Signature of nuclear matter properties on neutron star oscillations : equations of state representation and numerical simulations

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   - Context
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2. Relativistic hydrodynamics
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   - NS model and equations
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Context

- Neutron stars : high mass star collapse
- Mass $\sim 1.4\,M_\odot$, radius $\sim 10\,\mathrm{km}$
- First detection : 1967 pulsar by Jocelyn BELL
- First formation detection : 1987 supernova SN1987a (12 neutrinos)

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- Post-coalescence life ($\sim$few 0.1s) : hypermassive neutron star ($M > 3 - 4M_\odot$) then collapse into black hole

Goals

- Write 3D hypermassive neutron star evolution code : gravitational waves modes.

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[1] `https://lorene.obspm.fr`

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- Write 3D hypermassive neutron star evolution code : gravitational waves modes.
- Use different equations of state (EOS) in the code
- Code written using C++ and LORENE[1] (pseudospectral methods)
- Starting with spherical symmetry (1D) (no GW yet !)

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Content

General relativity : 3+1 decomposition

Let $(\mathcal{M}, \mathrm{g})$ be a spacetime associated with g a metric of signature (-,+,+,+) ; it is assumed that there exists a foliation of $\mathcal{M}$ i.e. a scalar field $\hat{t}$ so that iso-$\hat{t}$ hypersurfaces

$$\Sigma_t = \{p \in \mathcal{M},\ \hat{t}(p) = t\} \tag{1}$$

verify

$$t \neq t' \Rightarrow \Sigma_t \cap \Sigma_{t'} = \emptyset \text{ et } \bigcup_{t \in \mathbb{R}} \Sigma_t = \mathcal{M} \tag{2}$$

Anywhere a timelike vector n, orthogonal to Σ_t, is defined.

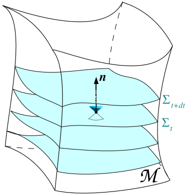

Figure: Illustration of a foliation

General relativity : 3+1 decomposition

Let us define an induced metric γ (3-metric) which signature is (+,+,+), its associated Riemann tensor R^i_{jkl}, Ricci, tensor $R_{ij} = R^k_{ijk}$, Ricci scalar $R = R^i_i$ (*intrinsic* curvature), the *extrinsic* curvature tensor K_{ij}, its trace $K = K^i_i$, the lapse function N, the shift vector β^i.

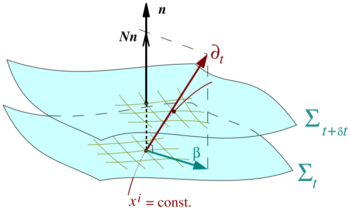

Figure: Illustration of the lapse and the shift

Knowing the lapse, the shift, and the 3-metric allows to fully determine $\mathcal{M}$'s geometric structure :

$$g_{\alpha\beta} = \begin{pmatrix} g_{00} & g_j \\ g_i & g_{ij} \end{pmatrix} = \begin{pmatrix} -N^2 + \beta_k\beta^k & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix} \tag{3}$$

→ Time and space are naturally separated

Model : spherically symmetric cold star

Assumptions

- Fast cooling : T is a fraction of T_F in a few minutes - $T = 0$ neutron star
- β equilibrium : $p + e^- \leftrightarrow n + \nu_e$
- Spherical symmetry : quantities depend on t and r only, all vectors are radial.

We use the perfect fluid model :

$$\boxed{T_{\alpha\beta} = (e+p)u_\alpha u_\beta + p g_{\alpha\beta}} \qquad (4)$$

u unitary timelike vector, e total energy density, p pressure.

- Code : 1D Chebyshev radial grid - 1 nucleus (star), 1 compactified domain.
- The boundary between the domains is comoving with the border of the star
- Hydrodynamical equations :

$$\nabla_\mu T^\mu_\nu = 0 \text{ and } \nabla_\mu \left(n_B u^\mu\right) = 0 \qquad (5)$$

$+$ Einstein equations :

$$^4R_{\alpha\beta} - \frac{1}{2}{}^4R g_{\alpha\beta} = 8\pi T_{\alpha\beta} \qquad (6)$$

written as PDE system $+$ equation of state to close the system i.e. relation between thermo variables

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Polytrope

A polytrope is a 1-parameter equation of state. For neutron stars it describes cold star at β-equilibrium. The parameter is the density of baryons in the star :

$$p(n_B) = \kappa n_B^\gamma \tag{7}$$

- κ : pressure coefficient
- γ : adiabatic index

$\gamma = 2$: approximation for nuclear matter ; $\gamma = 5/3$: non-relativistic Fermi gas ; $\gamma = 4/3$: ultra relativistic Fermi gas
Easy to use, analytical and convenient for numerical tests but not realistic.

Realistic EOS : instabilities

Realistic EOSs induce instabilities in the code

- Phenomenological models
- Nuclear experimental and astrophysical data are extrapolated
- Represented as discrete tables (p, e, ..., are tabulated)
- Insufficient precision on sound speed (numerical derivatives)
- Thermodynamical consistency is not always possible (crust/core)

→ Pseudo-polytropes fitting scheme (Jose PONS, Alicante, Spain)

Realistic EOS : representation

Drawbacks of tables :

- Lack of precision
- Lots of data for 2 or 3 parameters (thousands to millions of grid point)
- Longer computation time (interpolation)

Assets of a fitting scheme :

- Precision = machine accuracy
- No interpolation through tables
- Light storage (pseudo-polytropes : a dozen coefficients for 2-param EoS + a formula)

But :

- Crust ($n_B < 0.08\mathrm{fm}^{-3}$) difficult to handle : "removed"
- Losing physical features

Realistic EOS : pseudo-polytrope

Pseudo-polytropes (internal free energy parametrization) :

$$f(n_B) = m_B g(n_B) n_B^\alpha \tag{8}$$

$+$ thermo principles. In practice :

$$g(x = \ln n_B) = a + bx + cx^2 \tag{9}$$

Parameters a, b, c fitted on f/n_B, $\ln(p/n_B)$ and $\Gamma_1 = \frac{\mathrm{d}\ln p}{\mathrm{d}\ln n_B}$ on the interval $[n_1, n_{\mathrm{max}}]$. Below n_1 a polytrope is chosen, with parameters adjusted for thermo consistency. Polytrope recovered by : $g(x) = cst$ and $\gamma \equiv \alpha + 1$.

Generalization to two parameters x, y (β-eq relaxed) :

$$g(x, y) = a(y) + b(y)x + c(y)x^2 \tag{10}$$

$+$ simplified model for the crust

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APR EoS [APR98] : MR diagram = equilibrium sequence

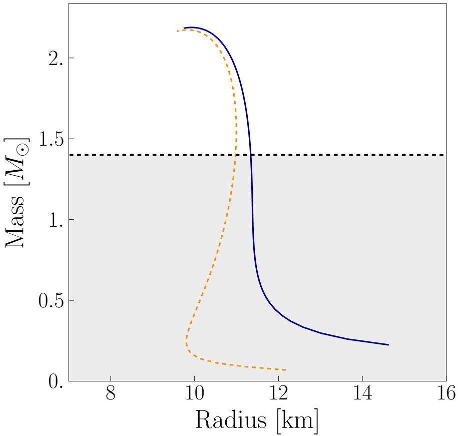

Figure: Mass-radius diagram for APR

Legend :

- Solid blue : 1-param version of APR in CompOSE
- Dashed orange : 1-param pseudo-polytropic fit

Features :

- Maximum mass
- Radii difficult to reproduce
- Different crust ($M < 1.4 M_\odot$)

Frequency extraction : principle for "$l = 0$"

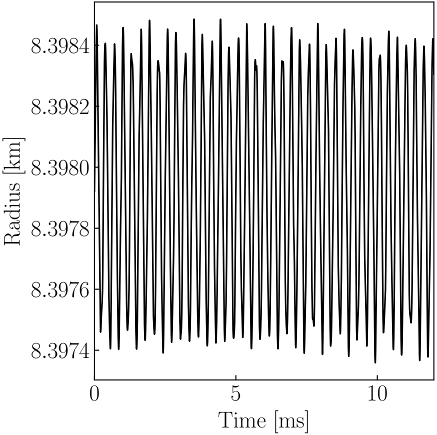

Figure: Radius vs time (EoS = APR)

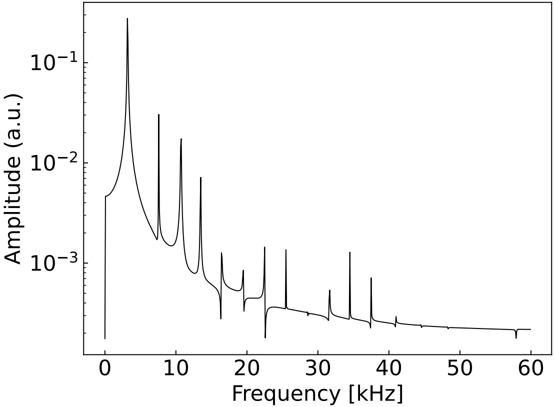

Figure: $\hat{R}(f)$ spectrum

EoS APR : Mf diagram.

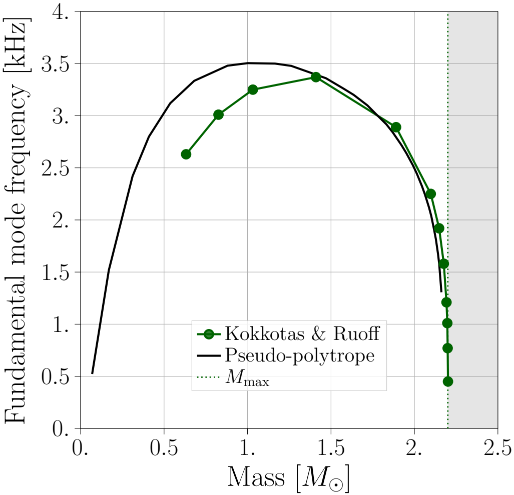

Figure: Mass-frequency diagram for APR. ref : Kokkotas & Ruoff [KR01]

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Conclusions, outlook

Recap :

- Fitting scheme that produces close MR and Mf diagrams for higher masses ($M > 1.4M_\odot$).
- Asset : economical storage and reduced computing time

Outlook :

- Paper on representation of 1- and 2-param EoS to be submitted
- EoS representation : 3-parameters i.e. hot NS
- 3D code
- Multi-domain inside the star

Thank you

Bibliography I

[APR98] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, *Equation of state of nucleon matter and neutron star structure*, Phys. Rev. C **58** (1998), no. 3, 1804–1828 (en).

[KR01] K. D. Kokkotas and J. Ruoff, *Radial oscillations of relativistic stars*, A&A **366** (2001), no. 2, 565–572.

Frequency extraction : principle

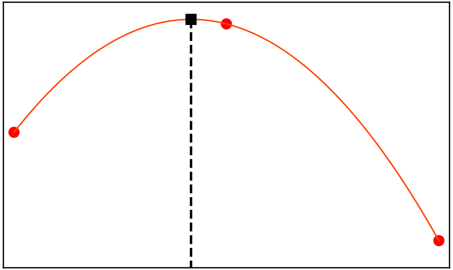

Figure: Frequency extraction principle

APR thermo : table vs fit