PARTICLE ACCELERATION

1. Astrophysical plasmas
2. Second-order Fermi acceleration (clouds)
3. First-order Fermi acceleration (shocks)
4. Relativistic shocks
5. What else?

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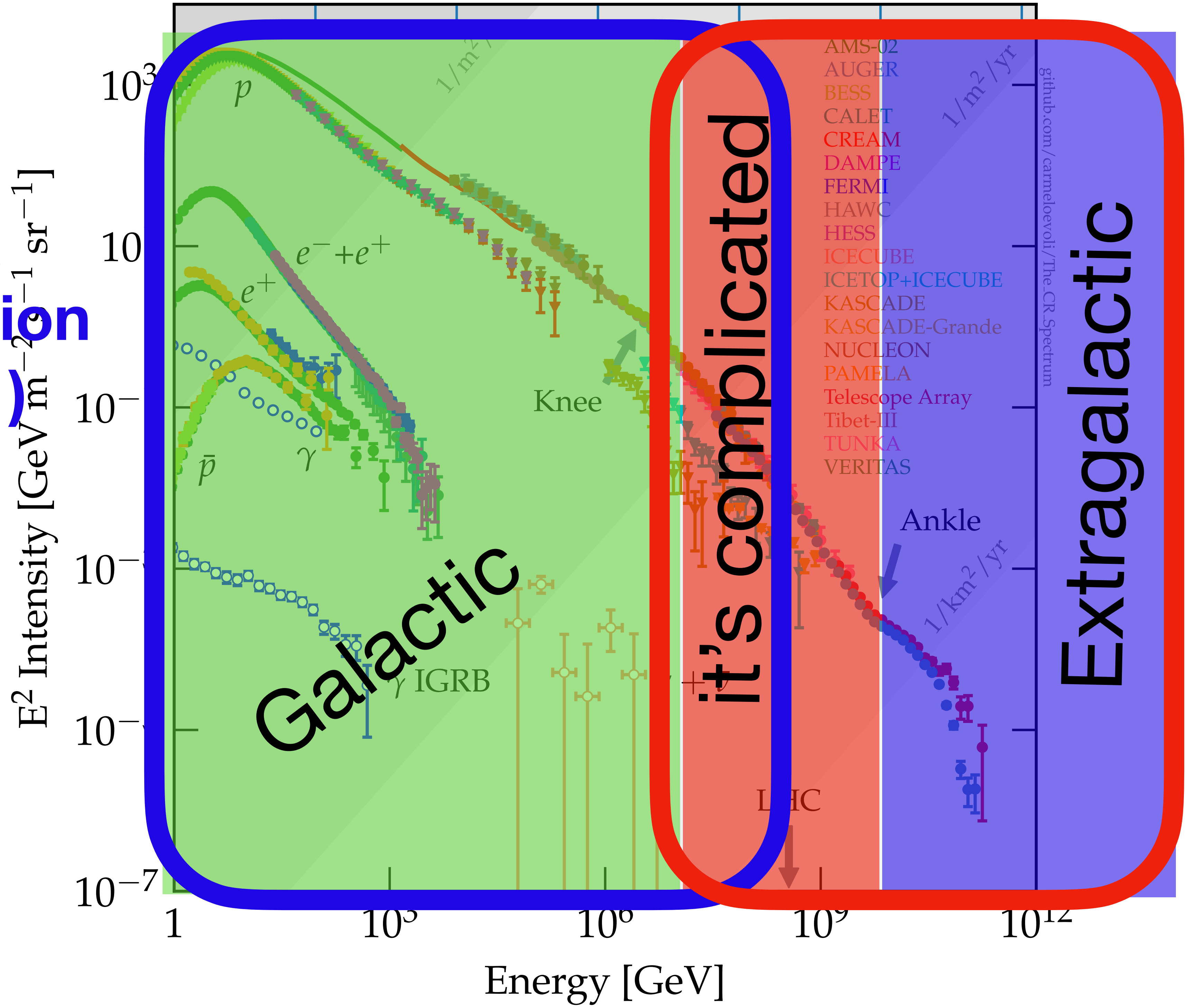
The cosmic ray spectrum

Shock acceleration (Fermi1)

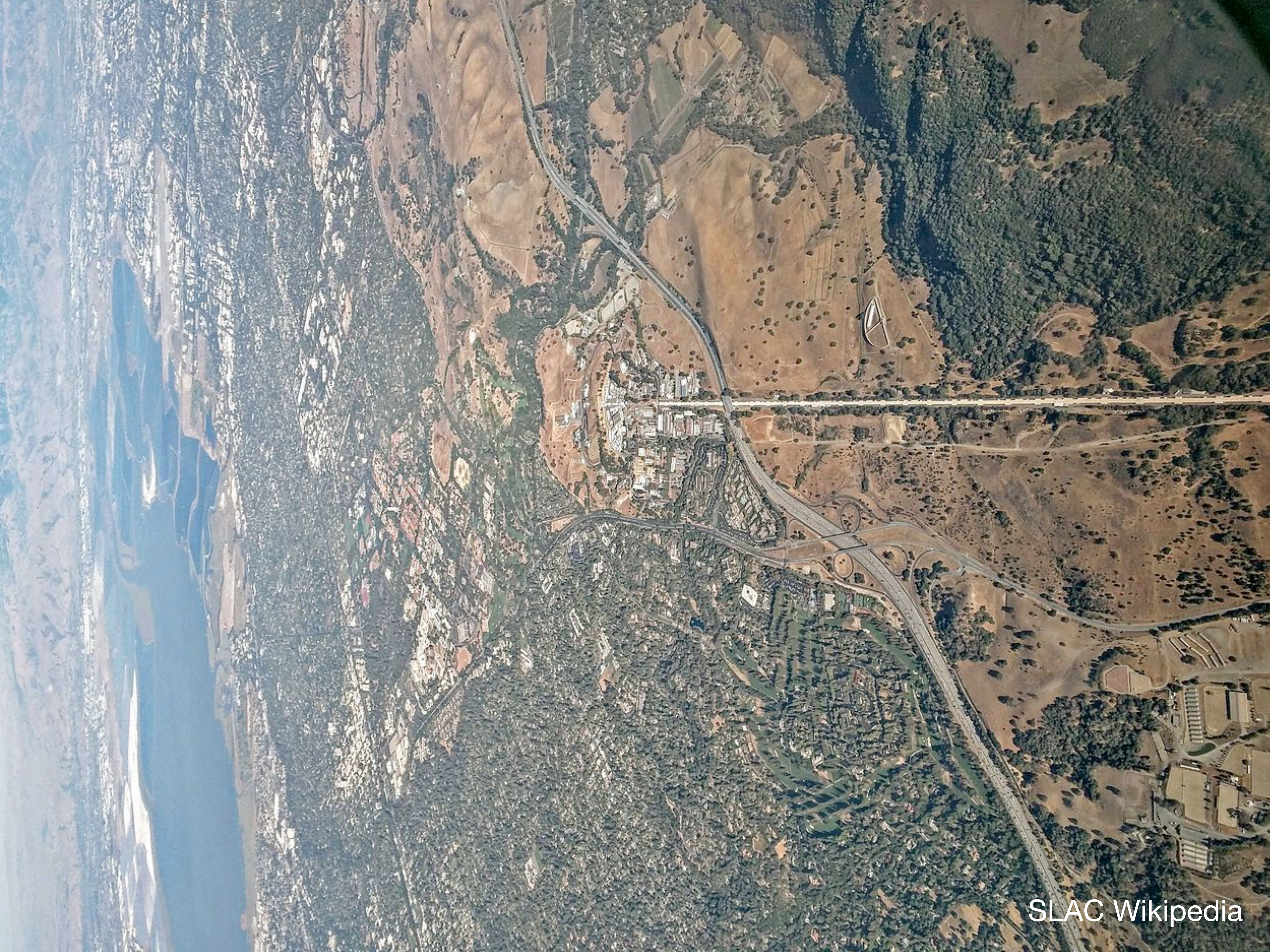


Multiple possibilities: Fermi 1,2, shear, reconnection, gaps …

Reviews: Blasi 2013, Kagan 2015, Sironi+ 2015, Lemoine 2019, Rieger 2019

Evoli 2021

SLAC Wikipedia



SLAC Wikipedia

LHC wikipedia

INTRODUCTION: ASTROPHYSICAL PLASMAS

« Collisionless » plasma: a gas of electrons and ions (protons for simplicity) that are not colliding, overall neutral.

Simple approach, particle « i » located at: (position and velocity) in phase space, the space made by these two vectors

$$\vec{x_i}(t), \vec{v_i}(t)) = r_i\vec{(t)}$$

Position:

$$\delta(\vec{x} - \vec{x_i}(t))\delta(\vec{v} - \vec{v_i}(t))$$

Particle distribution function in phase space:

$$f_s(\vec{x}, \vec{v}, t) = \sum_{i=1}^{N} \delta(\vec{x} - \vec{x_i}(t))\delta(\vec{v} - \vec{v_i}(t))$$

In general different components (at least electrons and protons), hence the subscript s=e,i that is electrons or protons.

Particle distribution function in phase space:

$$f_s(\vec{x},\vec{v},t) = \sum_{i=1}^{N} \delta(\vec{x}-\vec{x}_i(t))\delta(\vec{v}-\vec{v}_i(t))$$

Let’s differentiate with respect to time:

$$\frac{\partial f_s}{\partial t} = -\sum_{i=1}^{N} \dot{\vec{x}}_i \cdot \vec{\nabla}_{\vec{x}}\delta(\vec{x}-\vec{x}_i(t))\delta(\vec{v}-\vec{v}_i(t)) - \sum_{i=1}^{N} \boxed{\dot{\vec{v}}_i} \cdot \vec{\nabla}_{\vec{v}}\delta(\vec{x}-\vec{x}_i(t))\delta(\vec{v}-\vec{v}_i(t))$$

Acceleration, we know how to write this, providing we know the forces at play. Let consider a « non-relativistic » case first.

Equation of motion:

$$m_s\dot{\vec{v}}_i = q\vec{E}_M(\vec{x},t) + \frac{q}{c}\vec{v}_i \times \vec{B}_M(\vec{x},t)$$

M is for « microscopic », the field produce by all the other particles on one particle.

Dark matter: would have to include include gravitation!

Distribution:

$$\frac{\partial f_s}{\partial t} = -\sum_{i=1}^{N} \dot{\vec{x}}_i \cdot \vec{\nabla}_{\vec{x}} \delta(\vec{x} - \vec{x}_i(t)) \delta(\vec{v} - \vec{v}_i(t)) - \sum_{i=1}^{N} \dot{\vec{v}}_i \cdot \vec{\nabla}_{\vec{v}} \delta(\vec{v} - \vec{v}_i(t)) \delta(\vec{x} - \vec{x}_i(t))$$

Motion:

$$m_s \dot{\vec{v}}_i = q \vec{E}_M(\vec{x}, t) + \frac{q}{c} \vec{v}_i \times \vec{B}_M(\vec{x}, t)$$

Density of charge

Maxwell:

$$\vec{\nabla} \cdot \vec{E} = 4\pi \boxed{\zeta^M(\vec{x}, t)}$$

$$\vec{\nabla} \cdot \vec{B}^M(\vec{x}, t) = 0$$

$$\vec{\nabla} \times \vec{E}^M = -\frac{1}{c} \frac{\partial \vec{B}^M}{\partial t}$$

$$\vec{\nabla} \times \vec{B}^M = \frac{4\pi}{c} \boxed{\vec{J}^M(\vec{x}, t)} + \frac{1}{c} \frac{\partial \vec{E}^M}{\partial t}$$

current

Also:

$$\zeta^M(\vec{x}, t) = \sum_{s=i,e} q_s \int \mathrm{d}^3\vec{v} f_s(\vec{x}, \vec{v}, t)$$

$$\vec{J}^M(\vec{x}, t) = \sum_{s=i,e} q_s \int \mathrm{d}^3\vec{v} f_s(\vec{x}, \vec{v}, t) \vec{v}$$

MEANING: the dynamics of a particle of type I is becoming sensitive to the charge and current produced by all the other particles of all the types.

Distribution:

$$\frac{\partial f_s}{\partial t} = -\sum_{i=1}^{N} \boxed{\dot{\vec{x}}_i \cdot \vec{\nabla}_{\vec{x}}} \delta(\vec{x} - \vec{x}_i(t))\delta(\vec{v} - \vec{v}_i(t)) - \sum_{i=1}^{N} \boxed{\dot{\vec{v}}_i} \cdot \vec{\nabla}_{\vec{v}} \delta(\vec{v} - \vec{v}_i(t))\delta(\vec{x} - \vec{x}_i(t))$$

Motion: $m_s \dot{\vec{v}}_i = q\vec{E}_M(\vec{x}, t) + \frac{q}{c}\vec{v}_i \times \vec{B}_M(\vec{x}, t)$

Pulling out v x nabla in first term + injecting acceleration

$$\frac{\partial f_s}{\partial t} = -\vec{v} \cdot \vec{\nabla}_{\vec{x}} \sum_{i=0}^{N} \delta(\vec{x} - \vec{x}_i(t))\delta(\vec{v} - \vec{v}_i(t)) - \frac{q_s}{m_s} \sum_{i=0}^{N} \left[\vec{E^M}(\vec{x}_i, t) + \frac{\vec{v}_i}{c} \times \vec{B^M}(\vec{x}_i, t) \right] \cdot \vec{\nabla}_{\vec{v}} \delta(\vec{x} - \vec{x}_i(t))\delta(\vec{v} - \vec{v}_i(t))$$

Using the definition of fs

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla}_{\vec{x}} f_s + \frac{q_s}{m_s} \left[\vec{E^M}(\vec{x}_i, t) + \frac{\vec{v}_i}{c} \times \vec{B^M}(\vec{x}_i, t) \right] \cdot \vec{\nabla}_{\vec{v}} f_s = 0$$

Klimantovitch-Dupree equation

We can stop here already, but the problem is that the magnetic field and electric field with index M on them are microscopic, thus they are fluctuating in all directions, there are a mess, so not very convenient.

Klimantovitch-Dupree equation

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \vec{\nabla}_{\vec{x}} f_s + \frac{q_s}{m_s} \left[E^{\vec{M}}(\vec{x_i}, t) + \frac{\vec{v_i}}{c} \times B^{\vec{M}}(\vec{x_i}, t) \right] \cdot \vec{\nabla}_{\vec{v}} f_s = 0$$

not interested in the motion of a single particle: we care about is a statistical description. We want to integrate over a volume large enough, so that we have enough of them, but not too big, so that we do not average everything out. Thus we'll be able to retain the properties of the plasma.

Let's write the physical quantities as an average value + fluctuations.

Only first order

$$f_s \rightarrow F_s + \delta f_s$$

$$E^M \rightarrow E + \delta E$$

$$B^M \rightarrow B + \delta B$$

$$\frac{\partial F_s}{\partial t} + \vec{v} \cdot \vec{\nabla} F_s + \frac{q_s}{m_s} \left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right] \cdot \vec{\nabla} F_s = -\frac{q_s}{m_s} \langle \left(\delta\vec{E} + \frac{\vec{v}}{c} \times \delta\vec{B} \right) \delta f_s \rangle$$

Product of two perturbations, small compared to LHS

$$\frac{\partial F_s}{\partial t} + \vec{v}\cdot\vec{\nabla}F_s + \frac{q_s}{m_s}\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right]\cdot\vec{\nabla}F_s = -\frac{q_s}{m_s}\langle\left(\delta\vec{E} + \frac{\vec{v}}{c}\times\delta\vec{B}\right)\delta f_s\rangle$$

Product of two perturbations, small compared to LHS

This accounts for « collision-less collisions » in the plasma: what happens to particles due to fluctuations induced on small scales by other particles

not interested in the small scale, but to larger scales. So in the following, we'll equate the RHS to 0. (We throw away information, that is not really accessible anyway..)

VLASOV equation:

$$\frac{\partial F_s}{\partial t} + \vec{v}\cdot\vec{\nabla}_x F_s + \frac{q_s}{m_s}\left[\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right]\cdot\vec{\nabla}_v F_s = 0$$

Easy interpretation of this: this equation is derivative wrt time + derivative wrt first variable + derivative wrt second variable = conservation of particles in phase space.

$$\frac{\mathrm{D}F_s}{\mathrm{D}t} = 0$$

Remember: we have two species (electrons and ions), for each ones, we have an equation, but E and B are the same for both: E and B provide the coupling between the two equations.

Sources terms quite similar except: $f_s, E^M, B^M \rightarrow F_s, E, B$

SUMMARY

Each species in the plasma is defined by a Vlasov equation:

$$\boxed{\frac{\partial F_s}{\partial t}} + \vec{v} \cdot \boxed{\vec{\nabla} F_s} + \frac{q_s}{m_s} \left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B} \right] \boxed{\vec{\nabla} F_s} = 0$$

E and B satisfy Maxwell's equation, with source terms:

$$\zeta(\vec{x}, t) = \sum_{s=i,e} q_s \int \mathrm{d}^3 \vec{v} \, \boxed{F_s(\vec{x}, \vec{v}, t)}$$

$$\vec{J}(\vec{x}, t) = \sum_{s=i,e} q_s \int \mathrm{d}^3 \vec{v} \, \boxed{F_s(\vec{x}, \vec{v}, t)} \vec{v}$$

We can solve for F_s

APPLICATIONS

We can apply this to anything! as long as this is collision-less.

Exercise:

Let's consider plasma of electrons of protons, described by these equations.

Let's assume that there is no "net macroscopic" electric field E, but there is a magnetic field B. We can perturbe the system, knowking F_s at the zero order, and we shake it.

Just like we would do to find the dispersion relation of EM waves, or sound waves.

$$F_s = F_s^{(0)} + \delta F_s$$

$$B = B^{(0)} + \delta B$$

$$E = \delta E$$

$$\delta \vec{B} \to \delta B \exp\left[-i\omega t + i\vec{k}\cdot\vec{x}\right]$$

$$\delta F_s \to \delta F_s \exp\left[-i\omega t + i\vec{k}\cdot\vec{x}\right]$$

We need an assumption for $F_s{}^0$: for instance that the plasma is thermalized and that the temperature is 0. $F_s{}^0$ is then a delta function at p=0. If you inject this in the Vlasov equation, you get (after 10 pages of calculation):

General form: $\mathcal{F}(k, w) = 0$

$$\frac{k^2c^2}{\omega^2} = 1 + \sum_{s=i,e} \frac{4\pi^2 q_s^2}{\omega} \int \mathrm{d}p\mathrm{d}\mu \frac{p^2 v(1-\mu)}{\omega - kv_{//} \pm \Omega_s} \left[\frac{\partial F_s}{\partial p} + \frac{1}{p}\frac{\partial F_s}{\partial \mu}(\frac{k_\perp}{\omega} - \mu)\right]$$

$$\Omega_s = q_s B_0/(m_s c)$$

Simplifying assumptions: assuming that the particles are cold, then terms with p are simpler + isotropic (any mu)

Exercise:

Solve for a cold gas, with B oriented in the z direction (0, 0, z) with k=(0,0,k)

$$\omega = v_A k \qquad v_A = \frac{B_0}{\sqrt{4\pi\rho}}$$

This tells you the only modes allowed in the plasma.

RELATIVISTIC EQUATIONS

$$m\dot{V} \leftrightarrow \dot{p} \qquad (\vec{x_i}, \vec{v_i}), \leftrightarrow (\vec{x_i}, \vec{p_i})$$

$$\frac{\partial F_s}{\partial t}(\vec{x}, \vec{p}, t) + \vec{v} \cdot \vec{\nabla}_{\vec{x}} F_s + \frac{q_s}{m_s}\left[\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right] \cdot \vec{\nabla}_{\vec{p}} F_s = 0$$

What about CRs? very 'rare' in the ISM (in terms of numbers).

$$n_{\mathrm{CR}} \sim 10^{-9}\mathrm{cm}^{-3} \qquad n_{\mathrm{ISM}} \sim 1\mathrm{cm}^{-3}$$

You can do the same of CRs and solve the same equations, and you get that:

$$\omega = v_A k + j...$$

imaginary, then this implies a damping/or an exploding term in the exponential of B. The addition of CRs make the waves explode!
The Alfvèn waves, instead of propagating as waves, start bouncing out and become exploding: this is a crucial ingredient in the acceleration of CRs.

Average quantities from Vlasov

F_s

$$\frac{\partial f_s}{\partial t} + v_r \frac{\partial f_s}{\partial x_r} + q_s \left[E_r + \frac{1}{c} \epsilon_{rst} v_s B_t \right] \frac{\partial f_s}{\partial v_r} = 0$$

Levi-Civita
symbol

Let's consider a generic function of velocity $\Psi(\vec{v})$

$$\langle \Psi \rangle = \frac{\int \mathrm{d}^3 \vec{v} \Psi(\vec{x}, \vec{v}t) f(\vec{x}, \vec{v}, t)}{\int \mathrm{d}^3 \vec{v} f(\vec{x}, \vec{v}, t)}$$

$$= \frac{\int \mathrm{d}^3 \vec{v} \Psi(\vec{x}, \vec{v}, t) f(\vec{x}, \vec{v}, t)}{n}$$

Density

$$\int (1) \times (2) \mathrm{d}^3 \vec{v} \mathrm{d}^3 \vec{p} \longrightarrow$$

$$\text{First term} \rightarrow \int \mathrm{d}^3 \vec{v} \Psi \frac{\partial f}{\partial t} = \frac{\partial \int \mathrm{d}^3 \vec{v} \Psi f}{\partial t} = \frac{\partial n \langle \Psi \rangle}{\partial t}$$

$$\text{Second term} \rightarrow \int \mathrm{d}^3 \vec{v_r} \Psi \frac{\partial f}{\partial x_r} = \frac{\partial \int \mathrm{d}^3 \vec{v} \Psi v_r f}{\partial x_r} = \frac{\partial}{\partial x_r} \left[n \langle \Psi v_r \rangle \right]$$

The general equation we get is:

$$\frac{\partial}{\partial t}\left[n\langle\Psi\rangle\right]+\frac{\partial}{\partial x_r}\left[n\langle\Psi v_r\rangle\right]-\frac{qn}{m}E_r\langle\frac{\partial\Psi}{\partial v_r}\rangle-\frac{qn}{mc}\cdot\epsilon_{rst}B_t\langle\frac{\partial\Psi}{\partial v_s}v_s\rangle=0$$

Ψ is completely general, we can take whatever function of v we want!

$$\Psi=1 \qquad \langle v_r\rangle=u_r$$

$$\boxed{\frac{\partial n}{\partial t}+\frac{\partial nu_r}{\partial x_r}=0}$$

Mass conservation

one for ions + one for electrons

$$\Psi=v_r \qquad \boxed{\frac{\partial\rho u_r}{\partial t}+\frac{\partial\rho\langle v_rv_s\rangle}{\partial x_s}-\frac{qn}{c}\epsilon_{rst}B_tu_s=0}$$

Momentum

$$\Psi=\frac{1}{2}v_r^2 \qquad \frac{\partial\frac{1}{2}\rho\langle u_r^2\rangle}{\partial t}+\frac{\partial\frac{1}{2}\rho\langle v^2v_z\rangle}{\partial x_r}-qn\epsilon_{rst}E_ru_r=0$$

Energy

Goal: average quantities (Temperature, Pressure)

Let’s define:

$$\rho = n_i m_i + n_e m_e$$

$$\zeta = (n_i - n_e) e$$

Ions $u_{r,i} = \langle v_{r,i} \rangle$

$u_{r,e} = \langle v_{r,e} \rangle$

Electrons

Fluctuations

$$w_{r,i} = v_{r,i} - u_{r,i}$$

$$w_{r,e} = v_{r,e} - u_{r,e}$$

Pressure tensors:

$$P_{i,rs} = m_i \int \mathrm{d}^3\vec{v} f_i w_{ir} w_{is}$$

$$P_{e,rs} = m_i \int \mathrm{d}^3\vec{v} f_e w_{er} w_{es}$$

Now the trick: let's take the equation of mass for electrons and ions, and sum them together:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

General, no information on ions and electrons

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General, no information on ions and electrons

Sum for momentum:

$$\rho \frac{\mathrm{D}\vec{u}}{\mathrm{D}t} = \rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} \right] = -\vec{\nabla} P + \frac{1}{c} \vec{J} \times B (+\zeta \vec{E})$$

Pressure of the overall plasma

Usually 0

time derivative of density of velocity: equation of motion of the plasma

force due to pressure gradient that makes the plasma move.

Additional term if we had accelerated particles (pressure CR)

MHD equations

IDEAL MHD: assume that the conductivity of the plasma is infinite!

We did not do energy conservation because we'll assume that the gas is adiabatic, not losing energy towards the outside of the system. Then we only use an equation of state for the gas.

Resistivity

Plasma reference frame: $\vec{E}' = \eta \vec{J}$

Co moving field

Lorentz transform: $\vec{E}' = \vec{E} + \frac{1}{c}\vec{v} \times \vec{B} = \eta \vec{J}$

Maxwell $\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \implies \vec{J} = \frac{c}{4\pi}\left[\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t}\right]$

$$\frac{1}{c}\vec{J} \times B = \frac{1}{4\pi}\left[(\vec{\nabla} \times \vec{B}) \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \times \vec{B}\right]$$

Scales: size L, magnetic field B $\propto \frac{B^2}{L}$ $\propto \frac{BE}{cT} = \frac{B^2 V}{cL}$

Way Larger

$\eta \to 0$ $\vec{E} = -\frac{1}{c}\vec{v} \times \vec{B}$

The only field that remains in a plasma, is the one associated to the movement of the plasma (the induced magnetic field) = because the plasma is moving in a magnetized medium -> not the large scale field.

Ideal MHD

$$\frac{1}{c}\vec{J}\times\vec{B} = \frac{1}{4\pi}\left[(\vec{\nabla}\times\vec{B})\times\vec{B} - \frac{1}{c}\frac{\partial\vec{E}}{\partial t}\times\vec{B}\right] \approx \frac{1}{4\pi}\left[(\vec{\nabla}\times\vec{B})\times\vec{B}\right]$$

$$\rho\left[\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right] = \frac{1}{4\pi}\left[(\vec{\nabla}\times\vec{B})\times\vec{B}\right]$$

The current has disappeared!

Ideal MHD SUMMARY

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0$$

$$\rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right] = -\vec{\nabla} P + \frac{1}{4\pi} \left[(\vec{\nabla} \times \vec{B}) \times \vec{B} \right]$$

Momentum

$$\left(\frac{\partial}{\partial t} \cdot + \vec{v} \cdot \vec{\nabla} \right) p \rho^{-\boxed{\gamma}}$$

Adiabaticity

Adiabatic index (ratio of specific heats = 5/3 usually, when including CRs, usually goes close to 4/3)

$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

What are the perturbations allowed in this system?

We look for a relation in the form: $\mathcal{F}(\omega, k) = 0$

$$\rho \rightarrow \rho_0 + \delta \rho$$

$$\vec{B} \rightarrow = \vec{B}_0 + \delta \vec{B}$$

Assuming (we could in principle go to higher orders)

$$\langle \delta\rho \rangle = \langle \delta\vec{B} \rangle = 0$$

$$\rho \rightarrow \rho_0 + \delta\rho$$

$$\vec{B} \rightarrow= \vec{B}_0 + \delta\vec{B}$$

$$\delta B = \boxed{\delta B(k)} \exp\left[-i\omega t + i\vec{k}\cdot\vec{x}\right]$$

amplitude of the k mode
(at a specific frequency)

$$-i\omega\delta\rho + i\rho\vec{k}\cdot\delta\vec{u} = 0$$

$$-i\omega\delta P\rho^{-\gamma} + \gamma\rho^{-\gamma-1}i\omega p\delta\rho = 0$$

$$\boxed{\frac{\delta P}{\delta\rho} = \gamma\frac{P}{\rho} = c_s^2}$$

perturbing the adiabaticity has something to do with sound waves..

we are sitting in the frame in which u =0: $\omega = kv_A$

Otherwise: $\omega = kv_A \pm u$

Momentum conservation gives: $-i\omega\rho\delta\vec{v} = -i\vec{k}\delta P + \frac{i}{4\pi}(\vec{k}\times\delta\vec{B})\times\vec{B}_0$

tracking only the first order terms and ignoring higher orders. (quantities constant in space, without this assumption, the amplitude of the Fourier modes also has to depend on « z")

Adiabaticity gives: $\delta P = c_s^2\delta\rho$

-> re-injected in momentum. + Mass conservation

$$-i\omega\delta\vec{B} = -i\vec{k}\times(\delta\vec{v}\times\vec{B}_0)$$

$$\mathcal{F}(k,\omega)\delta v = 0$$

$$-i\omega\rho\delta\vec{v} = -i\vec{k}c_s^2\frac{\rho}{\omega}\vec{k}\cdot\delta\vec{v} - \frac{i}{4\pi\omega}\left[\vec{k}\times\left(\vec{k}\times(\delta\vec{v}\times\vec{B}_0)\right)\right]\times\vec{B}_0$$

Exercise: $\vec{B}_0 = (0,0,B_0) \quad k = (0,0,k)$

What are the modes that can propagate PARALLEL to B?

$$\begin{pmatrix}\delta u_x\\ \delta u_y\\ \delta u_z\end{pmatrix} = \frac{c_s^2}{\omega^2}k^2\delta v_z\begin{pmatrix}0\\0\\1\end{pmatrix} + \frac{k^2B_0^2}{4\pi\rho\omega^2}\begin{pmatrix}\delta u_x\\ \delta u_y\\ 0\end{pmatrix}$$

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$$\begin{pmatrix} \delta u_x \\ \delta u_y \\ \delta u_z \end{pmatrix} = \frac{c_s^2}{\omega^2} k^2 \delta v_z \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \frac{k^2 B_0^2}{4\pi\rho\omega^2} \begin{pmatrix} \delta u_x \\ \delta u_y \\ 0 \end{pmatrix}$$

What do we learn? The modes with k parallel to B0 are of two types.

Modes allowed are: 1) sound waves without perpendicular motions- longitudinal modes

$$\omega^2 = k^2 c_s^2$$

2) completely different modes, moving in the perpendicular direction (Alfven waves).

$$\omega^2 = k^2 \frac{B_0^2}{4\pi\rho} = k^2 v_A^2$$

(This is only the parallel field! = Rich phenomenology)

ALFVEN WAVES

$$\delta \vec{E} = -\frac{1}{c}\delta\vec{v} \times \vec{B}_0 \qquad \delta v \Rightarrow \vec{E}$$

So the Alfvén waves we found are:

1) transverse modes
2) perpendicular motion of the plasma (in the perp plane)
3) with Electric field associated

Why do we care about this type and not soundwaves: no Electric field or magnetic field associated with them!

1) Magnetic field is scattering particles, providing the k is right:

$k \approx k_{\rm res} = \frac{\Omega}{v\mu} \Rightarrow \delta\vec{B}$ Changes the pitch angle of particles

2) responsible for second order Fermi acceleration, even if $|\delta E| < |B_0|$

3) what is shaking the box? the accelerated particles in the plasma, by themselves excite perturbations. CRs, by themselves can excite at the correct scale.

Exercise:

What are the modes that can propagate PERPENDICULAR to B?

$$\omega^2 = k^2(c_s^2 \pm v_A^2)$$

(fast/slow modes), situations in between..

Acceleration mechanisms

Dynamic,
Hydrodynamic

$$\delta\vec{E} \neq \vec{0}$$

Regular acceleration

$$\delta\vec{E} = \vec{0}$$

$$\delta\vec{E}^2 \neq 0$$

Stochastic acceleration

Non-ideal MHD flows

$\vec{E}\cdot\vec{B} \neq 0$ or $\vec{E}^2 - \vec{B}^2 > 0$

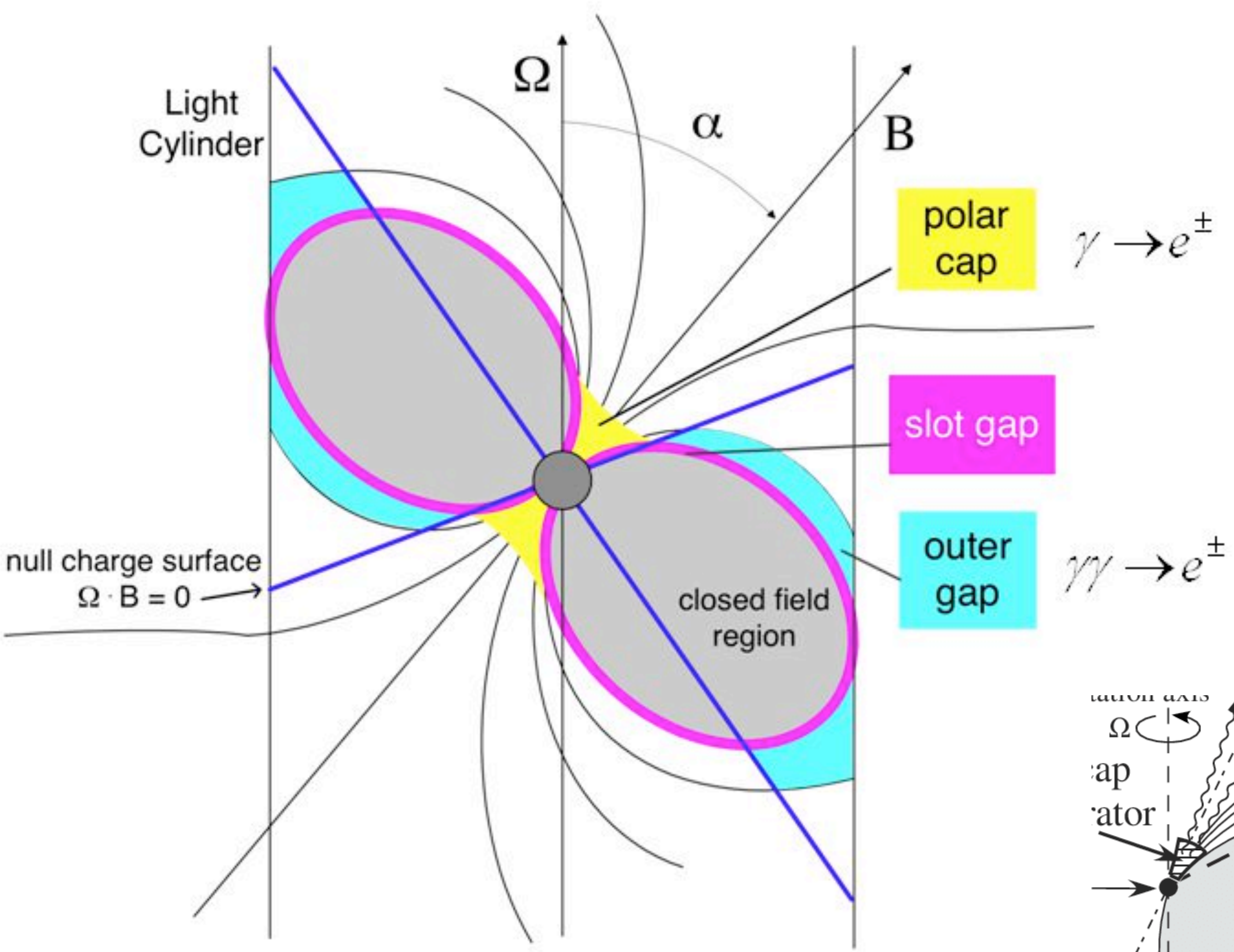
$\vec{E}^2 - \vec{B}^2 < 0$ and $\vec{E}\cdot\vec{B} = 0$

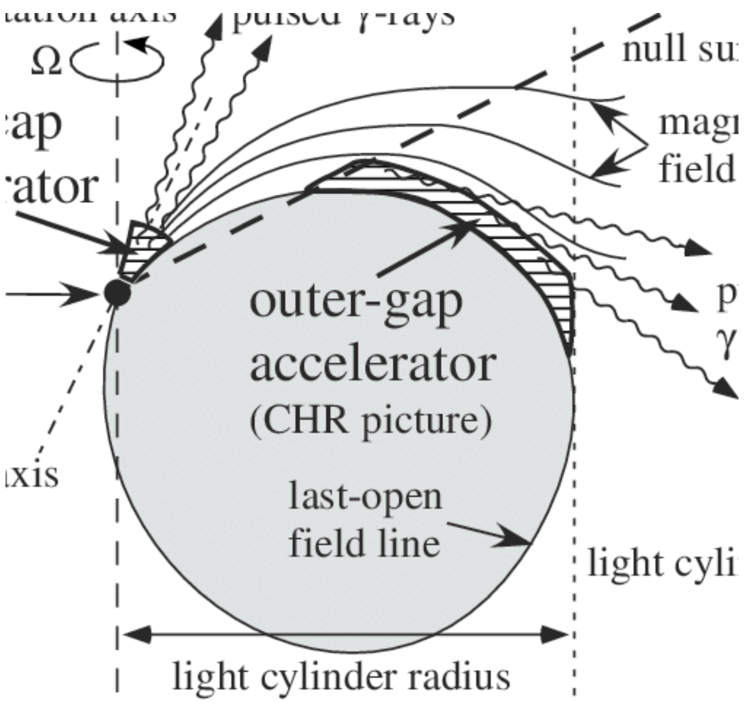
Difficult to achieve, electric field easily short-circuited!

Astrophysical situations?

1. « Gaps » pulsar caps for instance, regions where E and B are parallel
2. Magnetic reconnection

Pulsar gaps





Harding 2015

Magnetic reconnection

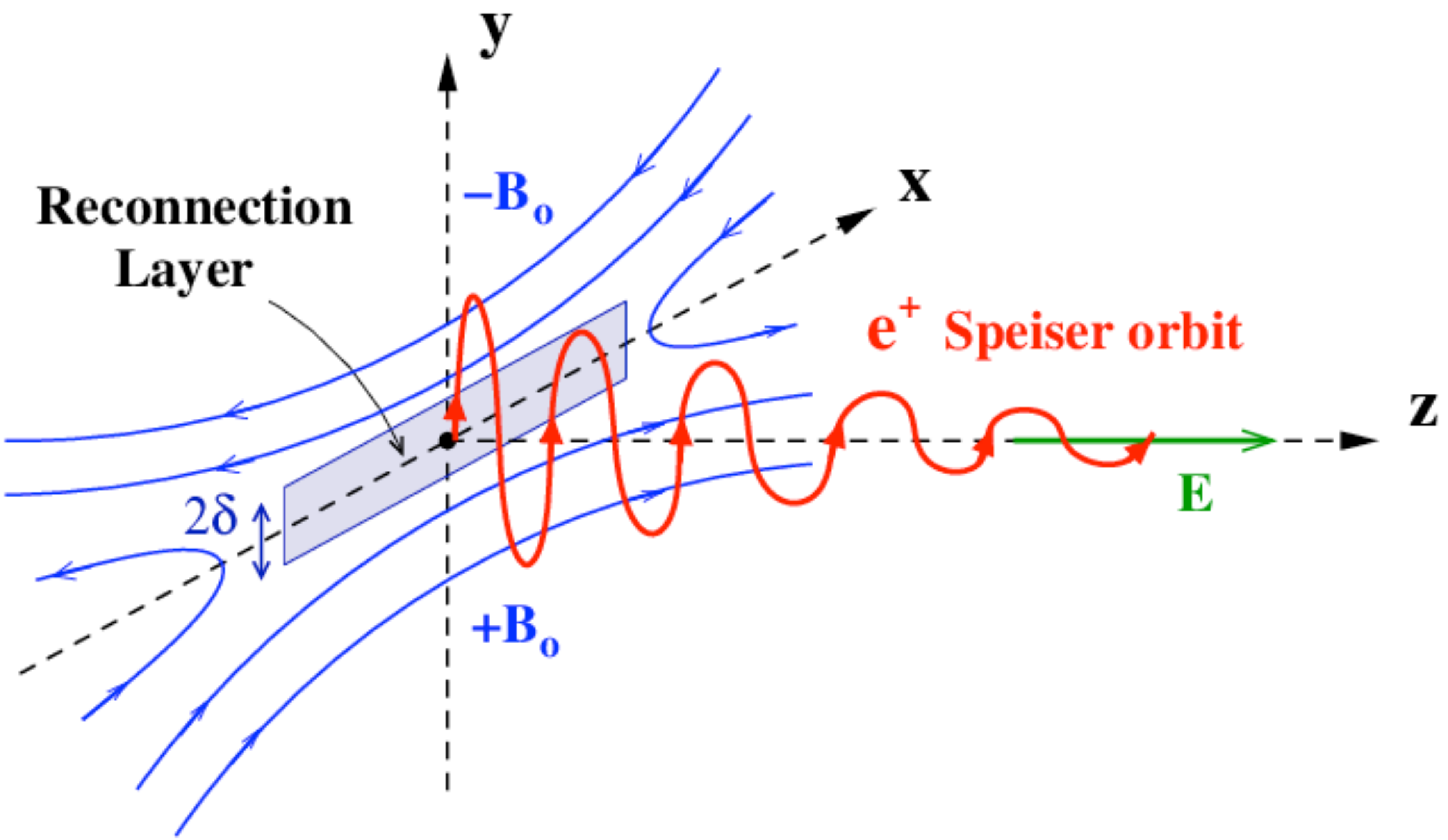
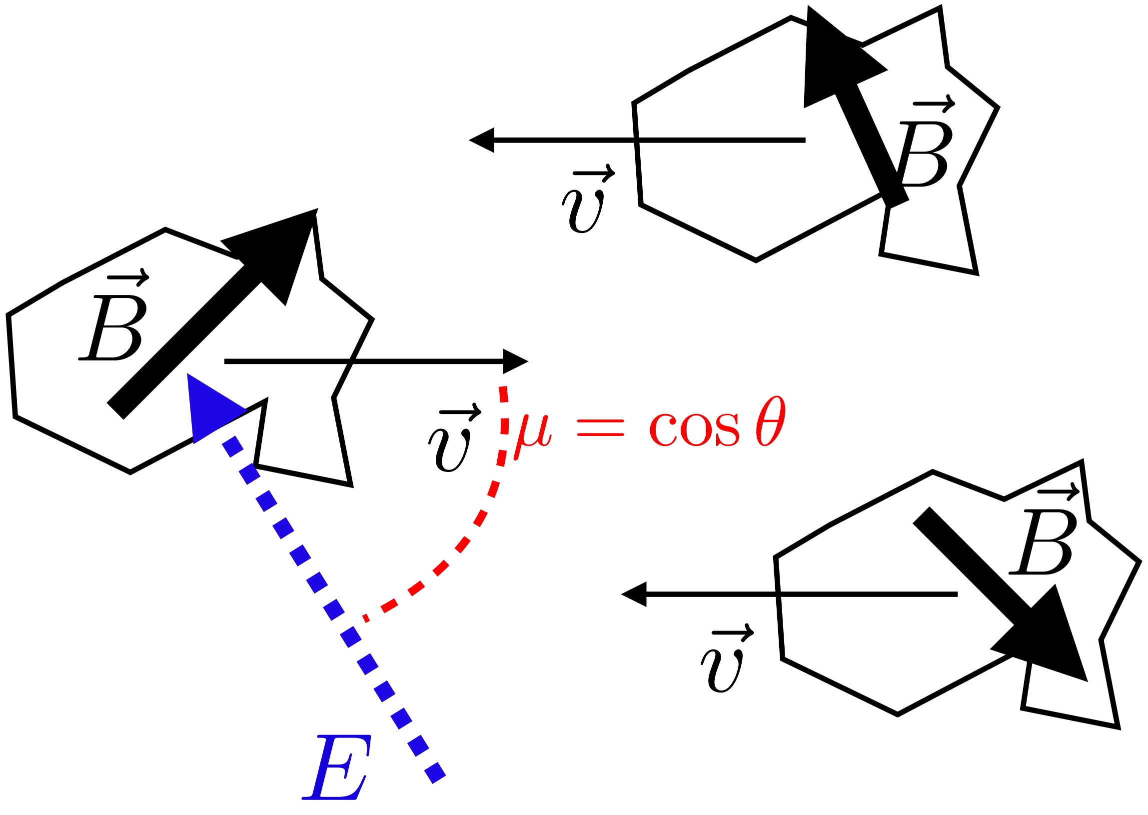


FIG. 1.— This diagram represents a relativistic Speiser orbit, i.e.,

Cerutti et al. 2013

Exercise: magnetized regions (clouds)



Each cloud carry a magnetic field that is an Alfvén wave (or collection of Alfvén waves)

A ‘test-‘particle enters the system with energy E

$\gamma, v =$ Lorentz factor and velocity of the cloud

For an observer sitting on this cloud (Lorentz transform):

$$E' = \gamma E + \beta\gamma p\mu \qquad \beta = \frac{v}{c}$$

Momentum in the x direction (let’s say for simplicity everything happens along x)

$$p'_x = \beta\gamma E + \gamma p\mu$$

Let’s say that once you are on the cloud, the observer doesn’t tell with microphysics, and the momentum is simply mirrored (inverted)

$$p'_x \to -p'_x$$

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$$p'_x \rightarrow -p'_x$$

In this setup, the cloud is a black box, we don't know what going on inside : just deflection by magnetic field and momentum reversed, thus the energy going out is:

$$E'' = \gamma E' + \beta\gamma p'_x$$

$$E'' = \gamma^2 E(1 + \beta^2 + 2\beta\mu \boxed{\frac{p}{E}})$$

Velocity of the particle

$$E'' = \gamma^2 E(1 + \beta^2 + 2\beta\mu v)$$

Question : where is the electric field here?

Answer: the cloud is moving, so there is an induced electric field. $\vec{V} \times \vec{B}$ is doing the work on the particle, from the microphysics point of view. So there is an electric field from a rest frame in motion (Lorentz transform take this into account)

How much do we gain in energy?

$$\Delta E = \frac{E'' - E}{E} = \gamma^2(1 + 2\beta v\mu + \beta^2) \approx 2\beta^2 + 2\beta v\mu$$

β is the velocity of the cloud, μ is between -1 and 1, so it means

ΔE Can be positive or negative !

This system is an acceleration AND a deceleration system

Fundamentally important! The second order nature of the process is due to the fact that some configuration lead to particle energization and some to particle de-energization

The probability of interaction is clearly depending of the velocity of the fluids (think about one street people going in opposite directions: more probable to interact with facing people)

$$P(\mu) = A\frac{\beta\mu + v}{1 + v\beta\mu}$$

$$v \to c$$

$$\approx A(1 + \beta\mu)$$

Relative velocity

We impose:

$$\int_{-1}^{+1} \mathrm{d}\mu P(\mu) = 1$$

$$A = 1/2$$

So that:

$$\langle\frac{\Delta E}{E}\rangle = \int_{-1}^{+1} \mathrm{d}\mu P(\mu)(2\beta^2 + 2\beta c\mu) = \frac{8}{3}\beta^2 = \frac{8}{3}\left(\frac{V}{c}\right)^2$$

$$\langle \frac{\Delta E}{E} \rangle = \int_{-1}^{+1} \mathrm{d}\mu P(\mu)(2\beta^2 + 2\beta c\mu) = \frac{8}{3}\beta^2 = \frac{8}{3}\left(\frac{V}{c}\right)^2$$

This is still positive ! Due to the fact that the head-on collisions are a bit more probable than head-tail collisions.

Keep in mind that the particle direction is changing, the vector is increasing/decreasing.increasing etc. A lot of step are needed to gain energy.

If you take as an estimate of V the Alfven velocity,

$$v_A = \frac{B_0}{\sqrt{4\pi\rho}} \sim 10\mathrm{km/s}$$

$$\langle \frac{\Delta E}{E} \rangle \sim 10^{-9}$$

gain very small!

Remark: 8/3 factor depends on simplifying ‘mirror’ assumption, in reality with have μ

μ' (Angle out) and the ‘correct’ result is 4/3

Spectrum - Fermi 2?

Given the average time between collision, an energy rate can be derived:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4}{3}\left(\frac{v^2}{cL}\right)E = \alpha E$$

mean free path between clouds

Solving diffusion-advection equation with escaping time, the spectrum obtained is:

$$N(E)\mathrm{d}E \propto E^{1+\frac{1}{\alpha\tau_{\mathrm{esc}}}}\mathrm{d}E$$

Characteristic time particles remain in acceleration region

Spectrum strongly depends on $\alpha\tau_{\mathrm{esc}}$

Very unconstrained!

CAN WE DO BETTER THAN CLOUDS?

With ideal MHD, we can ask additional questions: what happens to a plasma that is moving

Simplifying assumption: we'll assume that the magnetic field is « small »: we retain only the basic fluid terms in the RHS of the MHD equation $\vec{\nabla} \times \vec{B} \times \vec{B}$

Conservation of mass:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

Conservation of momentum:

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right] = -\vec{\nabla} P + \frac{1}{4\pi} \left[(\vec{\nabla} \times \vec{B}) \times \vec{B} \right]$$

Conservation of momentum + adiabaticity:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla}) \vec{v} = -\frac{\gamma}{\gamma - 1} \vec{\nabla} \left(\frac{P}{\rho} \right)$$

Let's make some assumptions :

1) stationarity
2) 1 Dimensional problem

$$\frac{\partial \rho v}{\partial x} = 0$$

$$\frac{\partial}{\partial x}\left(\rho v^2 + P\right) = 0$$

$$\frac{\partial}{\partial x}\left(\frac{1}{2}\rho v^3 + \frac{vP\gamma_{\rm ad}}{\gamma_{\rm ad} - 1}\right) = 0$$

Leads to 3 unknown / 3 equations

1) Constant solution -> not vey interesting

2) other non-trivial solutions?

YES, but it is not continuous

Something weird is going on in the plasma..

So somewhere in the plasma, something is happening. 1) and 2) the region before and after this weird thing that is happening

Rankine-Hugoniot relations

$$\rho_1 u_1 = \rho_2 u_2$$

$$\rho_1 u_1^2 + P_1 = \rho_2 u_2^2 + P_2$$

$$\frac{1}{2}\rho_1 u_1^3 + \frac{u_1\gamma_{\rm ad}P_1}{\gamma_{\rm ad} - 1} = \frac{1}{2}\rho_2 u_2^3 + \frac{u_2\gamma_{\rm ad}P_2}{\gamma_{\rm ad} - 1}$$

Non trivial solution, Provided that: $u_1 > c_s$

$$c_s = \sqrt{\gamma_{\rm ad}\frac{P_1}{\rho_1}}$$

Mach number

$$\mathcal{M} = \frac{u_1}{c_s} > 1$$

What is this solution?

We introduce compression factor: $r = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma_{\rm ad}+1)\mathcal{M}_1^2}{(\gamma_{\rm ad}-1)\mathcal{M}_1^2+2}$

The problem admits a solution in which the quantities are NOT continuous: SHARP TRANSITION

$r = \texttt{constant}$ $\quad r = \frac{(\gamma_{\rm ad}+1)}{(\gamma_{\rm ad}-1)}$

At this location : plasma slows down and compressed violently

In addition:

$$\mathcal{M}_1 \to \infty \quad \Longrightarrow \quad \frac{P_1}{P_2} \to \infty$$

$$\frac{P_1}{P_2} = \frac{2\gamma_{\rm ad}\mathcal{M}_1^2}{\gamma_{\rm ad}+1} - \frac{\gamma_{\rm ad}-1}{\gamma_{\rm ad}+1}$$

HEATING of the plasma! (Kinetic of the plasma transformed into presusre)

$$\mathcal{M}_1 \gg 1 \qquad \frac{T_2}{T_1} = \frac{2\gamma_{\rm ad}(\gamma_{\rm ad}-1)}{(\gamma_{\rm ad}+1)^2}\mathcal{M}_1^2$$

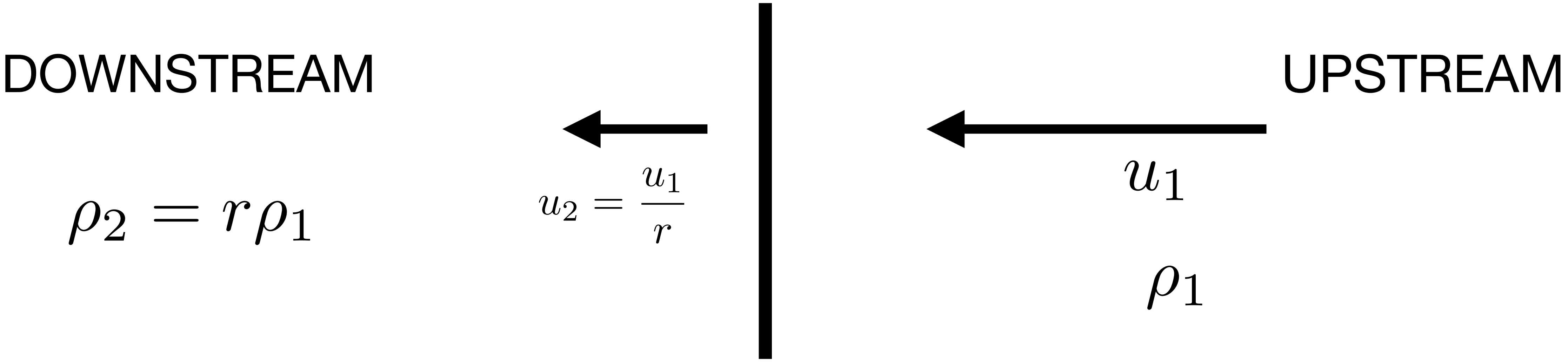
Using definition of Mach number:

$$kT_2 = \frac{2(\gamma_{\rm ad}-1)}{(\gamma_{\rm ad}+1)^2} m u_1^2$$

Simple fraction of kinetic energy into thermal energy!

Temperature downstream (kinetic energy per particle) = all the kinetic energy upstream with a factor in front of it

Now we can call this special location a SHOCK:



DOWNSTREAM

UPSTREAM

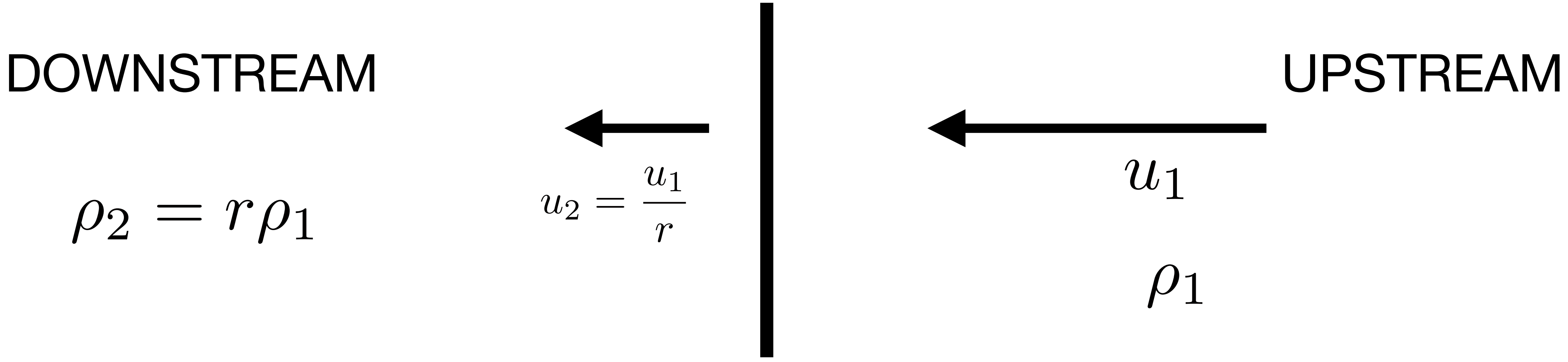
$\rho_2 = r\rho_1$

$u_2 = \frac{u_1}{r}$

u_1

ρ_1

With this configuration, we can eliminate the situations where the particle lose energy



DOWNSTREAM

UPSTREAM

$u_2 = \frac{u_1}{r}$

u_1

$\rho_2 = r\rho_1$

ρ_1

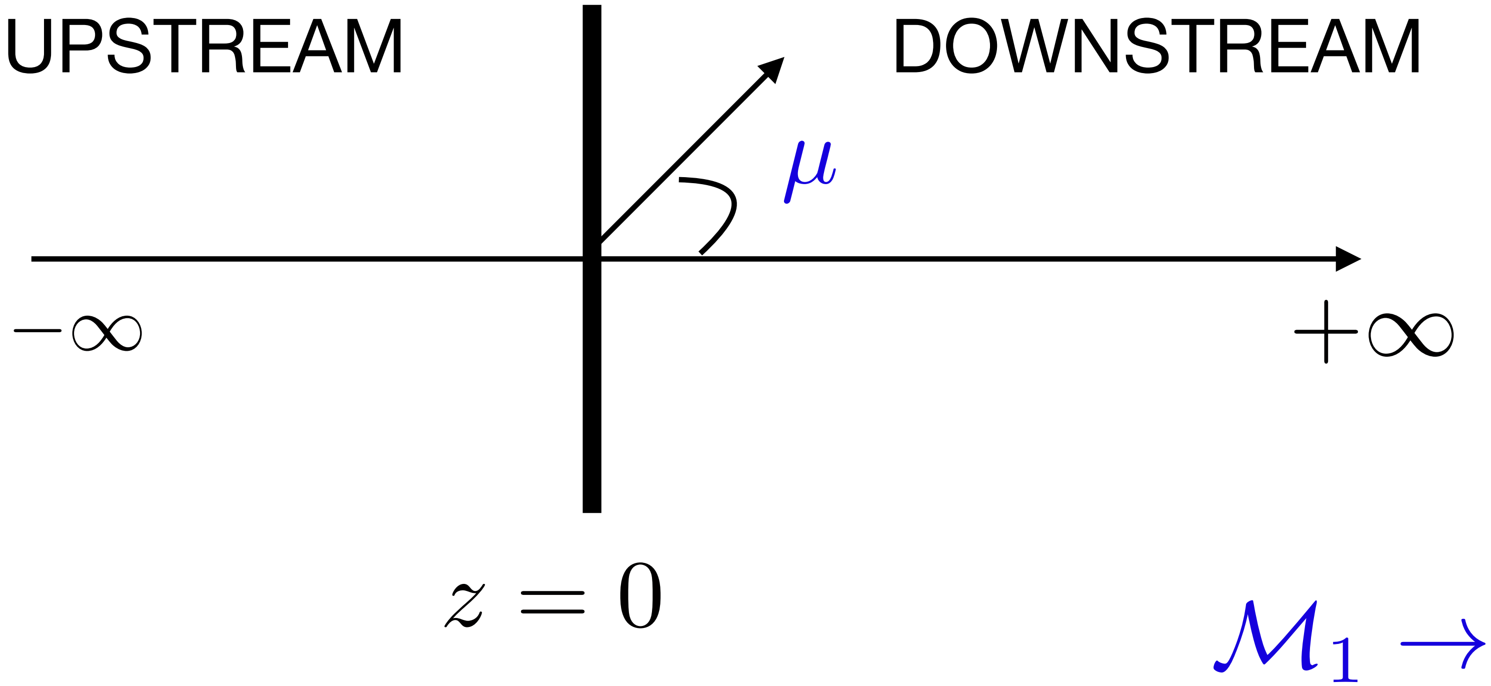
$$\gamma_{\rm ad} = \frac{5}{3} \qquad r = 4$$

Also, with the shock, the role played by the Alfven speed, is now played by u_1-u_2

$$v_A = 10 \mathrm{km/s} \rightarrow \Delta u \sim 10000 \mathrm{km/s}$$

More convenient for particle acceleration

We will easily be dealing with Mach numbers: $\mathcal{M} \gtrsim 100$



$$\rho_2 = r\rho_1$$

$$u_2 = \frac{u_1}{r}$$

$$\mathcal{M}_1 \to \infty \quad \Longrightarrow \quad r = 4$$

In the rest frame of the shock

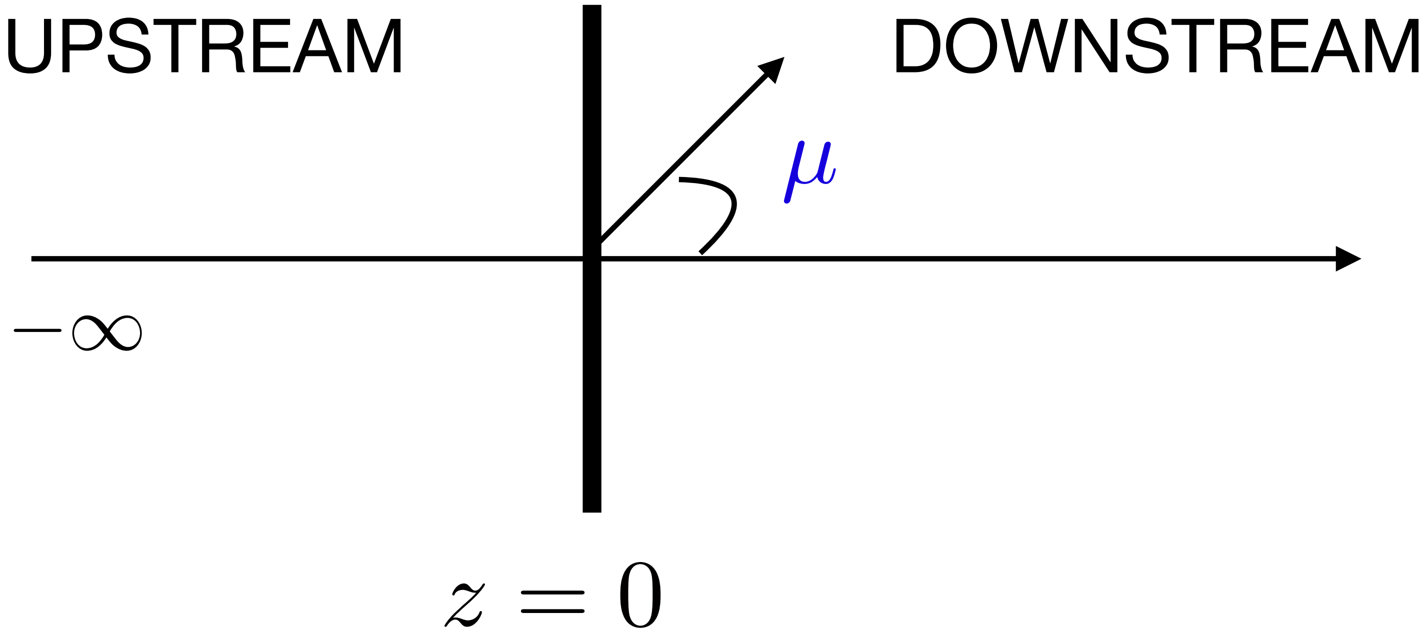
First-order Fermi mechanism (not proposed by Fermi,
~1970 and Fermi dead 1954)

We start with particle upstream energy E

$$\beta = \frac{u_1 - u_2}{c}$$

Downstream $E_{\rm d} = \gamma E(1 + \beta\mu)$

In order to arrive downstream: $0 \leq \mu \leq 1$



Particle diffuse due to inhomogeneities/fluctuations/Alfvén waves in the magnetic fields in the plasma: there is a finite probability that the particle goes back to the up with energy:

Downstream $E_{\rm d} = \gamma E(1+\beta\mu) \quad 0 \leq \mu \leq 1$

$$E_{\rm up} = \gamma^2 E(1+\beta\mu)(1-\beta\mu') \qquad -1 \leq \mu' \leq 0$$

Final energy is always increasing! You gain every time

Same that for clouds, let's calculate the probability of returning to shock (that up->down) $P(\mu)$

Total number of particles : $\int \mathrm{d}\boxed{\Omega \frac{N}{4\pi} v_\mu} = Nv/4$

N = number of particles sitting at the shock, assumed to be isotropized through diffusion

Current through the surface:

$$P(\mu)\mathrm{d}\mu = \frac{ANv_\mu}{\frac{Nv}{4}}\mathrm{d}\mu$$

$$\int_0^1 P(\mu)\mathrm{d}\mu = 1 \qquad \Longrightarrow \qquad P(\mu) = 2\mu$$

Repeating the same calculation in the other direction, same result with a « minus » sign.

Gain in energy average over mu:

$$\langle \frac{E_{\rm up} - E}{E} \rangle_\mu = \int_0^1 \mathrm{d}\mu \int_{-1}^0 \mathrm{d}\mu' P(\mu) P(\mu') \frac{\Delta E}{E}$$

$$= \int_0^1 \mathrm{d}\mu \int_{-1}^0 \mathrm{d}\mu' 2\mu(-2\mu') \left[\gamma^2(1+\beta\mu)(1-\beta\mu') - 1\right] = \frac{4}{3}\frac{u_1 - u_2}{c} = \frac{4}{3}\beta$$

$$\langle \frac{E_{\rm up} - E}{E} \rangle_\mu = \int_0^1 {\rm d}\mu \int_{-1}^0 {\rm d}\mu' P(\mu) P(\mu') \frac{\Delta E}{E}$$

$$= \int_0^1 {\rm d}\mu \int_{-1}^0 {\rm d}\mu' 2\mu(-2\mu') \left[\gamma^2 (1 + \beta\mu)(1 - \beta\mu') - 1\right] \; = \frac{4}{3} \frac{u_1 - u_2}{c} = \frac{4}{3}\beta$$

The gain is higher, but still small, it takes several cycles to gain substantial energy!

Remark: 1) we have only integrated over situations where particles GAIN energy (no energy decrease as with the clouds)
2) Diffusion : where is it here?

If coefficient diffusion is small particles diffusing upstream are forced to go back to the shock; downstream, different situation, the plasma motion is taking you AWAY from the shock!

THE ESCAPE PROBLEM

How do particles escape the accelerator? Link between cosmic rays and accelerated particles

What if the particle does one cycles and then escapes? It's useless. Let's calculate the probability that one particle stays in the accelerator.

$$P_{\mathrm{returnfromupstream}} = 1$$

$$P_{\mathrm{downstream}} = ?$$

Distribution of particles immediately downstream: f_d

We have to be slightly more subtle than before; the velocity of the fluid has to be here! $f_d(v\mu + u_2)$

Flux of particles: $\int_{\mu values\ so\ that\ v\mu+u_2>0} f_d(v\mu + u_2)\mathrm{d}\mu$

For simplicity, let’s assume that particles are relativistic $v = c$

Flux of particles going ‘in’ the positive direction

$$\phi_{\text{in}} = \int_{-u_2}^{1} f_d(\mu + u_2)\text{d}\mu$$

Flux down -> up

$$\phi_{\text{out}} = \int_{-1}^{-u_2} f_d(\mu + u_2)\text{d}\mu$$

Probability return:

$$P_{\text{ret}} = \frac{\phi_{\text{out}}}{\phi_{\text{in}}} = \frac{(1-u_2)^2}{(1+u_2)^2} \boxed{\rightarrow \approx 1 - 4\frac{u_2}{c}}$$

$$u_2 \ll 1$$

With typical numbers for SNR shock:

$$P_{\text{ret}} \approx 0.96$$

Rewriting with a ‘c’ to make it more clear

We now have two things:

1. Probability of return
2. Mean energy gain per cycle

THE ACCELERATED SPECTRUM

We start with one particle of energy E0:

First cycle:
$$\frac{\Delta E}{E} = \frac{4}{3}\frac{v}{c} \Rightarrow \frac{E_1 - E_0}{E_0} = \frac{4}{3}\frac{v}{c}$$

$$E_1 = E_0(1 + \frac{4}{3}\frac{v}{c})$$

Second cycle:
$$E_2 = E_0(1 + \frac{4}{3}\frac{v}{c})^2$$

K cycle:
$$E_k = E_0(1 + \frac{4}{3}\frac{v}{c})^k$$

$$\ln\left(\frac{E_k}{E_0}\right) = k\ln(1 + \frac{4}{3}\frac{v}{c})$$

If several particles are injected

$$N_0$$

$$N_1 = N_0 P_{\mathrm{ret}}$$

$$N_k = N_0 P_{\mathrm{ret}}^k$$

$$\ln\left(\frac{N_k}{N_0}\right) = k\ln(P_{\mathrm{ret}})$$

$$= k\ln(1 - 4\frac{u_2}{c})$$

$$\frac{\ln(\frac{E_k}{E_0})}{\ln(1+\frac{4}{3}\frac{v}{c})} = \frac{\ln(\frac{N_k}{N_0})}{\ln(1-4\frac{u_2}{c})}$$

Non-relativistic shock waves:

$$\frac{u_2}{c} \ll 1 \qquad \frac{N_k}{N_0} = \left(\frac{E_k}{E_0}\right)^{-\gamma}$$

$$\gamma = \frac{3}{r-1}$$

Physical interpretation of Nk: number of particles at a given energy

Normal quantities:

$$\boxed{n(E)}E \sim E^{-\frac{3}{r-1}}$$

Differential spectrum

$$n(E) \sim E^{-\frac{r+2}{r-1}} \qquad r = 4 \qquad n(E) \to E^{-2}$$

Remark: 1) here we assumed that the particles are relativistic, which is not correct because we are supposed to start with non-relativistic particles
2) still no diffusion coefficient in here (the microphysics is there)!

TRANSPORT OF PARTICLES

If you have a charged particles in Alfven waves, it diffuses in momentum (pitch angle), as a consequence, this is diffusion in space.

For ONE particle at a given k:

$$D_{\mu\mu} = \frac{1}{2}\langle\frac{\Delta\mu\Delta\mu}{\Delta t}\rangle = \left(\frac{q\delta B}{mc\gamma}\right)^2 (1-\mu^2)\frac{1}{2v\mu}\delta\left(k \pm \frac{\Omega}{v\mu}\right)$$

The diffusion process is an intrinsically RESONANT process, you must have waves with the right wavenumber to diffuse!

Usually, in the ISM, power spectrum: $\mathcal{F}(k) = \frac{\delta B^2}{B_0^2}(k)$

You can integrate the diffusion coefficient on the power spectrum (trivial integral), and get that the \delta B is at k

Diffusion coefficient in space: $D_{zz} = \frac{1}{3} r_L v \frac{1}{\mathcal{F}(k)|_{k=k_{\rm res}}}$

Diffusion coefficient in space is inversely proportional to the power spectrum. Less power -> particles go straighter

In space: in order for the D to be small, you have to pump energy into the turbulence!

Transport equation:

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[D \frac{\partial f}{\partial z}\right] + \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}z} p \frac{\partial f}{\partial p} + Q$$

1Dimenstionnal + the second order acceleration term has been thrown out

Transport Equation

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1Dimenstionnal + the second order acceleration term
has been thrown out (App)

Number of particles in phase space per volume : $f(z, \vec{p}, t)$

Number of particles (per volume): $N(p)\mathrm{d}p = 4\pi p^2 f(p)\mathrm{d}p$

Stationarity: $\frac{\partial f}{\partial t} = 0$

Everywhere except at the shock : $\frac{\mathrm{d}u}{\mathrm{d}z} = 0$

$$\frac{\mathrm{d}u}{\mathrm{d}z} = (u_2 - u_1)\delta(z)$$

Linear equation : hence the Q, injection term, in order to have
something at the end, you need to inject it

Transport Equation

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] + \frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}z}p\frac{\partial f}{\partial p} + Q$$

Definition: The accelerated particles are the ones which don't feel the discontinuity! (No compression): their gyroradius is gigantic compared to the size of the shock (radius of thermal particles). Thus f is continuous across the shock.

Injection at the shock surface: $Q \propto \delta(z)\delta(p - p_{\mathrm{inj}})$

Upstream:

$$\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z} - uf\right] = 0$$

$$D\frac{\partial f}{\partial z} - uf = \mathrm{cte} \quad \Longrightarrow$$

At infinity, both terms = 0

$$D\frac{\partial f}{\partial z} = uf$$

At the shock:

$$D\frac{\partial f}{\partial z}|_{z=0-} = u_1 f_0$$

Downstream: in principle can be infinite (if not constant,
it would then be blowing up..!)

$$D\frac{\partial f}{\partial z}|_{z=0+} = 0 \quad \Longrightarrow$$

F Homogeneous

Can be shown in more
formal way (takes longer)

Integrating across the shock (between 0- and 0+):

Transport equation:

$$\frac{\partial f}{\partial t}+u\frac{\partial f}{\partial z}=\frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right]+\frac{1}{3}\frac{\mathrm{d}u}{\mathrm{d}z}p\frac{\partial f}{\partial p}+Q$$

Integrating across the shock (between 0- and 0+):

(f is continuous)

$$0=D\frac{\partial f}{\partial z}|_{z=0+}-D\frac{\partial f}{\partial z}|_{z=0-}+\frac{1}{3}(u_2-u_1)p\frac{\mathrm{d}f_0}{\mathrm{d}p}+Q_0\delta(p-p_{\mathrm{inj}})$$

$$u_1f_0=-\frac{1}{3}(u_1-u_2)p\frac{\mathrm{d}f_0}{\mathrm{d}p}+Q_0\delta(p-p_{\mathrm{inj}})$$

Without the injection term (p>pinj), this equation is:

$$f_0(p)\propto p^{-\frac{3u_1}{u_1-u_2}}=p^{-\frac{3r}{r-1}}$$

This is different from the one we found before, because we have f0 in phase space, before, it was in energy space!

Spectrum in momentum/energy

$$N(p)\mathrm{d}p = 4\pi p^2 f(p)\mathrm{d}p$$

$$N(p) \propto p^2 p^{-\frac{3r}{r-1}} = p^{-\frac{r+2}{r-1}}$$

The spectrum accelerated at the shock is not E-2, it is p-4

	Non-relativistic	Relativistic
For a strong shock	$\propto E^{-3/2}$	$\propto E^{-2}$
	$\propto p^{-4}$	$\propto p^{-4}$

This is important, if for instance you are dealing with a relativistic jet, with Lorentz factor Gamma, in which particles are non-relativistic with respect to the jet, you have to remember this!

Also for low energy cosmic rays, important!

Universal spectrum in the limit of large Mach number >>1

Stationarity? Does it make sense?

The maximum energy is changing, so it does not make sense..

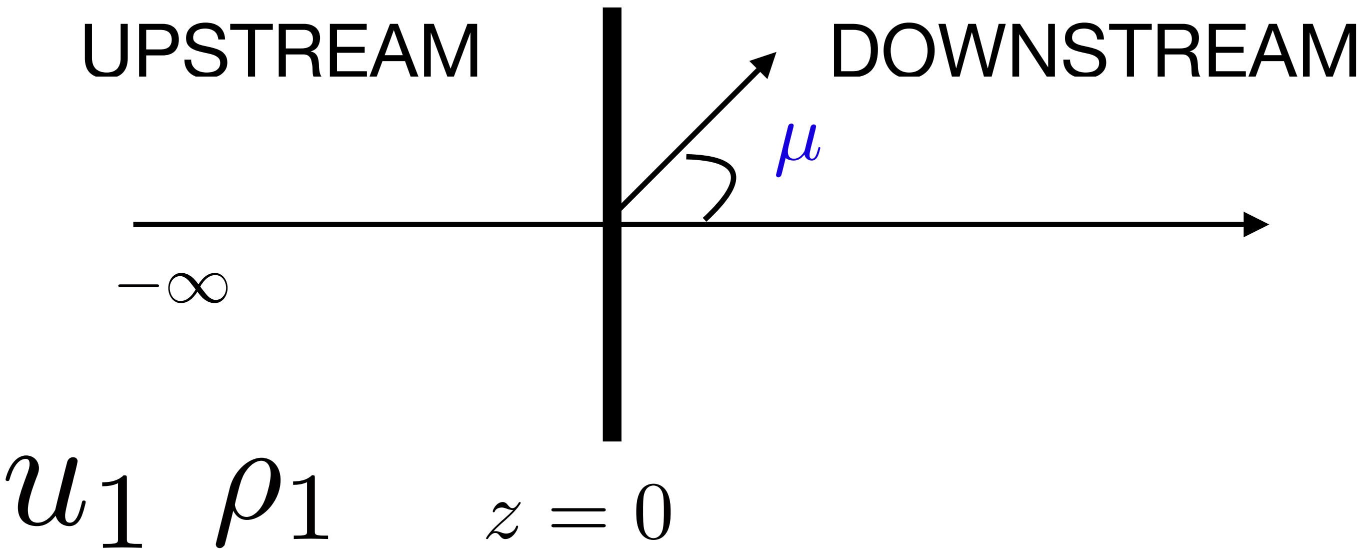
The stationarity carries the assumption that pmax is infinite!

DIFFUSION COEFFICIENT

Still not in the equation! We got to the power law with no ‘physical scale’ in the problem

The timescales of the problem are important

Ways to address the problem are long (tens of pages), easier way to get a sense:



UPSTREAM: you see the turbulence of the ISM coming in

Acceleration time: depends on upstream, how « easy » it is to come back

Diffusion length (how far particles leave) $l_d \approx \sqrt{4D\tau}$

How much the plasma is moving $u_1\tau$

Diffusion length (how far particles leave) $l_d \approx \sqrt{4D\tau}$ (

How much the plasma is moving $u_1\tau$

Competition between the two processes, to estimate how far particle move, we equate the two:

$$\tau \approx \frac{D}{u_1^2}$$

To get quick acceleration, D must be as small as possible

$$D_{zz} = \frac{1}{3} r_L v \frac{1}{\mathcal{F}(k)|_{k=k_{\rm res}}}$$

And push power into the turbulence!

$$r_{\rm L} = \frac{pc}{eB} \propto E$$

$$\mathcal{F} \propto k^{-\alpha} \propto r^{\alpha} \propto E^{\alpha}$$

The process is harder and harder at the highest energies!

F = energy density per unit logarithmic bandwidth of waves with wavenumber k

WEAK POINTS IN THE THEORY

Timescale efficient acceleration at SNR? (Few centuries)

$$D_{\mathrm{ISM}} \approx 3 \times 10^{28} E_{\mathrm{GeV}}^{1/3} \mathrm{cm}^2/\mathrm{s}$$

$$u_1 \sim 10000 \mathrm{km/s}$$

$$\Longrightarrow \; E_{\mathrm{max}} \sim 10 \mathrm{GeV}$$

Far from the energies we want (in cosmic rays), in gamma-rays!

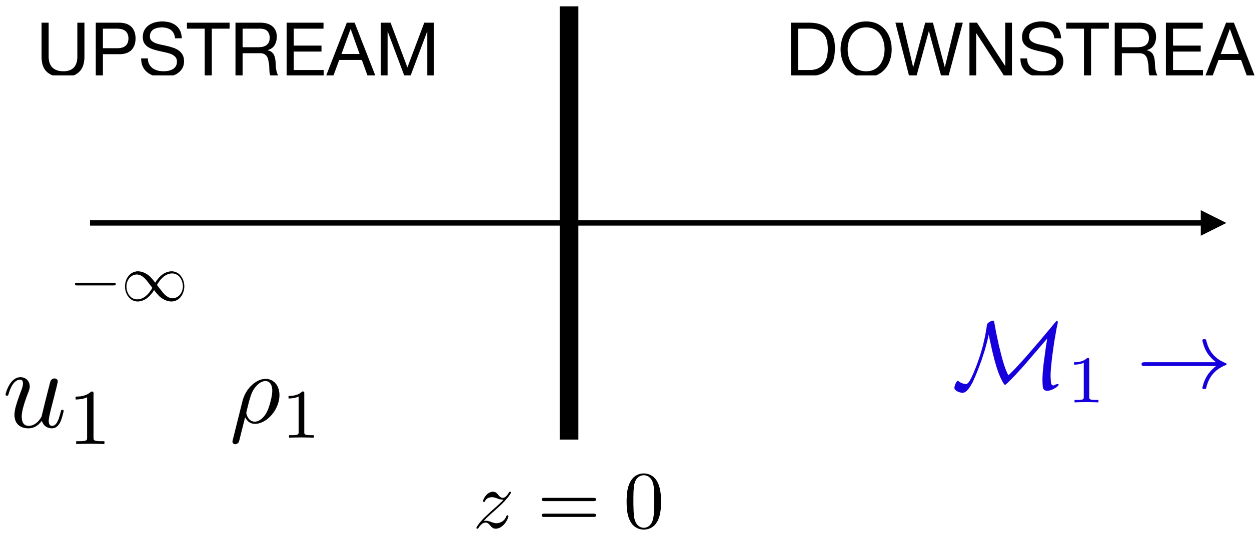
We need: $\mathcal{F} \gg \mathcal{F}_{\mathrm{Galaxy}}$

Need for more turbulence, the one produced by accelerated particles themselves -> non-linear problem!

Remark: $\epsilon_{\mathrm{CR}} = \int_{E_0}^{E_{\max}} E E^{-2} \mathrm{d}E = \ln\left(\frac{E_{\max}}{E_0}\right) \quad \sim \rho u^2$

If energy in CRs becomes comparable to ram pressure: problem! -> non-linear problem: the pressure of CRs cannot be neglected anymore in conservation equations

SUMMARIZING MAIN POINTS



UPSTREAM

DOWNSTREAM

$-\infty$

$u_1 \quad \rho_1$

$z = 0$

$$\rho_2 = r\rho_1$$

$$u_2 = \frac{u_1}{r}$$

$$\mathcal{M}_1 \to \infty \quad \Longrightarrow \quad r = 4$$

$$f(p) \propto p^{-\frac{3r}{r-1}}$$

All results are independent of the Diffusion coefficeient

Problems: stationarity, total energy in CRs can become large

$$\epsilon_{\rm CR} \int 4\pi T(p) f(p) p^2 {\rm d}p \propto \ln \frac{p_{\rm max}}{p_{\rm min}} \sim \rho u^2$$

Emax too small, assuming Galactic diffusion coefficient!

A MORE GENERAL THEORY (non-linear)

From conservation of momentum, we had: $\frac{\partial}{\partial z}\left(\rho v^2 + P\right) = 0$

Mass: $\frac{\partial \rho v}{\partial z} = 0$

We need to had the pressure of accelerated particles: $\frac{\partial}{\partial z}\left(\rho v^2 + P + P_{\rm CR}\right) = 0$

Is the CR pressure affecting the jump conditions?

In our approach, particles and thermal gas are distinct!
F is continuous across the shock, and so is Pcr

Let's apply the equations to what happens before the shock (upstream)

From infinity (0) to right before the shock (1):

$$\rho_0 u_0^2 + P_{\rm gas,0} + P_{\rm CR,0} = \rho_1 u_1^2 + P_{\rm gas,1} + P_{\rm CR,1}$$

From infinity (0) to right
before the shock (1):

$$\rho_0 u_0^2 + P_{\rm gas,0} + \cancel{P_{\rm CR,0}} = \rho_1 u_1^2 + P_{\rm gas,1} + P_{\rm CR,1}$$

No CRs upstream infinity

$$1 + \underset{\frac{1}{\gamma_{\rm ad}\mathcal{M}_0^2}}{\boxed{\frac{P_{\rm gas,0}}{\rho_0 u_0^2}}} = \frac{\rho_1 u_1^2}{\rho_0 u_0^2} + \frac{P_{\rm gas,1}}{\rho_0 u_0^2} + \frac{P_{\rm CR,1}}{\rho_0 u_0^2}$$

$$\cancel{\frac{1}{\gamma_{\rm ad}\mathcal{M}_0^2}} + 1 = \frac{u_1}{u_0} + \cancel{\frac{P_{\rm gas,1}}{\rho_0 u_0^2}} \boxed{+ \xi_{\rm CR}}$$

Small enough to neglect

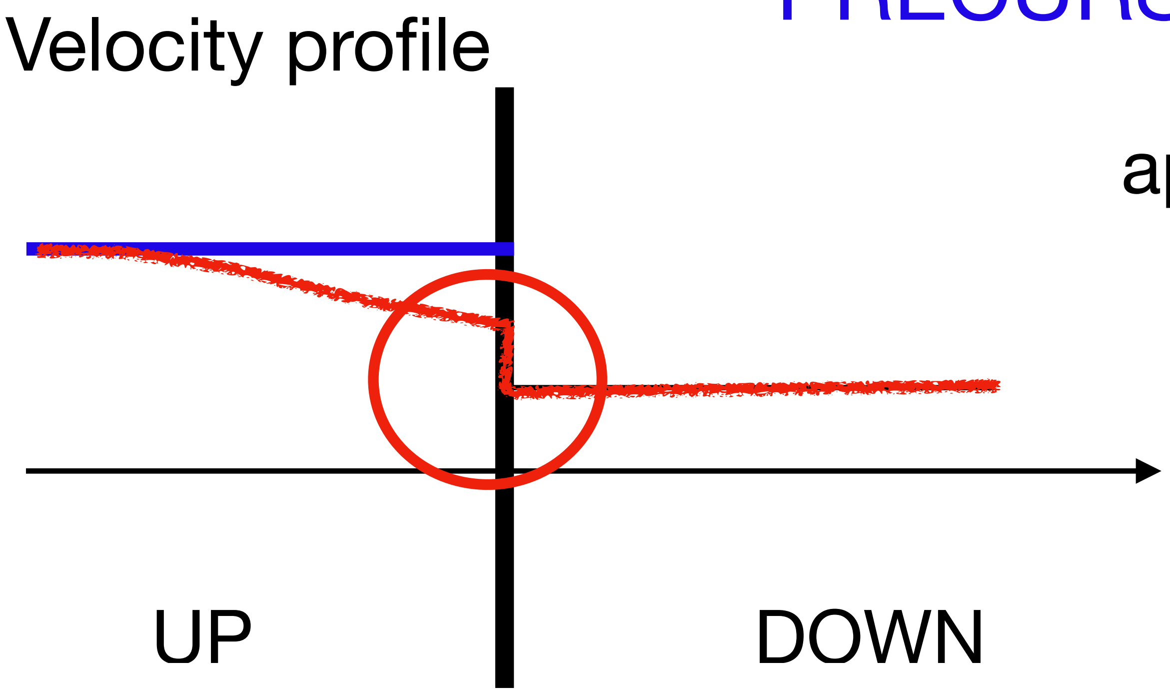
Efficiency of acceleration

$$\frac{u_1}{u_0} = 1 - \xi_{\rm CR}$$

The plasma is slowing down as it is
approaching the shock

D growing function of momentum, particles at high p get further from the shock, we see more particles as we get closer to the shock

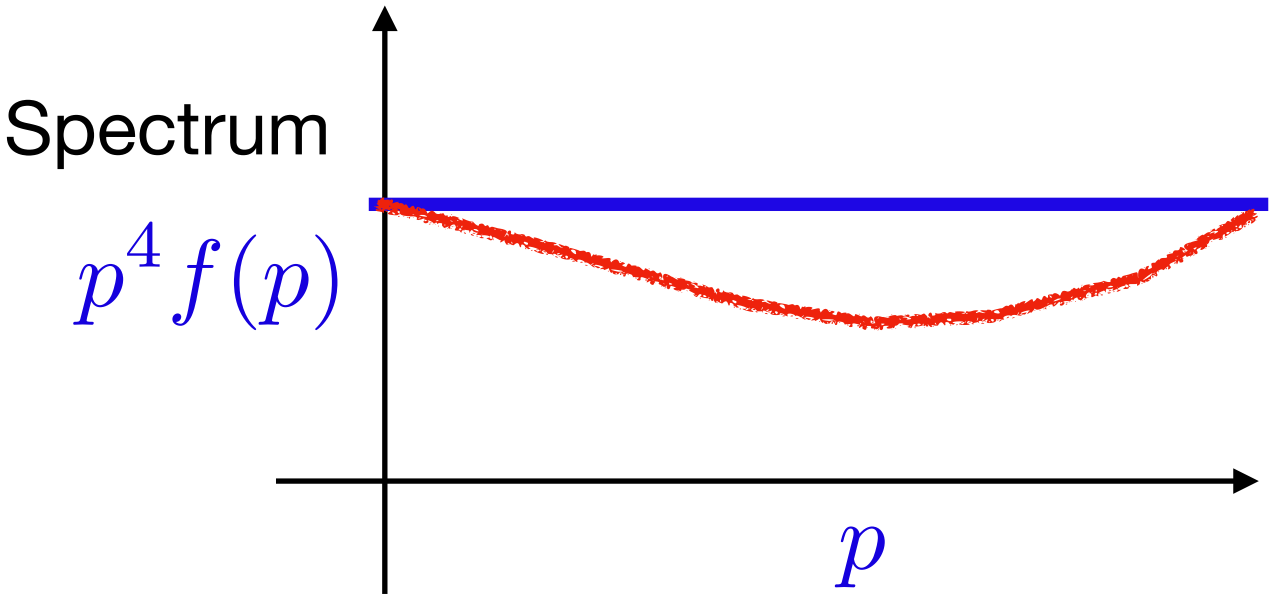
PRECURSOR



In this (simple) non-linear approach, the compression factor becomes function of p

Reducing the compression factor at small momenta

$p^{-\frac{3r}{r-1}}$



Increasing compression factor for high momenta

Concave spectrum, not power-law anymore!

Shock= heating machine, takes bulk plasma motion, and transform into internal energy, T2 behind the shock function of efficiency

PRECURSOR

Efficient accelerator -> less heating (will affect thermal X-rays from behind the shock)

$$T_2^{\mathrm{CR}} \ll T_2$$

It is possible to build a complete non-linear description, taking into account the back reaction of accelerated particles on the shock, etc. The system is then self-regulating: if you put too much accelerated CRs, then the subshock becomes smaller and ‘swhitches’ off the acceleration

OTHER ISSUES: EMAX TOO LOW

Let’s go back to perturbations: particles can scatter resonantly with perturbations, provided that there is power on the right wavenumber

Thought experiment: Shooting CRs in straight lines on Alfven waves: the beam will broaden due to Alfven waves; the initial momentum will turn out and seem to be disappearing; but we have to conserve momentum. IDEA: the particle diffusing in that background will ‘amplify’ the waves

$$f(p,\mu) = f_0(p)\left[1 + \frac{v_{\rm D}\mu}{c}\right]$$

Current $\vec{J} = -D\vec{\nabla} f$

$$f v_{\rm D} = -D\nabla f$$

v_D/c is also the level of anisotropy (in a diffusive regime): particle isotropized up to the level of v_D/c

Initial state

$$\int \mathrm{d}\mu \boxed{4\pi p^2 f_0(p) p} \left(1 + \frac{v_\mathrm{D}\mu}{c}\right) \boxed{p\mu} \qquad = \frac{2}{3} n(>p) m v \gamma \frac{v_\mathrm{D}}{c}$$

Total number of particles
in that momentum bin

Momentum in a given
direction

$$n(>p) = 4\pi p^3 f_0(p)$$

Particles isotropized in the rest frame of the waves

Final state $= \frac{2}{3} n(>p) m v \gamma \frac{v_\mathrm{A}}{c}$

Change in momentum: $\Delta p \approx n(>p) m v \gamma \frac{v_\mathrm{D} - v_\mathrm{A}}{c}$

This momentum is gained by the waves!

Typical timescale for this to happen?

$$D = \frac{1}{3} v r_\mathrm{L} \frac{1}{\mathcal{F}(k)} \qquad D \approx \frac{1}{3} c \lambda(p) \Rightarrow \lambda(p) = \frac{r_\mathrm{L}}{\mathcal{F}(k_\mathrm{res})}$$

$$\mathcal{F}(k) \sim \frac{\delta B^2(k)}{B_0^2} \qquad \tau = \frac{\lambda(p)}{c} = \frac{\gamma}{\Omega_\mathrm{cyc}} \frac{1}{\mathcal{F}(k)}$$

$$\Omega = \frac{\Omega_\mathrm{cyc}}{\gamma} = \frac{eB_0}{mc\gamma}$$

(With k=kres in all this)

We know how much momentum is lost, and the timescale for loss, we have:

Rate of momentum lost by the particles (to the waves!)

$$\frac{\Delta p}{\Delta t} = \frac{n(> p)m\gamma(v_{\rm D} - v_A)}{\gamma}\Omega_{\rm cyc}\mathcal{F}(k)$$

Rate of momentum gained by the waves!

$$\left(\frac{\Delta p}{\Delta t}\right)_{\rm waves} = \overbrace{\underbrace{\frac{\delta B^2}{8\pi}}_{\text{Energy density}}\frac{1}{v_A}}^{\text{Momentum}}\underbrace{\gamma_w}_{\text{Growth rate [1/Time]}}$$

Quick and dirty (dimension analysis)

At some sort of equilibrium:
momentum lost by particles =
momentum gained by waves

$$\left(\frac{\Delta p}{\Delta t}\right)_{\rm waves} = \left(\frac{\Delta p}{\Delta t}\right)_{\rm CRs}$$

$$\gamma_w = \frac{n(>p)}{n_{\rm gas}} \frac{v_{\rm D} - v_A}{v_A} \Omega_{\rm cyc}$$

This is the rate we get when the waves get momentum from accelerated particles

In the Galaxy:

$$n_{\rm gas} \sim 1{\rm cm}^{-3}$$

$$n(> 1{\rm GeV}) \sim 10^{-9}{\rm cm}^{-3}$$

$$v_{\rm D} \sim \text{few times} \times v_A$$

$$\Omega_{\rm cyc} \sim k{\rm Hz}$$

$$\Rightarrow \tau_w = \gamma_w^{-1} \sim 1000{\rm years}$$

Wave growth is very fast compared to timescales of residence/escape (10-100 Myr) in the Galaxy

Close to shock waves, v_D is way greater, since the bulk motion of particles is coming with the shock (in rest frame ISM)

$$\Rightarrow \tau_w = \gamma_w^{-1} \sim 0.1 - 1{\rm sec}$$

In less that second, waves are amplified! VERY IMPORTANT phenomena

IMPLICATION OF WAVE GENERATION/AMPLIFICATION

Increasing F, thus decreasing the diffusion coefficient, thus shorter acceleration time thus higher Emax!

Until when?

The resonant reasoning assumes B0 is ordered; by amplifying it, at some point fluctuations are greater than the initial magnetic field, things break down

$$\delta B \sim B_0$$

At SNe, with this effect: $E_{\max} \sim 10^4 \mathrm{GeV}$

That's still not enough to account for observations.. there are more things

CR NON-RESONANT STREAMING

CRs accelerated at the shock front.
Looking from far away, bunch of charged particles trying to come towards you -> current -> plasma in which a current is streaming.

Plasma: wants to stay neutral, high conductivity, doesn't like currents ->return current to stay neutral

UPSTREAM: In the plasma (gas) = protons + electrons

Charge: $n_p e - n_e e = 0$

Including accelerated particles: $(n_p + n_{\mathrm{CR}})e - n_e e = 0$

Large

Small

Current CRs: $e n_{\mathrm{CR}} v_{\mathrm{sh}} = e n_e v_{\mathrm{d}}$

Small

Electrons 2000 less massive than protons, they are the one moving, with velocity vd (relative drift with respect to ions)

Current CRs: $$e\,\underset{\text{Small}}{n_{\mathrm{CR}}}\,\overset{\text{Large}}{v_{\mathrm{sh}}} = e\,\overset{\text{Large}}{n_{\mathrm{e}}}\,\overset{\text{Small}}{v_{\mathrm{d}}}$$

A small drift velocity can thus help compensate the CR current

The « return » current is the opposite to the CR current! $-J_{\mathrm{CR}}$

How do the equations change?

Conservation of mass: $$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$

Conservation of momentum:

$$\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = -\vec{\nabla}P + \frac{1}{4\pi}\left[(\vec{\nabla}\times\vec{B})\times\vec{B}\right] - \frac{1}{c}\vec{J_{\mathrm{CR}}}\times\vec{B}$$

$$\frac{\partial}{\partial t}\left(P\rho^{-\gamma}\right) = 0$$

$$\frac{\partial\vec{B}}{\partial t} = \vec{\nabla}\times(\vec{v}\times\vec{B}) \quad \vec{\nabla}\cdot\vec{B} = 0$$

All the same except one!

Let’s consider a simple case: $\vec{B_0} = (0, 0, B_0) \quad \vec{k} = (0, 0, k)$

And perturb the equations:

$$\rho \rightarrow \rho_0 + \delta\rho$$

$$\vec{B} \rightarrow = \vec{B_0} + \delta\vec{B}$$

Boring arithmetics, but doable:

$$\delta v_x = \frac{k^2 V_A^2}{\omega^2}\delta v_x - i\frac{J_{\rm CR} k B_0}{\omega^2 \rho c}\delta v_y$$

$$\delta v_y = \frac{k^2 V_A^2}{\omega^2}\delta v_y + i\frac{J_{\rm CR} k B_0}{\omega^2 \rho c}\delta v_x$$

If you ignore the term with Jcr you get exactly the Alfen waves!
Interestingly : ux and uy are now coupled!

Dispersion relation:

$$\omega^4 + (k^2 v_A^2)^2 - 2\omega^2 k^2 v_A^2 = \alpha^2 k^2$$

$$\alpha = \frac{J_{\rm CR} B_0}{\rho c}$$

Alpha =0 -> standard relation!

Dispersion relation:

$$\omega^4 + (k^2 v_A^2)^2 - 2\omega^2 k^2 v_A^2 = \alpha^2 k^2 \qquad \alpha = \frac{J_{\rm CR} B_0}{\rho c}$$

Now, complex solutions are possible for \omega

$$\delta B \propto \exp(-i\omega t + \vec{k} \cdot \vec{x}) \qquad \omega^2 < 0 \Rightarrow \text{instability}$$

$$\omega^2 < 0 \Leftrightarrow k^2 v_A < \alpha k \qquad \Leftrightarrow k < \frac{4\pi}{c} \frac{J_{\rm CR}}{B_0} = k_{\rm max}$$

$$\omega = k_{\rm max} v_A$$

(Result of calculation)

Interestingly: Purely growing modes! No real part! No moving, just wild increase!

$$k_{\rm max} \gg r_{\rm L}$$

Modes grow (a lot!), but do not resonate with CRs! The growth of modes implies that there is a force on the plasma $\vec{J} \times \vec{B}$

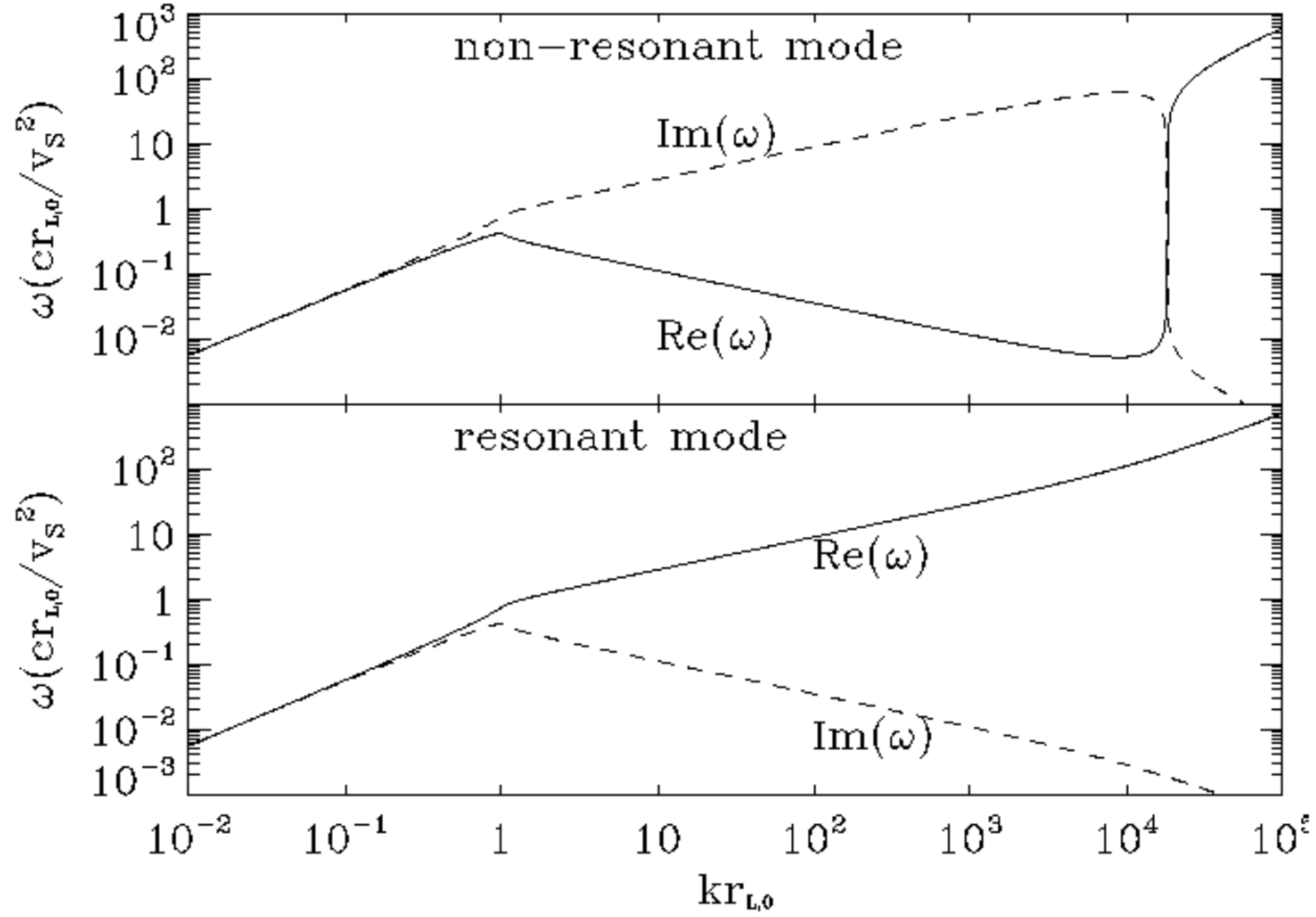


Fig. 9 Real and imaginary parts of the frequency as a function of wavenumber for the resonant (top panel) and non-resonant (bottom panel) modes, as calculated in (Amato and Blasi, 2009). Wavenumbers are in units of $1/r_{L,0}$, while frequencies are in units of $V_{sh}^2/(cr_{L,0})$. In each panel, the solid (dashed) curve represents the real (imaginary) part of the frequency. The values of the parameters are as follows: $V_{sh} = 10^9 cms^{-1}$, $B_0 = 1\mu G$, $n = 1\ cm^{-3}$, $\xi_{CR} = 10\%$ and $p_{max} = 10^5 m_p c$.

Correct way of getting the results: kinetics (Vlasov equation protons + electrons, Maxwell -> perturb and study unstable modes)

Blasi 2013

Modes grow substantially (and fast!), but do not resonate with CRs!

$k_{\rm max} \gg r_{\rm L}$

The growth of modes implies that there is a force on the plasma $\vec{J} \times \vec{B}$

This force stretches the loops of fields

When does it stop? Typically when the loops of fields become of the order of the Larmor radius of particles

$$\frac{\xi_{\rm CR} \rho v^2}{\ln(\frac{p_{\rm max}}{p_{\rm min}})} \frac{v}{c} \approx \frac{\delta B^2}{4\pi}$$

For SNR shocks, values for magnetic field 1000 times larger than the ISM! Field produced upstream

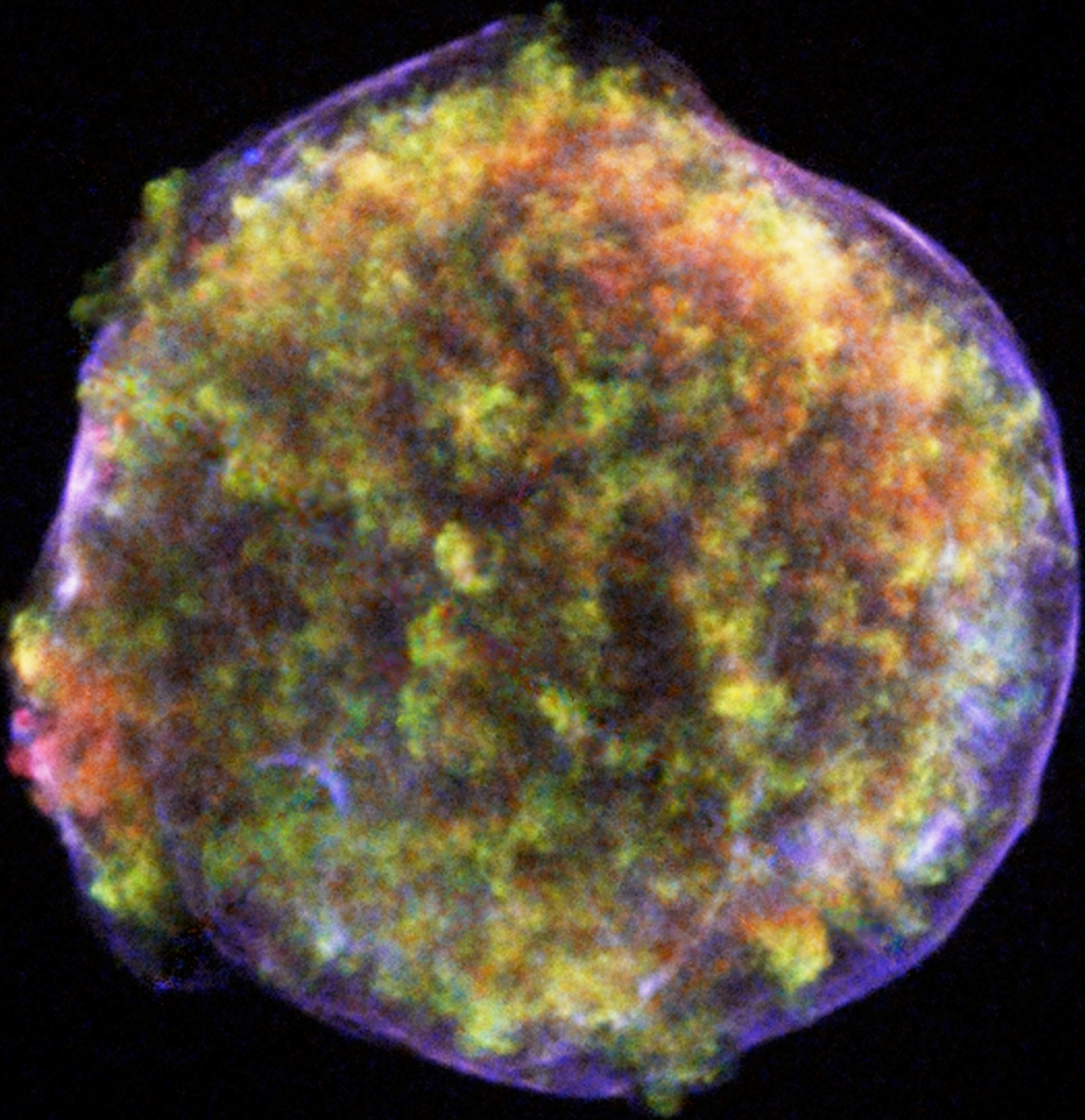
X-ray rims = non-thermal emission around the shock surface

Thickness = 0.01 0.001 parsec

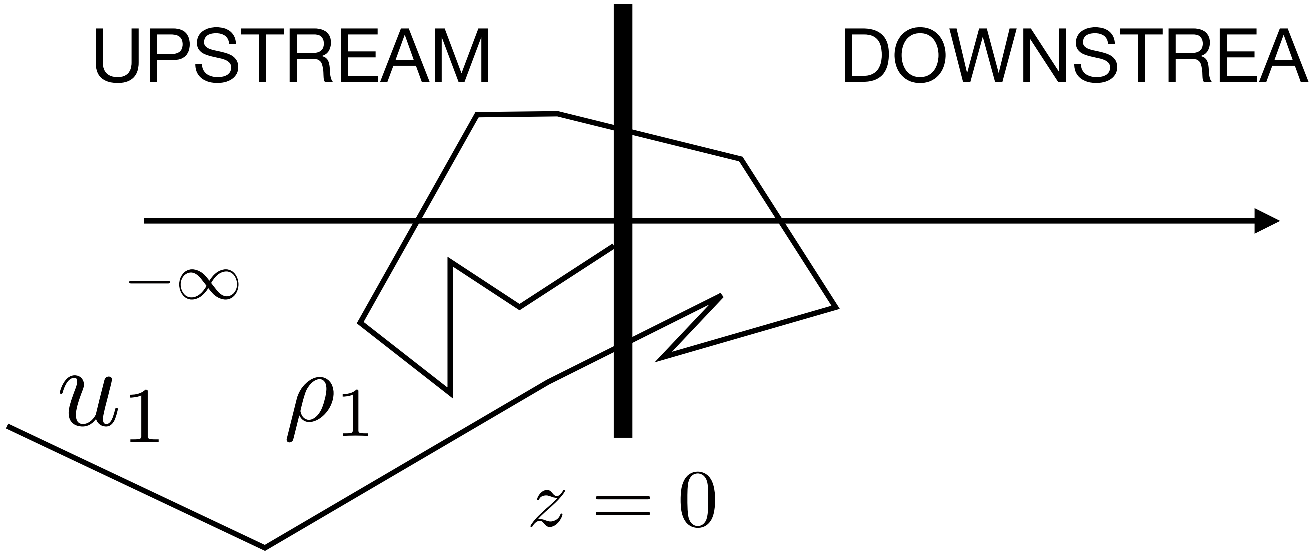
To produce this emission -> synchrotron of high energy electrons 10 TeV electrons

Thickness=

$$\Delta x = \sqrt{D \tau_{\rm loss}} \Rightarrow \delta B \sim 1 {\rm mG}$$

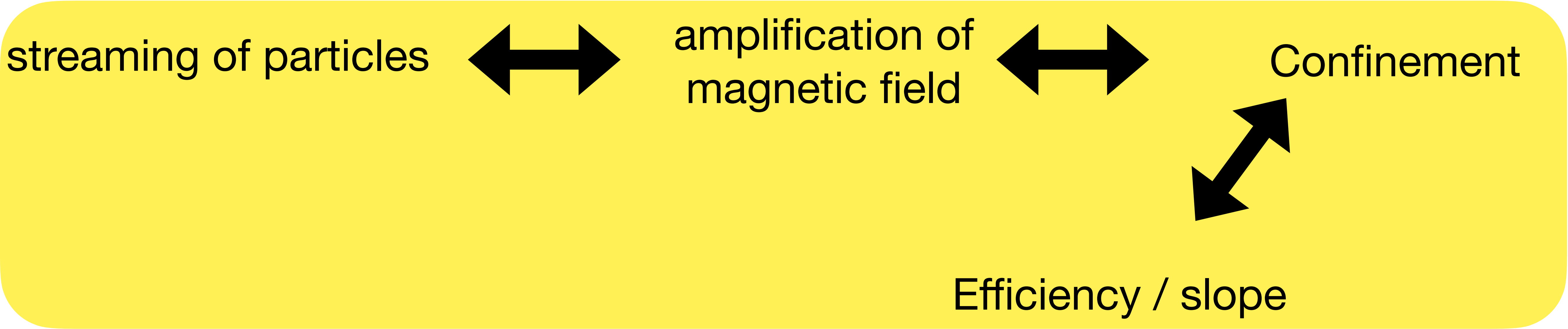


THE ESCAPE PROBLEM



Simple estimate, but hard to know what escapes exactly

$$P_{\rm ret} = \frac{\phi_{\rm out}}{\phi_{\rm in}} = \frac{(1-u_2)^2}{(1+u_2)^2} \quad \rightarrow \approx 1 - 4\frac{u_2}{c}$$



streaming of particles

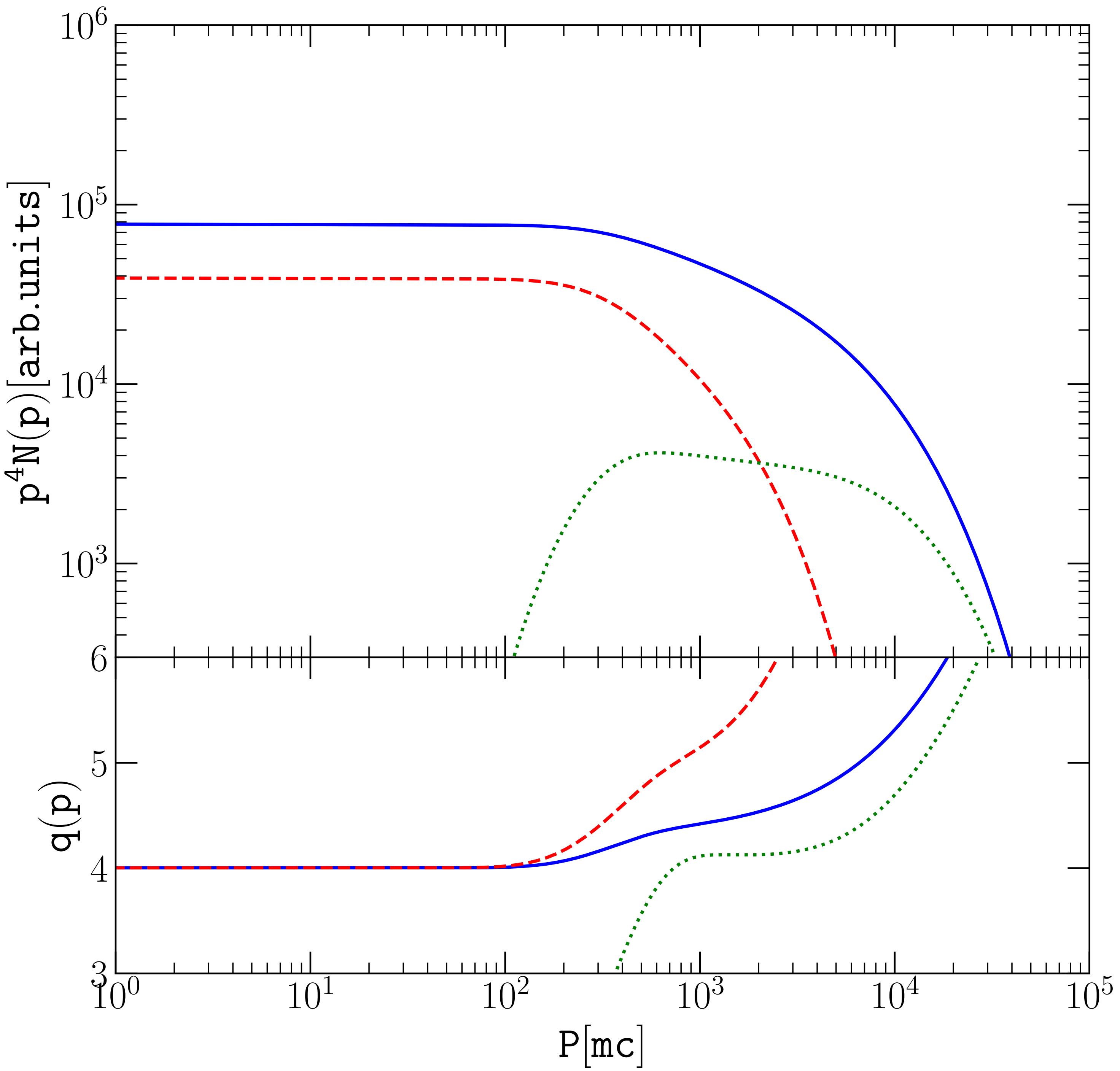
amplification of magnetic field

Confinement

Efficiency / slope

Particle content: accelerated vs. injected?

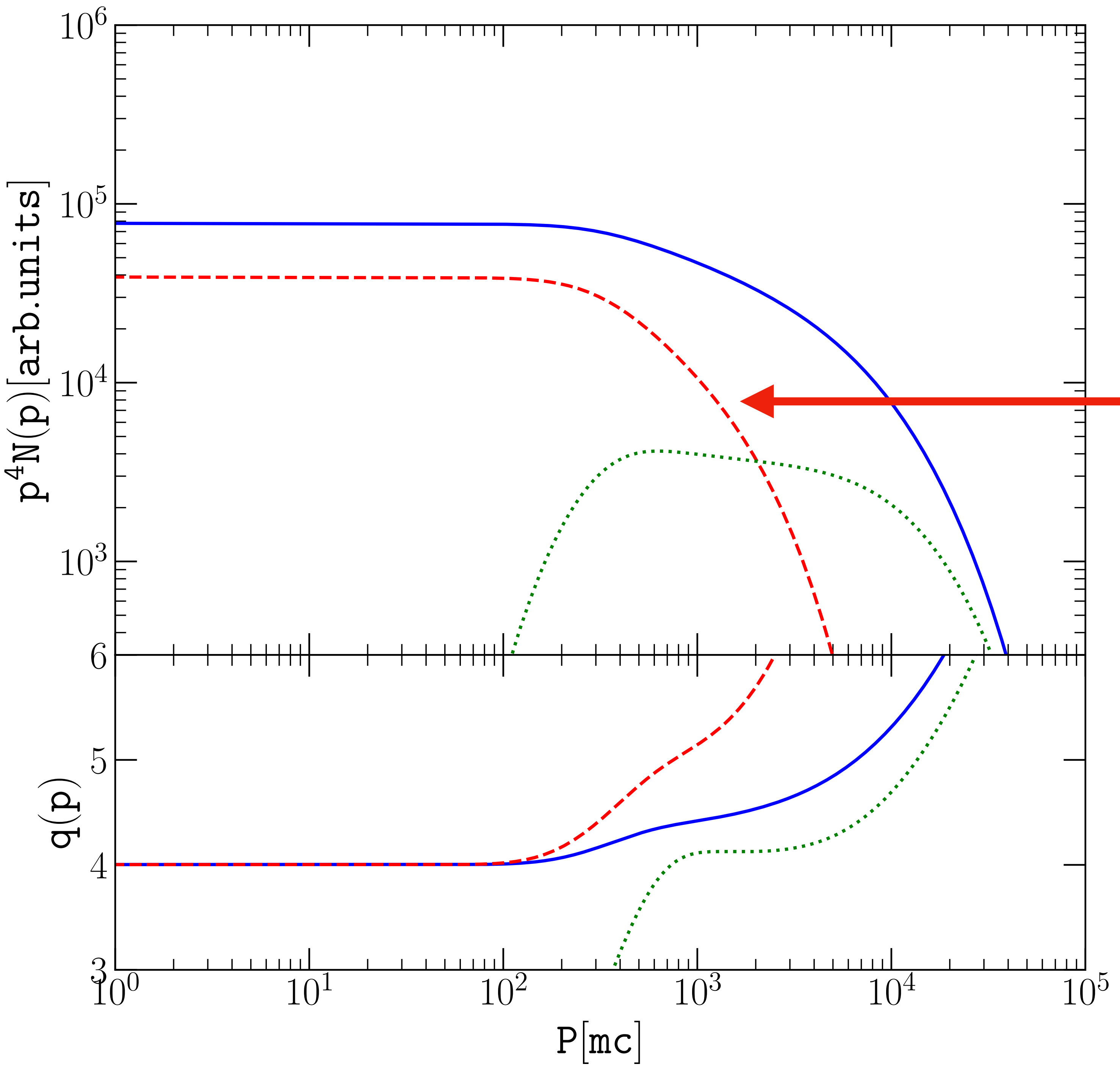
SNRs from Type Ia



Ptuskin & Zirakashvili 2008, Caprioli et al. 2010, Blasi 2013, PC et al. 2021, Morlino et al. 2021

Particle content: accelerated vs. injected?

SNRs from Type Ia

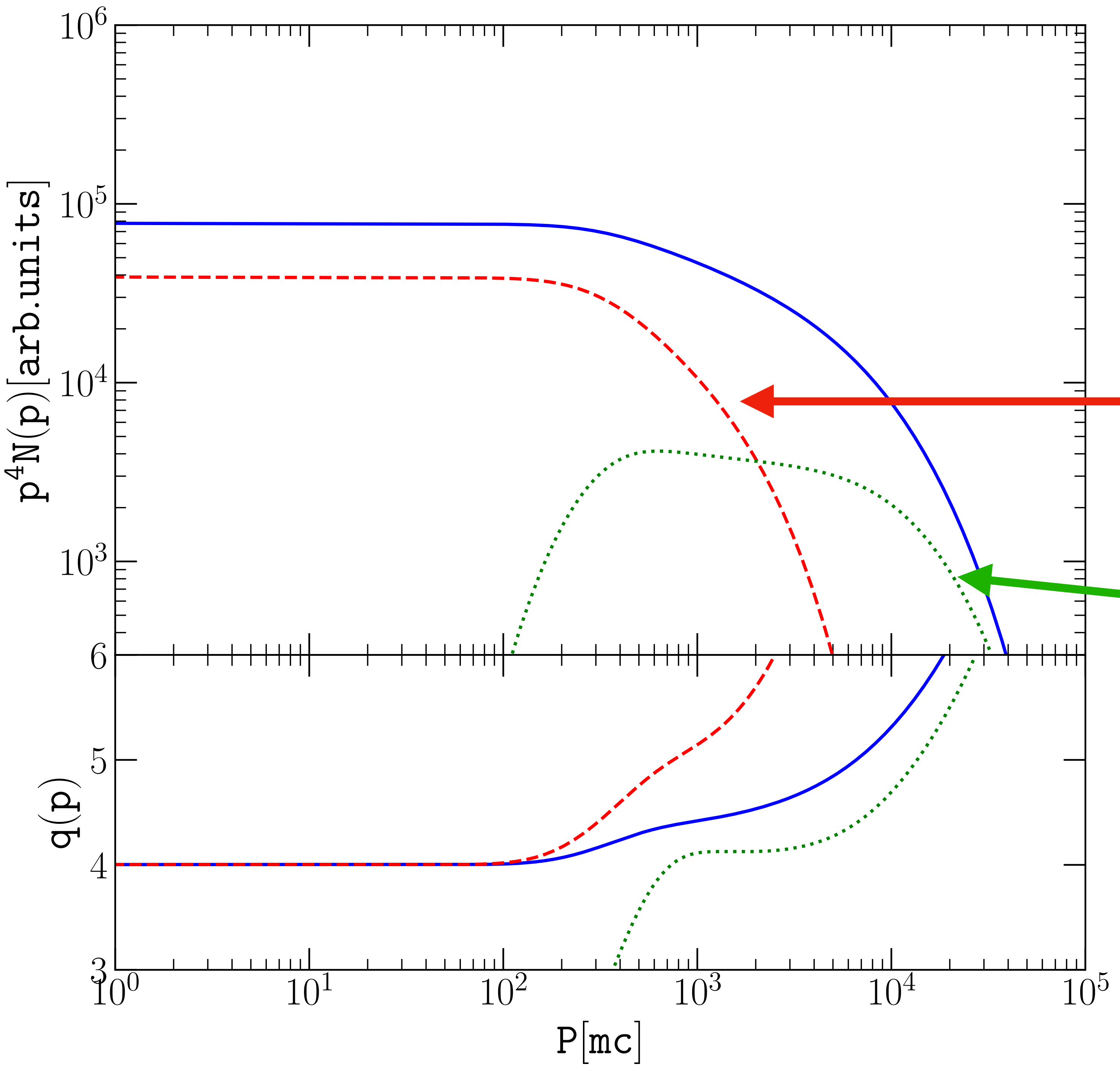


Particle trapped inside SNR + losses

Ptuskin & Zirakashvili 2008, Caprioli et al. 2010, Blasi 2013, PC et al. 2021, Morlino et al. 2021

Particle content: accelerated vs. injected?

SNRs from Type Ia



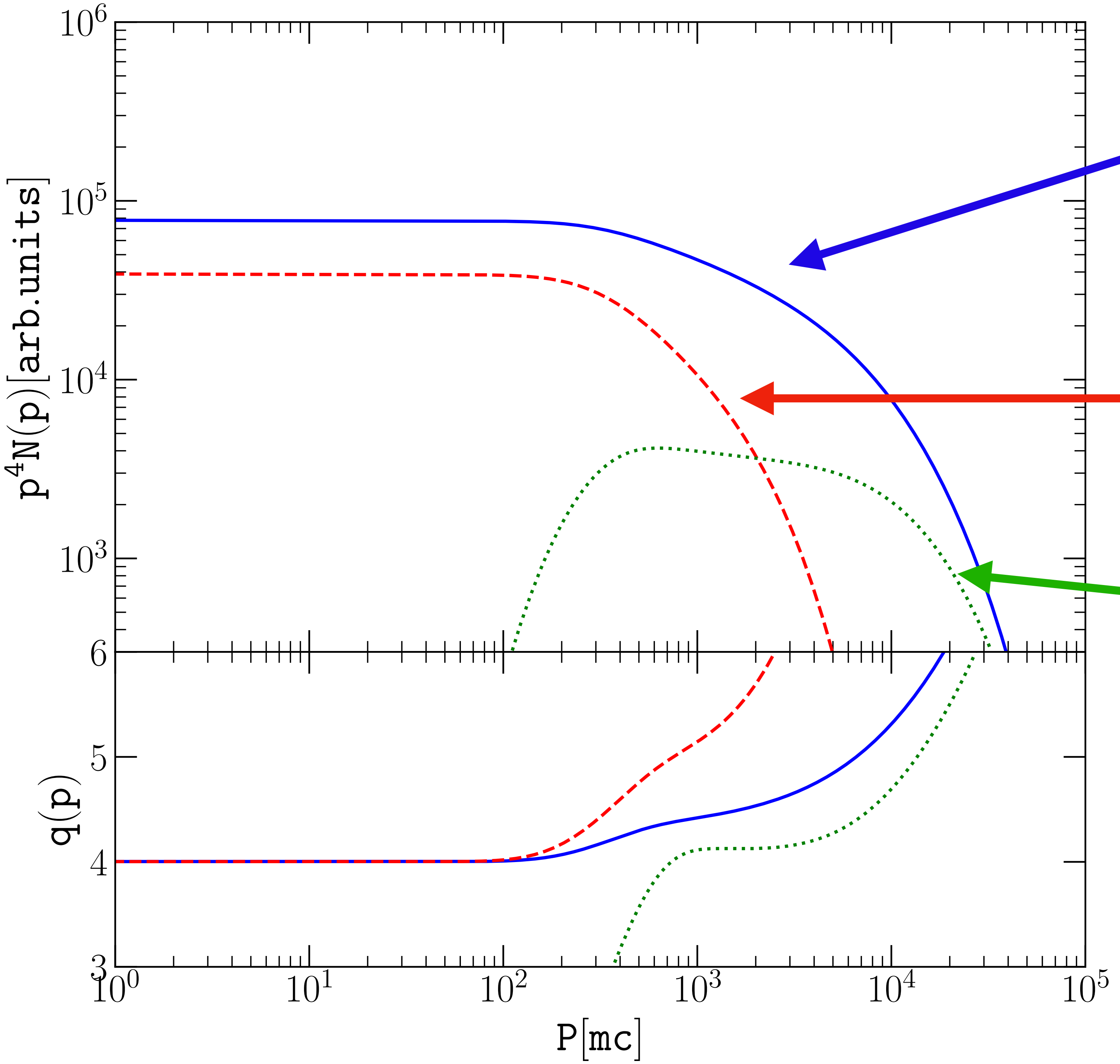
Particle trapped inside SNR + losses

Particle constantly escaping in the ISM

Ptuskin & Zirakashvili 2008, Caprioli et al. 2010, Blasi 2013, PC et al. 2021, Morlino et al. 2021

Particle content: accelerated vs. injected?

SNRs from Type Ia



Sum

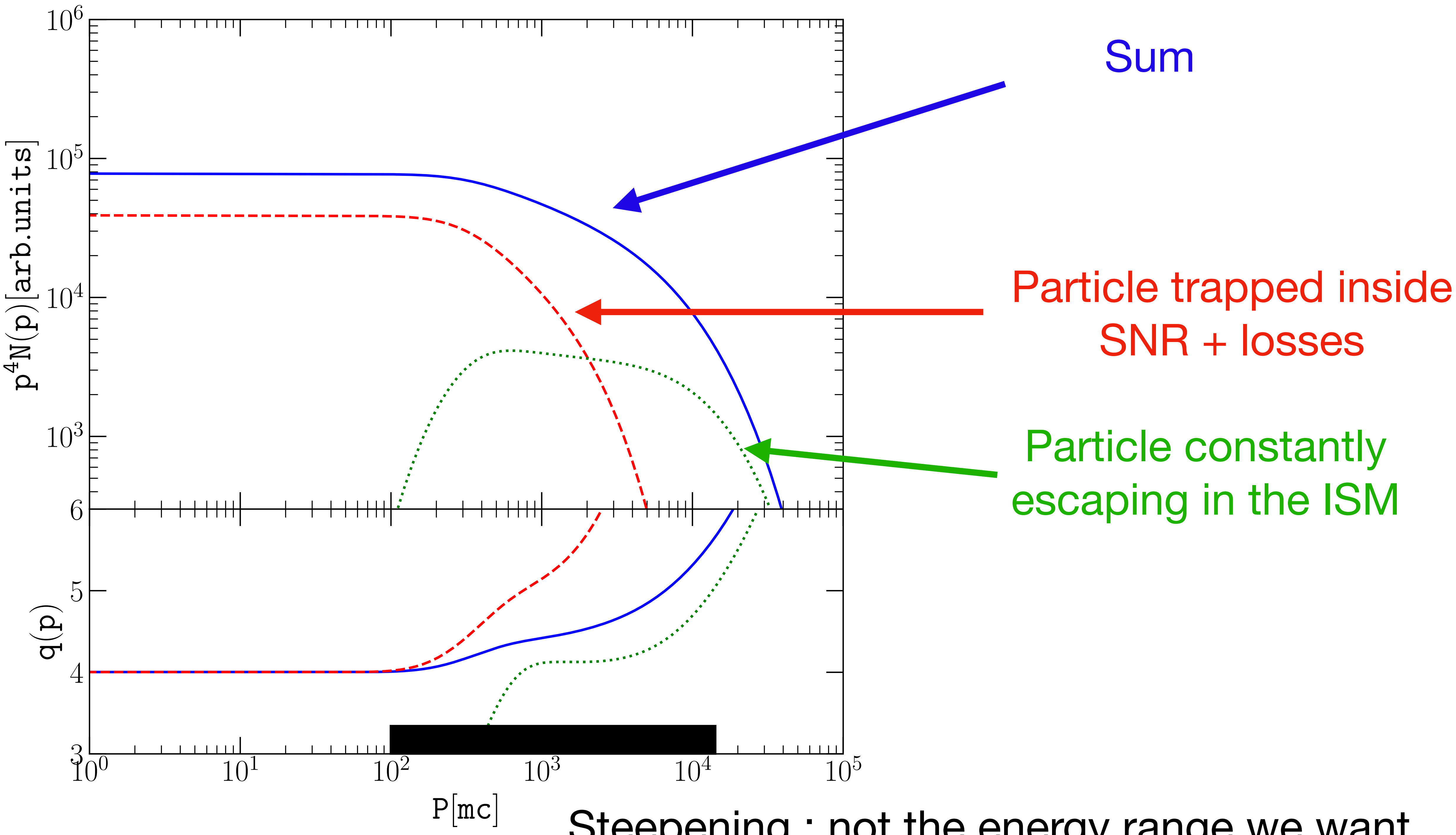
Particle trapped inside SNR + losses

Particle constantly escaping in the ISM

Ptuskin & Zirakashvili 2008, Caprioli et al. 2010, Blasi 2013, PC et al. 2021, Morlino et al. 2021

Particle content: accelerated vs. injected?

SNRs from Type Ia



Sum

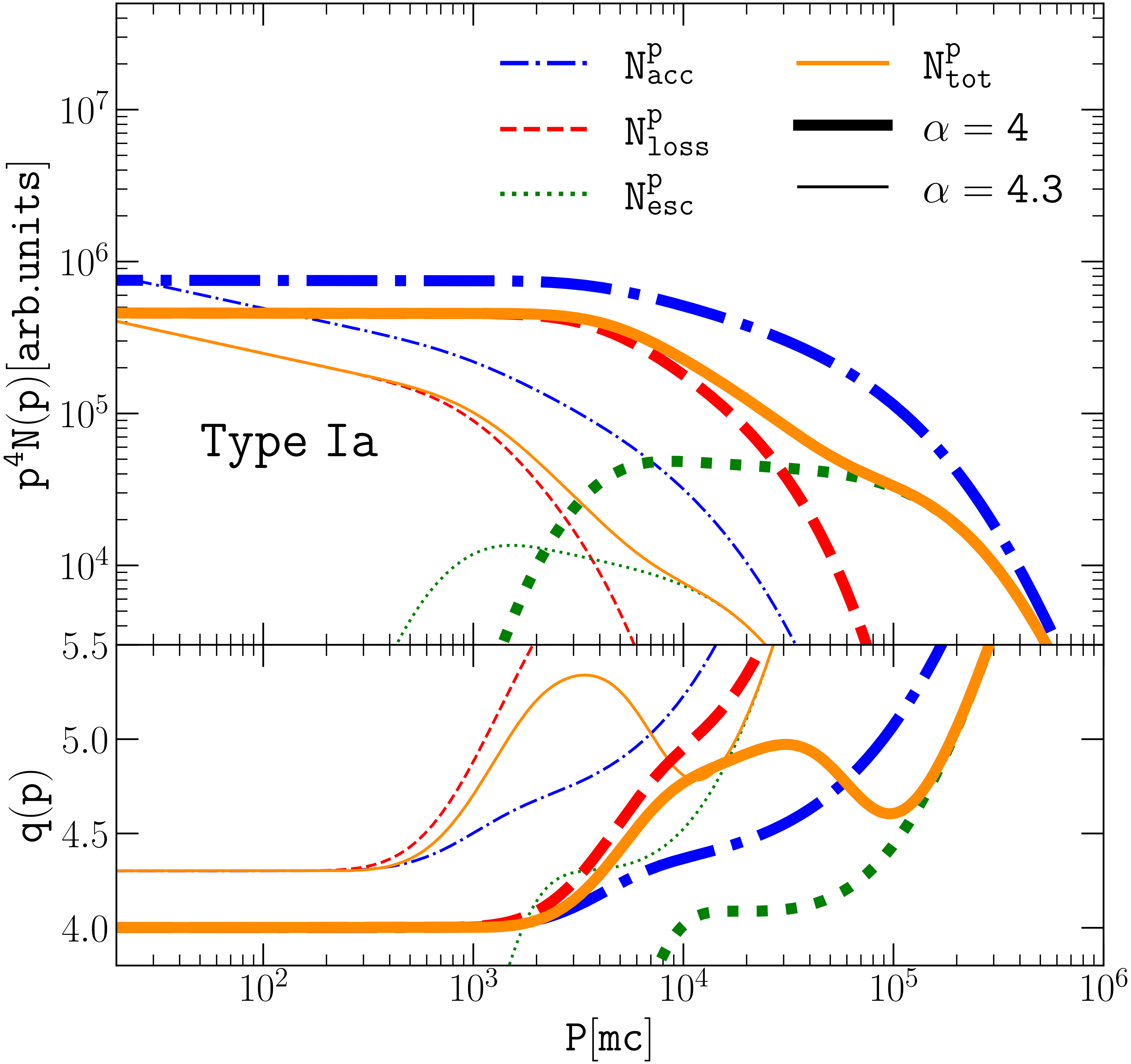
Particle trapped inside SNR + losses

Particle constantly escaping in the ISM

Steepening : not the energy range we want, same idea for other sources?

Ptuskin & Zirakashvili 2008, Caprioli et al. 2010, Blasi 2013, PC et al. 2021, Morlino et al. 2021

Particle content: accelerated vs. injected?



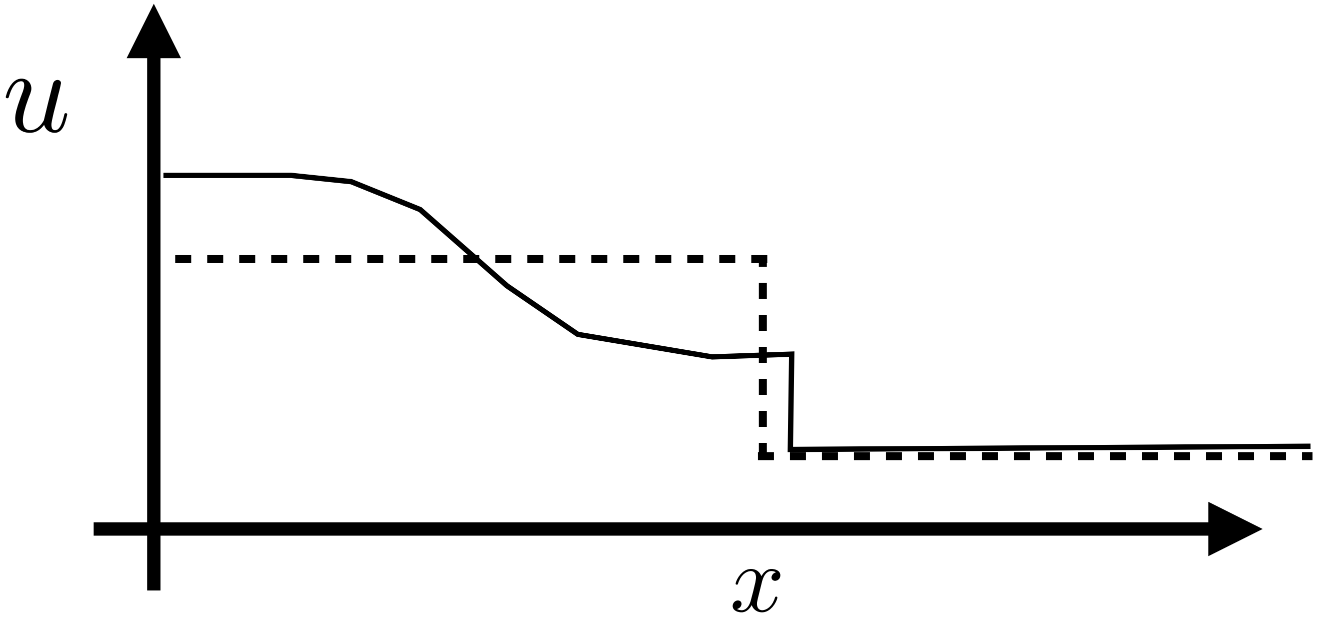
PC, Blasi, Caprioli 202

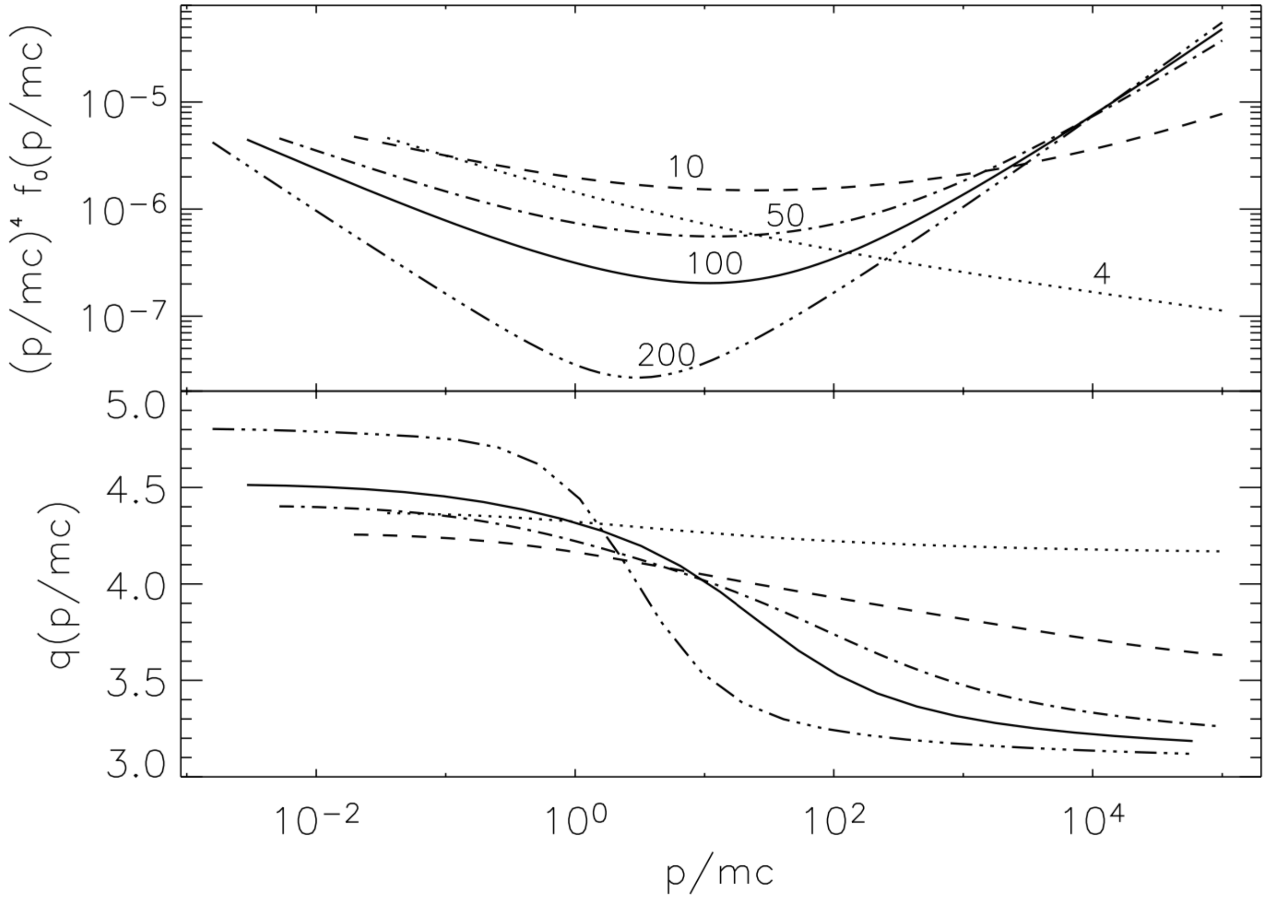
Spectrum at the shock?

Until now: fixed slope at the shock produced steeper summed injected spectrum.

$$f(p) \propto p^{-\alpha} \qquad\qquad f(p) \propto p^{-\alpha(t)} \quad \alpha \neq 4$$

Non-linear effects: efficient particle acceleration acting on the shock structure





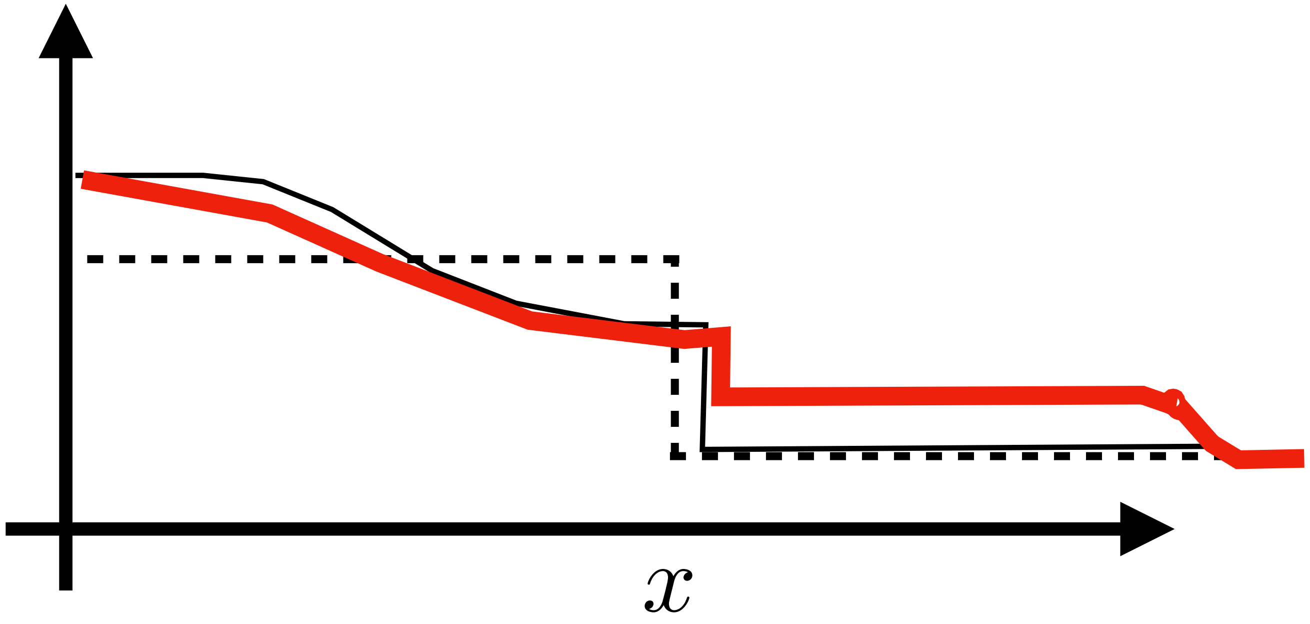
Drury& Völk (1980,1981), Bell (1987)
Jones & Ellison (1991), Ellison, Möbius & Paschamnn (1990), Ellison, Baring & Jones (1995, 1995) Kang & Jone
(1997, 2005) Kang, Jones & Gieseler (2002), Malkov (1997), Malkov, Diamond & Völk (2000)
Blasi (2002), Amato & Blasi (2005,2006)

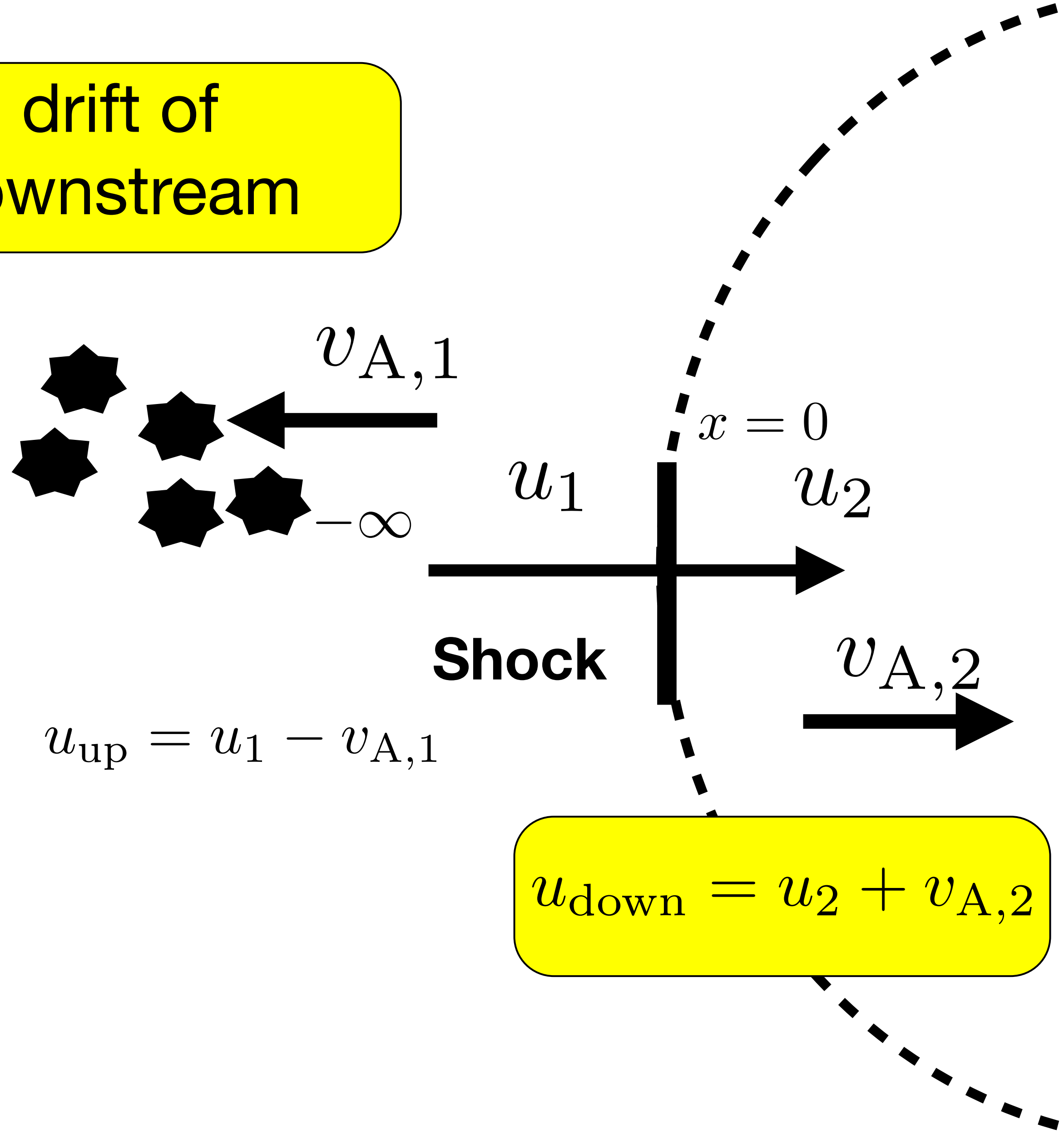
Spectrum at the shock?

Until now: fixed slope at the shock produced steeper summed injected spectrum.

$$f(p) \propto p^{-\alpha} \qquad f(p) \propto p^{-\alpha(t)} \quad \alpha \neq 4$$

Non-linear effects: drift of scattering centers downstream





$u_{\mathrm{up}} = u_1 - v_{\mathrm{A},1}$

$u_{\mathrm{down}} = u_2 + v_{\mathrm{A},2}$

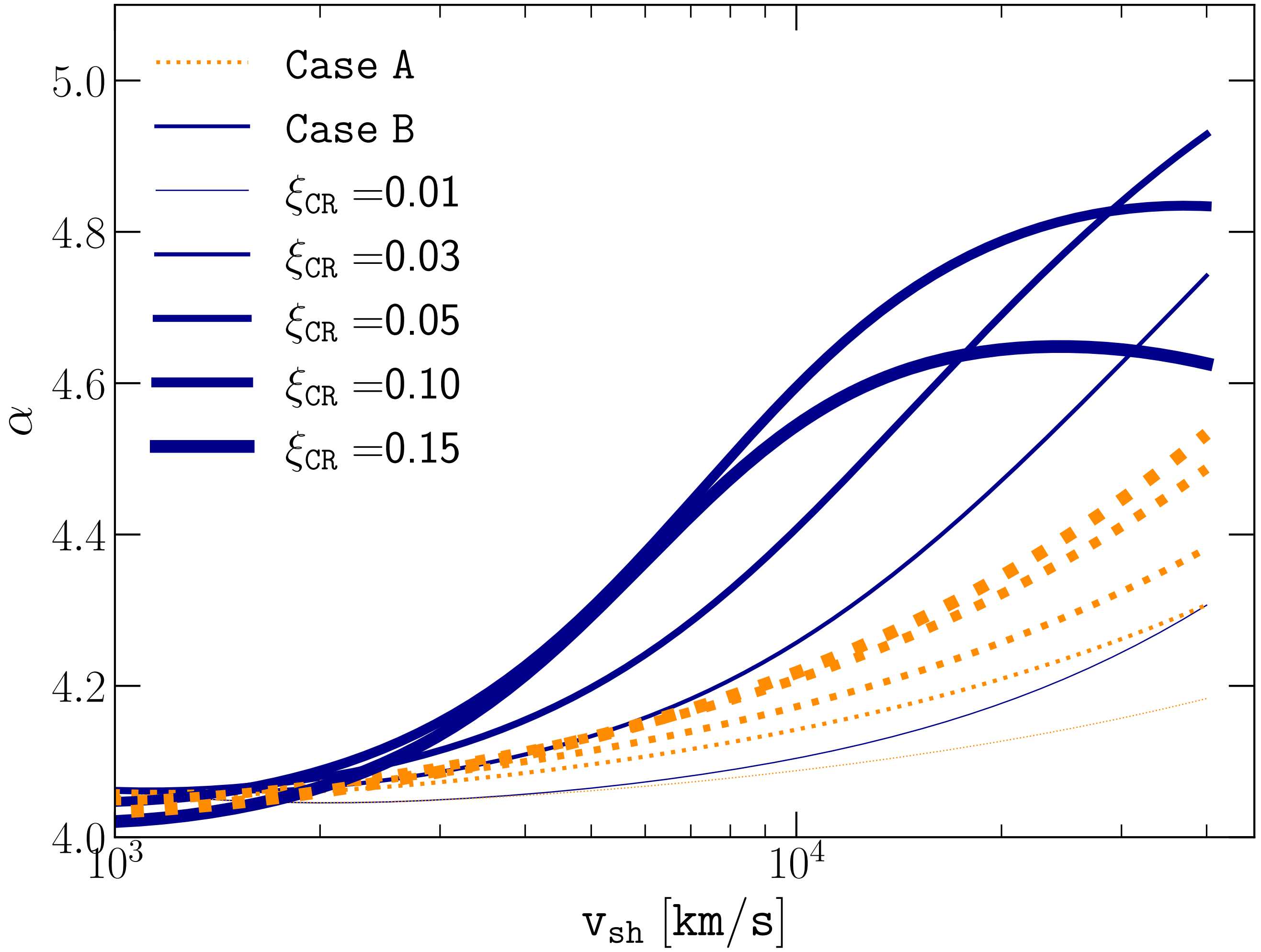
Zirakashvili & Ptuskin (2008)

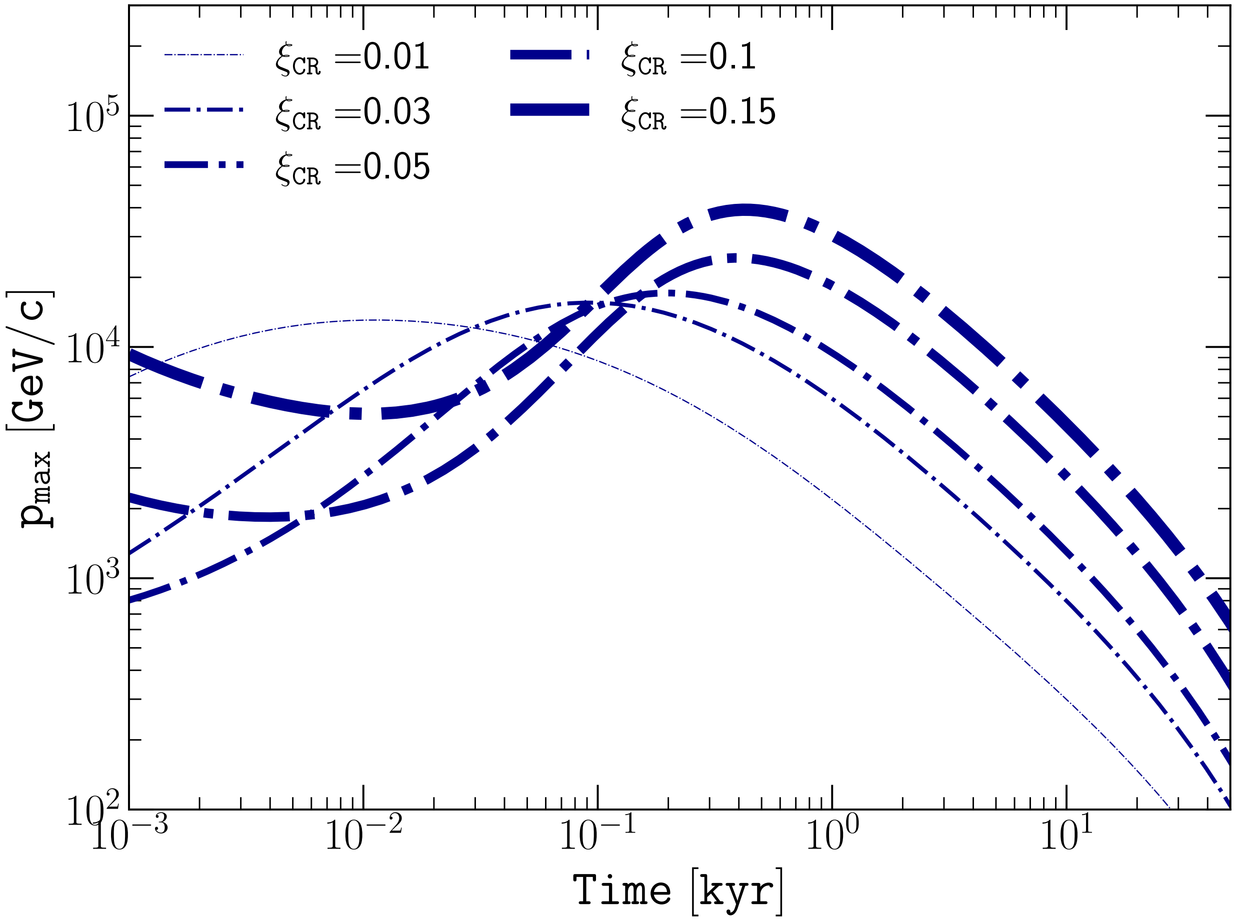
Drury (1983), Caprioli, Haggerty & Blasi (2020), Diesing & Caprioli (2021), PC, Blasi & Caprioli (submitted 2022)

Spectrum at the shock?

$$v_{\mathrm{A},2} = R_{\mathrm{tot}} \frac{\delta B_2}{\sqrt{4\pi\rho}}$$

Bell: current from all particles (maximum value B)



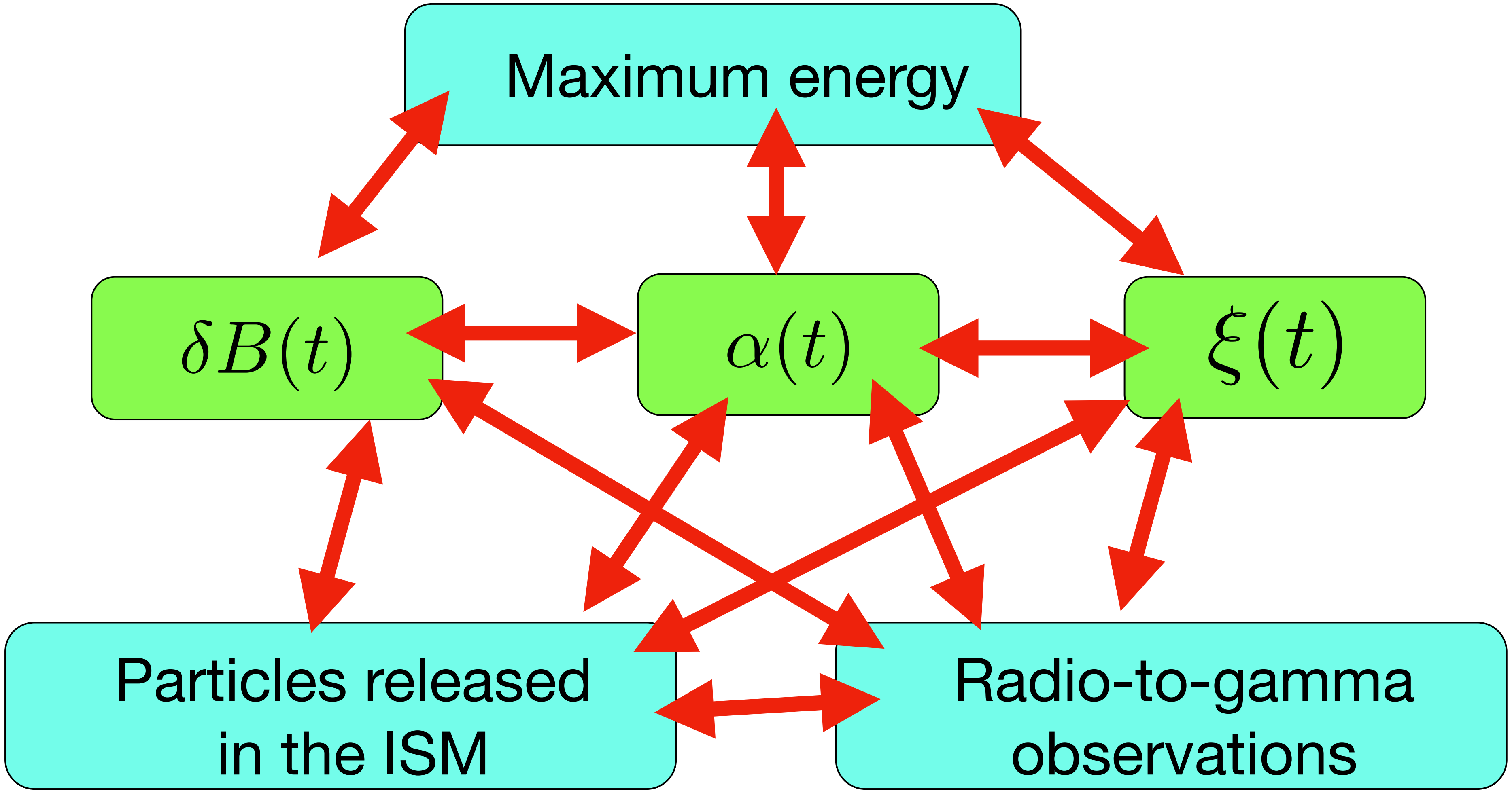


Bell: current escaping particles upstream infinity

Consequences on pmax!

Drury (1983), Caprioli, Haggerty & Blasi (2020), Diesing & Caprioli (2021), PC, Blasi & Caprioli (submitted 2022)

Summary



Maximum energy

$\delta B(t)$

$\alpha(t)$

$\xi(t)$

Particles released in the ISM

Radio-to-gamma observations

Classifying particle acceleration mechanisms

Invariants:

$$\vec{E}' = \gamma_E(\vec{E} - v_{\mathrm{E}}/c \times \vec{B})$$

$$\vec{B}' = \gamma_E(\vec{B} + v_{\mathrm{E}}/c \times \vec{E})$$

Lorentz transform from R’ to
R moving at v_E

$$\vec{E} \cdot \vec{B} = -\frac{1}{4} F_{\mu\nu} * F^{\mu\nu}$$

$$\vec{E}^2 - \vec{B}^2 = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu}$$

Regular acceleration (linear)

$$\vec{E} \cdot \vec{B} = 0$$

$$\vec{E}^2 - \vec{B}^2 > 0$$

$$\vec{v_{\mathrm{E}}} = c\frac{\vec{E} \times \vec{B}}{E^2}$$

$$\vec{E}' = \frac{1}{\gamma_E}\vec{E}$$

$$\vec{B}' = 0$$

There is a frame in
which B vanishes

Reconnection

$$\vec{E} \cdot \vec{B} \neq 0$$

$$\frac{\mathrm{d}\vec{p_{//}}}{\mathrm{d}t} = q\vec{E_{//}}$$

Parallel along B

Gaps at compact
objects

Stochastic

$$\vec{E} \cdot \vec{B} = 0$$

$$\vec{E}^2 - \vec{B}^2 < 0$$

deboost:

$$\vec{v_{\mathrm{E}}} = c\frac{\vec{E} \times \vec{B}}{B^2}$$

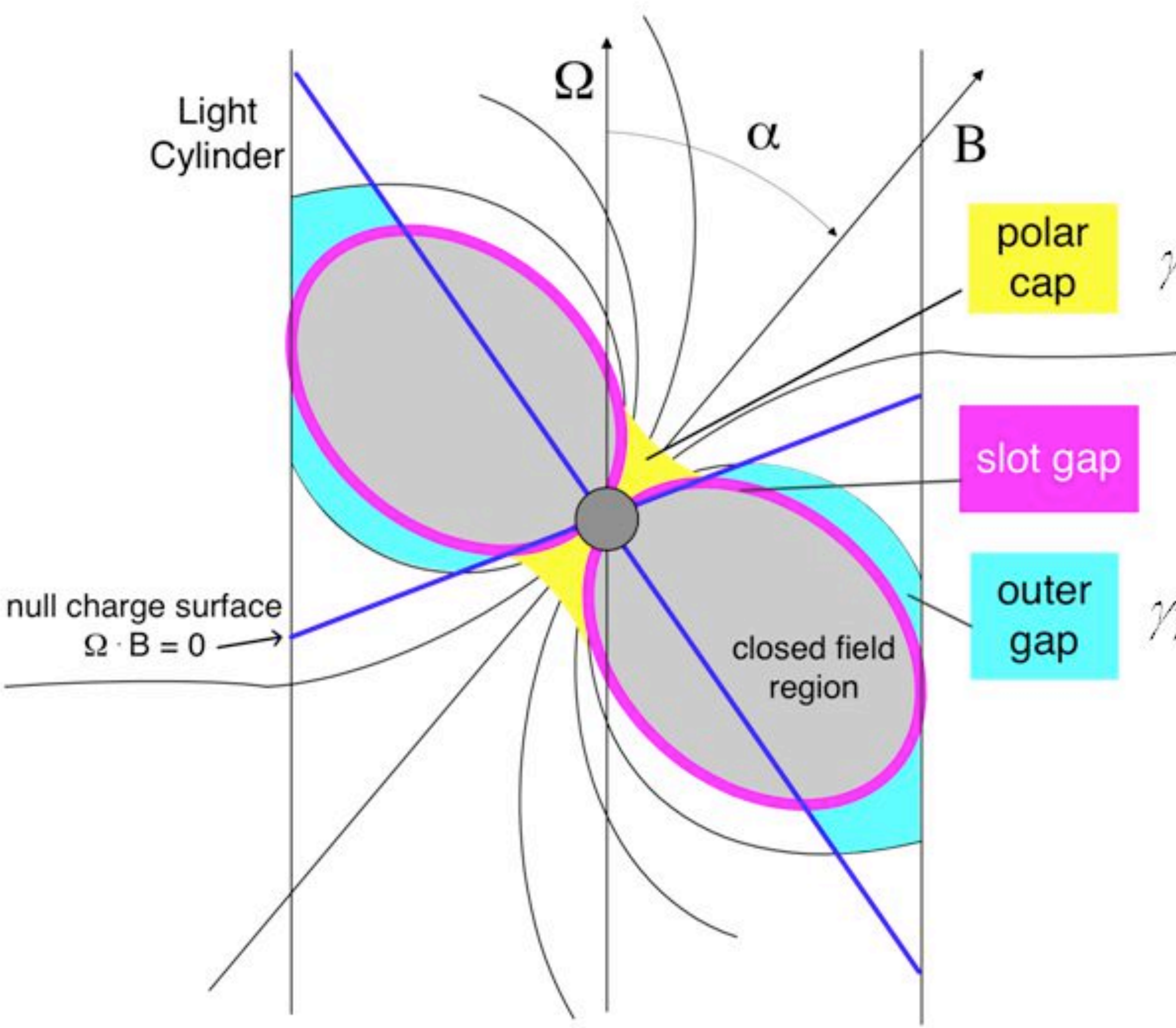
$$\vec{E}' = 0$$

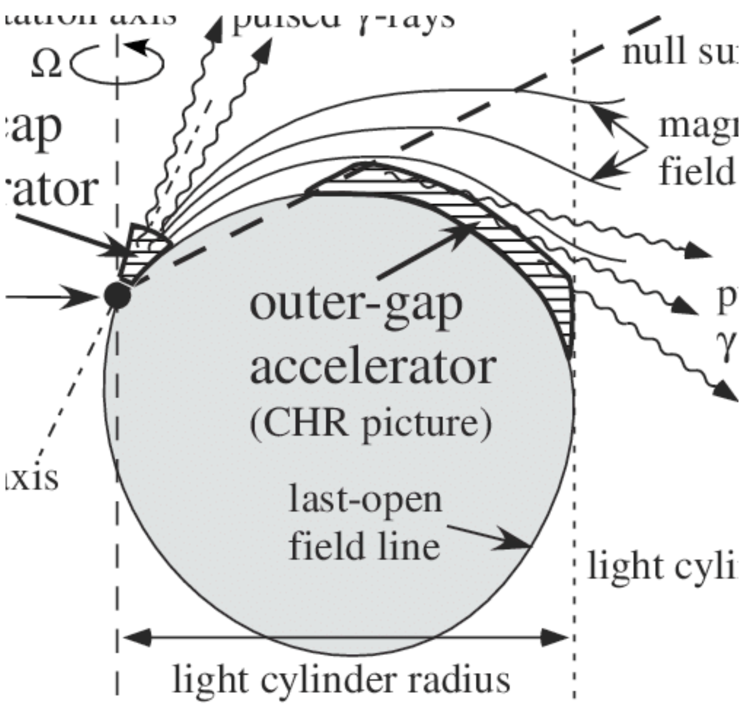
$$\vec{B}' = \frac{1}{\gamma_E}\vec{B}$$

There is a frame in
which E vanishes

Stochastic
acceleration in ideal
MHD plasma: Fermi,
shear, turbulence

Pulsar gaps





Harding 2015

Magnetic reconnection

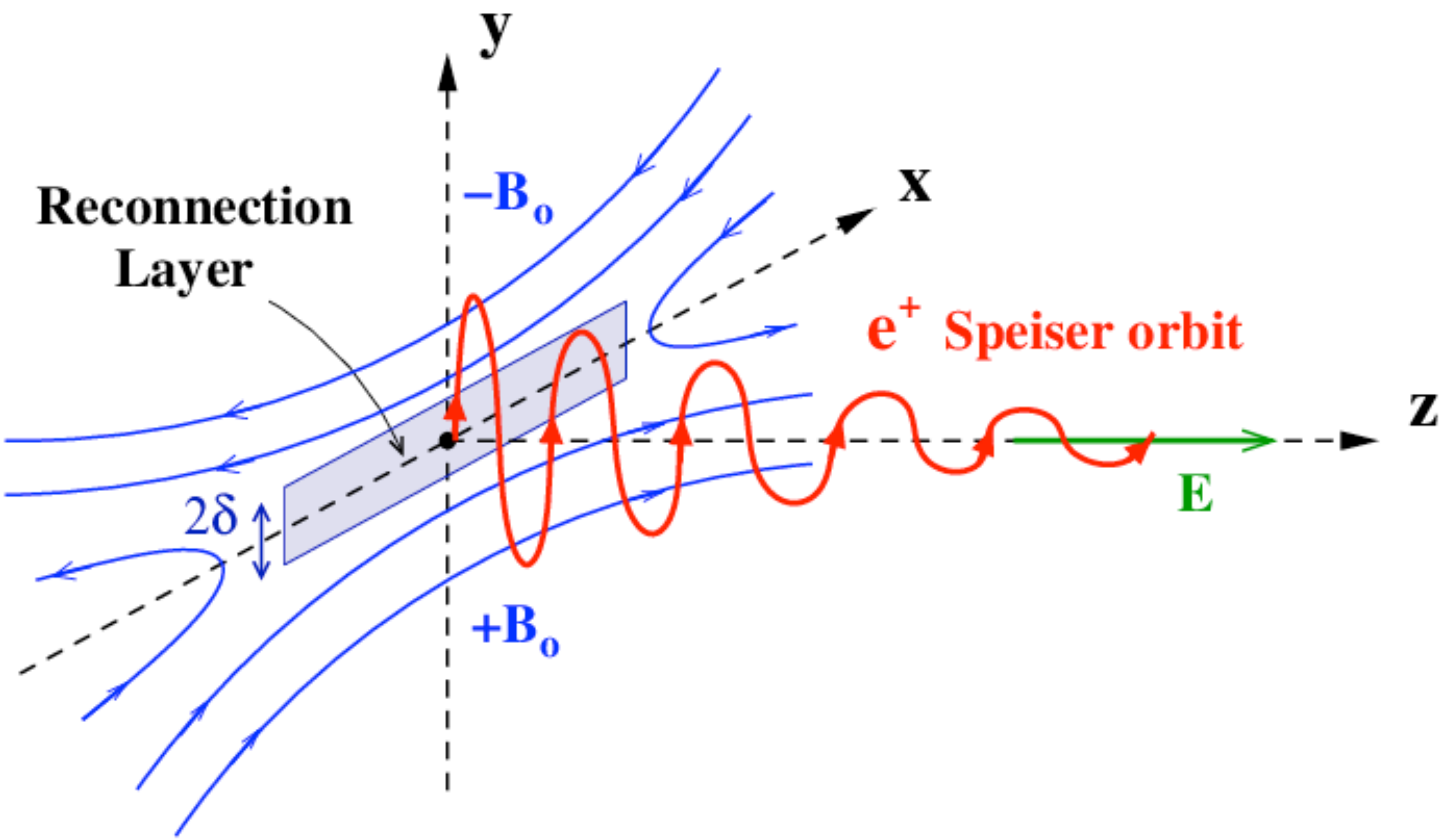
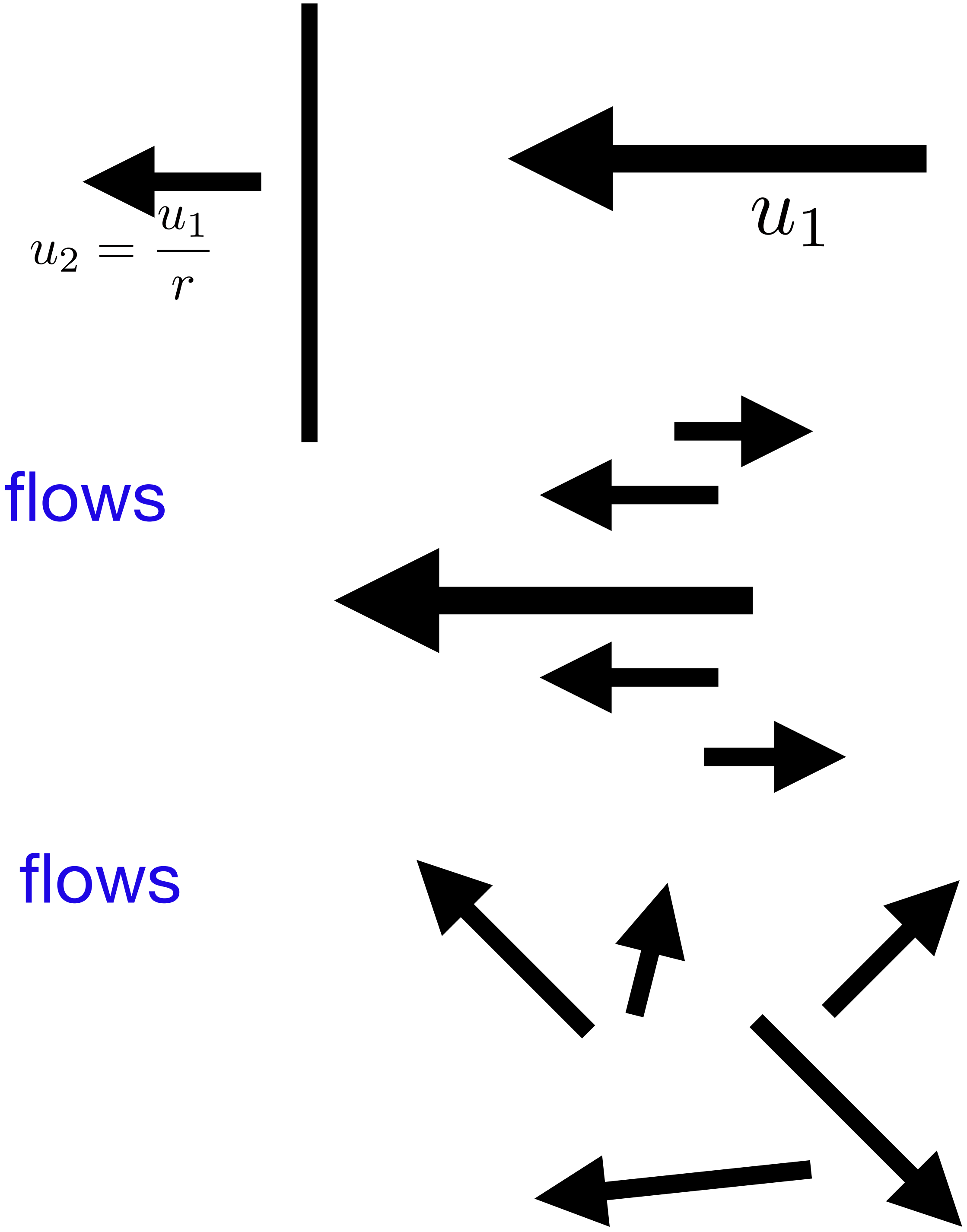


FIG. 1.— This diagram represents a relativistic Speiser orbit, i.e.,

Cerutti et al. 2013

Stochastic acceleration: Fermi 1, Fermi 2, shear, turbulent..



Shocks

$$\partial_x v_{\mathrm{E}}^x \neq 0$$

Sheared flows

$$\partial_x v_{\mathrm{E}}^y \neq 0$$

Turbulent flows

$$\partial_i v_{\mathrm{E}}^j \neq 0$$

In all cases, sheared velocity

$$\partial_i v_{\mathrm{E}}^j$$

E vanishes in the rest frame

$$\vec{v_{\mathrm{E}}} = c\frac{\vec{E} \times \vec{B}}{B^2}$$

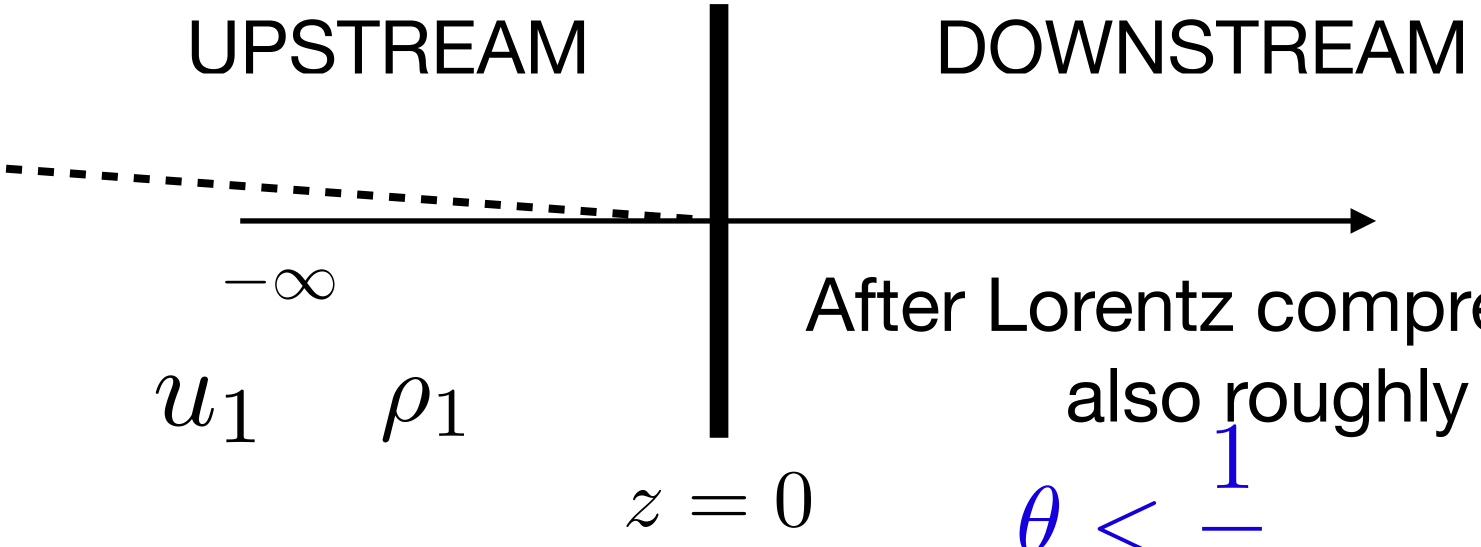
TOWARS RELATIVISTIC SHOCS

Extragalactic sources: gamma-ray bursts, etc, relativistic shocks

When the shock speed becomes close to c, at least two things fail:

1. Assumption that particles are isotropic at the shock not fulfilled (same speed, same direction)
2. It's harder for particle to go back to the shock, because they go at the same speed

Let's consider a relativistic shock, with magnetic field almost parallel to shock



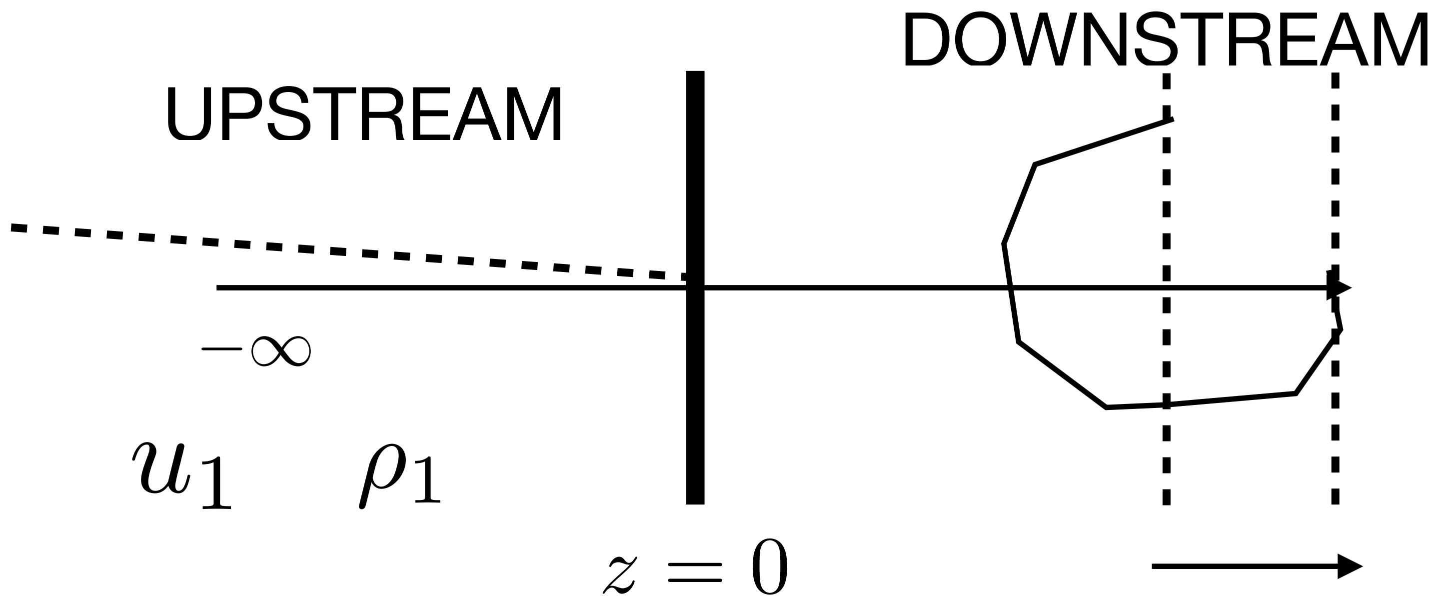
After Lorentz compresison, behind the shock, also roughly parallel, provided

$$\theta < \frac{1}{\Gamma}$$

GRB: $\Gamma \sim 300$

1) Unless completely parallel -> field becomes perpendicular

2) Rankine-Hugoniot relations: $u_2 \Rightarrow \frac{1}{3}c$



3) imagine you are at a location somewhere downstream. To do one gyration, it takes: $\tau = \frac{2\pi r_{\rm L}}{c}$

3/4 of gyration: (in order to get a chance to cross the shock again) $\tau = \frac{2\pi r_{\rm L}}{c}\frac{3}{4}$

In this time, the shock has moved by:

$$\boxed{l = \frac{1}{3}c\tau = \frac{\pi}{2}r_{\rm L} > r_{\rm L}}$$

$$l = \frac{1}{3}c\tau = \frac{\pi}{2}r_{\rm L} > r_{\rm L}$$

Problem! The particle will not make it back to the shock.
In other words,the turn probability is thus very small

To make relativistic shock acceleration efficient, need for strong turbulence

Entangled problems, thus interest for other mechanisms (reconnection)

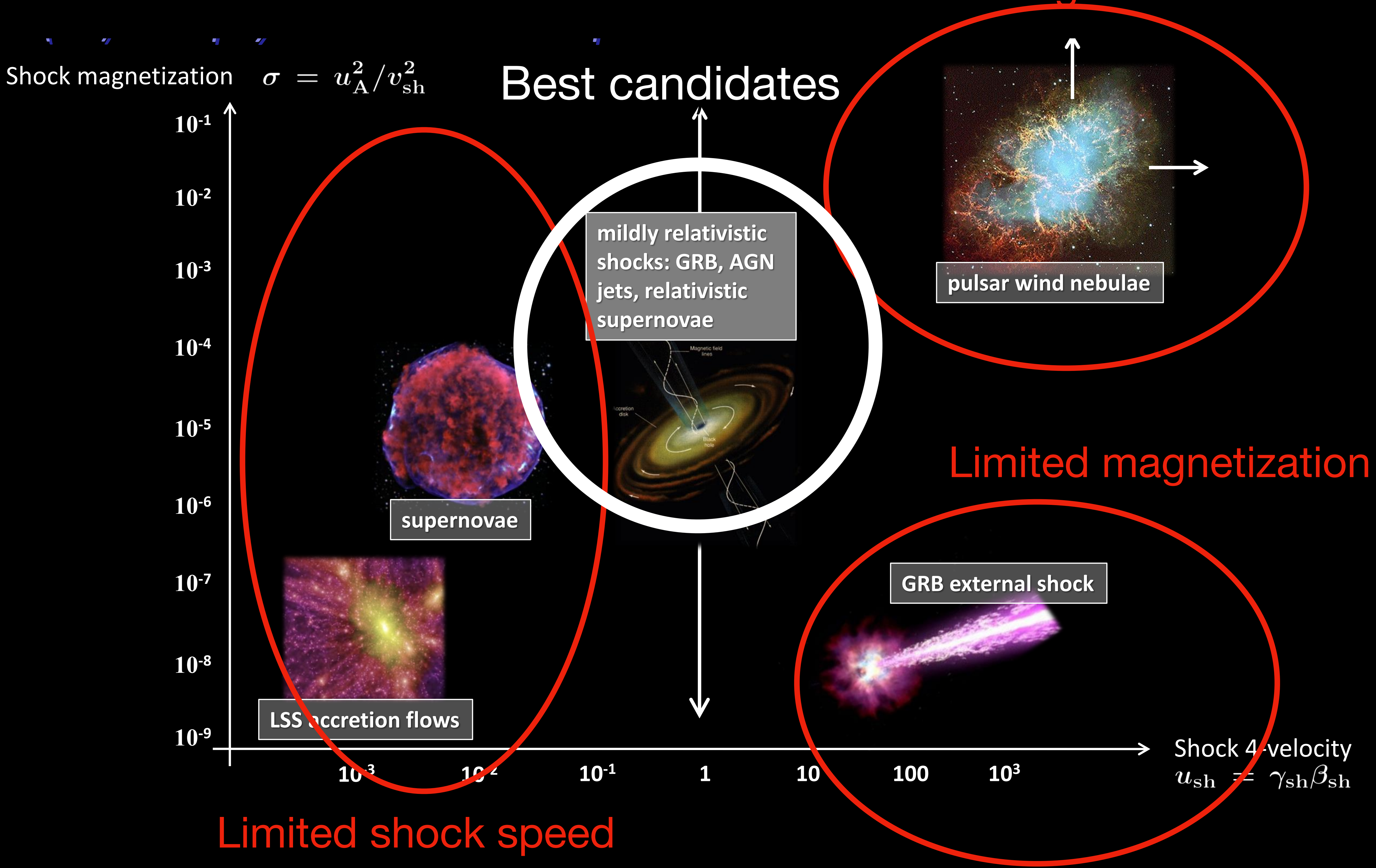
	Spectrum	Efficiency	Emax
Fermi 2			
Fermi1 non-relat.			
Fermi1 Relativistic			
Magnetic reconnection			
Gaps			
Shear			
…			

	Spectrum	Efficiency	Emax
Fermi 2	$\propto p^{\frac{1}{\alpha \tau_{\mathrm{esc}}}-1}$	$+$	$?$
Fermi1 non-relat.	$\propto p^{-4}$	$+++$	$?$
Fermi1 Relativistic	$\propto p^{-4.2}$	$+$	$?$
Magnetic reconnecti			
Gaps			
Shear		$?$	

Supposed to be « universal » (test-particle) and we easily get deviation from power-law + harder/steeper spectra!!

Limitations due to external magnetic fields

Best candidates



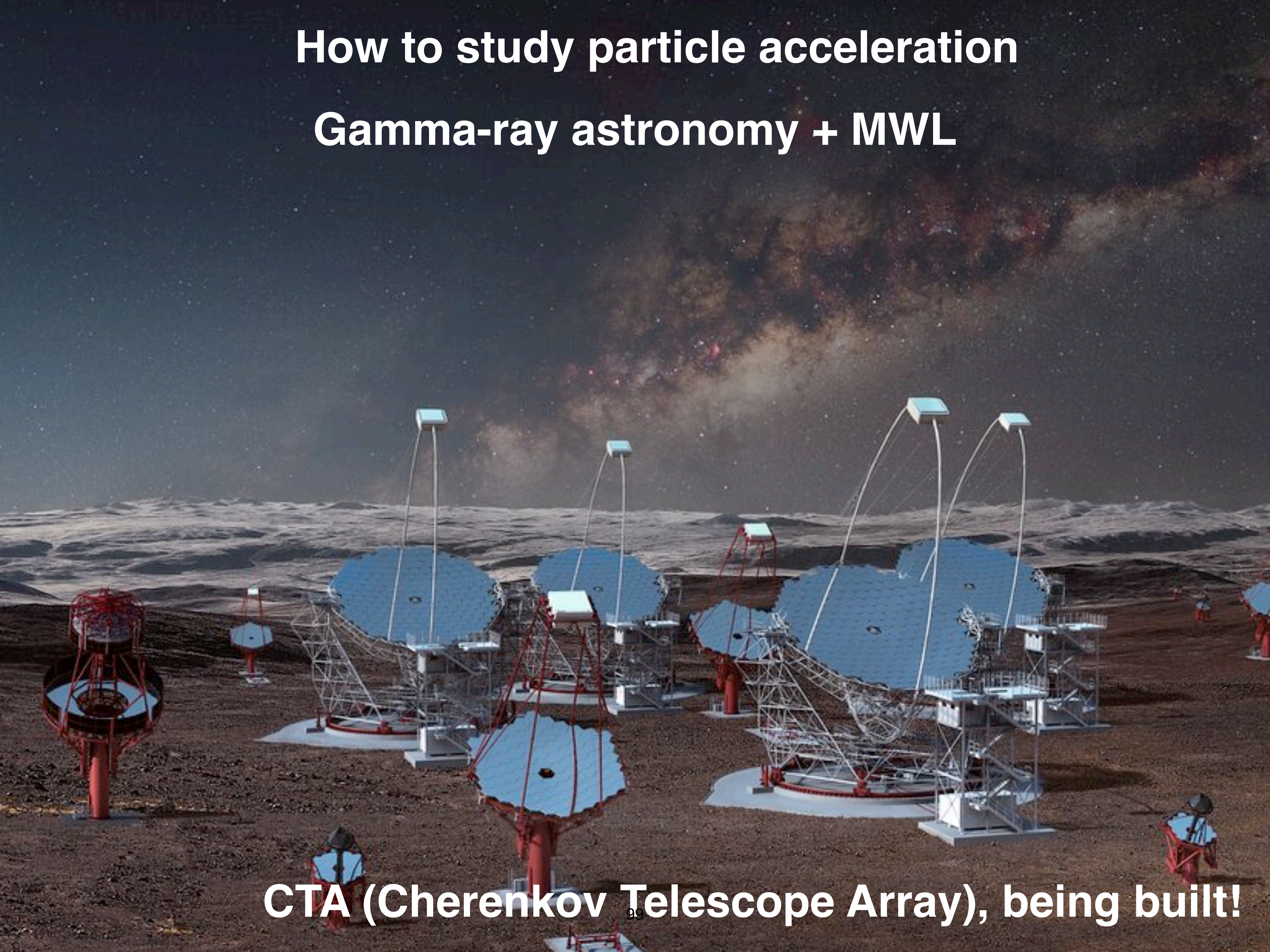
Limited magnetization

Limited shock speed and Emax

Lemoine 2022

How to study particle acceleration

Gamma-ray astronomy + MWL



CTA (Cherenkov Telescope Array), being built!

Large size telescope (LST) CTA (first light Dec. 2018)

Example: PIC simulations, acceleration in turbulence

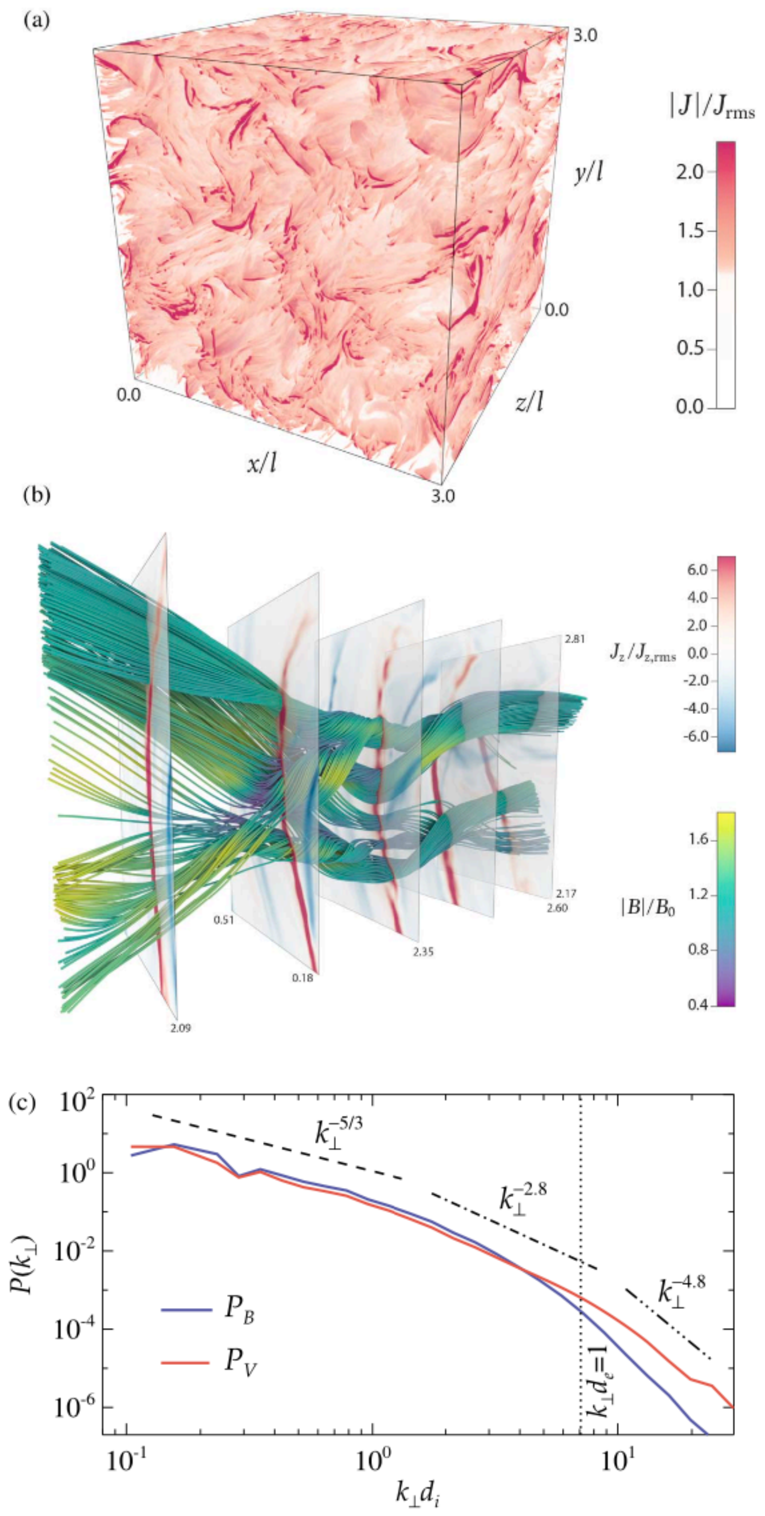
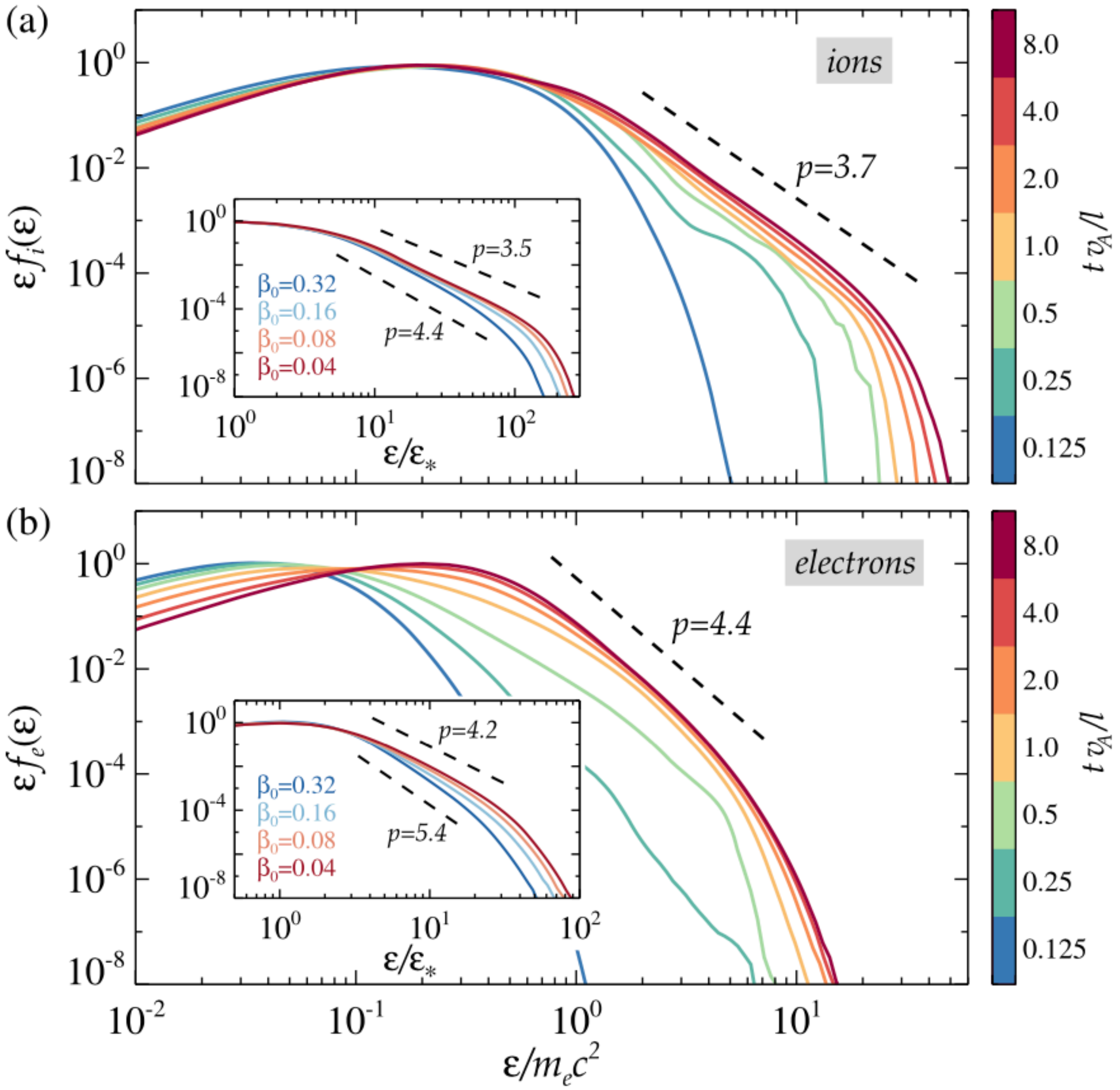


Figure 1. (a) Volume rendering of the current density $|\boldsymbol{J}|$ at $t = 1.25\, l/v_A$ from the fiducial simulation ($\beta_0 = 0.08$). (b) Zoomed-in subdomain with five x–y slices at different z showing the current density J_z, along with selected magnetic field lines illustrating the presence of magnetic flux ropes. (c) One-dimensional $k_\perp$ energy spectra of magnetic (blue) and velocity (red) fluctuations at $t = 1.25\, l/v_A$. Different spectral slopes are provided for reference.

The Astrophysical Journal Letters, 936:L27 (5pp), 2022 September 10



Problem of scale: from micro to very macro scales

$$l_{\text{injection}} \sim 10^7 - 10^8 \text{cm}$$
$$l_{\text{escape}} \sim 10^{22} \text{cm}$$

Comisso & Sironi 2022

Example: shocks in the lab, with LASERs

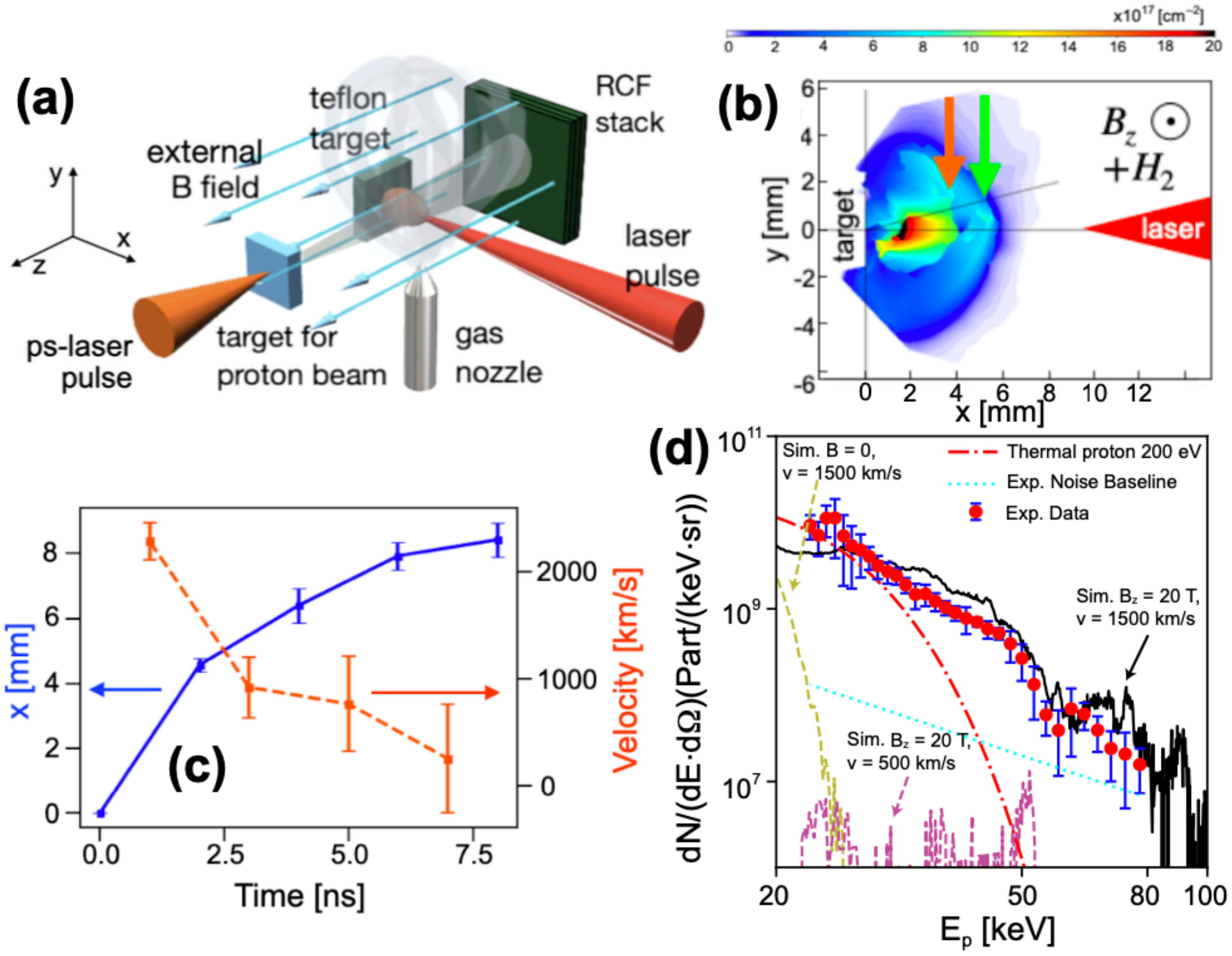


FIGURE 1. (a) Setup of the experiment to characterize a single magnetized shock (Yao *et al.* 2021, 2022). The whole scene is embedded in an H_2 gas of low density ($\sim 10^{18}$ cm^{-3}) emanating from a pulsed nozzle. Further, the whole assembly is embedded in a strong magnetic field (20 T). (b) Density measurement (integrated along the line of sight) 4 ns after the laser irradiation of the solid target. (c) Evolution of the shock front position along the x-axis and the corresponding velocity. (d) Evidence for the energization of protons picked up from the ambient medium. Proton energy spectra of both the experiment (red dots) and of three PIC simulations, the black solid line for the magnetized fast case with $B_z = 20$ T and initial shock velocity $v = 1500$ km/s, the yellow dashed line for the unmagnetized fast one with $B = 0$ and $v = 1500$ km/s, and the purple dashed line for the magnetized slow one with $B_z = 20$ T and $v = 500$ km/s, all measured at t=2.6 ns in the simulations. The analytical thermal proton spectrum is shown with the red dash-dotted line (200 eV); and the experimental noise baseline is shown in cyan dotted line.

Yao et al. 2022

Take away

1. With first/second order Fermi acceleration (or most mechanisms), we easily get power-laws

2. With first/second order Fermi acceleration, we easily get deviations from power-laws (or most mechanisms) -> interesting because we are entering an age where we can potentially probe this (radio to gamma-rays)

3. Usual questions of particles acceleration: mechanism, efficiency, slope, maximum energy

4. Usual questions of particles acceleration should also be: injection? Escape of particles? Level of turbulence

5. Don’t forget injection problem, deviations from power-law, and entangled problems

6. Mixed processes are very likely involved

References

Shock acceleration

Fermi, 1949, 54
Longair, High energy astrophysics
Drury, 1983
Bell, 2004, 2013
Blasi 2013, review
Sironi, Keshet, Lemoine, 2015, review

Fermi acceleration

Lemoine, 2019

Shear acceleration

Rieger 2019, review

Magnetic reconnection

Kagan 2015, review

Particle acceleration

Lectures by Blasi (2020), Gabici (2021) , Lemoine (Houches 2023)