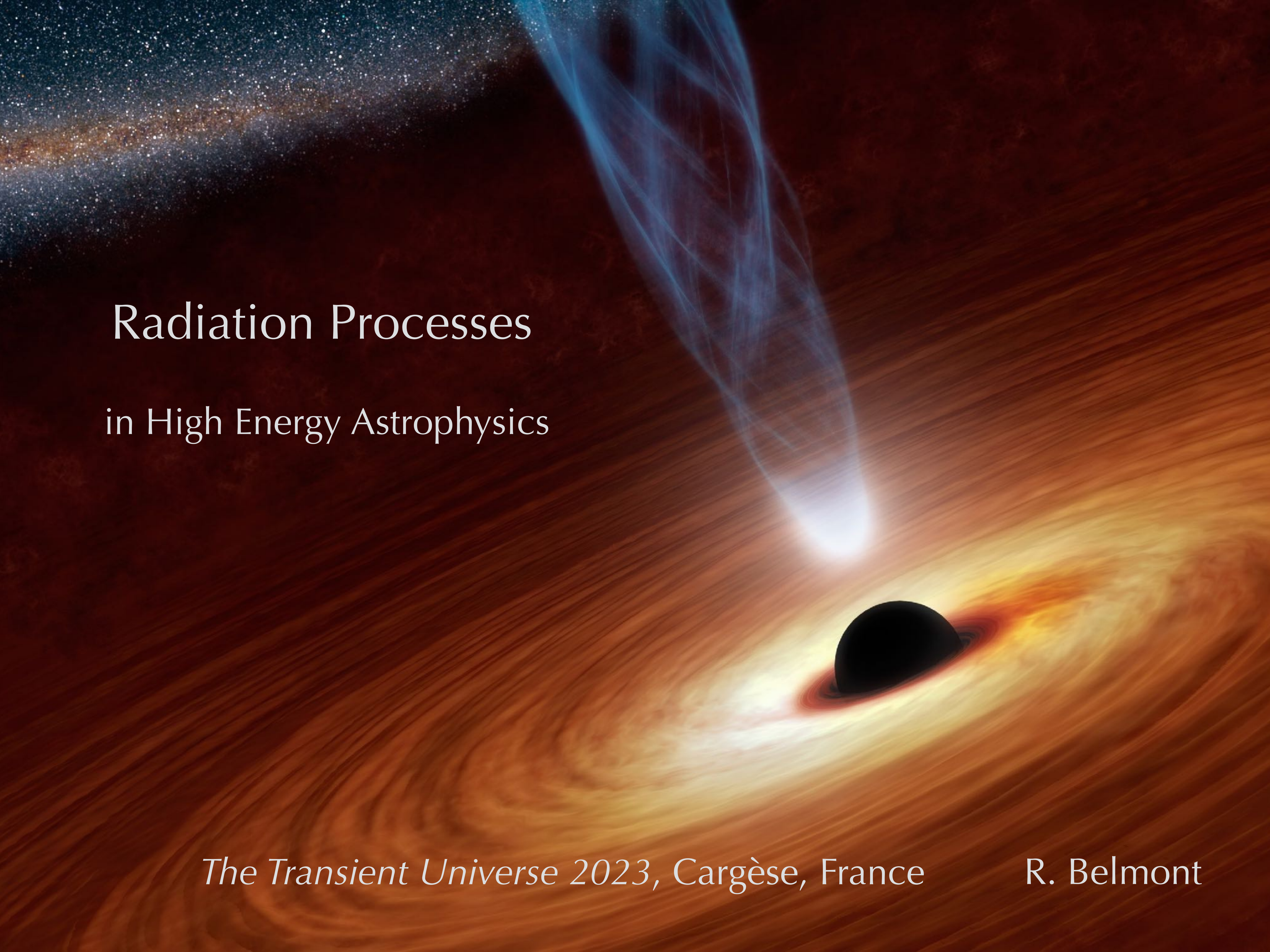


Radiation Processes

in High Energy Astrophysics

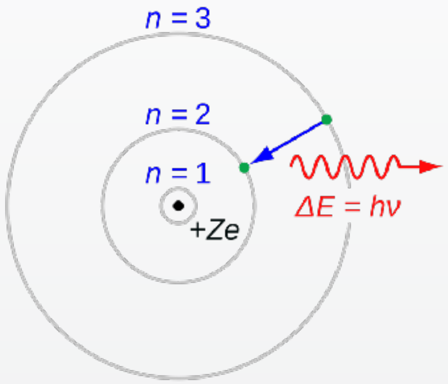
The Transient Universe 2023, Cargèse, France

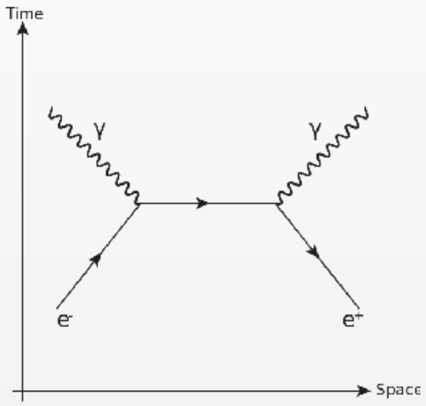
R. Belmont

I. Motivations

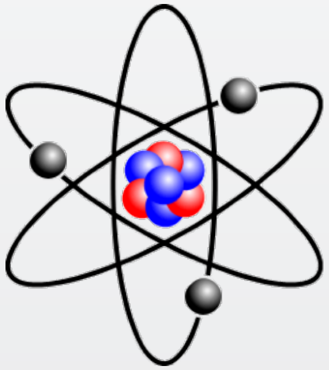
Photons as tracers

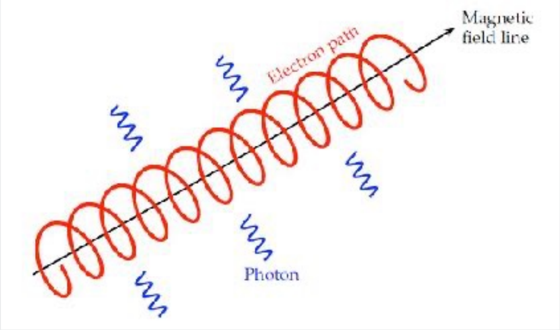
- **Interpreting observation :**
- **Many processes**
  - Atomic/nuclear lines, bremsstrahlung, synchrotron, Compton, pair production/annihilation, particle physics…

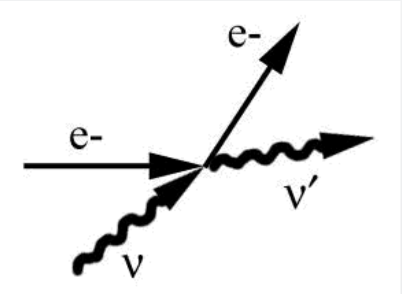




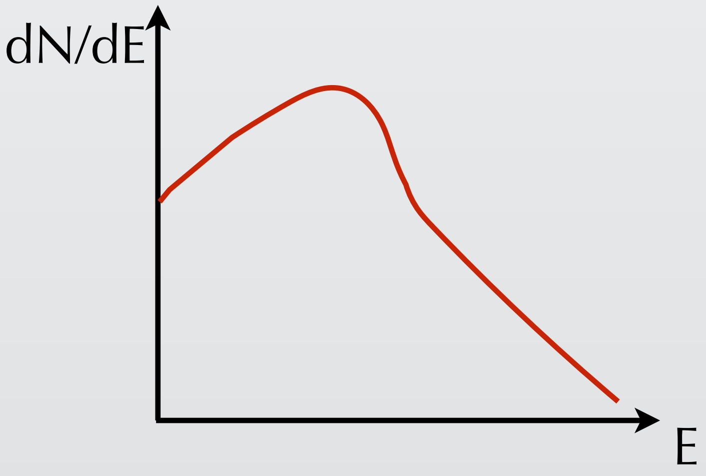
- **Particle nature**
  - leptons, protons, ions



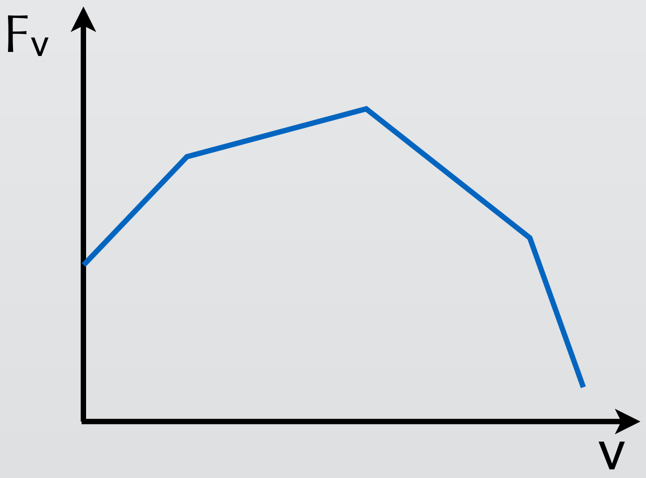




- **Transfer**
  - Thin/thick
- **Particle distributions**
  - Thermal, non-thermal, hybrid etc..

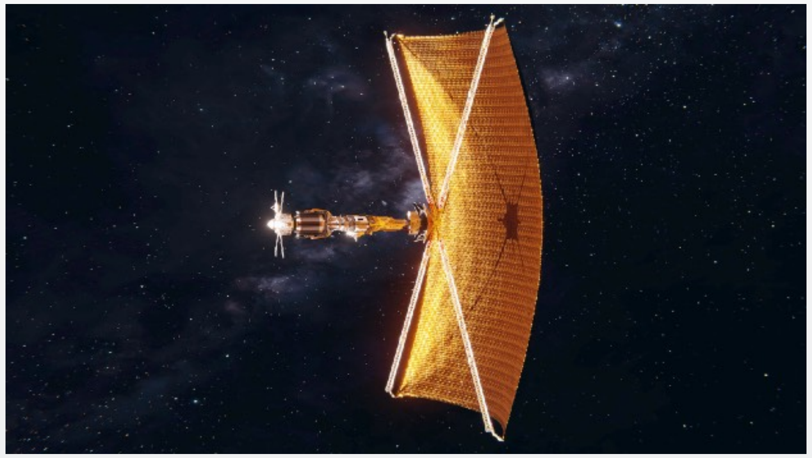


- **Homogeneous/inhomogeneous**
- **Isotropic/anisotropic medium**
- **Challenges:**
  - Identifying all these key aspects
  - Derive simple ways to model the emission



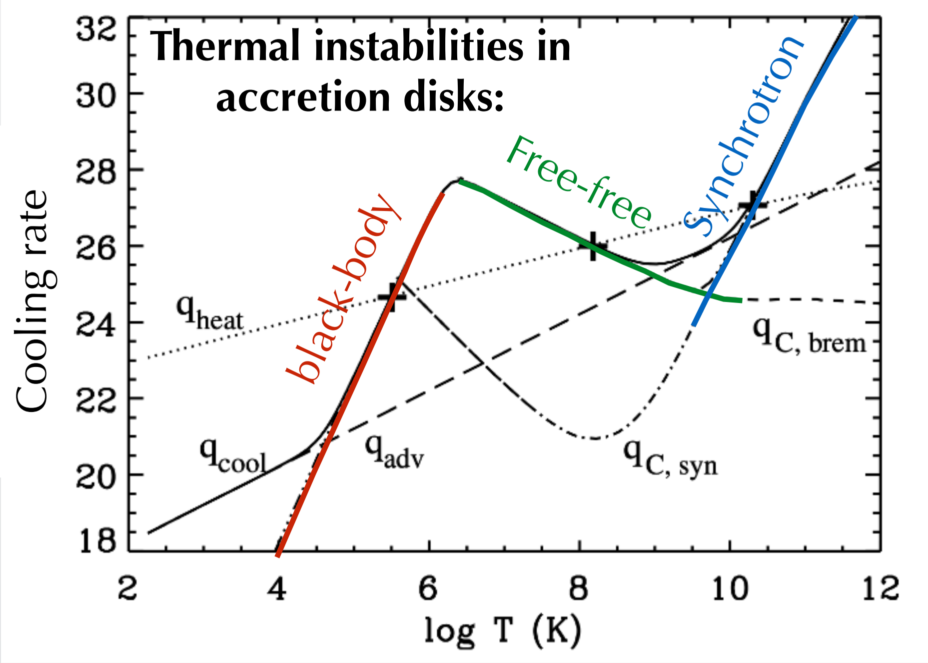
Photons as shapers

- **Shaping the properties of matter**



- **Dynamics**
  - Radiation pressure
  - -> e.g. Eddington Luminosity $L_{\mathrm{Edd}} = \dfrac{4\pi c G M m_p}{\sigma_T}$

- **Energetics:**
  - Cooling
    - e.g. thermal instabilities
  - heating
  - Thermalisation



Petrucci et al. 2010

- **Competition with many other processes**
  - acceleration, escape, adiabatic cooling…

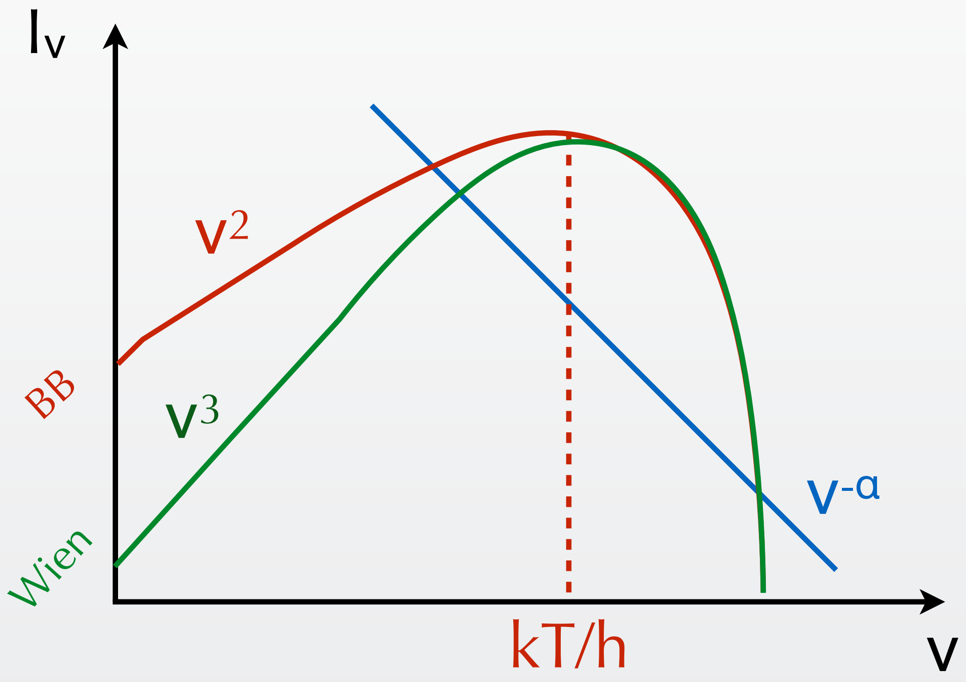
Outline

- **Back to basics**
    - Special relativity: Doppler and beaming
    - Emission in classical Electrodynamics
    - Emission/absorption
    - From one to many particles
- **Bremsstrahlung**
- **Magneto-bremsstrahlung**
    - Cyclo-synchrotron radiation
    - Curvature radiation
    - Diffuse Synchrotron radiation
    - Strong fields
- **Compton scattering**
- **Photon annihilation**

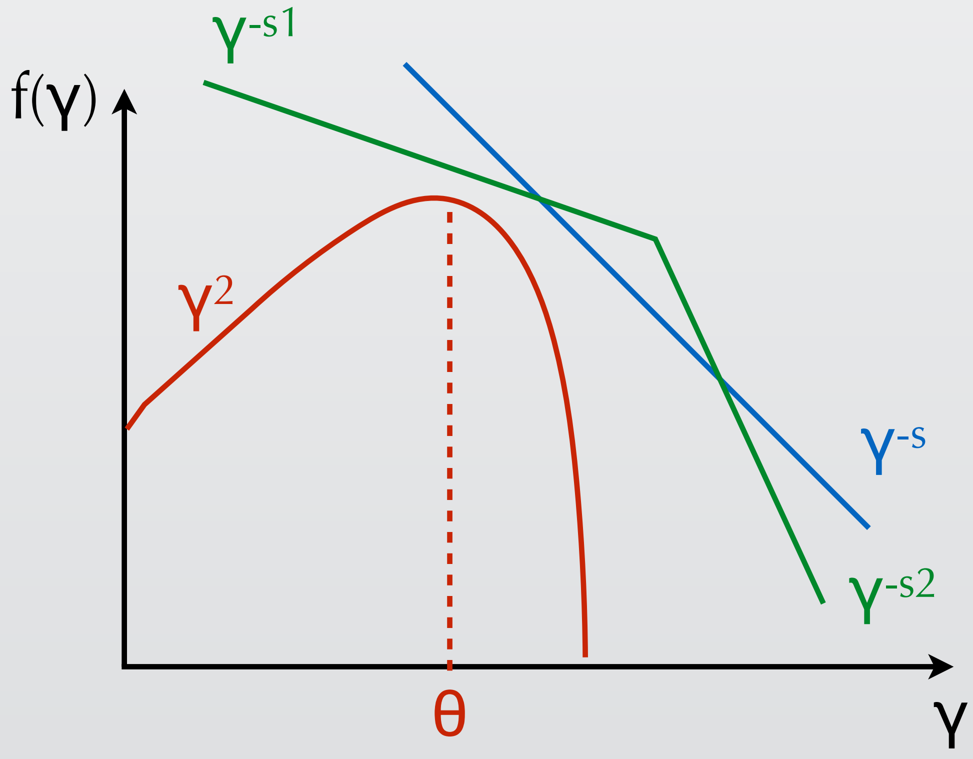
I. Back to Basics

Particles and Photons

- **Photon properties**
  - Energy: E=hν
  - Isotropic energy distributions:
    - Power-law $I_\nu \propto \nu^{-\alpha_i}$
    - Planck $I_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$
    - Wien $I_\nu = \frac{2h\nu^3}{c^2}e^{-h\nu/kT}$



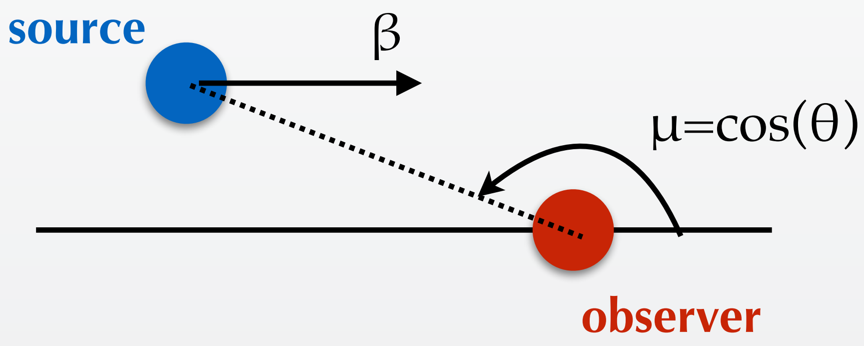
- **Particle properties**
  - Velocity: β=v/c
  - Energy: γ = E/mc²
  - Kinetic energy: γ-1 = (E-mc²)/mc²
  - Momentum: p = β γ = P/mc
  - Isotropic energy distributions:
    - Maxwell-Juttner (θ=kT/mc2): $f(\gamma) = \frac{\gamma p\ e^{-\gamma/\theta}}{\theta K_2(1/\theta)}$
    - Power-law: $f(\gamma) \propto \gamma^{-s}$
    - Hybrid
    - Broken power-laws



Special Relativity

- **Radiation properties are frame-dependent**

- **Two frames: source(‘)/observer**



- **Relativistic Doppler shift:** $\boxed{\nu = \nu' \delta}$
  - Doppler factor:

$$\frac{1}{2\gamma_0} \leq \delta = \frac{1}{\gamma_0(1 + \mu\beta_0)} \leq 2\gamma_0$$

Receding source

Approaching source

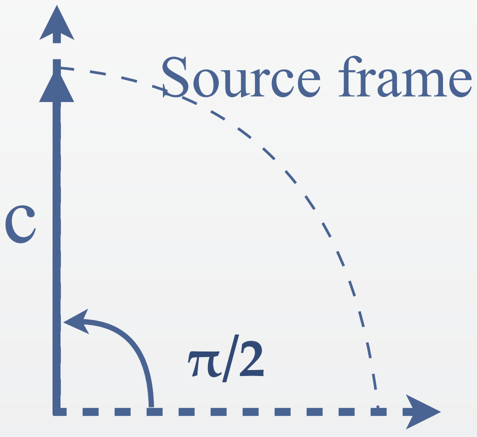
Time dilation

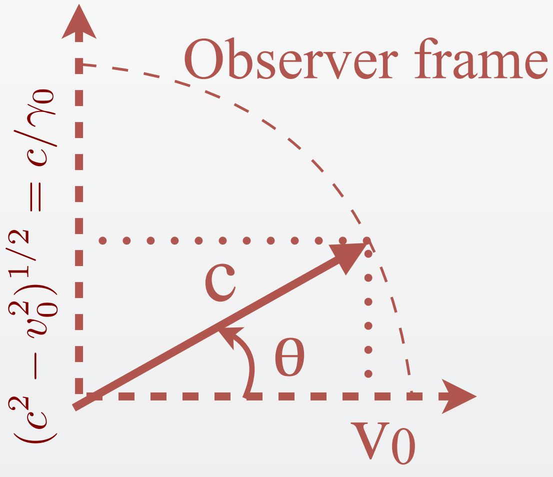
classical Doppler
(propagation)

  - Photon emitted in approaching sources are observed at higher energy
  - e.g. spectral features in Blazars, GRBs

Special Relativity

- **Relativistic Aberration:**





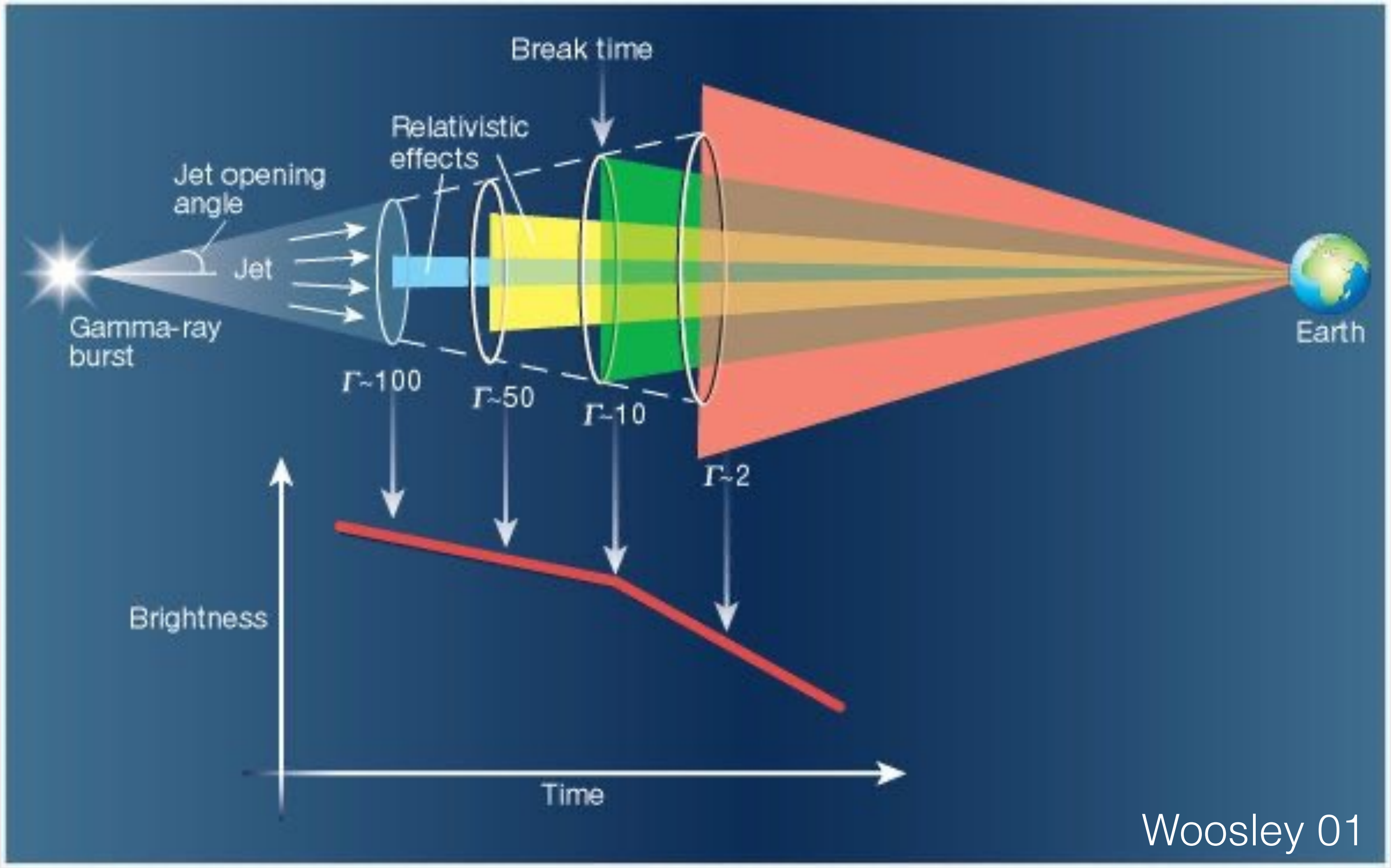
- Ex: Perpendicular photon emission
  - Observed with an angle: $\cos\theta = v_0/c$
  - For relativistic velocities: $\beta_0 \sim 1 - 1/(2\gamma_0^2)$ => $\theta \sim 1/\gamma_0$

- For any smooth angular distribution: $d\Omega = d\Omega'/\delta^2$
  - **Beaming**
  - The emission is beamed into a narrow cone

$$\boxed{\theta \leq \frac{1}{\gamma_0}}$$

Special Relativity

- **Example: Jet breaks in GRBs**

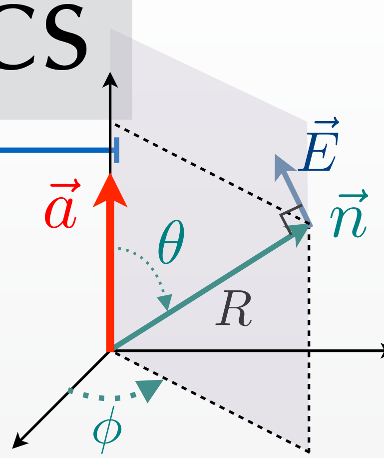


Woosley 01

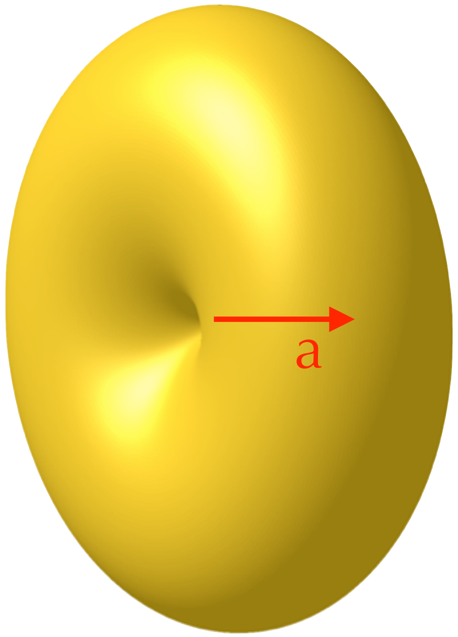
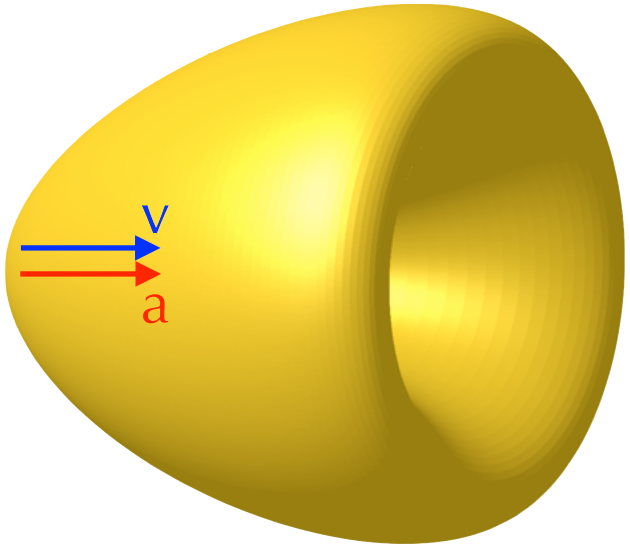
  - Geometry:
    - Constant jet opening angle θ_{jet}
    - Expansion
    - Slowing down (Γ_{jet} decreases)
    - Decrease of the emission
  - Relativistic beaming:
    - Local emission beamed in a cone of angle $\theta_{em}=1/\Gamma_{jet}$

- At early times: $\theta_{em} << \theta_{jet}$
  - Only a small fraction of the jet is visible
  - Deceleration => Increased visible area
  - Flux decrease partly compensated => Slow decrease of the received flux
- At later time : $\theta_{em} > \theta_{jet}$
  - The entire jet is seen
  - no more compensation
  - Steepening of the flux decrease

Emission in classical electrodynamics



- **Emission due to accelerated charges**
- **In the particle rest frame:** $\vec{E} = \dfrac{q}{Rc^2}\,\vec{n} \times (\vec{n} \times \vec{a})$

	Non Relativistic	**Any Regime**	
		Parallel v,a	Perpendicular v,a
Total Power	$P = \dfrac{2q^2a^2}{3c^3}$	$P = \dfrac{2q^2}{3c^3}\left(a_{\parallel}^2\gamma^6 + a_{\perp}^2\gamma^4\right)$	
Angular distribution	$\dfrac{dP}{d\Omega} = \dfrac{q^2a^2}{4\pi c^3}\sin^2\theta$	$\dfrac{dP}{d\Omega}(\theta,\phi)$	
	Dipolar emission	Beamed emission	
			
Spectrum	$\dfrac{dP_\nu}{d\Omega} \propto \left\lvert \mathrm{FT}[\vec{E}] \right\rvert^2$		

Emission and Absorption

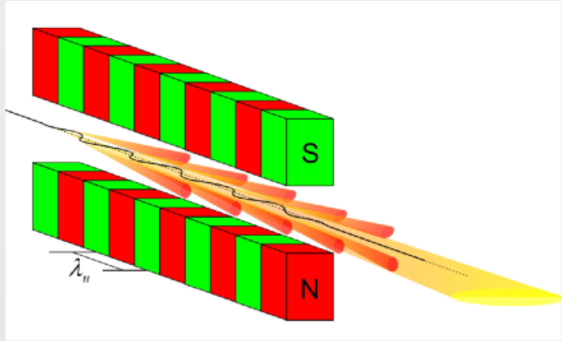
- **Any emission process is associated to a corresponding absorption process**
  - Emission (e.g. in erg/s/Hz/str/part.) : $P_\nu(\gamma, \Omega)$
  - Absorption cross-section (cm2/part) : $\sigma_\nu(\gamma, \Omega)$
  - e.g. free-free or synchrotron absorption

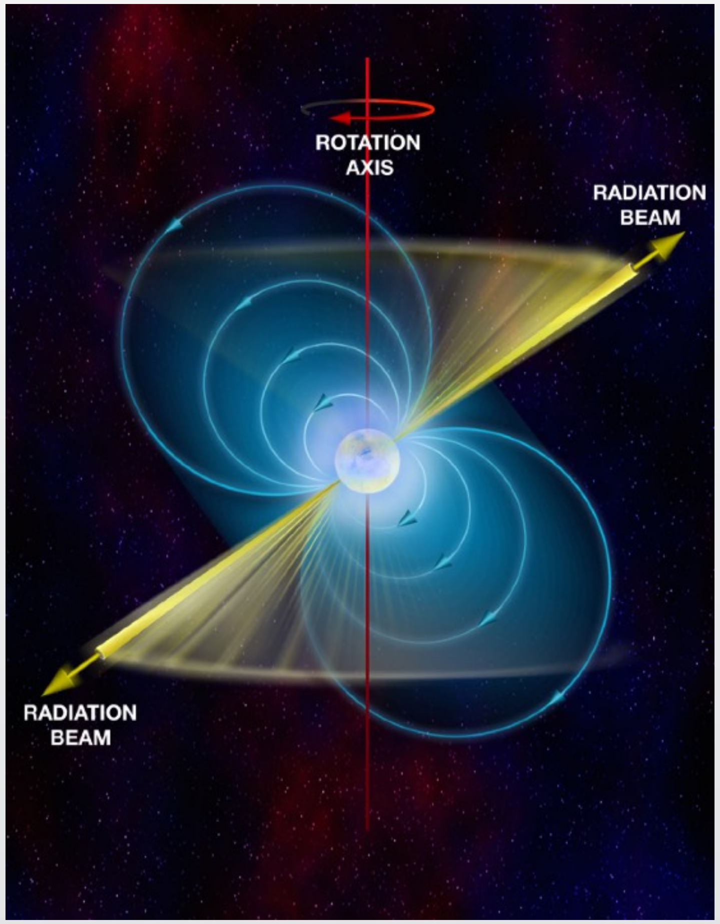
- **Both are linked by statistical properties in thermal equilibrium:**
  - For lines: Einstein coefficients
  - For continuum processes:

$$\sigma_\nu(\gamma, \Omega) = \frac{c^2}{2h\nu^3} \frac{1}{p\gamma} \left[\underbrace{(p\gamma P_\nu)_{\gamma + h\nu/mc2}}_{\text{True absorption}} - \underbrace{(p\gamma P_\nu)_\gamma}_{\text{Stimulated emission}} \right]$$

From one to many particles

- **Single particles:**
  - All radiation properties results from the acceleration properties
  - Different kinds of accelerations define different emission processes (e.g. free-free, Compton, synchrotron…)
- **Multiple particles:**
  - ***Coherent*** radiation
    - Particles have a collective motion, in a small area, on short time scales
    - E.g. antennae, synchrotron undulators, pulsar atmospheres (dipole emission, **FRBs**)…
    - The produced electric fields add up coherently: $\boxed{P_{\nu,\mathrm{tot}} = N^2 P_\nu}$





  - ***Incoherent*** radiation:
    - Particles motions are independant
    - e.g. free-free emission
    - The produced powers add-up: $\boxed{P_{\nu,\mathrm{tot}} = N P_\nu}$

This lecture

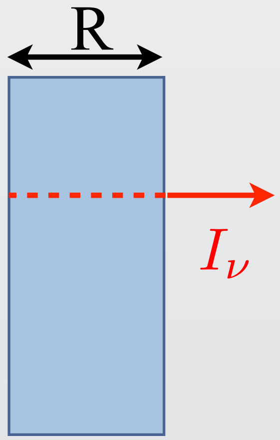
From one to many particles

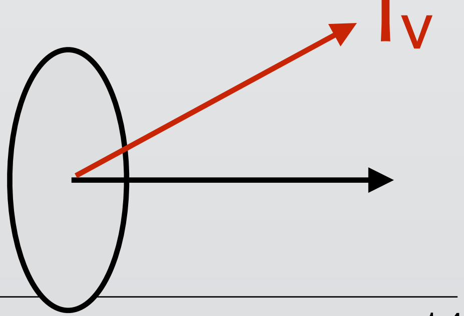
- **If isotropic media:**
  - Properties can be integrated over directions
  - Angle-integrated emission (erg/s/Hz/part) : $P_\nu(\gamma)$
  - Angle averaged absorption cross section (cm2/part) : $\sigma_\nu(\gamma)$

- **Once integrated over the particle distribution:**
  - Specific emissivity (erg/s/Hz/cm^3/str) $j_\nu = \frac{n}{4\pi}\int P_\nu(\gamma) f(\gamma) d\gamma$
  - Absorption/scattering coefficient (/cm) $\alpha_\nu = n\int \sigma_\nu(\gamma) f(\gamma) d\gamma$
  - Both are linked:
    - Thermal distribution: $\frac{j_\nu}{\alpha_\nu} = B_\nu$
    - Power-law distribution: $\frac{j_\nu}{\alpha_\nu} \sim \nu^{5/2}$

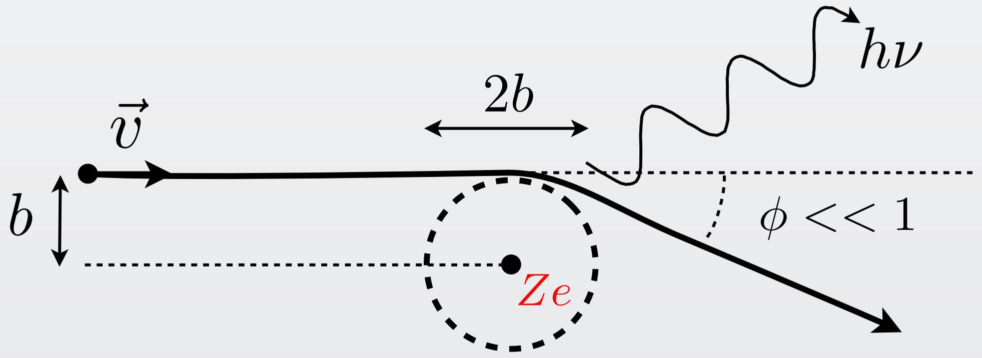
$$\int f(\gamma) d\gamma = 1$$

- **Transfer equation:** $\left(\frac{1}{c}\frac{\partial}{\partial t} + \vec{n}.\vec{\nabla}\right) I_\nu = j_\nu - \alpha_{a,\nu} I_\nu + \alpha_{s,\nu}(\langle I_\nu \rangle - I_\nu)$
  - I_ν (erg/s/Hz/cm^2/str) = Specific intensity
  - j_ν (erg/s/Hz/cm^3/str) = Specific emissivity
  - $\alpha_{a,\nu}$ (1/cm) = Absorption coefficient $\tau_{a,\nu} \sim \alpha_{a,\nu} R$
  - $\alpha_{s,\nu}$ (1/cm) = Scattering coefficient $\tau_s \sim \alpha_s R$
  - Effective optical depth: $\tau^* \sim \sqrt{\tau_a(\tau_a + \tau_s)}$
  - Optically thin/thick: τ=1



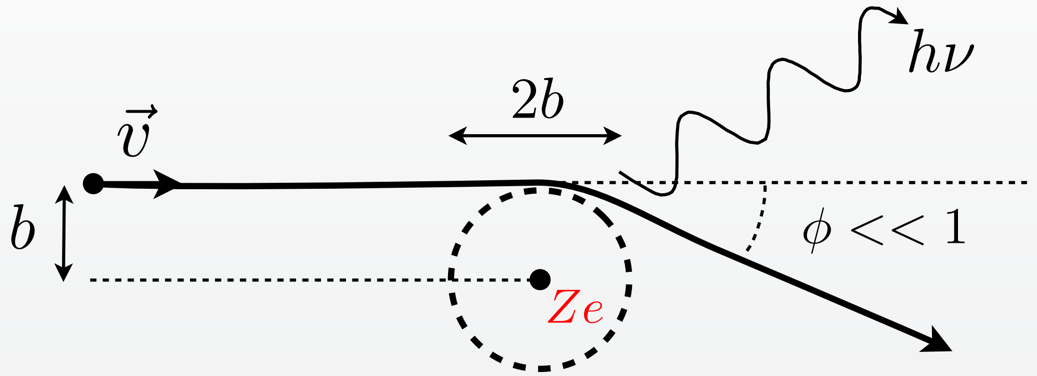


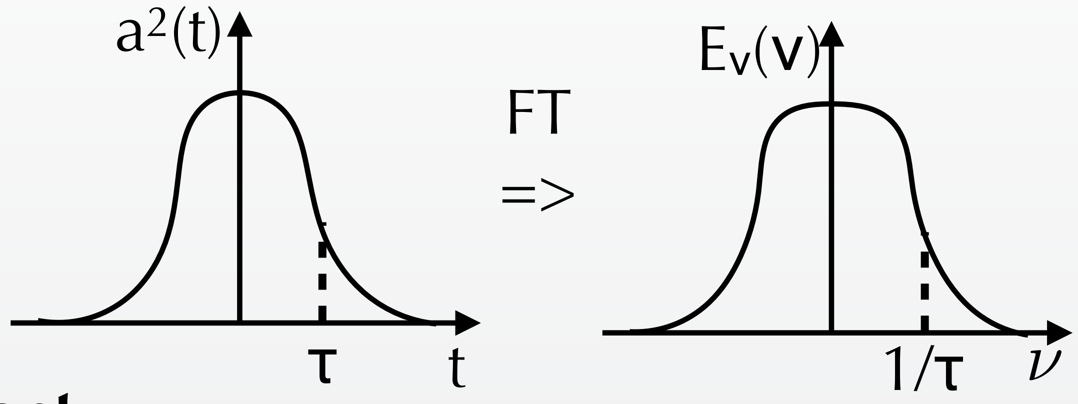
II. Bremsstrahlung



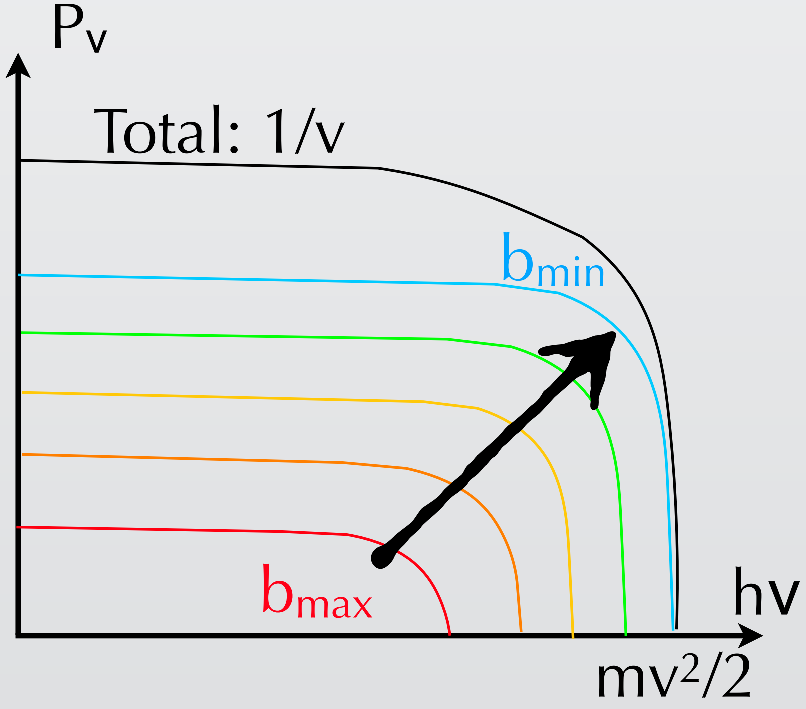
- **Radiation of charges accelerated by the Coulomb field of other particles**

II. Bremsstrahlung





- **Simple derivation for a single interaction event:**
  - Non-relativistic + small deflexion angle
  - Constant acceleration: $a = \dfrac{Ze^2}{mb^2}$ during interaction time $\tau = 2b/v$
  - Total energy radiated: $E(b,v) = \tau P_{int} = \tau \dfrac{2e^2a^2}{3c^3} = \dfrac{4Z^2e^6}{3m^2c^3vb^3}$
  - Differential energy rather uniform $E_\nu(b,v) = \dfrac{E}{\nu_c} = \dfrac{8Z^2e^6}{3m^2c^3v^2b^2}$
  - up to : $\nu_c = 1/\tau = v/2b$
  - Close particles produce more intense and more energetic radiation



- **Emission from many interactions**
  - Power radiated for many interactions:

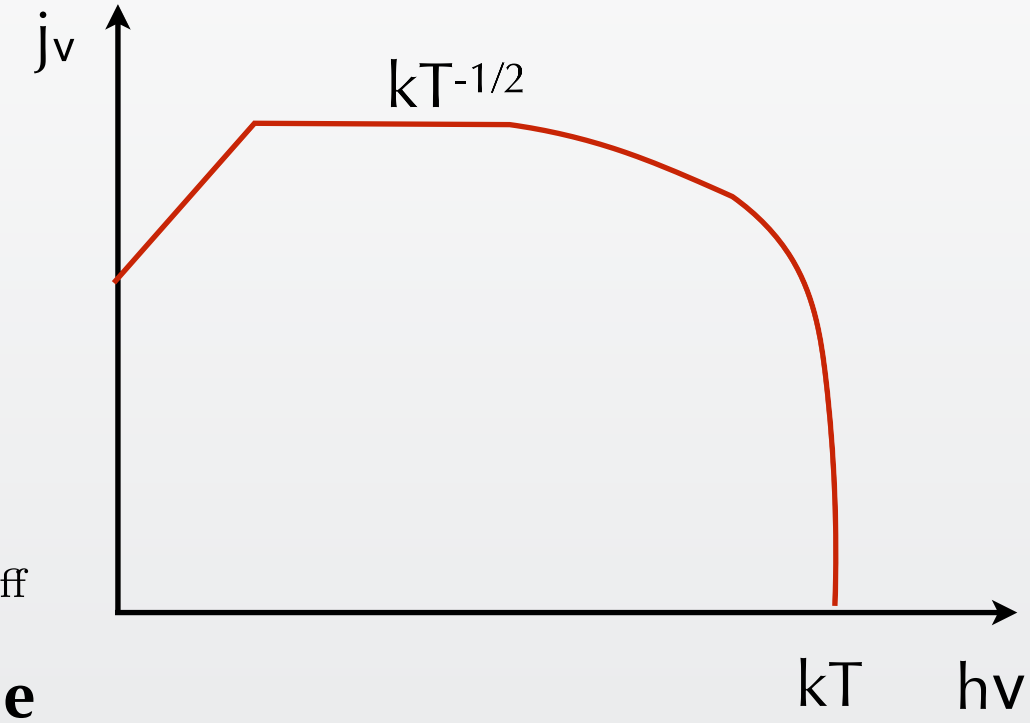
$$P_\nu(v) = n_e n_i v \int E_\nu 2\pi b db = n_e n_i \frac{16Z^2e^6}{3m^2c^3v} \log\left(\frac{b_{\max}}{b_{\min}}\right)$$

  - Cutoff at $h\nu_{\max} = mv^2/2$

Bremsstrahlung

- **For isotropic particle distributions:** $j_\nu^{\rm ff} = \frac{1}{4\pi}\int P_\nu f(v)dv$

- **For thermal distributions:**
  - Flat spectrum: $\boxed{j_\nu^{\rm ff} \propto n_i n_e Z^2 T^{-1/2} e^{-\frac{h\nu}{k_B T}}\ \bar{g}_{\rm ff}(\nu, T)}$
  - Gaunt factor $g_{\rm ff}$ close to unity
  - Cutoff frequency $\boxed{h\nu_c = k_B T}$
  - Total Power: $j^{\rm ff} \propto n_i n_e Z^2 T^{1/2}$
  - Absorption coefficient: $\alpha_\nu^{\rm ff} \propto n_i n_e Z^2 T^{-3/2} \nu^{-2}\ \bar{g}_{\rm ff}$

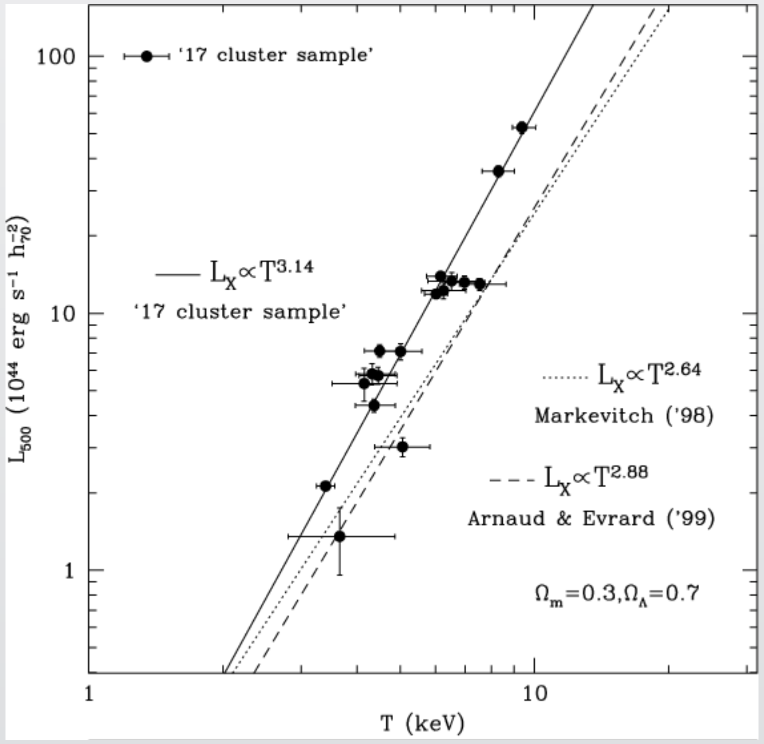


- **Exact calculations imply minor corrections in the Gaunt factor**

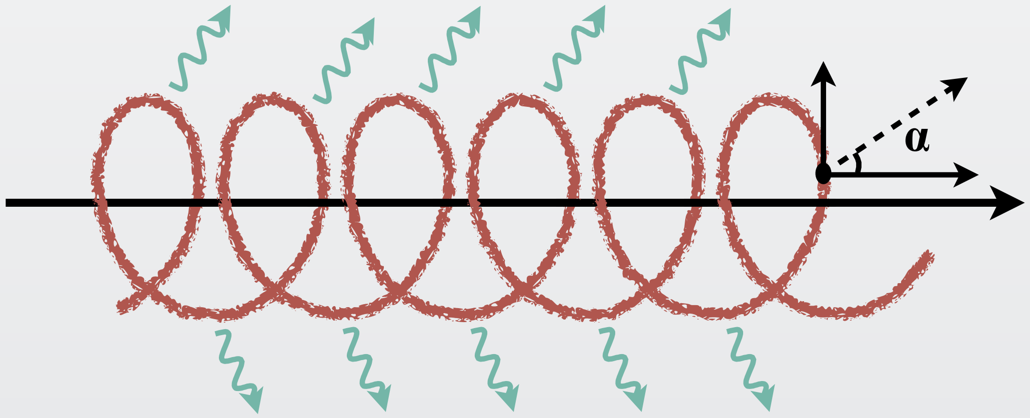
- **Observed in e.g.:**
  - Accretion column onto white dwarfs
  - Intra-cluster gas :

$$L_X \propto \underset{\big\downarrow}{n(T,R)^2}\ T^{1/2}\ R^3$$

Test cluster physics (viral, heating etc…)



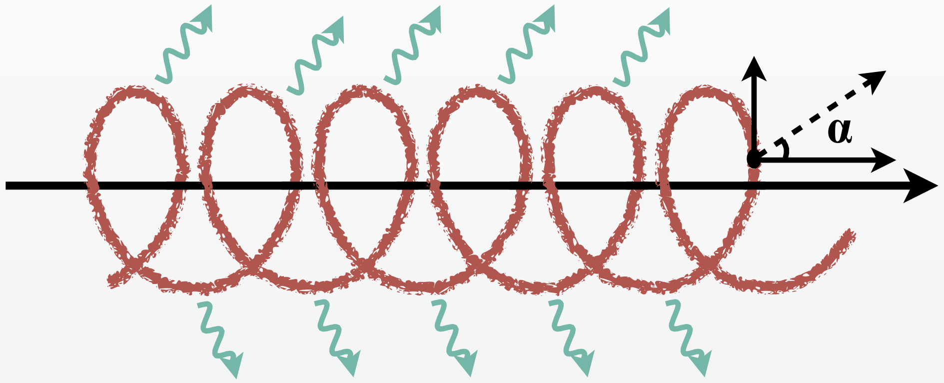
III. Magneto-Bremsstrahlung



- **Emission of charges defected by a magnetic field:**
  - Uniform field: cyclo-synchrotron radiation
  - Curved field: curvature radiation
  - Turbulent field: diffusive synchrotron radiation

Orbital motion

- **Orbital motion:**
  - Pitch angle α
  - Larmor frequency: $\boxed{\nu_B = \frac{\nu_L}{\gamma} = \frac{1}{\gamma}\frac{qB}{2\pi mc} \propto \frac{B}{\gamma}}$
  - Larmor radius: $\boxed{r_B = \frac{mc}{qB}p_\perp \propto \frac{\gamma}{B}}$

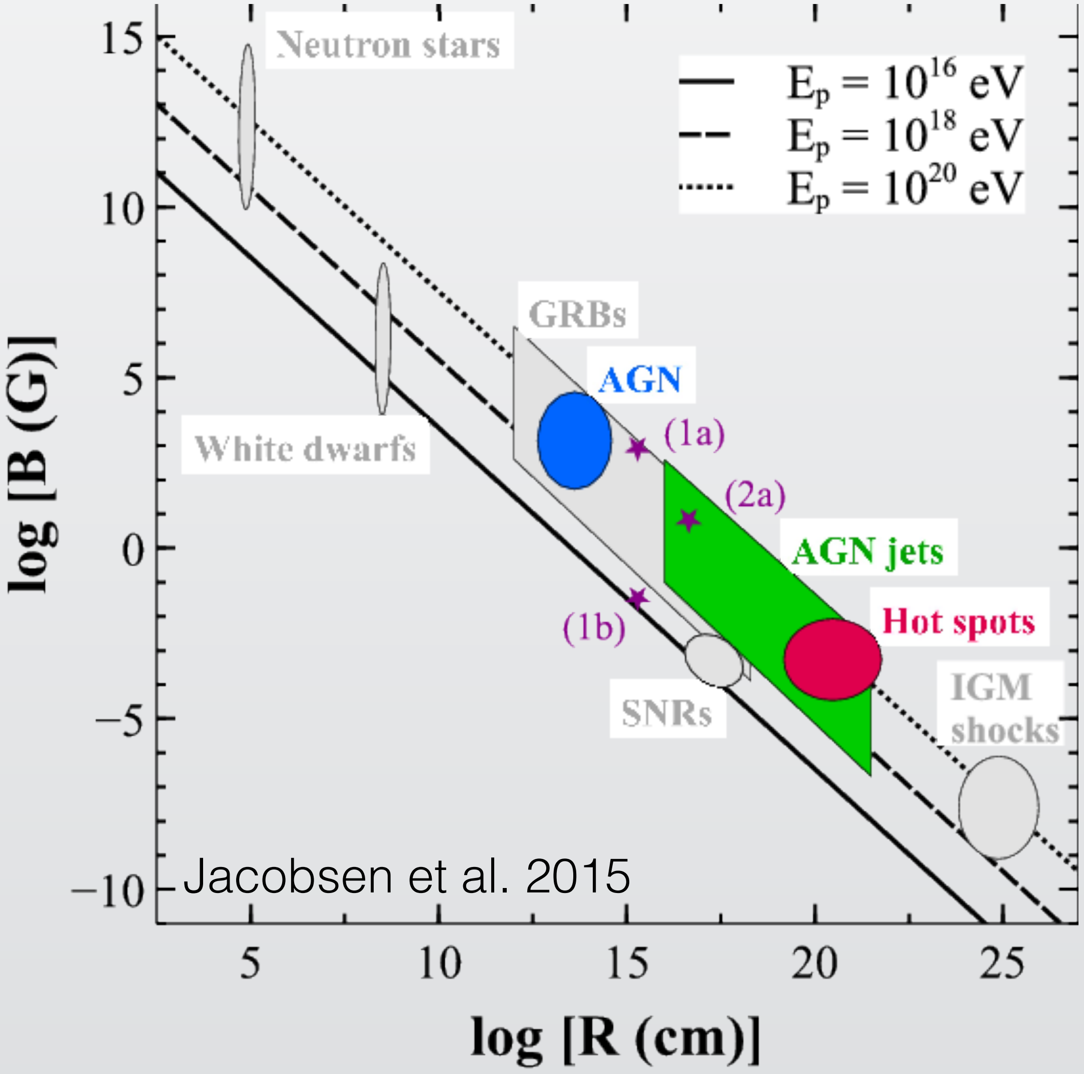


- **Hillas criterium for particle acceleration:**
  - Most acceleration processes require several gyroperiods
  - Particles must be confined within the acceleration site for several gyroperiods

$$r_B < R_{\text{source}}$$

  - Maximal acceleration energy

$$E_{\max} = qBR_{\text{source}}$$



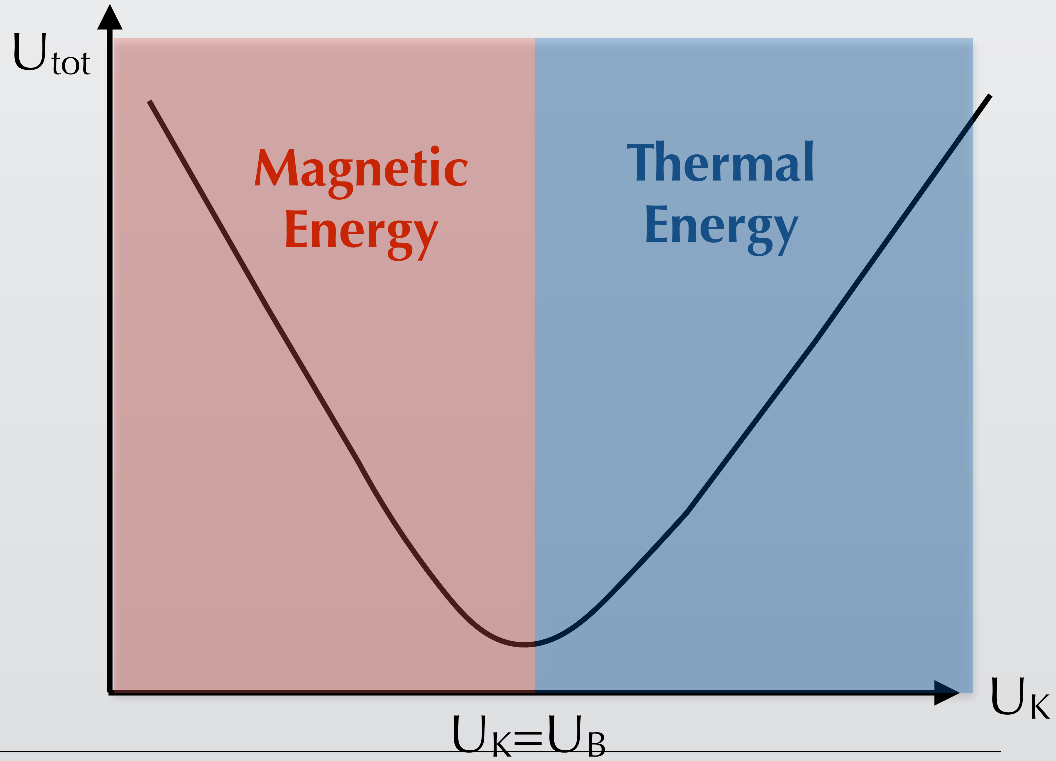
Jacobsen et al. 2015

II.1 Cyclo-Synchrotron Radiation

- **Assumptions:**
  - Uniform field + small energy losses + Classical mechanics
- **Total power radiated per particle:**
  - Perpendicular acceleration: $a_\perp = \frac{qB}{mc}\frac{v_\perp}{\gamma}$
  - Pitch-angle dependent power: $P = 2c\sigma_T U_B p_\perp^2$
  - Average over isotropic distributions: $P = \frac{4}{3}c\sigma_T U_B p^2 \propto B^2\gamma^2$
  - With the magnetic energy density: $U_B = \frac{B^2}{8\pi}$
- **Cooling time:**

$$t_s = \frac{(\gamma-1)mc^2}{P} = \frac{3mc^2}{4c\sigma_T U_B(\gamma+1)} \propto B^{-2}\gamma^{-1}$$

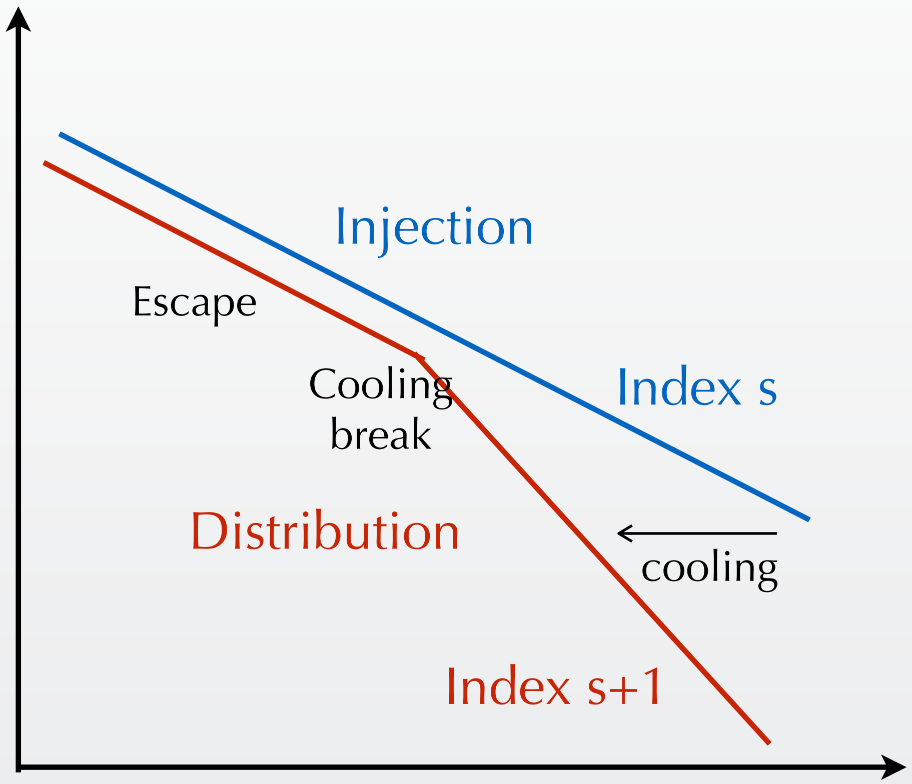
- **Minimal energy theorem:**
  - $U_{tot} = U_K + U_B$
  - $L = N\, U_B\, U_k^q$
  - $U_{tot} = U_K + L/(NU_k^q)$
  - $U_K = q\, U_B$ corresponds to the minimal energy



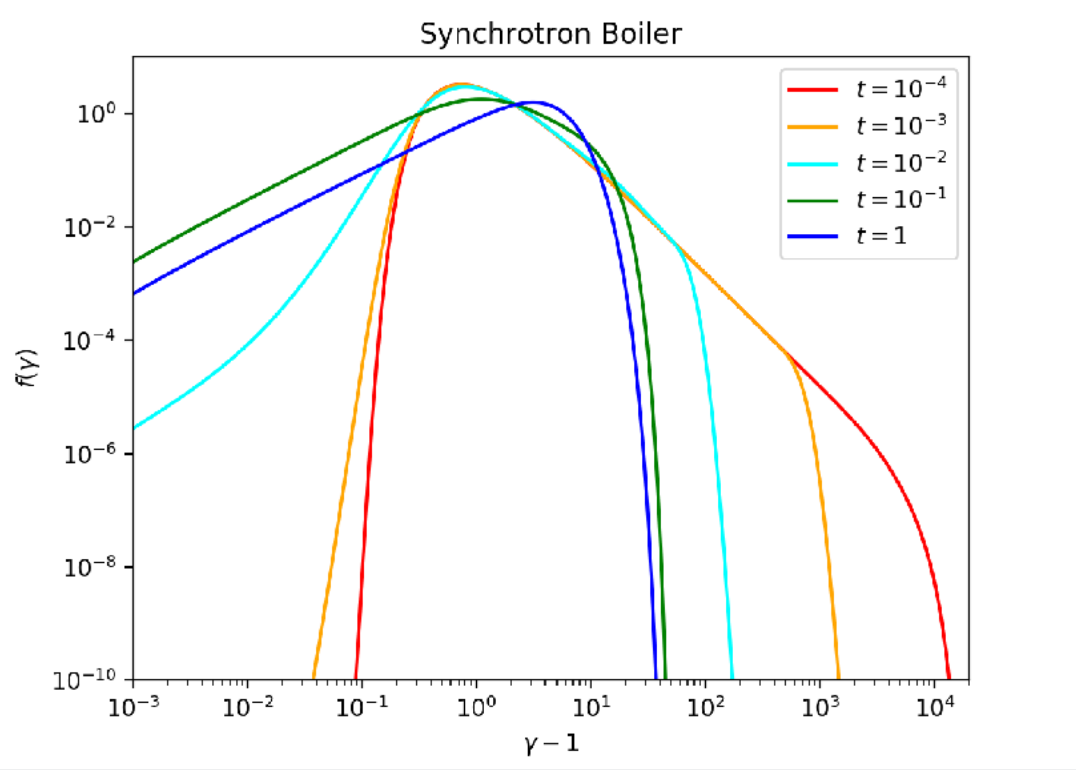
Synchrotron Interactions

- **Particle Cooling:** $$\frac{dE}{dt} = \frac{4}{3} c \sigma_T U_B p^2$$

- **Distribution slope:**
  - Cooling vs acceleration
  - Evolution equation: $$\frac{dN_\gamma}{dt} = \frac{d}{d\gamma}(-\dot{\gamma} N_\gamma) + Q_{\text{inj}} - \frac{N_\gamma}{t_{\text{esc}}}$$
  - Non thermal injection: $Q_{\text{inj}}(\gamma) \propto \gamma^{-s}$
  - => cooling break and steepening $N_\gamma \propto \gamma^{-(s+1)}$

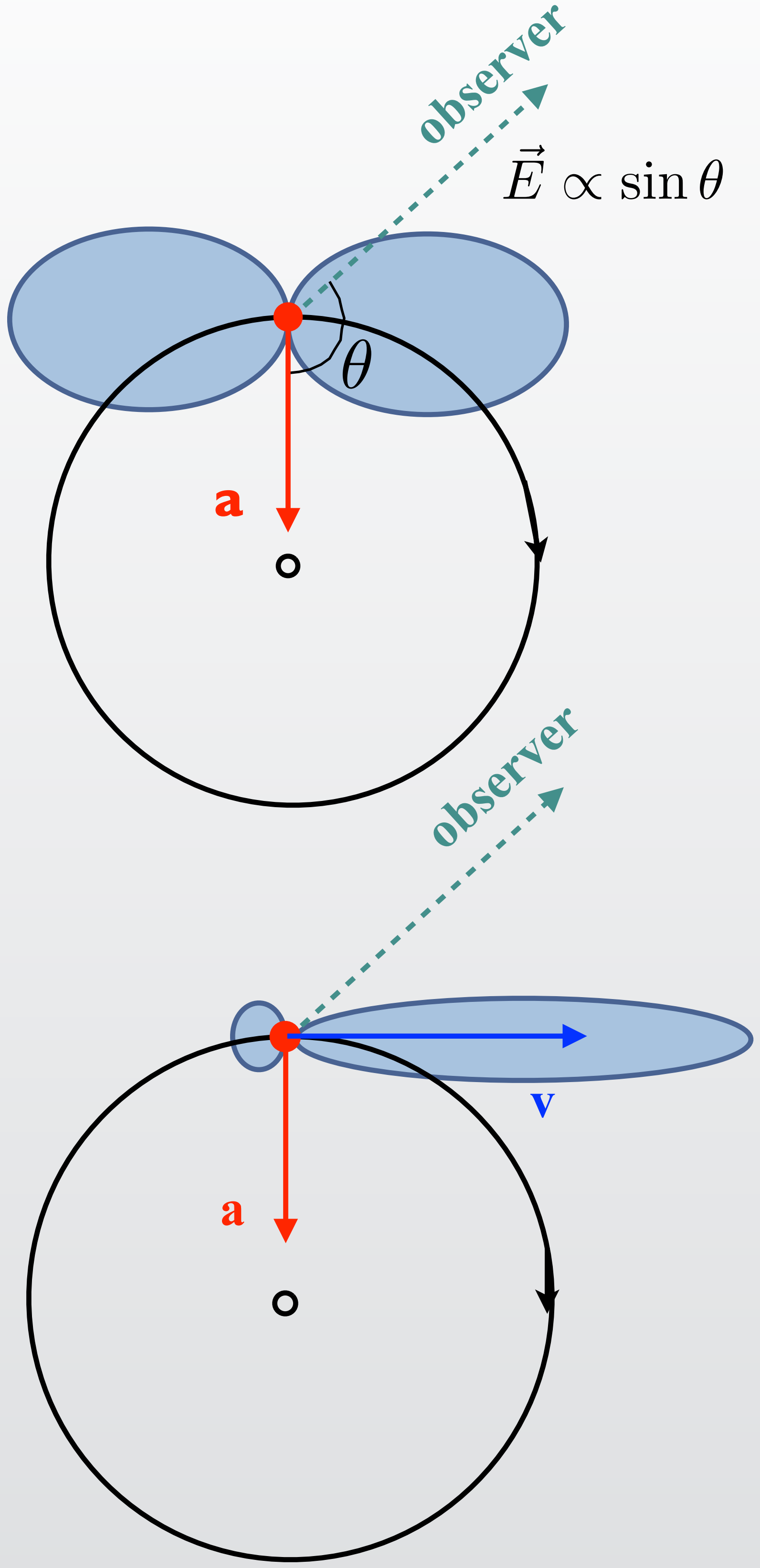


- **Thermalisation:**
  - Emission + Absorption => synchrotron boiler



Cyclo-Synchrotron Spectrum

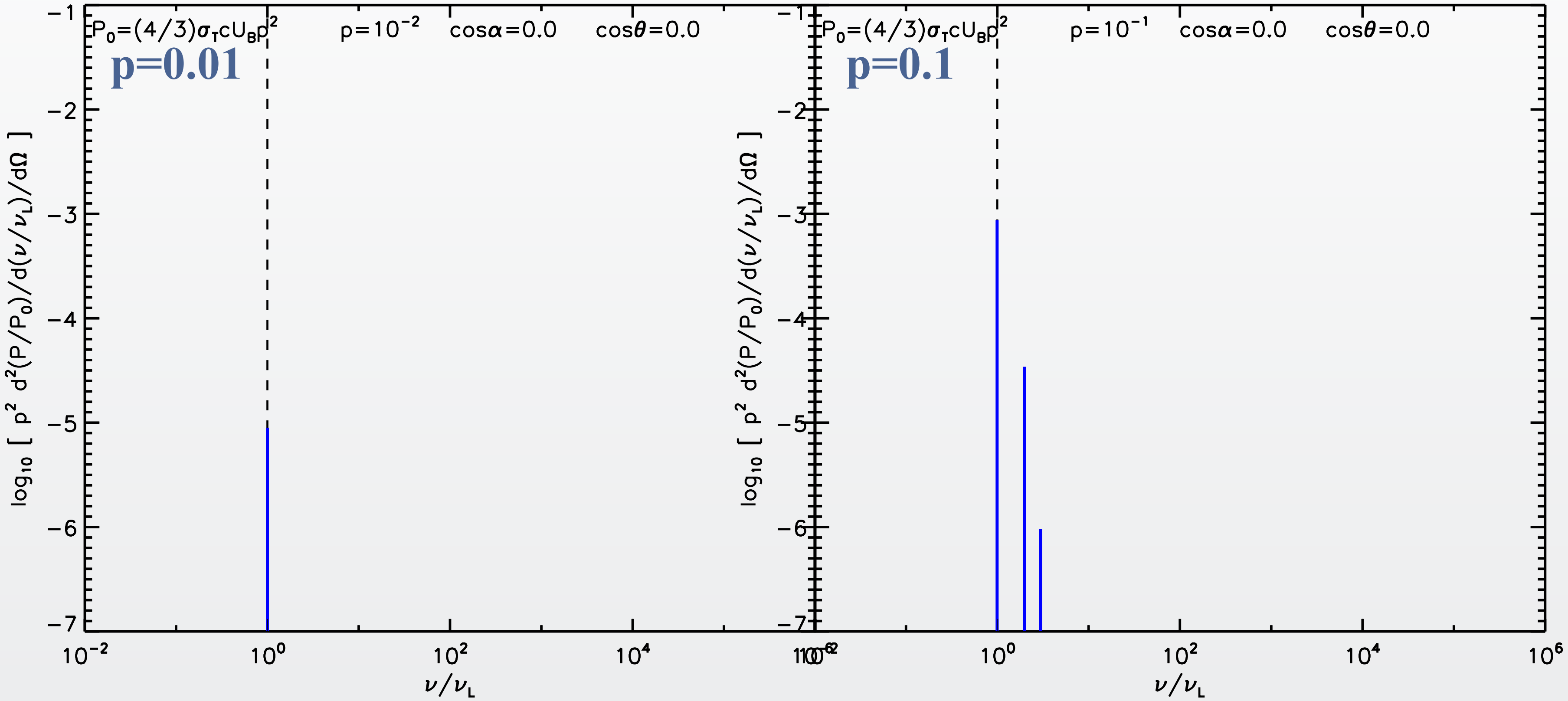
- **Assuming:**
  - Pure perpendicular motion
  - Observer in the orbital plane
- **Cyclotron line:**
  - Sinus electric field
  - Spectrum = single line at ν_L
- **Cyclo-synchrotron spectrum:**
  - Pulsed electric field
    - Pulse separation: $\Delta t = 1/\nu_B$
    - Pulse duration $\delta t \sim (1-\beta)\dfrac{\delta\theta}{\nu_B} \sim (\gamma^3\nu_B)^{-1}$
  - Spectrum:
    - many harmonic lines at $k\nu_B$
    - Enveloppe peaking at $\nu_c \sim \gamma^2\nu_L \propto \gamma^2 B$

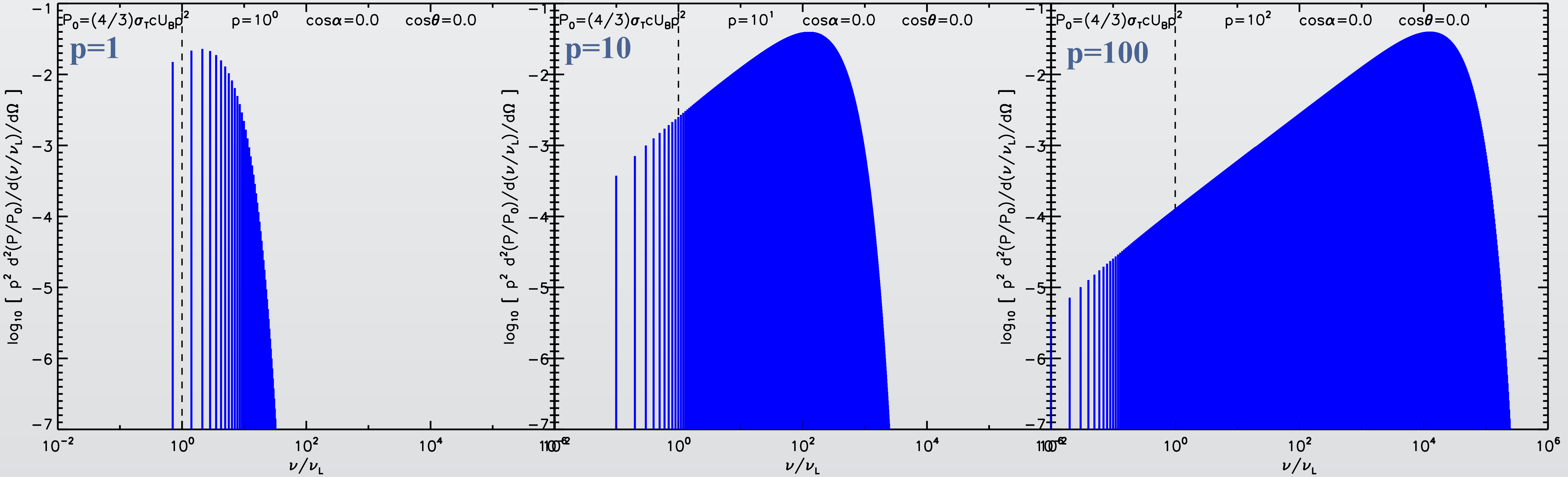


Cyclo-Synchrotron Spectrum

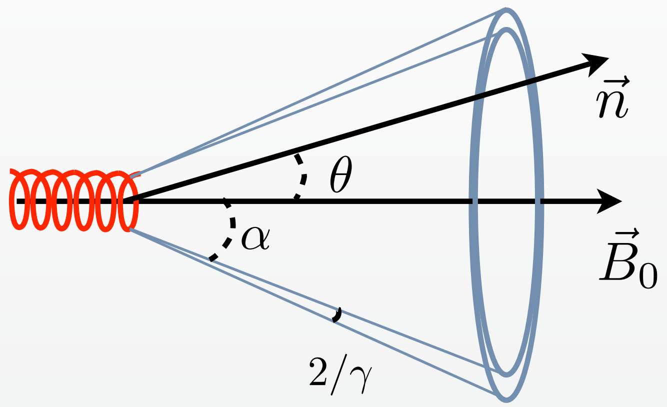
$\alpha=\pi/2$ (pure perpendicular motion)

$\theta=\pi/2$ (observer in the orbital plane)



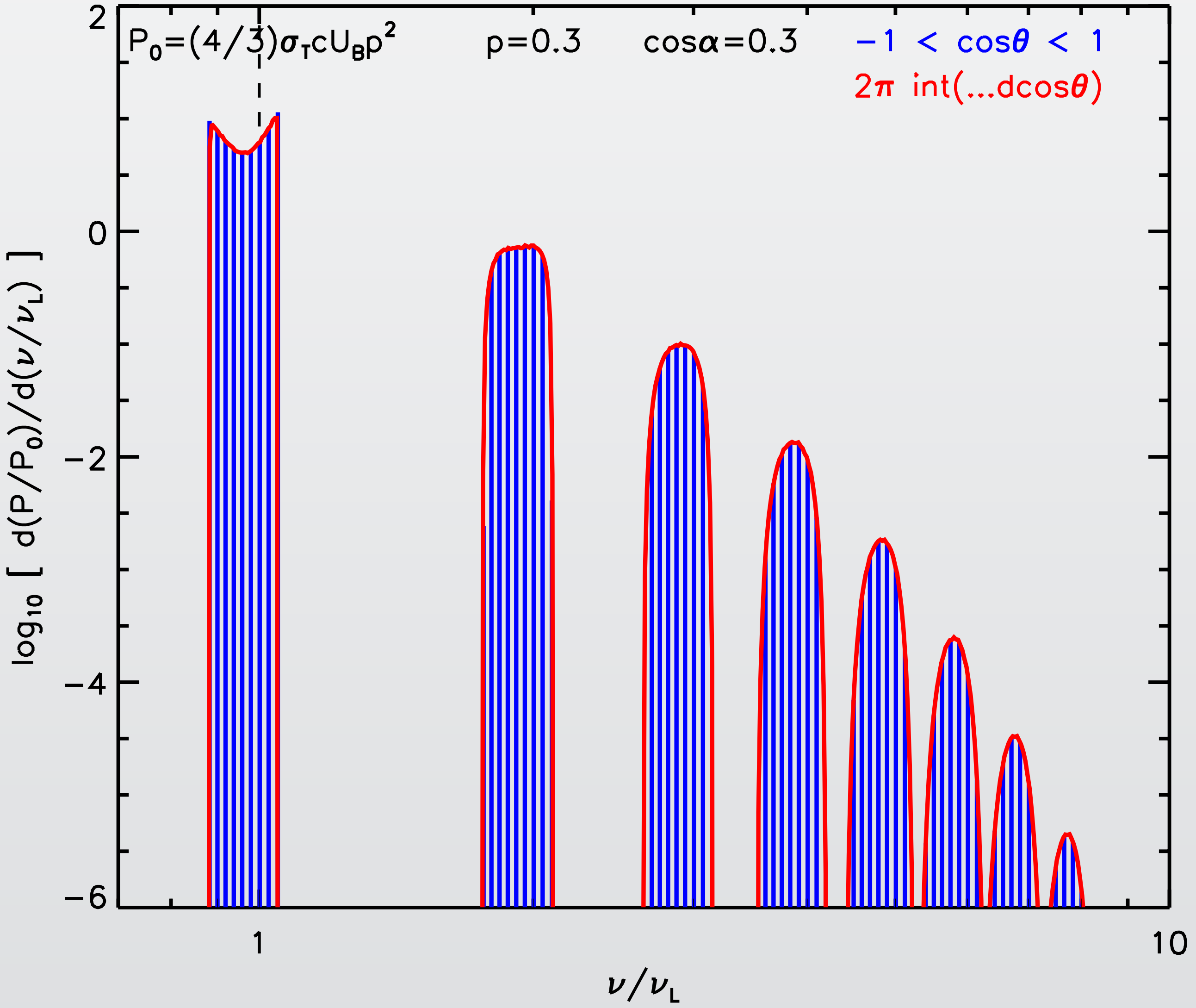


Cyclo-Synchrotron Spectrum

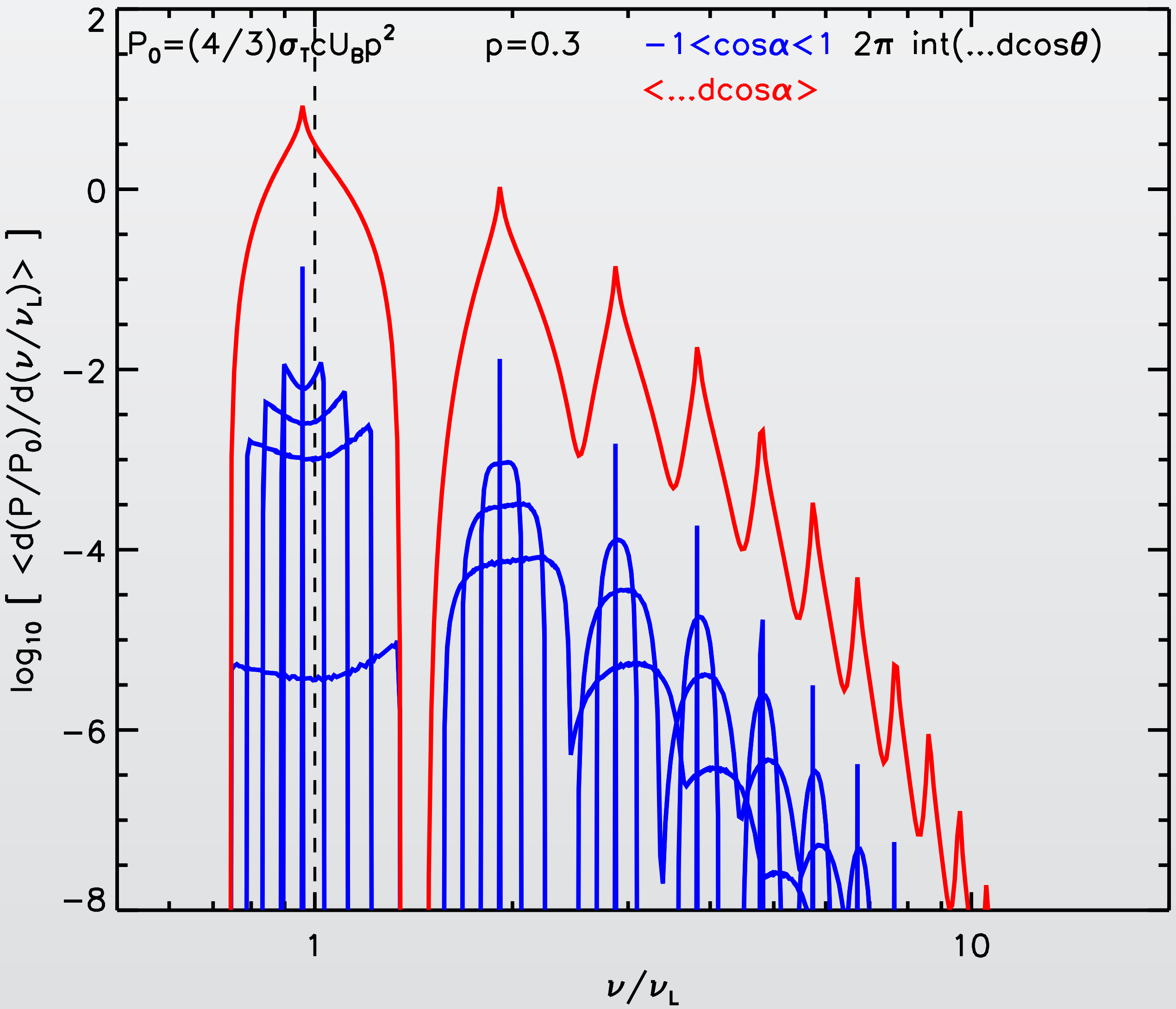


Line frequencies: $\nu_k = k\nu_B^* = k\,\dfrac{\nu_L/\gamma}{1-\beta\cos\theta\cos\alpha}$

Integration over observation angles



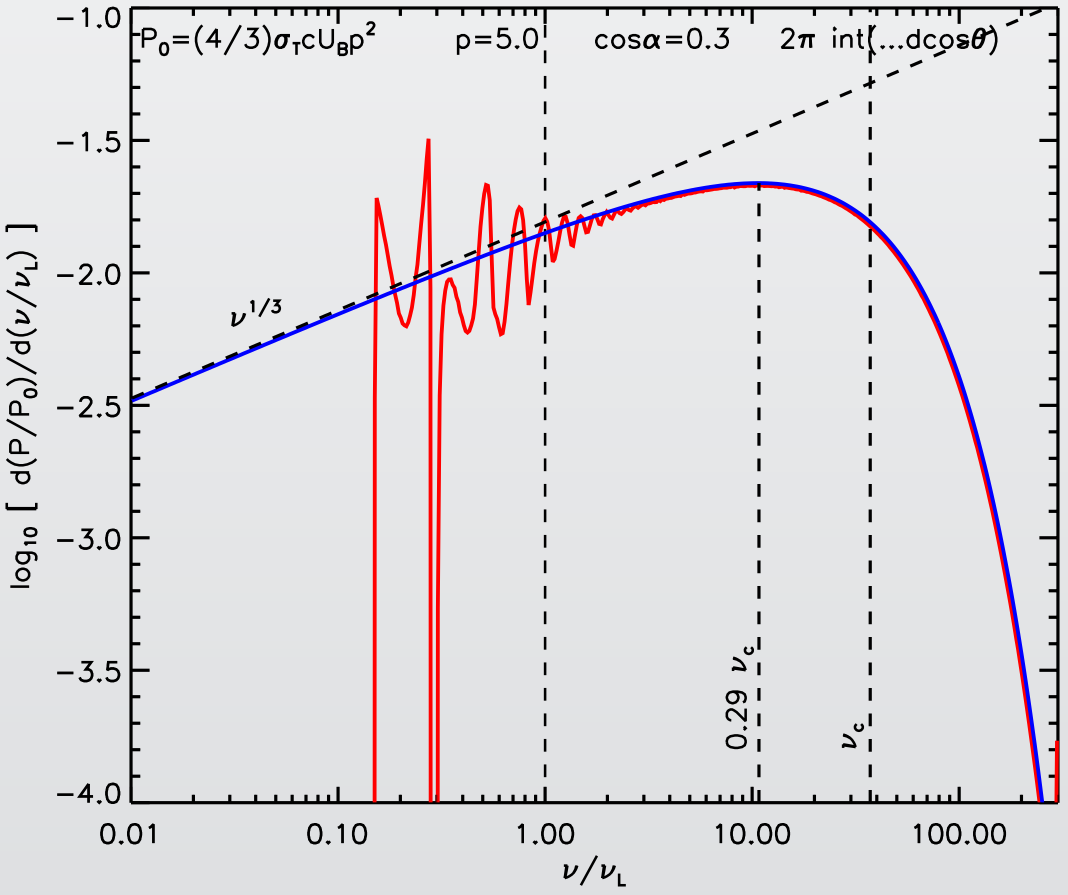
+ average over pitch angles

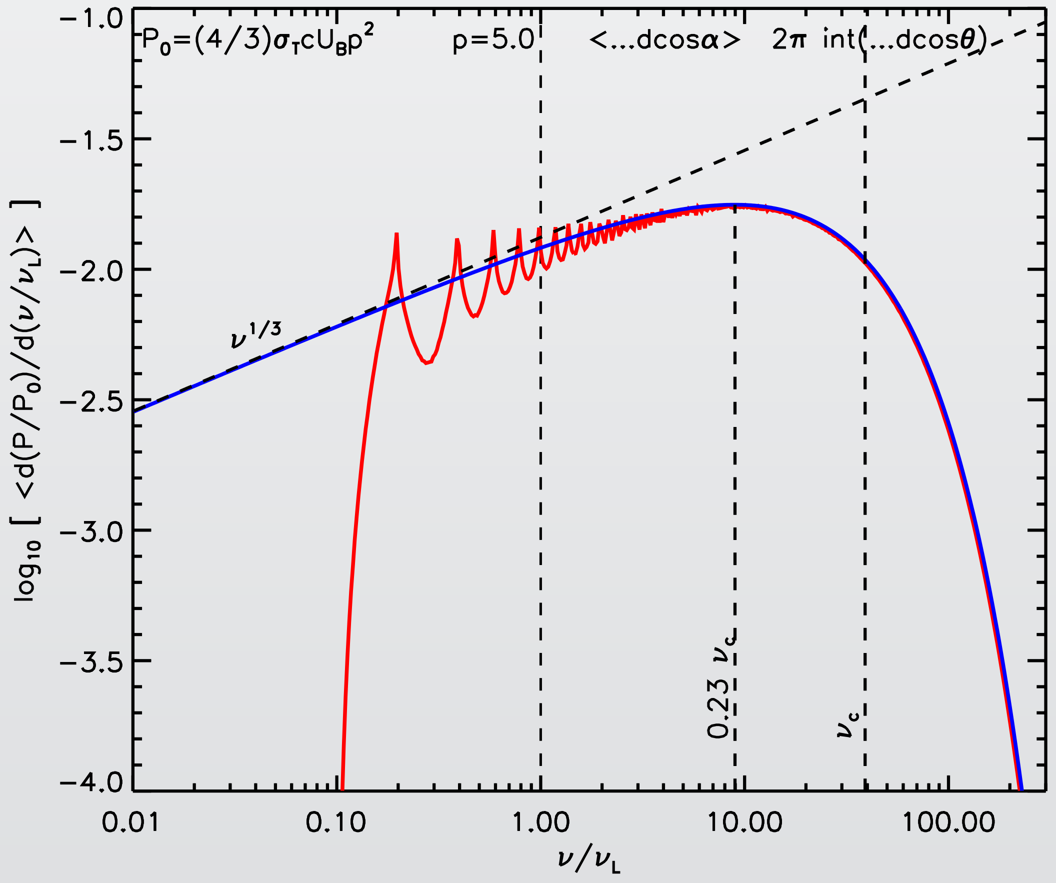


Synchrotron Spectrum

- **For ultra-relativistic particles (γ>>1), emission in a narrow shell**

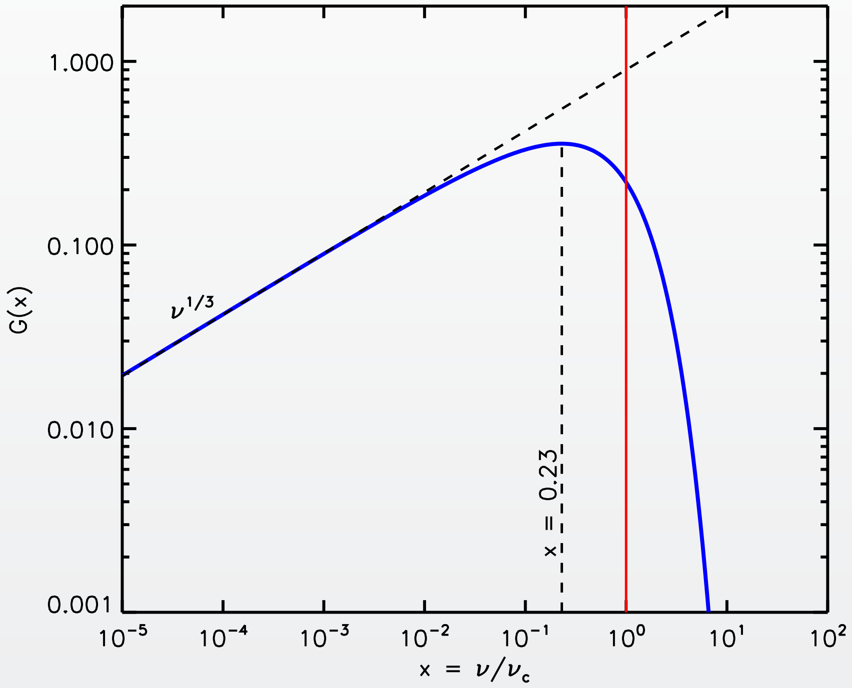
Pitch Angle	Anisotropic (dα>1/γ, aligned with los)	Isotropic (average)
Spectrum	$\frac{dP}{d\nu} = \frac{P_0}{\nu_c}\frac{9\sqrt{3}}{8\pi}F\left(\frac{\nu}{\nu_c}\right)$	$\frac{dP}{d\nu} = \frac{P_0}{\nu_c}\frac{27\sqrt{3}}{8\pi}H\left(\frac{\nu}{\nu_c}\right)$
Total Power	$P_0 = 2c\sigma_T U_B \gamma^2 \sin^2\alpha$	$P_0 = \frac{4}{3}c\sigma_T U_B \gamma^2$
Cutoff Frequency	$\nu_c = \frac{3}{2}\gamma^2 \nu_L \sin\alpha$	$\nu_c = \frac{3}{2}\gamma^2 \nu_L$
Functional	$F(x) = x\int_x^\infty K_{5/3}(x')dx'$	$H(x) = \left(\frac{x}{2}\right)^2\left[K_{4/3}(x/2)K_{1/3}(x/2) - \frac{3x}{10}\left(K_{4/3}^2(x/2) - K_{1/3}^2(x/2)\right)\right]$





Synchrotron Spectrum

- **Summary:**
  - Total power (erg/s): $\boxed{P \sim c\sigma_T U_B \gamma^2 \propto B^2\gamma^2}$
  - Cutoff frequency: $\boxed{\nu_c \sim \gamma^2 \nu_L \propto \gamma^2 B}$

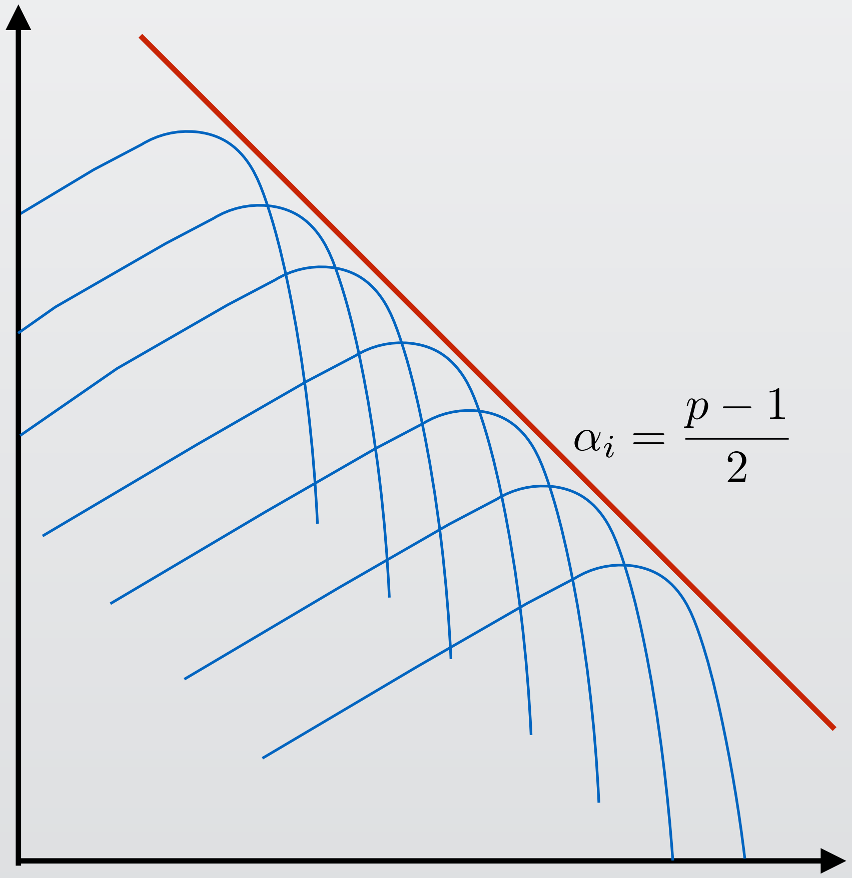


- **Maximal energy of synchrotron photons:**
  - Small loss approximation: $\nu_B t_s > 1$

$$B\gamma(\gamma+1) < \frac{3q}{\sigma_T} = 2.17 \times 10^{15}\mathrm{G}$$

$$h\nu_c < \frac{27}{16\pi}\frac{mc^2}{\alpha_f}$$

  - For electrons: $E_{e,\max} \sim 50\ \mathrm{MeV}$
  - For protons: $E_{p,\max} \sim 100\ \mathrm{GeV}$
  - Can be overcome by continuous injection of HE particles (production, reconnection)



- **Particle distributions:** $j_\nu = \frac{1}{4\pi}\int P_\nu(\gamma) f(\gamma) d\gamma$
  - Power-law distribution of particles (index p)
  - => Power-law spectrum $\boxed{j(\nu) \propto B^{\frac{p+1}{2}} \nu^{-\alpha_i}}$

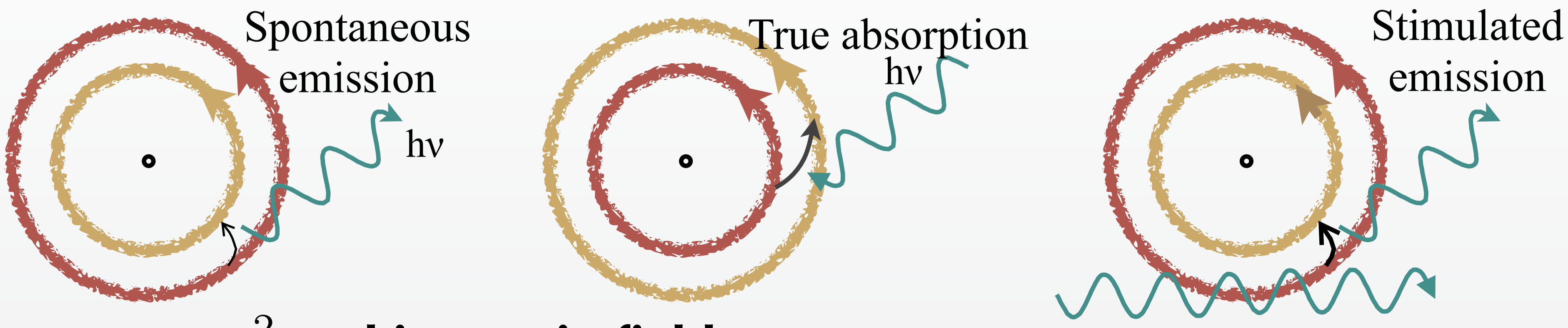
Synchrotron Polarisation

- **Ordered magnetic field:**
  - Linearly polarised perp to the projected field: $P_{\parallel} < P_{\perp}/3$
  - Polarisation degree: $\Pi = \dfrac{P_{\perp} - P_{\parallel}}{P_{\perp} + P_{\parallel}} > 50\%$
  - Power-law particle distribution: $\Pi(\alpha, \nu) = \dfrac{p+1}{p+7/3} \geq 70\%$

- **Tangled magnetic field:**
  - Net polarisation cancels out!

- **Polarisation helps constraining the emission mechanism**
  - GRBs
  - X-ray binaries outburst…

Synchrotron Self-Absorption



- For $h\nu << \gamma mc^2$ and isotropic fields:

$$\sigma_\nu = \frac{1}{2m\nu^2}\frac{1}{\gamma p}\frac{d\left(\gamma p P_\nu\right)}{d\gamma} \qquad \alpha_\nu = \frac{1}{4\pi}\int \sigma_\nu(\gamma) f(\gamma) d\gamma$$

- Strong absorption at low photon energy!

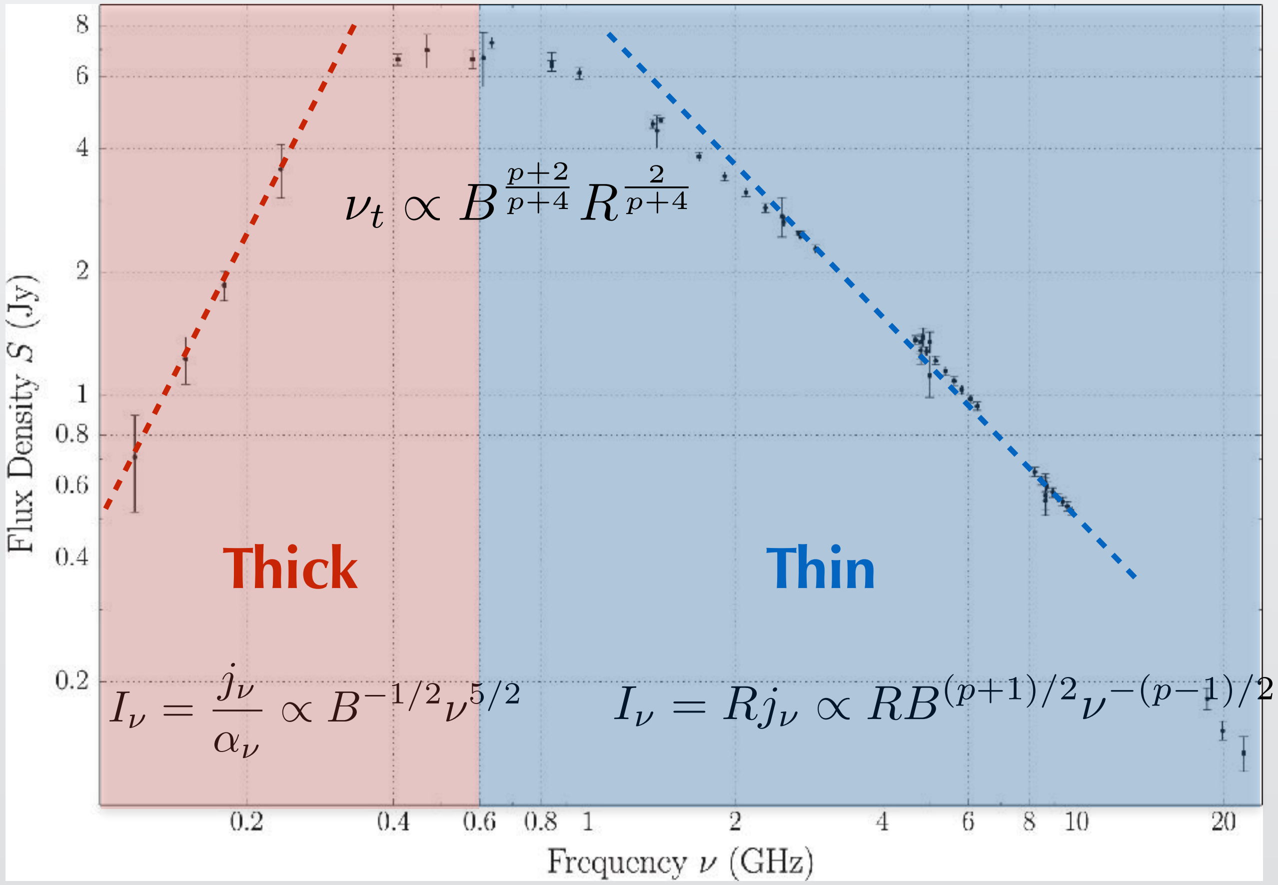
- Ex: for isotropic power-law distribution of particles:

$$j(\nu) \propto B^{\frac{1}{2}+\frac{p}{2}} \nu^{\frac{1}{2}-\frac{p}{2}}$$

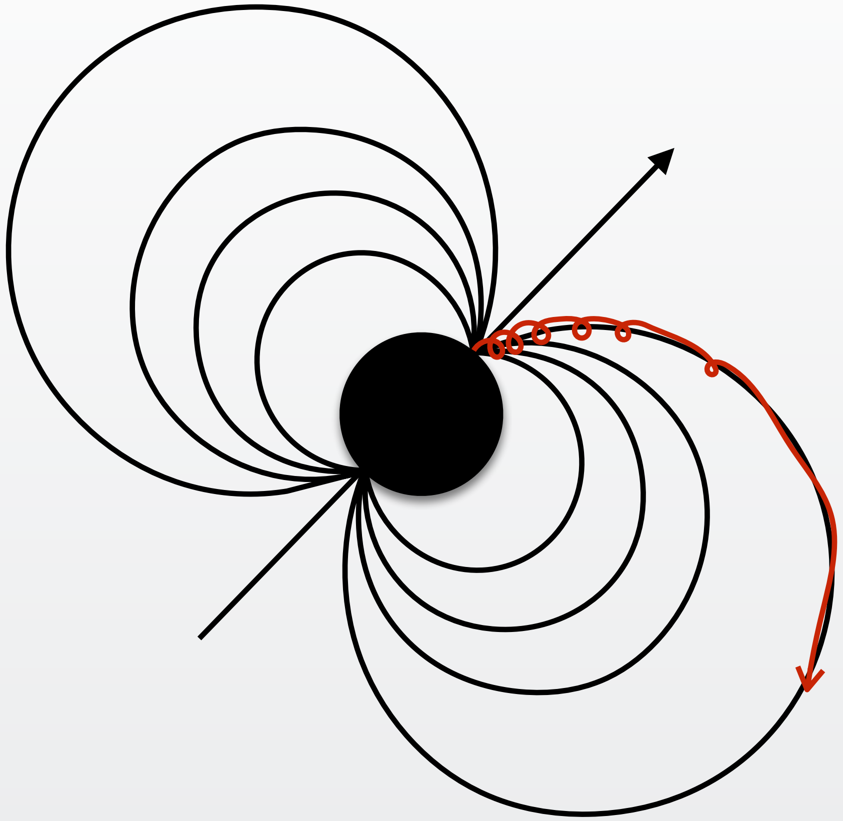
$$\alpha_\nu \propto B^{1+\frac{p}{2}} \nu^{2-\frac{p}{2}}$$

  - Turn-over frequency => B
  - Issues with inhomogeneous media
  - Other competing processes (free-free absorption, induced Compton)

PKS B0008-421, Callingham+15



II.2 Curvature Radiation

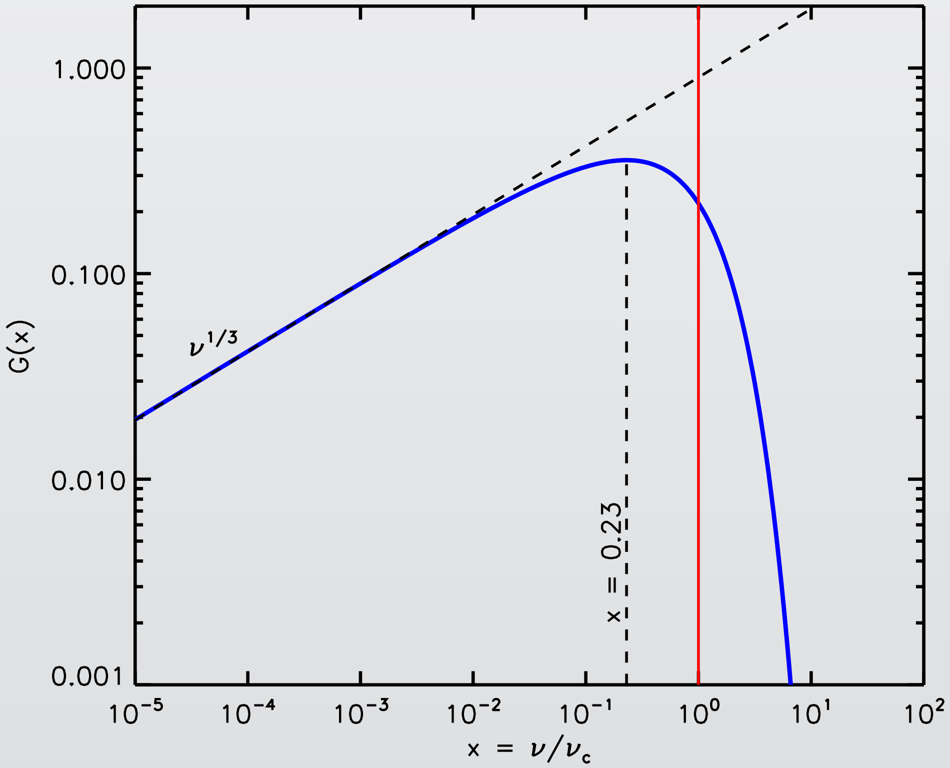


- **Emission of particles guided by circular magnetic field lines**

- **e.g Pulsar magnetosphere**
  - Strong electric acceleration // field lines
  - $\begin{cases} v_{\parallel} \to c \\ v_{\perp} \to 0 \end{cases}$ => weak synchrotron

- **Circular motion => results for synchrotron apply**
  - Total power: $P = \frac{2}{3}\frac{qc\gamma^4}{R^2}$ $\qquad P_{\rm synch} \propto \gamma^2 B^2$
  - Spectrum
  - Cutoff frequency: $\nu_c = \frac{3}{4\pi}\frac{c\gamma^3}{R}$ $\qquad \nu_{\rm synch} \propto \gamma^2 B \sin\alpha$
  - Can produce higher energy photons than pure synchrotron in the outer gaps

$$h\nu_c \approx 3\left(\frac{\gamma}{10^7}\right)^3\left(\frac{R}{10^6\mathrm{cm}}\right)^{-1}\mathrm{GeV}$$



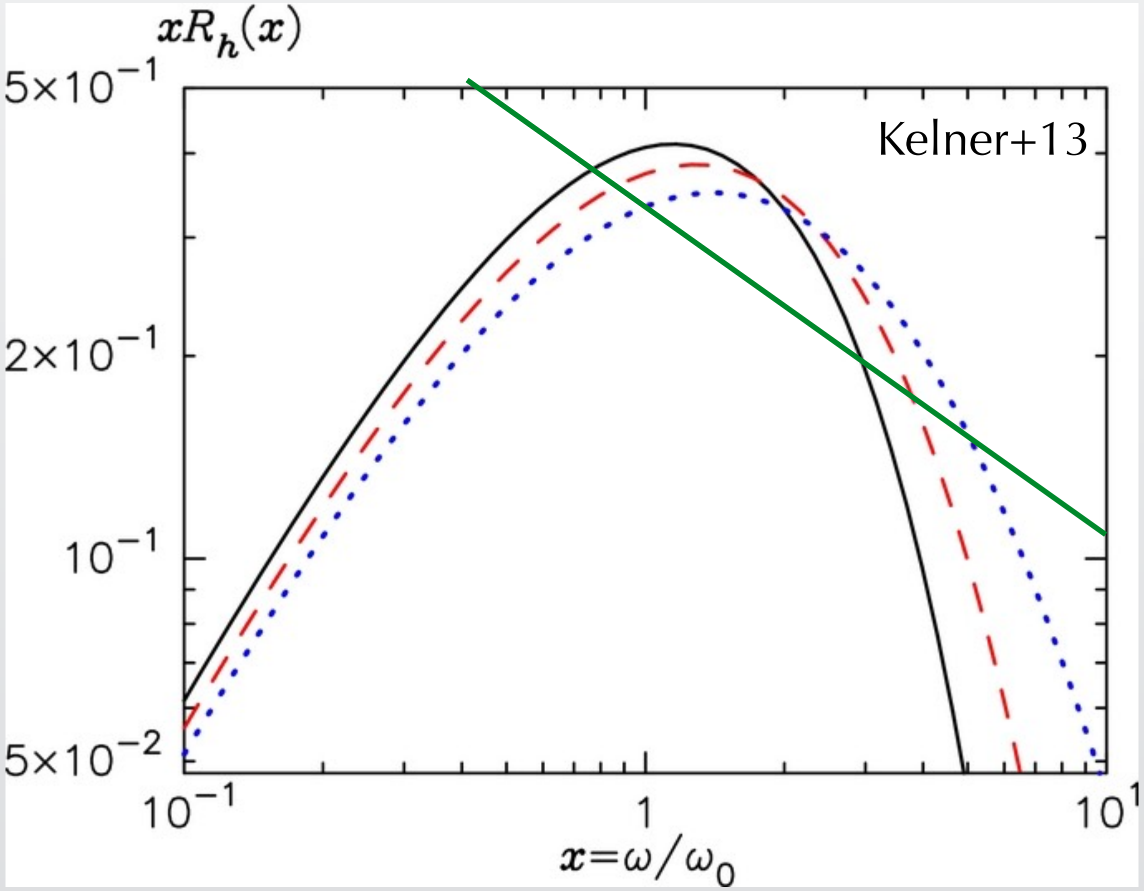
- **Most generally: *Synchro-curvature* radiation**

II.3 Diffuse Synchrotron Radiation

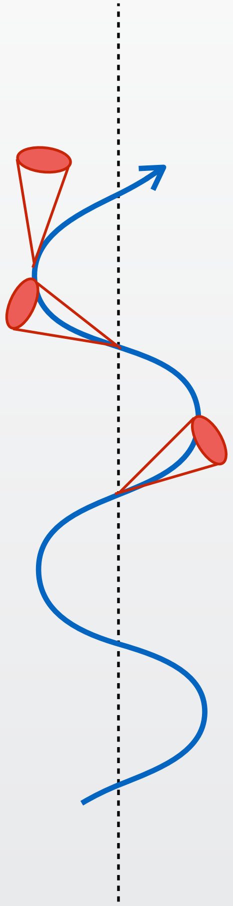
- **Turbulent magnetic field with perturbations of various scales and amplitude**

- **Large scale fluctuations ($\lambda \gg R_L$):**
  - => Inhomogeneous synchrotron radiation
  - Scale is irrelevant => turbulence described by amplitude distribution: $P_B(B)$
  - Simple convolution of the synchrotron spectrum: $P_\nu(\gamma,\nu) = P_\nu(\gamma,\nu,B) \circledast P_B(B)$
    - For peaked magnetic spectrum: shifted and broaden emission
    - For power-law magnetic spectrum: power-law spectrum $P_B(B) \propto B^{-s}$

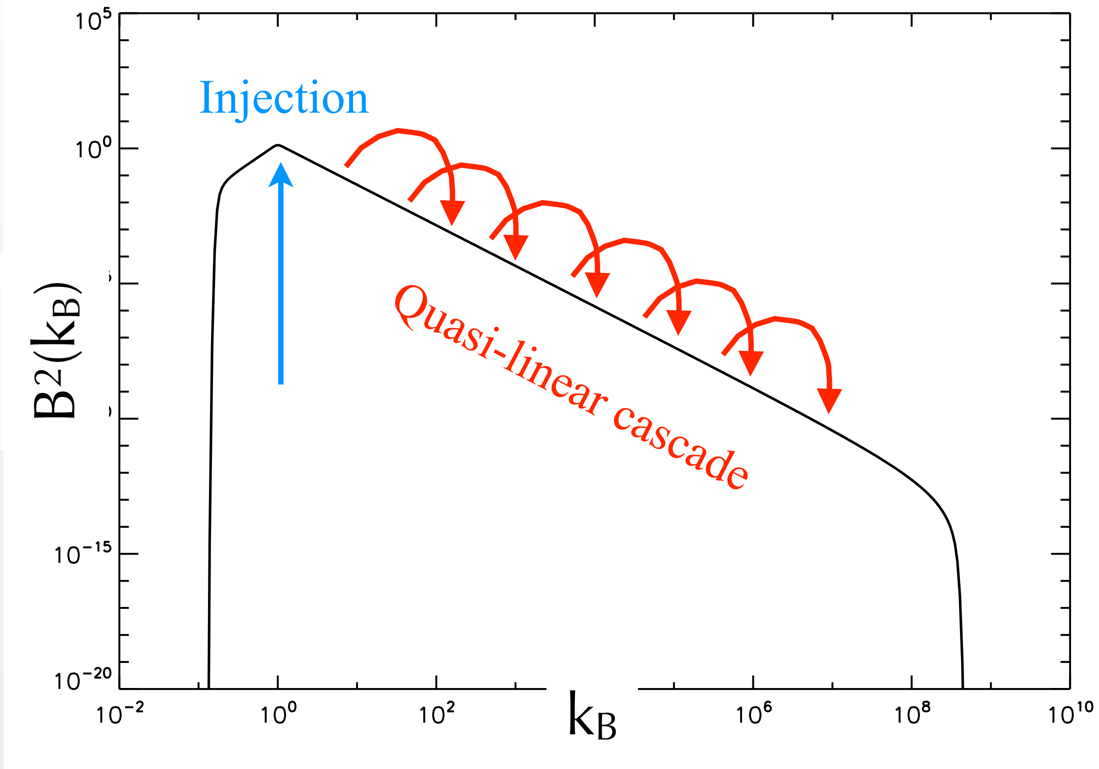
$$F_\nu \propto \nu^{-s+2}$$

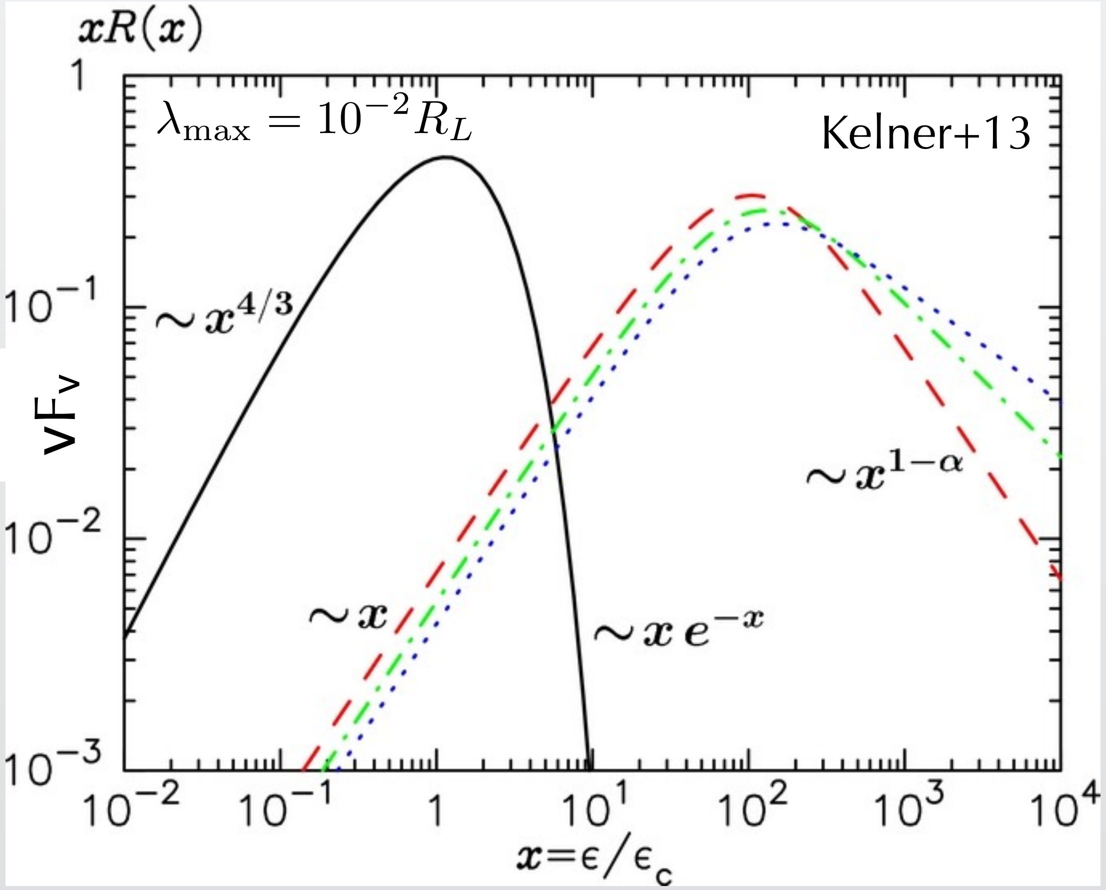


II.3 Diffuse Synchrotron Radiation



- **Small scale fluctuations (λ<<R$_L$):**
  - => ***Jitter*** radiation
  - For one of the mono-chromatic waves in the turbulent field: $k_B=2\pi/\lambda_B$
  - The observed maximal photon frequency is $\nu_j \sim \gamma^2 k_B c \sim \frac{R_L}{\lambda_B}\nu_{c,\mathrm{synch}} > \nu_{c,\mathrm{synch}}$
  - Small scale turbulence => large photon energy
  - Convolution with the field turbulent spectrum: $B^2(k_B) \propto k_B^{-\alpha}$
    - Slower rise
    - Higher energy cutoff
    - Power-law tail
  - Applications to
    - GRBs
    - Blazars
    - Crab flares

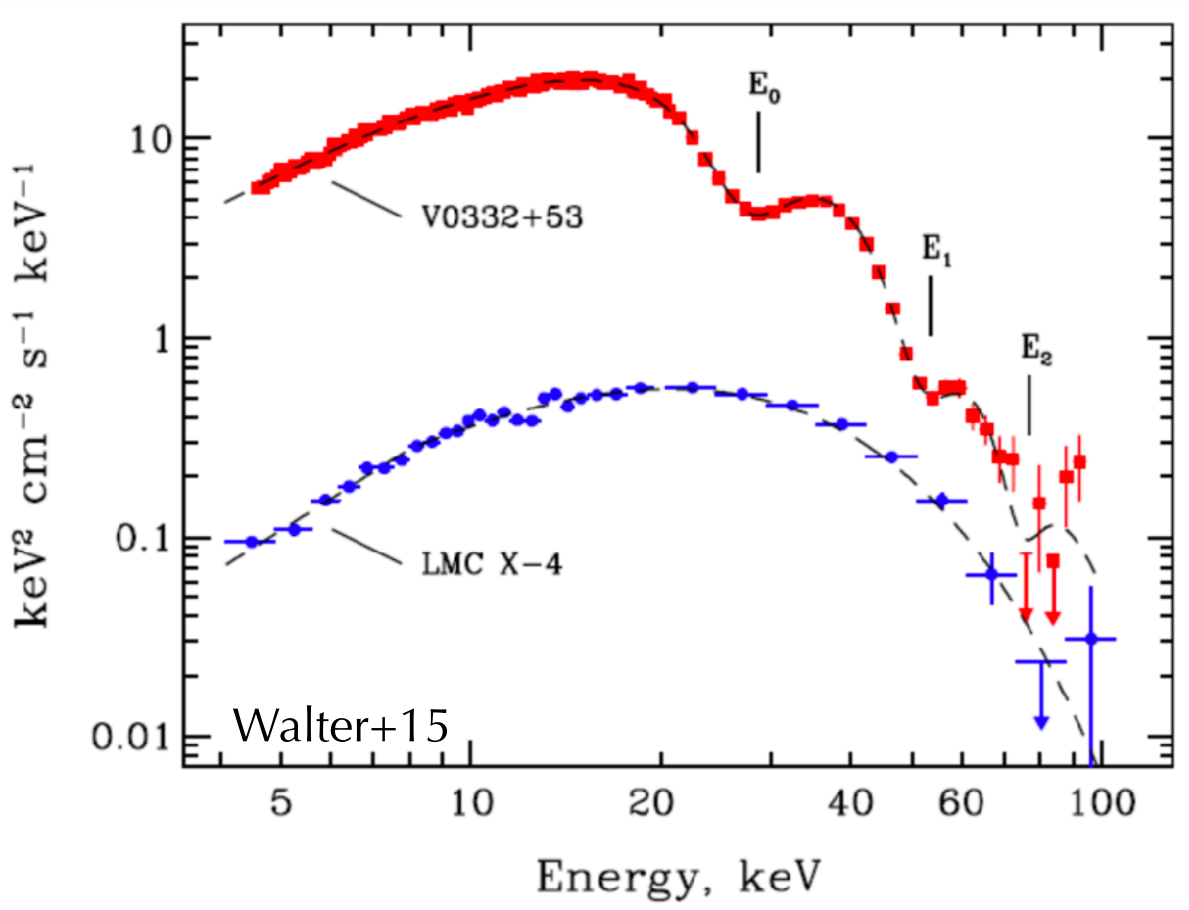




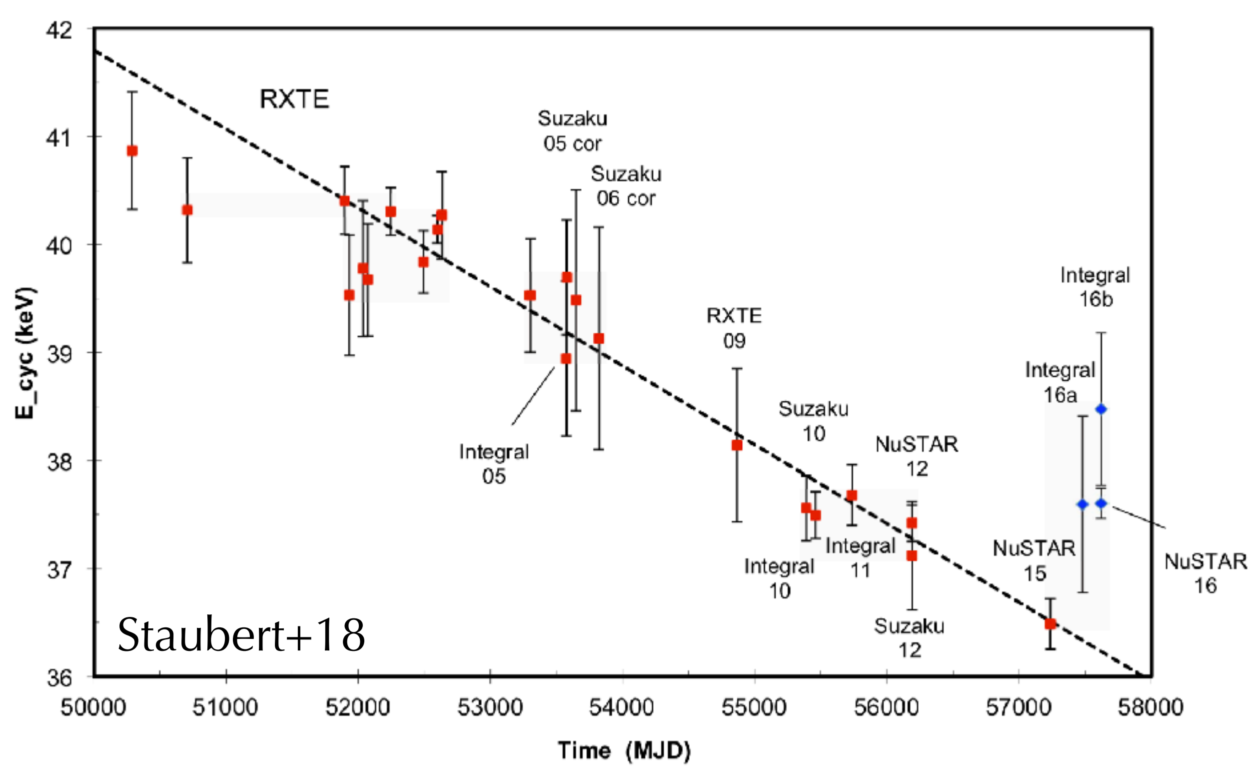
II.4 Strong Fields

- **Classical limit fails for strong fields**
- **Landau levels:** $p_n^2 = 2n\frac{B}{B_c} = 2n\frac{h\nu_L}{mc^2}$
- **Critical field:** $B_c = \frac{2\pi m^2 c^3}{qh} \sim 4\times10^{13}\mathrm{G}$

  (For leptons)
- **Significant quantification for n~1:**
  - Low energy and/or large field
- **In accretion onto magnetised neutron stars (Trumper+78):**
  - Sub-relativistic plasma (kT_e ~ 10 keV, p^2~0.06)
  - Cyclotron features observed above $h\nu_B$~30keV => B~10^{12}G => n~1
  - Only the ground Landau level is populated => absorption features
  - *Cyclotron resonant scattering features* (CRSF)

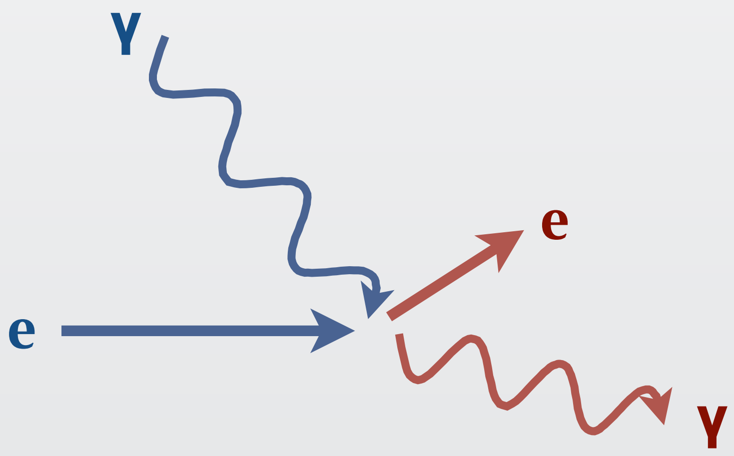


Walter+15



Staubert+18

IV. Compton Scattering

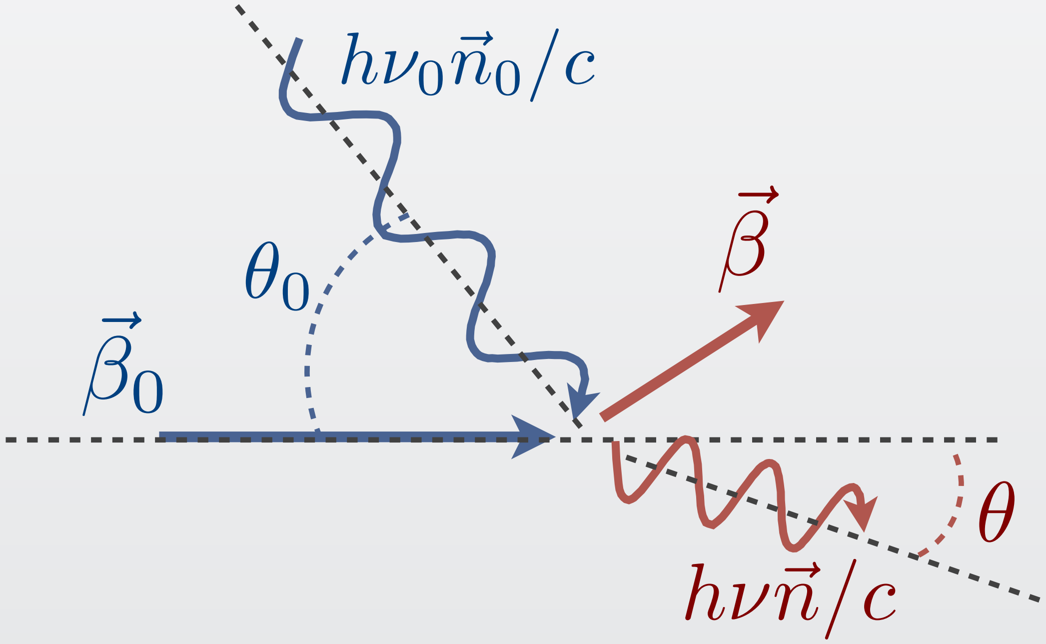


- **Scattering of photons by charged particles**

Compton Scattering

- **Echange of energy and momentum**

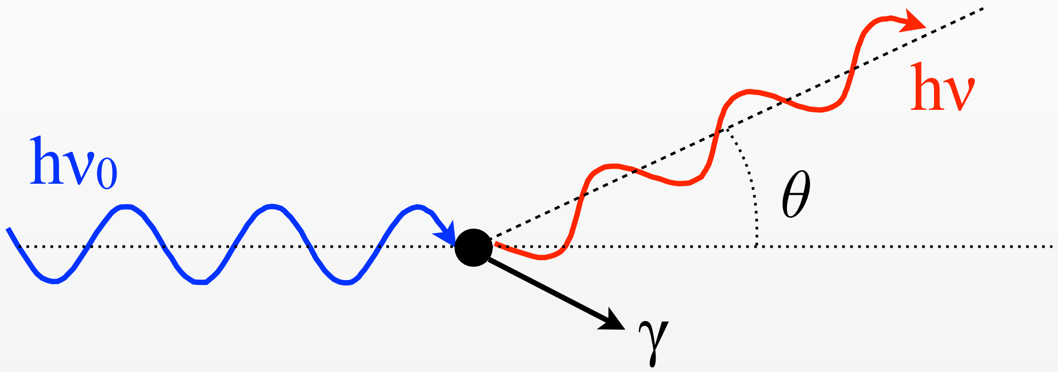
- **The outcome**
  - Depends on energies of incoming photons/particles
  - Depends on interaction angle
  - Is described by distributions (angle, energy)



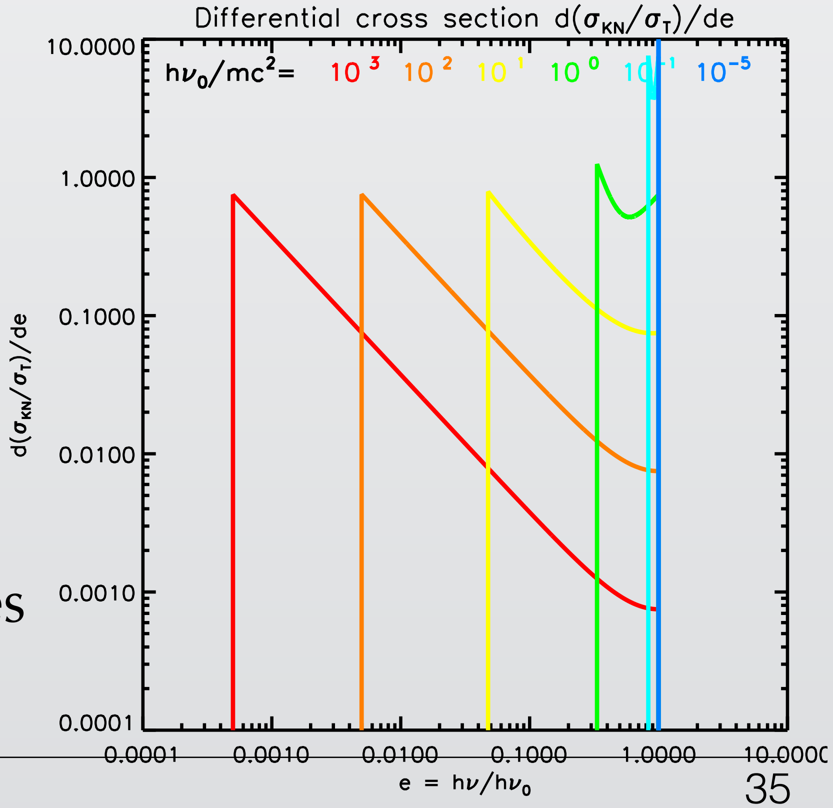
- **In most cases:**
  - If $E_{part} < E_{phot}$: Compton down-scattering
  - If $E_{part} > E_{phot}$: Compton up-scattering

- **Two main regimes:**
  - Thomson: classical mechanics, no particle recoil
  - Klein-Nishina: quantum mechanics, particle recoil

In the particle rest frame

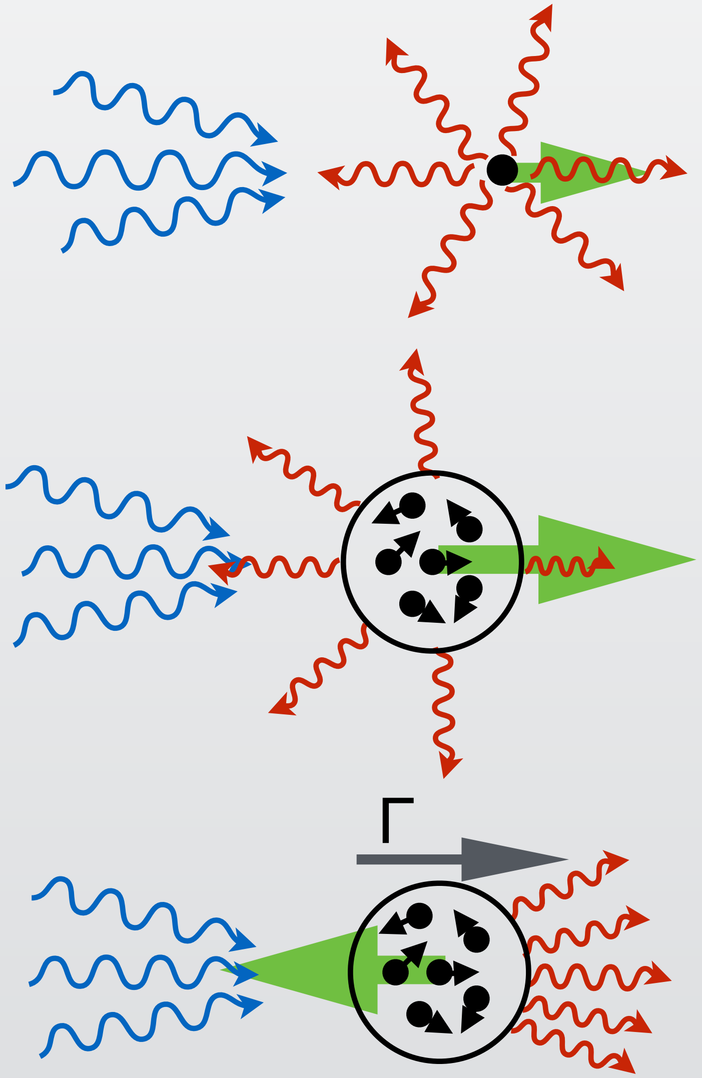


- **Depends only on the photon energy E_{phot}**
- **The Thomson regime ($E_{phot} < mc^2$):**
  - Particle recoil can be neglected
  - Particle oscillates as a response to a linearly polarised wave => radiation
  - Monochromatic emission: coherent scattering
    - Mean acceleration: $< a^2 >= \frac{1}{2}\left(\frac{eE}{m}\right)^2$
    - Emitted Power $P = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \frac{cE^2}{8\pi} = \sigma_T S$
    - Thomson cross section: $\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2 \approx 6.65 \times 10^{-25}\mathrm{cm}^2$
    - Dipolar emission
  - For unpolarised radiation:
    - quasi-Isotropic emission
- **The Klein-Nishina regime ($E_{phot} > mc^2$):**
  - Anisotropic radiation
  - Energy transfer photon->particle => scattered spectrum
  - Drop of the cross section => barely relevant to physical cases



In the source frame

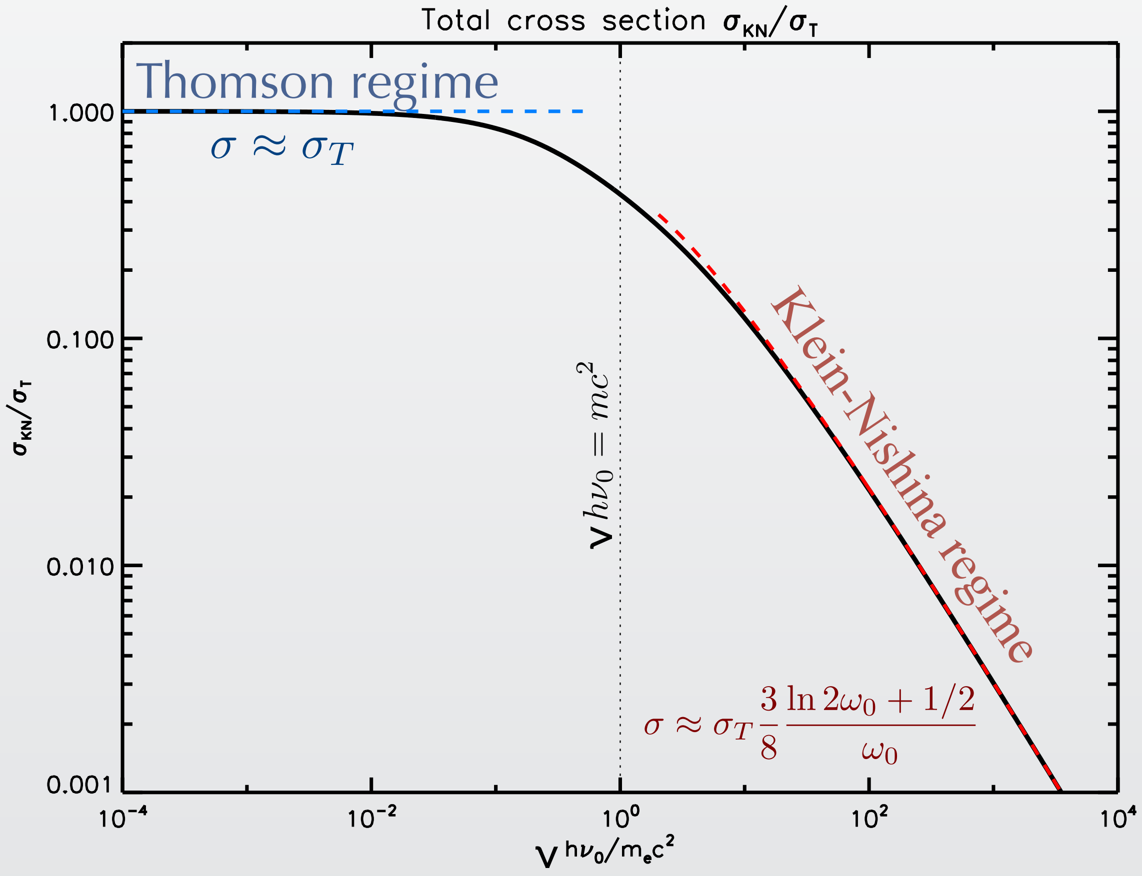
- **Also depends on the interaction angle and the particle energy**
- **Example of a single scattering event:**
  - Head-on collision: $\nu_0'=2\gamma\,\nu_0$
  - Coherent Thomson scattering: $\nu'=\nu_0'$
  - Backward scattering: $\nu = 2\gamma\nu'$
  - Up-scattering by a factor $A=4\gamma^2$
- **Example of anisotropic interaction: Compton on bulk motion (AGN, GRBs)**
  - Cold matter: radiation pressure from an anisotropic field: $f = (\sigma_T/c)S$
  - Hot plasma: more efficient by about γ_{th}^2 (*Compton rocket*)
  - Relativistic bulk motion: emission beamed in forward direction => recoil force (*Compton drag*) => Γ saturates.



Isotropic distributions

- **Average over interaction and scattering angles**
- **Total cross section drops at:** $\boxed{\gamma_0 \frac{h\nu_0}{mc^2} = 1}$

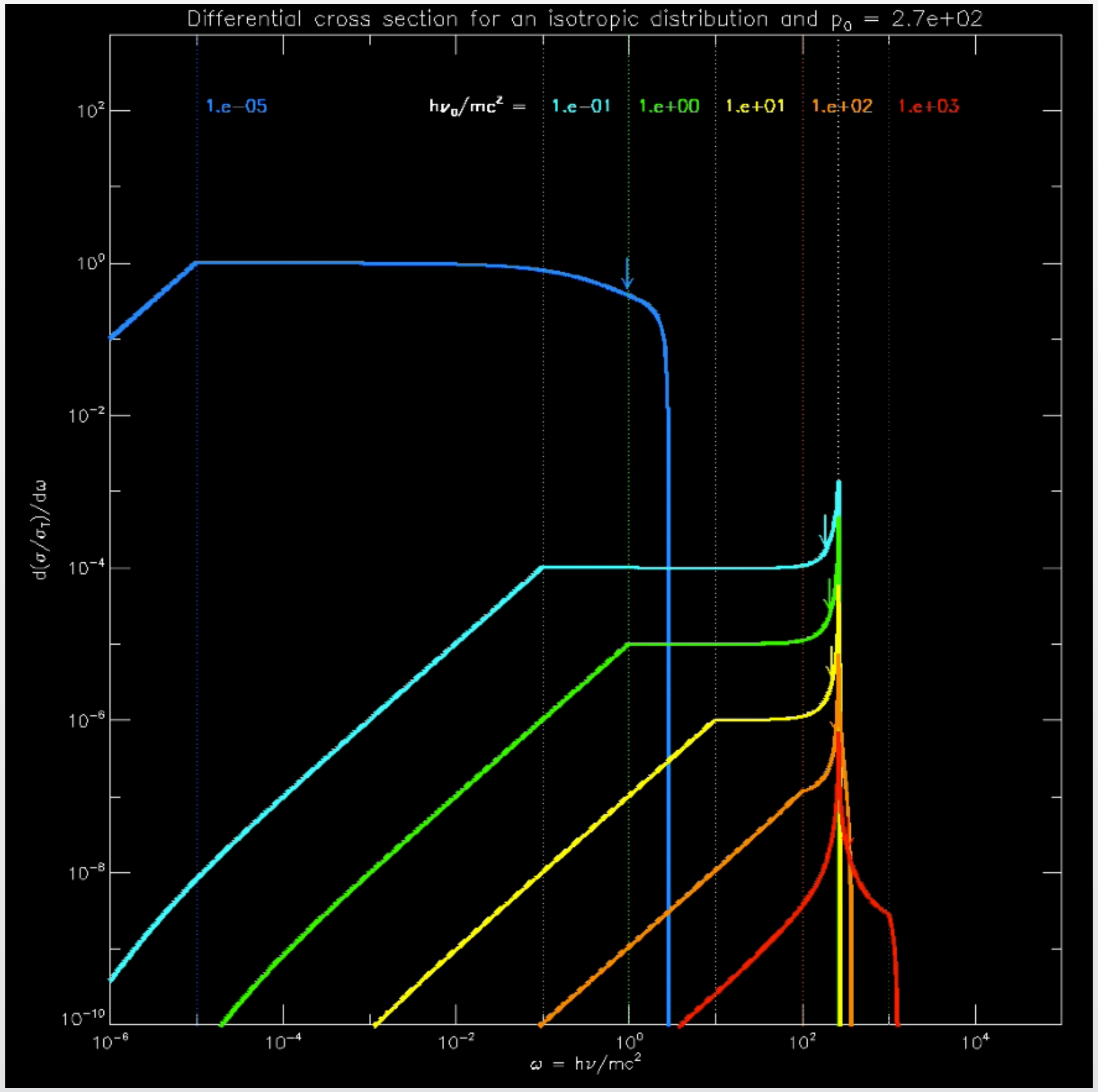
	$h\nu_0$	$E_{max} = \gamma_{max} mc^2 = (mc^2)^2/h\nu_0$
CMB	1 K	PeV
Radio jet of blazars	10^{13} Hz	10 TeV
Star	10 000 K	100 GeV
AGN accretion disk	10 eV	10 GeV
NS/SMBH accretion disk	1 keV	100 MeV



  - KN rarely relevant…

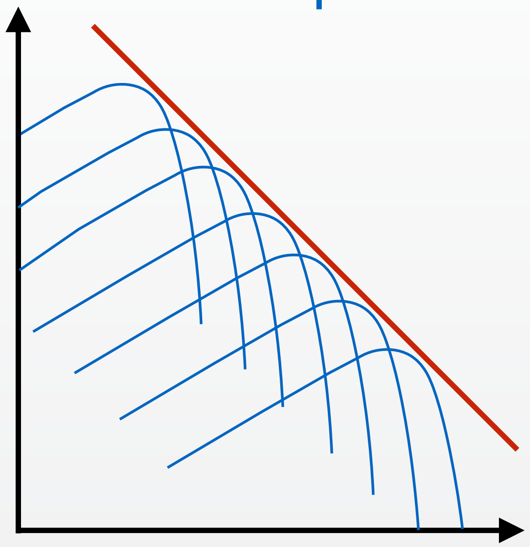
Isotropic distributions

- **Scattered spectrum in the Thomson regime:**
    - No simple expression $\frac{d\sigma}{d\nu'}(\nu, \gamma \to \nu')$

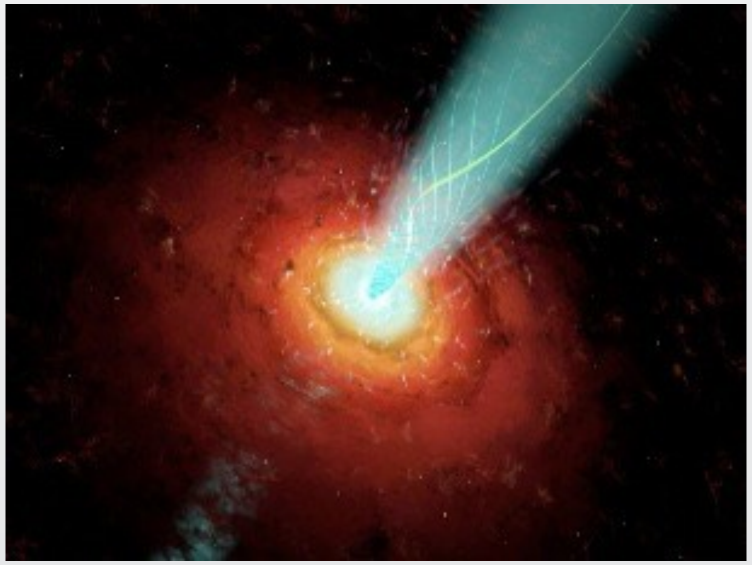


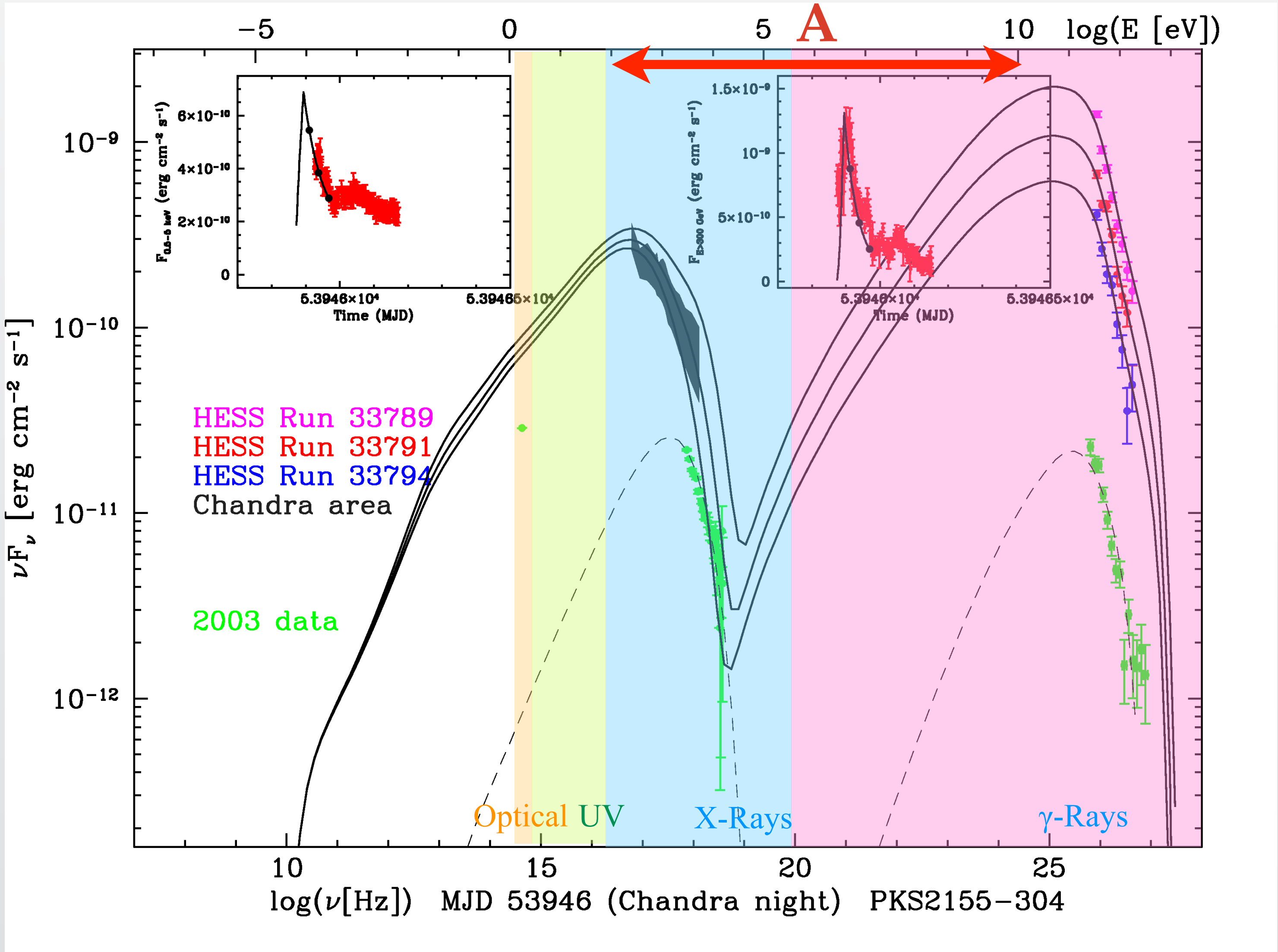
-
    - Average amplification factor in Thomson regime, for up-scattering : $\langle A \rangle = 1 + \frac{4}{3}p^2$
    - If γ>> 1: $\boxed{\langle A \rangle \sim \gamma^2}$ ($h\nu \sim \gamma^2 h\nu_0$ comparable to synchrotron: $h\nu_c \sim \gamma^2 h\nu_L$)

Isotropic distributions



- **Convolved with the seed photon spectrum**
- **Convolution with the particle energy distribution:**
  - Thermal: substitution for $\langle p^2 \rangle \approx 3\theta(1+4\theta)$
  - Power law of index s => power-law spectrum with index $\alpha_i = \dfrac{p-1}{2}$
- **Ex: Blazar spectrum**





- Synchrotron self-Compton
- A ~10^8
- If no Doppler: γ~10^4
- $h\nu_0$=100 eV => $\gamma h\nu_0/mc^2>1$
- KN regime

Effect on particles

- **Particle Energy variation in the Thomson regime in up-scattering cases:**

$$\langle \Delta E \rangle = (\langle A \rangle - 1) h\nu_0 = \frac{4}{3} h\nu_0 p^2$$

- **With soft photon density U$_{\mathrm{ph}}$:** $\frac{dE}{dt} = c\sigma_T \int \langle \Delta E \rangle \, dN_\nu = \frac{4}{3} c\sigma_T U_{\mathrm{ph}} p^2$

- **Comparable results to the synchrotron cooling rate** $\frac{dE}{dt} = \frac{4}{3} c\sigma_T U_B p^2$

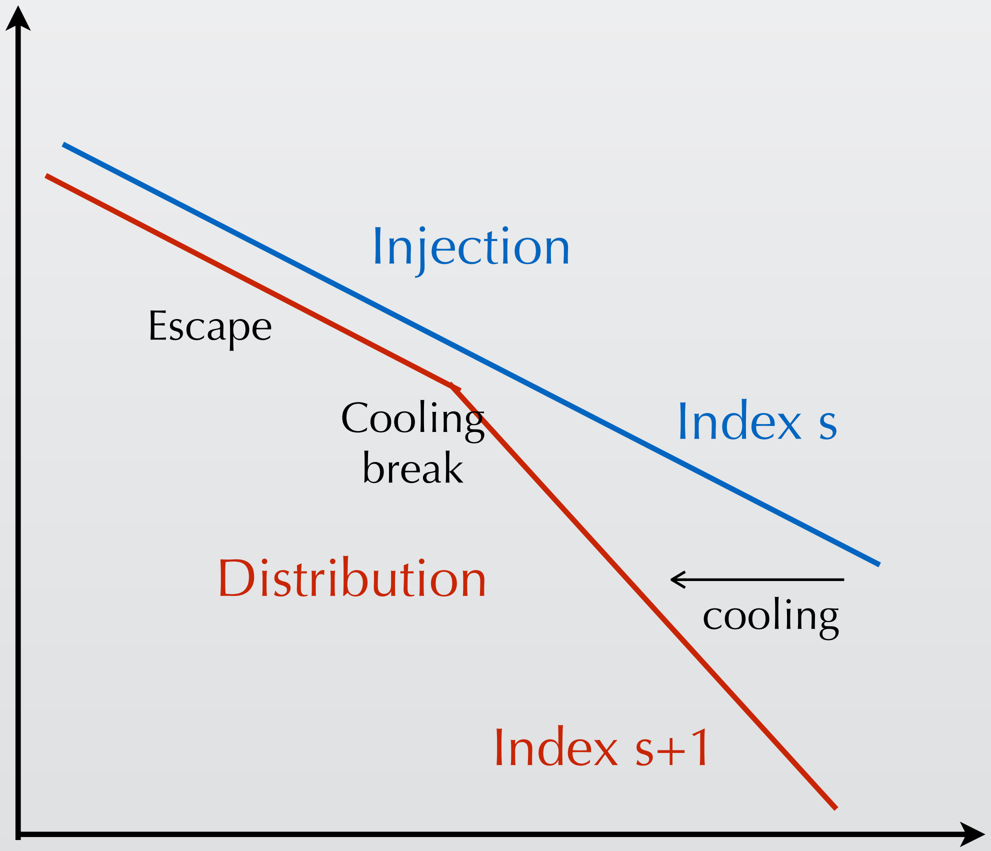
- **Cooling time:** $t_c = \frac{3}{4} \frac{mc^2}{c\sigma_T U_{\mathrm{ph}}} \frac{1}{\gamma + 1}$

- **Example of kinetic equation for particles:**

$$\frac{dN_\gamma}{dt} = \frac{d}{d\gamma} (\dot{\gamma} N_\gamma) + S(\gamma) - p_{esc} N_\gamma$$

  - For Power-law injection:
    - Cooling break
    - Steepening when cooling dominates over escape

$$S(\gamma) \propto \gamma^{-s} \;\Rightarrow\; N_\gamma \propto \gamma^{-(s+1)}$$



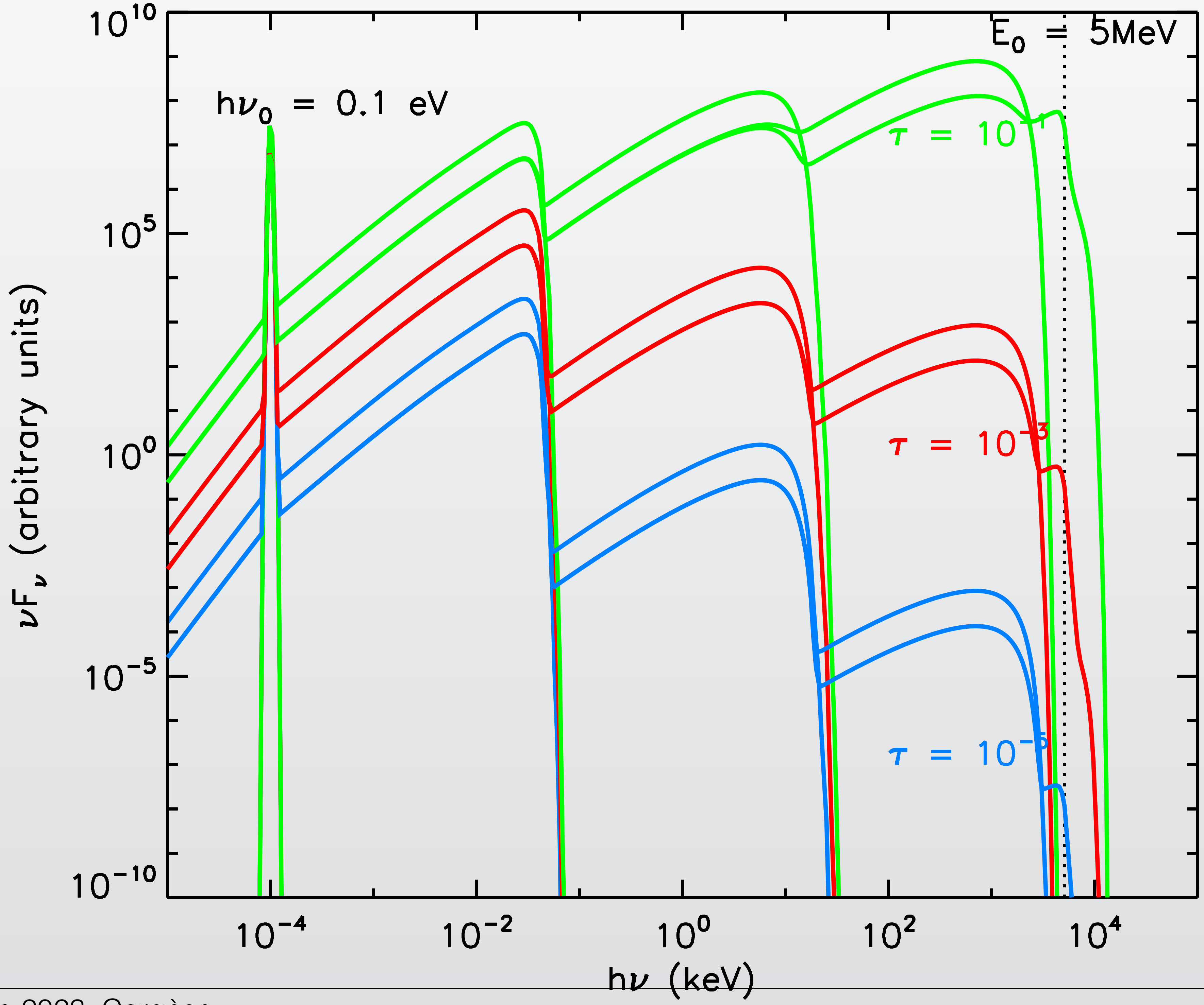
Multiple Scattering

- **Radiation transfer problem for a source of given size R**
- **Characterised by 2 quantities:**
  - Optical depth: $\tau_T = \sigma_T n_e R$
  - The y-parameter: $y = N_{\text{scat}} \dfrac{\Delta E}{E} \sim \dfrac{4}{3} p^2 N_{\text{scat}}$

	Thomson Thin $\tau_T < 1$	Thomson thick $\tau_T > 1$
Photon scattering rate per unit volume	$\dot{N}_{\text{scat}} \sim c\sigma_T n_e$	$\dot{N}_{\text{scat}} \sim c\sigma_T n_e$
Escape time	R/c	$N_{\text{scat}}^{1/2} \ R/c$
Mean number of scatterings	τ_T	τ_T^2
y	$4\theta(1+4\theta)\tau$	$4\theta(1+4\theta)\tau^2$

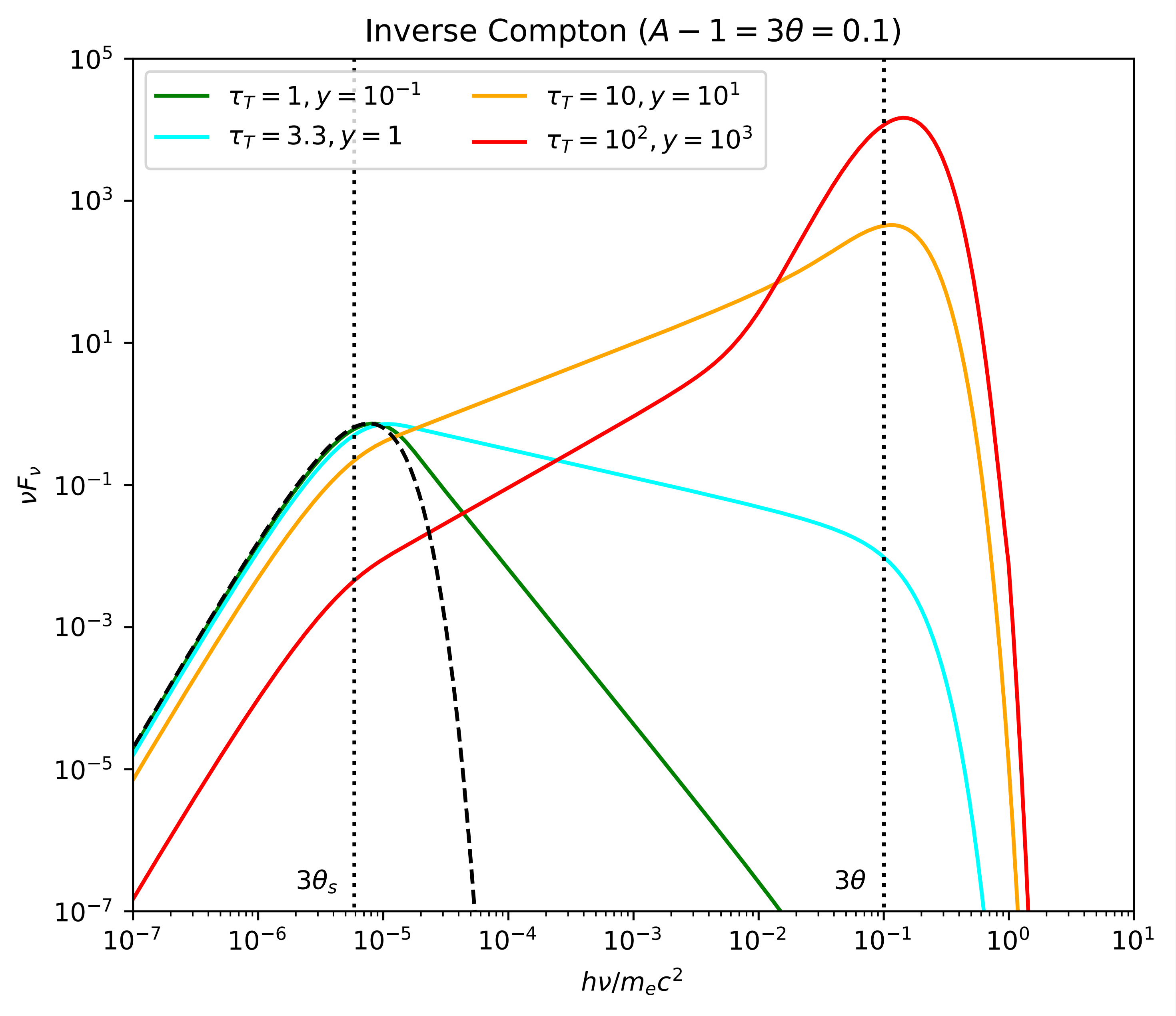
Example of Multiple up-scattering

- **γ=10, hν_0=0.1eV, Thomson**



Regimes of Multiple up-scattering

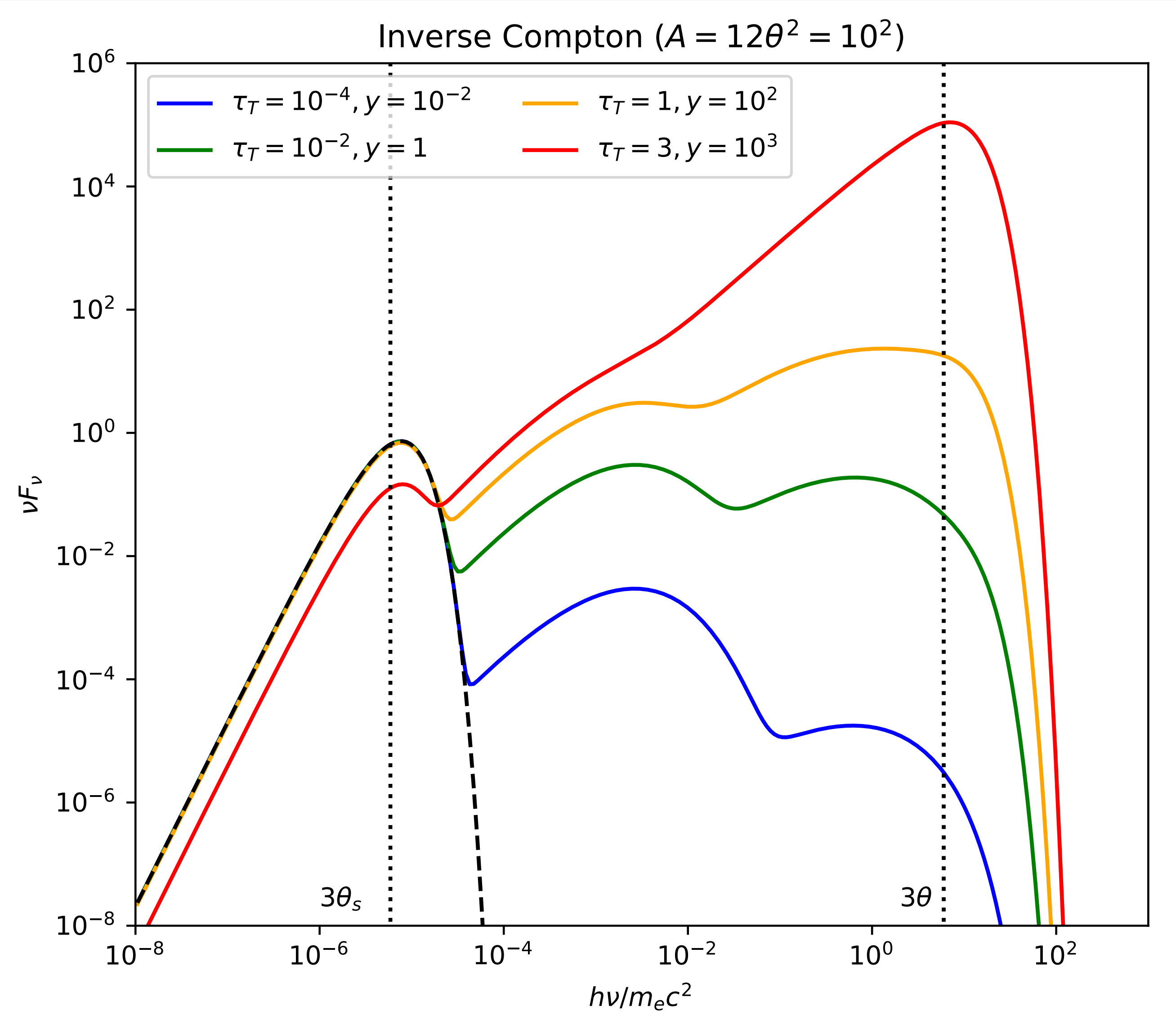
- **$\gamma\, h\nu_0/mc^2 < \gamma(\gamma\text{-}1) < 1$:**
  - Thomson
  - Small A => Power-law
  - Small tau (y<<1) => cutoff at kT
  - Large tau (y>>1) => saturation at kT

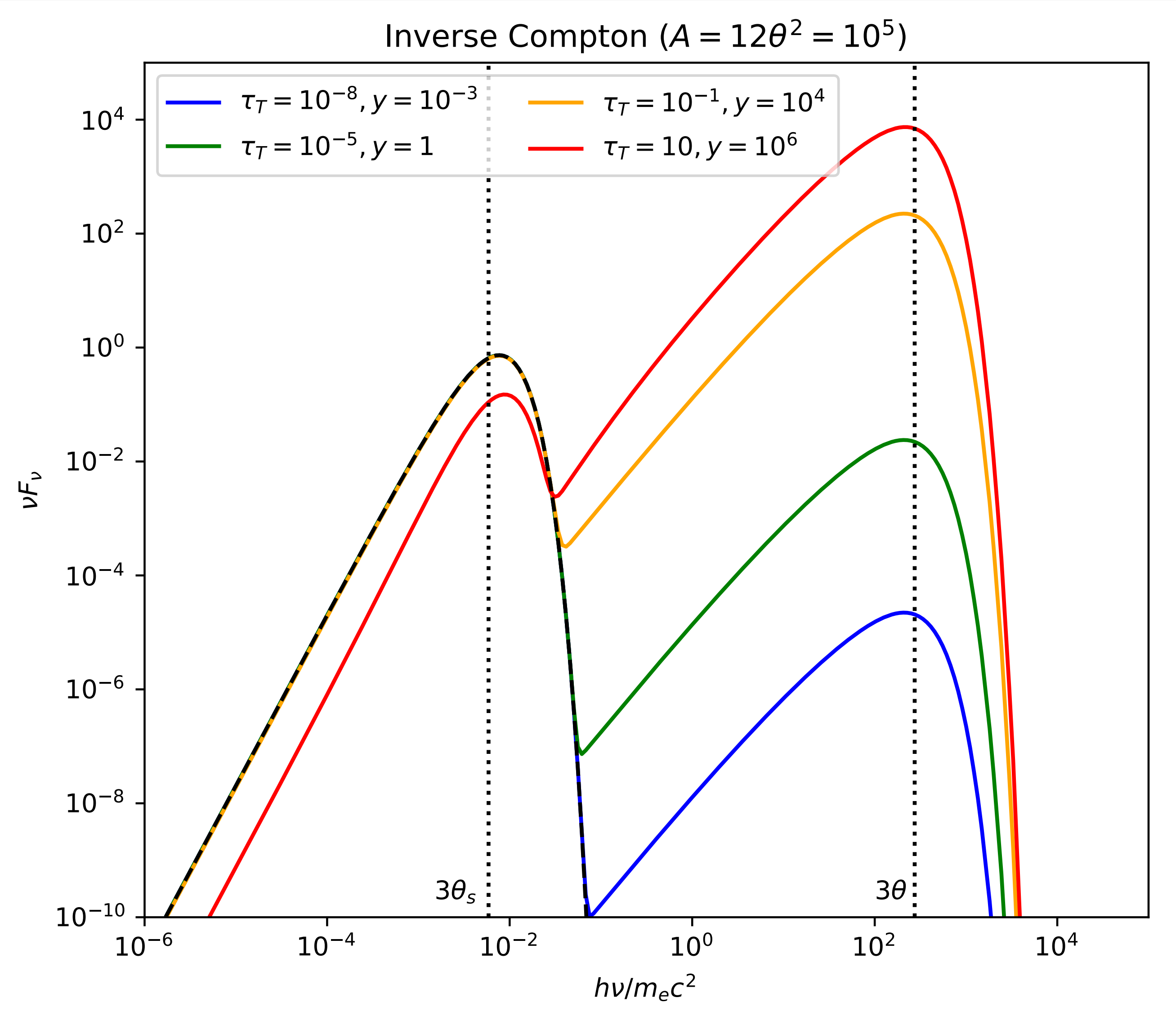


Regimes of Multiple up-scattering

- **$\gamma h\nu_0/mc^2 < 1 < \gamma(\gamma-1)$:**
  - Thomson
  - large A => Several Compton orders

- **$1 < \gamma h\nu_0/mc^2 < \gamma(\gamma-1)$:**
  - KN, large A => Only one Compton order





Kinetic Equation

- **General kinetic equation for photons including :**

$$\frac{dN_\nu}{dt} = c\iint \left[\frac{d\sigma}{d\nu}(\nu',\gamma\to\nu) - \frac{d\sigma}{d\nu'}(\nu,\gamma\to\nu')\right] dN_{\nu'}dN_\gamma$$

  - Steady state solution for thermal distribution: Wien spectrum $N_\nu \propto \nu^2 e^{-\frac{h\nu}{k_BT}}$

- **In the sub-relativistic regime, thermal particles: The Kompaneets equation**

$$\frac{dN_\omega}{dt} = n_e c\sigma_T\left[\frac{d}{d\omega}\left(\omega(\underbrace{\omega-4\theta)N_\omega}_{\text{cooling}}\right) + \theta\frac{d^2(\omega^2N_\omega)}{d\omega^2}\right] \qquad \omega = \frac{h\nu}{mc^2}$$

- **Must be included in some sort of transfert equation**
  - Source and escape terms
  - Other processes

Induced Compton Scattering

- **Total scattering rate:** $(1+n(\nu'))\,\dfrac{d\sigma}{d\nu'}(\nu,\gamma\to\nu')$
  - With occupation number: $n(\nu') = \dfrac{c^3}{8\pi}\dfrac{N'_\nu}{\nu'^2}$

- **n is a strongly decreasing function of ν**
  - ex: Planck spectrum: $n \propto T/\nu$
  - => favours down-scattering

- **Steady state solution for thermal particle distributions:**
  - Bose-Einstein distribution $N_\nu = \dfrac{8\pi}{c^3}\dfrac{\nu^2}{\lambda_N e^{\frac{h\nu}{kT}} - 1} + C_N\delta(\nu)$
  - Fugacity and chemical potential: $\lambda_N = e^{\mu_N} > 1$
  - Harder than pure Compton scattering at low frequency ($F_\nu \propto \nu^3$ instead of $F_\nu \propto \nu^2$)

- **Implied in high brightness-temperature radio sources…**

Two-Photon Scattering

- **Particle physics also predicts multiple-photon scattering** $\gamma + e \rightarrow \gamma + \gamma + e$

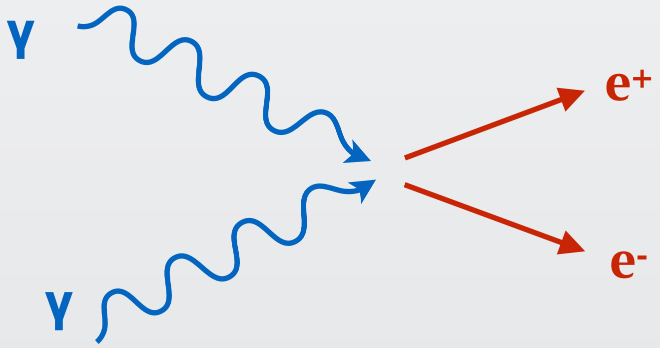
  $\gamma + e \rightarrow \gamma + \gamma + \gamma + e$

- **Cross-section decreases as** α_f^n

- **However, it becomes a sources of photons that can compete with other emission mechanisms**

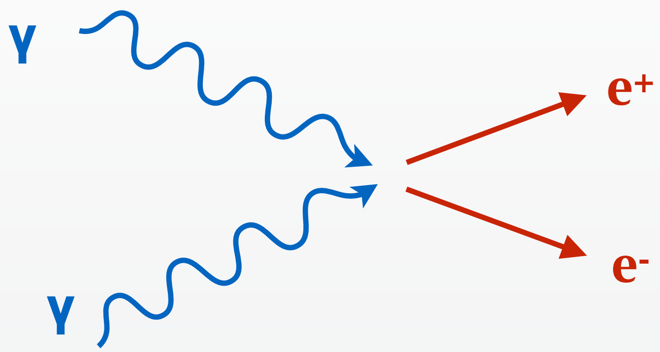
- **Ex:**
  - Pulsar magnetosphere:
    - Need for photons to produce pairs and fill the magnetosphere
    - In competition with photon splitting $\gamma + B \rightarrow \gamma + \gamma$
  - GRB photosphere:
    - Photon production rates decreases as the jet expands until it freezes
    - Double Compton might play a role in transition thermal->non-thermal
    - In competition with bremsstrahlung

V. Pair production



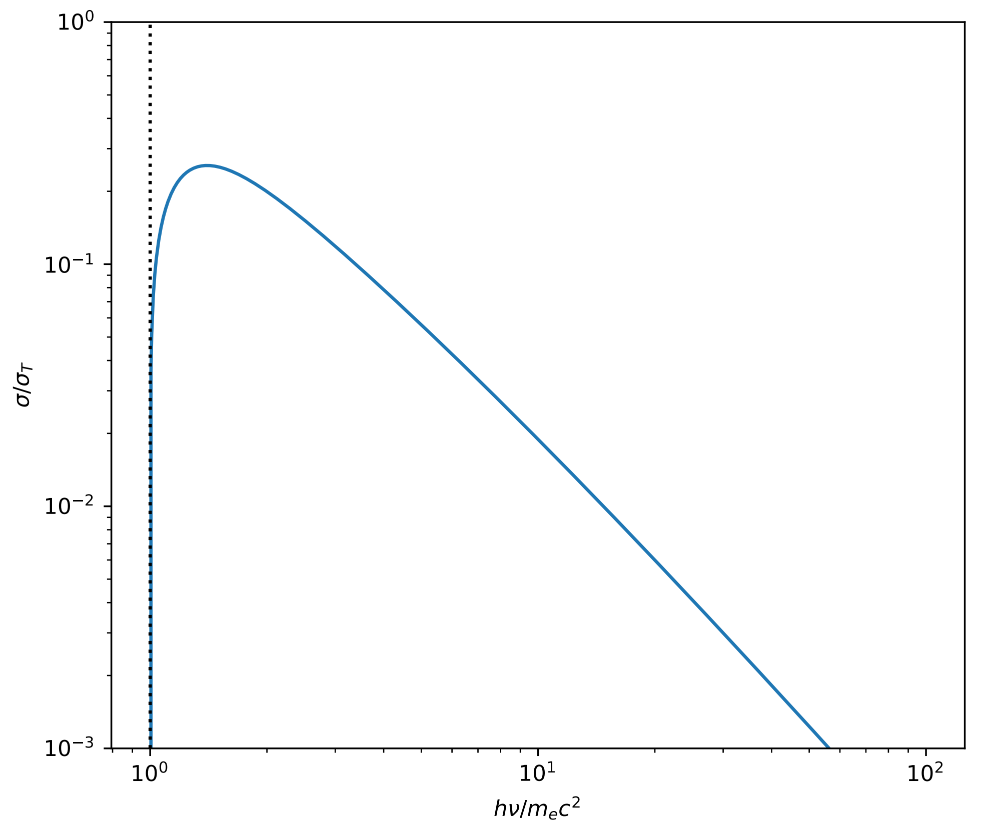
- **Photon-photon annihilation**

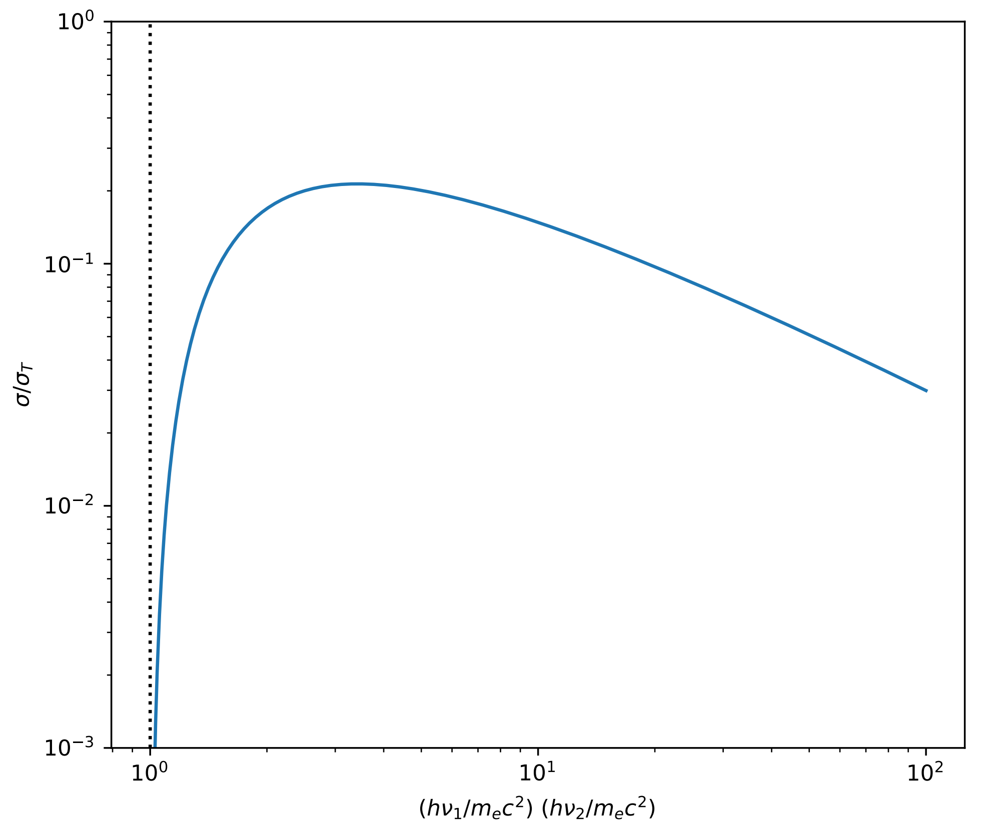
Cross Section



- **In the center of momentum frame:**
  - 2 photons of same energy $h\nu'$
  - 2 leptons of same energy γ'
  - Energy threshold: $h\nu' > m_e c^2$
  - Energy conservation: $h\nu' = \gamma' m_e c^2$
  - Annihilation requires MeV photons

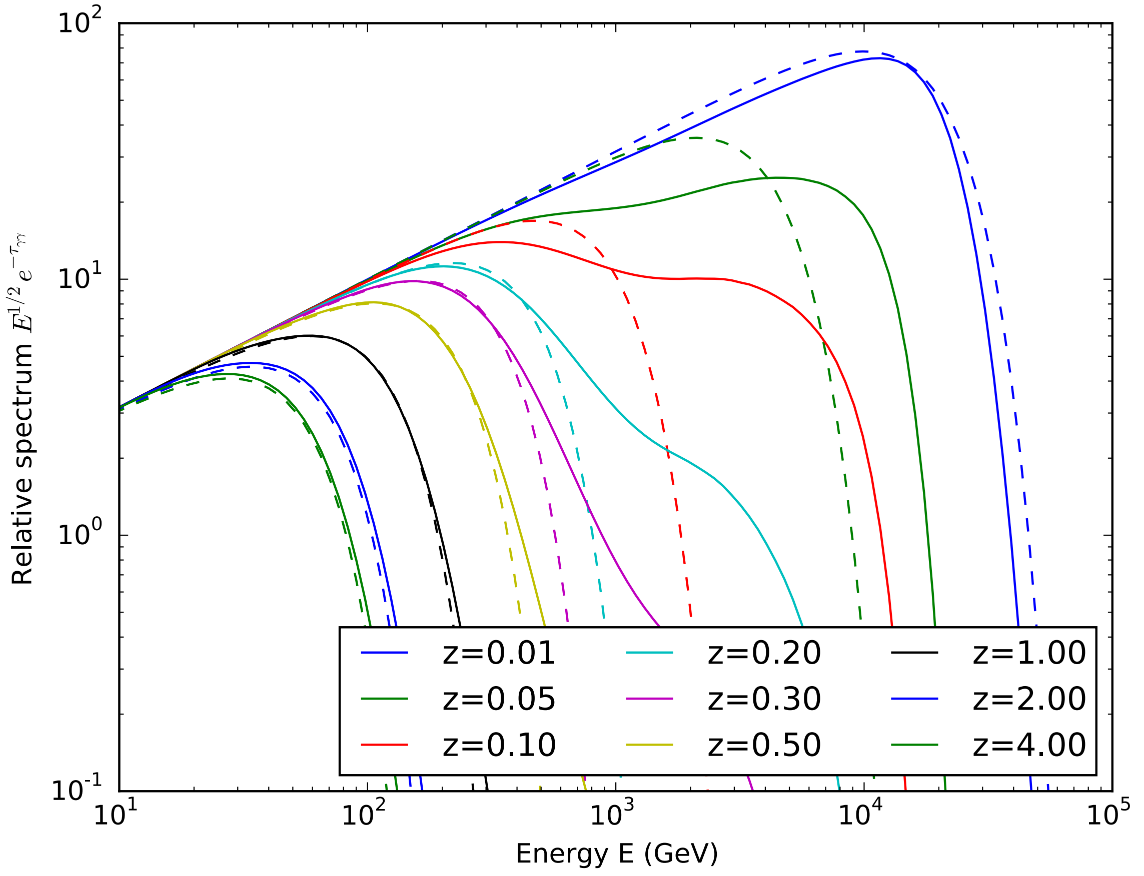
- **In the source frame:**
  - Results depends on the angles
  - Threshold: $\frac{h\nu_1}{m_e c^2} \frac{h\nu_2}{m_e c^2} \frac{1-\cos\theta}{2} \geq 1$
    - Head-on collisions: softer threshold
    - Trailing collisions: harder threshold
  - Average for isotropic photons fields:
    - Efficiency peaks at $\boxed{\frac{h\nu_1}{m_e c^2} \frac{h\nu_2}{m_e c^2} \sim 1}$

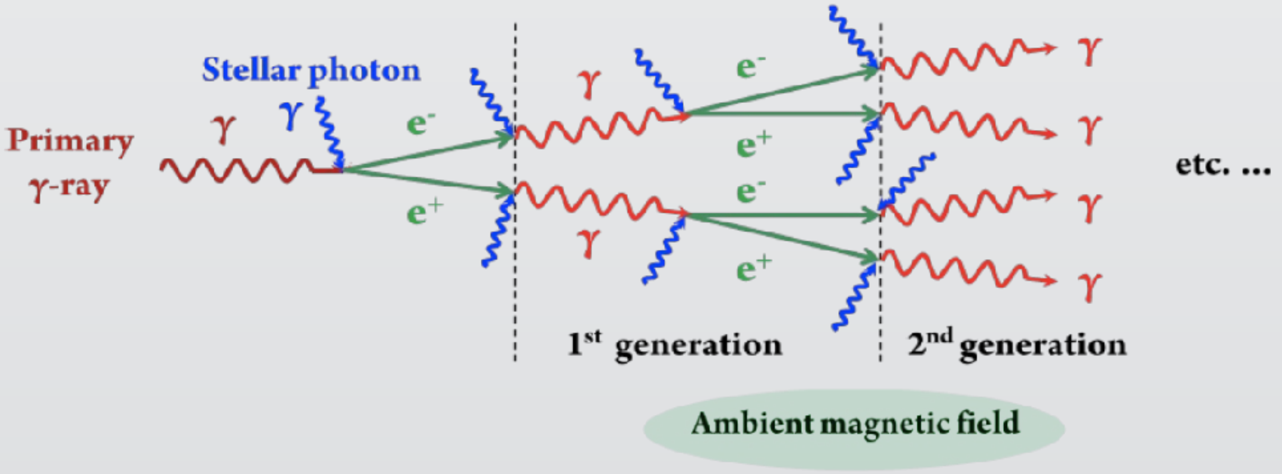




Photon absorption

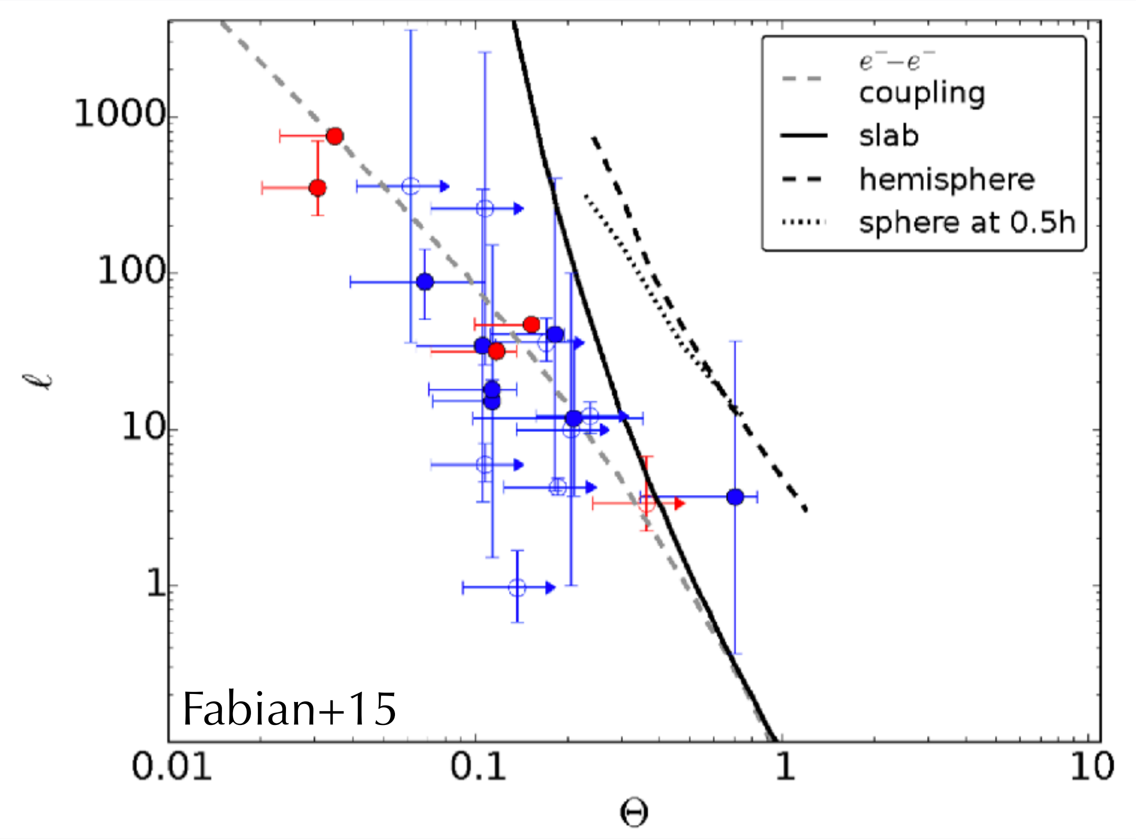
- **Photon-photon optical depth:** $\tau_{\gamma\gamma}(\nu_1) \sim \frac{\sigma_T}{5} n_\gamma(\nu_2) R$ $\quad h\nu_2 = \frac{(m_e c^2)^2}{h\nu_1}$
- **Absorption:** $I_\nu = I_\nu^0 e^{-\tau_{\gamma\gamma}}$
  - Optically thick/thin: $\tau_{\gamma\gamma} = 1$
- **Absorption along the line of sight: pair cascades**
  - Extragalactic TeV photon are absorbed by EBL photons
  - Produced pairs up-scatter CMB photons
  - etc… => shower



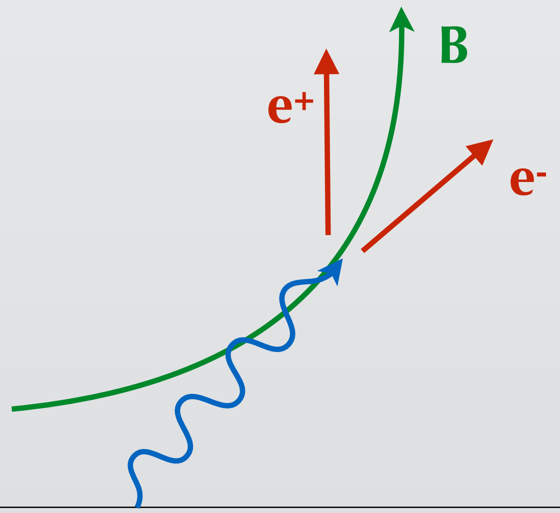


Pair production

- **Matter production and temperature decrease**
- **Pair thermostat in AGN** **(e.g. Svensson+84):**
  - For kT > $m_e c^2$: MeV photons (e.g. Compton)
  - For high photon density sources ($l = \dfrac{\sigma_T L}{R m_e c^3} > 1$): strong pair production
  - => Temperature saturation



- **Pair instability supernovae** **(e.g. Gilmer+17)**
  - For M~100 M_o
  - Pair production => temperature and pressure decrease
  - Collapse trigger
- **Pulsar magnetosphere**
  - Pair plasma
  - Production by
    - double photon annihilation
    - Single photon annihilation in strong field
    - Curvature radiation



References

- **Main text books:**
  - Rybicki G.B. & Lightman A.P., Radiative processes in astrophysics (1979)
  - Aharonian F. A., Very High energy cosmic gamma radiation (2004)
  - Ghisellini, Radiative processes in high energy astrophysics (2012)
  - Shu F.H., The physics of astrophysics, vol.II: radiation (1992)
  - Jackson J.D., Classical Electrodynamics (1998)

- **Detailed cross sections**
  - Jauch J.M. & Rohrlich F., The theory of photons and electrons (1980)
  - Heitler W., Quantum theory of radiation (1954)
  - Blumenthal G.R. & Gould R.J., Reviews of Modern Physics (1970)