MBHB/SBHB parameter estimation with lisabeta

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LPC CAEN

- Status of lisabeta
- LISA response
- Example PE for MBHBs
- Example PE for SBHBs
- Improving the search and sampling for MBHBs
- Demonstration notebooks

Objectives and scope

- Science prospective
- Prototyping real analysis (LDC)
- Source types: MBHBs, SBHBs for now — GBs soon
- Consortium-available (full members, public soon)

https://gitlab.in2p3.fr/marsat/lisabeta https://gitlab.in2p3.fr/marsat/lisabeta_release

Levels of approximation

MBHBs:

- Fisher: for high SNR limit (depends on signal !) √
- Set noise realization to 0 \checkmark
- Initialize MCMC from Fisher \checkmark
- Full run with initialization from priors \checkmark
- Full run with noise \checkmark (refactoring)
- Superposition of sources, unknown noise, noise artifacts... X

(SBHBs less advanced)

Tools implemented

- SNR computations
- MCMC: ensemble sampler with parallel tempering (ptemcee)
- Nested sampling: pymultinest
- Informed proposals to deal with sky degeneracies
- Fast likelihoods
- Waveforms: PhenomD, PhenomHM

Costs

- SNR: few ms
- Fisher: <100ms at high-M, worse at low-M
- Likelihood: MBHB 2-3ms, SBHB 3-5ms
- Inference with Fisher init. (best case):
 ~ICPUh
- Inference of complicated posterior, with noise: 100-200 CPUh

LISA response

LISA orbits



LISA frame

Short-lived signals: use frame based on LISA plane



Response

From spacecraft s to spacecraft r through link s:

$$y_{slr} = \frac{1}{2} \frac{1}{1 - \hat{k} \cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l \quad \text{interferometry}$$
(TDI)

Fourier-domain (separation of timescales [Marsat-Baker 2018])

$$\mathcal{T}_{slr} = \frac{i\pi fL}{2} \operatorname{sinc} \left[\pi fL \left(1 - k \cdot n_l\right)\right] \exp\left[i\pi f \left(L + k \cdot \left(p_r + p_s\right)\right)\right] n_l \cdot P \cdot n_l(\boldsymbol{t_f})$$

Time and frequency-dependency Time: motion of LISA on its orbit Frequency: departure from long-wavelength



Parameter estimation in practice: ptemcee

Ensemble sampling

- Evolve an ensemble of chains (walkers)
- Choose randomly another chain, propose new point with stretch-move
- Idea: the proposal adapts to the current state of the chains

Parallel tempering

- Evolve an ensemble of chains (walkers)
- Choose randomly another chain, propose new point with stretch-move
- Idea: the proposal adapts to the current state of the chains

Tailored proposals and map

- Known degeneracies in the sky position
- Branch with a jump proposal for degenerate locations
- Parameter map for most efficient sampling





Response decomposed

$$\mathcal{T}_{slr} = \frac{i\pi fL}{2} \operatorname{sinc} \left[\pi fL \left(1 - k \cdot n_l\right)\right] \exp\left[i\pi f \left(L + k \cdot \left(p_r + p_s\right)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

+ Doppler phase (delay to the center of constellation):

 $\exp\left[2i\pi \boldsymbol{f}\boldsymbol{k}\cdot\boldsymbol{p}_0(\boldsymbol{t}_{\boldsymbol{f}})\right]$

Time and frequency-dependency in transfer functions Time: motion of LISA on its orbit Frequency: departure from longwavelength approx.

Response:

- 'Full': keep all terms
- 'Frozen': ignore LISA motion
- 'Low-f': ignore f-dependency
- 'Frozen Low-f': ignore both



MBHB signal: Higher harmonics in the waveform



MBHB signal: heterodyned likelihood

Decomposing the likelihood:

$$\ln \mathcal{L} = -\frac{1}{2}(s - d|s - d)$$

= $-\frac{1}{2}(s - s_0|s - s_0) + (s - s_0|d - s_0) - \frac{1}{2}(s_0 - d|s_0 - d)$

Residuals from reference waveform:

 $s_{\ell m} = r_{\ell m} e^{i \Phi^0_{\ell m}}$

Implementation:

$$(s - s_0 | s - s_0) = \sum_{\ell m} \sum_{\ell' m'} (r_{\ell m} r^*_{\ell' m'} | e^{i(\Phi^0_{\ell' m'} - \Phi^0_{\ell m})})$$
$$(s - s_0 | d - s_0) = \sum_{\ell m} (r_{\ell m} | e^{-i\Phi^0_{\ell m}} (d - s_0))$$

- Fix a sparse frequency grid (~128)
- Linear interpolation of the residuals, mode-by-mode
- Precompute 0-th and 1st polynomial inner products against phase and data terms, with a fine resolution



PE for MBHBs: degeneracies and role of HM



PE for MBHBs: LDC-I result



SBHB signal: Fourier-domain signal and response



PE results: SBHBs



MBHB burn-in: need for an efficient search

Likelihood of individual walkers in the ensemble (each color a walker):



Ideal sampling, no burn-in

Using parameter estimation settings for search - non optimal



LISA data - band-passing, whitening



- **Band-passing**: select frequencies below 2mHz
- Whitening: work with signal/noise, so that all frequencies/times contribute equally

LISA data - band-passed, whitened in time domain



LISA data - band-passed, whitened in time domain

Whitened, band-passed data 2010TDI A 0 -10-201.11 1.12 1.13 1.14 1.15 1.101.16t (s) $\times 10^7$ sigma-thresholding for detection 100 -104.7954.7854.7904.8054.800 $\times 10^{6}$ 5.02.50.0 -2.5-5.08.746 8.742 8.744 8.750 8.748

MBHB initial search: F-statistic on small data segments



MBHB initial PE: sampling with low frequencies



MBHB final PE: sampling with all frequencies



- Reminder about LISA sources and signals: MBHBs, GBs
- Data analysis the basics
- Status of lisabeta
- Example PE for MBHBs
- Improving the search for MBHBs

• Improving the sampling for MBHBs

MBHB sampling with degeneracies: need efficient method



MBHB sampling with degeneracies: parameter map



- Ignoring motion and high-frequency effects: response becomes very simple
- Use variables as close as possible to what we really observe, to make the posterior look Gaussian
- Sampling can be done in any set of parameters, with Jacobian of the transformation analytic here

Response variables: 2 complex (+ 2 sky angles)

$$\sigma_{+} = \rho e^{2i\varphi} \left[t_{\theta}^{4} e^{-2i\psi_{L}} + t_{\iota}^{4} e^{2i\psi_{L}} \right] e^{-2i\lambda_{a}} ,$$

$$\sigma_{-} = \rho e^{2i\varphi} \left[e^{-2i\psi_{L}} + t_{\theta}^{4} t_{\iota}^{4} e^{2i\psi_{L}} \right] e^{2i\lambda_{a}} ,$$

$$\rho(d,\iota,\theta_L) = \frac{1}{4d} \sqrt{\frac{5}{\pi}} \frac{1}{\left(1 + t_{\iota}^2\right)^2 \left(1 + t_{\theta}^2\right)^2}$$

MBHB sampling with degeneracies: parameter map

Toy problem, completely degenerate extrinsic 22 likelihood without motion and high-f effects



MBHB sampling with degeneracies: parameter map

Transformed parameters



Original (physical) parameters

Demonstration notebooks

- <u>https://gitlab.in2p3.fr/marsat/lisabeta/-/blob/master/</u> <u>examples/example_mbhb.ipynb</u>
- <u>https://gitlab.in2p3.fr/marsat/lisabeta/-/blob/master/</u> examples/ldc1_mbhb_demo.ipynb

Noise properties

Noise PSD

- Noise autocorrelation function: (stationarity: depends only on τ)
- Noise PSD formal definition:
- Stationarity: independance in FD $\langle \tilde{n}(f)\tilde{n}^*(f')\rangle = \frac{1}{2}S_n(f)\delta(f-f')$
- Gaussianity: noise in freq. bins is Gaussian



$$K(\tau) = \langle n(t)n(t+\tau) \rangle$$

$$S_n(f) = 2 \int d\tau e^{2i\pi f\tau} K(\tau)$$
$$\mathsf{D}_{-} \langle \tilde{\varphi}(f) \tilde{\varphi}^*(f') \rangle = \frac{1}{2} S_{-}(f) \delta(f)$$

$$\tilde{n}(f) \sim \mathcal{N}(0, \frac{1}{2\Delta f}S_n(f))$$



Less-than ideal assumptions for LISA ! Non-stationarity, glitches...

Likelihood

- Likelihood: $\mathcal{L} = p(\text{data}|\text{signal params})$
- PDF of the noise: collection of independent $\ln p(n = n_i) = \text{const} \frac{1}{2} \sum_i \Delta f \frac{2}{S_n(f)} |\tilde{n}_i|^2$ Gaussian noise variables in each bin
- Likelihood is the probability that the noise makes up for the difference between observed data and theoretical signal: $d = h(\theta) + n$ $\ln p(d|\theta) = \ln p(n = d - h(\theta)) = -\frac{1}{2}(d - h(\theta)|d - h(\theta))$

Bayesian formalism

• Matched-filtering overlap:
$$(h_1|h_2) = 4 \operatorname{Re} \int df \, \frac{\tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)}$$

• For Gaussian, stationary noise, for independent channels:

$$\ln \mathcal{L}(d|\theta) = -\sum_{\text{channels}} \frac{1}{2} (h(\theta) - d|h(\theta) - d)$$

$$d = h(\theta_0) + n_0$$

• Bayes theorem defines the posterior:

$$\theta|d) = \frac{\mathcal{L}(d|\theta)p_0(\theta)}{p(d)}$$

- h GW signal
- θ parameters
- d data stream
- θ_0 signal params.
- n_0 noise real.
- S_n noise PSD

 $\begin{array}{c} p_0(\theta) \text{ prior} \\ p(d) \text{ evidence} \end{array}$

p(

Signal-to-noise (SNR)

Measures loudness of signal:

$$\mathrm{SNR}^2 = (h|h) = 4 \int \frac{df}{S_n} |h|^2$$

Simple detection statistics: SNR>8-10 (true detection statistics LIGO/Virgo more complicated)

Fisher matrix analysis

• Quadratic expansion of log-likelihood around true signal, approx. likelihood as a Gaussian

$$h(\theta) = h(\theta_0) + \Delta \theta_i \partial_i h + \dots$$

$$\ln \mathcal{L} = -\frac{1}{2} \Delta \theta_i F_{ij} \Delta \theta_j + \mathcal{O}(\Delta \theta^3)$$

$$F_{ij} = (\partial_i h | \partial_j h)$$

- Matrix inversion to get to the covariance of the Gaussian $C = F^{-1}$
- Valid at high SNR, and misses degeneracies

Bayesian sampling tools

- MCMC methods, nested sampling
- MCMC proposals: ensemble samplers (emcee), differential evolution, ...
- Parallel tempering: explore full parameter space
- Informed proposals to deal with degeneracies

Levels of approximation

- Fisher: for high SNR limit (depends on signal !)
- Set noise realization to 0
- Initialize MCMC from Fisher
- Full run with initialization from priors
- Full run with noise
- Superposition of sources, unknown noise, noise artifacts...

GW signals seen by LISA - the basics



LISA: different BHB signals

- **MBHBs**: very loud, mergerdominated (mostly short)
- **SBHBs**: early inspiral, some chirping during LISA obs. (multiband ?)
- **GBs**: quasi-monochromatic, superposed
- **EMRIs**: long-lived, many harmonics
- Stochastic backgrounds
- TDEs !

Phases of the signal:

- **Inspiral**: covered by post-Newtonian (PN) perturbative series
- **Merger**: covered only by numerical relativity (NR)
- **Ringdown**: NR, superposition of Quasi-Normal Modes (QNM)



Contrasting LIGO/Virgo and LISA responses: LISA

LISA-frame



Low-f approximation: **two LIGO-type detectors** in motion [Cutler 1997]



High-f: **three channels** with complicated frequency-dependence

Sky localisation from the modulations induced by the orbits for long-lived signals

Sky localization can also come from high-f effects.

Degeneracies - multimodality in the sky possible !

Massive black holes: signals and challenges

- Very loud sources, SNRs of several thousands !
- Detection of merger easy, but detection as early as possible ?
- Advance localization for multimessenger observations ?
- Signals can be short (< I day) and degenerate
- Waveform model systematics for such loud signals ? Biases, residuals for other sources ?
- Subdominant features in the signal are important



Galactic binaries: signals and challenges

- Mostly WD-WD, some other compact objects
- Full galaxy: ~20 million systems !
- About ~20000 individually resolvable
- Form a (non-stationary) background
- Verification binaries
- Quasi-monochromatic GW emitters
- Modulation by LISA motion (sidebands in Fourier-domain)
- Superposition of signals in Fourier-domain



Extreme mass ratio inspirals, stellar-mass black holes

EMRIs

- Long-lived, complex signals, large number of wave cycles $(10^4 10^5)$
- Strong precession and eccentricity features, orbits in the relativistic regime around Kerr
- Exquisite determination of some parameters also means that the signals are hard to find !
- Theoretical work on waveform models needed

Stellar-mass BHs

- Quiet signals: a few detections in the LISA band
- Inspiral regime far from merger, very large number of cycles $(10^5-10^6)\,$
- Challenge of detection: template banks impossible
- Multiband analysis, archival searches ?





LISA data - LDC-2 Sangria



- **MBHBs**: chirping signals, emerging from low-f noise
- **GBs**: quasi-monochromatic, horizontal lines

LISA data - LDC-2 Sangria Time-Domain



- **MBHBs**: loudest ones clearly visible by eye above the noise
- **GBs**: superposed signals, annual modulation due to the LISA motion

LISA data - LDC-2 Sangria Frequency-Domain



- **MBHBs**: loudest ones visible in the spectrum, subdominant
- **GBs**: signals local in frequency, both individually resolvable and building up a background

LISA Fourier-domain response

Response

Laser frequency shift, spacecrafts s to r through link I: $y = \Delta \nu / \nu$

$$y_{slr} = \frac{1}{2} \frac{1}{1 - \hat{k} \cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$



Transfer function for modulated and delayed signal $FT[F(t)h(t + d(t))] = \mathcal{T}(f)\tilde{h}(f)$

Fourier-domain for **chirping signals** (separation of timescales):

$$\mathcal{T}_{slr} = \frac{i\pi fL}{2} \operatorname{sinc} \left[\pi fL \left(1 - k \cdot n_l\right)\right] \exp\left[i\pi f \left(L + k \cdot \left(p_r + p_s\right)\right)\right] n_l \cdot P \cdot n_l(t_f)$$

Time and frequency-dependency Time: motion of LISA on its orbit Frequency: departure from long-wavelength

+ Time-delay interferometry (TDI)

linear combinations of y_{slr} with more delays

LISA mission - 2034







LISA sources 10⁻¹⁶ Galactic Background $\underset{{}^{\bullet}}{\overset{\rm hour}{10^7}}M_{\odot}$ MBHBs at z = 3day 10⁻¹⁷ Verification Binaries EMRI Harmonics month Characteristic Strain day _ LIGO-type BHBs hour 10⁻¹⁸, $10^{6} M_{\odot}$ GW150914 Gal. Bin. (SNR > 7)ear mont $10^{5} M_{\odot}$ 10⁻¹⁹ 10⁻²⁰ Observatory Characteristic Strain 10⁻²¹ Total 10⁻³ 10⁻² 10⁻⁴ 10⁰ 10⁻⁵ 10⁻¹ Frequency (Hz)

Terminology:

- Massive black holes binaries (MBHBs)
- Stellar-mass black hole binaries (SBHBs): masses observable by ground-based detectors [Sesana 2016]
- Galactic Binaries (GBs): mostly WD-WD
- Extreme Mass Ratio Inspirals (EMRIs)
- Sochastic backgrounds (GBs, cosmo.)
- TDEs !



Contrasting LIGO/Virgo and LISA responses: LIGO/Virgo

Pattern functions

Simple multiplicative response

 $s = F_+ h_+ + F_\times h_\times$

Angular dependence:

$$F_{+} = \frac{1}{2} \left(1 + \cos^{2} \theta \right) \cos \left(2\phi \right) ,$$

$$F_{\times} = \cos \theta \sin \left(2\phi \right)$$

Time-of-arrival triangulation

- Two detectors: ~ring on the sky
- Better localization for 3 or more detectors (even low SNR!)



One-arm frequency observables

From spacecraft s to spacecraft r through link s: $y = \Delta \nu / \nu$

$$y_{slr} = \frac{1}{2} \frac{1}{1 - \hat{k} \cdot n_l} n_l \cdot (h(t_s) - h(t_r)) \cdot n_l$$

$$t_s = t - L - \hat{k} \cdot p_s, \quad t_r = t - \hat{k} \cdot p_r$$

$$h = h_+ P_+(\hat{k}) + h_\times P_\times(\hat{k}) \quad \text{GW at SSB}$$

Time-delay interferometry (TDI)

- Crucial to cancel laser noise
- First generation: unequal arms
- Second generation: propagation and flexing
- Michelson X,Y,Z Uncorrelated noises A,E,T

Approximations

- Long-wavelength approximation: two moving LIGOs rotated by $\,\pi/4\,$ + orbital delay
- Rigid approximation (order of the delays does not matter, delay=L simple in Fourier domain)



$$-\underbrace{\left[(y_{31}^{\text{GW}} + y_{13,2}^{\text{GW}}) + (y_{21}^{\text{GW}} + y_{12,3}^{\text{GW}})_{,22} - (y_{21}^{\text{GW}} + y_{12,3}^{\text{GW}}) - (y_{31}^{\text{GW}} + y_{13,2}^{\text{GW}})_{,33}\right]_{,2233}}_{X^{\text{GW}}(t-2L_2-2L_3)\simeq X^{\text{GW}}(t-4L)}$$