# LVK cosmological pipelines

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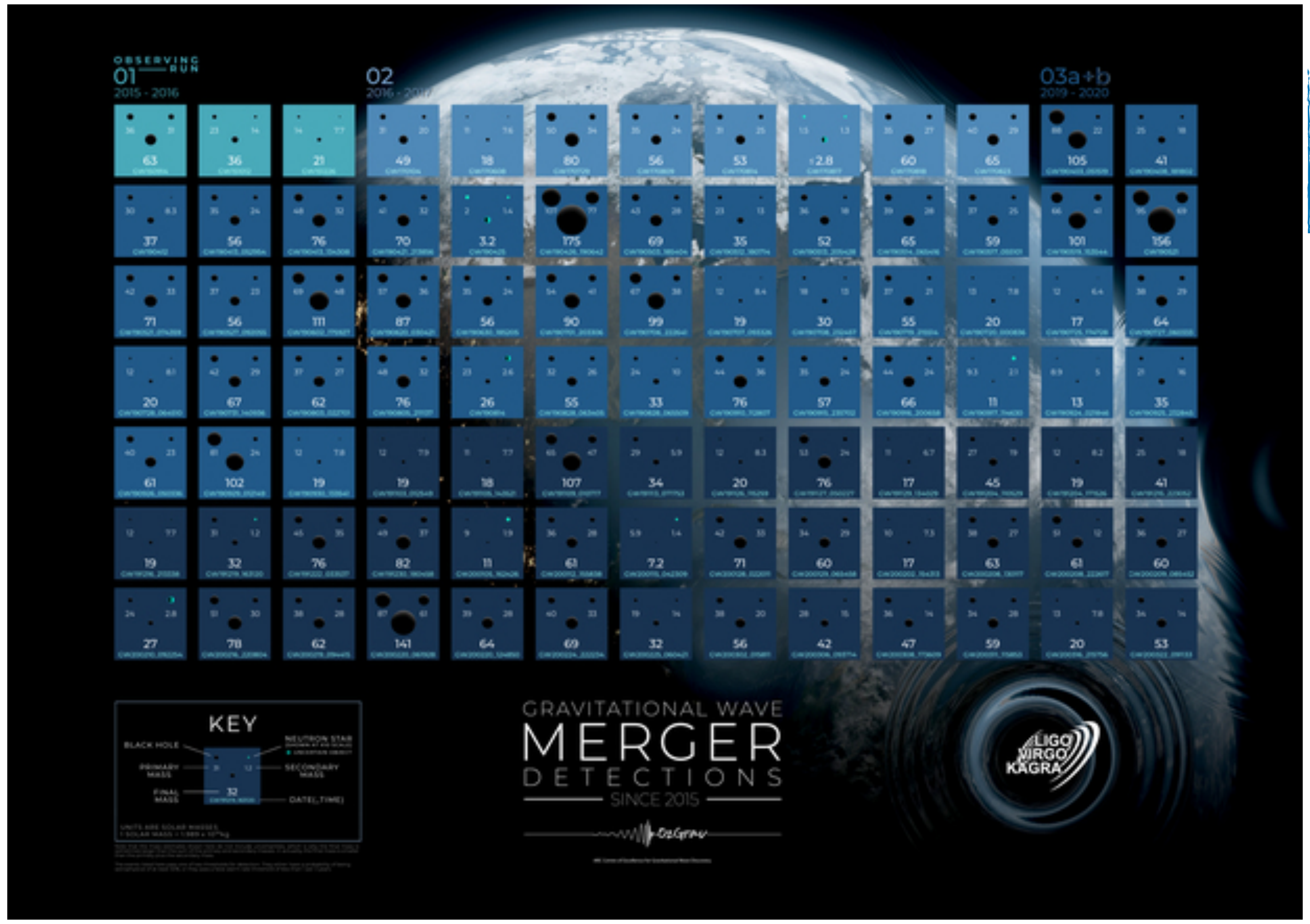
*2111.03604*

# Cosmology with gravitational waves



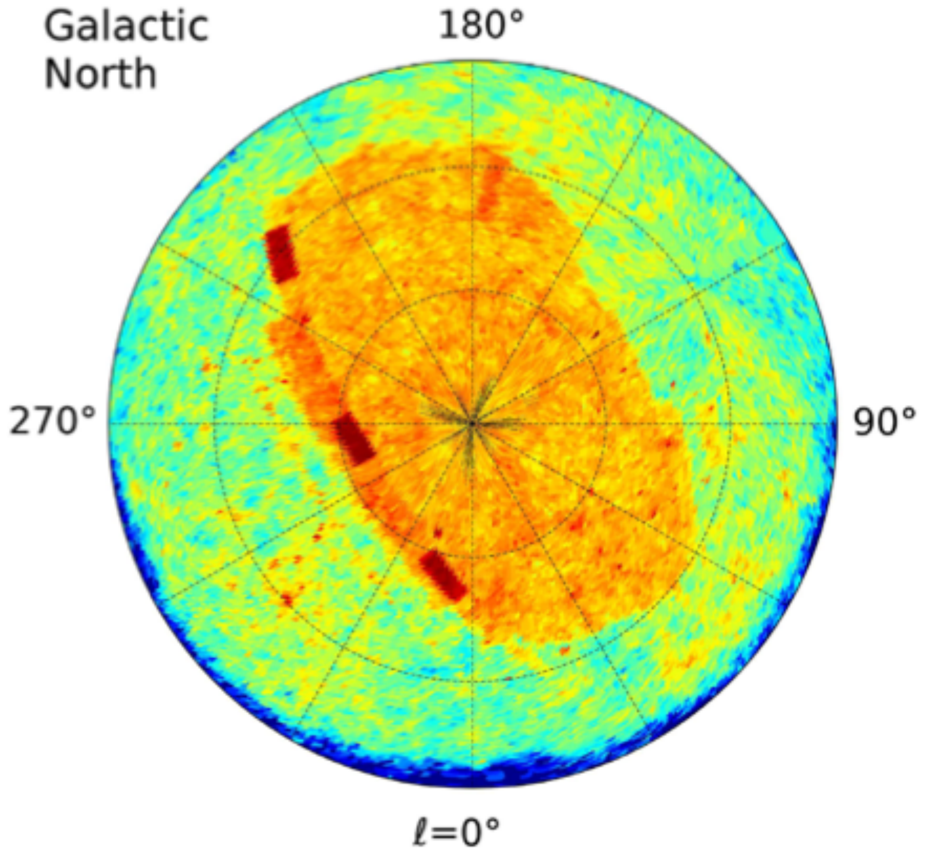

Carl Knox (OzGrav, Swinburne University of Technology)

GW data

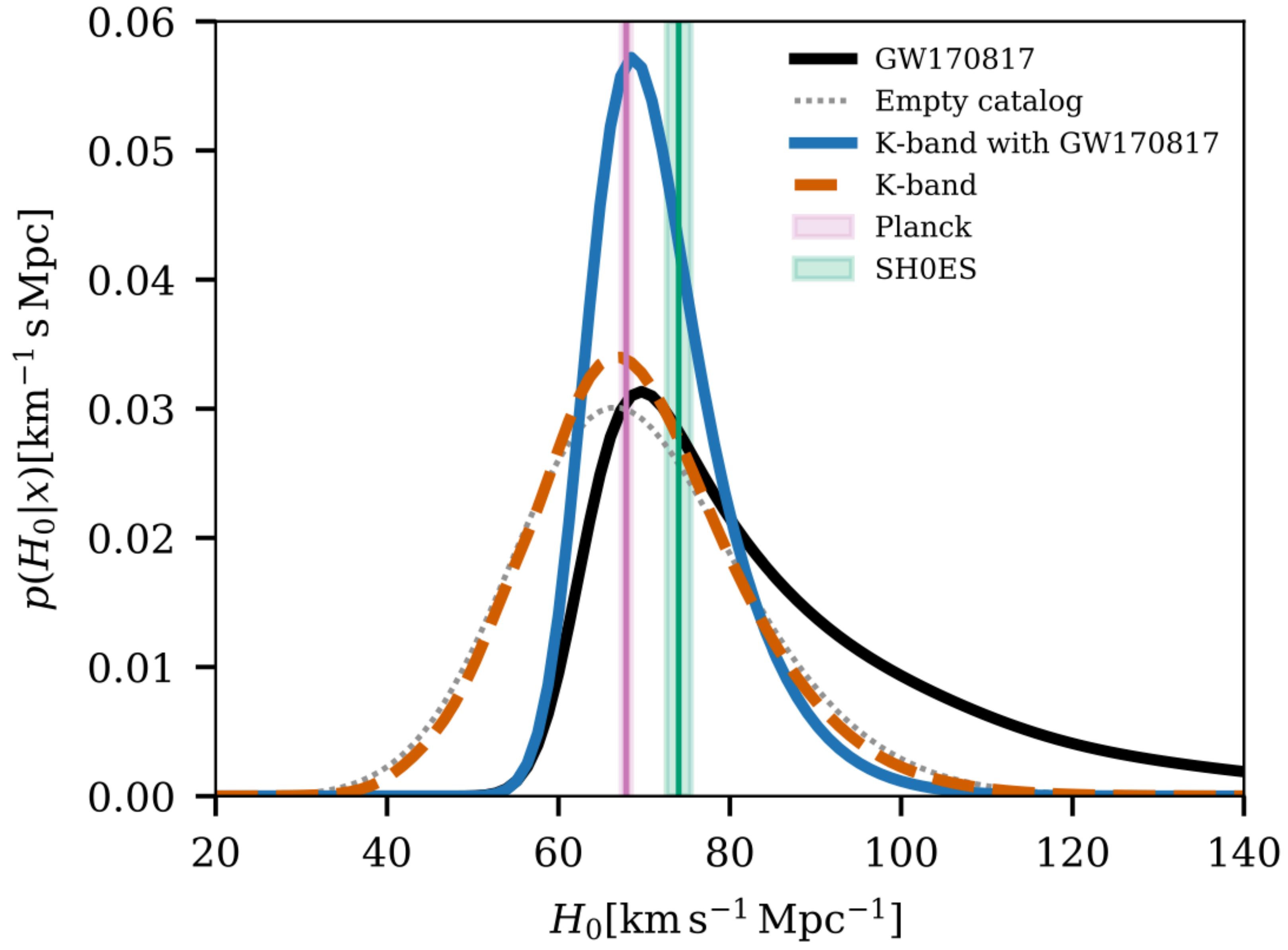
Pipelines

GWcosmo
IcaroGW

Galaxy Catalog Data




Dálya et al. 1804.05709

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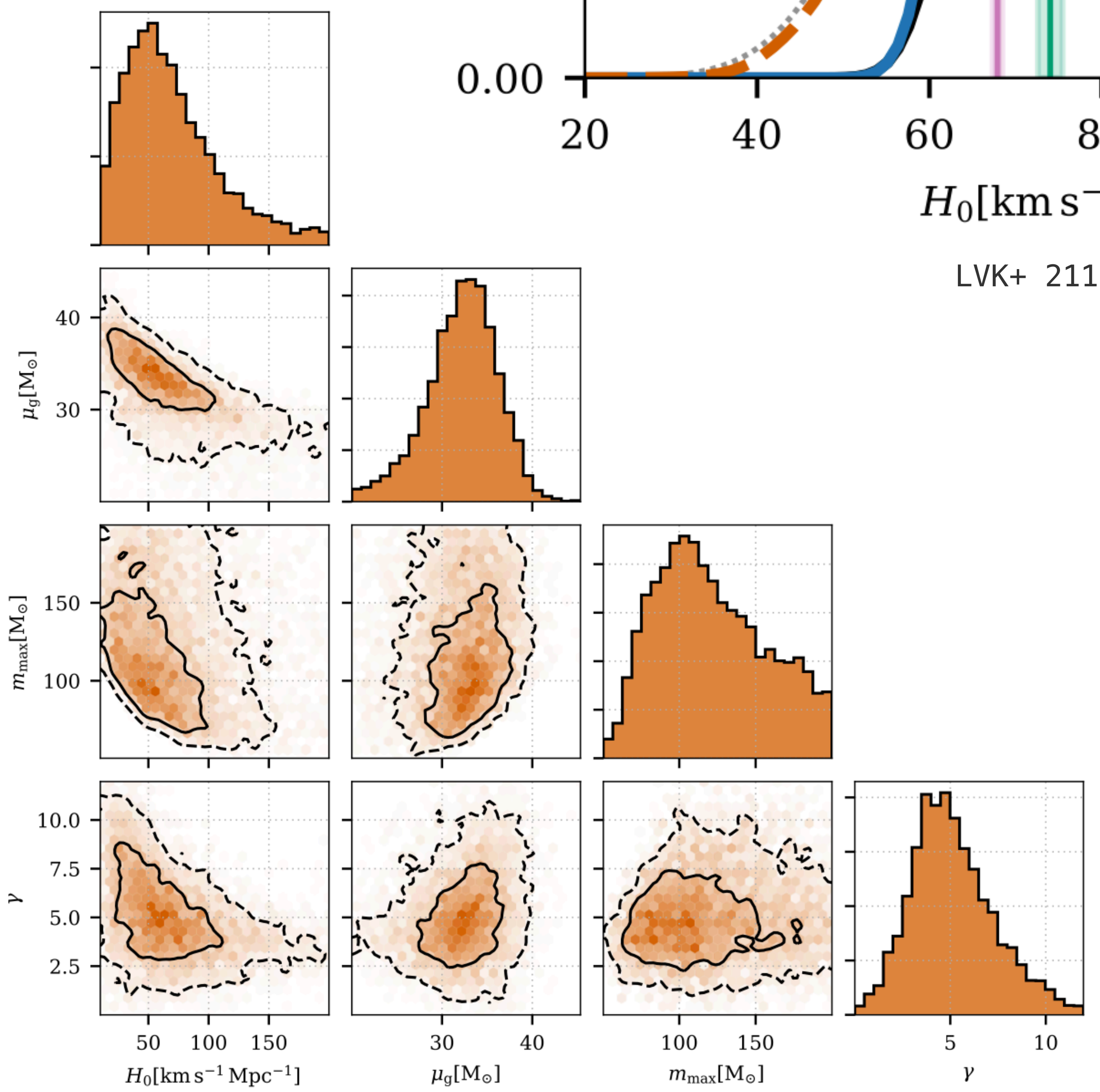


# Cosmology with gravitational waves

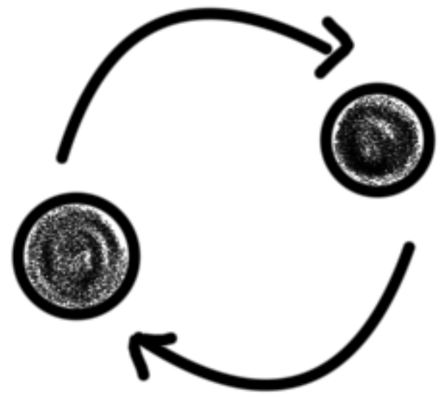
- IcaroGW
  - Redshift information from source frame mass distribution
  - Marginalize over mass population

- GWcosmo
  - Assumption: GW sources in galaxies
  - Statistical redshift association from galaxy catalogs
  - Fixed mass distribution

Redshift information

GW data

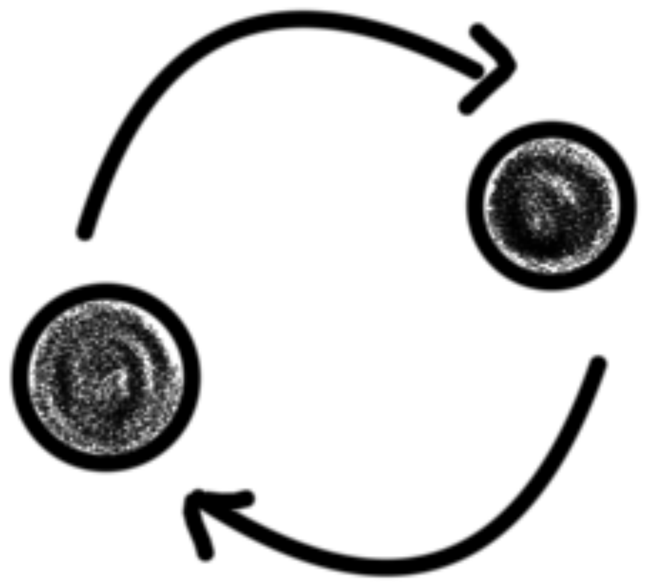
$$d_L(z) = \frac{(1+z)c}{H_0}\int_0^z \frac{dz'}{[\Omega_m(1+z')^3+\Omega_\Lambda]^{1/2}}$$



# Gravitational wave parameters

Source frame masses

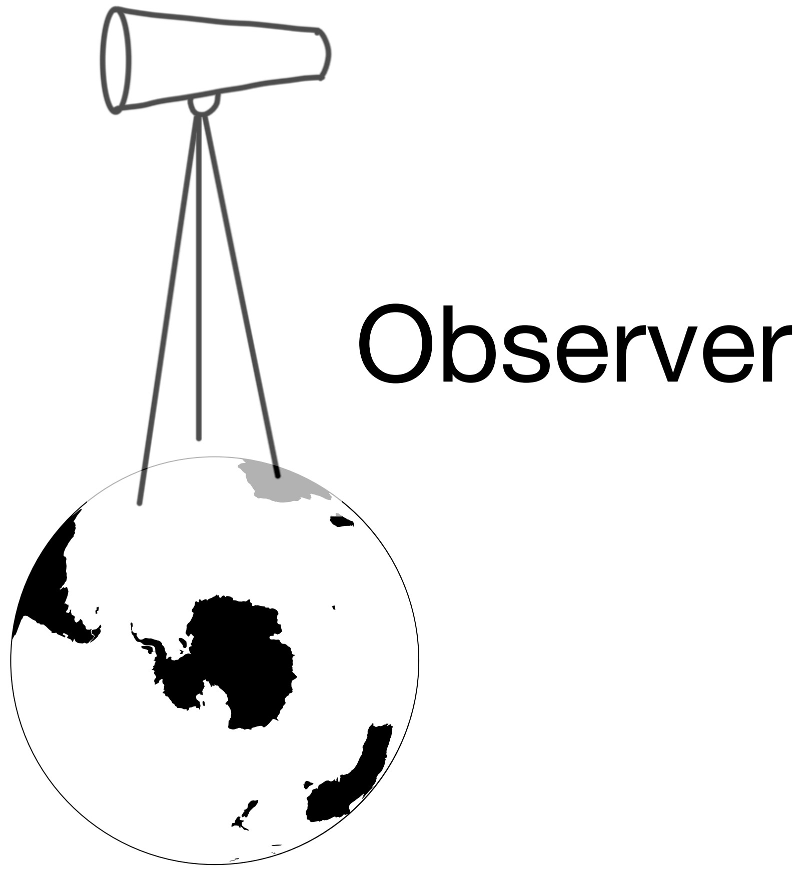
$m_1^{(s)}, m_2^{(s)}$



Expansion $(H_0, \Omega_\mathrm{m}, \ldots)$

Detector frame masses

$m_1^{(d)}, m_2^{(d)}$




- GW frequency is shifted to lower values by the expansion
- Individual GW signal carries no redshift information

$$m^{(d)} = (1+z)m^{(s)}$$

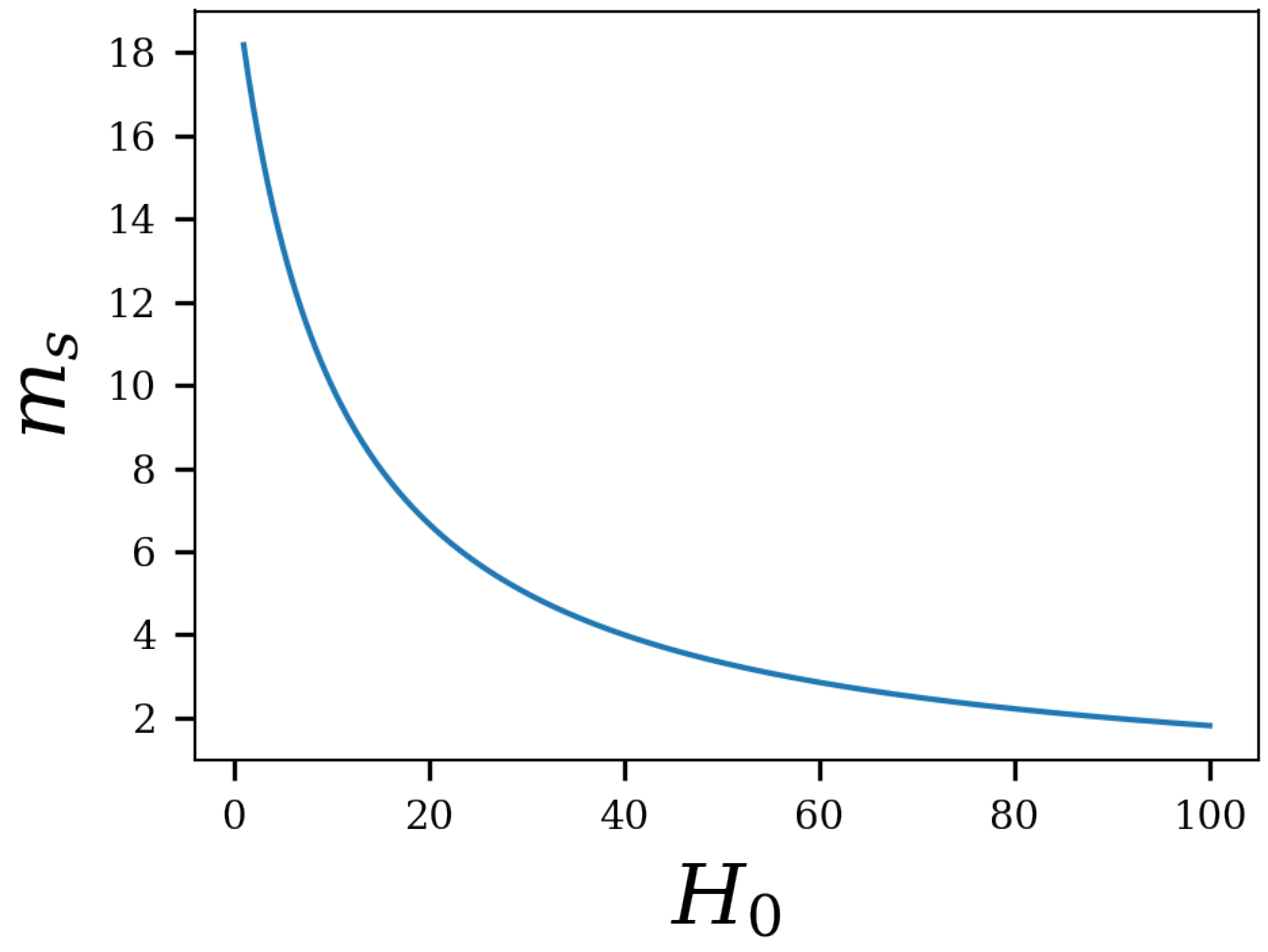
# Source frame population

- Assumption of mass model → statistical measurement of redshift

$$m^{(d)} = (1+z)m^{(s)} \quad \rightarrow \quad z = \frac{m^{(d)}}{m^{(s)}} - 1$$

- Joint fit of cosmological parameters and mass population models (*Taylor et al. 2012, Taylor and Gair 2012, Farr et al. 2019, You et al. 2020*)

- Strong correlation between $H_0$ and the characteristic mass scales




$$m^{(s)} = \frac{m^{(d)}}{1 + d_L H_0 / c} \qquad z \approx \frac{d_L H_0}{c}$$

# Statistical framework of IcaroGW (Mastrogiovanni et al. 2103.14663)

- Bayesian analysis with selection effects (Mandel et al. 1809.02063, Thrane and Talbot 1809.02293, Vitale et al. 2007.05579)

$$p(\Lambda|\{x\}) \propto p(\Lambda) \prod_{j=1}^{N_{\rm obs}} \frac{\int p(x_j|\theta_j)\, p_{\rm pop}(\theta_j|\Lambda) \mathrm{d}\theta_j}{\int p_{\rm det}(\theta_j)\, p_{\rm pop}(\theta_j|\Lambda) \mathrm{d}\theta_j}$$

- Metaparameters $\Lambda$ : population parameters, cosmological parameters, …
- GW data $\{x\}$
- Source parameters $\theta = \{m_{1,2}^{(d)}, d_L^{\rm GW}, \dots\}$
- GW likelihood $p(x_i\,|\,\Lambda, \theta)$, obtained from posterior samples
- Population assumption $p_{\rm pop}(\theta\,|\,\Lambda)$
- Detection probability $p_{\rm det}(\theta)$

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Here: Only $\theta = \{m_{1,2}^{(d)}, d_L\}$

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# Statistical framework of IcaroGW

*(Mastrogiovanni et al. 2103.14663)*

Bayesian analysis with selection effects

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- Only events passing threshold (on signal to noise ratio or false alarm rate) are considered
- Numerical evaluation of $p_{\rm det}(\theta)$: produce a set of events and label them either "detected" or "undetected" (passing SNR threshold and IFAR threshold)

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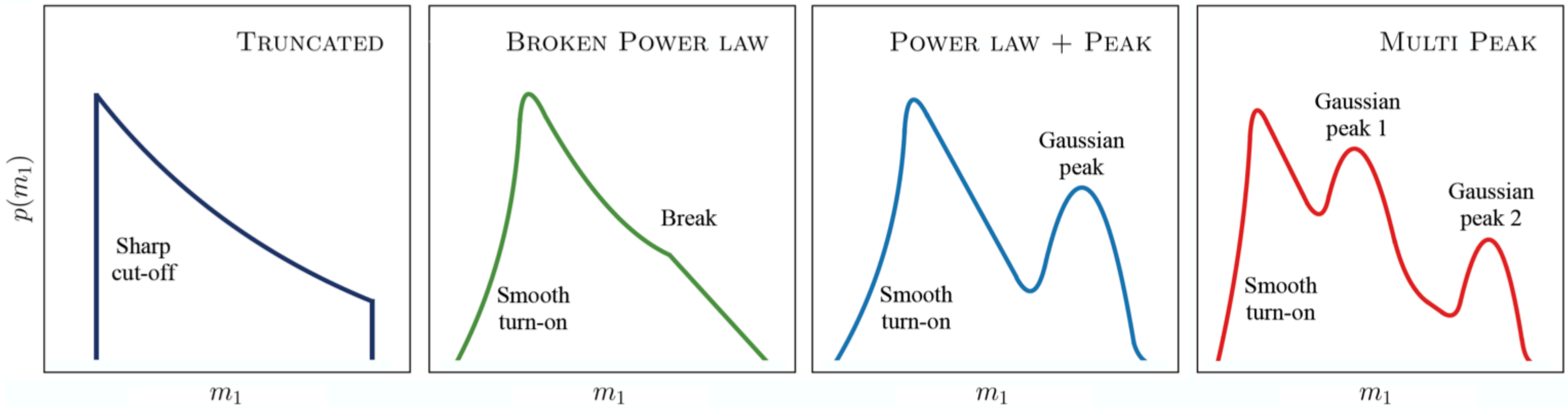
# The source mass population model $p_{\rm pop}(\theta \,|\, \Lambda)$




LVK+ 2010.14533

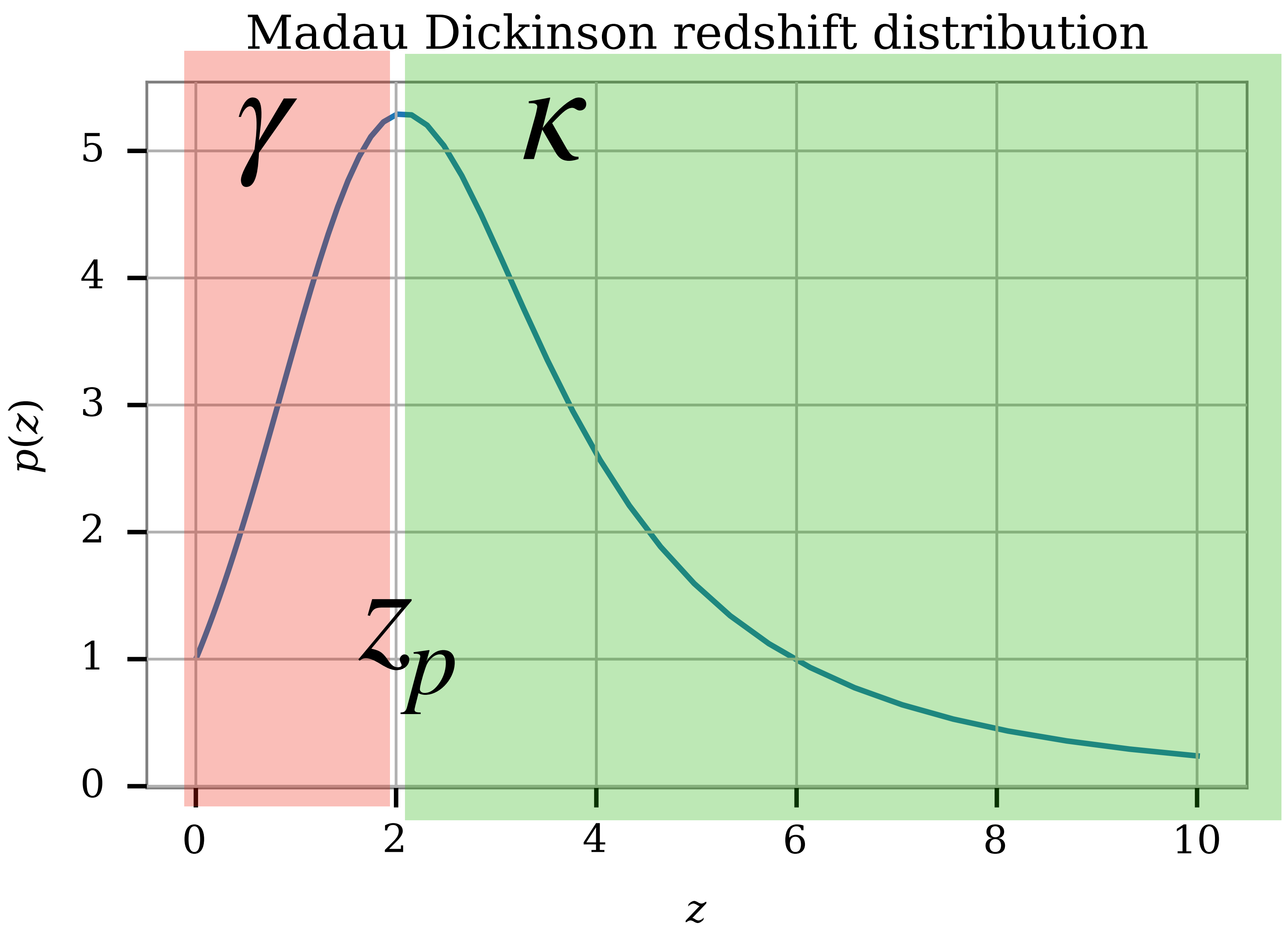
# Redshift distribution

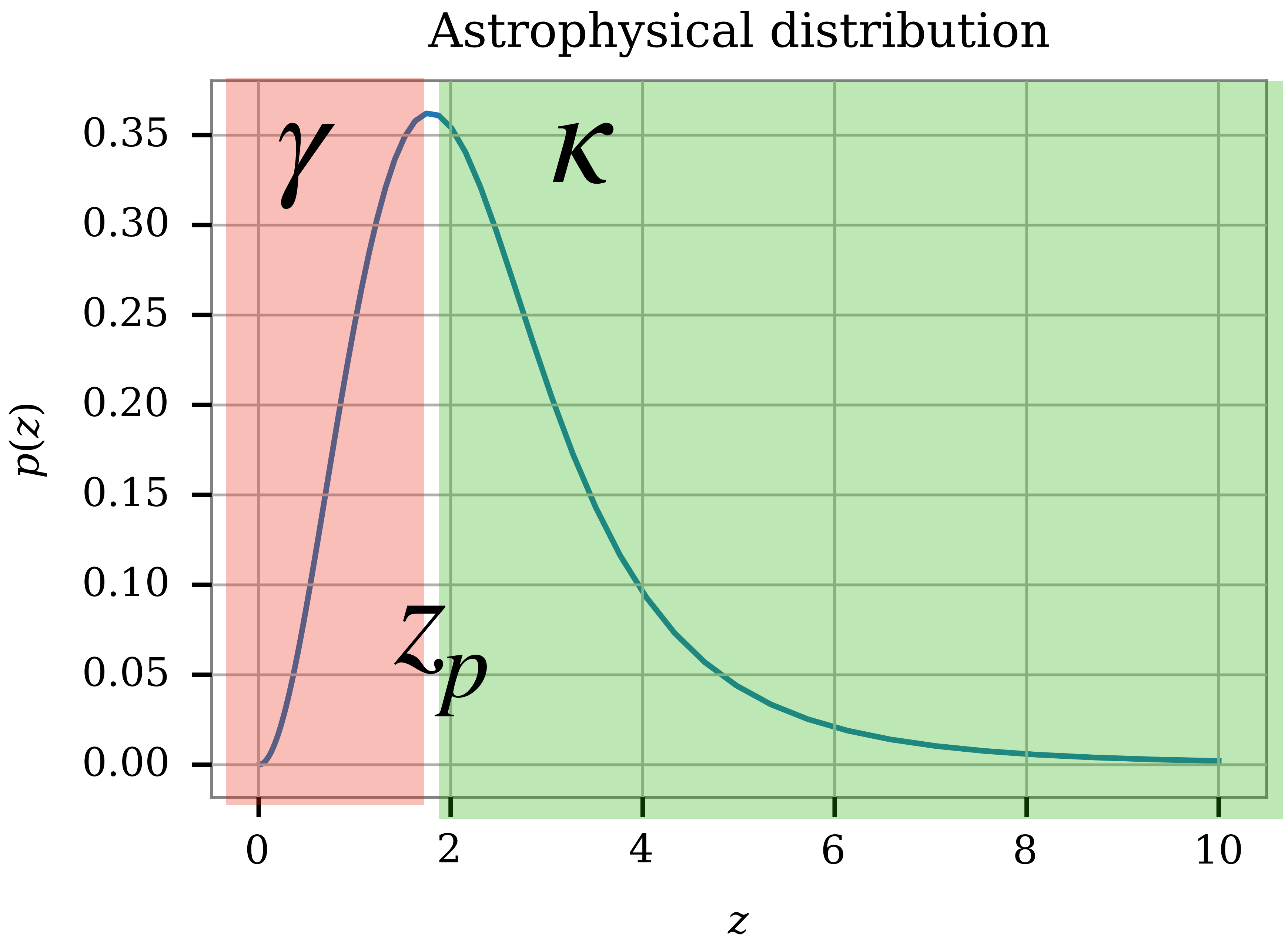
$p_{\mathrm{pop}}(\theta \,|\, \Lambda)$

Madau Dickinson model

$$f(z \,|\, \gamma, \kappa, z_p) = \left(1 + \frac{1}{(1+z_p)^{\gamma+\kappa}}\right) \frac{(1+z)^{\gamma}}{1 + \left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}}$$

- Assumption: binary black hole distribution follows the star formation rate described by a Madau Dickinson model



# Using galaxy catalogs

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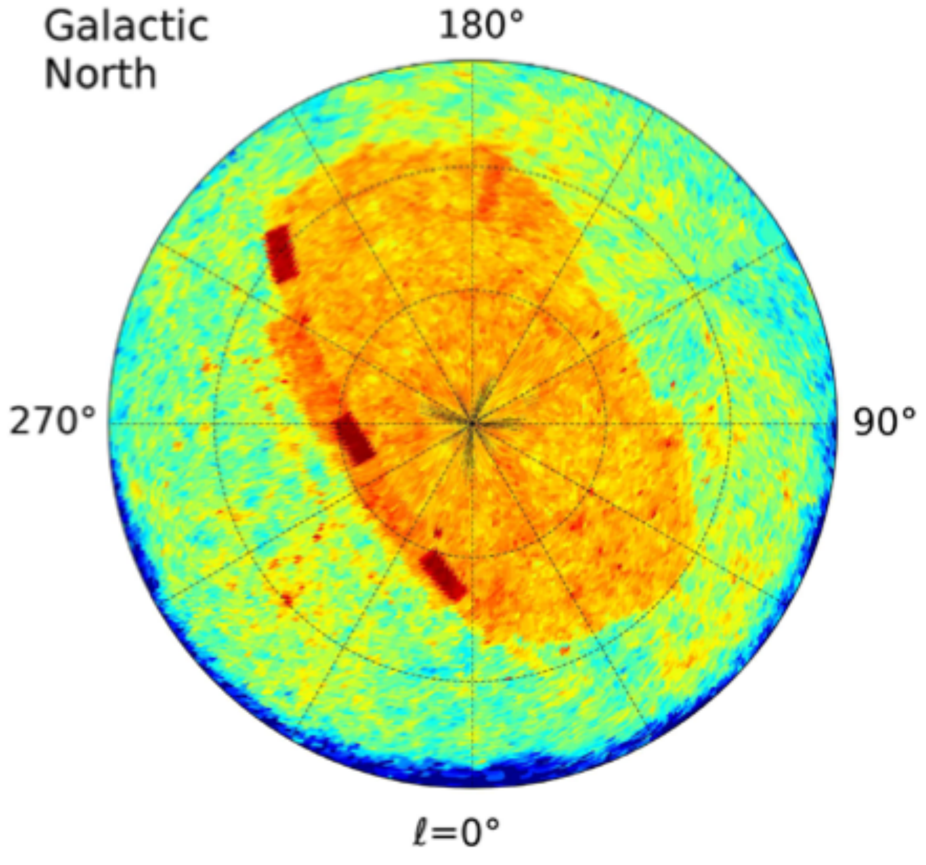
GW data

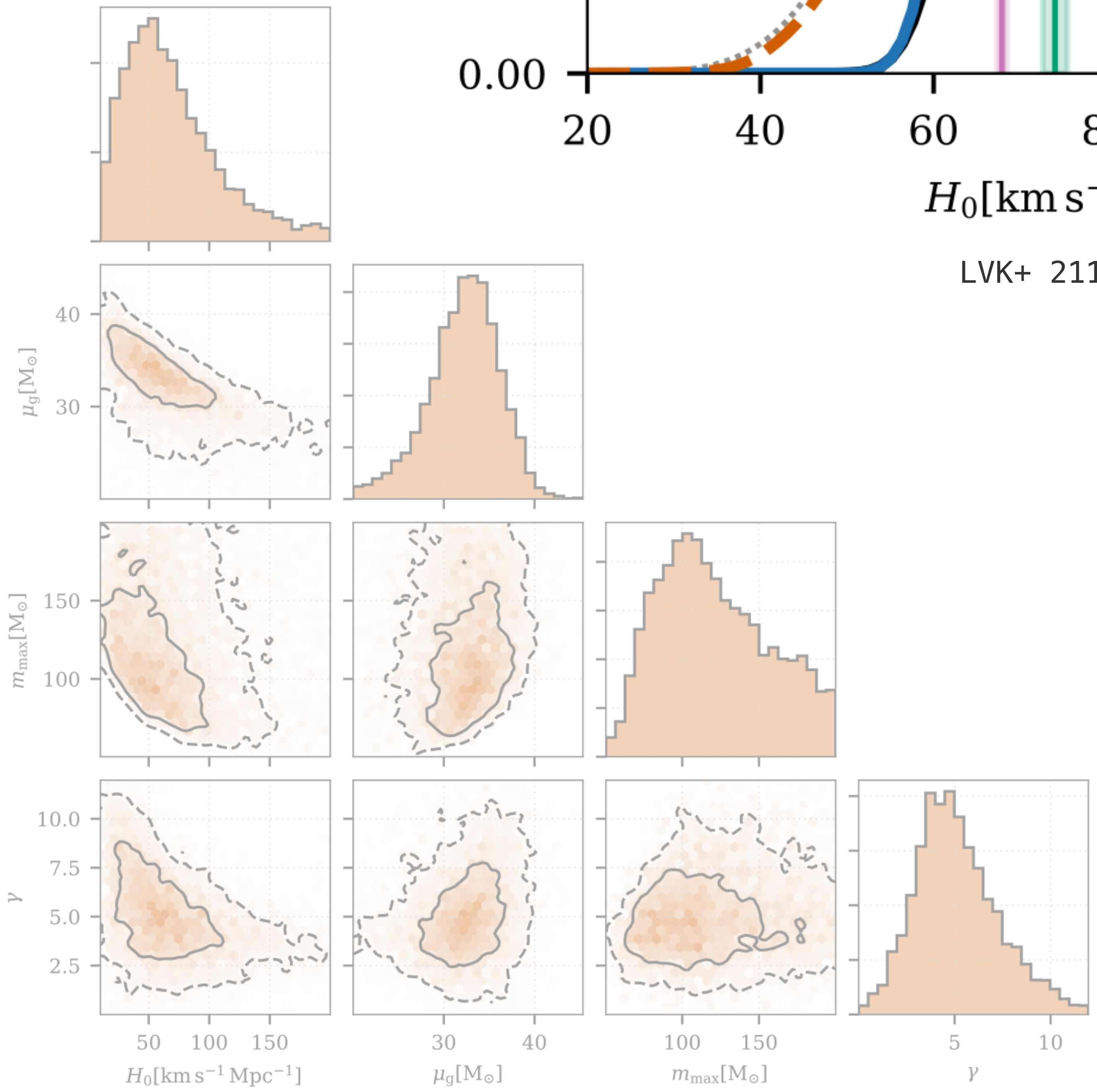
Pipelines

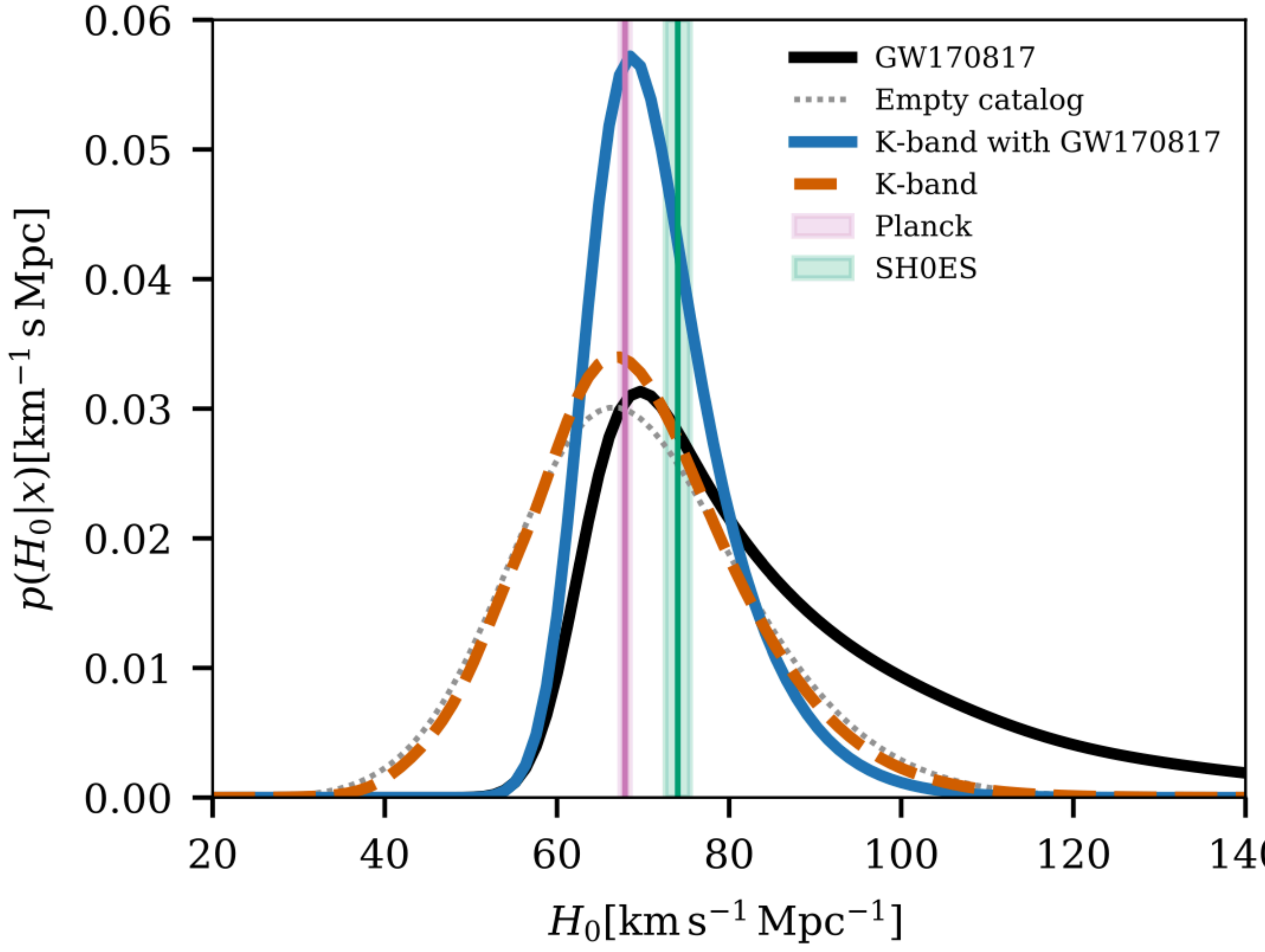
**GWcosmo**
IcaroGW

Galaxy Catalog Data




Dálya et al. 1804.05709



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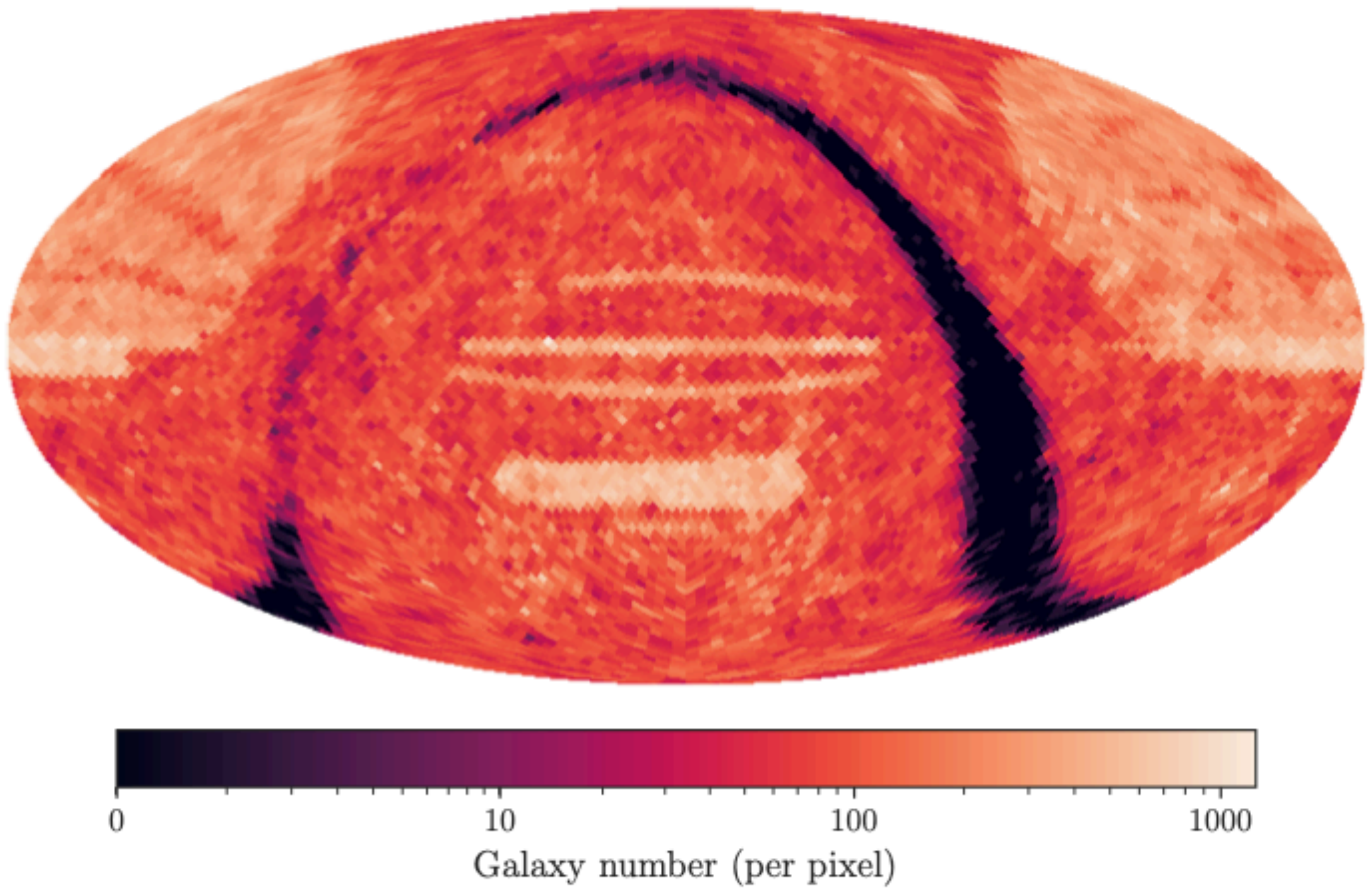
# GWcosmo *(Gray et al. 1908.06050)*

- Assumption: GW sources are found in galaxies
  - Possible GW hosts from galaxy catalogs *(Schutz 1986)*
  - *Weigh each galaxy according to luminosity*
- **Input:**
  - **Sky position**
  - Luminosity distance
  - Masses
- **Challenge:** Galaxy catalogs incompleteness
  - Calculate selection effects:
    - Host galaxy is observed (in catalog) or not




Gray et al. 2111.04629

# Statistical framework of GWcosmo (Gray et al. 1908.06050)

Bayesian analysis with selection effects

Prior $H_0$ | Observed events

$$p(H_0 \mid x_{\rm GW}, N_{\rm obs}, \Lambda_m, \Lambda_z) = p(H_0) \prod_{i=1}^{N_{\rm obs}} \sum_{g \in [G, \bar{G}]} p(x_{\rm GW,i} \mid H_0, \Lambda_m, g) p(g \mid H_0, \Lambda_m, \Lambda_z, \hat{d})$$

Posterior on $H_0$ | Host galaxy in catalog? | One event posterior

Hidden here are the selection effects

- Assumes a fixed mass and redshift distribution
- $\hat{d}$ : “event is detected”
- $N_{\rm obs}$ number of observed events

# Statistical framework of GWcosmo *(Gray et al. 1908.06050)*

Host galaxy in catalog?

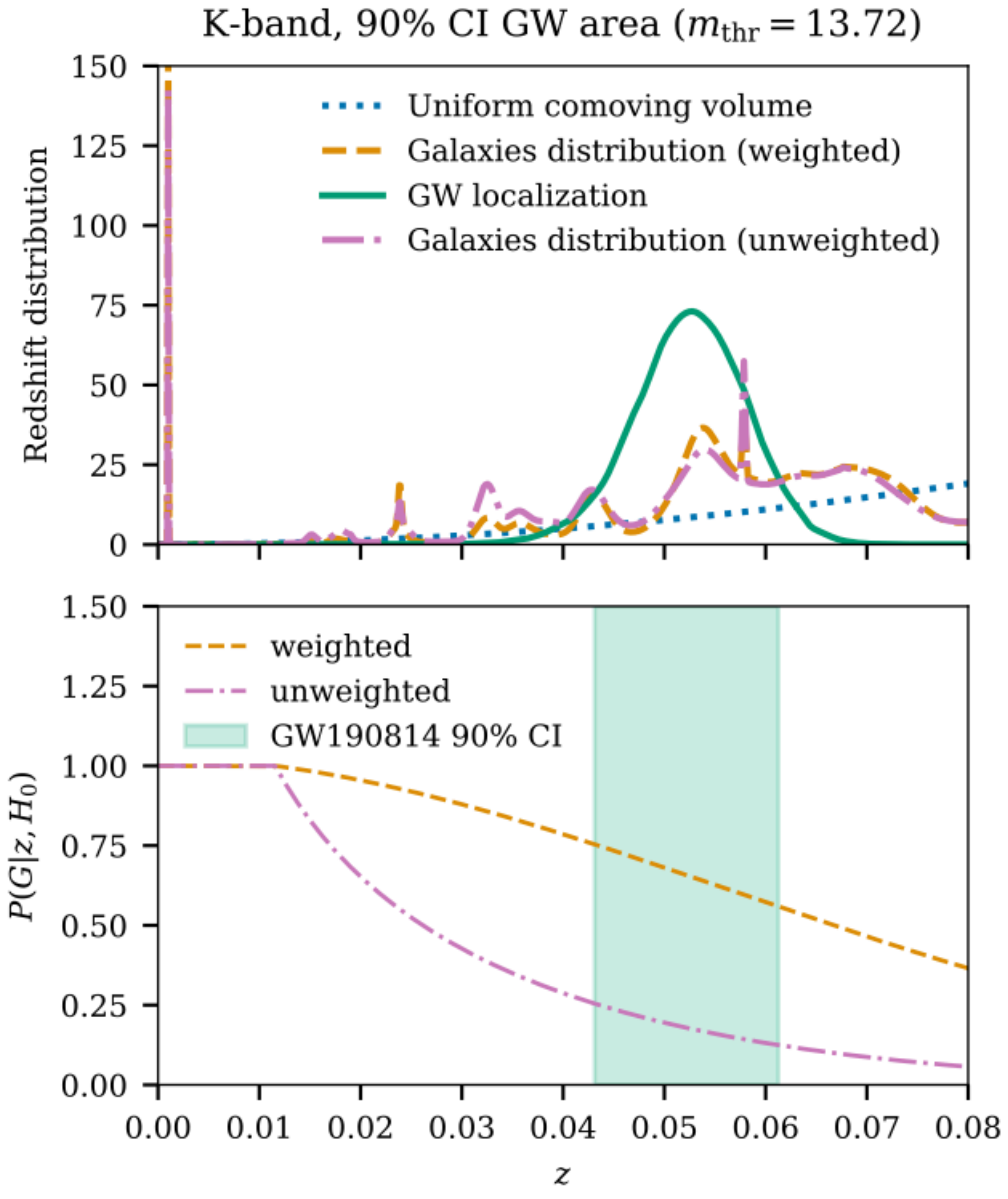
Probability of being in catalog

Probability of **not** being in catalog

$$\sum_{g\in[G,\bar{G}]} p(x_{\rm GW,i}|H_0,\Lambda_{m,z},g)p(g|H_0,\Lambda_{m,z},\hat{d}) = p(x_{\rm GW,i}|H_0,\Lambda_{m,z},G)p(G|H_0,\Lambda_{m,z},\hat{d}) + p(x_{\rm GW,i}|H_0,\Lambda_{m,z},\bar{G})p(\bar{G}|H_0,\Lambda_{m,z},\hat{d})$$

One event posterior

- Assumptions for selection effects
  - Apparent magnitude threshold of the galaxy catalog
  - Redshift distribution of galaxies
  - Luminosity distribution of galaxies (e.g. Schechter function)
- Pixelated approach: Treat selection effects as non-uniform in the sky

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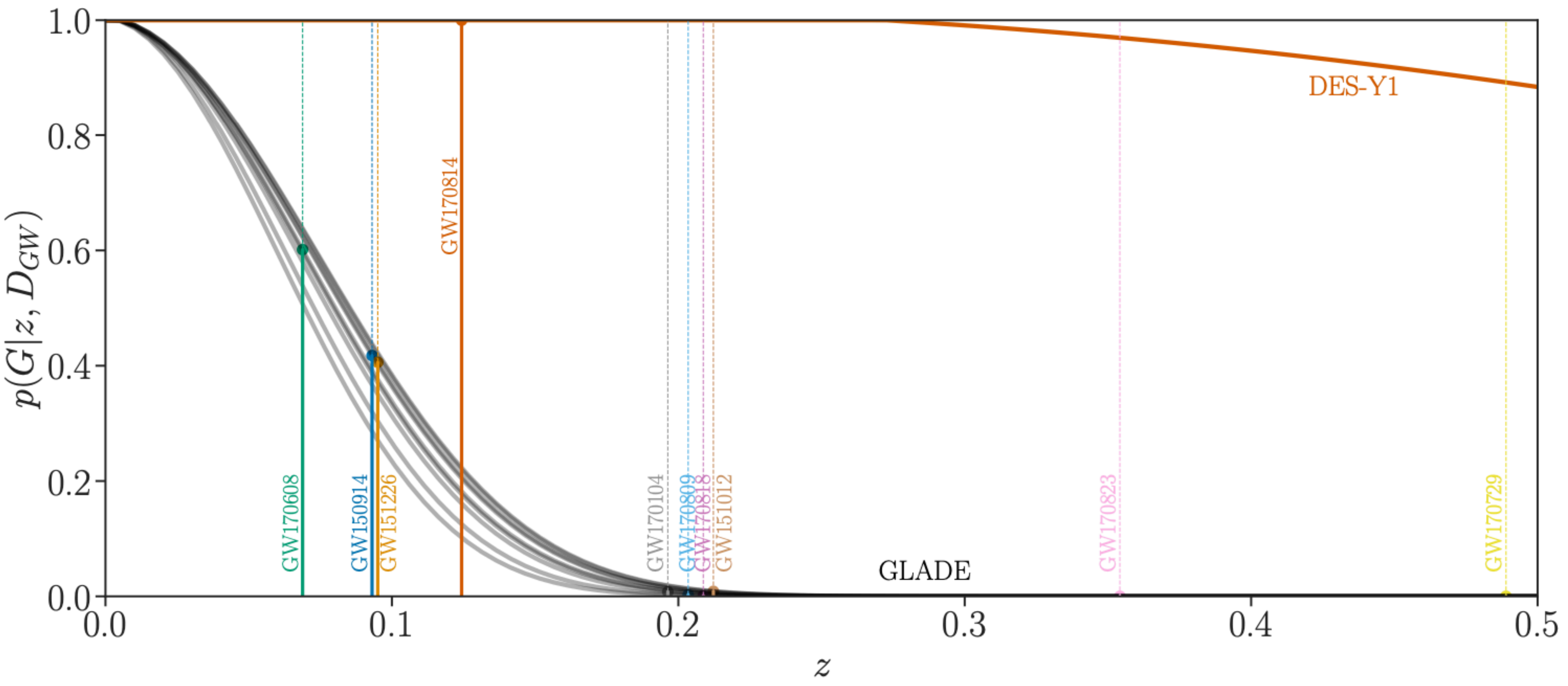
# Galaxy catalog used for GWcosmo

- All-sky catalog: Glade *(Dályа et al. 2018)*
- Partial coverage, but deeper in redshift: DES *(Drlica-Wagner et al. 2018)*

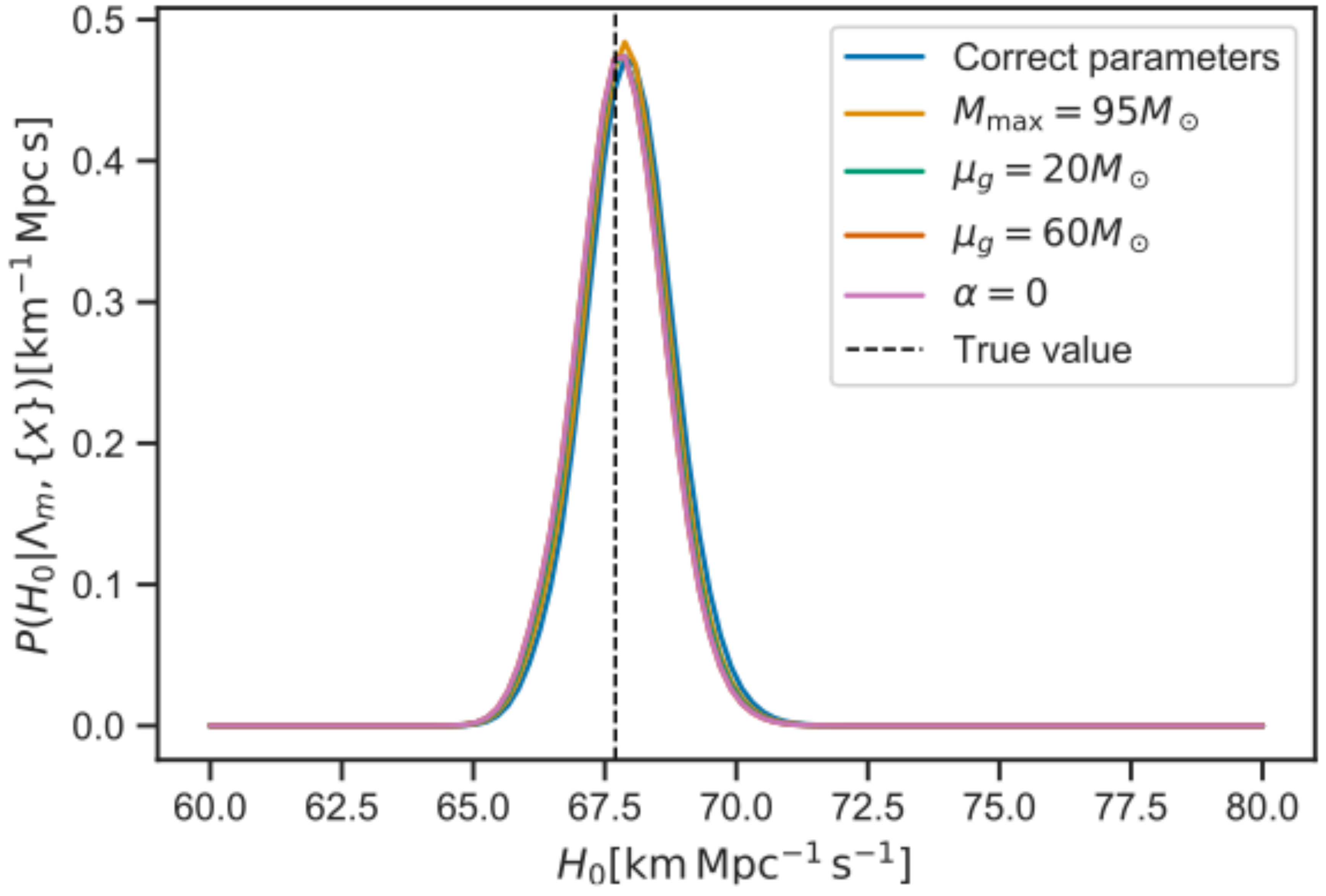
Probability of being in the catalog



# Two limits…

“GWcosmo”

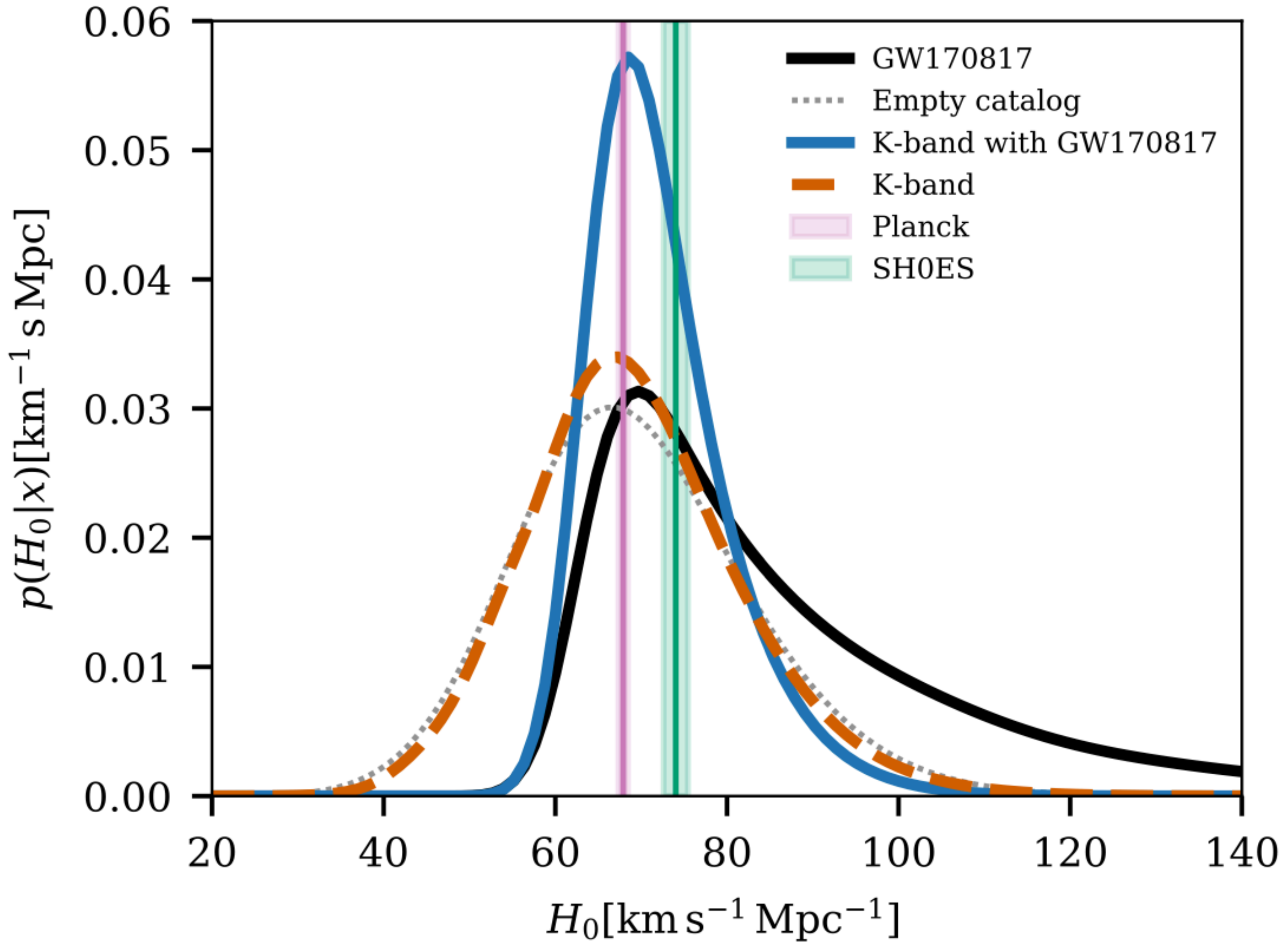
- Close-by signals, well-localised (very few compatible galaxies)
  - $H_0$ posterior is independent of the mass distribution assumed




2103.14663

“IcaroGW”

- Far signals, not well-localised
  - Redshift information from source frame mass distribution

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# Conclusions

- IcaroGW (no EM information) method allows to simultaneously constrain cosmological and population parameters
- GWcosmo uses galaxy catalog information to constrain $H_0$
- Strong degeneracies between the rate evolution $\gamma$, the overall rate of events $R_0$, the Hubble constant $H_0$
  - $\rightarrow$ Marginalize over population assumptions
- Help for O4 preparations is very welcome
- Active development

"GWcosmo"

- Marginalize over mass and redshift distribution

"IcaroGW"

- Modifications of gravity
- Evolving dark energy equation of state

# Thank you!

Questions?