Constraining spontaneous black hole scalarization with gravitational waves

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I. Hairy black holes

A stationary, asymptotically flat solution that is not Kerr and has a nontrivial scalar-field profile

$$S = \frac{1}{16\pi}\int \sqrt{-g}\, \mathrm{d}^4x \left(R - 2(\partial\phi)^2 - V(\phi)\right)$$

The only suitably regular, stationary, asymptotically flat vacuum black hole solutions are those for which the metric is Kerr and the scalar is everywhere a constant.

[Hawking 1972; Sotiriou and Faraoni 2012]

$$\phi = \phi_0 : V'(\phi_0) = 0, \quad V''(\phi) \geq 0$$

Scalar–Gauss–Bonnet theories

$$S = \frac{1}{16\pi}\int \sqrt{-g}\,\mathrm{d}^4x \left(R - 2(\partial\phi)^2 + \lambda^2 f(\phi)\mathcal{G}\right)$$

$\mathcal{G} = R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$ is the Gauss–Bonnet invariant

λ is a coupling constant with dimensions of length

The equation of motion for the scalar field is

$$\nabla^\mu \nabla_\mu \phi = -\frac{1}{4}\lambda^2 f'(\phi)\mathcal{G}$$

I If $f'(\phi) \neq 0 \; \forall \phi$ finite

Any spacetime with $\mathcal{G} \neq 0$ has $\phi \neq$ constant

All black holes have scalar hair

$\lambda \lesssim 4.4 \text{ km}$ [Lyu, Jiang and Yagi 2022]

II If $\exists \phi_0 : f'(\phi_0) = 0$

$(g_{\text{Kerr}}, \phi_0)$ is a valid solution, but not unique

Tachyonic instability for some (M, S, λ)

Non-Kerr solutions are “spontaneously scalarized”

Scalar–Gauss–Bonnet theories

$$S = \frac{1}{16\pi} \int \sqrt{-g}\, \mathrm{d}^4x \left(R - 2(\partial\phi)^2 + \lambda^2 f(\phi)\mathcal{G} \right)$$

Dilatonic
[Kanti et al. 1996]

$f(\phi) = \frac{1}{\beta} e^{\beta\phi}$

Shift symmetric
[Sotiriou and Zhou 2014]

$f(\phi) = \phi$

I $f'(\phi) \neq 0 \;\forall \phi$
All black holes hairy

Quadratic
[Silva et al. 2018]

$f(\phi) = \frac{1}{2}\phi^2$

Gaussian
[Doneva and Yazadjiev 2018]

$f(\phi) = \frac{1}{2\beta}(1 - e^{-\beta\phi^2})$
fix $\beta = 6$

II $f'(\phi_0) = 0,\, f''(\phi_0) = 1$
with $\phi_0 = 0$
Spont. scalarization

Because the field equations are invariant under the rescaling

$$\lambda \mapsto b\lambda, \quad r \mapsto br \quad \text{for any constant } b > 0,$$

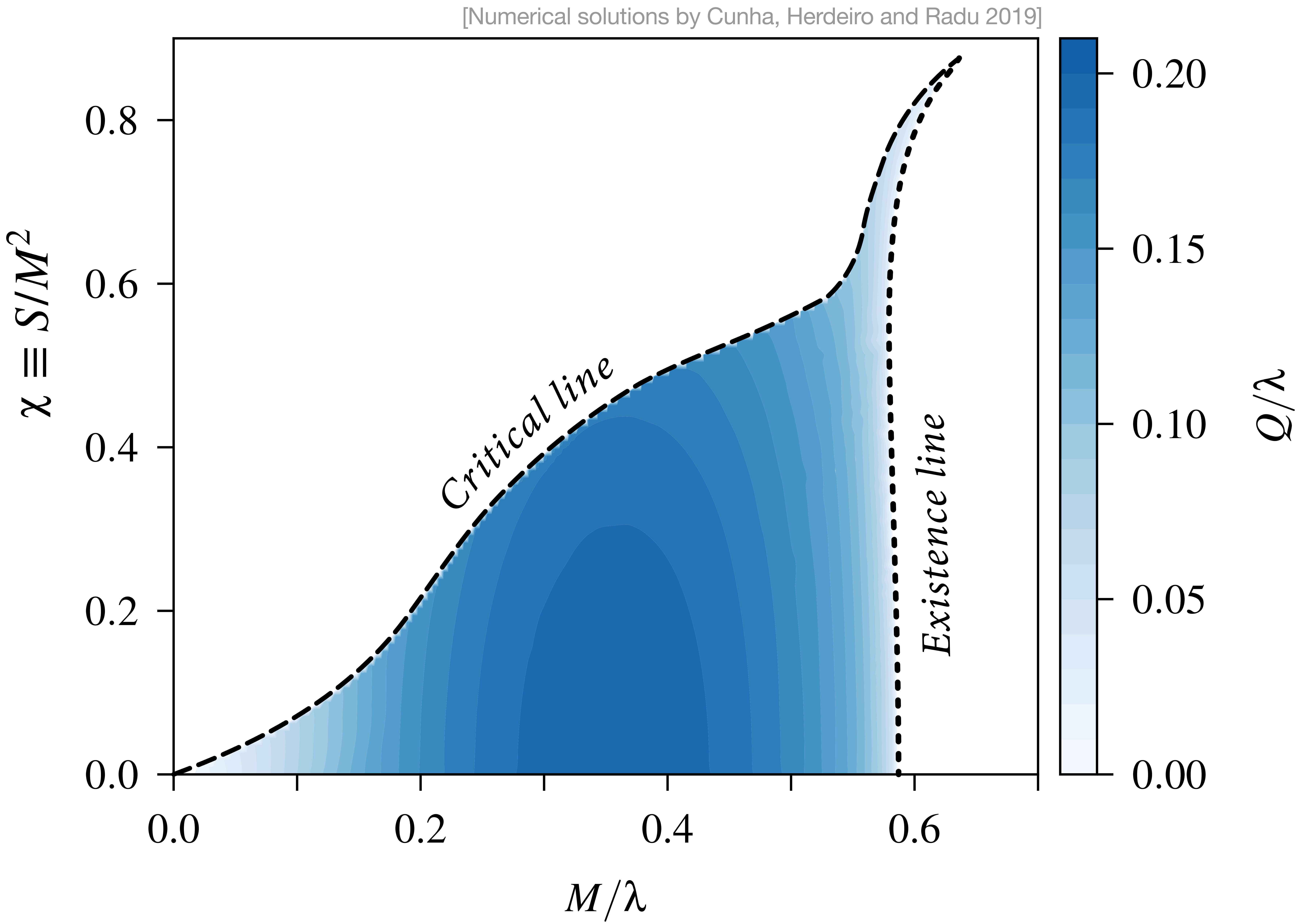
the stationary, axisymmetric solutions $\Xi = (g_{\mu\nu}, \phi)$ are such that

$$\Xi \equiv \Xi\left(r, \theta; \lambda, \frac{M}{\lambda}, \frac{S}{M^2}\right)$$

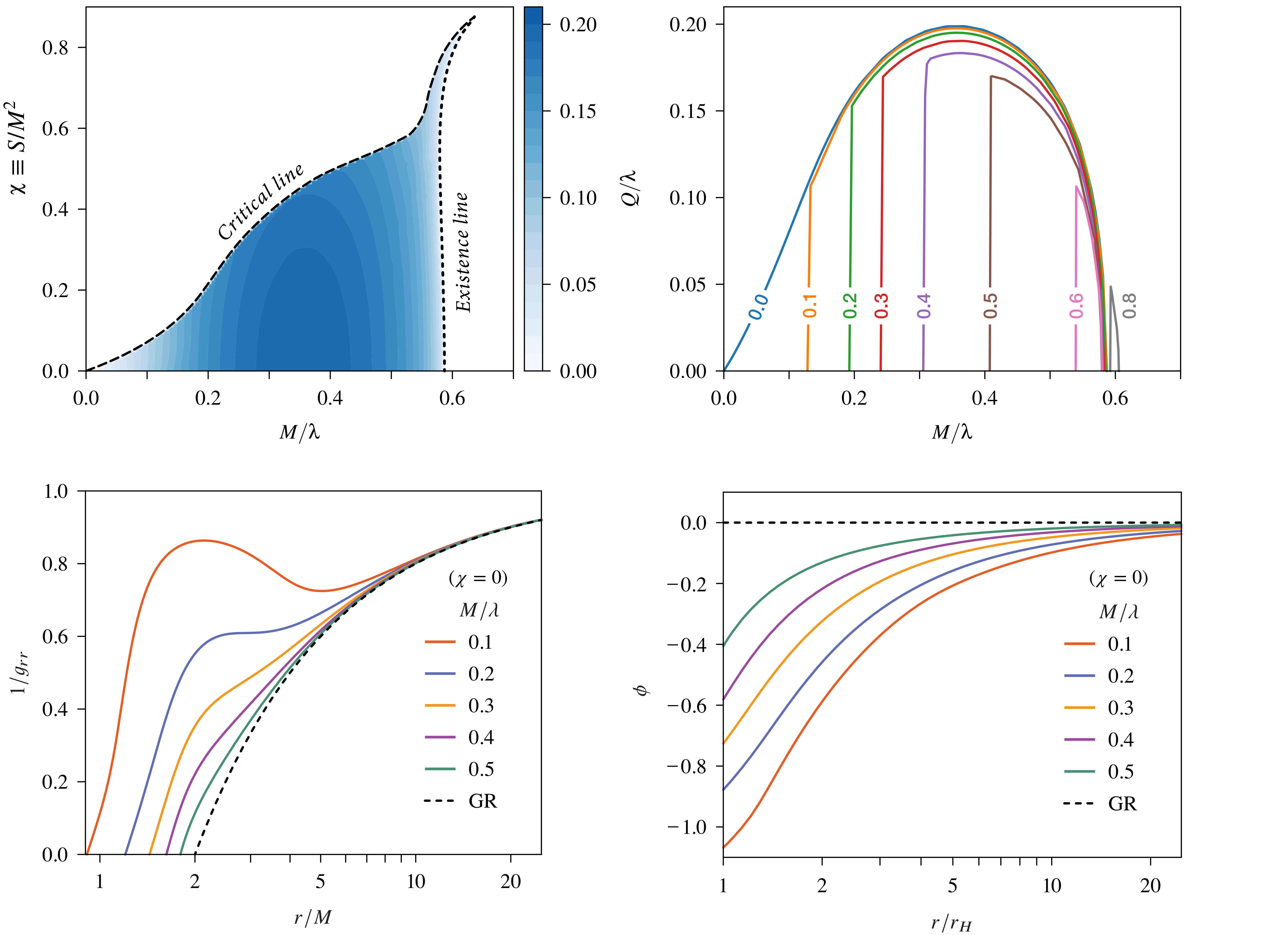
The *scalar charge* Q is read off from the asymptotic expansion

$$\phi = \phi_0 - \frac{Q}{r} + O(r^{-2}), \quad Q = \lambda \times f\left(\frac{M}{\lambda}, \frac{S}{M^2}\right)$$

Scalar hair is of *secondary* type [Coleman, Preskill and Wilczek 1992]



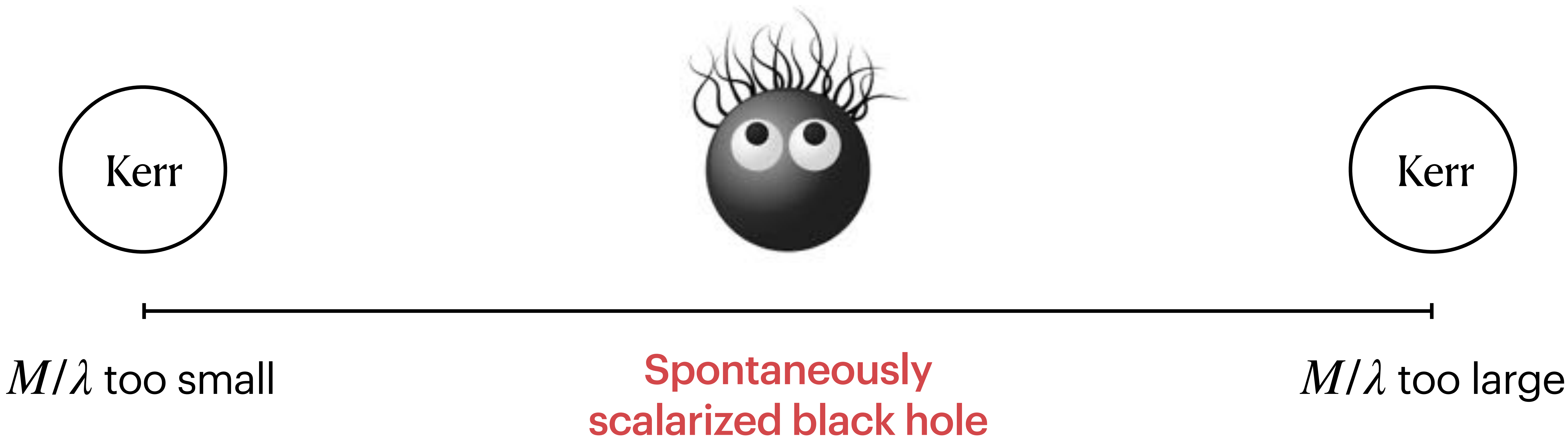
[Numerical solutions by Cunha, Herdeiro and Radu 2019]
Critical line
Existence line
$\chi \equiv S/M^2$
M/λ
Q/λ
0.0
0.2
0.4
0.6
0.8
0.00
0.05
0.10
0.15
0.20



χ ≡ S/M²
M/λ
Critical line
Existence line
0.00
0.05
0.10
0.15
0.20
Q/λ
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.8
1/g_rr
r/M
(χ = 0)
M/λ
0.1
0.2
0.3
0.4
0.5
GR
ϕ
r/r_H

II. Gravitational-wave constraints

Hairy black holes = scalar waves



A black hole of mass M probes a certain range of values of λ

Slowly spinning black holes probe a larger range

$$p(\lambda \,|\, d) = \int \mathrm{d}\theta \; p(\lambda, \theta \,|\, d)$$

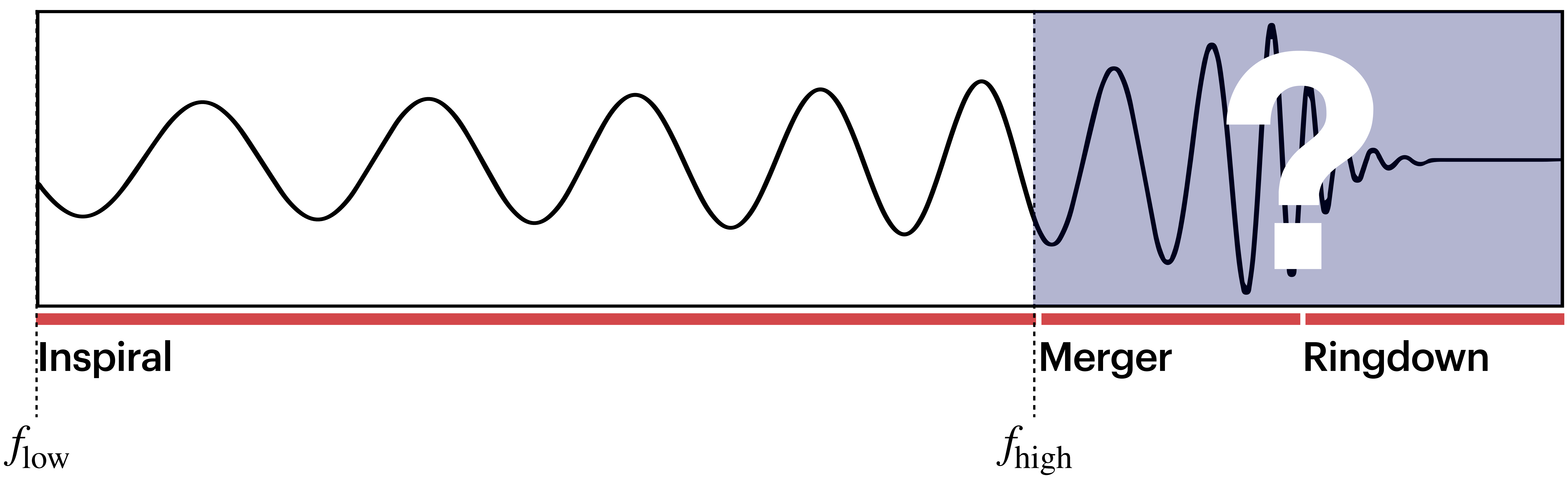
$$p(\lambda, \theta \,|\, d) \;\propto\; p(d \,|\, \lambda, \theta)\, \pi(\lambda, \theta)$$

Assume detector noise is stationary, Gaussian, and uncorrelated

[Cutler and Flanagan 1994]

$$p(d\,|\,\lambda,\theta) \propto \prod_{a \in \{\text{detectors}\}} \exp\left(-2\int_{f_{\text{low},a}}^{f_{\text{high},a}} \mathrm{d}f \, \frac{|\tilde{d}_a(f) - \tilde{h}_a(f;\lambda,\theta)|^2}{S_{n,a}(f)} \right)$$

Signal → $\tilde{d}_a(f)$; Waveform → $\tilde{h}_a(f;\lambda,\theta)$; Noise PSD → $S_{n,a}(f)$



Higher-order terms

1 Binding energy

$$E = -\frac{M^2\nu}{r} - \frac{Q_1 Q_2}{r} + \cdots \qquad (M = M_1 + M_2,\ \nu = M_1 M_2 / M^2)$$

Newton's inverse square law

Extra attraction mediated by the scalar

2 Outgoing flux

$$\mathscr{F} = \frac{32}{5}\nu^2 (M\Omega)^{10/3} + \frac{2}{3}\Delta\alpha^2 \nu^2 (M\Omega)^{8/3} + \cdots$$

Gravitational radiation (Quadrupolar)

Scalar radiation (Dipolar)

$$\Delta\alpha = \frac{Q_1}{M_1} - \frac{Q_2}{M_2}$$

3 Balance equation

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \mathscr{F}$$

Decompose the waveform into spherical harmonics

$$\tilde{h}_{\ell m}(f) = \mathscr{A}_{\ell m}(f)\, e^{i\Psi_{\ell m}(f)}$$

Phase: $\Psi_{\ell m} = \Psi^{(\mathrm{GR})}_{\ell m} + \delta\Psi_{\ell m}$

$Q_A \equiv Q_A(M_A, S_A, \lambda)$

$$\delta\Psi_{\ell m} = \underbrace{\frac{5m}{14\,336\,\nu}\left(\frac{Q_1}{M_1} - \frac{Q_2}{M_2}\right)^2 \left(\frac{2\pi M f}{m}\right)^{-7/3}}$$

–1PN scalar dipole radiation

[Sennett, Marsat and Buonanno 2016]

Amplitude: $\mathscr{A}_{\ell m} = \mathscr{A}^{(\mathrm{GR})}_{\ell m}$

Log uniform

(Restricted to $\lambda \in [1, 10^3]\ M_\odot$)

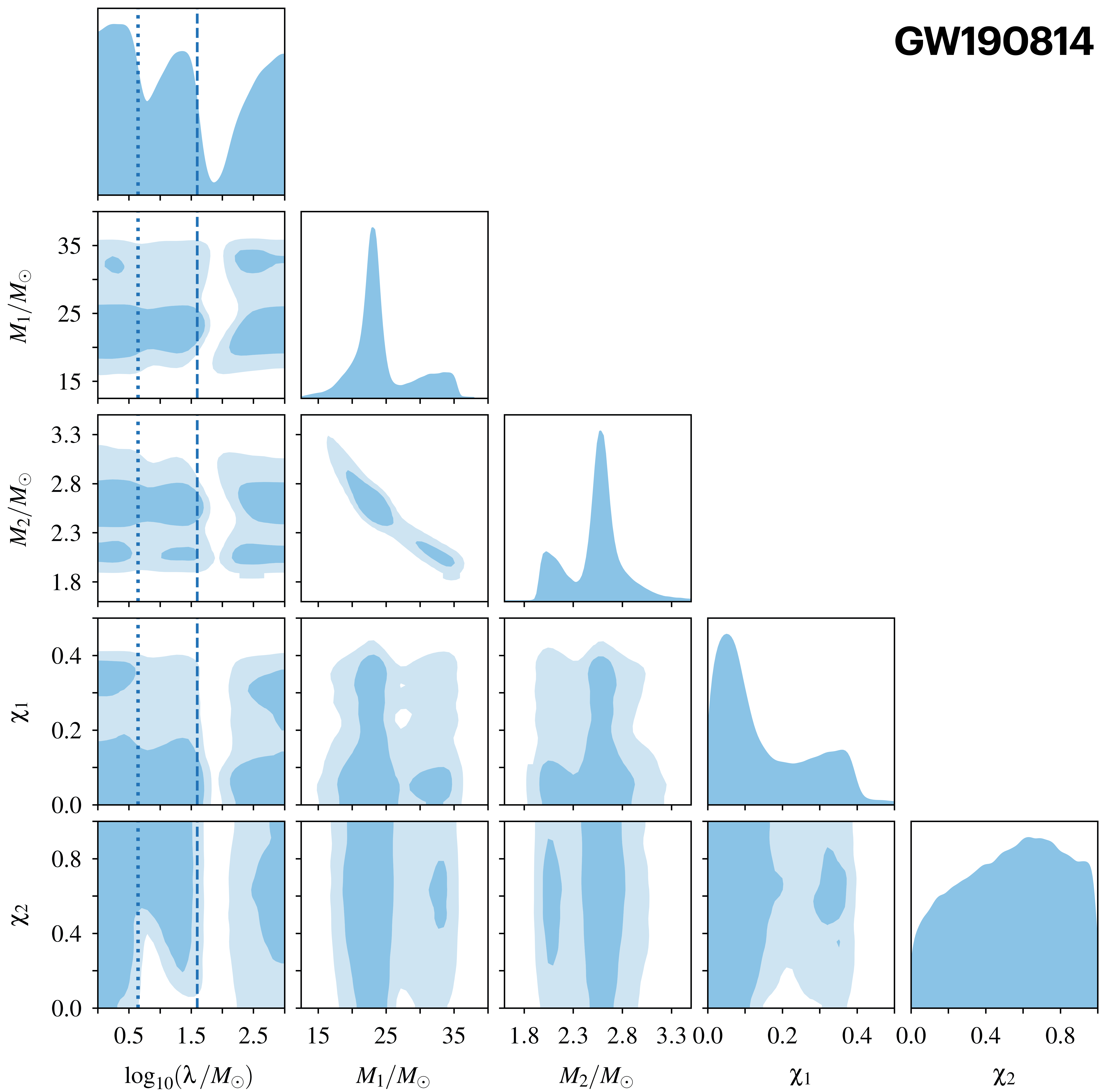
$$p(\lambda, \theta \,|\, d) \propto p(d \,|\, \lambda, \theta)\, \pi(\lambda, \theta)$$

GW190814 + GW151226

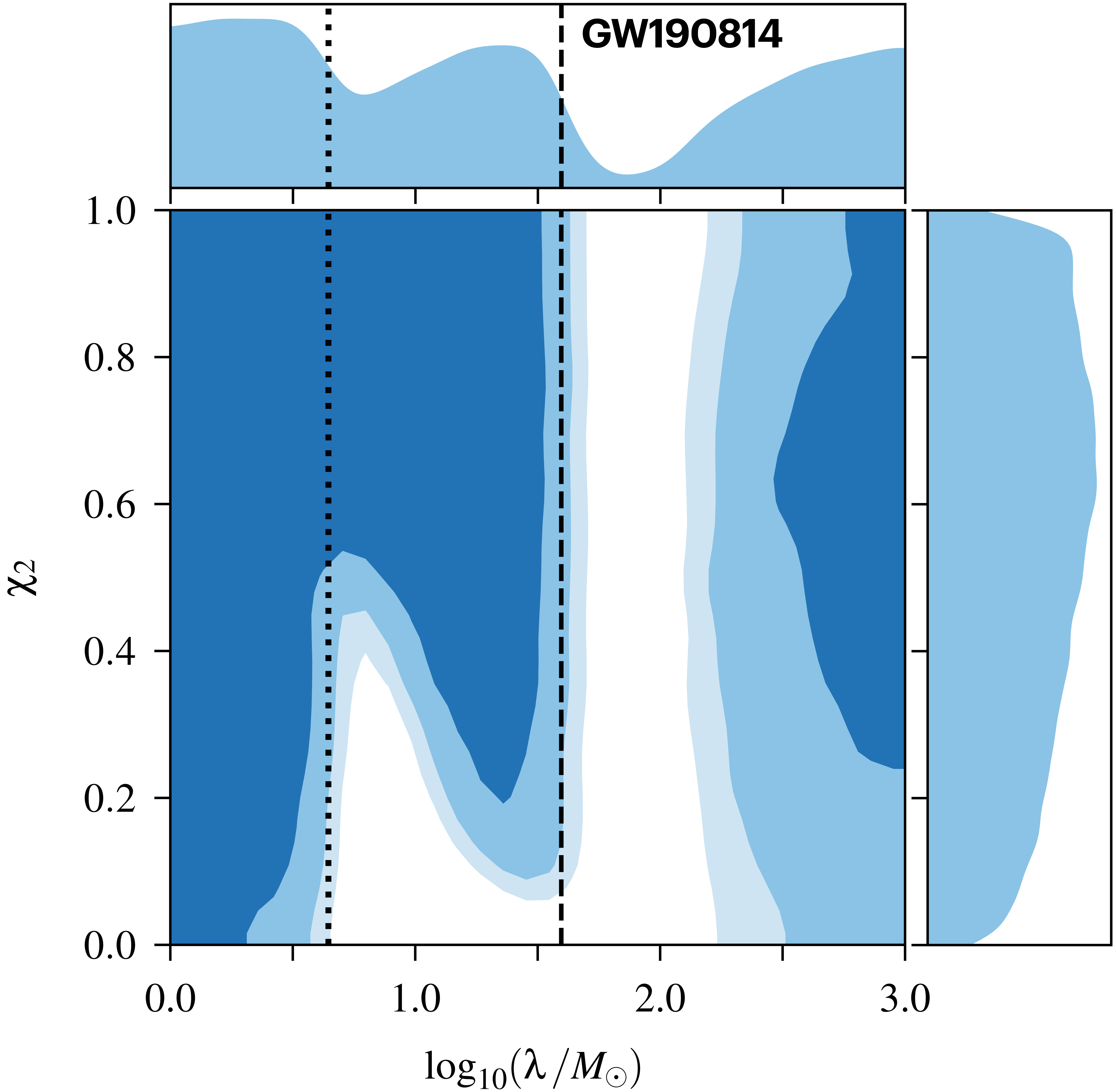
$(\chi_1 \ll 1)$

Same as in GR

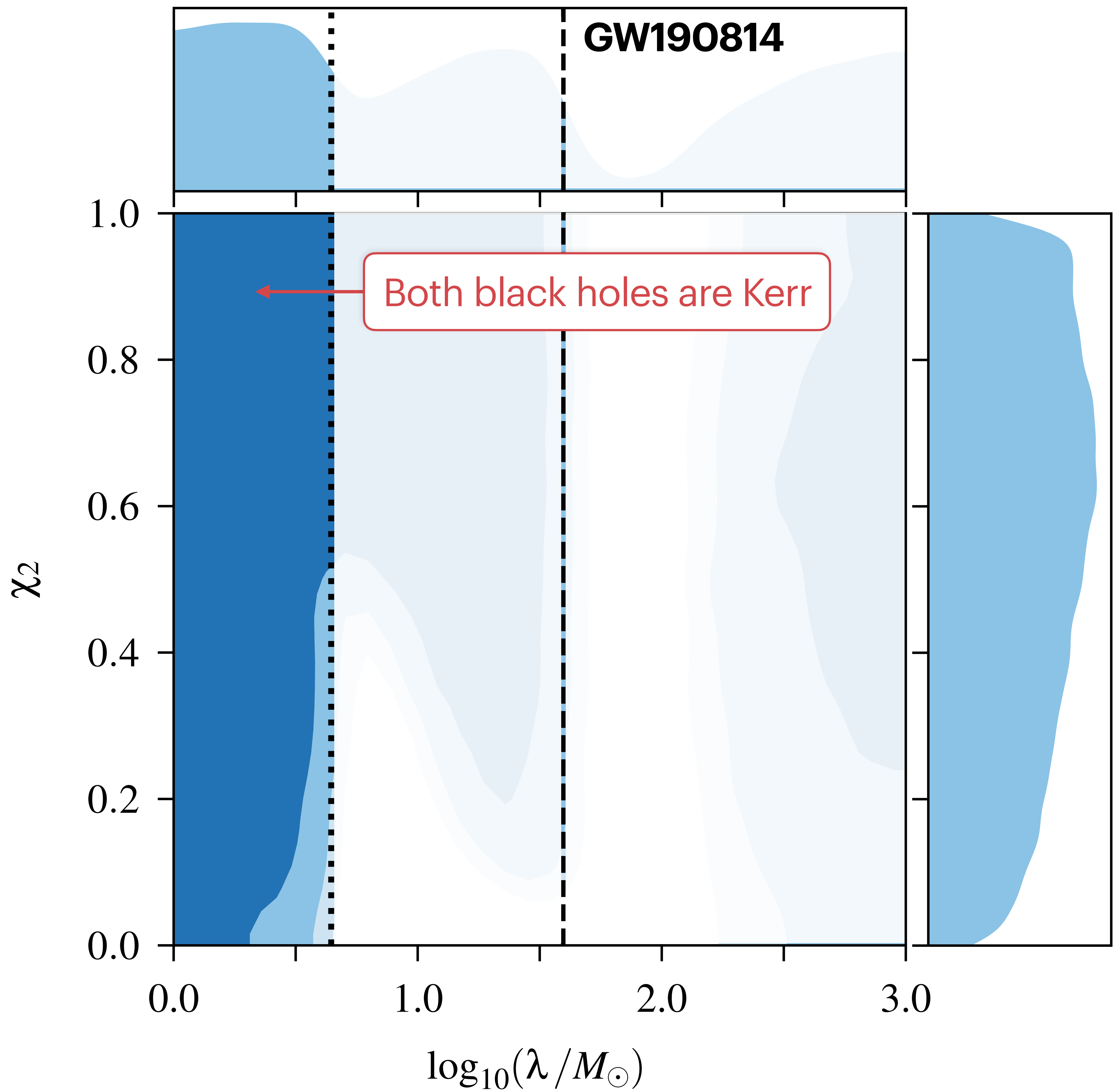
$\theta = \{M_1, M_2, \chi_1, \chi_2, \ldots\}$

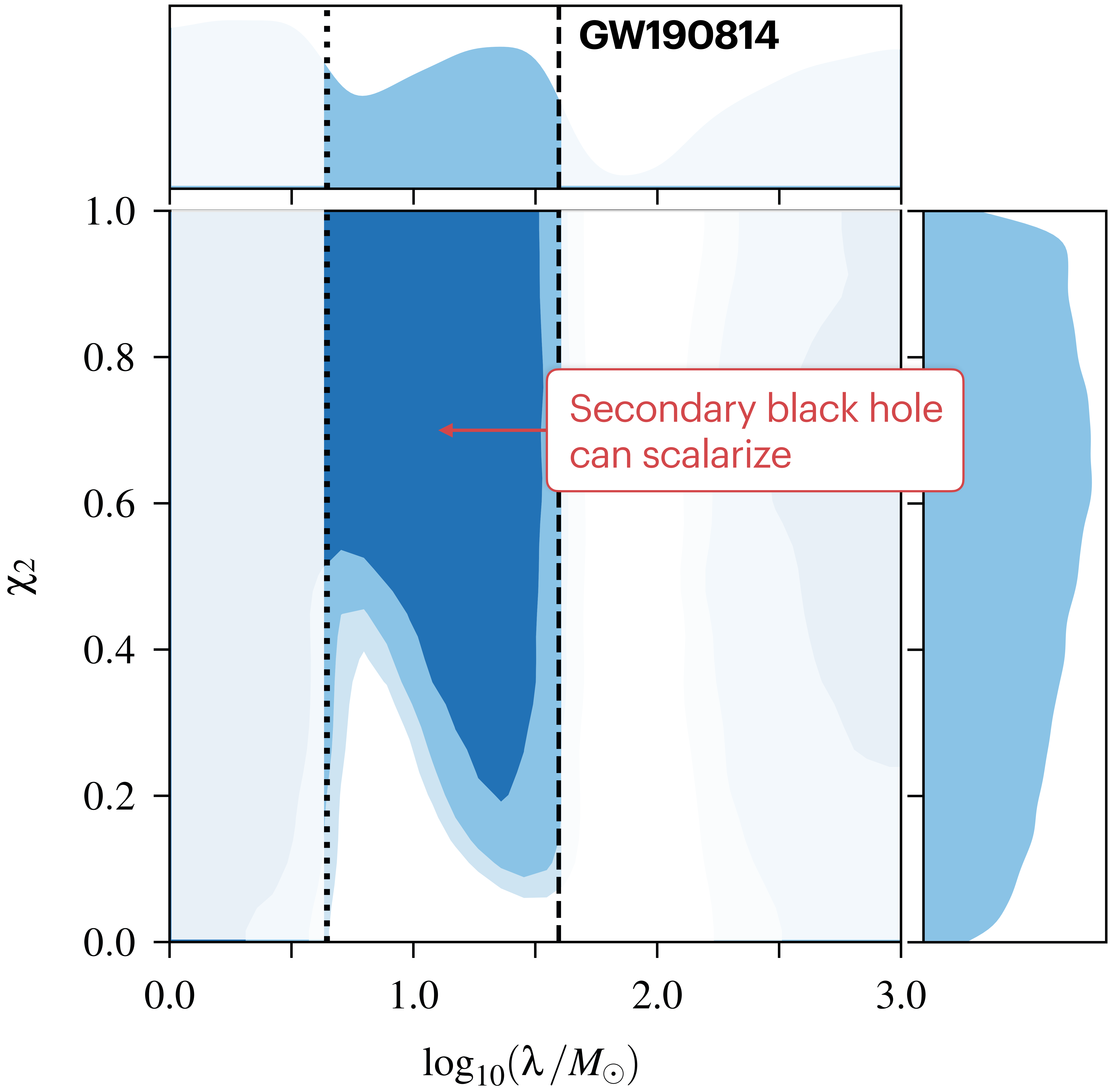


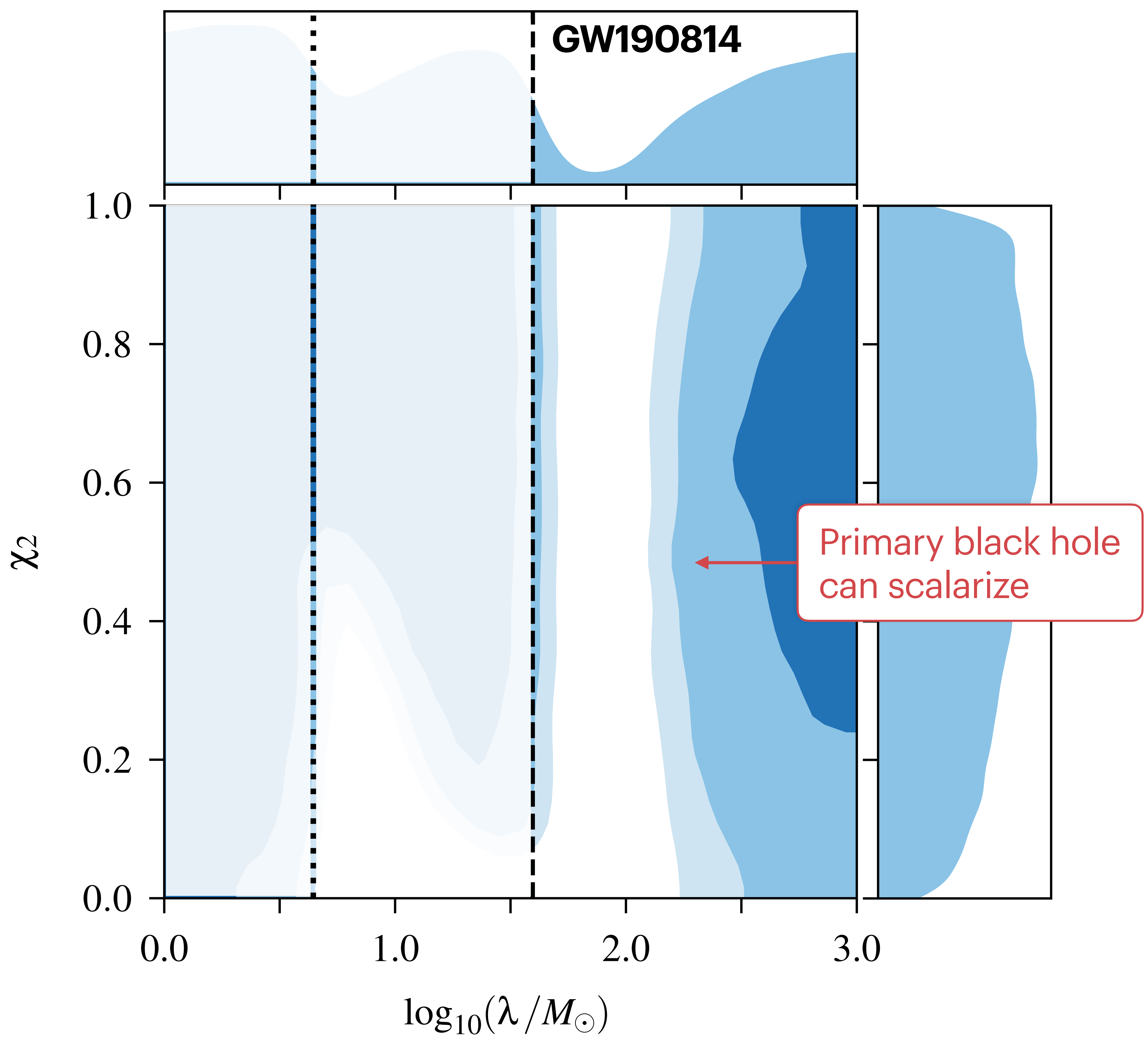
GW190814
$M_1/M_\odot$
$M_2/M_\odot$
χ_1
χ_2
$\log_{10}(\lambda/M_\odot)$
35
25
15
3.3
2.8
2.3
1.8
0.4
0.2
0.0
0.8
0.5
1.5
2.5



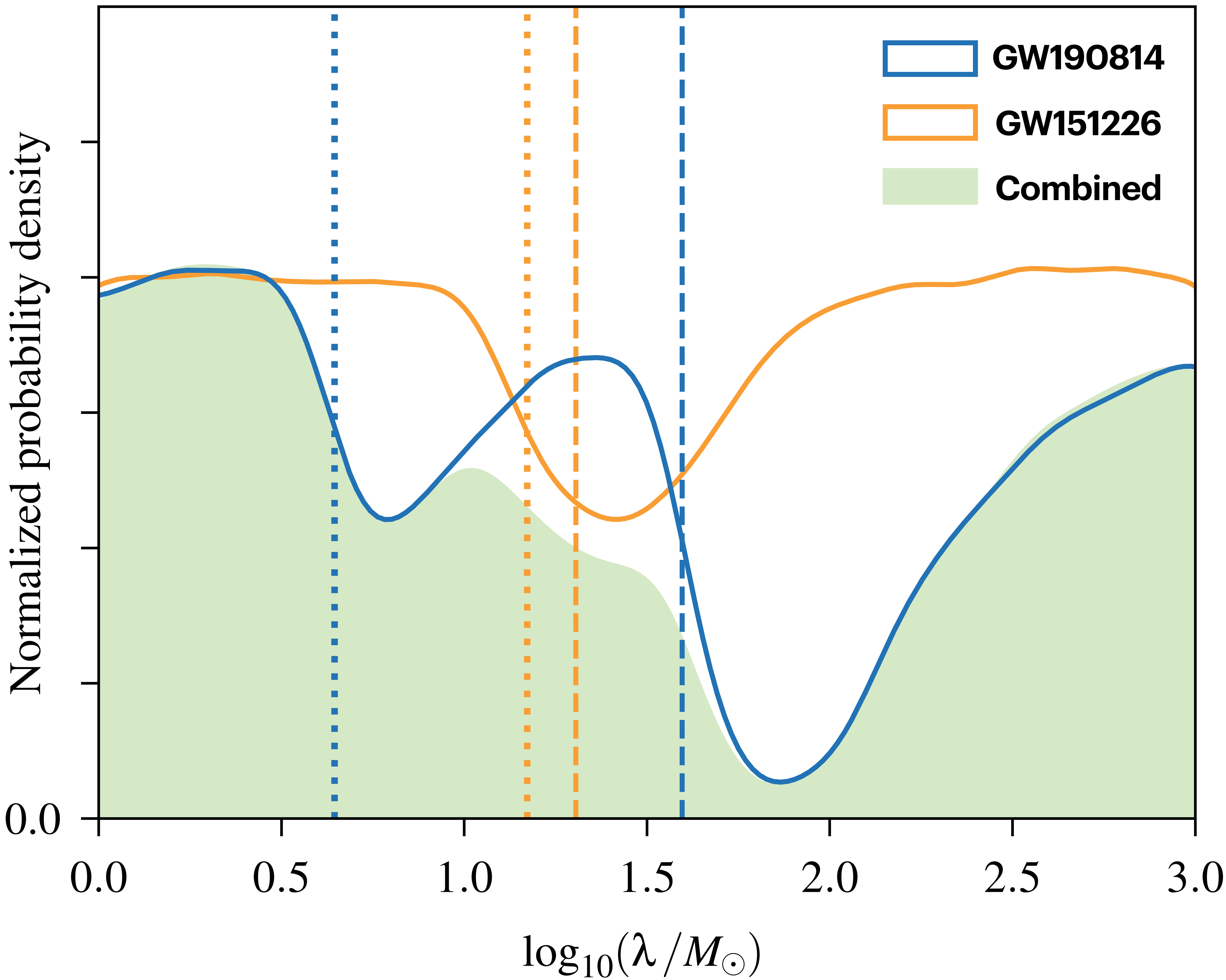
GW190814
1.0
0.8
0.6
0.4
0.2
0.0
χ_2
0.0
1.0
2.0
3.0
$\log_{10}(\lambda/M_\odot)$


GW190814
Both black holes are Kerr
1.0
0.8
0.6
0.4
0.2
0.0
χ_2
0.0
1.0
2.0
3.0
$\log_{10}(\lambda/M_\odot)$


GW190814
Secondary black hole can scalarize
1.0
0.8
0.6
0.4
0.2
0.0
χ_2
0.0
1.0
2.0
3.0
$\log_{10}(\lambda/M_{\odot})$



GW190814
1.0
0.8
0.6
0.4
0.2
0.0
χ_2
0.0
1.0
2.0
3.0
$\log_{10}(\lambda/M_\odot)$
Primary black hole can scalarize


GW190814
GW151226
Combined
Normalized probability density
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\log_{10}(\lambda/M_\odot)$

Compare theories with different values of λ by computing

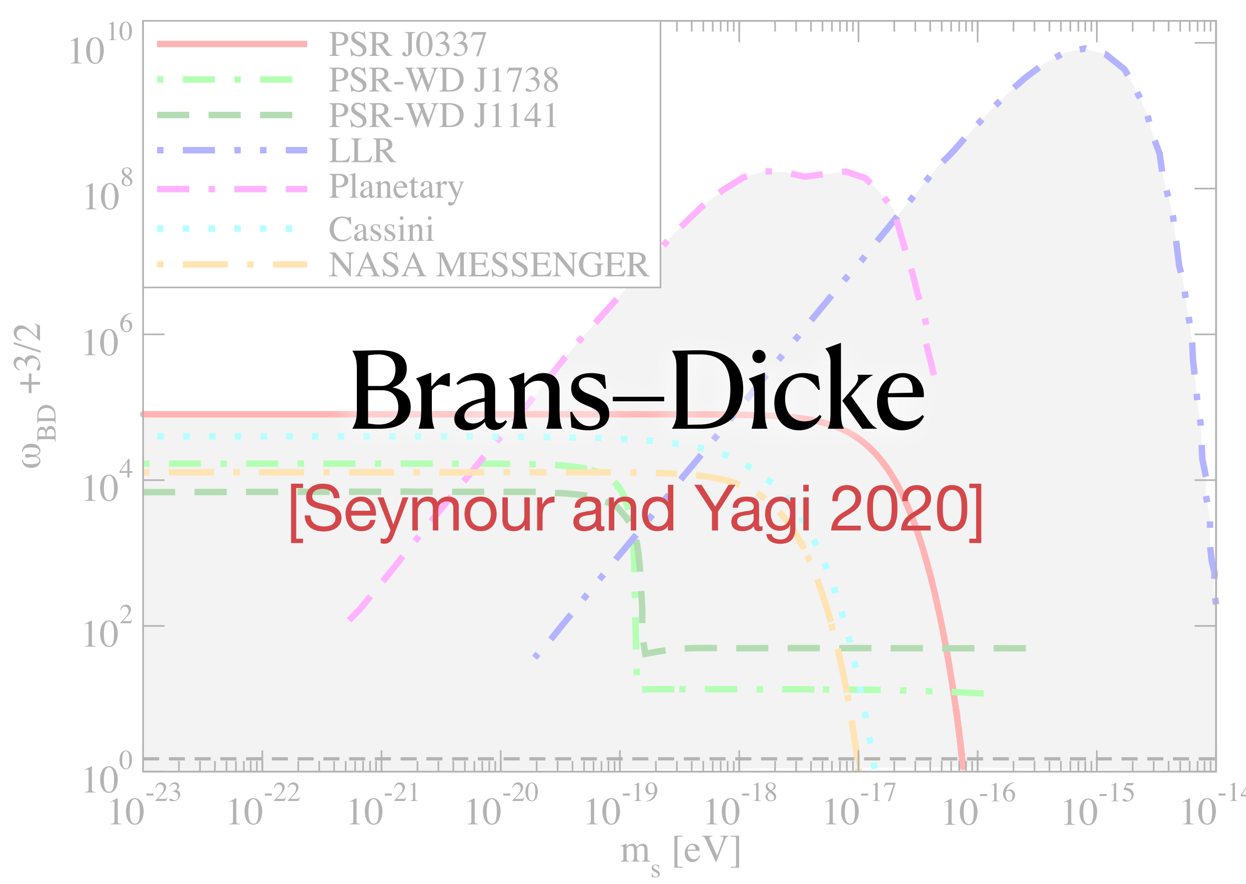
$$B(\log \lambda) = \lim_{\lambda_0 \to 0} \frac{p(\log \lambda \,|\, d)}{p(\log \lambda_0 \,|\, d)}$$

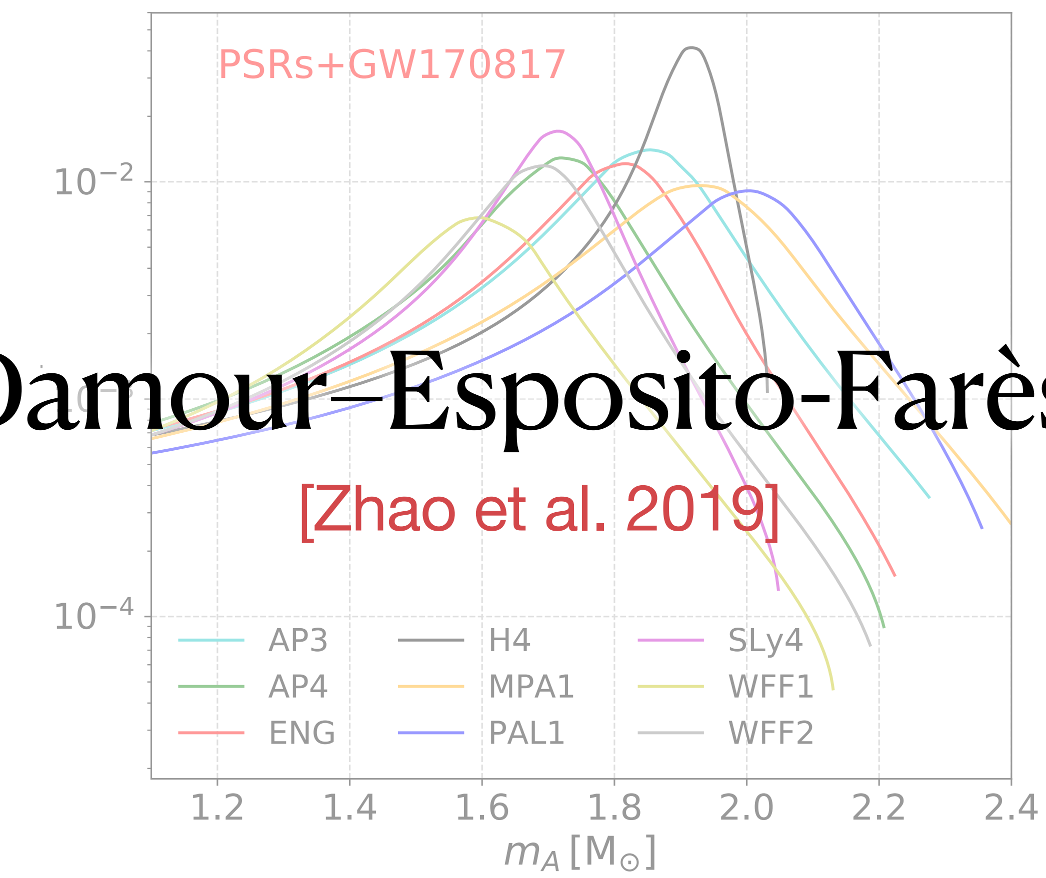
A theory with Bayes factor B is $1/B$ times less likely than GR is at being the correct underlying description of the signal

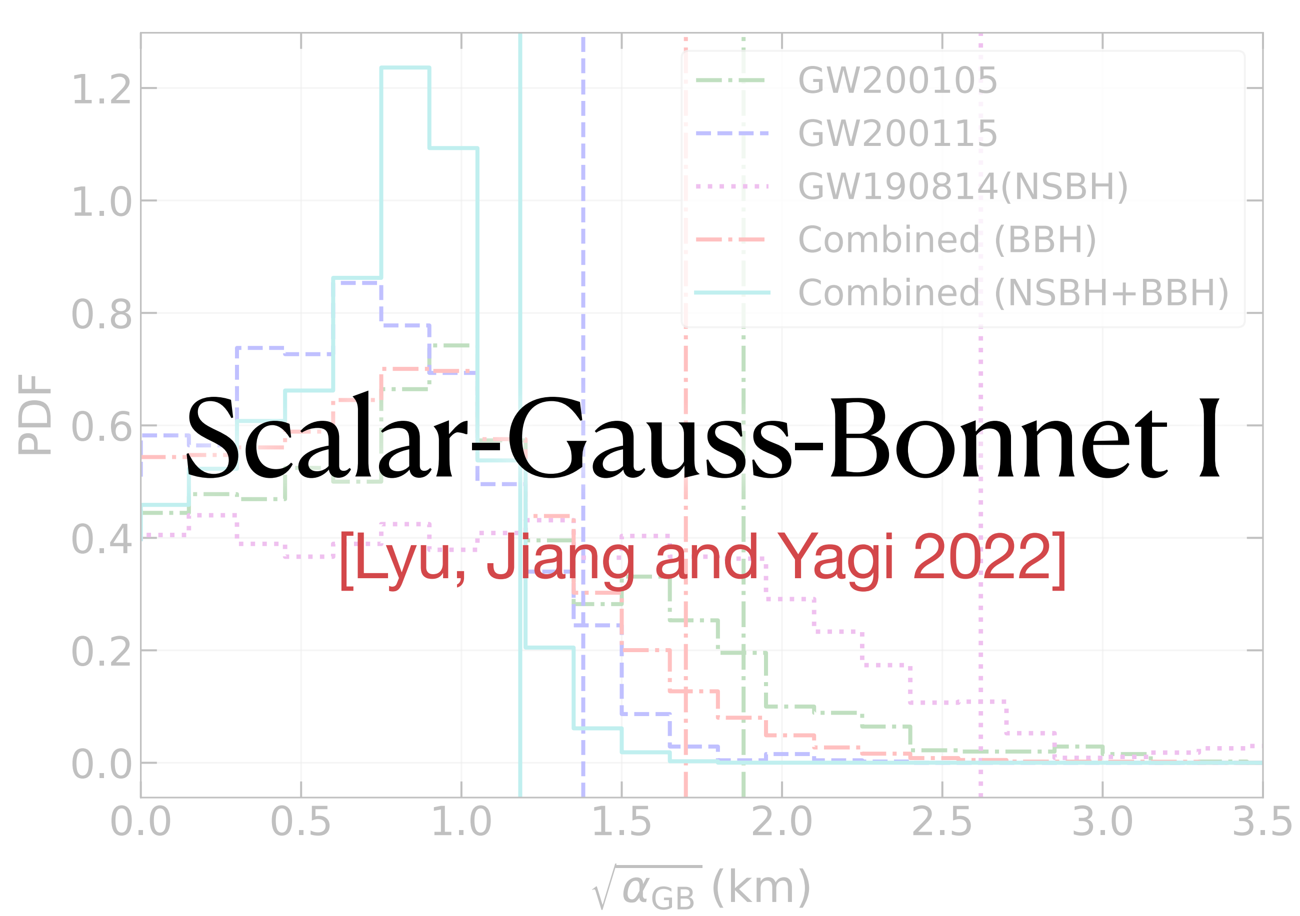
$$B \leq 0.1 : \qquad 56\ M_\odot \lesssim \lambda \lesssim 96\ M_\odot$$

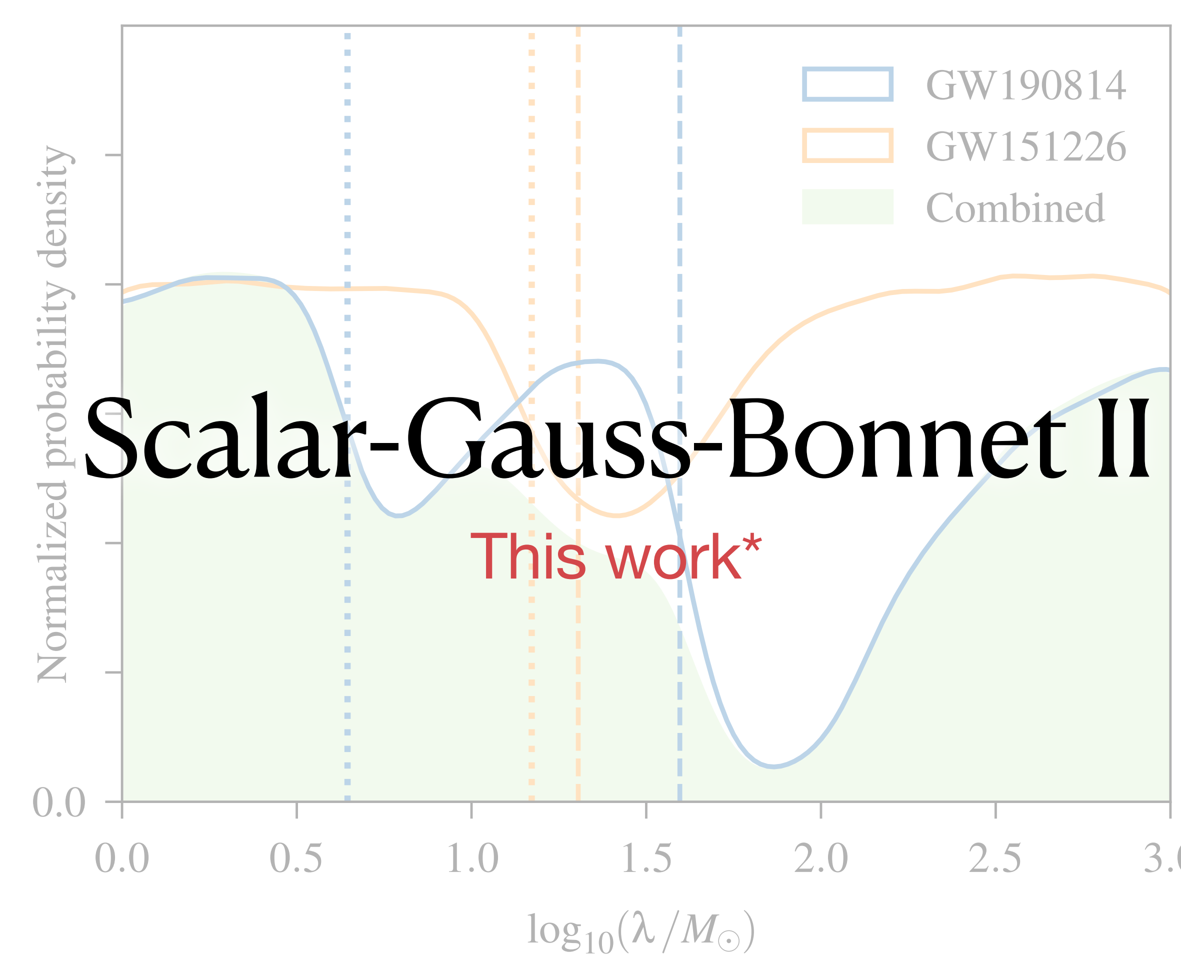
Massless scalar-tensor theories

	All objects have scalar charges	Spontaneous scalarization
No hairy black holes	Brans–Dicke [Seymour and Yagi 2020]	Damour–Esposito-Farèse [Zhao et al. 2019]
Hairy black holes	Scalar-Gauss-Bonnet I [Lyu, Jiang and Yagi 2022]	Scalar-Gauss-Bonnet II This work*

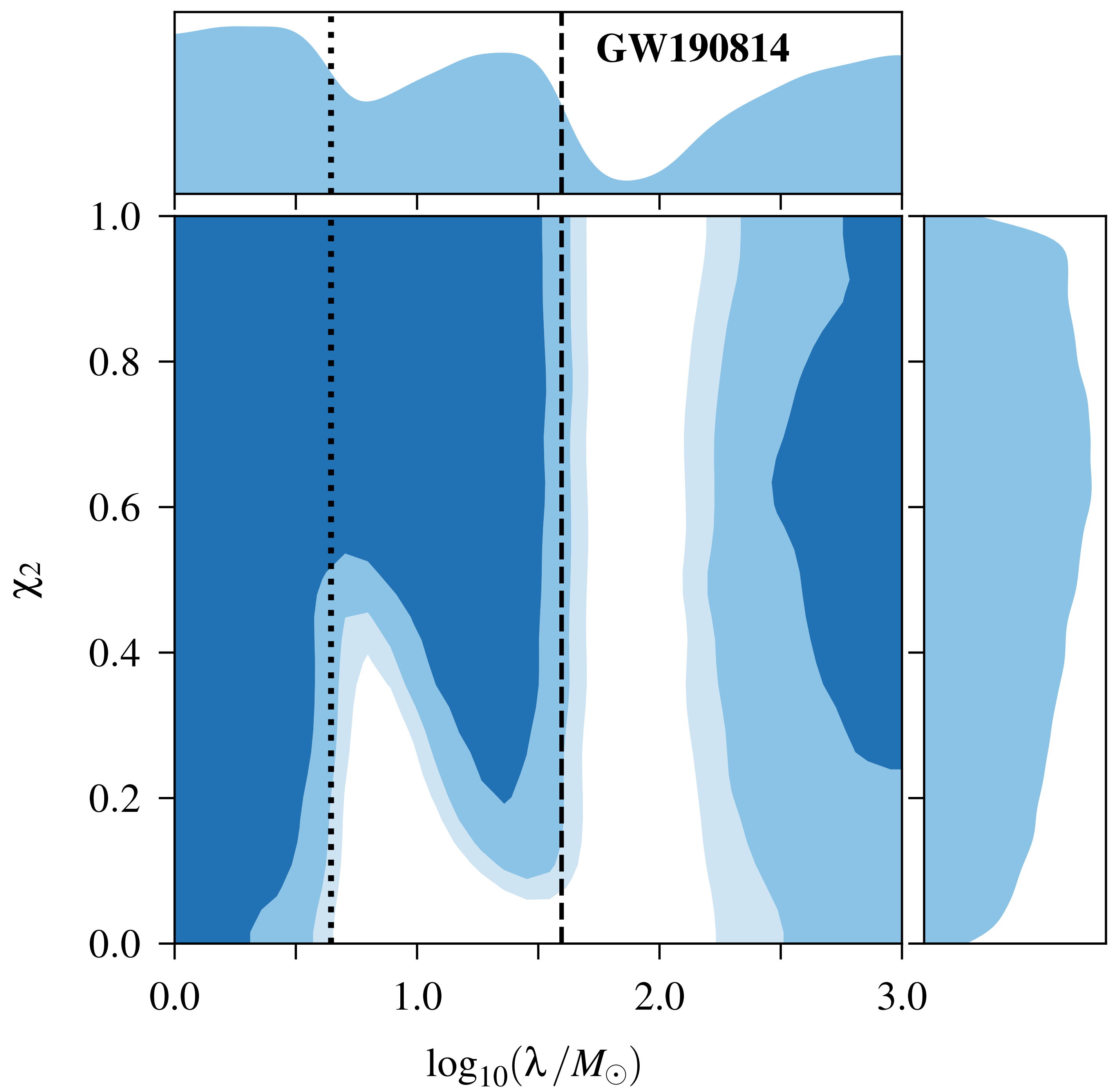


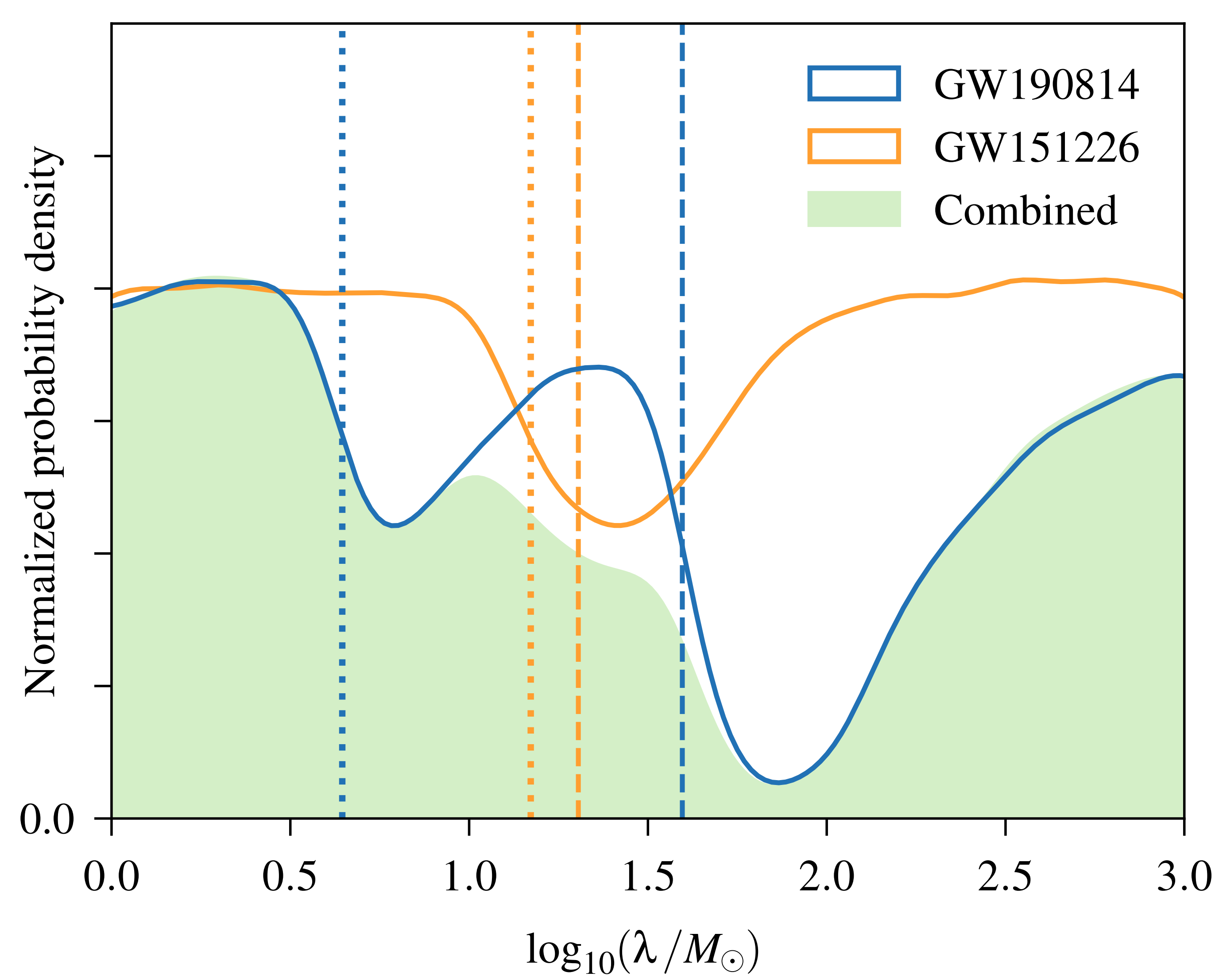






*See also [Danchev, Doneva and Yazadjiev 2022] for binary pulsar constraints





Future directions

- Improved waveform templates with higher PN effects; merger–ringdown portion?
- Update constraints with more GW events, especially from BHNS systems
- Constrain related models like spin-induced scalarization