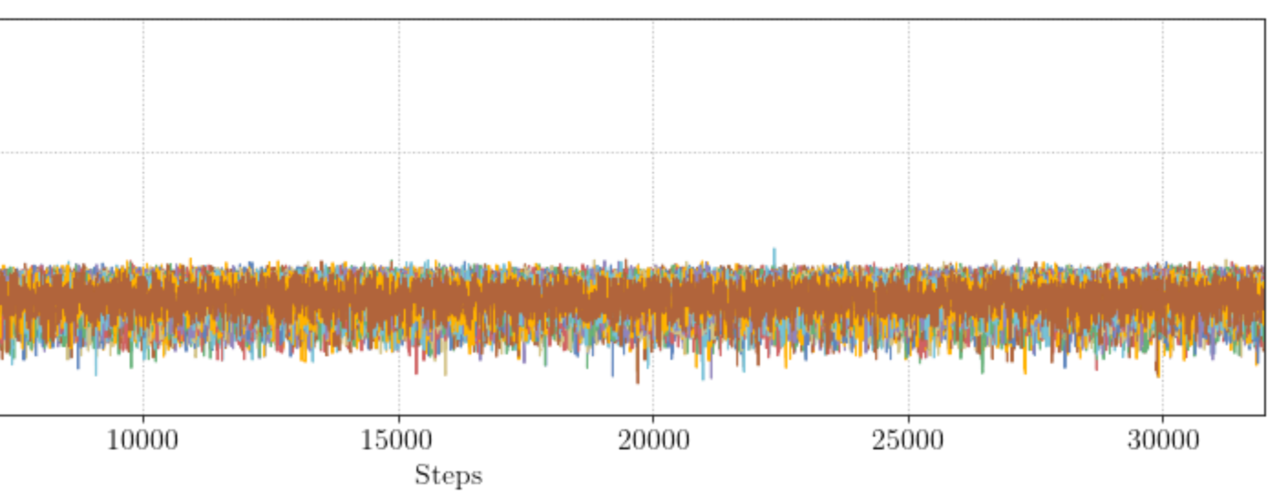
MBHB example: II) initial PE with low frequencies



- Initialize from F-statistic samples, draw randomly extrinsic parameters
- Restrict to low frequencies
- Sample and get a first guess of all params

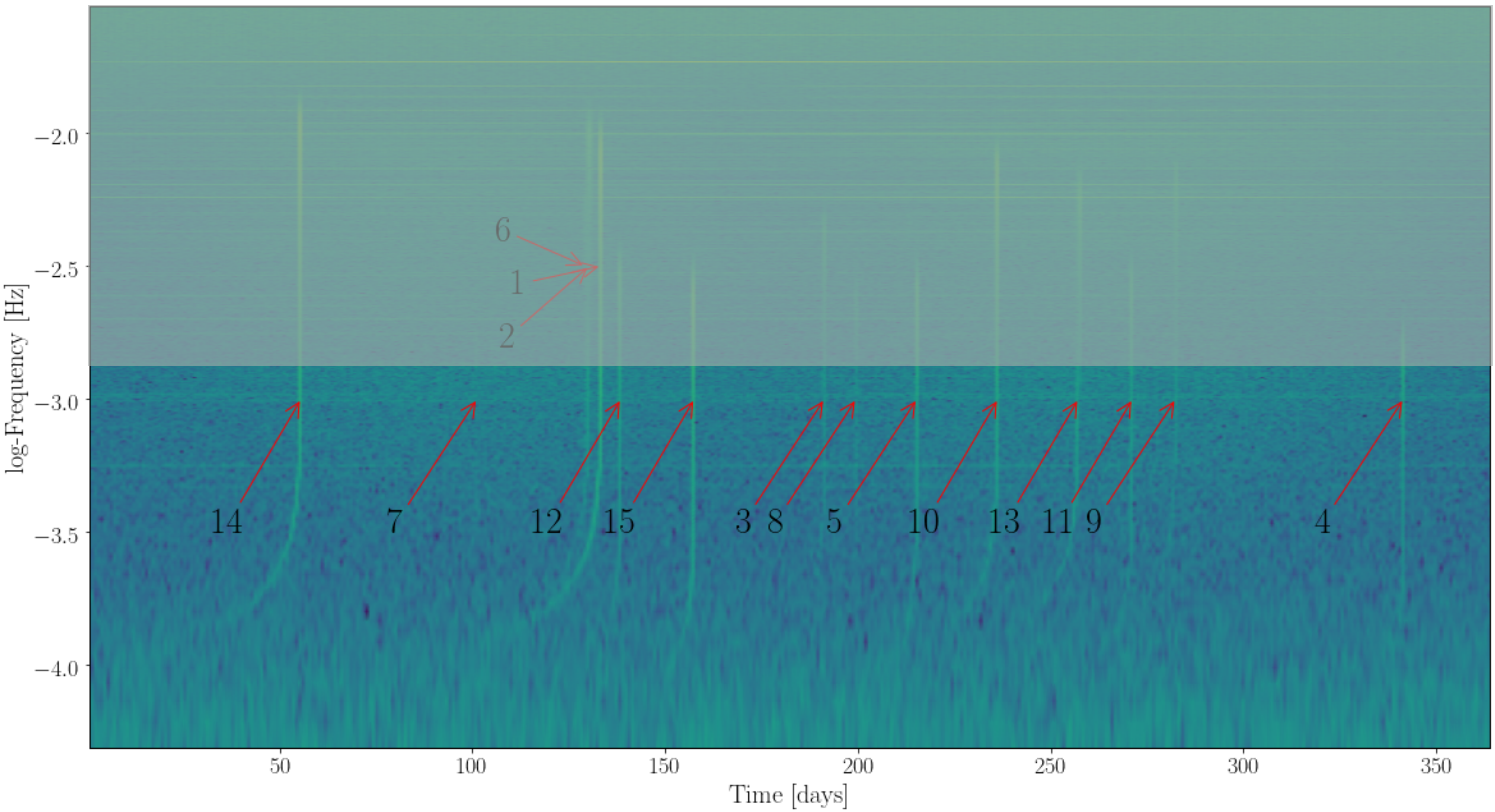
MBHB example: III) sampling with all frequencies





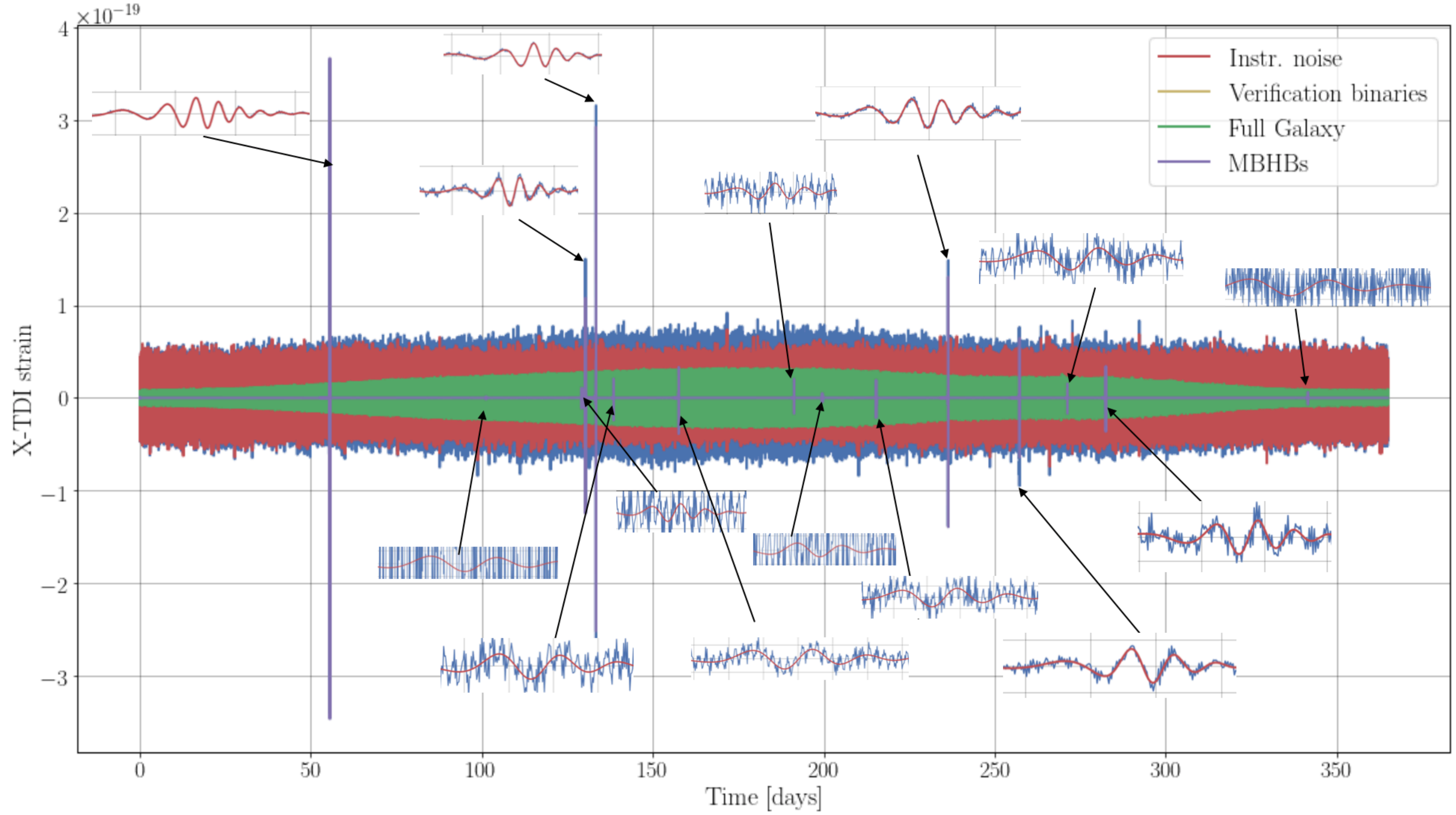
- Initialize from samples obtained with low-frequencies only
- Include all frequencies
- Sample and get final posterior

LISA data - band-passing, whitening

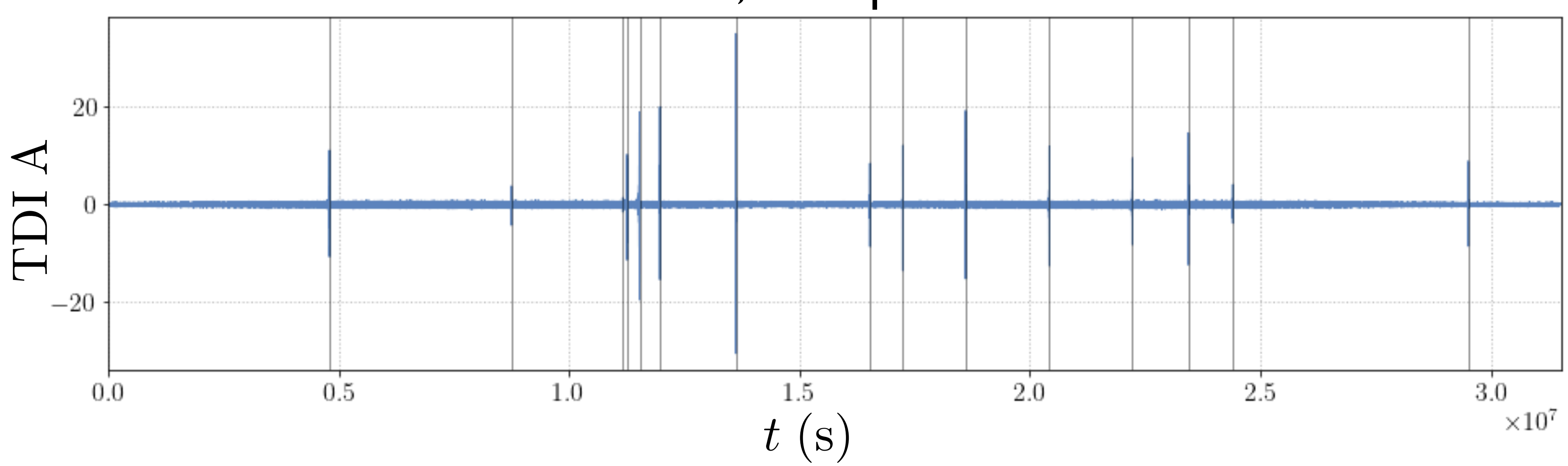


- **Band-passing**: select frequencies below 2mHz
- **Whitening**: work with signal/noise, so that all frequencies/times contribute equally

LISA data - band-passed, whitened in time domain

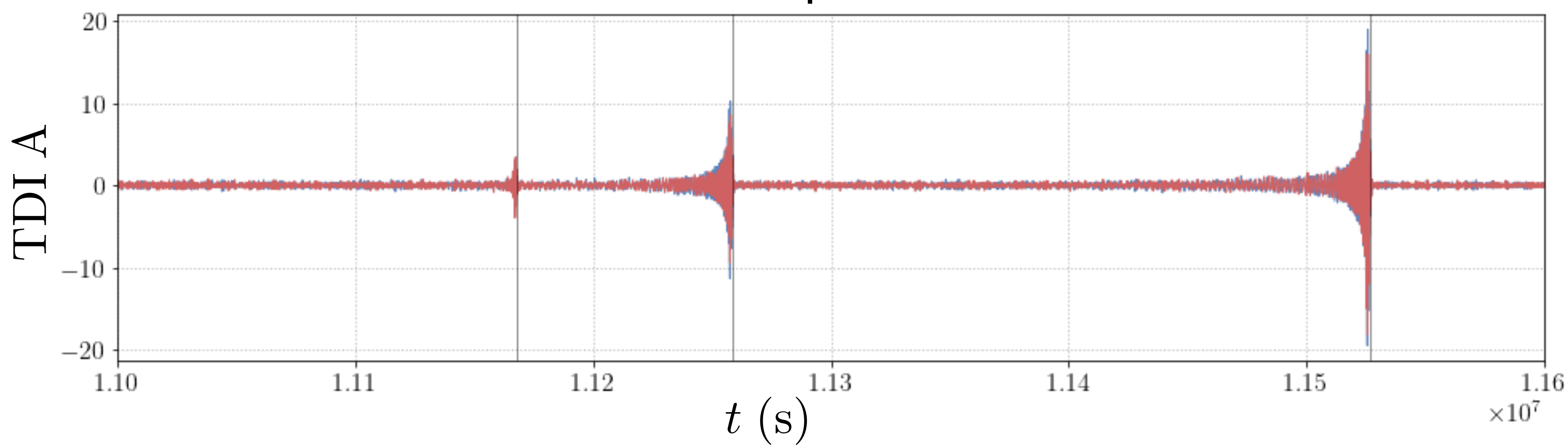


Whitened, band-passed data

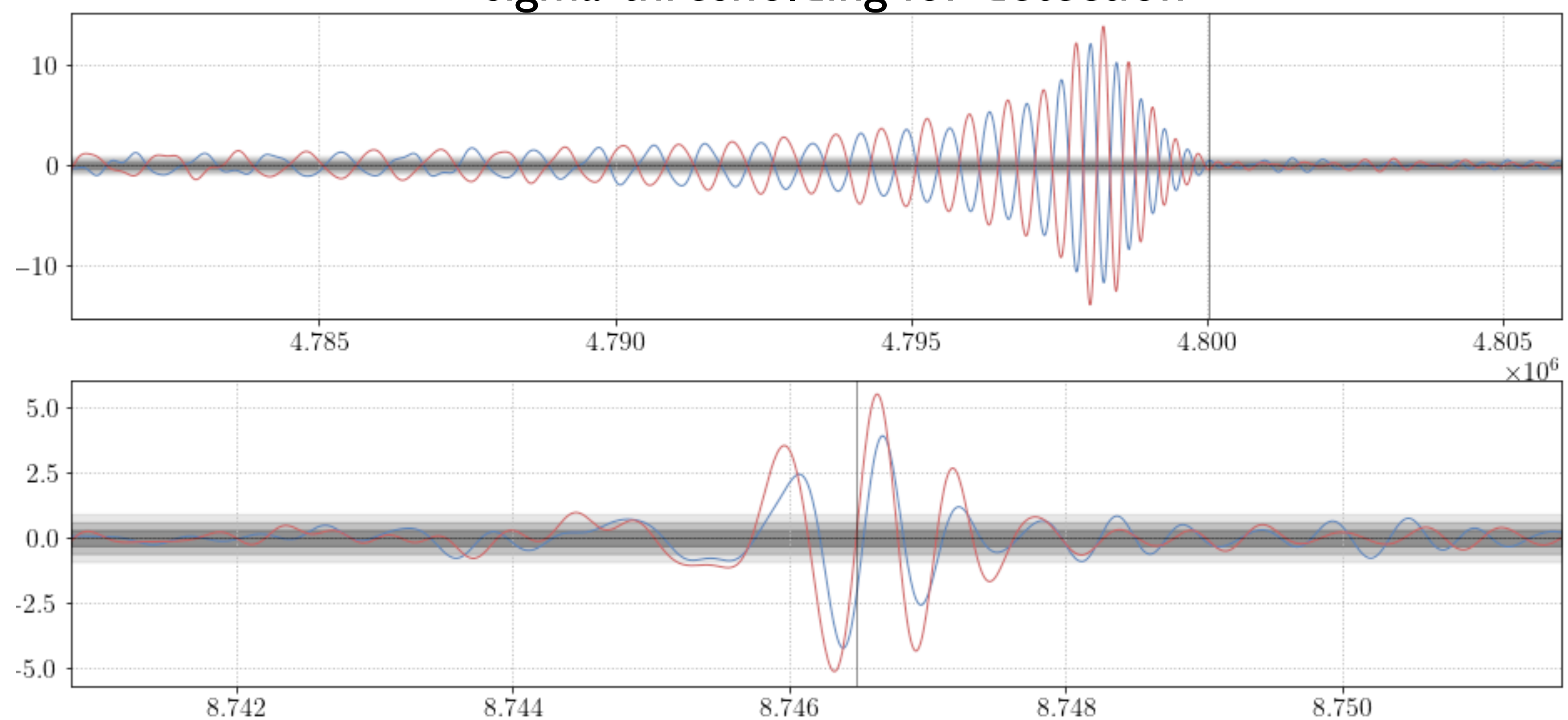


LISA data - band-passed, whitened in time domain

Whitened, band-passed data



sigma-thresholding for detection



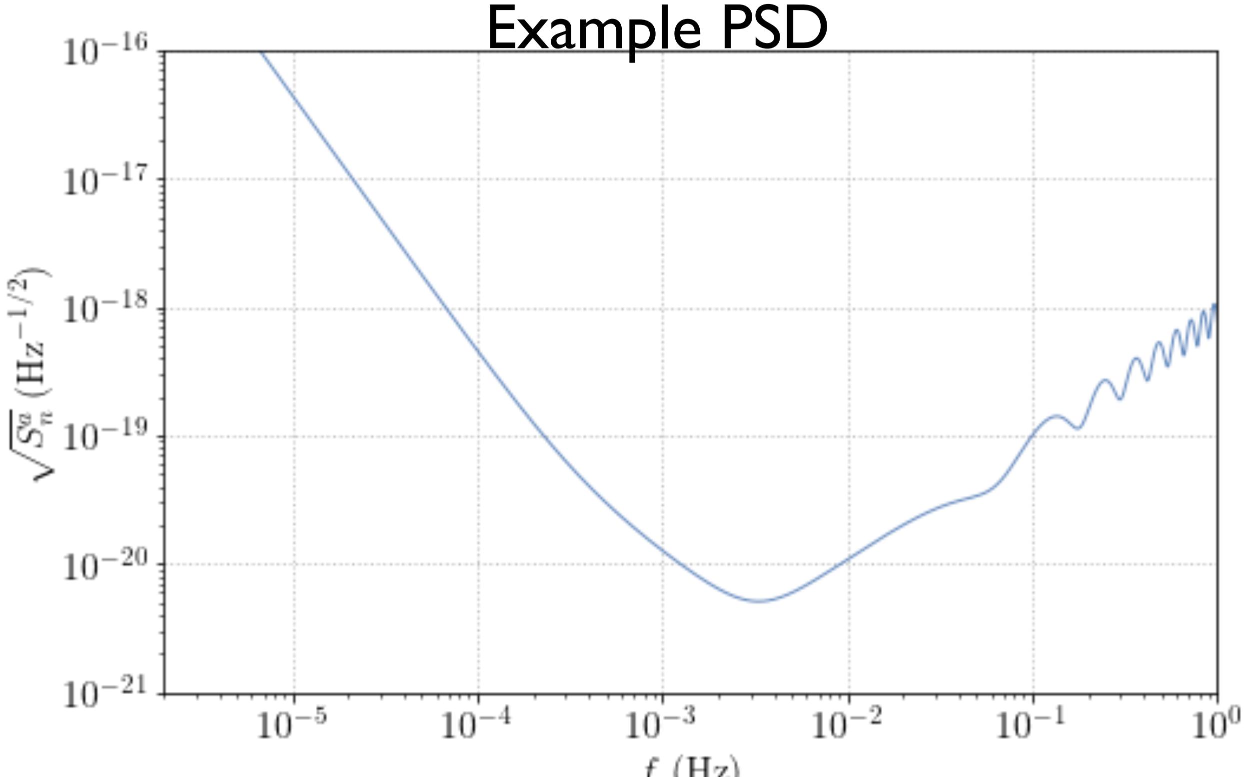
Noise properties

We are making probabilistic statements…
Assuming properties of the random noise !

Noise PSD

- Noise autocorrelation function: $K(\tau) = \langle n(t) n(t+\tau) \rangle$
(stationarity: depends only on τ)
- Noise PSD formal definition: $S_n(f) = 2 \int d\tau e^{2i\pi f \tau} K(\tau)$
- Stationarity: **independance** in FD $\langle \tilde{n}(f) \tilde{n}^*(f') \rangle = \frac{1}{2} S_n(f) \delta(f - f')$
- Gaussianity: noise in freq. bins is Gaussian $\tilde{n}(f) \sim \mathcal{N}(0, \frac{1}{2\Delta f} S_n(f))$

Example PSD



Less-than ideal assumptions for LISA !
Non-stationarity, glitches…

Likelihood and Bayesian analysis

Likelihood

- Likelihood: $\mathcal{L} = p(\mathrm{data}|\mathrm{signal\ params})$
- PDF of the noise: collection of independent Gaussian noise variables in each bin

$$\ln p(n = n_i) = \mathrm{const} - \frac{1}{2}\sum_i \Delta f \frac{2}{S_n(f)}|\tilde{n}_i|^2$$

$$= -\frac{1}{2}4\int \frac{df}{S_n(f)}|\tilde{n}(f)|^2 = -\frac{1}{2}(n|n)$$

- Likelihood is the probability that the noise makes up for the difference between observed data and theoretical signal: $d = h(\theta) + n$

$$\ln p(d|\theta) = \ln p(n = d - h(\theta)) = -\frac{1}{2}(d - h(\theta)|d - h(\theta))$$

Bayesian formalism

- Matched-filtering overlap: $(h_1|h_2) = 4\mathrm{Re}\int df \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$
- For Gaussian, stationary noise, for independent channels:

$$\ln \mathcal{L}(d|\theta) = -\sum_{\mathrm{channels}} \frac{1}{2}(h(\theta) - d|h(\theta) - d)$$

$$d = h(\theta_0) + n_0$$

- Bayes theorem defines the posterior: $p(\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$

h GW signal
θ parameters
d data stream
θ_0 signal params.
n_0 noise real.
S_n noise PSD

$p_0(\theta)$ prior
$p(d)$ evidence

Parameter estimation tools

Signal-to-noise (SNR)

Measures loudness of signal:

$$\mathrm{SNR}^2 = (h|h) = 4 \int \frac{df}{S_n} |h|^2$$

Simple detection statistics: SNR>8-10 (true detection statistics LIGO/Virgo more complicated)

Fisher matrix analysis

- Quadratic expansion of log-likelihood around true signal, approx. likelihood as a Gaussian

$$h(\theta) = h(\theta_0) + \Delta\theta_i \partial_i h + \dots$$
$$\ln \mathcal{L} = -\frac{1}{2} \Delta\theta_i F_{ij} \Delta\theta_j + \mathcal{O}(\Delta\theta^3)$$
$$F_{ij} = (\partial_i h | \partial_j h)$$

- Matrix inversion to get to the covariance of the Gaussian $C = F^{-1}$
- Valid at high SNR, and misses degeneracies

Bayesian sampling tools

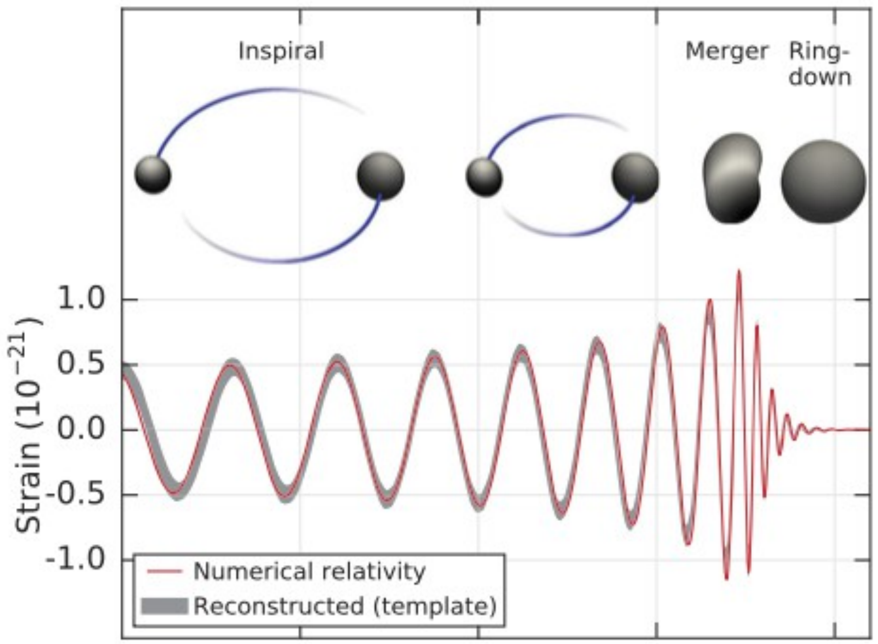
- MCMC methods, nested sampling
- MCMC proposals: ensemble samplers (emcee), differential evolution, …
- Parallel tempering: explore full parameter space
- Informed proposals to deal with degeneracies

Levels of approximation

- Fisher: for high SNR limit (depends on signal !)
- Set noise realization to 0
- Initialize MCMC from Fisher
- Full run with initialization from priors
- Full run with noise
- Superposition of sources, unknown noise, noise artifacts…

GW signals seen by LISA - the basics

IMR Signal

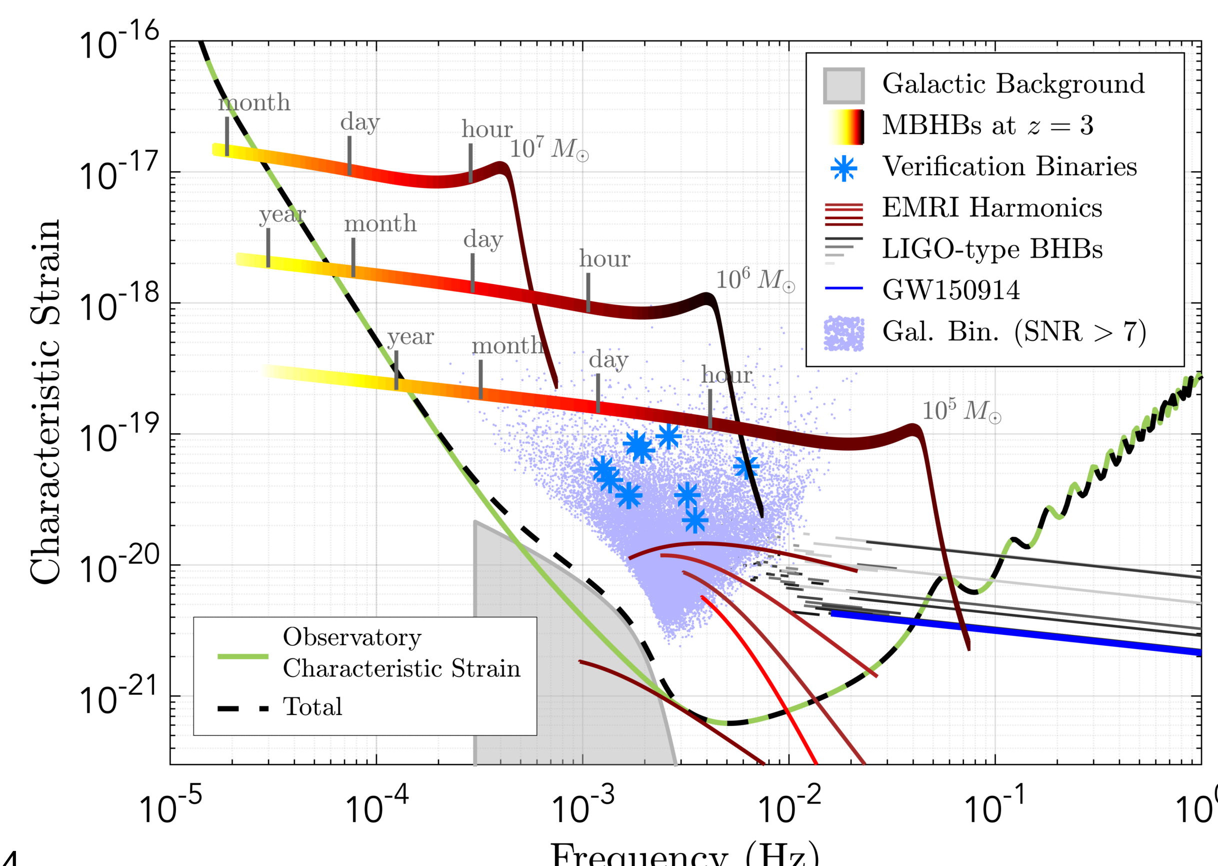


Phases of the signal:

- **Inspiral**: covered by post-Newtonian (PN) perturbative series
- **Merger**: covered only by numerical relativity (NR)
- **Ringdown**: NR, superposition of Quasi-Normal Modes (QNM)

LISA: different BHB signals

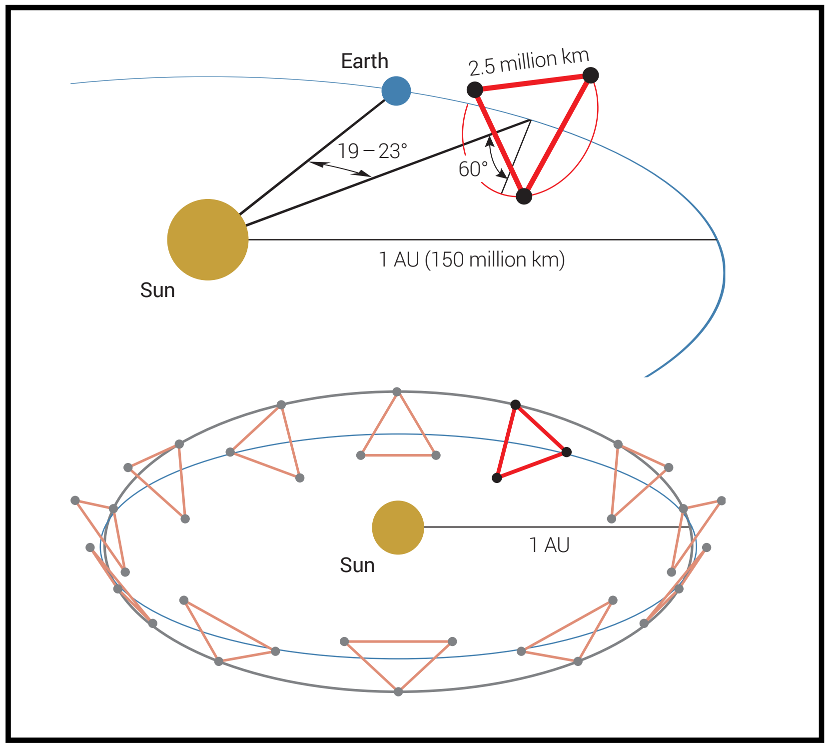
- **MBHBs**: very loud, merger-dominated (mostly short)
- **SBHBs**: early inspiral, some chirping during LISA obs. (multiband ?)
- **GBs**: quasi-monochromatic, superposed
- **EMRIs**: long-lived, many harmonics
- **Stochastic backgrounds**
- **TDEs !**

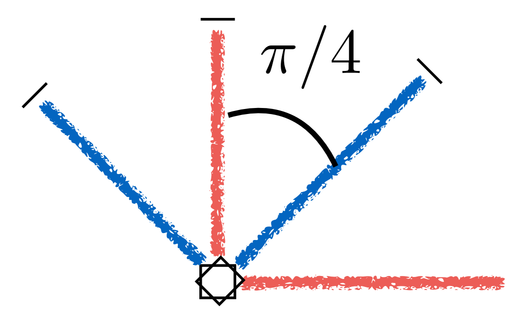


Contrasting LIGO/Virgo and LISA responses: LISA

LISA-frame

SSB-frame: global view of the orbits

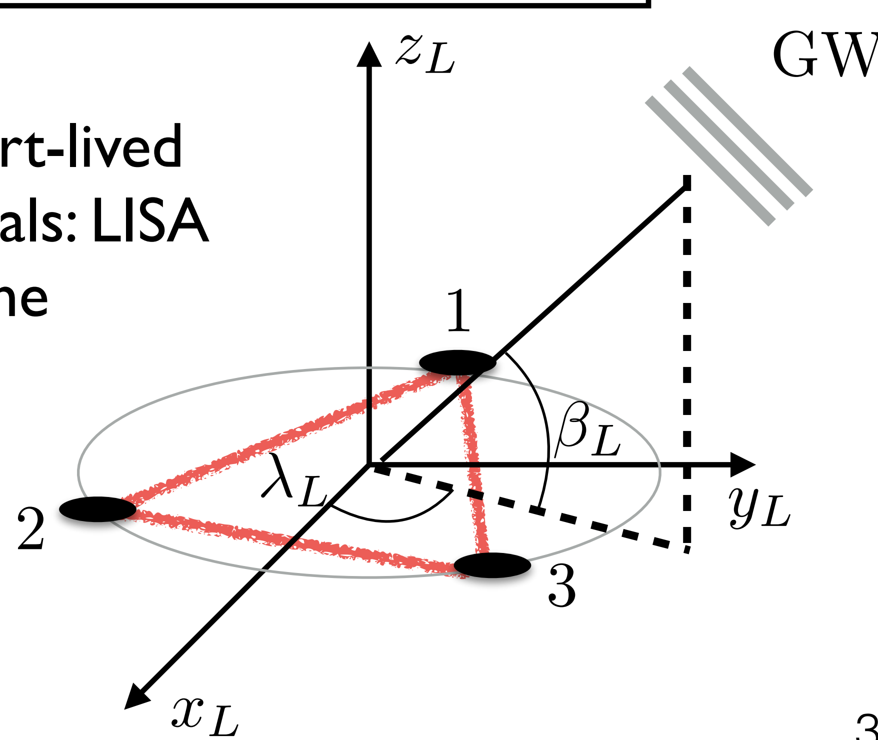




Low-f approximation: **two LIGO-type detectors** in motion [Cutler 1997]

High-f: **three channels** with complicated frequency-dependence

Short-lived signals: LISA frame



Sky localisation from the modulations induced by the orbits for long-lived signals

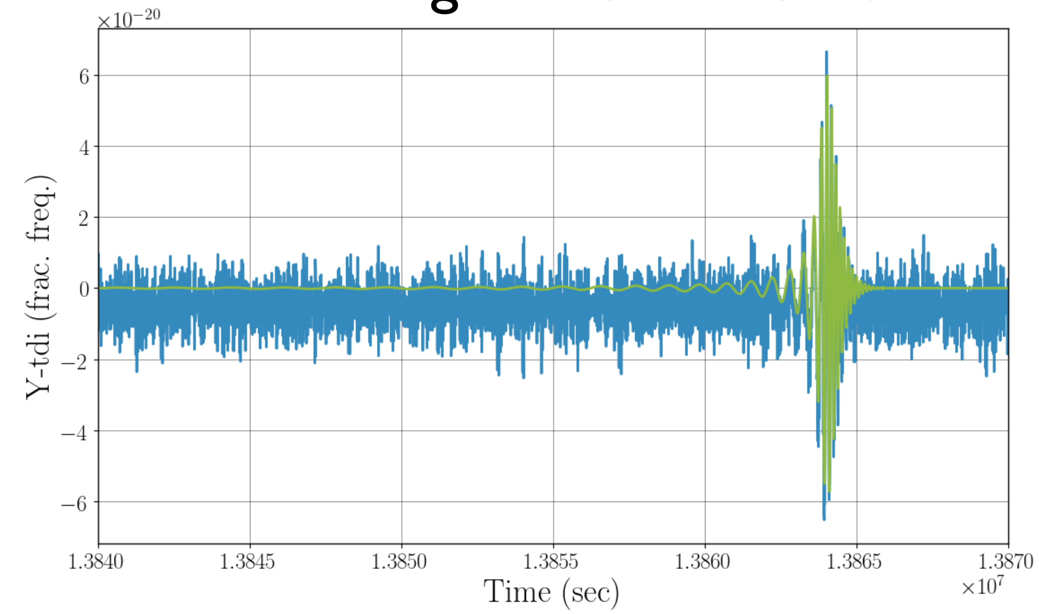
Sky localization can also come from high-f effects.

Degeneracies - multimodality in the sky possible !

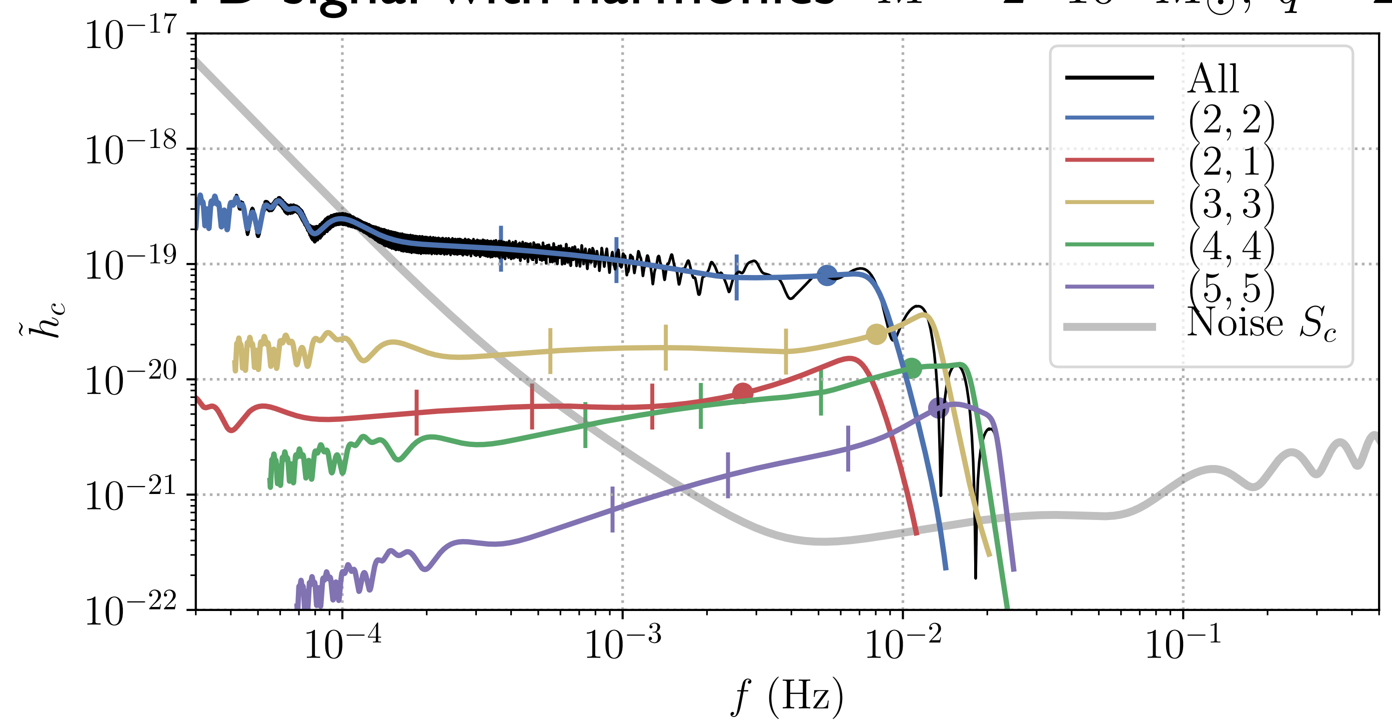
Massive black holes: signals and challenges

- Very loud sources, SNRs of several thousands !
- Detection of merger easy, but detection as early as possible ?
- Advance localization for multimessenger observations ?
- Signals can be short (< 1day) and degenerate
- Waveform model systematics for such loud signals ? Biases, residuals for other sources ?
- Subdominant features in the signal are important

MBHB merger in time-domain



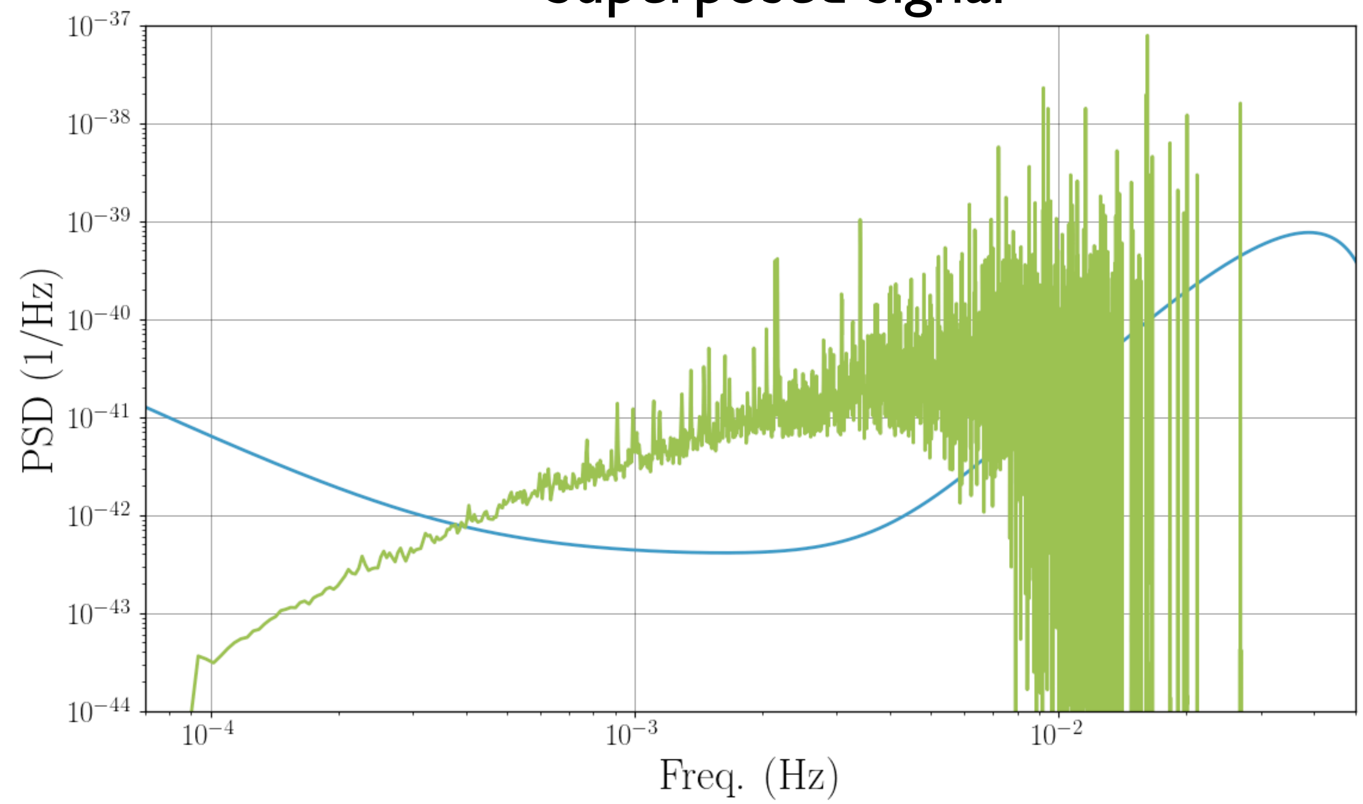
FD signal with harmonics $M = 2 \cdot 10^6\ M_\odot,\ q = 2$



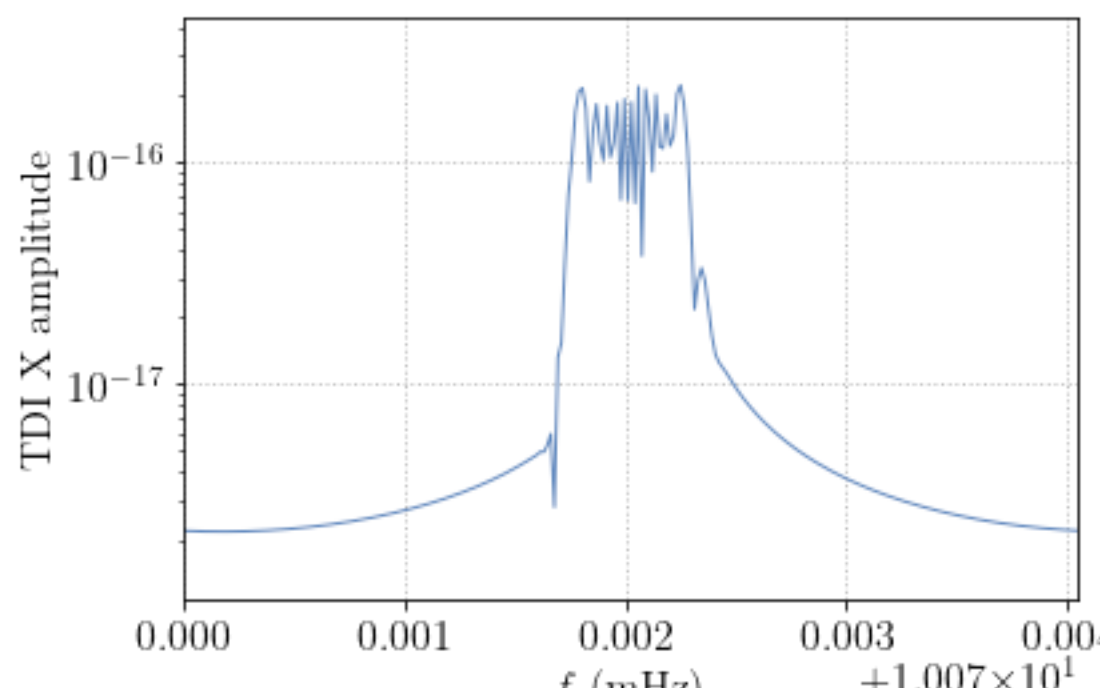
Galactic binaries: signals and challenges

- Mostly WD-WD, some other compact objects
- Full galaxy: ~20 million systems !
- About ~20000 individually resolvable
- Form a (non-stationary) background
- Verification binaries
- Quasi-monochromatic GW emitters
- Modulation by LISA motion (sidebands in Fourier-domain)
- Superposition of signals in Fourier-domain

Superposed signal



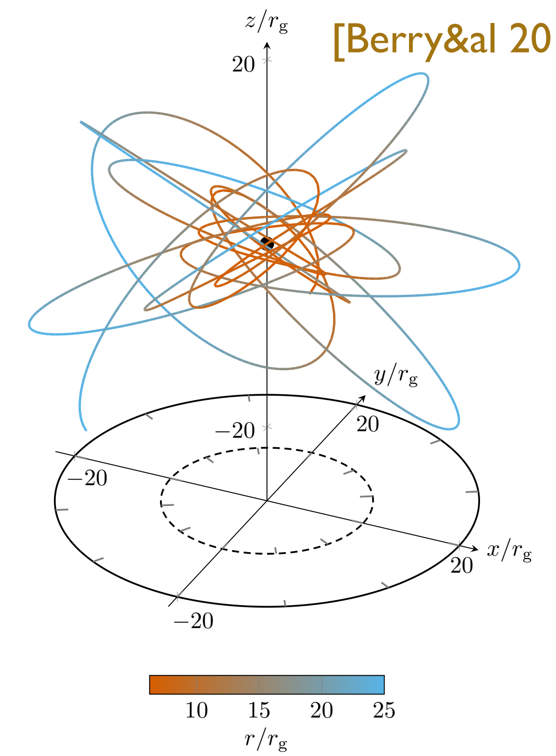
Individual signal



Extreme mass ratio inspirals, stellar-mass black holes

EMRIs

- Long-lived, complex signals, large number of wave cycles $(10^4 - 10^5)$
- Strong precession and eccentricity features, orbits in the relativistic regime around Kerr
- Exquisite determination of some parameters — also means that the signals are hard to find !
- Theoretical work on waveform models needed

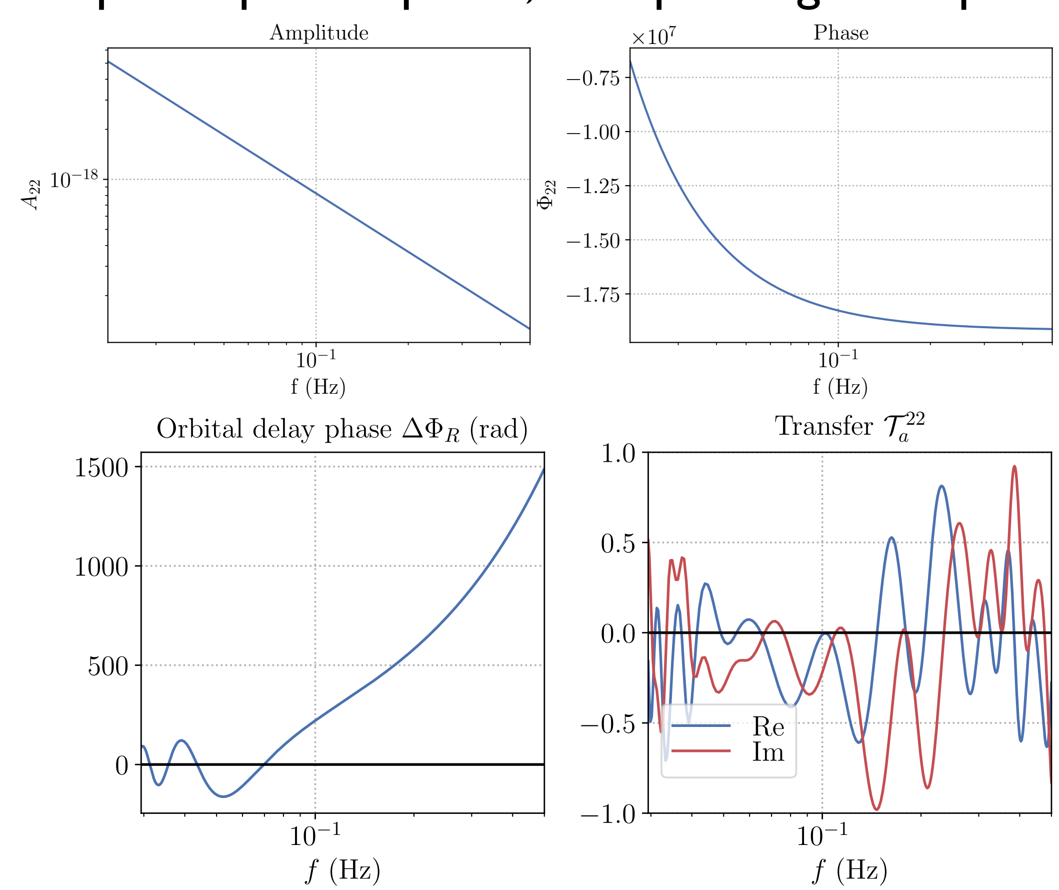


[Berry&al 2019]

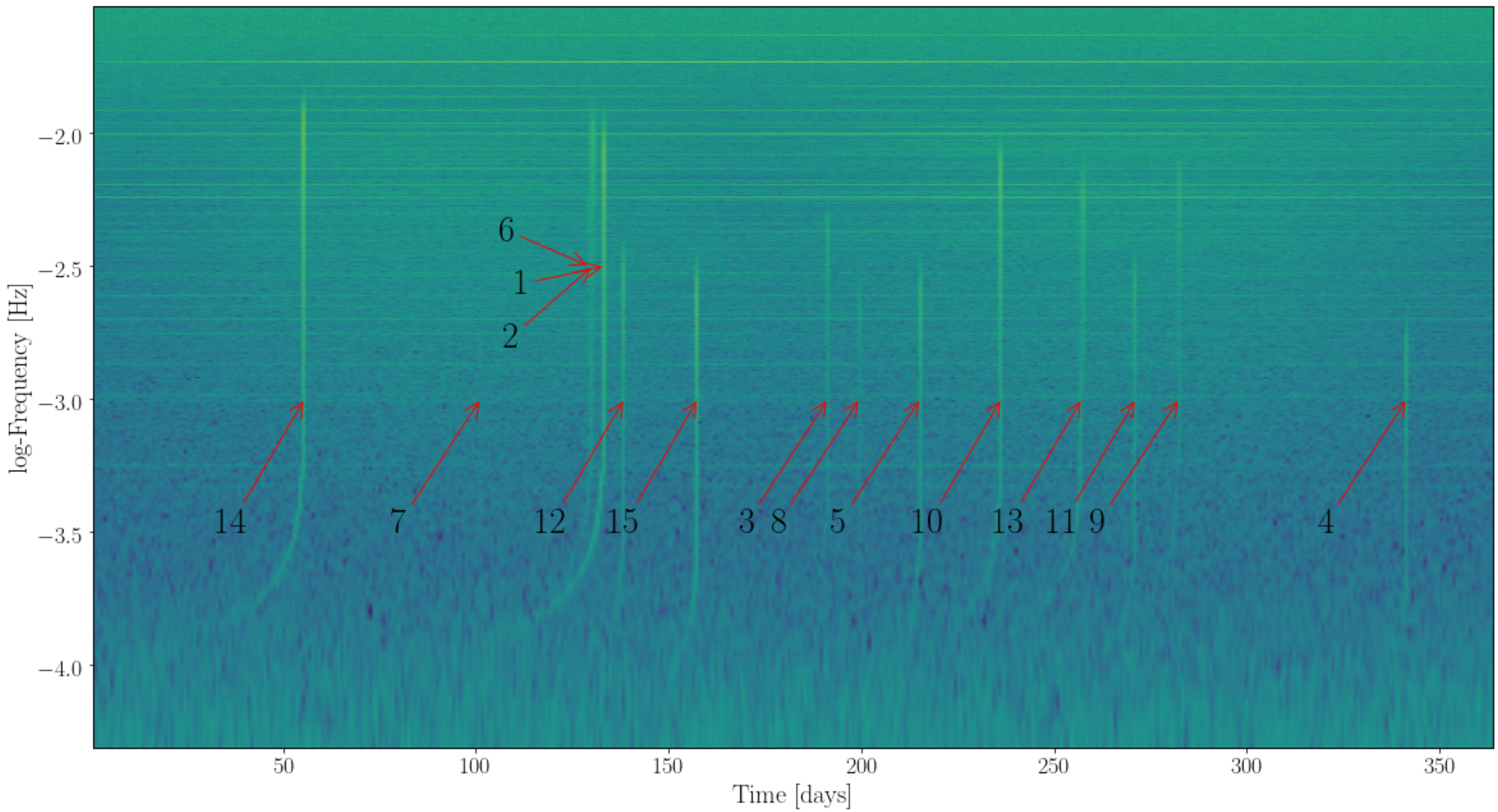
Stellar-mass BHs

- Quiet signals: a few detections in the LISA band
- Inspiral regime far from merger, very large number of cycles $(10^5 - 10^6)$
- Challenge of detection: template banks impossible
- Multiband analysis, archival searches ?

Simple amplitude/phase, Complex high-f response

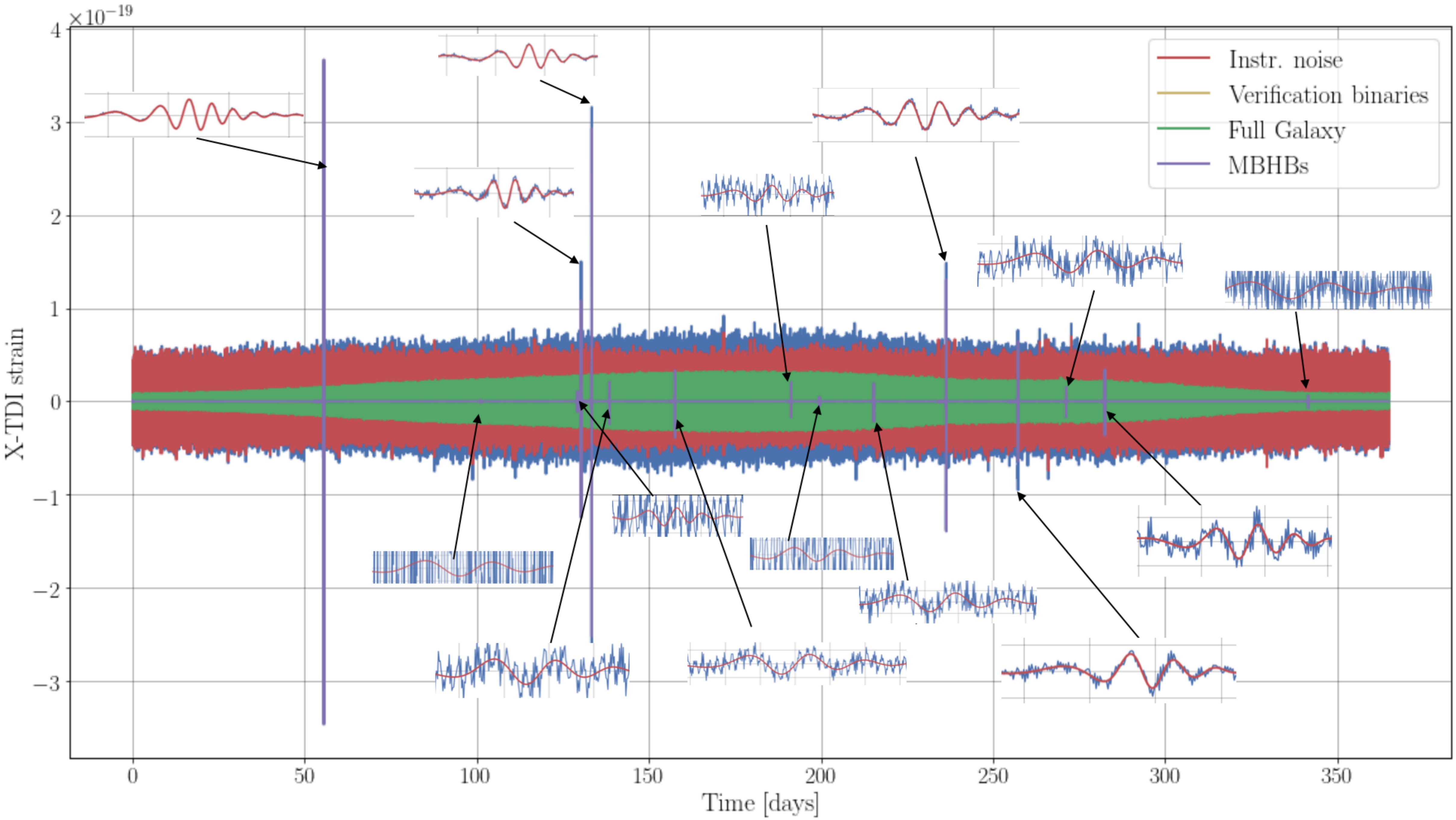


LISA data - LDC-2 Sangria



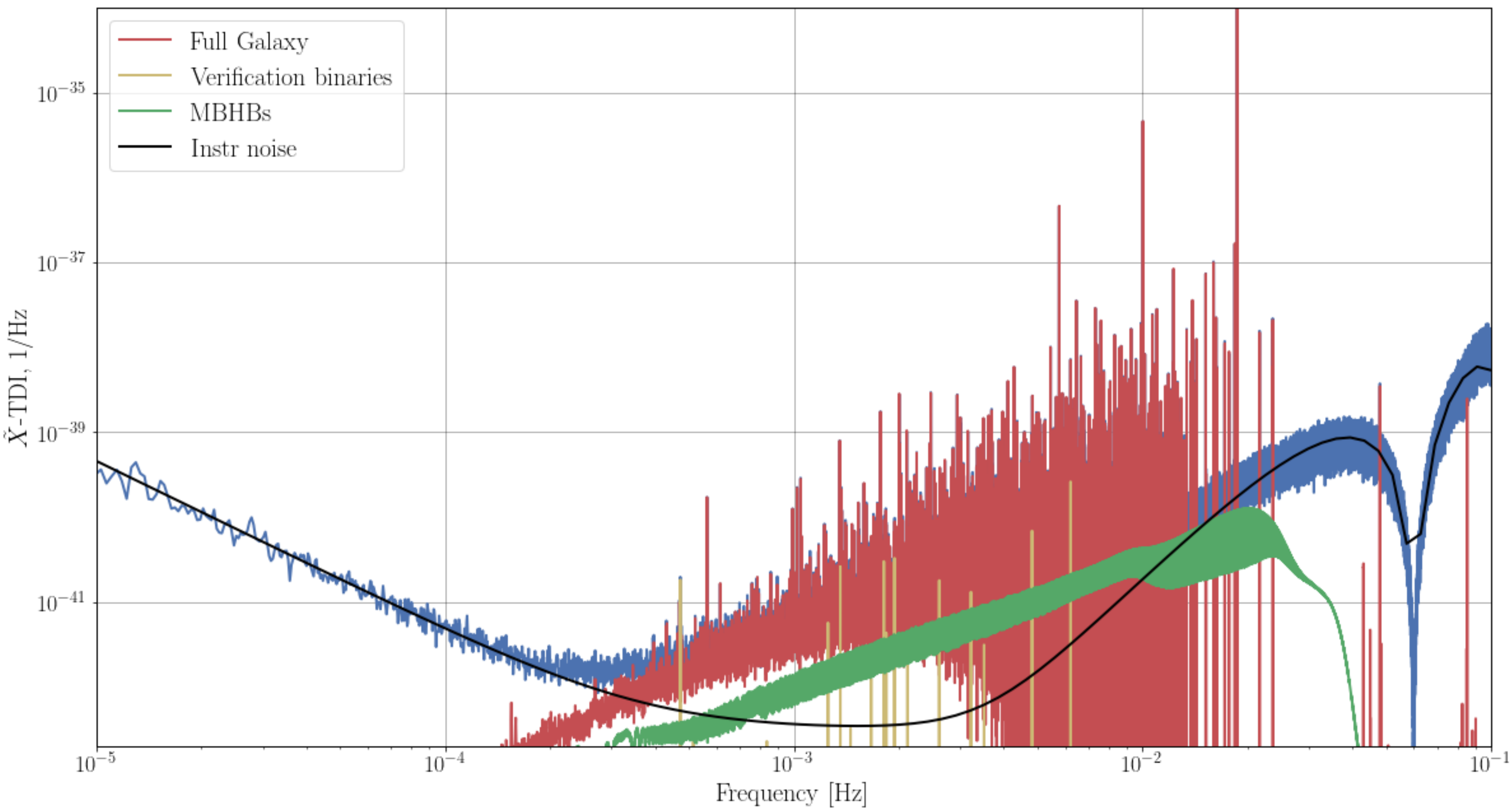
- **MBHBs**: chirping signals, emerging from low-f noise
- **GBs**: quasi-monochromatic, horizontal lines

LISA data - LDC-2 Sangria Time-Domain



- **MBHBs**: loudest ones clearly visible by eye above the noise
- **GBs**: superposed signals, annual modulation due to the LISA motion

LISA data - LDC-2 Sangria Frequency-Domain



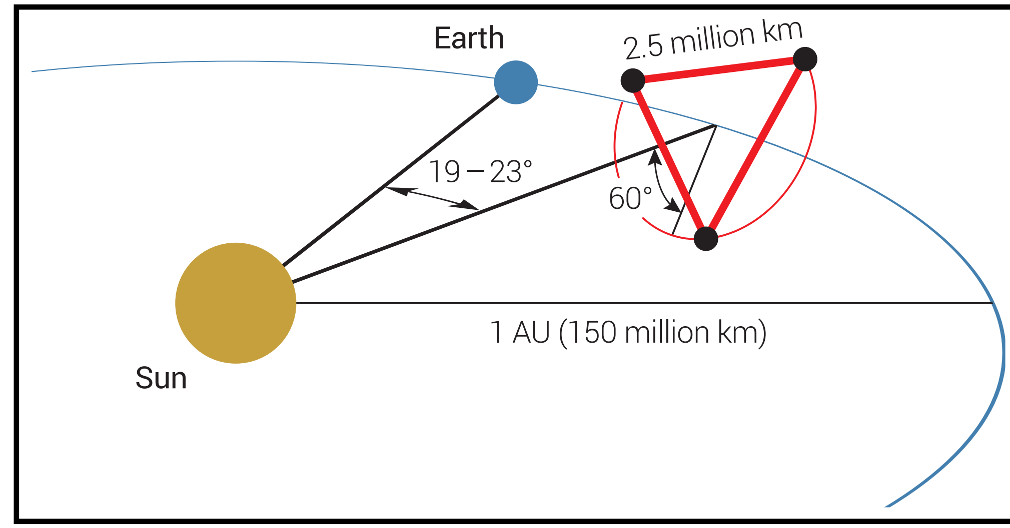
- **MBHBs**: loudest ones visible in the spectrum, subdominant
- **GBs**: signals local in frequency, both individually resolvable and building up a background

LISA Fourier-domain response

Response

Laser frequency shift, spacecrafts s to r through link l: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1-\hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$



Transfer function for modulated and delayed signal

$$\mathrm{FT}\left[F(t)h(t+d(t))\right] = \mathcal{T}(f)\tilde{h}(f)$$

Fourier-domain for **chirping signals** (separation of timescales):

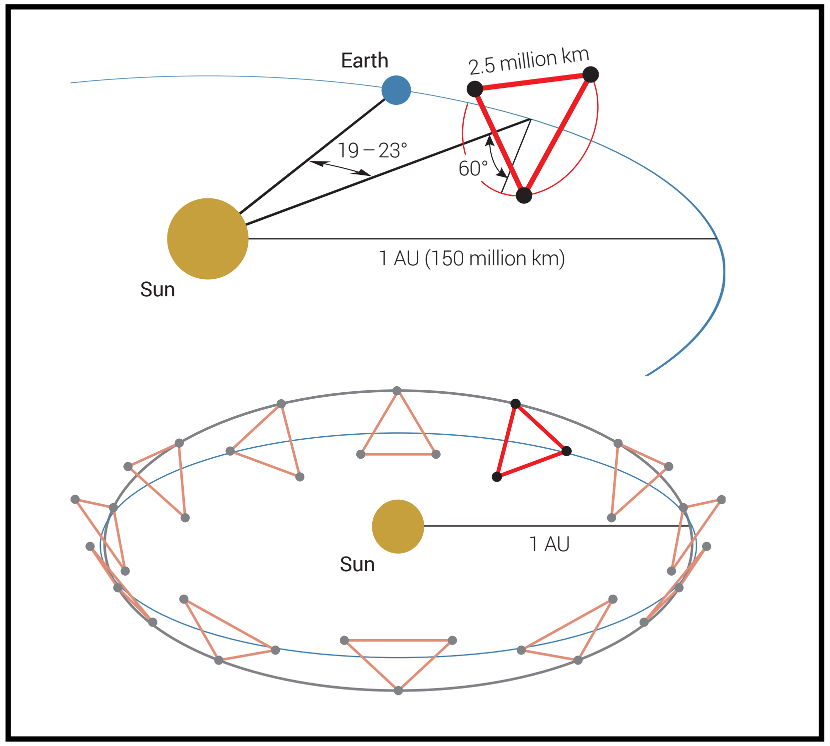
$$\mathcal{T}_{slr} = \frac{i\pi f L}{2}\mathrm{sinc}\left[\pi f L\left(1 - k\cdot n_l\right)\right]\exp\left[i\pi f\left(L + k\cdot(p_r + p_s)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

Time and frequency-dependency
Time: motion of LISA on its orbit
Frequency: departure from long-wavelength

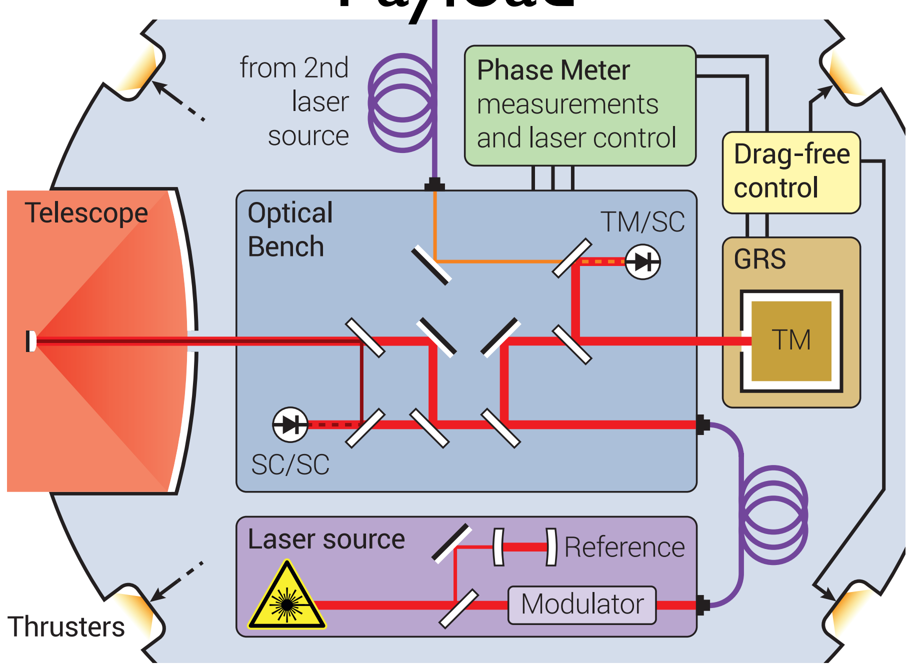
+ Time-delay interferometry (TDI)
linear combinations of y_{slr} with more delays

LISA mission - 2034

Orbits

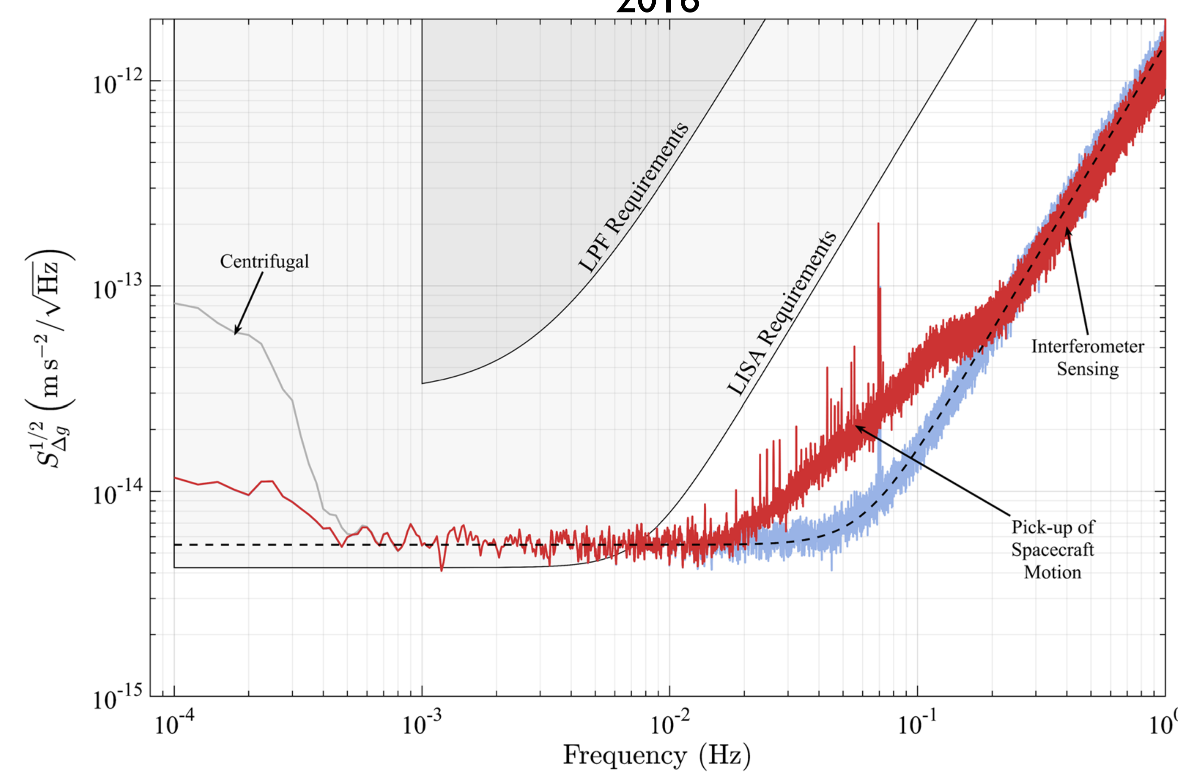


Payload

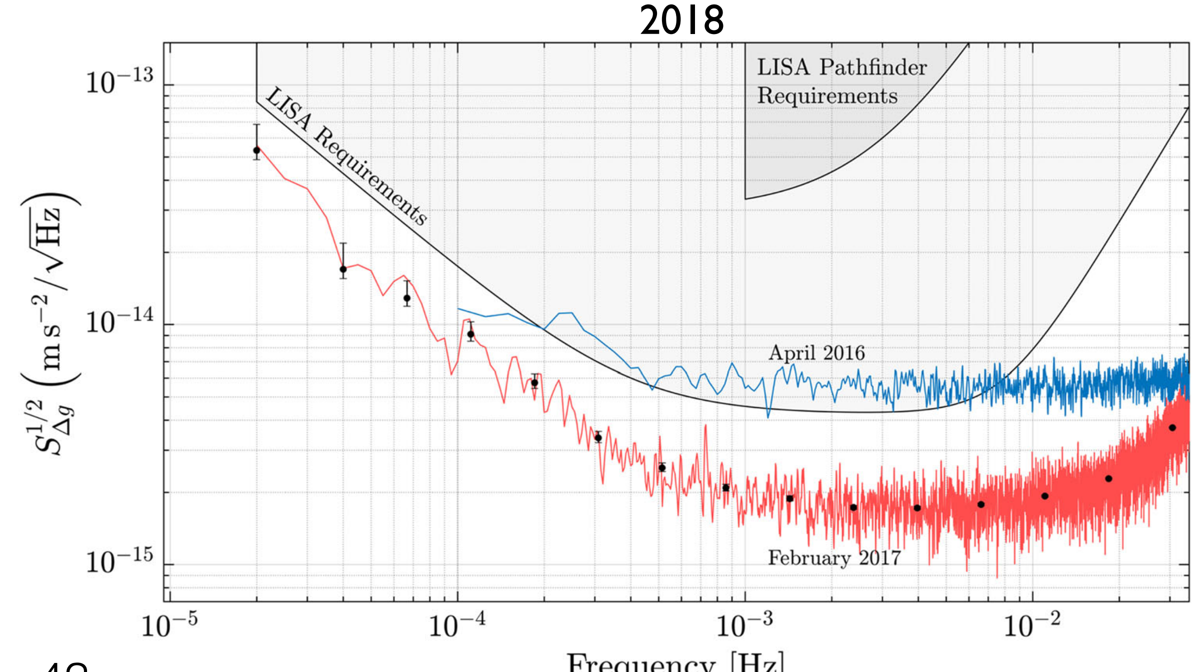


LISA Pathfinder success !

2016

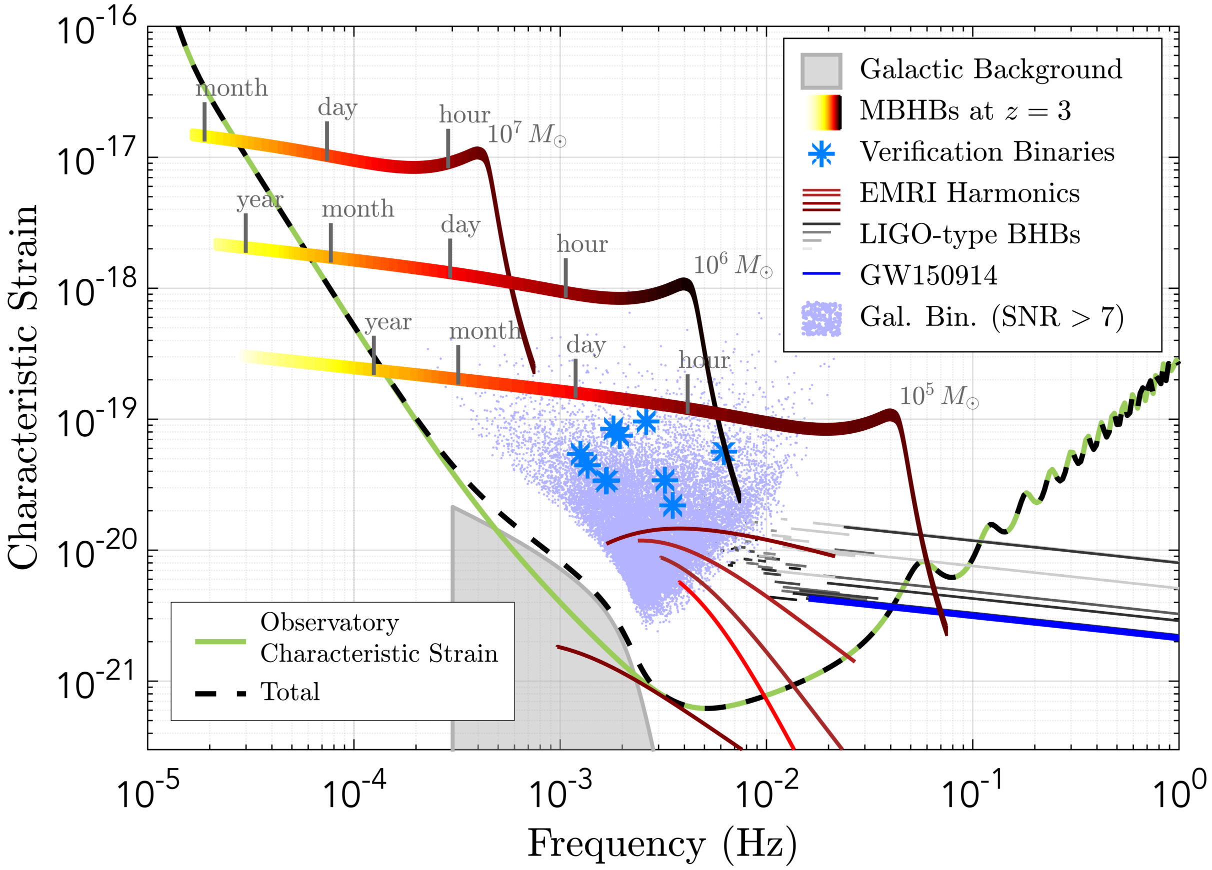


2018

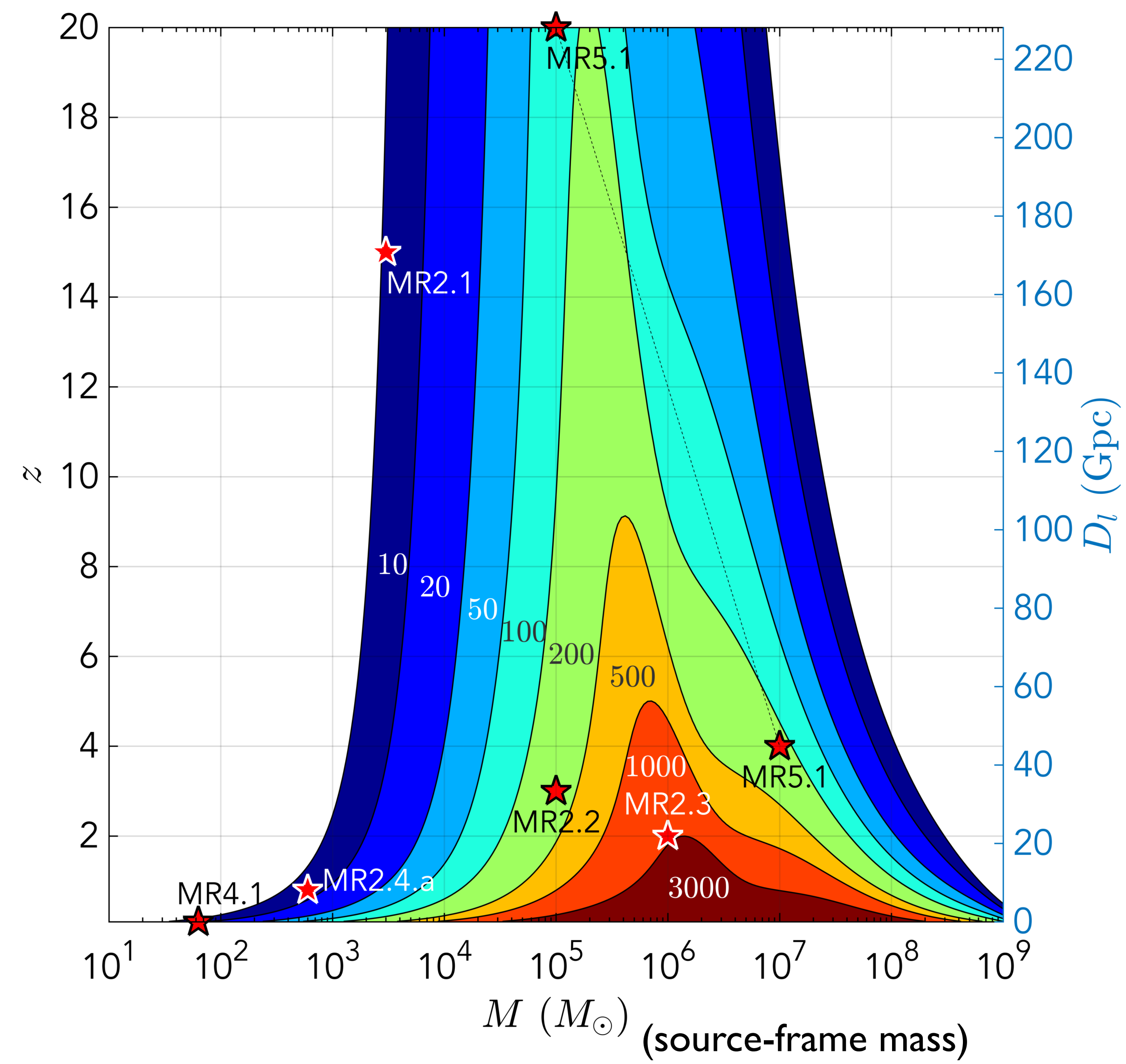


LISA sources

LISA sources



MBHBs SNR



Terminology:

- Massive black holes binaries (MBHBs)
- Stellar-mass black hole binaries (SBHBs): masses observable by ground-based detectors [Sesana 2016]
- Galactic Binaries (GBs): mostly WD-WD
- Extreme Mass Ratio Inspirals (EMRIs)
- Sochastic backgrounds (GBs, cosmo.)
- TDEs !

Contrasting LIGO/Virgo and LISA responses: LIGO/Virgo

Pattern functions

Simple multiplicative response

$$s = F_+ h_+ + F_\times h_\times$$

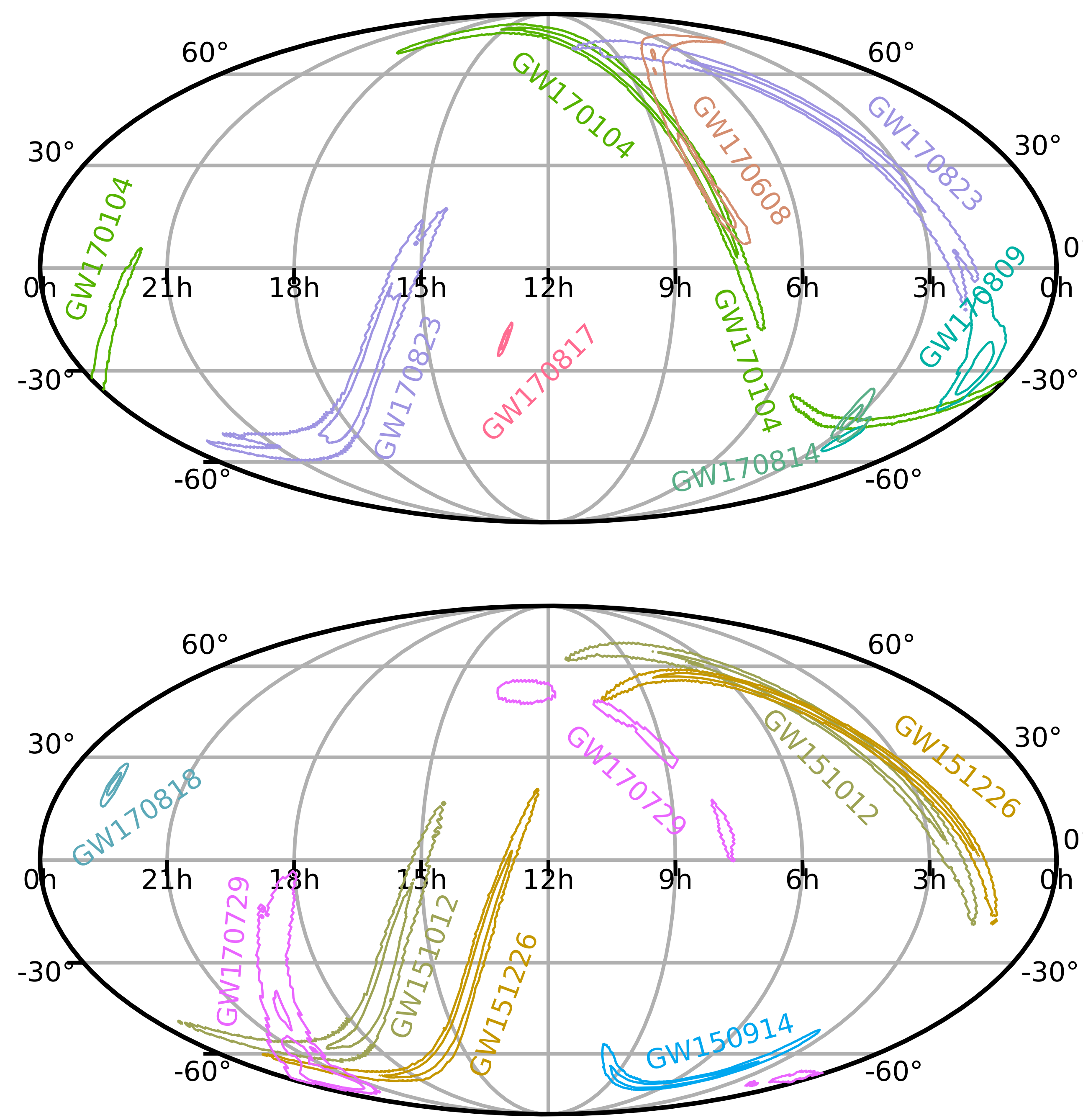
Angular dependence:

$$F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos\left(2\phi\right) \ ,$$
$$F_\times = \cos\theta\sin\left(2\phi\right)$$

Time-of-arrival triangulation

- Two detectors: ~ring on the sky
- Better localization for 3 or more detectors (even low SNR!)

GWTC-1 sky localisation



LISA instrument response

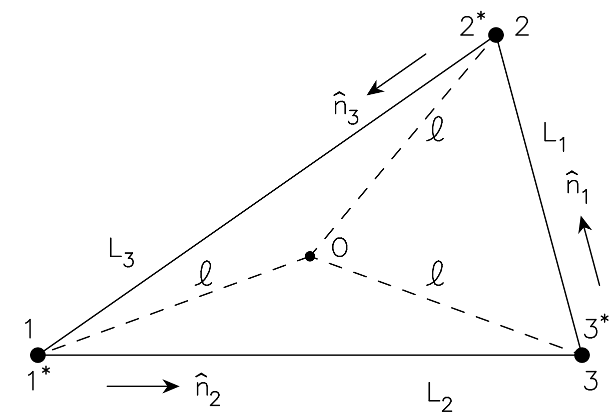
One-arm frequency observables

From spacecraft s to spacecraft r through link s: $y = \Delta\nu/\nu$

$$y_{slr} = \frac{1}{2}\frac{1}{1 - \hat{k}\cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

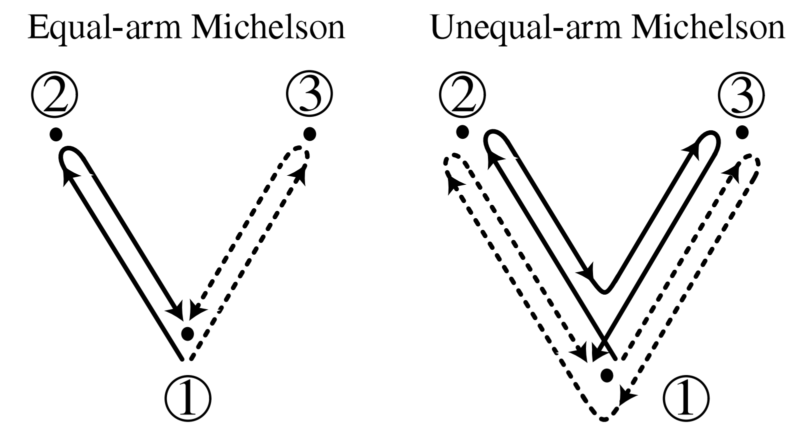
$$t_s = t - L - \hat{k}\cdot p_s, \quad t_r = t - \hat{k}\cdot p_r$$

$$h = h_+ P_+(\hat{k}) + h_\times P_\times(\hat{k})$$ GW at SSB



Time-delay interferometry (TDI)

- Crucial to cancel laser noise
- First generation: unequal arms
- Second generation: propagation and flexing
- Michelson X,Y,Z - Uncorrelated noises A,E,T



$$X_1^{\rm GW} = \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]}_{X^{\rm GW}(t)}$$
$$- \underbrace{\left[(y_{31}^{\rm GW} + y_{13,2}^{\rm GW}) + (y_{21}^{\rm GW} + y_{12,3}^{\rm GW})_{,22} - (y_{21}^{\rm GW} + y_{12,3}^{\rm GW}) - (y_{31}^{\rm GW} + y_{13,2}^{\rm GW})_{,33}\right]_{,2233}}_{X^{\rm GW}(t-2L_2-2L_3)\simeq X^{\rm GW}(t-4L)} .$$

Approximations

- Long-wavelength approximation: two moving LIGOs rotated by $\pi/4$ + orbital delay
- **Rigid approximation** (order of the delays does not matter, delay=L simple in Fourier domain)