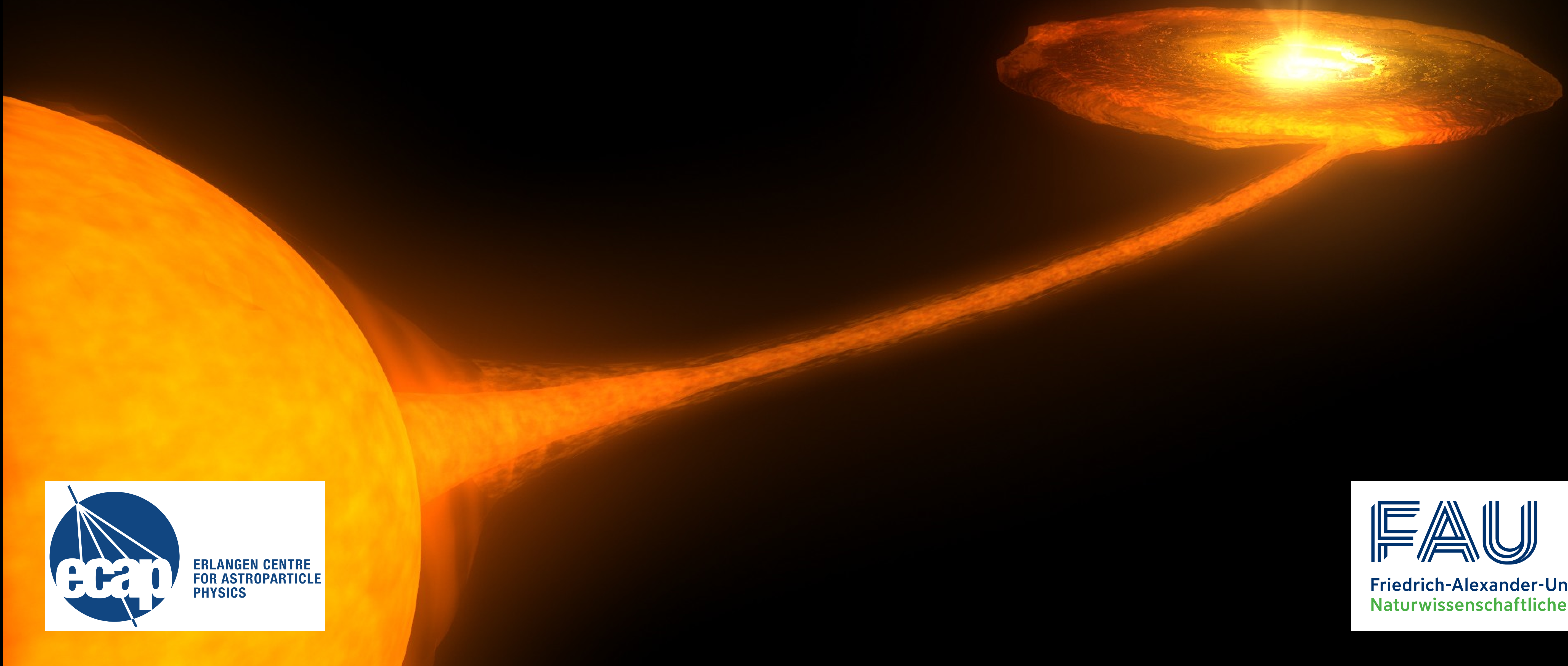
Accretion:
An Introduction

Jörn Wilms



ERLANGEN CENTRE FOR ASTROPARTICLE PHYSICS

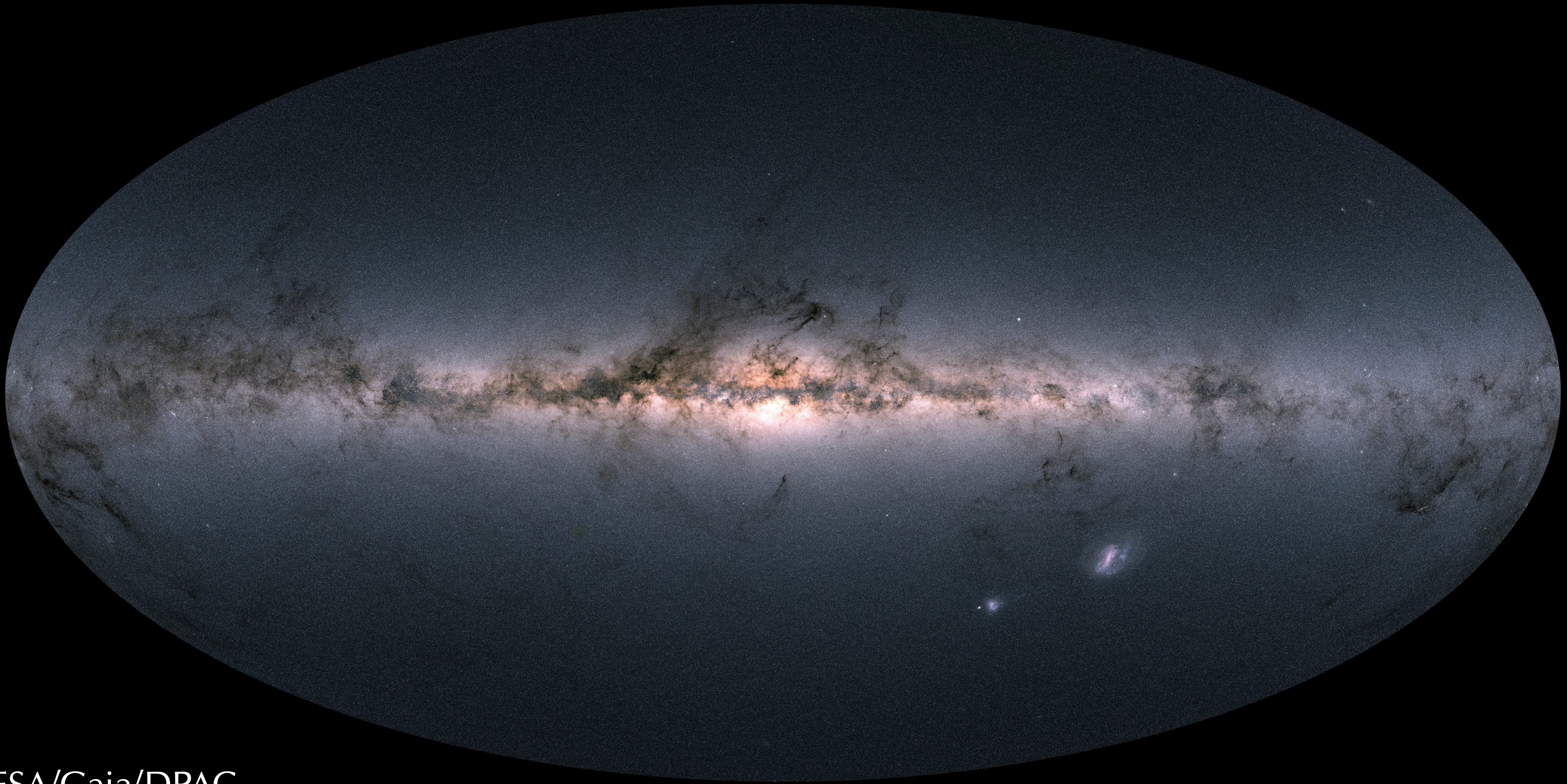
Friedrich-Alexander-Universität
Naturwissenschaftliche Fakultät

Contents

Structure of this presentation:

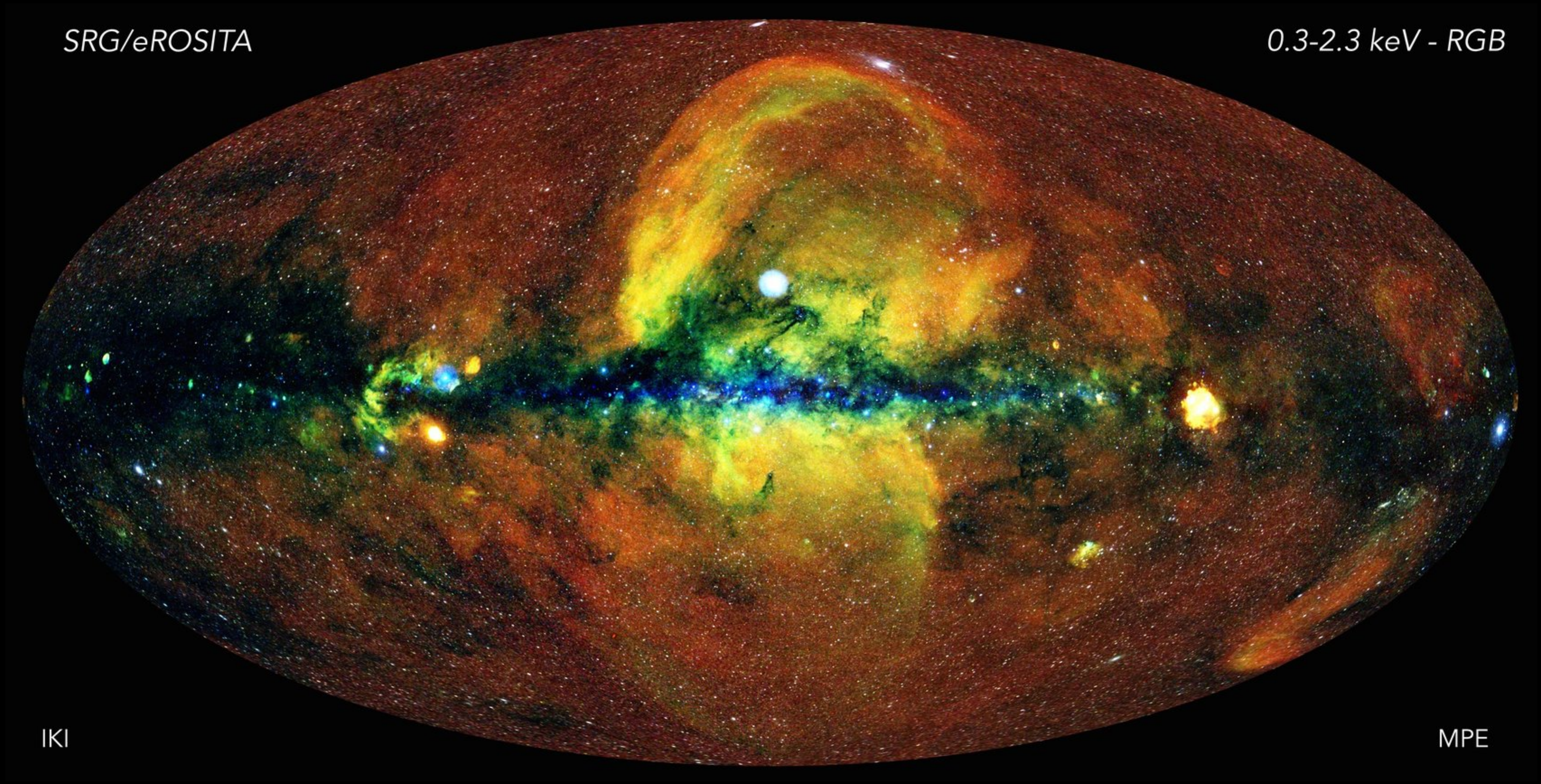
- Why study accretion?
  Observational motivation
- Theory of thin accretion disks
  ...too much math
- Confrontation with observation
  some examples, see remainder of the school for the gory details
- Spherical accretion in X-ray binaries
  ...if there is time
- Outlook

The visual Sky

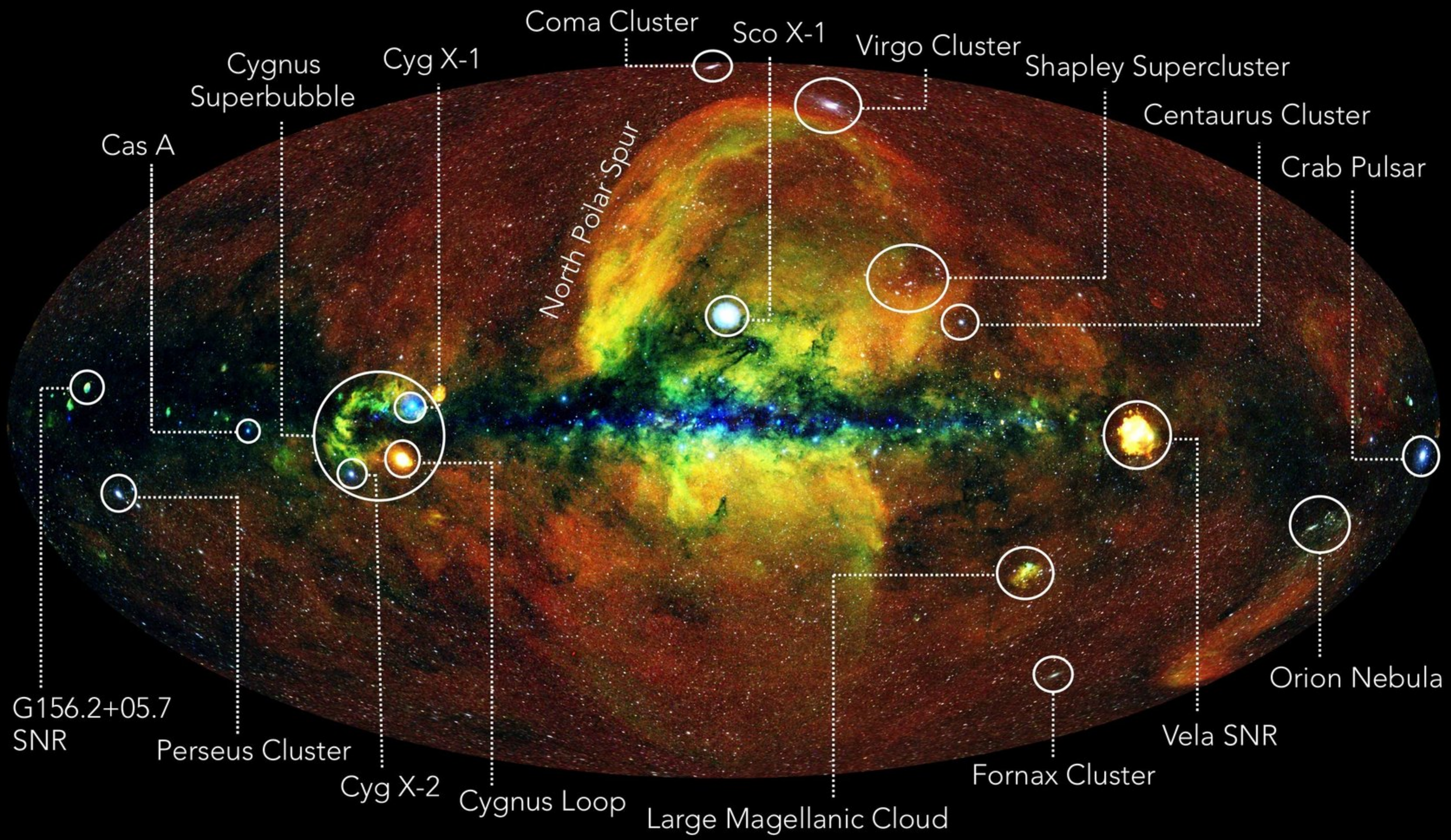


ESA/Gaia/DPAC

The first eROSITA All Sky Survey

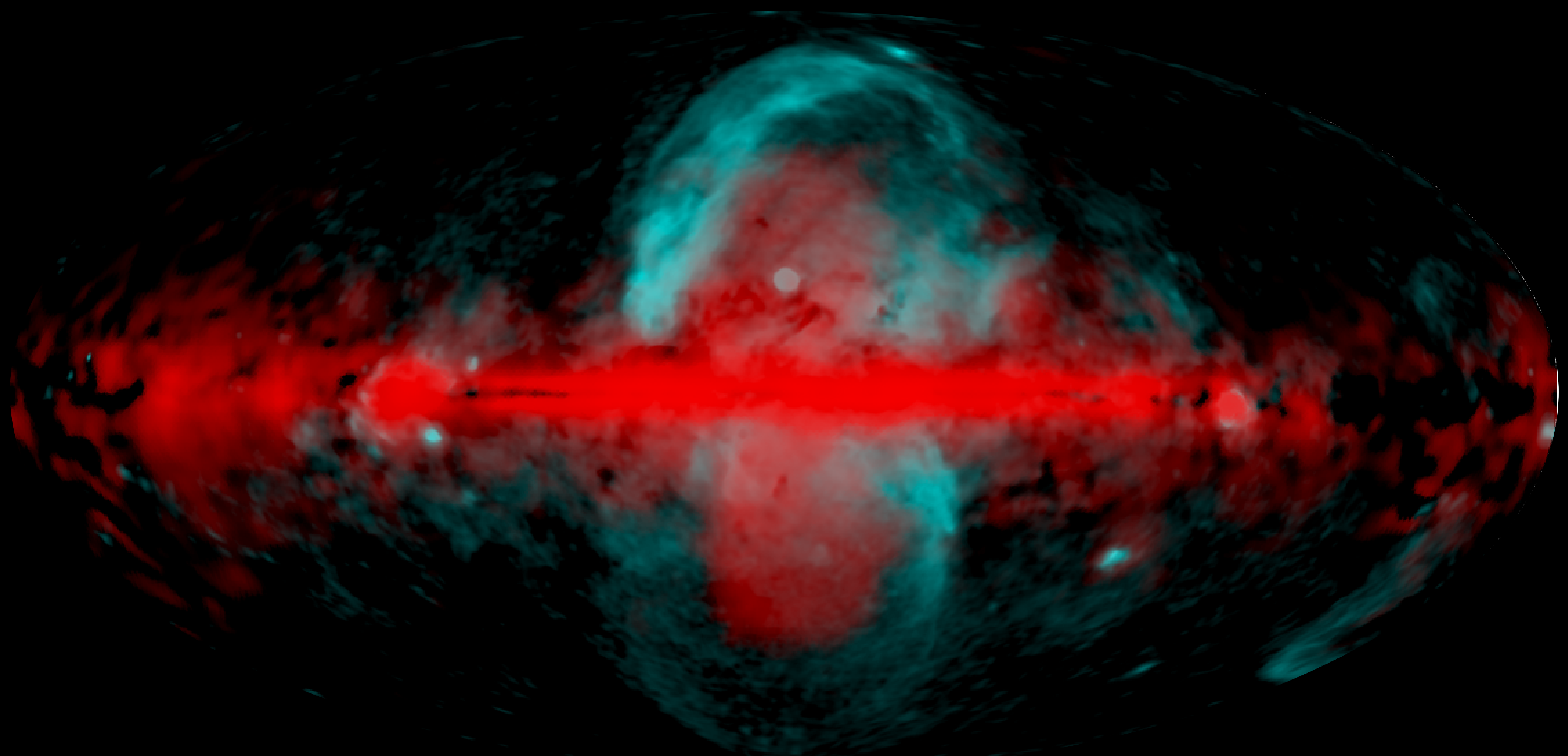


Navigating the eROSITA X-ray sky



- 1.1 M sources
- >5k galaxy clusters

eROSITA and the Fermi bubbles



Predehl et al., 2020, Nature 588, 227 blue: X-rays (0.3–2.3 keV), red: γ-rays (20 MeV–300 GeV)

$L_X \sim 1 \times 10^{39}\,\mathrm{erg\,s^{-1}} \sim 25 \times 10^6\,L_\odot$

need $10^{41}\,\mathrm{erg\,s^{-1}}$ for a few Myr (Star Burst or Activity in Sgr A*)

Astrophysical energy sources:

1. Nuclear fusion (stars)

Reactions à la

$$4p \longrightarrow {}^{4}He + \Delta E_{nuc}$$

Energy released:

Fusion produces $\sim 6 \times 10^{11}\,J\,g^{-1}$

(i.e., $\Delta E_{nuc} \sim 0.007 m_p c^2$)

How to make X-rays

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2. Gravitation (X-ray astro)

Accretion of mass m from ∞ to R_S on black hole with mass M gives

$$\Delta E_{\text{acc}} = \frac{GMm}{R_S} \quad \text{where } R_S = \frac{2GM}{c^2}$$

Accretion produces $\sim 10^{13}\,\text{J}\,\text{g}^{-1}$

(i.e., $\Delta E_{\text{acc}} \sim 0.1\, m_p c^2$)

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Accretion produces $\sim 10^{13}\,\mathrm{J\,g^{-1}}$

(i.e., $\Delta E_\mathrm{acc} \sim 0.1\, m_p c^2$)

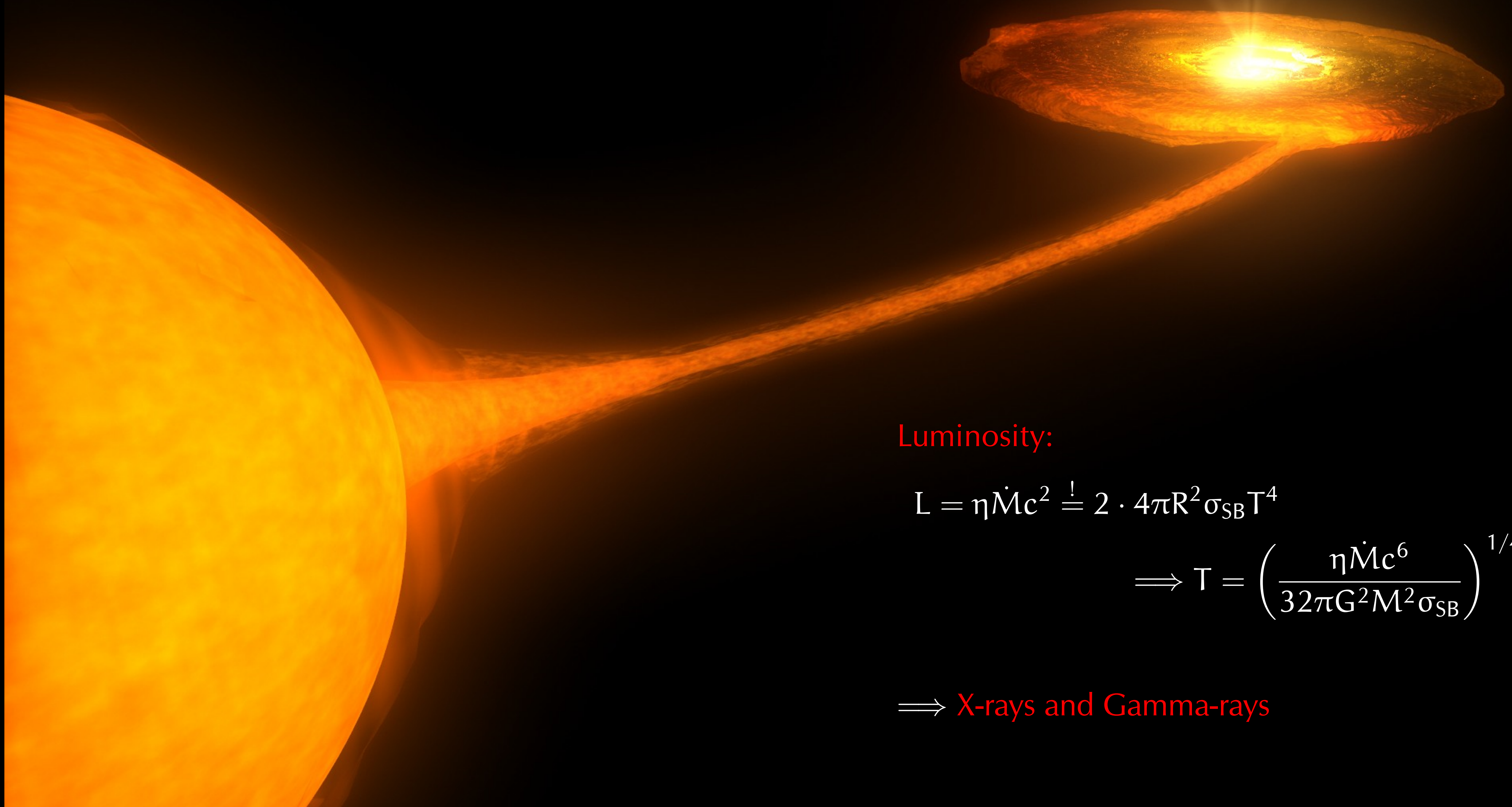
$\Longrightarrow$ Accretion of material is the most efficient astrophysical energy source.

…thus accreting objects are the most luminous in the whole universe, and crucial for its evolution.

Note: energy gets radiated away from *outside* the Schwarzschild radius!

X-ray Binary: Material flows from normal star via inner Lagrange point onto Black Hole

$\Longrightarrow$ Formation of an accretion disk.



Luminosity:

$$L = \eta \dot{M} c^2 \overset{!}{=} 2 \cdot 4\pi R^2 \sigma_{SB} T^4$$

$$\Longrightarrow T = \left(\frac{\eta \dot{M} c^6}{32\pi G^2 M^2 \sigma_{SB}} \right)^{1/4} \sim 10^7\,\mathrm{K}$$

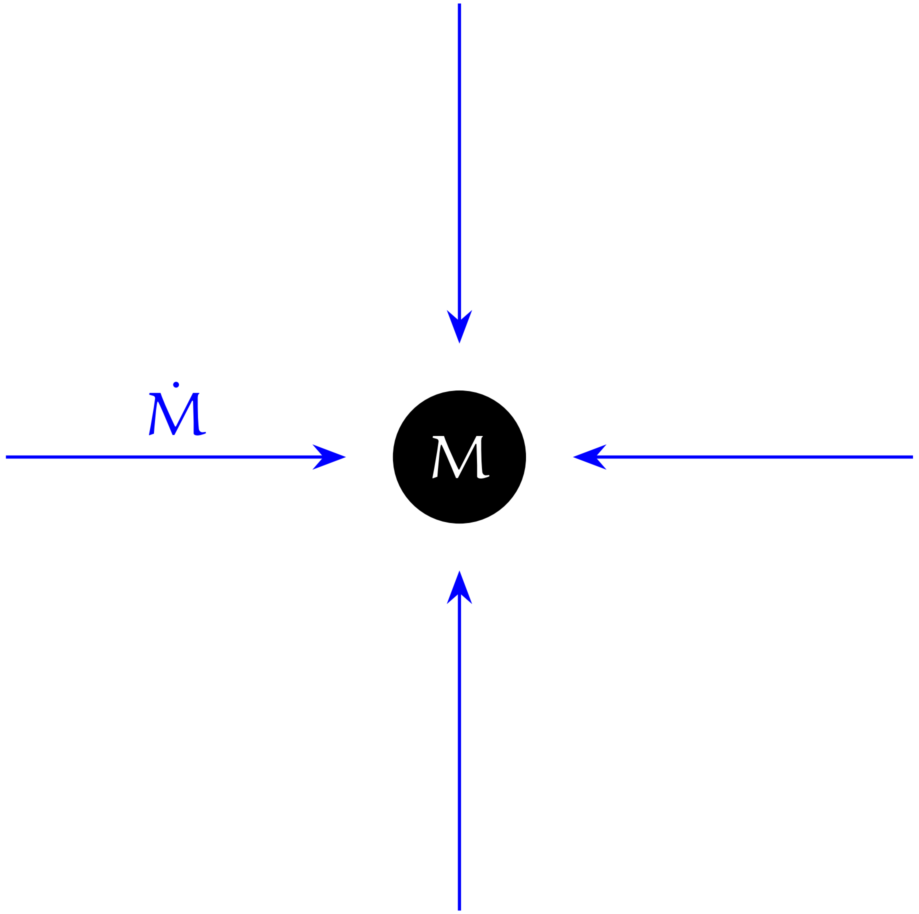
$$\Longrightarrow kT \sim 4\,\mathrm{keV}$$

$\Longrightarrow$ X-rays and Gamma-rays

Eddington luminosity

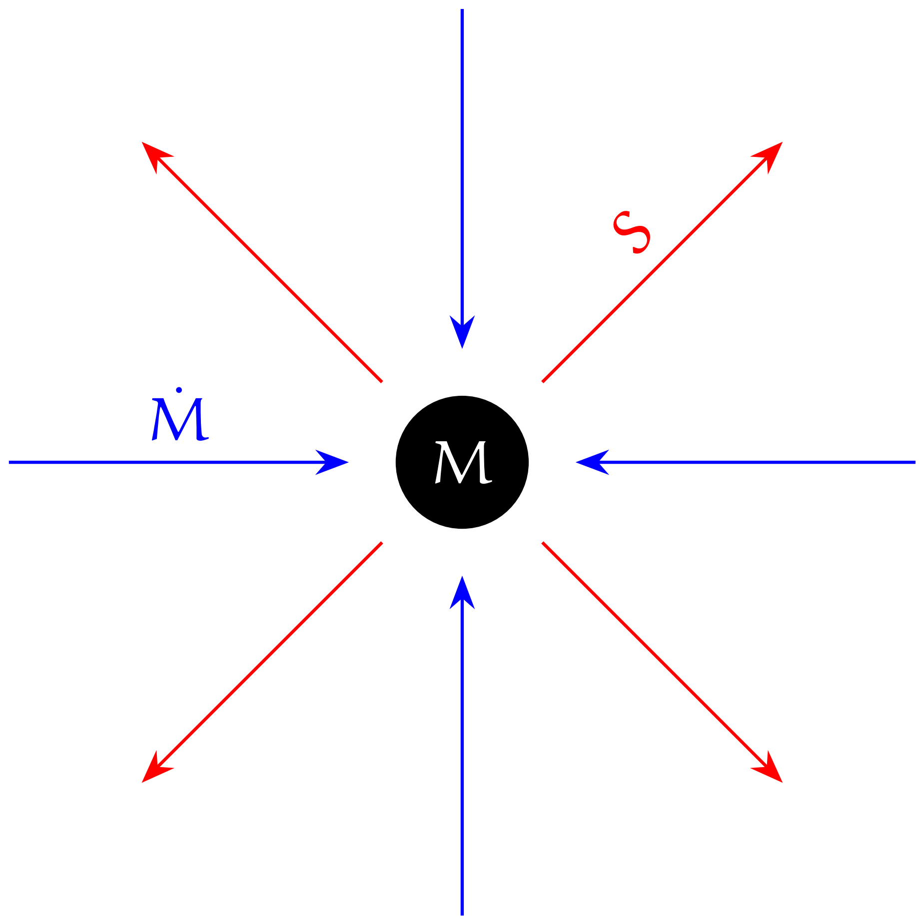
Assume mass M

Eddington luminosity



Assume mass M spherically symmetrically accreting ionized hydrogen gas, rate: $\dot{M}$.

Eddington luminosity



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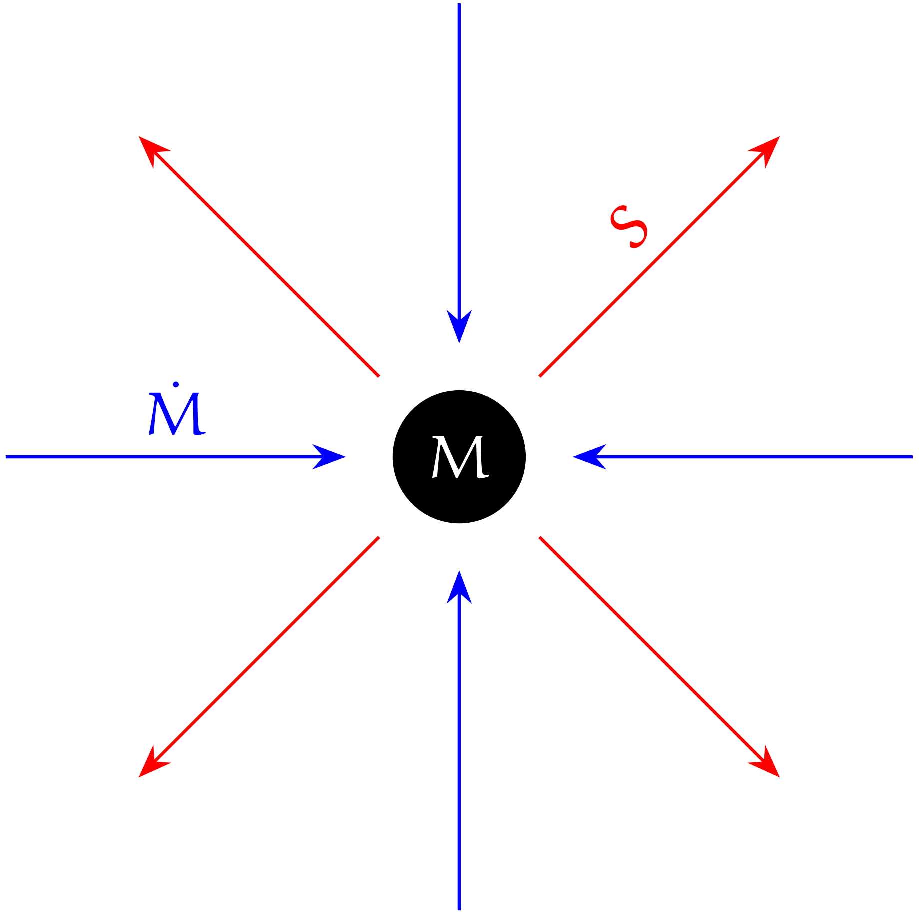
At radius r, accretion produces energy flux S.

Eddington luminosity



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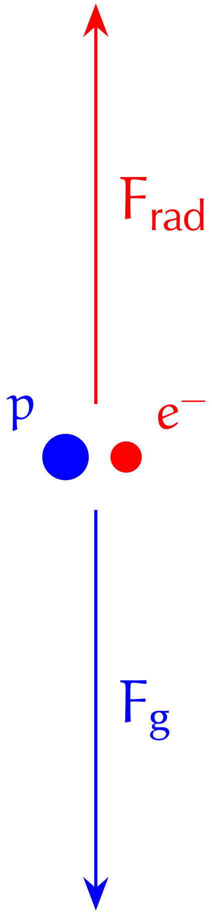
At radius r, accretion produces energy flux S.

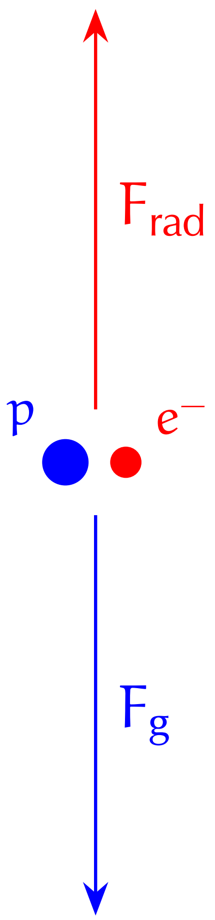
Important: Interaction between accreted material and radiation!

Eddington luminosity

Force balance on accreted electrons and protons:



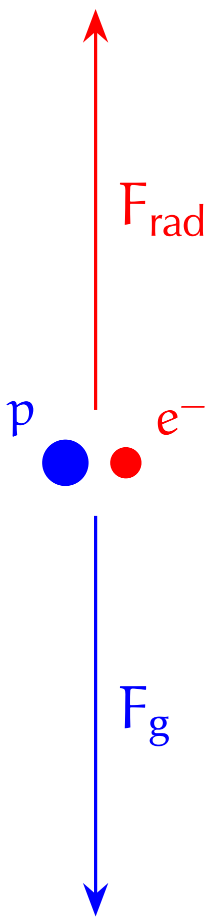
Eddington luminosity



Force balance on accreted electrons and protons: Inward force: gravitation:

$$F_g = \frac{GMm_p}{r^2}$$

Eddington luminosity



Force balance on accreted electrons and protons: Inward force: gravitation:

$$F_g = \frac{GMm_p}{r^2}$$

Outward force: radiation force:

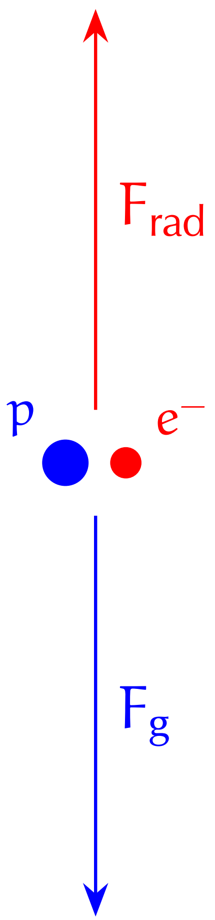
$$F_{rad} = \frac{\sigma_T S}{c}$$

where energy flux S is given by

$$S = \frac{L}{4\pi r^2}$$

where L: luminosity.

Eddington luminosity



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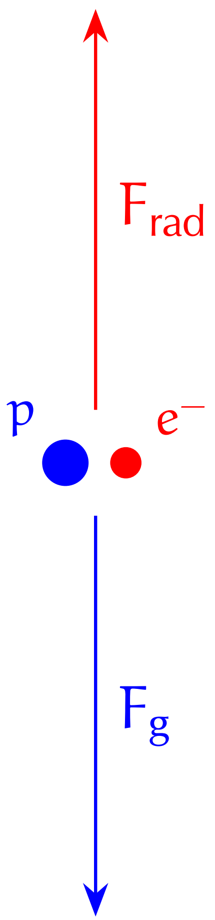
Note: $\sigma_T \propto (m_e/m_p)^2$, so negligible for protons.

But: strong Coulomb coupling between electrons and protons $\Longrightarrow$ F_{rad} also has effect on protons!

Eddington luminosity



Accretion is only possible if gravitation dominates:

$$\frac{GMm_p}{r^2} > \frac{\sigma_T S}{c} = \frac{\sigma_T}{c} \cdot \frac{L}{4\pi r^2}$$

and therefore

$$L < L_{Edd} = \frac{4\pi G M m_p c}{\sigma_T}$$

or, in astronomically meaningful units

$$L < 1.3 \times 10^{38}\,\mathrm{erg\,s^{-1}} \cdot \frac{M}{M_\odot}$$

where L_{Edd} is called the Eddington luminosity.

But remember the assumptions entering the derivation: spherically symmetric accretion of fully ionized pure hydrogen gas.

Eddington luminosity

Characterize accretion process through the accretion efficiency, η:

$$L = \eta \cdot \dot{M} c^2$$

where $\dot{M}$: mass accretion rate (e.g., $\mathrm{g\,s^{-1}}$ or $M_\odot\,\mathrm{yr}^{-1}$).

Therefore maximum accretion rate ("Eddington rate"):

$$\dot{m} = \frac{L_{\mathrm{Edd}}}{\eta c^2} \sim 2 \times 10^{-8} \cdot \left(\frac{M}{1\,M_\odot}\right) M_\odot\,\mathrm{yr}^{-1}$$

(for $\eta = 0.1$)

Typical mass accretion rate in a X-ray binary:

$$\dot{M} = 10^{-8}\,M_\odot\,\mathrm{yr}^{-1} = 6.3 \times 10^{18}\,\mathrm{g\,s}^{-1}$$

Eddington luminosity

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$$L_{X,XRB} = 0.1\dot{M}c^2 = 5.7 \times 10^{37}\,erg\,s \sim 10^4\,L_\odot$$

$M_\odot = 2 \times 10^{33}\,g$, $L_\odot = 4 \times 10^{33}\,erg\,s^{-1}$

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$\Longrightarrow$ for XRB at $d = 1000\,\mathrm{pc}$

$$N_{ph,XRB} = \frac{1}{\langle E_X \rangle} \cdot \frac{L}{4\pi d^2} = 80\,\mathrm{ph\,cm}^{-2}\,\mathrm{s}^{-1}$$

assuming $\langle E_X \rangle = 1\,\mathrm{keV} = 1.06 \times 10^{-9}\,\mathrm{erg}$; in reality count rates are lower due to spectral shape

Eddington luminosity

Typical mass accretion rate for a black hole in an Active Galactic Nucleus:

$$\dot{M} = 0.1\,M_\odot\,\mathrm{yr}^{-1}$$

Therefore:

$$L_{AGN} \sim 10^{44}\,\mathrm{erg\,s^{-1}} \sim 10^{10}\,L_\odot$$

about the same as a whole galaxy

$\Longrightarrow$ for nearby AGN at $d = 1\,\mathrm{Mpc}$, 10% in X-rays:

$$N_{ph,X,AGN} = \frac{1}{\langle E_X \rangle} \cdot \frac{0.1 \cdot L_{AGN}}{4\pi d^2} \sim 80\,\mathrm{ph\,cm^{-2}\,s^{-1}}$$

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$\Longrightarrow$ Accreting objects are luminous – X-ray and gamma-ray astronomy necessary to understand this physical process

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Literature

- J. Frank, A. King, D. Raine, 2002, Accretion Power in Astrophysics, 3rd edition, Cambridge Univ. Press

  The standard textbook on accretion, covering all relevant areas of the field. Buy it.

- T. Padmanabhan, 2001, Theoretical Astrophysics, II. Stars and Stellar Systems, Cambridge Univ. Press

  The Landau & Lifshitz of Theoretical Astrophysics. Buy it.

- J.E. Pringle, 1981, Accretion Disks in Astrophysics, Ann. Rev. Astron. Astrophys. **19**, 137

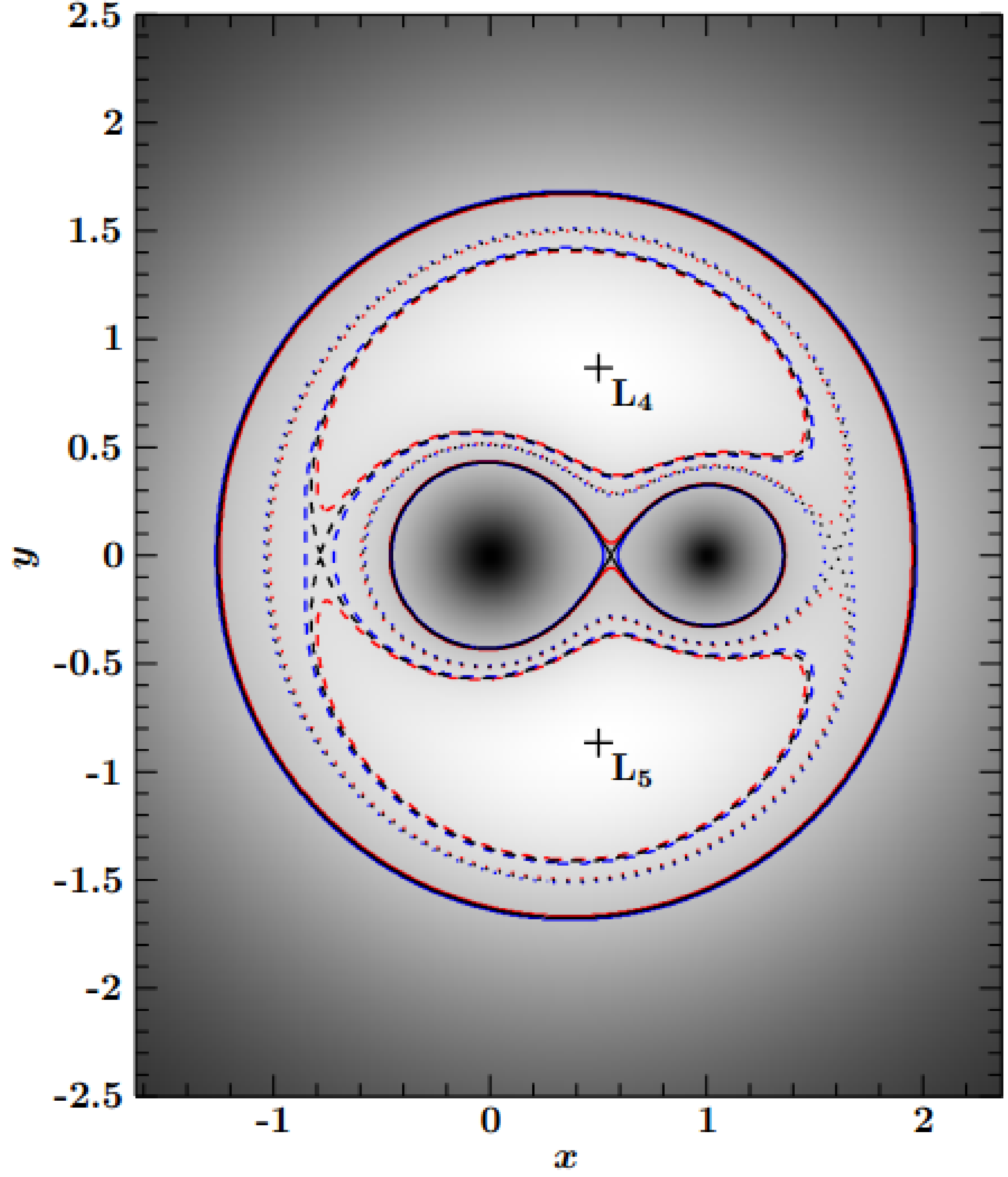
  Concise review of classical accretion disk theory.

- N.I. Shakura & R. Sunyaev, 1973, Black Holes in Binary Systems. Observational Appearance. Astron. Astrophys. **24**, 337 and

  J.E. Pringle & M. Rees, 1972, Accretion Disc Models for Compact X-Ray Sources, Astron. Astrophys., 22(1), 1

  The fundamental papers, which *really* started the field.

Roche Geometry



Motion of gas in corotating frame around masses M_1, M_2 given by

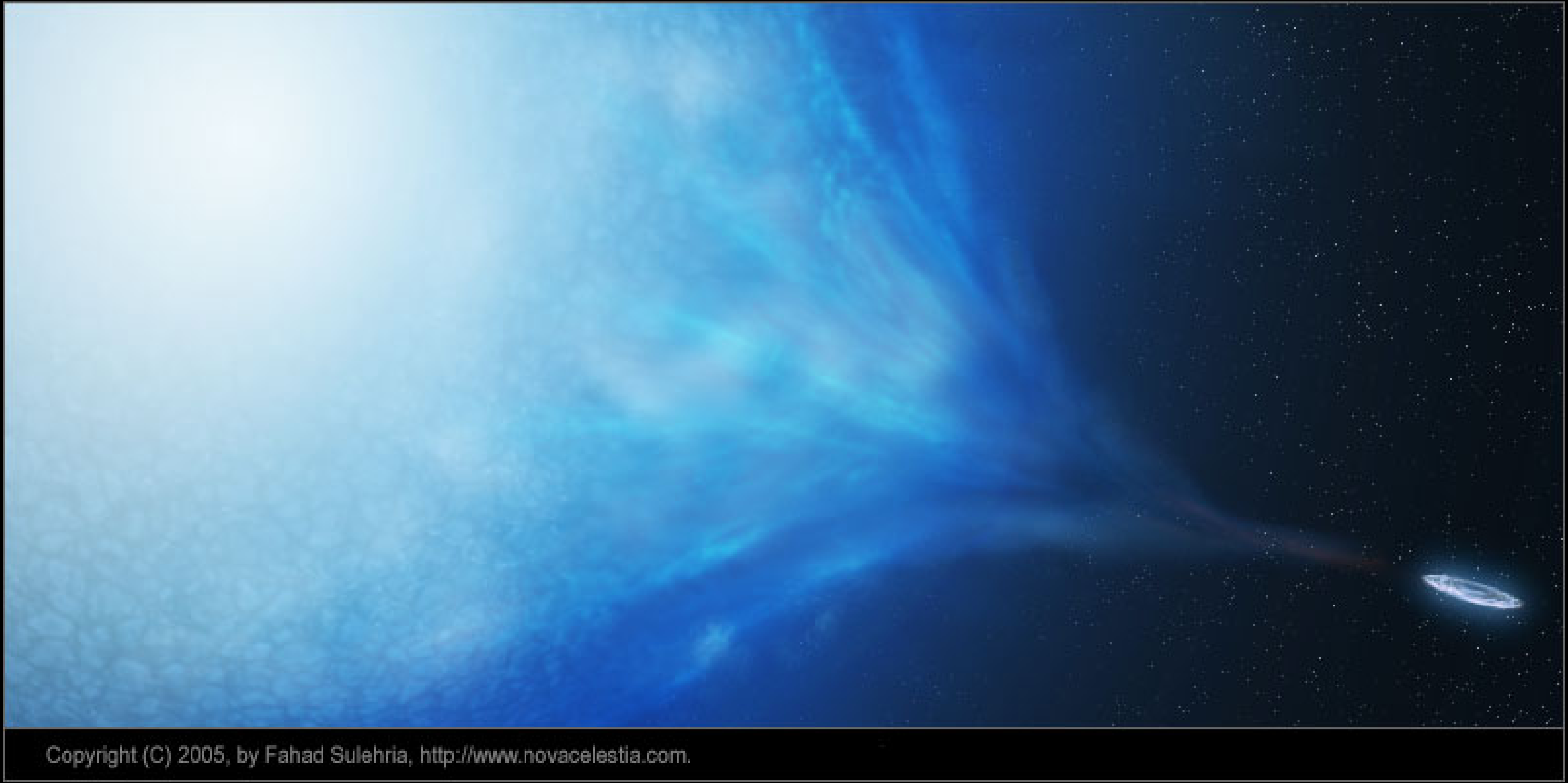
$$\frac{d^2\mathbf{r}}{dt^2} + 2\omega \times \frac{d\mathbf{r}}{dt} = -\frac{1}{\rho}\nabla P - \nabla \Phi_R$$

where the Roche potential:

$$\Phi_R(\mathbf{r}) = -\frac{GM_1}{|\mathbf{r}-\mathbf{r}_1|} - \frac{GM_2}{|\mathbf{r}-\mathbf{r}_2|} - \frac{1}{2}(\omega \times \mathbf{r})^2$$

and where

$$\omega = \left(\frac{GM}{a^3}\right)^{1/2} \hat{e}$$

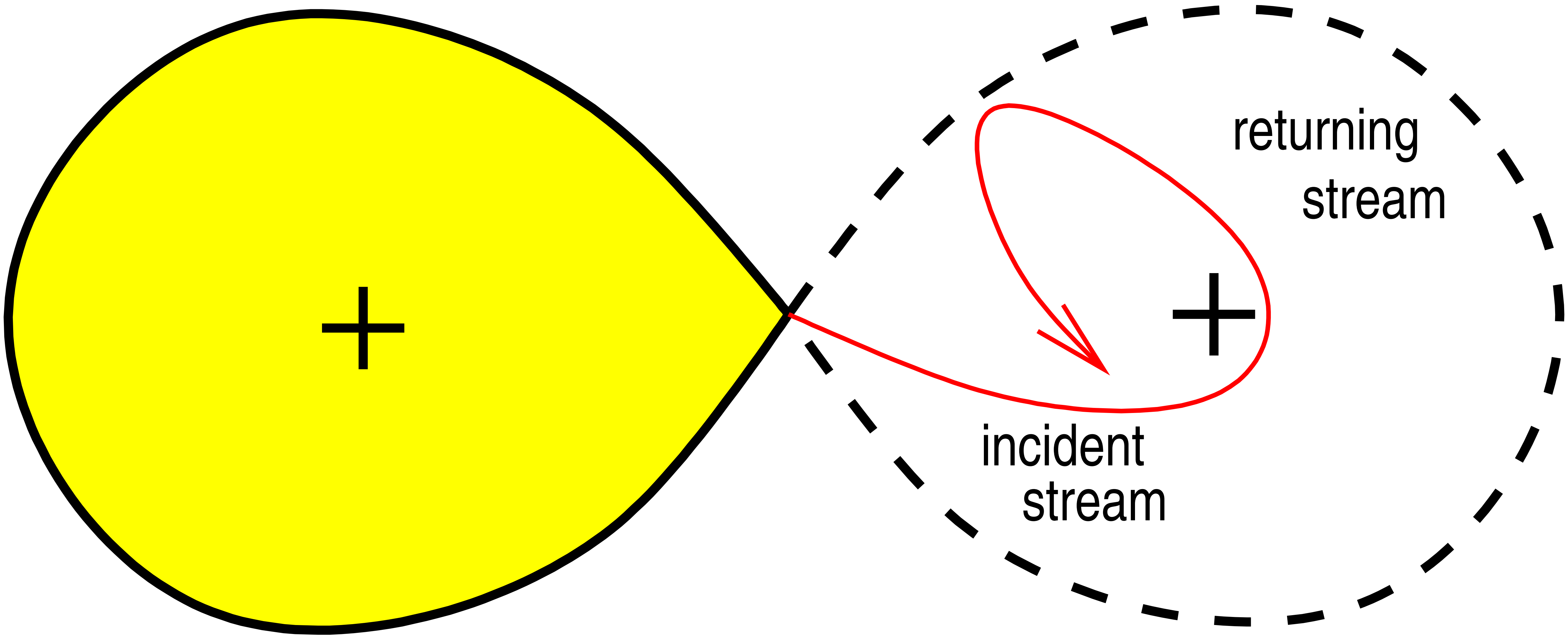


Matter comes from companion star

$\Longrightarrow$ accreted matter has angular momentum

$\Longrightarrow$ accretion disk forms.

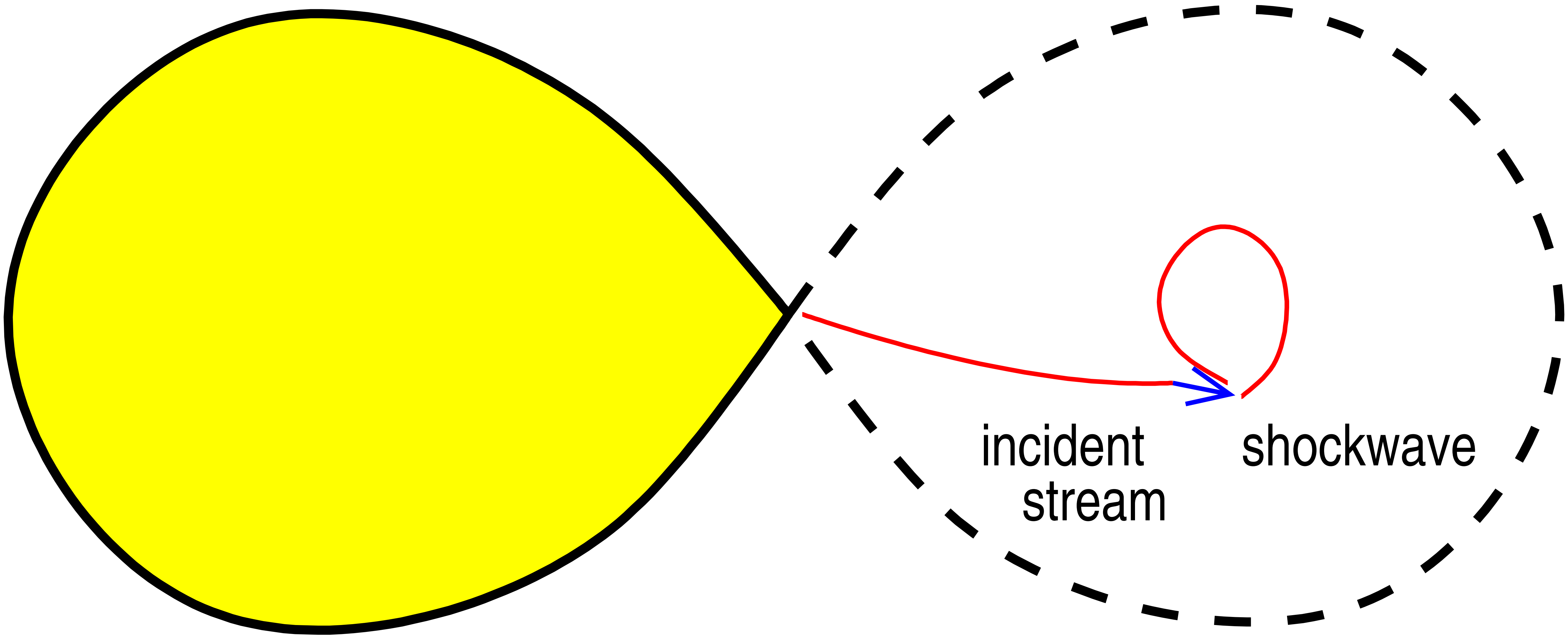
Roche Lobe Overflow



(after Lubow & Shu, 1975, Fig. 4)

Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Roche Lobe Overflow



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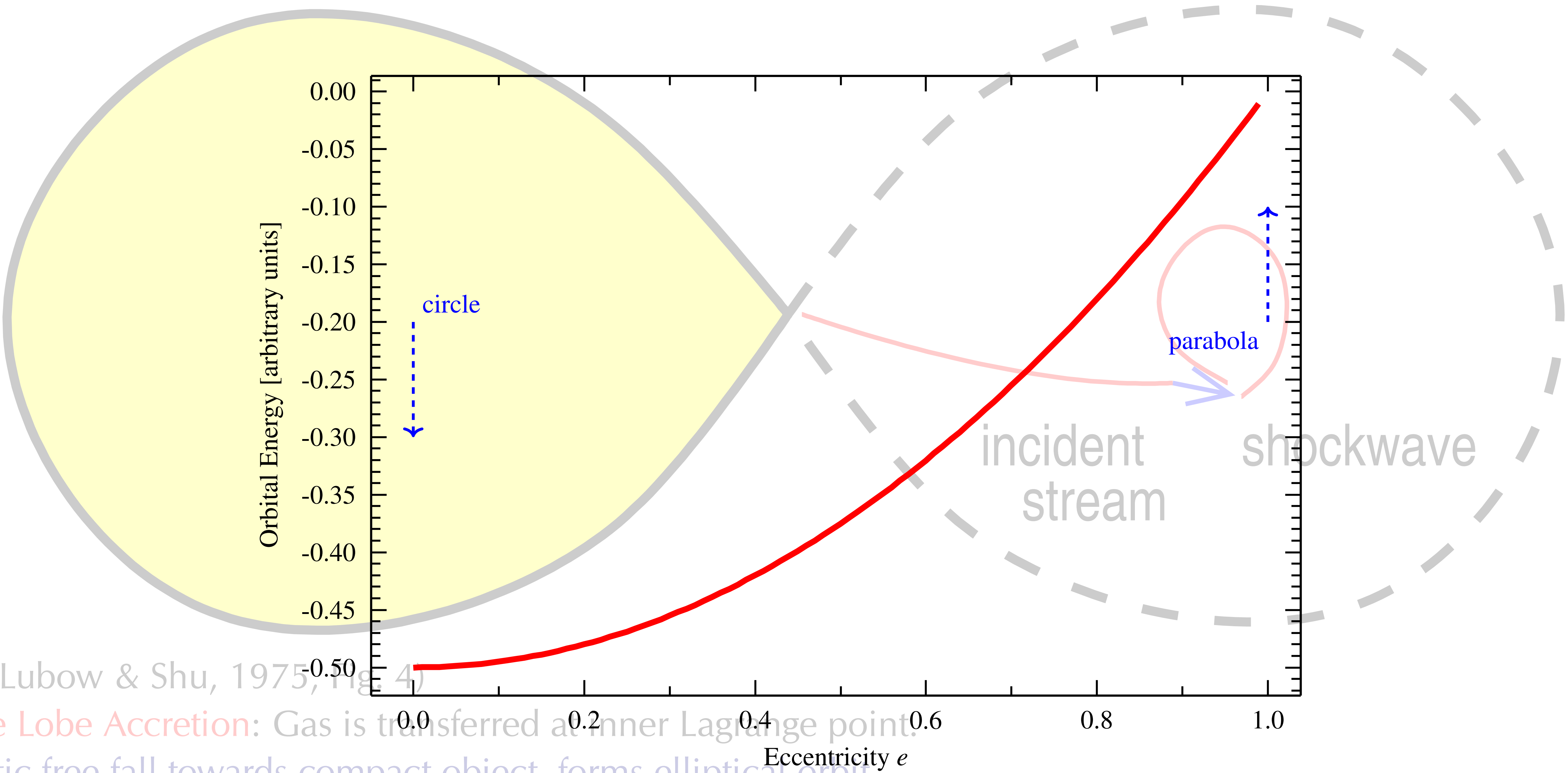
Roche Lobe Accretion: Gas is transferred at inner Lagrange point.

Ballistic free fall towards compact object, forms elliptical orbit

Note: ellipse rotates because of Coriolis force!

Stream intersects $\Longrightarrow$ shock $\Longrightarrow$ randomization $\Longrightarrow$ circular orbit forms.

Roche Lobe Overflow



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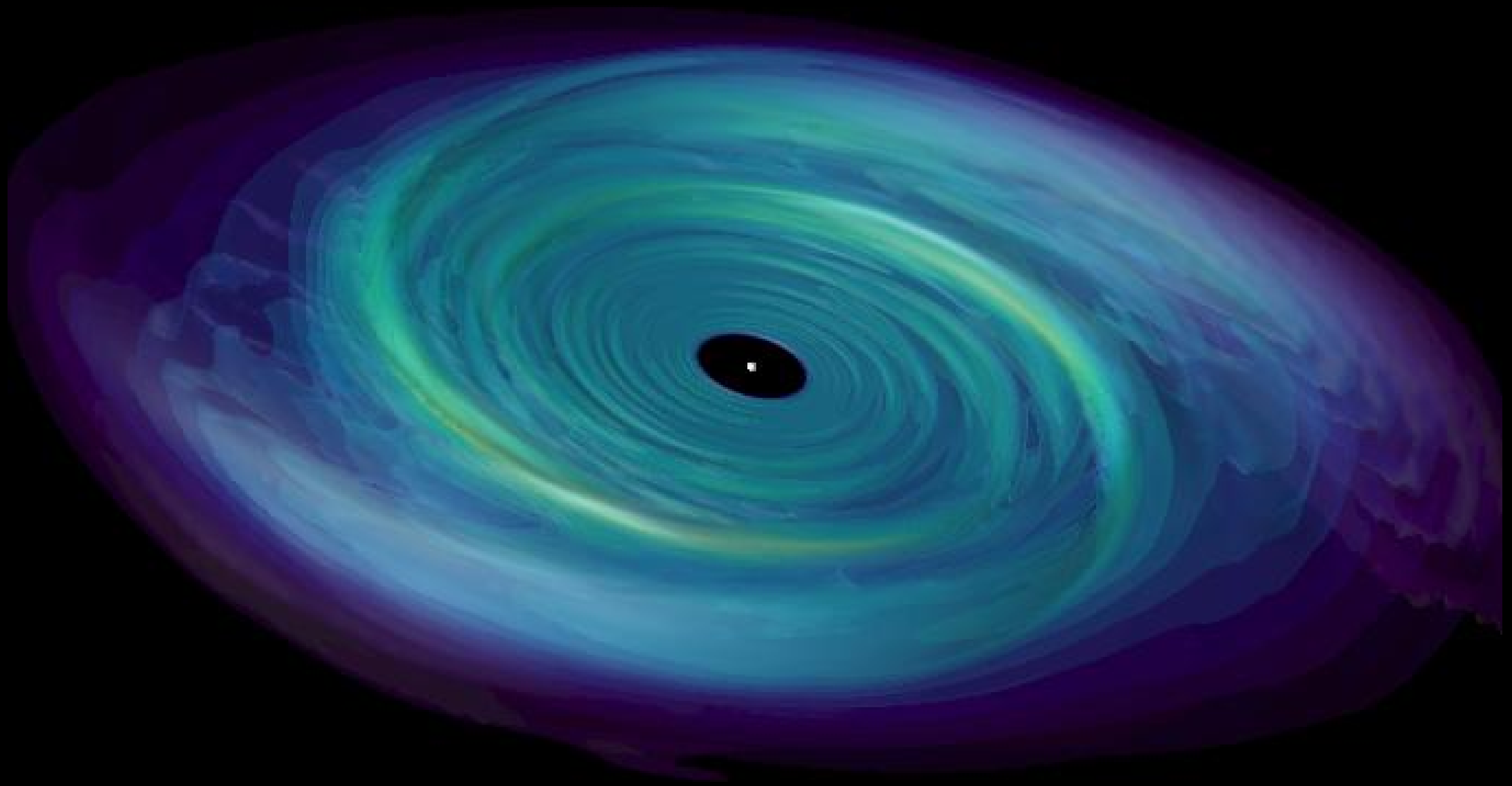
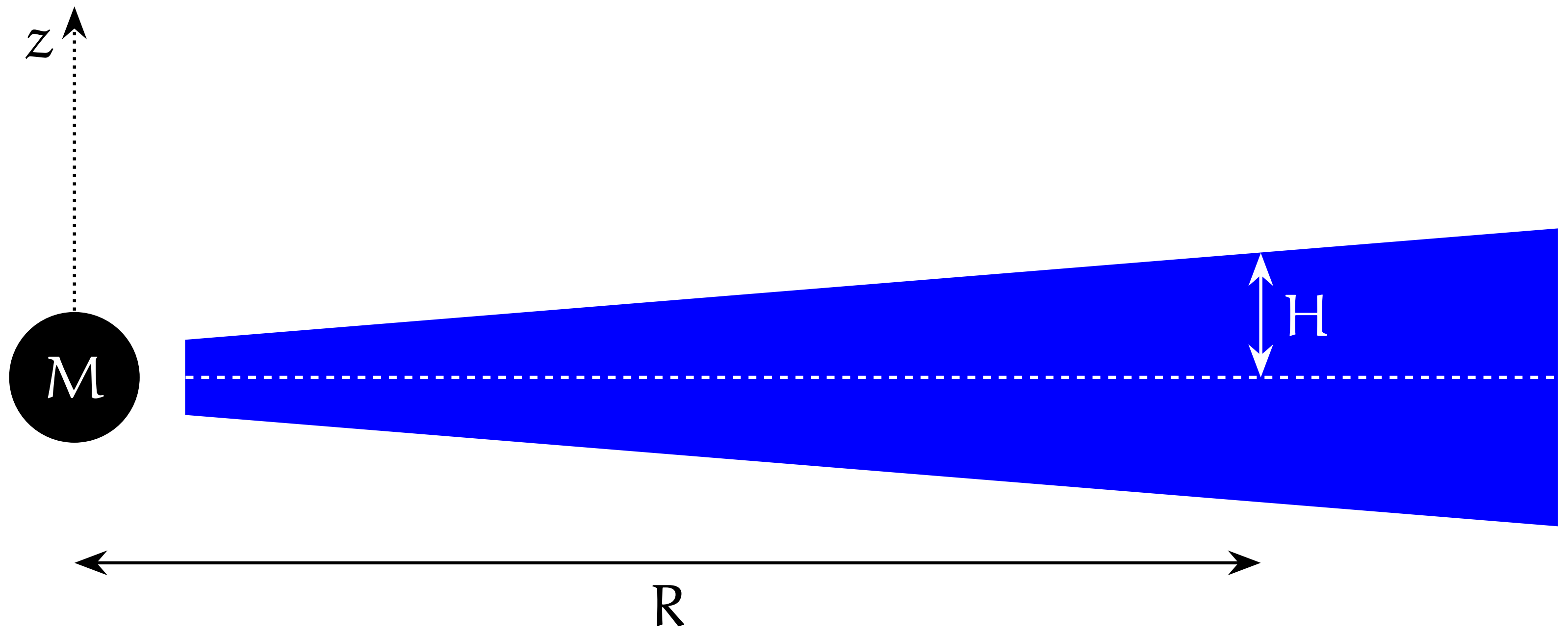


Owen & Blondin

Thin Disks: Vertical Structure



Most important case: $L \ll L_{Edd}$

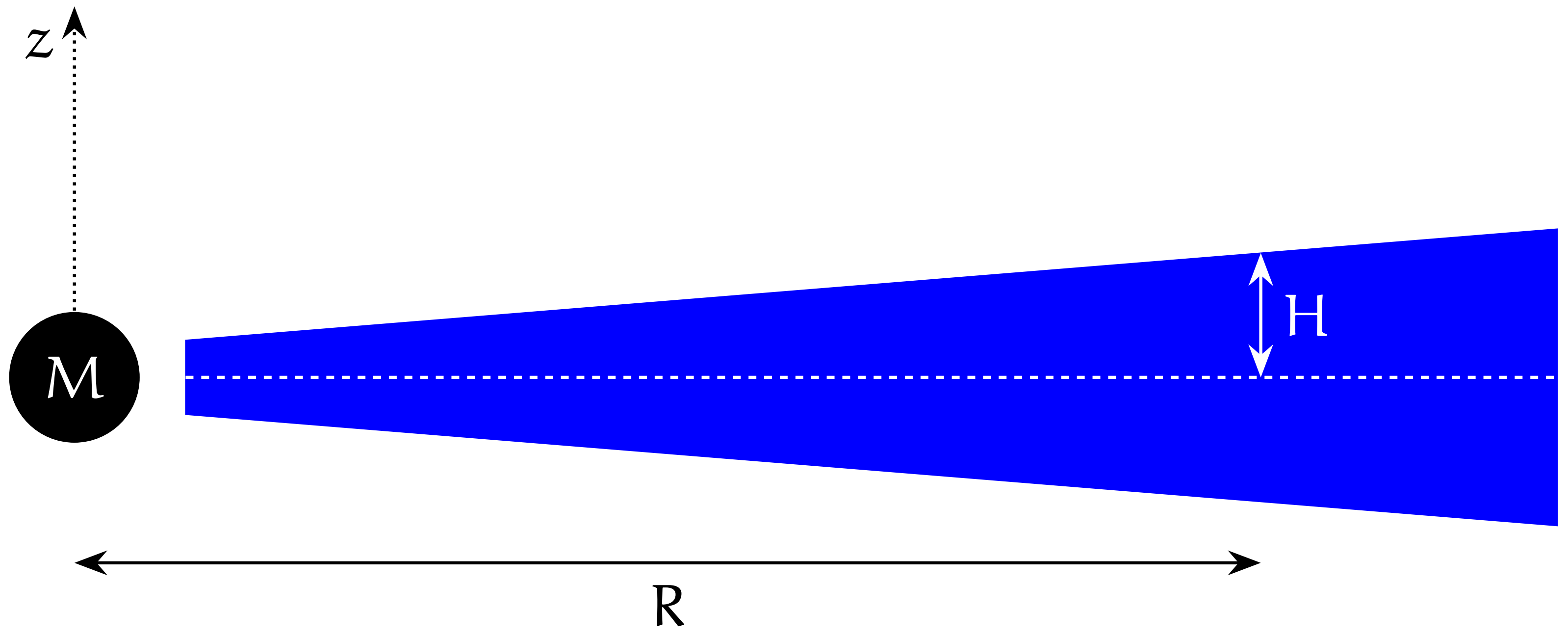
$\Longrightarrow$ Radiation pressure negligable

$\Longrightarrow$ Horizontal structure supported only by gas pressure

$\Longrightarrow$ thin accretion disks, i.e., vertical thickness, H, $\ll$ radius R:

$$H \ll R$$

Thin Disks: Vertical Structure

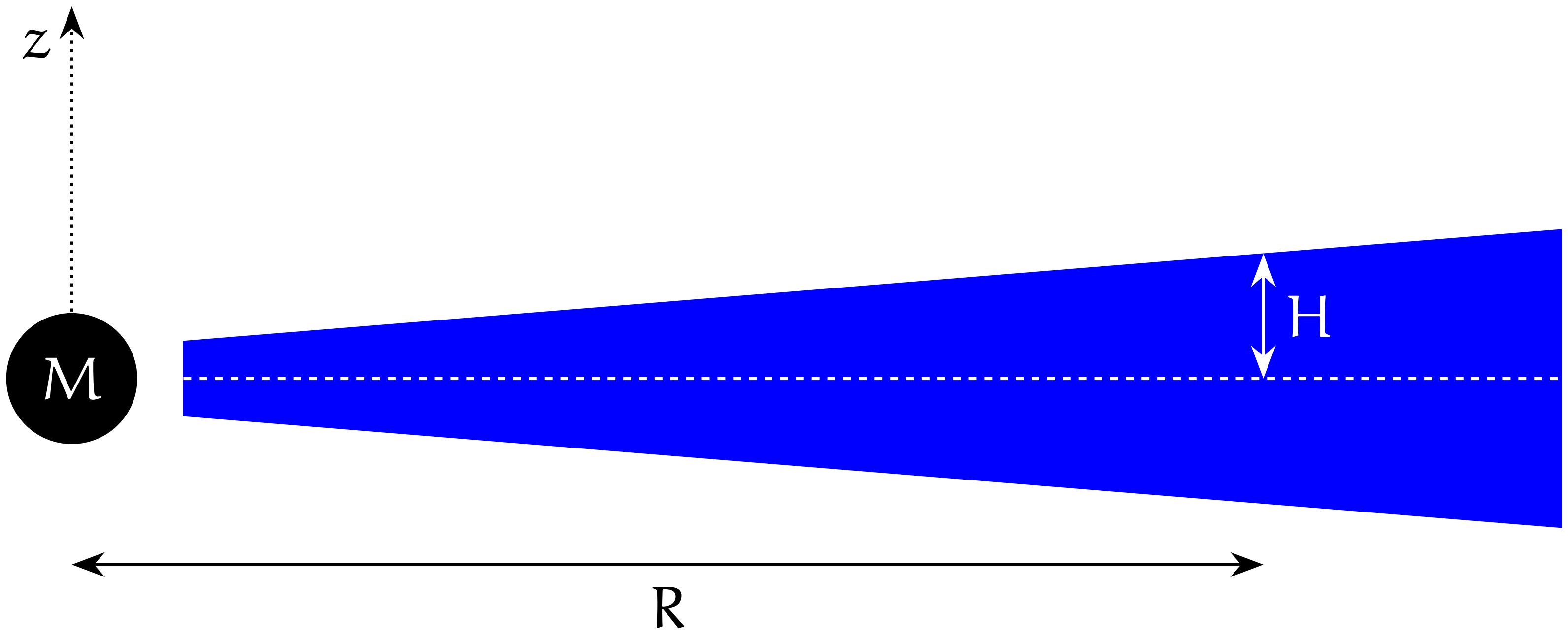


Gravitational acceleration in z-direction:

$$a_{z,\mathrm{grav}} = \frac{GM}{R^2}\frac{z}{R}$$

often called “force per unit mass”…

Thin Disks: Vertical Structure



Gravitational acceleration in z-direction:

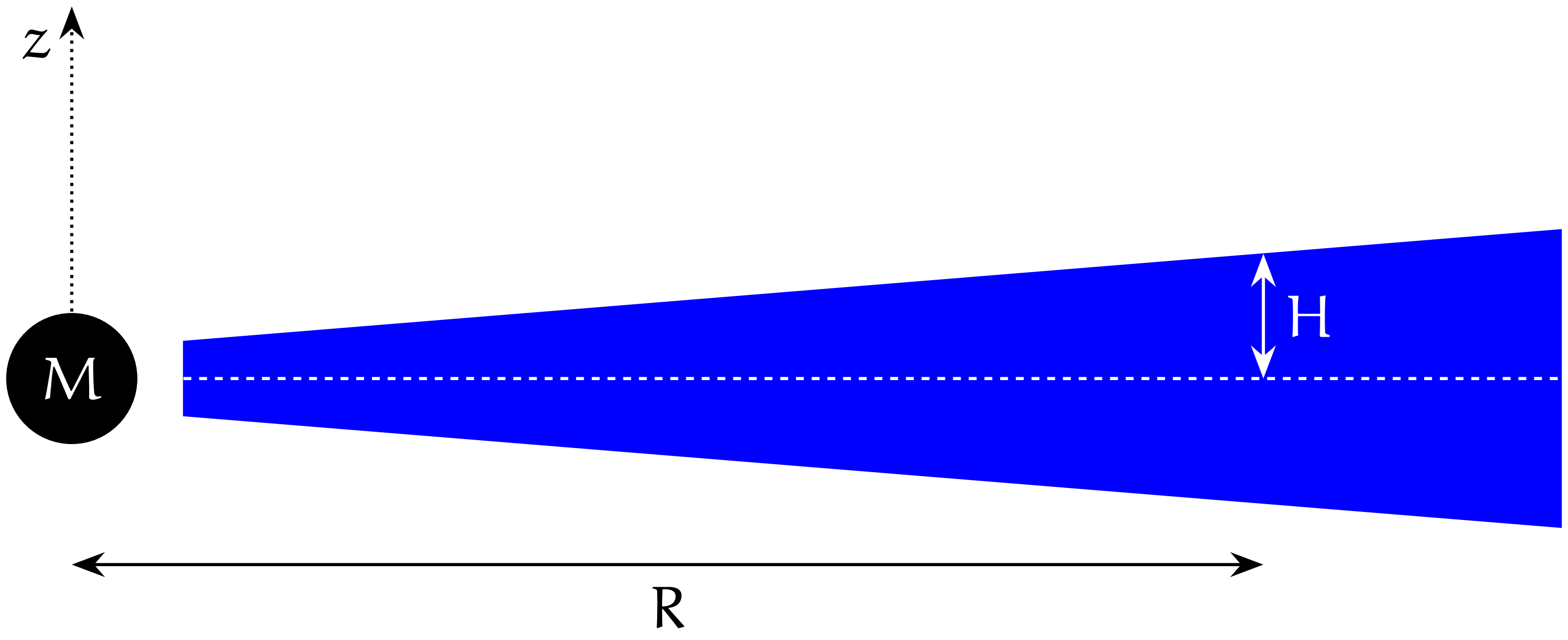
$$a_{z,\mathrm{grav}} = \frac{GM}{R^2}\frac{z}{R} \propto z$$

vertical density profile

$$n(z) \propto \exp\left(-\frac{z}{H}\right)$$

where H: scale height

exact value depends on details of accretion disk theory



Gas pressure must support disk vertically against gravitation

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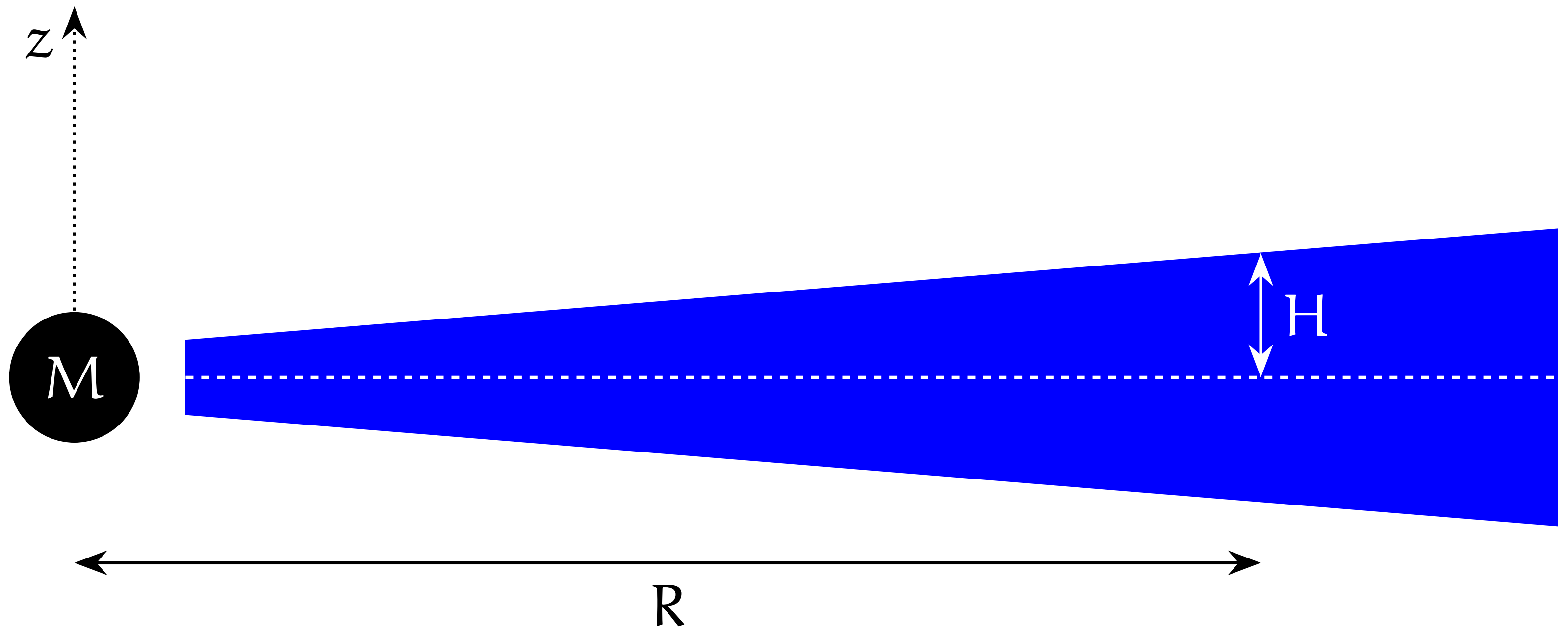

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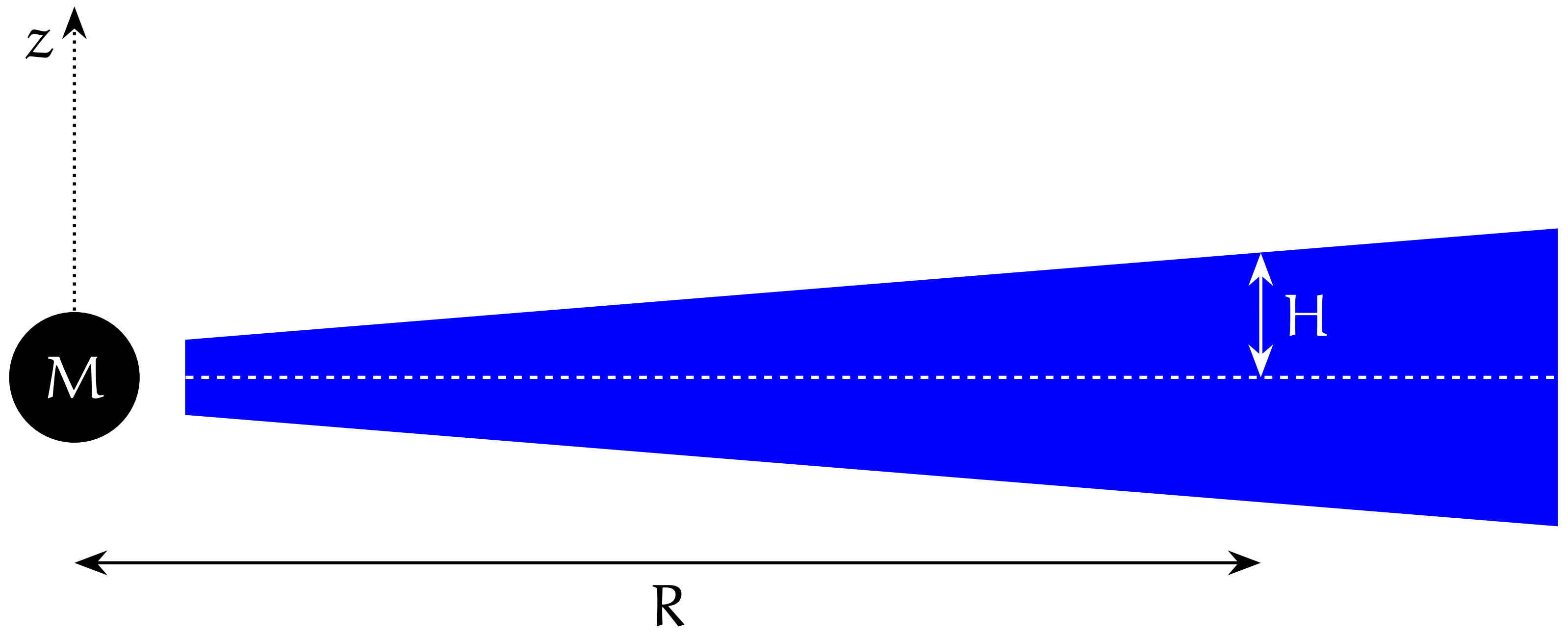
Acceleration due to gas pressure:

$$a_{z,\mathrm{gas}} = \frac{1}{\rho}\left|\frac{\partial P}{\partial z}\right| \sim \frac{1}{H}\frac{P_c}{\rho_c} \quad \text{since} \quad \frac{\partial P}{\partial z} \sim \frac{P_c}{H}$$

P_c: characteristic pressure, ρ_c: characteristic density

Thin Disks: Vertical Structure



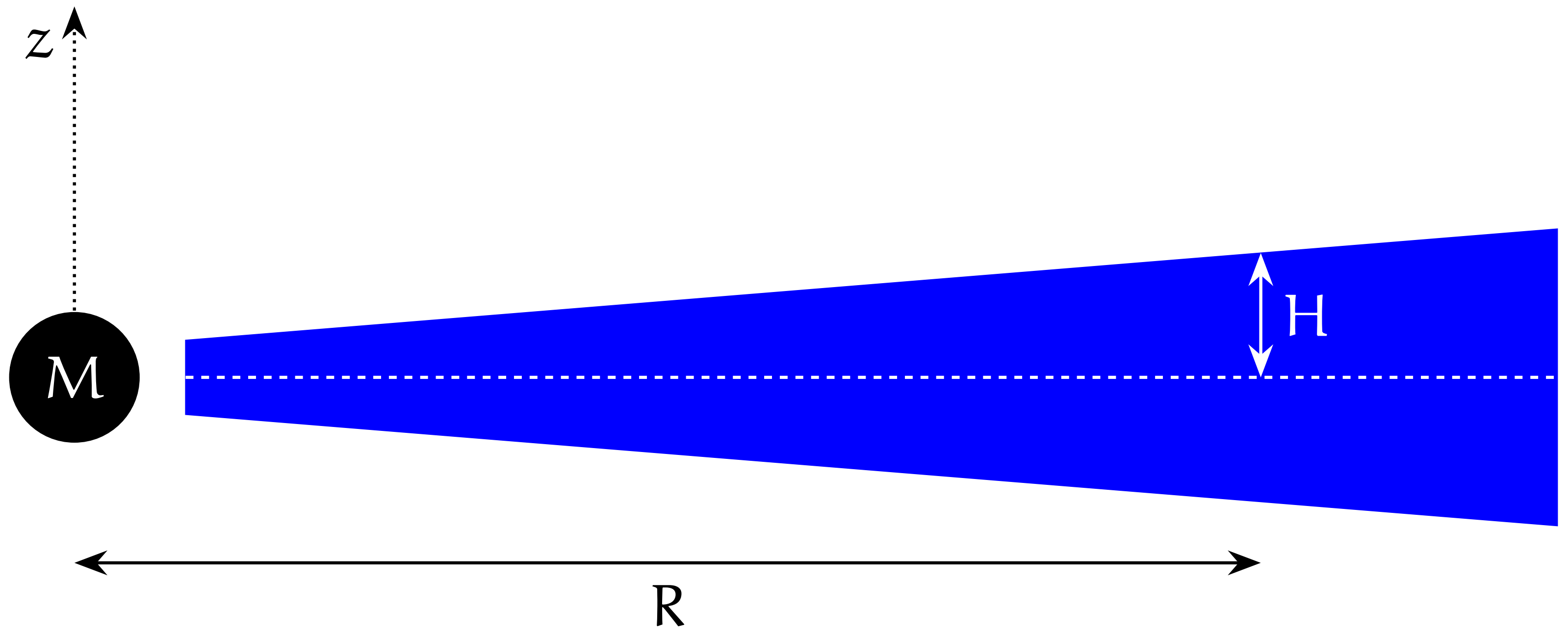
For vertically stationary disk need:

$$a_{z,\mathrm{grav}} = a_{z,\mathrm{gas}}$$

or

$$\frac{GM}{R^2}\frac{H}{R} \sim \frac{P_c}{\rho_c H}$$

Thin Disks: Vertical Structure



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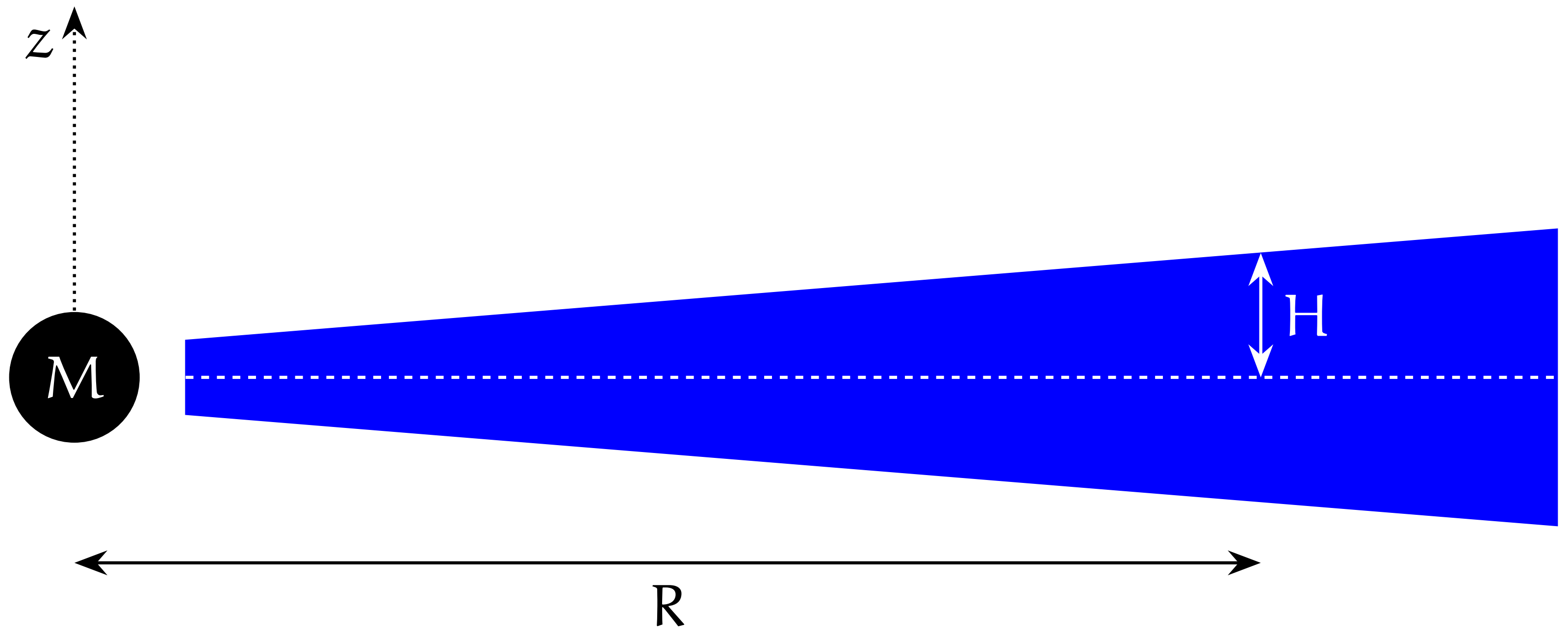
Kepler speed: $v_\phi^2 = GM/R = 1.2 \times 10^{10}\,(M/M_\odot)(R/10^6\,\mathrm{cm})^{-1}\,\mathrm{cm\,s^{-1}}$
Speed of sound: $c_s^2 = P/\rho$

Thin Disks: Vertical Structure



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or

$$\frac{v_\phi^2}{R}\frac{H}{R} \sim \frac{c_s^2}{H} \quad \Longrightarrow \quad c_s^2 = v_\phi^2 \cdot \frac{H^2}{R^2}$$

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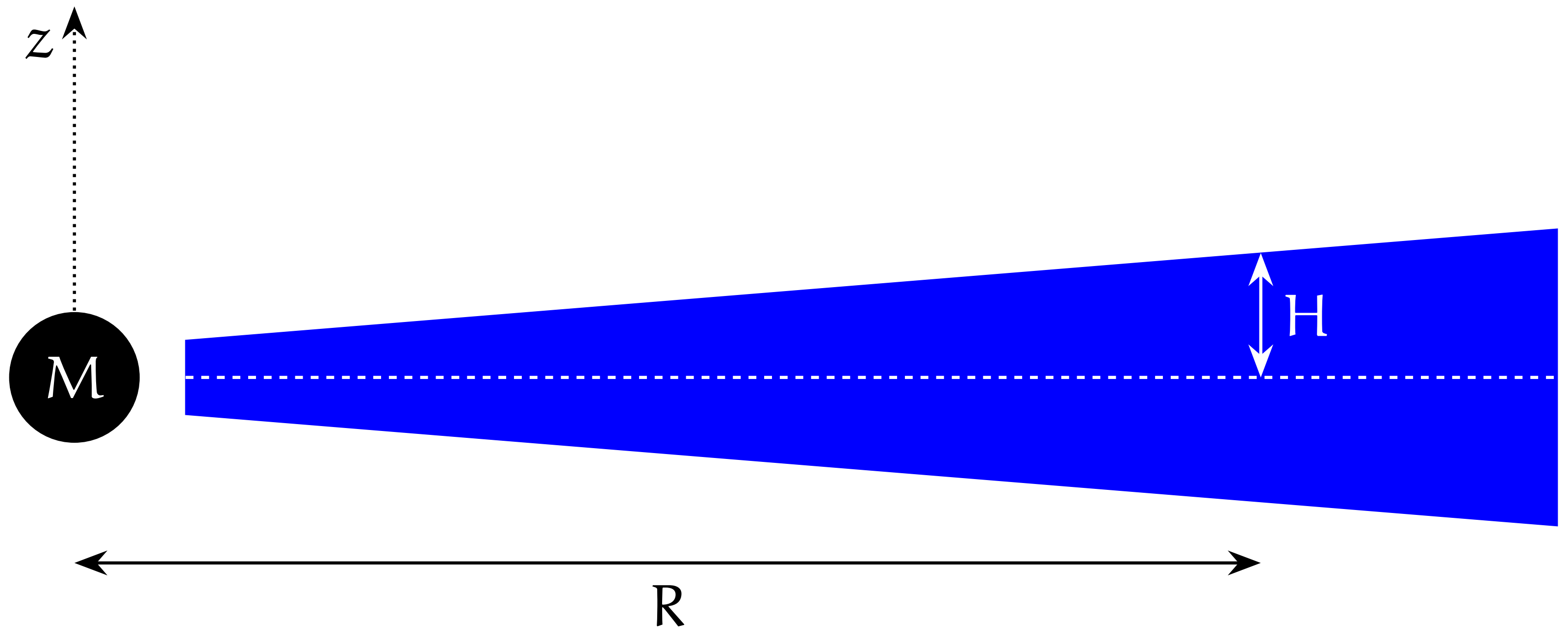


For vertically stationary disk need:

$$a_{z,\mathrm{grav}} = a_{z,\mathrm{gas}}$$

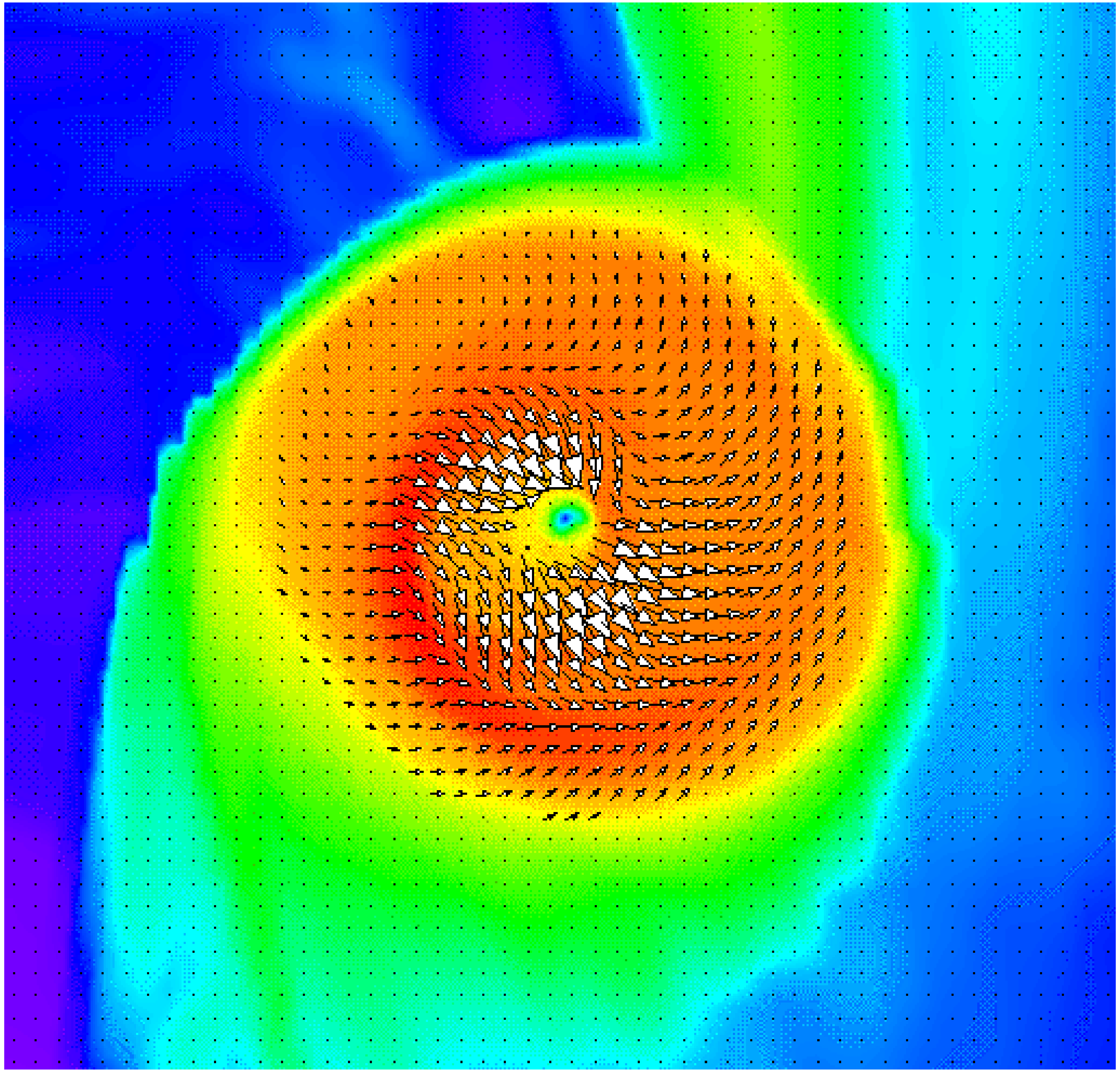
or

$$c_s^2 = v_\phi^2 \cdot \frac{H^2}{R^2} \quad \Longrightarrow c_s \ll v_\phi \quad \text{since } H \ll R$$

Thin accretion disks are highly supersonic.

Equation above also implies $H = c_s R / v_\phi$, and since $c_s \propto T^{1/2}$: disk must be cold!

Thin Disks: Radial Structure



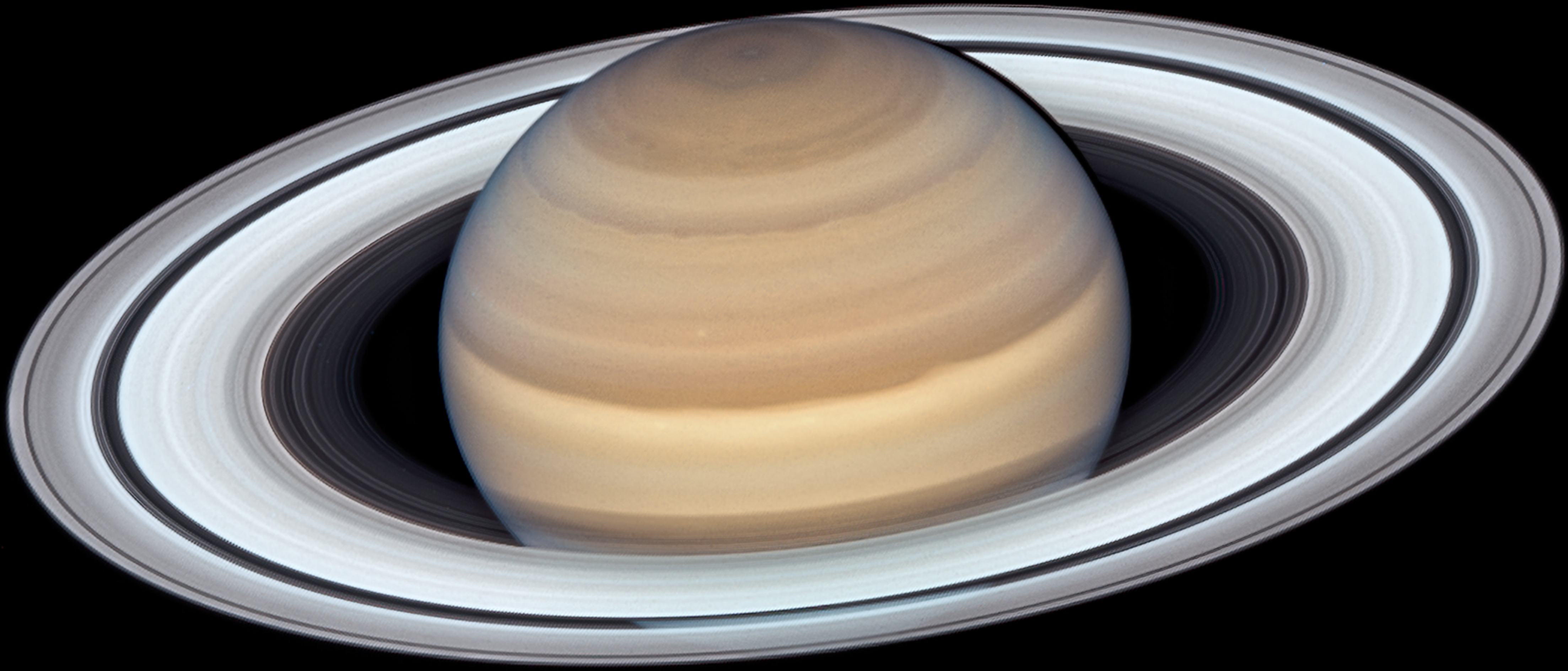
Radial acceleration due to pressure:

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \sim \frac{P_c}{\rho_c R} \sim \frac{c_s^2}{R} \sim \frac{v_\phi^2}{R}\frac{H^2}{R^2} = \frac{GM}{R^2}\frac{H^2}{R^2} \ll \frac{GM}{R^2}$$

analoguous reasoning to vertical structure

$\Longrightarrow$ radial acceleration due to pressure is negligable compared to gravitational acceleration

$\Longrightarrow$ (almost) no radial motion



STScI

A disk with (almost not) radial flow

Thin Disks: Angular Momentum Transport

Although $v_R \ll v_\phi$, there is some radial flow.

Problem: Angular Momentum Conservation

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Problem: Angular Momentum Conservation

Angular velocity in Keplerian disk:

$$\omega(R) = \frac{v_R}{R} = \sqrt{\frac{GM}{R}}\frac{1}{R} = \left(\frac{GM}{R^3}\right)^{1/2}$$

$\Longrightarrow$ "differential rotation"

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Angular momentum of a matter ring:

$$L(R) = R \times M(R) v_\phi = M(R) R \times R \omega R$$

Do not confuse angular momentum with luminosity!

Thin Disks: Angular Momentum Transport

Although $v_R \ll v_\phi$, there is some radial flow.

Problem: Angular Momentum Conservation

Angular velocity in Keplerian disk:

$$\omega(R) = \frac{v_R}{R} = \sqrt{\frac{GM}{R}}\frac{1}{R} = \left(\frac{GM}{R^3}\right)^{1/2}$$

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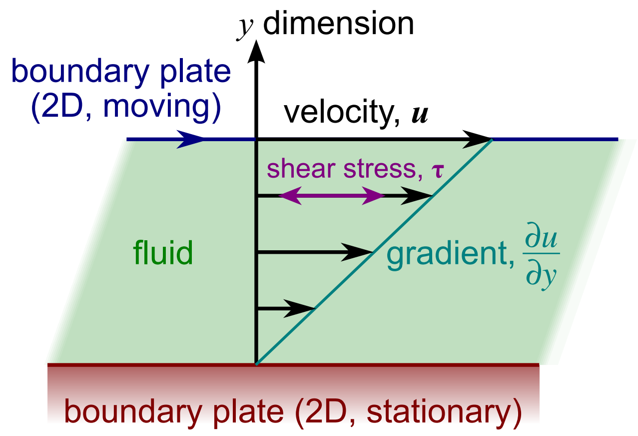
Total angular momentum lost when mass moves in unit time from $R + dR$ to R:

$$\frac{d^2L}{dR\,dt} = \dot{M} \cdot \frac{d(R^2\omega(R))}{dR}$$

reminder: $\dot{M}$ =mass accretion rate

Viscosity

Since $L = L(R)$ decreases w/R: accreting matter needs to lose angular momentum.
This is done by viscous forces excerting torques:
Reminder: viscosity



Viscous force (Newton's law of viscosity):

$$\frac{F}{A} = \mu \frac{\partial v}{\partial y}$$

where μ: coefficient of dynamic viscosity (sometimes called shear viscosity).
Often, use kinematic viscosity, i.e.,

$$\nu = \mu/\rho$$

where ρ: density of fluid.
units: $\nu = \lambda u$ where λ: typical length, u: typical velocity

Viscous force between two accretion disk rings:

$$dF_{\mathrm{visc}} = 2\pi R\, dz\, \nu\rho\, R \frac{d\omega}{dR} \quad \Longrightarrow \text{integrating over } z \quad \Longrightarrow F_{\mathrm{visc}} = 2\pi\nu\Sigma R^2 \frac{d\omega}{dR}$$

where Σ is the surface density of the disk, i.e.,

$$\dot{M} = -2\pi R \cdot \Sigma \cdot v_R$$

Do not confuse v_R and ν!

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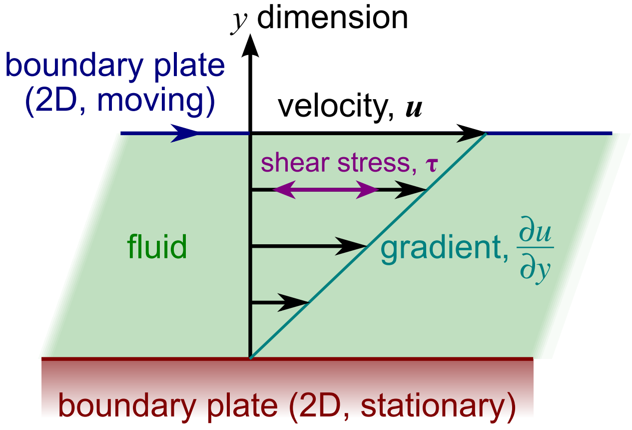

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Total torque

$$G(R) = F \cdot R = \nu \Sigma 2\pi R^3 \frac{d\omega}{dR}$$

and the net torque acting on a ring in disk is

$$\frac{\partial G}{\partial R} dR$$

$\Longrightarrow$ This net torque needs to balance change in specific angular momentum in disk.
Work done leads to energy release ("viscous dissipation rate"): $\propto F\Delta v$

Balancing net torque and angular momentum loss gives:

$$\dot{M}\frac{d(R^2\omega)}{dR} = -\frac{d}{dR}\left(\nu\Sigma 2\pi R^3 \frac{d\omega}{dR}\right)$$

Insert $\omega(R) = (GM/R^3)^{1/2}$ and integrate:

$$\nu\Sigma R^{1/2} = \frac{\dot{M}}{3\pi}R^{1/2} + \text{const.}$$

where const. obtained from no torque boundary condition at inner edge of disk at $R = R_*$: $dG/dR(R_*) = 0$, such that

$$\nu\Sigma = \frac{\dot{M}}{3\pi}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]$$

Therefore the viscous dissipation rate per unit area is

$$D(R) = \nu\Sigma\left(R\frac{d\omega}{dR}\right)^2 = \frac{3GM\dot{M}}{4\pi R^3}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right] \tag{1}$$

Thin Disks: Temperature Profile

The viscous dissipation rate was

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If disk is optically thick: Thermalization of dissipated energy

$\Longrightarrow$ Temperature from Stefan-Boltzmann-Law:

$$2\sigma_{SB}T^4 = D(R)$$

(disk has *two* sides!) and therefore

$$T(R) = \left\{\frac{3GM\dot{M}}{8\pi R^3\sigma_{SB}}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4}$$

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Inserting astrophysically meaningful numbers:

$$\begin{aligned} T(R) &= \left\{\frac{3GM\dot{M}}{8\pi R^3\sigma_{SB}}\left[1 - \left(\frac{R_*}{R}\right)^{1/2}\right]\right\}^{1/4} \\ &= 6.8\times 10^5\,\mathrm{K}\cdot\eta^{-1/4}\left(\frac{L}{L_{Edd}}\right)^{1/2} L_{46}^{-1/4}\mathcal{R}^{1/4}x^{-3/4} \end{aligned}$$

where $\eta = L_{Edd}/\dot{M}_{Edd}c^2$, $x = c^2R/2GM$, $\mathcal{R} = (1-(R_*/R)^{1/2})$.

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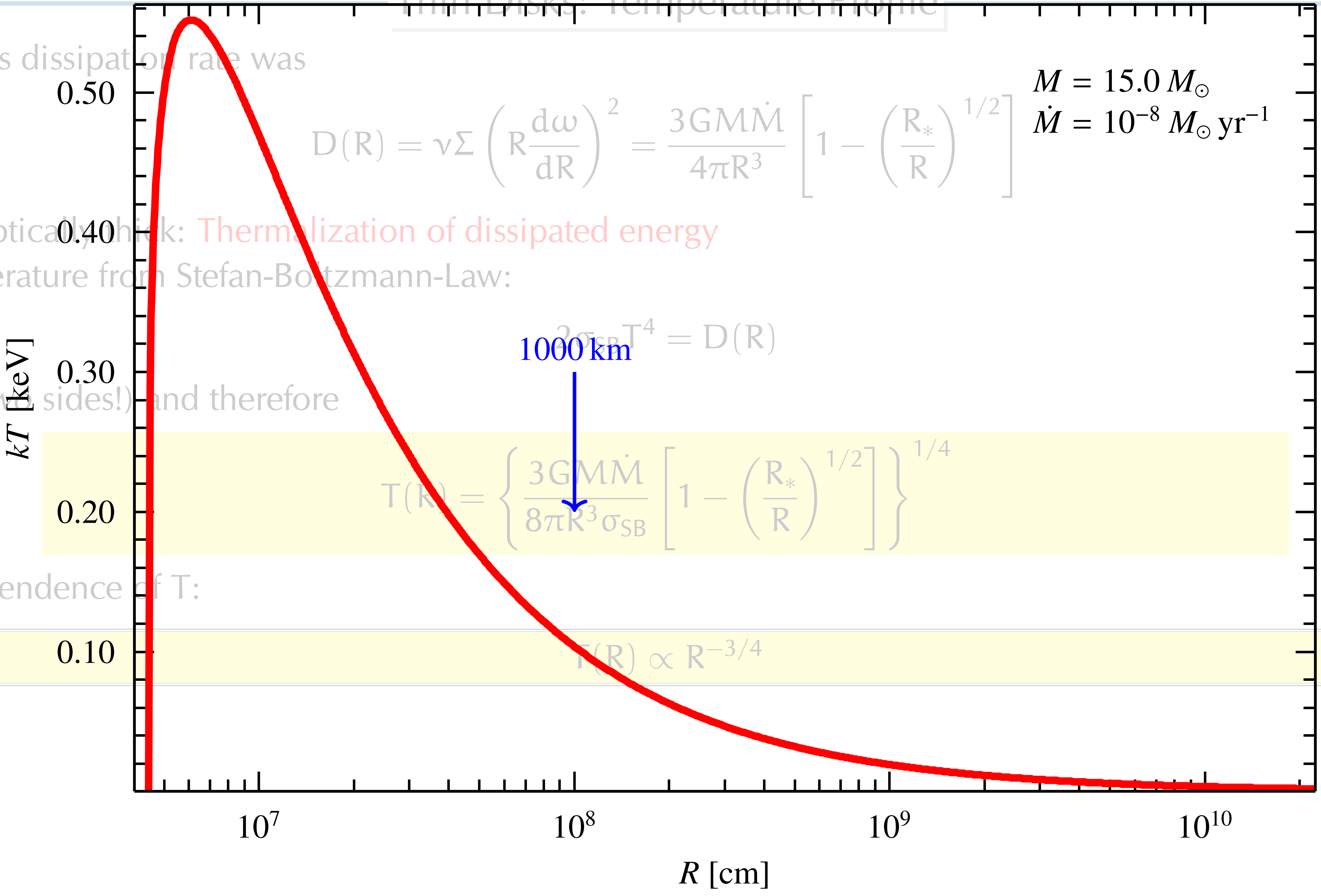

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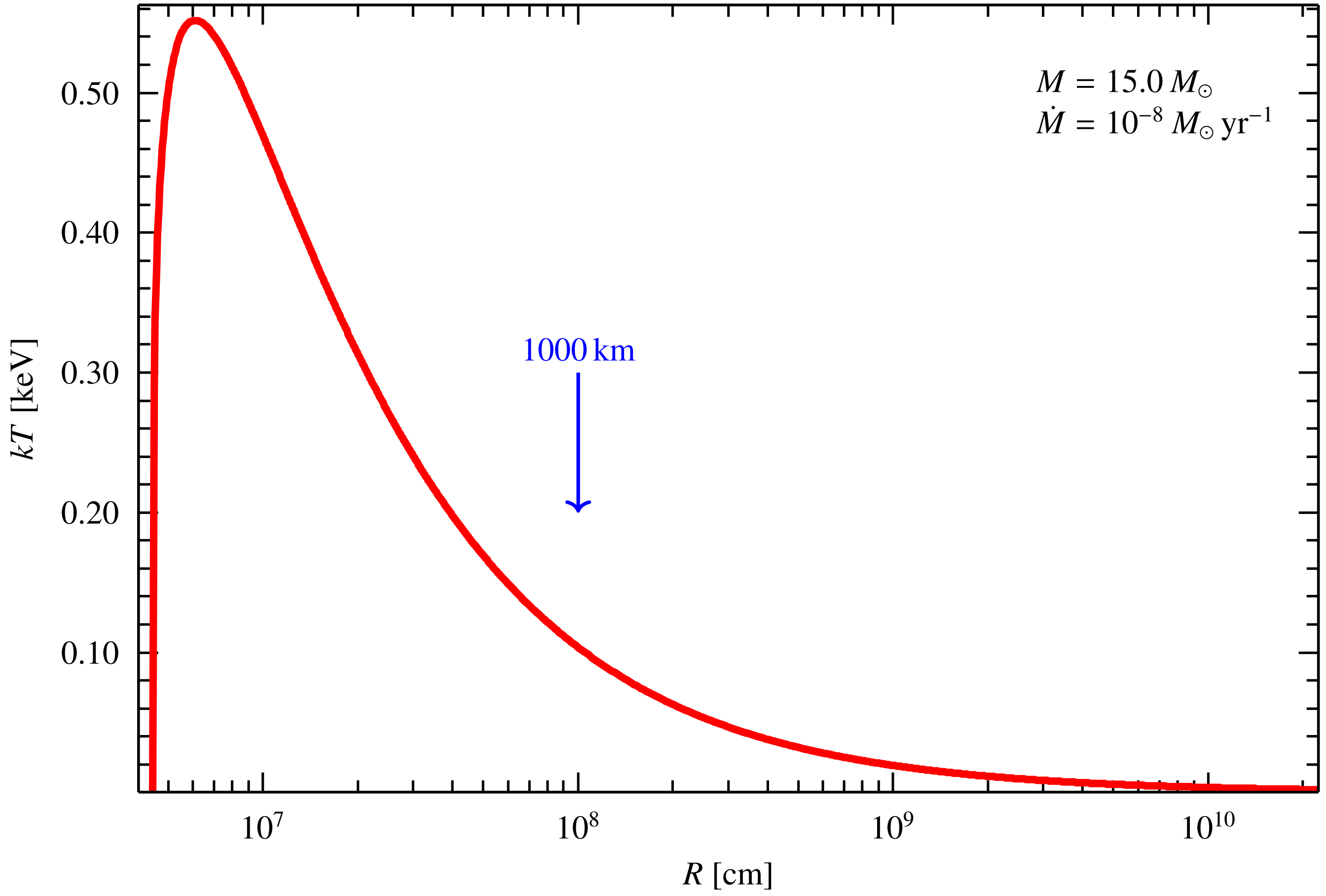
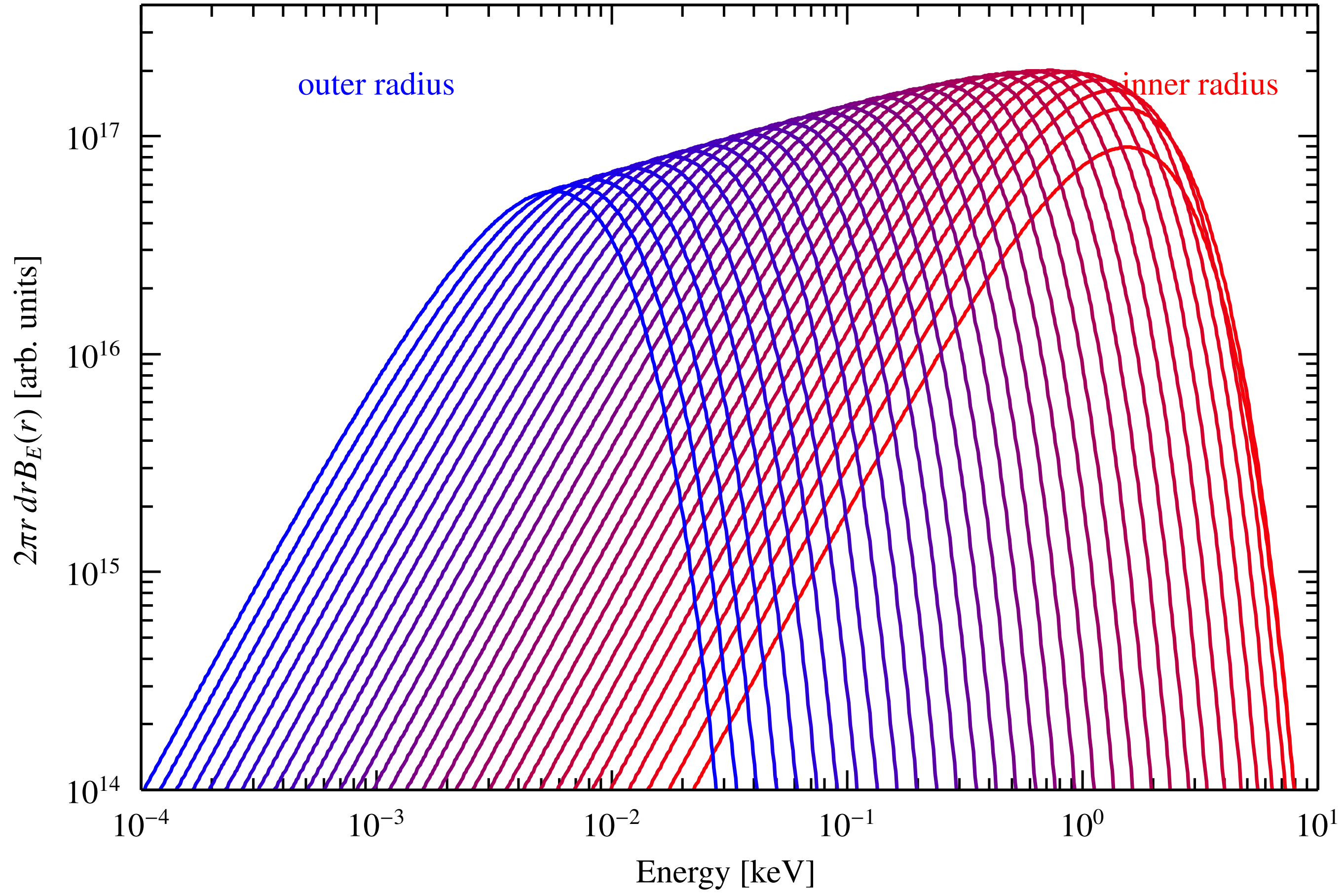
Dependence on mass (note: for NS/BH inner radius $R_* \propto M$!):

inner disk temperature $T_{in} \propto (\dot{M}/M^2)^{1/4}$

Emitted Spectrum



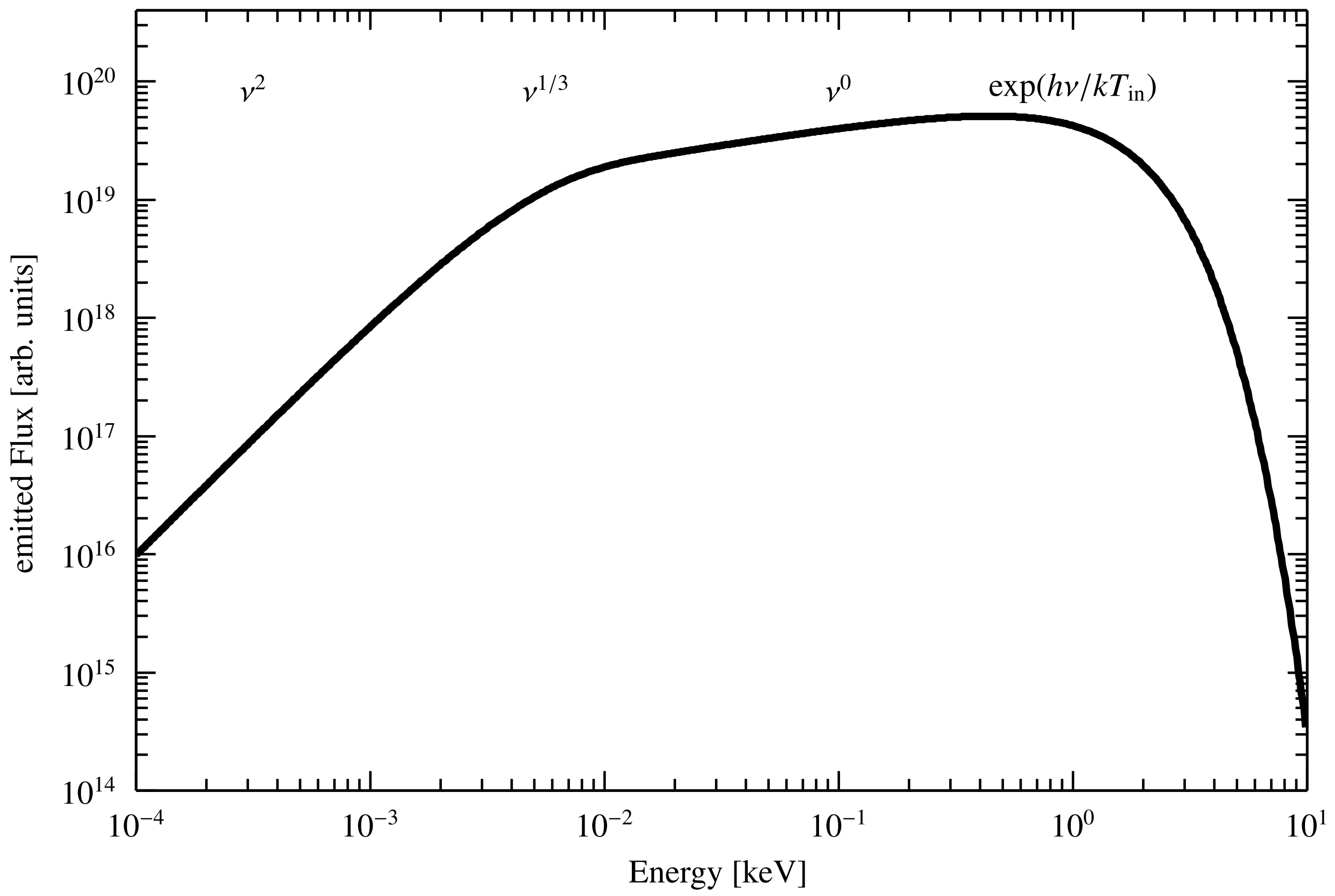


If disk is optically thick, then locally emitted spectrum is black body.

Total emitted spectrum obtained by integrating over disk

$$F_\nu = \int_{R_*}^{R_{out}} B(T(R))\,2\pi R\,dR$$

Emitted Spectrum



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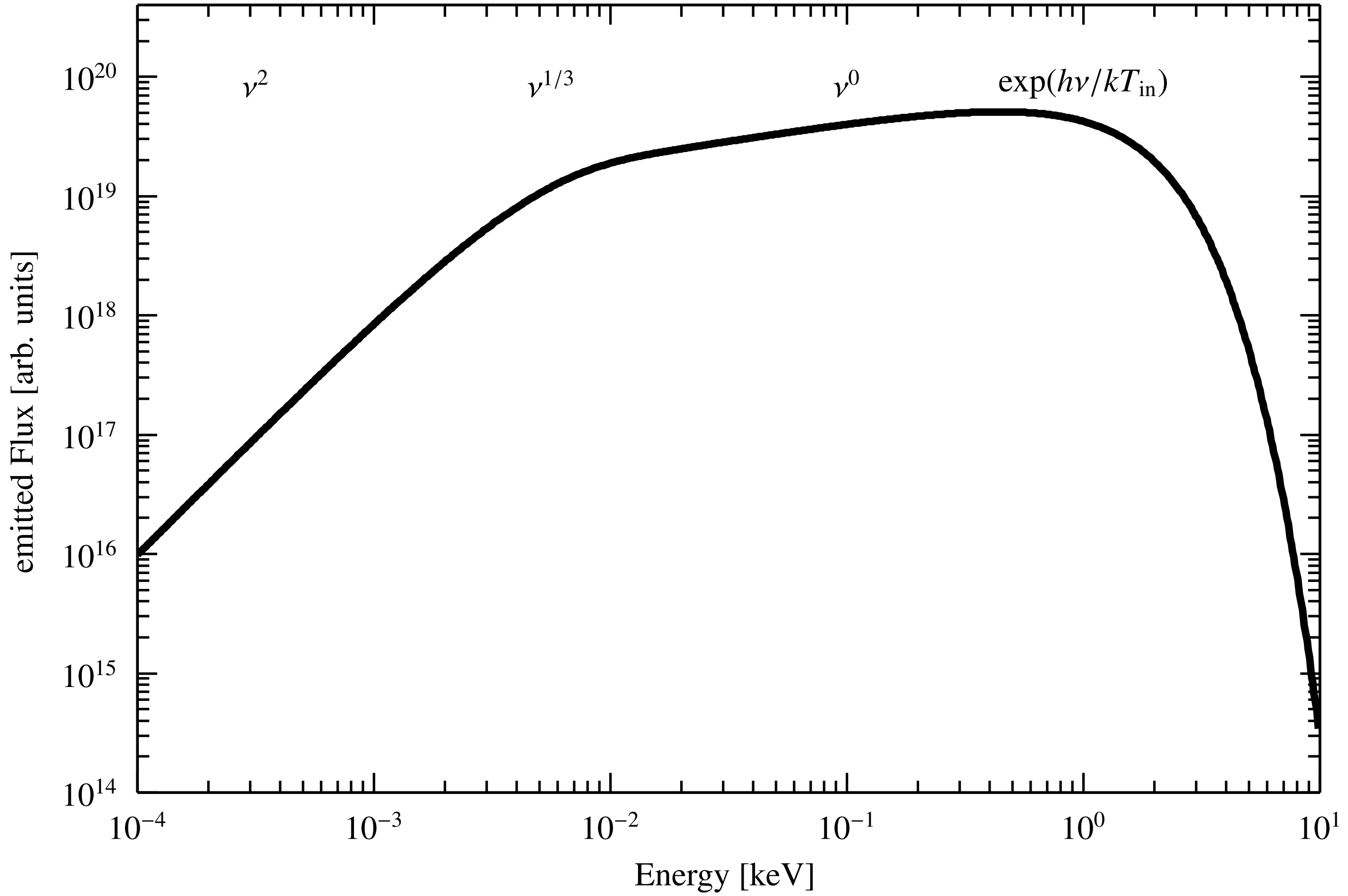
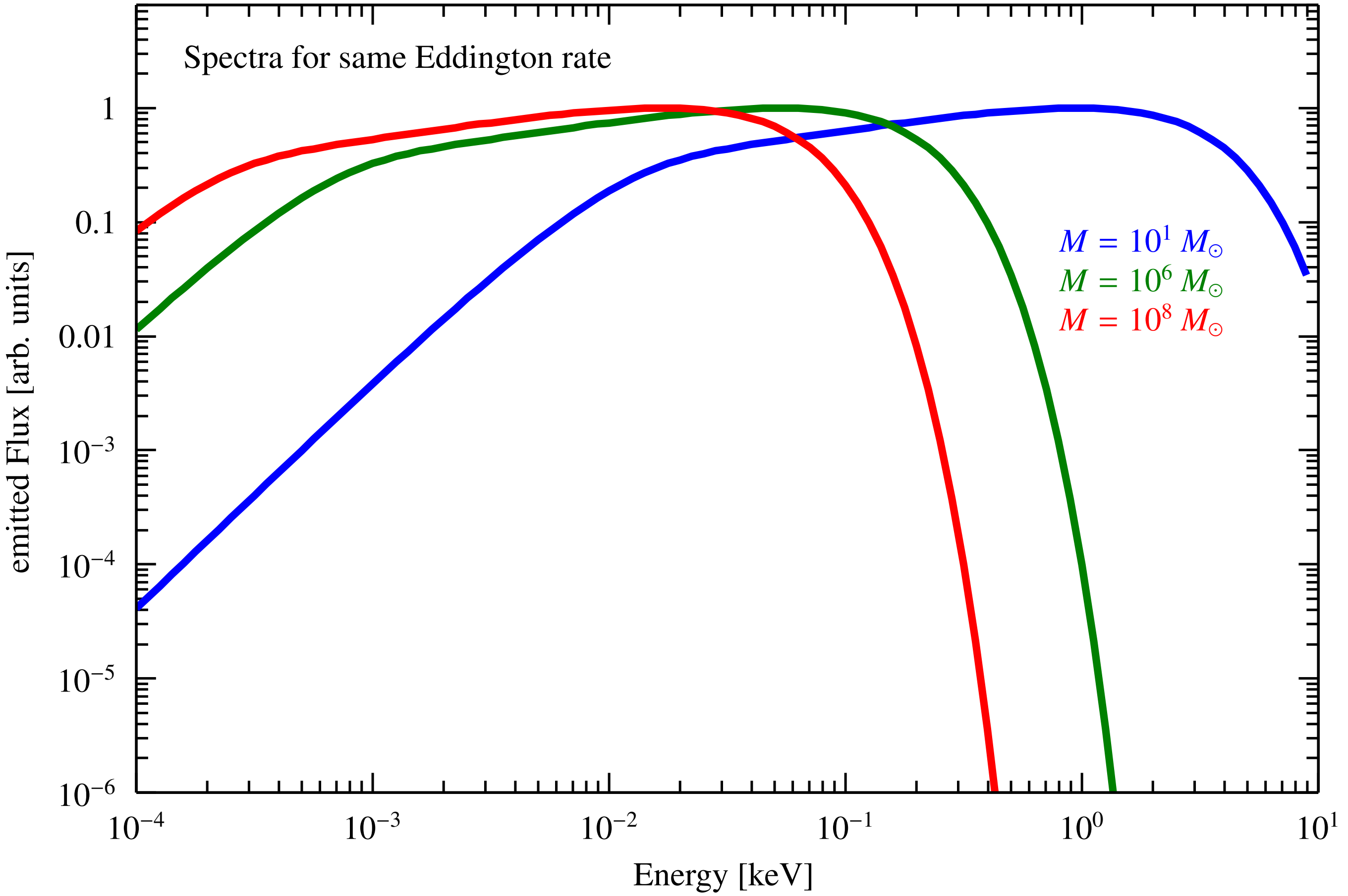
Resulting spectrum looks essentially like a stretched black body.

Emitted Spectrum





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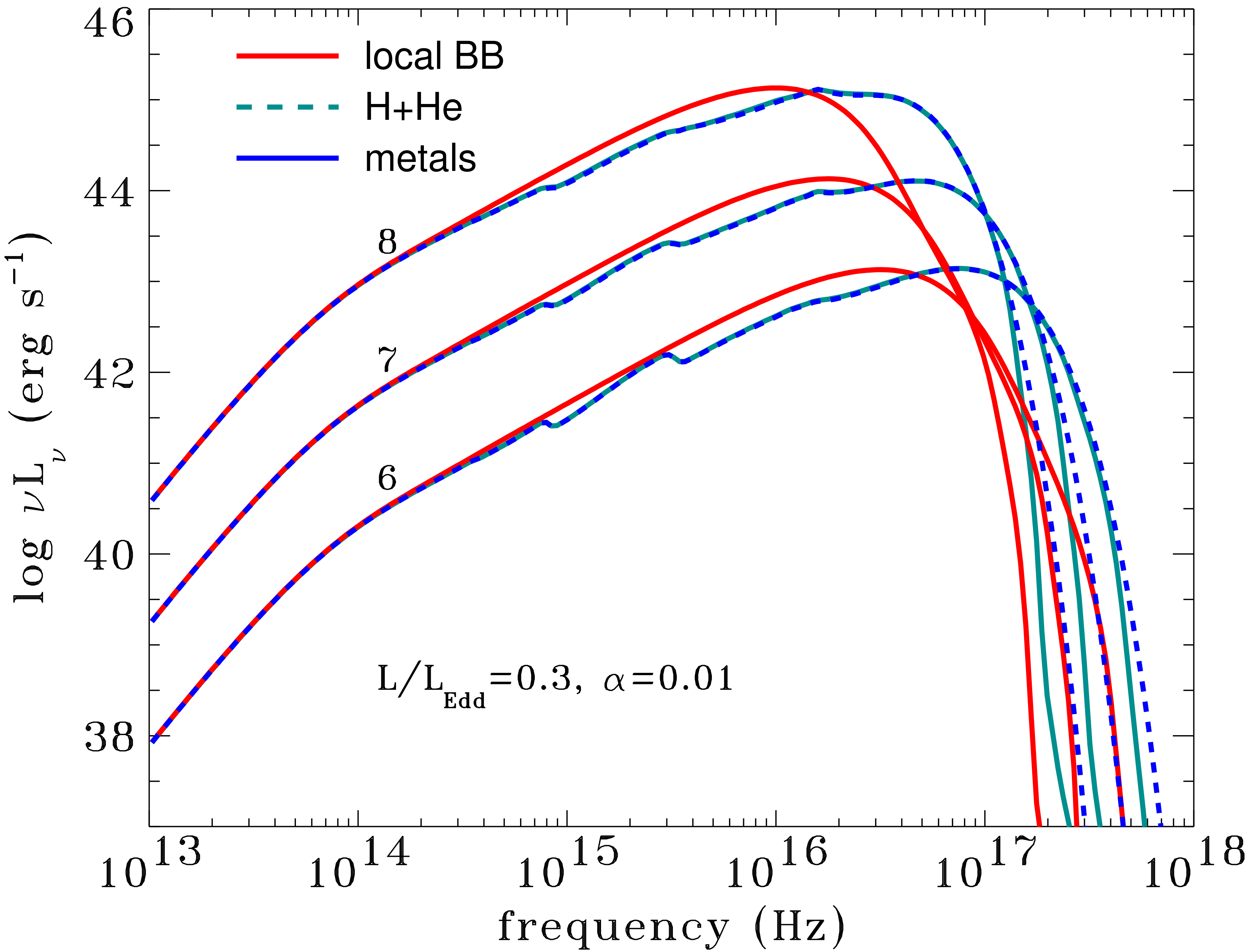
Total emitted spectrum obtained by integrating over disk

$$F_\nu = \int_{R_*}^{R_{out}} B(T(R))\, 2\pi R\, dR$$

Peak moves to lower energies for higher mass.

Emitted Spectrum



Hubeny et al., 2001, Fig. 13

In reality: accretion disk spectrum depends on

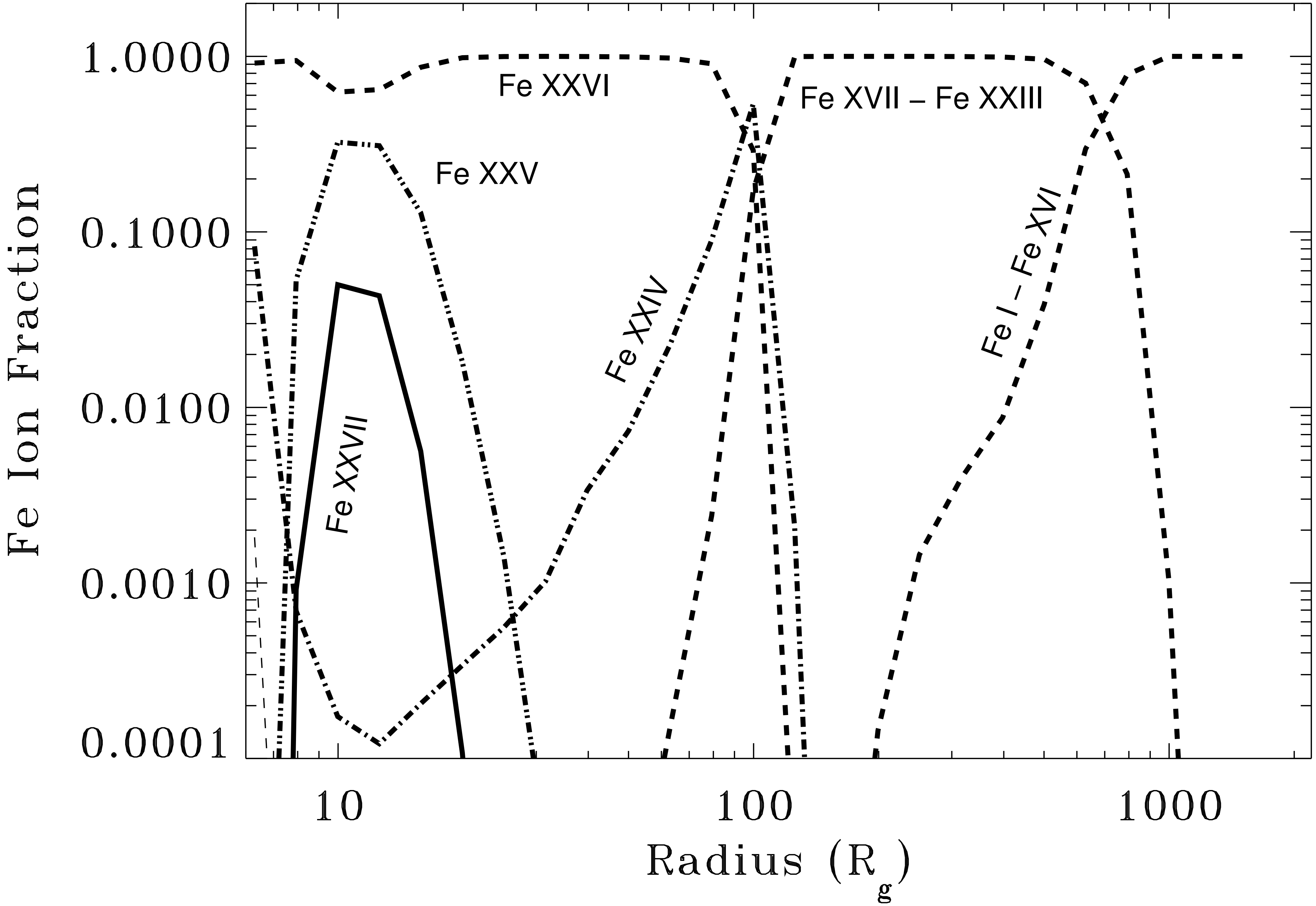
- elemental composition ("metallicity")
- viscosity ("α-parameter")
- ionization state and luminosity of disk ($\dot{M}$)
- properties of compact object

and many further parameters

Until today: no really satisfactory disk model available.

this is even true for cataclysmic variables or young stars

Emitted Spectrum



Fe species in a disk around a Galactic BH (Davis et al., 2005, Fig. 6)

Viscosity

Most important unknown in accretion disk theory: viscosity

even though it dropped out of $T(R)$!

Earth: viscosity of fluids typically due to molecular interactions (molecular viscosity).

Kinematic viscosity:

$$\nu_{\text{mol}} \sim \lambda_{\text{mfp}} c_s$$

where the mean free path

$$\lambda_{\text{mfp}} \sim \frac{1}{n\sigma} \sim 6.4 \times 10^4 \left(\frac{T^2}{n}\right) \text{ cm}$$

and the speed of sound

$$c_s \sim 10^4 T^{1/2} \text{ cm s}^{-1}$$

such that

$$\nu_{\text{mol}} \sim 6.4 \times 10^8 \, T^{5/2} n^{-1} \text{ cm}^2 \text{ s}^{-1}$$

Viscosity

Viscosity important when Reynolds number small ("laminar flow"), where

$$\mathrm{Re} = \frac{\text{inertial force}}{\text{viscous force}} \sim \frac{\rho R v}{\rho \nu} = \frac{R v}{\nu}$$

Follows from Navier-Stokes Equations

Using typical accretion disk parameters:

$$\mathrm{Re}_{\mathrm{mol}} \sim 2 \times 10^{14} \left(\frac{M}{M_\odot}\right)^{1/2} \left(\frac{R}{10^{10}\,\mathrm{cm}}\right)^{1/2} \left(\frac{n}{10^{15}\,\mathrm{cm}^{-3}}\right) \left(\frac{T}{10^4\,\mathrm{K}}\right)^{-5/2}$$

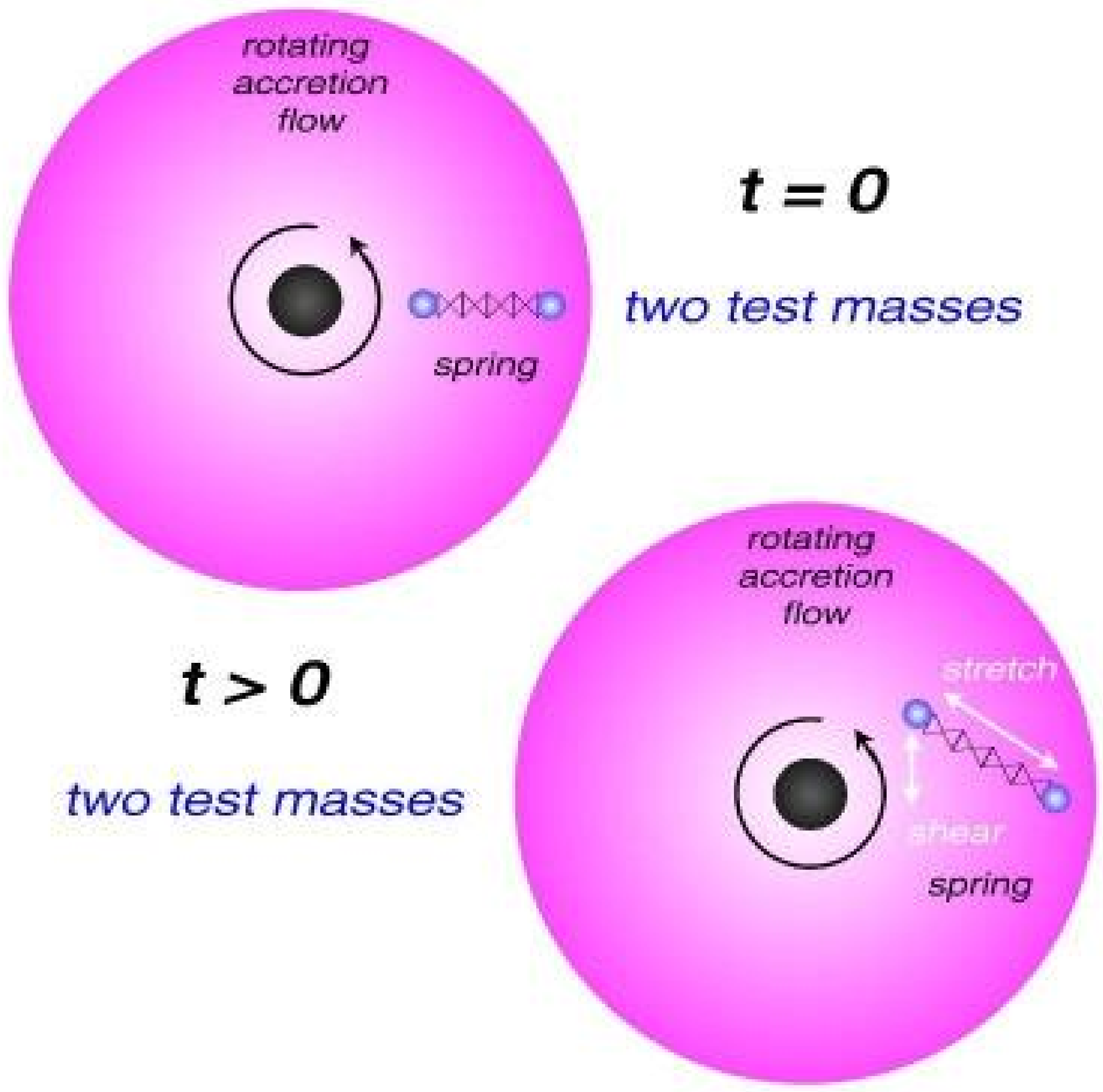
$\Longrightarrow$ Molecular viscosity is irrelevant for astrophysical disks!

since Re $\gtrsim 10^3$: turbulence $\Longrightarrow$ Shakura & Sunyaev: turbulent viscosity

$$\nu_{\mathrm{turb}} \sim v_{\mathrm{turb}} \ell_{\mathrm{turb}} \sim \alpha\, c_s \cdot H$$

where $\alpha \lesssim 1$ and $\ell_{\mathrm{turb}} \lesssim H$ typical size for turbulent eddies.

This is a *recipe*, no physics! Also, does not say that $\alpha = \text{const.}$!

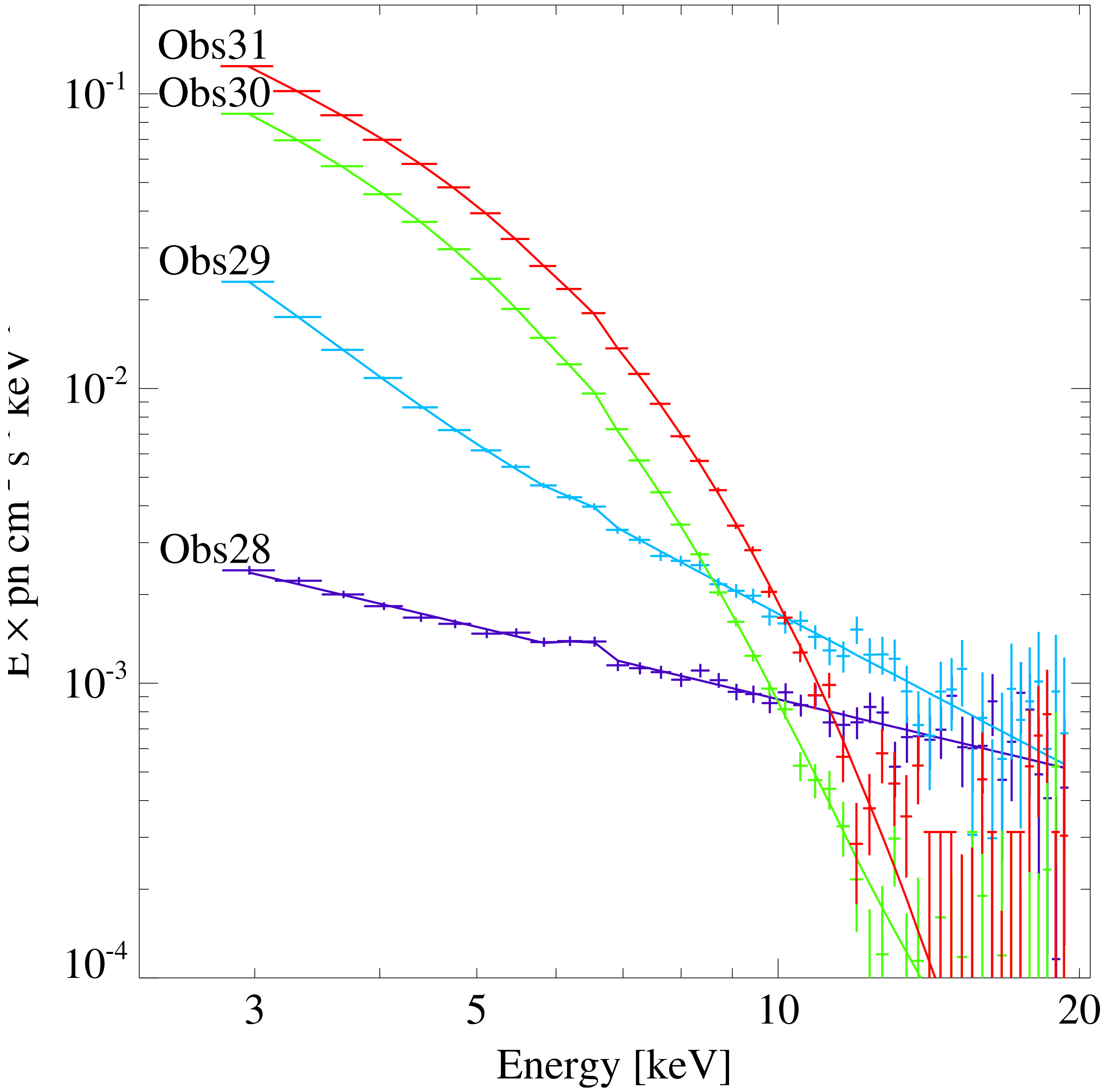


R. Müller
Mechanical analogy of MRI: spring in differentially rotating medium.

Physics of turbulent viscosity is unknown, however, α prescription yields good agreement between theory and observations.

Possible origin: Magnetorotational instability (MRI): MHD instability amplifying B-field inhomogeneities caused by small initial radial displacements in accretion disk
$\Longrightarrow$ angular momentum transport

(Balbus & Hawley 1991, going back to Velikhov 1959 and Chandrasekhar 1961)

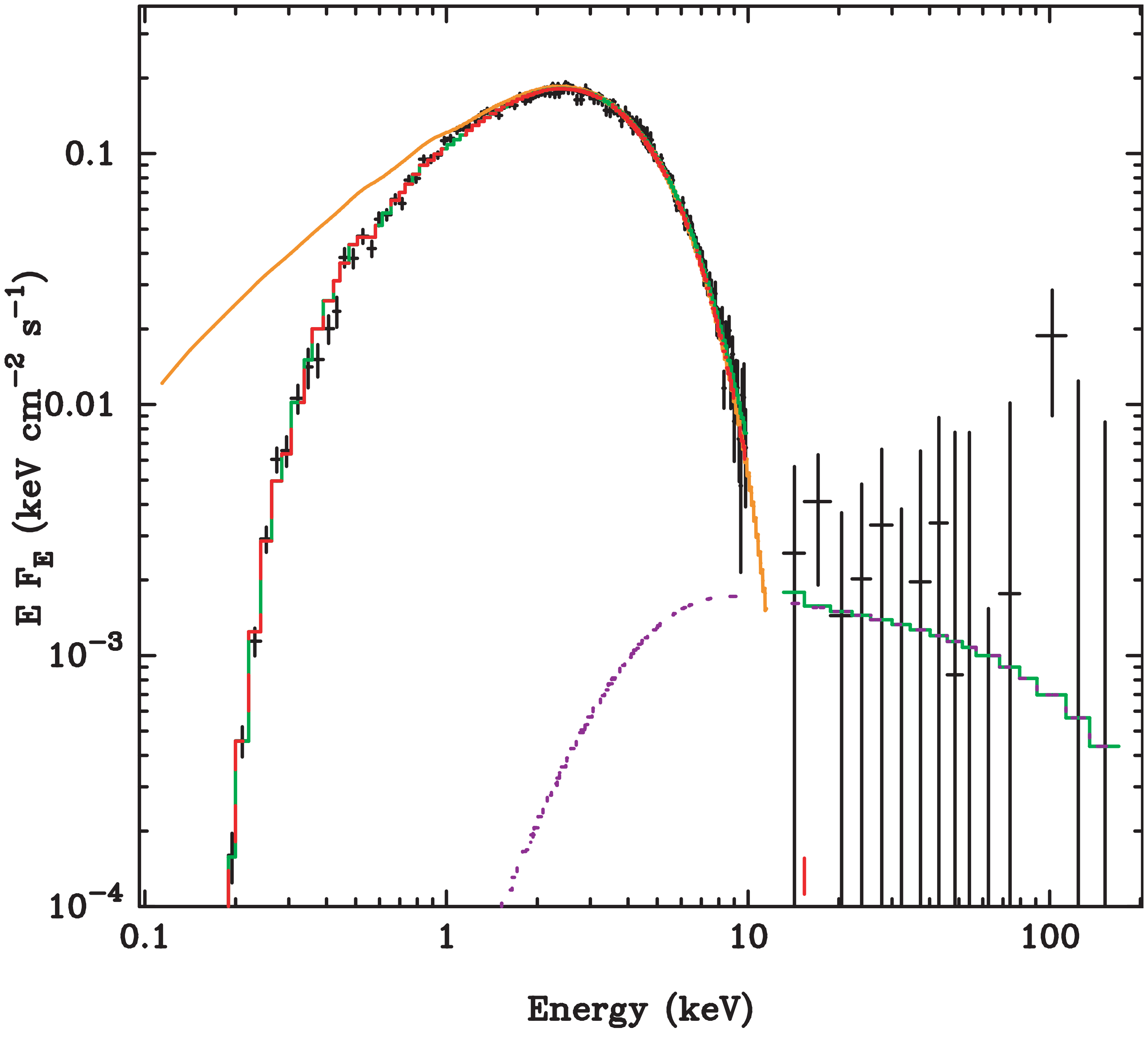


LMC X-3, (Wilms et al., 2001)

Problem with AGN: peak of disk in UV

$\Longrightarrow$ Galactic Black Holes: T is higher

Find ok agreement between accretion disk models and theory.
In general: models with just $T \propto r^{-3/4}$ and no additional (atomic) physics seem to work best?!?

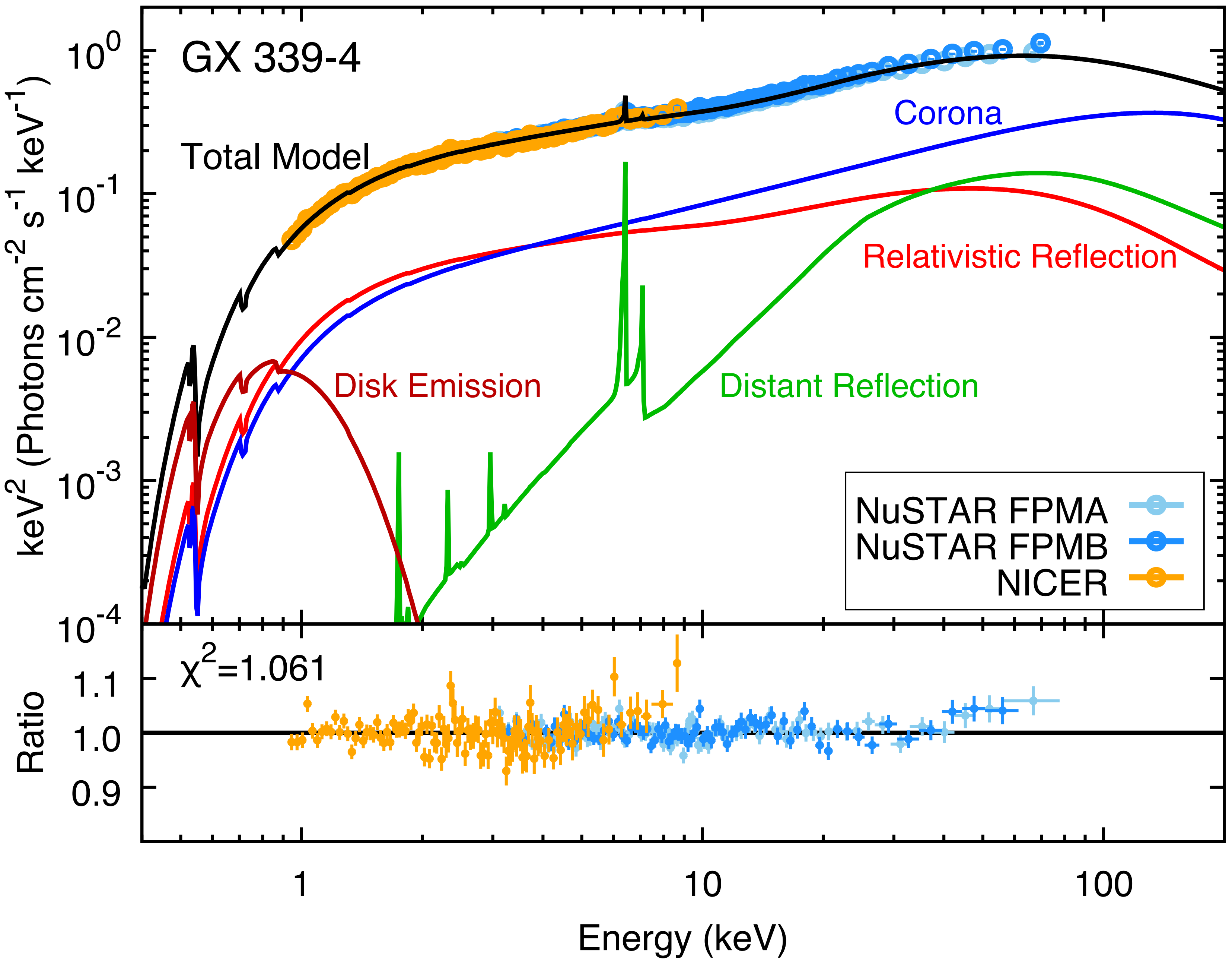


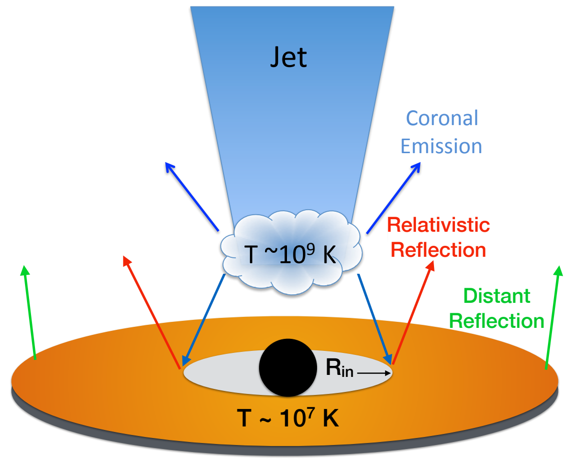
(Davis et al., 2006)

Comparison of self-consistent accretion disk model with LMC X-3 data $\Longrightarrow$ good agreement, although values of α smaller than expected (fits find $0.01 < \alpha < 0.1$ instead of 0.1–0.8).

Top red line: inferred accretion disk spectrum without interstellar absorption.

The "corona"





Beware: X-ray spectra observed from accretion flows onto black holes are *not* dominated by accretion flow!

even less so in AGN!!

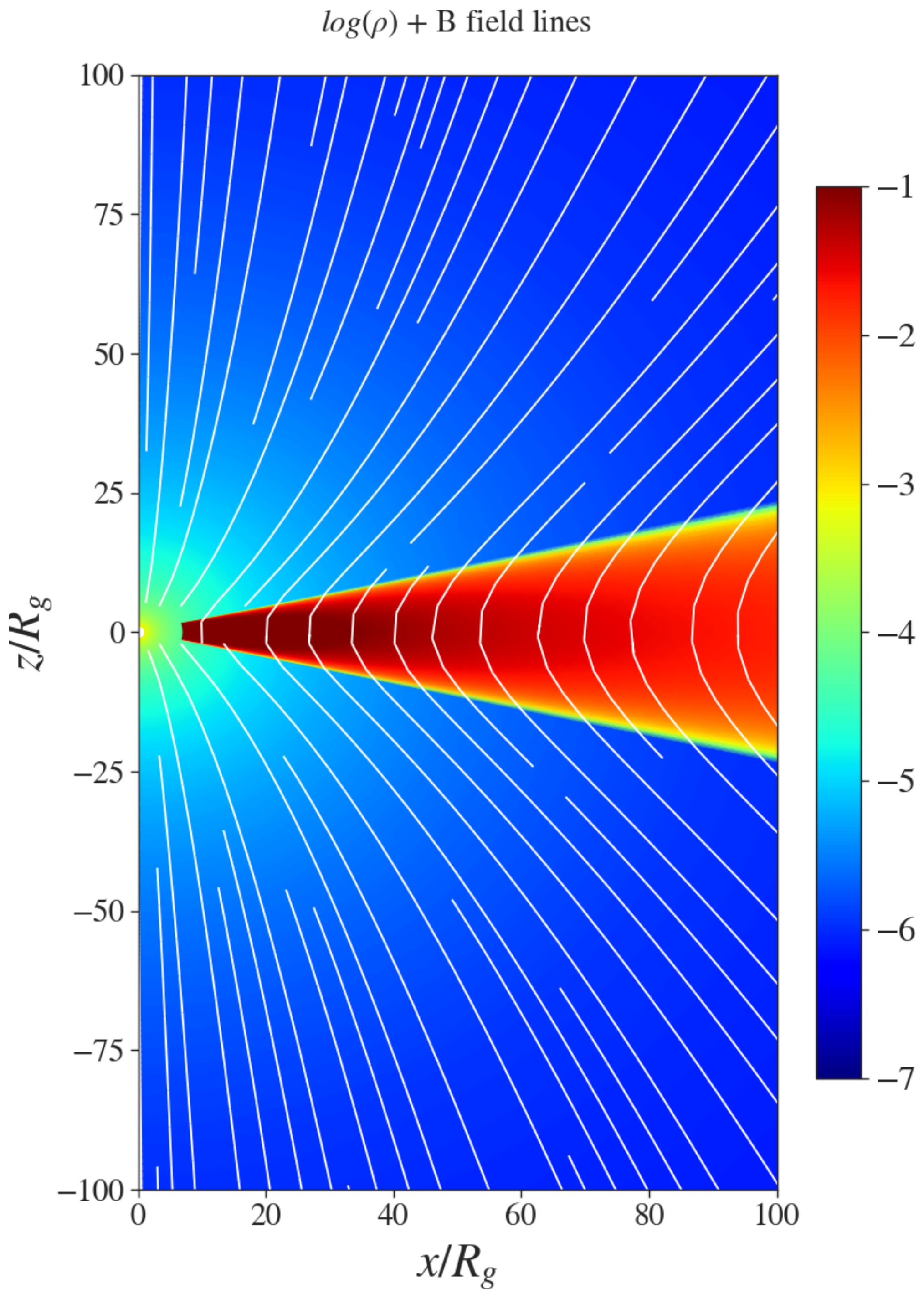
$\Longrightarrow$ Dominated by Comptonization and reflection.

Disk photons Comptonized in hot electron gas

sometimes called the "corona" – worst name in all of astrophysics

Geometry of disk and Comptonizing plasma is 100% unclear

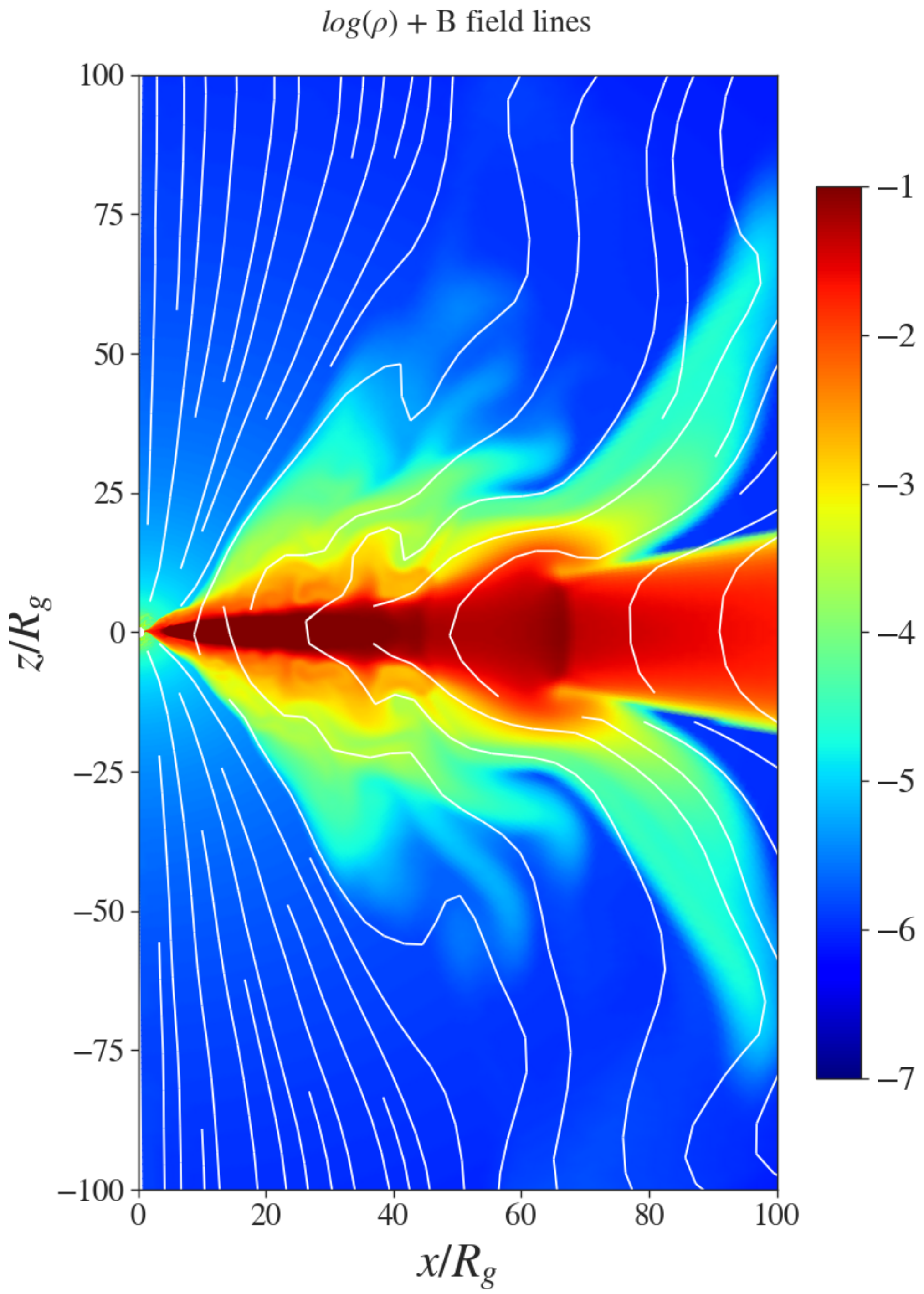
A rant: Do not believe anybody who tells you otherwise! The only thing that is clear is that simple "sandwich geometries" do not work – open question in Galactic and extragalactic BHs since $\gtrsim$ 25 years.



Many accretion flows are associated with jets and Outflows.

Jet formation:
$t = 0\,s$

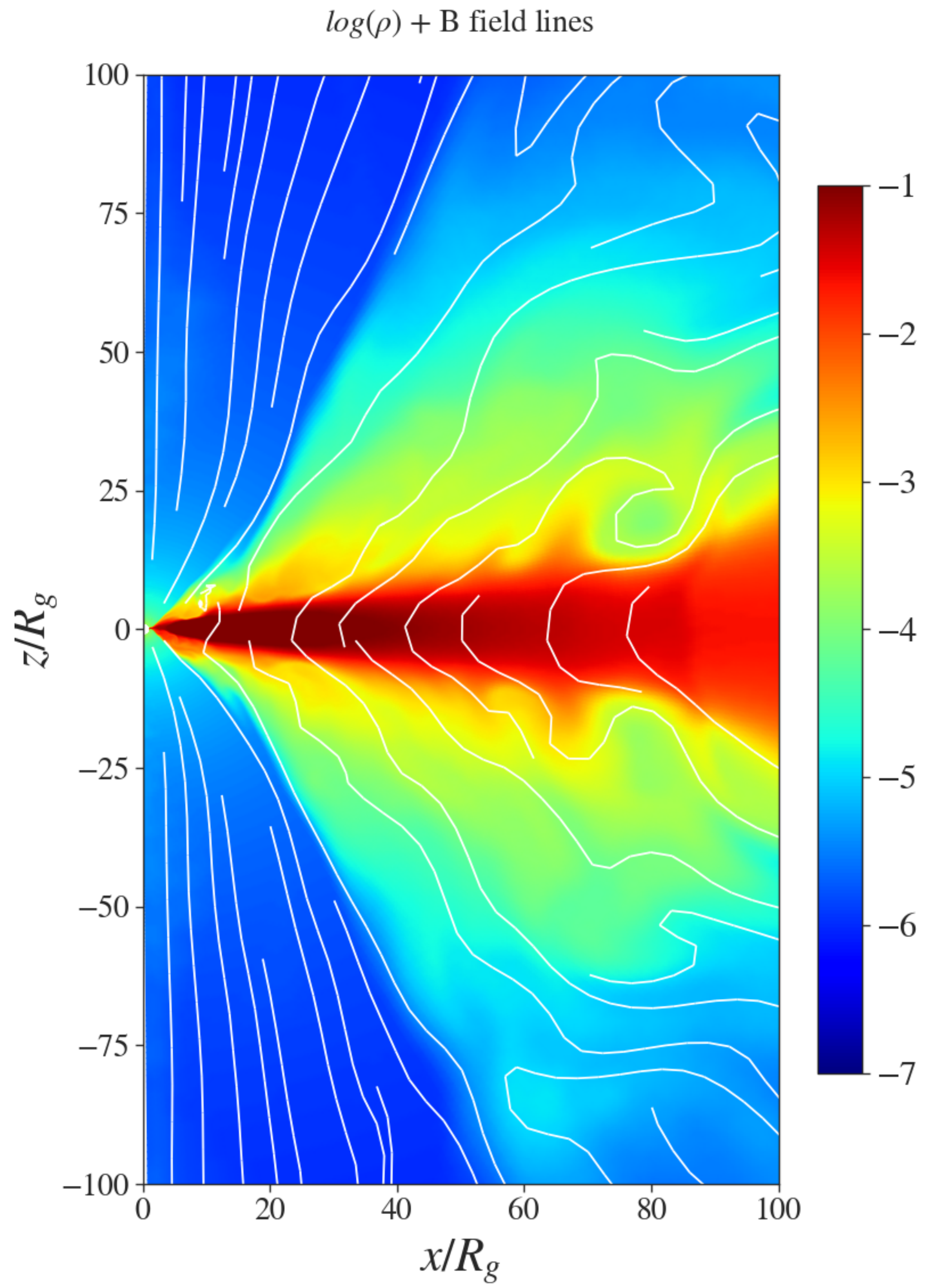
C. Fendt (MPIA Heidelberg), priv. comm.



Many accretion flows are associated with jets and Outflows.

Jet formation:
t = 2000 s

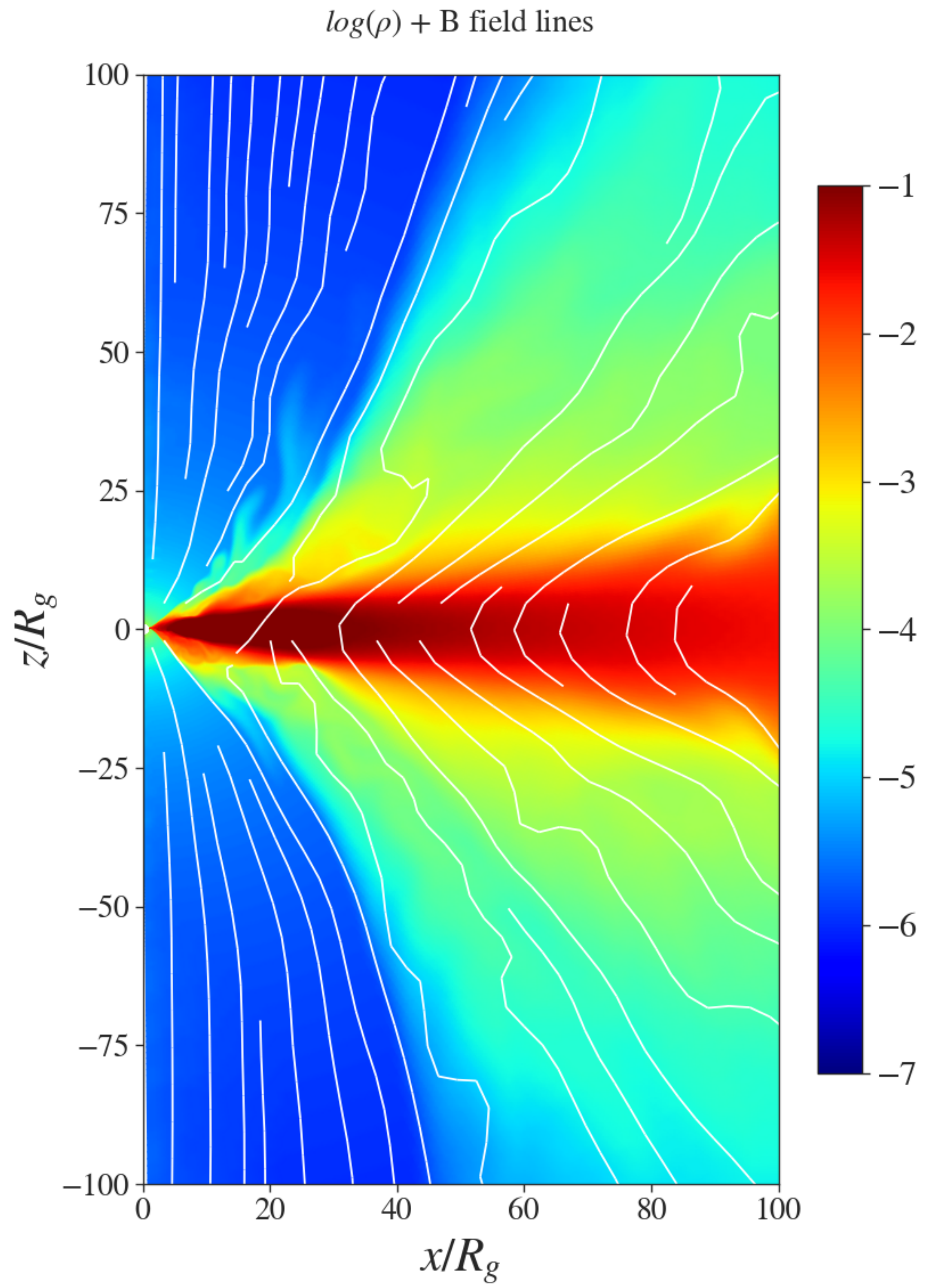
C. Fendt (MPIA Heidelberg), priv. comm.



Many accretion flows are associated with jets and Outflows.

Jet formation:
t = 5000 s

C. Fendt (MPIA Heidelberg), priv. comm.

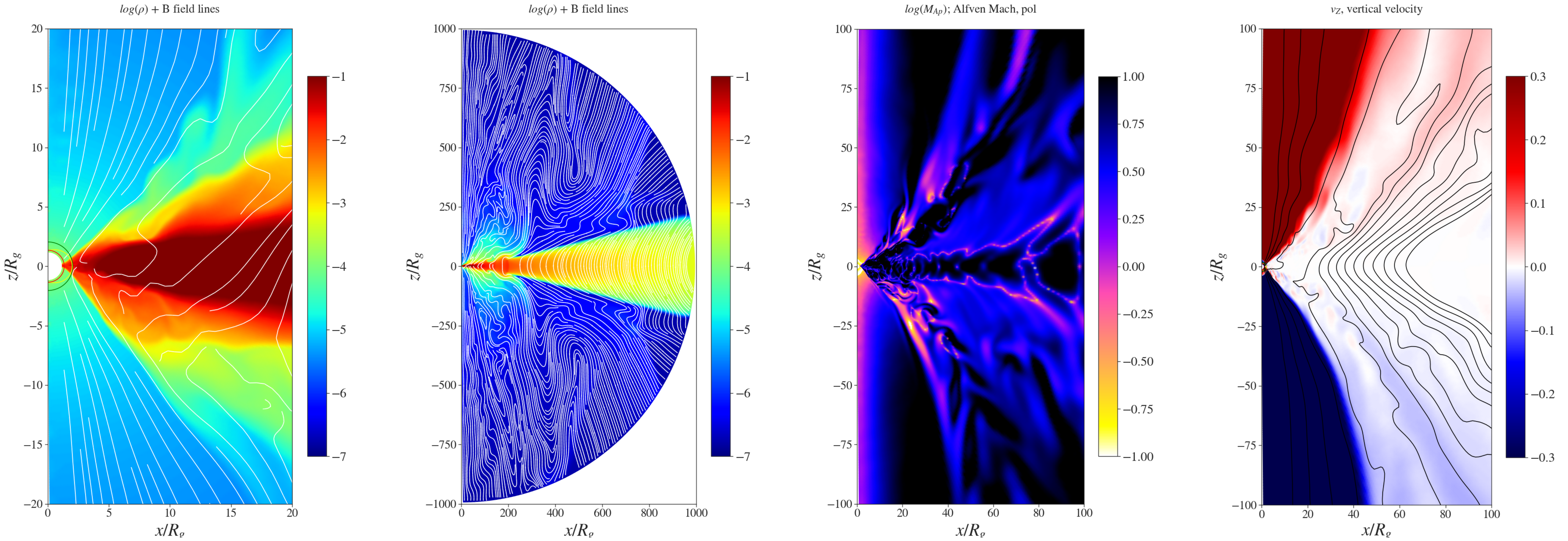


Many accretion flows are associated with jets and Outflows.

Jet formation:
$t = 10000\,s$

C. Fendt (MPIA Heidelberg), priv. comm.

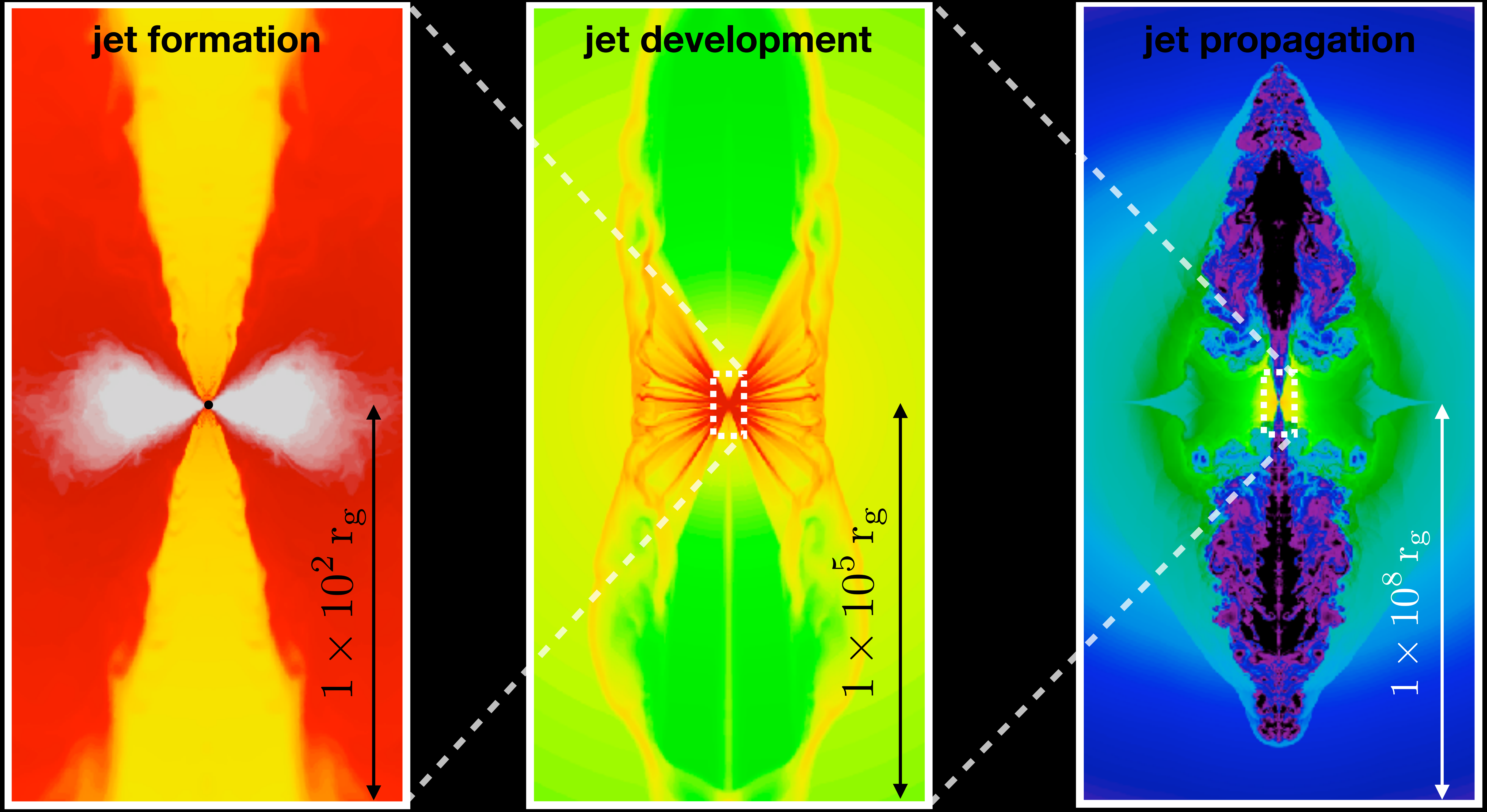
Jets



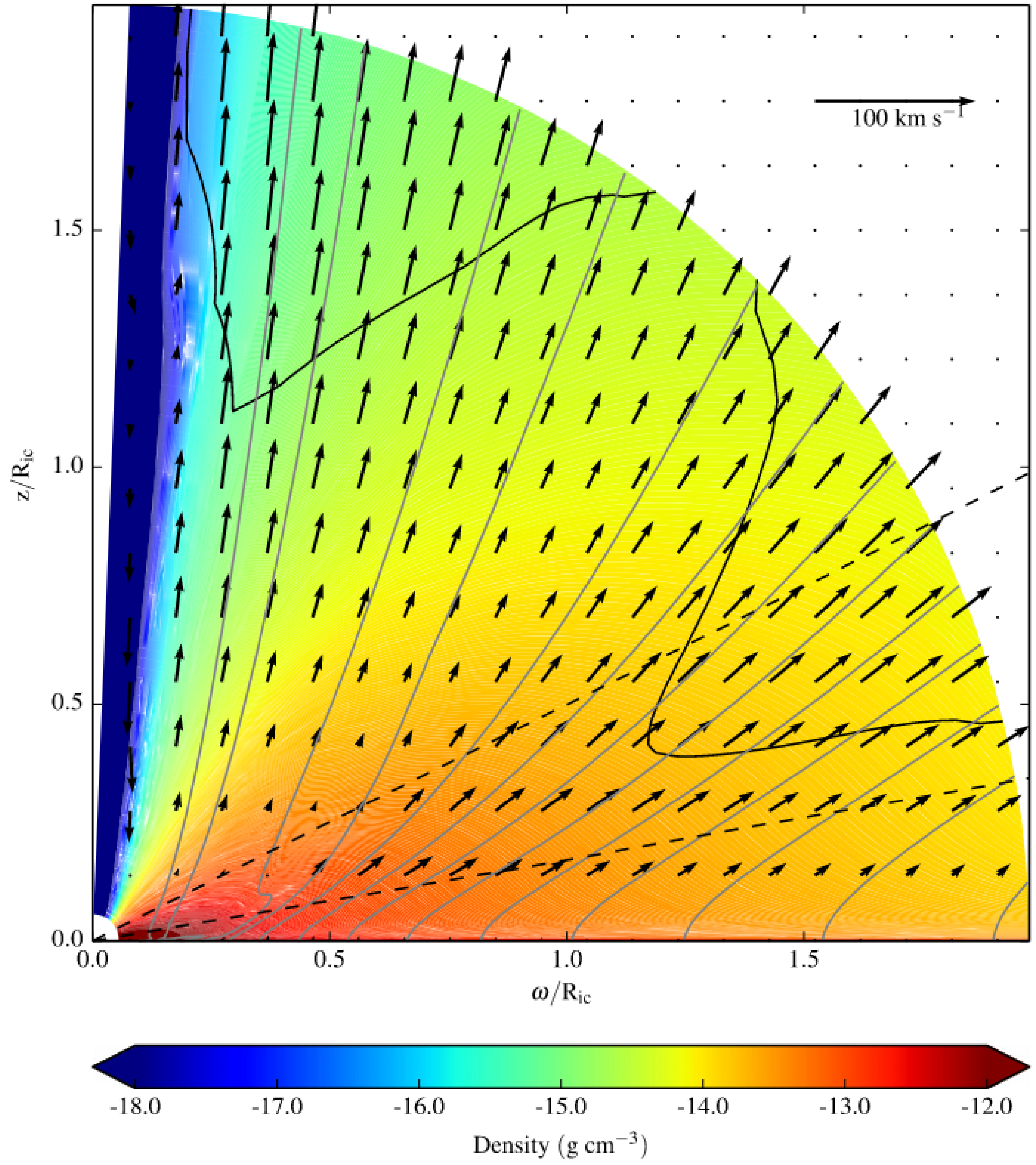
C. Fendt, priv. comm.

Modern MHD models now manage to launch jets, but still severe challenges due to large scales to cover.

Black hole binaries: presence of "dark jets" even in radio quiet states $\Longrightarrow$ existence of a jet does not always imply radio emission!



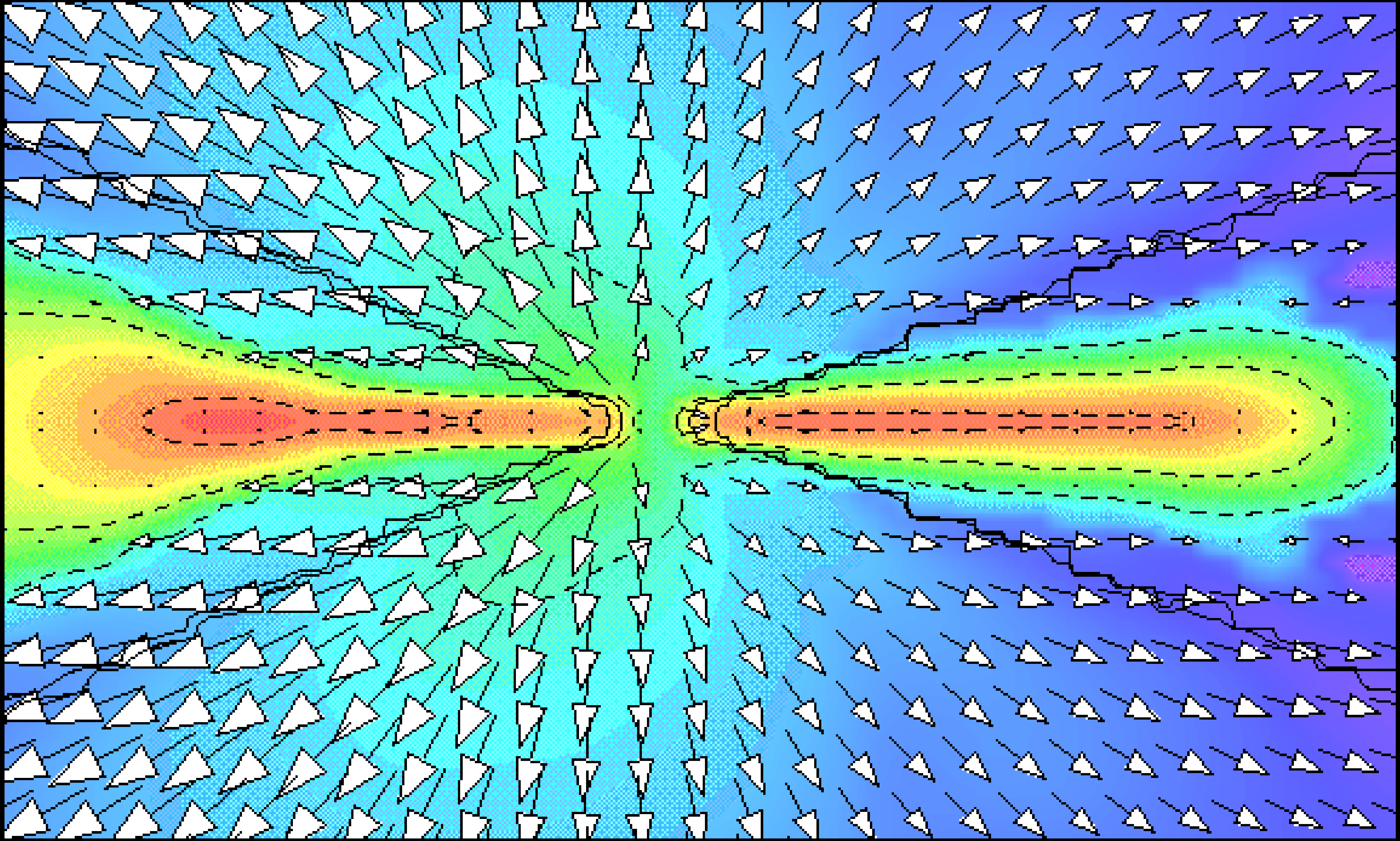
jet formation
jet development
jet propagation
$1 \times 10^2\ r_g$
$1 \times 10^5\ r_g$
$1 \times 10^8\ r_g$



Recent years: Disk winds and outflows are important

- Driven by magnetic fields and/or radiation
- Speeds observed of $\sim$ few1000 km s^{-1}
- Additional source of angular momentum loss

Higginbottom & Proga, 2015



courtesy J. Blondin

X-rays from central source heat disk surface, drive a strong wind.

Bondi-Hoyle and Wind Accretion

Early type stars (O, B, mass $\gtrsim 10\,M_\odot$):

- strong winds, driven by radiation pressure in absorption lines
- mass loss rates 10^{-10} to $10^{-6}\,M_\odot\,\mathrm{yr}^{-1}$
- Wind velocity

$$v(r) \sim v_\infty \left(1 - \frac{R_\star}{r}\right)^\beta$$

with $v_\infty \sim 2000\,\mathrm{km\,s^{-1}}$ and $\beta \sim 0.5 \ldots 1.0$

A fraction of the wind can be accreted by a compact object

$\Longrightarrow$ ~spherical accretion

$\Longrightarrow$ Bondi-Hoyle accretion

(Bondi & Hoyle, 1944)

The simplest case of wind accretion is spherically symmetric accretion.

For spherically symmetric accretion, we can derive the exact solution for the gas flow from the equations of gas dynamics:

Conservation of mass is described by the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{2}$$

while the conservation of momentum is described by the Euler equation

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \mathbf{f} \tag{3}$$

where $\mathbf{f}$ is a force density (force per unit volume).

By definition, in the spherically symmetric case the flow has only a radial component. Furthermore, if the flow is steady, then all time derivatives vanish. This means that the equation of continuity now reads

$$\nabla \cdot (p\mathbf{v}) = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \rho v \right) = 0 \tag{4}$$

and therefore

$$r^2 \rho v = \text{const.} = C \tag{5}$$

The constant is related to the mass accretion rate: Since the inward flux of mass is given by $\rho|v|$, the mass accretion rate is

$$\dot{M} = 4\pi r^2 \rho |v| \tag{6}$$

and therefore

$$C = \frac{\dot{M}}{4\pi} \tag{7}$$

To obtain the velocity profile, we use the Euler equation (Eq. 3). Because of Newton's law of gravitation

$$\mathbf{F} = \frac{GMm}{r^2} \frac{\mathbf{r}}{r} \tag{8}$$

the force density has a radial component only and is given by

$$f = -\frac{GM\rho}{r^2} \tag{9}$$

Inserting this into Euler's equation then results in

$$\rho v \frac{dv}{dr} = -\frac{dP}{dr} - \frac{GM\rho}{r^2} \tag{10}$$

which simplifies to

$$v \frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho} \frac{dP}{dr} \tag{11}$$

This differential equation can be solved under the boundary condition of some velocity at infinity. Furthermore, we need to know the equation of state, i.e., how the pressure relates to other quantities in the system. Here, we will be using the polytropic equation of state

$$P = K\rho^{\gamma} \tag{12}$$

where K is some constant. As shown in lectures on thermodynamics, if the gas is isothermal, then $\gamma = 1$, if the flow is adiabatic instead, then $\gamma = 5/3$ (γ is the ratio of specific heats).
With this equation of state, the speed of sound is

$$c_s^2 = \frac{\partial P}{\partial \rho} = K\gamma\rho^{\gamma-1} \tag{13}$$

We now insert the equation of state into Eq. (11):

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\gamma K\rho^{-\gamma-1}\frac{d\rho}{dr} = -\frac{GM}{r^2} - c_s^2\frac{1}{\rho}\frac{d\rho}{dr} \tag{14}$$

But because of Eq. (4)

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\rho v\right) = 0 \tag{4}$$

we have

$$\frac{1}{r^2}\left(\frac{d\rho}{dr}\left(r^2 v\right) + \rho\frac{d}{dr}\left(r^2 v\right)\right) = 0 \quad \Longleftrightarrow \quad \frac{1}{\rho}\frac{d\rho}{dr} = -\frac{1}{vr^2}\frac{d}{dr}\left(r^2 v\right) \tag{15}$$

Inserting this into Eq. (14) gives

$$v\frac{dv}{dr} = -\frac{GM}{r^2} + c_s^2\frac{1}{vr^2}\frac{d}{dr}\left(r^2 v\right) = -\frac{GM}{r^2} + c_s^2\left(\frac{2}{r} + \frac{1}{v}\frac{dv}{dr}\right) \tag{16}$$

Multiplying by v then results in

$$v^2\frac{dv}{dr} = -\frac{GMv}{r^2} + \frac{2v}{r}c_s^2 + c_s^2\frac{dv}{dr} \tag{17}$$

and therefore

$$\left(v^2 - c_s^2\right)\frac{dv}{dr} = v\left(\frac{2c_s^2}{r} - \frac{GM}{r^2}\right) \tag{18}$$

Spherical symmetric accretion:

$$\left(v^2 - c_s^2\right) \frac{dv}{dr} = v \left(\frac{2c_s^2}{r} - \frac{GM}{r^2}\right) \tag{18}$$

For r large: right hand side is positive.

Since $dv/dr < 0$ for accretion, this means that for large r: $v < c_s$.

Similarly, for small r: $v > c_s$

$\Longrightarrow$ sonic point for $v = c_s$ at

$$r_{\text{sonic}} = \frac{GM}{2c_s^2}$$

$\Longrightarrow$ *If* the flow goes supersonic, it does so at $r = r_{\text{sonic}}$

Note that c_s depends on r, several other solutions are possible, but the above one is the most common one for the objects we're looking at. See Holzer & Axford (1970) for details.

To finish the discussion of Bondi-Hoyle accretion, we now explicitly integrate Euler's equation

$$v\frac{dv}{dr} + \frac{GM}{r^2} + \frac{1}{\rho}\frac{dP}{dr} = 0 \tag{11}$$

over r:

$$\int v\frac{dv}{dr}\,dr + \int \frac{GM}{r^2}\,dr + \int \frac{dP}{\rho} = 0$$

inserting $dP = K\gamma\rho^{\gamma-1}d\rho$ and integrating then gives the Bernoulli integral

$$\frac{1}{2}v^2 + \frac{\gamma}{\gamma-1}K\rho^{\gamma-1} - \frac{GM}{r} = \text{const.}$$

which obviously is related to energy conservation and can be written as

$$\frac{1}{2}v^2 + \frac{c_s^2}{\gamma-1} - \frac{GM}{r} = \text{const.} = \frac{c_{s,\infty}}{\gamma-1} \tag{19}$$

where $c_{s,\infty}$ is the speed of sound at $r = \infty$.

This follows since $v(r \to \infty) = 0$.

From Eq. (19) we can now determine the speed of sound at the sonic point

$$c_s^2(r_{\rm sonic}) = c_{s,\infty}\left(\frac{2}{5-3\gamma}\right)^{1/2}$$

and the mass accretion rate is

$$\dot{M} = 4\pi r^2 \rho |v| = 4\pi r_{\rm sonic}^2 \rho(r_{\rm sonic}) c_s(r_{\rm sonic})$$

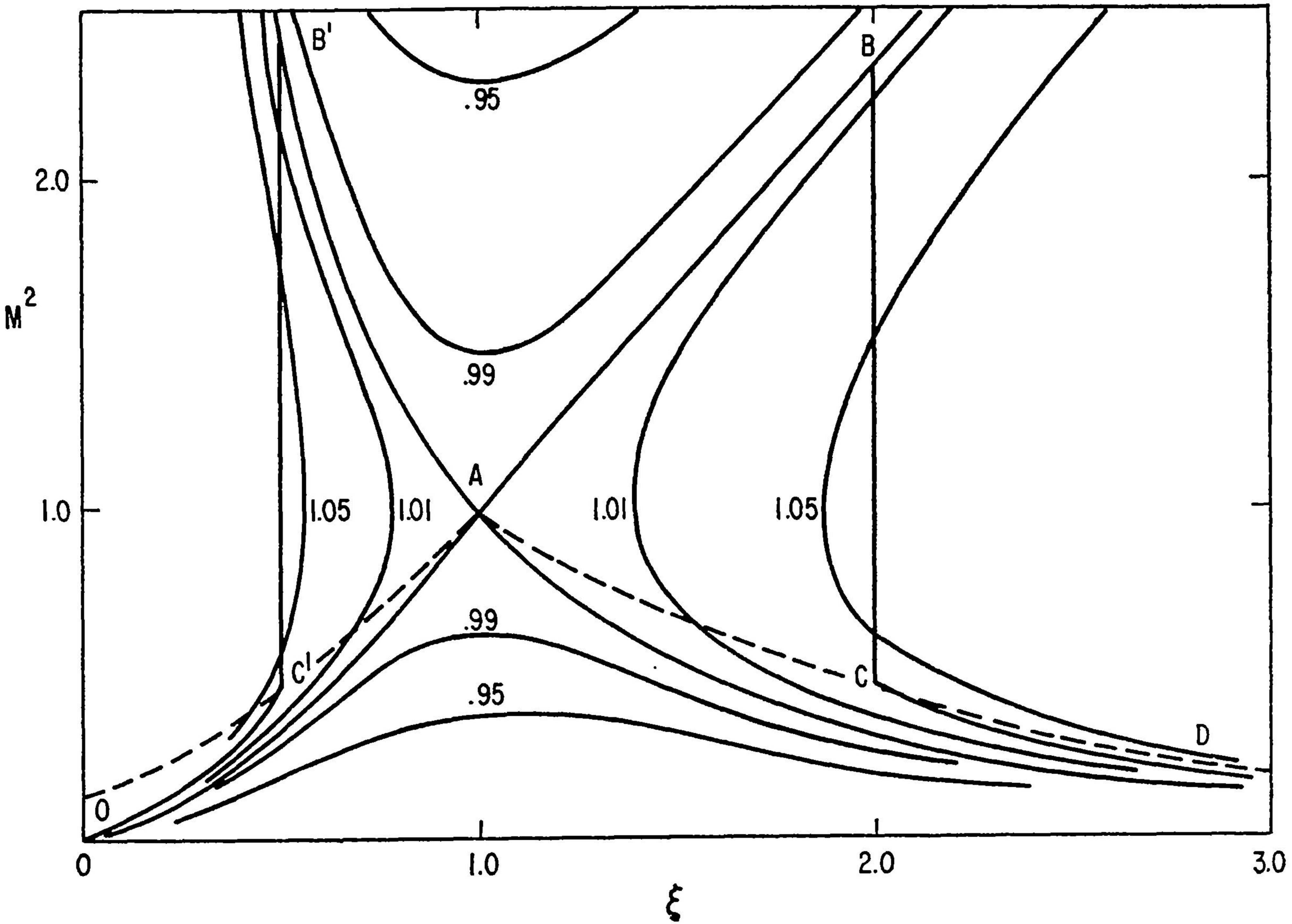
Since $c_s^2 \propto \rho^{\gamma-1}$,

$$\rho(r_{\rm sonic}) = \rho_\infty \left(\frac{c_s(r_{\rm sonic})}{c_{s,\infty}}\right)^{2/(\gamma-1)}$$

Therefore

$$\dot{M} = \pi G^2 M^2 \frac{\rho_\infty}{c_{s,\infty}^3}\left(\frac{2}{5-3\gamma}\right)^{(5-3\gamma)/2(\gamma-1)} \qquad (20)$$

Bondi-Hoyle Accretion



Mach number ($M = v(r)/c_s(r)$) as a function of radial distance, $\xi = r/r_{sonic}$, for all possible solutions of the spherical accretion problem.

(Holzer & Axford, 1970, Fig. 1)

Taking $\gamma = 5/3$, Eq. (20) becomes

$$\begin{aligned} \dot{M} &= \pi G^2 M^2 \frac{\rho_\infty}{c_{s,\infty}^3} \\ &= \pi \left(\frac{GM}{c_{s,\infty}^2} \right)^2 \rho_\infty c_{s,\infty} \\ &= \pi r_{acc}^2 \rho_\infty c_{s,\infty} \end{aligned} \tag{21}$$

where the accretion radius is defined as

$$r_{acc} = \frac{GM}{c_{s,\infty}^2}$$

Often, r_{acc} is defined as $r_{acc} = 2GM/c_s$, see next slide for the reason why.

r_{acc} defines the approximate radius of influence of an accreting body.

Wind accretion

If the ambient medium is not at rest: wind accretion.

In principle: do a similar calculation as for Bondi-Hoyle accretion. However, this would take too long, so let's do an approximate treatment here.

Let the wind's velocity be v_∞. The material in the wind is captured once

$$\frac{1}{2}v_\infty^2 = \frac{GM}{r_\mathrm{acc}}$$

such that the accretion radius for wind accretion is

$$r_\mathrm{acc} = \frac{2GM}{v_\infty^2}$$

...explaining why many people like to have a factor 2 also in the definition of r_acc for Bondi-Hoyle accretion.

Therefore, analoguously to Eq. (21),

$$\dot{M} = \pi r_\mathrm{acc}^2 \rho_\infty v_\infty = \frac{4\pi G^2 M^2 \rho_\infty}{v_\infty^3}$$

Wind accretion

To estimate the typical parameters of a wind accretor, we need to estimate v_∞ for a compact object at a distance a from the donor star

The typical velocity consists of two contributions:

1. The stellar wind velocity profile

$$v_{\rm wind}(a) \sim v_{\rm wind,\infty}\left(1-\frac{R_\star}{a}\right)^\beta \tag{8.1}$$

2. The orbital velocity of the compact object

$$v_{\rm compact}(a) = \sqrt{\frac{GM}{a}}$$

Therefore

$$v_\infty^2 \sim v_{\rm wind}^2 + v_{\rm compact}^2 = \frac{GM}{a} + v_{\rm wind,\infty}^2\left(1-\frac{R_\star}{a}\right)^{2\beta} \sim \frac{GM}{a} + v_{\rm wind,\infty}^2$$

the last is true assuming that the compact object is outside of the wind acceleration zone

Wind accretion

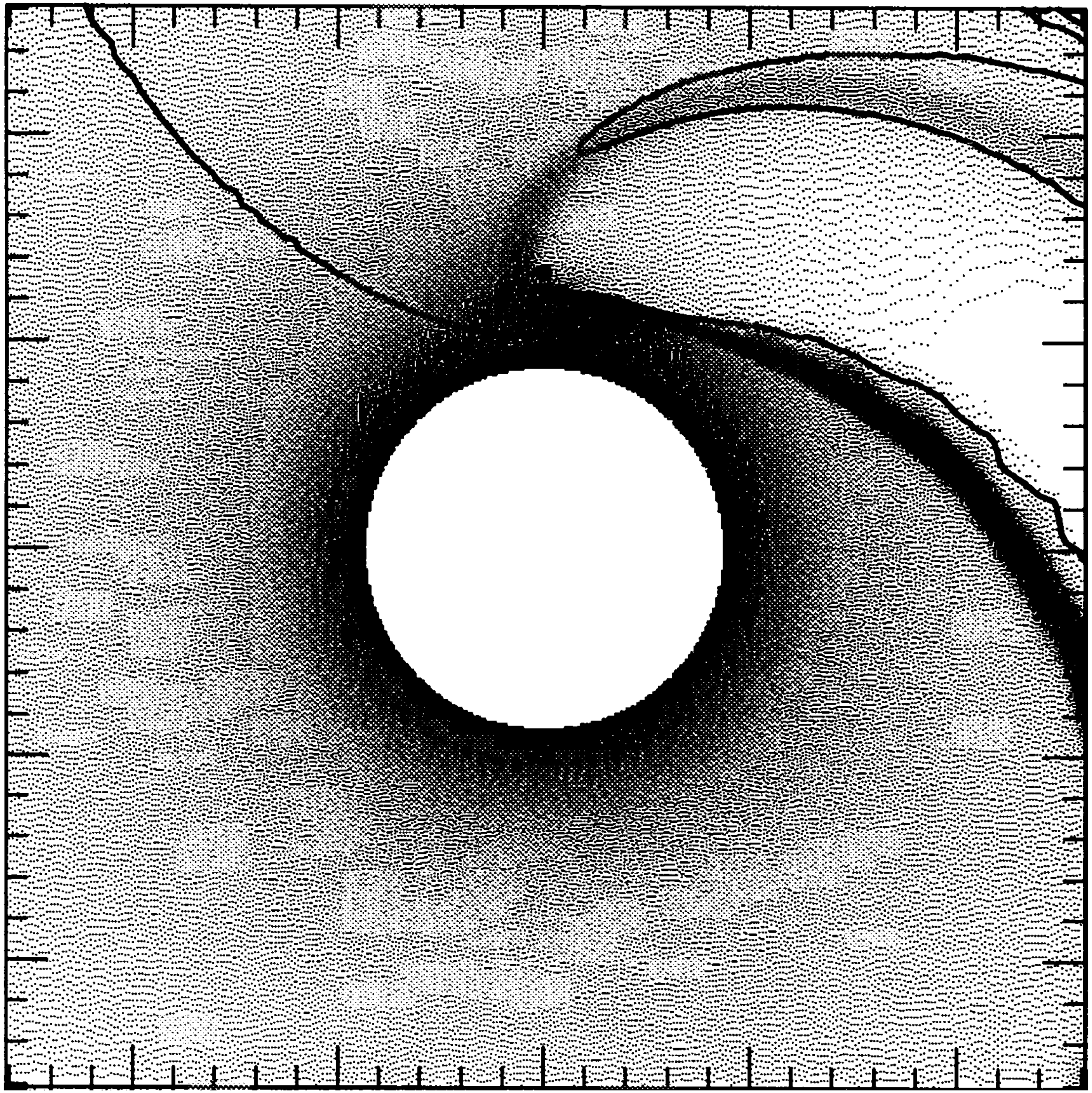
Finally, making use of the fact that the wind density is

$$\rho_\infty = \frac{\dot{M}_W}{4\pi a^2 v_{\text{wind},\infty}}$$

where $\dot{M}_W$ is the wind loss rate of the donor.
Therefore, the accretion rate of the compact object is

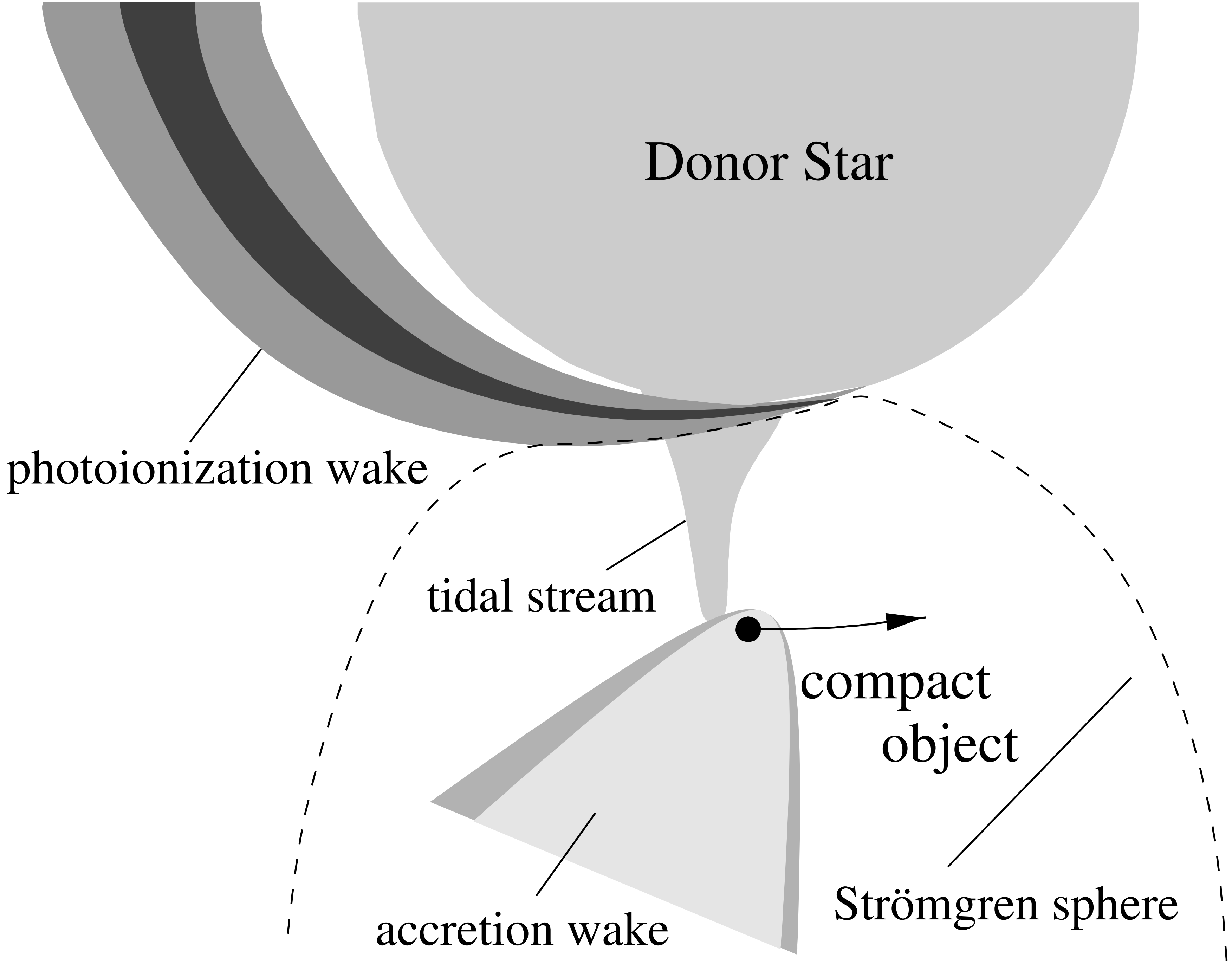
$$\dot{M} = \frac{G^2M^2}{a^2 v_{\text{wind},\infty}\left(\frac{GM}{a} + v^2_{\text{wind},\infty}\right)^{3/2}}\dot{M}_W$$
$$= \begin{cases} \left(\frac{GM}{a v^2_{\text{wind},\infty}}\right)^{1/2} \dot{M}_W & \text{for } v_{\text{orbit}} \gg v_{\text{wind},\infty} \\ \frac{G^2M^2}{a^2 v^4_{\text{wind},\infty}}\dot{M}_W & \text{for } v_{\text{orbit}} \ll v_{\text{wind},\infty} \end{cases}$$

So, for $M = 1.44\,M_\odot$, $v_{\text{wind},\infty} = 500\,\text{km}\,\text{s}^{-1}$, $a = 10^7$ km, $\dot{M} = 6 \times 10^{-3}\dot{M}_W$, i.e., the Eddington rate ($\dot{M}_{\text{Edd}} = 2.9 \times 10^{-8}\,M_\odot\,\text{yr}^{-1}$ for $1.44\,M_\odot$) is reached for $\dot{M}_W = 4.8 \times 10^{-6}\,M_\odot\,\text{yr}^{-1}$, which is very realistic.



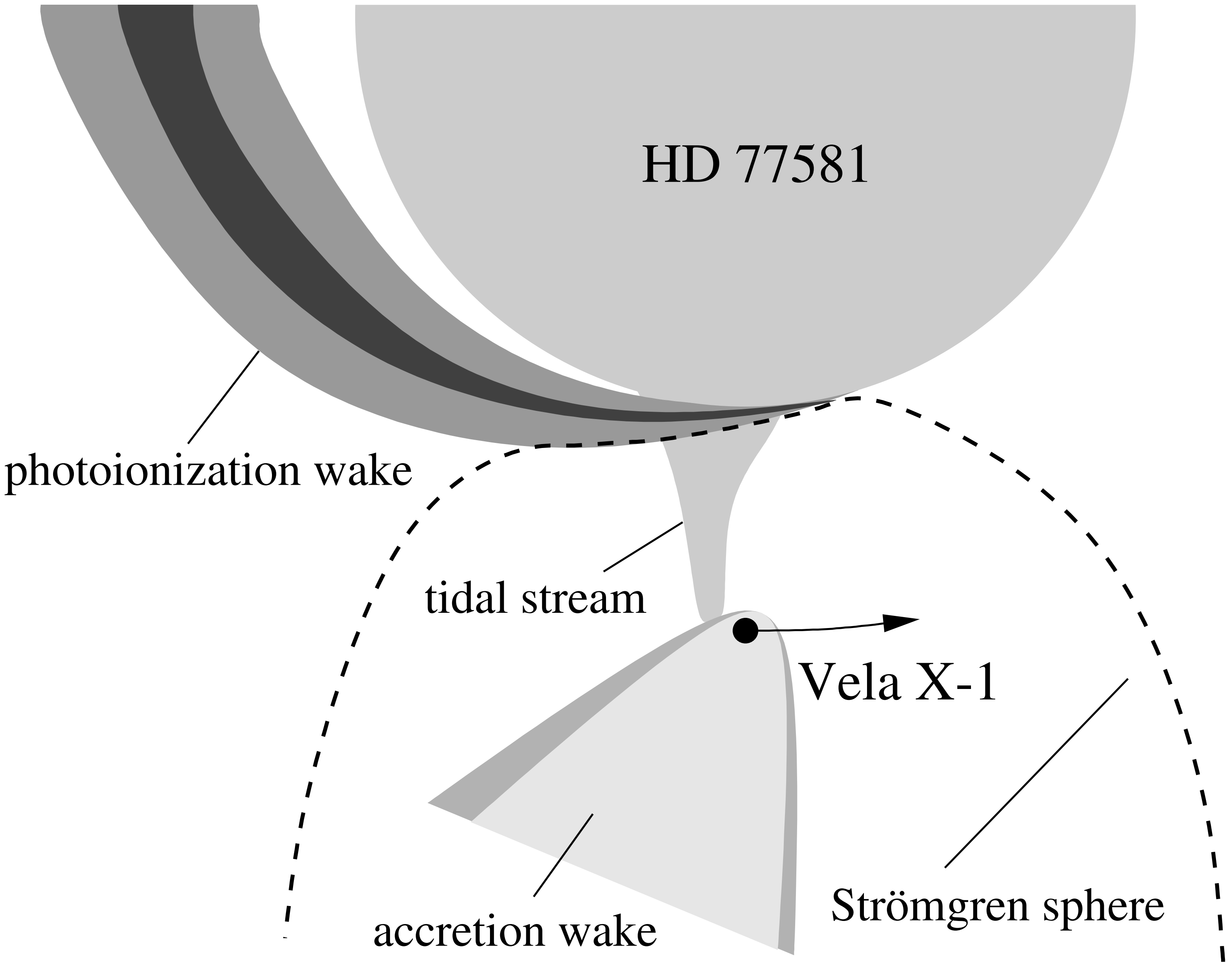
(Blondin 1994, Fig. 4)

Realistic hydrodynamical computations are very difficult (asymmetry of accretion process, ionization of wind, large range of length-scales involved,. . .).



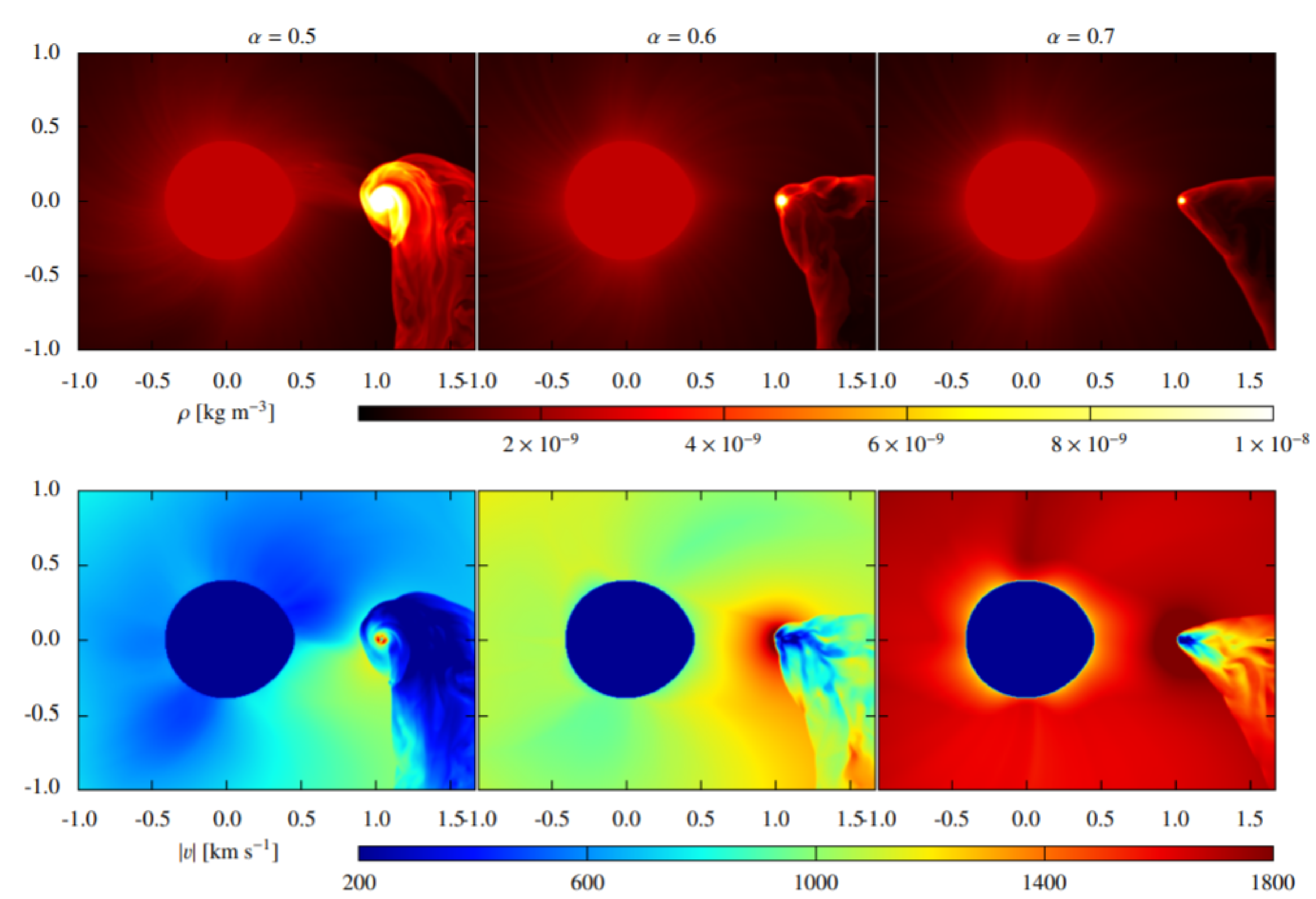
Principal components for wind-accretion:

- Ionized Strömgren region (wind ionized by X-rays from compact object).
- Accretion shock around compact object (since $v_{\mathrm{orb}} > c_s$).
- Ionization wake where material is overdense.



In realistic HMXB, because the accreted material still has some angular momentum, a small accretion disk still forms.

J. Blondin: "The disk is being BASHED by the stellar wind, BATTERED by the tidal stream, and BLASTED by X-rays"



(Čechura & Hadrava, 2015, Fig. 2; parameters for Cyg X-1)

Dependency of structure of wind accretion for changes in α

Summary

Accretion is one of the most important physical processes in universe

Could only touch some of the topics of accretion physics here

Topics missed:

- magnetospheric accretion
- outburst behavior of binaries
  can be used to measure α
- thick disks
- supercritical accretion
  ULX, radiation driven warping. . .
- radiatively inefficient accretion flows (“ADAFs”) and accretion driven inflow/outflow solutions (ADIOS)
- time variability
  sorry ;)

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