New model of Planck polarized dust maps using Cross Wavelet Scattering Transform

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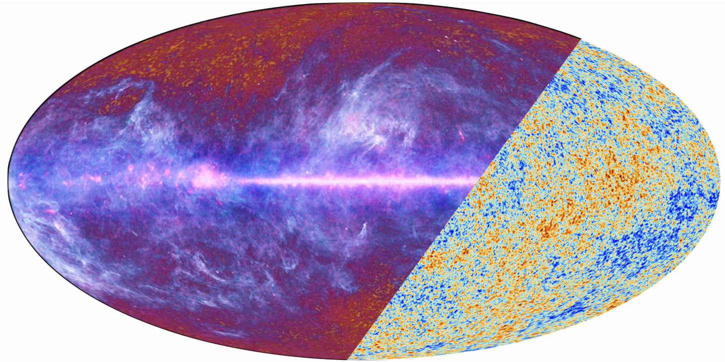
Introduction



Realistic foregrounds models necessary for B-modes detection
→ Construct such models directly from observational data?

Introduction

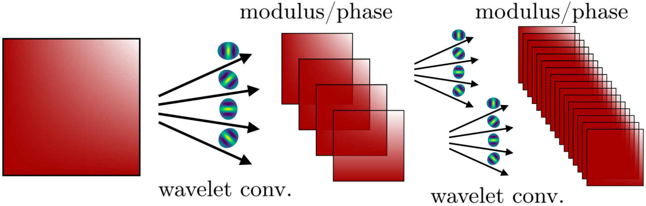
- **Polarized dust - noise separation**
  - As a 2-components separation
  - From observational data
  - Using $\neq$ non-Gaussian signatures
  - Multi-λ added later on

Goal: obtain realistic non-Gaussian foreground model

Outline

Scattering transform and generative models

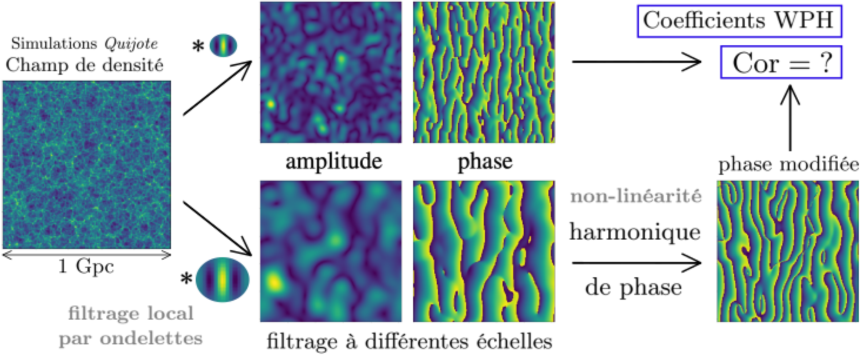
- **Scattering transform (ST) statistics** **(Mallat+, 2010+)**
  - Collaboration with data science (S. Mallat)
  - Inspired from neural networks constructions
    $\rightarrow$ efficient characterization and reduced variance
  - Do not need any training stage
    $\rightarrow$ explicit mathematical form and interpretability



$\rightarrow$ Wavelet filters separating the different scales
$\rightarrow$ Coupling between scales with non-linearities

Scattering transform and generative models

- **Wavelet Phase Harmonics and phase alignment**



→ **Efficient characterization of scale interactions**

Scattering transform and generative models

- **Generative model from ST statistics** *(Bruna, Mallat, 2019)*
  - Maximum entropy model under statistical constraints
  - New realizations of a process from its ST statistics
  - Non-gaussian properties quantitatively reproduced

Scattering transform and generative models

- **Generative model from ST statistics** *(Bruna, Mallat, 2019)*
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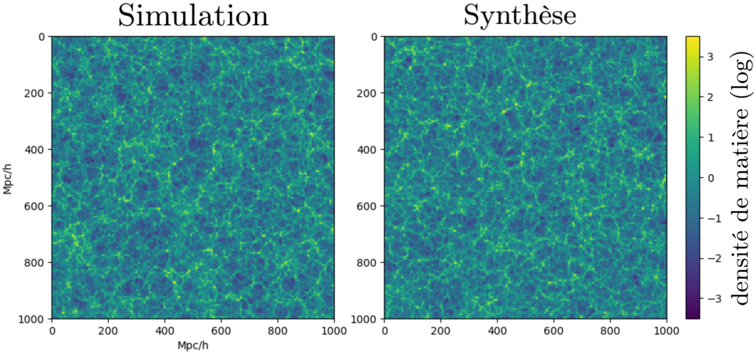
- **Practical implementation**
  - Constraints $\Phi(x)$ from a (set of) realization(s) x
  - Sampled with a gradient-descent algorithm
    $\rightarrow$ from a white noise
    $\rightarrow$ optimizing $\tilde{x}$ such that $\Phi(\tilde{x}) \simeq \Phi(x)$
  - 10-30 seconds on a GPU for a 256^2 map

Scattering transform and generative models

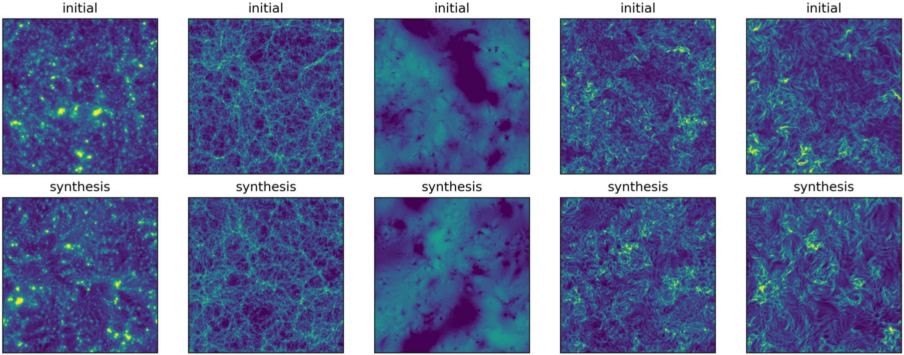
- **Quantitative validation of syntheses** *(Allys, Marchand+, 2020)*
  - Wavelet Phase Harmonics (WPH)
  - Matter density field of the large scale structures



→ **Usual statistics very well reproduced (up to 1-10 %)**
→ **Allowed CMB fgd removal from 1 dust map** *(Jeffrey+, 2021)*

Scattering transform and generative models

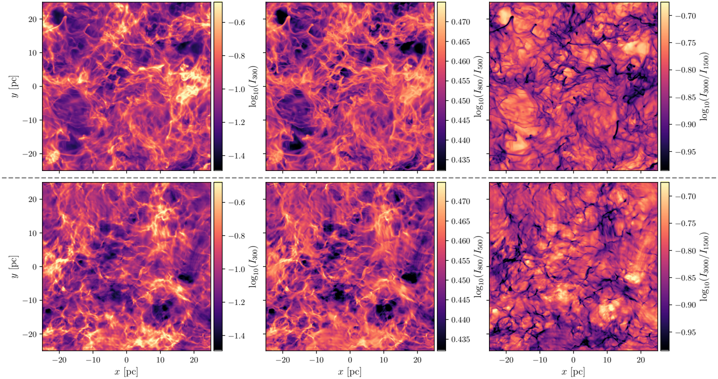
- **Syntheses from a single image** *(Cheng+, in prep.)*
  - Scattering covariances + dimensionality reduction



→ **Realistic syntheses from a single input image!**

Scattering transform and generative models

- **Multi-channel syntheses** *(Regaldo+, in prep.)*
  - From cross Wavelet Phase Harmonics



$\rightarrow$ **Multifrequency generative models can be constructed!**

Component separation from ST statistics

- **Generative model from direct constraint case**
  - $\Phi(x)$ known
  - Generate $\tilde{x}$ such that

$$\Phi(x) \simeq \Phi(\tilde{x})$$

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- **Comp. sep. from indirect constraints (example)**
  - $d = x + y$, assume $\Phi(d)$ and $\Phi(y)$ known
  - Generate $\tilde{x}$ such that

$$\Phi(d) \simeq \left\langle \Phi(\tilde{x} + y_i) \right\rangle_i$$

$\rightarrow$ Recover a map $\tilde{x}$ with correct statistics ?

First application on polarized dust

- **Separation of dust polarized emission and noise** **(Regaldo+ 21)**
  - *Planck* Chamaeleon-Musca region at 353 GHz
  - Flat-sky approx., $20^\circ \times 20^\circ$ patch
  - Treated as a 2 components signal $d = s + n$
  - 300 *Planck* instrumental noises $\tilde{n}$

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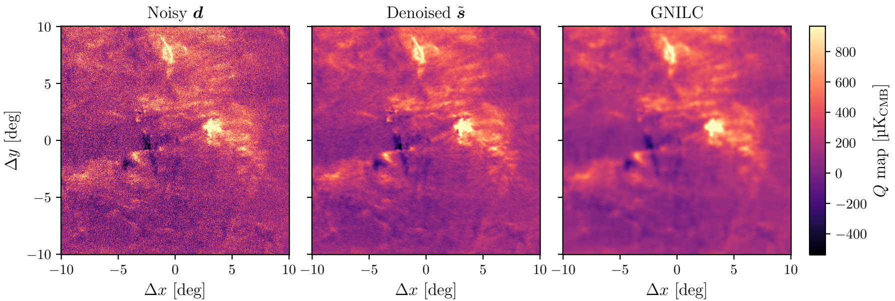
- **Application on Q/U data**
  - One single indirect constraint:

$$\Phi(d) \simeq \left\langle \Phi(u + \tilde{n}) \right\rangle_{\tilde{n}}$$

  - Start from the noisy map

Separation of polarized dust emission and noise

- Results on observational data (Régaldo+ 21)



→ Transition btw. deterministic and statistical
→ Conceptual validation of the method
→ Clear room for improvement

Outline

1. Scattering Transform and components separation
2. **FoCUS component separation algorithm**
3. Validation and application to Sroll2 data

Problem formulation

- **Input data for the algorithm**
  - Full-mission d and two $d_{1,2}$ half-missions (353 GHz)

$$d = s + n, \qquad d_{1,2} = s + n_{1,2}$$

  - Similar treatment for E and B
  - 500 full-sky noises $\{\tilde{n}, \tilde{n}_1, \tilde{n}_2\}$
  - A clean T temperature map (from 857 GHz)

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- **Key elements for dust/noise separation** **(Delouis+, in prep.)**
  - Work directly on the sphere
  - Inhomogeneous statistics of the dust
  - Introduce cross-statistics btw. components

  → **Allow for much stronger constraints!**

Cross Wavelet Scattering Transform (CWST)

- **Cross Wavelet Scattering Transform** **(Delouis+, in prep.)**
  - Simplified wavelets ψ_λ on the sphere (3x3 kernels!)
  - Directly on HealPix spherical data
  - Two layers of coefficients: (simplified)

$$
\begin{aligned}
S_1(I_a, I_b)_{\lambda_1} &\propto \sqrt{|I_a \star \psi_{\lambda_1} \cdot I_b \star \psi_{\lambda_1}|} \\
S_2(I_a, I_b)_{\lambda_1,\lambda_2} &\propto \left| \sqrt{|I_a \star \psi_{\lambda_1} \cdot I_b \star \psi_{\lambda_1}|} \star \psi_{\lambda_2} \right|
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$$

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→ Recover usual WST for auto-statistics

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→ Successive convolutions/modulus + cross-terms
→ Recover usual WST for auto-statistics

Non-linearities ⇒ have to deal with biased cross-terms

FoCUS Components separation algorithm

- **Set of constraints (for E or B)**

  1. **From the half-missions cross-term**

$$\Phi(d_1, d_2) \simeq \left\langle \Phi(u + \tilde{n}_1, u + \tilde{n}_2) \right\rangle_{\tilde{n}}$$

$\rightarrow$ Constraints the statistics of dust

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  2. **Cross between denoised and noisy**

     $$\Phi(d, u) \simeq \left\langle \Phi(u + \tilde{n}, u) \right\rangle_{\tilde{n}}$$

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3. **Cross with temperature map**

$$\Phi(T, d) \simeq \left\langle \Phi(T, u + \tilde{n}) \right\rangle_{\tilde{n}}$$

→ Constraints TE/TB statistical dependency
→ Independent on the TE/TB exact knowledge!

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  → Constraints TE/TB statistical dependency
  → Independent on the TE/TB exact knowledge!

→ **Very simple framework to impose strong constraints!**

FoCUS components separation algorithm

- **Spatial inhomogeneity of dust signal**
  - Assume latitude-only dependency
  - 3 constraints for 5 standard *Planck* masks
    $\rightarrow f_{\text{sky}} \in [1.0, 0, 73, 0.63, 0.43, 0.27]$
  - 15 loss terms in total

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  - 3 constraints for 5 standard *Planck* masks
    → $f_{\text{sky}} \in [1.0, 0, 73, 0.63, 0.43, 0.27]$
  - 15 loss terms in total

- **Final FoCUS algorithm (Delouis+, in prep.)**
  - Obtain a map from these 15 constraints
  - Gradient descent from the initial noisy map
    → biases computed every 500 iterations
  - Final map $\tilde{s}$ is the denoised map

Outline

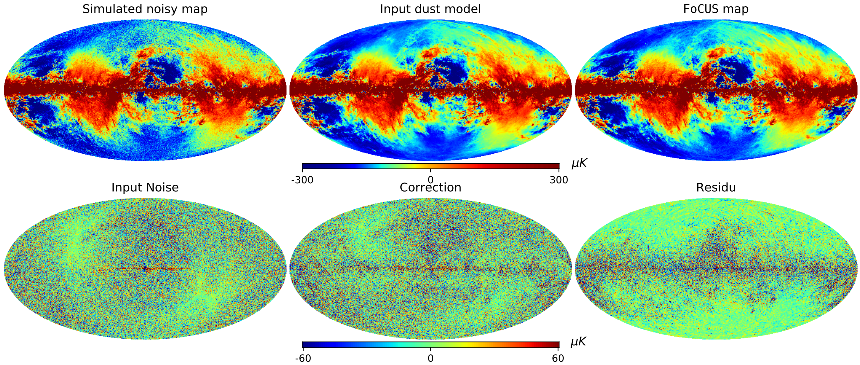
Validation on mock data

- **Set of data used**
  - T (857 GHz), E/B (353 GHz) from **(Vansyngel+, 17)**
  - 300 noise realizations from Sroll2 **(Delouis+, 19)**
  - Healpix maps with $N_{\mathrm{Side}} = 256$

→ **Do we reproduce the correct statistics?**
→ **Do we reproduce the correct map?**

Validation on mock data

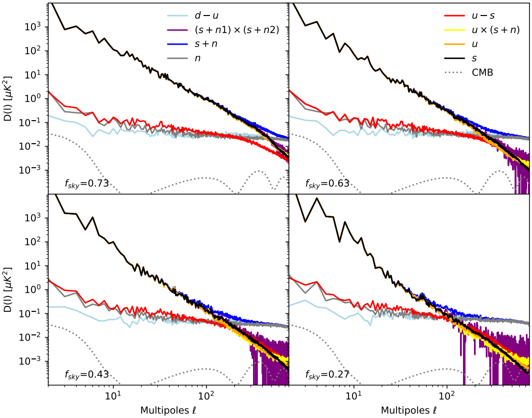
- **Analysis in real space**



→ **Consequent level of denoising**
→ **Different structures at high latitude**

Validation on mock data

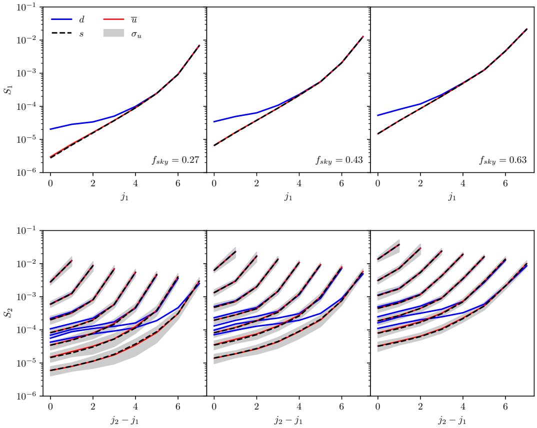
- **Analysis in power/cross-spectrum space**



→ **Transition from deterministic to statistics**

Validation on mock data

- **Analysis in CWST space**



→ Non-Gaussian CWST are well recovered!

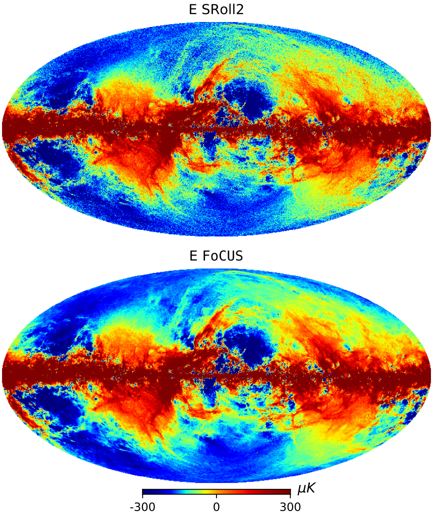
Application to *Planck* data

- **Maps from Sroll2 dataset (Delouis+, 19)**
  - T (857 GHz), E/B (353 GHz)
  - 300 noise realizations
  - Healpix maps with $N_{\mathrm{Side}} = 256$
- **Comparison with Vansyngel, PSM, and NILC**

→ Correct statistics expected, not a deterministic map

Application to *Planck* data

- Analysis in real space



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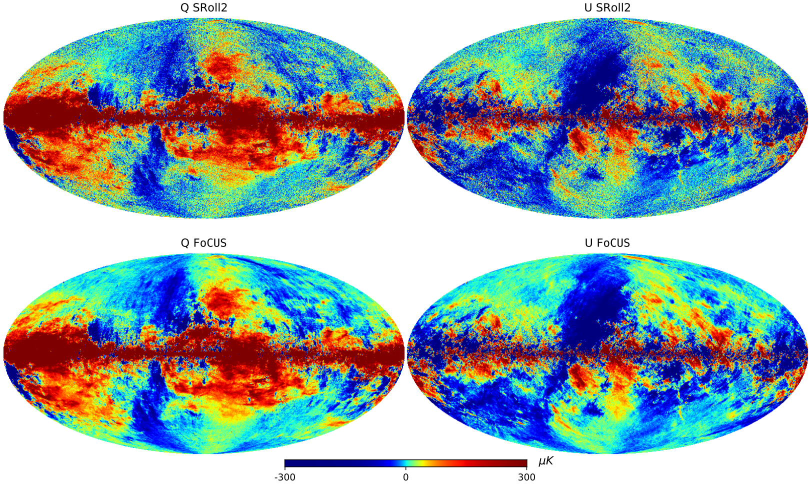

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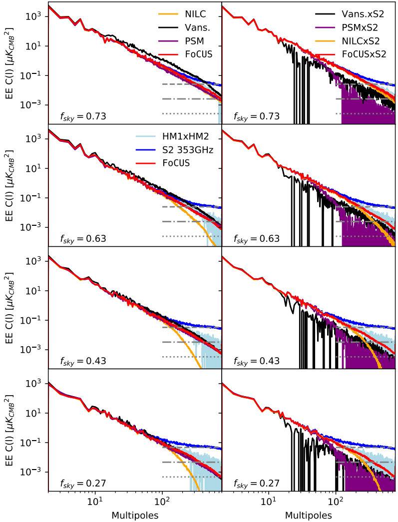

Application to *Planck* data

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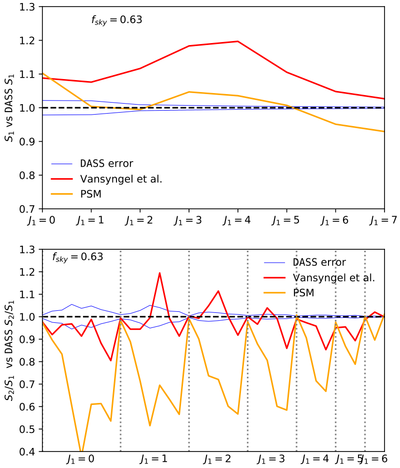

Conclusion

- **FoCUS denoised E/B maps at 353 GHz (Delouis+, in prep.)**
  - Obtained for $N_{\text{Side}} = 256$, $\ell \lesssim 800$
  - Expected correct Power/Cross-Spectrum
  - Expected correct non-Gaussian statistics (CWST)
  - No deterministic structures at small scales/high latitude

  → Extension to multi-λ **(Regaldo+, in prep.)**

  → Proper ST on the sphere **(Mousset+, w.i.p.)**

  → Include in a bayesian framework

- **Components separation from ST**
  - Diverse physical constraints easily implemented
  - Strong non-Gaussian statistical contraints
  - Can be applied directly on observations

  → See C. Auclair's talk

Thanks for your attention!