Black holes and wormholes in scalar-tensor gravity

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Outline

Scalar-tensor theories

Black holes solutions

Disformal transformations

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Lovelock theory of gravity

Most general theory of gravity in D dimensions which give covariant, conserved, second-order field equations in terms of the metric only

$$S = \int \mathrm{d}^D x \sqrt{-g} \sum_{n=0}^{\lfloor (D-1)/2 \rfloor} \alpha_n R^{(n)}$$

$$R^{(0)} = 1, \quad R^{(1)} = R, \quad R^{(2)} \equiv \mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$$

$\mathcal{G}$ = Gauss-Bonnet invariant (natural higher order term, links with string theory)

Horndeski theories

Most general scalar-tensor action yielding 2nd order equations of motion

$$S\left[g_{\mu\nu},\phi\right] = \int \mathrm{d}^4x\sqrt{-g}\left\{\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5\right\}$$

$$\mathcal{L}_2 = G_2, \quad \mathcal{L}_3 = -G_3\Box\phi, \quad \mathcal{L}_4 = G_4 R + G_{4X}\left[(\Box\phi)^2-(\phi_{\mu\nu})^2\right]$$

$$\mathcal{L}_5 = G_5\, G_{\mu\nu}\phi^{\mu\nu} - \frac{1}{6}G_{5X}\left((\Box\phi)^3 - 3\Box\phi\,(\phi_{\mu\nu})^2 + 2\phi_{\mu\nu}\phi^{\nu\rho}\phi^{\mu}_{\rho}\right)$$

where $G_k = G_k\left(\phi, X\right)$ with $X = -\frac{1}{2}\phi_\mu\phi^\mu$, $\phi_\mu = \nabla_\mu\phi$, etc

Usual assumptions: parity-symmetry $\phi\to-\phi$ or shift-symmetry $\phi\to\phi+c$ ($\Rightarrow$ Noether current)

Conformally-coupled scalar field

Most general action with conformal symmetry $g_{\mu\nu} \to \mathrm{e}^{2\sigma} g_{\mu\nu}$, $\phi \to \phi - \sigma$ of the scalar sector[1]

$$S = \int \mathrm{d}^4 x \frac{\sqrt{-g}}{16\pi} \left\{ R - 2\lambda e^{4\phi} - \beta \mathrm{e}^{2\phi} \left(R + 6 \left(\nabla\phi\right)^2 \right) - \alpha \left[\phi \mathcal{G} - 4 G^{\mu\nu} \phi_\mu \phi_\nu - 4 \Box\phi \left(\nabla\phi\right)^2 - 2 \left(\nabla\phi\right)^4 \right] \right\}$$

- Three parameters α, β, λ
- Geometric equation $R + \frac{\alpha}{2}\mathcal{G} = 0$

[1] P. G. S. Fernandes, Phys. Rev. D **103** (2021) no.10, 104065

Static spherically-symmetric solutions

Ansatz $\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2, \quad \phi = \phi(r)$

$$R + \frac{\alpha}{2}\mathcal{G} = 0 \Rightarrow f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + 4\alpha\left(\frac{2M}{r^3} - \frac{q}{r^4}\right)}\right)$$
$$\approx 1 - \frac{2M}{r} + \frac{q}{r^2}, \quad r \to \infty$$

Two possibilities $q = 0$, $q = -2\alpha$. Focus on the latter: $\lambda = \beta^2/4\alpha$,

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + 8\alpha\left(\frac{M}{r^3} + \frac{\alpha}{r^4}\right)}\right), \quad \phi(r) = \ln\left(\frac{\sqrt{-2\alpha/\beta}}{r}\right)$$

Nature of spacetimes

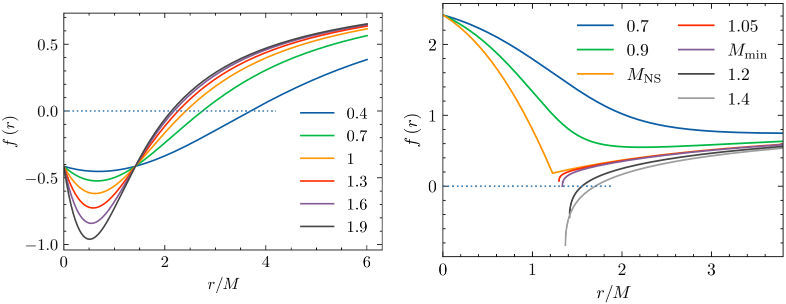

Figure 1: Metric function $f(r)$ for different $M/\sqrt{|\alpha|}$ for $\alpha > 0$ (left) and $\alpha < 0$ (right). Left: black hole with horizon $r_+ > r_{\text{Schwarzschild}} = 2M$. Right: naked singularity or black hole with horizon $r_+ < r_{\text{Schwarzschild}}$

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Going beyond Horndeski by deforming the metric

$(g_{\mu\nu}, \phi) \mapsto (\tilde{g}_{\mu\nu}, \phi)$ with $\tilde{g}_{\mu\nu} = g_{\mu\nu} + D(\phi, X)\,\phi_\mu\phi_\nu$

$\rightsquigarrow S[g,\phi] = \tilde{S}[\tilde{g},\phi]$ where $\tilde{S}$ is beyond Horndeski:

$$\tilde{S}[\tilde{g},\phi] = \int \mathrm{d}^4x\sqrt{-g}\left\{\tilde{\mathcal{L}}_2 + \tilde{\mathcal{L}}_3 + \tilde{\mathcal{L}}_4 + \tilde{\mathcal{L}}_5 + \tilde{\mathcal{L}}_{4b} + \tilde{\mathcal{L}}_{5b}\right\}$$

$\Rightarrow$ If $(g_{\mu\nu}, \phi) \mapsto (\tilde{g}_{\mu\nu}, \phi)$ is invertible, then any solution $\left(g^0_{\mu\nu}, \phi^0\right)$ to the field equations of S gives a solution $\left(\tilde{g}^0_{\mu\nu}, \phi^0\right)$ to the field equations of $\tilde{S}$

Applications

- Construction of healthy non-singular cosmological solutions (bouncing Universe, Universe with Genesis) only possible beyond Horndeski[2]

- Deformation of a stealth Kerr solution to a "deformed Kerr spacetime" with $D(\phi, X) = D = \text{cst.}$[3]

- Construction of wormholes[4]

[2] M. Libanov, S. Mironov and V. Rubakov, JCAP **08** (2016), 037
[3] T. Anson, E. Babichev, C. Charmousis and M. Hassaine, JHEP **01** (2021), 018
[4] A. Bakopoulos, C. Charmousis and P. Kanti, JCAP **05** (2022) no.05, 022

Deformation in spherical symmetry

Seed solution $\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\mathrm{d}\Omega^2, \quad \phi = \phi(r)$

$\rightsquigarrow \mathrm{d}\tilde{s}^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)W^{-1}(\phi,X)} + r^2\mathrm{d}\Omega^2$

where $W(\phi, X) \equiv 1 - 2D(\phi, X)\,X$ must satisfy:

- W^{-1} vanishes at a radius r_0 greater than the horizon, such that $\tilde{g}^{rr}(r_0) = 0$ while $\tilde{g}_{tt}(r) > 0$ for any $r \geq r_0$ $\rightsquigarrow r_0 \equiv$ wormhole throat
- $W \to 1$ as $r \to \infty$ (asymptotic flatness)
- W such that $(g_{\mu\nu}, \phi) \mapsto (\tilde{g}_{\mu\nu}, \phi)$ is invertible

Wormhole construction

In our case: $\phi(r) = \ln\left(\sqrt{-2\alpha/\beta}/r\right)$ and $X = -\frac{f(r)}{2r^2}$

$\rightsquigarrow$ define $\psi = \sqrt{\frac{-2\alpha}{\beta}}\mathrm{e}^{-\phi} \underset{\text{on-shell}}{=} r$ and choose

$$W^{-1}(\psi, X) \propto 1 + \frac{2\psi^2 X}{A\left(\psi/\sqrt{|\alpha|}\right)} \underset{\text{on-shell}}{=} 1 - \frac{f(r)}{A\left(r/\sqrt{|\alpha|}\right)}$$

so that

- Throat $f(r_0) = A\left(r_0/\sqrt{|\alpha|}\right)$
- Singularity $f(r_*) = \frac{1}{2}A\left(r_*/\sqrt{|\alpha|}\right)$

$\Rightarrow$ choose A so that $r_* < r_0$

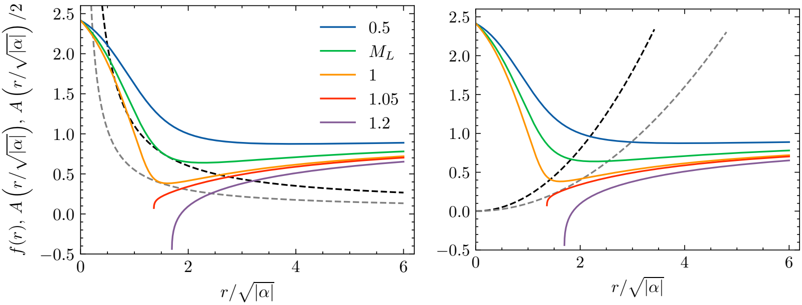

Figure 2: A (black curve) and $A/2$ (grey curve), and metric function $f(r)$ for several values of $M/\sqrt{|\alpha|}$. Left and right plots correspond to distinct choices for A. The throat radius r_0 (the singular radius r_*) is the largest intersection of $f(r)$ with the black (grey) curve. The singularity r_* is hidden by the throat r_0 on the left, not on the right

Regular coordinates

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{h(r)} + r^2\mathrm{d}\Omega^2, \quad h(r) = \frac{f(r)}{1-1/a}\left(1 - \frac{f(r)}{a + \frac{\sqrt{|\alpha|}}{r}}\right)$$

Change $r^2 = l^2 + r_0^2$, $\quad \mathrm{d}s^2 = -F(l)\,\mathrm{d}t^2 + \frac{\mathrm{d}l^2}{H(l)} + \left(l^2 + r_0^2\right)\mathrm{d}\Omega^2$

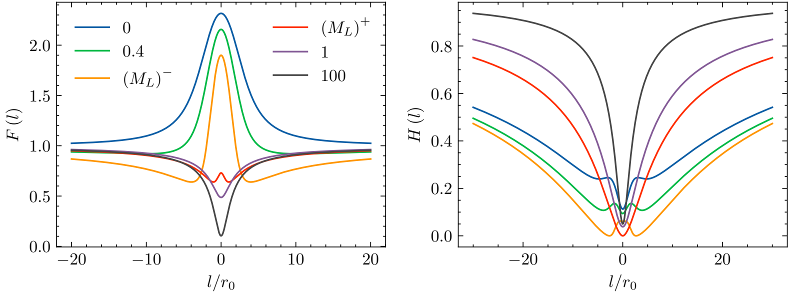

Figure 3: Functions $F(l)$ and $H(l)$ (with $a = 0.1$)

Conclusion

- Conformally-coupled scalar field $\rightsquigarrow$ black hole and naked singularity solutions, with well-defined scalar field and canonical kinetic term

- Generating new solutions through disformal transformations

- Scalar-tensor (beyond Horndeski) theory admitting regular wormhole solutions