Primordial gravitational waves from excited states

Sébastien Renaux-Petel

CNRS - Institut d'Astrophysique de Paris

Mostly based on 2111.14664 (with J. Fumagalli, G. Palma, S. Sypsas, L. T Witkowski, C. Zenteno)

(also 2004.08369, 2012.02761, 2105.06481, 2110.09480, 2112.06903, 2112.10163)

GDR Ondes Gravitationnelles, Orsay 20.06.2022

European Research Council
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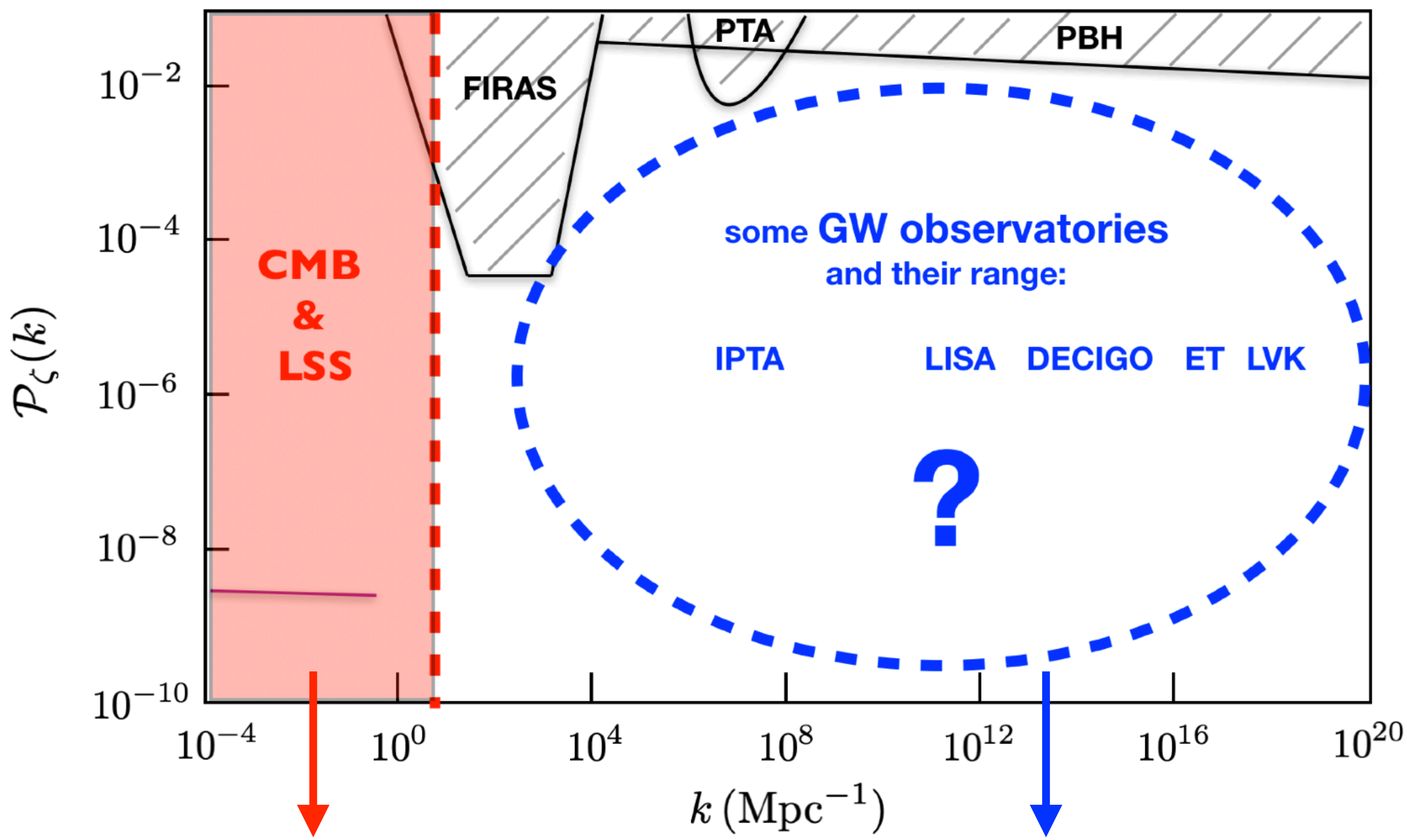
GEODESI

Probing inflation



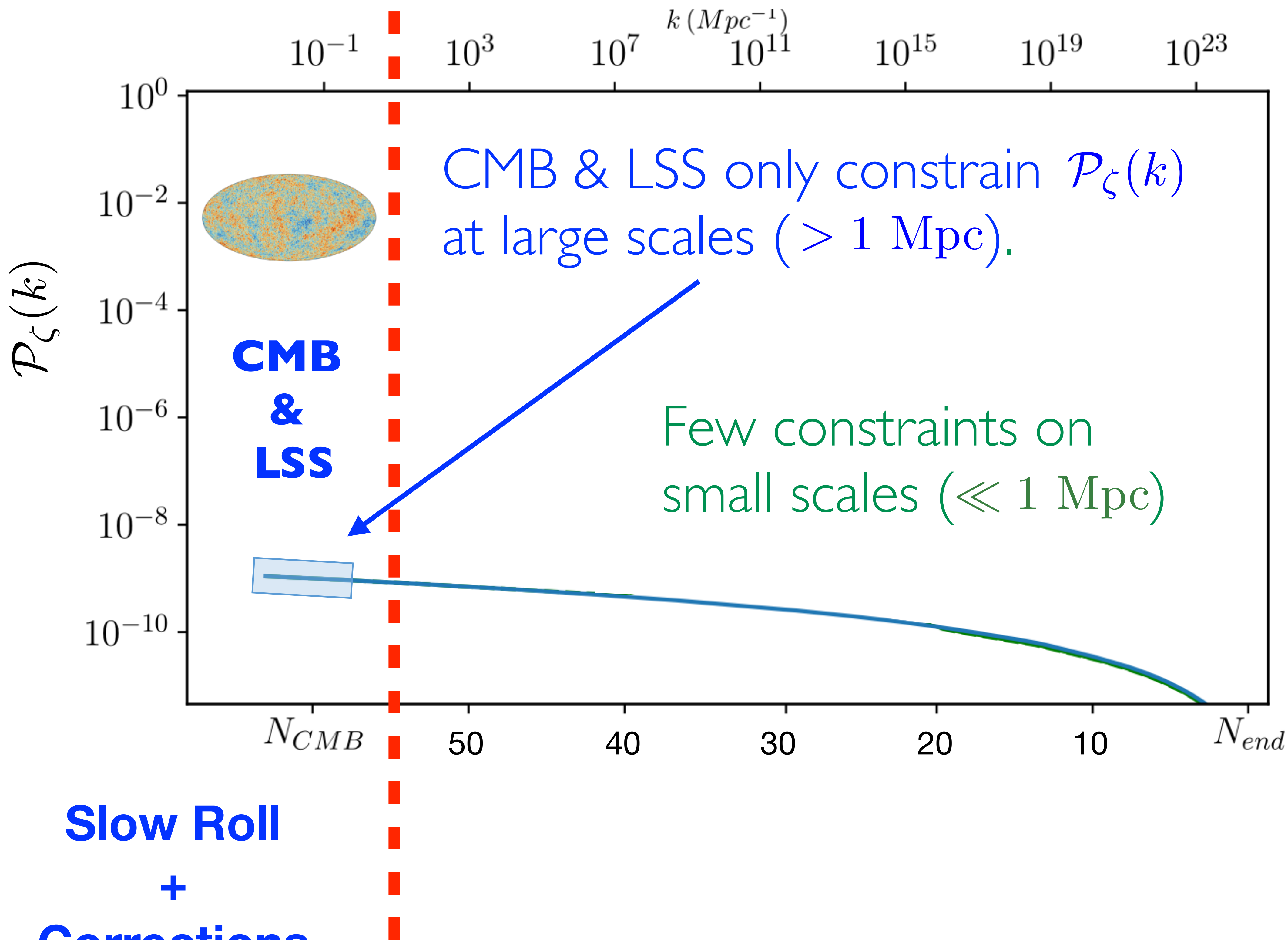
Precision physics

Non-Gaussianity, features

Exploratory physics

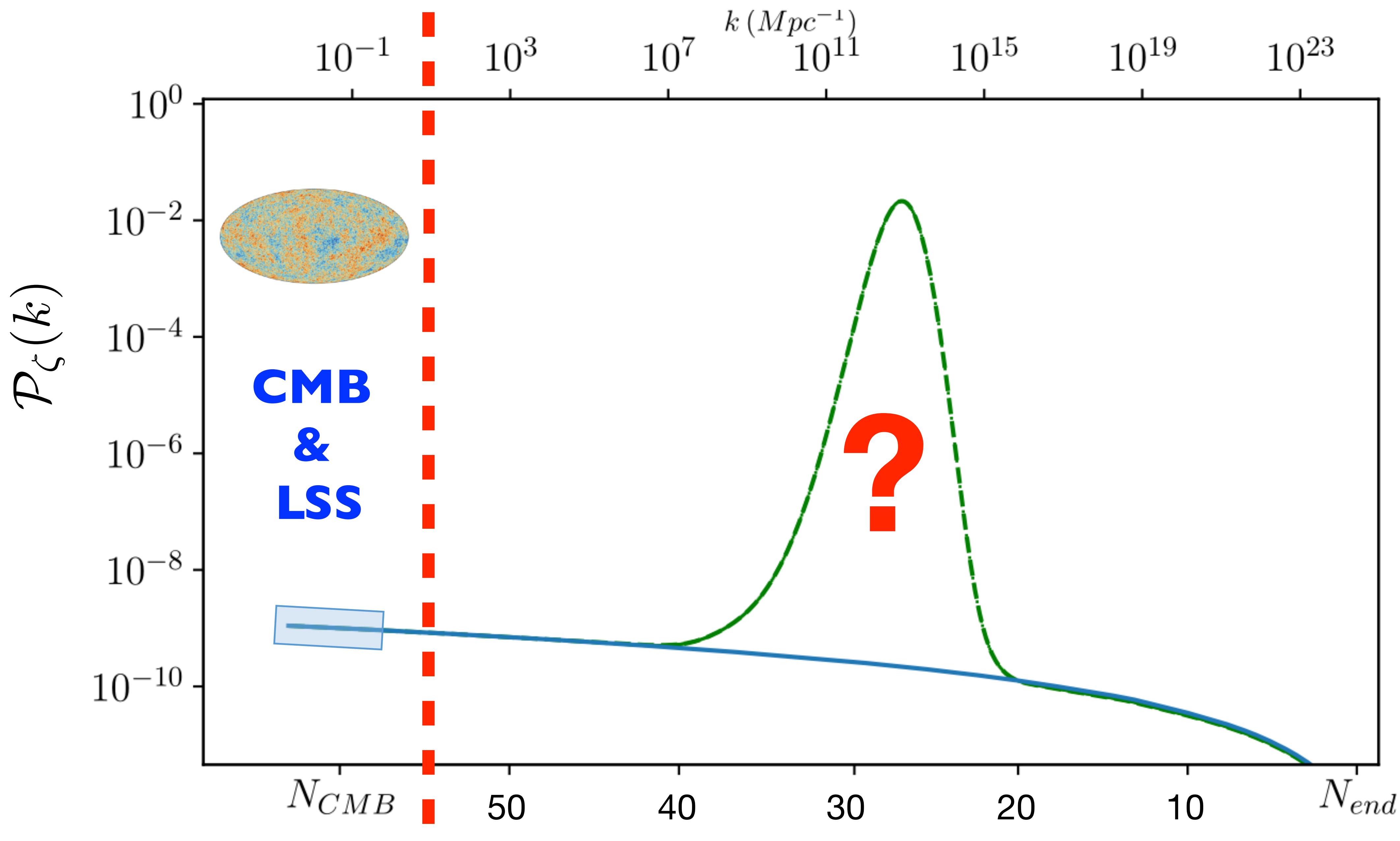
Probing dark inflationary era with gravitational waves

Inflation on small scales?



Slow Roll
+
Corrections

Inflation on small scales?



Slow Roll

+

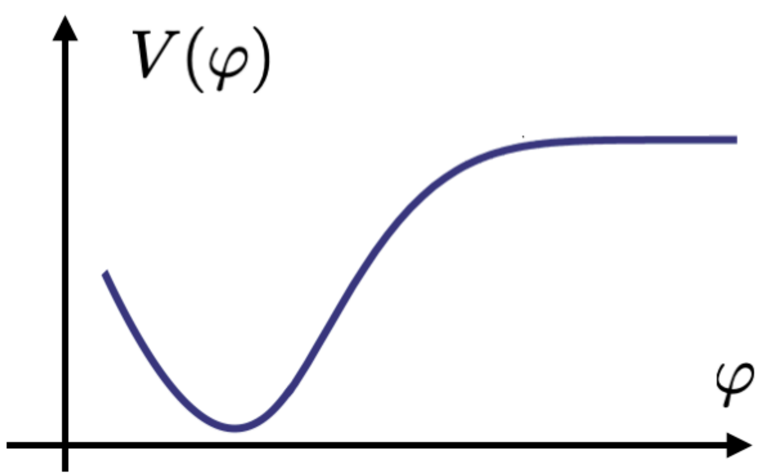
Corrections

Drastically different?

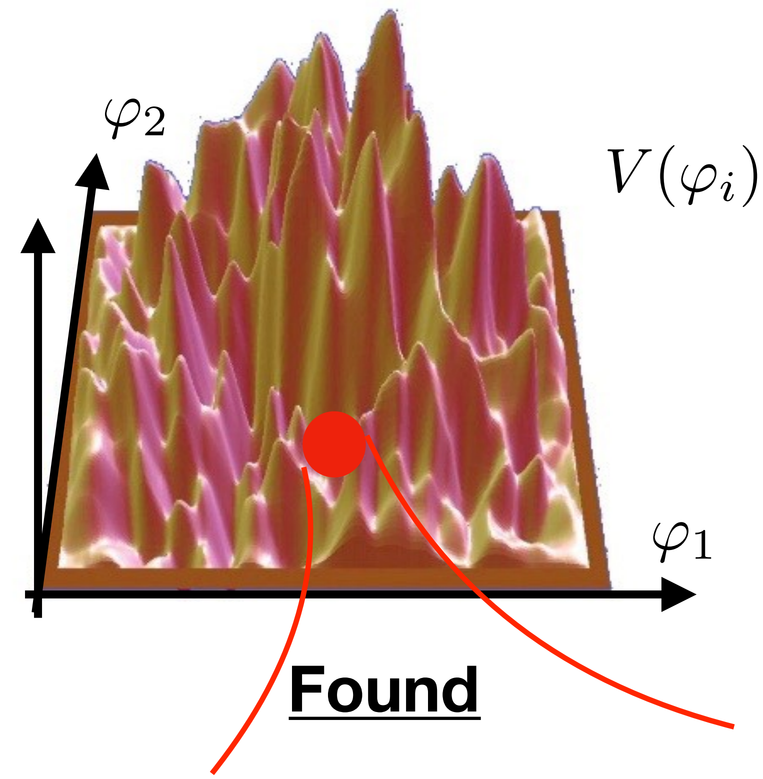
Naturally unnatural

Inflation on small scales?

A prolonged phase of 60 e-folds of inflation is not natural (eta-problem)



Hoped

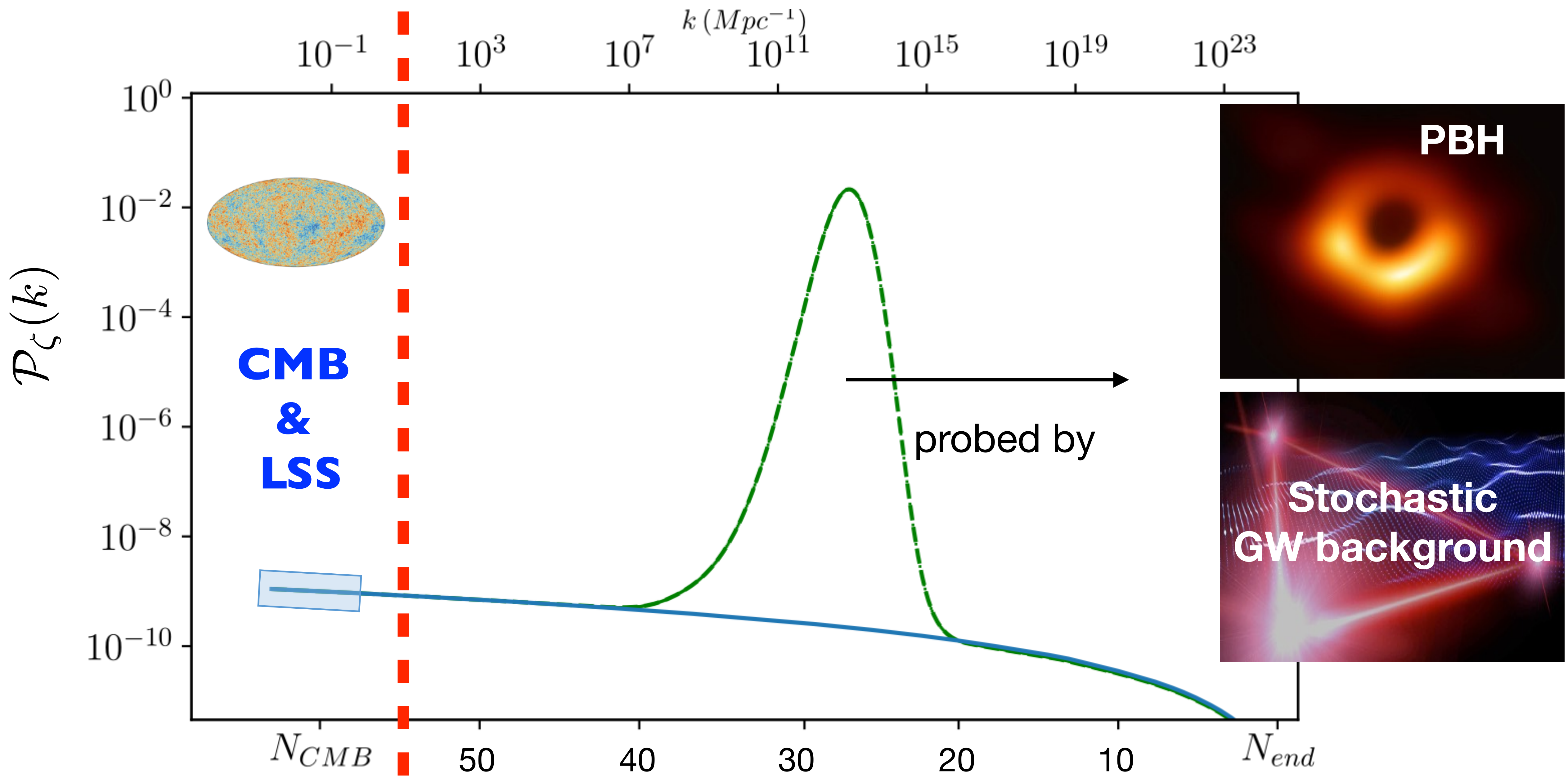


Found

More natural for inflation to
have occurred in successive phases

Non-trivial physics at transitions: *features*

Inflation on small scales?



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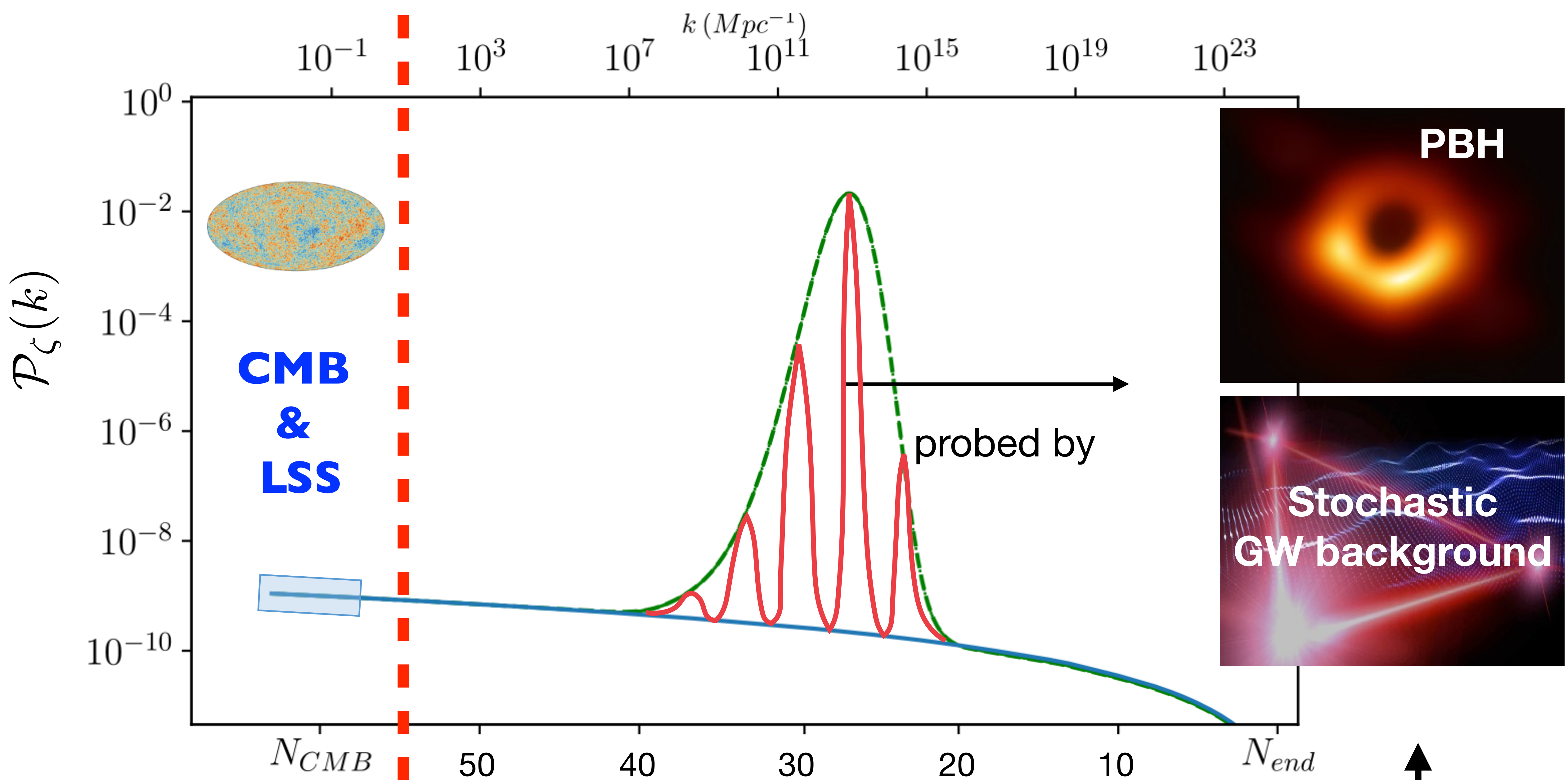


Slow Roll
+
Corrections

A sharp peak comes with large oscillations

Fumagalli, RP, Witkowski 2012.02761

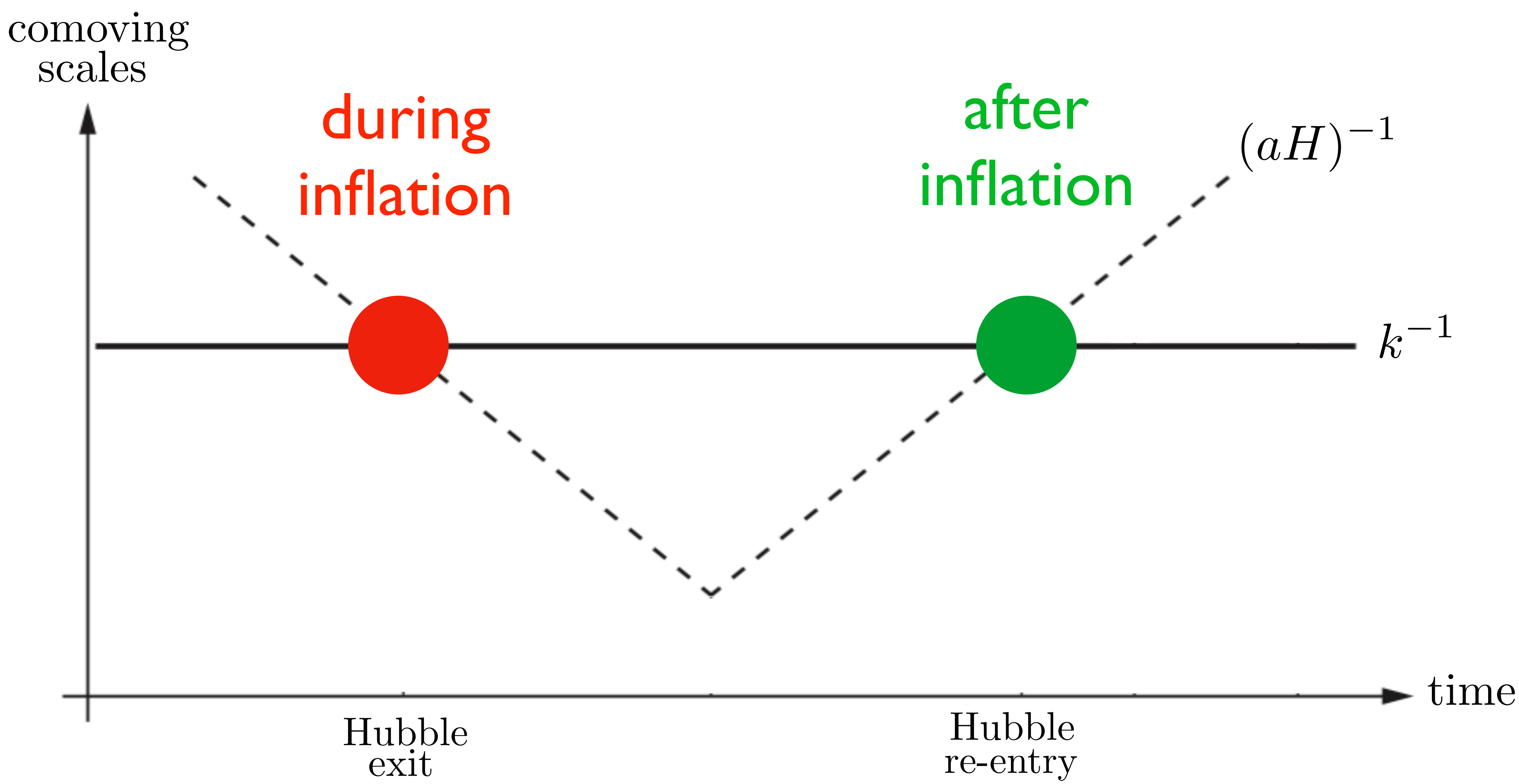
transferred

GW from inflation, which ones?

Vacuum quantum fluctuations $\Box h = 0$

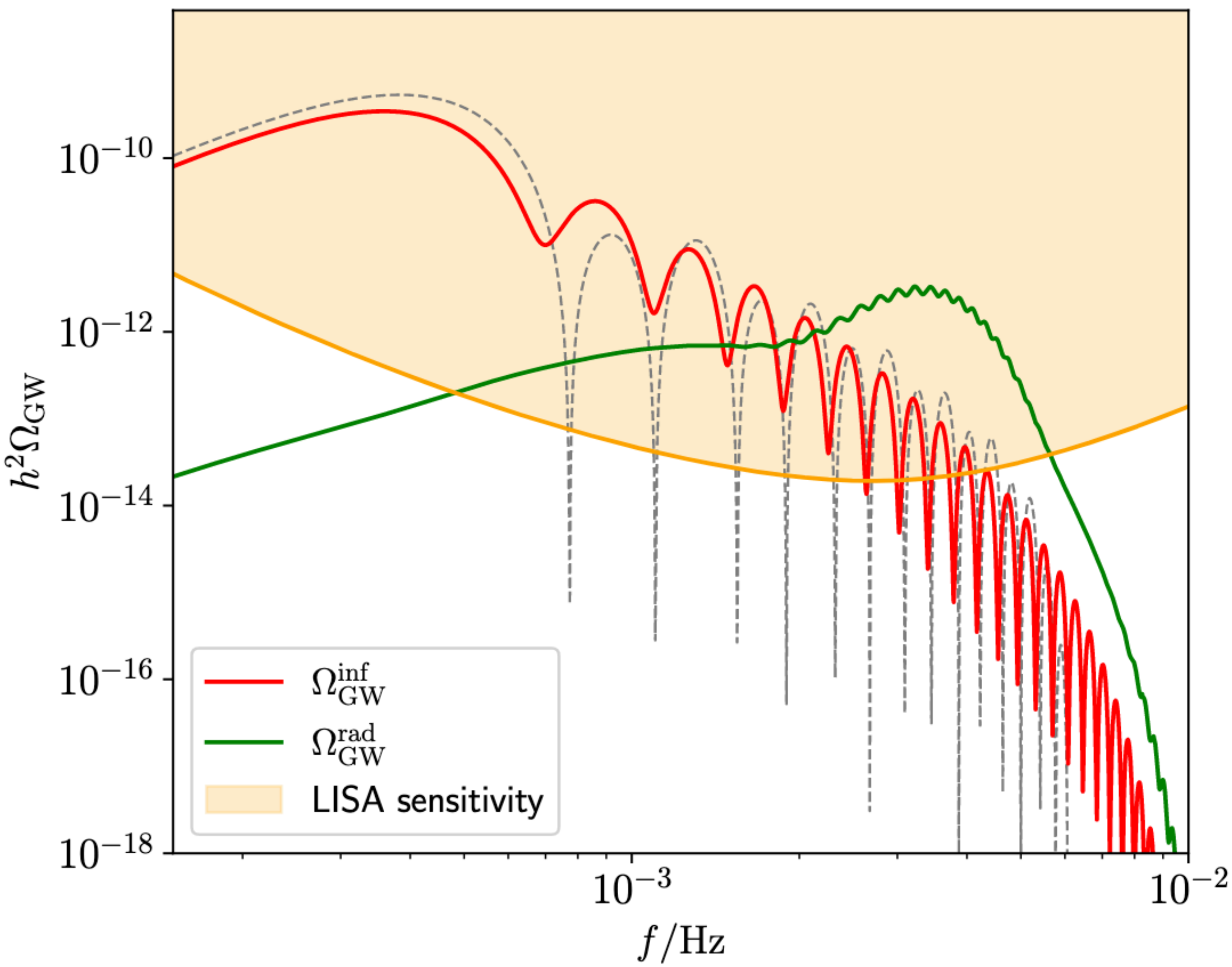
well understood, looked for in CMB polarization, tiny for interferometers

Here: GW sourced by scalar fluctuations $\Box h \sim (\partial\zeta)^2$



SGWB signature of sharp features

A sharp event during inflation leads to smoking gun oscillatory signatures for the two types of scalar-induced GWs

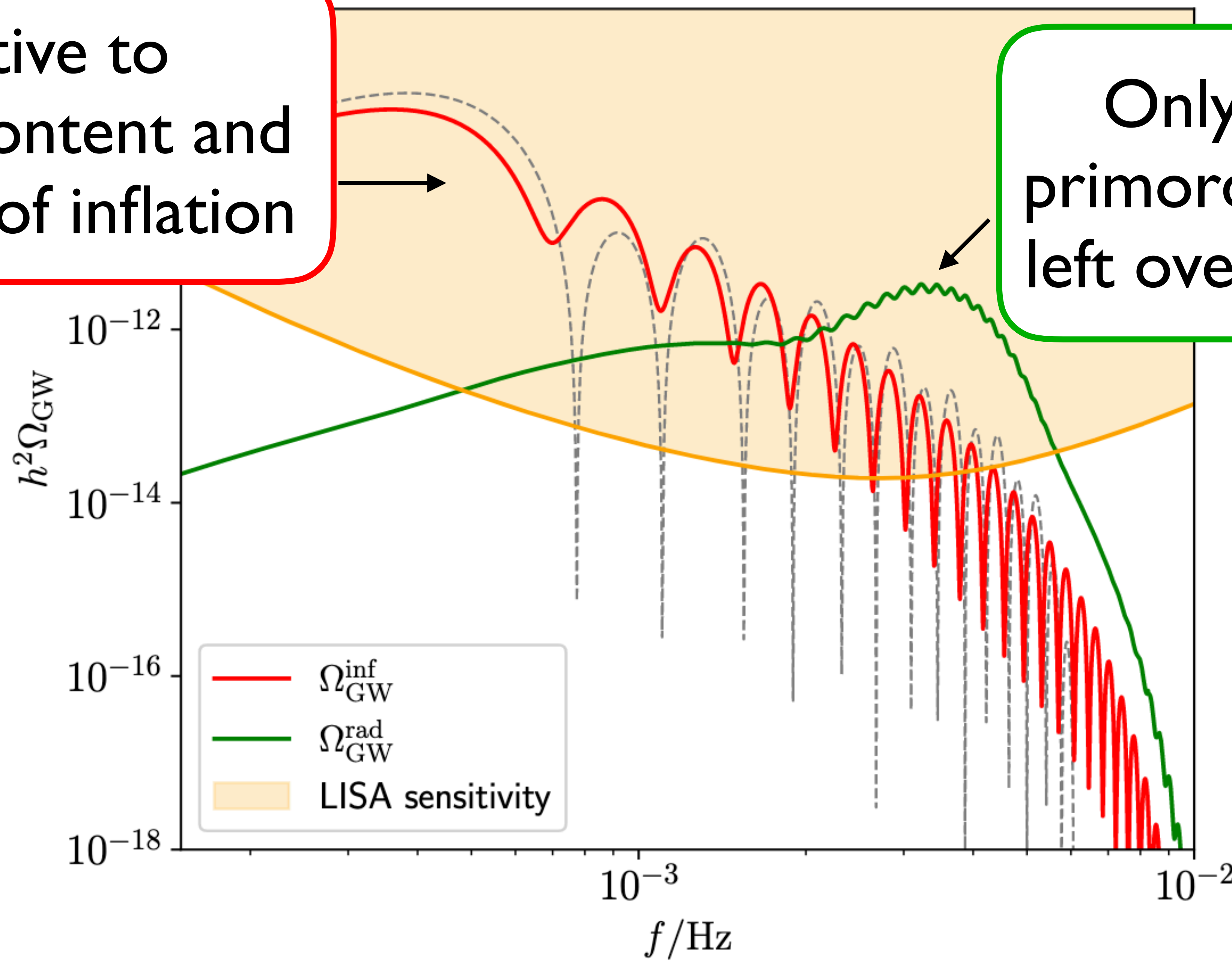


SGWB signature of sharp features

A sharp event during inflation leads to smoking gun oscillatory signatures for the two types of scalar-induced GWs

Sensitive to particle content and dynamics of inflation

Only sensitive to primordial fluctuations left over after inflation



Sharp feature → ***Excited state***

excitation of
sub-Hubble modes

production
of particles

Bunch-Davies | **Sharp feature** | **Excited state**

$$\zeta_k^{\rm BD}(\tau) = \left(\frac{k^3}{2\pi^2}\right)^{-1/2} \mathcal{P}_0^{1/2} e^{-ik\tau}(1+ik\tau)$$

$$\mathcal{P}_0 = \frac{H^2}{8\pi^2 \epsilon M_{\rm Pl}^2}$$

$$\zeta_k(\tau) = \alpha_k \zeta_k^{\rm BD}(\tau) + \beta_k \zeta_k^{*\rm BD}(\tau)$$

quantisation: $|\alpha_k|^2 - |\beta_k|^2 = 1$

number density: $n_k = |\beta_k|^2 \sim |\alpha_k|^2 \gtrsim 1$

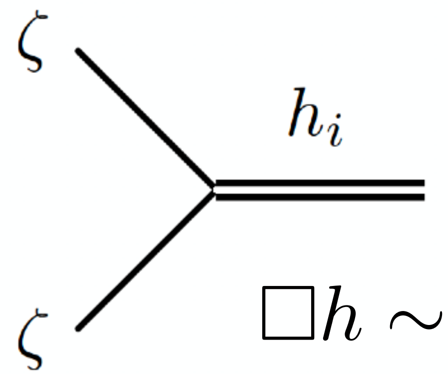
for large particle production

τ

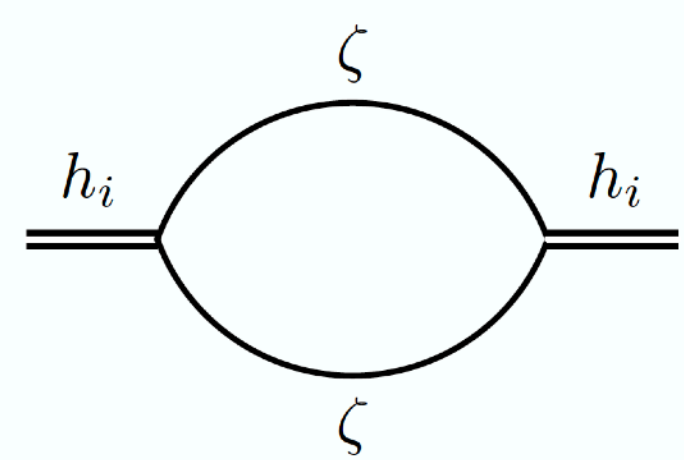
$\tau_{\rm f} = -1/k_{\rm f}$

Scalar-induced GWs after inflation

review
Domenech 2021



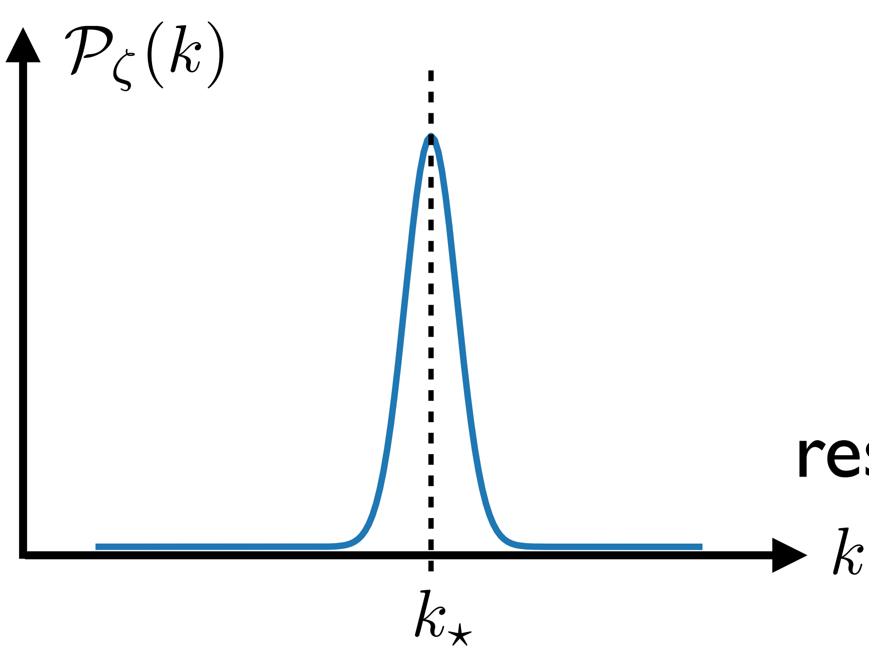
$\Box h \sim (\partial\zeta)^2$



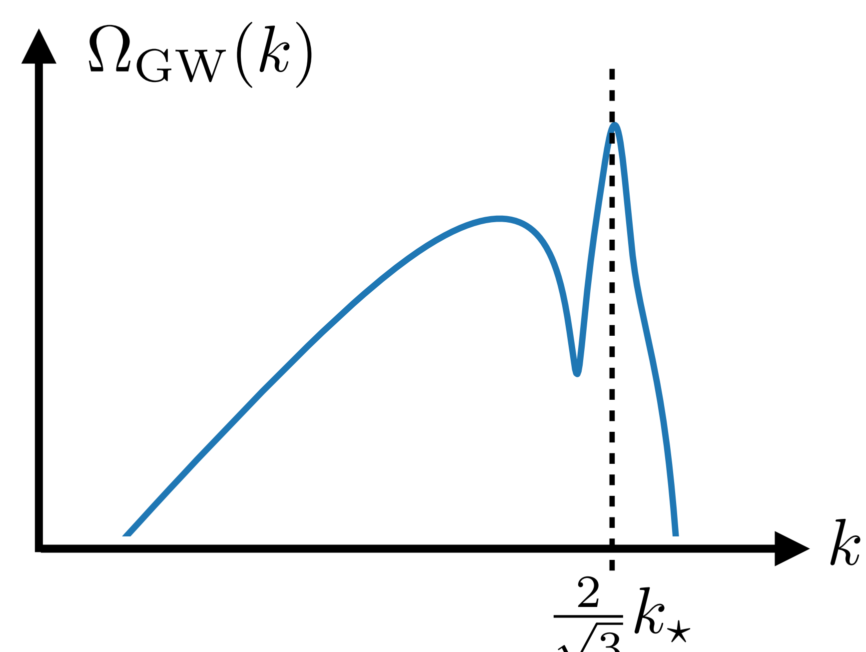
Enhanced $\delta\rho$ → Enhanced GWs at horizon re-entry after inflation

energy density per $\log(k)$ -interval:

$$\Omega_{\rm GW}(k) = \int\int T(u,v)\mathcal{P}_\zeta(ku)\mathcal{P}_\zeta(kv) \quad \sim 10^{-5}\,\mathcal{P}_\zeta^2$$

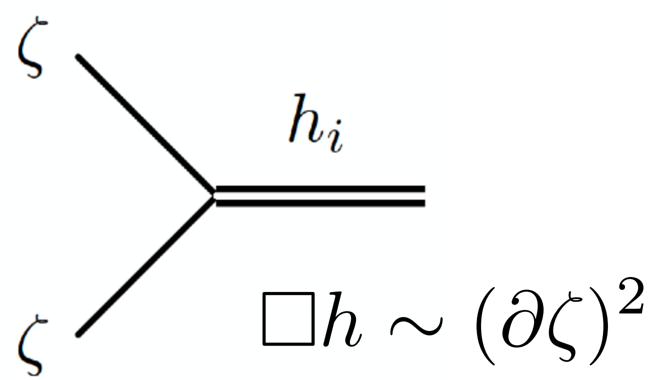


peak from
resonant amplification

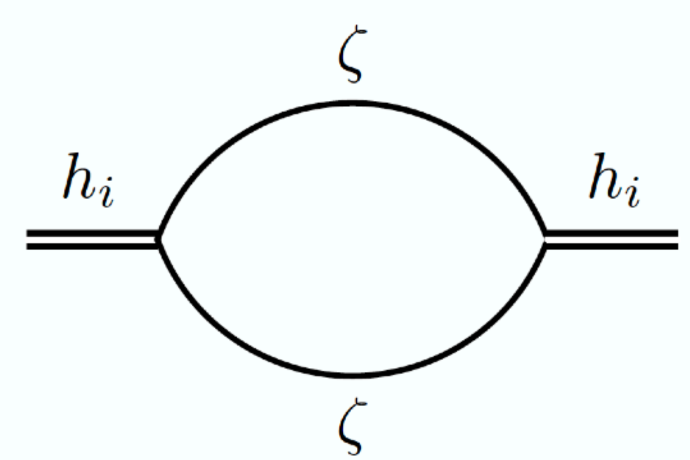


Scalar-induced GWs after inflation

review
Domenech 2021



$\longrightarrow$



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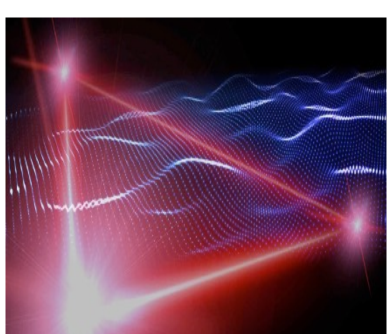
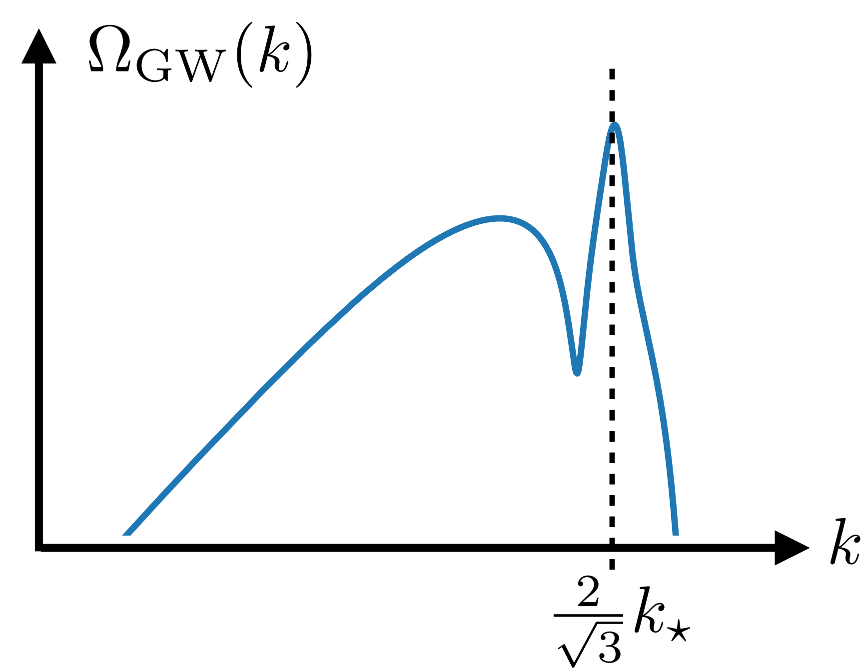
$$\log\left(\frac{f}{10^{-3}{\rm Hz}}\right) \simeq \log\left(\frac{k}{10^{12}{\rm Mpc}^{-1}}\right) \simeq N_{\rm after\ CMB} - 30$$



GW observatories probe inflation on small scales



Scalar-induced GWs after inflation

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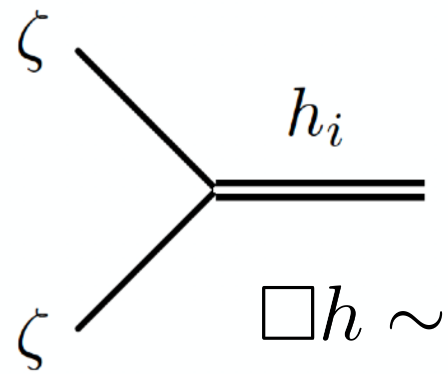

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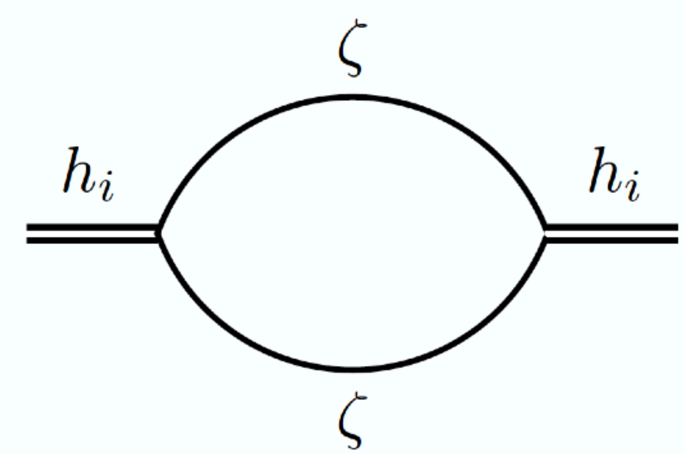

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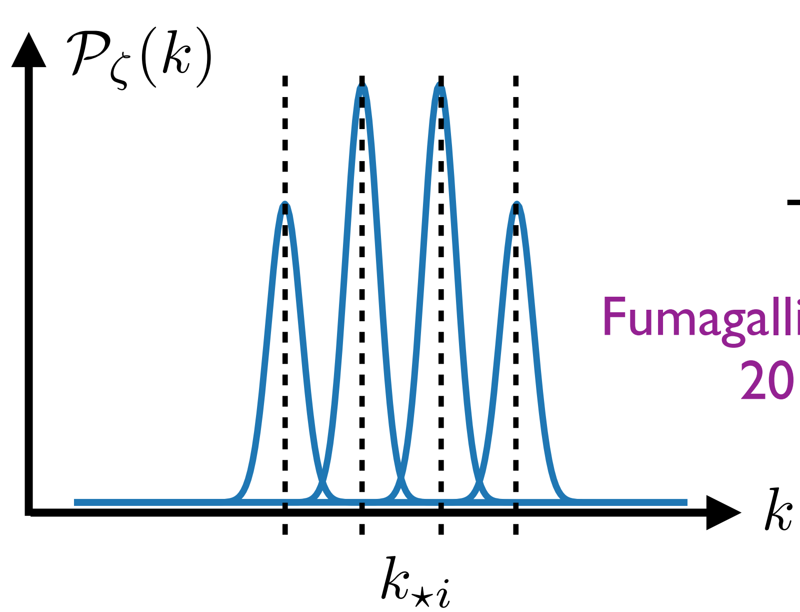
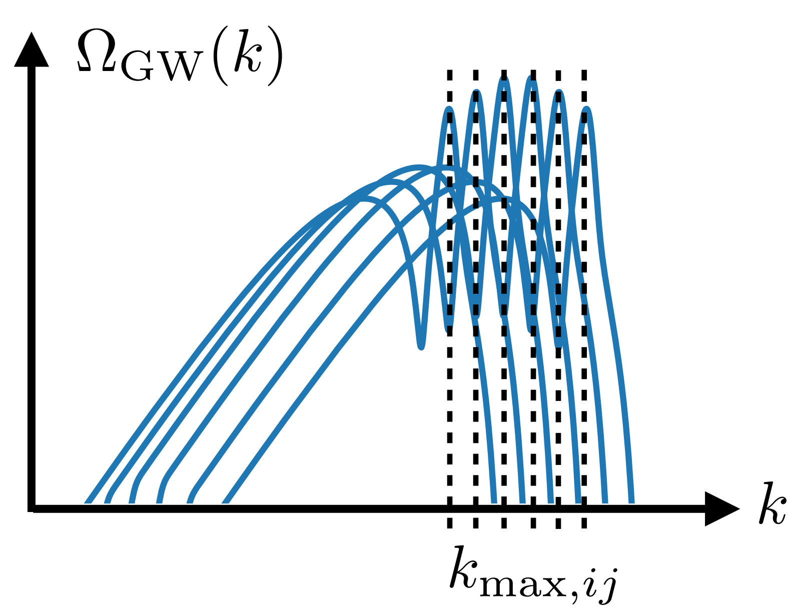


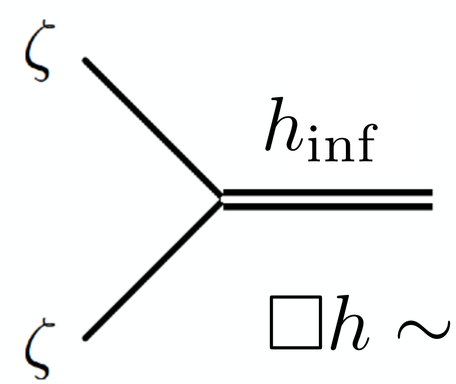
Fumagalli, RP, Witkowski
2012.02761



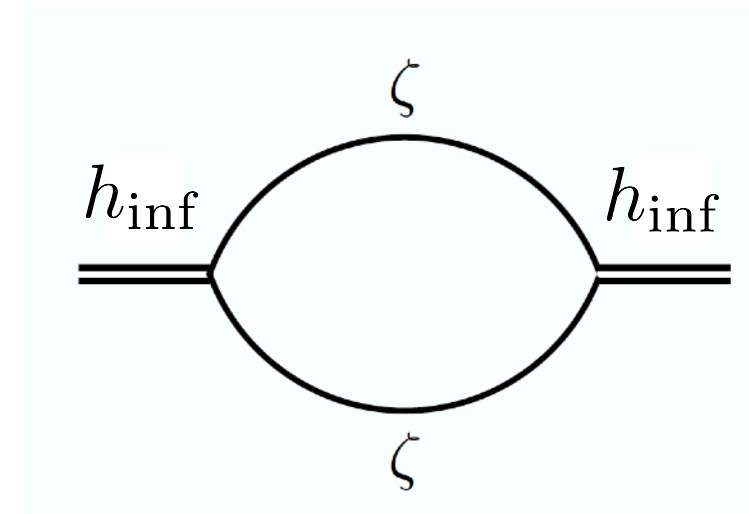
Scalar-induced GWs during inflation

$$\Omega_{\rm GW}^{\rm inf}(k) \sim 10^{-6}\mathcal{P}_t(k)$$

Formally similar
to post-inflationary
induced GWs:



$$\Box h \sim (\partial\zeta)^2$$



But:

- the source is different
- the Green function is different
- fields are a priori quantum

Scalar-induced GWs during inflation

$$\Box h_{\boldsymbol{k}} = S_{\boldsymbol{k}} \sim \int d^3\boldsymbol{p}\, \zeta_{\boldsymbol{p}} \zeta_{\boldsymbol{k}-\boldsymbol{p}} \quad \longrightarrow \quad h_{\boldsymbol{k}}(t) \sim \int^t G_k(t,t') S_{\boldsymbol{k}}(t)$$

Classical Green function method:

suitable approximation to full quantum ‘in-in’ computation, as large particle production guarantees that classical effects are dominant

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Classical Green function method:

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$$\langle h_{\boldsymbol{k}} h_{-\boldsymbol{k}} \rangle' \sim \int dt' \int dt'' G_k(t,t') G_k(t,t'') \langle S_{\boldsymbol{k}}(t') S_{-\boldsymbol{k}}(t'') \rangle$$

$$\sim \int d^3\boldsymbol{p} \langle \zeta_{\boldsymbol{p}}(t') \zeta_{-\boldsymbol{p}}(t'') \rangle \langle \zeta_{\boldsymbol{k}-\boldsymbol{p}}(t') \zeta_{-(\boldsymbol{k}-\boldsymbol{p})}(t'') \rangle$$

$$\longrightarrow \quad \mathcal{P}_h(k,t) \sim \int d^3\boldsymbol{p} \left| \int dt' G_k(t,t') \zeta_p(t') \zeta_{|\boldsymbol{k}-\boldsymbol{p}|}(t') \right|^2$$

Scalar-induced GWs during inflation

Full result (correct for first time)

$$\mathcal{P}_t(k,\tau) = \frac{k^3}{2\pi^4 M_{\rm Pl}^4} \sum_{i,j} \int_0^\infty \mathrm{d}p\, p^6 \int_0^\pi \mathrm{d}\theta\, \sin^5\theta \times \left| \int^\tau \mathrm{d}\tau_1\, g_k(\tau,\tau_1) \sum_X Q_{Xi}(p,\tau_1) Q_{Xj}(|\mathbf{k}-\mathbf{p}|,\tau_1) \right|^2$$

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Sum over all types of independent quanta
(all dofs are in general quantum-mechanically correlated)

Sum over all types of scalar dofs during inflation
(contrast with post-inflationary GWs governed by final zeta)

$$\hat{Q}_X(\boldsymbol{k},\tau) = \sum_{i=1}^{\mathcal{N}} Q_{Xi}(k,\tau)\hat{a}_i(\boldsymbol{k}) + \mathrm{h.c.}(-\boldsymbol{k})$$ with $$\left[\hat{a}_i(\boldsymbol{k}), \hat{a}_j^\dagger(\boldsymbol{k}')\right] = (2\pi)^3 \delta^{ij} \delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$$

Scalar-induced GWs during inflation

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$$\left| \int^\tau {\rm d}\tau_1\, g_k(\tau,\tau_1) \sum_X Q_{Xi}(p,\tau_1) Q_{Xj}(|\mathbf{k}-\mathbf{p}|,\tau_1) \right|^2$$

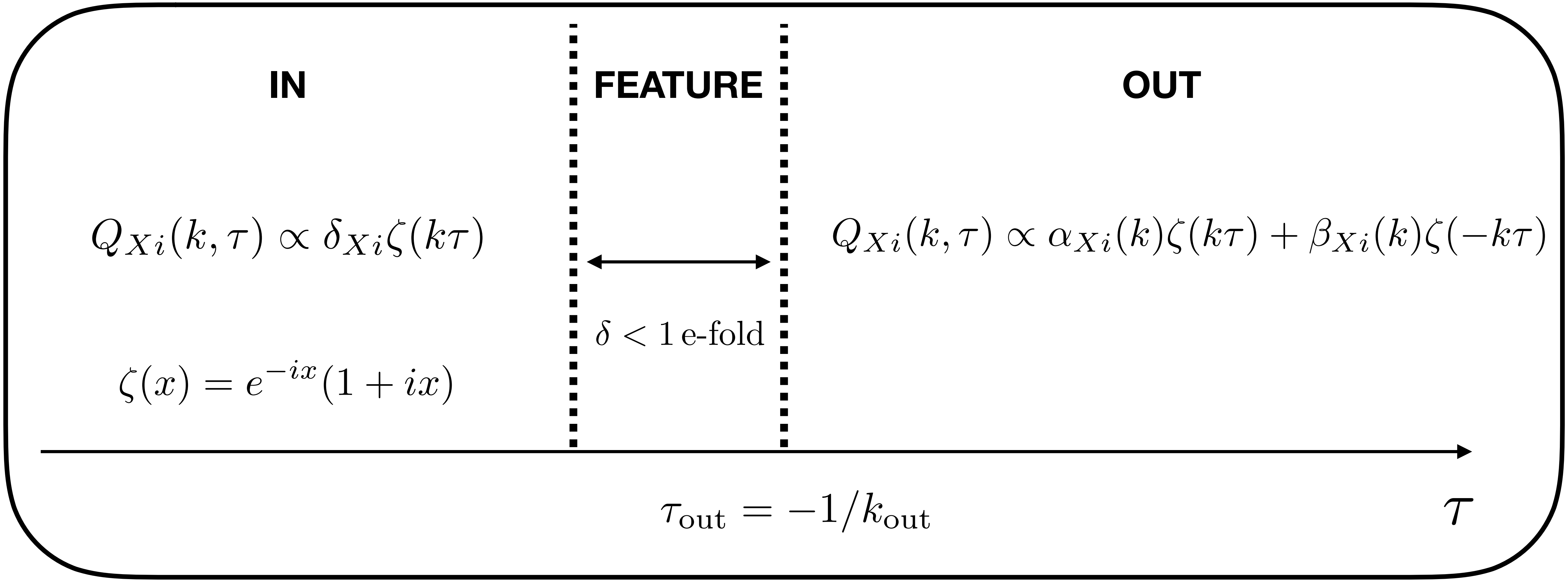
'Divergence' in large p limit

'Divergence' in infinite past limit

Standard Bunch-Davies divergences, renormalized away

Dynamically generated excited state: starting from a given time, some modes experience particle production → natural regulators to all integrals: dominated by classical effects

GWs from excited states

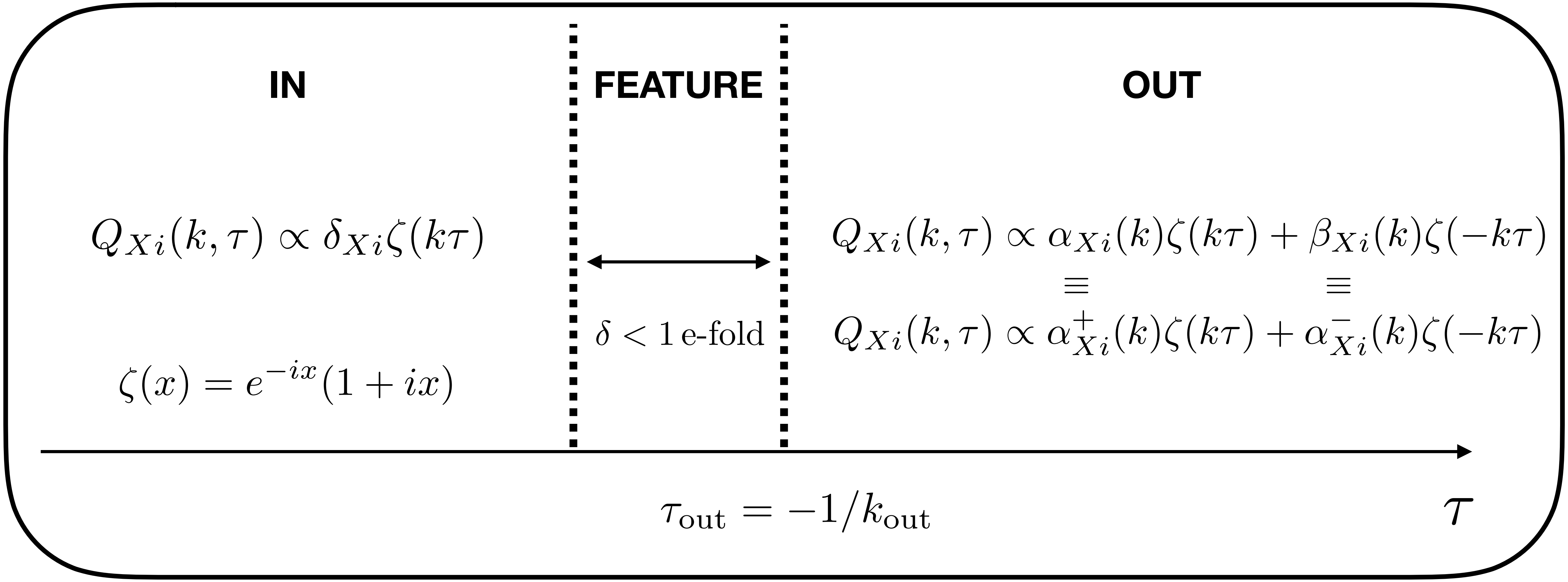


\`feature region' negligible under motivated assumptions

out region: green function + mode functions known

→ time integration can be performed analytically

GWs from excited states



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GWs from excited states

$$\mathcal{P}_t(k) = \frac{H^4}{8\pi^4 M_{\rm Pl}^4}\int {\rm d}x \int {\rm d}y\ \mu(x,y)\ \times$$

$$\sum_{i,j}\left|\sum_{X;\,{\rm s}_{1,2}=\pm} \alpha^{{\rm s}_1}_{Xi}(xk)\alpha^{{\rm s}_2}_{Xj}(yk)\mathcal{G}({\rm s}_1 x, {\rm s}_2 y)\right|^2$$

Kernels:

$$\mathcal{G}(x,\pm y) = \int_{-k/k_{\rm out}}^{0} dz e^{-i(x\pm y)z}(1+ixz)(1\pm iyz)\frac{z\cos z - \sin z}{z^2}$$

GWs from excited states

integration over momenta and geometrical factor

$$x = \frac{p}{k}, \quad y = \frac{|\boldsymbol{k} - \boldsymbol{p}|}{k}$$

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Overal amplitude w/o excited states

Sum over positive and negative frequency modes

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++ (or - -): highly oscillatory exponential damps integrals

+- (or - +): for $x \simeq y$ constructive interferences between + and - frequency modes

$$\mathcal{G}(x, y) \ll \mathcal{G}(x, -y) \supset e^{ik/k_{\rm out}}$$

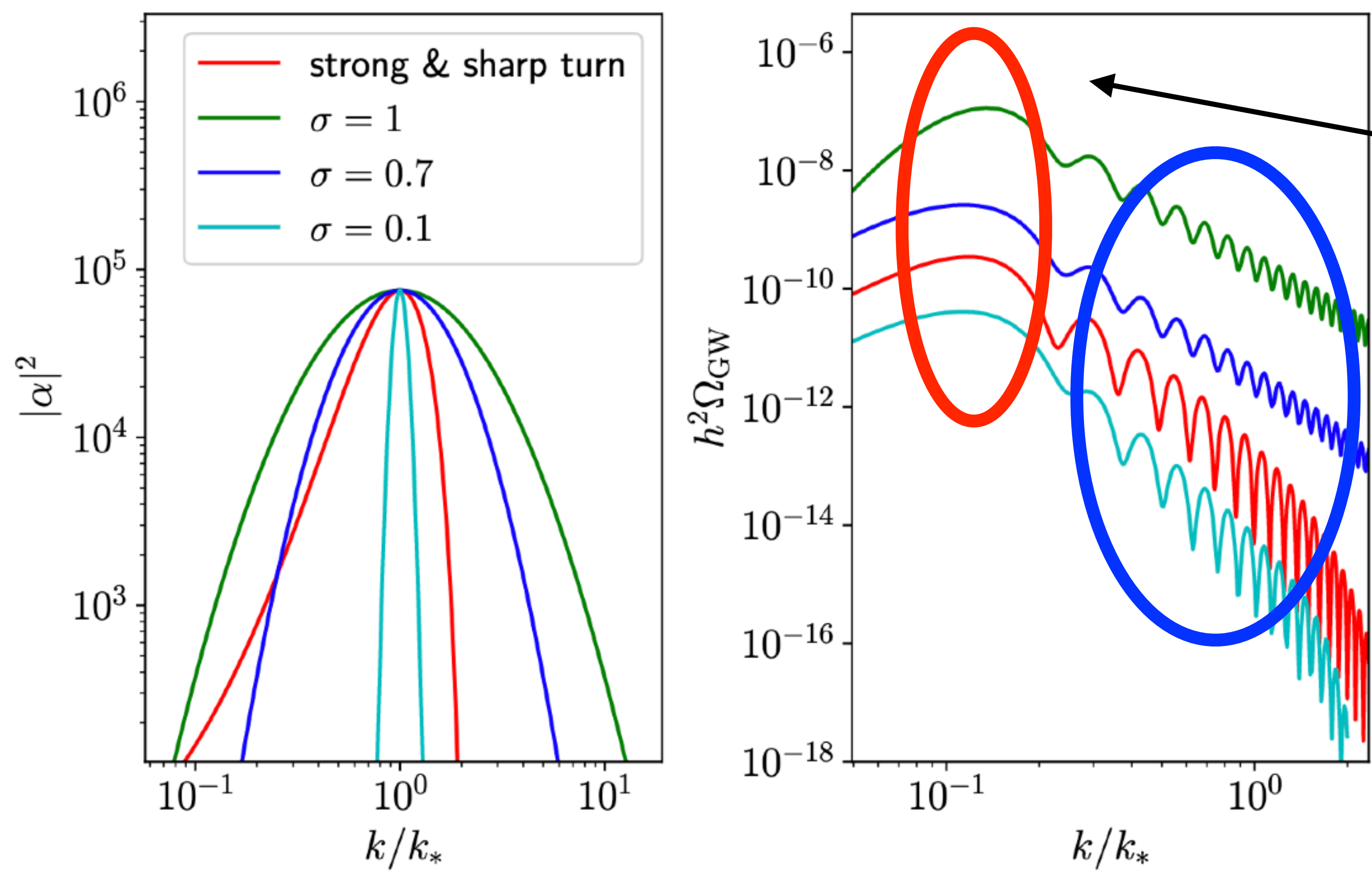
Similar to enhancement of NGs for excited states (here tensor-scalar-scalar)

GWs from excited states

One field for simplicity + large particle production (prerequisite) $\beta \simeq \alpha e^{i\theta}$

$$\mathcal{P}_t(k) = \frac{H^4}{4\pi^4 M_{\rm Pl}^4}\int_0^\infty \mathrm{d}y \int_{|1-y|}^{1+y} \mathrm{d}x \; \mu(x,y) \; |\alpha(xk)|^2 \, |\alpha(yk)|^2 \times \left(|\mathcal{G}(x,-y)|^2 + \mathrm{Re}\left[e^{i(\theta(xk)-\theta(yk))} \mathcal{G}^2(x,-y) \right] \right)$$

Universal features independent of precise Bogoliubov coefficients

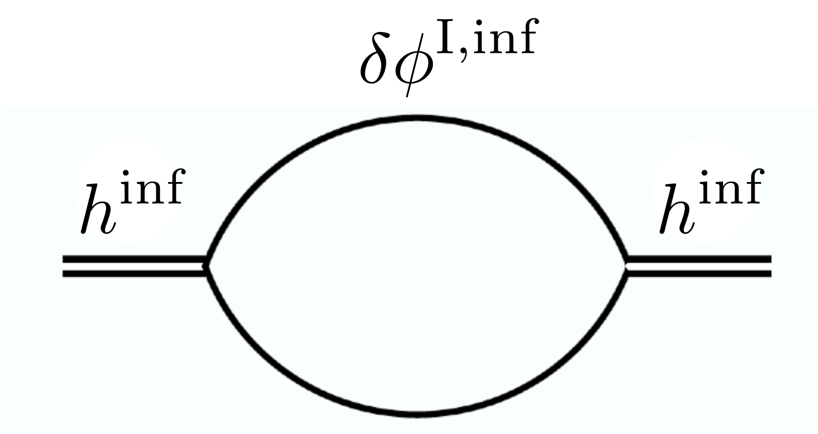


Max at $k \simeq 3.5 k_{\rm out}$

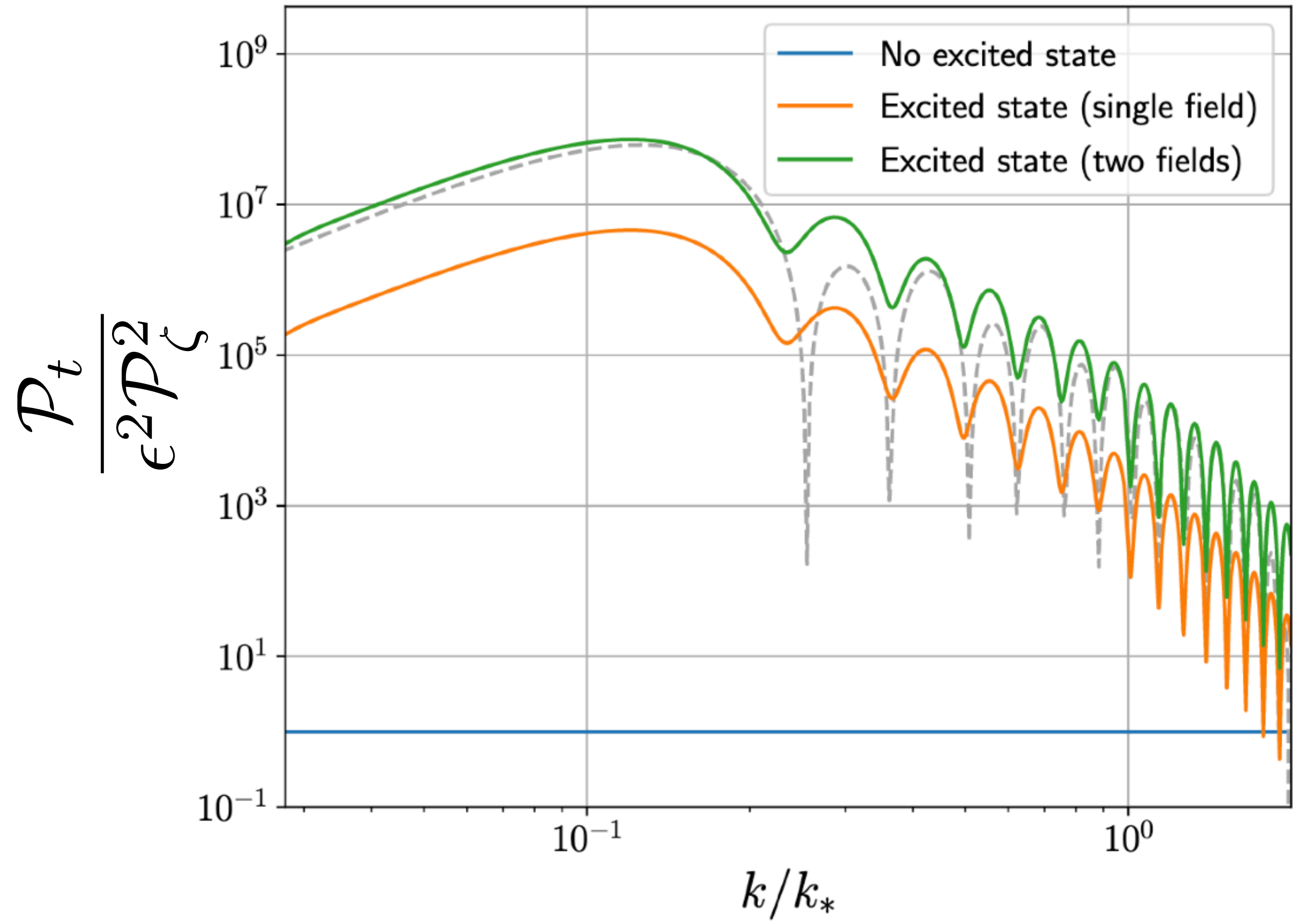
Order one oscillations on UV-tail, at frequency $\omega = 2/k_{\rm out}$

e.g. $|\alpha(k)|^2 = \overline{\alpha}^2 \exp\left[-\frac{1}{2\sigma^2}\ln^2(k/k_*)\right], \theta = 0$

GWs from excited states



One-loop tensor power spectrum



Scalars in vacuum: $\mathcal{P}_t \sim \epsilon^2\mathcal{P}_\zeta^2$

Constructive interferences between positive and negative frequency modes

⟶ Scalar excited states **amplify the signal** by orders of magnitude + **order one oscillations**

GWs from excited states

Narrowly peaked Bogoliubov coefficients

$$\Omega_{\rm GW}^{\rm inf}(k) = \bar{\Omega}_{\rm GW}^{\rm inf} \frac{1}{(\omega k)^3} \left(1 - \left(\frac{k}{2k_*}\right)^2\right)^2 \left(\sin(\omega k/2) - 4\frac{(1-\cos(\omega k/2))}{\omega k}\right)^2$$

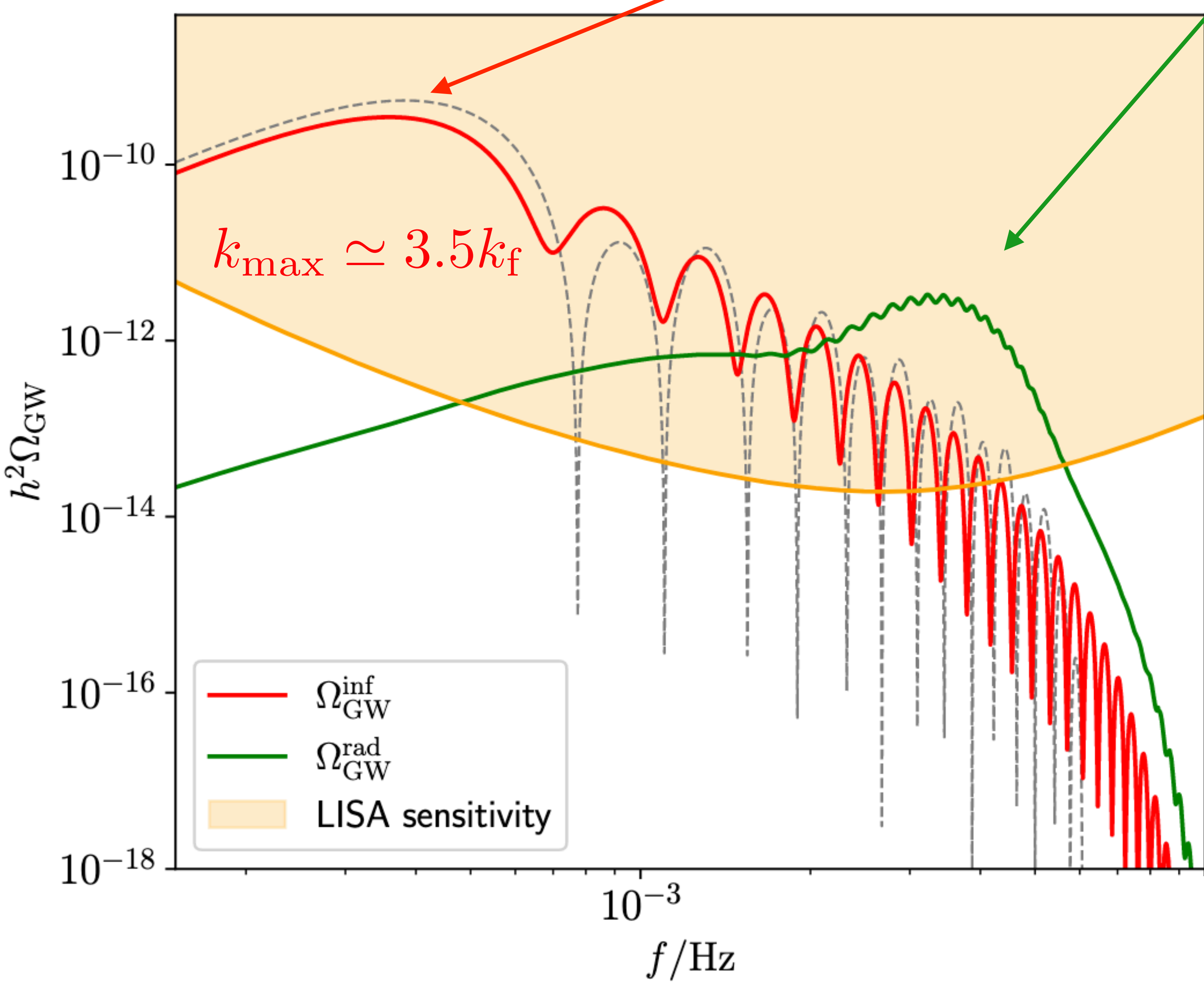
$\omega = 2/k_{\rm out}$

Inflationary-era GW spectrum due to dynamically generated scalar excited states with large occupation numbers that peak around k_*

GWs from sharp features

Dynamically generated excited state, after time corresponding to k_f

Maximum of scalar power spectrum:
$k_* = \gamma k_\mathrm{f} \gg k_\mathrm{f}$



$$\frac{\Omega_\mathrm{GW}^\mathrm{inf}|_{k_\mathrm{max}}}{\Omega_\mathrm{GW}^\mathrm{rad}|_{2k_*/\sqrt{3}}} = \mathcal{O}(1)10^{-2}\mathcal{N}^2\epsilon^2\gamma^5$$

Amplification by excited state

Amplification by number of fields excited during inflation

Order one oscillations

Perturbative control $\gamma^4\mathcal{P}_\zeta \lesssim 1$

Conclusion

Generic formalism for GW sourced during inflation

Scalar excited states lead to enhanced primordial GWs, with universal and unique characteristics

Proof of principle of data reconstruction with PCA analysis if signal sufficiently high   Fumagalli, Pieroni, RP, Witkowski, 2112.06903

Interesting to consider:

- more realistic and complex dynamics

- mixed contribution inflation / post-inflation to work out

- beyond perturbative treatment