Fixing the dynamics in scalar-Gauss-Bonnet gravity

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Based on the works made in collaboration with Miguel Bezares, Enrico Barausse and Luis Lehner
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Why gravity beyond GR in a nutshell

usually based on two main arguments:

- no description of gravitational effects in quantum environments

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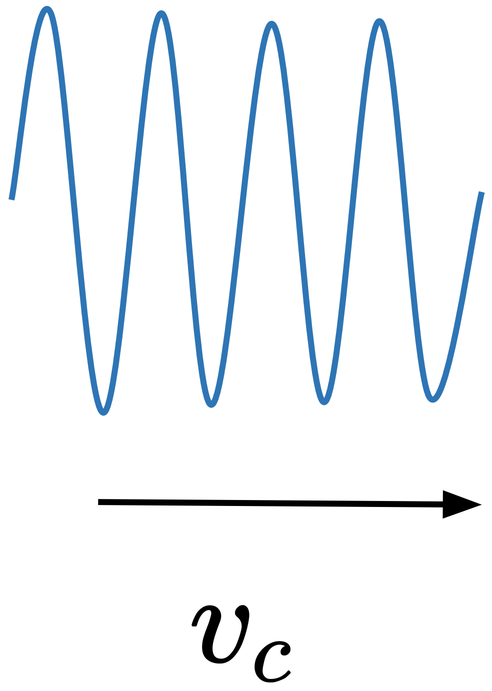
a different perspective: testing GR with black holes

Well-posedness beyond GR

- necessary to make prediction with numerical evolutions
- complex mathematical problem (not entering into details)
- focus on characteristic speeds
- derivative interactions can make them diverge or become imaginary dynamically

Well-posedness beyond GR

Characteristic speeds



- Take a wave
- Let it propagate in the theory
- Characteristic speeds set by feature of the theory: most important is the highest-derivative operators
- Depends on the dynamical evolution of the system under consideration

Example: Scalar Gauss-Bonnet gravity

$$\mathcal{G} = R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2$$

The action

$$S_{GB} = \frac{1}{2}\int \sqrt{-g}\left[R - \frac{1}{2}\partial_a\phi\partial^a\phi + f(\phi)\mathcal{G}\right]$$

The equations of motion

$$R_{ab} - \frac{1}{2}g_{ab}R = T^{(\phi)}_{ab} + T^{(\mathcal{G})}_{ab}$$

$$\Box\phi = -f'(\phi)\mathcal{G}$$

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Scalar Gauss-Bonnet gravity

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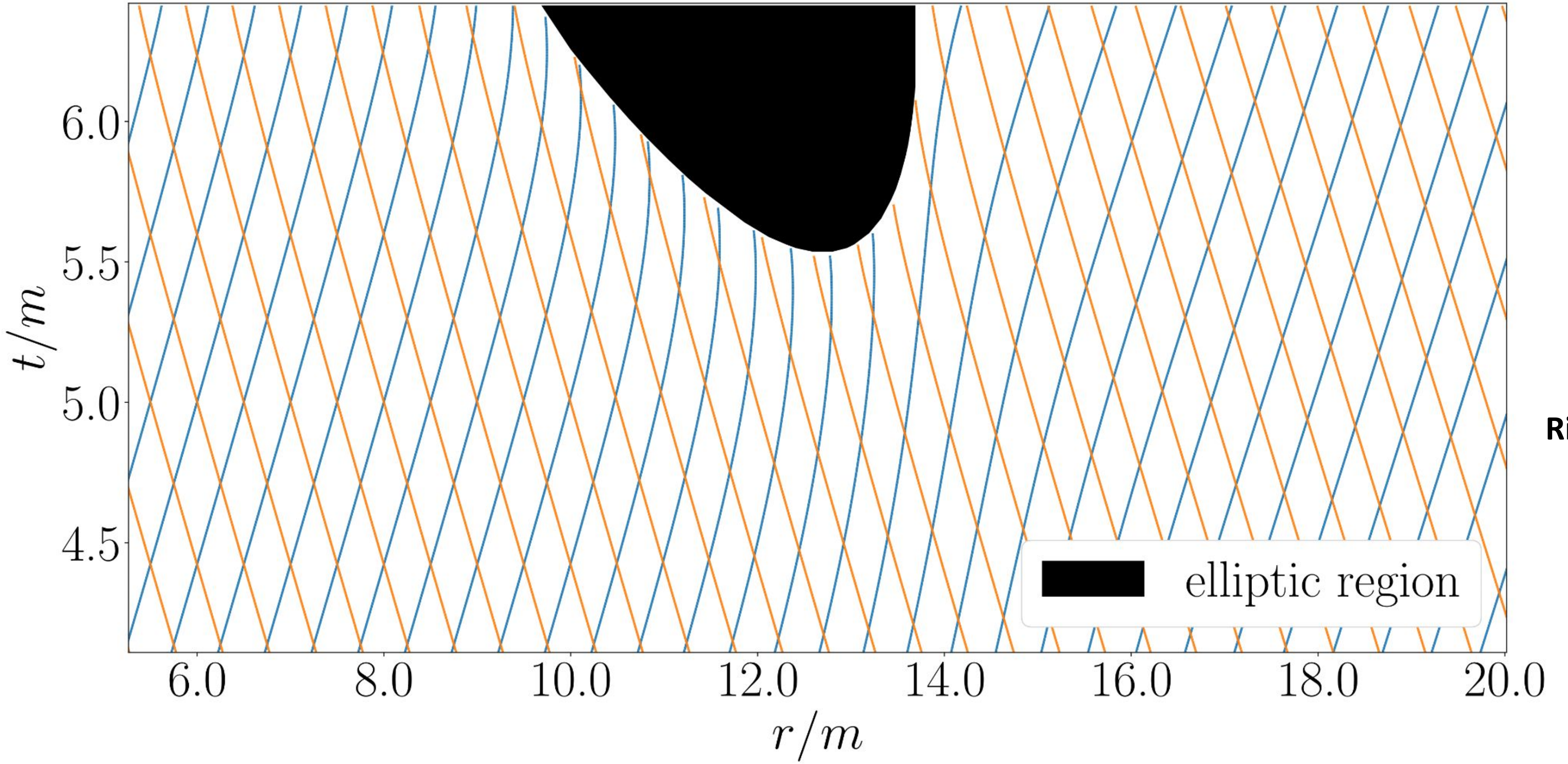

Ripley, Pretorius 2019

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The equations of motion

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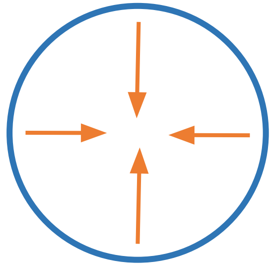
$$\Box \phi = \cancel{-f'(\phi)\mathcal{G}} + \Sigma$$

$$\xi \Box \Gamma_{ab} = \Gamma_{ab} - T^{(\mathcal{G})}_{ab}$$

$$\xi \Box \Sigma = \Sigma + f'(\phi)\mathcal{G}$$

Spherical collapse in fixed sGB

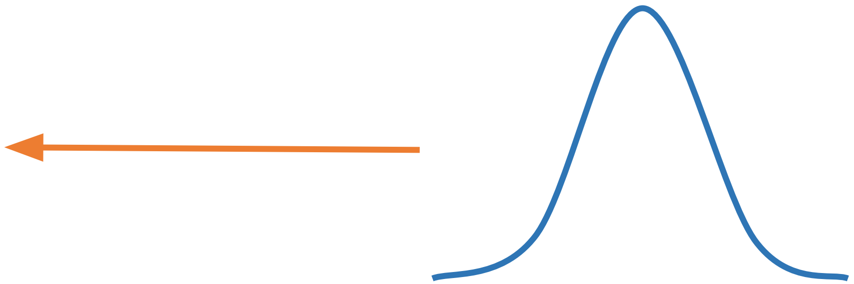


Assume spherical symmetry for metric and scalar

$$ds^2 = -e^{2A(t,r)}dt^2 + e^{2B(t,r)}dr^2 + r^2 d\Omega^2$$

$$\phi = \phi(t,r)$$

Let evolve a gaussian scalar profile under gravity to form an apparent horizon



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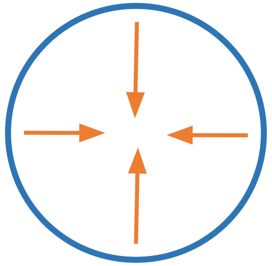

Characteristic speeds

Full theory

$$v_{\pm} = \mathcal{C} \pm \sqrt{\mathcal{D}}$$

Complicated functions with derivatives of metric and scalar

Fixed theory

$$v_{\pm} = \pm e^{A-B}$$

Always real and finite

Spherical collapse in fixed sGB

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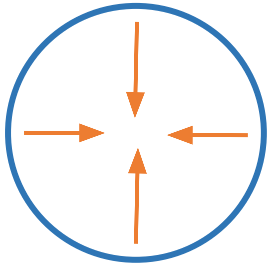

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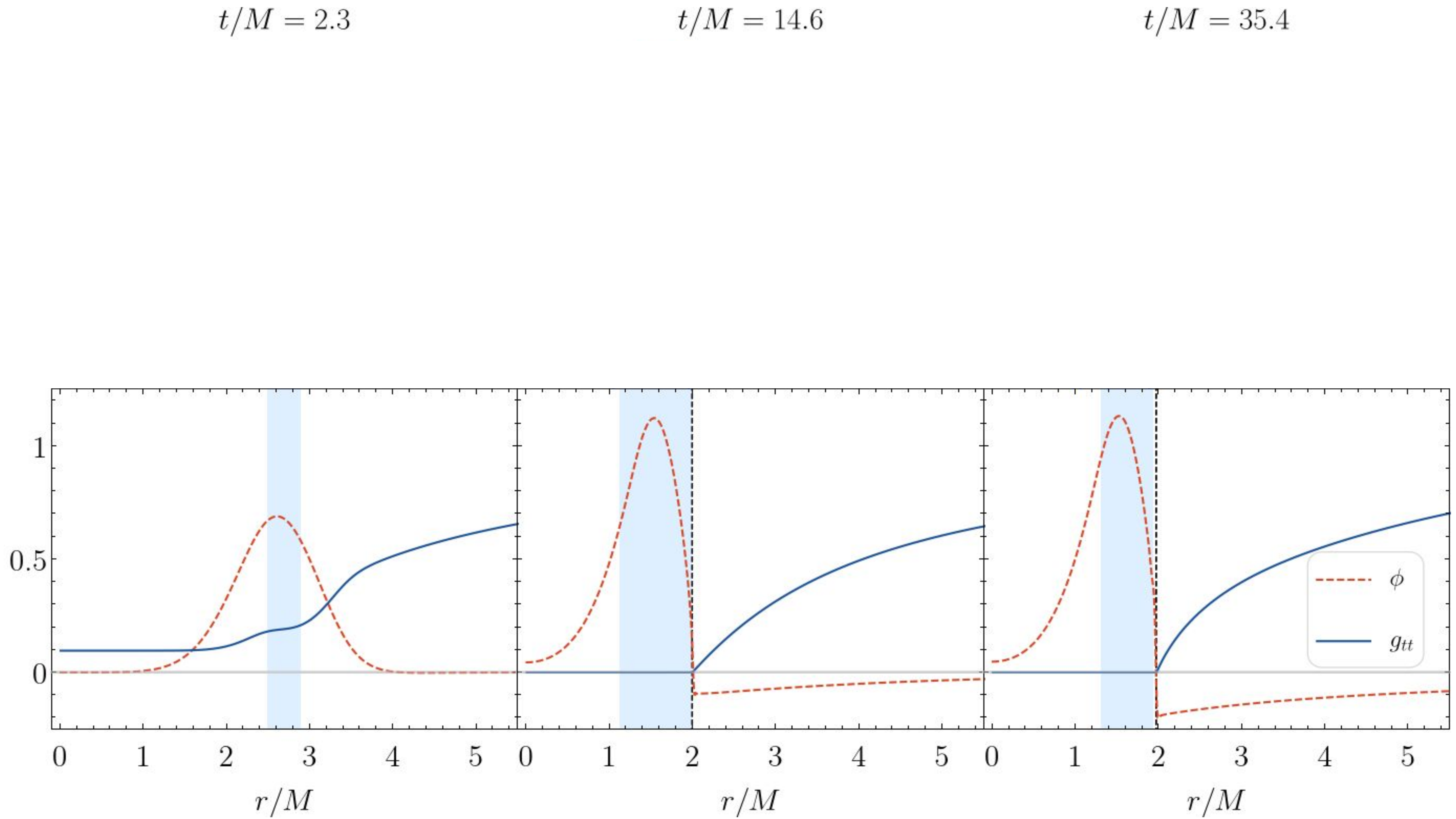
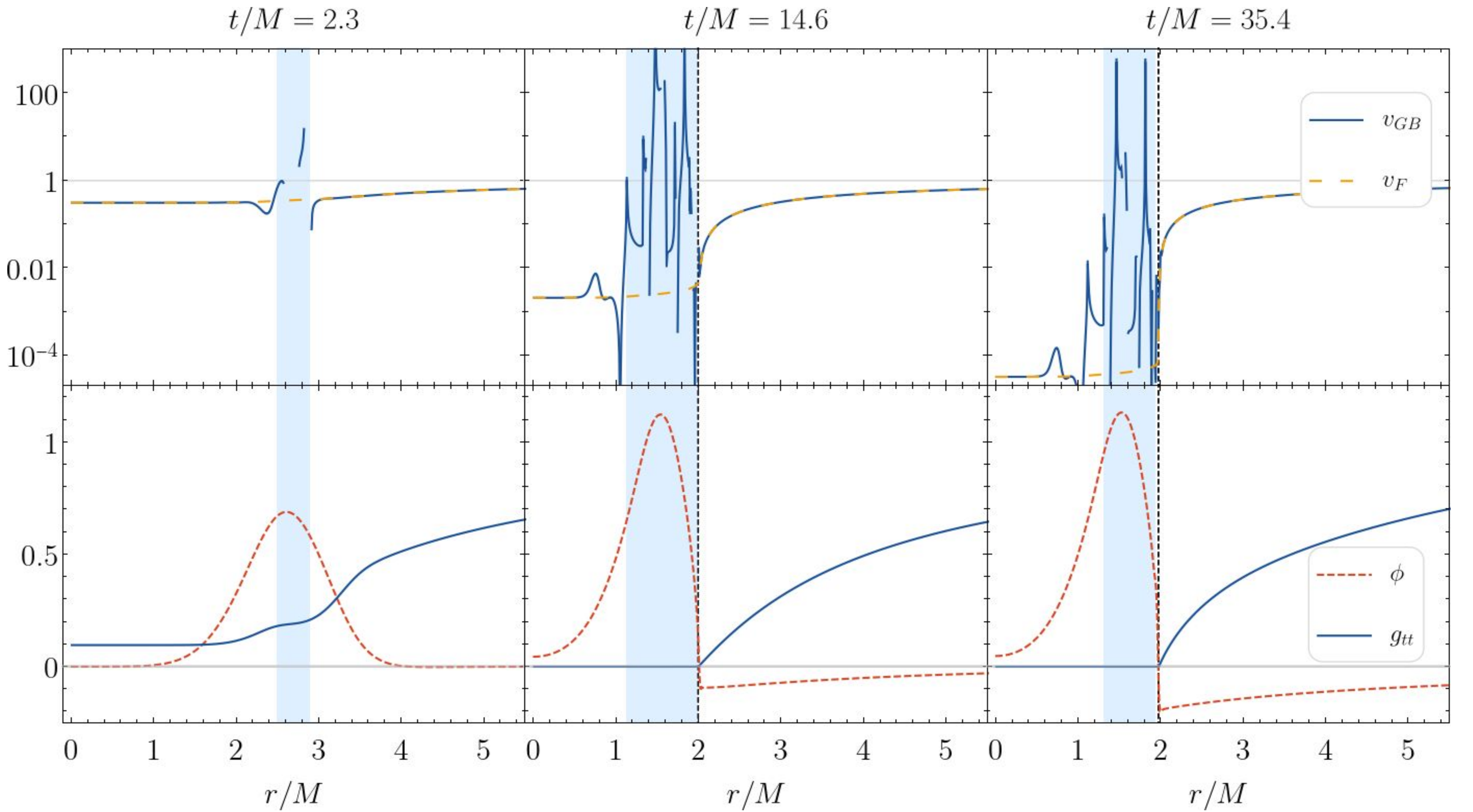
Some visual results from NF, Bezares, Barausse, Lehner 2022



t/M = 2.3
t/M = 14.6
t/M = 35.4
1
0.5
0
0 1 2 3 4 5
r/M
φ
g_tt



t/M = 2.3
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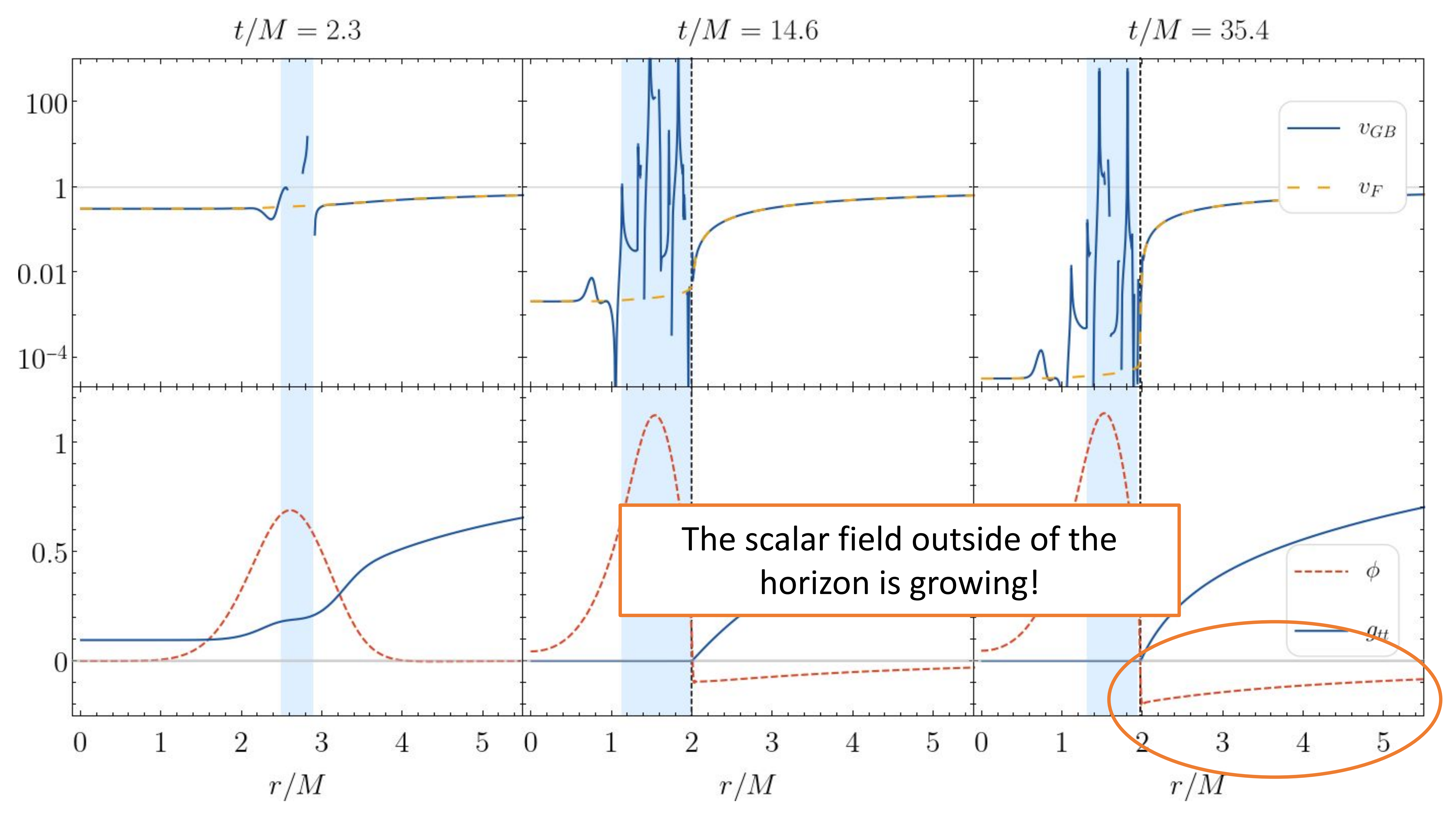

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The scalar field outside of the horizon is growing!

Spherical collapse in fixed sGB

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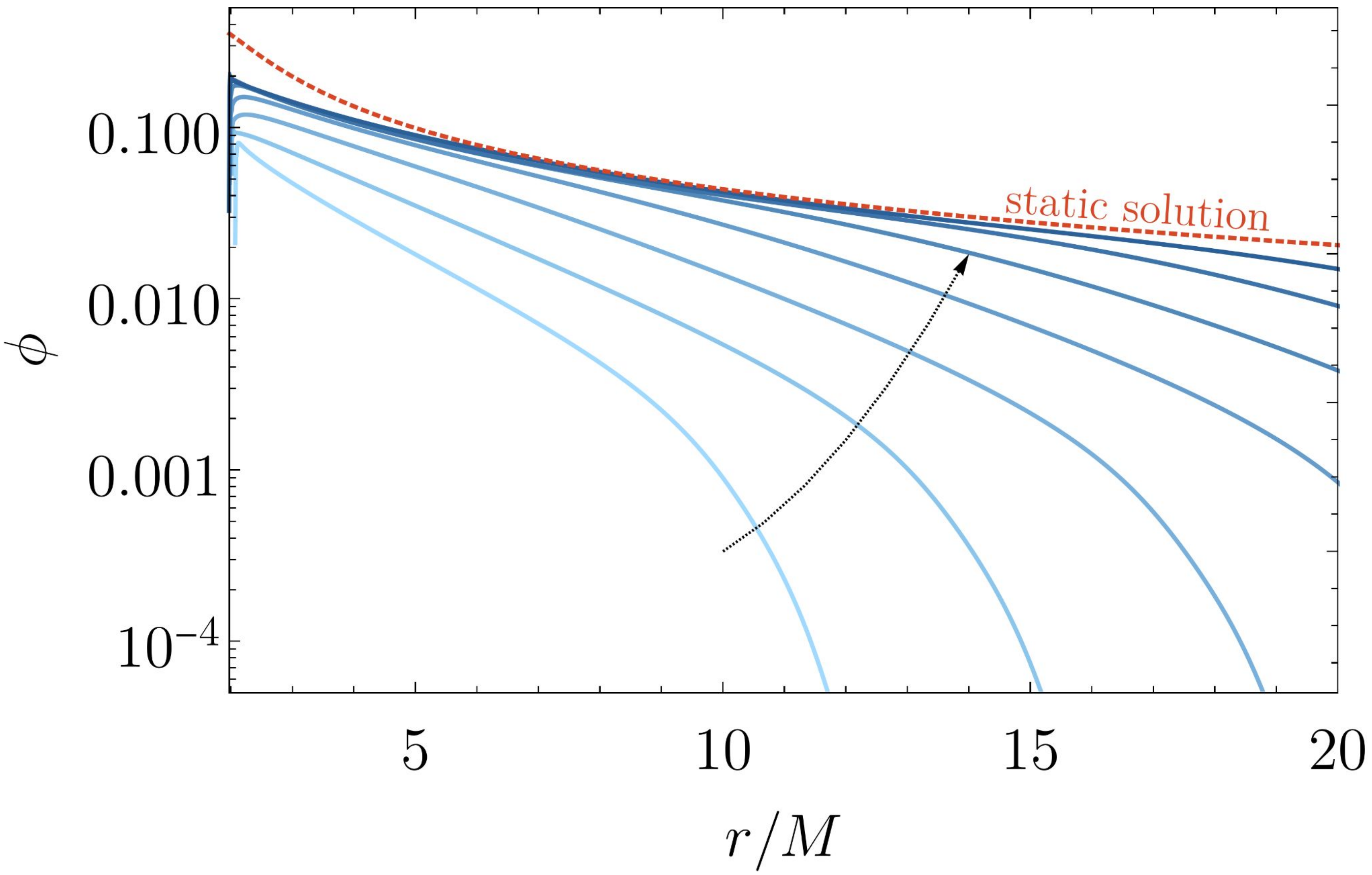

$$f(\phi) = \frac{1}{8}\left[\eta\phi^2 + \frac{1}{2}\zeta\phi^4\right]$$

BH scalarization

[Silva, Macedo, Sotiriou, Gualtieri, Sakstein, Berti 2019]

$$\eta = 6$$

$$\zeta = -6\eta$$

$$\xi = \eta$$

Conclusions

- Beyond GR theories can be ill-posed
- Possible solution $\longrightarrow$ *fixing-the-equations* [Cayuso, Ortiz, Lehner 2017]
- Spherical collapse faithful in the beginning and in the endstate

Open issues

- Application of the method to 3+1 problems (BH-BH or NS-NS binaries) for the generation of GW templates

- Can we trust the *fixing* in the dynamical region?

- Are all the *fixing* the same?

Thanks for the attention =D