Čerenkov radio pulses from electromagnetic showers in the time-domain

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Why study electromagnetic pulse production of showers in the time-domain?

Currently many experiments are attempting to detect the
strong radio emission from neutrino induced particles showers
in dense dielectric media.

Radio measurements are performed in the time-domain.

Frequency domain computations, although technically
equivalent to td, do not illuminate understanding of
the pulse phase.

Simulation results show it is possible to map time-domain
pulse behavior to the shower charge profile.
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The LPM effect has distinct manifestations in the time-domain.

Other pulse properties of interest manifest themselves in the
time-domain pulse shape.

Computation of Electromagnetic Fields in the Time-Domain from First Principles.

Maxwell's equations in the transverse gauge produce wave equations for the potentials ϕ and **A**

$$\nabla^2 \phi = -\frac{\rho}{\epsilon}$$

$$\nabla^2 \mathbf{A} - \mu\epsilon \frac{\partial^2 \mathbf{A}}{\partial^2 t} = -\mu \mathbf{J}_\perp$$

Only transverse currents contribute to radiation fields

In the far field

$$\mathbf{J}_\perp = -\hat{\mathbf{u}} \times (\hat{\mathbf{u}} \times \mathbf{J})$$

Direction of observation

The gauge fields can be solved using Green's functions

$$\phi = \frac{1}{4\pi\epsilon} \int \frac{\rho(\mathbf{x}', t')}{|\mathbf{x} - \mathbf{x}'|} d^3\mathbf{x}'$$

Contribution vanishes in the far-field.

$$\mathbf{A}(t, \theta) = \frac{\mu}{4\pi} \int \frac{\mathbf{J}_\perp(\mathbf{x}', t')}{|\mathbf{x} - \mathbf{x}'|} \delta(\sqrt{\mu\epsilon}|\mathbf{x} - \mathbf{x}'| - (t - t'))d^3\mathbf{x}' dt'$$

Strategy: compute all contributions to the vector potential and take a derivative to obtain the electric field

$$\mathbf{E}(t, \mathbf{x}) = -\frac{\partial}{\partial t}\mathbf{A}(t, \mathbf{x})$$

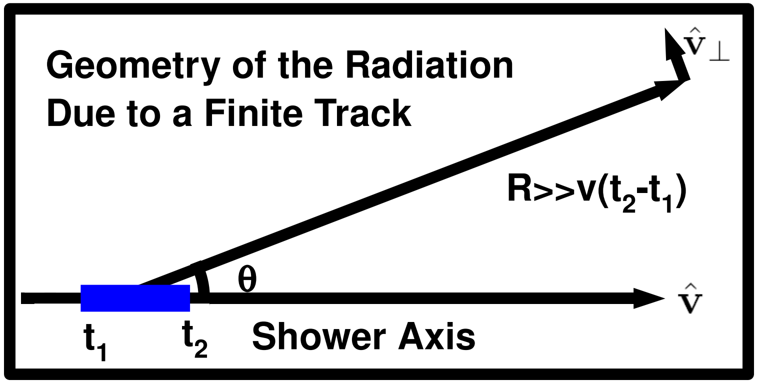
Radiation from a charged particle track

Current density

$$\mathbf{J}_{\perp}(\mathbf{x}', t') = e\mathbf{v}_{\perp}\,\delta^3(\mathbf{x}' - \mathbf{x_0} - \mathbf{v}t')[\Theta(t' - t_1) - \Theta(t' - t_2)]$$

Apply the far-field approximation

$$|\mathbf{x} - \mathbf{x_0} - \mathbf{v}t'| \simeq R - \mathbf{v}\cdot\hat{\mathbf{u}}\,t'$$



Vector Potential

$$\mathbf{A}\,(t,\theta) = \frac{\mu e}{4\pi R}\mathbf{v}_{\perp}\frac{\Theta(t - \frac{nR}{c} - (1 - n\beta\cos\theta)t_1) - \Theta(t - \frac{nR}{c} - (1 - n\beta\cos\theta)t_2)}{(1 - n\beta\cos\theta)}$$

With limit $\theta \to \theta_C$

$$R\mathbf{A}(t,\theta_C) = \left[\frac{e\mu_r}{4\pi\epsilon_0 c^2}\right]\delta\left(t - \frac{nR}{c}\right)\mathbf{v}_{\perp}\,\delta t$$

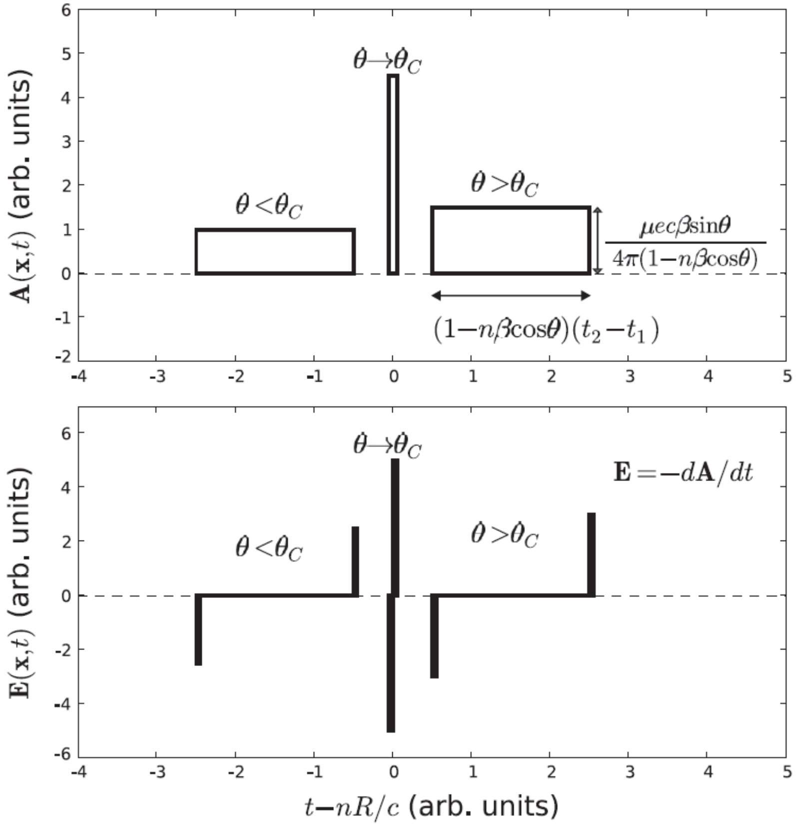
Detector sampling time bin width

The electric field

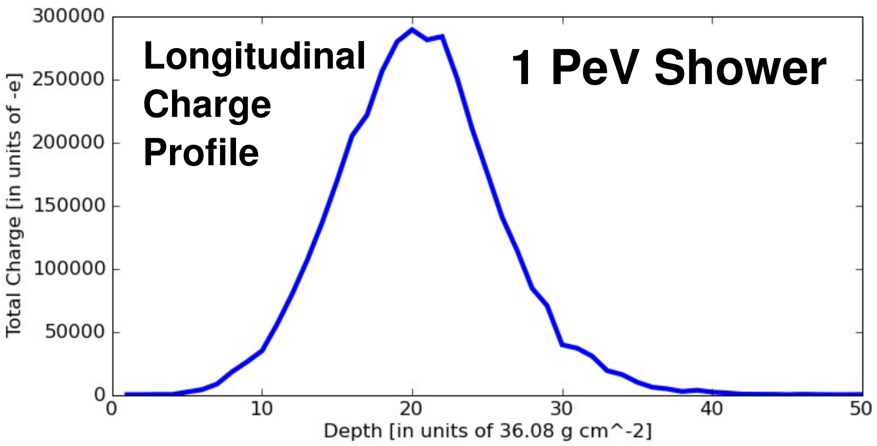
$$\mathbf{E}(t,\mathbf{x}) = -\frac{\partial}{\partial t}\mathbf{A}(t,\mathbf{x})$$

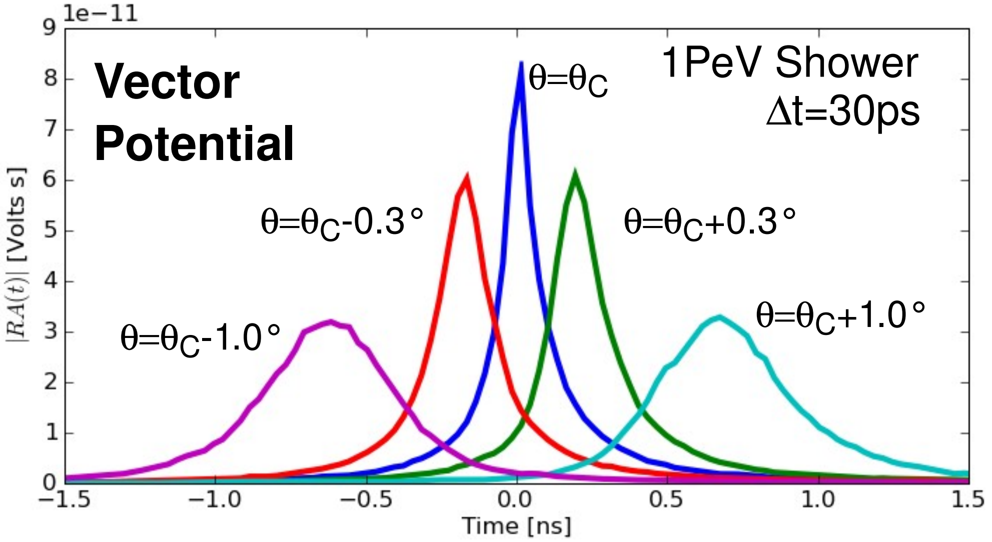
$$R\mathbf{E}(t,\theta) = -\frac{e\mu_r}{4\pi\epsilon_0 c^2}\mathbf{v}_{\perp}\frac{\delta(t - \frac{nR}{c} - (1 - n\beta\cos\theta)t_1) - \delta(t - \frac{nR}{c} - (1 - n\beta\cos\theta)t_2)}{(1 - n\beta\cos\theta)}$$

Fourier transform gives RE(w) used in ZHS frequency-domain calculations.



Time-domain radiation from electromagnetic showers in ice.

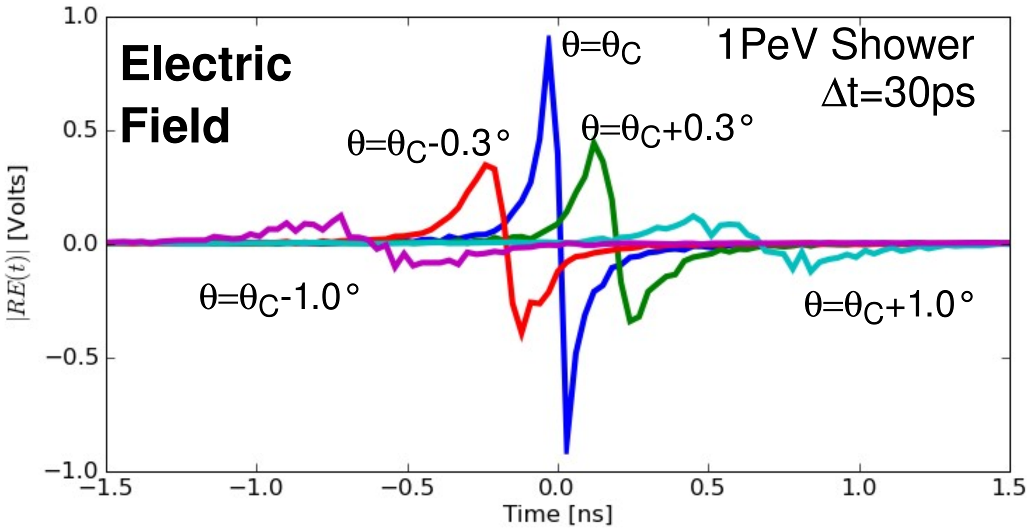




Time-domain calculation added to ZHS code.

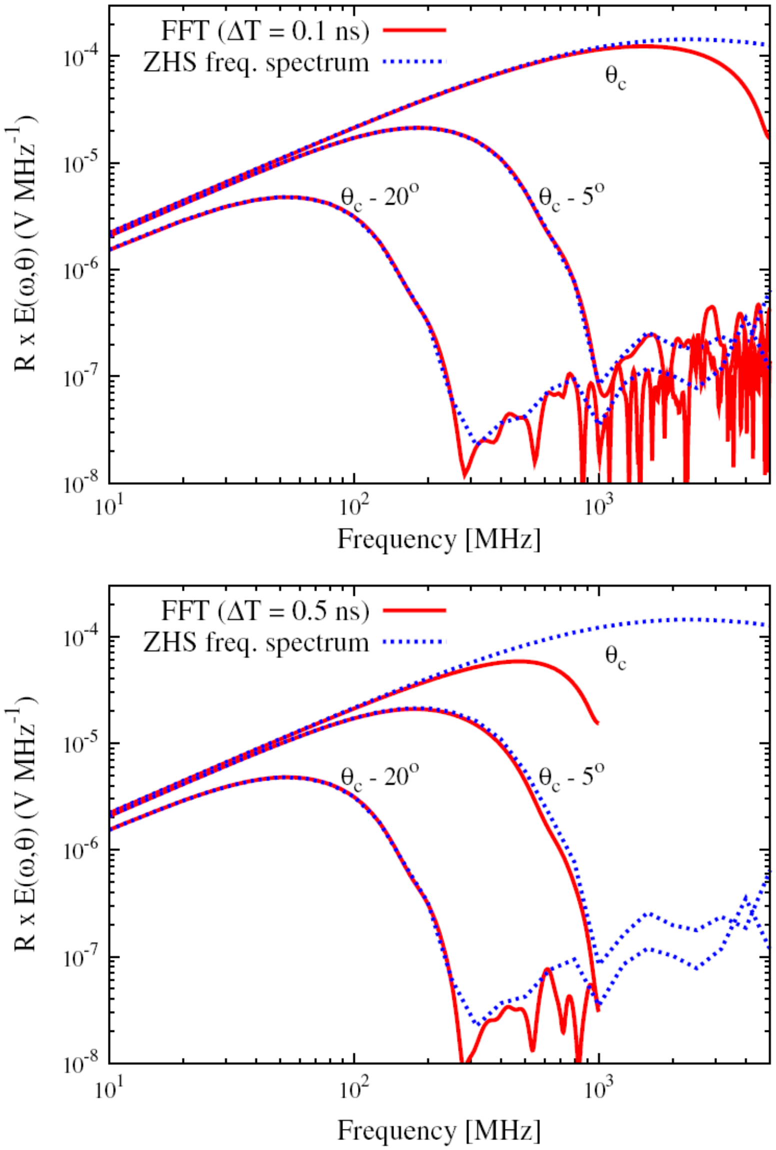
Vector potential traces the lateral charge profile.

Electric field has interesting causal relations.

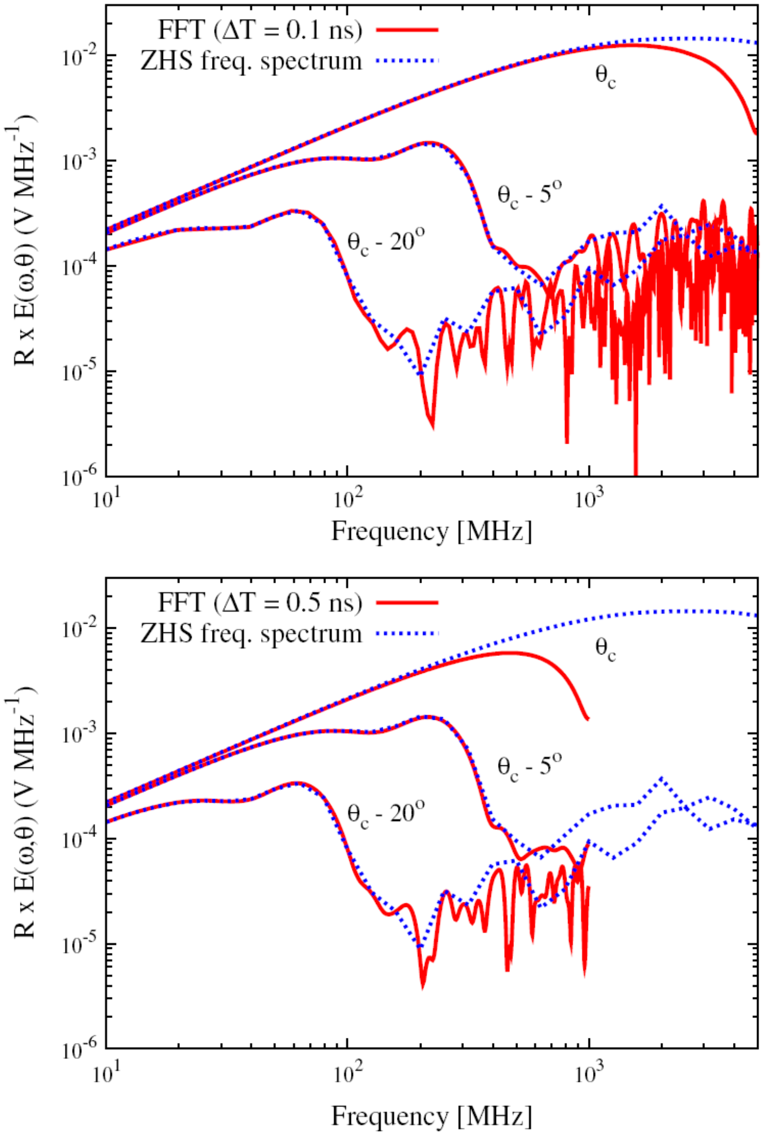


Consistency with Frequency Domain Calculations

1 PeV Shower



100 PeV Shower



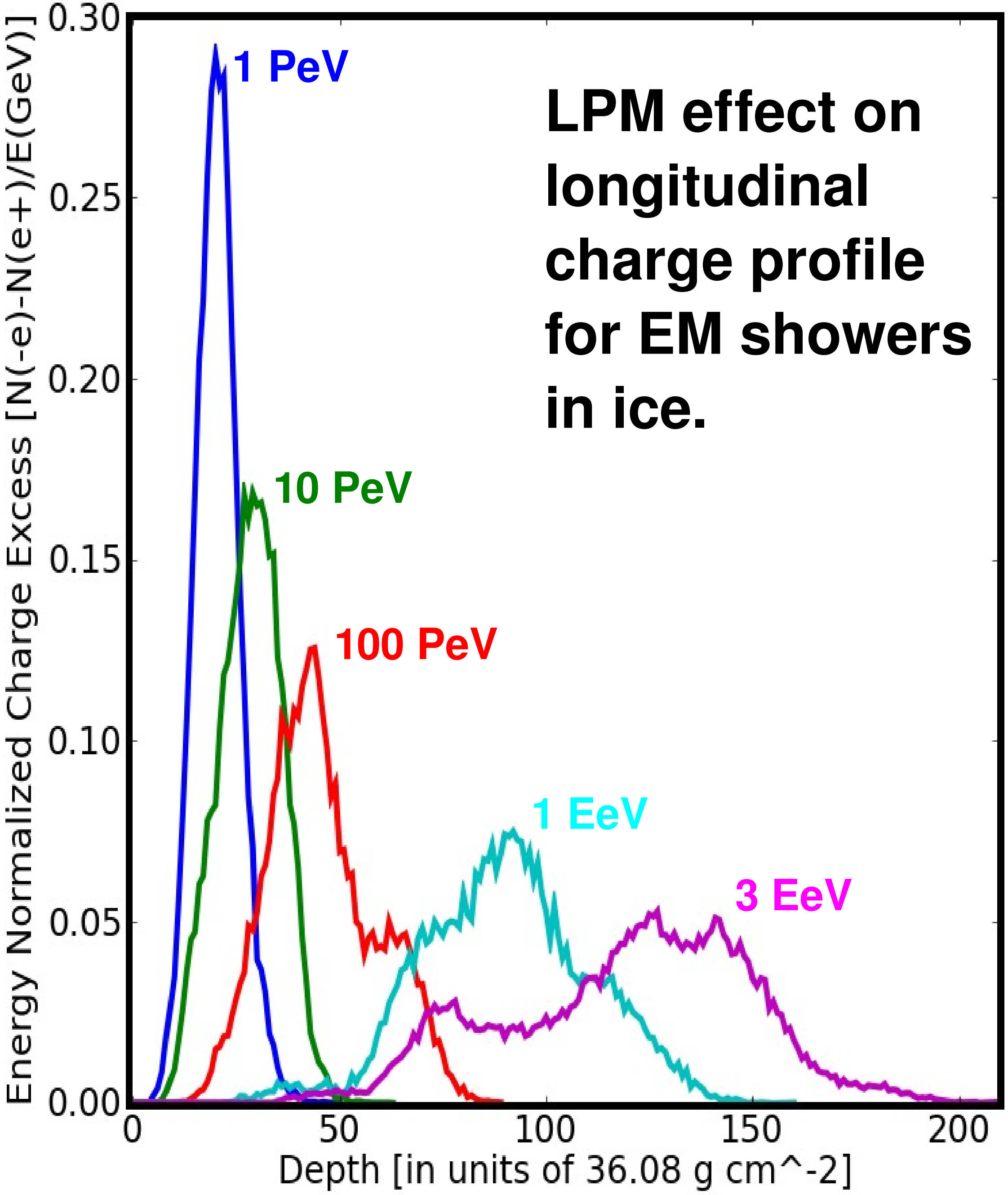
The LPM Effect

Relativistic screening effect on electron-photon interaction reduces the electromagnetic cross-section.

Effect turns on at E>1 PeV.

Produces shower elongation.

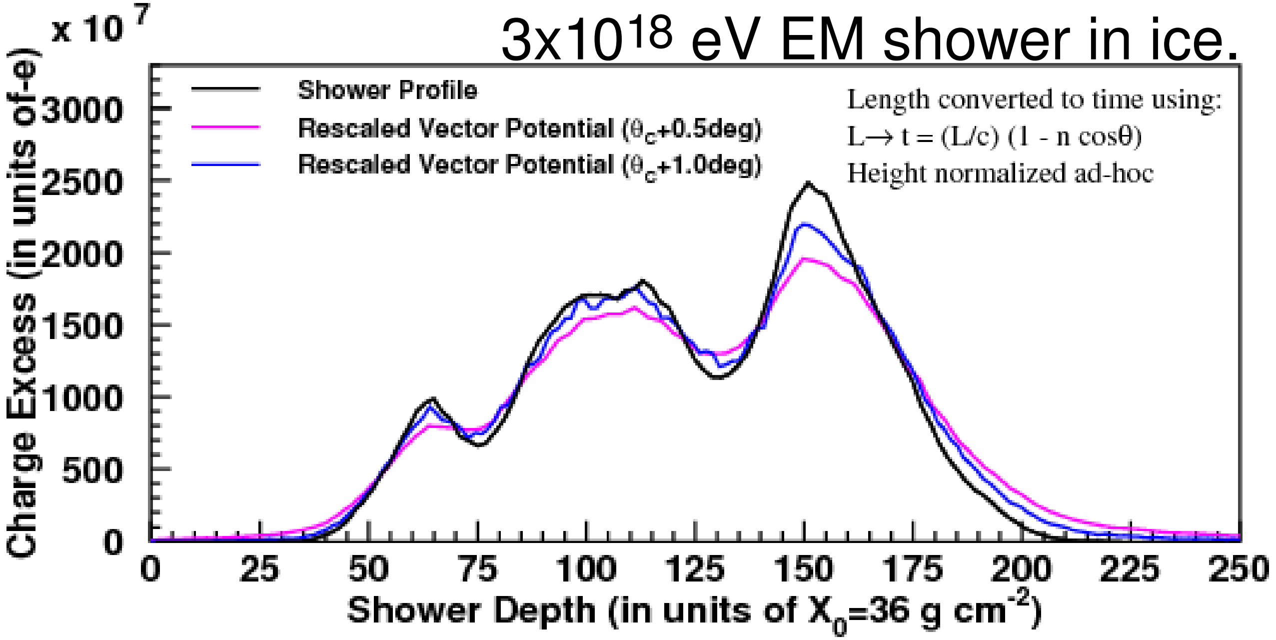
At E>100PeV it produces multiple lumps on the longitudinal charge profile.

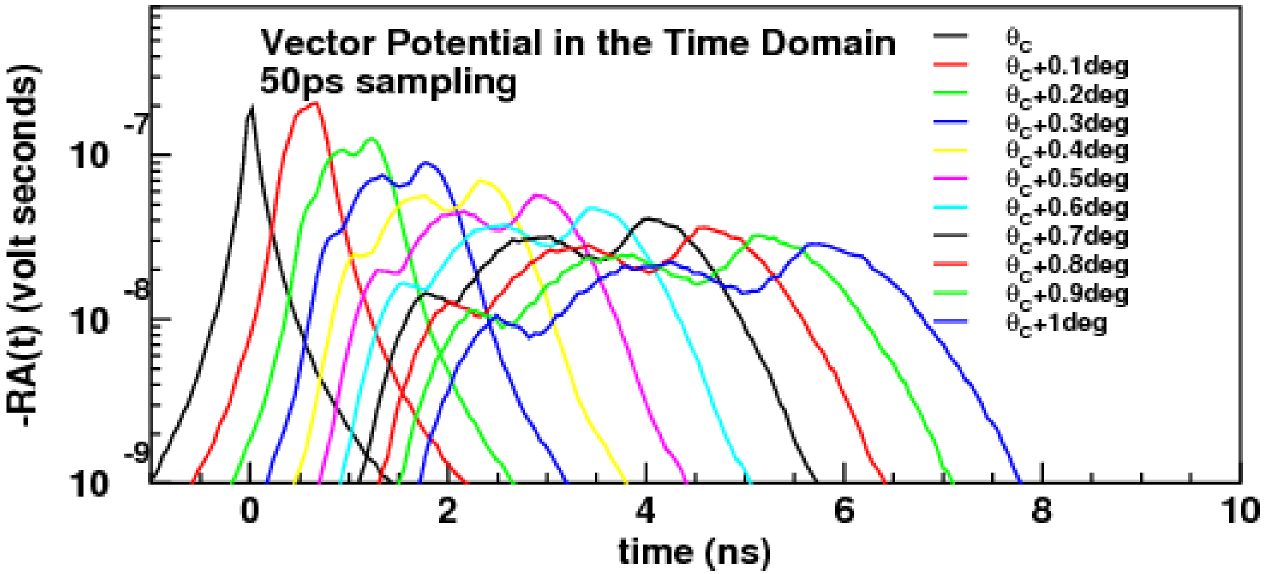


Relation between vector potential and longitudinal charge profile

For observations away from the Cerenkov angle the vector potential traces the longitudinal charge profile.

$3\text{x}10^{18}$ eV EM shower in ice.





1-Dimensional Model

Model shower excess charge as a current density

$$\mathbf{J}(\mathbf{x}', t') = \mathbf{v}\, f(z', \rho')Q(z')\delta(z' - vt')$$

Assume no lateral profile, i.e., f(z,ρ')=δ(x)δ(y)

Salient Features:

Δt~Δz(1-n cos θ)

A~Qsinθ/|1-n cos θ|

Q(z') is the charge excess profile
(well modeled as ~20% of the shower profile).

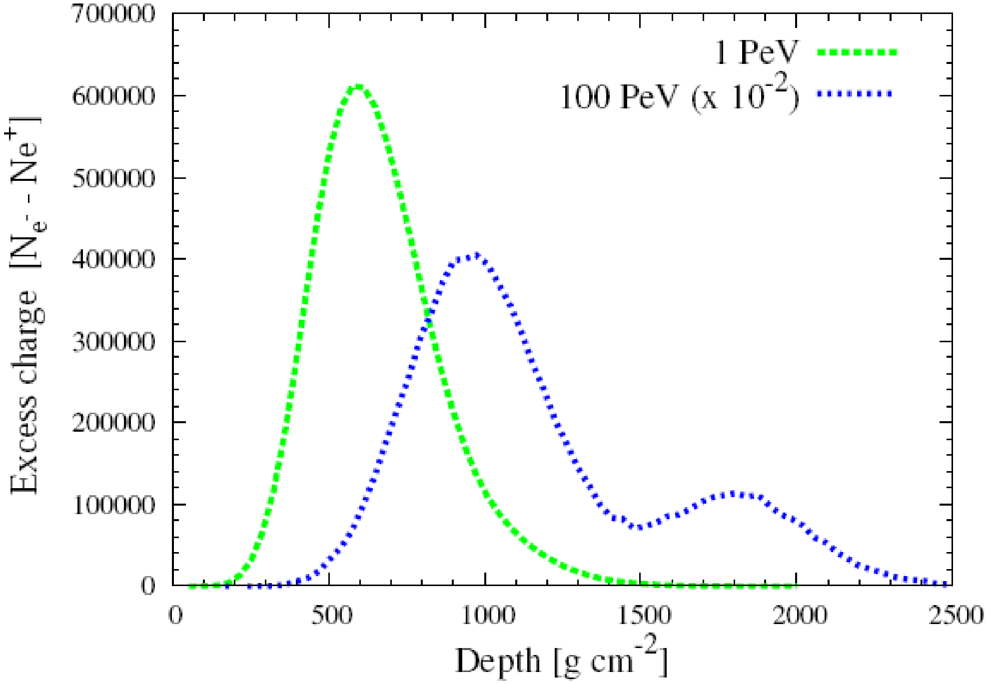
$$\mathbf{A}_{rad}(R, \theta, t) = \mathbf{v}_{\perp} \frac{\mu}{4\pi R} \frac{Q(\zeta(t))}{|1 - n\beta\cos\theta|}$$

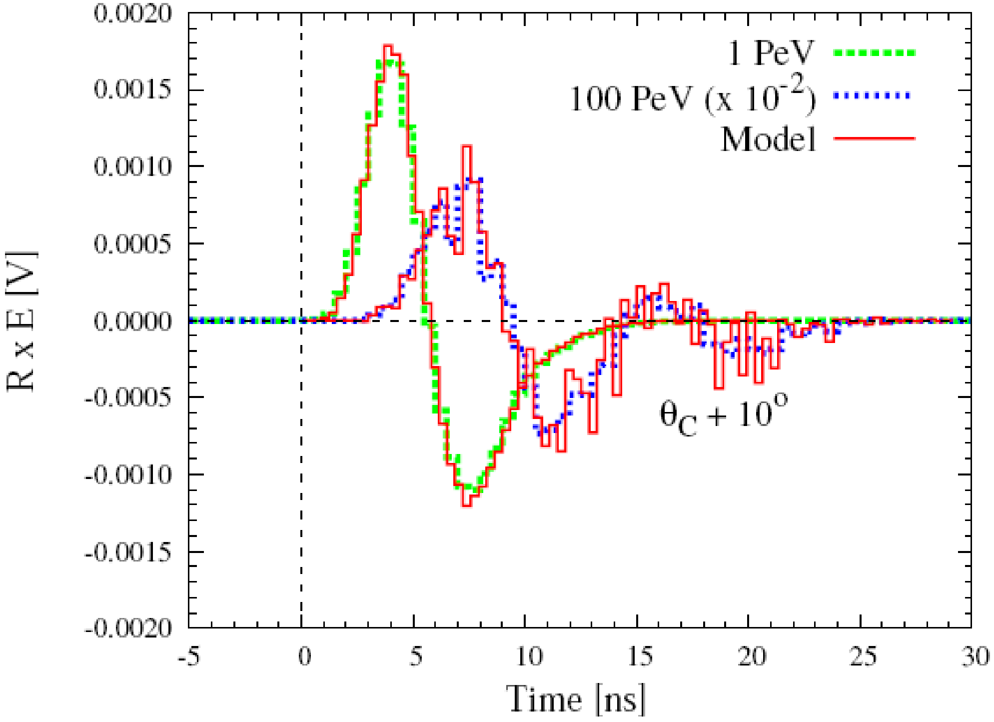
ζ maps the time coordinate
to z' via the linear relation

$$\zeta(t) = \beta \frac{ct - nR}{1 - n\beta\cos\theta}$$

$$\mathbf{E}_{rad}(R, \theta, t) = -\mathbf{v}_{\perp} \frac{\mu}{4\pi v R} \frac{1}{|1 - n\beta\cos\theta|(1 - n\beta\cos\theta)} \left.\frac{dQ(\zeta)}{d\zeta}\right|_{\zeta = \beta\frac{ct - nR}{1 - n\beta\cos\theta}}$$

1-Dimensional Model


Longitudinal Charge Excess Profile


Electric Field

Very accurate model for observation angles θ_C>2.5°

Overshoots high frequency contents for θ_C<2.5°

$\Delta t \sim \Delta z(1\text{-}n \cos \theta)$ relation has no cutoff as $\theta \rightarrow \theta_C$

Convolution Model

1-D model linearly rescales the longitudinal charge profile and length to get vector potential.

$$\mathbf{A}_{rad}(R,\theta,t) = \hat{\mathbf{z}}_{\perp}\frac{\mu v}{4\pi R}\frac{Q(\zeta(t))}{|1-n\beta\cos\theta|}$$

$$\zeta(t) = \beta\frac{ct-nR}{1-n\beta\cos\theta}$$

Full model contains a considerably harder integral

$$\mathbf{A}_{rad}(R,\theta,t) = \frac{\mu}{4\pi R}\int_{-\infty}^{\infty} dz' Q(z') \underbrace{\int_0^{\infty} r'dr' \int_0^{2\pi} d\phi' \hat{\mathbf{v}}_{\perp} f(z',r')\delta\left(z'\left[\frac{1}{v}-\frac{n}{c}\cos\theta\right]-\left[t-\frac{n}{c}(R-r'\sin\theta\cos(\phi-\phi'))\right]\right)}_{g\left(z'\frac{1-n\beta\cos\theta}{n\beta\sin\theta}-\frac{vt-n\beta R}{n\beta\sin\theta}\right)}$$

At the Cerenkov angle
$A_C(t) \propto g(t)$

$$\mathbf{A}_C(R,\theta_C,t) = \frac{\mu v}{4\pi R}\hat{\mathbf{z}}_{\perp}\frac{1}{n\beta\sin\theta} g\left(-\frac{vt-n\beta R}{n\beta\sin\theta}\right)\int_{-\infty}^{\infty} dz' Q(z')$$

Total Charge

Complete Model

$$A(t) = N\, A_{1D} * A_C$$

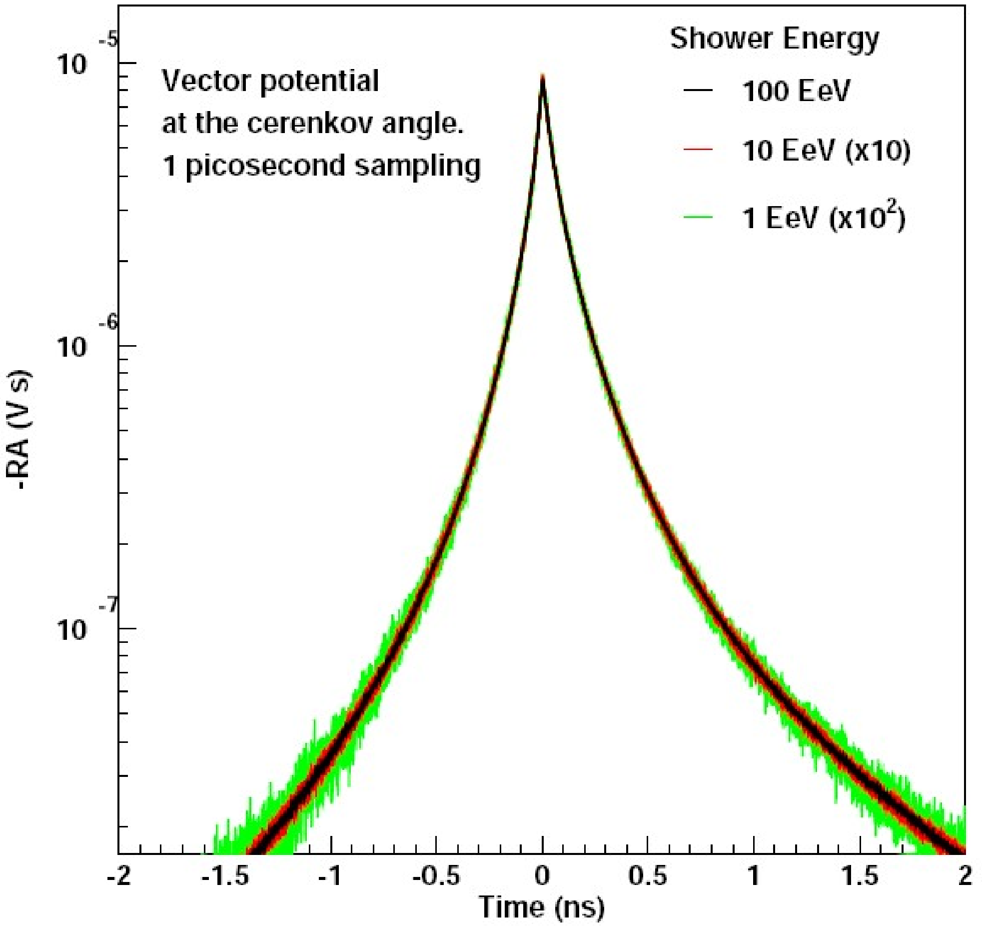
normalization

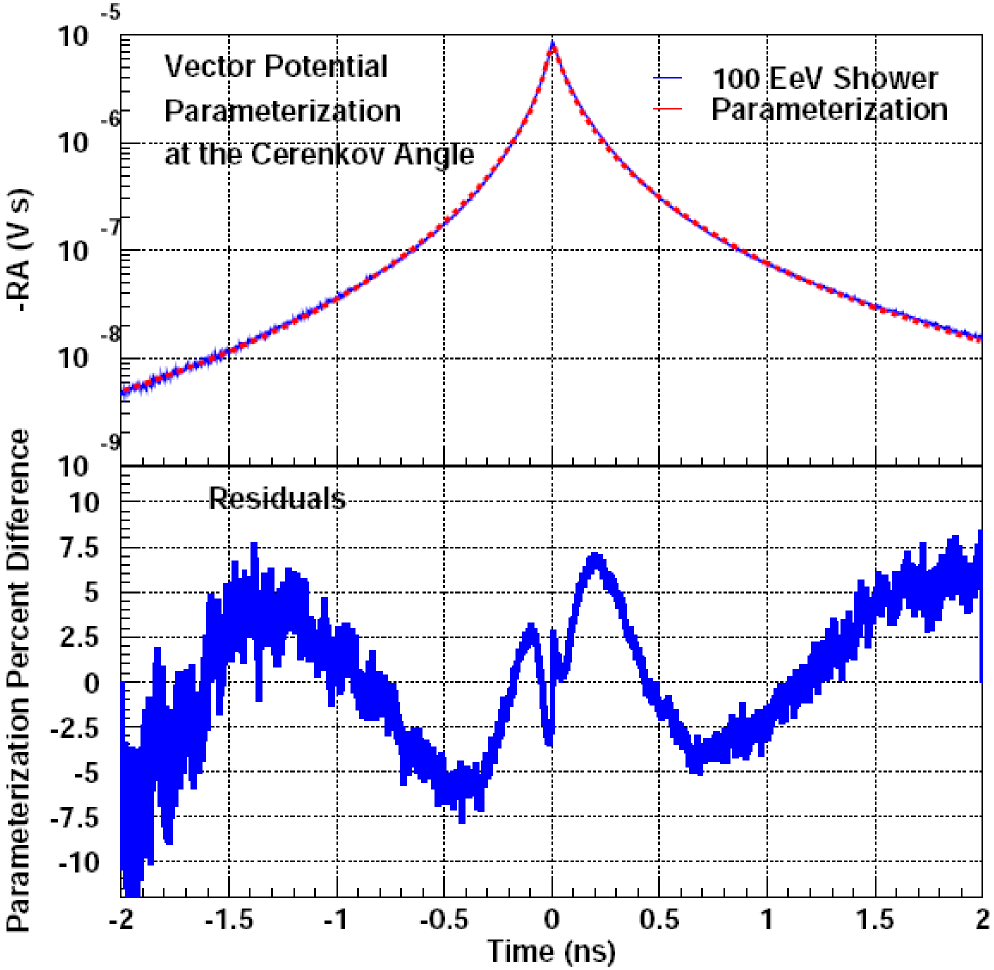
1D model from charge profile

convolution

Obtain $A_C(t)$ from simulations

Vector Potential Parametrization

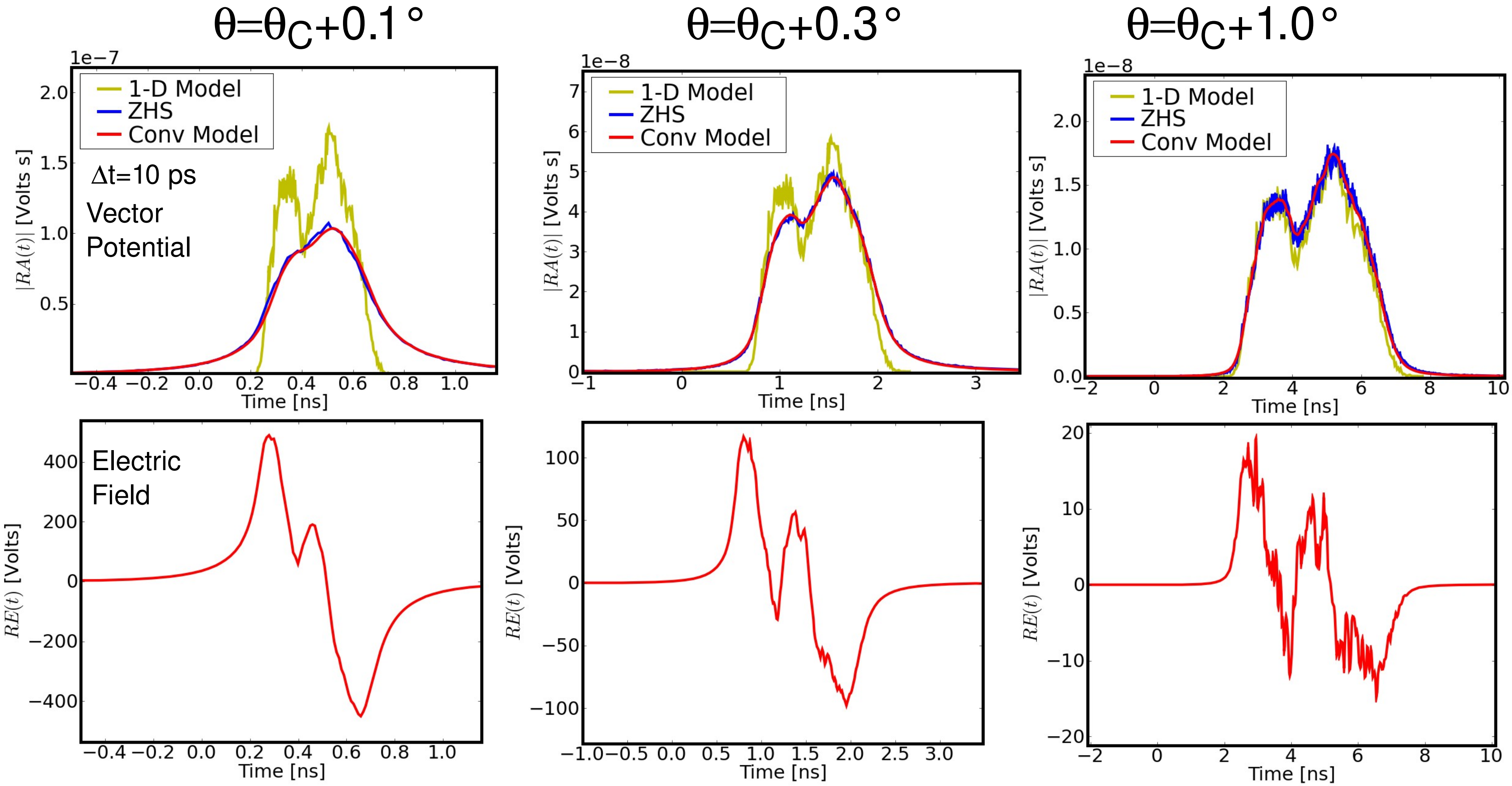




Paremeterization

$$RA(\theta_C, t) = -8.85 \times 10^{-14} [\mathrm{V\ s}] \frac{E}{[1\mathrm{TeV}]} \begin{cases} 0.45 \exp\left(-|t|/0.05\right) + 0.55(1+3|t|)^{-3} & \text{if } t > 0 \\ 0.47 \exp\left(-|t|/0.03\right) + 0.53(1+3.05|t|)^{-3.5} & \text{if } t < 0 \end{cases}$$

Convolution Model Results



$\theta=\theta_C+0.1°$ $\theta=\theta_C+0.3°$ $\theta=\theta_C+1.0°$

Causality and Pulse Shape

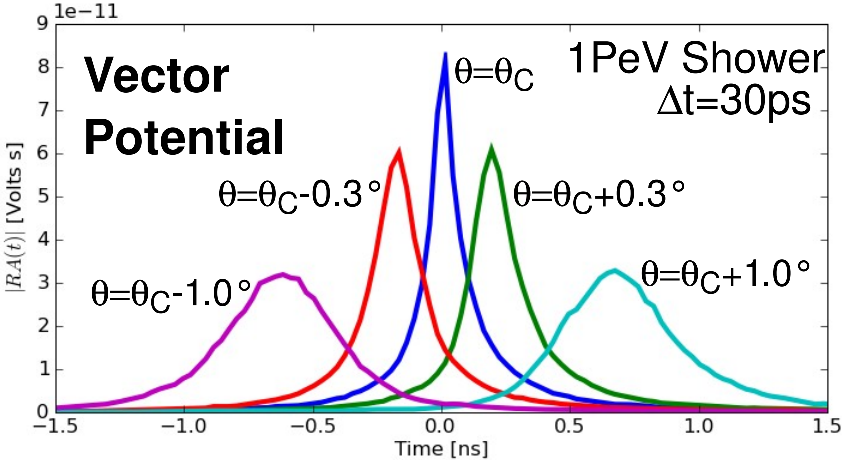
Solution shows that when observing Čerenkov radiation at angles

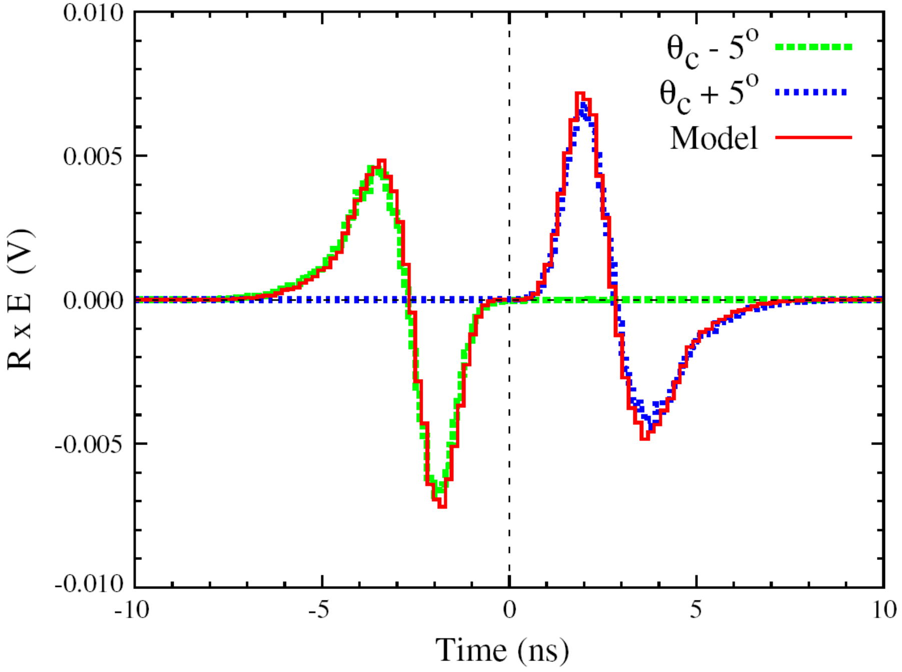
$\theta<\theta_C$ apparent particle speeds are faster than c/n.

$\theta=\theta_C$ all particle tracks appear to emit simultaneously

$\theta>\theta_C$ apparentparticle speeds are smaller than c/n.

Solutions predict that pulses above and below θ_C are reflections of each other.





$$\mathbf{E}_{rad}(R,\theta,t) = -\mathbf{v}_{\perp}\frac{\mu}{4\pi v R}\frac{1}{|1-n\beta\cos\theta|(1-n\beta\cos\theta)}\left.\frac{dQ(\zeta)}{d\zeta}\right|_{\zeta=\beta\frac{ct-nR}{1-n\beta\cos\theta}}$$

Applications of time-domain algorithm

Neutrino simulations in ice with ZHAireS

ZHAireS=ZHS+Aires

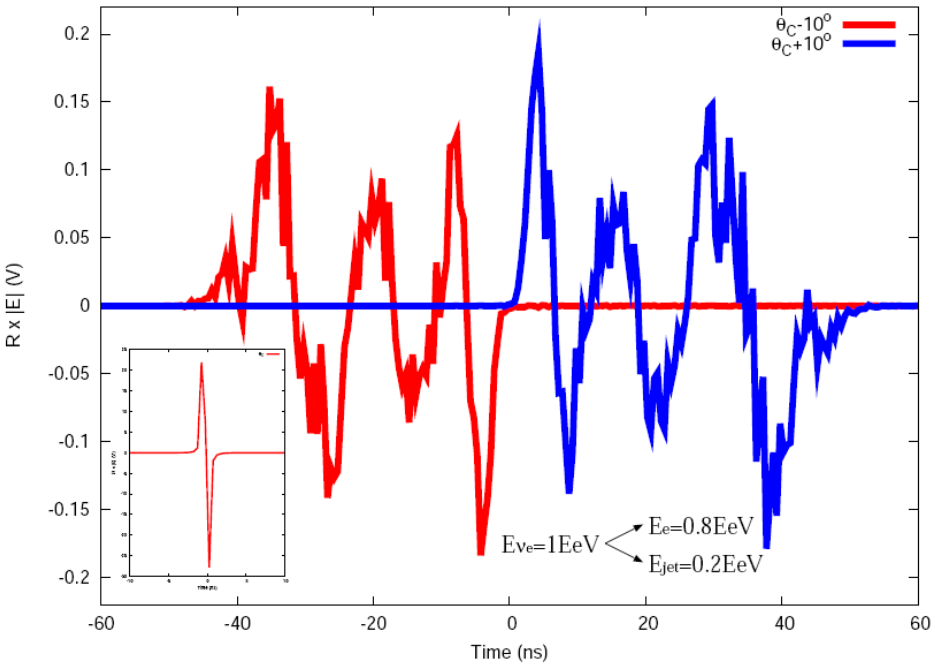
simulates both EM and hadronic showers

Proton showers in air

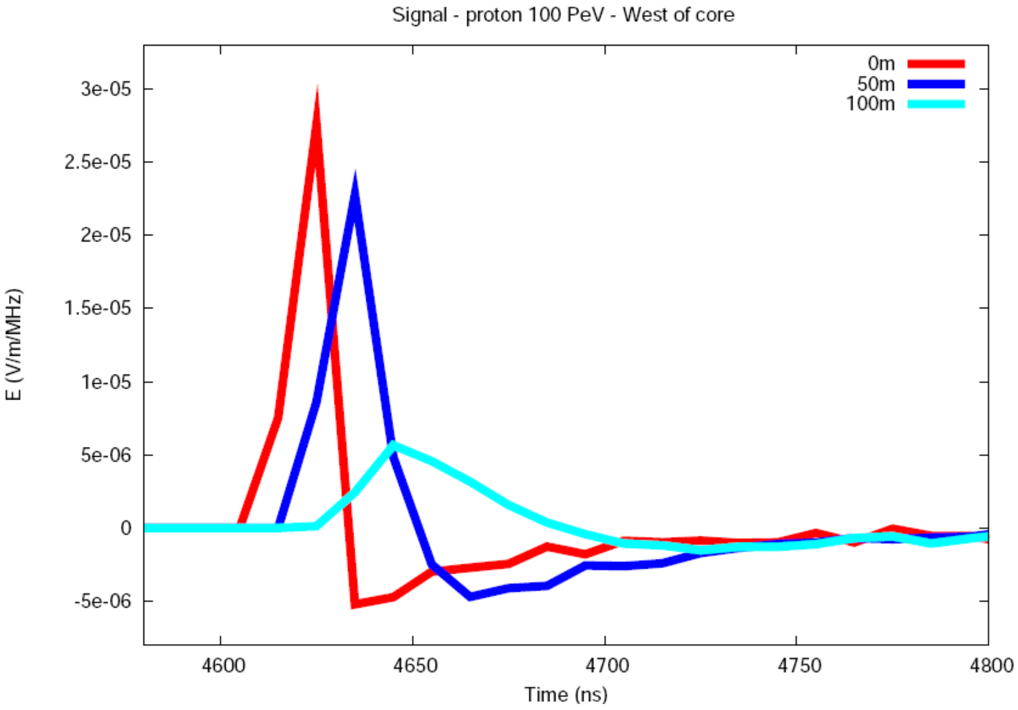
Algorithm reproduces synchrotron radiation in air.

BIPOLAR pulses

ν_e in ice–> e^-+ hadronic jet



proton shower in air at θ=0



More details in W. Carvalho et al. in the poster session

Conclusion

Time-domain Čerenkov pulse structure has a wealth of information.

Time-domain and previous frequency domain calculations are consistent.

LPM shower structure can possible be measured for events observed tenths of degrees away from the Cerenkov angle.

Causal structure of the pulse shape can be used to constrain at what angle from θ_C the observation is made.

We have accurate models for pulse structure given the shower's charge excess longitudinal profile. This is potentially useful for data analysis.