Credits: Geoff Chen

Building SPT-3G 2019/2020 likelihood

Etienne Camphuis (IAP)

with Silvia Galli, Karim Benabed and Eric Hivon

Colloque national CMB-France #3 - IAP - June 20th, 2022

Outline

A. **Overview of SPT-3G 2019/2020**

B. Improving the likelihood pipeline

    1. Accurate covariance matrices
       [EC, Galli, Benabed, Hivon, Lilley 2022]

    2. How to treat point sources

        i. Inpainting (Gaussian constrained realization)

        ii. Analytical expansion of the point source contribution [Gratton, Challinor, …, Camphuis *in prep*]

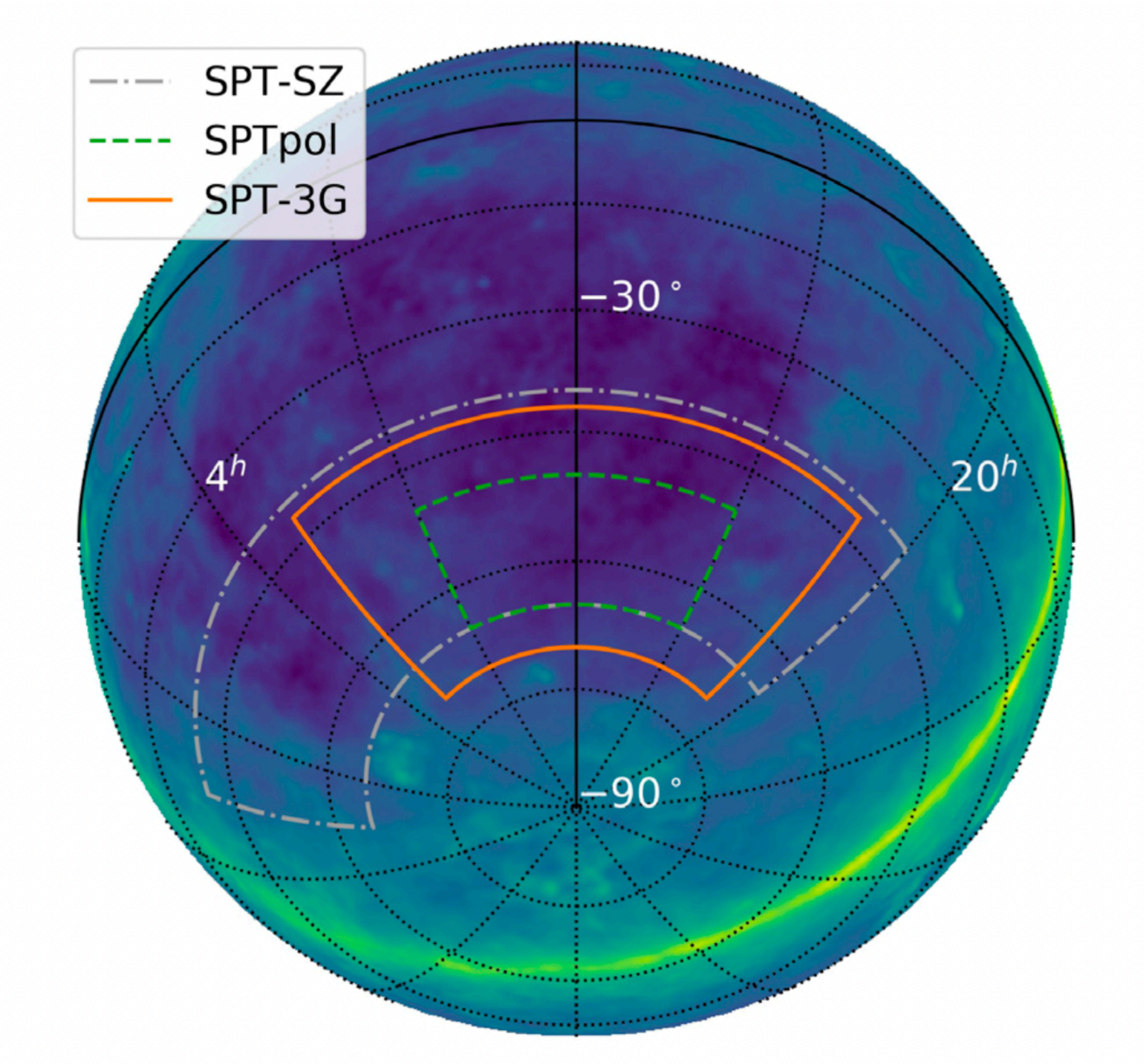
    3. CarPool (Accelerated simulations) [Chartier, Camphuis *in prep*]

South Pole Telescope

Ground-based experiment

- 10-meter diameter telescope observing the CMB anisotropies in T and P
- State of the art detector SPT-3G, observing during **5 years (2019-2023) in the winter -** sky patch: 4% of the sky
- 3 frequencies 90, 150, 220 GHz
- FWHM : 1.7, 1.4, 1.2 arcmin (at 95, 150, 220 GHz) vs *Planck* 5 arcmin
- Final map depth:
  - 2.8, 2.6, 6.6 μK-arcmin (T) vs *Planck* 40 μK-arcmin

SPT-3G « winter field » (4% vs *Planck* fullsky)



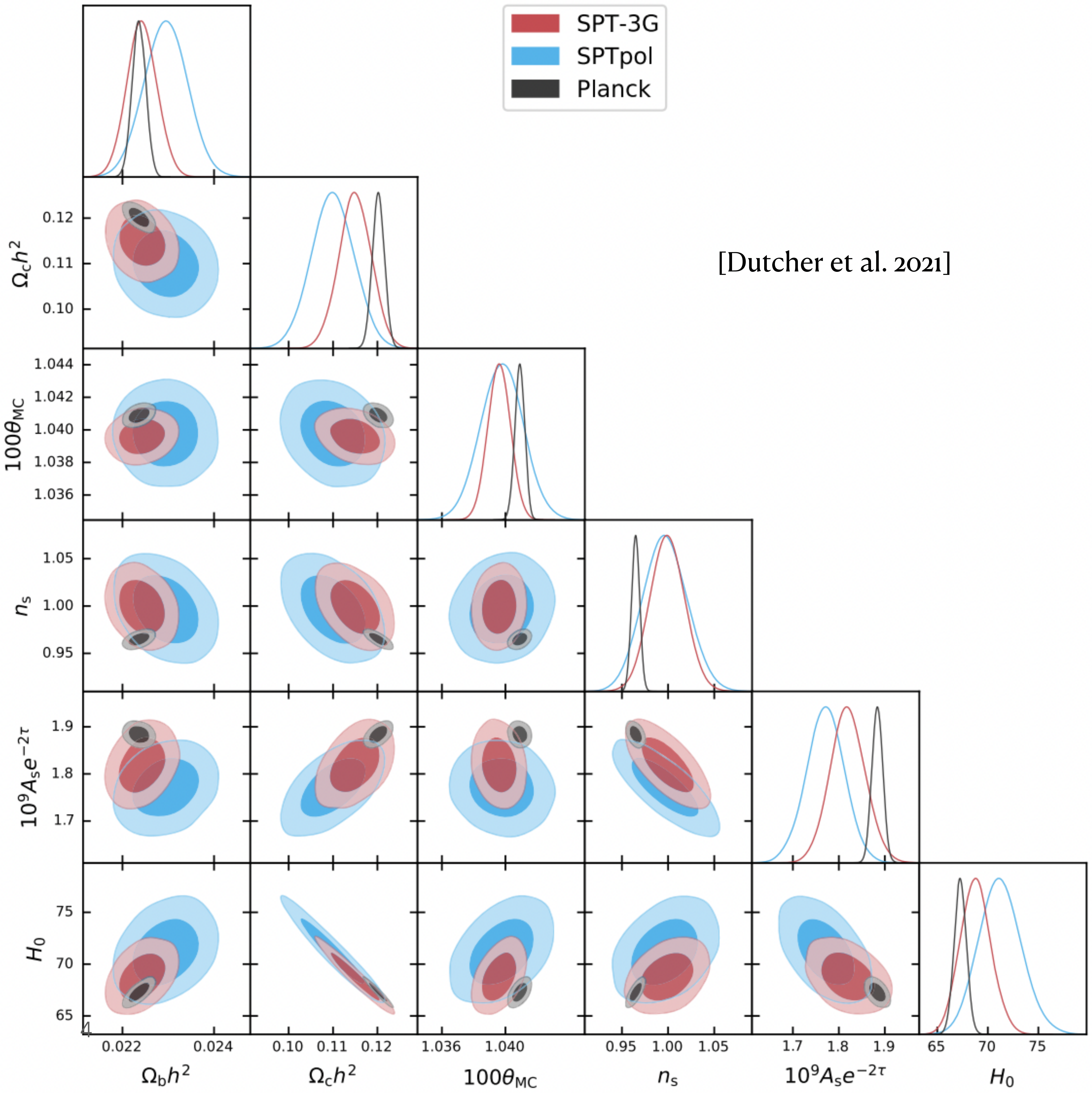
Sky patch overlaid over thermal dust emission [Dutcher et al. 2021]

SPT-3G 2019/2020

Next data release

- Analysis of 2019+2020 winter maps
  - factor ~ 4 lower noise than in SPT-3G 2018
  - Map depth:
    - ~ 5/4/15 μK-arcmin (T)
    - ~ 7/6/21 μK-arcmin (pol)
- Observations will continue through at least 2023 (total of 5 years)
  - Goal noise: 2.8, 2.6, 6.6 μK-arcmin (T)
  - ΛCDM constraints comparable with Planck from SPT-3G alone !

Contour plot and posterior distribution of parameters for SPT-3G 2018 data



[Dutcher et al. 2021]

Challenges for the future

- Data gets better ! So **we need to improve the pipeline,** as we want to trust our cosmological parameters.

- As is stands, the current pipeline requires a lot of computing ressources to run mock-observations: simulations that mimic telescope observation of a CMB + foregrounds sky. **We would like to find alternatives to these very expensive mock-observations.**

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Accurate covariance matrices

Core component of the likelihood

- Accurate CMB covariance matrices are required for a unbiased estimation of the cosmological parameters and their error bars. [Sellentin&Starck 2019]

- The relative accuracy on the cosmological parameters is that of the inverse of the covariance matrix [Taylor, Joachimi, Kitching 2012]

Power spectrum gaussian likelihood :

$$-\ln \mathscr{L}(\hat{C}\,|\,\Lambda\mathrm{CDM}) \propto \frac{1}{2}(\hat{C} - C^{\mathrm{th}})^T \Sigma^{-1} (\hat{C} - C^{\mathrm{th}}))$$

Accurate covariance matrices

Core component of the likelihood

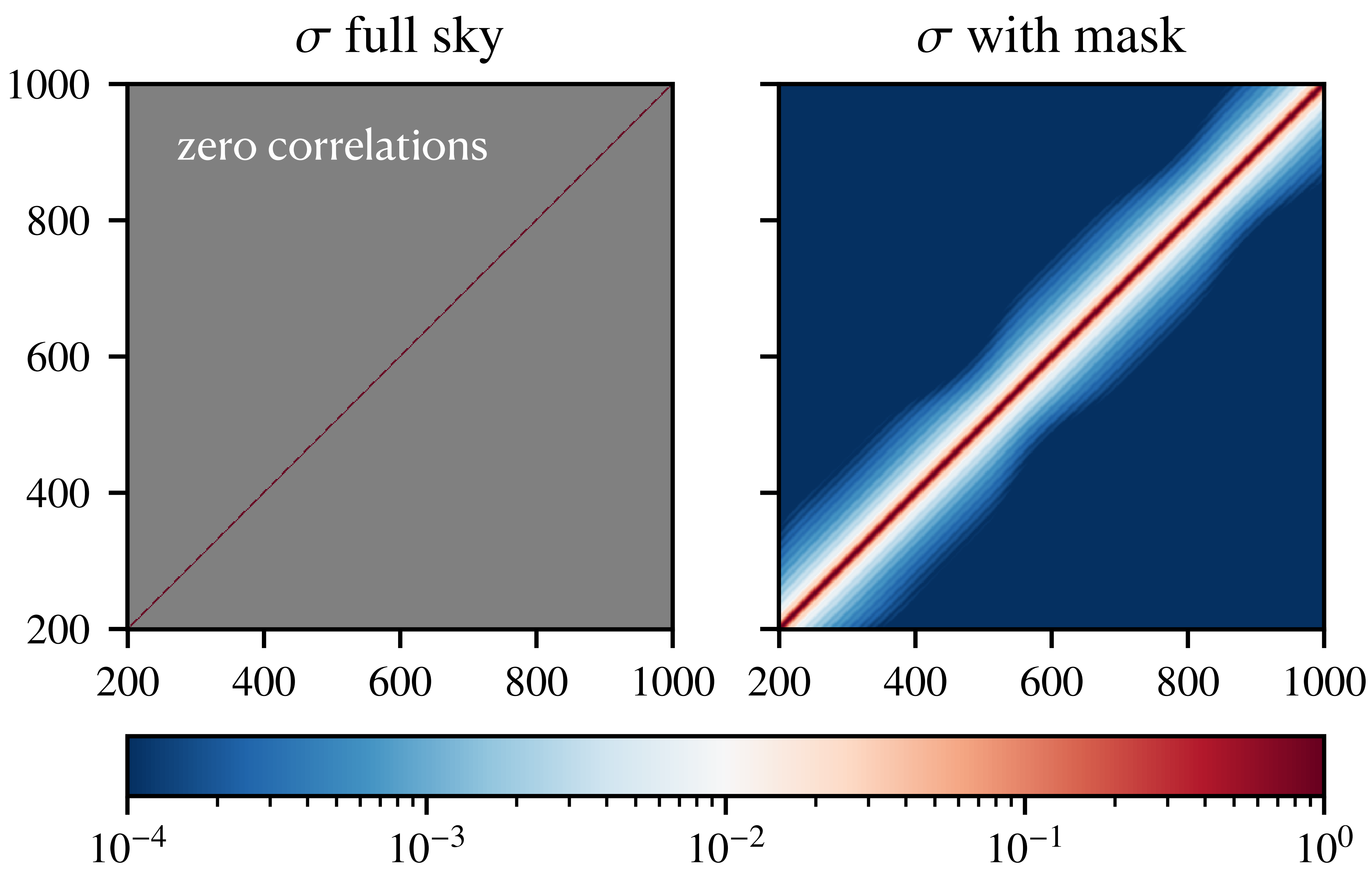
- Previous data release: mock-observations + estimate of noise through data, which requires computing resources and regularization [Balkenhol et al. 2021]
- Next data release: we would like to have a (semi-)**analytical computation,** precision and no need for regularization [EC et al. 2022] https://arxiv.org/abs/2204.13721. **Curved-sky analysis**
- Ingredients: mask (introduces coupling) W and fiducial spectrum C_ℓ^{th}

Power spectrum gaussian likelihood :

$$-\ln \mathscr{L}(\hat{C}\,|\,\Lambda\mathrm{CDM})$$

$$\propto \frac{1}{2}(\hat{C} - C^{\mathrm{th}})^T \Sigma^{-1} (\hat{C} - C^{\mathrm{th}}))$$

Unbinned correlation matrices full sky vs masked sky



Formalism

Covariance matrix of the pseudo-power spectrum

pseudo-power spectrum
(on the masked sky)

$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1 \ell_2} C^{\mathrm{th}}_{\ell_1} \, \bar{\Theta}^{\ell_1 \ell_2}_{\ell\ell'}[W] \; C^{\mathrm{th}}_{\ell_2}$$

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Pure geometric coupling - MASTER matrix

Well known [Hivon et al. 2002]

Scales as $\mathcal{O}(\ell^3_{\mathrm{max}})$ (or even $\mathcal{O}(\ell^2_{\mathrm{max}})$ using [Louis et al. 2020])

Formalism

Covariance matrix of the pseudo-power spectrum

Fiducial power spectrum from model

Can include beam, transfer function, noise, pixel window function.

pseudo-power spectrum
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Covariance coupling kernel

Scales as $\mathcal{O}(\ell^6_{\max})$ and $\ell_{\max} \sim 4000$

Always approximated in the literature

UNTIL NOW!

Pure geometric coupling - MASTER matrix

Well known [Hivon et al. 2002]

Scales as $\mathcal{O}(\ell^3_{\max})$ (or even $\mathcal{O}(\ell^2_{\max})$ using [Louis et al. 2020])

Exact covariance

- **I implemented for the first time an exact computation, with a x1000 speedup**
- **This code allows to compute any row of covariance at any multipole**

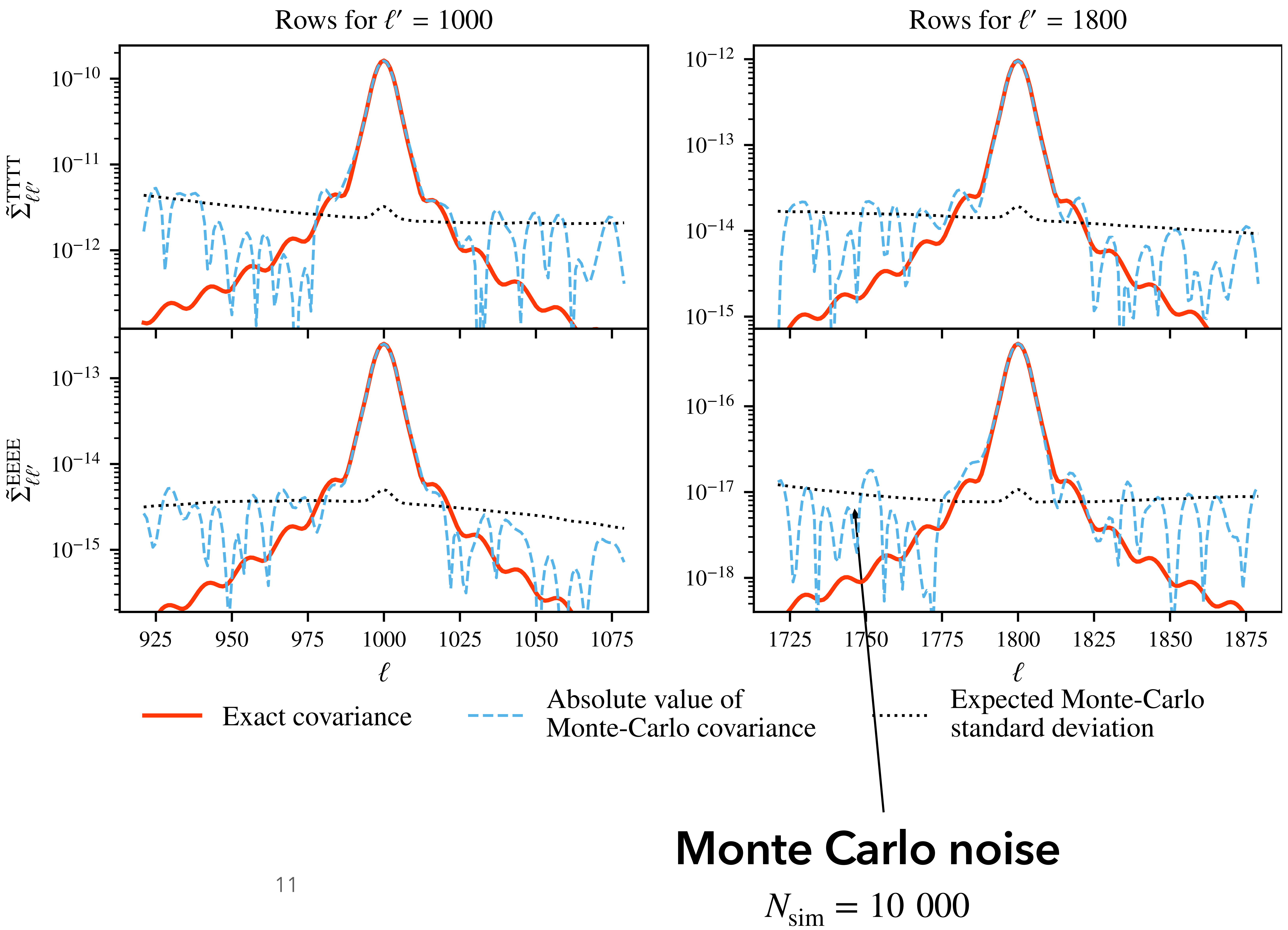
$$\text{For a given } \ell,\ \tilde{\Sigma}_{\ell\ell'} \forall \ell' \ \sim\ \mathcal{O}(\ell^4)$$

$$\Longrightarrow \tilde{\Sigma}_{\ell\ell'} \forall (\ell, \ell') \ \sim \mathcal{O}(\ell_{\max}^5)$$

$$\text{instead of } \mathcal{O}(\ell_{\max}^6)$$

Exact covariance

- How does this code compare to simulations ?
- It is still expensive: 300h CPU time for a row at $\ell = 1000$



Approximations

- **To use less computing ressources, we will use approximations of the covariance matrix. They are expected to be precise on large sky fraction, but their accuracy need to be evaluated for small area (SPT3G~4%)**

$$\tilde{\Sigma}_{\ell\ell'} \approx \tilde{\Sigma}^{\mathrm{APP}}_{\ell\ell'} \sim \mathcal{O}(??) < \mathcal{O}(\ell^5_{\mathrm{max}})$$

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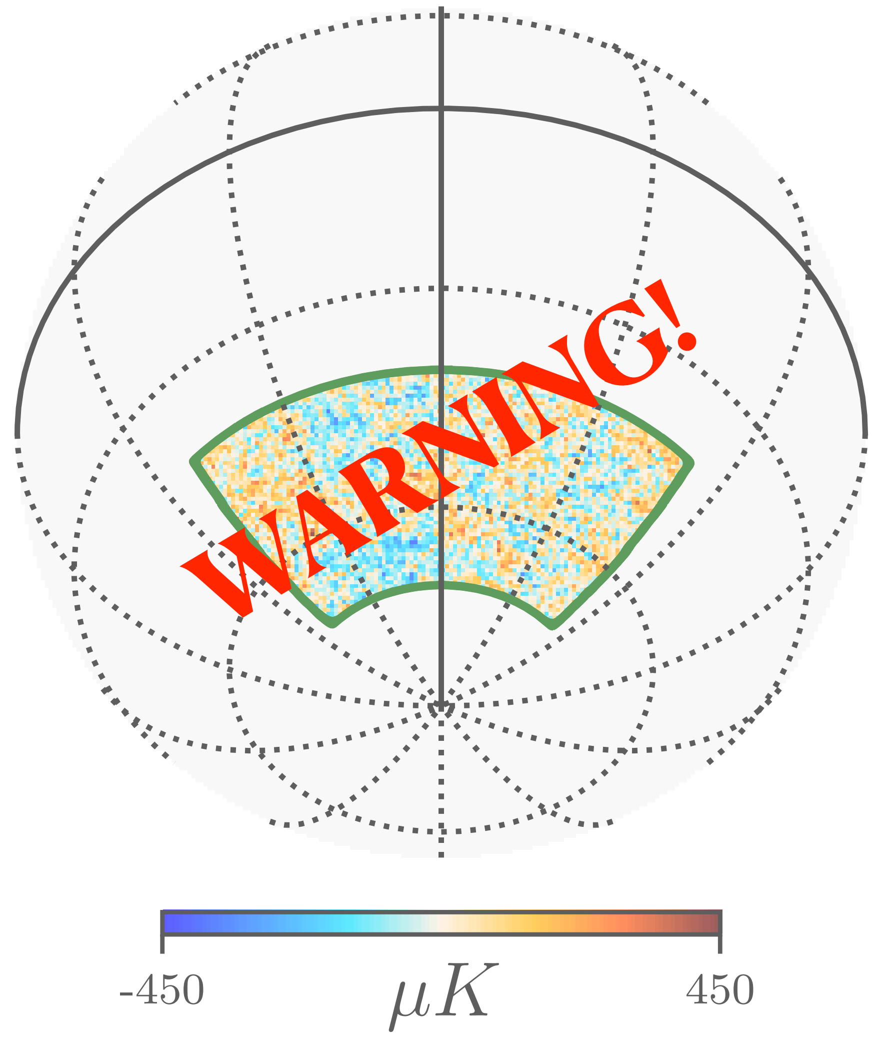
No source masking !!



CMB temperature anisotropies

Approximations

It is not realistic to run the exact computation for our analysis => we use approximations that work for every multipole !

- [Efstathiou 2004]+[Challinor&Chon 2004] **NKA - *Planck*** and others => $\mathcal{O}(\ell_{\max}^3)$
- [Friedrich et al. 2021] **FRI** => $\mathcal{O}(\ell_{\max}^3)$ **-** *DESY3*
- [Nicola et al. 2021] **INKA** => $\mathcal{O}(\ell_{\max}^3)$

- [EC et al. 2022] **ACC** - obtained with our knowledge from the exact computation

Scales as $\mathcal{O}(d_{\max} n_{\mathrm{side}}^4) \gg \mathcal{O}(\ell_{\max}^3)$ (~100h of CPU-time vs few minutes) but it has to be computed only once per mask

Approximations

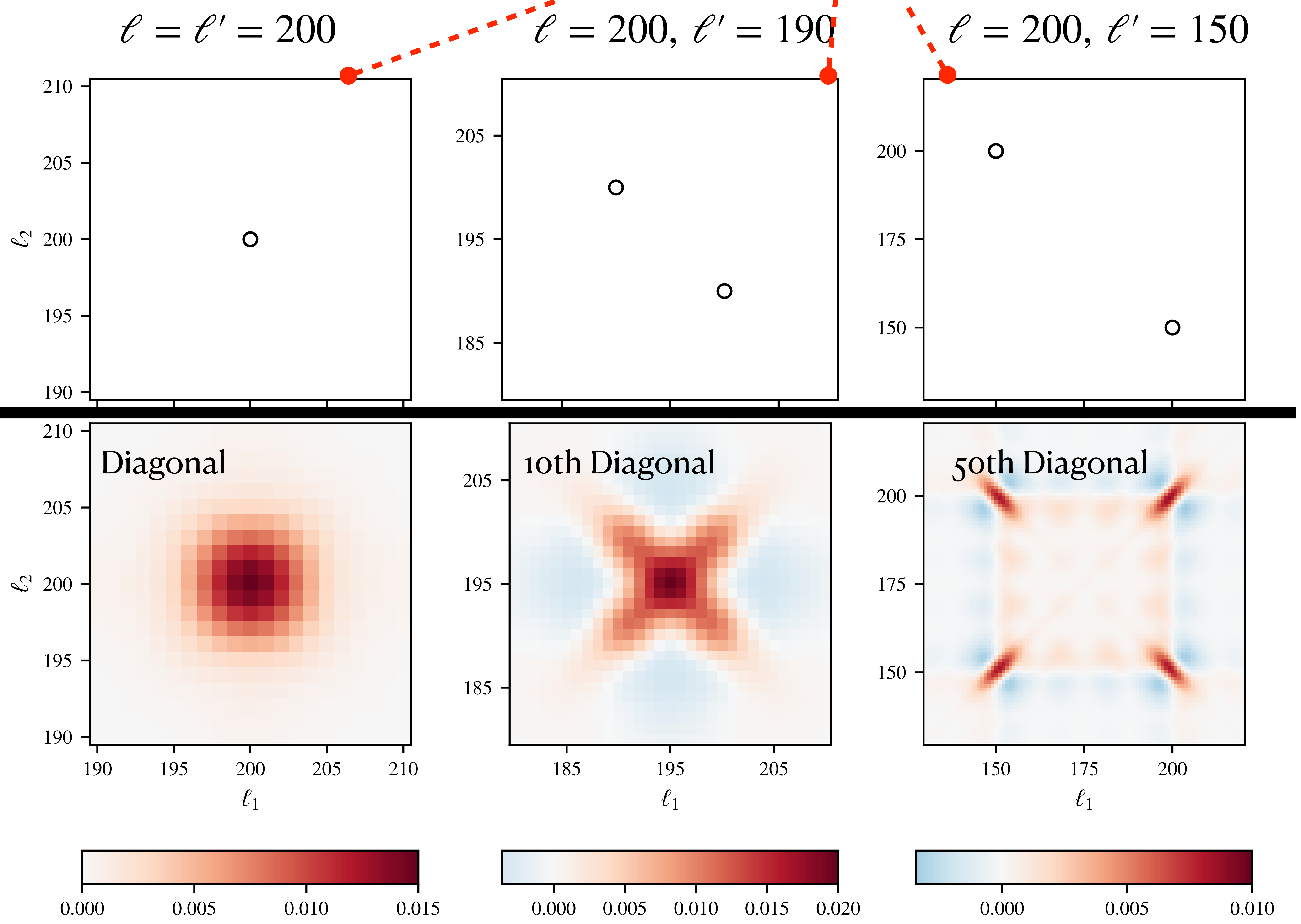
$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W]\, C^{\mathrm{th}}_{\ell_2}$$

Comparing the covariance coupling kernels

<u>Approximations:</u>

- NKA (Planck) (o)

<u>Exact</u>



Approximations $\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1}\, \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W]\, C^{\mathrm{th}}_{\ell_2}$

Comparing the covariance coupling kernels

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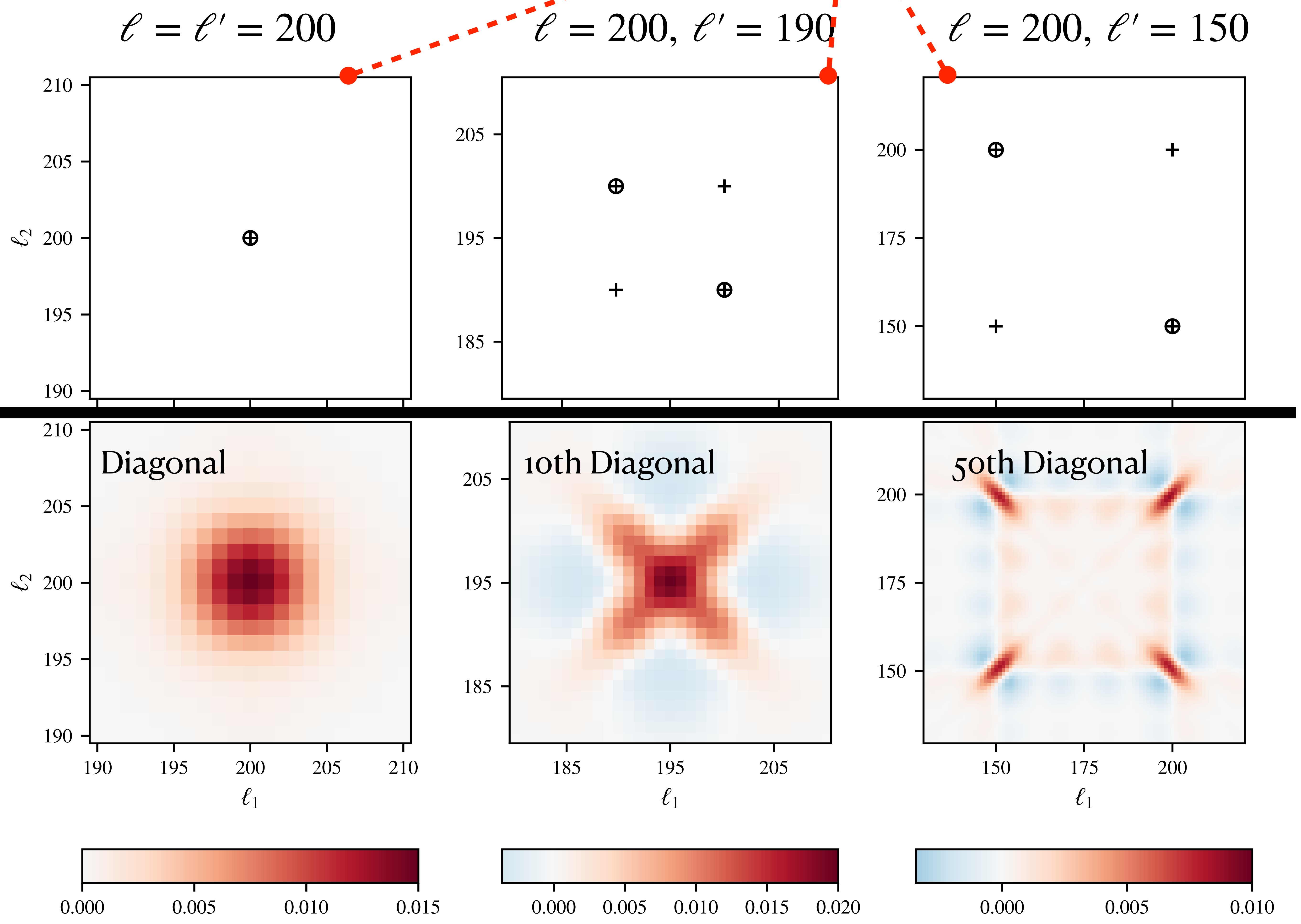


Approximations

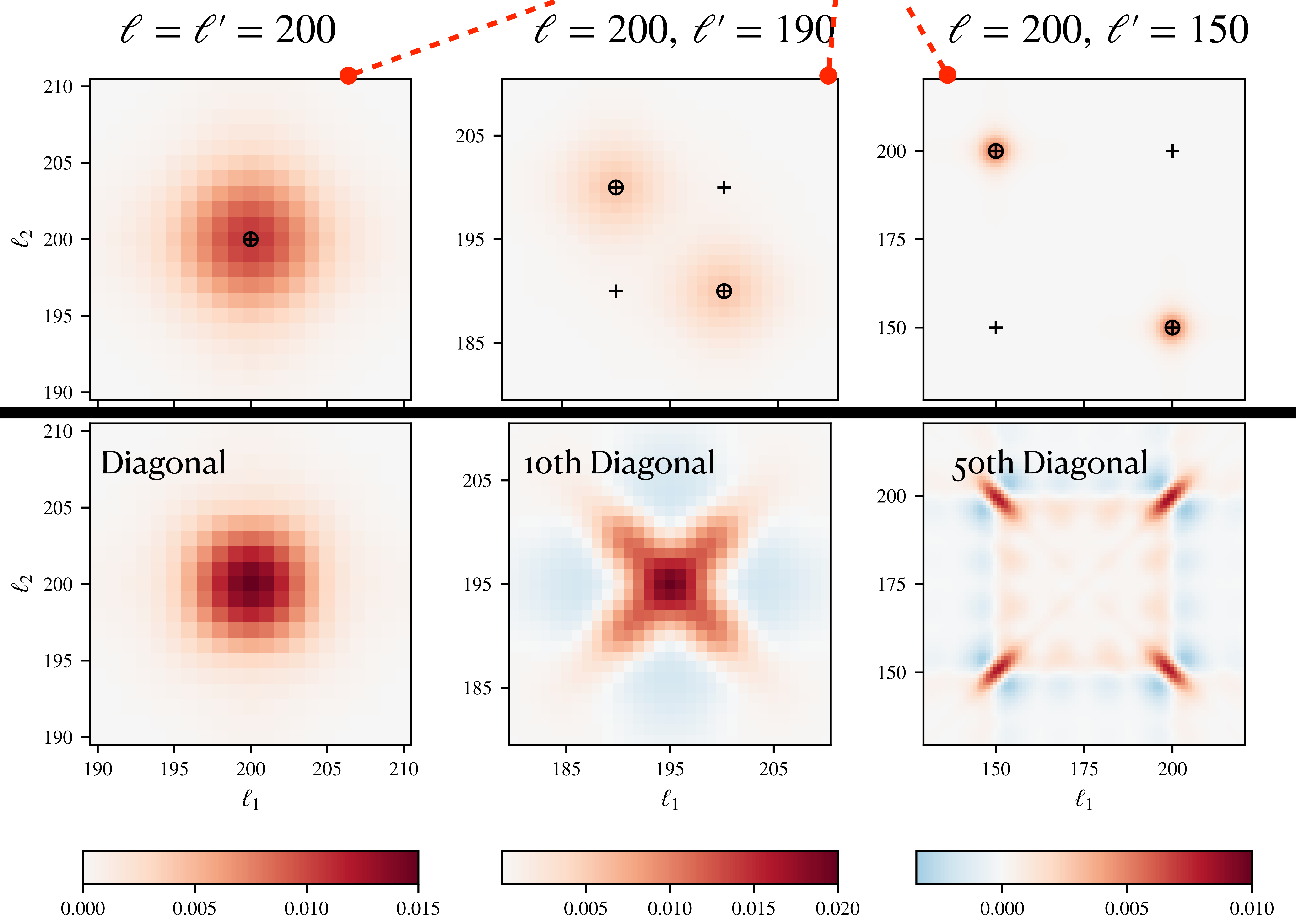
$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \, C^{\mathrm{th}}_{\ell_2}$$

Comparing the covariance coupling kernels

<u>Approximations:</u>

- NKA (Planck) (o)
- FRI (+)
- INKA (image)

<u>Exact</u>



Approximations

$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1}\, \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W]\, C^{\mathrm{th}}_{\ell_2}$$

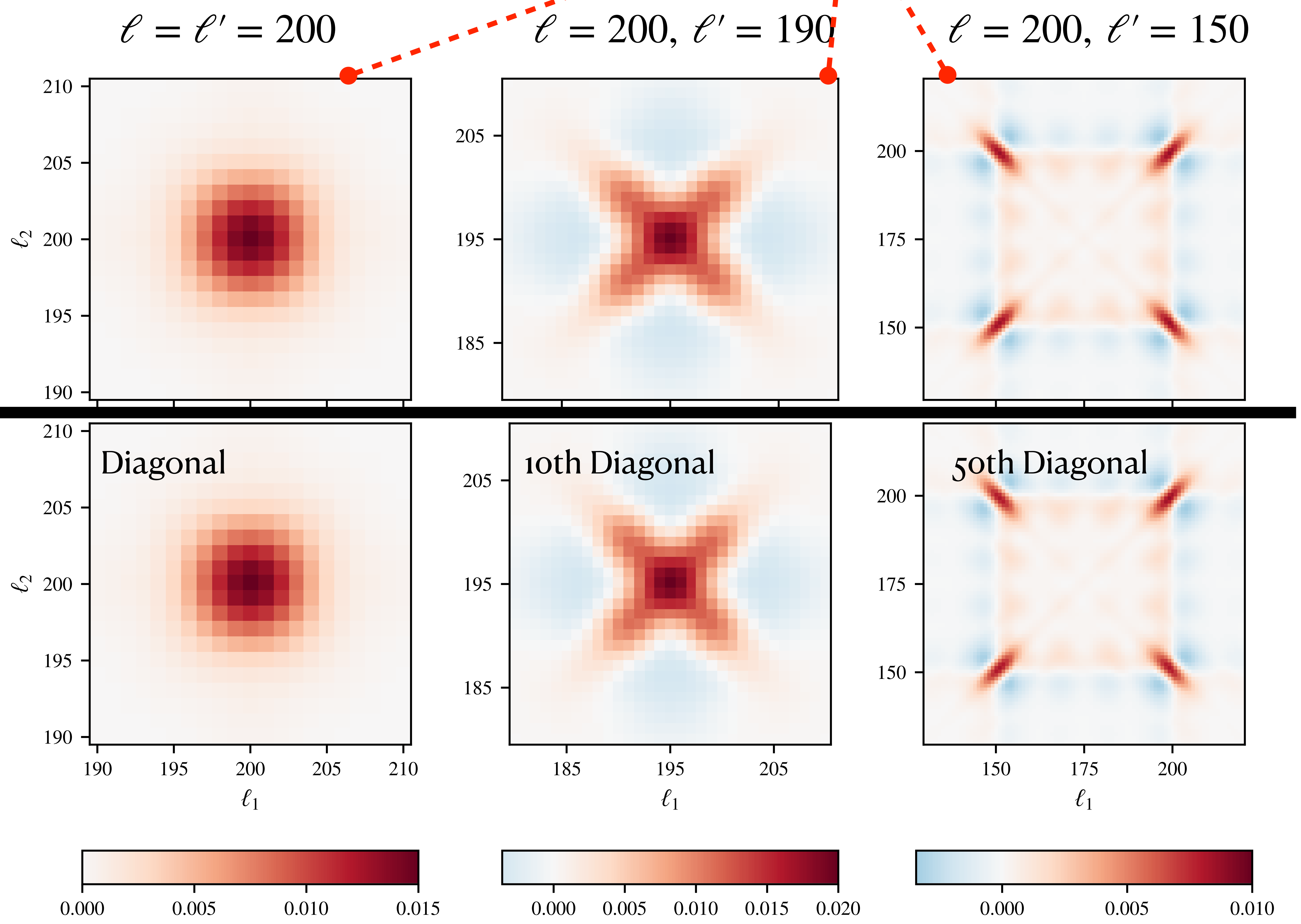
Comparing the covariance coupling kernels

Approximations:

- **ACC (this work)**

Using the same $\bar{\Theta}$ for identical multipole separation $|\ell - \ell'|$

Exact

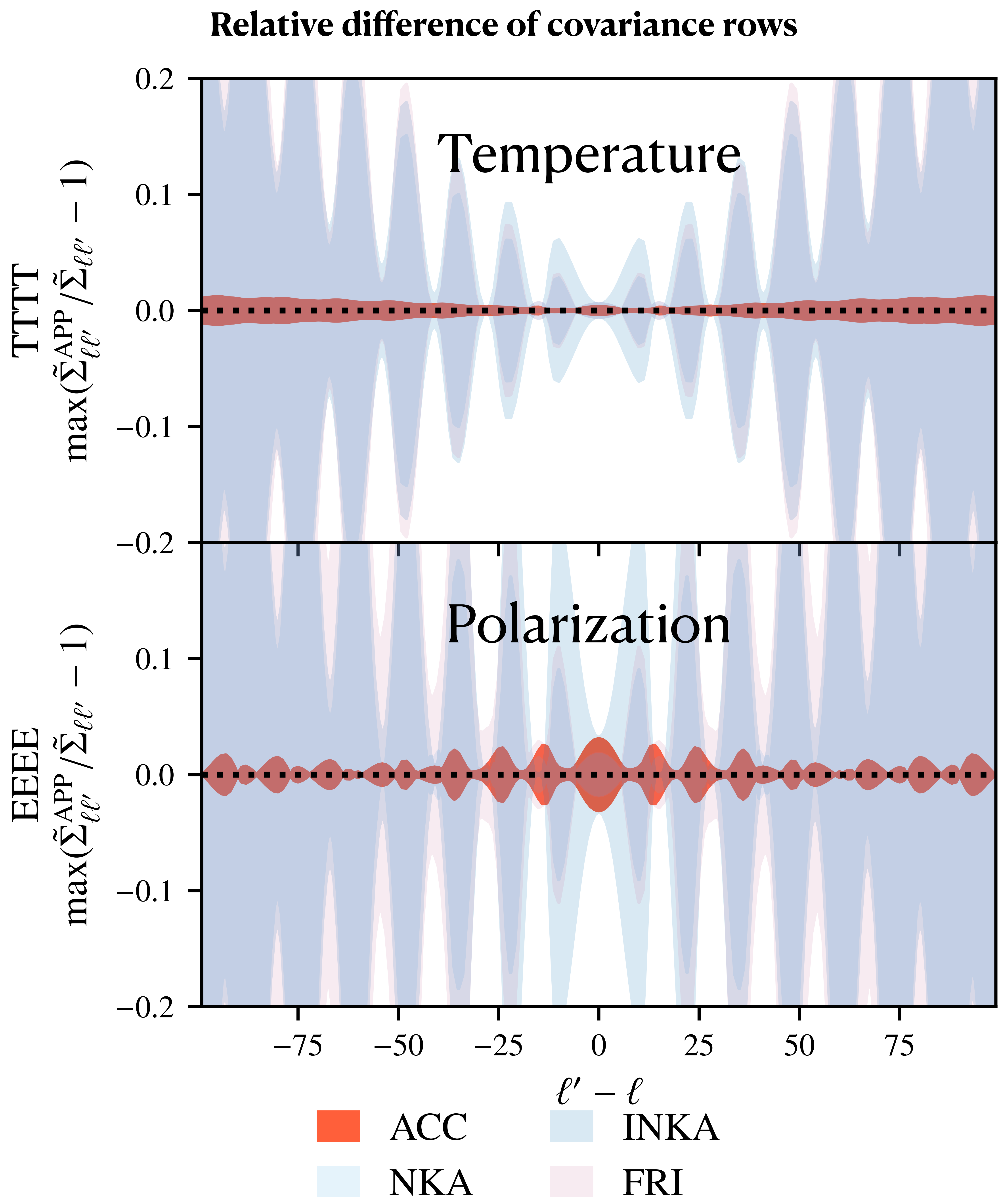


Results

Accuracy of approximations

- We look at the relative difference of rows of the covariance centered on the diagonal

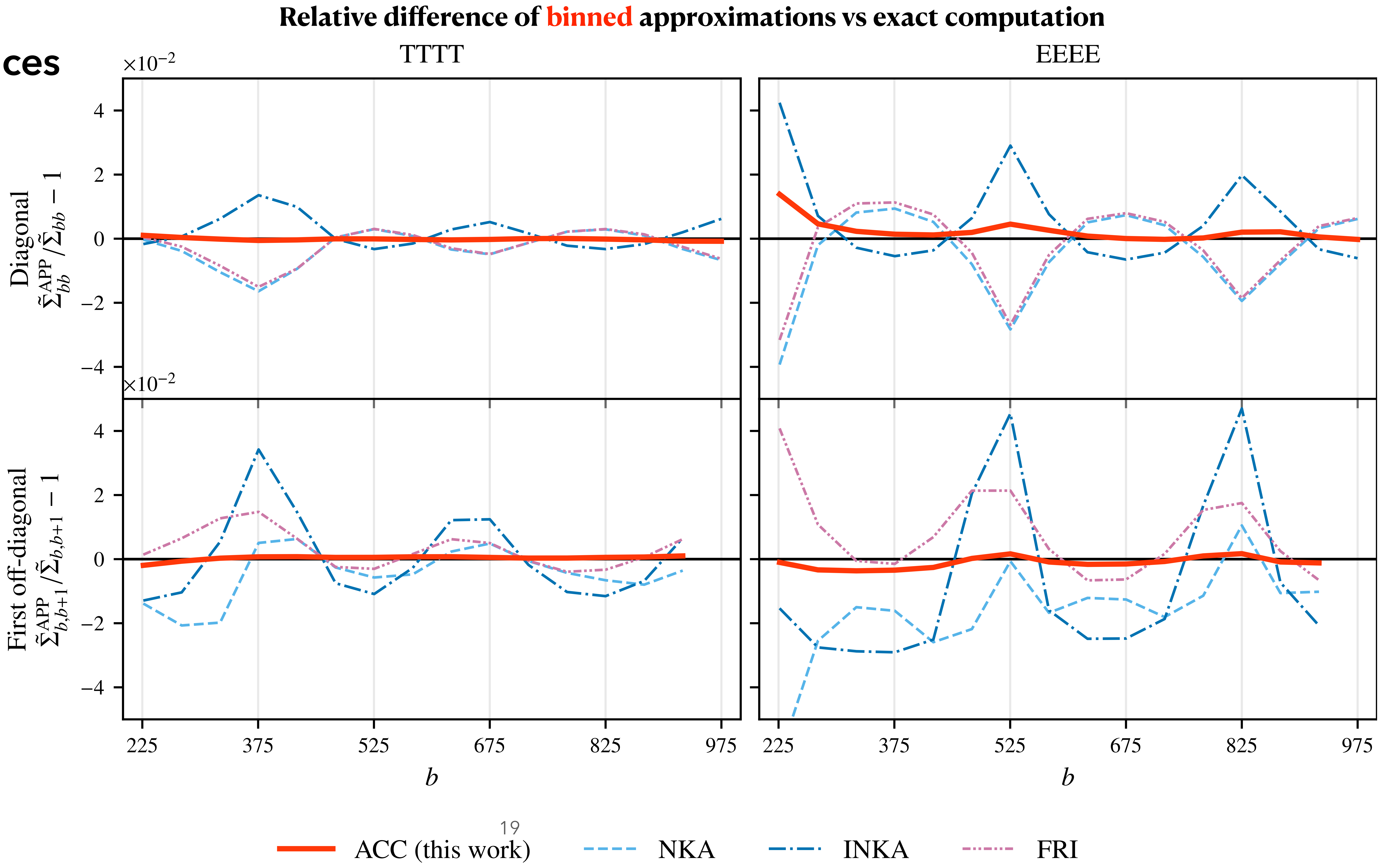
- In red **ACC**



Results

Binned covariances

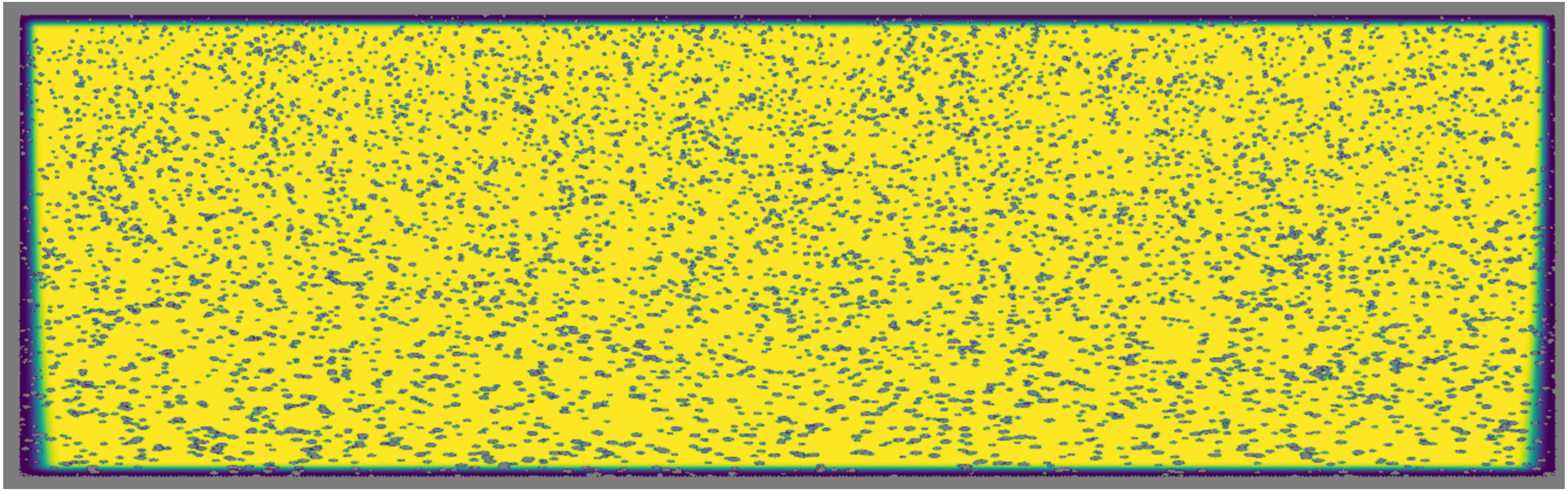
- Looking at binned covariance ($\Delta\ell = 50$)
- Literature approximations work with precision up to 5%.
- ACC is more precise, percent level



Caveat

Approximations are known to fail when we mask the sources!

Cartesian view of apodized mask with holes



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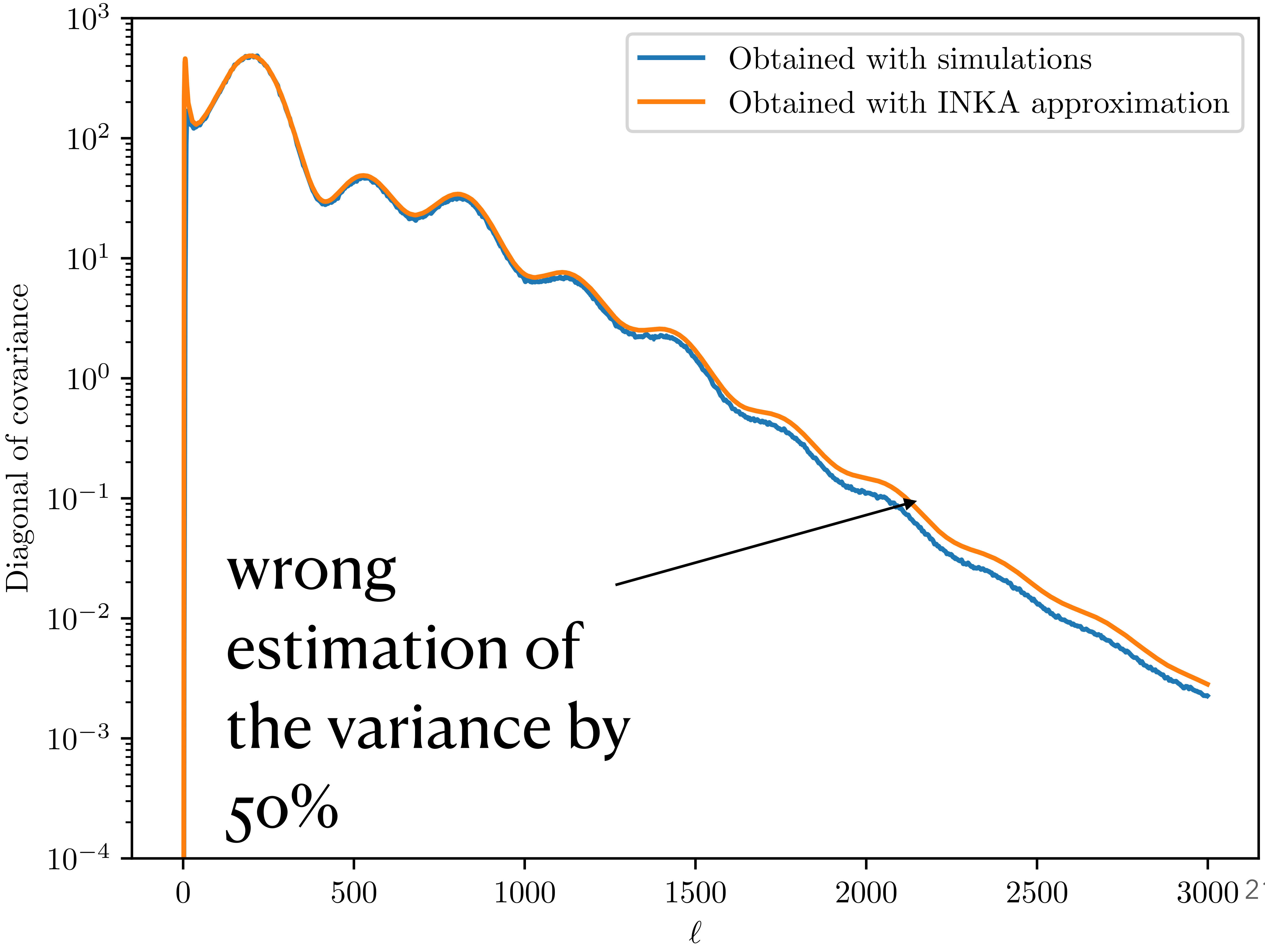
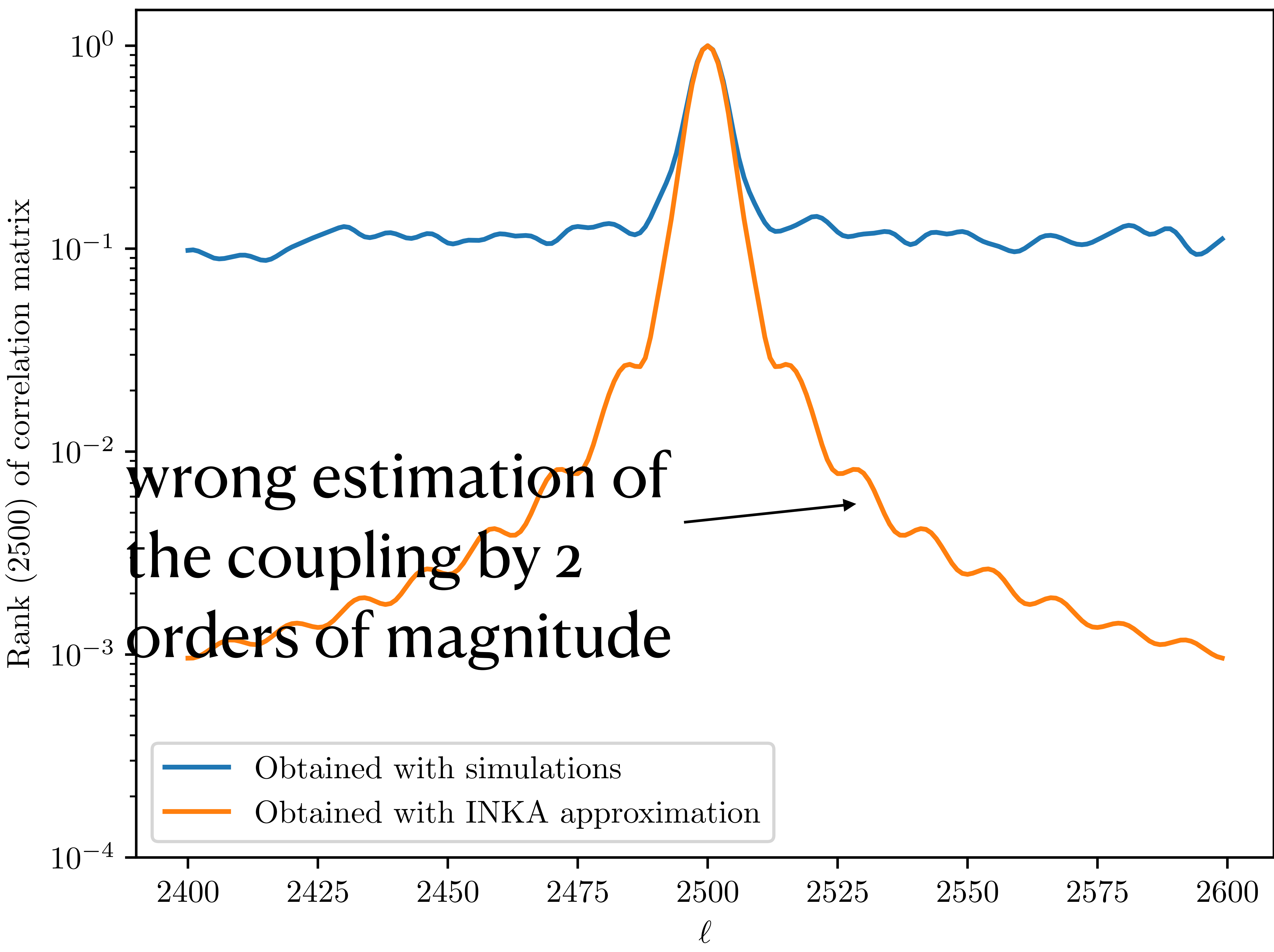
Mask with holes: simulations vs INKA approx ; diagonals



Mask with holes: simulationss vs INKA approx ; rank of correlation



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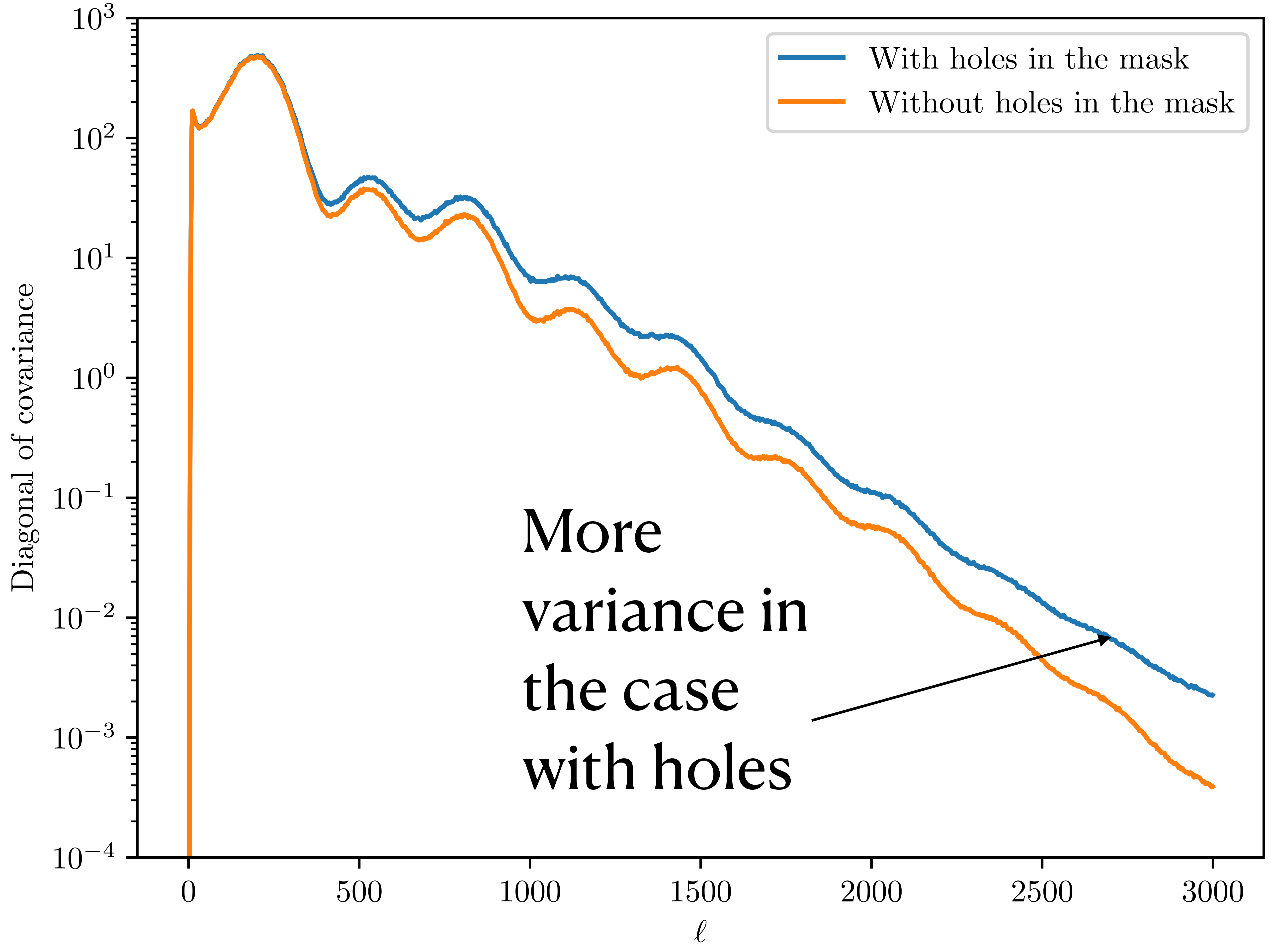
(2.i.) Inpainting

Why ?

- Bright clusters or sources need to be masked
- This will create spurious correlations between the modes
- What if we did not mask them ?

Diagonal of covariance matrices (correctly normalized) in the case with(out) holes

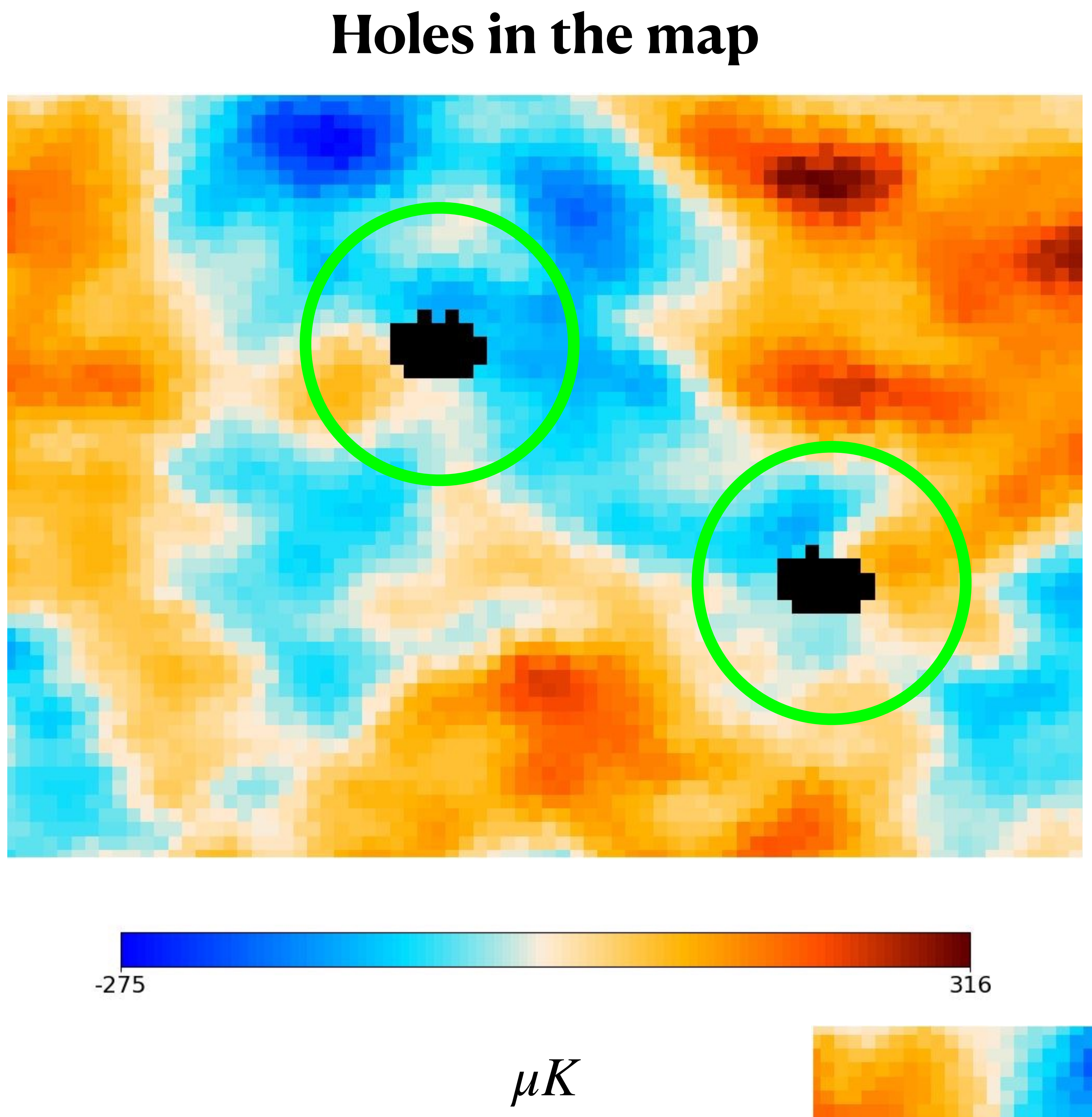


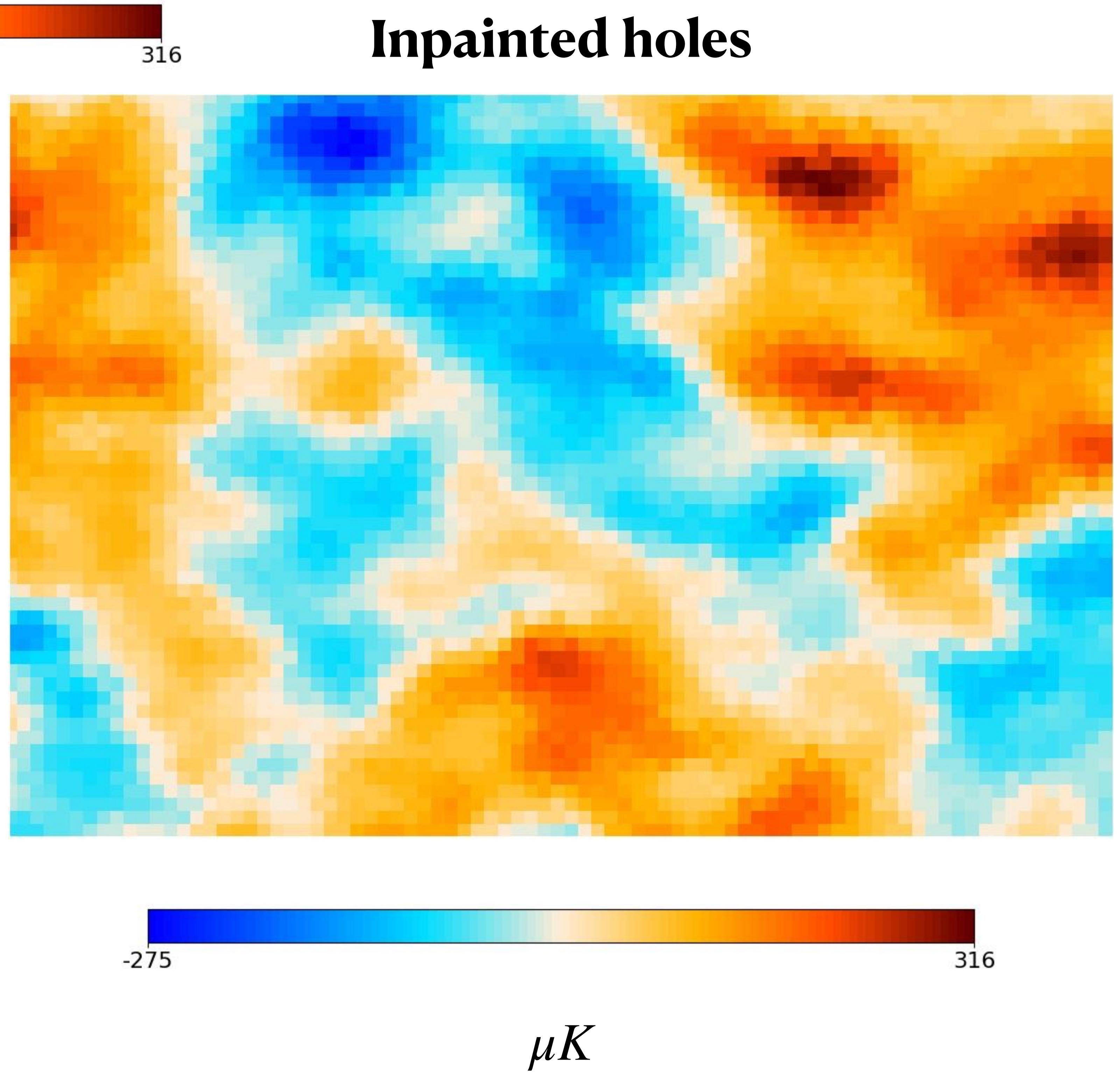
(2.i.) Inpainting

Idea

- One can fill the signal inside the holes with a gaussian constrained realization of the CMB anisotropies.

- Challenges: very high resolution maps ($n_{\text{side}} = 8192$) with many sources ($N_{\text{sources}} \sim 2000$) of varying radii.





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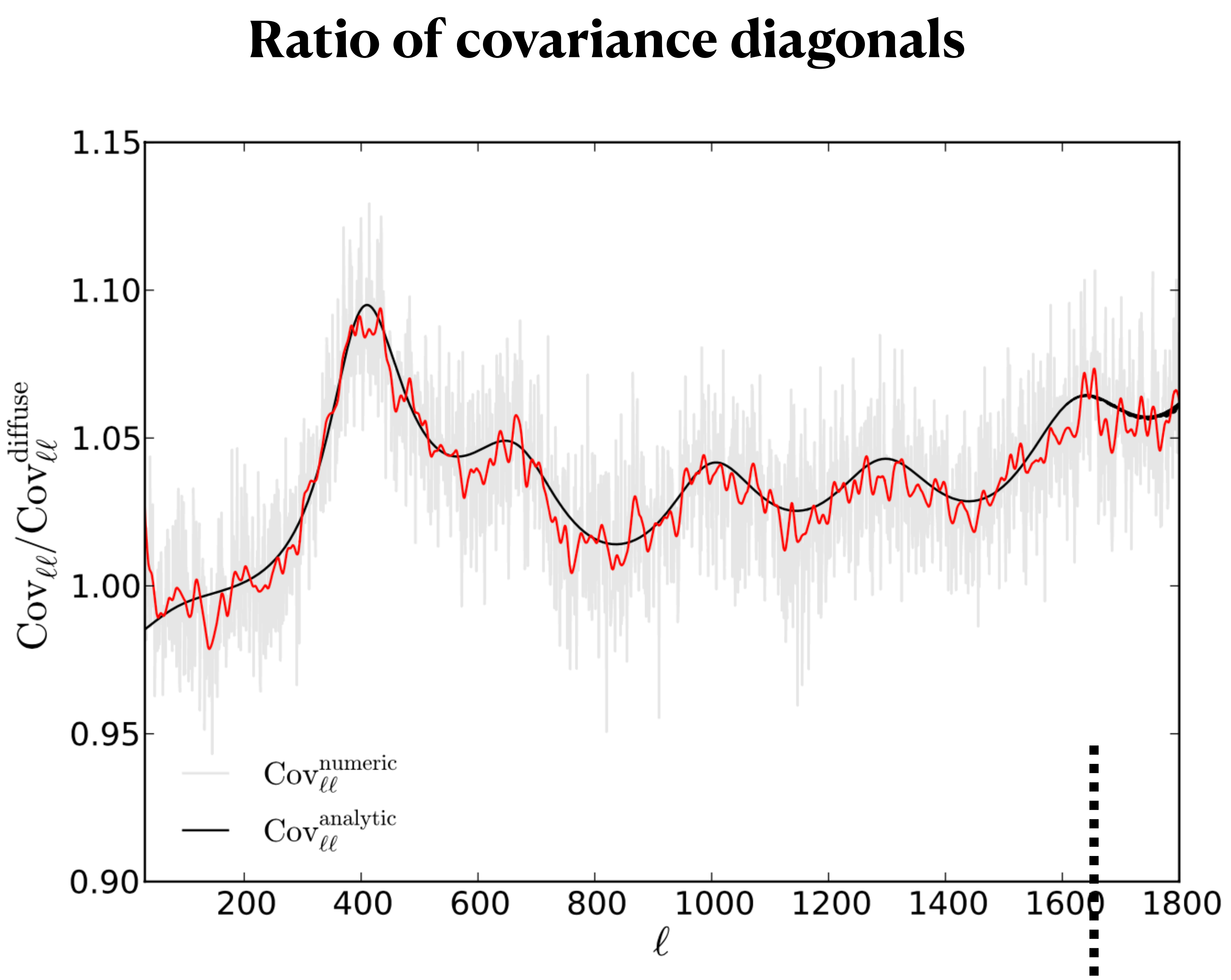
(2.ii.) Analytical CMB covariance with sources

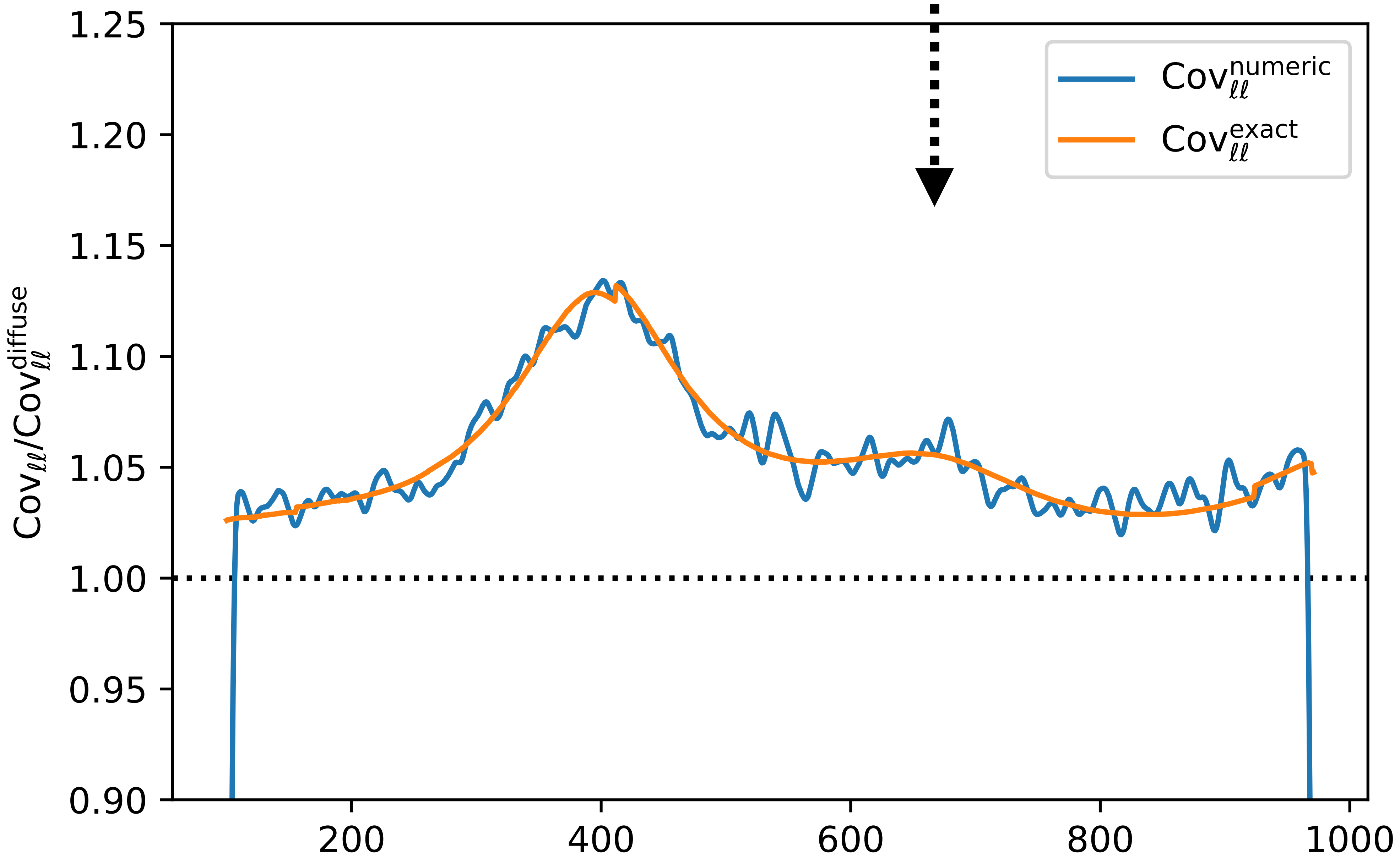
[Gratton, Challinor, Migliaccio, Hivon, Lilley, Camphuis]

- Idea: separate mask contributions in diffuse + sources
- Use expansion of the covariance into cumulants
- Here an example for *Planck*

Preliminary



My contribution



Quick summary

- We have been working on improving the likelihood pipeline by building an accurate covariance matrix. We are now able to compute exactly the covariance matrix.
- We also have determined the accuracy of existing/new approximations on small survey area.

- We have solutions to deal with the point source masking problem

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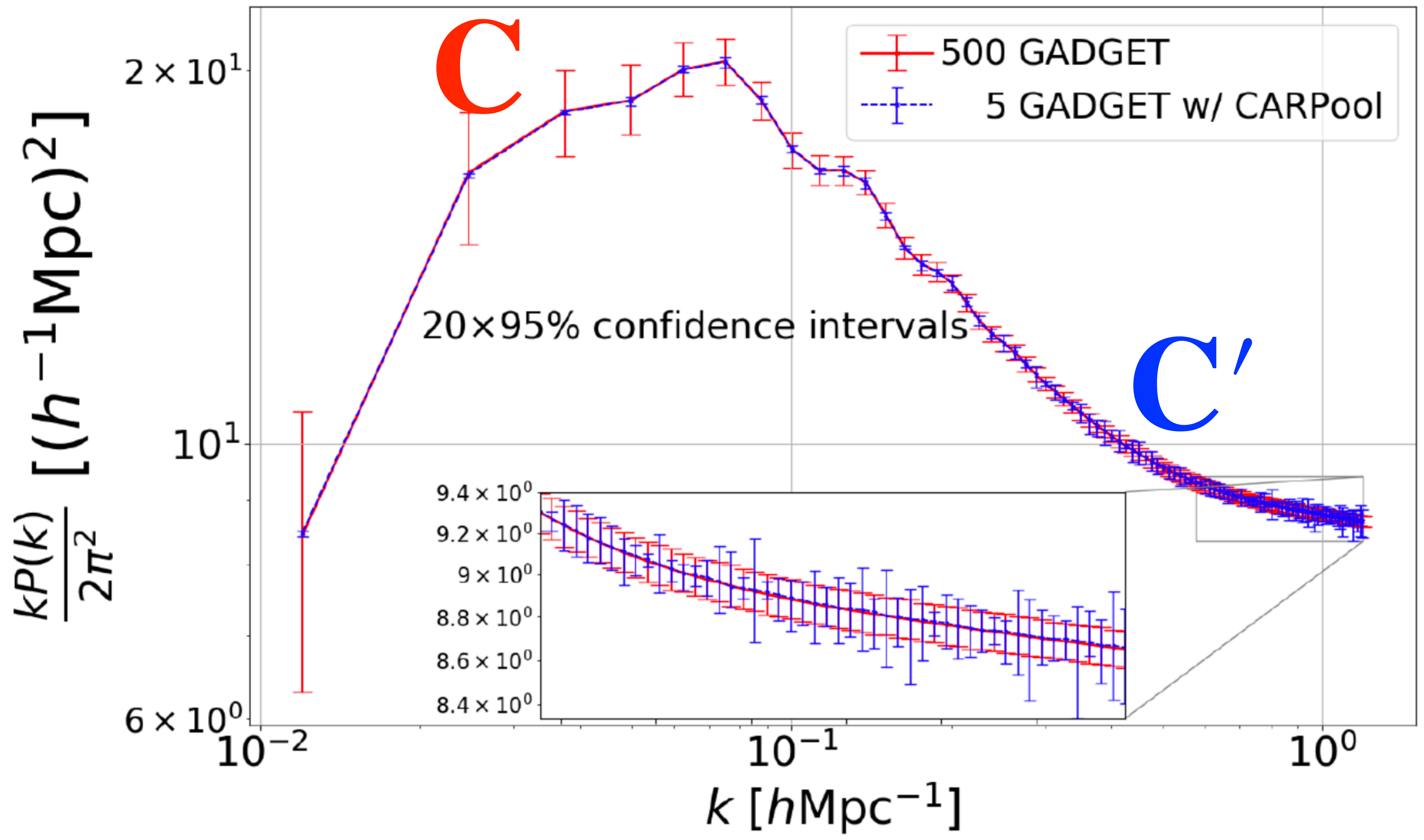
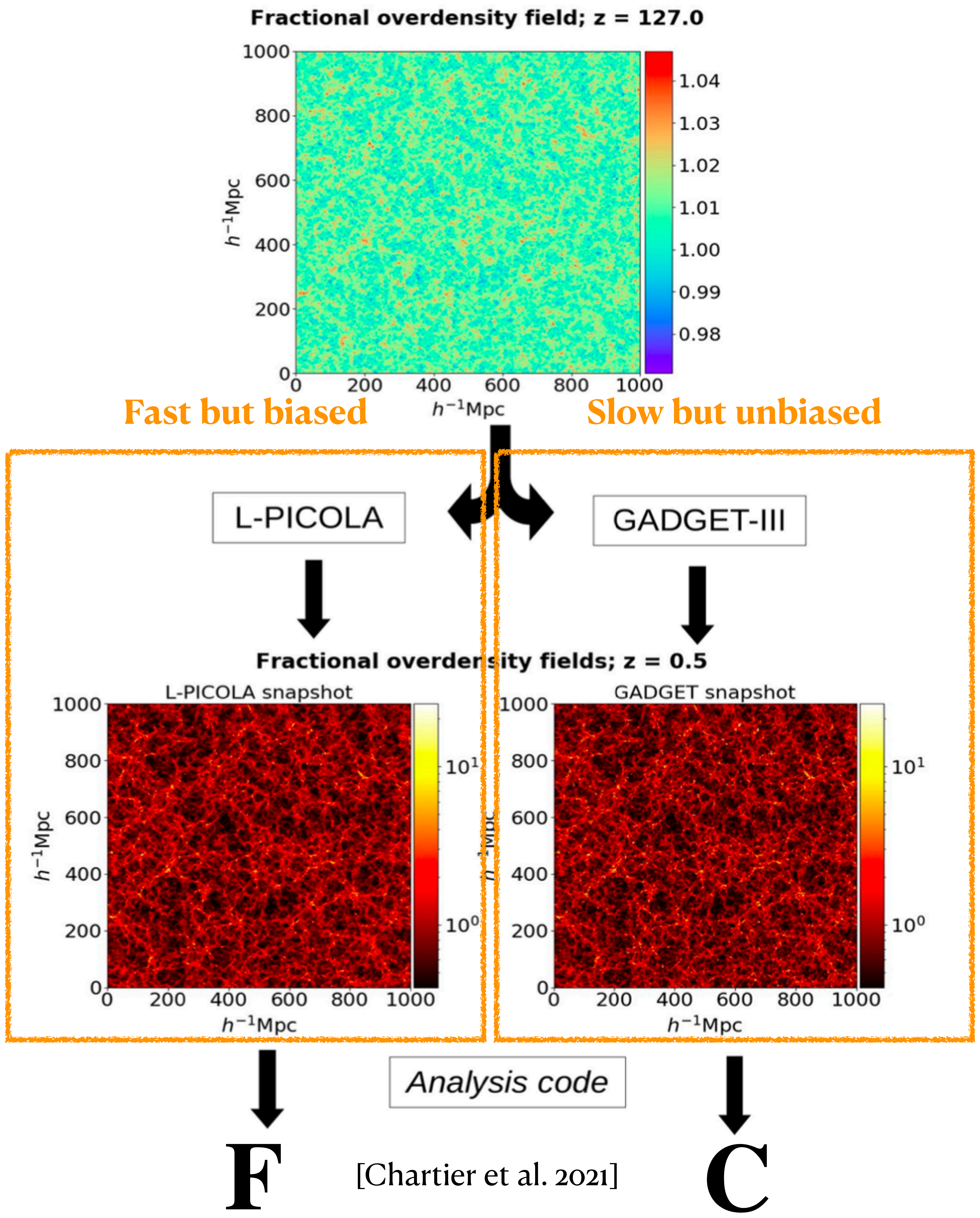
CarPool?

[Chartier, Wandelt et al., 2021]

- An example with matter power spectrum

Very precise power spectrum with 100 times less simulations !





[Chartier et al. 2021]

CarPool?

Accelerated simulations using correlations between estimators

- [Chartier, Wandelt et al., 2021]
- Two ingredients:
  - **surrogates** F : a fast or well-known estimator.
  - **simulations** C : a slow or poorly-known estimator. We want to estimate its mean or its covariance using variance reduction method.

They need to be correlated (start from same random seed!): $\rho = \text{Pearson}(C, F) \sim 0.8 - 1.$

$$C' = C - \beta(F - \langle F \rangle)$$

$$\text{var}[C'] = \text{var}[C] - 2\beta\text{cov}[C, F] + \beta^2\text{var}[F]$$

$$\text{if } \beta = \frac{\text{cov}[F, C]}{\text{var}(F)} \text{ then } \frac{\text{var}(C')}{\text{var}(C)} = 1 - \rho$$

Covariance with holes in the mask

Method

- Motivation: analytical approximations of the covariance matrix fail when including sources in the computation. A solution could be to use simulations to obtain the covariance. But what if you update your point source mask ?
- Pipeline: $(T, Q, U) \begin{array}{l} \xrightarrow{W^{3600}} C_\ell \\ \xrightarrow{W^{3100}} F_\ell \end{array}$
- Surrogate: F_ℓ because **we have already computed the covariance**
- Simulations: C_ℓ because **we want to estimate the covariance (with a few simulations only!)**

Covariance with holes in the mask

Method

- <u>Motivation</u>: analytical approximations of the covariance matrix fail when including sources in the computation. A solution could be to use simulations to obtain the covariance. But what if you update your point source mask ?
- Pipeline: $(T, Q, U) \xrightarrow{W^{3600}} C_\ell$, $\xrightarrow{W^{3100}} F_\ell$ Obtained on a mask with 3100 holes
- Surrogate: F_ℓ because **we have already computed the covariance**
- Simulations: C_ℓ because **we want to estimate the covariance (with a few simulations only!)**

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Method

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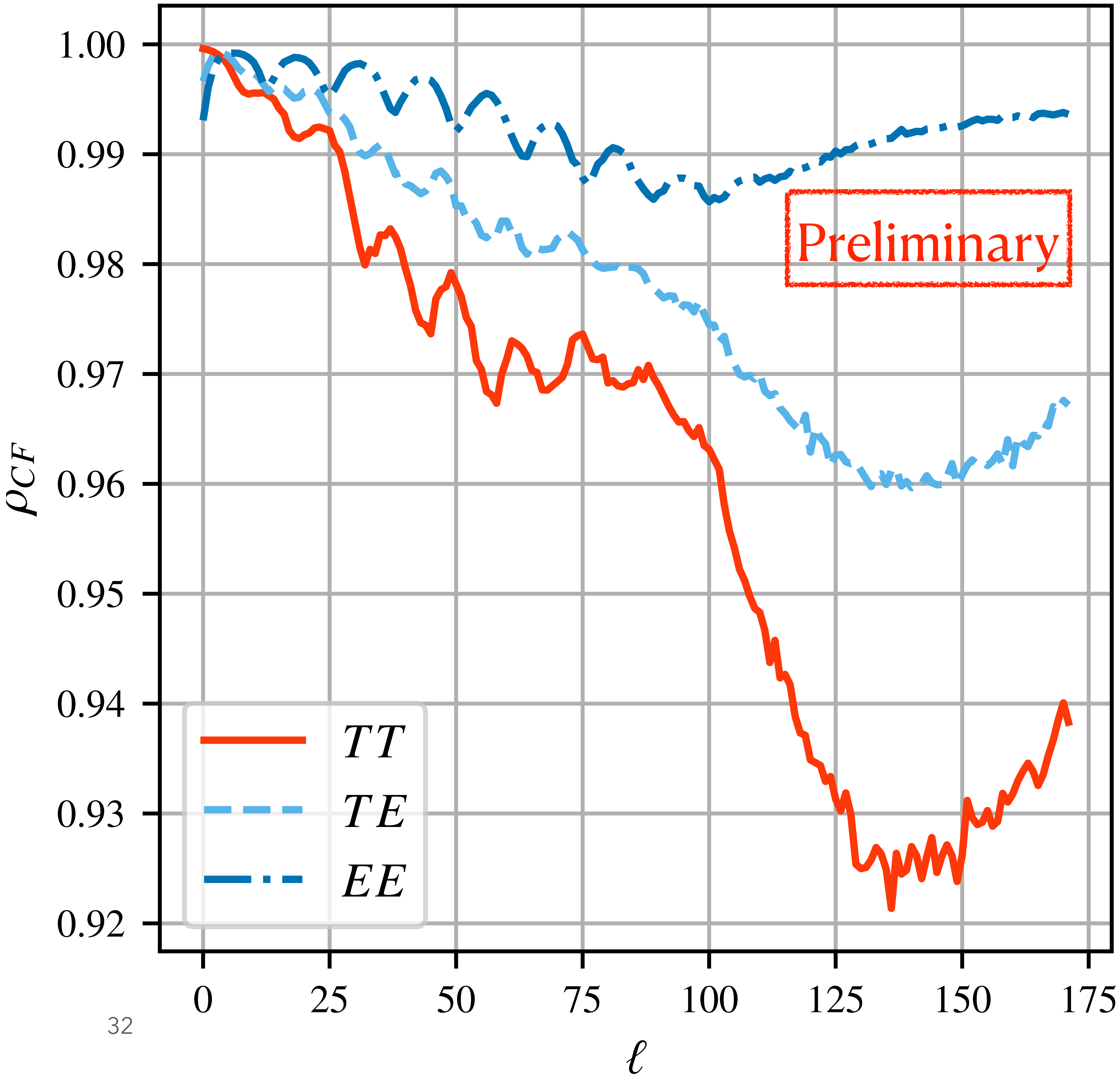
- Pipeline: $(T, Q, U) \xrightarrow{W^{3600}} C_\ell$ Obtained on a mask with 3600 holes

  $(T, Q, U) \xrightarrow{W^{3100}} F_\ell$ Obtained on a mask with 3100 holes
- Surrogate: F_ℓ because **we have already computed the covariance**
- Simulations: C_ℓ because **we want to estimate the covariance (with a few simulations only!)**

Covariance with holes in the mask

Correlation

- Very good correlation between simulation/surrogate!

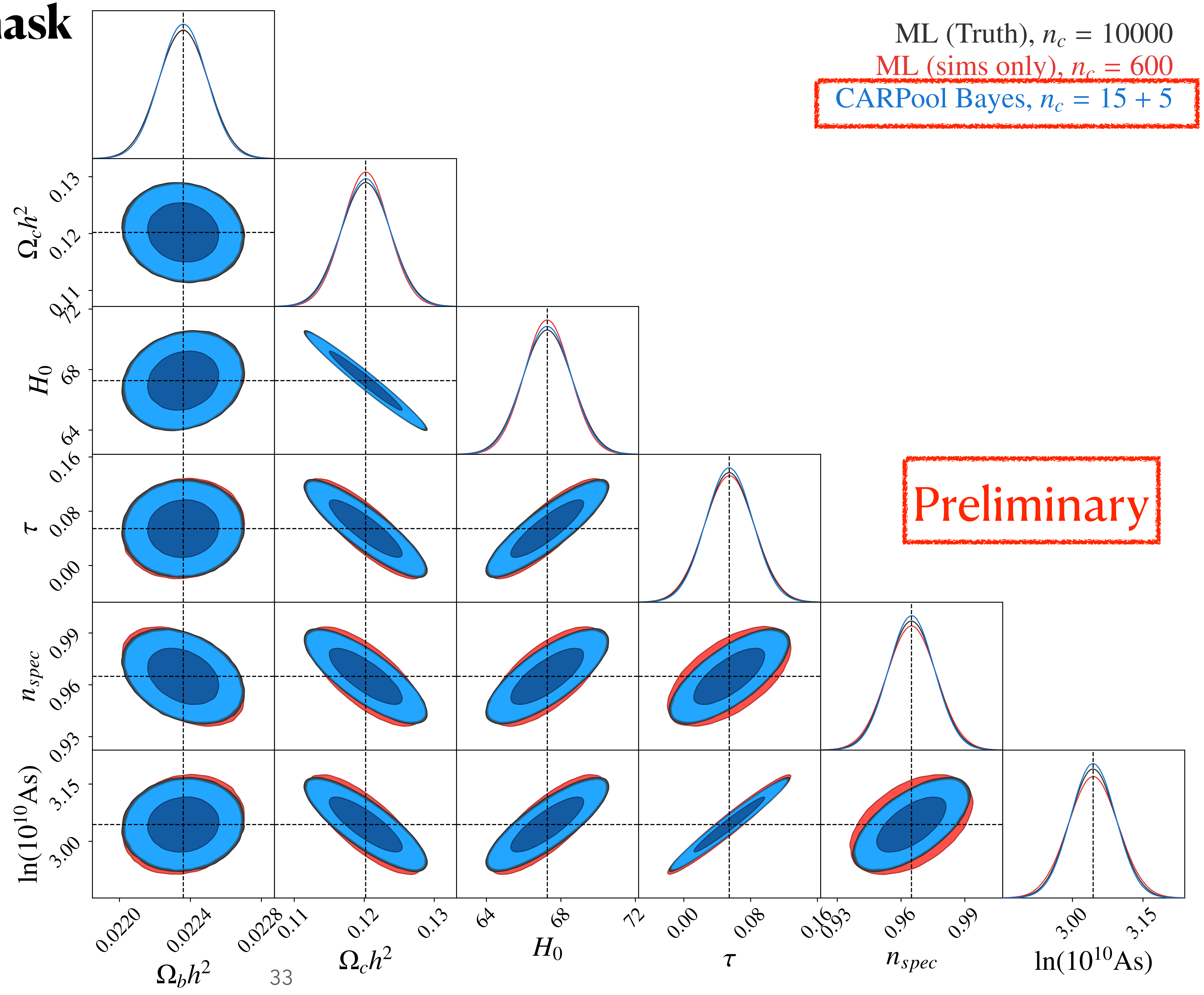


Covariance with holes in the mask

Results

Fisher plots with 3 covariances:

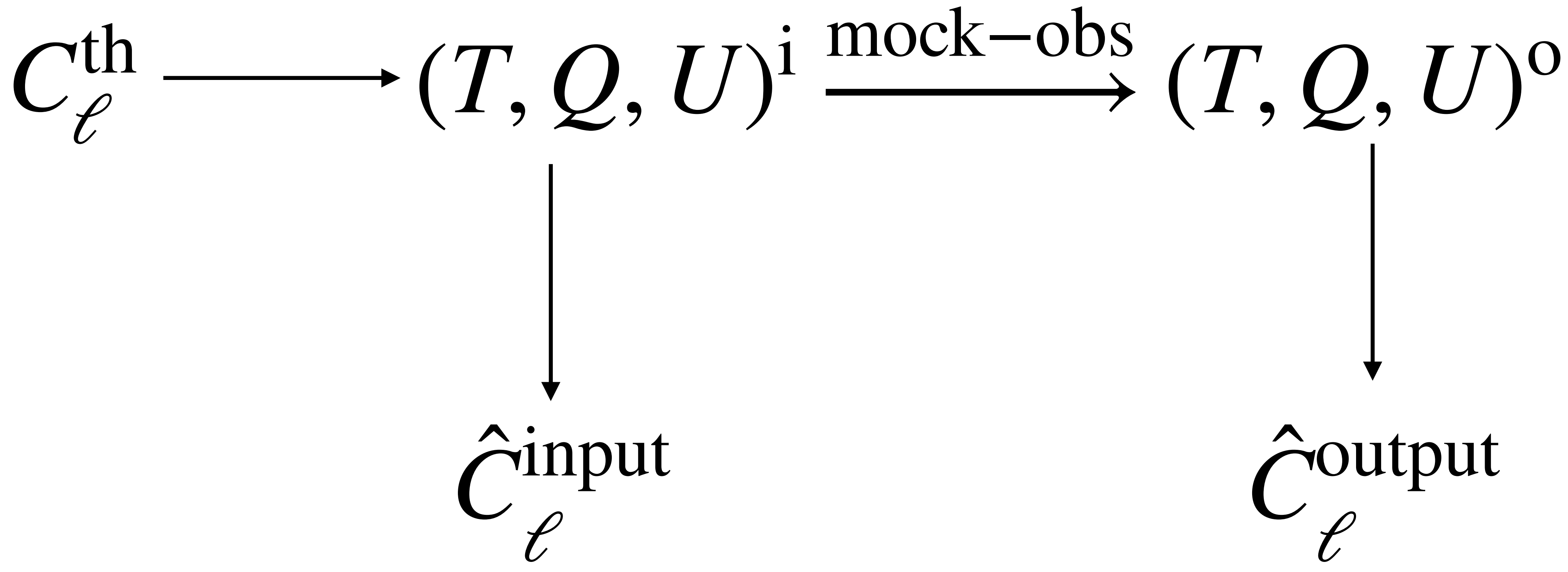
- ML: using 10 000 simulations, considered as « truth »
- ML 600: same but with only 600 simulations
- CarPool Bayes: using Carpool, and only 15+5 simulations (training + testing)



Carpool for mock-observations ?

Applying this technique to our mock-observations pipeline

- Motivation: we want to use mock-observations to compute the transfer functions or to validate our analytical computation.



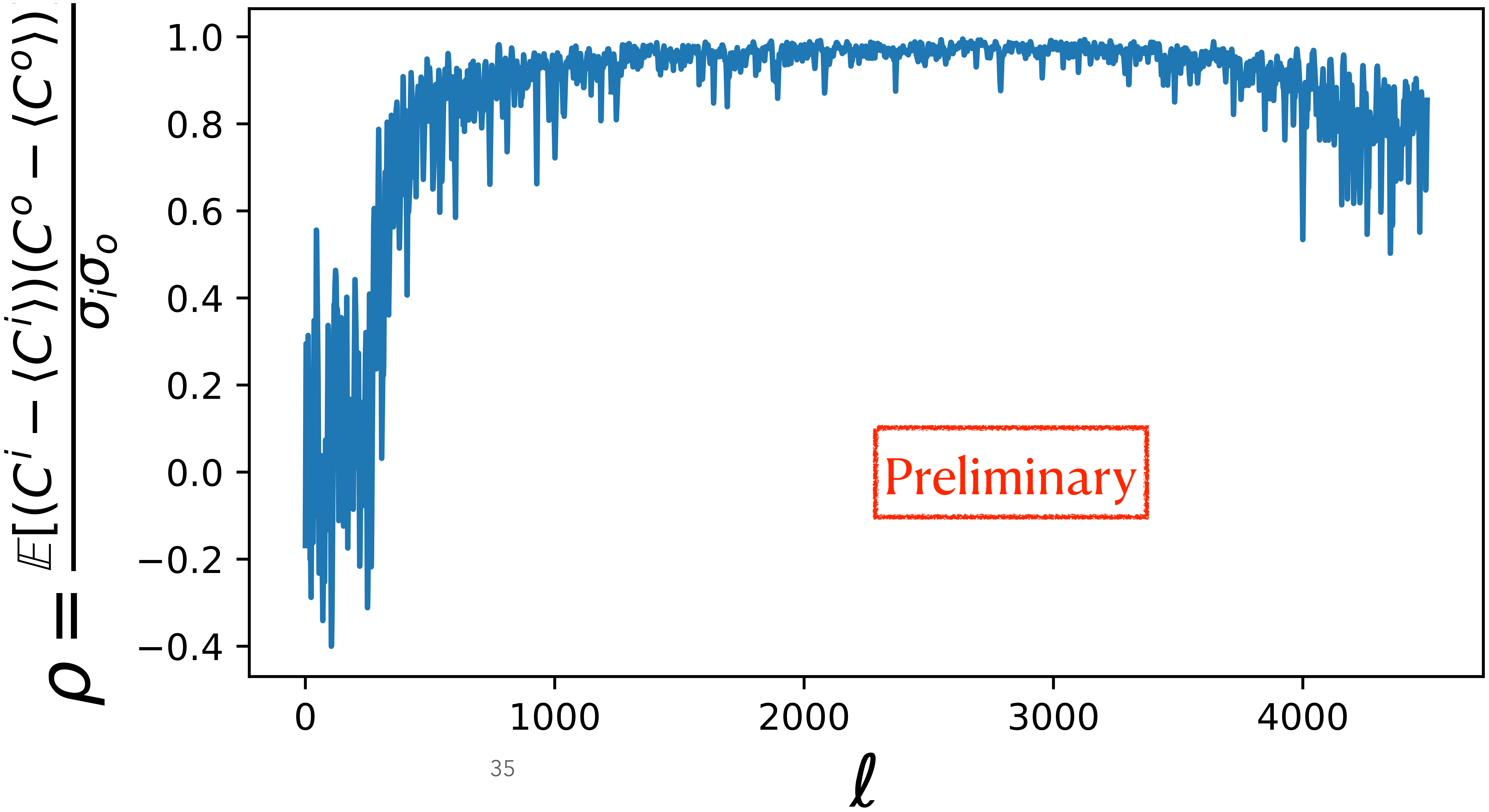
- Surrogate: $F_\ell \equiv \hat{C}_\ell^{\mathrm{input}}$ because **we know precisely the covariance (only CMB !)**
- Simulations: $C_\ell \equiv \hat{C}_\ell^{\mathrm{output}}$ because **we want to estimate the covariance (which includes filtering)**

Carpool for mock-observations ?

Applying this technique to our mock-observations pipeline

- Very high correlations between features of input vs features of output.
- This could help us to do « effectively » 10 or 100 times more simulations

Pearson correlation coefficient of input vs output



Credits: Aman Choski

Conclusions

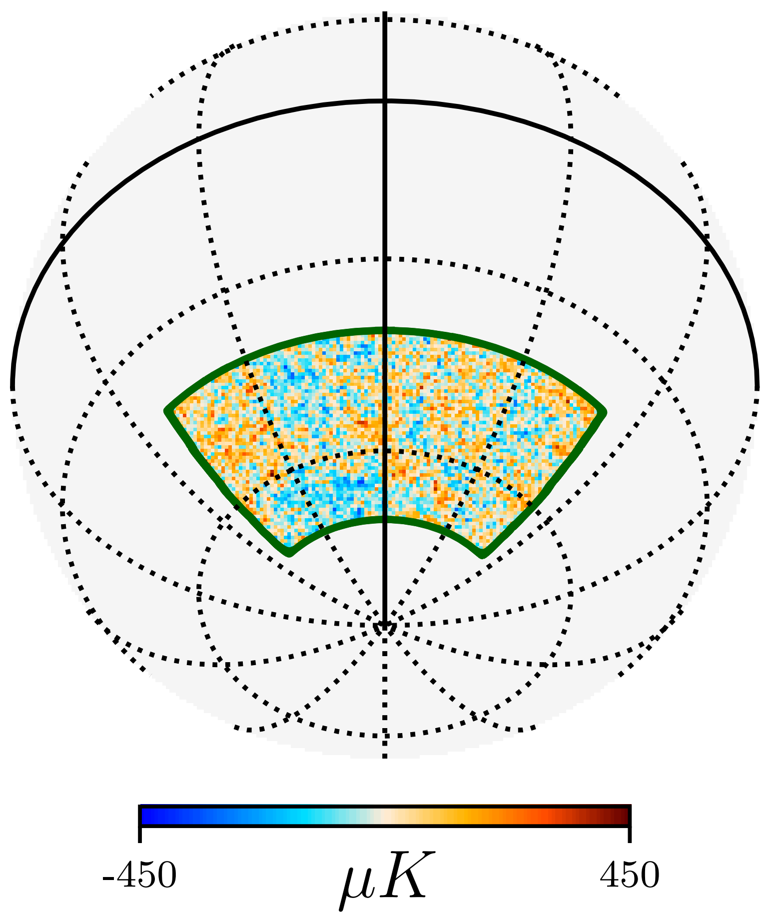
- SPT-3G will allow us to do great science.
- We have been working on improving the likelihood pipeline by building an accurate covariance matrix.
- We are currently working on approaches to improve the pipeline, with promising results.

Credits: Aman Choski

General method

Power spectrum based gaussian likelihood

Masked CMB maps



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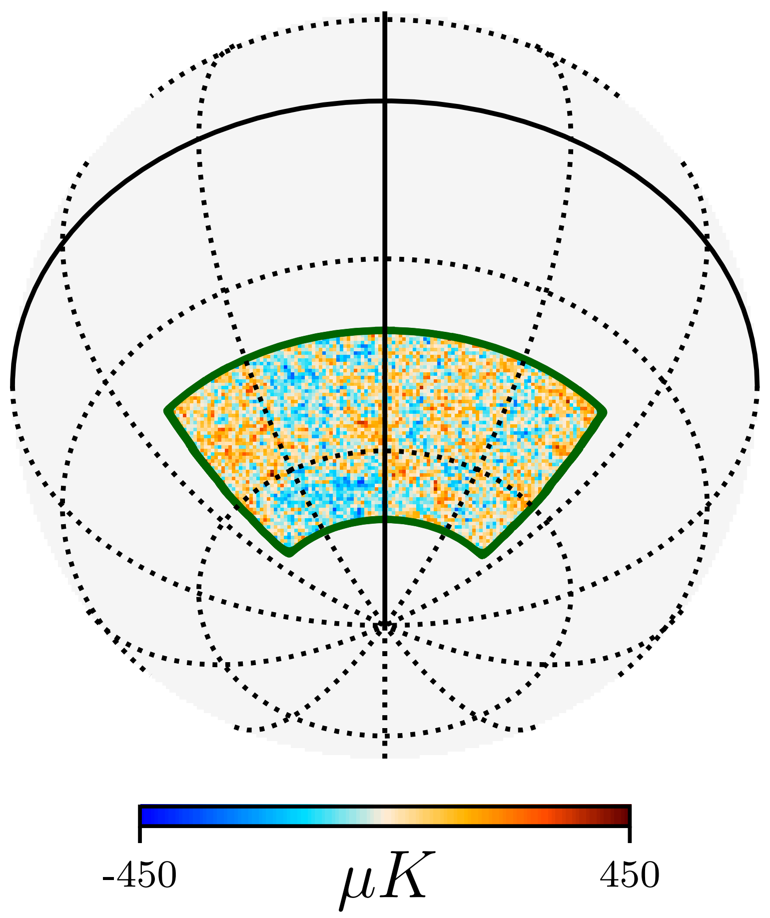

$$\hat{C}_\ell$$

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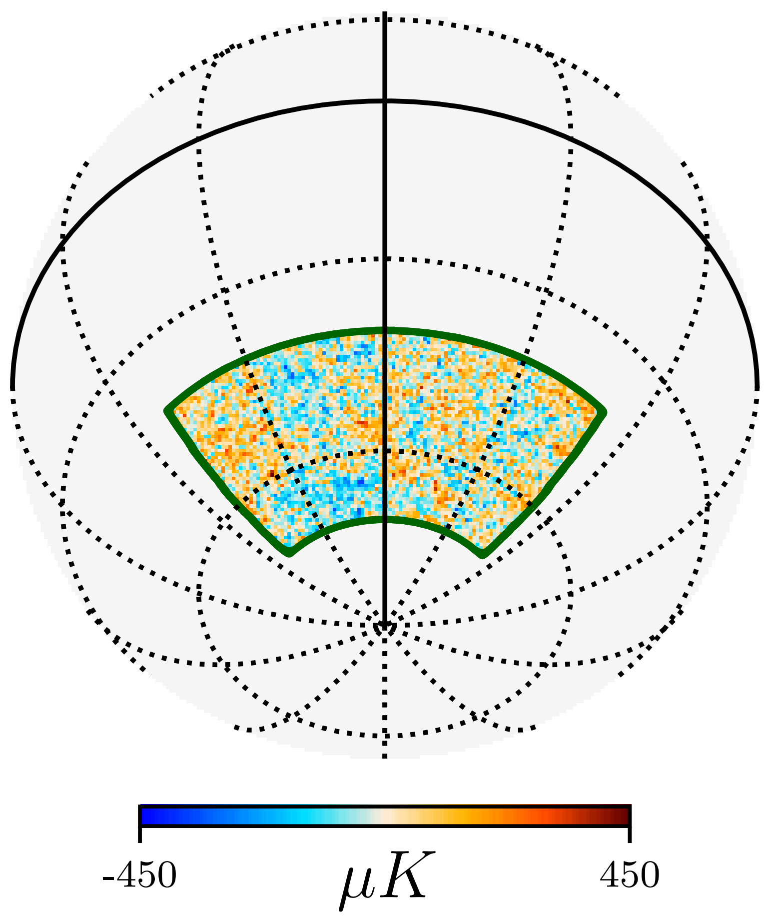

$\hat{C}_\ell$

C_ℓ^{th}

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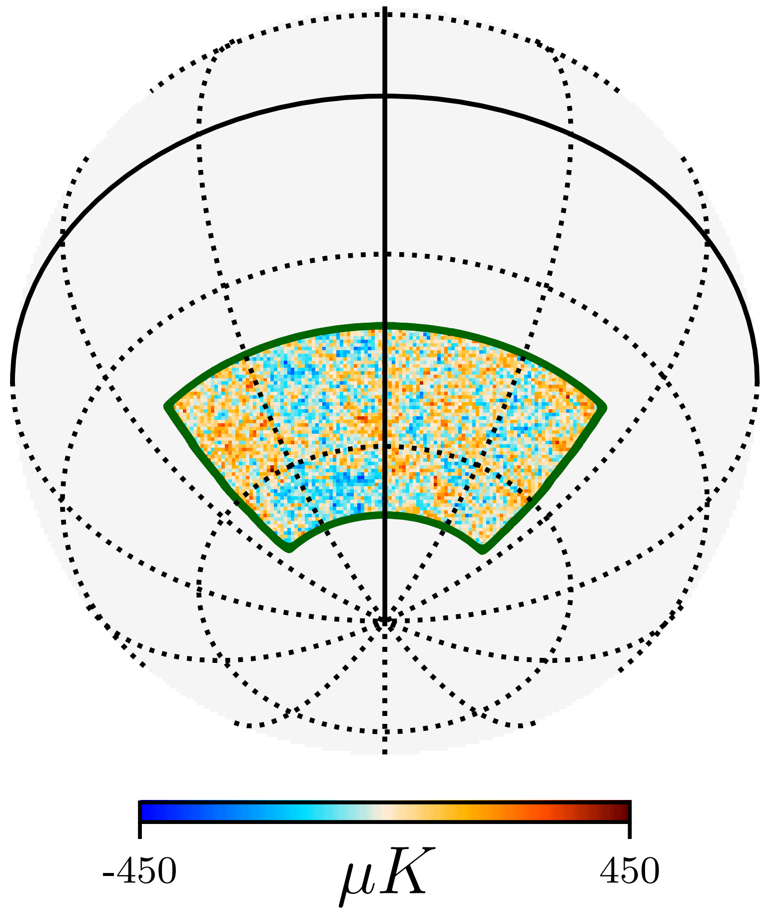
Power spectrum based gaussian likelihood

Masked CMB maps



$$\hat{C}_\ell$$

Gaussian likelihood :

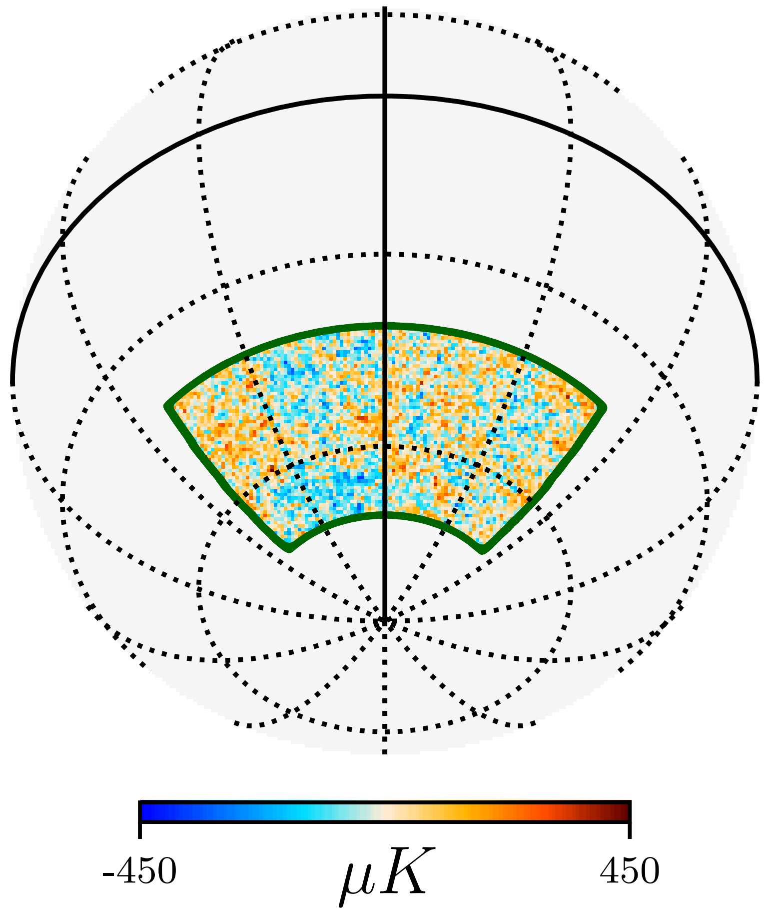
$$-\ln \mathscr{L}(\hat{C} \,|\, \Lambda\mathrm{CDM}) \propto \frac{1}{2}(\hat{C} - C^{\mathrm{th}})^T \Sigma^{-1} (\hat{C} - C^{\mathrm{th}})$$

$$C_\ell^{\mathrm{th}}$$

General method

Power spectrum based gaussian likelihood

Masked CMB maps



$\hat{C}_\ell$

Gaussian likelihood :

$$-\ln \mathscr{L}(\hat{C}\,|\,\Lambda\mathrm{CDM}) \propto \frac{1}{2}(\hat{C} - C^{\mathrm{th}})^T \Sigma^{-1} (\hat{C} - C^{\mathrm{th}})$$

Parameters

$$\begin{pmatrix} \Omega_c \\ \Omega_b \\ A_s e^{-2\tau} \\ n_s \\ \theta_{MC} \end{pmatrix}$$

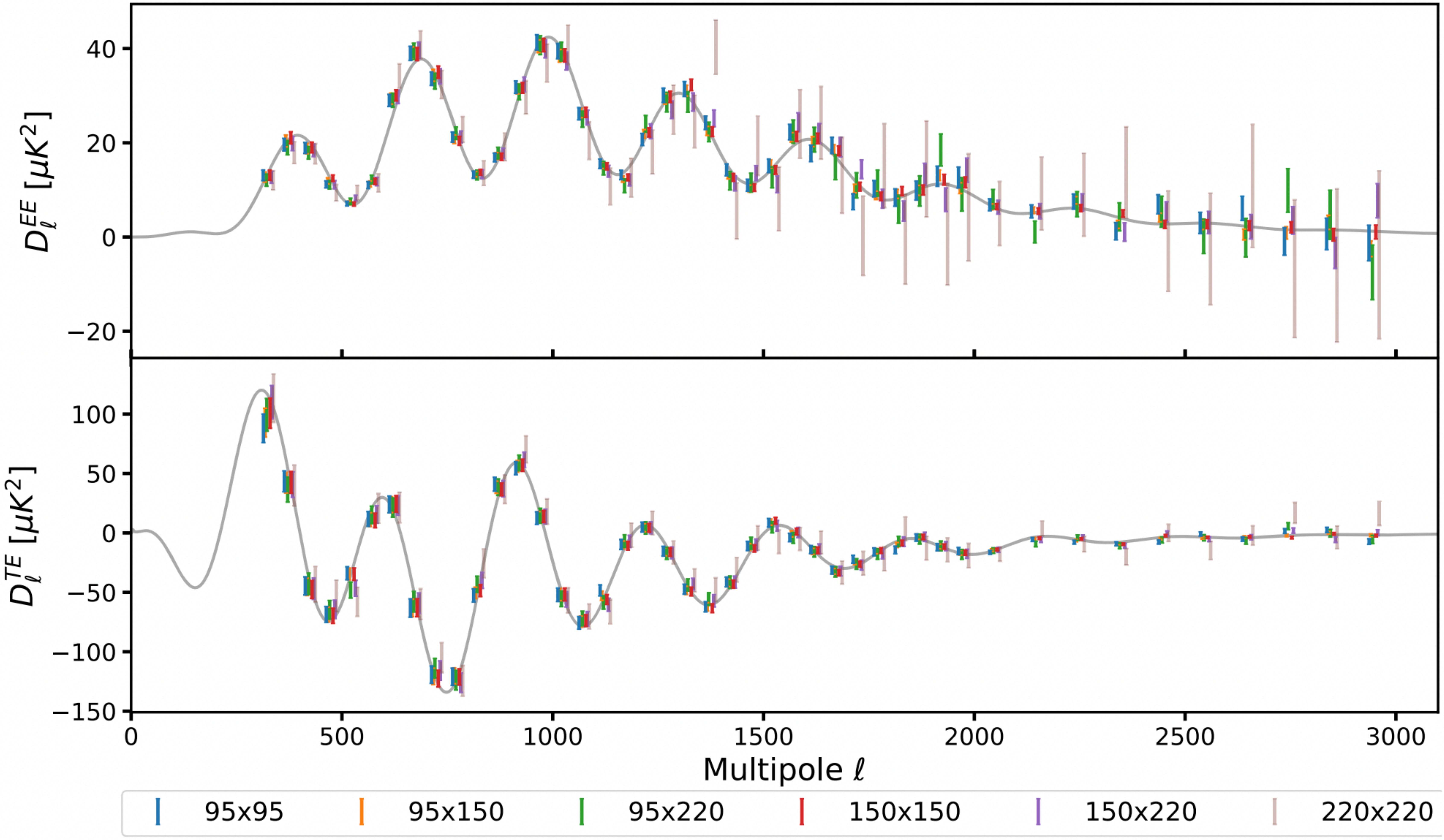
Explore with MCMC

C_ℓ^{th}

We explore the likelihood, in order to measure the parameters and obtain their posterior distributions

SPT-3G 2018

Power spectra

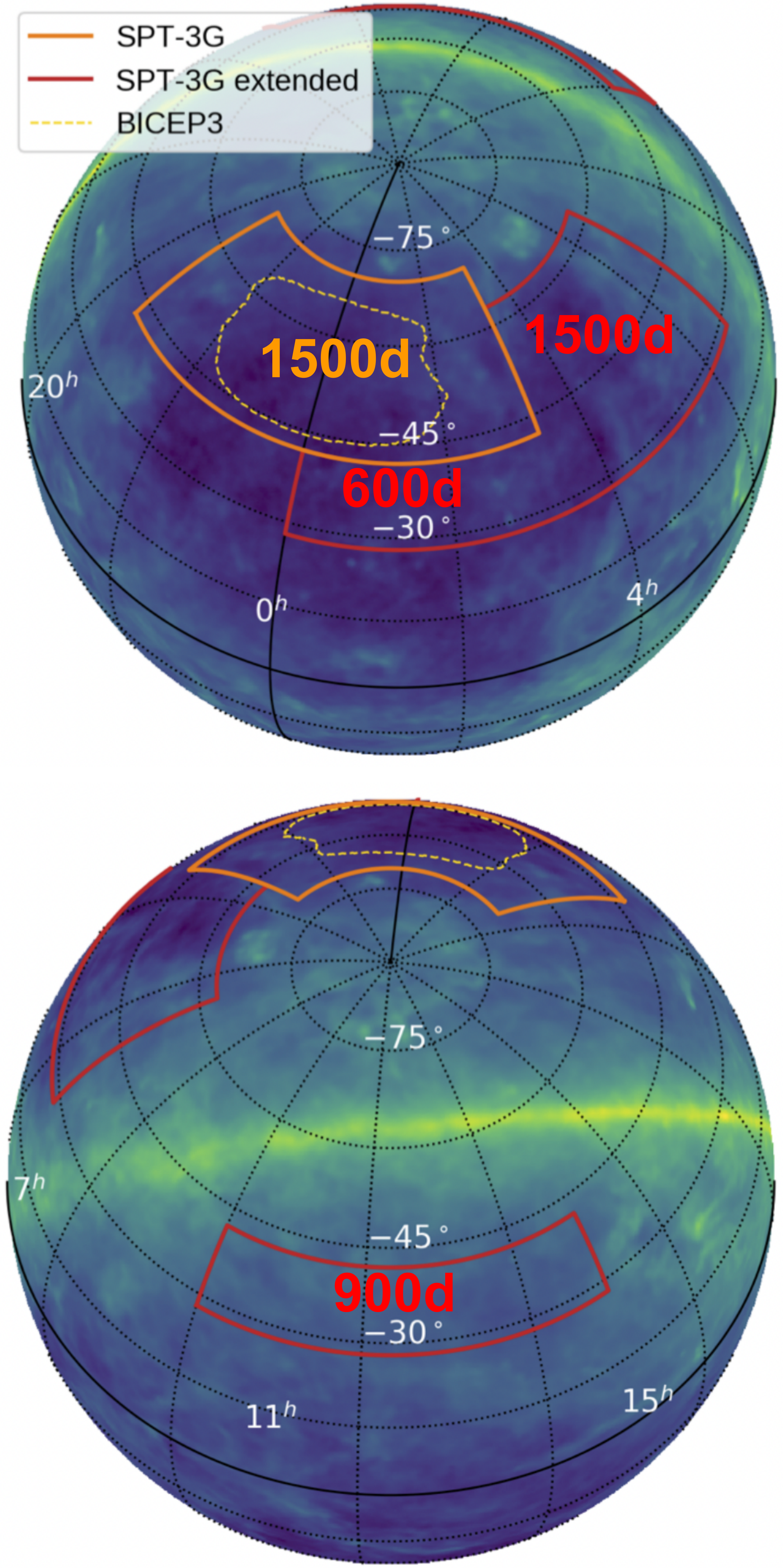


Auto/cross-frequencies spectra [Dutcher et al. 2021]

SPT-3G - Summer fields

Prospects - work by Federica Guidi

- In addition to the winter fields:
  3000 deg2 = 1500 (3.1%) + 600 (1.4%) + 900 (2.1%)
- Observing ~4 months per year
- Noise levels for summer 19/20 + 20/21:
  ~ 11, 10, 38 μK-arcmin (T)
  ~ 16, 14, 54 μK-arcmin (pol)
- Map depth of 2 years of summer observations is
- ~1.4 times better than the 2018 winter field maps
- ~2.5 times worse than the 2019+2010 winter fields
- 3 times larger sky fraction than winter
  → reduce sample variance



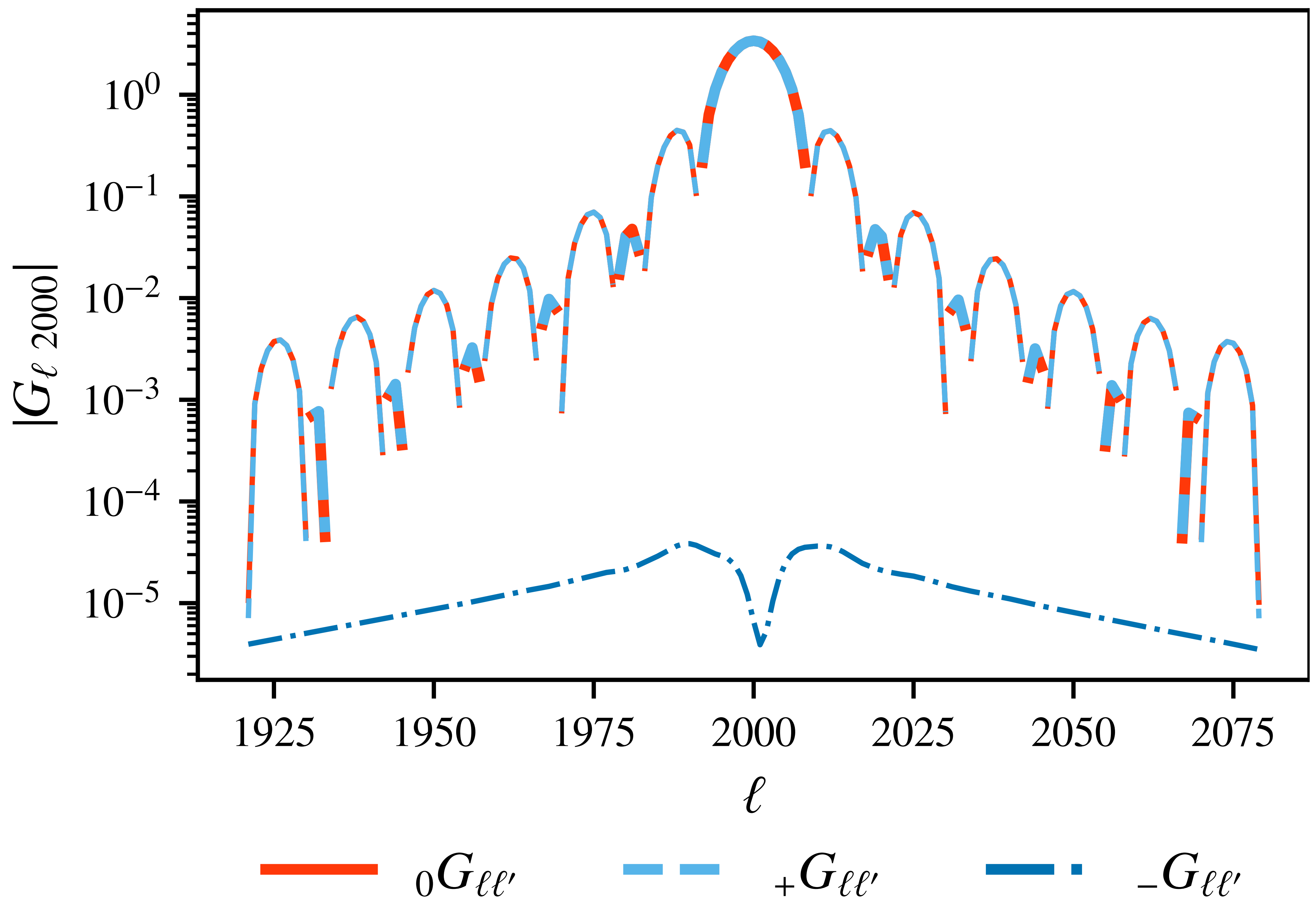
Slide by F. Guidi

PolSpice

- For this analysis we will use `PolSpice` estimator $\hat{C}_\ell$ [Szapudi et al. 2001] [Chon et al. 2004]
- $\hat{C}_\ell^{TT} = \sum_{\ell'} {}_0G_{\ell\ell'}\tilde{C}_{\ell'}^{TT}$
- and little more sophisticated for polarization, with kernels ${}_+G$, ${}_-G$, ${}_\times G$
- Accuracy of approximations extend to that case

PolSpice convoluting kernels

Rows for $\ell' = 2000$



In thick lines are positive values, narrow lines are negative values

Exact covariance

Algorithm

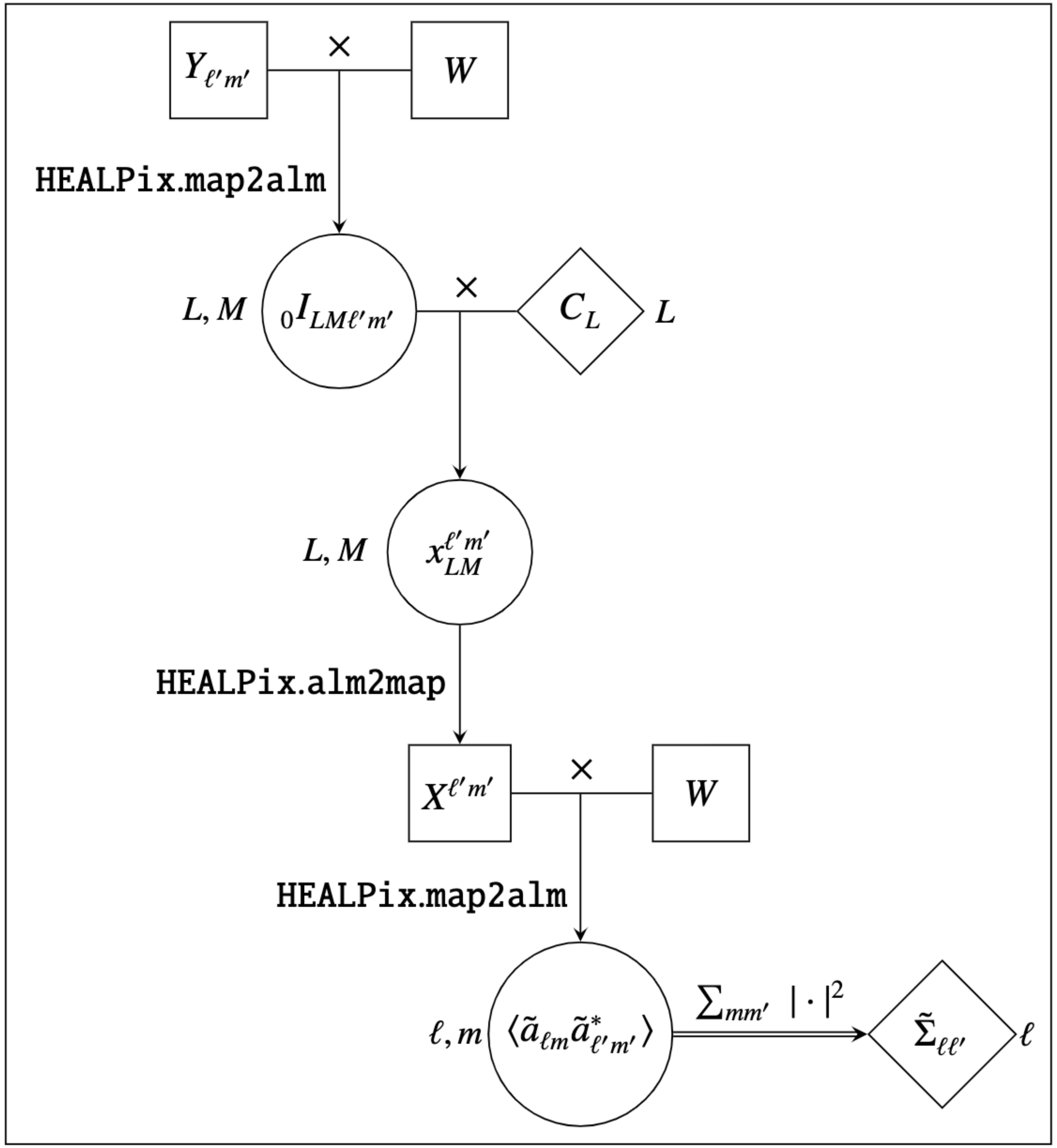


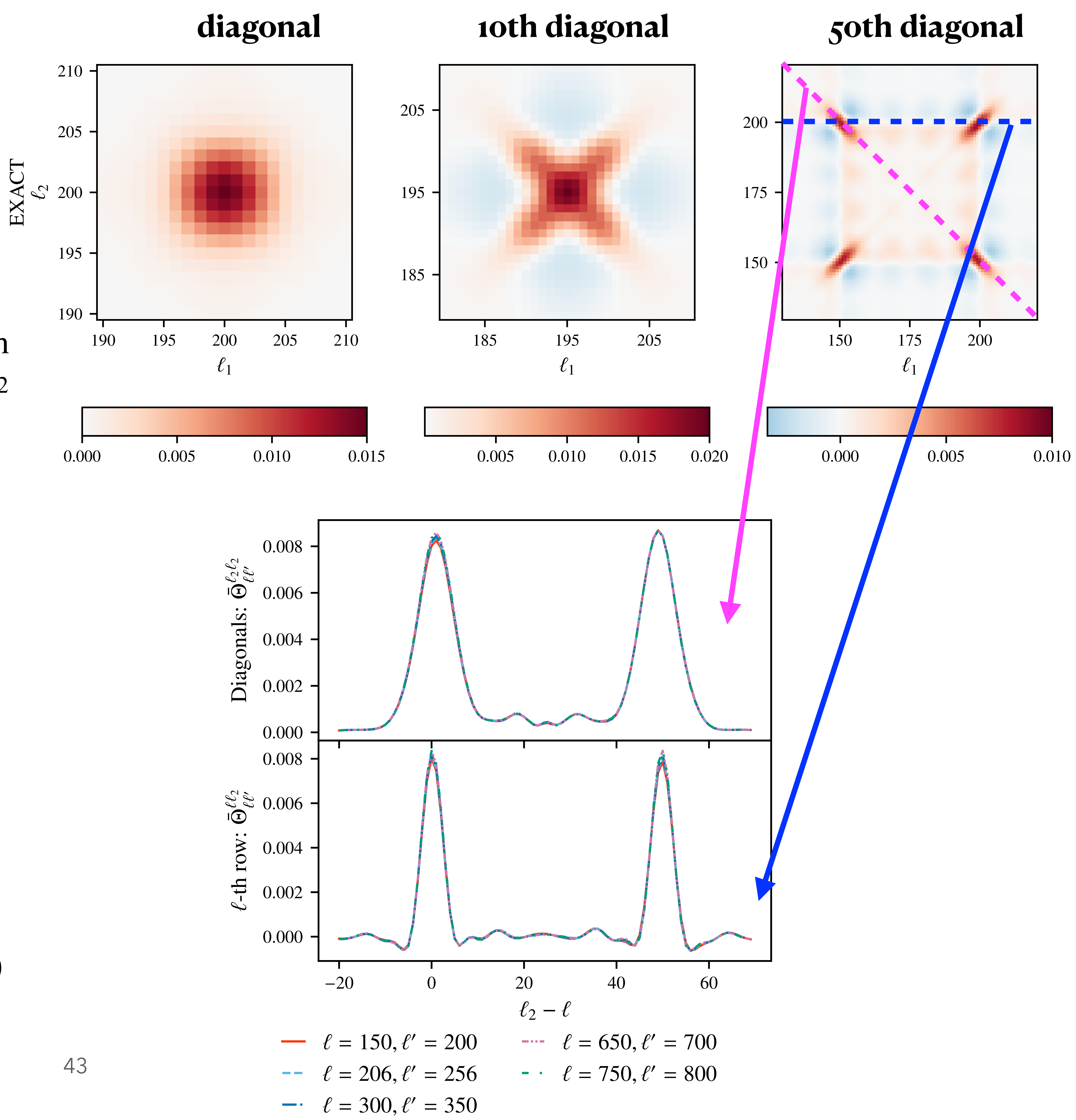
Fig. 1: Diagram showing how to compute one rank of the covariance using `HEALPix` tools. Square boxes (resp. diamond boxes,

ACC

New approximation

$$\mathrm{Cov}(C_\ell, C_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1}\ \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W]\ C^{\mathrm{th}}_{\ell_2}$$

- **ACC (approximated covariance coupling)**
- Scales in $\mathcal{O}(d_{\max} n_{\mathrm{side}}^4)$
- $d_{\max}$: maximum diagonal that you want to compute
- n_{side} : map resolution used to compute $\bar{\Theta}$

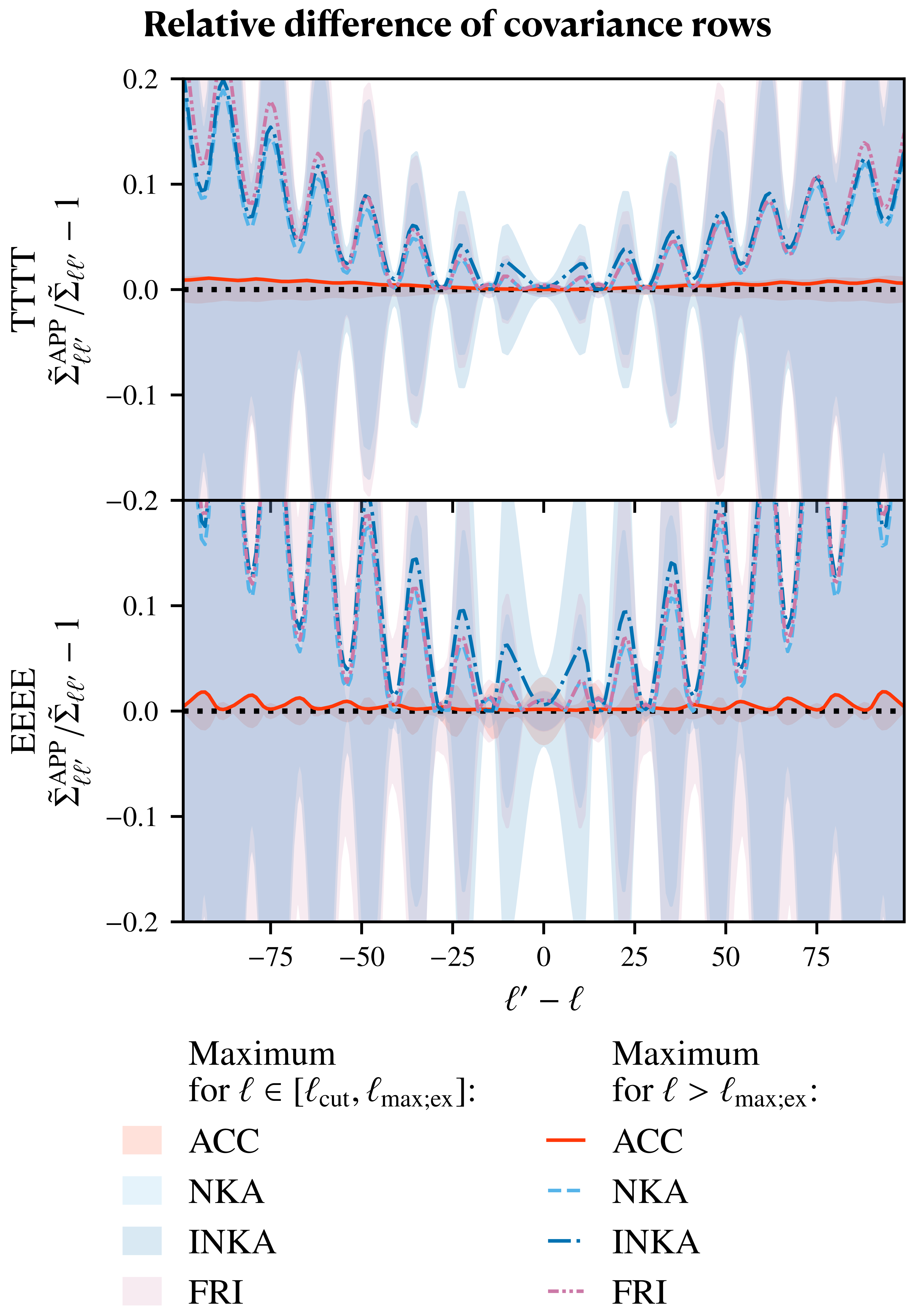


Results

Higher multipoles ?

- We look at the relative difference of rows of the covariance centered on the diagonal

- I add to the previous plot the result for larger rows but among a sparse number of them



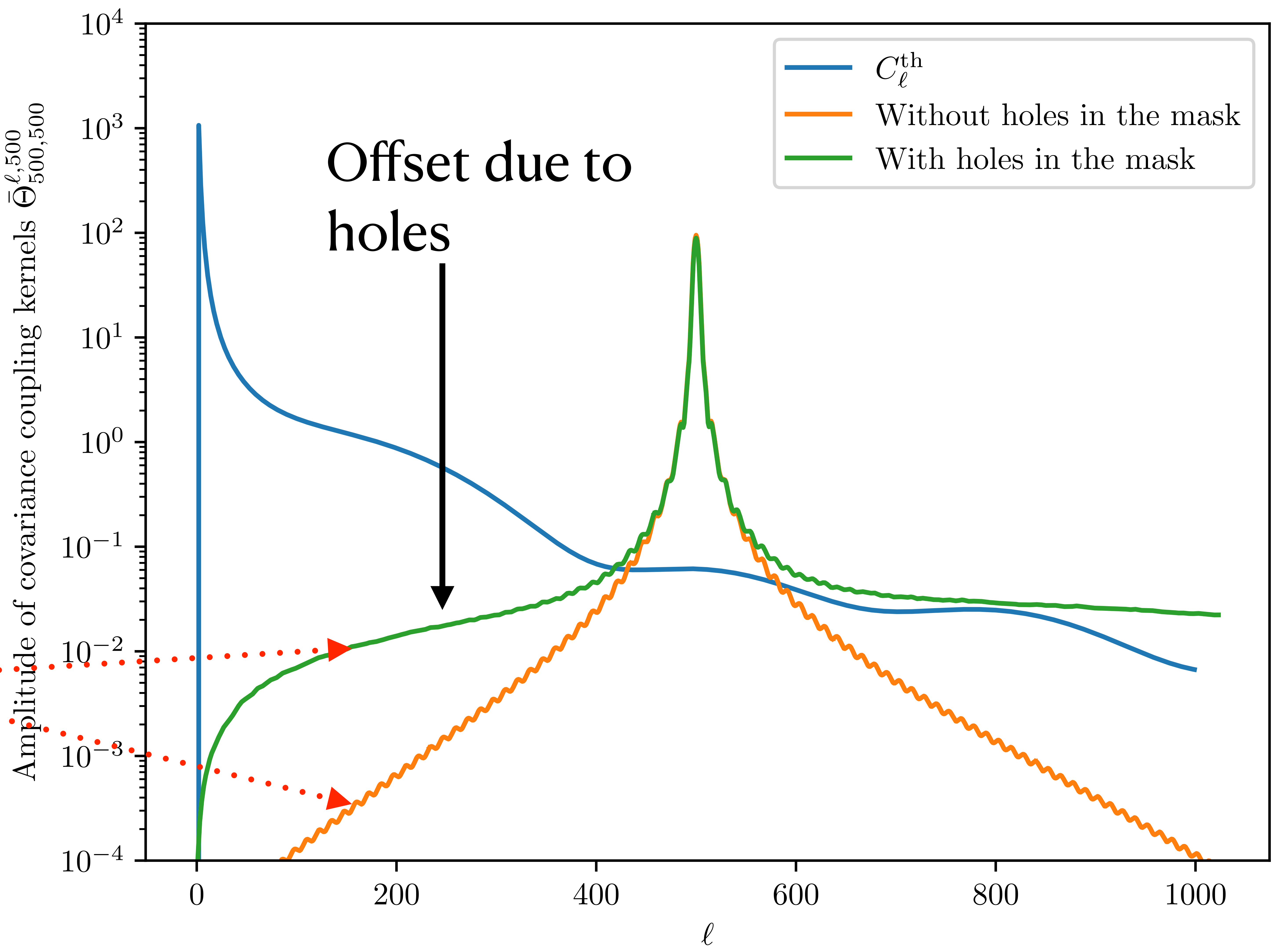
Caveat

Why ?

- The holes in the mask gives $\bar{\Theta}$ an offset
- This will be convoluted with the CMB power spectrum

$$\mathrm{Cov}(\tilde{C}_\ell, \tilde{C}_{\ell'}) = 2\Xi_{\ell\ell'}[W^2] \sum_{\ell_1\ell_2} C^{\mathrm{th}}_{\ell_1} \bar{\Theta}^{\ell_1\ell_2}_{\ell\ell'}[W] \; C^{\mathrm{th}}_{\ell_2}$$

- (In the plot, $\bar{\Theta}$s have been renormalized)



Inpainting

Final bias on the spectrum

- $C_\ell \rightarrow T, Q, U \xrightarrow{W} \hat{C}_\ell^{\text{bare}}$
- $T, Q, U \xrightarrow{\text{Inpainting}} [T, Q, U]^{\text{filled}} \xrightarrow{W} \hat{C}_\ell^{\text{filled}}$
- Here we plot $\frac{\langle \hat{C}_\ell^{\text{filled}} \rangle}{\langle \hat{C}_\ell^{\text{bare}} \rangle} - 1$
- Error bars are $\frac{\sigma(\hat{C}_\ell^{\text{bare}})}{\sqrt{N_{\text{sim}} = 100}}$
- Correction for the covariance ? only f_{sky} ?