



Perfectly parallel cosmological simulations using spatial comoving Lagrangian acceleration



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and the Aquila Consortium

www.aquila-consortium.org

17 November 2022

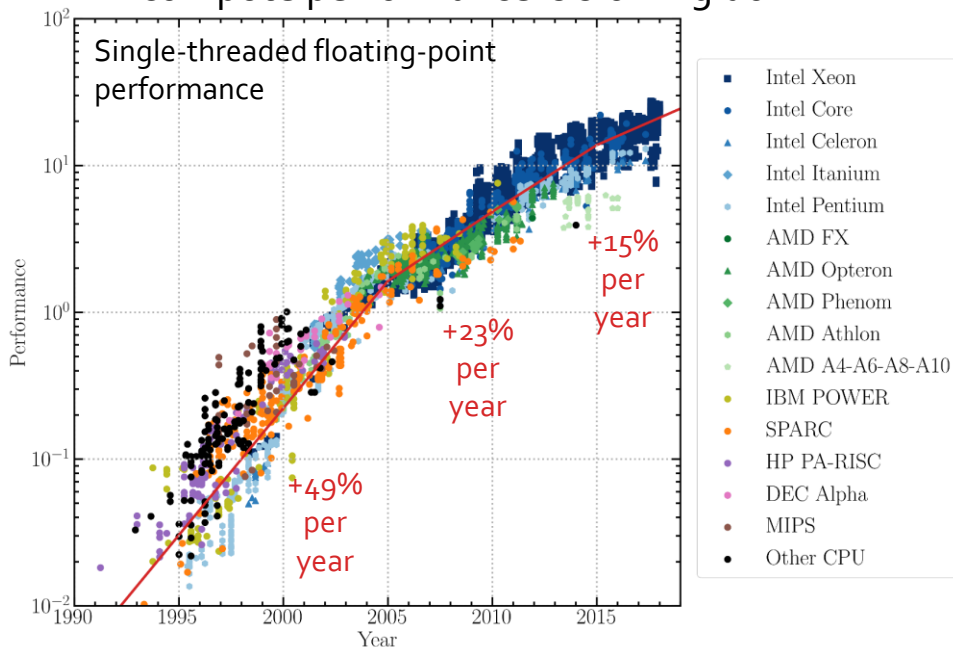
Why do we (still) need N -body codes?

- N -body simulations remain a basic ingredient for many **cosmological modelling problems**: galaxy clustering, ray-tracing, 21cm intensity mapping, Lyman- α .
- Frequentist approach: **mock surveys** (e.g. DESI, Euclid, LSST) are used for measurements of summaries and their covariances.
- Bayesian approach: **forward numerical data models** are the new way to express the theory:
 - ... embedded into a field-level likelihood: Bayesian large-scale structure inference (BORG),
 - ... or in a simulator-based approach: likelihood-free inference (ABC, DELFI, BOLFI, SELFI, etc.).



Numerical simulations in the exascale world

- Traditional hardware architectures are reaching their physical limit: per-core compute performance is slowing down.



Based on adjusted SPECfp® results, <http://spec.org>

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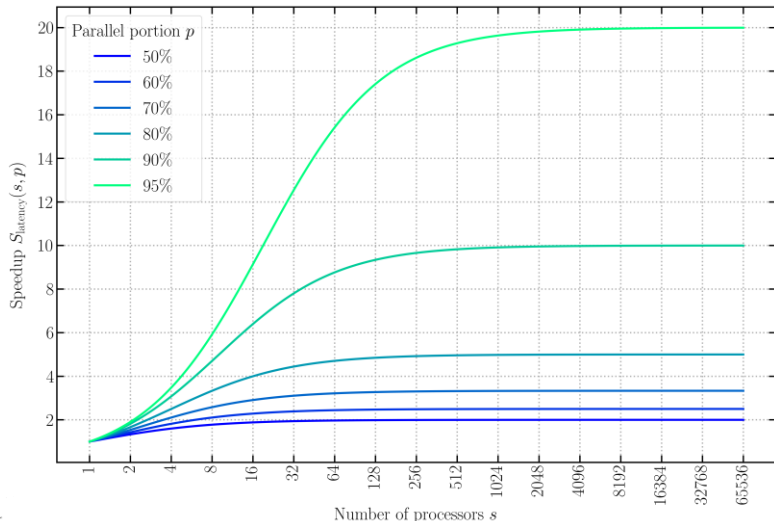
- Current hardware development focuses on:
 - Packing a larger number of cores into each CPU: currently $\mathcal{O}(10^5)$, soon $\mathcal{O}(10^6-7)$ in systems that are currently being built.
 - Developing hybrid architectures with cores + accelerators: GPUs, reconfigurable or dedicated chips (FPGAs/ASICs).



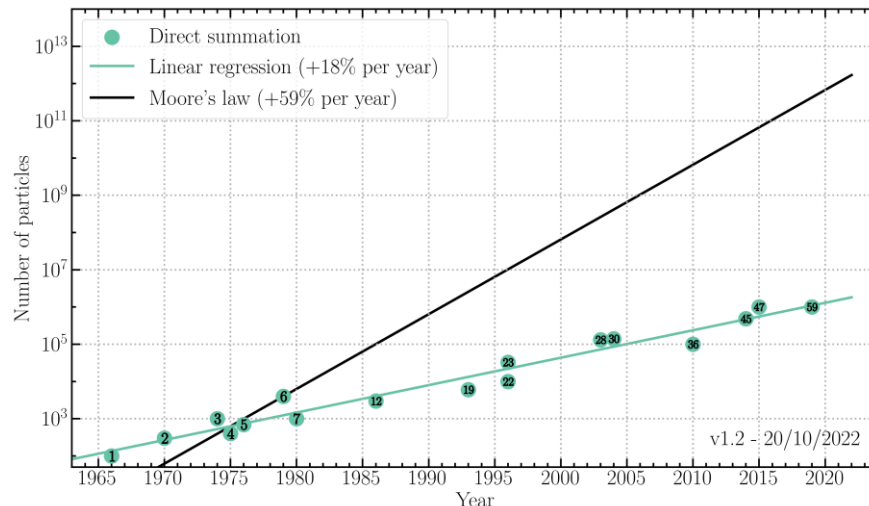
Parallelisation of N -body codes: the challenge

- Compute cycles are no longer the scarce resource. The cost is driven by **interconnections**.
- Amdahl's law: **latency kills the gains of parallelisation**.

Amdahl 1967, doi:10.1145/1465482.1465560



- The main issue preventing the easy parallelisation of N -body codes is the **long-range nature of gravitational interactions**.
- “Exact” gravity requires $\mathcal{O}(N^2)$ all-to-all communications between N particles.

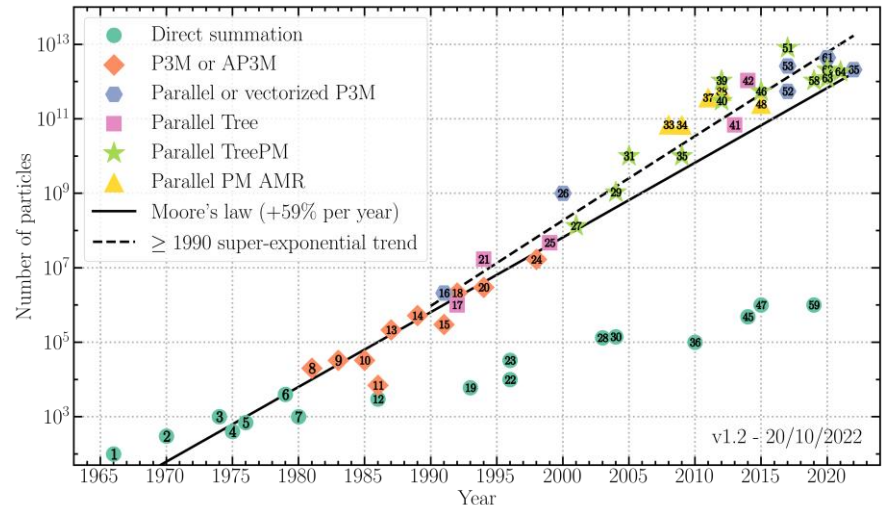


All references at [Github:florent-leclercq/Moore_law_cosmosims](https://github.com/florent-leclercq/Moore_law_cosmosims)



Parallelisation of N -body codes: the challenge

- Most of the work on numerical cosmology so far has focused on algorithms (such as tree, multipole, and mesh methods) that **reduce the need for communications** across the full computational volume.
- Since 1990, we observe a **super-exponential trend** that cannot be explained only by increase in computer speed.
- N -body codes cannot merely rely on computers becoming faster to reduce the computational time in the future.



All references at [Github:florent-leclercq/Moore_law_cosmosims](https://github.com/florent-leclercq/Moore_law_cosmosims)



tCOLA: Comoving Lagrangian Acceleration (temporal domain)

- Write the displacement vector as:

$$\Psi = \Psi_{\text{LPT}} + \Psi_{\text{res}} \quad (\mathbf{x} = \mathbf{q} + \Psi)$$

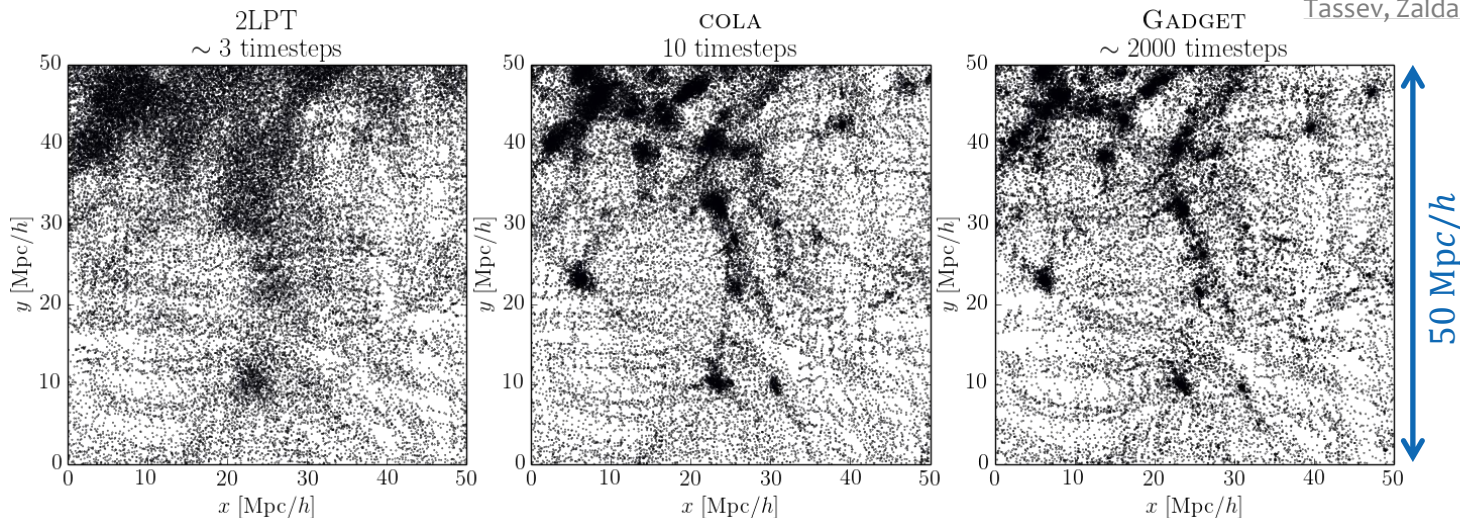
Tassev & Zaldarriaga, 1203.5785

Analytical solutions!

- Time-stepping (omitted constants and Hubble expansion):

$$\begin{array}{l} \text{Standard:} \\ \partial_a^2 \Psi = -\nabla_{\mathbf{x}} \Phi \end{array} \quad \Rightarrow \quad \begin{array}{l} \text{Modified:} \\ \partial_a^2 \Psi_{\text{res}} = \partial_a^2 (\Psi - \Psi_{\text{LPT}}) = -\nabla_{\mathbf{x}} \Phi - \partial_a^2 \Psi_{\text{LPT}} \end{array}$$

Tassev, Zaldarriaga & Eisenstein, 1301.0322



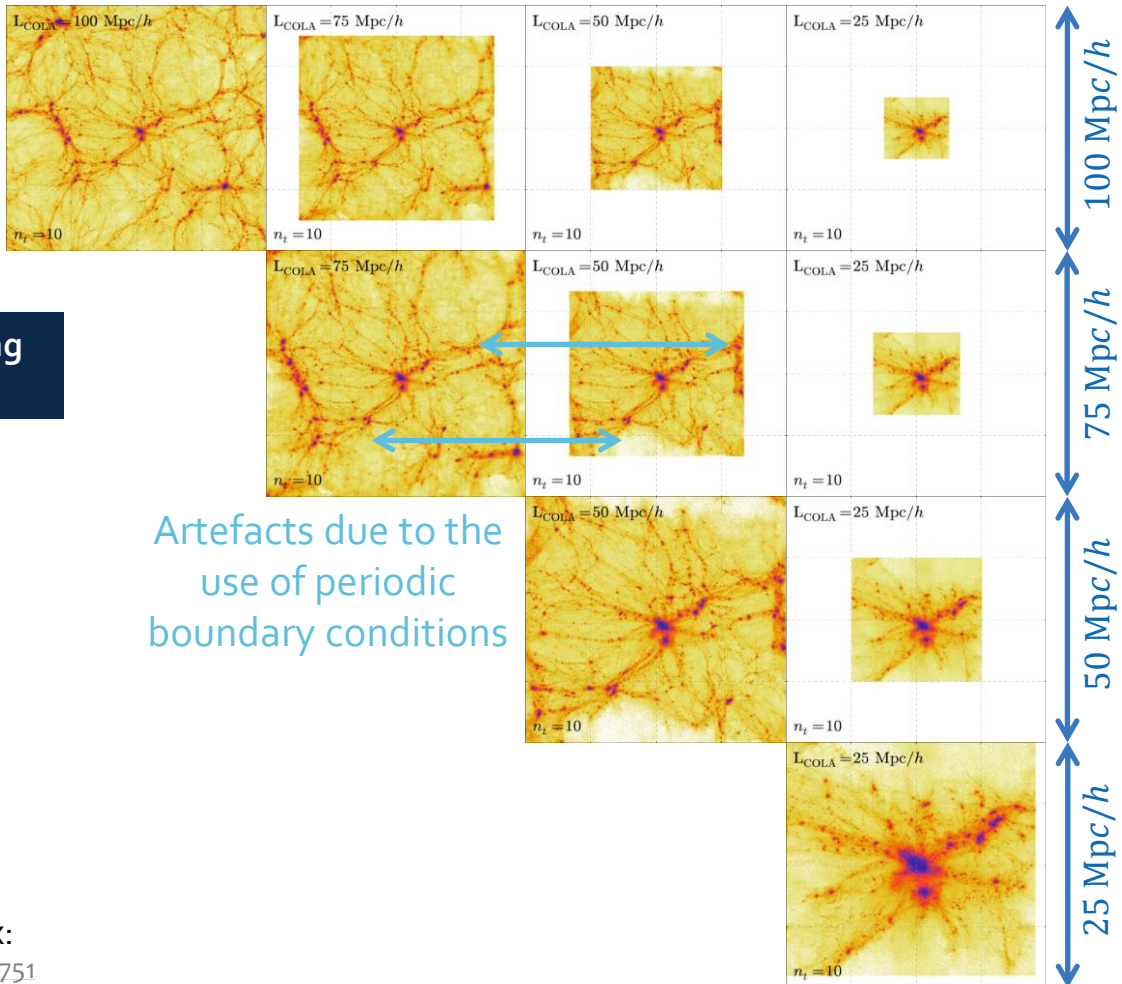
Beneficial gain of efficiency... but the real problem is not CPU-hours, but the inability to run on a very large number of cores due to latencies/parallelisation overhead.



sCOLA: Extension to the spatial domain

- Computing the LPT reference frame suggests a new strategy:

Can we decouple sub-volumes by using the large-scale analytical solution?



Artefacts due to the use of periodic boundary conditions

Proof of concept using one sub-box embedded into a larger simulation box:

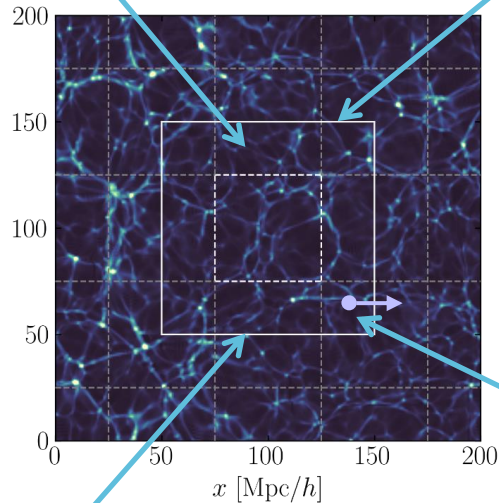
Tassev, Eisenstein, Wandelt & Zaldarriaga, 1502.07751

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The solution to boundary artefacts

1. A buffer region around each tile

2. Appropriate Dirichlet boundary conditions for the potential



- It is necessary to oversimulate some of the volume.
- The Poisson solver uses discrete sine transforms (DSTs) instead of FFTs.
- Overall, two approximations (to ensure no communication between tiles):

1. Linearly-evolving potential (LEP) at the boundaries: $\Phi_{BCs}(\mathbf{x}, a) \approx D_1(a)\phi^{(1)}(\mathbf{x})$

2. Outgoing particles do not deposit mass

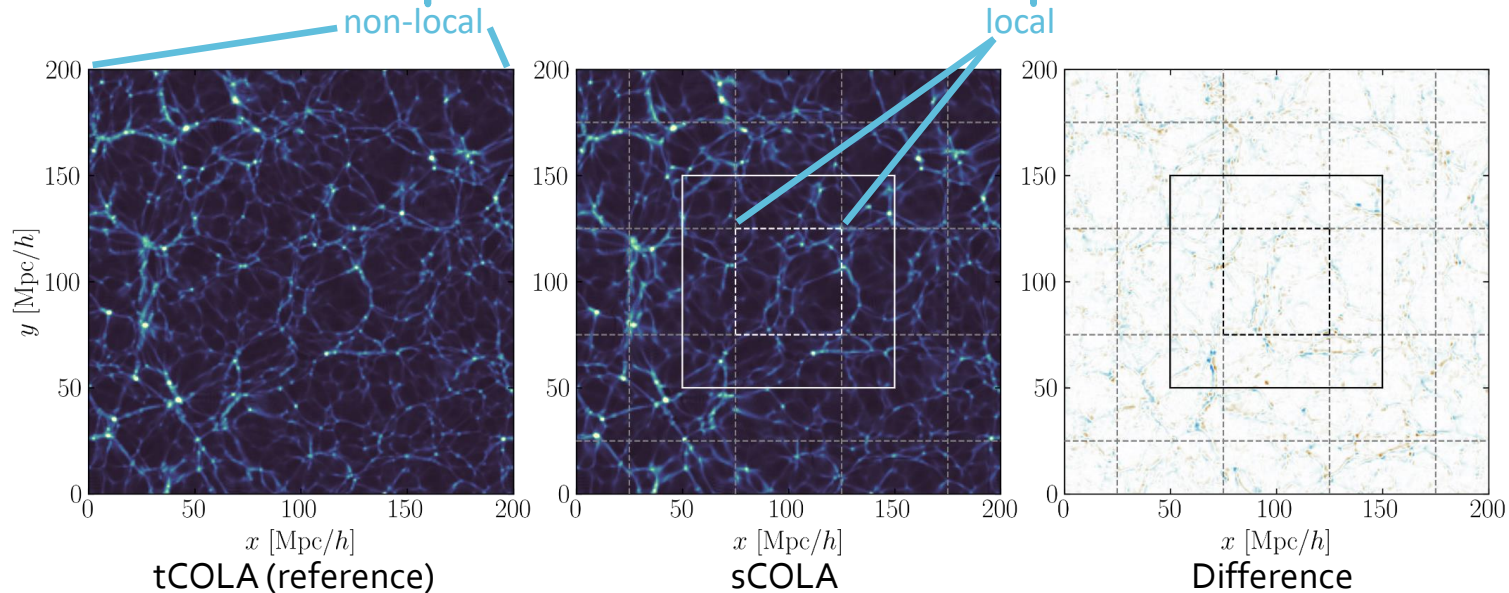


Perfectly parallel cosmological simulations using spatial comoving Lagrangian acceleration (sCOLA)

- Can we decouple sub-volumes by using the large-scale analytical solution?

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} = -\nabla \left[\underbrace{\Delta^{-1} \delta}_{\text{non-local}} \right]$$

$$\frac{\partial^2 (\mathbf{x} - \mathbf{x}_{1.s.})}{\partial t^2} = -\nabla \left[\underbrace{\Delta^{-1} (\delta - \delta_{1.s.})}_{\text{local}} \right]$$



Publicly available implementation:
Bitbucket:[florent-leclercq/simbelmyne/](https://bitbucket.org/florent-leclercq/simbelmyne/)



FL, Faure, Lavaux, Wandelt, Jaffe, Heavens, Percival & Noûs, 2003.04925

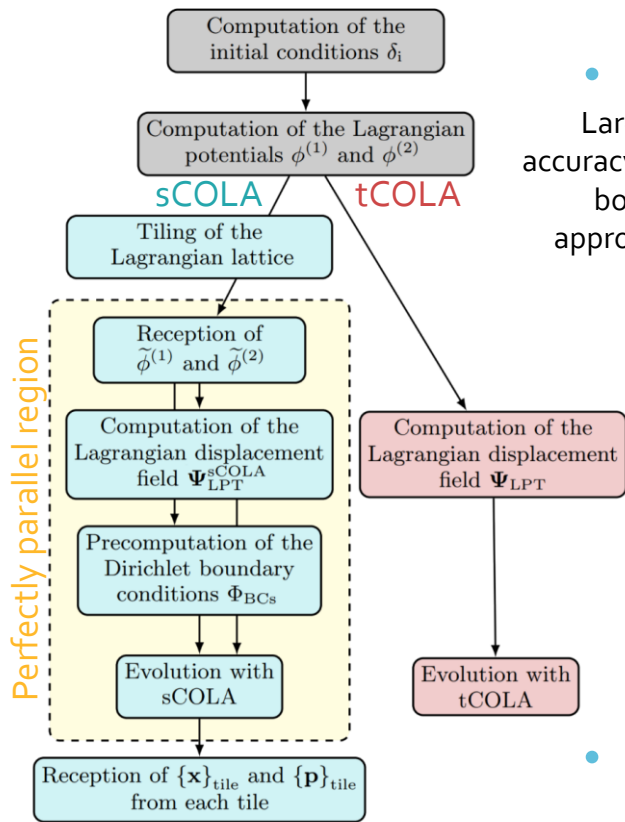
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Perfectly parallel cosmological simulations using sCOLA

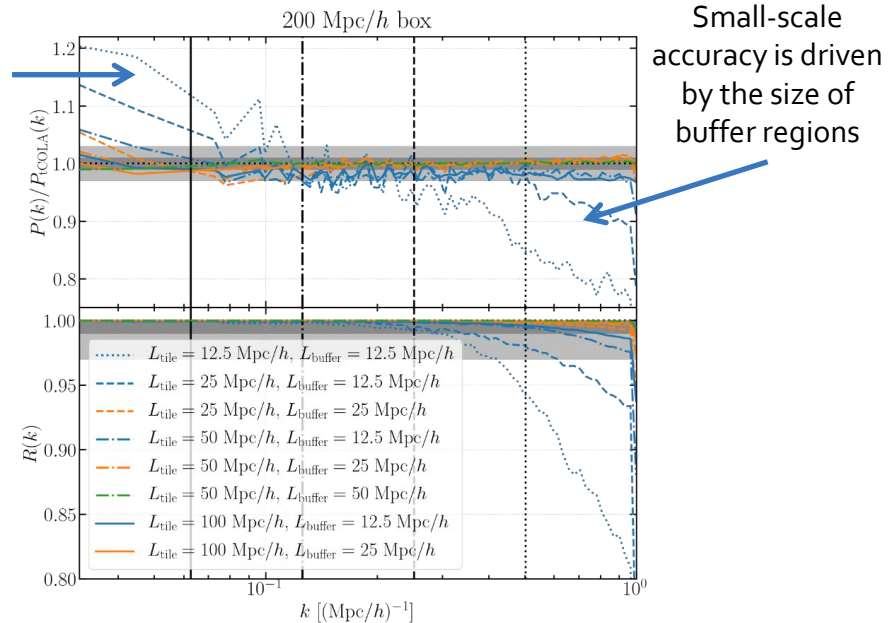
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The perfectly parallel algorithm and its accuracy



- Parameter investigation (size of tiles and buffer regions): Large-scale accuracy is driven by boundary approximations



Small-scale accuracy is driven by the size of buffer regions

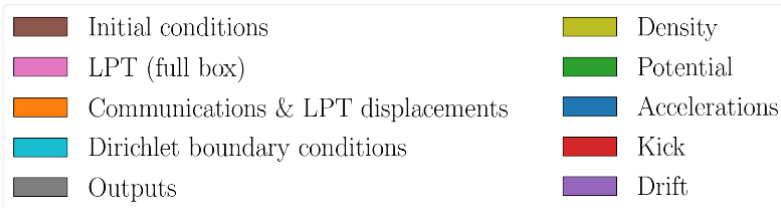
- Some setups reach 3 to 1% accuracy at all scales, as required for the next-generation of surveys.



Memory requirements, parallelisation potential & speed

- Buffer regions require to oversimulate the volume by some factor r .
- But small N -body simulations can be run in the L3 cache of CPUs, on GPUs or FPGAs: hardware speed-up factor of s .
- Parallelisation potential factor:

$$p = s \frac{N_{\text{tiles}}}{r} = s \left(\frac{L}{L_{\text{sCOLA}}} \right)^3$$



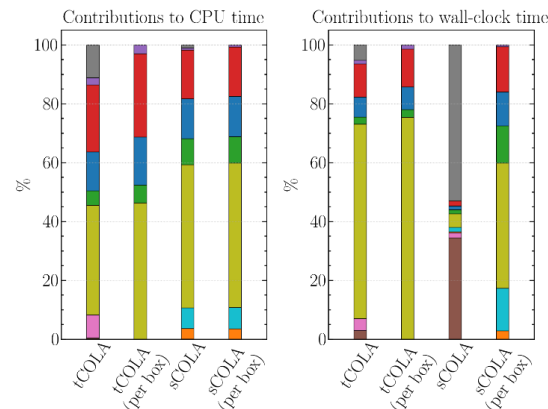
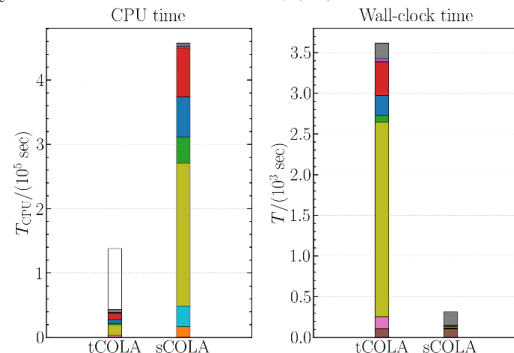
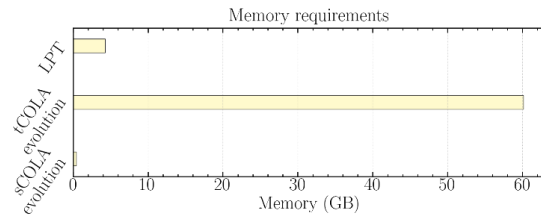
$$L = 1 \text{ Gpc}/h$$

$$L_{\text{tile}} = 125 \text{ Mpc}/h$$

$$L_{\text{buffer}} = 29.3 \text{ Mpc}/h$$

$$r \approx 3.17$$

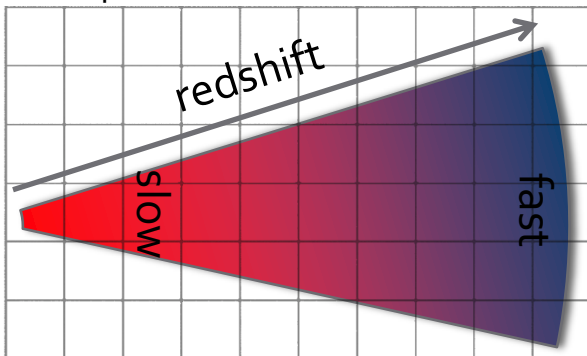
$$p \approx 161.59$$



Additional benefits

- **Light-cones and mock catalogues:**

- sCOLA boxes only need to run until they intersect the observer's past light-cone.
- Most of the high- z volume will run faster than $z = 0$.
- Many unobserved sCOLA boxes do not even have to run!
- The wall-clock time limit is the time for running a single sCOLA box to $z = 0$ at the observer's position.



- **Gravity and physics models:** any gravity model (e.g. P₃M, tree, or AMR) and non-gravitational physics (hydrodynamics) can be used within tiles.
- **Grid computing:** the algorithm is suitable for inexpensive, strongly asynchronous networks.
- **Robustness to node failure.**

Conclusions

- In the age of peta-/exa-scale computing, we introduced a **perfectly parallel** and easily applicable algorithm for cosmological simulations using sCOLA, a hybrid analytical/numerical technique.
- The approach is based on a tiling of the full simulation box, where **each tile is run independently**.
- Resulting larger and higher-resolution cosmological simulations can be used in the context of DESI, Euclid, LSST and upcoming **extremely large-scale surveys**.
- The algorithm can benefit from a variety of **hardware architectures**. It is suitable for participatory computing platforms such as Cosmology@Home (with potential visibility/outreach benefits).

<https://www.cosmologyathome.org>

- The algorithm is implemented in the **Simbelmynë** code, publicly available at

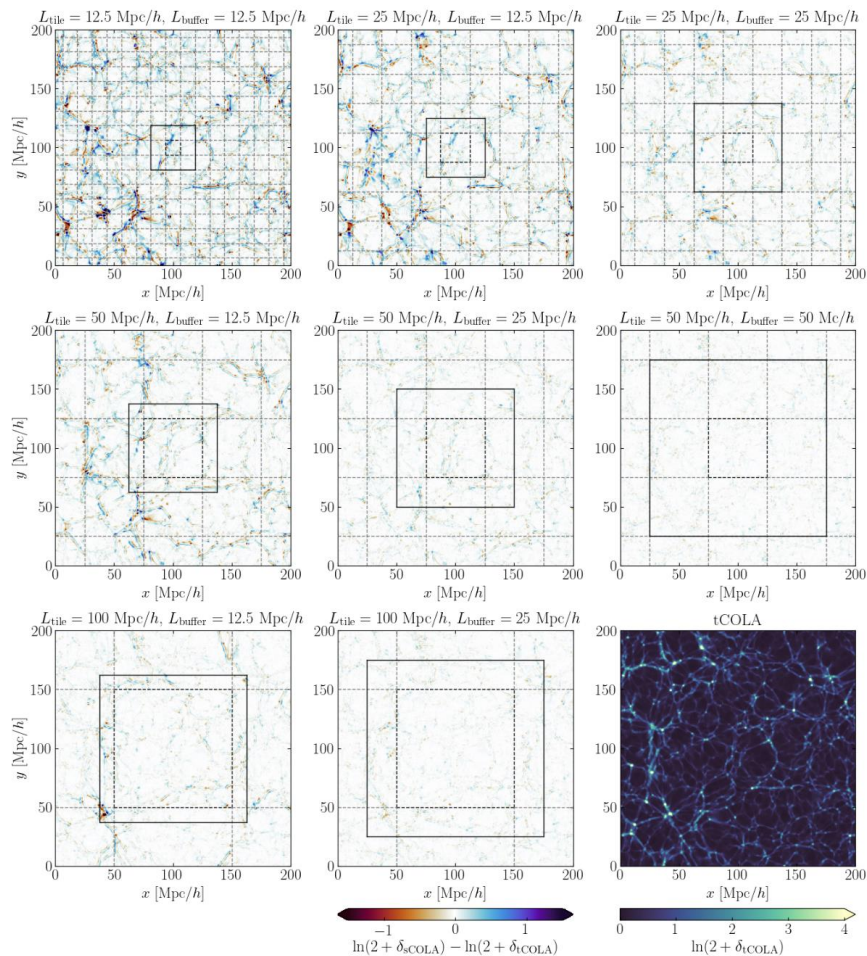
<https://simbelmyne.florent-Leclercq.eu> – [Bitbucket: florent-leclercq/simbelmyne](https://bitbucket.org/florent-leclercq/simbelmyne)



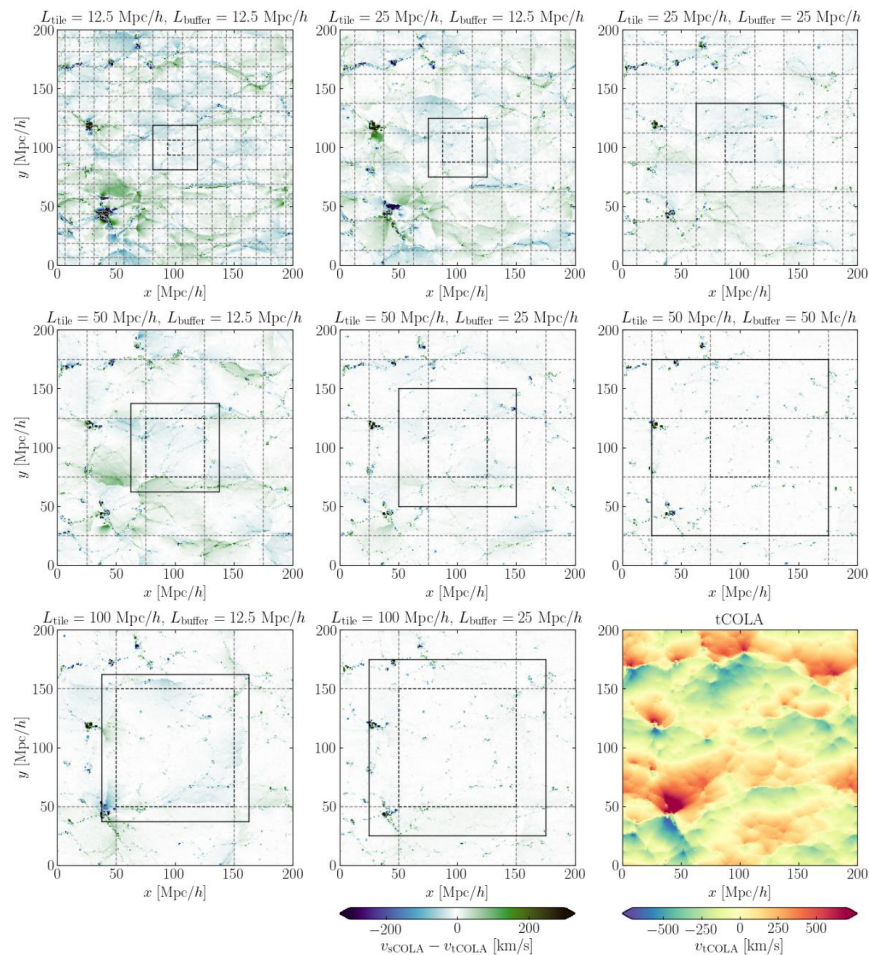
Additional slides



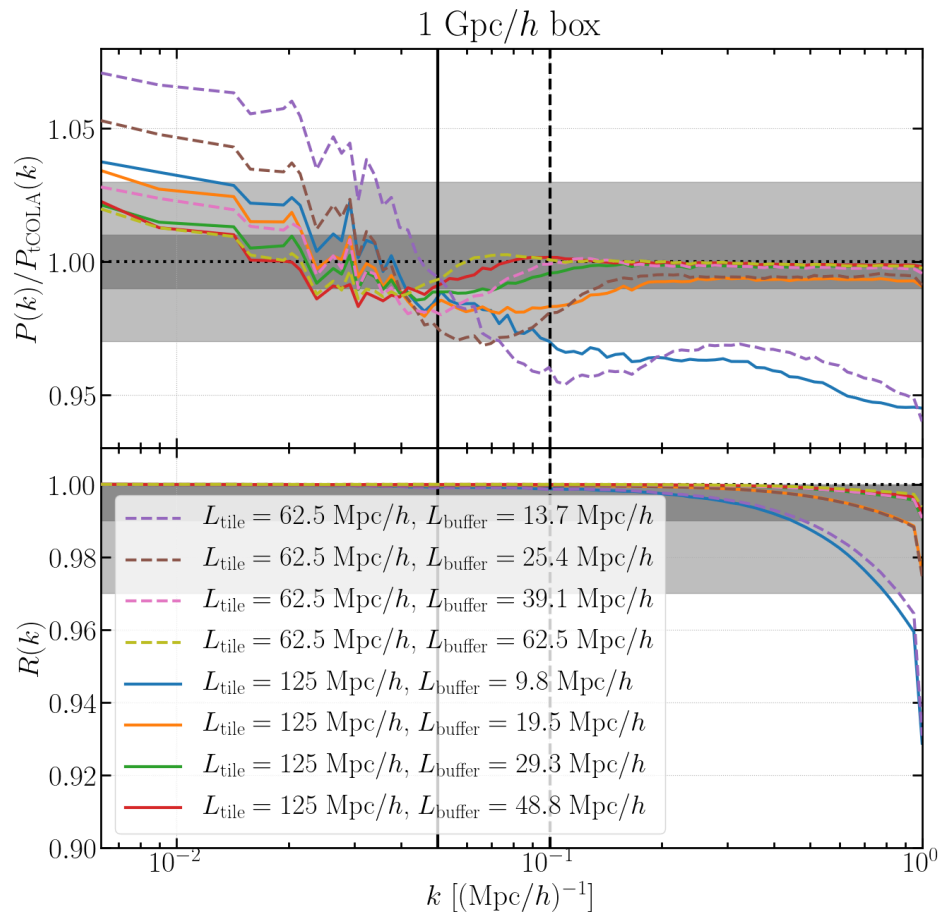
Density field



Velocity field

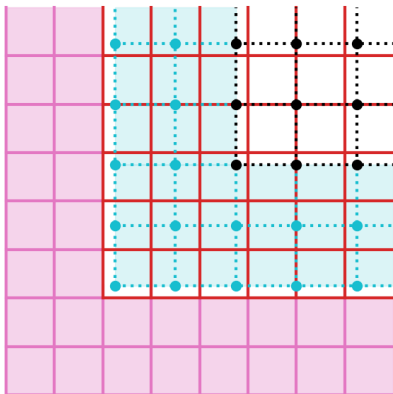


1 Gpc/h box

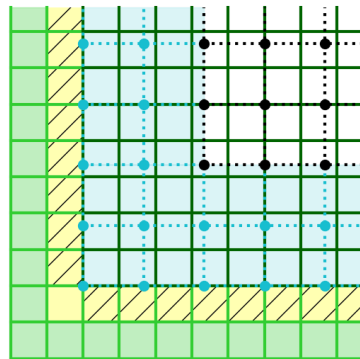


Test of the approximations

Lagrangian lattice and LPT grid



Lagrangian lattice and PM grid



200 Mpc/h box, $L_{\text{tile}} = 50 \text{ Mpc}/h$, $L_{\text{buffer}} = 12.5 \text{ Mpc}/h$

