Subadditive average distance and quantum promptness

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With A. Tolley, to appear.
see also 2108.12362

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In this talk:

$$\text{Quantum gravity} \neq \text{UV}$$

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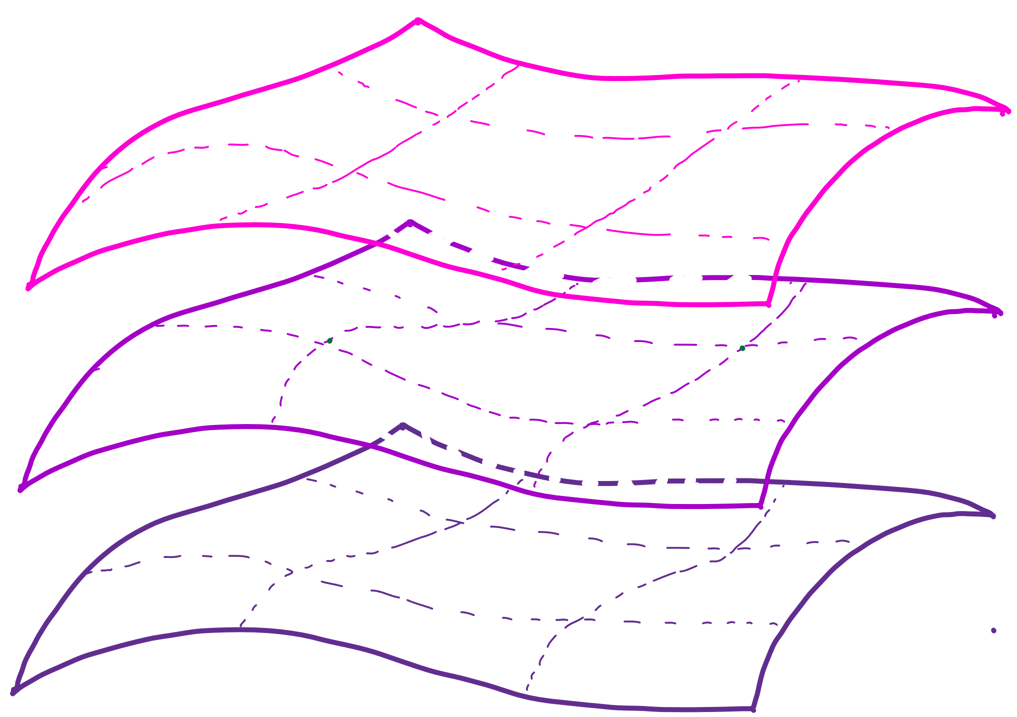
Quantum gravity $\neq$ UV

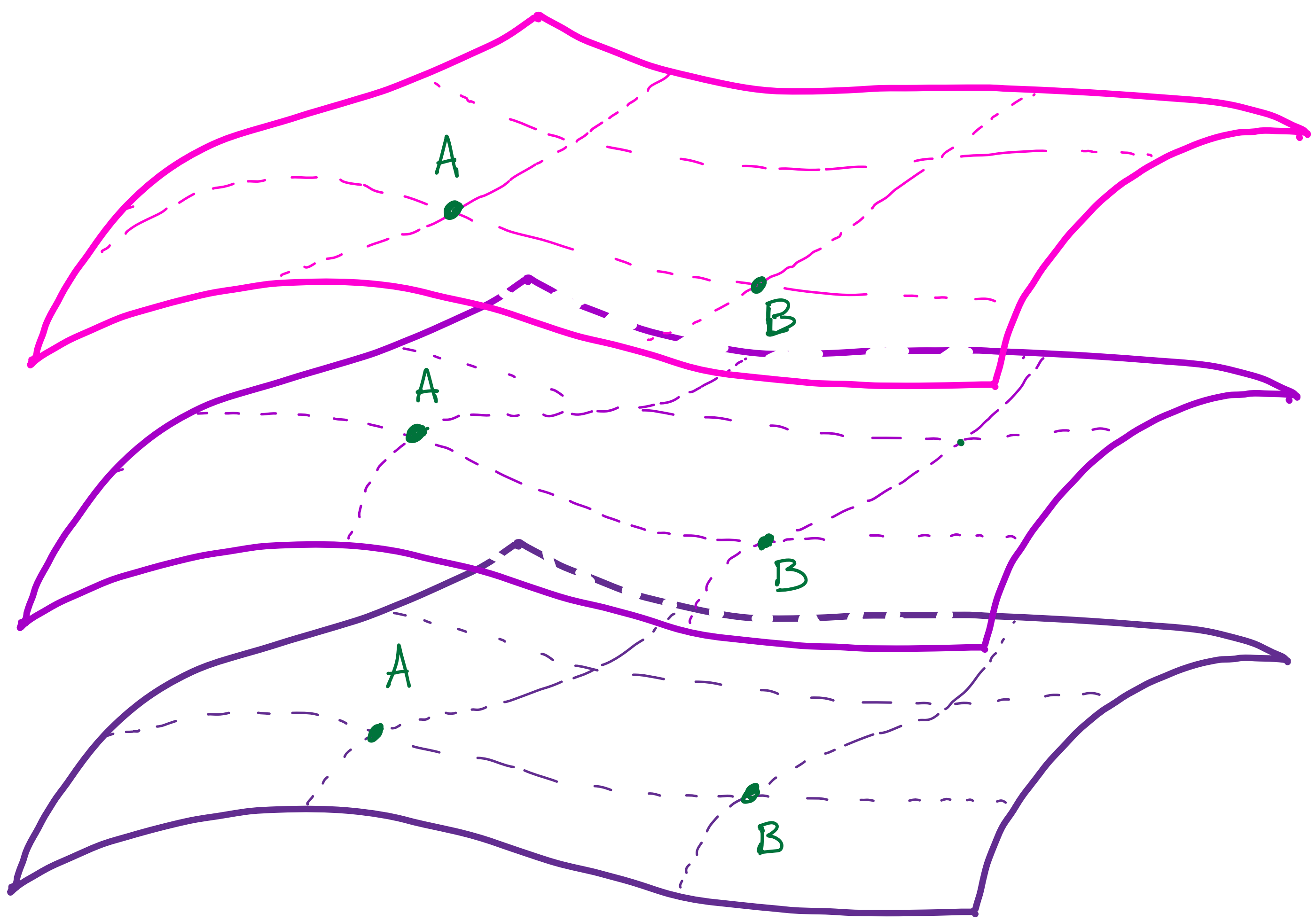
Simply, I want to consider superpositions of metrics

$$\Psi[h_{ij}(x)]$$



Local operators in quantum gravity

One difficulty (``gauge invariance”) is to find a prescription for identifying ``which point corresponds to which”



E.g. calculate $\langle R(A) \rangle$

Use Macroscopic Observers!

Example: say that Minkowski space is traversed at t=z by a gravitational wave of some polarization

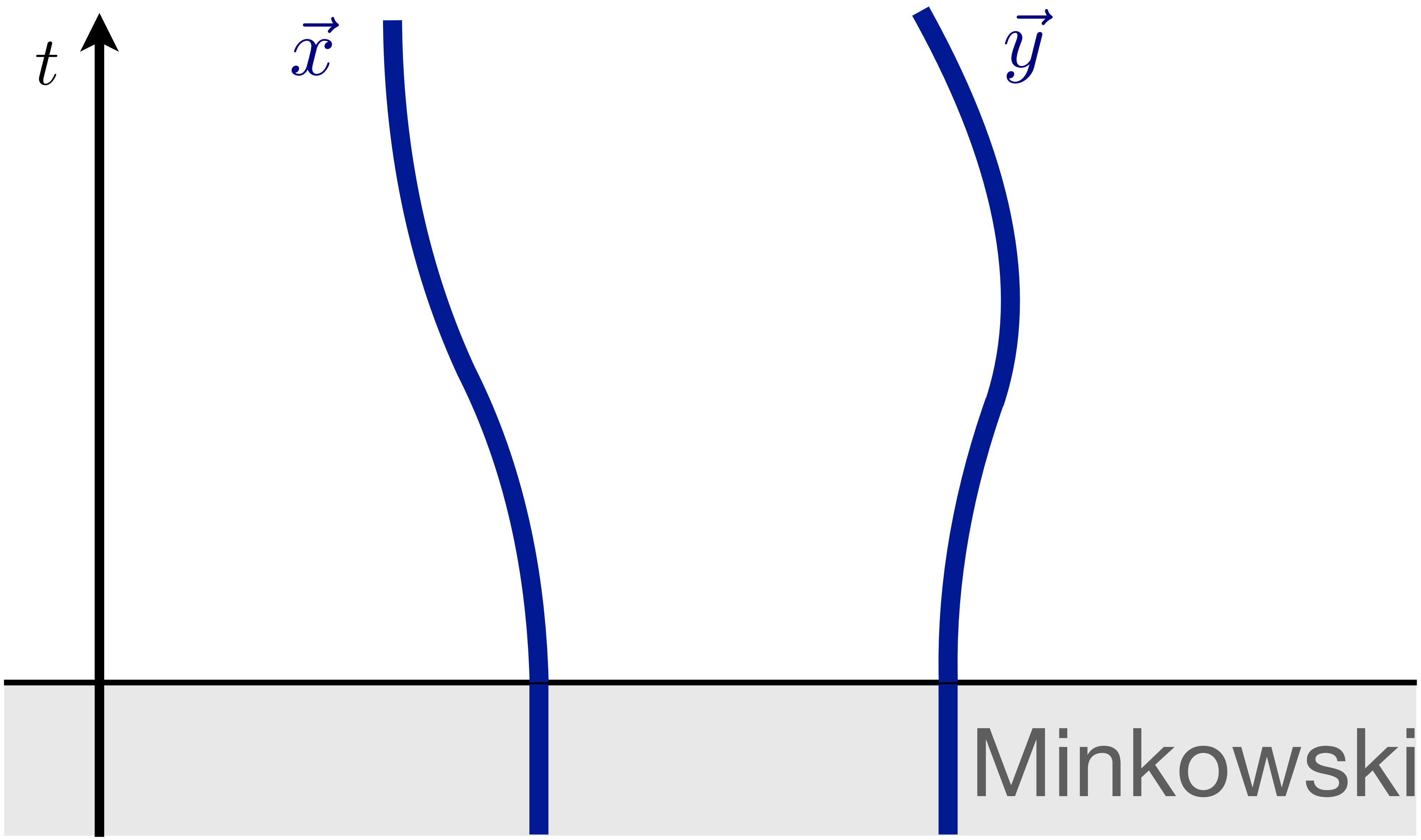
Classical solution $\simeq$ coherent state $|\psi_1\rangle$

Consider now a wave with e.g. different polarization $|\psi_2\rangle$
Etc.

Let's make sense of $|\psi\rangle = c_1|\psi_1\rangle + c_2|\psi_2\rangle + \dots$!!

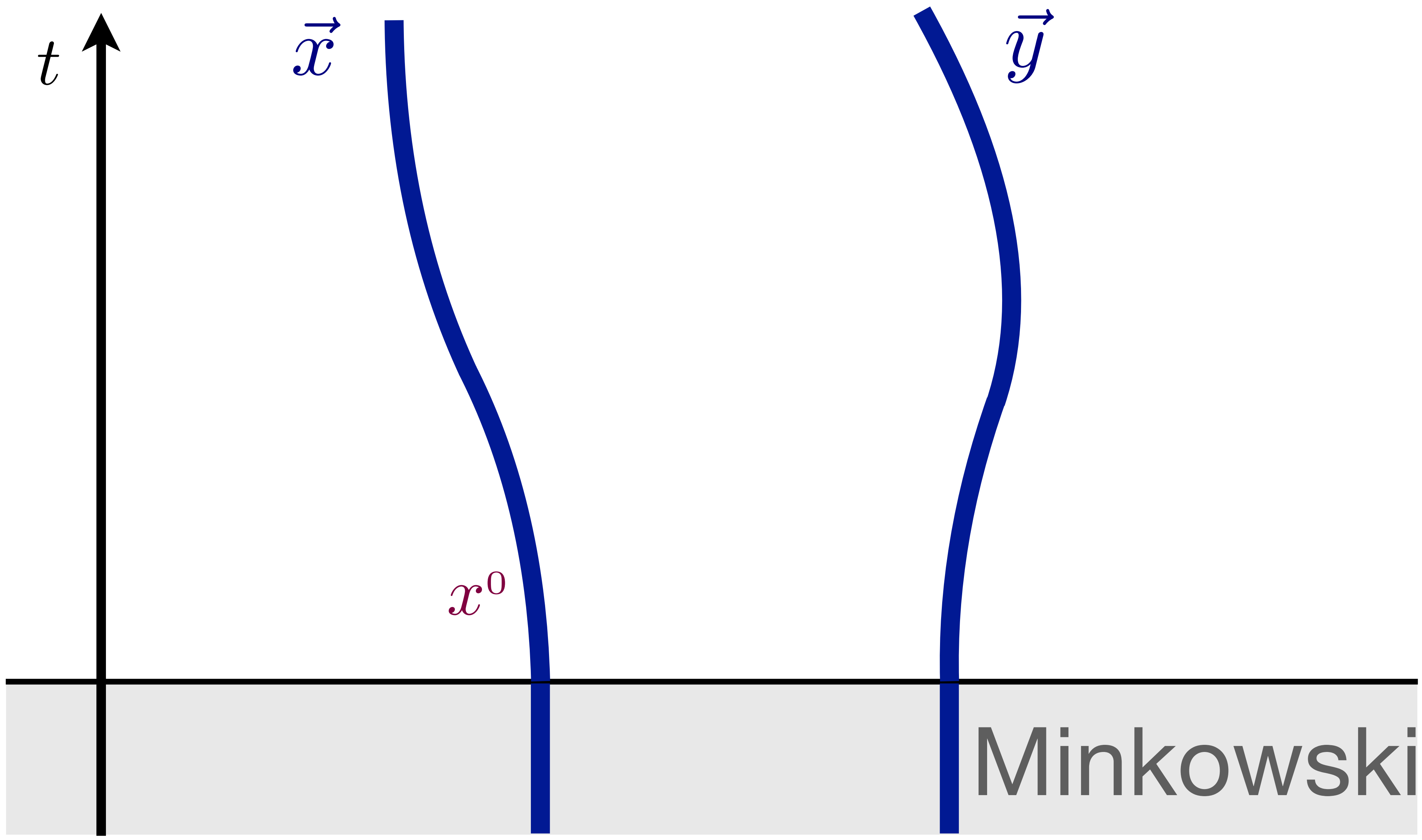
Use Macroscopic Observers

- Fill Minkowski initially with $\overrightarrow{x}$ = const. observers
- Define an event in $|\psi\rangle$ with $\overrightarrow{x}$ and proper time x^0



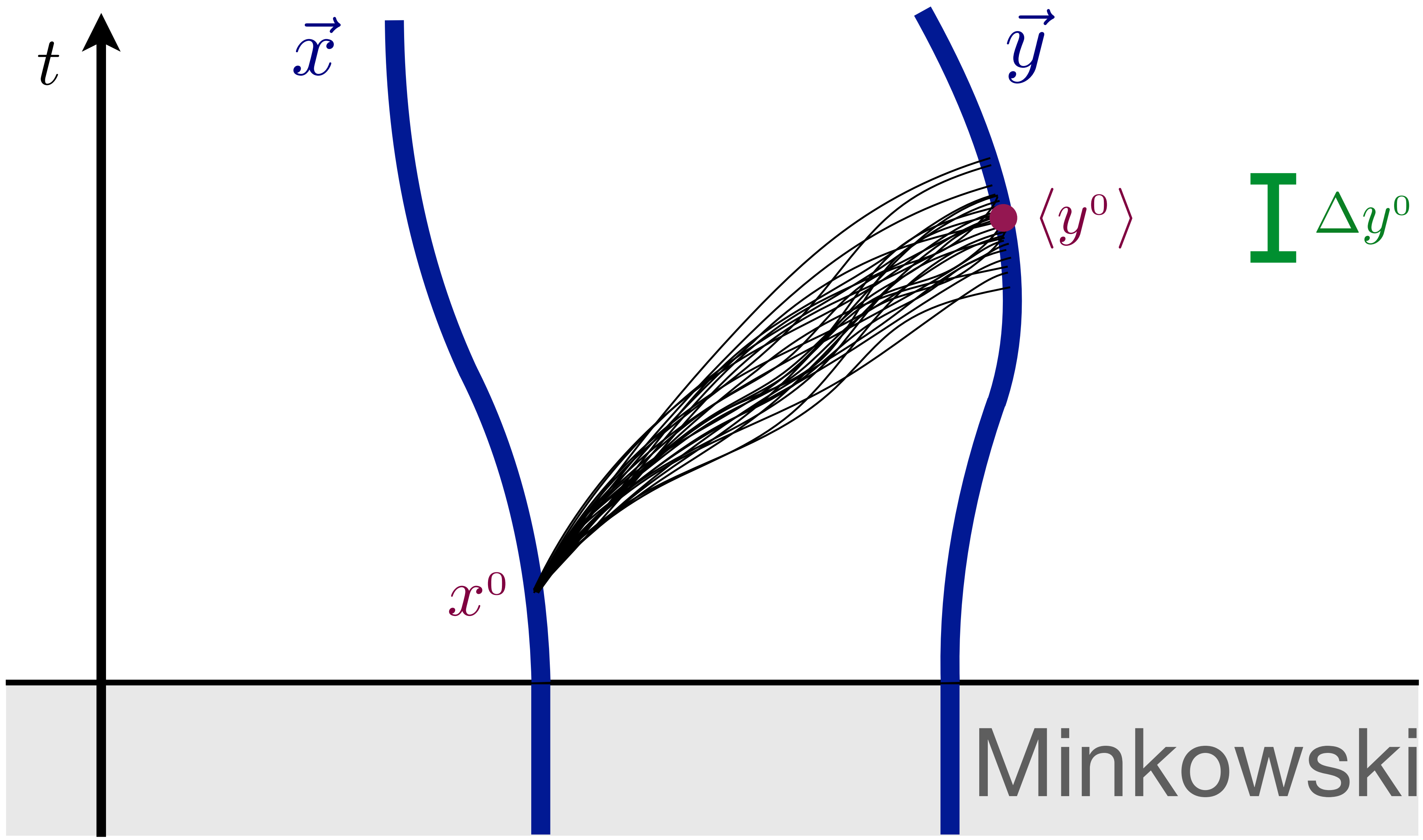
Gedanken experiments!

- $\overrightarrow{x}$ sends a photon at time x^0
- What's the probability that $\overrightarrow{y}$ detects it at time y^0?



Gedanken experiments!

Geometrical optics approx: photons follow geodesics
But here we have a "superposition of geodesics"



A proxy for causality experiments/observables

$$\bar{d}(x, y) \equiv \sqrt{\langle d^2(x, y) \rangle}$$

"physical coordinates"

The idea is that $\langle y^0 \rangle$ is well approximated by $\bar{d}(x, y) = 0$

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i.e. it is not *additive*

Additivity and lack thereof

Basic idea: geodesic distances are integrals

Third point problem (Euclidean signature):

given $d(x, z)$ and $0 < R < d(x, z)$: Find a third point y s.t.

$$d(x, y) = R, \qquad d(y, z) = d(x, z) - R$$

z

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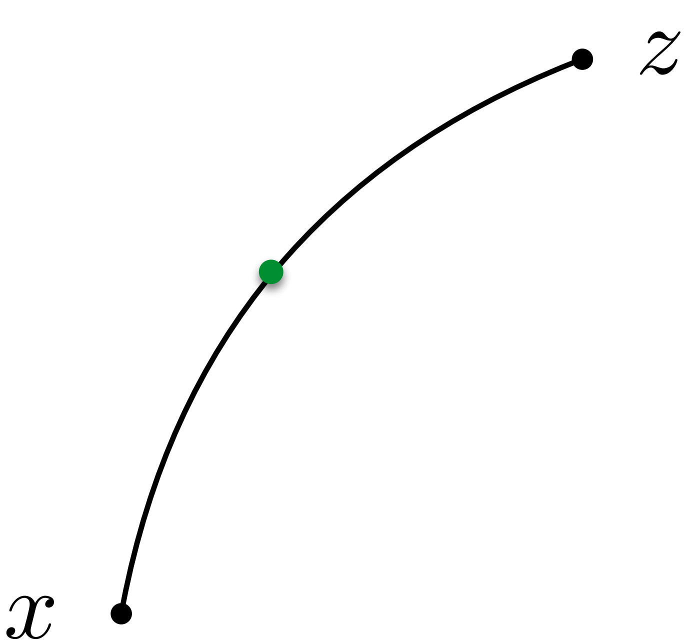
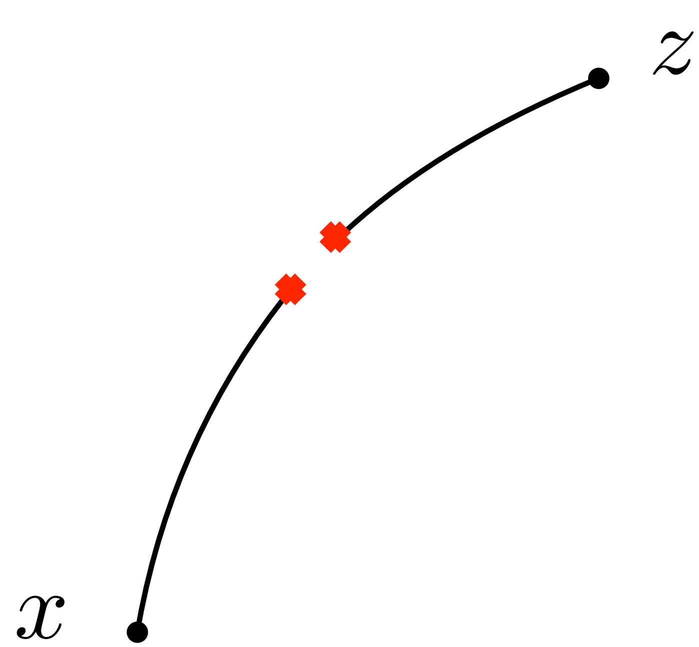
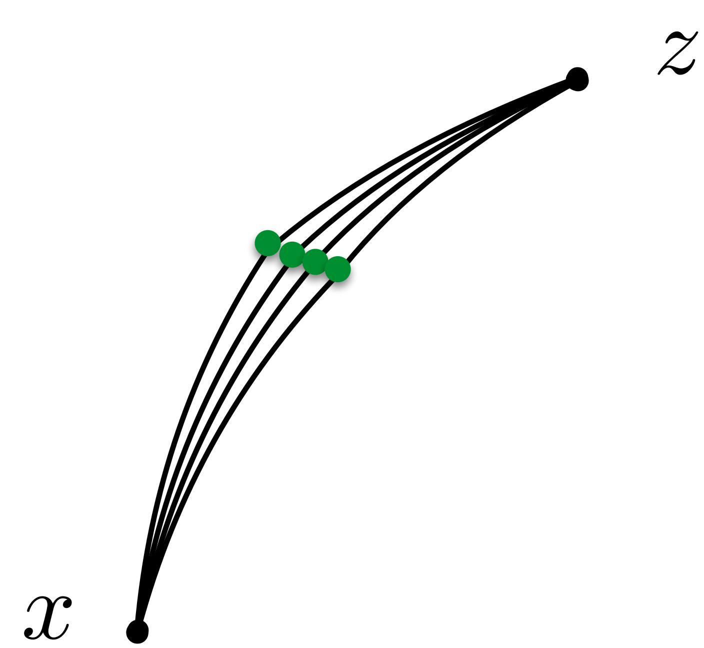


Additive:
only one solution



Subadditive:
no solution



Superadditive:
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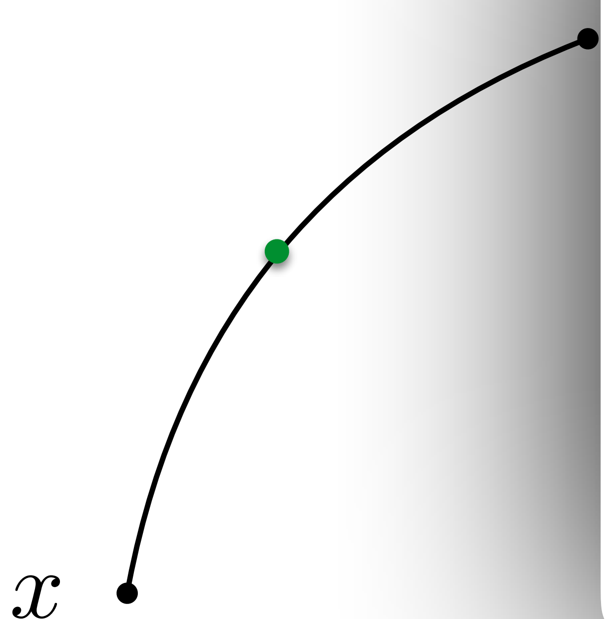
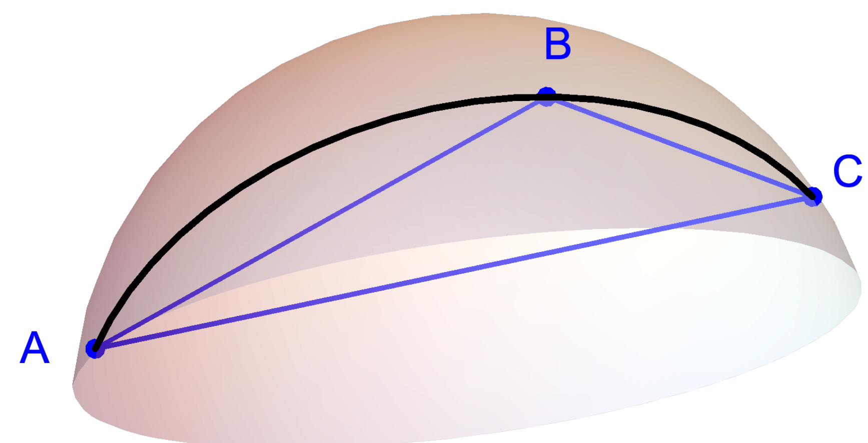
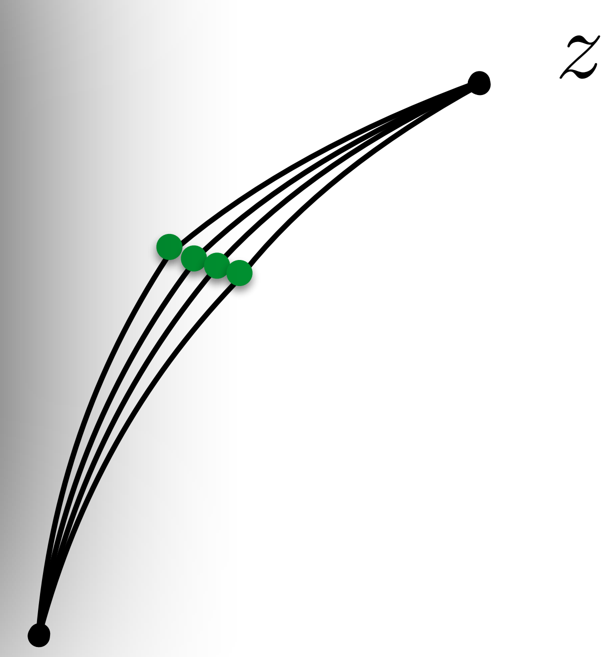
Similar to chordal distances







Additive:
only one solution

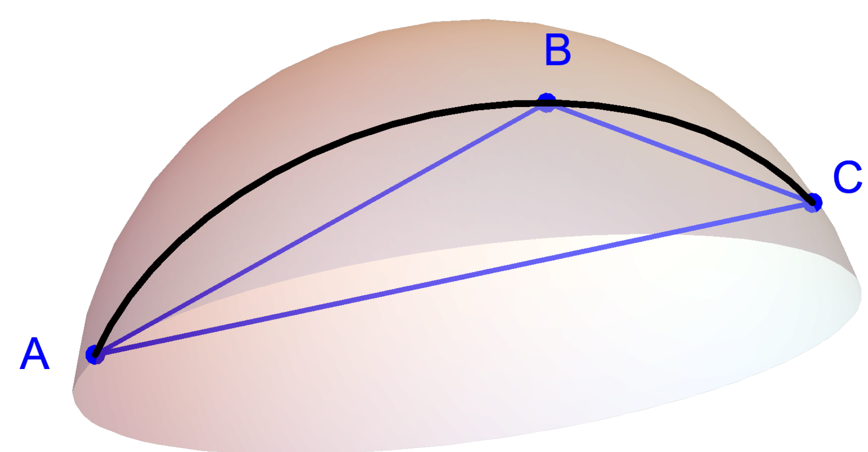
Subadditive:
~~no solution~~

Superadditive:
infinite solutions

Result in Euclidean signature:

Average distances always *subadditive*

Similar to chordal distances



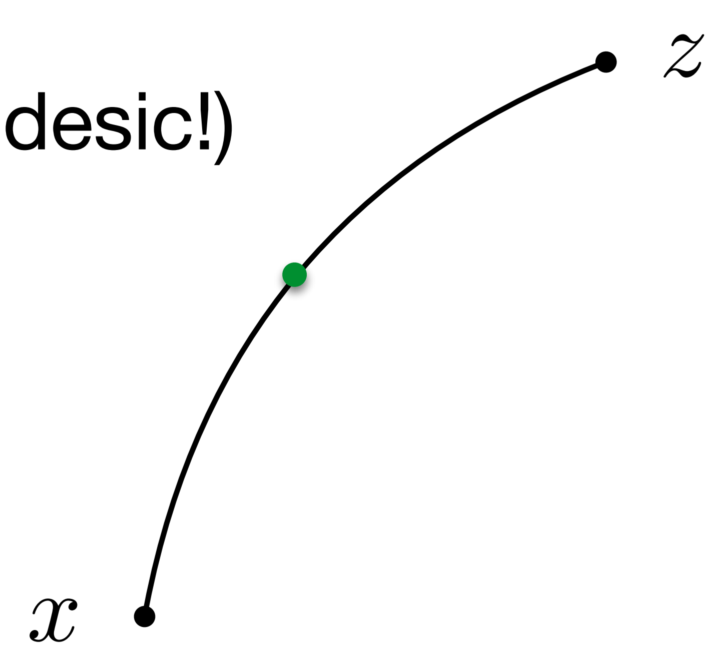
Relevance for causality

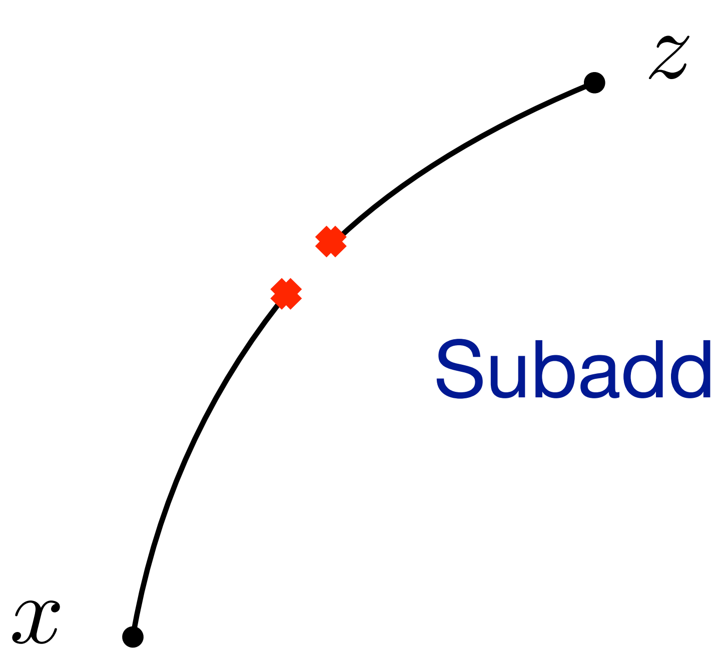
Third point problem (Lorentzian signature):

given $d(x, z) = 0$ find a third point y s.t.

$$d(x, y) = 0, \qquad d(y, z) = 0$$

Additive: one dimensional solution (the null geodesic!)





Subadditive: no solution

Local characterization:

inverse of $\langle g_{\mu\nu} \rangle$

$$C(x,y) \equiv \frac{1}{4} \frac{\partial \langle d^2(x,y) \rangle}{\partial y^\mu} \frac{\partial \langle d^2(x,y) \rangle}{\partial y^\nu} \bar{g}^{\mu\nu}(y) - \langle d^2(x,y) \rangle$$

Additive:	Subadditive:	Superadditive:
$C = 0$	$C < 0$	$C > 0$

Coordinate expansion:

$$C(0,x) = \frac{1}{4} \left(\bar{g}^{\alpha\beta} \langle \Gamma_{\alpha\mu\nu} \rangle \langle \Gamma_{\beta\rho\sigma} \rangle - \langle g_{\alpha\beta} \Gamma^\alpha_{\mu\nu} \Gamma^\beta_{\rho\sigma} \rangle \right) x^\mu x^\nu x^\rho x^\sigma + \mathcal{O}(x^5)$$

Effect building up at large separation

We can actually calculate it!

Example: thermal state of gravitons at temperature T

$$C(0,x) \simeq \frac{T^4}{M_P^2} \Delta x^4$$

$\Leftarrow$ effect important at $\ell \sim \frac{M_P}{T^2}$

Conjecture: Average distances are generally subadditive in QG

Implications for causality?

Given $\langle d^2(x,y)\rangle$ one can define a metric tensor $\langle g_{\mu\nu}\rangle = \bar{g}_{\mu\nu}$.

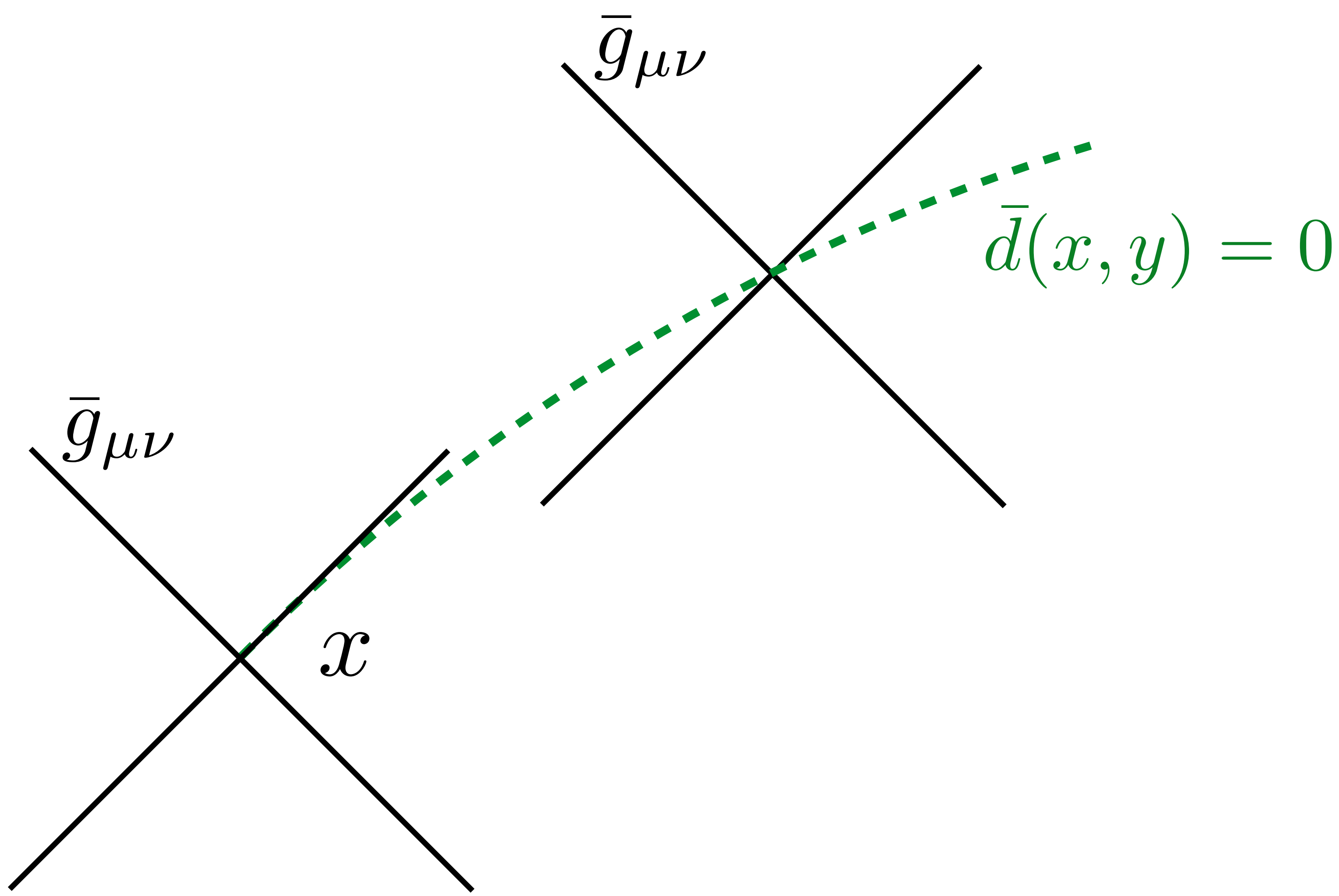
$$\bar{g}_{\mu\nu}(x) \;\equiv\; -\frac{1}{2}\lim_{y\to x}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\langle d^2(x,y)\rangle$$

But there is more to $\langle d^2(x,y)\rangle$ than $\langle g_{\mu\nu}\rangle$!

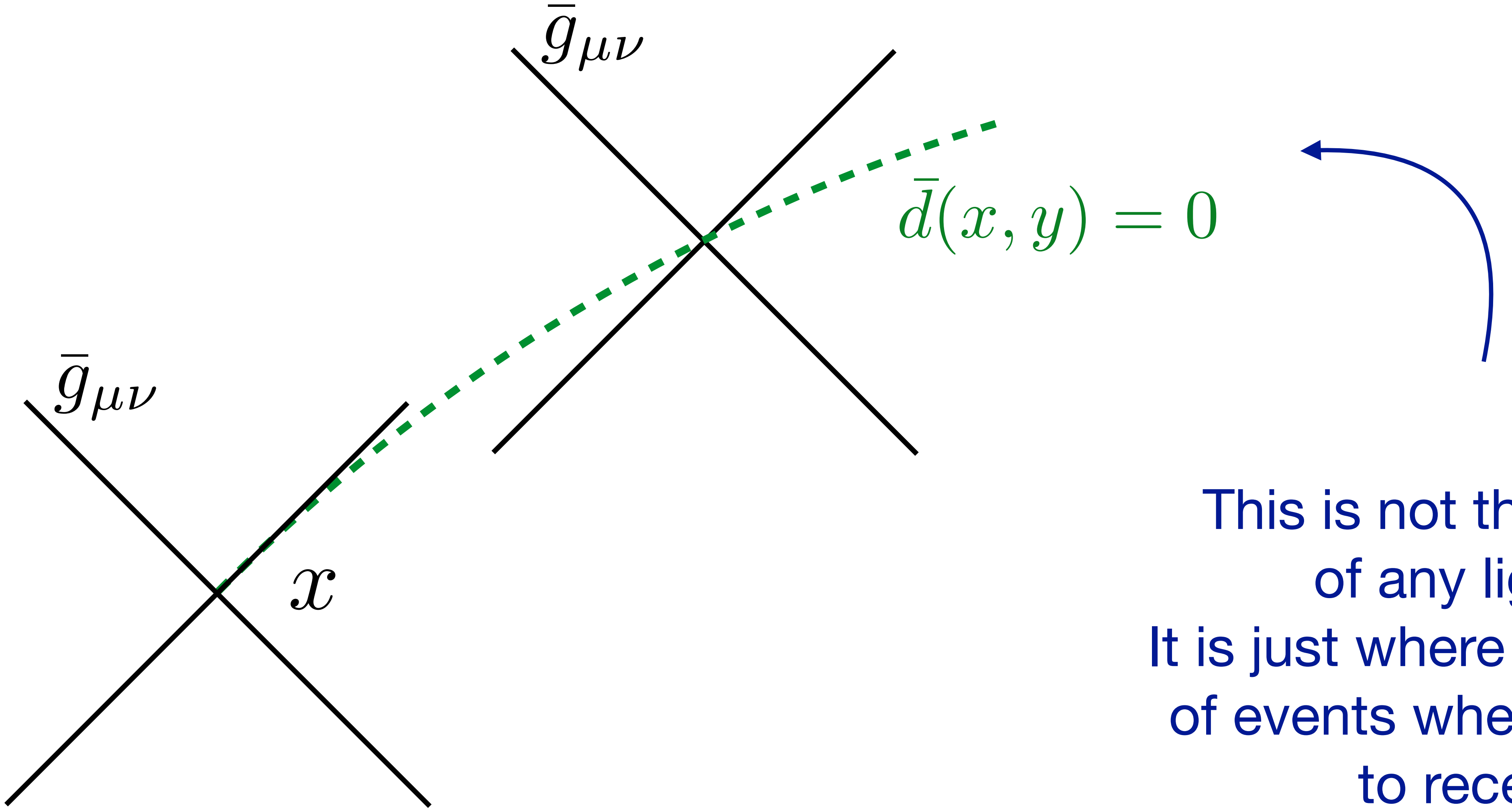
$\langle g_{\mu\nu}\rangle \Delta x^\mu \Delta x^\nu = 0$: where we expect the photon to be detected in the immediate vicinity of the emission.

Further away: see where $\langle d^2(x,y)\rangle = 0$

Implications for causality?



Implications for causality?



This is not the trajectory
of any light ray.
It is just where the ensemble
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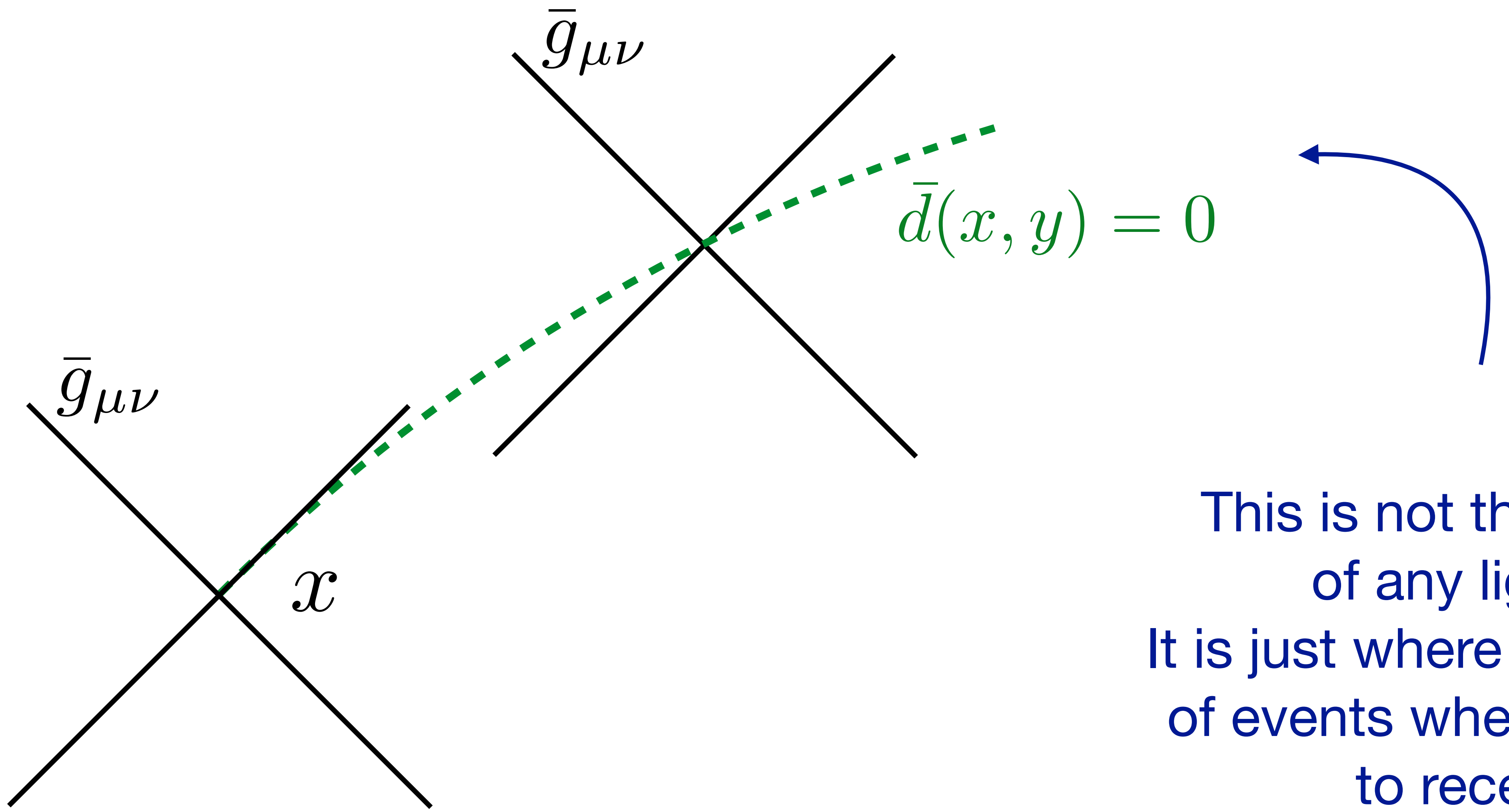

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Two causal structures at play. One *rigid* defined at each point. One dependent on the two extremes x and y.

Photons are ``prompt" wrt the rigid structure given by $\bar{g}_{\mu\nu}$