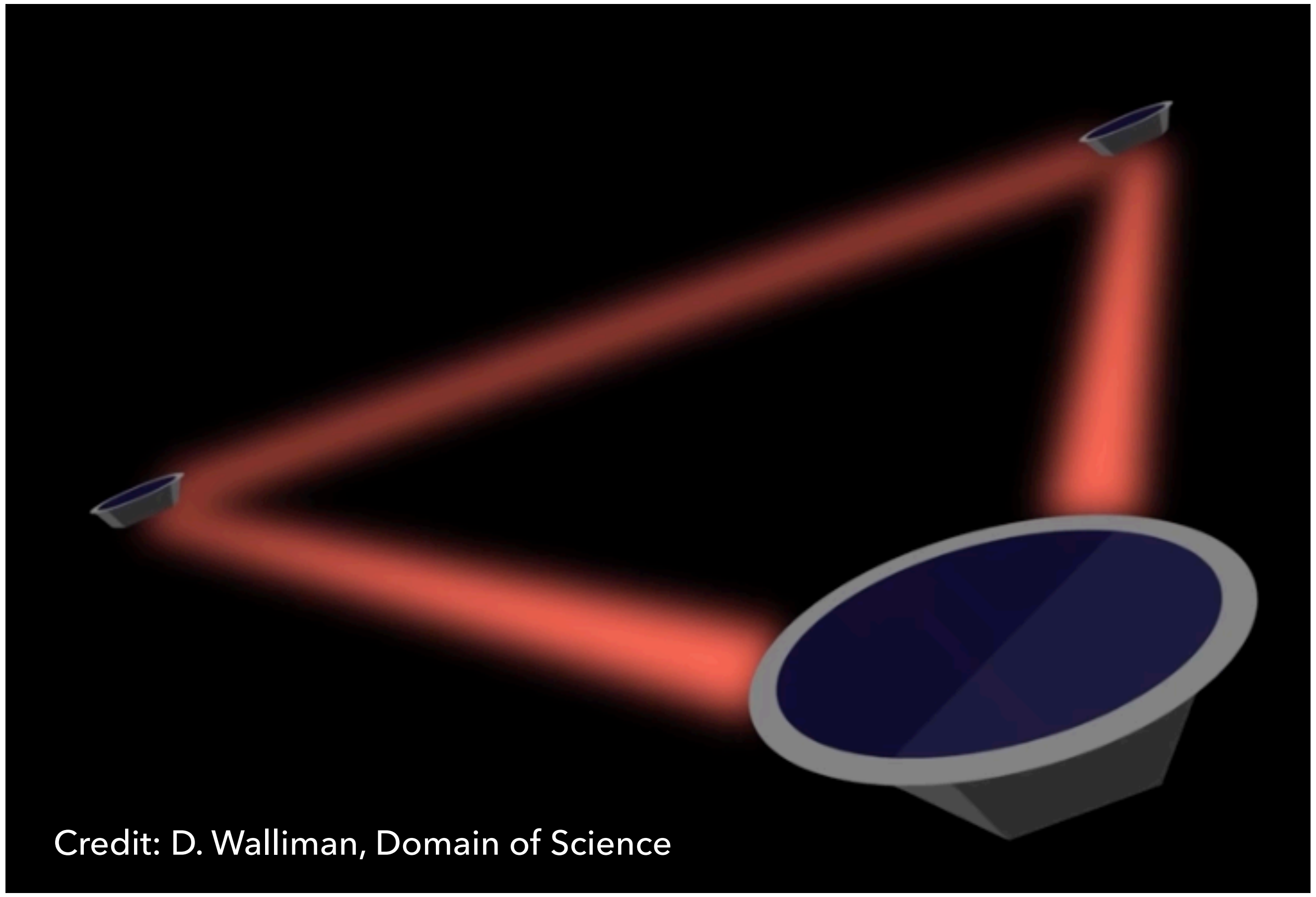
BAYESIAN INFERENCE FOR LISA DATA ANALYSIS

Quentin Baghi, CEA Saclay

Tuesday, November 15th, 2022

Rencontre du groupe de travail "méthodes d'analyse des données" du GdR oG

1. Challenges of LISA data analysis
2. Overview of used Bayesian concepts
3. Towards the future



CHALLENGES OF LISA DATA ANALYSIS

- The analysis of LISA data will be drastically **different from current ground-based detection**:
  - ✦ Numerous superimposed sources $\neq$ isolated events
  - ✦ Different time scales, larger waveform cycles observed
  - ✦ Signal-dominated measurement $\neq$ noise-dominated

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  - ✦ Instrumental transients (glitches)
  - ✦ Non-stationarities
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  Disturbances

- What kind of data will LISA measure?
  - ✦ Fractional frequency deviations (relative doppler shits) from 27 interferometers
  - ✦ Times series sampled at 4 Hz, observed over 4+ years with 89% duty cycle
  - ✦ Dominated by laser frequency noise
  - ✦ After pre-processing, obtain 3 time-delay interferometry (TDI) data streams (X, Y, Z)

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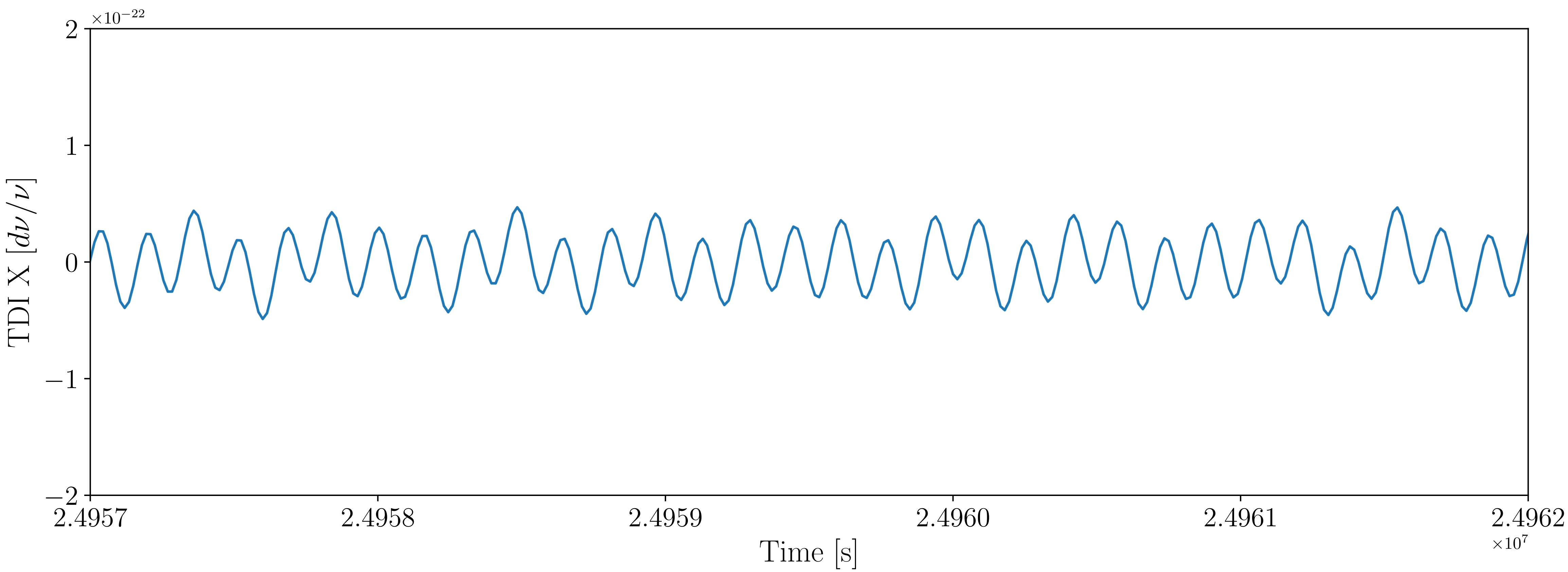

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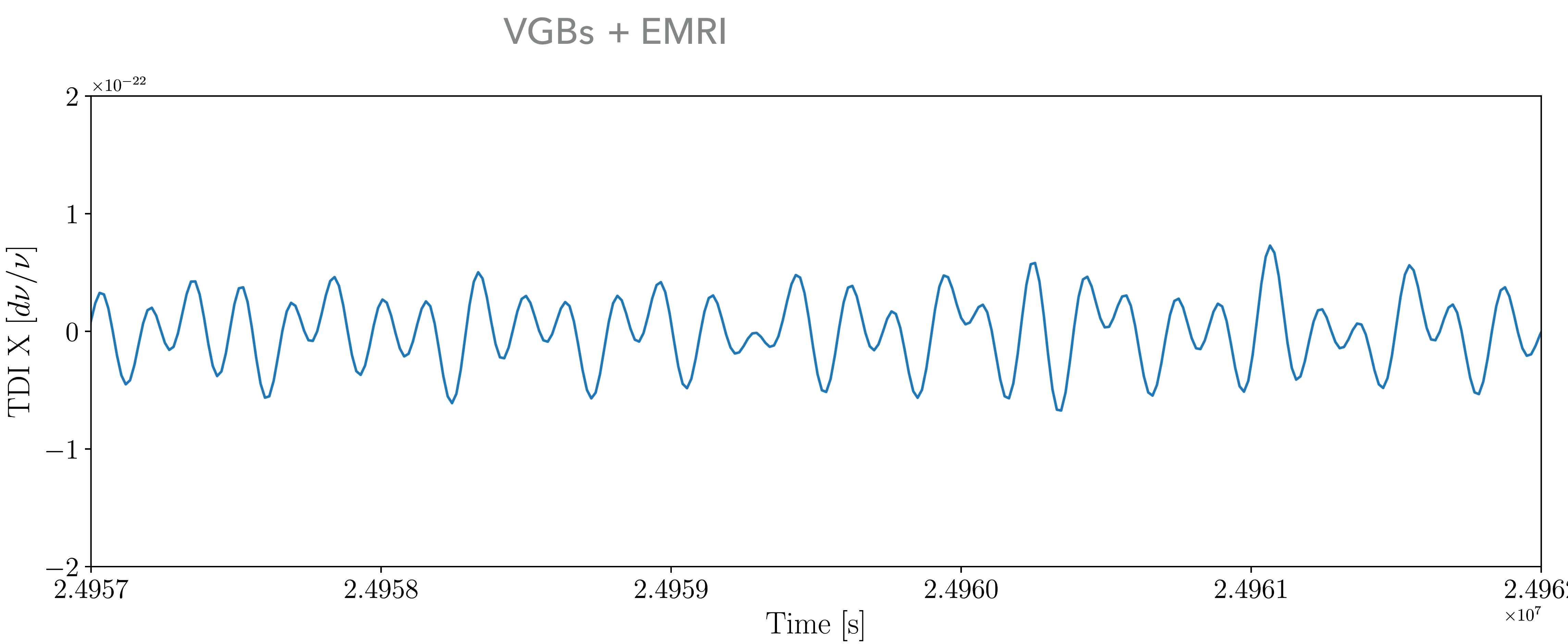

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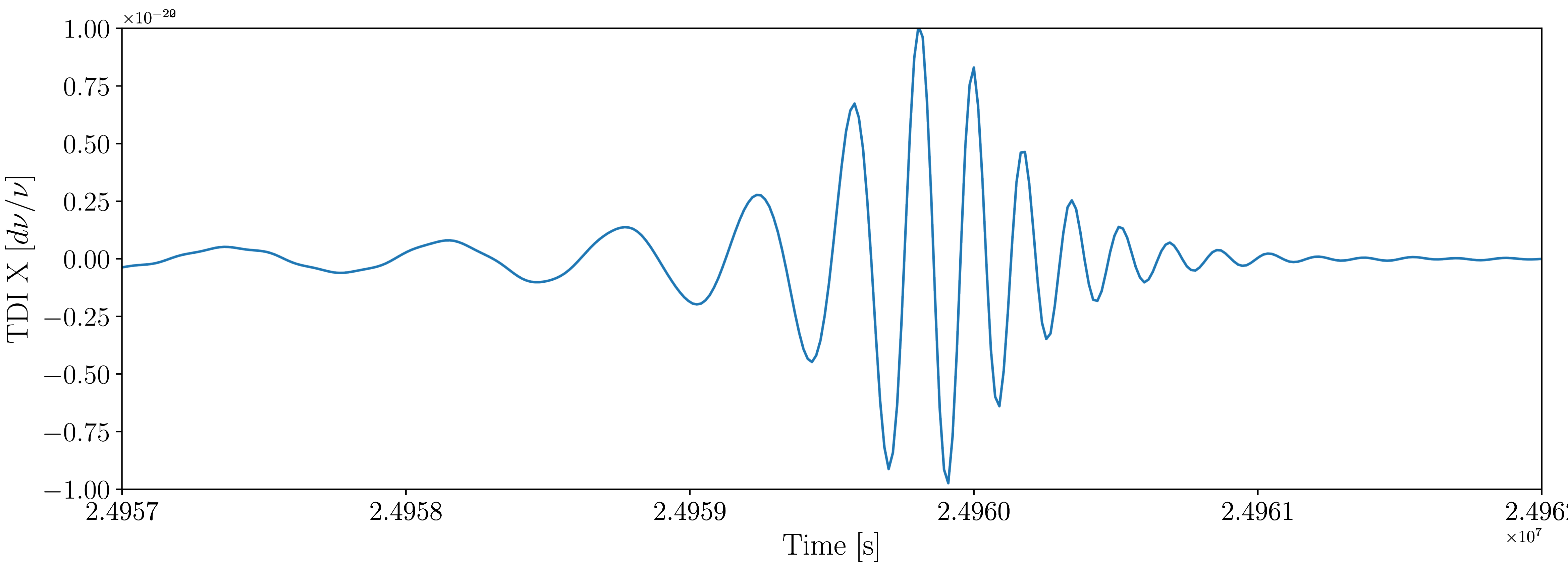

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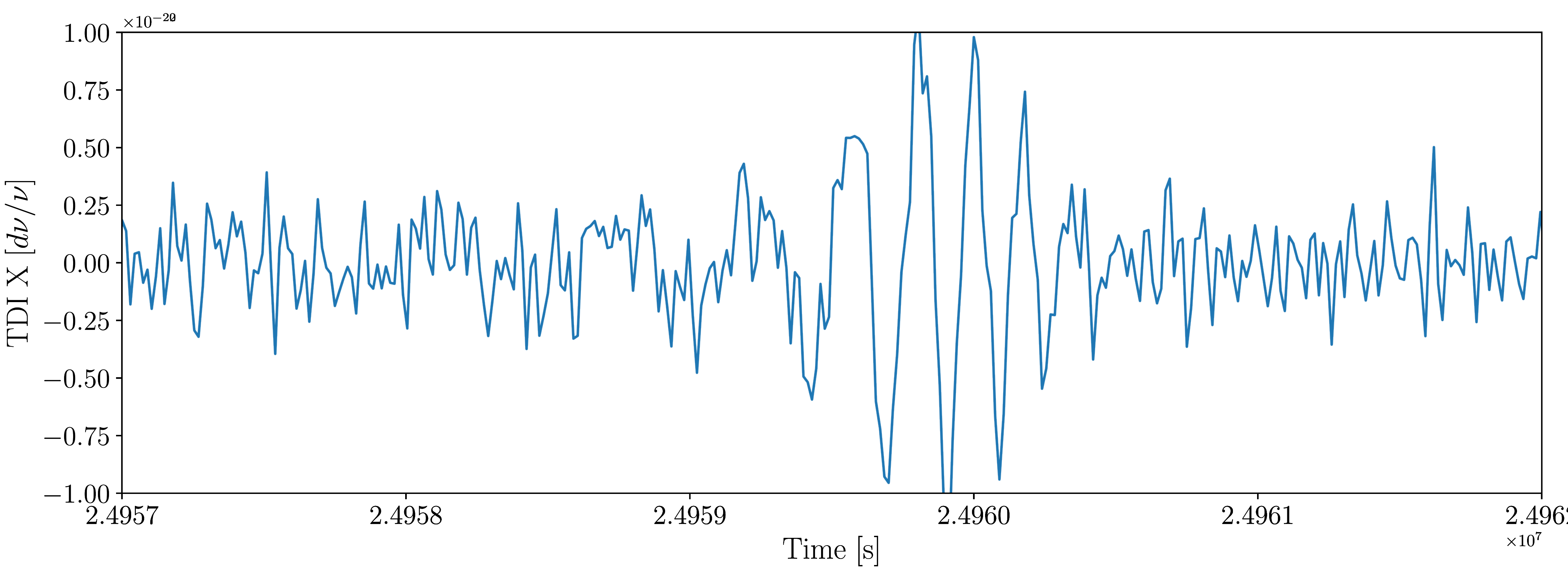

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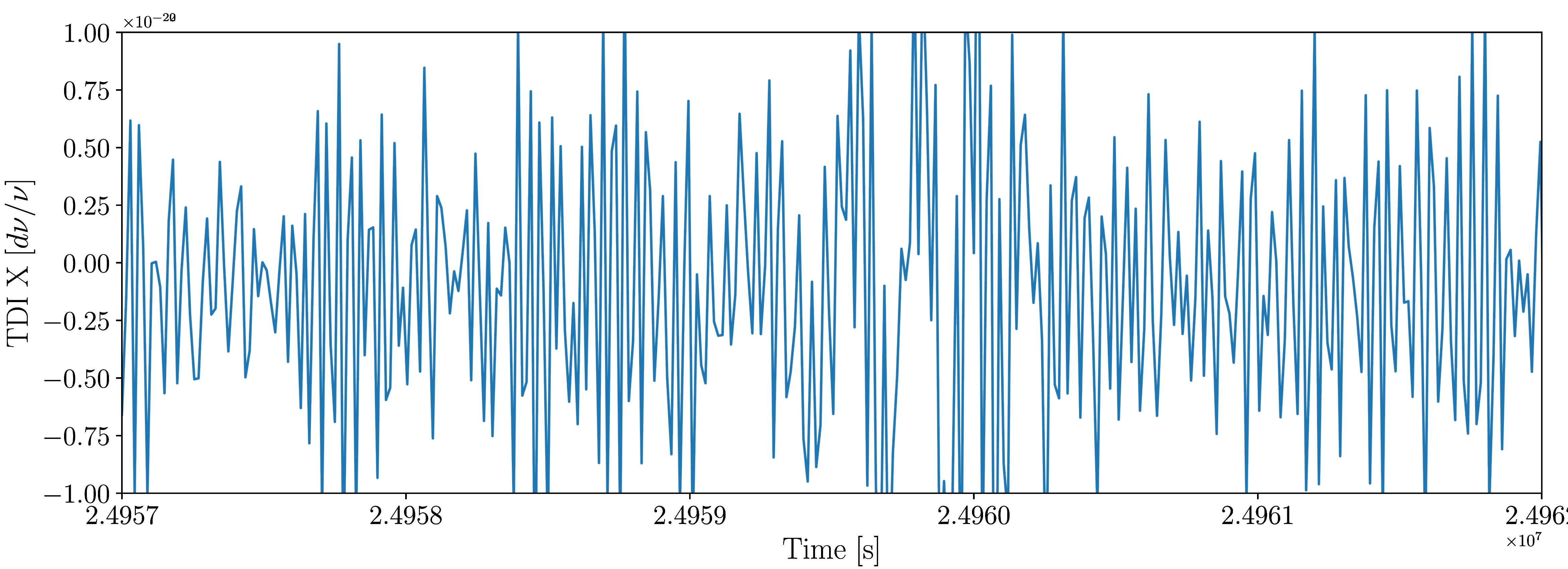

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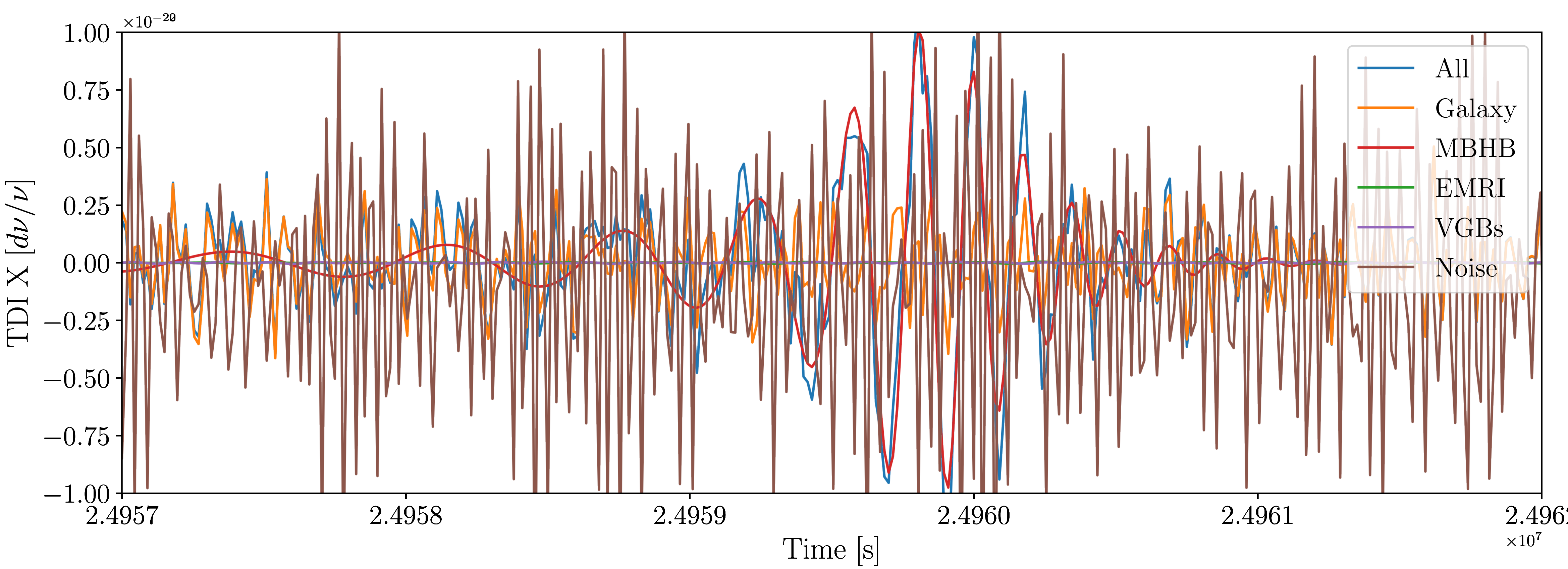

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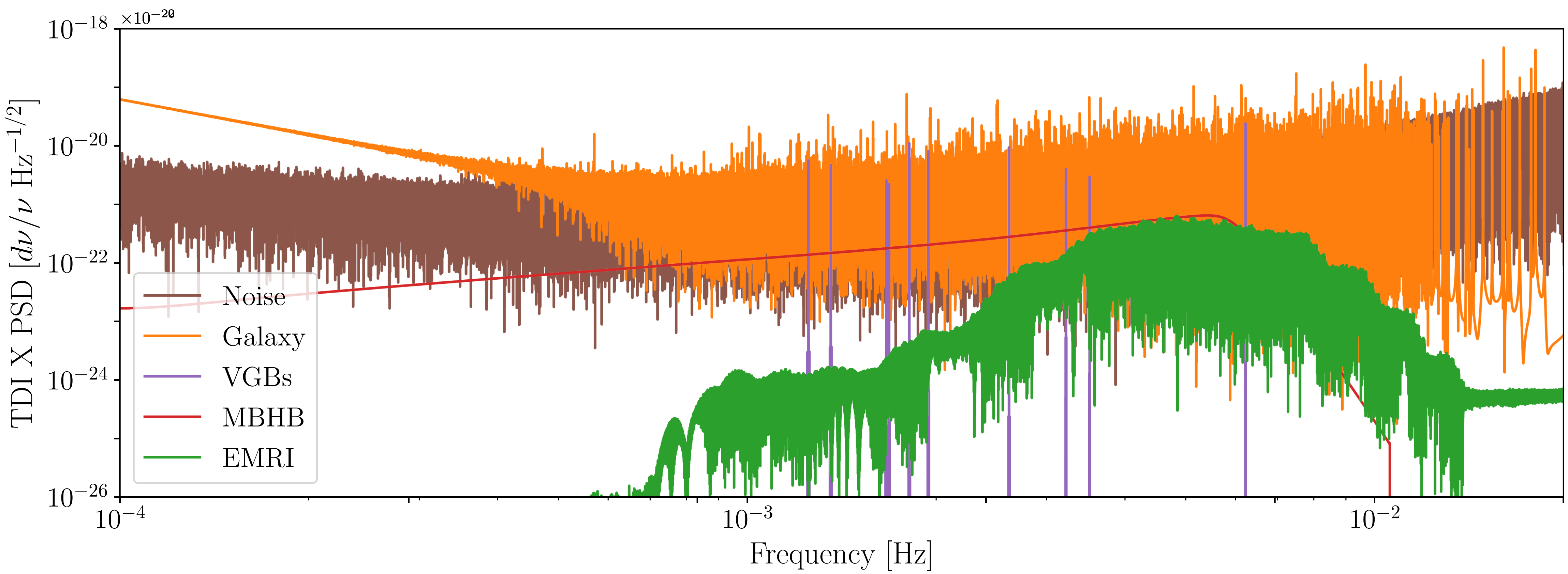

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  ✦ Bayesian framework: probe the parameters + number of model components posterior

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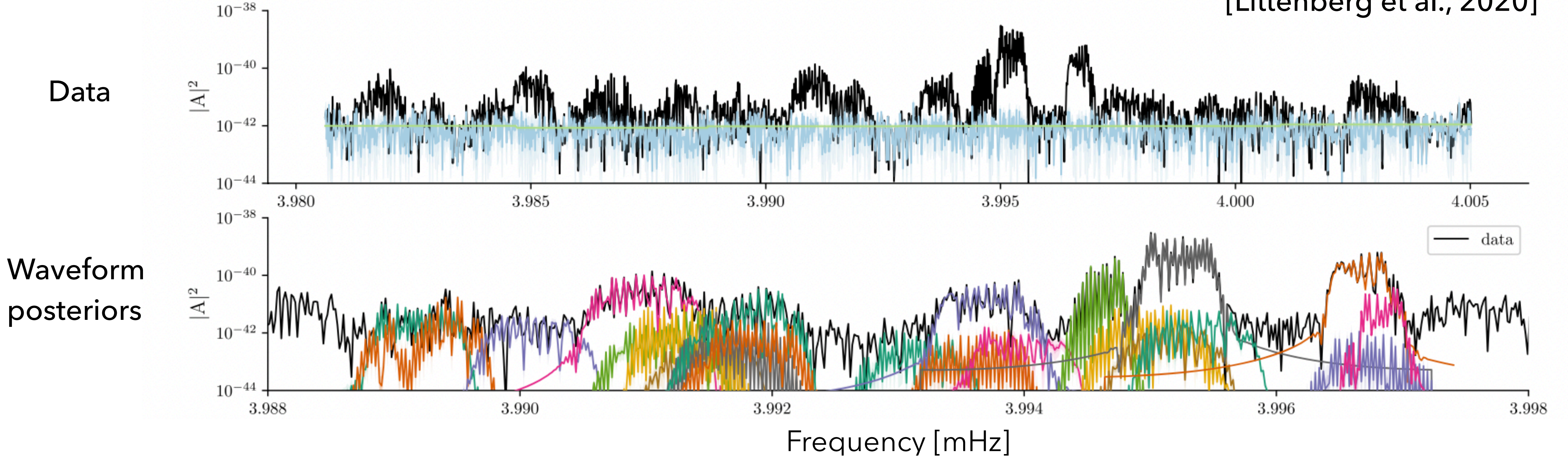
- The number of overlapping sources (especially Galactic binaries) is not know in advance
- Need to estimate the optimal number of sources

[Littenberg et al., 2020]



- Algorithm: reverse-jump Markov-chain Monte Carlo (RJMCMC)
- Allow for parallel computing by splitting the frequency-domain data into segments

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- Example: interrupted science data due to antenna repointing
- Consequence: both the signal and the covariance become expensive to compute
- One strategy is data augmentation [Baghi et al, 2019]

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BAYESIAN CONCEPTS: DATA AUGMENTATION

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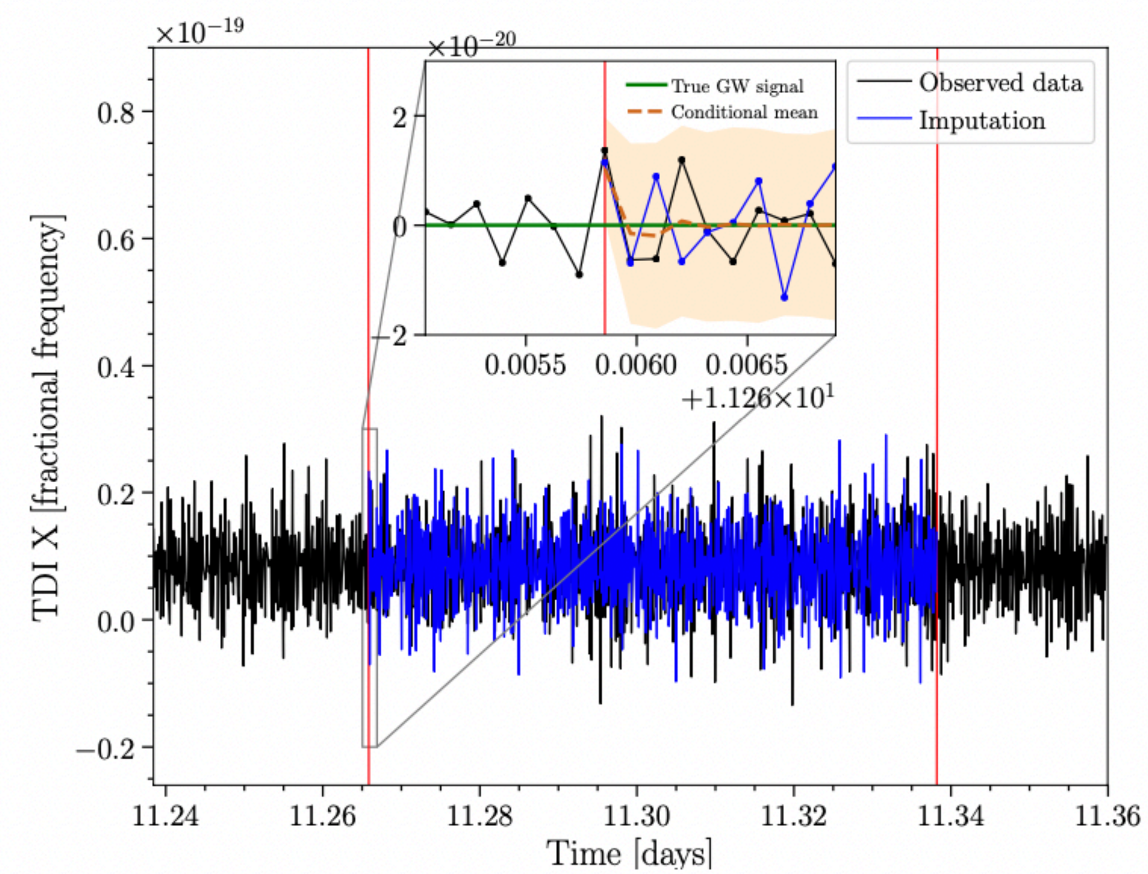

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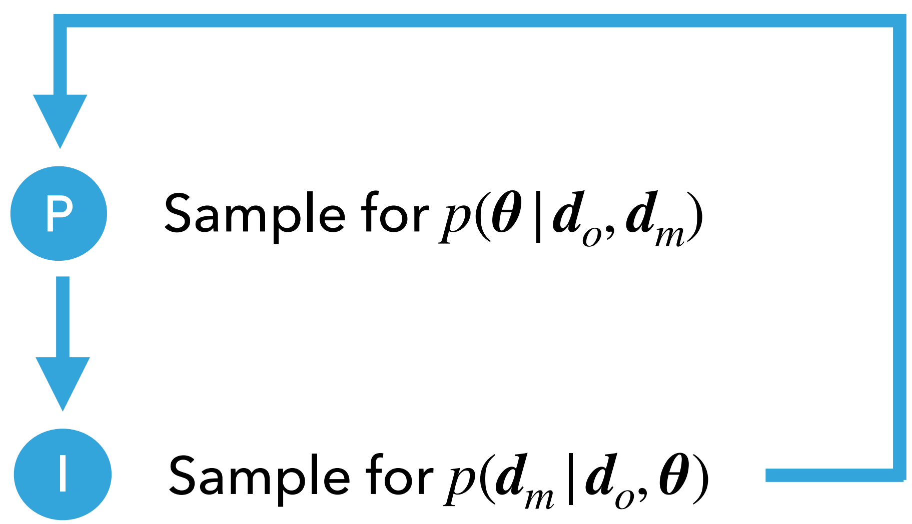

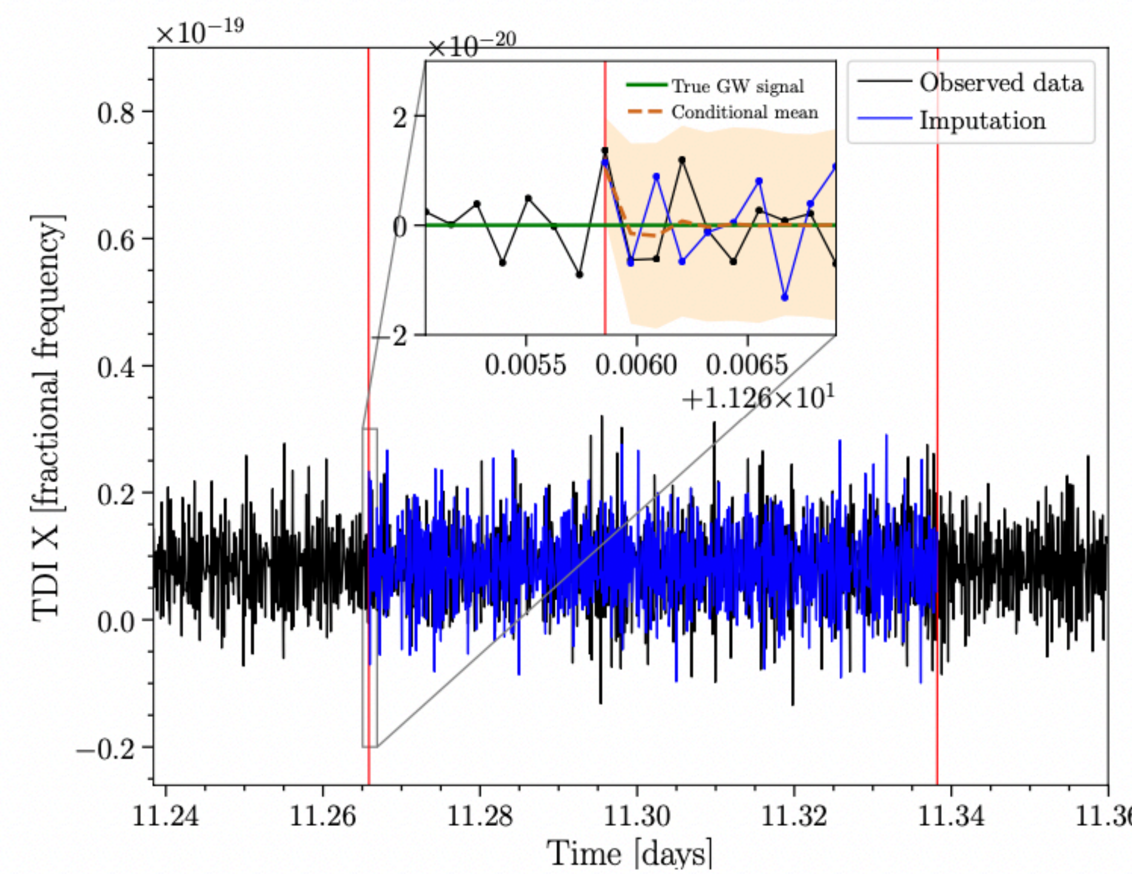

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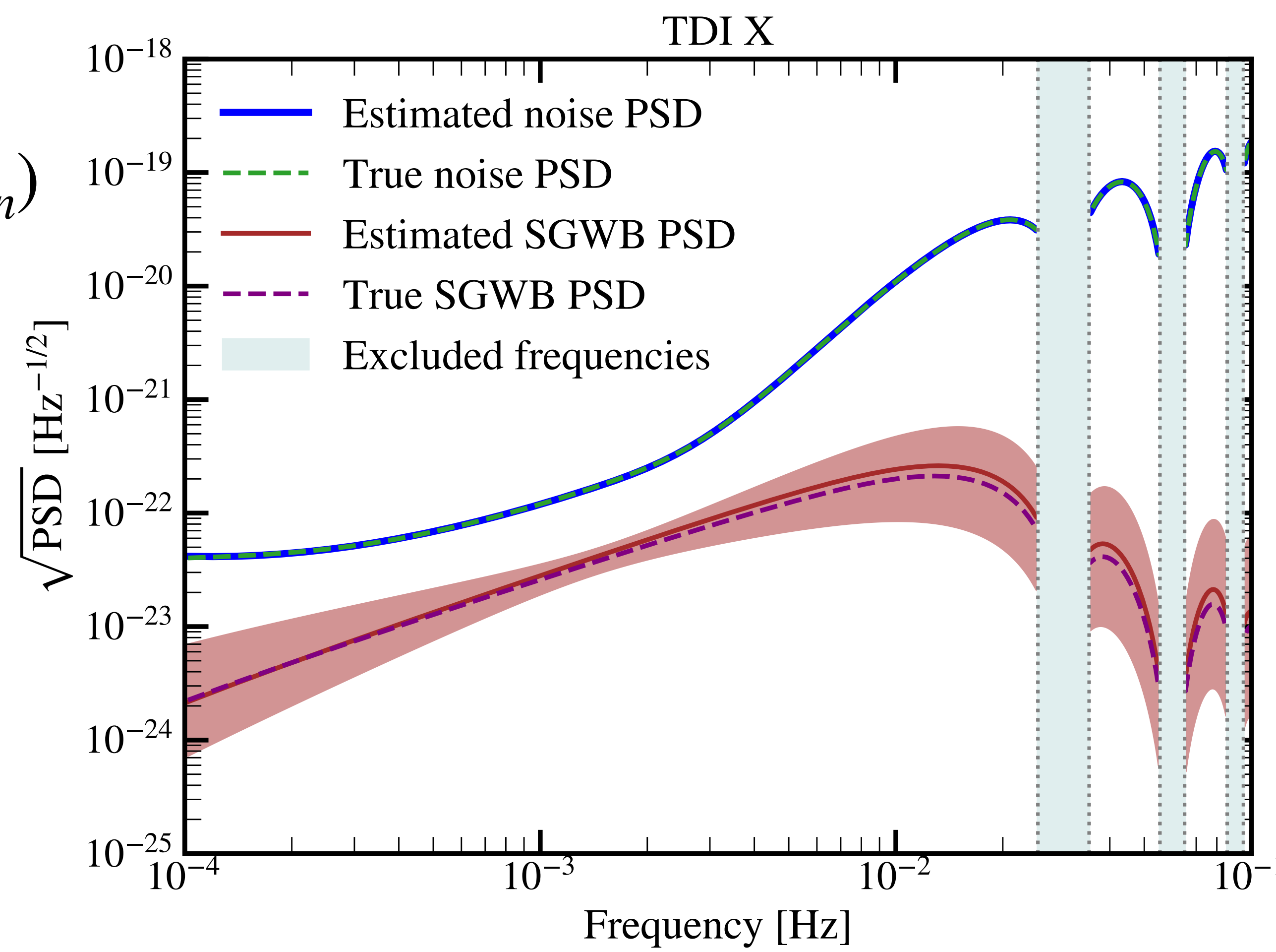

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Thank you for your attention !

BACKUP SLIDES

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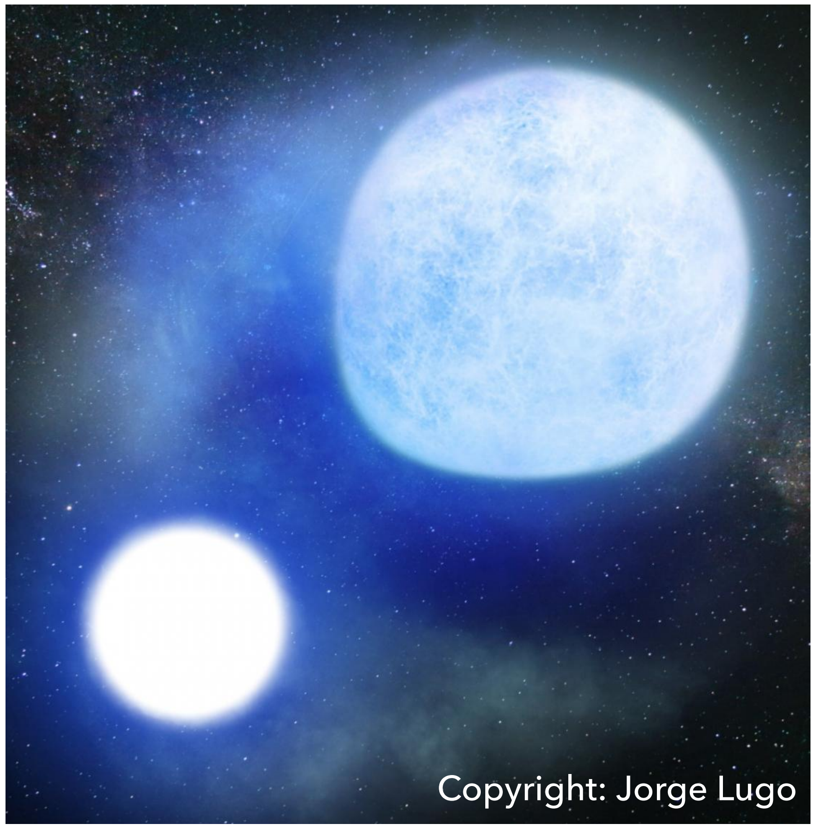

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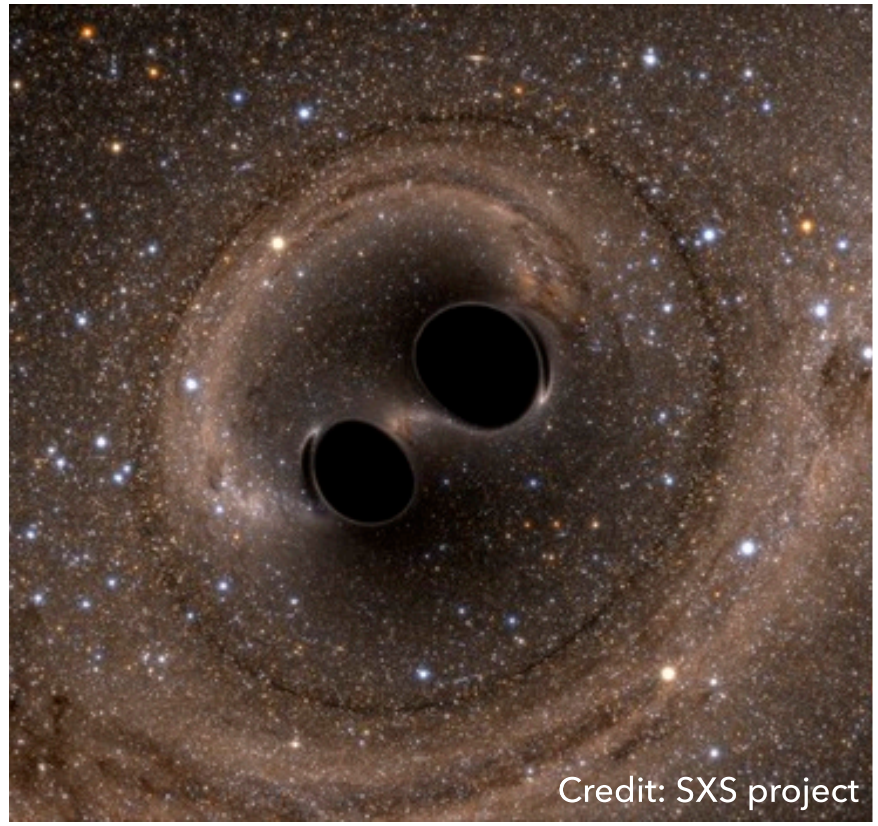

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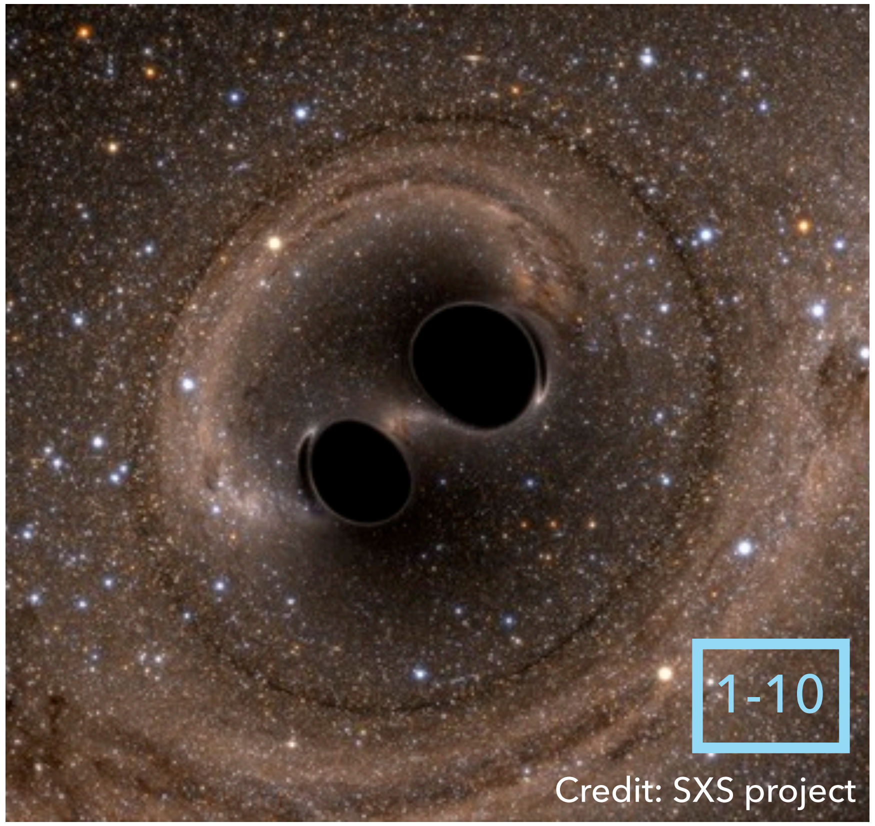

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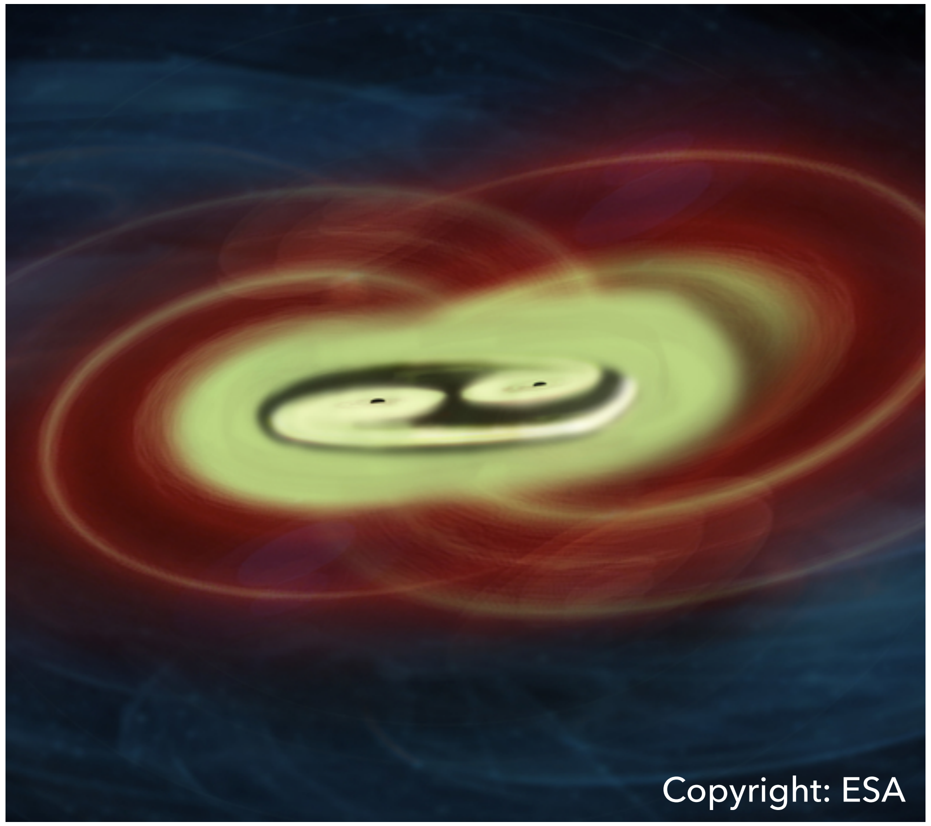

Possibly farther away: merging supermassive black holes

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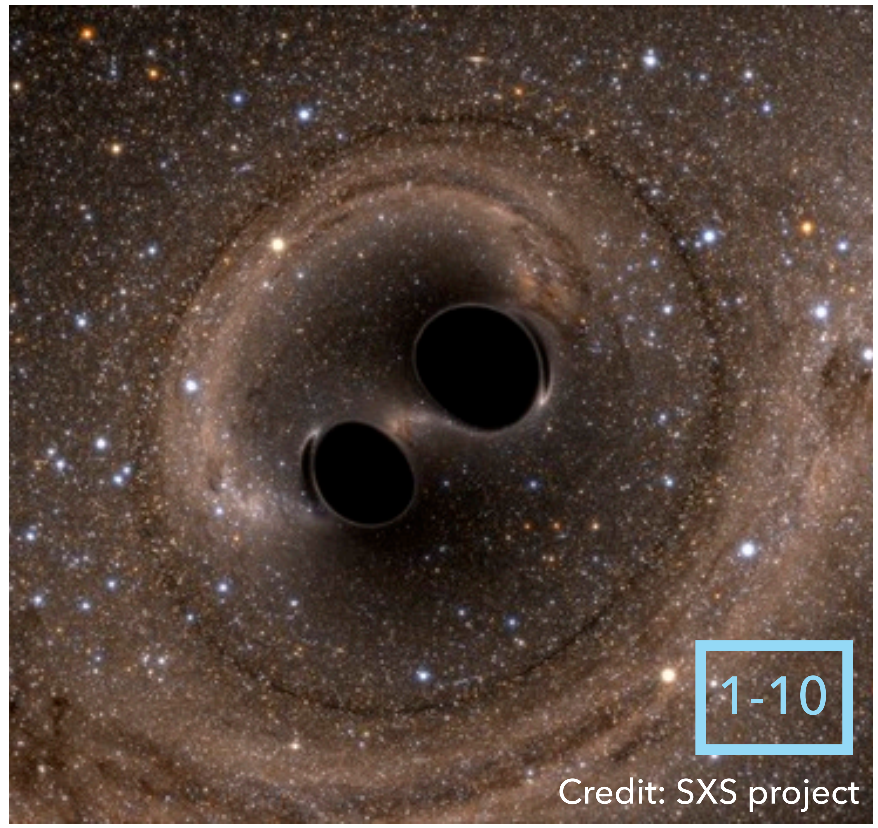

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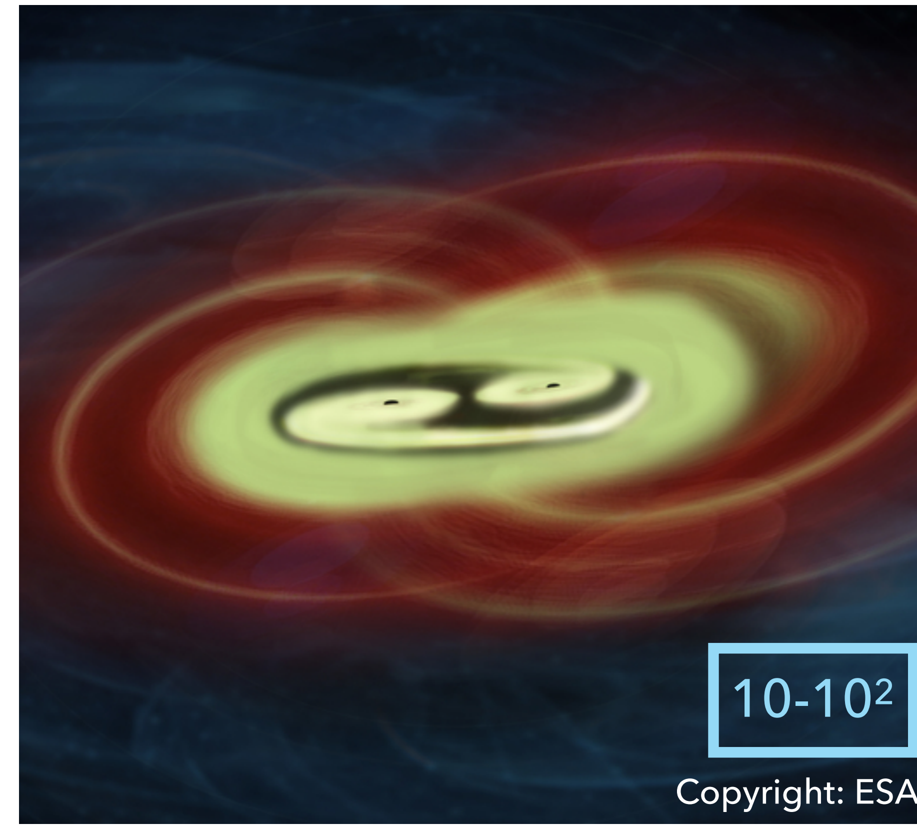

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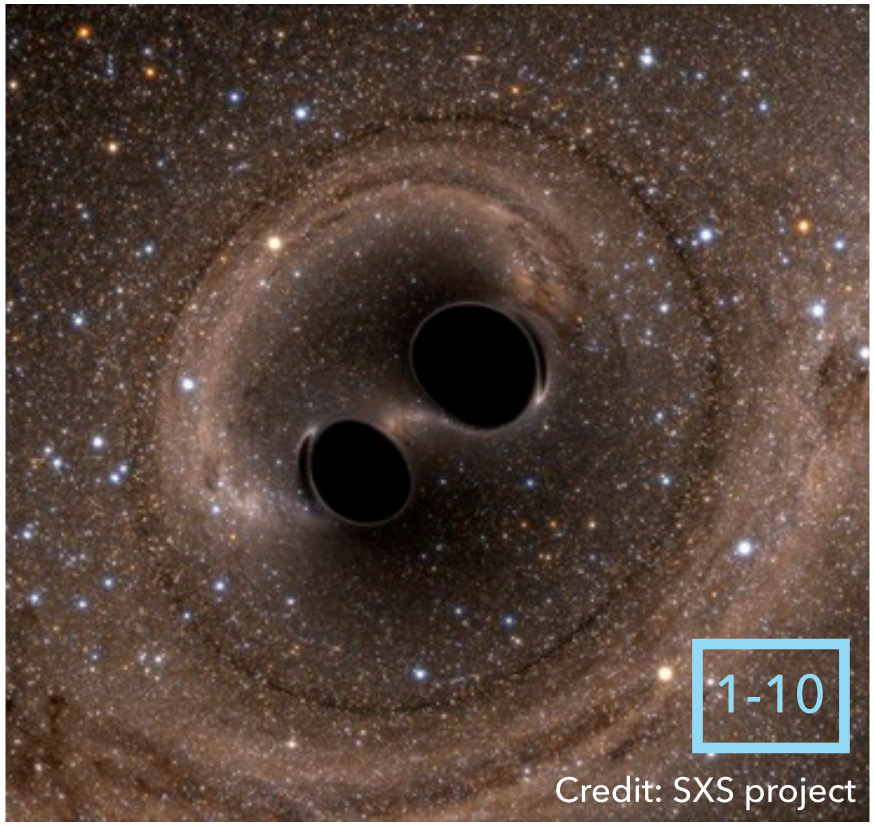

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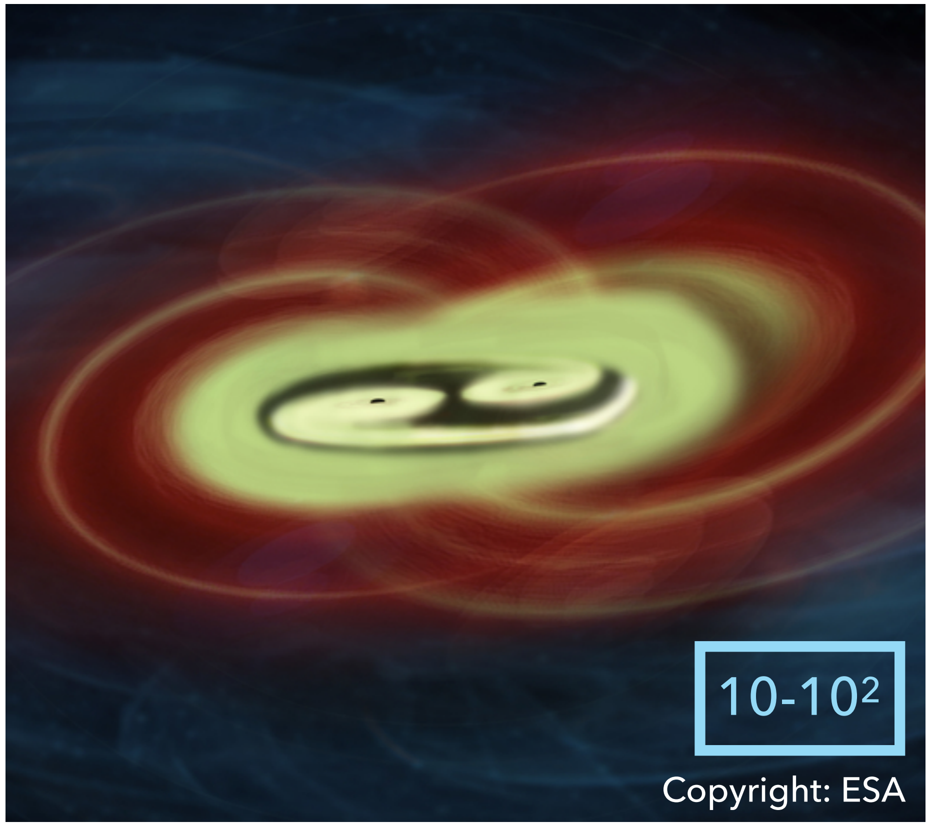

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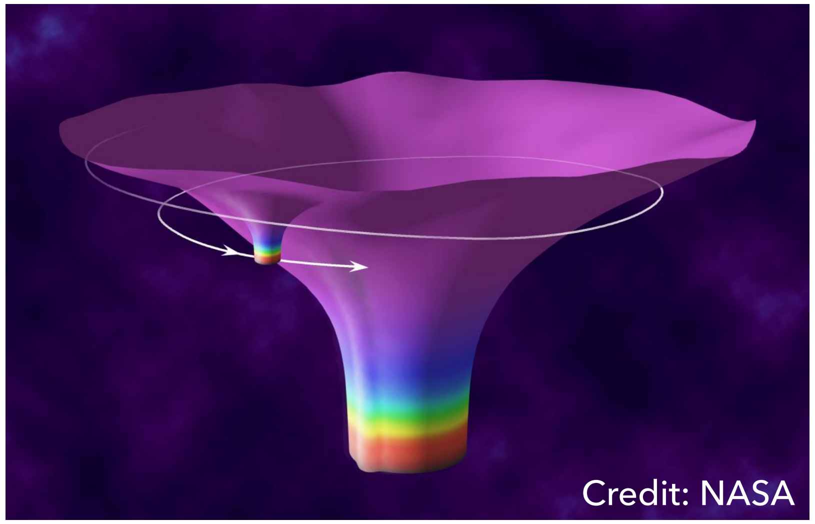

◀ Extreme-mass-ratio inspirals: a smaller compact object orbiting a supermassive black hole

What makes gravitational noise in the milihertz band?

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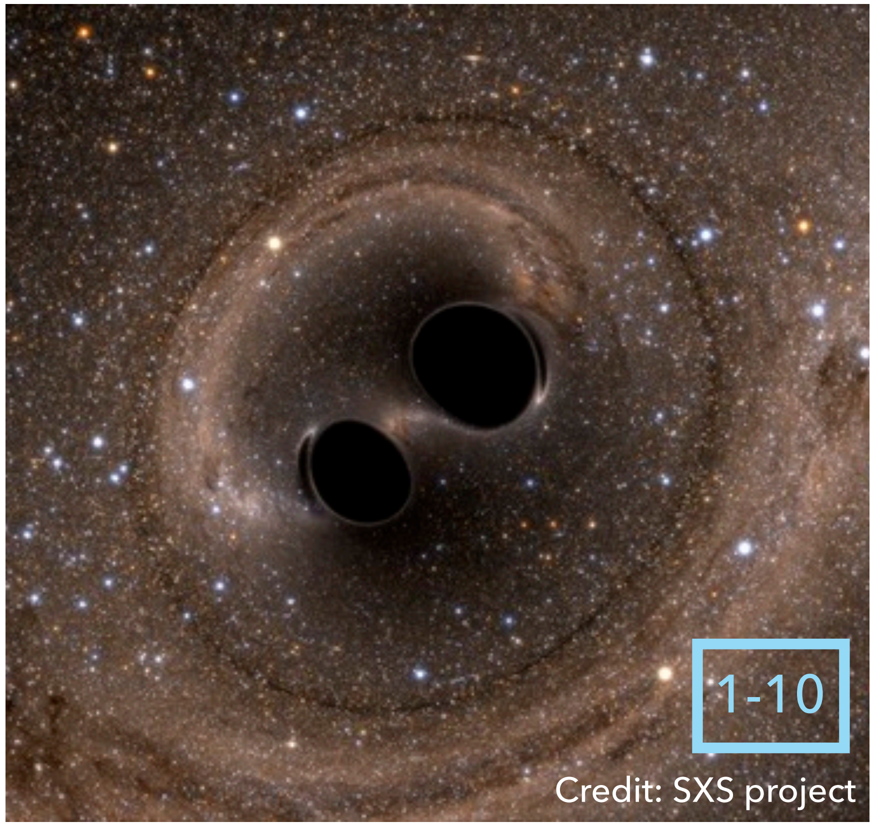
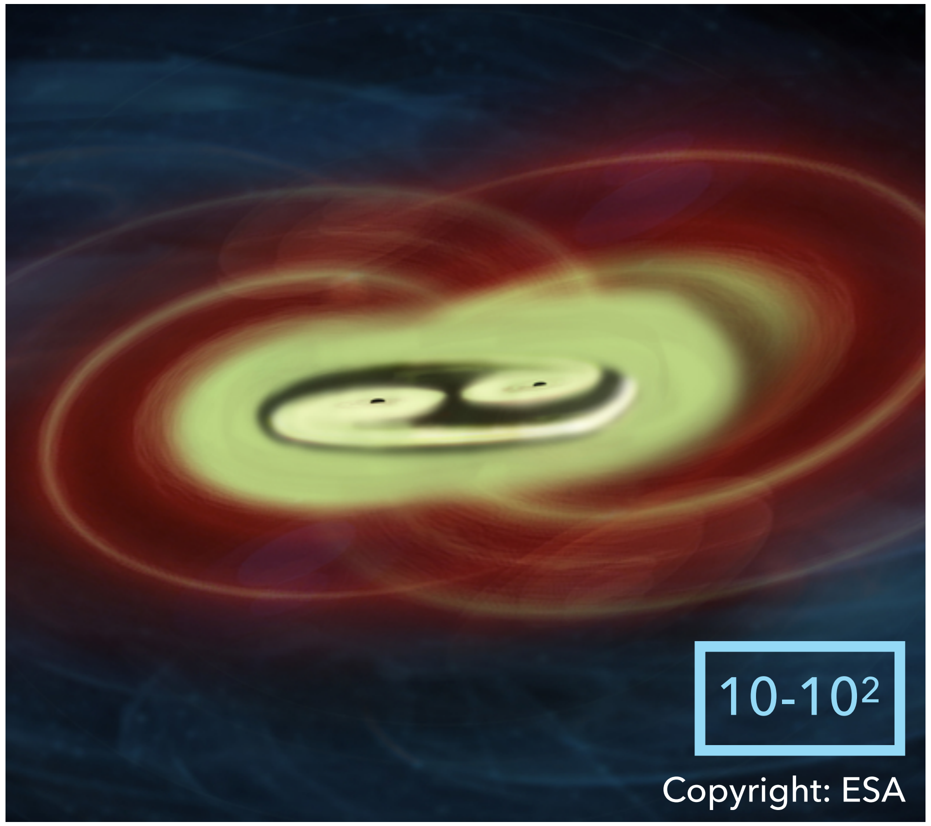
In our galaxy: pairs of orbiting white dwarfs



Credit: SXS project

One billion light-years away: collision of black holes



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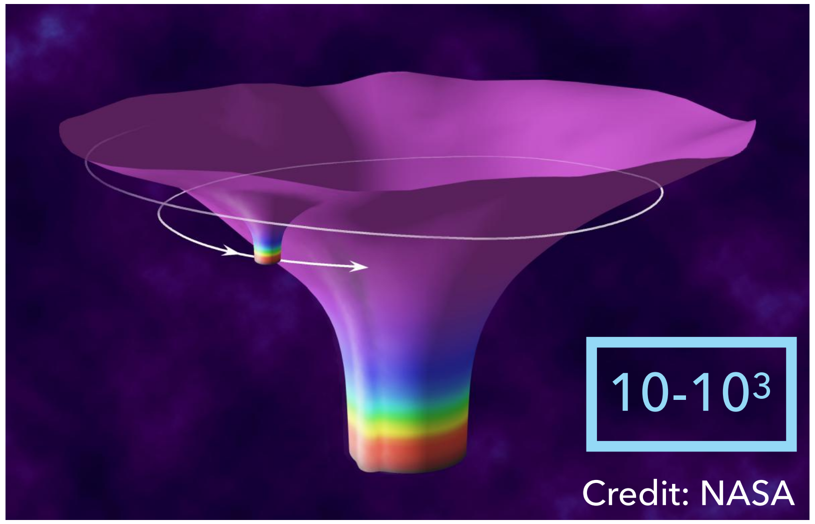
Possibly farther away: merging supermassive black holes



Credit: NASA

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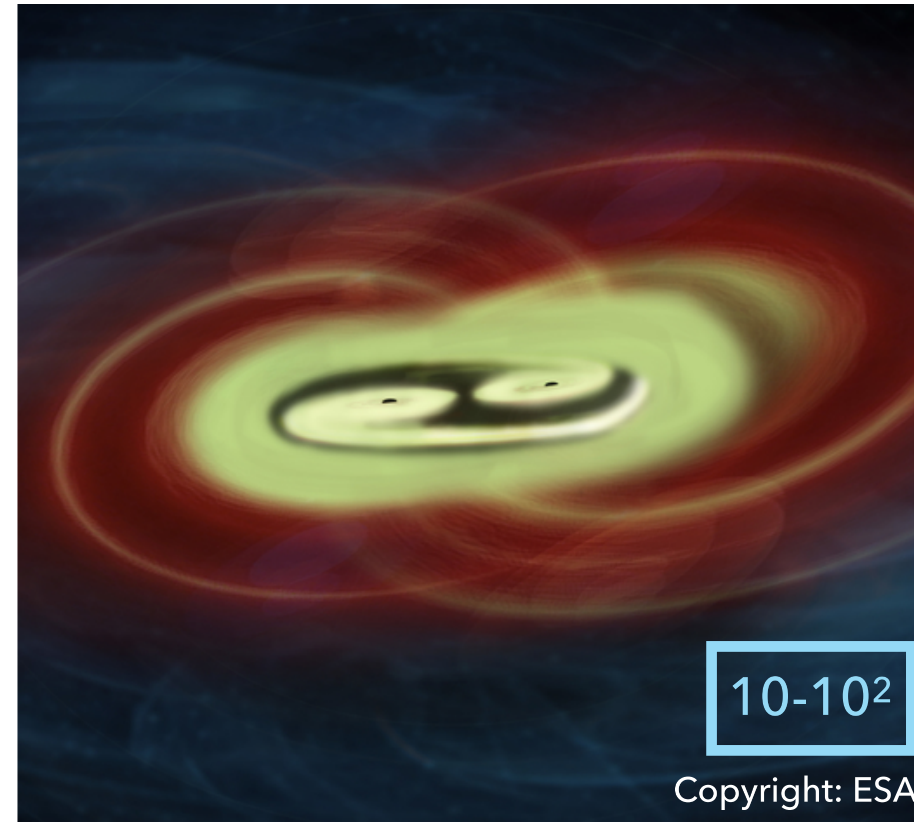

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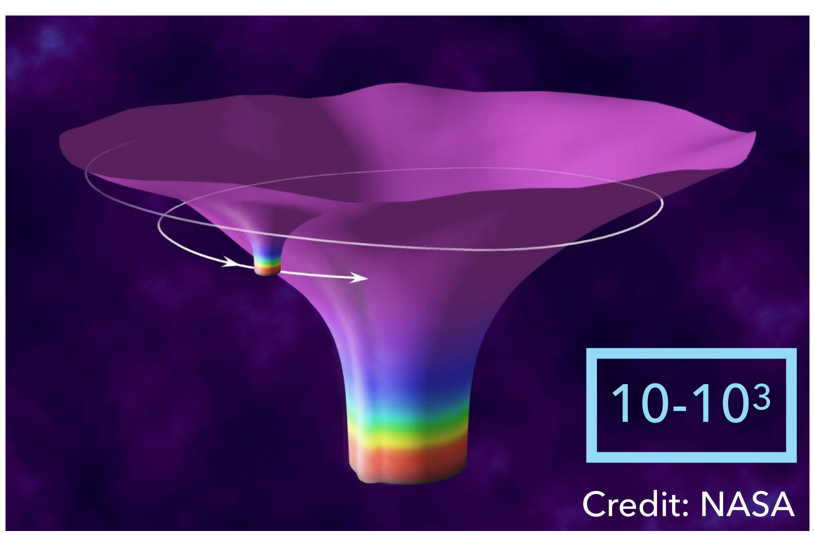
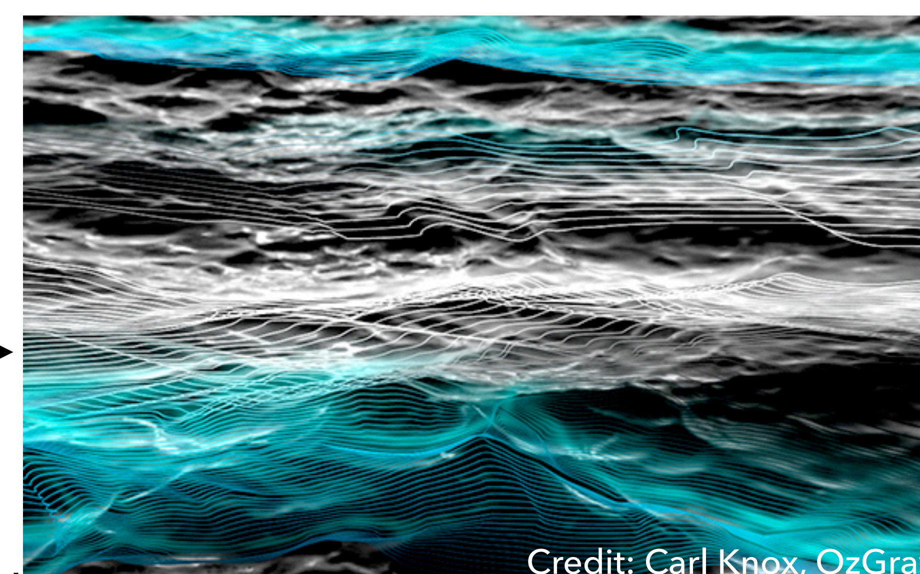
Possibly farther away: merging supermassive black holes



Credit: NASA

◀ Extreme-mass-ratio inspirals: a smaller compact object orbiting a supermassive black hole

In the entire universe: a cosmic gravitational wave background? ▶


Credit: Carl Knox, OzGrav