

Dark siren population analysis with modified gravity

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Modified propagation equation for gravitational waves

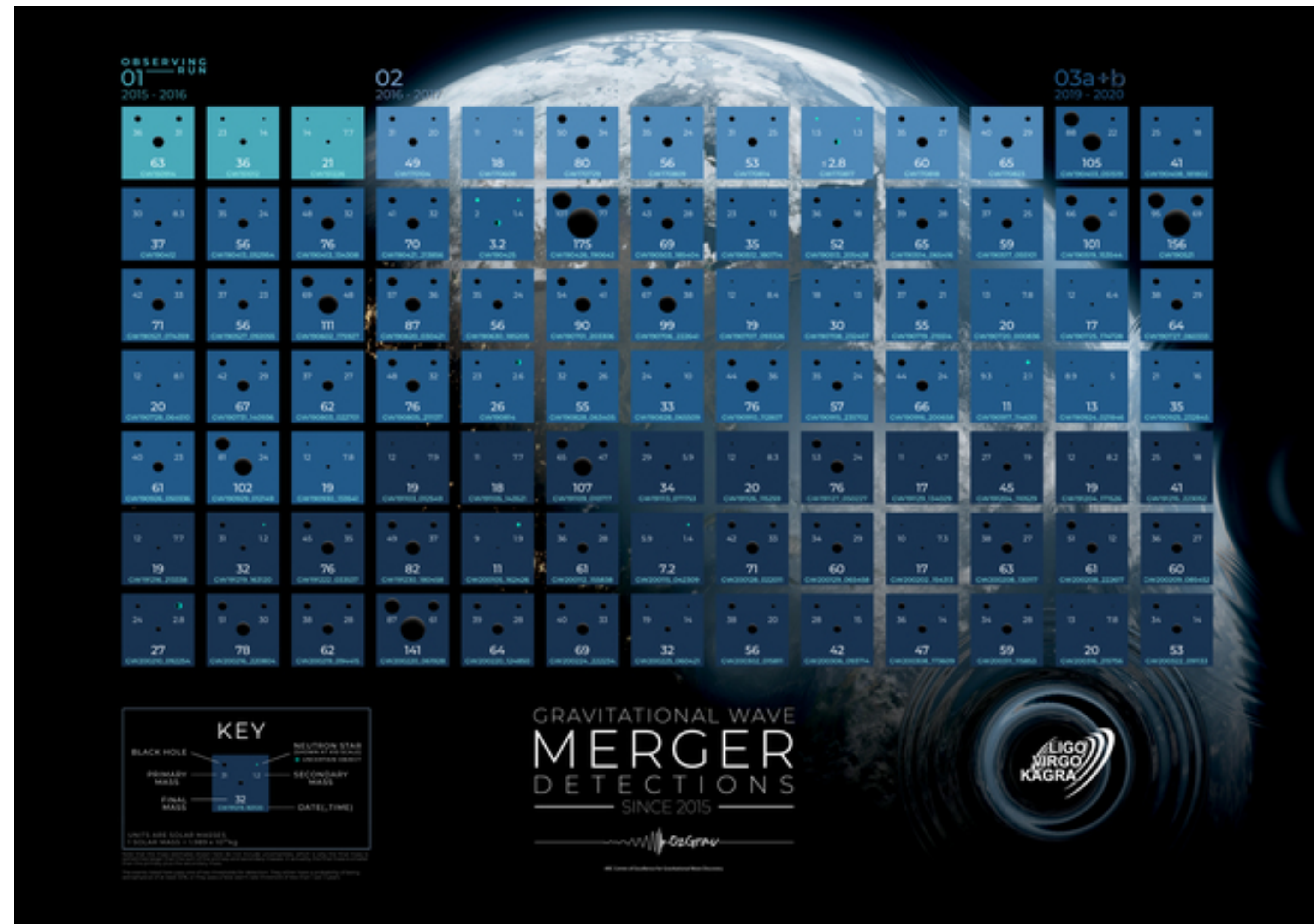
$$h_A'' + 2\mathcal{H}(1 - \delta(\eta))h_A' + k^2 h_A = 0$$

- k the wave vector, A the GW polarisation, η the conformal time, $\mathcal{H} = \frac{a'}{a}$ and δ the **friction term**
- Appears in some modified gravity theories (e.g. beyond Horndeski [1404.6495](#), DHOST, [1510.06930](#), [1703.03797](#), [1707.03625](#))
- **Results in a modified gravitational wave distance** (gravitational wave and electromagnetic distance do not coincide)
- **Testable** with gravitational wave observations

Observed events

2111.03606

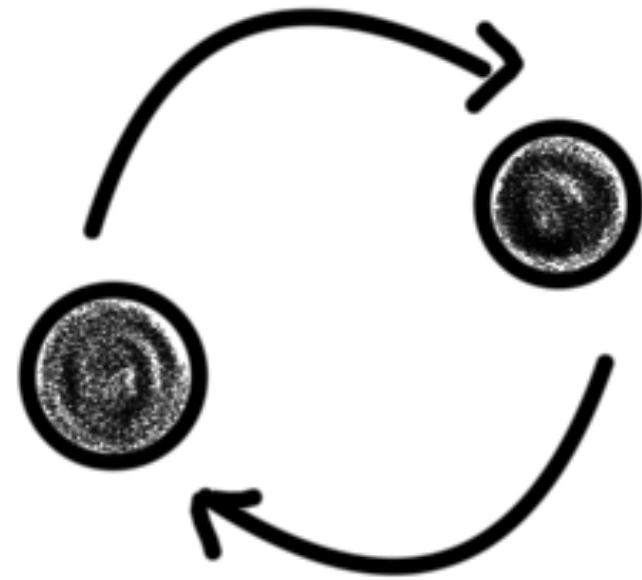
- 90 compact binary coalescences
- Majority are BBH systems
- Deduce mass population
- Infer cosmological parameters



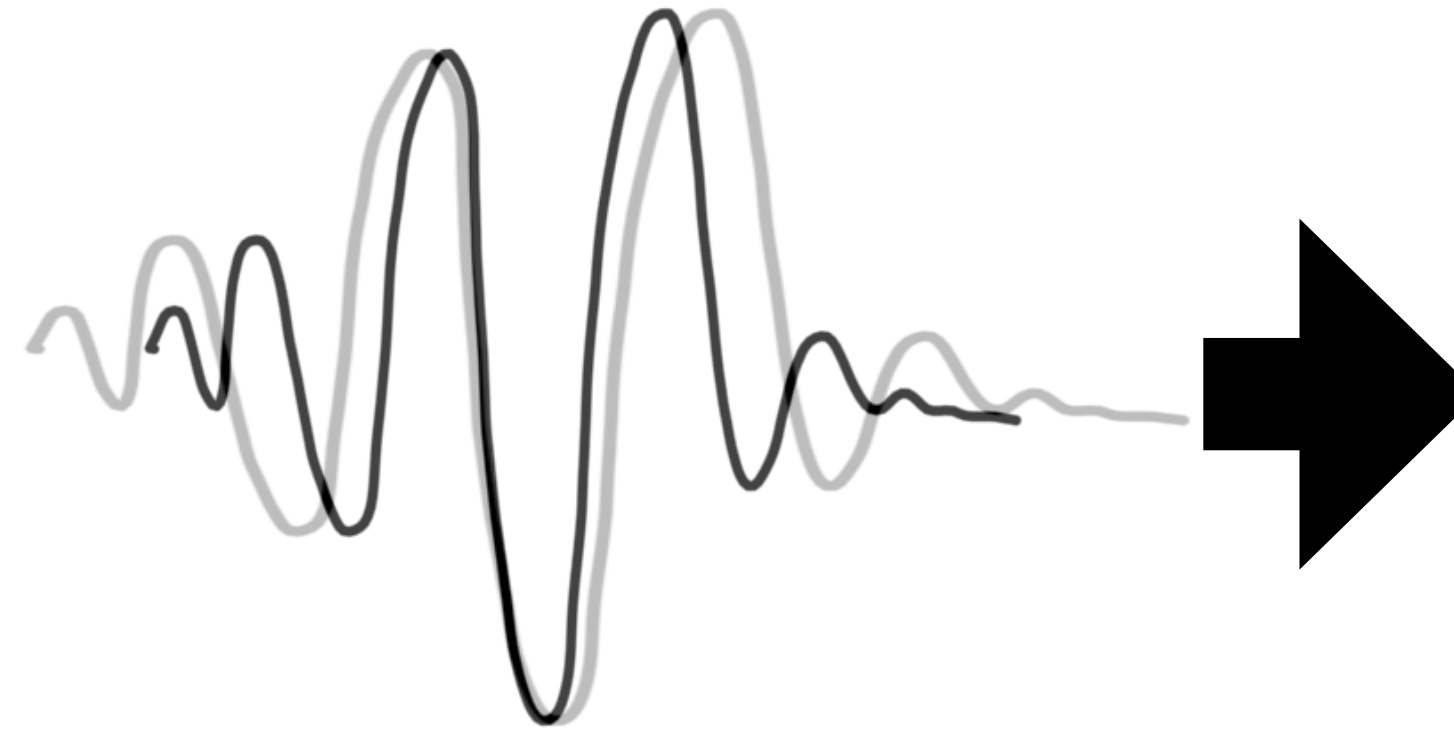
Information carried by a gravitational wave

Source frame masses

$$m_1^{(s)}, m_2^{(s)}$$



Expansion



Detector frame masses

$$m_1^{(d)}, m_2^{(d)}$$



Observer

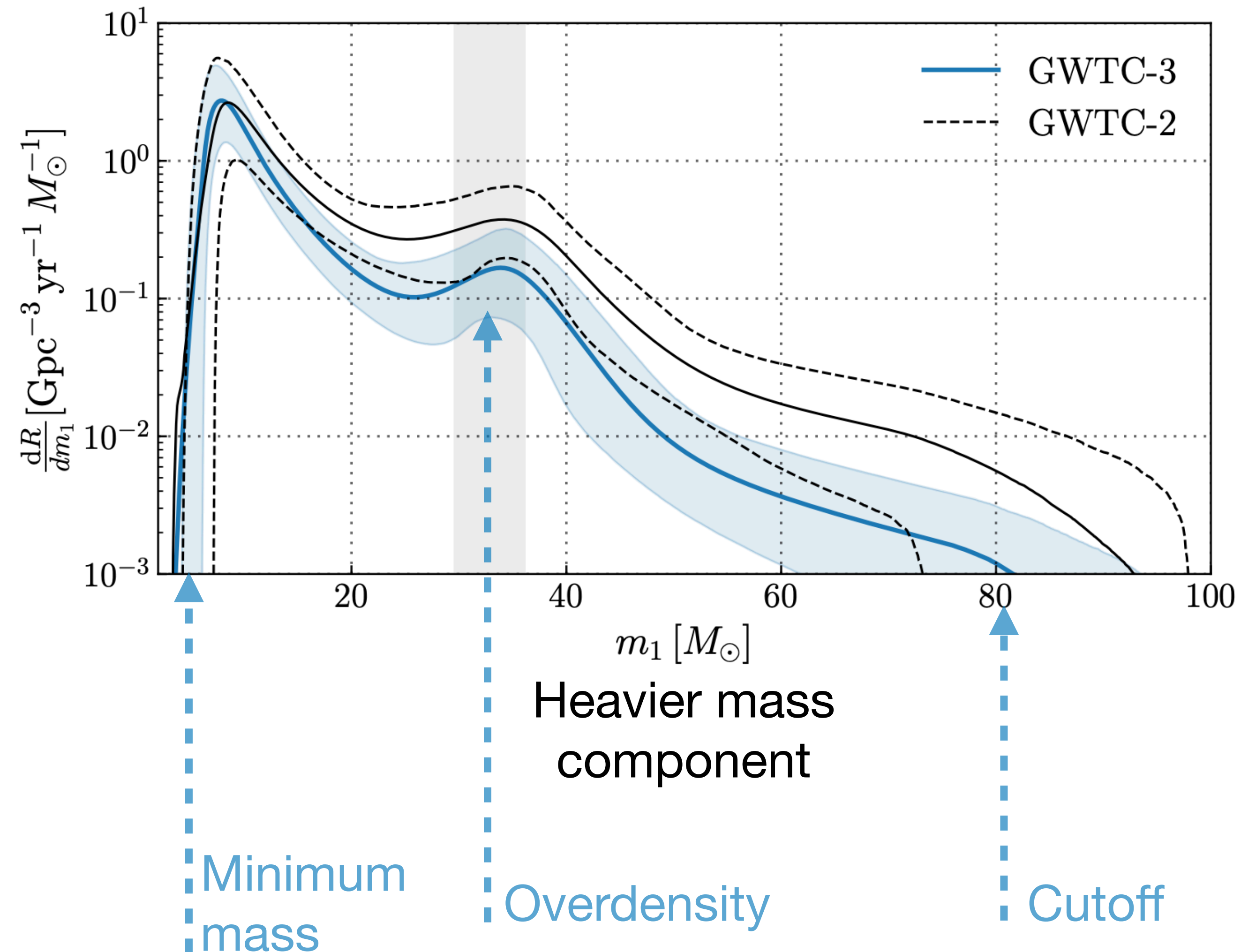
- GW frequency is **shifted to lower values by the expansion**
- Individual GW signal **carries no redshift information**

$$m^{(d)} = (1 + z)m^{(s)}$$

Population properties

2111.03634

- Collective event analysis
- Fix cosmological parameters
- Infer the **distribution** of BBH in
 - Source frame mass
 - Redshift



- Need redshift information

$$d_L^{\text{GW}}(z) = \frac{(1+z)c}{H_0} \int_0^z \frac{dz'}{[\Omega_m(1+z')^3 + \Omega_\Lambda]^{1/2}},$$

- **Several approaches** to GW cosmology with EM information (*Schutz 1986*):
 - Electromagnetic counterpart ([GW170817](#))
 - Statistical association of redshift from galaxy catalogs
- Current BBH horizon (1200 Mpc): **Beyond the completeness** of galaxy catalogs
- **For many events, no EM information** → **Dark sirens**

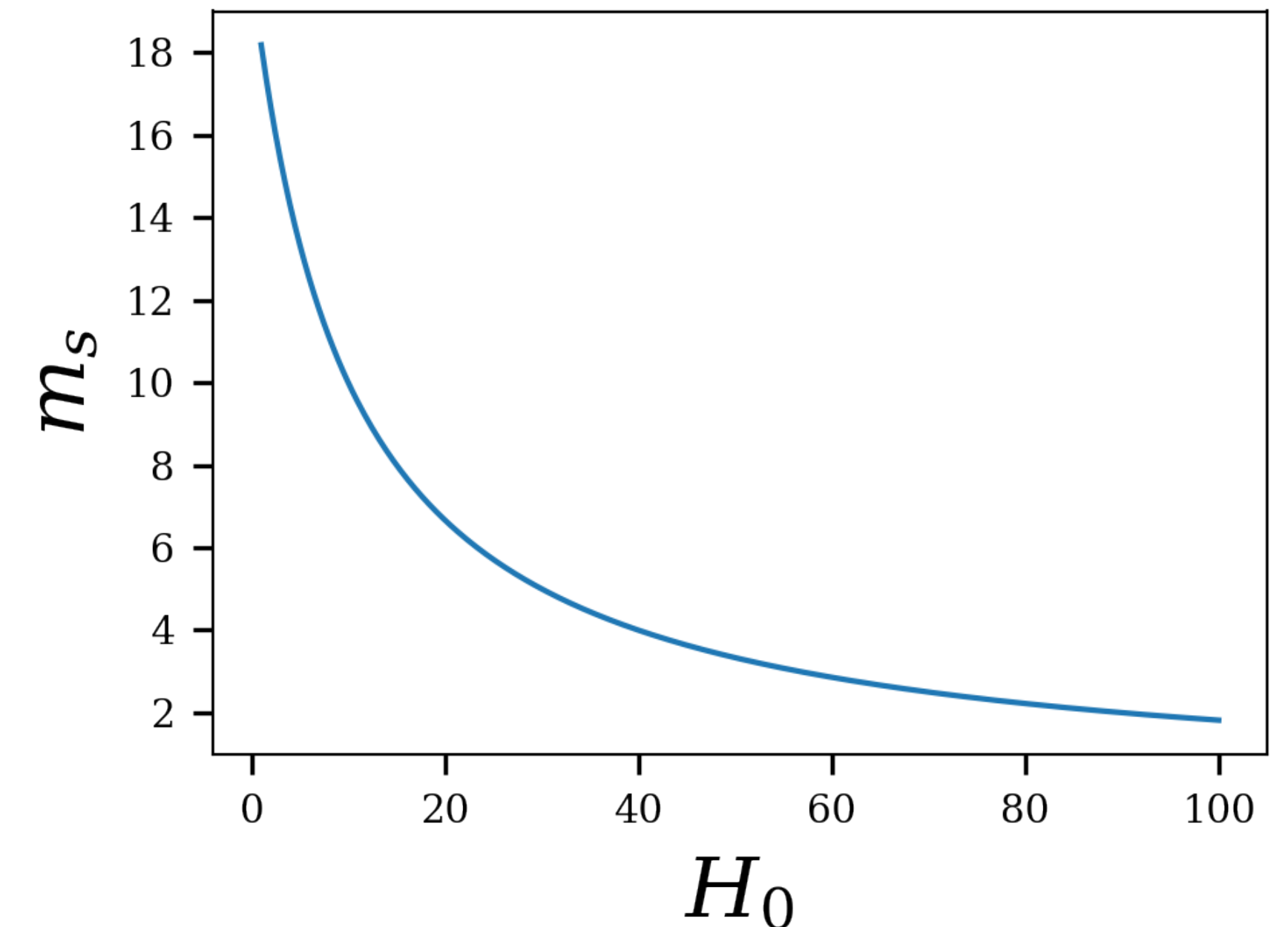
Source frame population and redshift measurement

- Redshift information **from a set of GW** detections:
assumption of mass model

$$m^{(d)} = (1 + z)m^{(s)} \quad \rightarrow \quad z = \frac{m^{(d)}}{m^{(s)}} - 1$$

- **Joint fit** of cosmological parameters and mass population models (*Taylor et al. 2012, Taylor and Gair 2012, Farr et al. 2019, You et al. 2020*)
- Strong correlation between H_0 and the characteristic mass scales

$$m^{(s)} = \frac{m^{(d)}}{1 + d_L H_0 / c} \quad z \approx \frac{d_L H_0}{c}$$



Framework

Assumption on the modifications of GR

$$h_A'' + 2\mathcal{H}(1 - \delta(\eta))h_A' + k^2 h_A = 0$$

- Phenomenological model [1906.01593](#)

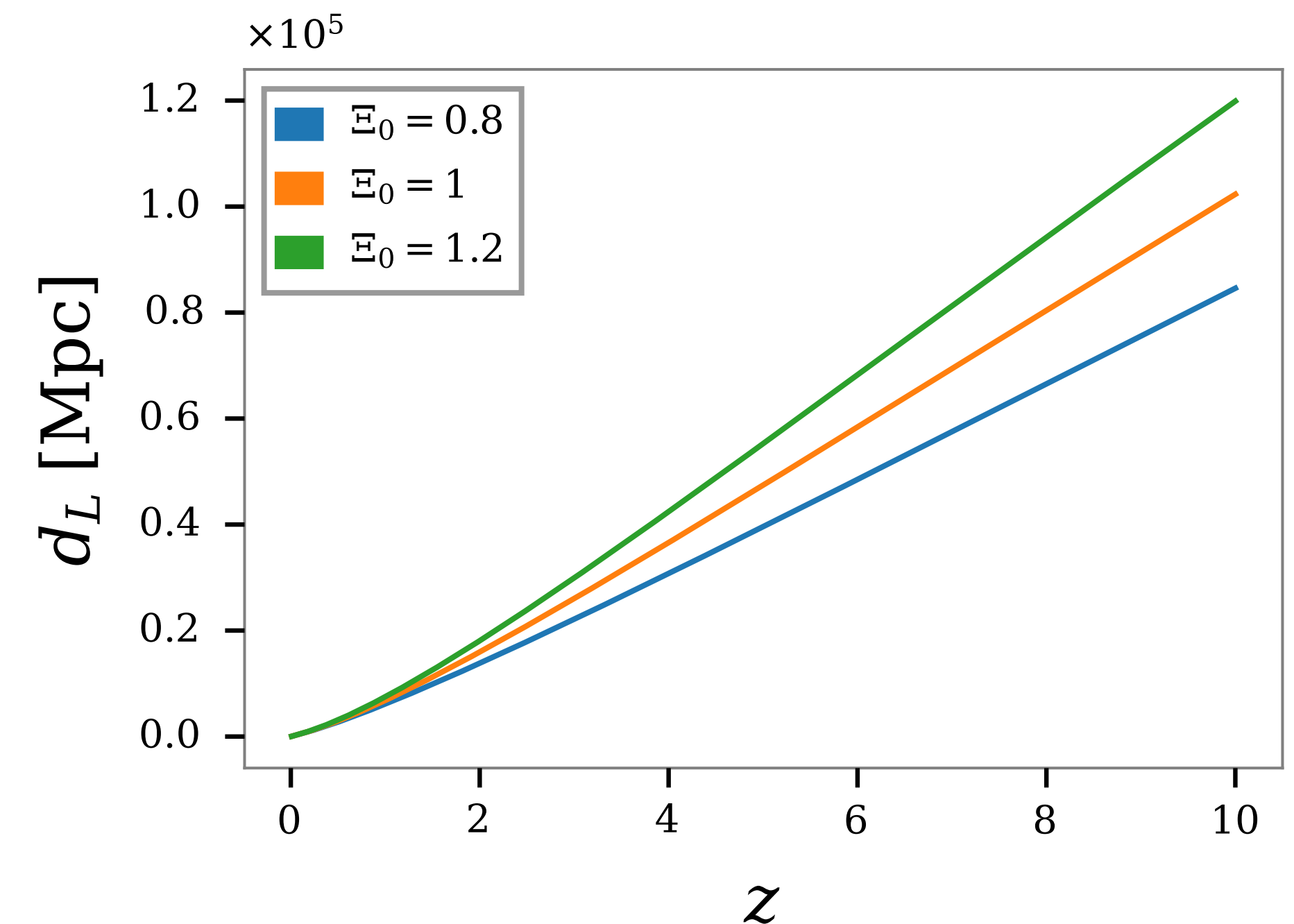
Ξ_0 characterises
early time behaviour

GR: $\Xi_0 = 1$

$$d_L^{\text{GW}} = d_L^{\text{EM}} \left(\Xi_0 + \frac{1 - \Xi_0}{(1+z)^n} \right)$$

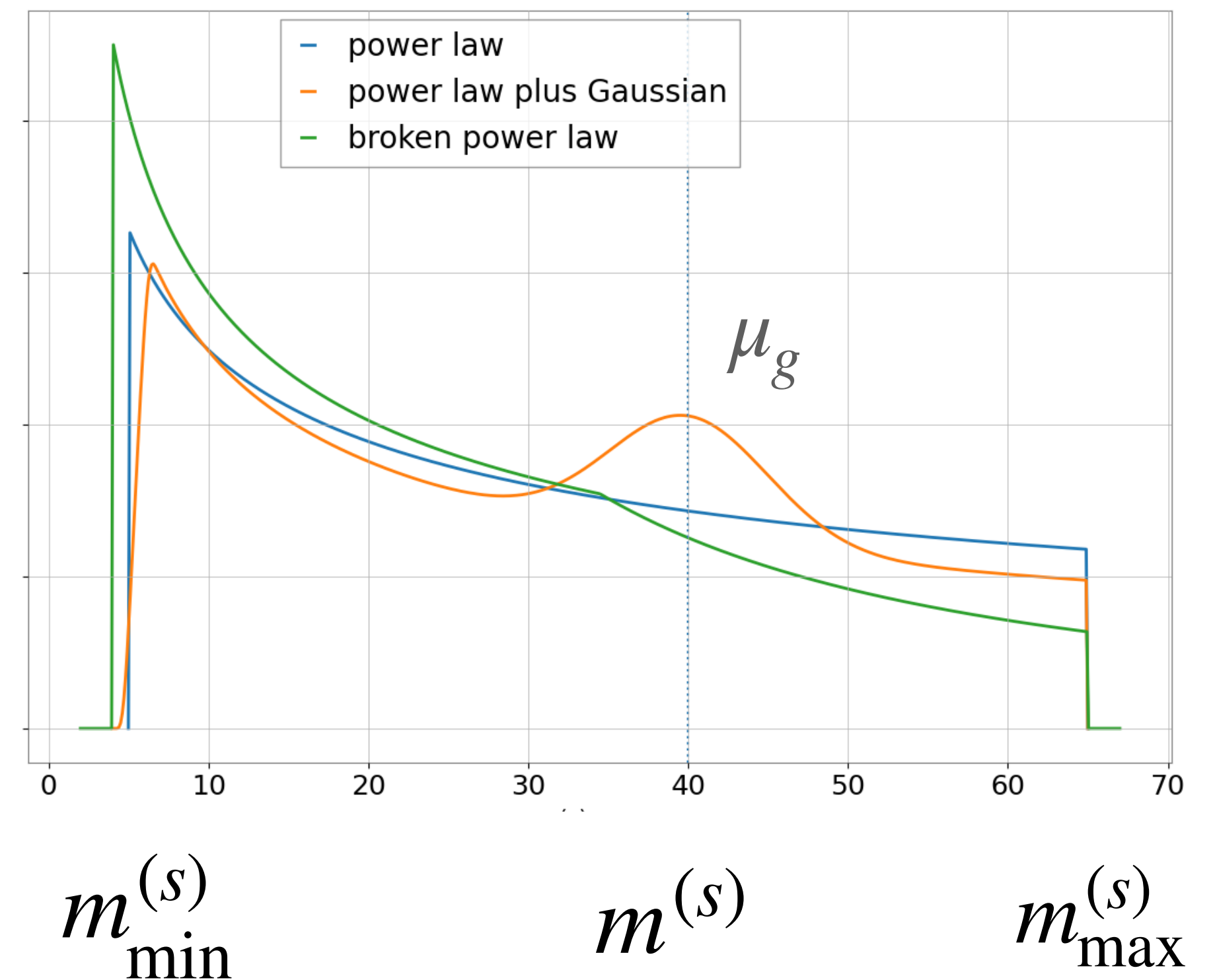
n characterises the
transition from early to
late times

- Assumptions: No modifications of the waveform during the inspiral phase and cosmological background is unchanged



The source mass population model

- **Various astrophysical mechanisms** shape the BH source mass distribution
 - **Pair instability supernova** (PISN, *J. R. Bond, W. D. Arnett, and B. J. Carr 1984*) → $m_{\max}^{(s)}$
 - **Pulsational PISN** (*Barkat et al. 1967; Woosley & Weaver 1986; Woosley 2017*) → Accumulation in a Gaussian peak μ_g
 - **X-ray observations** → No BHs $< m_{\min}^{(s)}$
- **Models** from *LVK Population properties: 2010.14533*
 - **Power law**: mass range and two power law slopes (PL)
 - **Power law + Gaussian peak** (PLG)
 - **Broken power law**
 - Power law + 2 Gaussians (Multi Peak)



Statistical framework

- Bayesian analysis with selection effects (*Mandel et al. 1809.02063, Thrane and Talbot 1809.02293, Vitale et al. 2007.05579*)

$$p(\Lambda|\{x\}) \propto p(\Lambda) \prod_{j=1}^{N_{\text{obs}}} \frac{\int p(x_j|\theta_j) p_{\text{pop}}(\theta_j|\Lambda) d\theta_j}{\int p_{\text{det}}(\theta_j) p_{\text{pop}}(\theta_j|\Lambda) d\theta_j}$$

- Metaparameters Λ : population parameters, cosmological and modified gravity parameters, ...
- GW data $\{x\}$
- Source parameters $\theta = \{m_{1,2}^{(d)}, d_L^{\text{GW}}, \dots\}$
- GW likelihood $p(x_i|\Lambda, \theta)$, obtained **from posterior samples**
- Population assumption $p_{\text{pop}}(\theta|\Lambda)$
- Detection probability $p_{\text{det}}(\theta)$

Statistical framework

Bayesian analysis with selection effects

$$p(\Lambda|\{x\}) \propto p(\Lambda) \prod_{j=1}^{N_{\text{obs}}} \frac{\int p(x_j|\theta_j) p_{\text{pop}}(\theta_j|\Lambda) d\theta_j}{\int p_{\text{det}}(\theta_j) p_{\text{pop}}(\theta_j|\Lambda) d\theta_j}$$

- Only events **passing threshold** (on signal to noise ratio or false alarm rate) are considered
- Numerical evaluation of $p_{\text{det}}(\theta)$: produce a set of events and label them either “**detected**” or “**undetected**” (here: passing SNR threshold)

Aim of the analysis

- **Joint parameter estimation**

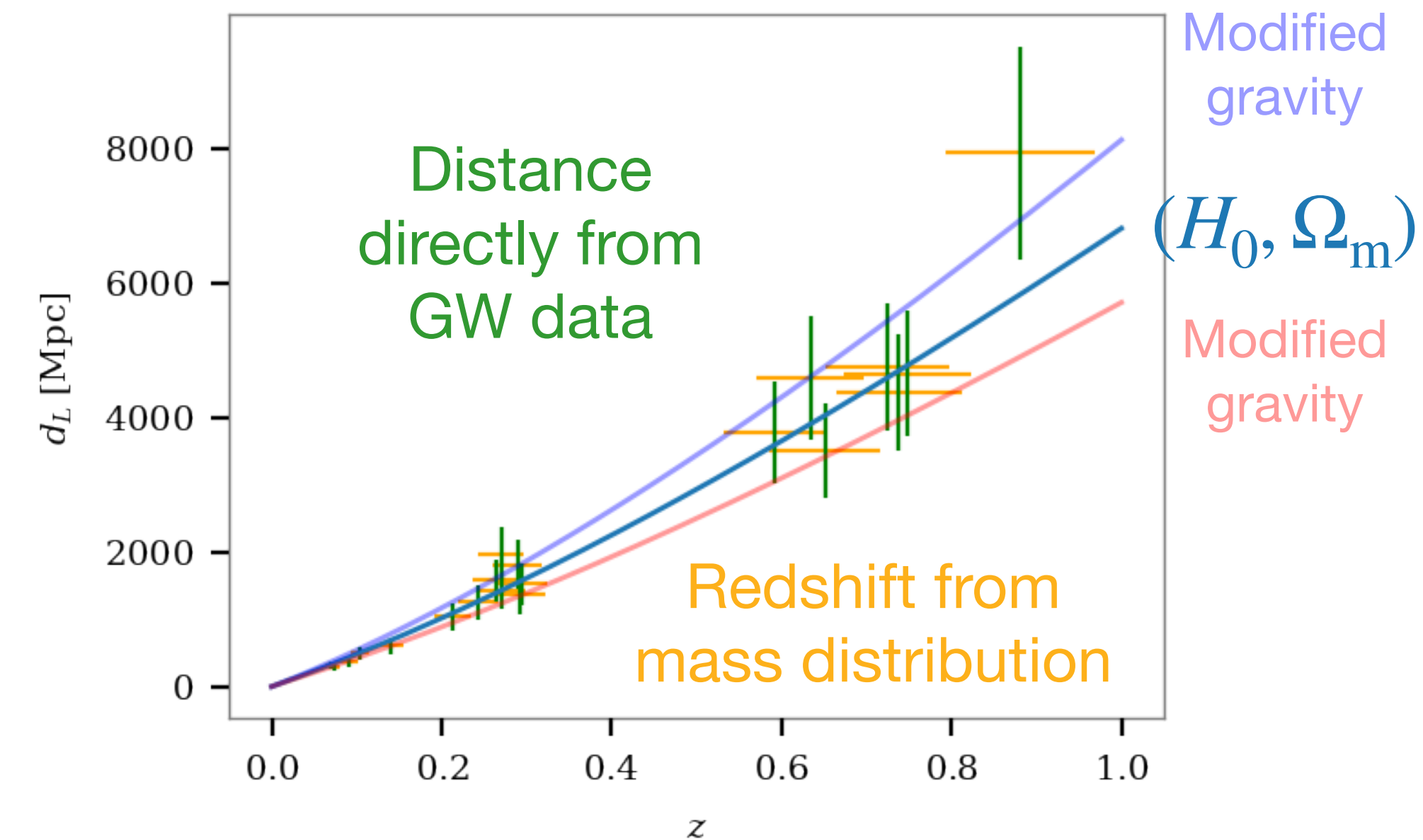
- GW luminosity distance parametrization
- Source mass parameters ($m_{\min}^{(s)}, m_{\max}^{(s)}, \dots$)
- Cosmological parameters (H_0, Ω_m)
- Rate evolution of sources

- Run this analysis **for O3 data**

- **Identify factors that impact the uncertainties of the final Ξ_0 posterior** for future observation runs (O4 and O4+O5). **Precision?**

- Based on Icarogw [2103.14663](#) (used for O3 in the last LVK cosmo paper)
- See also: [2104.05139](#), [2112.07650](#)

$$z = \frac{m^{(d)}}{m^{(s)}} - 1$$



Results (with GWTC-3)

Results with O3 data

Bayes factor: $\frac{p(\text{data} | \text{model}_1)}{p(\text{data} | \text{model}_2)}$

- GR: $\Xi_0 = 1$
- Compare Bayes factors
 → **Multi Peak + General Relativity** is preferred
- Consistent results for all 3 SNR cuts

60 BBH events, SNR > 10, IFAR > 4 yr

	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
GR	-2.4	0.0	-1.2	-6.3
<i>D</i>	-2.0	-0.2	-1.7	-6.4
Ξ_0	-3.2	-0.9	-2.1	-6.8
<i>c_M</i>	-3.0	-1.0	-2.1	-6.5

42 BBH events, SNR > 11, IFAR > 4 yr

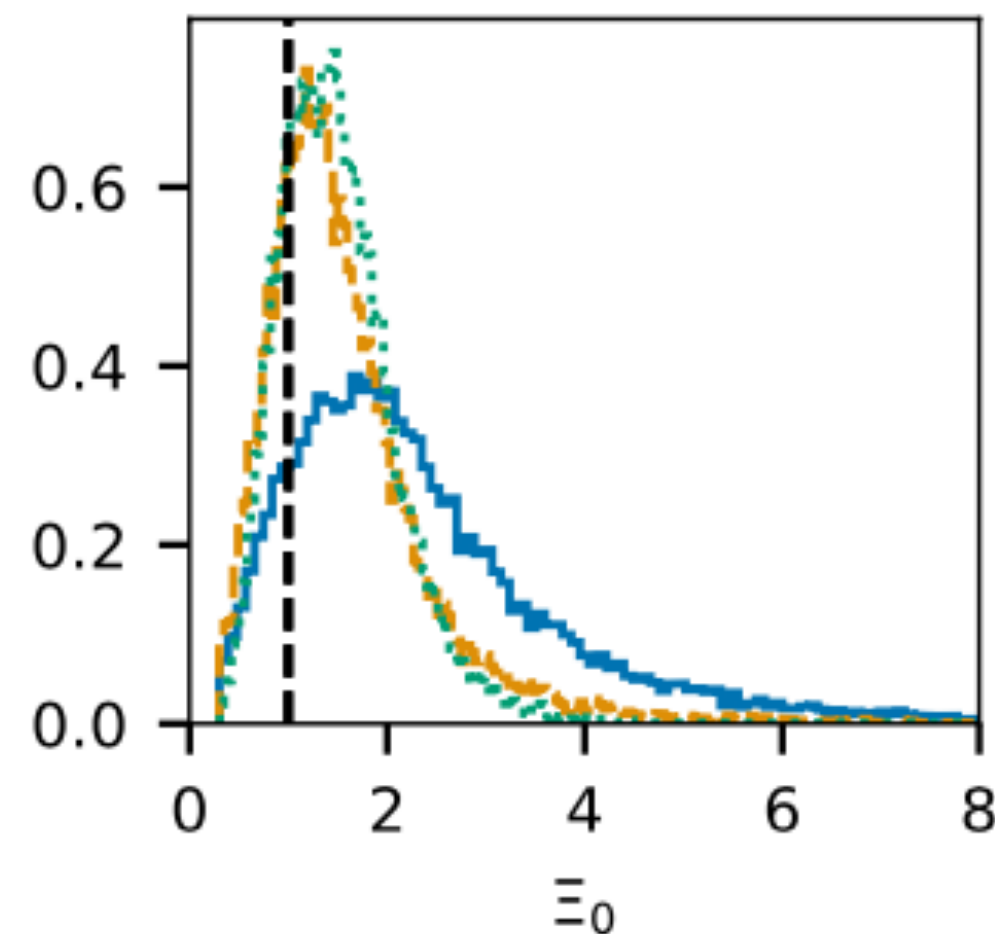
	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
GR	-1.5	0.0	-0.8	-3.2
<i>D</i>	-1.5	-0.0	-0.9	-3.4
Ξ_0	-1.9	-0.6	-1.4	-3.9
<i>c_M</i>	-1.9	-0.9	-1.7	-3.4

35 BBH events, SNR > 12, IFAR > 4 yr

	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
GR	-1.2	0.0	-1.1	-2.6
<i>D</i>	-1.1	-0.4	-1.2	-2.8
Ξ_0	-2.1	-1.0	-1.9	-3.3
<i>c_M</i>	-1.9	-1.2	-1.9	-3.1

Results with O3 data

- GR: $\Xi_0 = 1$
- For all modified gravity models: **compatible with their GR values** at 90% confidence level (for **Multi Peak**)



Multi peak mass model, varying SNR cut

60 BBH events, SNR > 10, IFAR > 4 yr

	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
D	6_{-2}^{+2}	5_{-1}^{+3}	5_{-1}^{+3}	$4.5_{-0.8}^{+3.1}$
Ξ_0	$1.6_{-0.8}^{+1.3}$	$1.4_{-0.7}^{+1.1}$	$1.3_{-0.7}^{+1.2}$	$0.6_{-0.2}^{+1.4}$
c_M	$1.0_{-2.6}^{+2.3}$	$0.5_{-2.4}^{+2.5}$	$0.1_{-2.1}^{+2.7}$	-2_{-1}^{+3}

42 BBH events, SNR > 11, IFAR > 4 yr

	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
D	$4.7_{-0.9}^{+2.9}$	$4.6_{-0.8}^{+2.6}$	$4.7_{-0.9}^{+2.7}$	5_{-1}^{+3}
Ξ_0	2_{-1}^{+3}	2_{-1}^{+4}	2_{-1}^{+3}	$0.7_{-0.4}^{+3.0}$
c_M	$0.5_{-4.2}^{+4.1}$	1_{-5}^{+4}	1_{-4}^{+4}	-3_{-2}^{+5}

35 BBH events, SNR > 12, IFAR > 4 yr

	Broken Power Law	Multi Peak	Power Law + Peak	Truncated
D	5_{-1}^{+3}	$4.6_{-0.9}^{+2.9}$	$4.8_{-1.0}^{+2.9}$	5_{-1}^{+3}
Ξ_0	$1.2_{-0.7}^{+1.4}$	$1.4_{-0.8}^{+1.8}$	$1.4_{-0.8}^{+1.8}$	$0.8_{-0.5}^{+2.0}$
c_M	$-0.1_{-3.0}^{+2.8}$	$0.3_{-3.3}^{+3.2}$	$0.4_{-3.0}^{+3.2}$	-2_{-3}^{+5}

See also: [Mancarella et al. 2112.05728](#)

Degeneracies

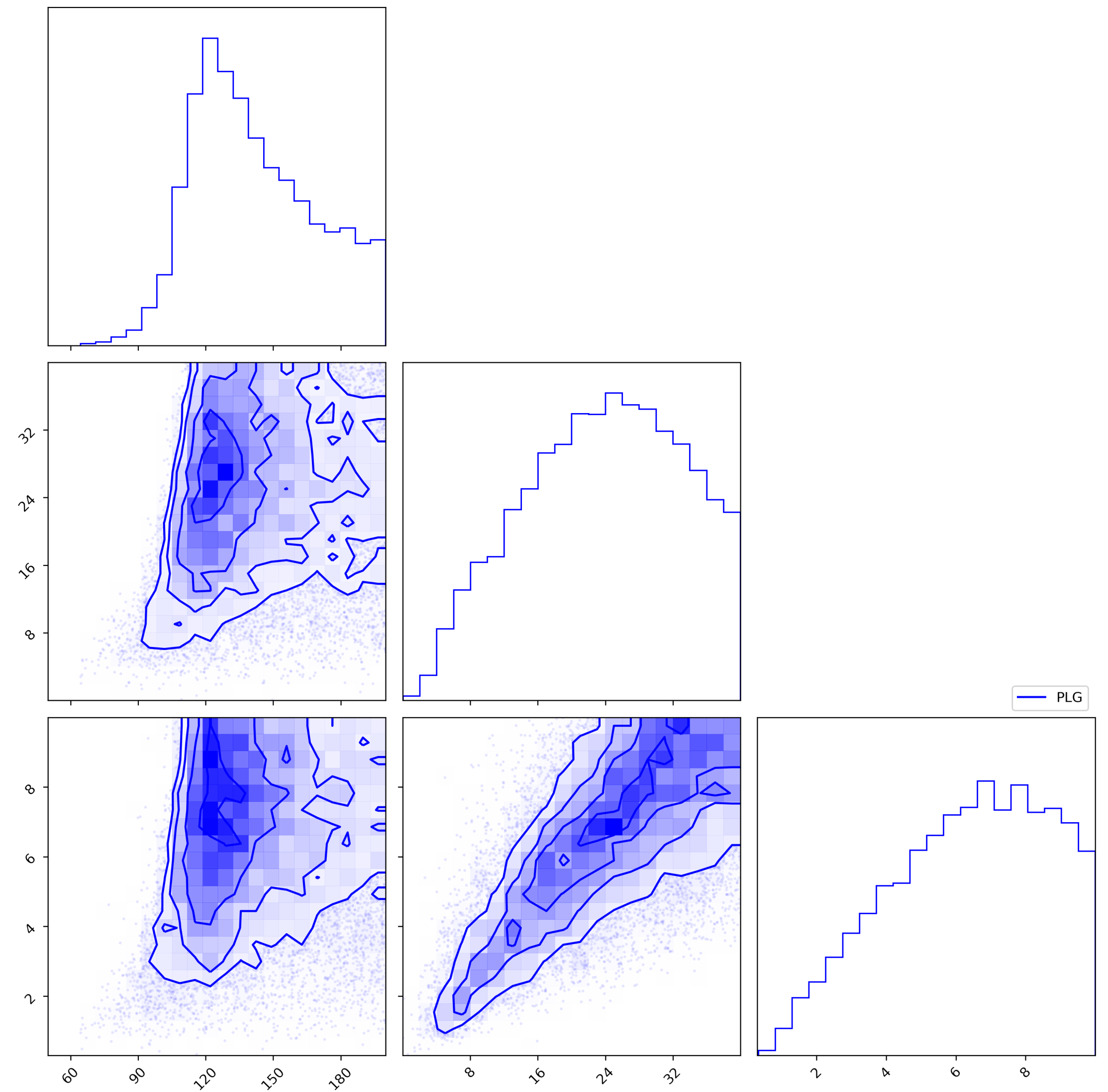
- GR: $\Xi_0 = 1$
- Gravity deviation parameter Ξ_0 strongly degenerate with the redshift distribution parameter γ

Redshift distribution parameter γ

Gravity deviation parameter Ξ_0

$m_{\max} [M_{\odot}]$
Maximum mass

Redshift distribution parameter



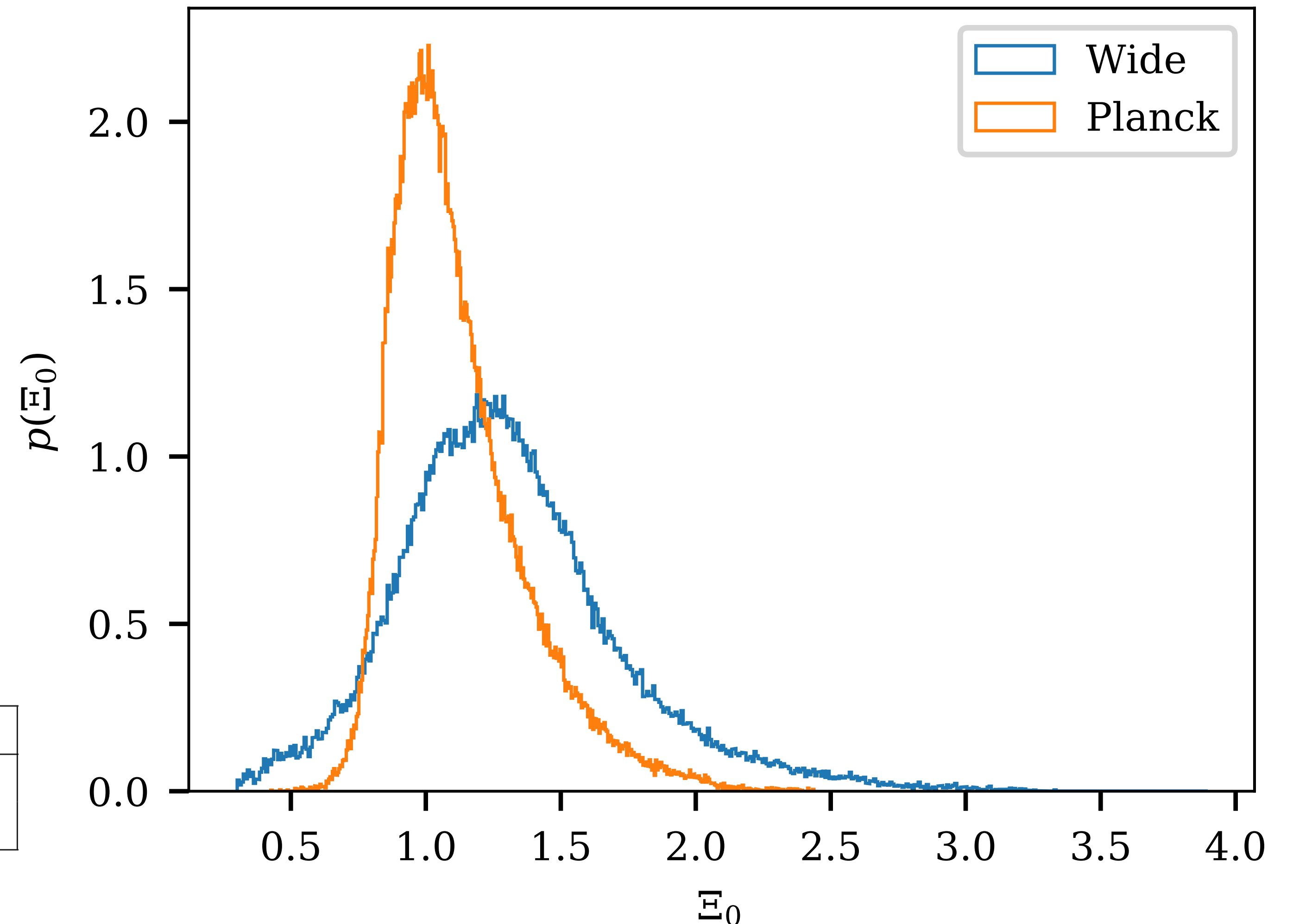
Forecast (with O4 + O5)

The effect of a prior on the cosmological values

Wide: Agnostic priors for the cosmological parameters

Planck: Priors from the Planck estimate for the cosmological parameters

	<i>Agnostic</i>	<i>From Planck</i>
H_0	$\mathcal{U}(30, 130)$	$\mathcal{U}(66.07, 68.47)$
Ω_M	$\mathcal{U}(0.05, 0.4)$	$\mathcal{U}(0.3082, 0.3250)$



Conclusions

- Method allows to **simultaneously constrain modified gravity, cosmological and population parameters**
 - Implication of O3 : bright sirens are rare
- **O3 data favours GR** over all modified gravity models investigated
- **Study impact of mass models on the measurement of Ξ_0 and on H_0**
- Strong degeneracies between the rate evolution γ , the overall rate of events R_0 , the Hubble constant H_0 and friction amplitude Ξ_0
 - Assumptions on astrophysics can bias this measurement
 - → Marginalize over population assumptions
- Constrain Ξ_0 to **50 %** with O4 and **20 %** with O4+O5