High precision modeling of polarized signals: moment expansion method generalized to spin-2 fields

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Vacher et al, arXiv:2205.01049

astrophysique & planétologie

Spin-moments

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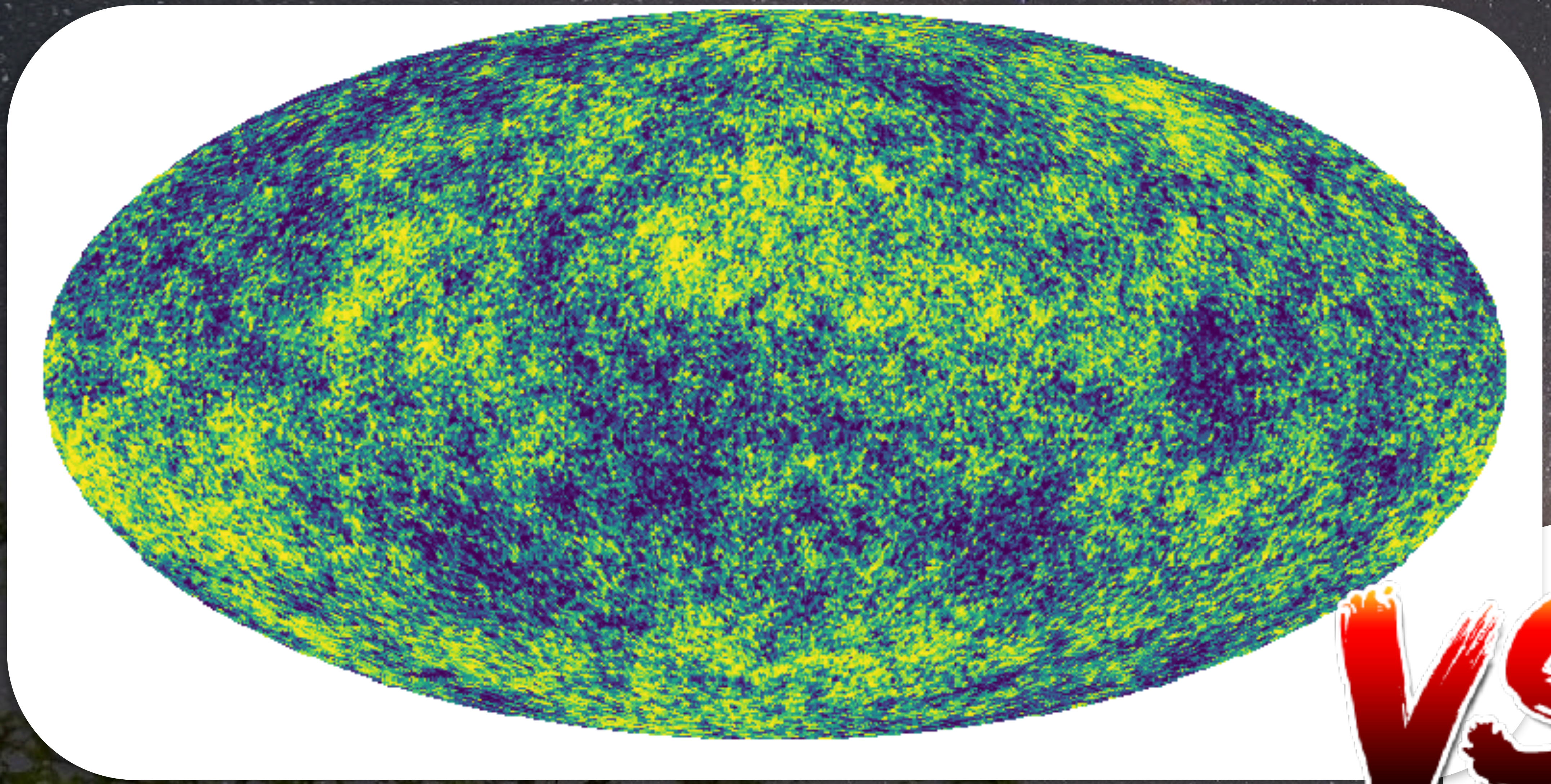
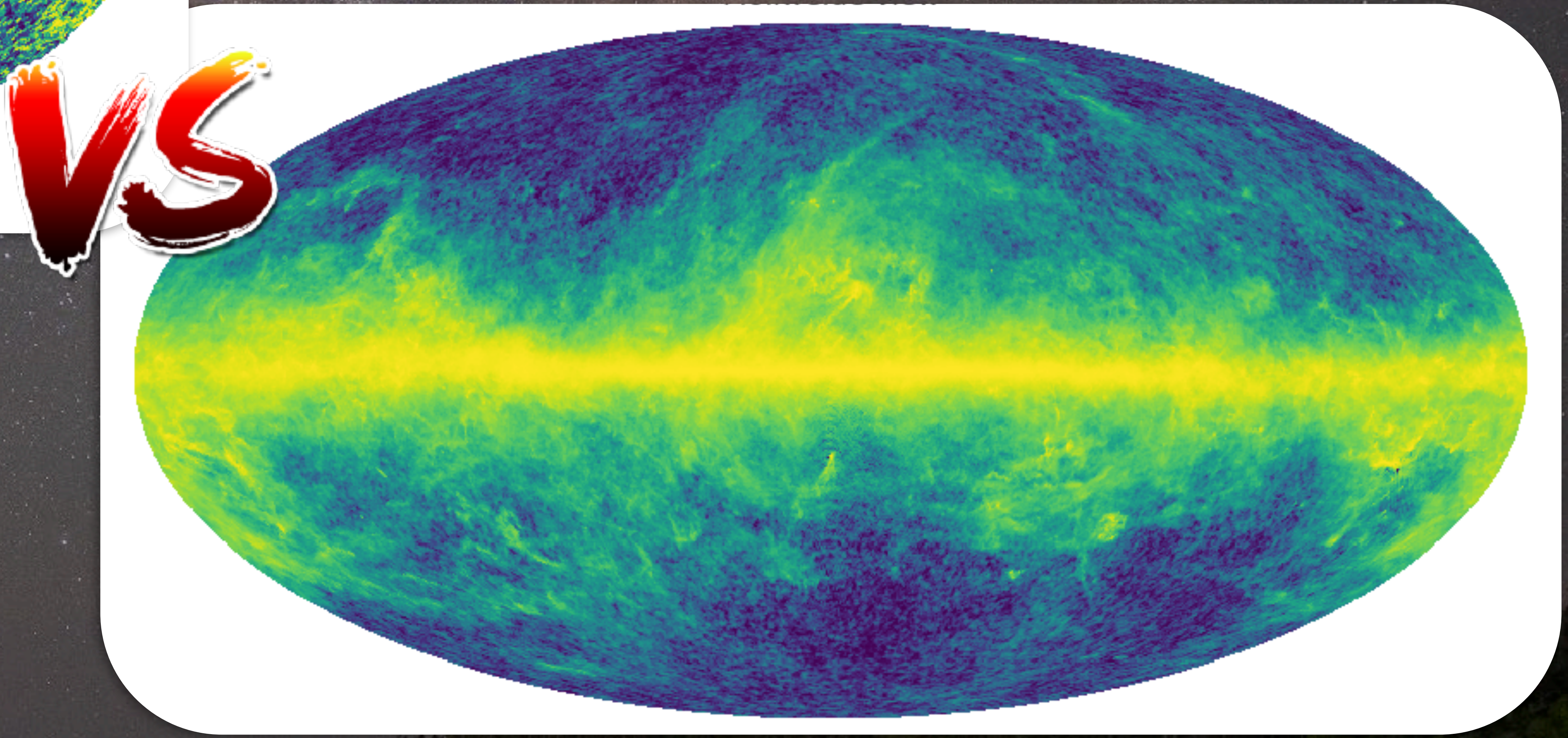
Galactic foregrounds

Multiple polarized astrophysical sources emitting mainly in CMB's wavelength interval





Galactic foregrounds

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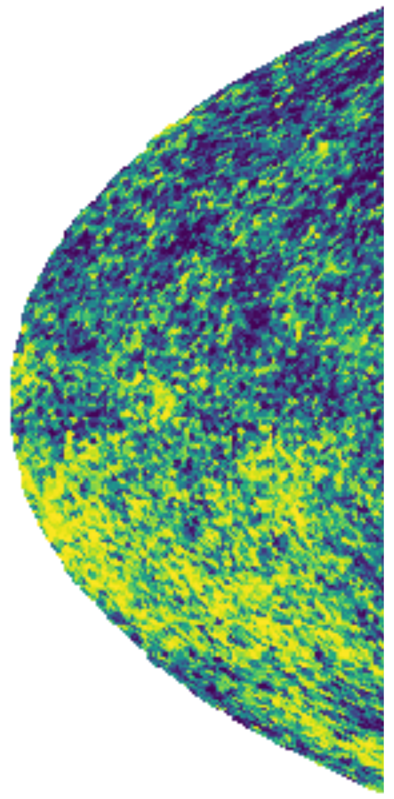
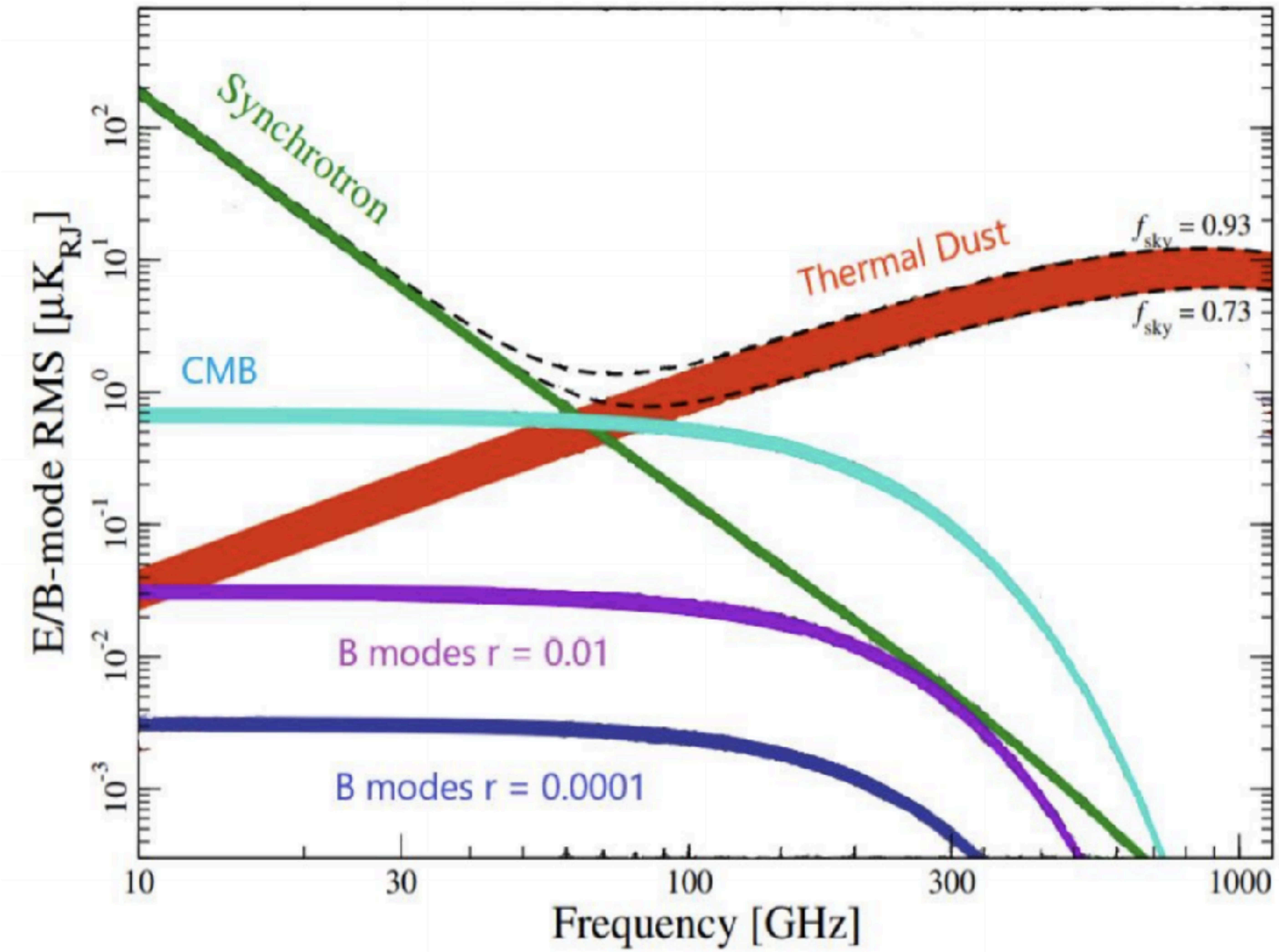
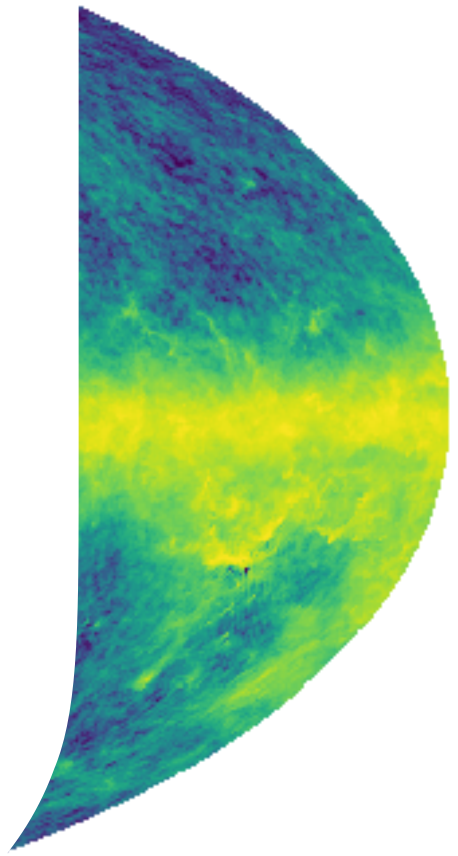






[Planck 2018]

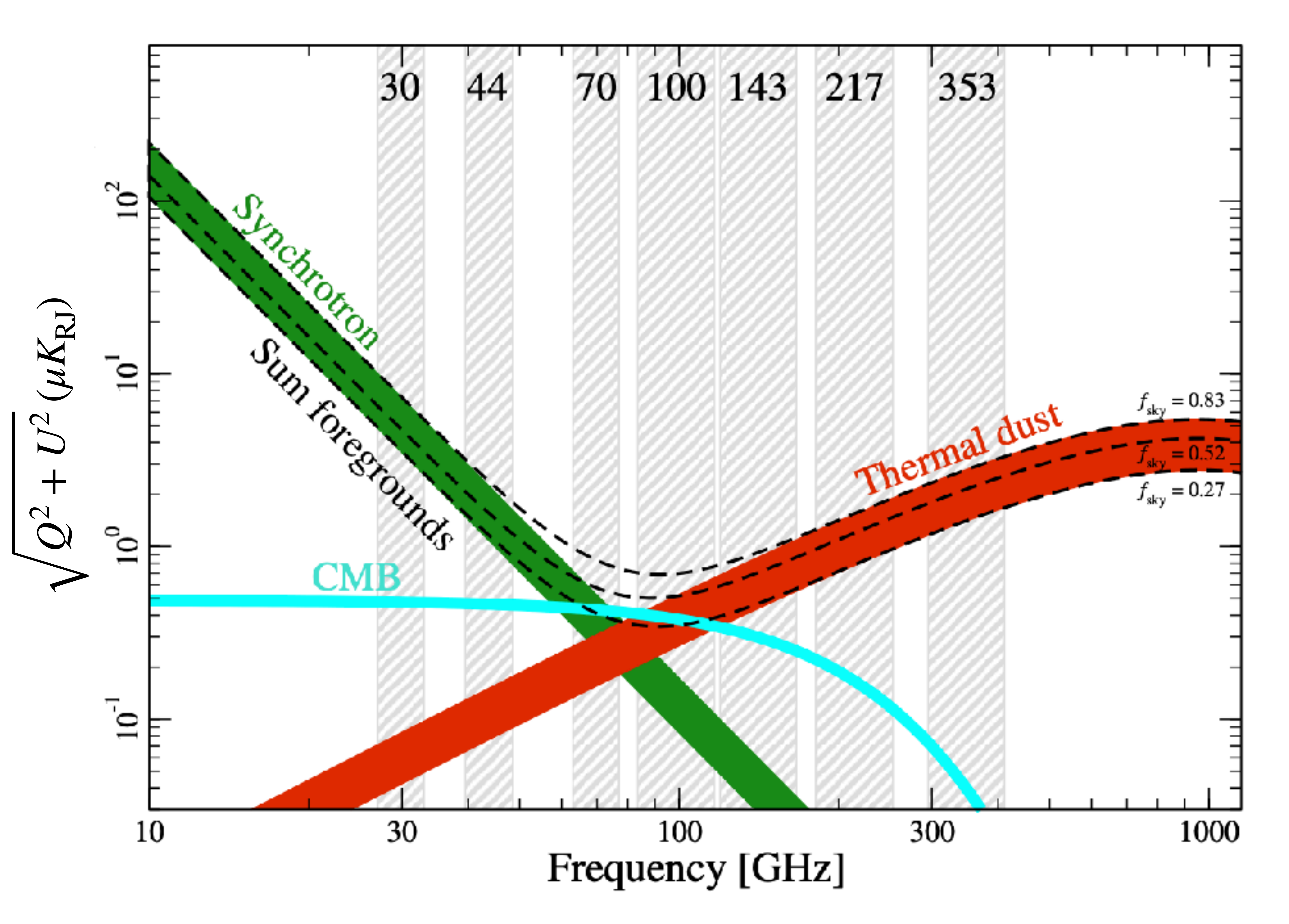
The diffuse polarized components of the ISM in the microwave:

Two main contributions:

- Low frequencies (≤ 100 GHz): Synchrotron radiation

- High frequencies (≥ 100GHz): Thermal dust radiation

+ AME …



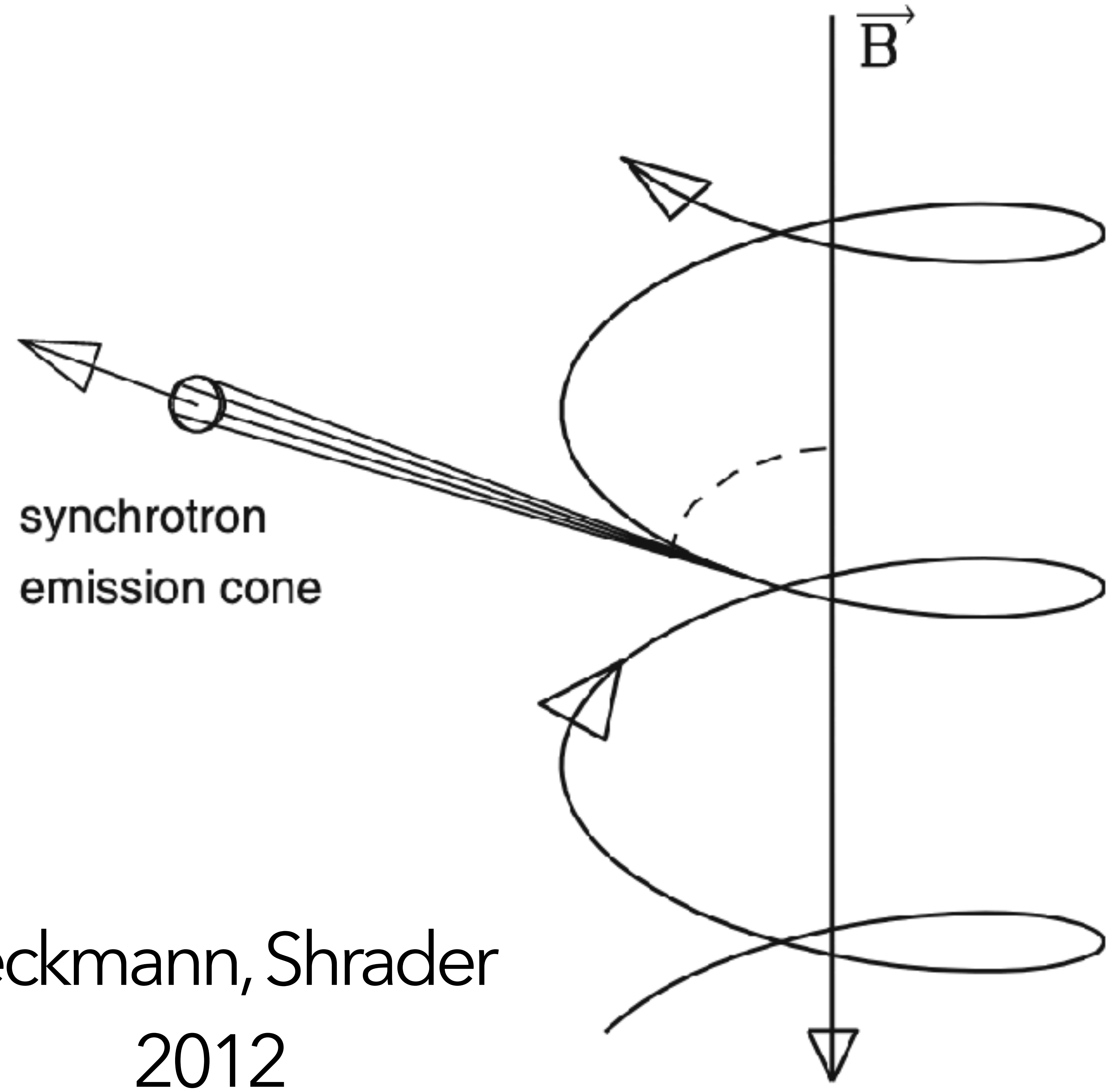
[Planck 2018]

Polarized synchrotron signal

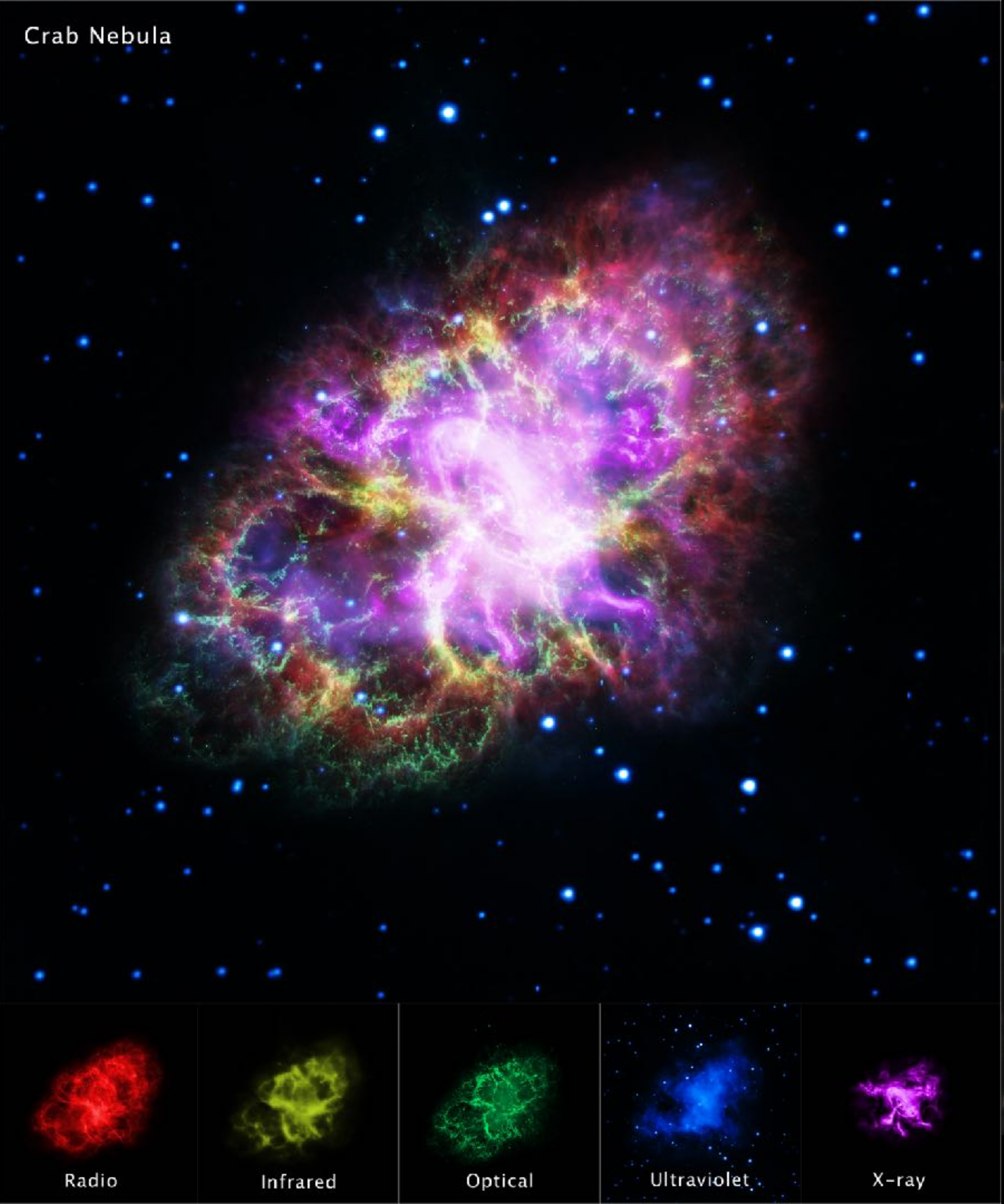
Canonical spectral energy distribution (SED), the **power-law:**

$$I_\nu(\beta_s) = A_s \nu^{\beta_s}$$

With typically $\beta_s \sim -3$



Beckmann, Shrader 2012



M1, the crab nebula

https://apod.nasa.gov

Thermal dust signal

Canonical spectral energy distribution (SED), the **modified black-body:**

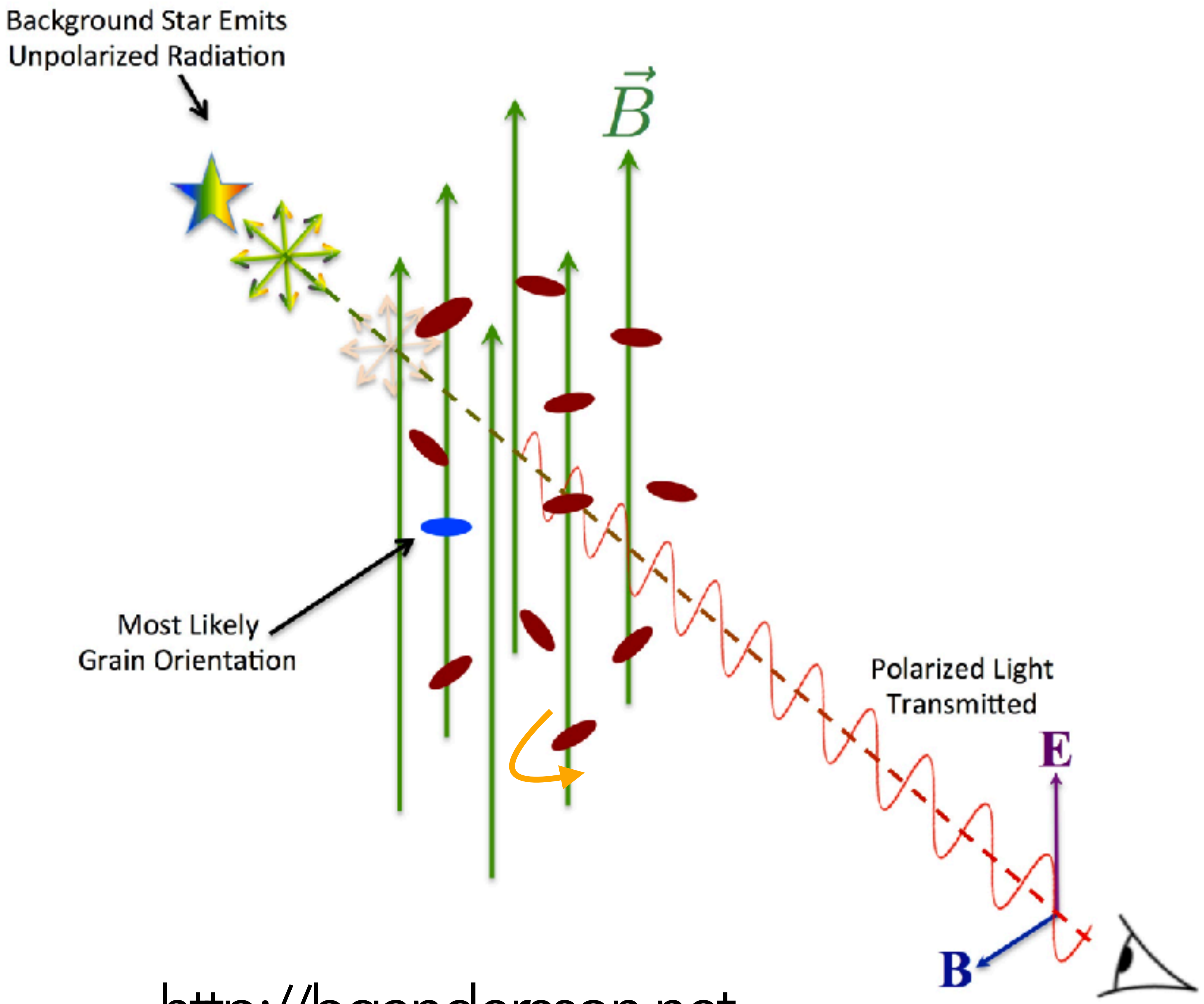
$$I_\nu(\beta_d, T) = A_d \times B_\nu(T_d) \times \nu^{\beta_d}$$

With typically $\beta_d \sim 1.5$ and $T_d \sim 20$ K

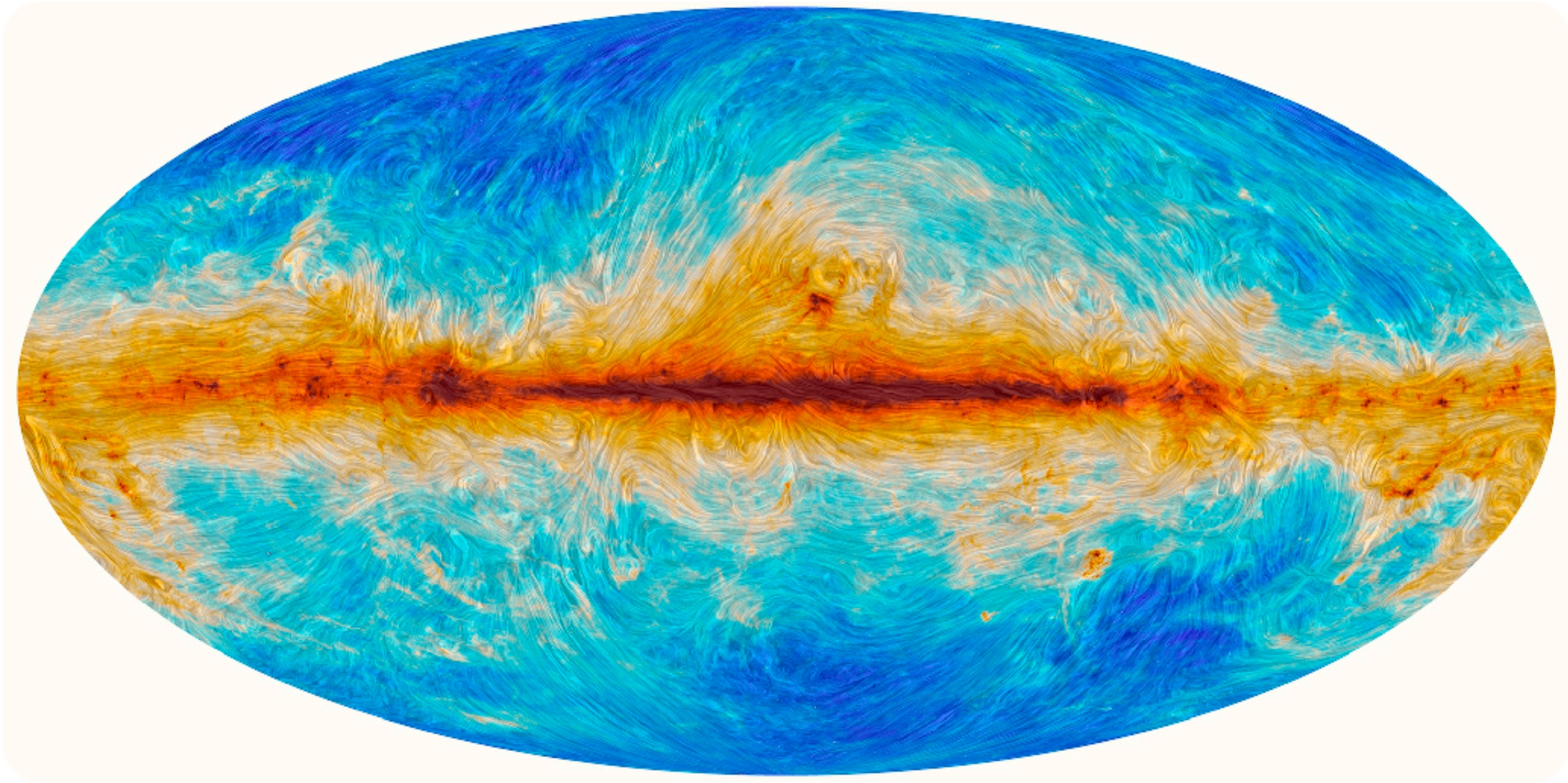
L. Vacher- 2022

[Hubble collaboration]

Thermal dust polarized signal



http://bgandersson.net



[Planck 2018 IV]

M77

How to properly model the diffuse polarized astrophysical signal on large scales?

II -How to describe properly the polarized signal ?

Stokes parameters

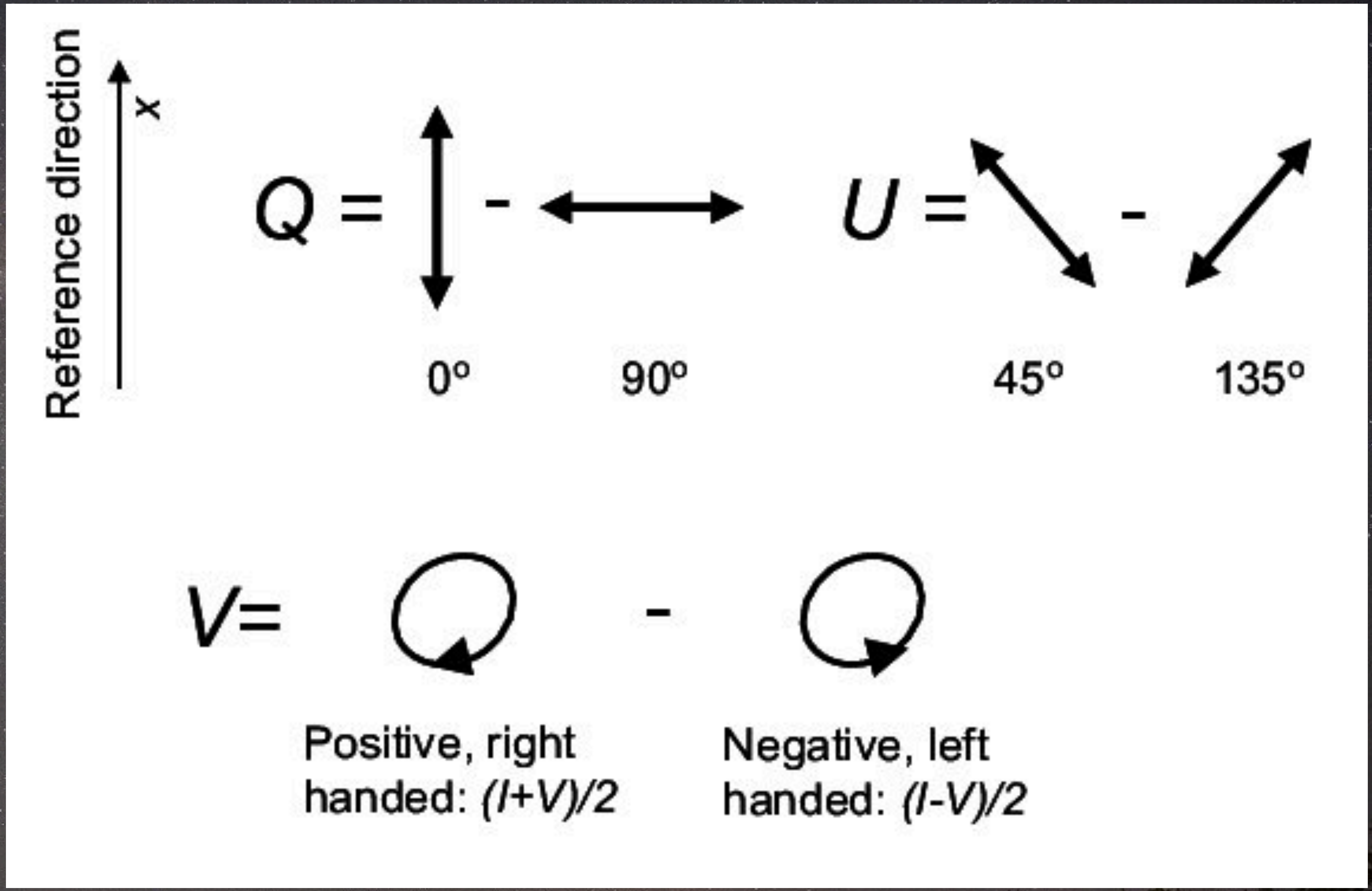
I, Q, U, V

I **intensity**

Q,U **linear** polarization

V **circular** polarization

$$I^2 \geq \mathscr{P}^2 = Q^2 + U^2 + V^2$$



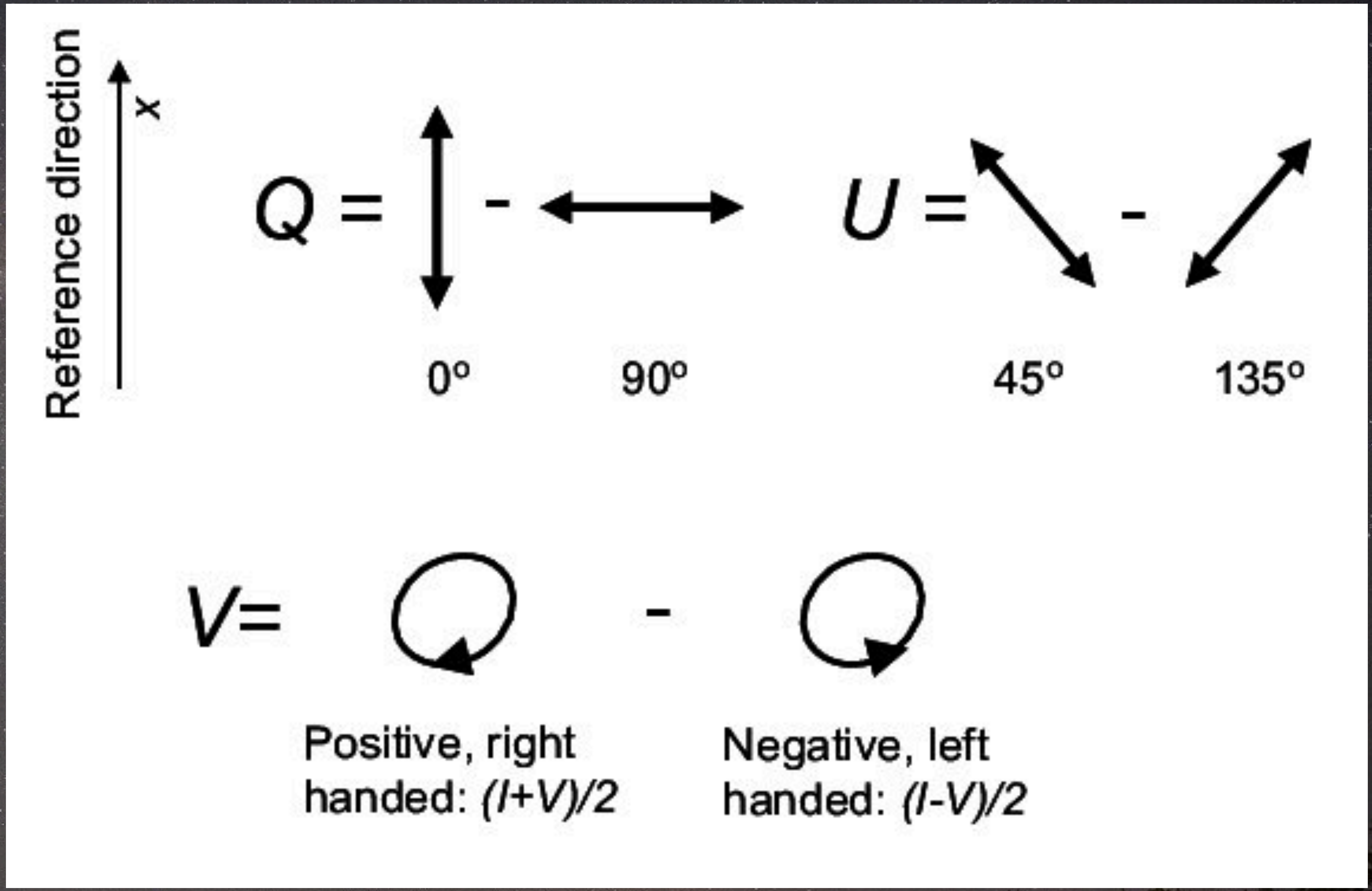
[Degl'Innocenti (2006)]

II -How to describe properly the polarized signal ?

While *I* and *V* are **frame independent** (scalar field)

Q and *U* are **not,** they are components in a given basis of a more complex object, equivalently:

- a 2x2 (STF) tensor
- a spin-2 spinor



[Degl'Innocenti (2006)]

II -How to describe properly the polarized signal ?

Q and U can be united to form the complex number (**spinor**)

$$\mathscr{P}_\nu := Q_\nu + iU_\nu = P_\nu e^{2i\gamma}$$

Q and U can be united to form the complex number (spinor)

- It's module, P_ν is called the **polarized intensity**
  Under reasonable assumption, $P_\nu \propto I_\nu$ is the SED.
- It's phase, γ is called the **polarization angle**

II -How to describe properly the polarized signal ?

Q and U can be united to form the complex number (**spinor**)

$$\mathscr{P}_\nu := Q_\nu + iU_\nu = P_\nu e^{2i\gamma}$$

The «**spin-2**» nature of $\mathscr{P}_\nu$ is hidden in the way it transforms under a right-handed **rotation of angle** θ around the line of sight:

$$\mathscr{P}'_\nu = e^{-2i\theta}\mathscr{P}_\nu$$

II -How to describe properly the polarized signal ?

You expect that every voxel (3D Pixel) of the galaxy emits with a linear polarized SED:

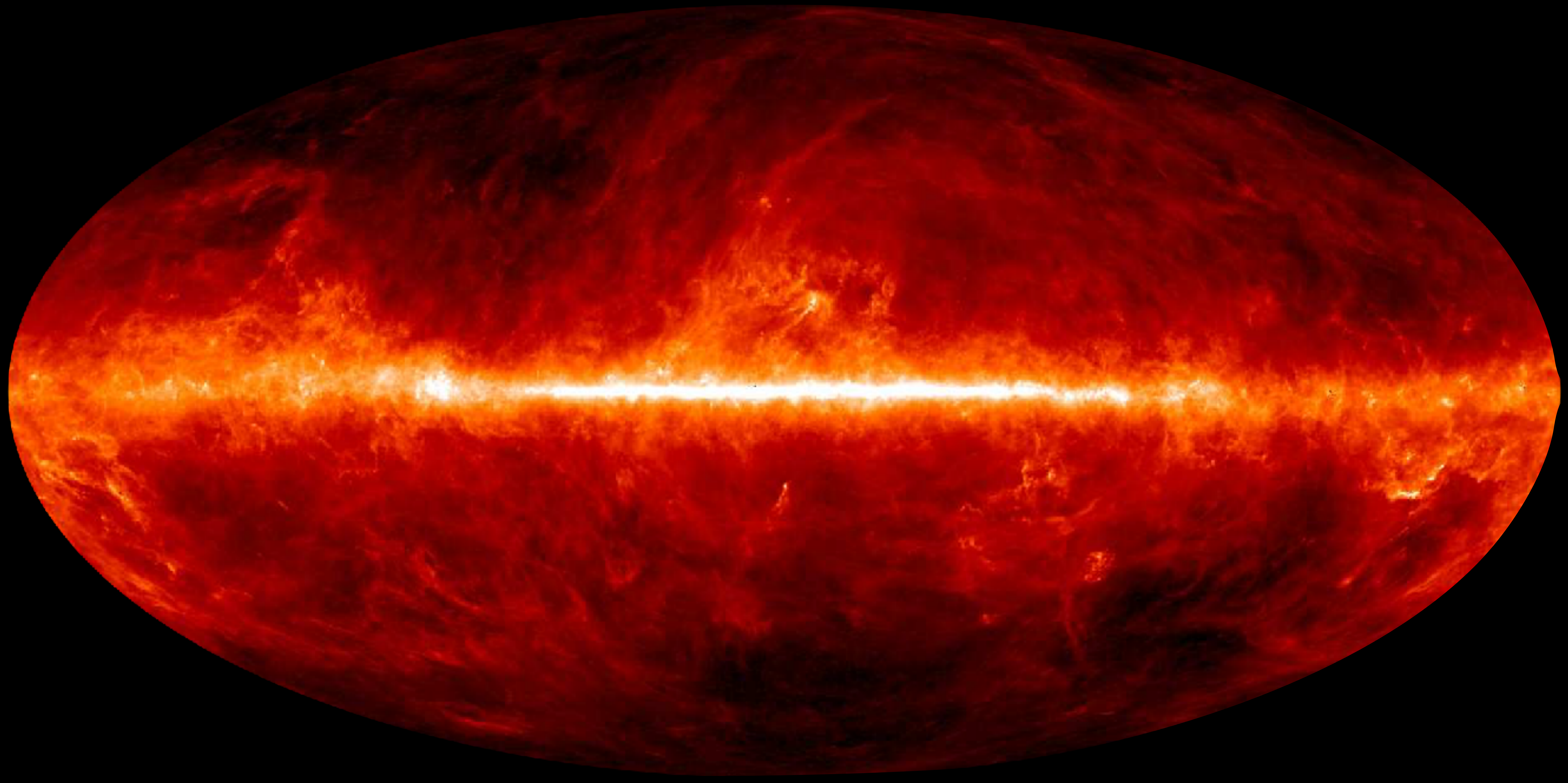
$$\mathscr{P}^s_\nu \simeq A \left(\frac{\nu}{\nu_0}\right)^{\beta_s} e^{2i\gamma}$$ *Synchrotron*

$$\mathscr{P}^d_\nu \simeq A \left(\frac{\nu}{\nu_0}\right)^{\beta_d} B_\nu(T_d)\, e^{2i\gamma}$$ Dust

...

(even when dropping this assumption, what I will present still holds)

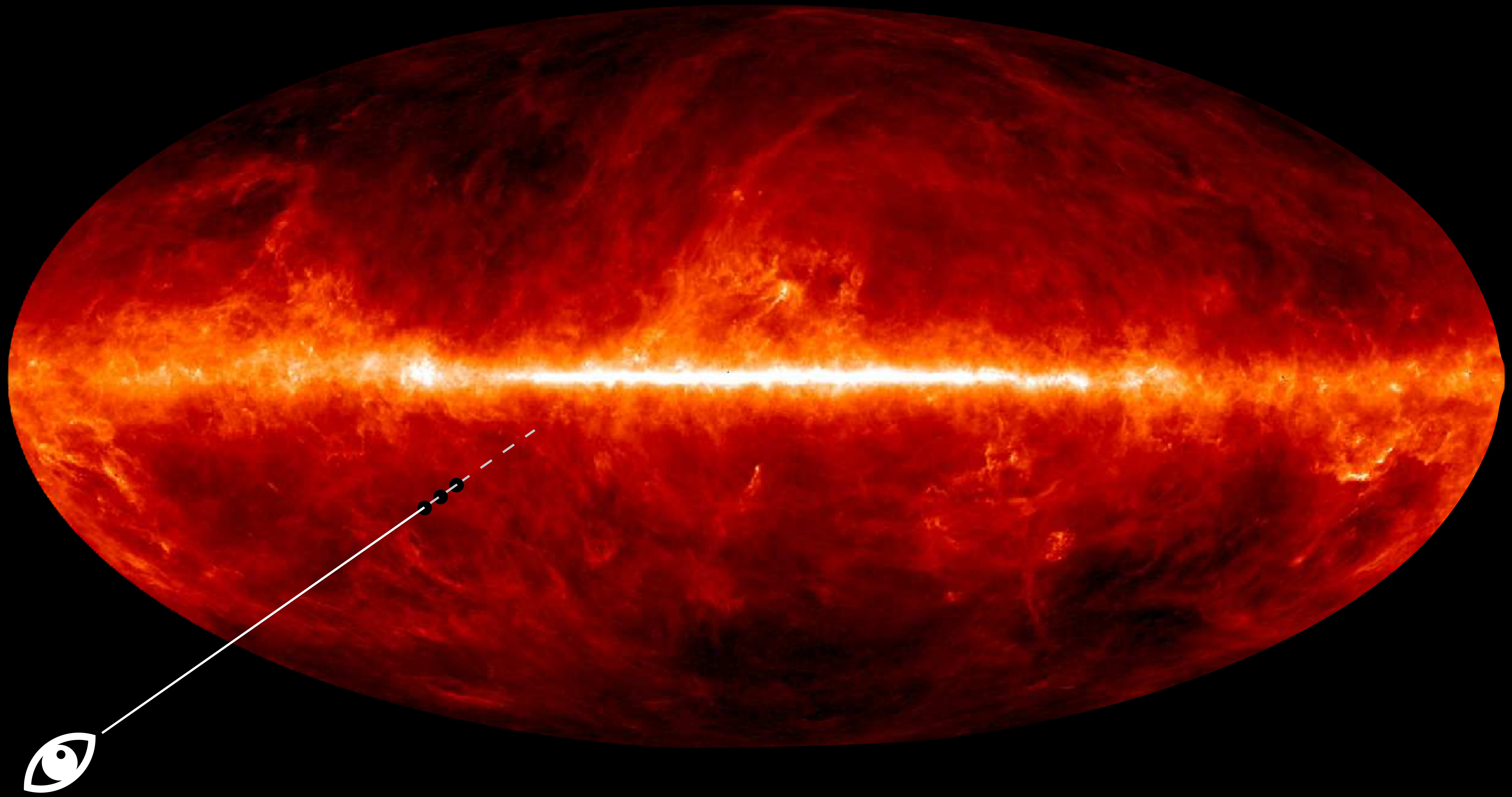
III - The problem of averaging



[Planck 2018 IV]

Spectral parameters (e.g. β, T for the MBB) of SEDs change with physical conditions across the sky/galaxy
(Predicted theoretically and verified observationally e.g. [Pelgrims 2021]

Averaging SEDs (spectral energy distribution $I(\nu)$)

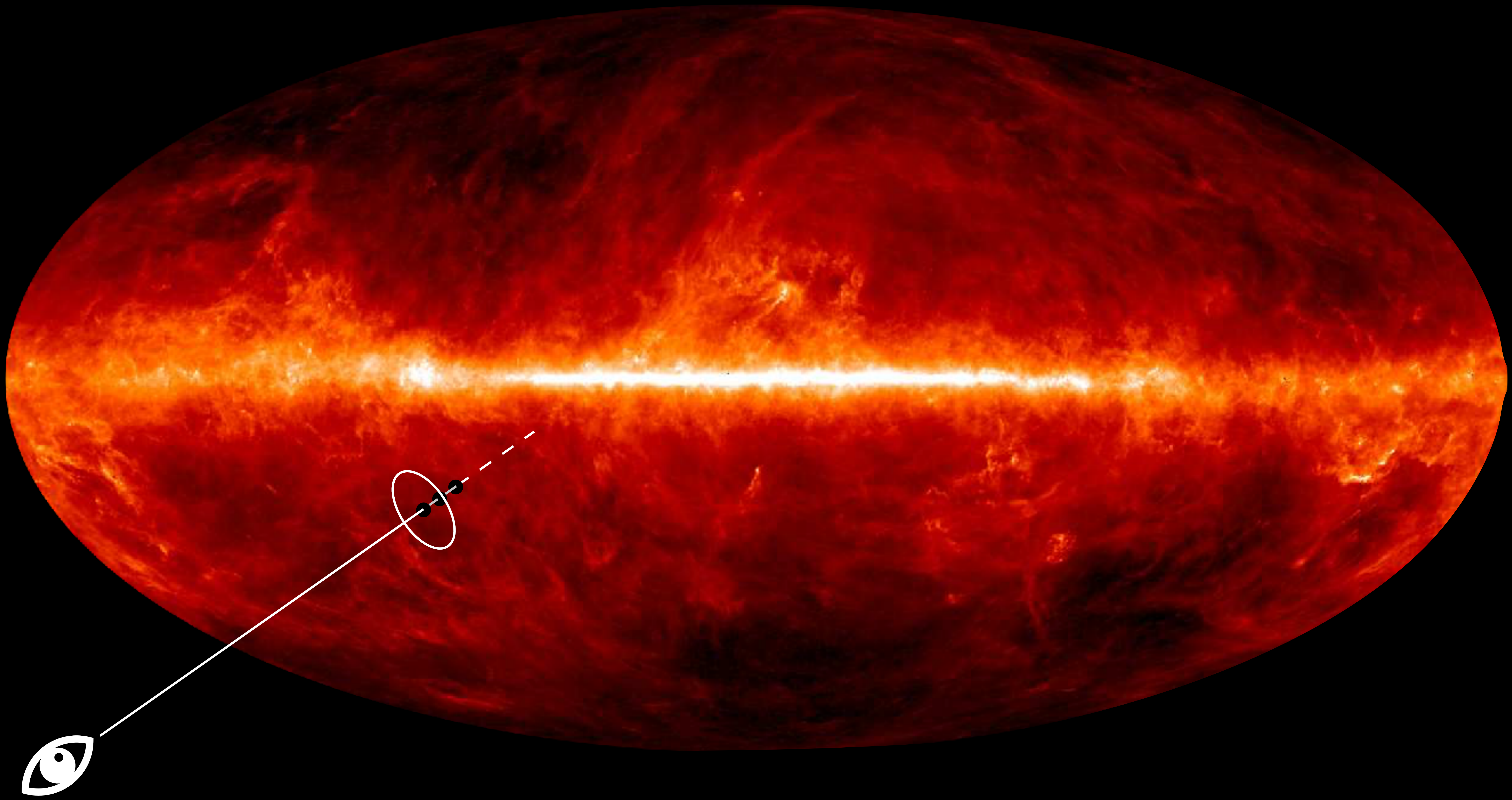


[Planck 2018 IV]

Fixed SED in *every* volume element

★ Line-of-sight average (*always there*!)

Averaging SEDs (spectral energy distribution $I(\nu)$)

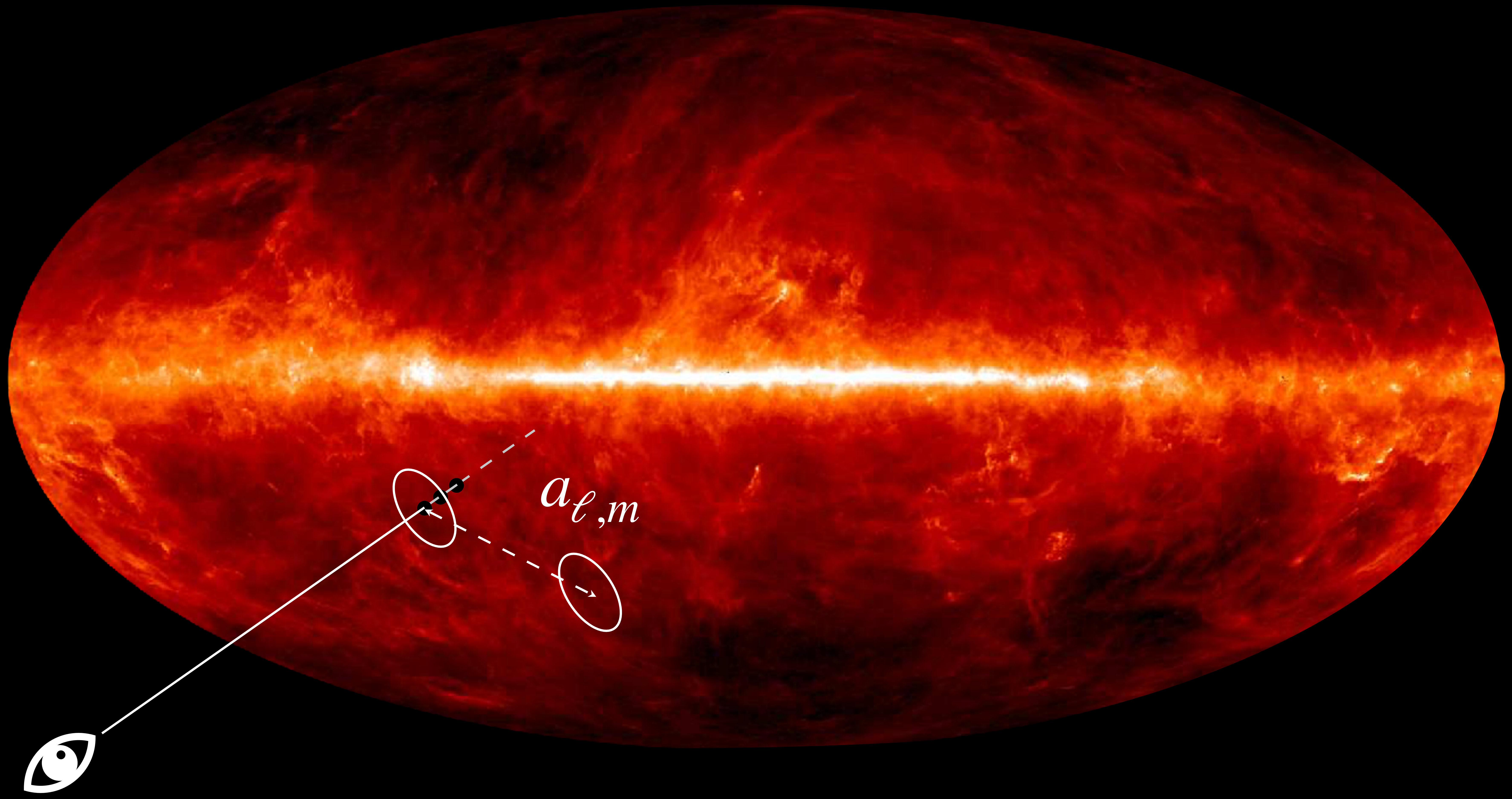


[Planck 2018 IV]

Fixed SED in *every* volume element

- ★ Line-of-sight average (*always there*!)
- ★ Experimental beam and frequency average

Averaging SEDs (spectral energy distribution $I(\nu)$)



[Planck 2018 IV]

Fixed SED in *every* volume element

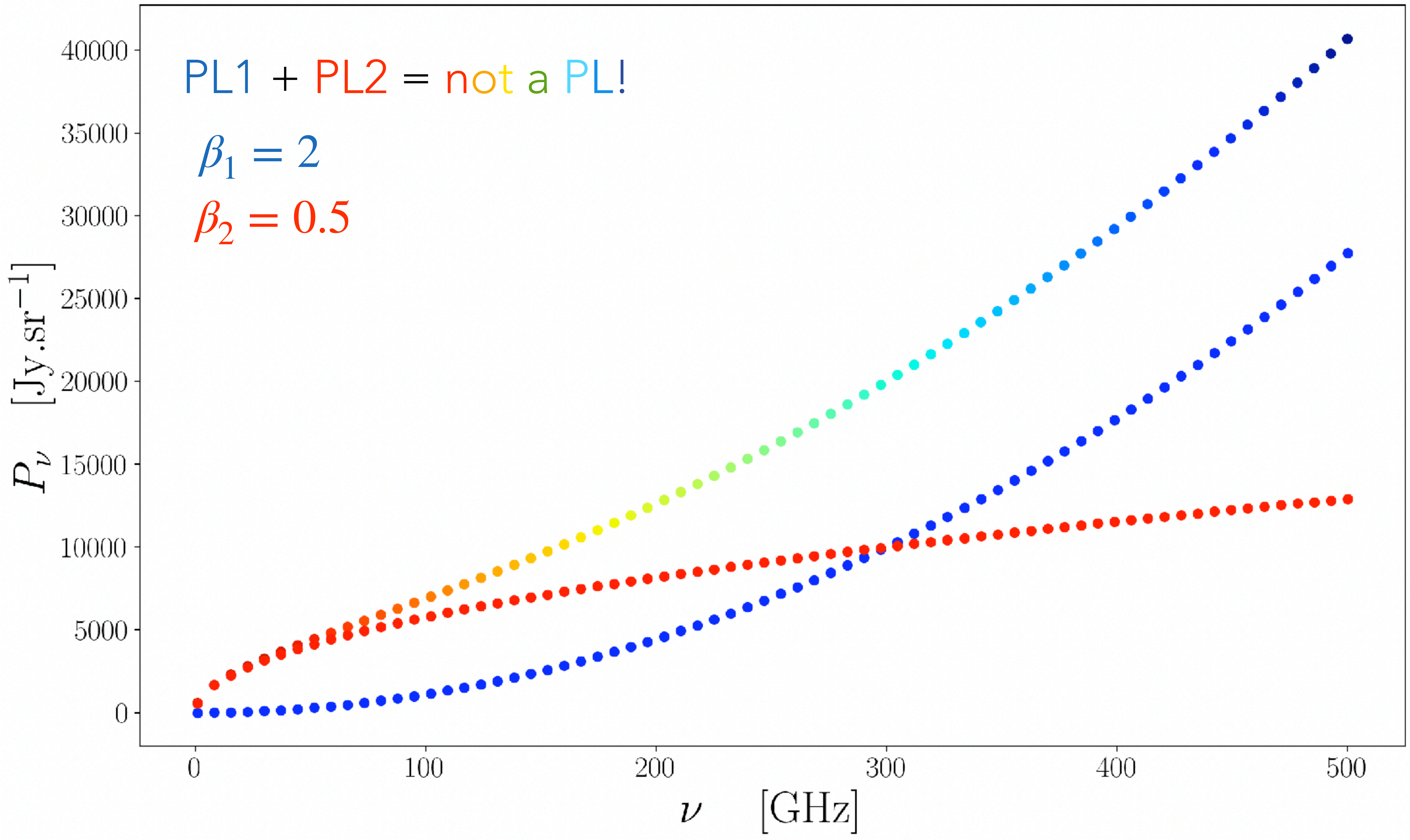
★ Line-of-sight average (*always there*!)

★ Experimental beam and frequency average

★ Map operations average (e.g., spherical harmonic expansion)

The consequences are:

- **SED distortions:** SEDs are not linear (e.g. MBB), so the sum of two canonical SEDs is not a canonical SED anymore.


PL1 + PL2 = not a PL!
$\beta_1 = 2$
$\beta_2 = 0.5$
P_ν [Jy.sr^{-1}]
40000
35000
30000
25000
20000
15000
10000
5000
0
0
100
200
300
400
500
ν [GHz]

The consequences are:

- **SED distortions:** SEDs are not linear (e.g. MBB), so the sum of two canonical SEDs is not a canonical SED anymore.
- **Frequency decorrelation:** SED are distorted differently at every point of the sky. One can not extrapolate a map at a given frequency to another frequency anymore (different bands becomes decorrelated) [see e.g. Pelgrims 2021]

The consequences are:

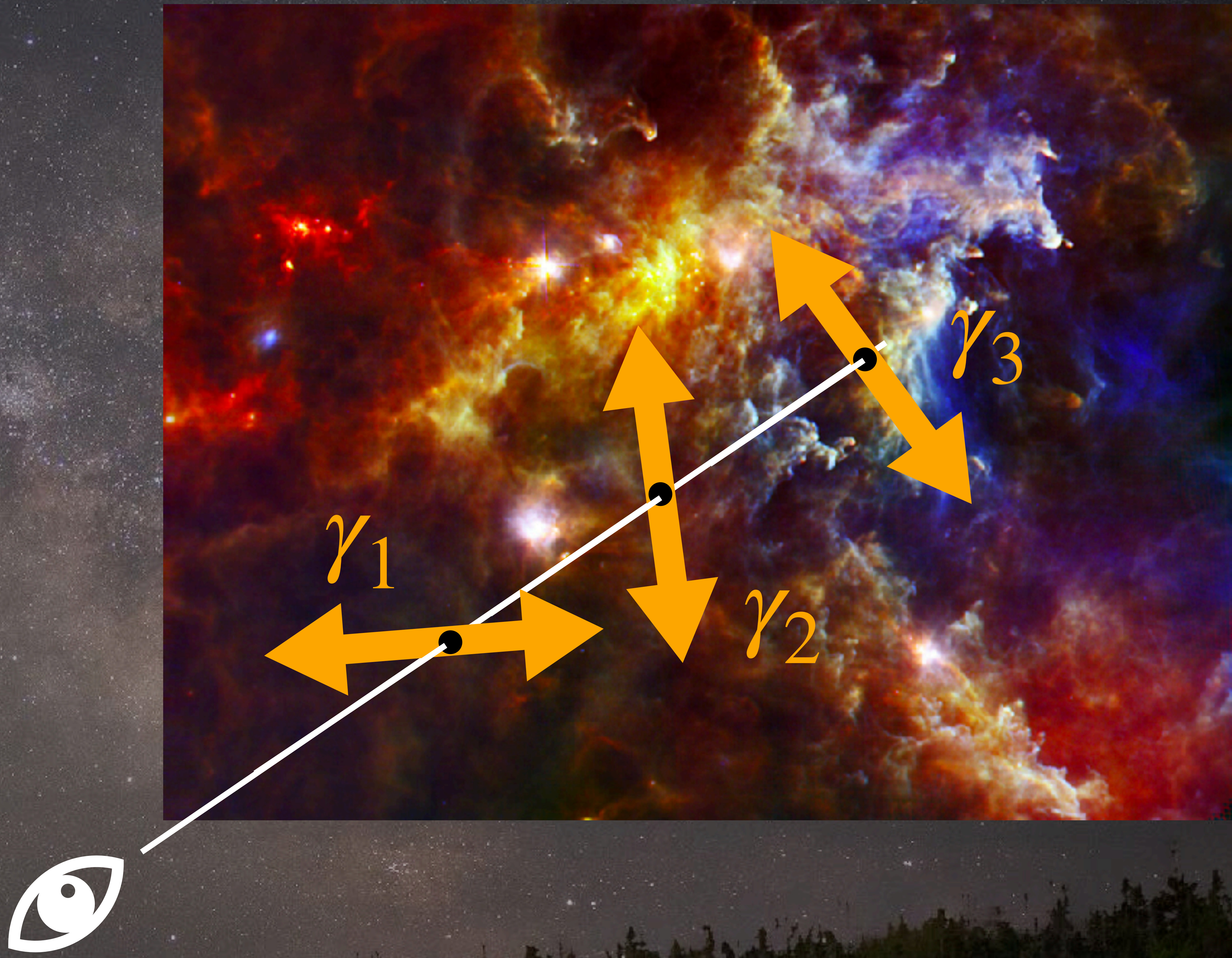
- **SED distortions:** SEDs are not linear (e.g. MBB), so the sum of two canonical SEDs is not a canonical SED anymore.
- **Frequency decorrelation:** SED are distorted differently at every point of the sky. One can not extrapolate a map at a given frequency to another frequency anymore (different bands becomes decorrelated) [see e.g. Pelgrims 2021]
- **Polarisation angle mixing:** Summing polarized SEDs with different (constant) polarization angles and spectral parameters, lead to a resulting frequency dependent pol. angle $\gamma \rightarrow \gamma_\nu$

Polarization angle mixing
And SED distortions

Let's now look at the power-law sum:

$$\mathscr{P}_\nu = P_\nu \, e^{2i\gamma_\nu}$$

$$= A_1(\nu/\nu_0)^{\beta_1} e^{2i\gamma_1} + A_2(\nu/\nu_0)^{\beta_2} e^{2i\gamma_2}$$



Polarization angle mixing And SED distortions

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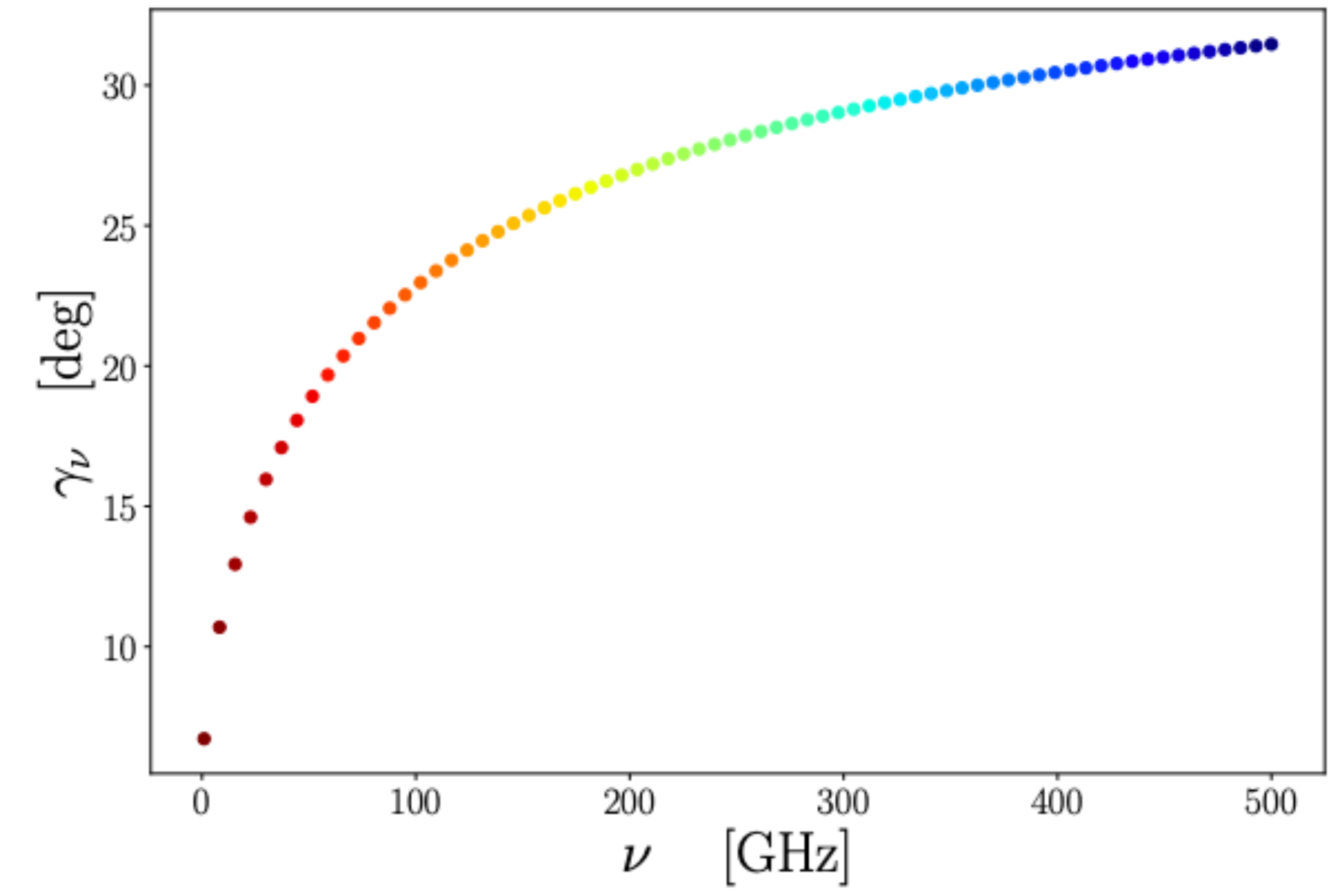
$$\mathscr{P}_\nu = P_\nu e^{2i\gamma_\nu}$$

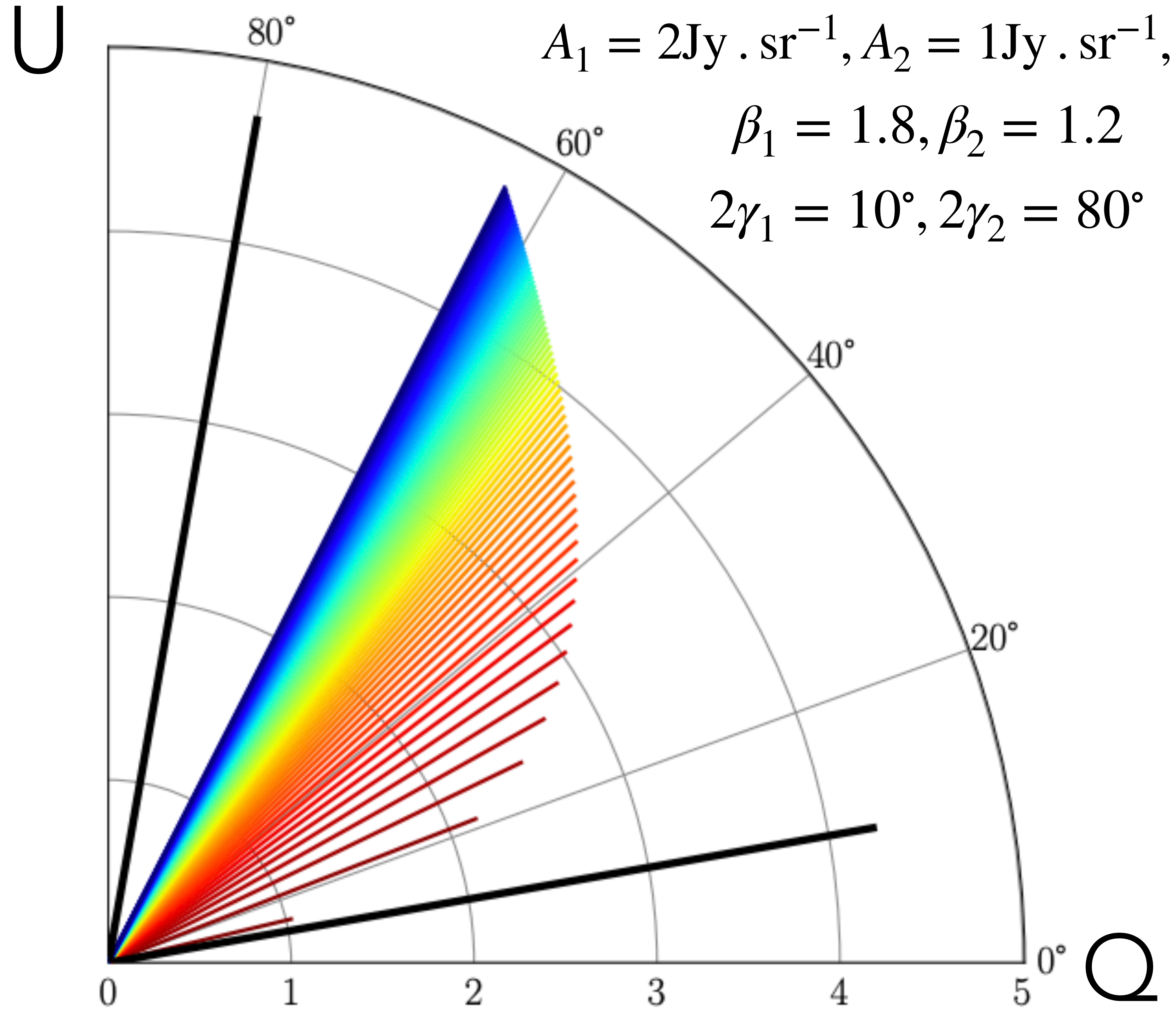
$$= A_1(\nu/\nu_0)^{\beta_1} e^{2i\gamma_1} + A_2(\nu/\nu_0)^{\beta_2} e^{2i\gamma_2}$$

a) $P_\nu \neq A'\nu^{\beta'} e^{2i\gamma'}$ not a power law

b) You can witness: $\gamma \rightarrow \gamma_\nu$!

HARD TO MODEL!





Bottom line

Even if one knows the SED in every voxel (e.g. power-law for synchrotron, MBB for dust …), it is not enough to model the averaged/large-scale signal.

IV- The moment expansion for intensity

Moment (Taylor inspired) expansion of I_ν in p:

$$I_\nu(p) = I_\nu(p_0) + \sum_i \omega_1^{p_i} \langle \partial_{p_i} I_\nu(p) \rangle_{p=p_0} + \frac{1}{2} \sum_{i,j} \omega_2^{p_i p_j} \langle \partial_{p_i} \partial_{p_j} I_\nu(p) \rangle_{p=p_0} + \ldots$$

[Chluba et al., 2017]

Moment expansion around the MBB in β :

$$I_{\mathrm{D}}(\nu, \vec{n}) = \frac{I_\nu(\beta_0, T_0)}{I_{\nu_0}(\beta_0, T_0)} \left[A(\vec{n}) + \omega_1(\vec{n}) \ln\left(\frac{\nu}{\nu_0}\right) + \frac{1}{2} \omega_2(\vec{n}) \ln^2\left(\frac{\nu}{\nu_0}\right) + \ldots \right]$$

IV- The moment expansion for intensity

Moment (Taylor inspired) expansion of I_ν in p:

$$I_\nu(p) = I_\nu(p_0) + \omega_1^p \frac{\partial I_\nu}{\partial p} + \omega_2^p \frac{\partial^2 I_\nu}{\partial p^2} + \dots$$

Moment expansion around the MBB in β :

$$I_\mathrm{D}(\nu, \vec{n}) = \text{MBB} \left[A(\vec{n}) + \beta \text{ order } 1 + \beta \text{ order } 2 + \dots \right]$$

[Chluba et al., 2017]

IV- The moment expansion for intensity

- Allows to model very accurately **SED distortions** and **frequency decorrelation** due to averaging of non linear **intensities**

- Applied successfully for **component separation** and spectral CMB distortions at the **map level** see e.g. Rotti et al (2021) Remazeilles et al (2021)

- And at the **power-spectra** level see e.g. Mangilli et al (2021), Azzoni et al (2021), Vacher et al (2022)

V- How to generalize this expansion to polarized signal i.e. to spinor fields?

Not so easy question but surprisingly easy answer:

Make moments spin-2 fields!

Let's skip the mathematical derivation shall we?
(If you are curious, see Vacher et al 2022 arXiv:2205.01049)

Generalizing to polarization with the spin-moments

Moment (Taylor inspired) expansion of I_ν in p:

$$\mathscr{P}_\nu(p) = \mathscr{P}_\nu(p_0) + \sum_i \mathscr{W}_1^{p_i} \langle \partial_{p_i} P_\nu(p) \rangle_{p=p_0} + \frac{1}{2} \sum_{i,j} \mathscr{W}_2^{p_i p_j} \langle \partial_{p_i} \partial_{p_j} P_\nu(p) \rangle_{p=p_0} + \dots$$

[Vacher et al., 2022]

Generalizing to polarization with the spin-moments

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$$\mathscr{P}_\nu(p) = \mathscr{P}_\nu(p_0) + \mathscr{W}_1^p \frac{\partial P_\nu}{\partial p} + \frac{1}{2} \mathscr{W}_2^p \frac{\partial^2 P_\nu}{\partial p^2} + \dots$$

[Vacher et al., 2022]

Moment coefficients becomes complex number (spinors)

$$\mathscr{W}_\alpha^p = Q[\mathscr{W}_\alpha^p] + iU[\mathscr{W}_\alpha^p] = \Omega_\alpha^p e^{2i\varpi_\alpha^p}$$

« Spin-moments »

Generalizing to polarization with the spin-moments

Moment (Taylor inspired) expansion of I_ν in p:

$$\mathscr{P}_\nu(p) = \mathscr{P}_\nu(p_0) + \mathscr{W}_1^p \frac{\partial P_\nu}{\partial p} + \mathscr{W}_2^p \frac{\partial^2 P_\nu}{\partial p^2} + \dots$$

They can be calculated analytically from the parameter distribution:

$$\mathscr{W}_\alpha^{p_j \dots p_l} = \frac{\left\langle A\, e^{2i\gamma}(p_j - \bar{p}_j)\dots(p_l - \bar{p}_l) \right\rangle}{\langle A \rangle}$$

[Vacher et al., 2022]

VI - Applications : power-laws

The spin-moment expansion for power-laws take the form:

$$\langle \mathscr{P}_\nu^{\rm PL} \rangle = P_\nu^{\rm PL}(\bar{A}, \bar{\beta}) \times \left\{ \mathscr{W}_0 + \mathscr{W}_1^{\beta} \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_2^{\beta^2}}{2} \ln\left(\frac{\nu}{\nu_0}\right)^2 + \frac{\mathscr{W}_3^{\beta^3}}{6} \ln\left(\frac{\nu}{\nu_0}\right)^3 + \cdots \right\}.$$

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Can be interpreted as a correction of a

Complex correction to β !!!???

$$\boldsymbol{\Delta\beta} = \frac{\mathscr{W}_{\mathbf{1}}^{\beta}}{\mathscr{W}_{\mathbf{0}}} \in \mathbb{C}$$

VI - Applications : power-laws

In the perturbative regime $\mathscr{W}_0 \gg \mathscr{W}^p_{\alpha'}$, the leading order can be rewritten

$$\langle \mathscr{P}_\nu^{\mathrm{PL}} \rangle = \bar{A} \left(\frac{\nu}{\nu_0} \right)^{\bar{\beta}} e^{2i\gamma_0} \times \left\{ 1 + \boldsymbol{\Delta}\beta \ln \left(\frac{\nu}{\boldsymbol{\nu_0}} \right) + \ldots \right\} \simeq \bar{\mathbf{A}} \left(\frac{\nu}{\boldsymbol{\nu_0}} \right)^{\bar{\beta} + \boldsymbol{\Delta}\beta} \mathbf{e}^{\mathbf{2i}\gamma_0}$$

VI - Applications : power-laws

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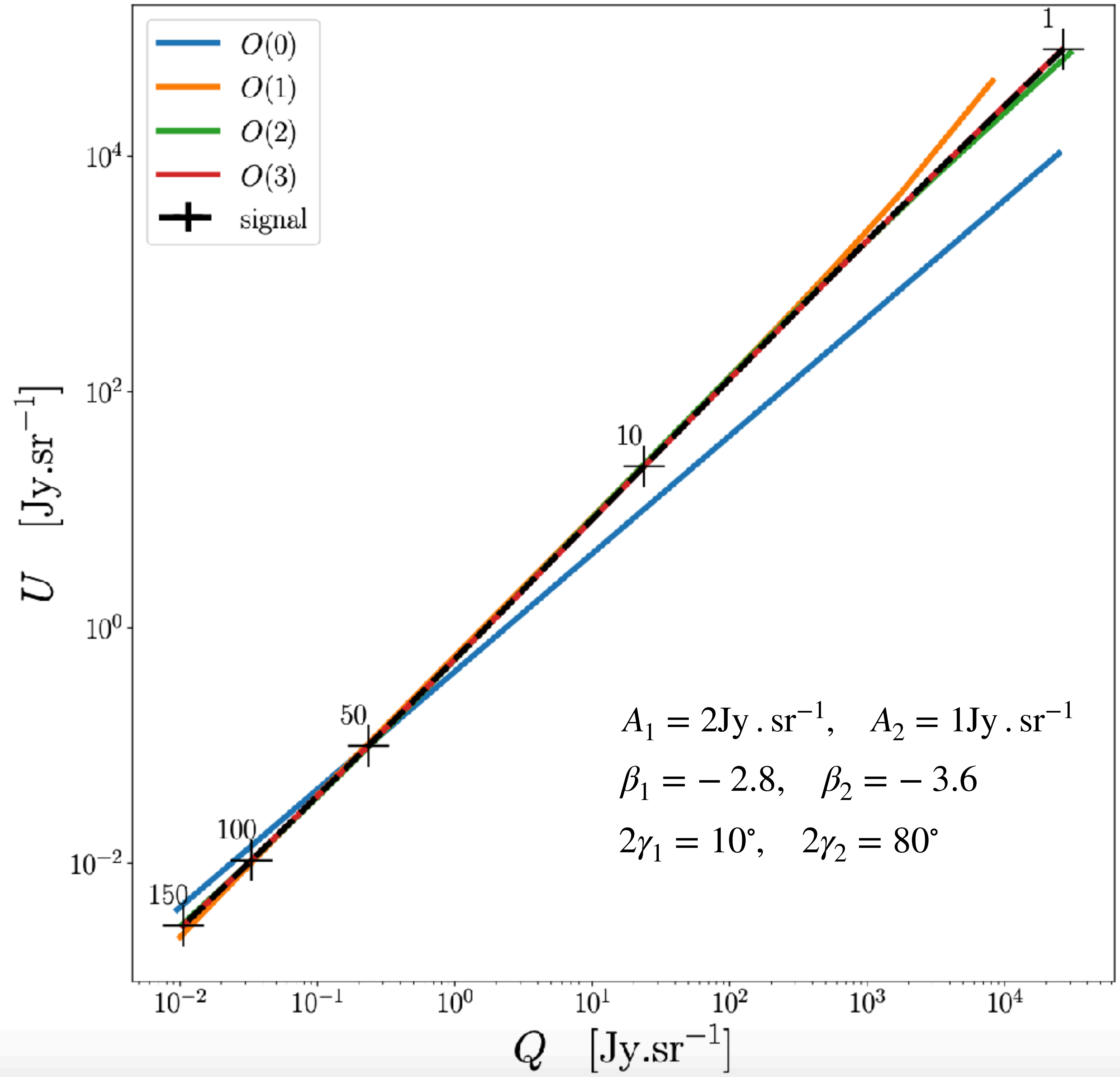
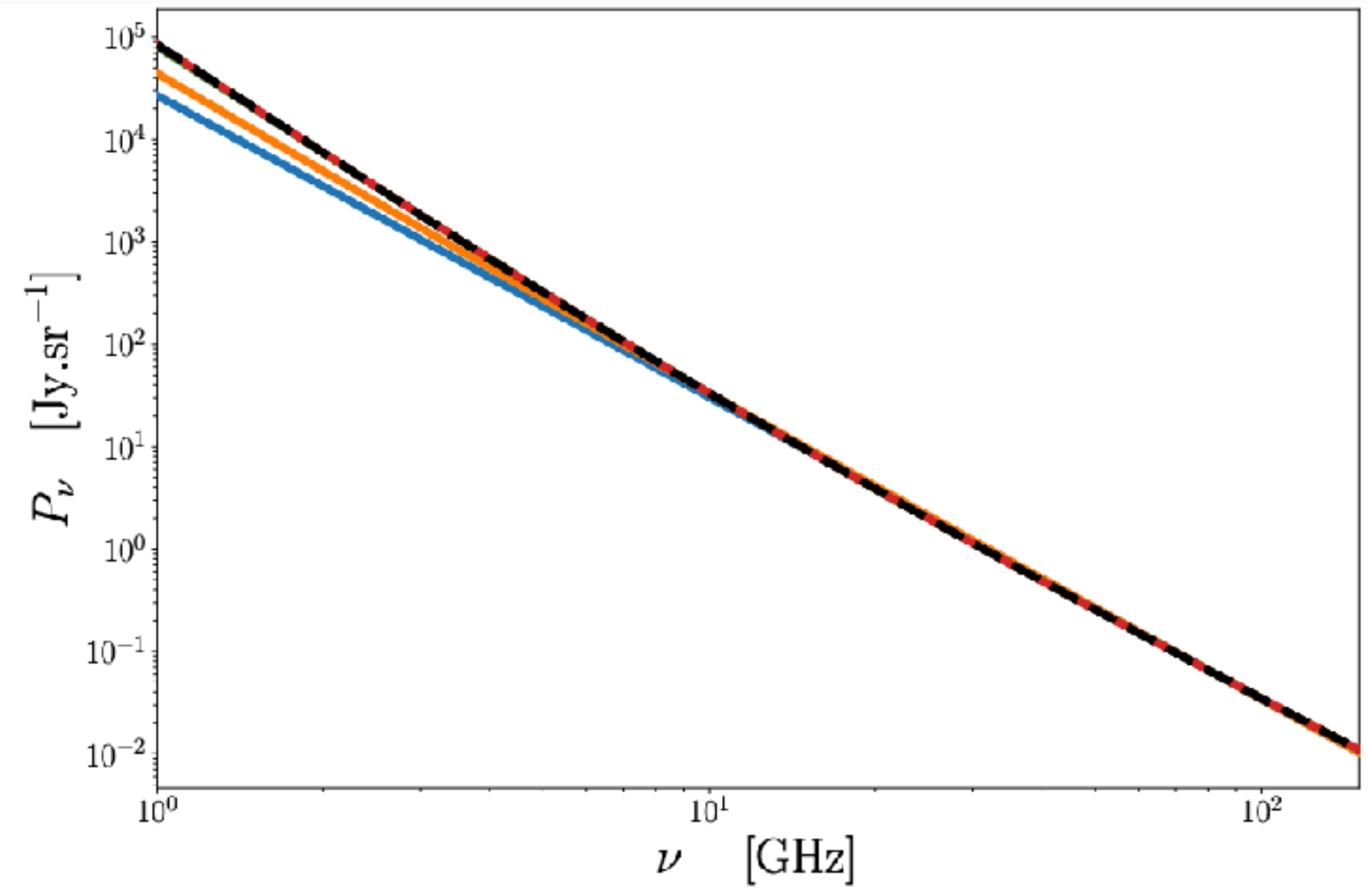
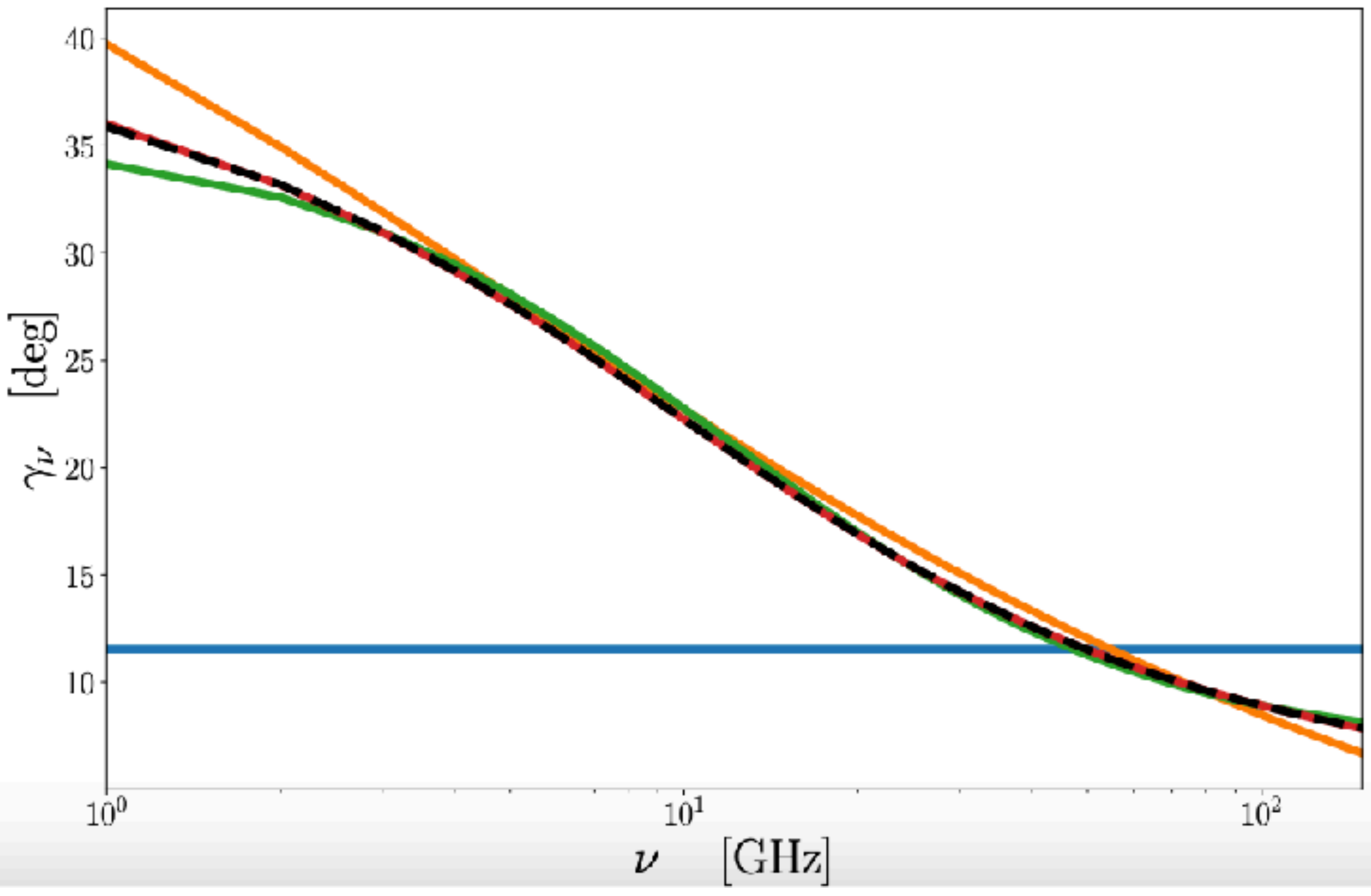
$$\beta^{\mathrm{PL}} = \bar{\beta} + \mathrm{Re}(\boldsymbol{\Delta}\beta)$$

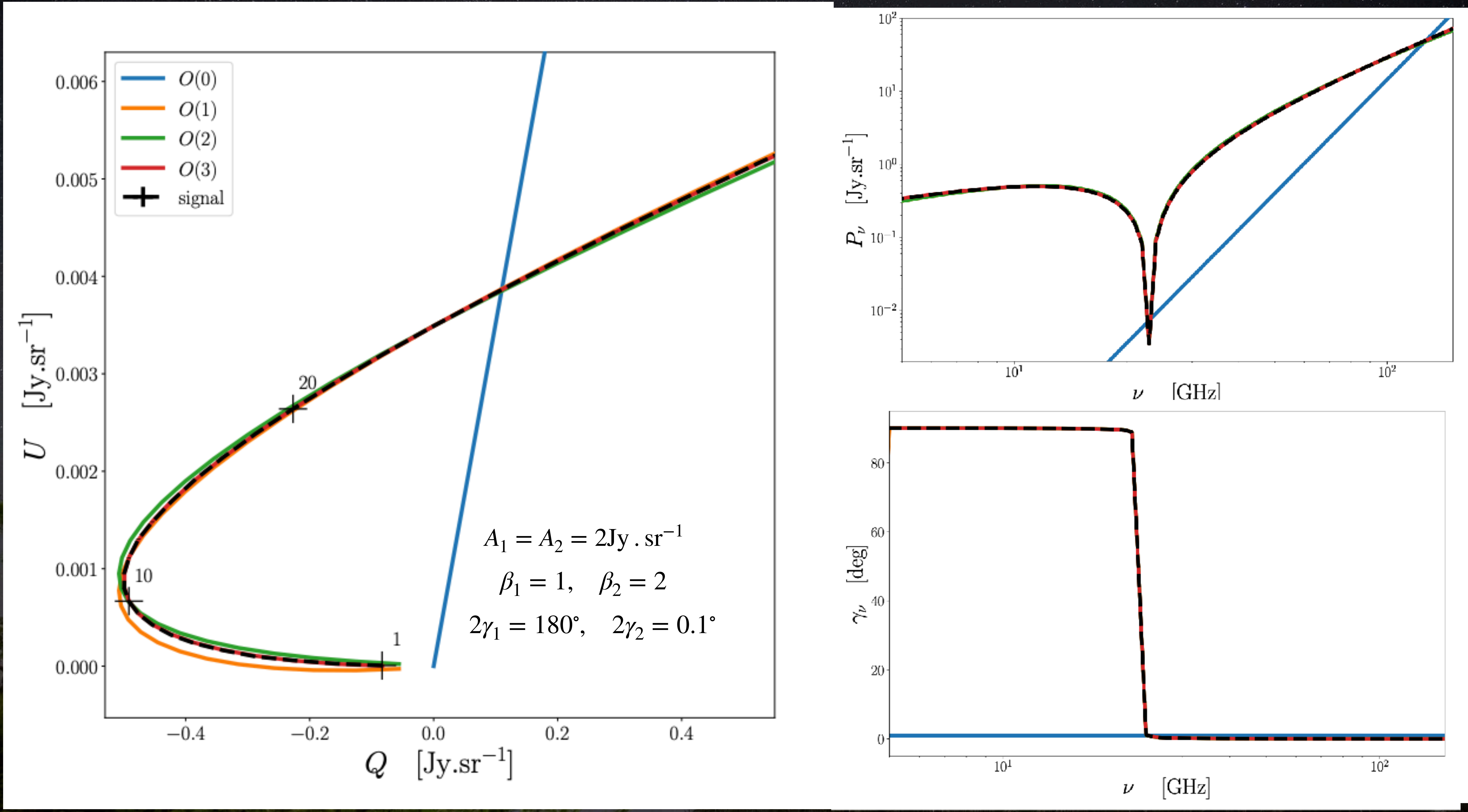
$$\gamma_\nu^{\mathrm{PL}} \approx \gamma_0 + \frac{1}{2} \mathrm{Im}\left(\boldsymbol{\Delta}\beta\right) \ln \left(\frac{\nu}{\nu_0} \right)$$

analytical expression for γ_ν at order 1!


O(0)
O(1)
O(2)
O(3)
signal
1
10
50
100
150
$A_1 = 2\text{Jy.sr}^{-1}, \quad A_2 = 1\text{Jy.sr}^{-1}$
$\beta_1 = -2.8, \quad \beta_2 = -3.6$
$2\gamma_1 = 10^\circ, \quad 2\gamma_2 = 80^\circ$
$U \quad [\text{Jy.sr}^{-1}]$
$Q \quad [\text{Jy.sr}^{-1}]$


$P_\nu \quad [\text{Jy.sr}^{-1}]$
$\nu \quad [\text{GHz}]$


$\gamma_\nu \quad [\text{deg}]$
$\nu \quad [\text{GHz}]$



O(0)
O(1)
O(2)
O(3)
signal
U [Jy.sr−1]
Q [Jy.sr−1]
A1 = A2 = 2Jy . sr−1
β1 = 1, β2 = 2
2γ1 = 180°, 2γ2 = 0.1°
Pν [Jy.sr−1]
ν [GHz]
γν [deg]
ν [GHz]

VI - Applications : Gray-bodies

Gray-bodies:

$$P_\nu^{\mathrm{GB}}(A,T) = AB_\nu(T)$$

The spin-moment expansion for power-laws take the form:

$$\langle \mathscr{P}_\nu^{\mathrm{PL}} \rangle = P_\nu^{\mathrm{GB}}(\bar{A},\bar{T}) \times \left\{ \mathscr{W}_0 + \mathscr{W}_1^T \Theta_1 + \frac{\mathscr{W}_2^{T^2}}{2}\Theta_2 + \frac{\mathscr{W}_3^{T^3}}{6}\Theta_3 + \cdots \right\}.$$

Complex temperature correction !

$$\Delta \mathbf{T} = \frac{\mathscr{W}_1^{\mathbf{T}}}{\mathscr{W}_0}$$

VI - Applications : Gray-bodies

In the perturbative regime $\mathscr{W}_0 \gg \mathscr{W}^p_{\alpha'}$, the leading order can be rewritten

$$\langle \mathscr{P}_\nu^{\rm PL} \rangle = B_\nu(\bar{T}) e^{2i\gamma_0} \times \left\{ 1 + \mathbf{\Delta T \Theta_1} + \dots \right\} \simeq \frac{2h\nu^3}{c^2} \frac{\bar{A}\, |\mathscr{W}_0|\, e^{2i\gamma_\nu}}{\sqrt{(e^{x_{\rm R}}-1)^2 + 2e^{x_{\rm R}}\left[1-\cos(x_{\rm I})\right]}} + \dots$$

$B_\nu(\tilde{T})$

With:

$$x_{\rm R} = h\nu \left(k\bar{T} + k{\rm Re}\left(\mathbf{\Delta T}\right)\right)^{-1}$$

$$x_{\rm I} = h\nu \left(k{\rm Im}\left(\mathbf{\Delta T}\right)\right)^{-1}$$

$$\gamma_\nu^T = \gamma_0 + \frac{1}{2}\tan^{-1}\left(\frac{e^{x_{\rm R}}\sin(x_{\rm I})}{e^{x_{\rm R}}\cos(x_{\rm I}) - 1}\right)$$

analytical expression for γ_ν at order 1! + Spectral modulation of the SED!

×10⁻⁹
O(0)
O(1)
O(2)
O(3)
signal
1000
2000
3000
4000
$A_1 = A_2 = 10^6 \mathrm{Jy\,.\,sr}^{-1}$
$T_1 = 13\mathrm{K}, \quad T_2 = 20\mathrm{K}$
$2\gamma_1 = 72^\circ, \quad 2\gamma_2 = -90^\circ$
U [Jy.sr^{-1}]
Q [Jy.sr^{-1}]
×10⁻¹⁰
P_ν [Jy.sr^{-1}]
ν [GHz]
γ_ν [deg]
ν [GHz]

VI - Applications : Modified black-bodies

Modified black-bodies = power-law x black-body

$$\langle \mathscr{P}_\nu^{\mathrm{mBB}} \rangle = P_\nu^{\mathrm{mBB}}(\bar{A}, \bar{\beta}, \bar{T}) \times \left\{ \mathscr{W}_0 \right.$$

Leading order

VI - Applications : Modified black-bodies

Modified black-bodies = power-law x black-body

$$\langle \mathscr{P}_\nu^{\mathrm{mBB}} \rangle = P_\nu^{\mathrm{mBB}}(\bar{A}, \bar{\beta}, \bar{T}) \times \Bigg\{ \mathscr{W}_0 + \mathscr{W}_1^{\beta} \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_2^{\beta^2}}{2} \ln\left(\frac{\nu}{\nu_0}\right)^2 + \frac{\mathscr{W}_3^{\beta^3}}{6} \ln\left(\frac{\nu}{\nu_0}\right)^3 + \cdots$$

Leading order

β PL expansion

VI - Applications : Modified black-bodies

Modified black-bodies = power-law x black-body

$$\langle \mathscr{P}_\nu^{\rm mBB} \rangle = P_\nu^{\rm mBB}(\bar{A},\bar{\beta},\bar{T}) \times \left\{ \mathscr{W}_0 \right.$$

Leading order

β PL expansion

$$+\mathscr{W}_1^{\beta} \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_2^{\beta^2}}{2}\ln\left(\frac{\nu}{\nu_0}\right)^2 + \frac{\mathscr{W}_3^{\beta^3}}{6}\ln\left(\frac{\nu}{\nu_0}\right)^3 + \cdots$$

T GB/BB expansion

$$+\mathscr{W}_1^{T}\Theta_1 + \frac{\mathscr{W}_2^{T^2}}{2}\Theta_2 + \frac{\mathscr{W}_3^{T^3}}{6}\Theta_3 + \cdots$$

VI - Applications : Modified black-bodies

Modified black-bodies = power-law x black-body

$$\langle \mathscr{P}_\nu^{\mathrm{mBB}} \rangle = P_\nu^{\mathrm{mBB}}(\bar{A}, \bar{\beta}, \bar{T}) \times \begin{cases} \mathscr{W}_0 \\ +\mathscr{W}_1^{\beta} \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_2^{\beta^2}}{2} \ln\left(\frac{\nu}{\nu_0}\right)^2 + \frac{\mathscr{W}_3^{\beta^3}}{6} \ln\left(\frac{\nu}{\nu_0}\right)^3 + \cdots \\ +\mathscr{W}_1^{T}\Theta_1 + \frac{\mathscr{W}_2^{T^2}}{2}\Theta_2 + \frac{\mathscr{W}_3^{T^3}}{6}\Theta_3 + \cdots \\ +\mathscr{W}_1^{T\beta}\Theta_1 \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_2^{T^2\beta}}{2}\Theta_2 \ln\left(\frac{\nu}{\nu_0}\right) + \frac{\mathscr{W}_3^{T\beta^2}}{6}\Theta_1 \ln\left(\frac{\nu}{\nu_0}\right)^2 + \cdots \end{cases}$$

Leading order

β PL expansion

T GB/BB expansion

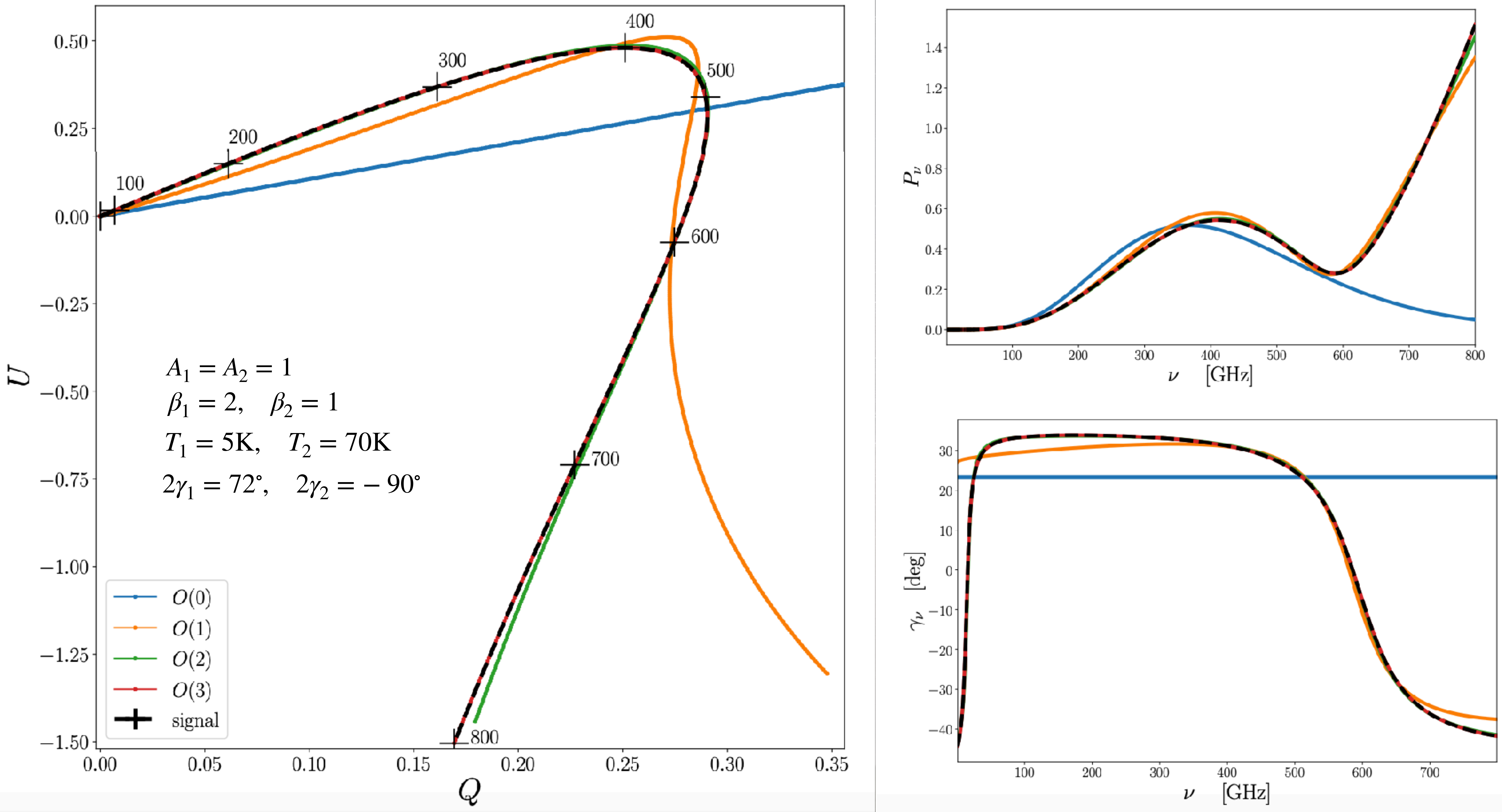
Cross-terms $T\beta$

VI - Applications : Modified black-bodies

In the perturbative regime $\mathscr{W}_0 \gg \mathscr{W}^p_{\alpha'}$, the leading order can be rewritten
As a sum of the PL + GB corrections for P_ν and γ_ν

$$\mathscr{P}^{mBB}_\nu \simeq \frac{2h\nu^3}{c^2} \frac{\bar{A}\,|\mathscr{W}_0|\,(\nu/\nu_0)^{\bar{\beta}^{\rm PL}}\, e^{2i\gamma_\nu}}{\sqrt{(e^{x_{\rm R}}-1)^2 + 2e^{x_{\rm R}}\,[1-\cos(x_{\rm I})]}} + \dots$$

$$\gamma_\nu = \gamma_0 + \gamma_\nu^{\rm PL} + \gamma_\nu^T$$



$A_1 = A_2 = 1$
$\beta_1 = 2, \quad \beta_2 = 1$
$T_1 = 5\text{K}, \quad T_2 = 70\text{K}$
$2\gamma_1 = 72^\circ, \quad 2\gamma_2 = -90^\circ$
O(0)
O(1)
O(2)
O(3)
signal
U
Q
100
200
300
400
500
600
700
800
P_ν
ν [GHz]
γ_ν [deg]
ν [GHz]

VI - Other averaging processes

- Instrumental effects
- Spherical harmonics
- Faraday rotation
- $P_\nu^Q \neq P_\nu^U$

The **formalism** applies the same way everywhere
But the **interpretations** of the spin-moments are different

What's next?

- **Theoretical** extensions (E,B ?)
- Application to **galactic physics** (ongoing on *Planck* data + *LiteBIRD* simulations…)
- Application to **component separation** (ongoing *LiteBIRD*)
- Application to **spectral distortions** (CMB)
- Applications to **cosmic birefringence**
- **SZ** effect …

Thanks for listening !

Centaurus A - Hubble Collaboration;

Back-up

Derivation : intensity

$$\langle I_\nu(A\,,\boldsymbol{p})\rangle = \int \frac{\mathrm{d}A(s)}{\mathrm{d}s}\,\hat{I}_\nu(\boldsymbol{p}(s))\,\mathrm{d}s \equiv \int \mathbb{P}(\boldsymbol{p},\hat{\boldsymbol{n}})\,\hat{I}_\nu(\boldsymbol{p})\,\mathrm{d}^N p. \qquad (2)$$

$$\begin{aligned}\hat{I}_\nu(\boldsymbol{p}) = \hat{I}_\nu(\bar{\boldsymbol{p}}) &+ \sum_j (p_j - \bar{p}_j)\,\partial_{\bar{p}_j}\hat{I}_\nu(\bar{\boldsymbol{p}}) \\ &+ \frac{1}{2}\sum_{j,k}(p_j - \bar{p}_j)(p_k - \bar{p}_k)\,\partial_{\bar{p}_j}\partial_{\bar{p}_k}\hat{I}_\nu(\bar{\boldsymbol{p}}) \\ &+ \frac{1}{3!}\sum_{j,k,l}(p_j - \bar{p}_j)(p_k - \bar{p}_k)(p_l - \bar{p}_l)\,\partial_{\bar{p}_j}\partial_{\bar{p}_k}\partial_{\bar{p}_l}\hat{I}_\nu(\bar{\boldsymbol{p}}) \\ &+ \dots\,.\end{aligned}$$

Derivation : intensity

$$\langle I_\nu(A,\boldsymbol{p})\rangle = I_\nu(\bar{A},\bar{\boldsymbol{p}}) + \sum_j^N \omega_1^{p_j}\partial_{\bar{p}_j} I_\nu(\bar{A},\bar{\boldsymbol{p}})$$
$$+ \frac{1}{2}\sum_{j,k}^N \omega_2^{p_j p_k}\partial_{\bar{p}_j}\partial_{\bar{p}_k} I_\nu(\bar{A},\bar{\boldsymbol{p}})$$
$$+ \frac{1}{3!}\sum_{j,k,l}^N \omega_3^{p_j p_k p_l}\partial_{\bar{p}_j}\partial_{\bar{p}_k}\partial_{\bar{p}_l} I_\nu(\bar{A},\bar{\boldsymbol{p}}) + \dots$$

$$\omega_\alpha^{p_j \dots p_l} = \frac{\langle A\,(p_j-\bar{p}_j)\dots(p_l-\bar{p}_l)\rangle}{\bar{A}} = \frac{\int \mathbb{P}(\boldsymbol{p},\hat{\boldsymbol{n}})\,(p_j-\bar{p}_j)\dots(p_l-\bar{p}_l)\,\mathrm{d}^N p}{\int \mathbb{P}(\boldsymbol{p},\hat{\boldsymbol{n}})\,\mathrm{d}^N p},$$

Derivation : Polarization

$$\langle \mathcal{P}_\nu \rangle = \left\langle P_\nu(A, \boldsymbol{p})\, \mathrm{e}^{2\mathbb{i}\gamma} \right\rangle \equiv \int \mathbb{P}(\boldsymbol{p}, \gamma, \hat{\boldsymbol{n}})\, \hat{P}_\nu(\boldsymbol{p})\, \mathrm{e}^{2\mathbb{i}\gamma}\, \mathrm{d}^N p\, \mathrm{d}\gamma.$$

$$\langle \mathcal{P}_\nu(A, \boldsymbol{p}, \gamma) \rangle = \hat{P}_\nu(\bar{\boldsymbol{p}}) \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma} \right\rangle + \sum_j^N \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma}(p_j - \bar{p}_j) \right\rangle \partial_{\bar{p}_j} \hat{P}_\nu(\bar{\boldsymbol{p}})$$
$$+ \frac{1}{2} \sum_{j,k}^N \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma}(p_j - \bar{p}_j)(p_k - \bar{p}_k) \right\rangle \partial_{\bar{p}_j} \partial_{\bar{p}_k} \hat{P}_\nu(\bar{\boldsymbol{p}}) + \dots ,$$

$$\mathcal{W}_\alpha^{p_j \dots p_l} = \frac{\left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma}(p_j - \bar{p}_j) \dots (p_l - \bar{p}_l) \right\rangle}{\bar{A}}$$

No pivot for polarization

$$\langle \mathcal{P}_\nu(A, \boldsymbol{p}, \gamma) \rangle = \hat{P}_\nu(\bar{\boldsymbol{p}}) \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma} \right\rangle + \sum_j^N \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma} (p_j - \bar{p}_j) \right\rangle \partial_{\bar{p}_j} \hat{P}_\nu(\bar{\boldsymbol{p}})$$

$$+ \frac{1}{2} \sum_{j,k}^N \left\langle A\, \mathrm{e}^{2\mathbb{i}\gamma} (p_j - \bar{p}_j)(p_k - \bar{p}_k) \right\rangle \partial_{\bar{p}_j} \partial_{\bar{p}_k} \hat{P}_\nu(\bar{\boldsymbol{p}}) + \dots ,$$

Leading to no leading order in the expansion and $\mathscr{W}_1^p$

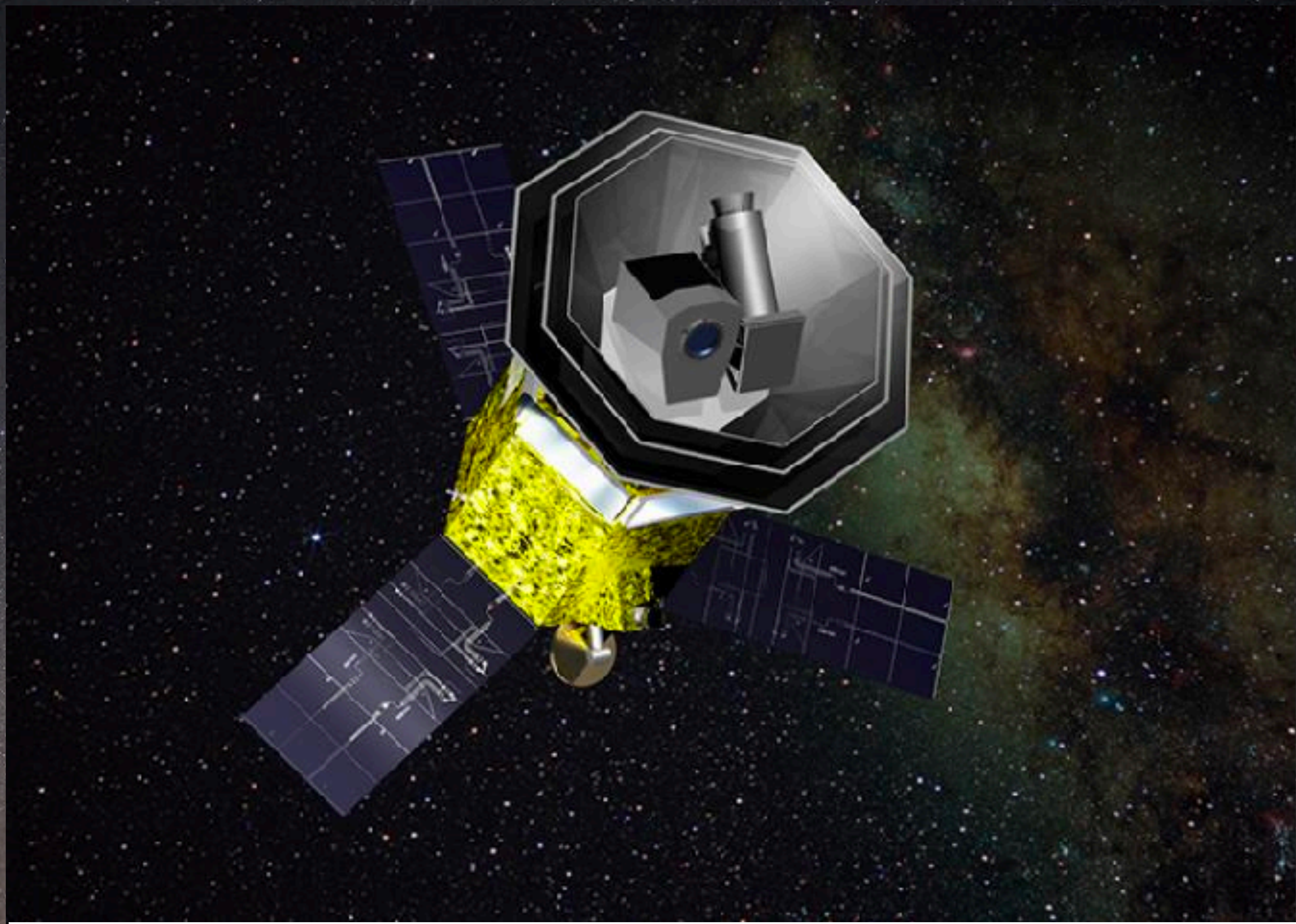
Representing the signal. One can not choose $\mathscr{W}_1^p = 0$

As a general condition to determine $\bar{p}$

—-> however possible in the perturbative regime $\mathscr{W}_0 \gg \mathscr{W}_1^p$

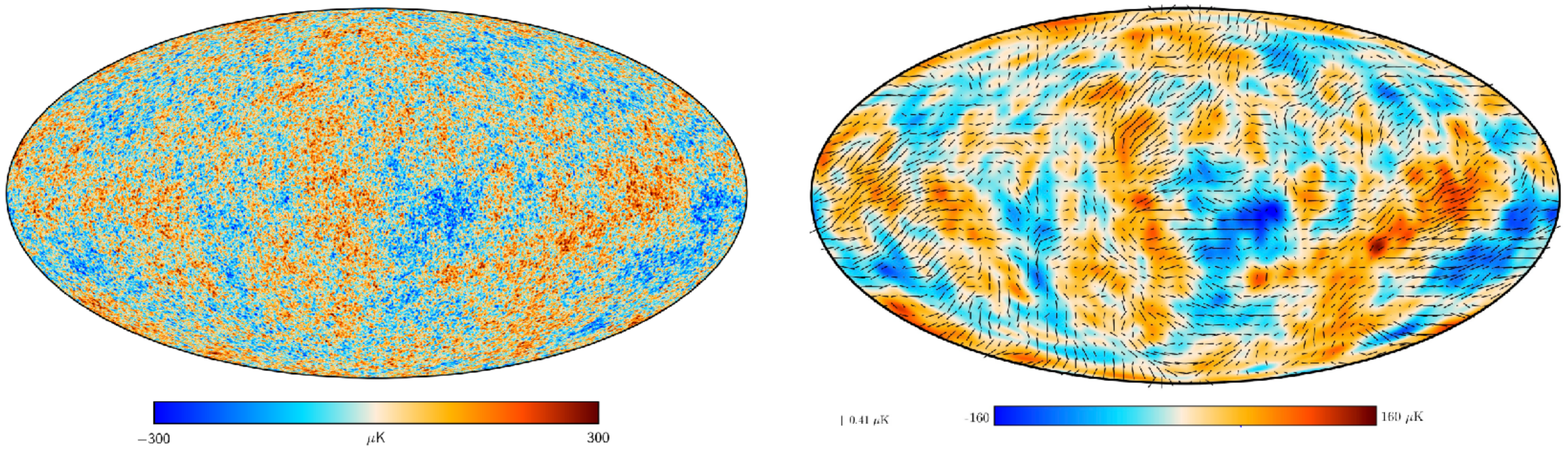
LiteBIRD and the *B*-modes quest

- JAXA project. Phase A CNES. ESA,NASA,CSA involved
- ***Lite*** *(Light) satellite for the studies of* ***B****-mode polarization and* ***I****nflation from cosmic background* ***R****adiation* ***D****etection*
- Build to reach $\delta r = 1 \times 10^{-3}$
- 3 telescopes LFT, MFT, HFT
- Expected in 2029 at L2 for more than 3 years of observation
- Good news also for galactic science !



LiteBIRD

Cosmic microwave background

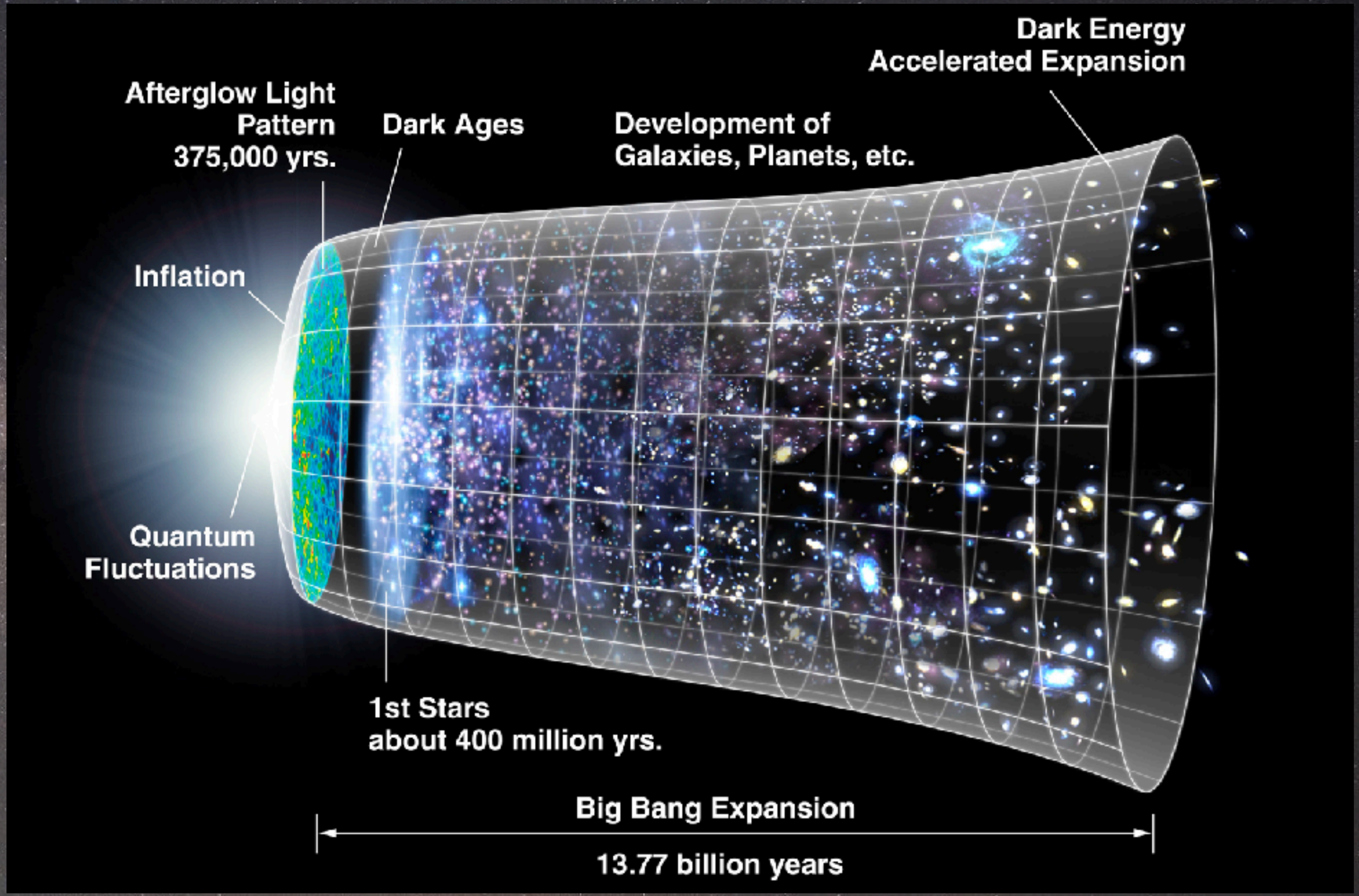


Cosmic Inflation and the *B* modes

Puzzles with Big-Bang cosmology :

- Flatness
- Horizon
- Extremely low entropy
- Cosmological defects
- formation of structures

} Fine tuning



New mechanism : **inflation**
New scalar degree of freedom : **Inflaton**

Primordial Universe expansion :
x 10^{26} in 10^{-35} s after primordial singularity

Photo-credit : NASA/WMAP Science Team

Cosmic Inflation and the *B* modes

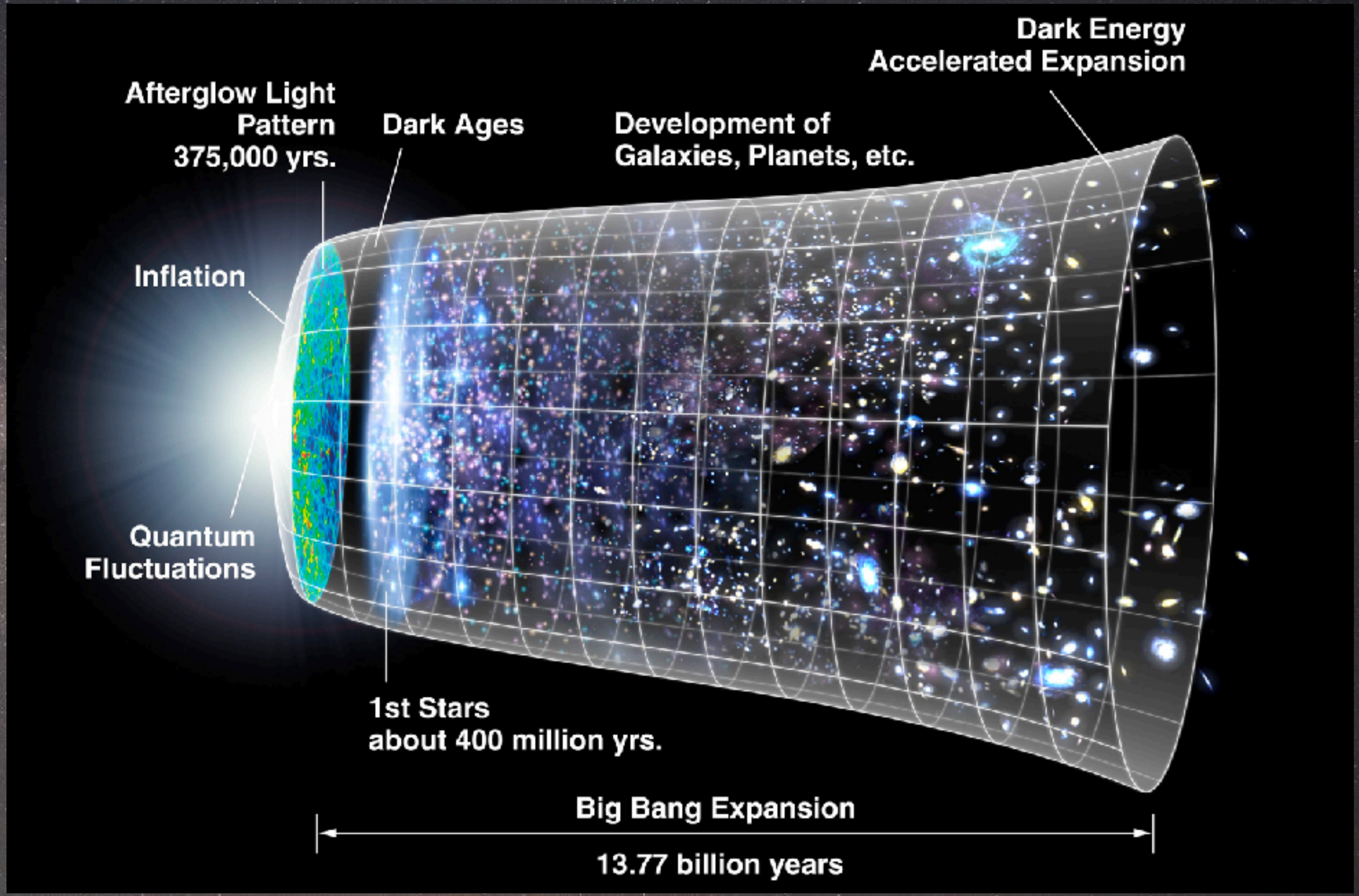
- Expected imprint on the CMB polarisation (propagation of gravitational waves in plasma)

- *B*-modes

- Parameter *r* proportional to energy scale

- *Today best constraint on r :*

$$r < 0.044$$



Ref : Tristram et al 2021, Planck + BICEP2/Keck data arXiv:2010.01139

Photo-credit : NASA/WMAP Science Team

I - Polarized signals in astrophysics and cosmology

Physics with a **preferred direction** will tend to produce light with a preferred **direction of oscillation (i.e. Polarization)**

Common in astrophysics (magnetic fields, grain shapes …)

My interest here will be focused on: **Large scale** polarized emission in the **Microwave/IR**

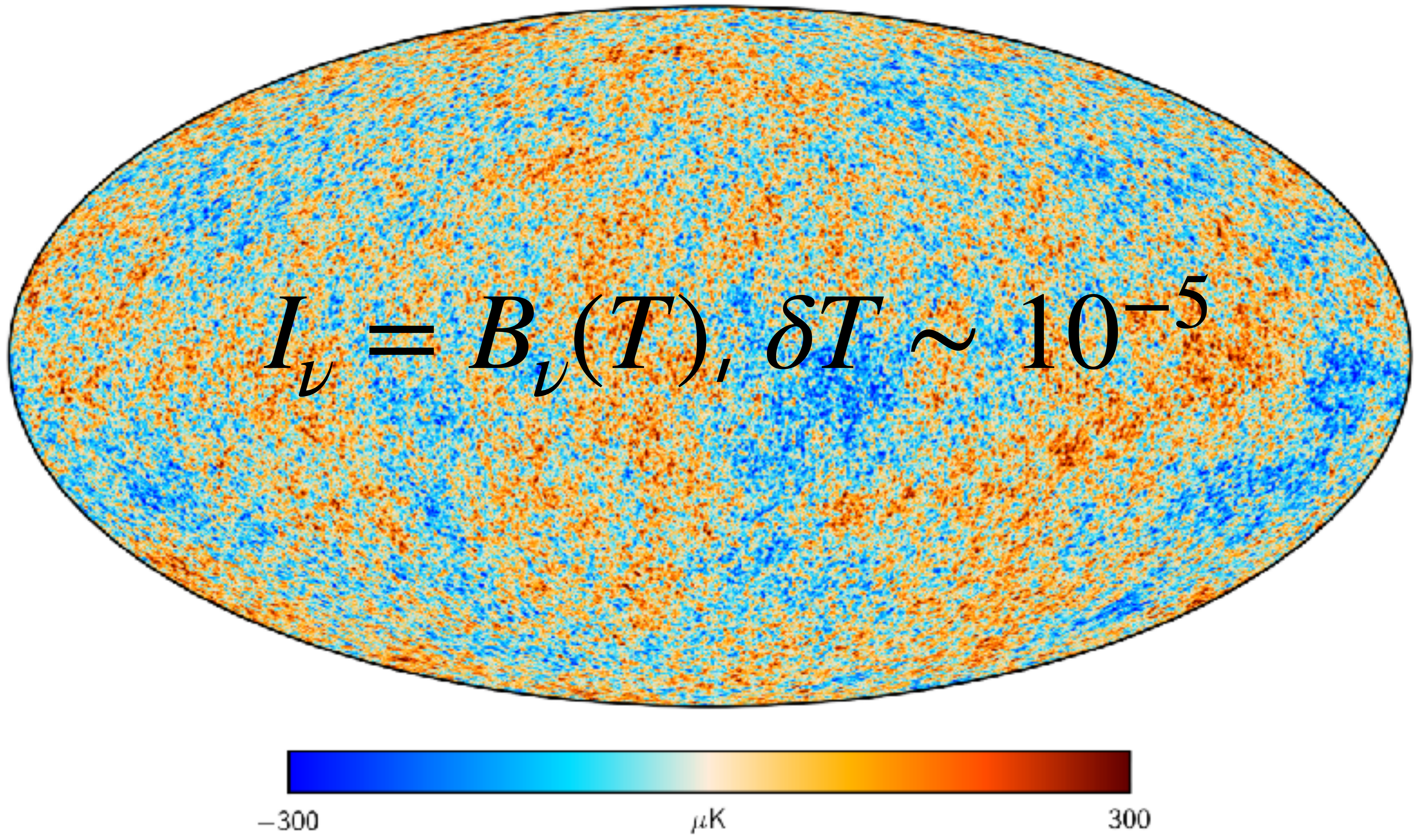
My point is …

Understanding better complicated **astrophysical polarized signal** is crucial to:

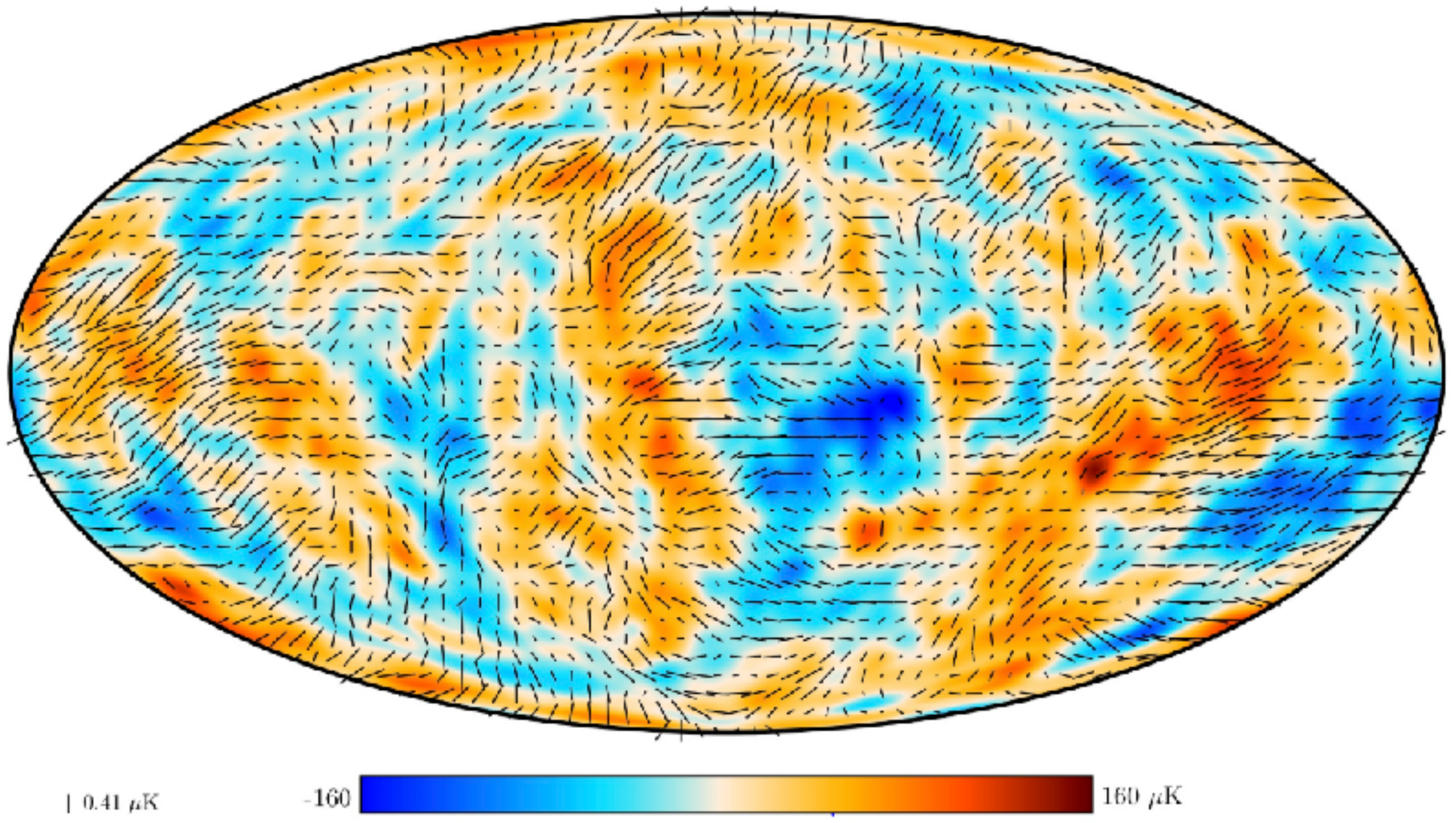
- Understand the **physics of emitting points** (critical for galactic physics, cosmology, high energy physics …)
- « Clean » the **polarized foregrounds** from CMB signal (or else)

CMB Polarization

Intensity



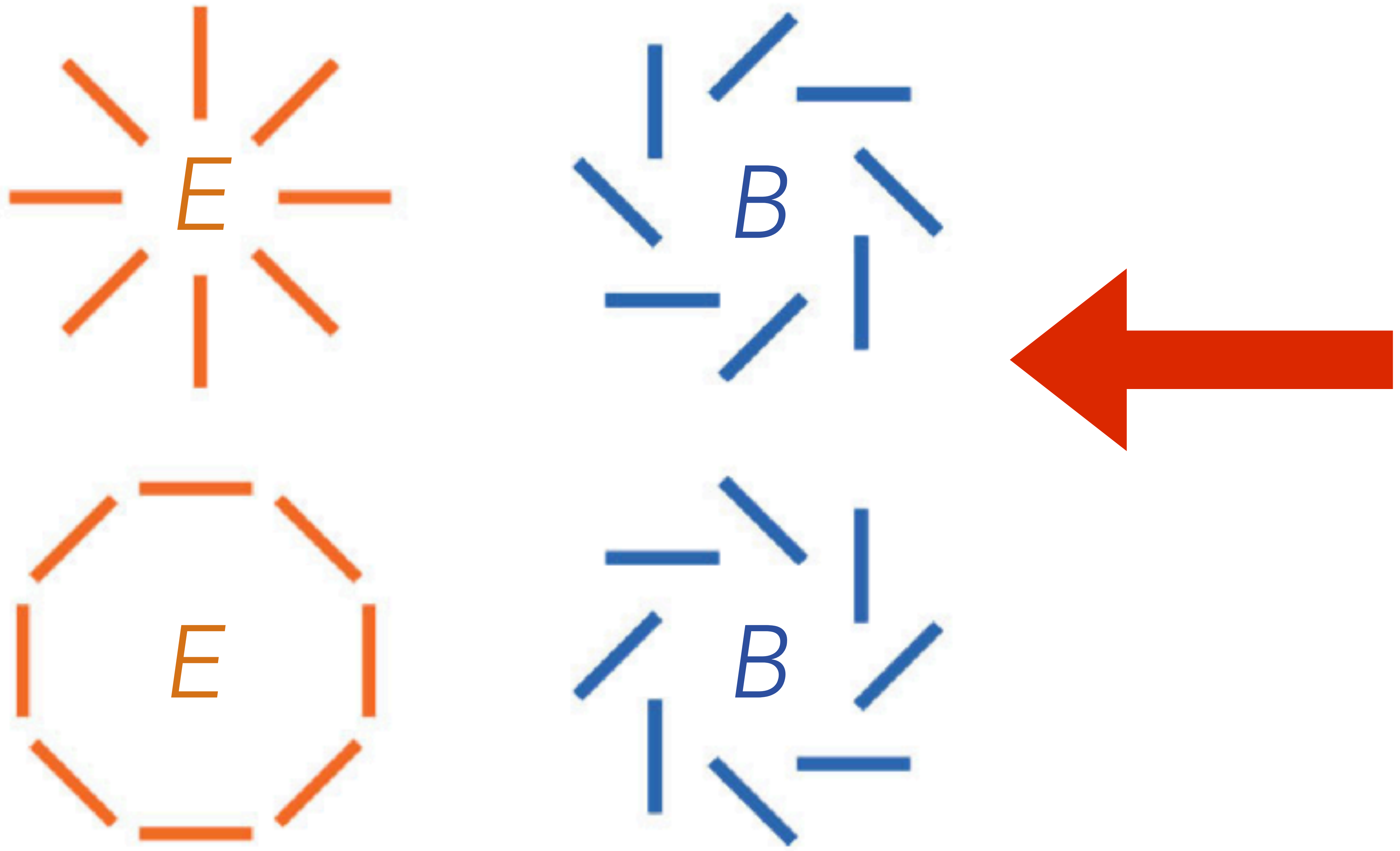
Polarization



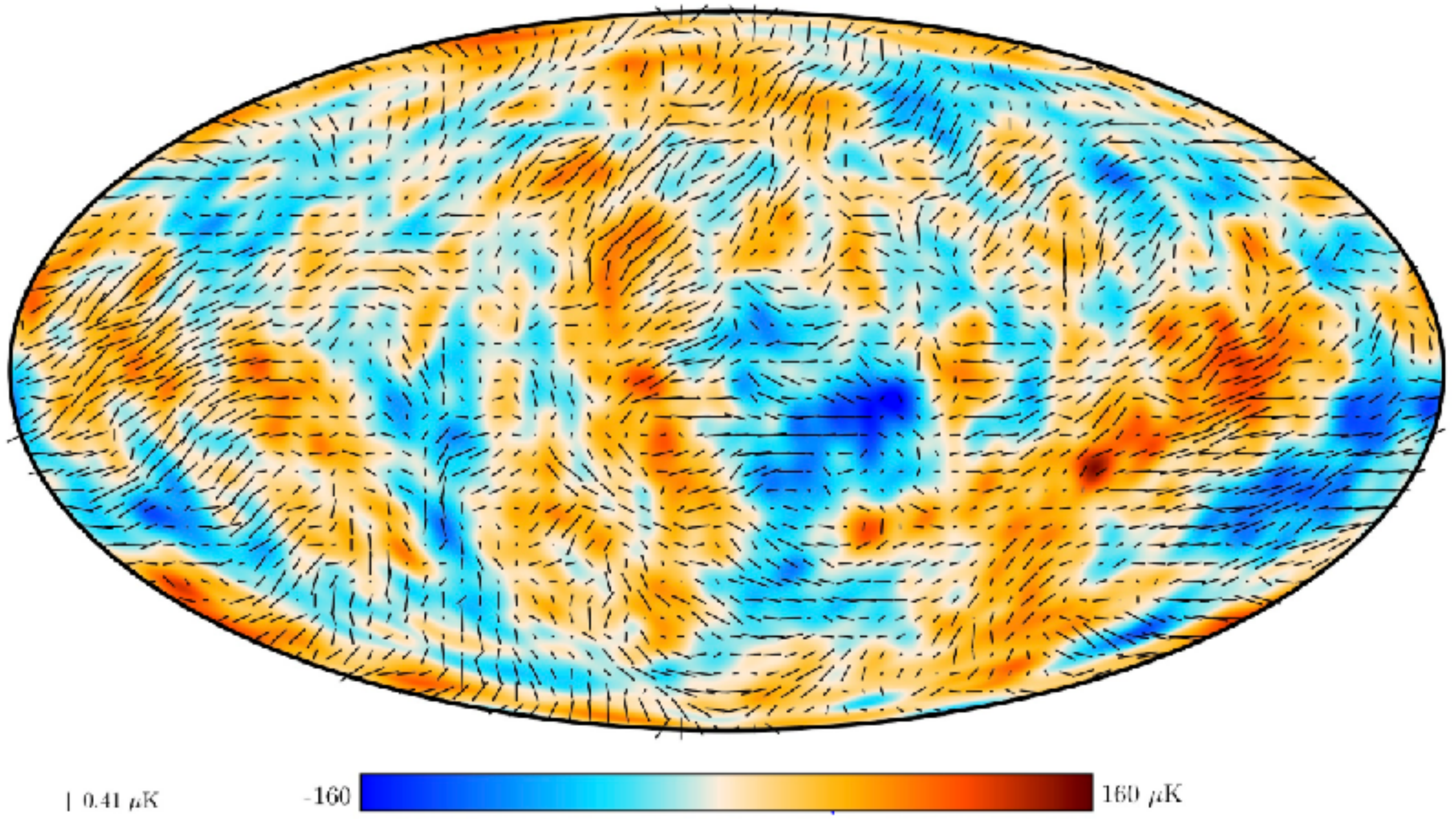
[Planck 2018]

CMB Polarization

E- and *B*- modes



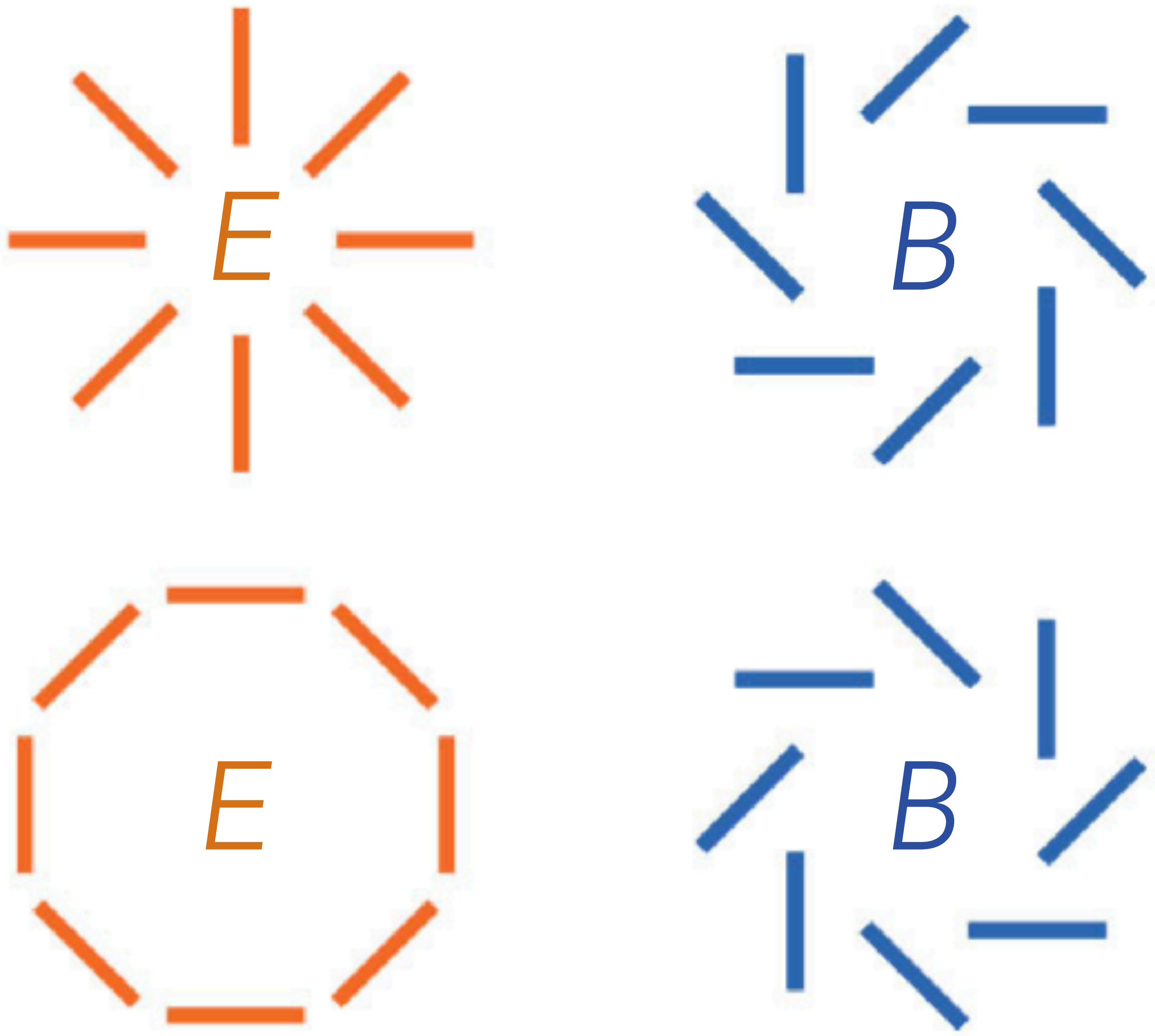
Polarization



[Planck 2018]

CMB Polarization

E- and *B*- modes



Ω_i Densities

H_0 Hubble parameter

τ Reionization

r, n_s Inflation

And much much more!

[Planck 2018]

Cosmic Inflation and the *B* modes

Puzzles with Big-Bang cosmology :

- Flatness
- Horizon
- Extremely low entropy
- Cosmological defects
- Formation of structures

} Fine tuning (Flatness, Horizon, Extremely low entropy)

Calls for a new mechanism : **inflation**
New scalar degree of freedom : **Inflaton**

Primordial Universe expansion :
x 10^{26} in 10^{-35} s after primordial singularity

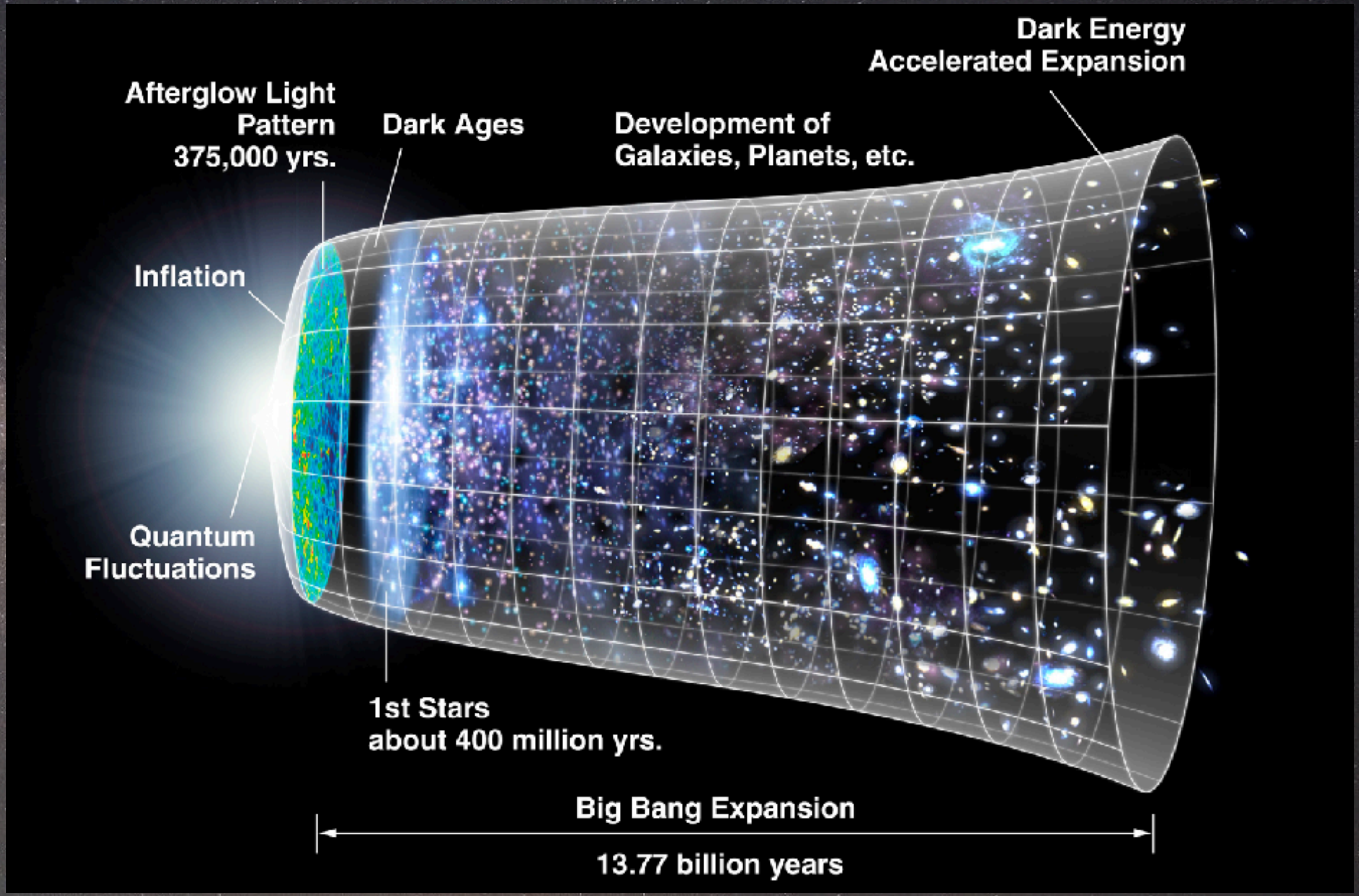


Photo-credit : NASA/WMAP Science Team

Cosmic Inflation and the *B* modes

Now inflation is part of the **standard model of cosmology**

Several hints but **no direct observation**

Would be the only source of primordial ***B-modes***

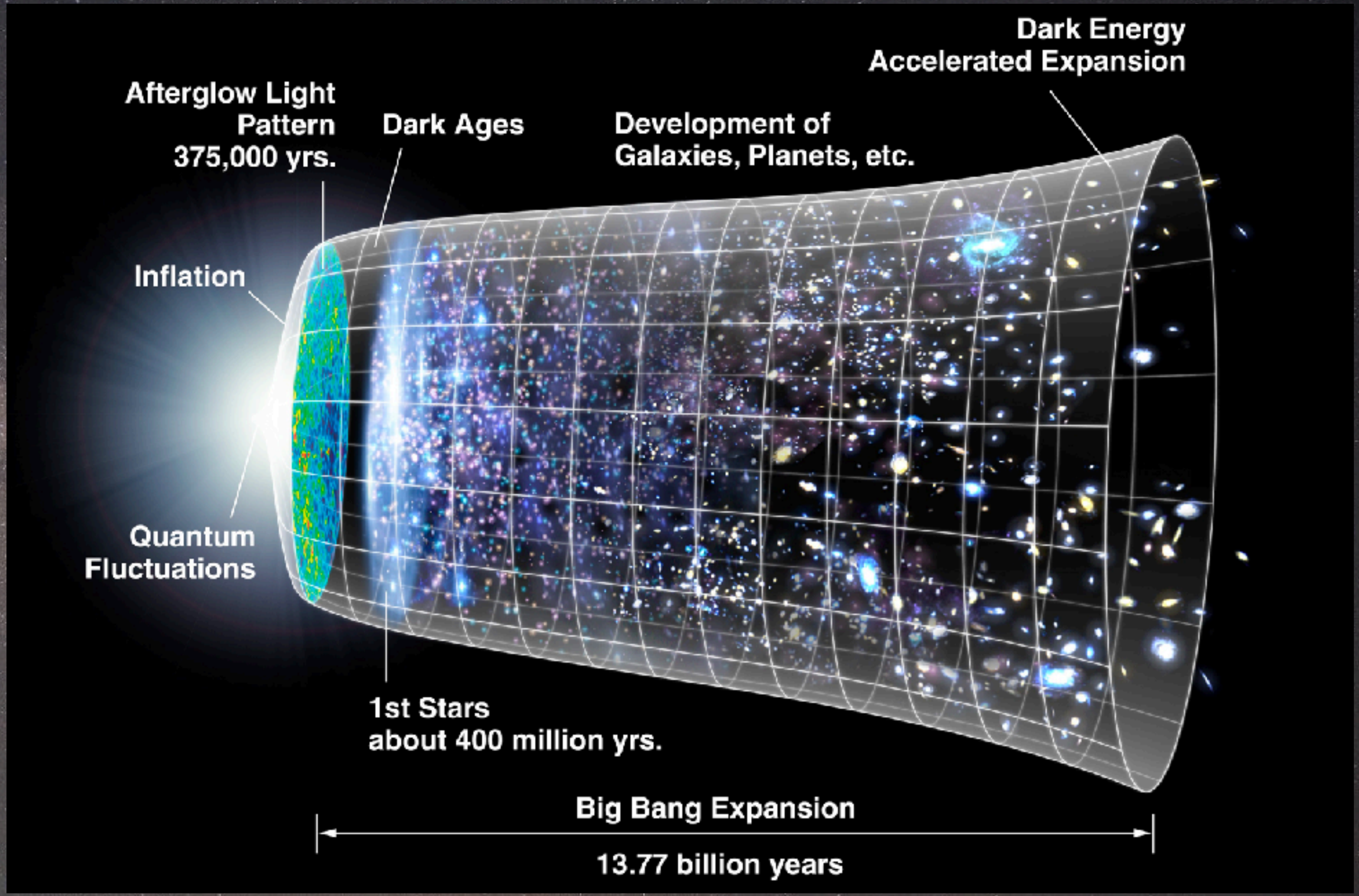
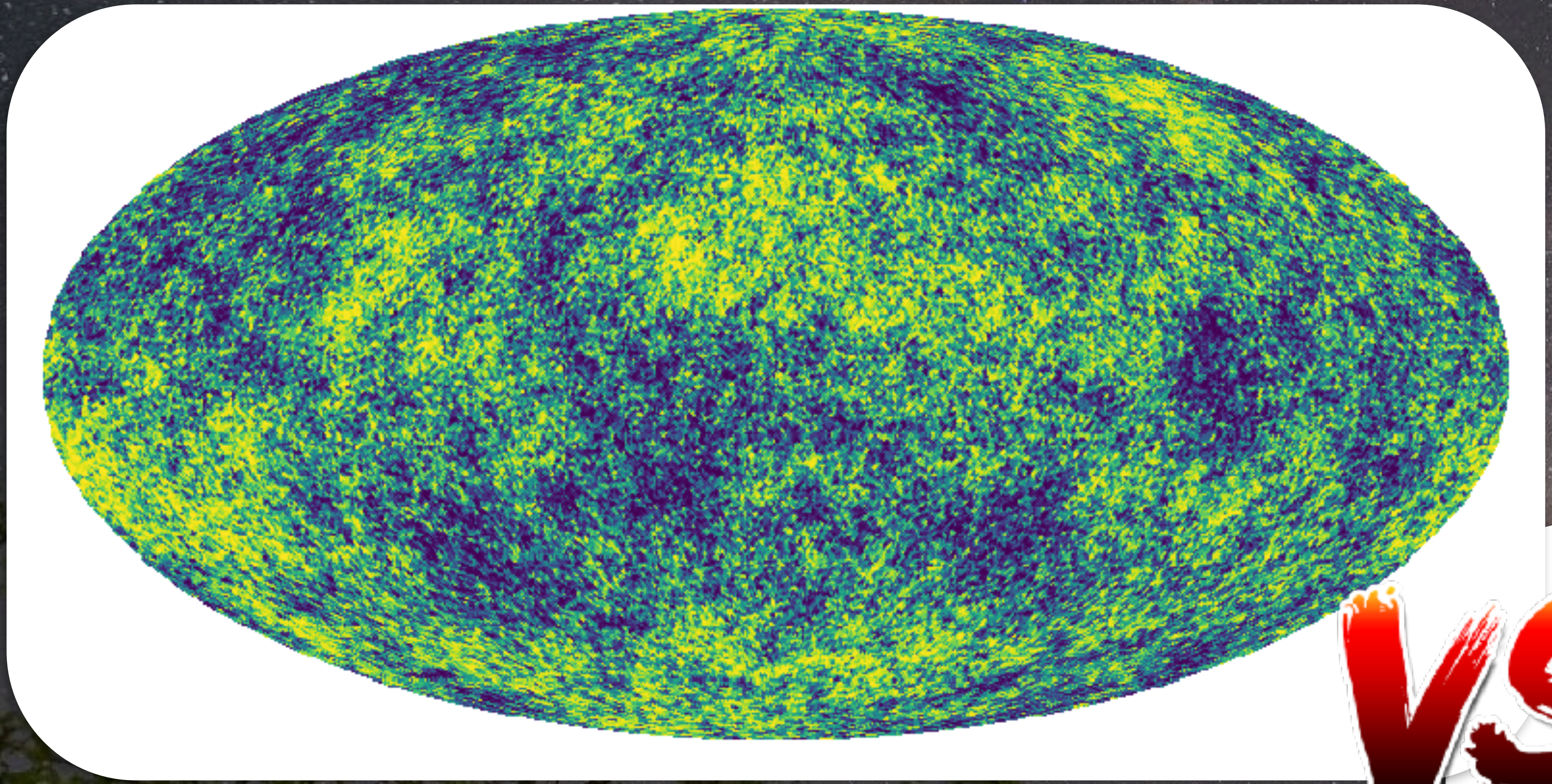


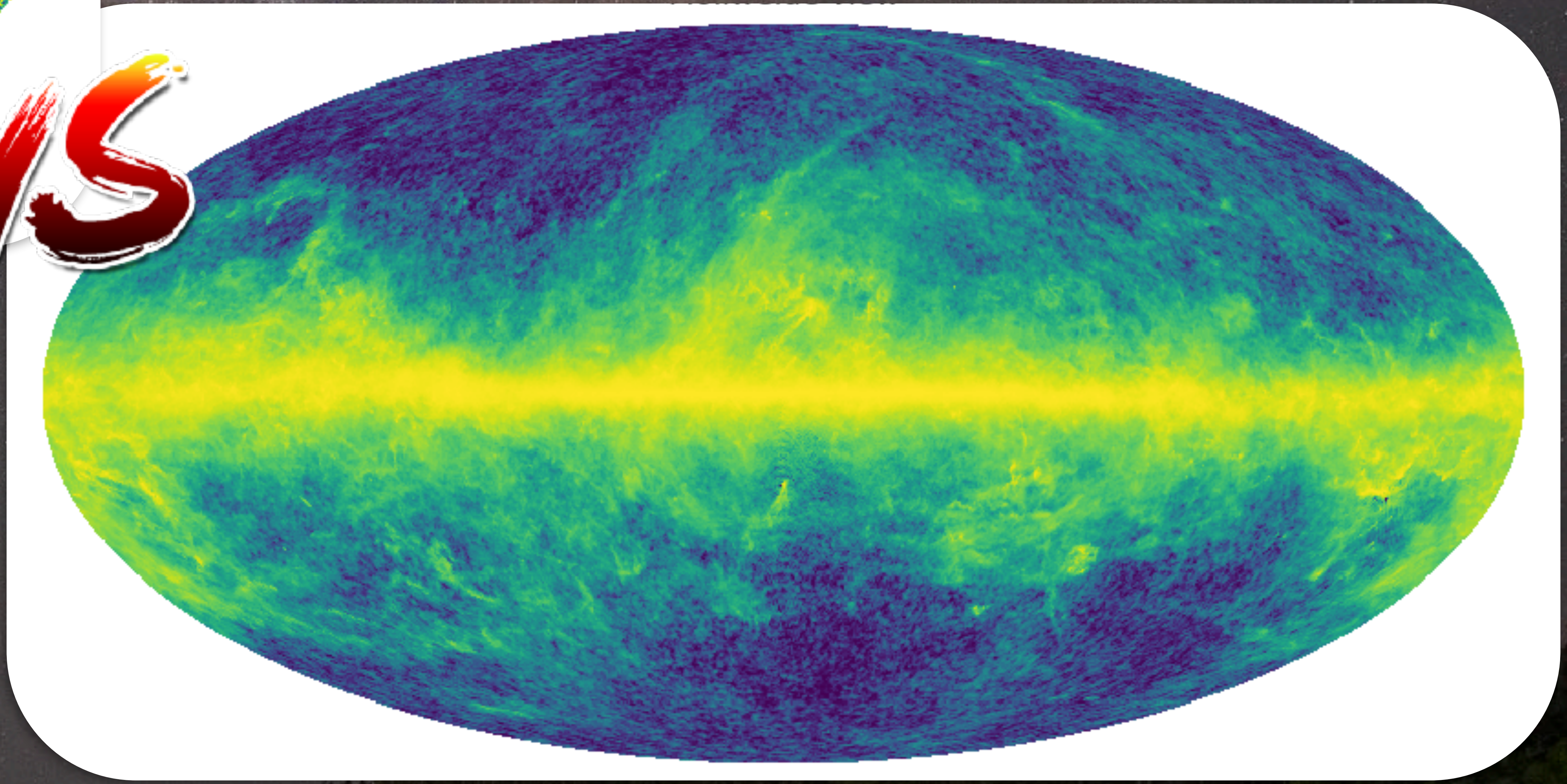
Photo-credit : NASA/WMAP Science Team

Galactic foregrounds



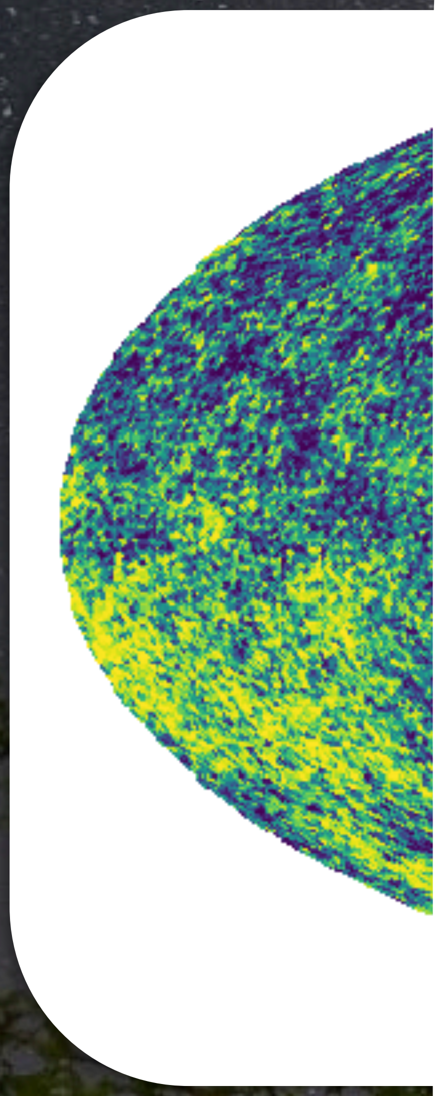
Polarized astrophysical sources emitting mainly in CMB's wavelength interval:

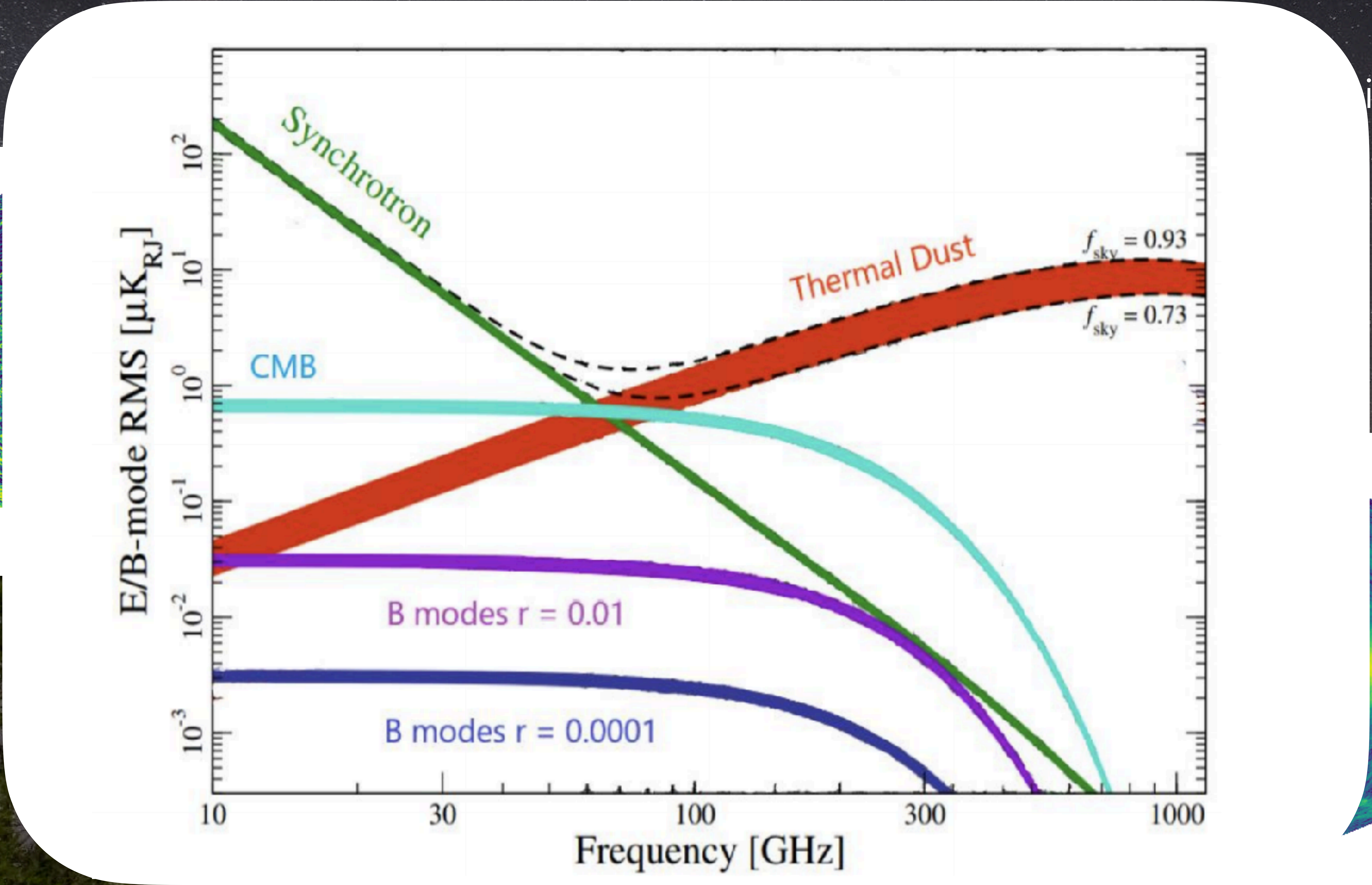
- ★ Dust thermal emission
- ★ Synchrotron
- ★ Spining dust (AME)

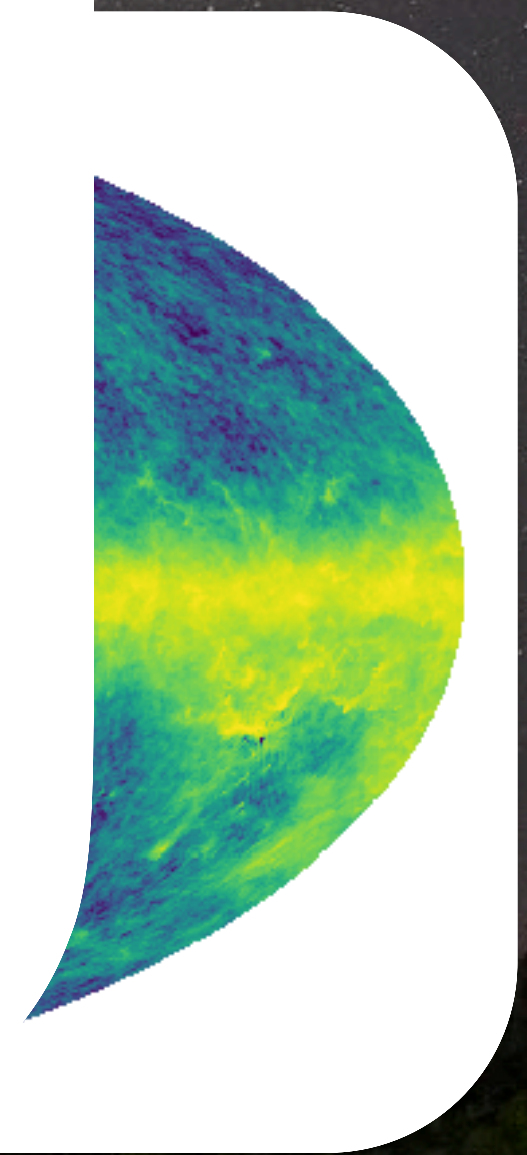


Galactic foregrounds

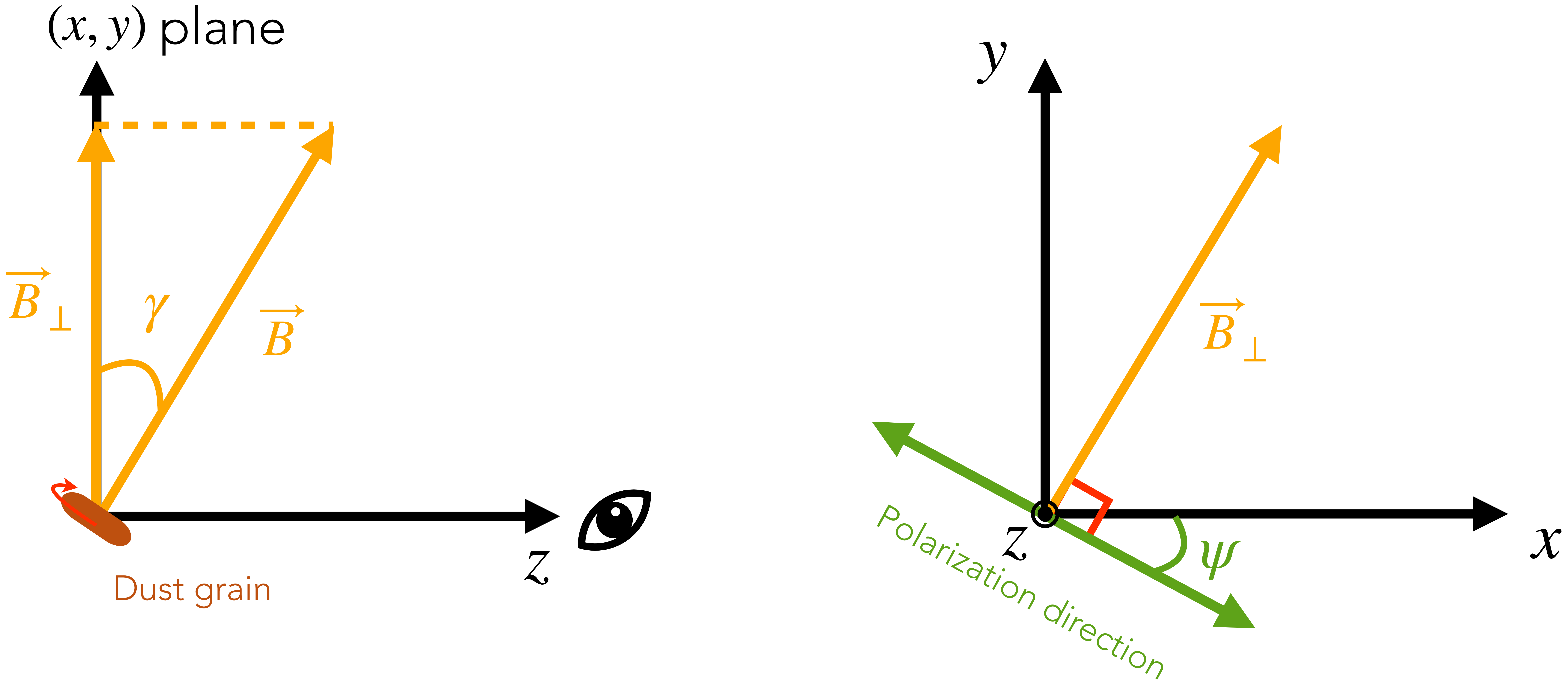
inly in



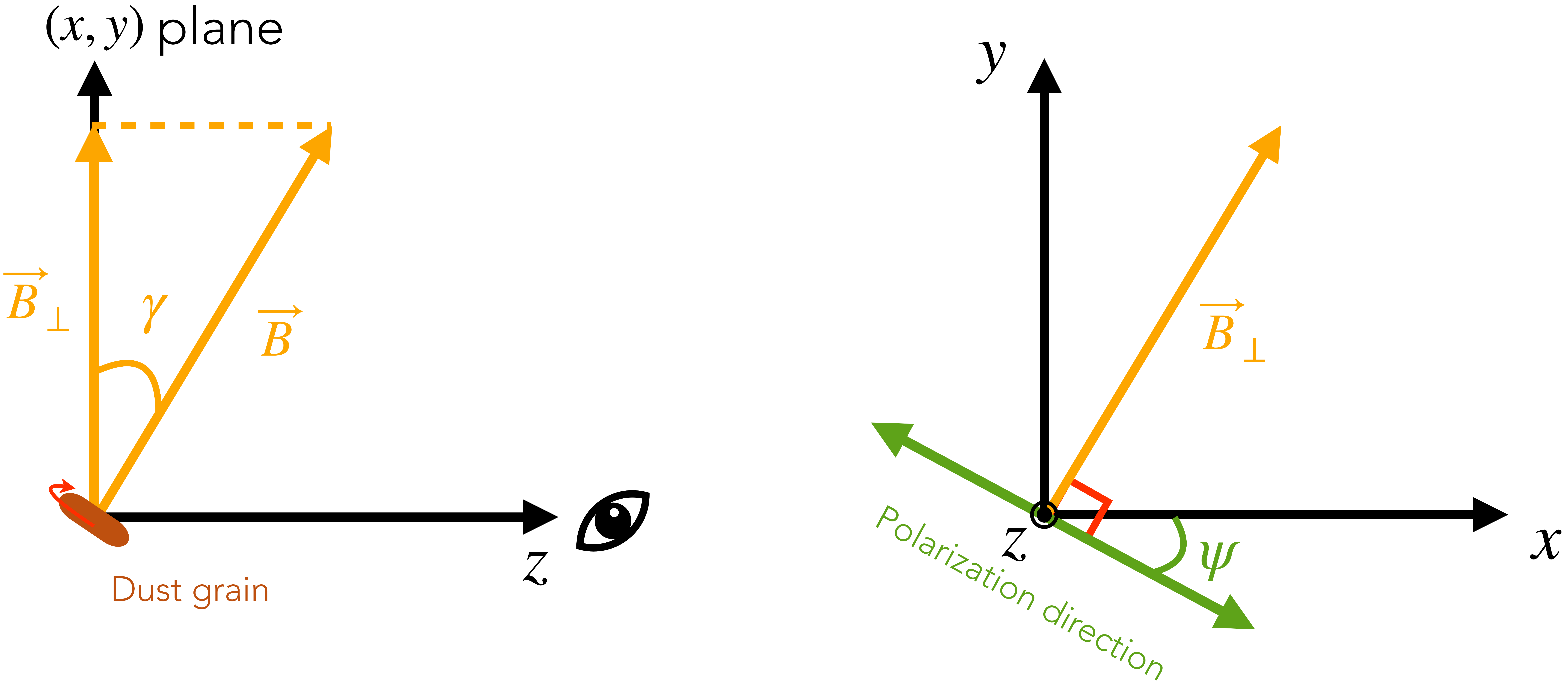




[Planck 2018]



(x, y) plane
$\overrightarrow{B}_\perp$
γ
$\overrightarrow{B}$
z
Dust grain
y
$\overrightarrow{B}_\perp$
z
ψ
x
Polarization direction


(x, y) plane
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Polarization direction