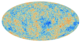
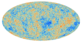
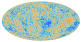
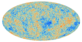
Cosmological master equations

$\frac{\mathrm{d}}{\mathrm{d}t}|$  $\rangle\langle$  $| = \mathcal{V}\Big(|$  $\rangle\langle$  $|\Big)$

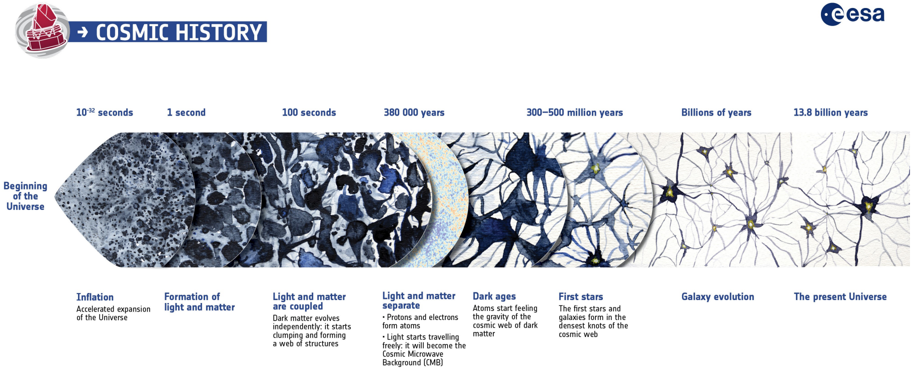
Thomas Colas

GdR OG

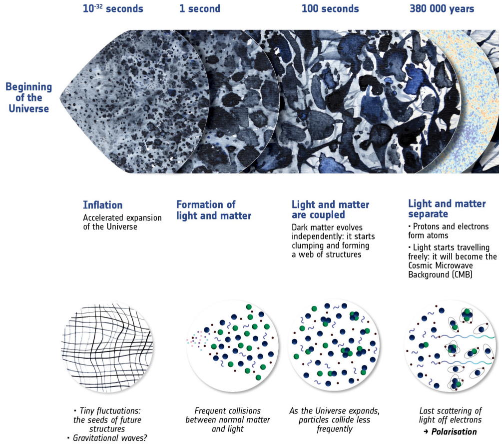
Outline

1. From cosmology to quantum optics
2. Master equations in cosmology
3. Benchmarking cosmological master equations

The standard model of cosmology



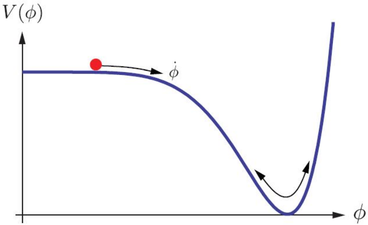
Quantum origin of cosmic inhomogeneities



Quantum fluctuations of the primordial vacuum seed all the structures of the Universe ⇒ Understanding this mechanism is crucial.

Simplest attempt: slow-roll inflation

- A **single scalar field**, the inflaton ϕ, slowly rolling along its potential for a sufficiently long time:



$\Rightarrow$ exponentially expanding universe (quasi de-Sitter)

1. shrinking Hubble sphere: **solve the Hot Big Bang puzzles**;
2. quantum fluctuations amplification: **seed structure formation**.

The primordial cosmology curse

1. All **observations** are consistent with single-field inflation.
2. All early universe **models** rely on multifield settings.

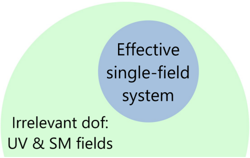
As early universe physicists, we need to understand:

- The emergence of single-field slow-roll pheno from multifield models;
- The **role of extra fields** during inflation.

A minimal approach

Three observations:

1. **Single-field slow-roll inflation** provides an excellent fit of the data.
2. At some point, inflation must end: couple to **SM fields**.
3. **UV-completions** of inflation often introduce new degrees of freedom.



What is the quantum description of the effective single-field system?

Outline

An example of extra ingredient: spectator field

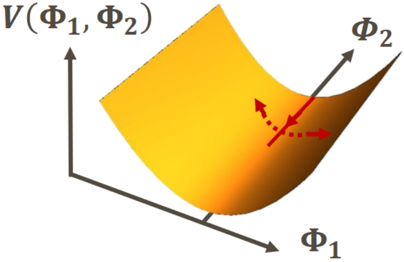
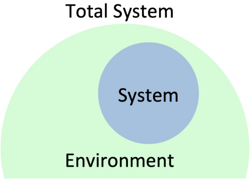


Figure: Adiabatic and entropic perturbations [Credits: L. Pinol]

- *WEFT result*: field stabilised, just a speed of sound at linear order.
- *OQS result*: curvature perturbations decohere while interacting with isocurvature modes [Prokopec & Rigopoulos, 2007].

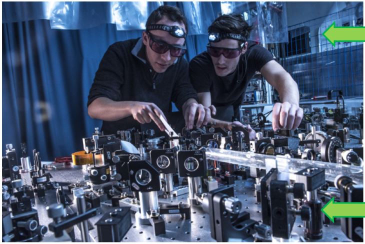
The early universe as an Open Quantum System (OQS)



- By integrating out the environment, the system dynamics becomes non-unitary.
- Cosmological perturbations are described by an OQS with dissipation and decoherence.
- They experience energy exchange and information loss into the environment.

Can we build an effective formalism which encompasses WEFT unitary results and OQS non-unitary evolution ?

The lab-based experiments wisdom



Credits: LKB

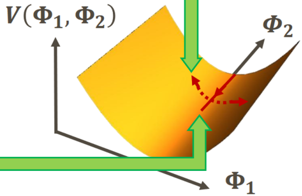
Environment:

- weakly specified

Interactions

System:

- measurable



Modelled by
Master Equations

Outline

The master equation zoo

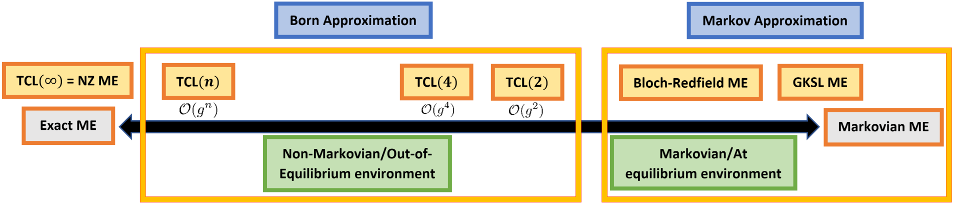
- Fundamental observables are correlators

$$\left\langle \widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \right\rangle (t) \equiv \mathrm{Tr}\left[\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \widehat{\rho}_{\mathrm{red}}(t)\right]$$

- **Master Equations** (ME) are dynamical equations for $\widehat{\rho}_{\mathrm{red}}(t)$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}\left(\widehat{\rho}_{\mathrm{red}}\right)$$

- There exists a **whole bestiary** of MEs [Breuer & Petruccione, 2002]



At which level should we work in cosmology ?

Master equations in cosmology

In **quantum optics**, Markovian MEs are ubiquitous:

1. Environments are *large*;
2. Environments are *stationary*;
3. Environments are at *thermal equilibrium*.

In **cosmology**, these assumptions must be reassessed:

1. Background is *symmetric, curved and dynamical*;
2. There is *no stationary* $|\mathrm{out}\rangle$ *state*;
3. Cosmological environments can be *out-of-equilibrium*.

Assessing cosmological master equations

- ME have already been applied in cosmology, see [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Hollowood & McDonald, 2017], [Martin & Vennin, 2018], [Brahma, Berera & Calderón-Figueroa, 2021], $\cdots$

- ME were **designed in a specific context** and **need some adaptations**
  $\Rightarrow$ Working in curved-space implies to reassess:
  - approximation schemes;
  - regimes of validity.

- We **benchmark the ME program** on an exactly solvable model:
  - We have analytic control on the system dynamics;
  - We compare exact and ME results.

Outline

The curved-space Caldeira-Leggett model

- Action for the field sector:

$$S = -\int \mathrm{d}^4x \sqrt{-\det g}\Bigg(\overbrace{\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{1}{2}m^2\varphi^2\right]}^{\text{System}} + \underbrace{\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}M^2\chi^2\right]}_{\text{Environment}} + \underbrace{\lambda^2\varphi\chi}_{\text{Interaction}}\Bigg)$$

- **Field redefinition**: rotation in field space of

$$\theta = -\frac{1}{2}\arctan\left(\frac{2\lambda^2}{M^2 - m^2}\right)$$

decouple the two sectors: fully integrable model.

- **Gaussian system**:
  - All information contained in the system covariance $\boldsymbol{\Sigma}_{\varphi\varphi}$;
  - Quantum information properties: $\gamma = \det\left[\boldsymbol{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Integrating out the environment

- In the **Fock space**: a master equation, with $\widehat{\boldsymbol{z}} = (\widehat{v}_\varphi, \widehat{p}_\varphi)^{\mathrm{T}}$

$$\underbrace{\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\widehat{H}_0(\eta) + \widehat{H}^{(\mathrm{LS})}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right]}_{\text{Unitary evolution}}$$
$$\underbrace{+\sum_{i,j}\mathcal{D}_{ij}(\eta)\left[\widehat{\boldsymbol{z}}_i\widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{z}}_j - \frac{1}{2}\left\{\widehat{\boldsymbol{z}}_j\widehat{\boldsymbol{z}}_i, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]}_{\text{Non-unitary evolution}}$$

- In the **phase space**: a Fokker-Planck equation, with $\boldsymbol{z} = (v_\varphi, p_\varphi)^{\mathrm{T}}$

$$\frac{\mathrm{d}W_{\mathrm{red}}}{\mathrm{d}\eta} = \underbrace{\left\{\widetilde{H}_0(\eta) + \widetilde{H}^{(\mathrm{LS})}(\eta), W_{\mathrm{red}}(\eta)\right\}}_{\text{Unitary evolution}}$$
$$\underbrace{+\boldsymbol{\Delta}_{12}(\eta)\sum_i\frac{\partial}{\partial\boldsymbol{z}_i}\left[\boldsymbol{z}_iW_{\mathrm{red}}(\eta)\right] - \frac{1}{2}\sum_{i,j}\left[\boldsymbol{\omega}\boldsymbol{D}(\eta)\boldsymbol{\omega}\right]_{ij}\frac{\partial^2W_{\mathrm{red}}(\eta)}{\partial\boldsymbol{z}_i\partial\boldsymbol{z}_j}}_{\text{Non-unitary evolution}}$$

Transport equation and non-perturbative resummation

- ME generates an effective transport equation:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega} - 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}$$

Unitary evolution — Non-unitary evolution

- ME studied in cosmology for its ability to resum late-time secular effects

[Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Brahma, Berera & Calderón-Figueroa, 2021]

- Resumation obtained when solving the transport equation **non-perturbatively**, considering ME as a *bona fide* dynamical map.

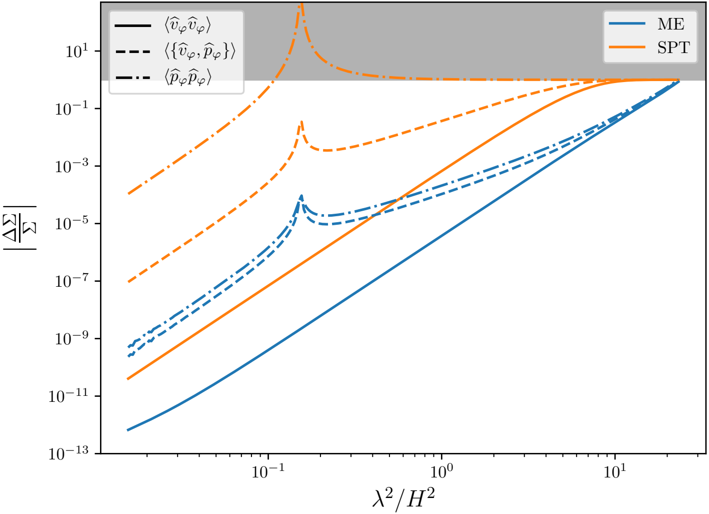
$$\boldsymbol{\Sigma}_{\varphi\varphi}^{(\text{pert})} = \left[1 + \alpha\lambda^4 \log\left(-k\eta\right) + A + \cdots\right]\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)}$$
$$\boldsymbol{\Sigma}_{\varphi\varphi}^{(\text{non-pert})} = A^{(\text{ren})} e^{\alpha\lambda^4 \log(-k\eta)}\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)} + \cdots$$

- **Important result**: if apply the ME program directly, the resummation is spurious. Meaningful resummation requires to first remove terms that vanish in the perturbative limit.

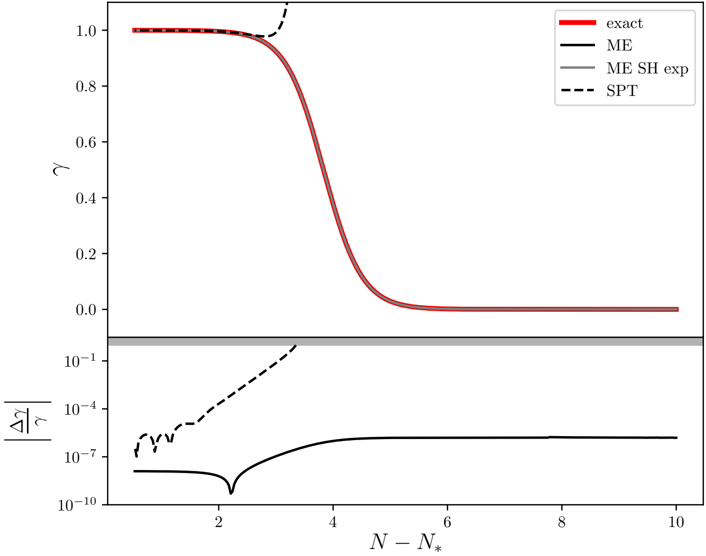
Benchmarking cosmological ME

- For the ME to be an interesting tool, it **needs to do better than standard techniques**.
- Benchmark against standard perturbation theory (SPT) results: **in-in formalism at linear order**:
  1. Write down mode functions equations of motion;
  2. Solve them perturbatively order by order;
  3. Compute observables from mode functions decomposition.
- We compare ME and SPT against exact results on:
  1. Accuracy on the **system covariance** $\boldsymbol{\Sigma}_{\varphi\varphi}$;
  2. Ability to recover the **purity** $\gamma = \det\left[\boldsymbol{\Sigma}_{\varphi\varphi}\right]^{-1}/4$.

Results on the power spectra



Results on the purity



Summary and outlook

- In cosmology, we need to deal with elusive environments.
    - **EFT** and **OQS** separate **things we know** from **things we don't**.

- ME may allow us to go beyond standard tools.
    - Approximation schemes et regime of validity must be **reassessed**.

- We benchmarked cosmological ME on an integrable model:
    1. **Non-perturbative resummation** is non trivial to implement.
    2. **Improved precision** on observables and QI properties.

Future directions:

- **Non-linear theory**: Self interactions in the environment (eg: $\mu\sigma^3$)
  $\Rightarrow$ Which relation with PNG generation ? [Assassi *et al.*, 2013]

- **Quantum corrections to the primordial tensor spectrum**:
  $\Rightarrow$ Computed perturbatively using ME formalism by [Brahma *et al.*, 2022]

Thank you for your attention !

Outline

Flat vs curved-space Caldeira-Leggett model

Flat space

- System: a harmonic oscillator of frequency

$$\omega^2 = k^2 + m^2$$

- Environment : large number of harmonic oscillators
- Linear interaction:

$$\widehat{H}^{\mathrm{int}}_{\boldsymbol{k}} = \sum_{\boldsymbol{q}} \lambda^2_{\boldsymbol{q}} \widehat{v}^{(S)}_{\boldsymbol{k}} \widehat{v}^{(E)}_{\boldsymbol{q}}$$

$\Rightarrow$ system interacts with infinitely many dof.

Curved space

- System: a parametric oscillator of frequency

$$\omega^2 = k^2 + m^2 a^2 - a''/a$$

- Environment : large number of parametric oscillators BUT
- Linear interaction + symmetries

$$\widehat{H}^{\mathrm{int}}_{\boldsymbol{k}} = \lambda^2 a^2 \widehat{v}^{(S)}_{\boldsymbol{k}} \widehat{v}^{(E)}_{-\boldsymbol{k}}$$

$\Rightarrow$ system only interacts with ONE environmental dof.

TCL_2 coefficients

$$\boldsymbol{D}_{11}(\eta) = -4\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \, \mathrm{Im}\left\{ p_\varphi(\eta) v_\varphi^*(\eta') \right\} \mathrm{Re}\left\{ v_\chi(\eta) v_\chi^*(\eta') \right\}$$

$$\boldsymbol{D}_{12}(\eta) = 2\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \, \mathrm{Im}\left\{ v_\varphi(\eta) v_\varphi^*(\eta') \right\} \mathrm{Re}\left\{ v_\chi(\eta) v_\chi^*(\eta') \right\}$$

$$\boldsymbol{\Delta}_{11}(\eta) = -4\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \, \mathrm{Im}\left\{ p_\varphi(\eta) v_\varphi^*(\eta') \right\} \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta') \right\}$$

$$\boldsymbol{\Delta}_{12}(\eta) = 2\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \, \mathrm{Im}\left\{ v_\varphi(\eta) v_\varphi^*(\eta') \right\} \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta') \right\}$$

- Can we compare them with exact counterparts ?
  - Fundamental object: system covariance $\boldsymbol{\Sigma}_{\varphi\varphi}$
  - Look at the *exact* and *effective* **transport equation**:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega} - 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi}$$

Comparison of the coefficients

- Exact coefficients:

$$\boldsymbol{\Delta}_{\mathrm{ex},11} = -\frac{\lambda^4}{M^2 - m^2} a^2, \qquad \boldsymbol{\Delta}_{\mathrm{ex},12} = 0$$
$$\boldsymbol{D}_{\mathrm{ex},11} = -\frac{2\lambda^4}{M^2 - m^2} a^2 \boldsymbol{\Sigma}_{\chi\chi,12}, \qquad \boldsymbol{D}_{\mathrm{ex},12} = \frac{\lambda^4}{M^2 - m^2} a^2 \boldsymbol{\Sigma}_{\chi\chi,11}$$

- TCL_2 coefficients
  1. In the super-Hubble regime $|-k\eta| \ll 1$;
  2. When the environment is heavy $M \gg H$;

$$\boldsymbol{\Delta}_{11} = \boldsymbol{\Delta}_{\mathrm{ex},11} + \boldsymbol{\Delta}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}, \qquad \boldsymbol{\Delta}_{12} = \boldsymbol{\Delta}_{\mathrm{ex},11} + \boldsymbol{\Delta}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}$$
$$\boldsymbol{D}_{11} = \boldsymbol{D}_{\mathrm{ex},11} + \boldsymbol{D}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}, \qquad \boldsymbol{D}_{12} = \boldsymbol{D}_{\mathrm{ex},12} + \boldsymbol{D}_{12}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}$$

where the matching is at order $\mathcal{O}(\lambda^4)$.

Spurious terms in the super-Hubble regime

When $M \gg H$,

$$\boldsymbol{D}_{11}^{\text{spur}} = \frac{1}{2\mu_\chi} \frac{1}{\nu_\varphi^2 + \mu_\chi^2} \frac{\lambda^4}{H^4} \frac{k^2}{z^2} \left(\frac{z_0}{z}\right)^{3/2} \left(\nu_\varphi - \frac{3}{2}\right)$$

$$\boldsymbol{D}_{12}^{\text{spur}} = -\frac{1}{4\mu_\chi} \frac{1}{\nu_\varphi^2 + \mu_\chi^2} \frac{\lambda^4}{H^4} \frac{k}{z} \left(\frac{z_0}{z}\right)^{3/2}$$

$$\boldsymbol{\Delta}_{11}^{\text{spur}} = \frac{1}{2\nu_\varphi} \frac{1}{\nu_\varphi^2 + \mu_\chi^2} \frac{\lambda^4}{H^4} \frac{k^2}{z^2} \left(\frac{z_0}{z}\right)^{3/2} \left(\nu_\varphi - \frac{3}{2}\right)$$

$$\boldsymbol{\Delta}_{12}^{\text{spur}} = -\frac{1}{4\nu_\varphi} \frac{1}{\nu_\varphi^2 + \mu_\chi^2} \frac{\lambda^4}{H^4} \frac{k}{z} \left(\frac{z_0}{z}\right)^{3/2}$$

with $z = -k\eta$ and

$$\nu_\varphi \equiv \frac{3}{2}\sqrt{1 - \left(\frac{2m}{3H}\right)^2}, \quad \text{and} \quad \mu_\chi = \frac{3}{2}\sqrt{\left(\frac{2M}{3H}\right)^2 - 1}$$

.

Analytic results on the covariance

Integrating the transport equation:

$$\boldsymbol{\Sigma}_{\varphi\varphi}(\eta) = e^{-2\int_{\eta_0}^{\eta} \mathrm{d}\eta' \boldsymbol{\Delta}_{12}(\eta')} \boldsymbol{g}_{\mathsf{LS}}(\eta,\eta_0)\boldsymbol{\Sigma}_{\varphi\varphi}(\eta_0)\boldsymbol{g}_{\mathsf{LS}}^{\mathrm{T}}(\eta,\eta_0) - \int_{\eta_0}^{\eta} \mathrm{d}\eta' e^{-2\int_{\eta'}^{\eta} \mathrm{d}\eta'' \boldsymbol{\Delta}_{12}(\eta'')} \boldsymbol{g}_{\mathsf{LS}}(\eta,\eta') \left[\boldsymbol{\omega} \boldsymbol{D}(\eta')\boldsymbol{\omega}\right] \boldsymbol{g}_{\mathsf{LS}}^{\mathrm{T}}(\eta,\eta').$$

SPT results (1)

Mode function decomposition

$$\widehat{v}_\varphi(\eta) = v_{\varphi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\varphi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\varphi\chi}(\eta)\widehat{a}_\chi + v^*_{\varphi\chi}(\eta)\widehat{a}^\dagger_\chi$$
$$\widehat{v}_\chi(\eta) = v_{\chi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\chi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\chi\chi}(\eta)\widehat{a}_\chi + v^*_{\chi\chi}(\eta)\widehat{a}^\dagger_\chi$$

which obey **equations of motion**

$$v''_{\varphi\varphi} + \omega^2_\varphi(\eta)v_{\varphi\varphi} = -\lambda^2 a^2(\eta)v_{\chi\varphi}$$
$$v''_{\chi\varphi} + \omega^2_\chi(\eta)v_{\chi\varphi} = -\lambda^2 a^2(\eta)v_{\varphi\varphi}$$

and

$$v''_{\chi\chi} + \omega^2_\chi(\eta)v_{\chi\chi} = -\lambda^2 a^2(\eta)v_{\varphi\chi}$$
$$v''_{\varphi\chi} + \omega^2_\varphi(\eta)v_{\varphi\chi} = -\lambda^2 a^2(\eta)v_{\chi\chi}$$

SPT results (2)

Solution **order by order**

- Zeroth order:

$$v^{(0)}_{\varphi\varphi}(\eta) = v_\varphi(\eta)$$
$$v^{(0)}_{\chi\chi}(\eta) = v_\chi(\eta)$$

  and $v^{(0)}_{\varphi\chi}(\eta) = v^{(0)}_{\chi\varphi}(\eta) = 0$.

- First order:

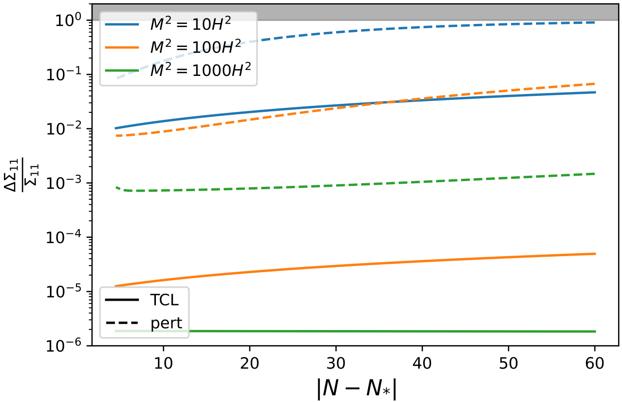
$$v^{(1)}_{\varphi\chi}(\eta) = -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\varphi(\eta) v^*_\varphi(\eta_1) \right\} v_\chi(\eta_1)$$
$$v^{(1)}_{\chi\varphi}(\eta) = -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\chi(\eta) v^*_\chi(\eta_1) \right\} v_\varphi(\eta_1).$$

- ...

Correlators are evaluated in the Heisenberg picture

$$\boldsymbol{\Sigma}(\eta) = \frac{1}{2} \mathrm{Tr}\left[\left\{ \widehat{\boldsymbol{z}}(\eta), \widehat{\boldsymbol{z}}^{\mathrm{T}}(\eta) \right\} \widehat{\rho}_0 \right]$$

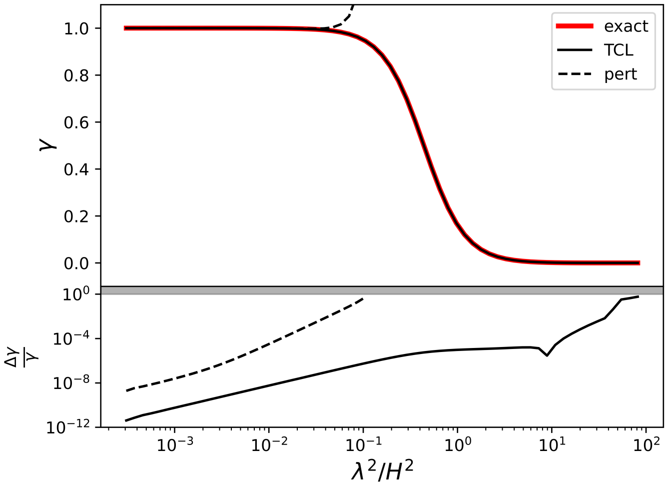
Power spectra and late-time resummation



Late-time resummation:

$$\boldsymbol{\Sigma}_{\varphi\varphi}^{\mathrm{TCL}} \supset e^{\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}|N-N_*|}\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)}$$

Purity and coupling



Outline

The environment as noises

Classical Brownian motion

- Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

- Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

- Wiener path integral [Wiener, 1923]

$$P = \int \mathcal{D}q e^{-S_0(q)}$$

Quantum Brownian motion

- Stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right]\mathrm{d}t + \mathrm{d}\widehat{\xi}$$

- Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}[\widehat{\rho}_{\mathrm{red}}]$$

- Influence functional [Vernon, 1959]

$$\widehat{\rho}_{\mathrm{red}} = \int \mathcal{D}\phi_0^{\pm}\mathcal{I}\left[\phi_0^{\pm}\right]\widehat{\rho}_{\mathrm{red},0}$$

TCL_2 Fokker-Planck equation

The reduced Wigner function evolves according to

$$\frac{\mathrm{d} W_{\text{red}}}{\mathrm{d}\eta} = \left\{ \widetilde{H}_0 + \widetilde{H}^{(\mathrm{LS})}, W_{\text{red}} \right\} + \boldsymbol{\Delta}_{12} \sum_i \frac{\partial}{\partial \boldsymbol{z}_i} \left(\boldsymbol{z}_i W_{\text{red}} \right) - \frac{1}{2} \sum_{i,j} \left[\boldsymbol{\omega} \boldsymbol{D} \boldsymbol{\omega} \right]_{ij} \frac{\partial^2 W_{\text{red}}}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_j},$$

Outline

Example 1: an exact ME

Master equation: dynamical equation for the quantum state of the system.

1. Start with Liouville-von Neumann equation

$$\frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}\eta} = -ig\left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \widetilde{\rho}(\eta)\right] \equiv g\mathcal{L}(\eta)\widetilde{\rho}(\eta)$$

2. Introduce projectors $\widetilde{\rho} \mapsto \mathcal{P}\widetilde{\rho} = \widetilde{\rho}_{\mathsf{red}} \otimes \rho_{\mathrm{E}}$ and $\mathcal{Q}\widetilde{\rho} = \widetilde{\rho} - \mathcal{P}\widetilde{\rho}$

3. Rewrite dynamics as

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \mathcal{K}(\eta, \eta')\mathcal{P}\widetilde{\rho}(\eta)$$

$\mathcal{K}(\eta, \eta')$: **memory kernel** which depends on **the coupling** and **the environment**.

Example 2: an effective ME

1. Expand in powers of **the coupling** constant

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \sum_{n=0}^{\infty} g^n \mathcal{K}_n(\eta)\mathcal{P}\widetilde{\rho}(\eta)$$

2. Lowest order leads to the non-Markovian ME

$$\frac{\mathrm{d}\widetilde{\rho}_{\text{red}}}{\mathrm{d}\eta} = -g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathrm{Tr}_E \left[\widetilde{\mathcal{H}}_{\text{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\text{int}}(\eta'), \widetilde{\rho}_{\text{red}}(\eta) \otimes \rho_{\mathrm{E}}\right]\right]$$

3. Error function $e_r^{(2)} \sim g^2 ||\mathcal{K}_4(\eta)|| / ||\mathcal{K}_2(\eta)||$.

Example 3: a Markovian ME

1. When the environment is a **bath** (large number of dof, thermal equilibrium), the dynamics is **Markovian**, the system admits a **semi-group evolution**

$$V(\eta_1)V(\eta_2) = V(\eta_1 + \eta_2)$$

.

2. It implies a specific form for the ME [Lindblad 1976]

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\widehat{H}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right] + \sum_k \gamma_k \left[\widehat{\boldsymbol{L}}_k \widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{L}}_k^\dagger - \frac{1}{2}\left\{\widehat{\boldsymbol{L}}_k^\dagger \widehat{\boldsymbol{L}}_k, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]$$

3. It relies on a fast decay of temporal correlations in the environment.

Question: At which level should we work in cosmology ?

The emergence of Markovianity

- Fast decay of environmental correlations

$$\mathcal{K}^{>}(\eta,\eta') \xrightarrow[\text{graining}]{\text{coarse-}} \delta(\eta-\eta')$$

ME reduces to a GKSL equation for which the dynamical map reads

$$\mathcal{L}\left[\widehat{\rho}_{\text{red}}\right] = -i\left[\widehat{\mathcal{H}},\widehat{\rho}_{\text{red}}\right] + \gamma\left(\widehat{L}\widehat{\rho}_{\text{red}}\widehat{L}^{\dagger} - \frac{1}{2}\left\{\widehat{L}^{\dagger}\widehat{L},\widehat{\rho}_{\text{red}}\right\}\right)$$

GKSL equation is CPTP: physical consistency of the solutions ensured.

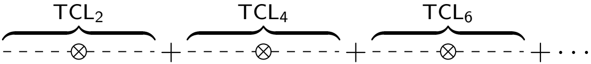
- Non-Markovian evolution/non-semigroup dynamical map implies dissipator matrix non-positive semi-definite.
- Non-positive semi-definite dissipator matrix is a generic feature of Non-Markovian OQS: not directly related to CPTP properties.
- Curved-space Caldeira-Leggett model ME belongs to the class of Gaussian non-Markovian ME ⇒ CPTP ensured by [Diósi & Ferialdi, 2014].

Outline

What has been resummed ?

- In the **exact theory**, there is only one 1PI:

- In the **effective theory**, there is an infinite tower of 1PI:



one for each of the TCL cumulant.

- Moreover, there are **non-unitary contributions** from diffusion and dissipation which do not have diagrammatic representation.
- Hence, the question of knowing **which diagram has been resumed** is **ill-posed**. This feature is shared with WEFT and the DRG.

Late-time resummation technique

Following [Boyanovsky, 2015], [Brahma *et al.*, 2021],

$$\langle\widetilde{v}_\varphi(\eta)\widetilde{v}_\varphi(\eta)\rangle = v_-(\eta)v_-(\eta)\left\langle\widehat{P}^2_\varphi\right\rangle + v_-(\eta)v_+(\eta)\left\langle\widehat{Q}_\varphi\widehat{P}_\varphi + \widehat{P}_\varphi\widehat{Q}_\varphi\right\rangle$$
$$+v_+(\eta)v_+(\eta)\left\langle\widehat{Q}^2_\varphi\right\rangle \to v_+(\eta)v_+(\eta)\left\langle\widehat{Q}^2_\varphi\right\rangle$$

with

$$\frac{\mathrm{d}\left\langle\widehat{Q}^2_\varphi\right\rangle}{\mathrm{d}\eta} = \Gamma(\eta)\left\langle\widehat{Q}^2_\varphi\right\rangle$$

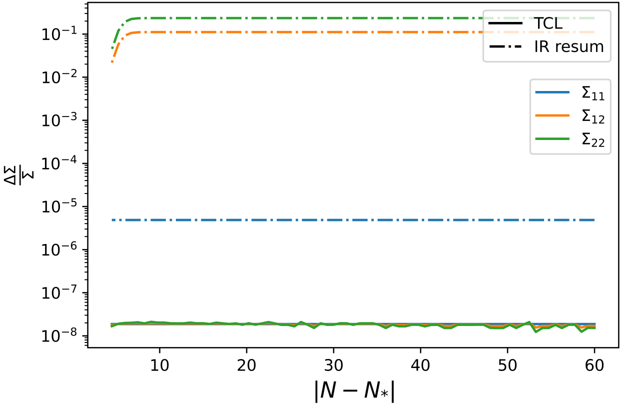
obtained from the TCL_2 ME.

In the *curved-space Caldeira-Leggett model*, leads to

$$\boldsymbol{\Sigma}^{\mathrm{TCL}}_{\varphi\varphi} \supset e^{-\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}\ln(-k\eta)}\boldsymbol{\Sigma}^{(0)}_{\varphi\varphi}$$

where late-time secular effects have been resummed.

Late-time resummation and the DRG



This resummation technique shares many features with the DRG [Burgess *et al.*, 2009].

Are they equivalent ?

Outline

TCL$_4$ generator

$$\mathcal{K}_4(\eta) = \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \int_{\eta_0}^{\eta_2} \mathrm{d}\eta_3 \Big[\mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{P}\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_2)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_3)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{P}\Big]$$

TCL$_4$ master equation

$$\frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_4}}{\mathrm{d}\eta} = \frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_2}}{\mathrm{d}\eta} - 4\lambda^8 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 a^2(\eta_1) \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 a^2(\eta_2) \int_{\eta_0}^{\eta_2} \mathrm{d}\eta_3 a^2(\eta_3)$$

$$\times \Big\{ \mathrm{Im}\big\{ v_\chi(\eta) v_\chi^*(\eta_2) \big\} \mathrm{Re}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_3) \big\} \mathrm{Im}\big\{ v_\varphi(\eta_1) v_\varphi^*(\eta_2) \big\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_3), \widetilde{\rho}_{\mathrm{red}}(\eta)]]$$

$$+ i\, \mathrm{Im}\big\{ v_\chi(\eta) v_\chi^*(\eta_2) \big\} \mathrm{Im}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_3) \big\} \mathrm{Im}\big\{ v_\varphi(\eta_1) v_\varphi^*(\eta_2) \big\} [\widetilde{v}_\varphi(\eta), \{\widetilde{v}_\varphi(\eta_3), \widetilde{\rho}_{\mathrm{red}}(\eta)\}]$$

$$+ \mathrm{Im}\big\{ v_\chi(\eta) v_\chi^*(\eta_3) \big\} \mathrm{Re}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \big\} \mathrm{Im}\big\{ v_\varphi(\eta_1) v_\varphi^*(\eta_3) \big\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_2), \widetilde{\rho}_{\mathrm{red}}(\eta)]]$$

$$+ i\, \mathrm{Im}\big\{ v_\chi(\eta) v_\chi^*(\eta_3) \big\} \mathrm{Im}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \big\} \mathrm{Im}\big\{ v_\varphi(\eta_1) v_\varphi^*(\eta_3) \big\} [\widetilde{v}_\varphi(\eta), \{\widetilde{v}_\varphi(\eta_2), \widetilde{\rho}_{\mathrm{red}}(\eta)\}]$$

$$+ \Big[- \mathrm{Re}\big\{ v_\chi(\eta) v_\chi^*(\eta_3) \big\} \mathrm{Im}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \big\} + \mathrm{Im}\big\{ v_\chi(\eta) v_\chi^*(\eta_3) \big\} \mathrm{Re}\big\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \big\} \Big]$$

$$\mathrm{Im}\big\{ v_\varphi(\eta_2) v_\varphi^*(\eta_3) \big\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_1), \widetilde{\rho}_{\mathrm{red}}(\eta)]] \Big\}.$$

Equivalence between perturbative TCL and in-in formalism

Cosmologists are used to compute correlators using the **in-in formalism**.

- At linear order, it is similar to the perturbative results presented above.

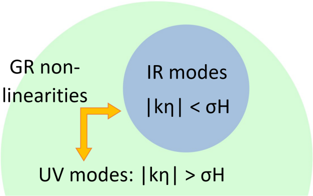
We have shown that:

- Perturbative TCL_2 is equivalent to $\mathcal{O}(\lambda^4)$ in-in.
- Perturbative TCL_4 is equivalent to $\mathcal{O}(\lambda^8)$ in-in.

Probably the proof **extend at all order**. Indeed, from the TCL cumulant expansion, all terms at a given order are included. It should ensure the matching with the in-in formalism at a given order.

Outline

The OpenEFT formalism



In [Brahma *et al.*, 2020], the leading cubic contribution is

$$H_{\text{int}} = \frac{M_{\text{Pl}}^2}{2} \int \mathrm{d}^3x \varepsilon_H^2 a \zeta^2 \partial^2 \zeta$$

- UV modes backreact on the IR dynamics.
- They induce decoherence of the IR sector.