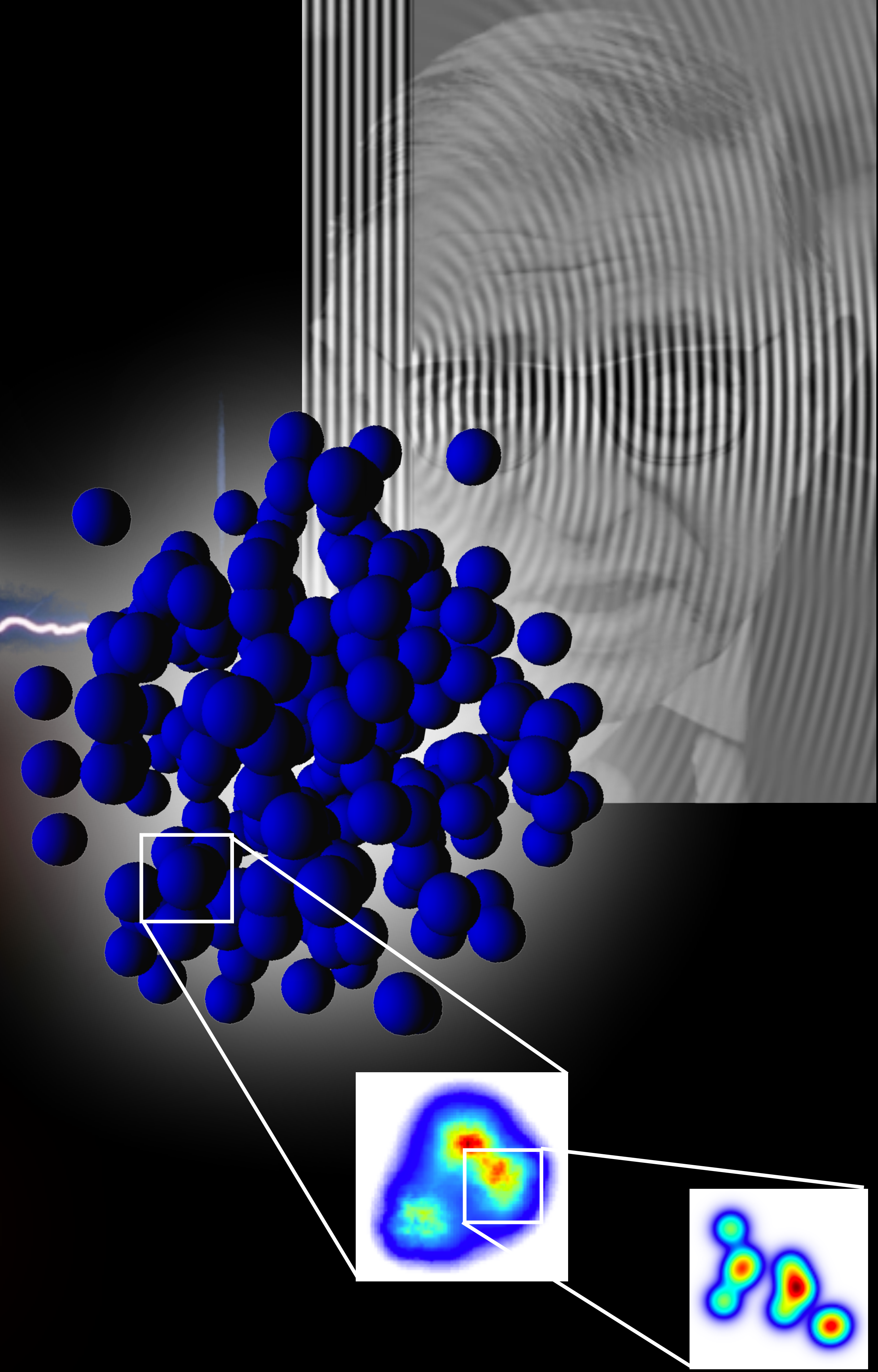
“Event generators of nuclear exclusive reactions: Small-*x* fluctuations in Sar*t*re” Gluodynamics

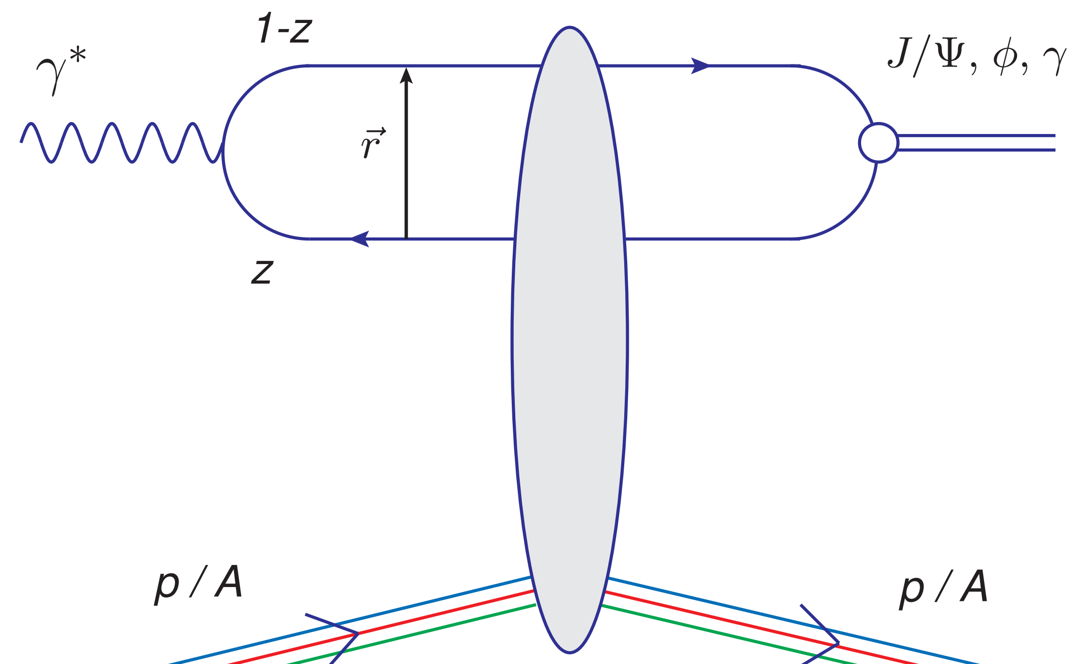
October 24, 2022

Tobias Toll
Indian Institute of Technology Delhi



H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

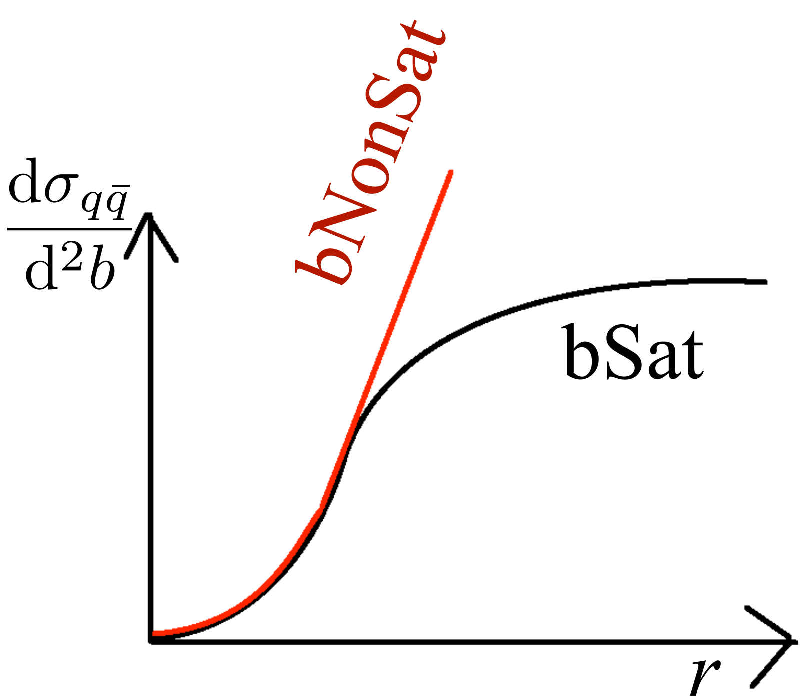
Exclusive diffraction Sartre



$$t = -\Delta^2$$

$$\frac{\mathrm{d}\sigma^{\gamma^* p \to Vp}}{\mathrm{d}t} = \frac{1}{16\pi} \left\langle \left| \mathcal{A}_{T,L}^{\gamma^* p \to Vp} \right|^2 \right\rangle_{\text{initial state}}$$

$$\mathcal{A}_{T,L}^{\gamma^* p \to Vp}(x_{I\!P}, Q^2, \Delta) = i \int 2\pi r \mathrm{d}r \int \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\vec{b} \left(\Psi_V^* \Psi\right)(r,z) J_0([1-z]r\Delta) e^{-\vec{b}\cdot\vec{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b})$$

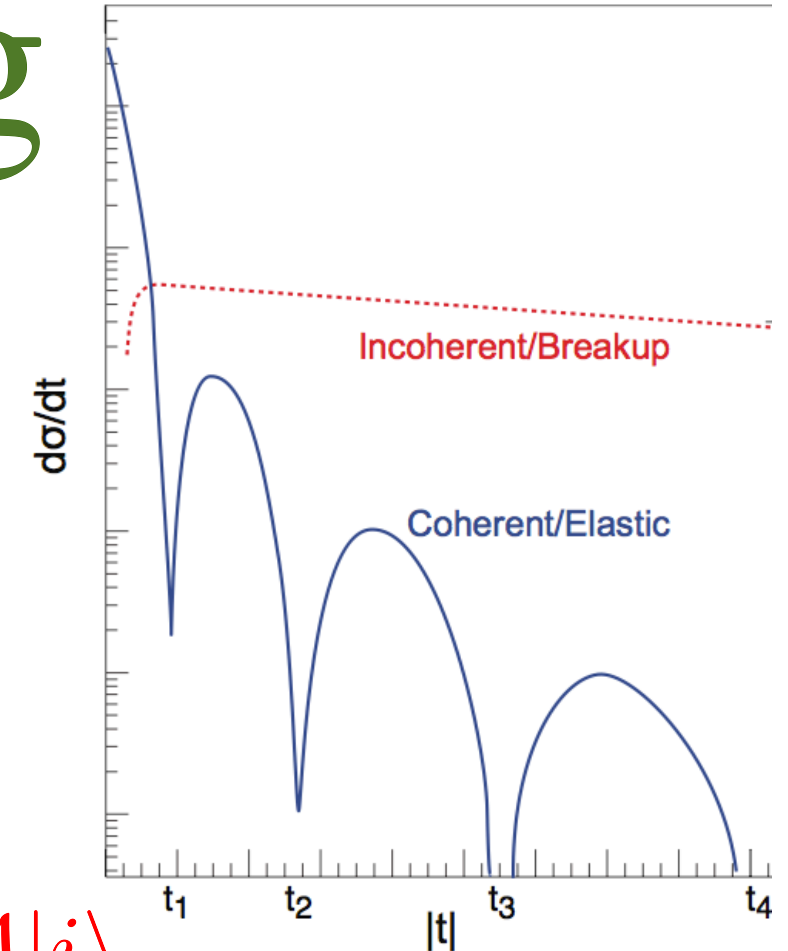


$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$

$$\frac{\mathrm{d}\sigma^{\text{nosat}}_{\mathrm{q\bar{q}}}}{\mathrm{d}\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)$$

2

Incoherent Scattering



Good, Walker:

Nucleus dissociates ($f \neq i$):

$$\sigma_{\text{incoherent}} \propto \sum_{f \neq i} \langle i|\mathcal{A}|f\rangle^\dagger \langle f|\mathcal{A}|i\rangle$$

complete set

$$= \sum_f \langle i|\mathcal{A}|f\rangle^\dagger \langle f|\mathcal{A}|i\rangle - \langle i|\mathcal{A}|i\rangle^\dagger \langle i|\mathcal{A}|i\rangle$$

$$= \langle i \,||\mathcal{A}|^2|\, i\rangle - |\langle i|\mathcal{A}|i\rangle|^2 = \langle |\mathcal{A}|^2\rangle - |\langle \mathcal{A}\rangle|^2$$

The incoherent CS is the variance of the amplitude!!

$$\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi}\left\langle |\mathcal{A}|^2 \right\rangle \qquad \frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi}|\langle \mathcal{A}\rangle|^2$$

The nucleus as a collection of nucleons

TT, Thomas Ullrich
Phys.Rev.C 87 (2013) 2, 024913, arXiv: 1211.3048
Comput.Phys.Commun. 185 (2014) 1835-1853 arXiv:1307.8059

Independent scattering approximations:

$$1 - \frac{1}{2}\frac{\mathrm{d}\sigma^{(p)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = \prod_{i=1}^{A}\left(1 - \frac{1}{2}\frac{\mathrm{d}\sigma^{(A)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, |\vec{b} - \vec{b}_i|)\right)$$

$$\frac{1}{2}\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = 1 - \exp\left(\frac{\pi^2}{2N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) \sum_{i=1}^{A} T_p(|\vec{b} - \vec{b}_i|)\right)$$

$$T_A(\vec{b}) = \int \mathrm{d}z \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{\vec{b}^2 + z^2} - R_0}{d}\right)}$$

$\Rightarrow$

$$T_A(\vec{b}) = \sum_{i=1}^{A} T_p(|\vec{b} - \vec{b}_i|)$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

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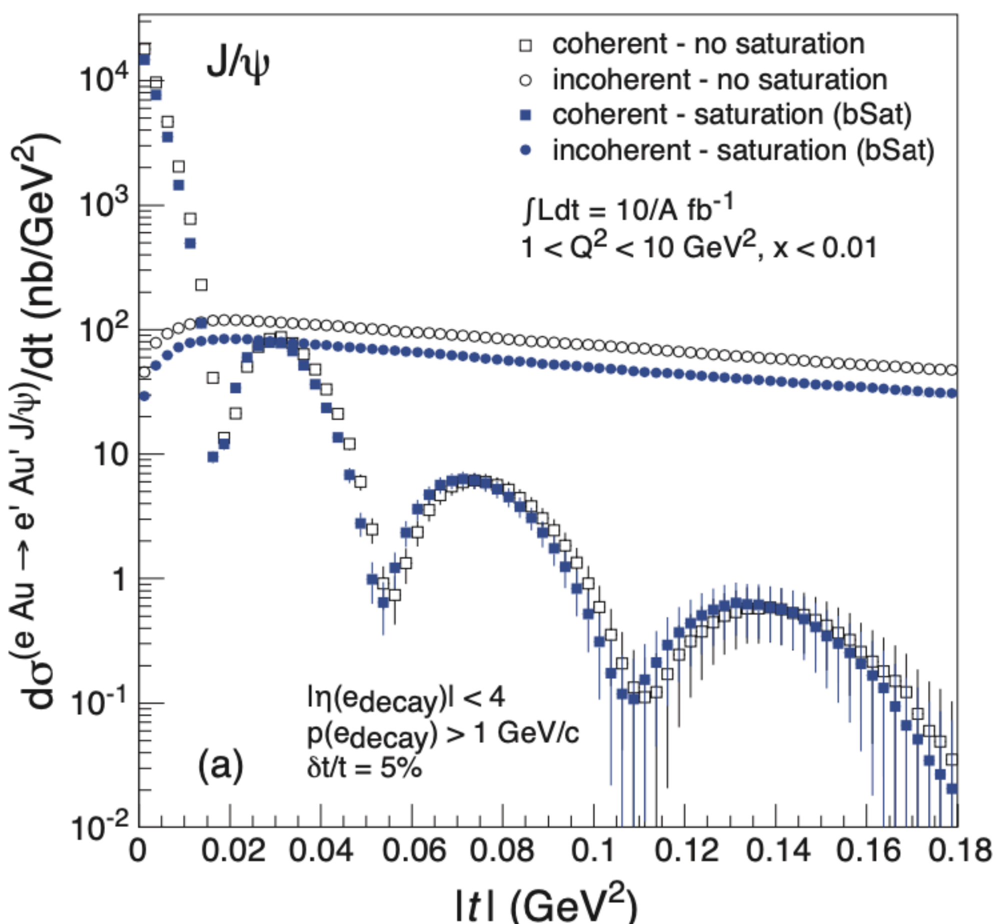

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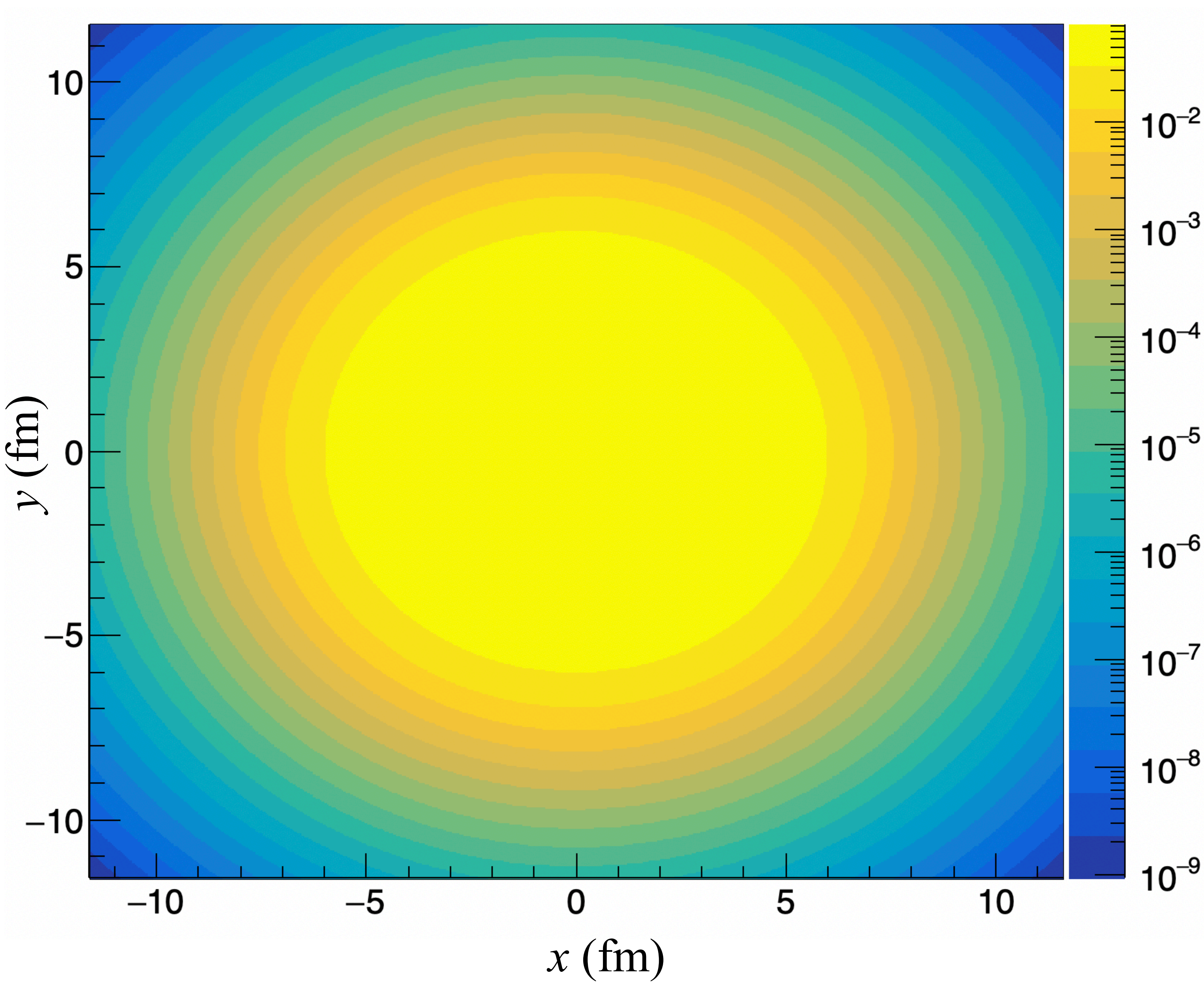
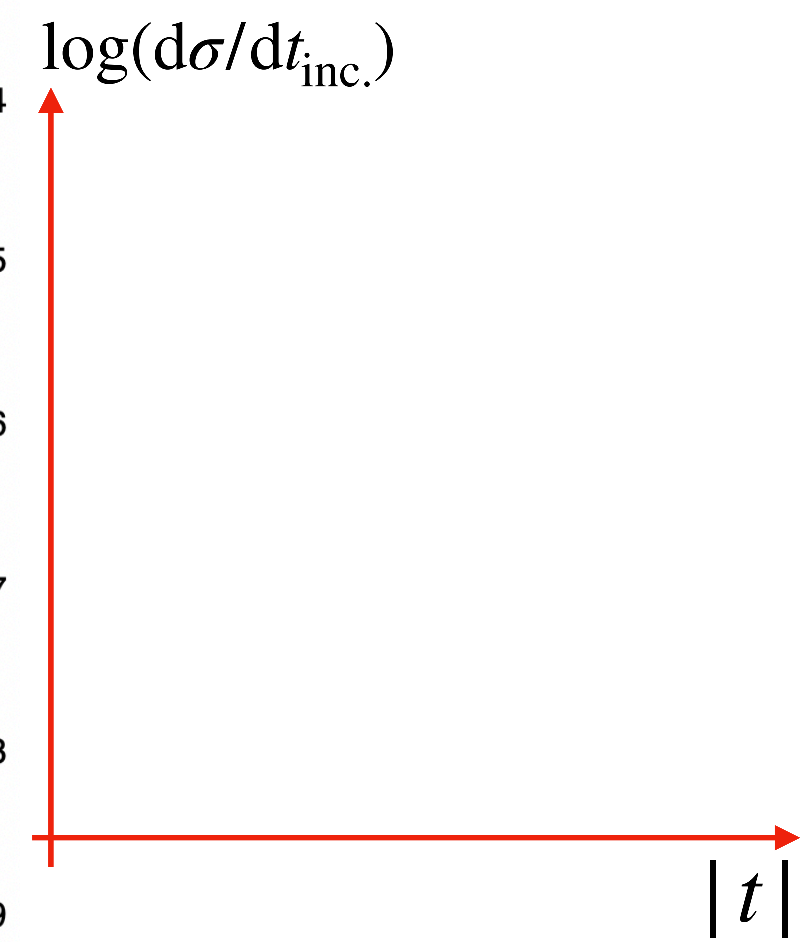
Into the heavy nucleus



$|t| = 0$



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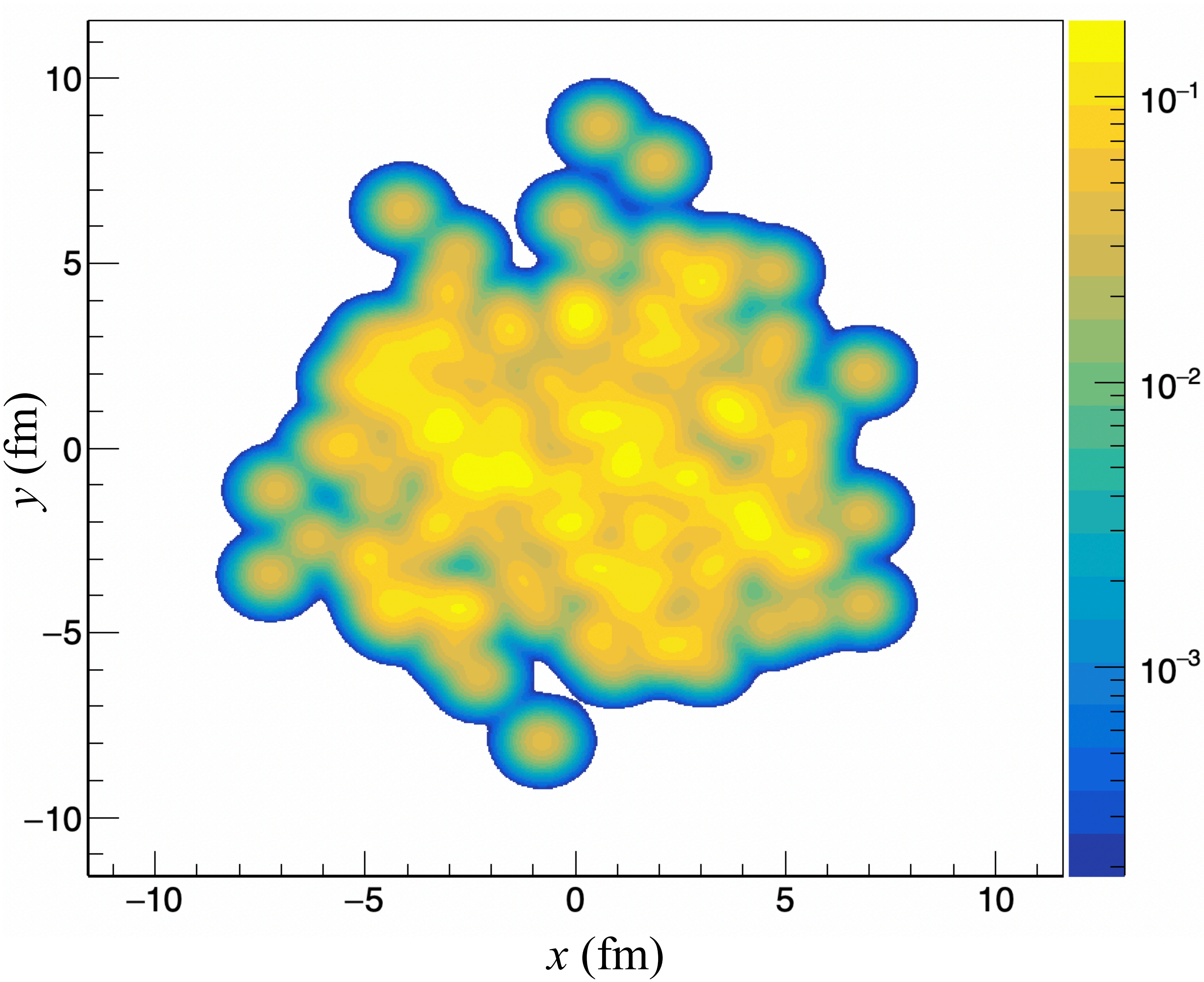
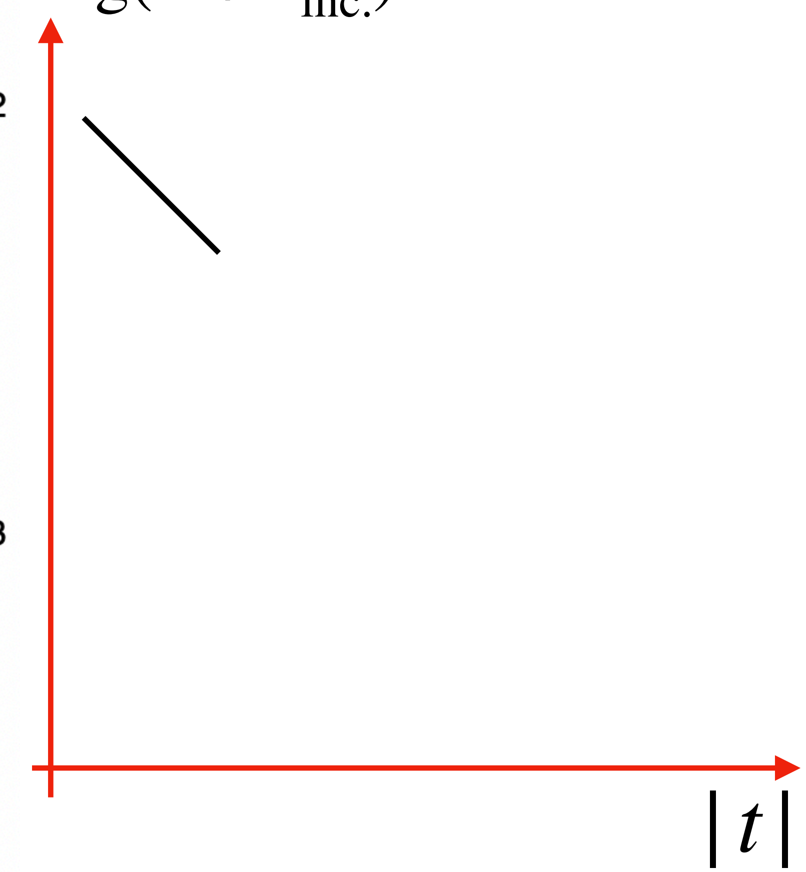
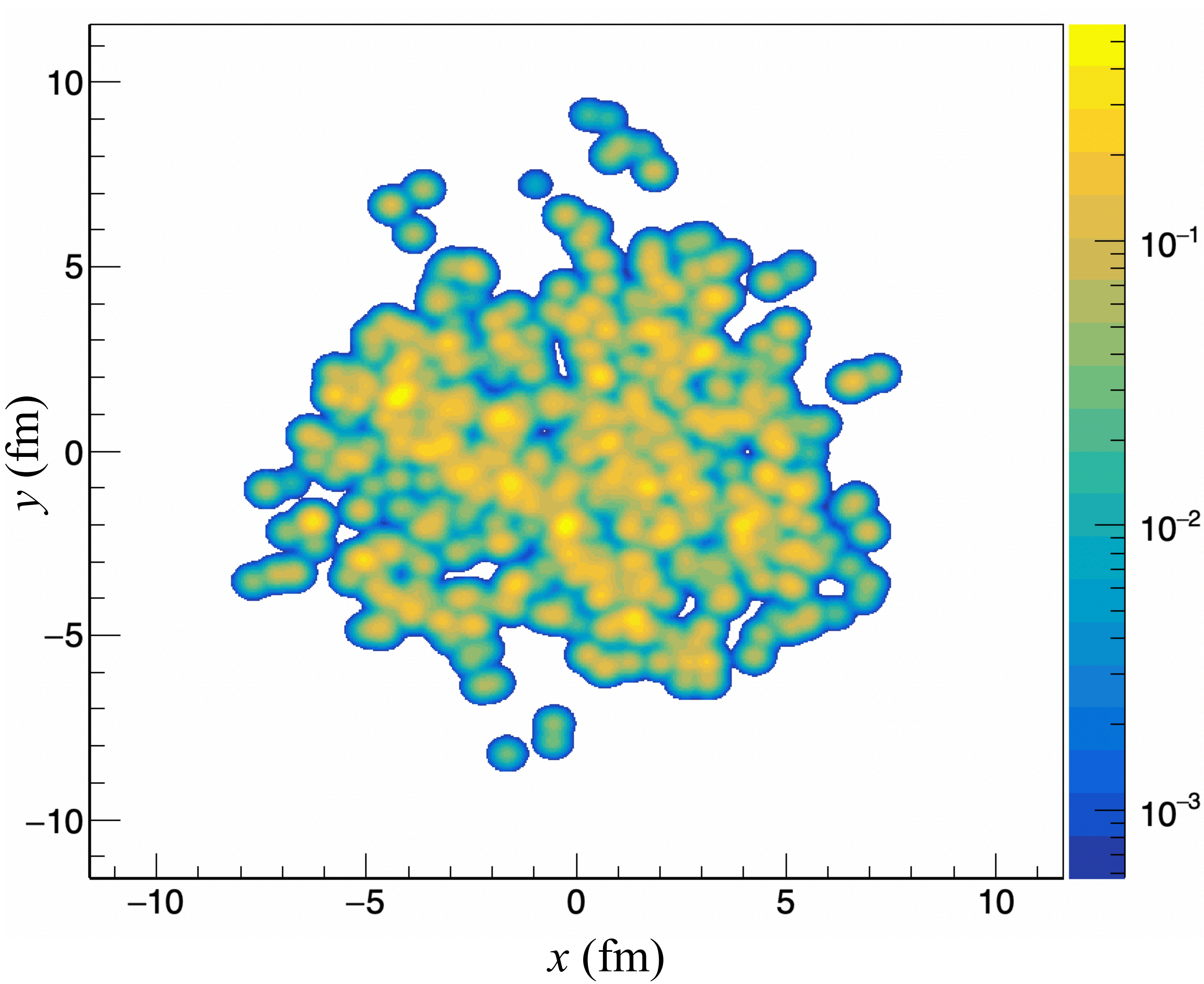


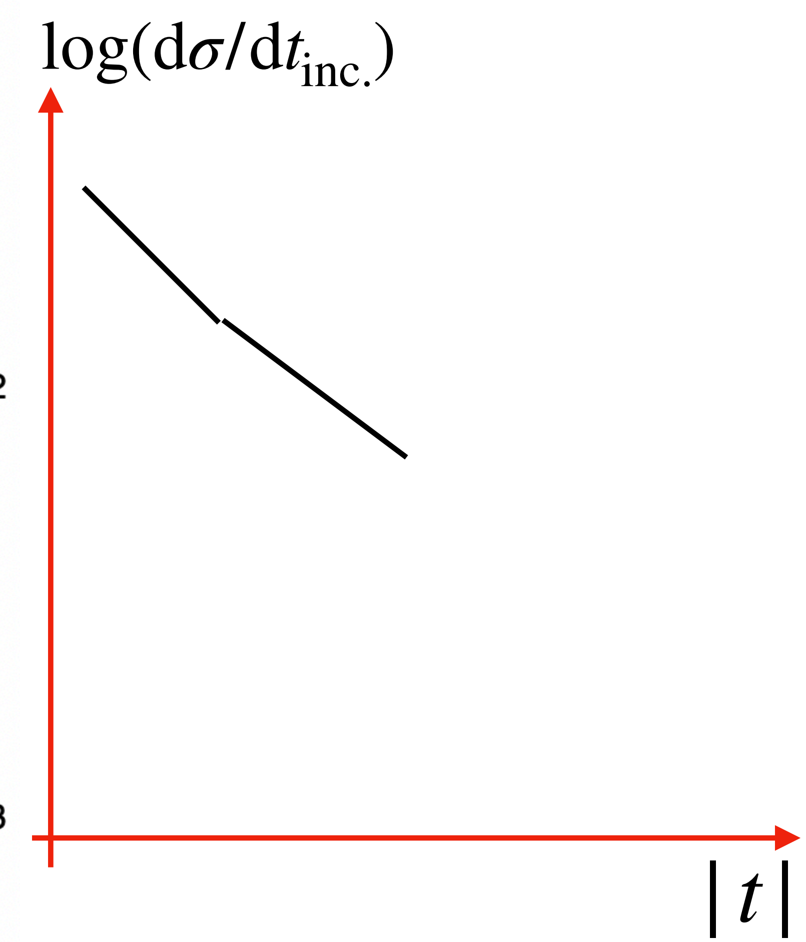
$|t| \lesssim 0.2\ \mathrm{GeV}^2$



Into the heavy nucleus



$0.2 \lesssim |t| \lesssim 2\ \mathrm{GeV}^2$



Hotspot model for incoherent *ep*-scattering

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

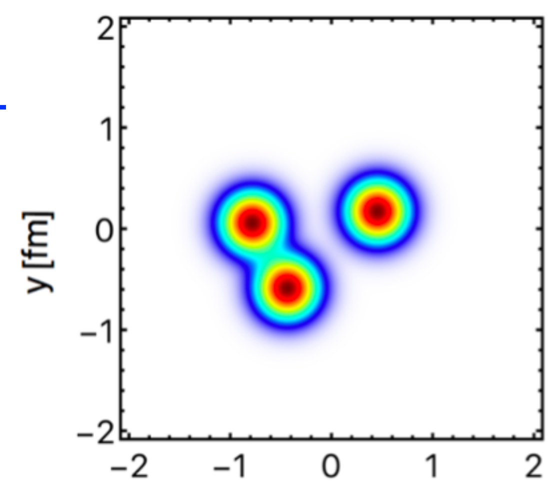
$$xg(x,\mu_0^2) = A_g x^{-\lambda_g}(1-x)^6$$

$$\mu^2 = \mu_0^2 + \frac{C}{r^2}$$

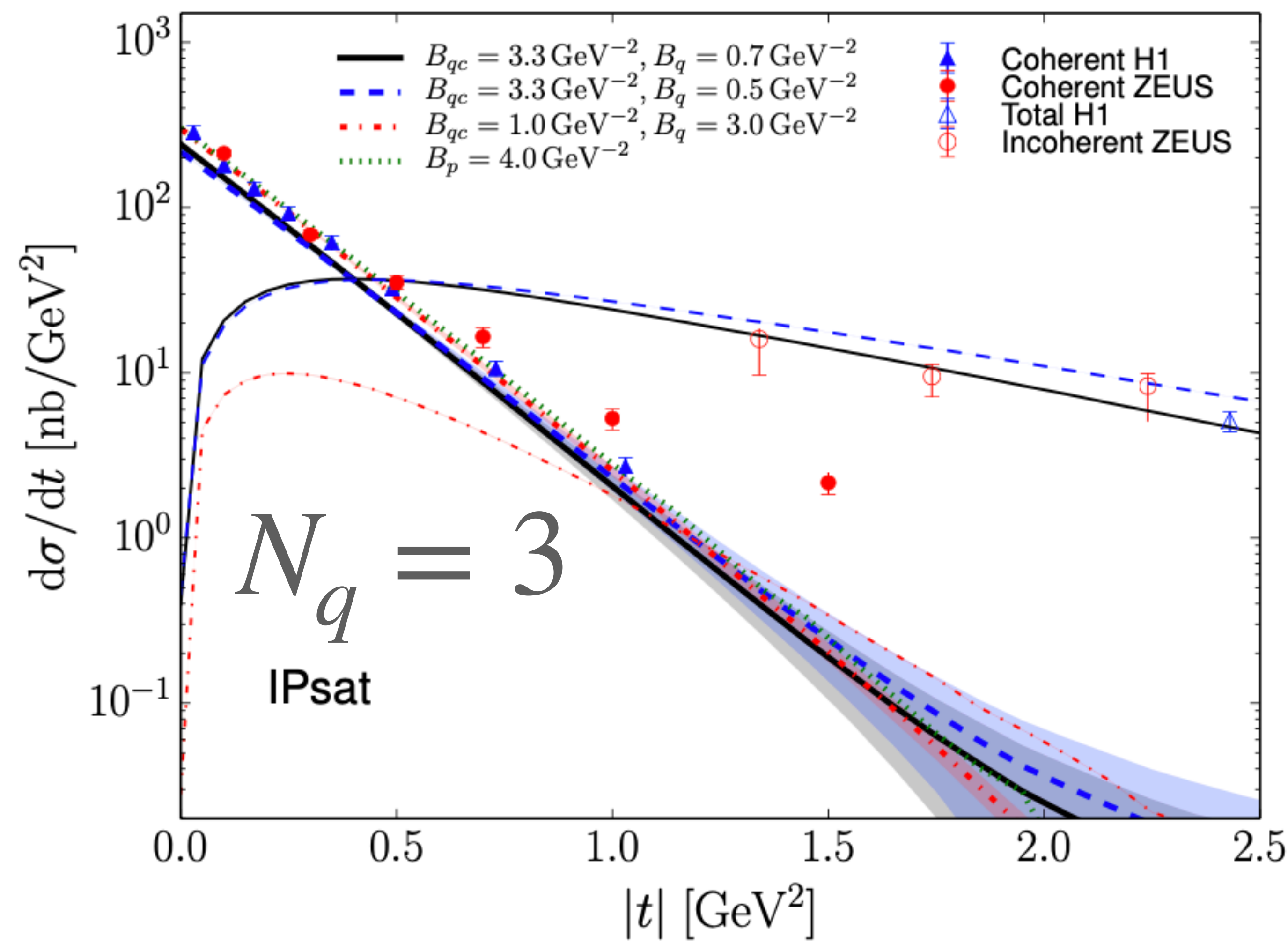
$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$\downarrow$$

$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q}}$$



$\vec{b}_i$ with a Gaussian distribution of width B_{qc}



H. Mäntysaari and B. Schenke Phys. Rev. Lett., 117(5):052301, 2016.

Also: large scale (small $|t|$) saturation scale fluctuations. Affects small $|t|$, one more parameter.

Incoherent Scattering in *ep*

$$\frac{d\sigma^{\text{nosat}}_{q\bar{q}}}{d\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)$$

$$xg(x,\mu_0^2) = A_g x^{-\lambda_g}(1-x)^6$$

$$\mu^2 = \mu_0^2 + \frac{C}{r^2}$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

↓

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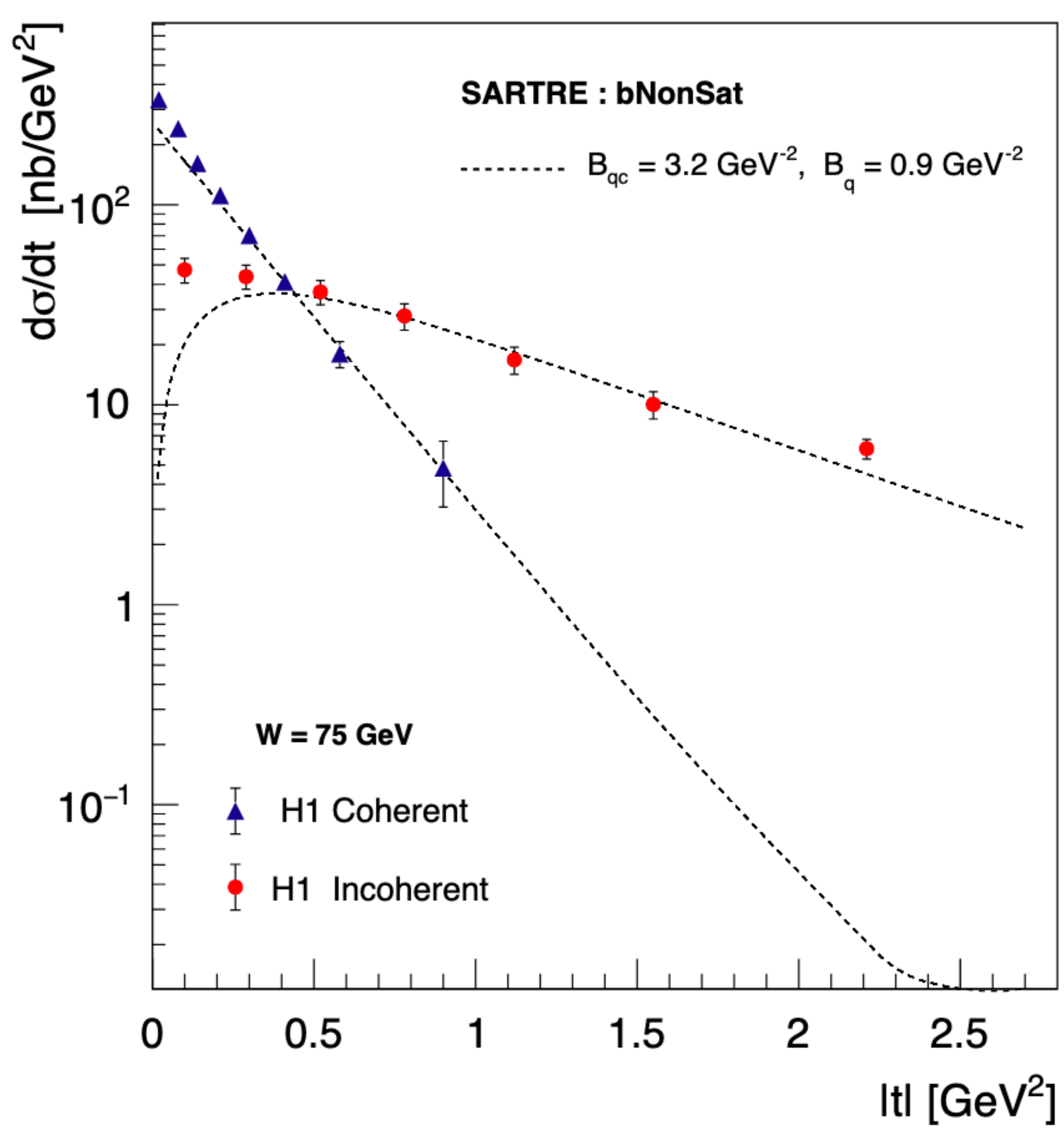
$\vec{b}_i$ with a Gaussian distribution of width B_{qc}

Incoherent J/ψ photoproduction



Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837, arXiv: 2106.12855

Incoherent Scattering in *ep*

$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$

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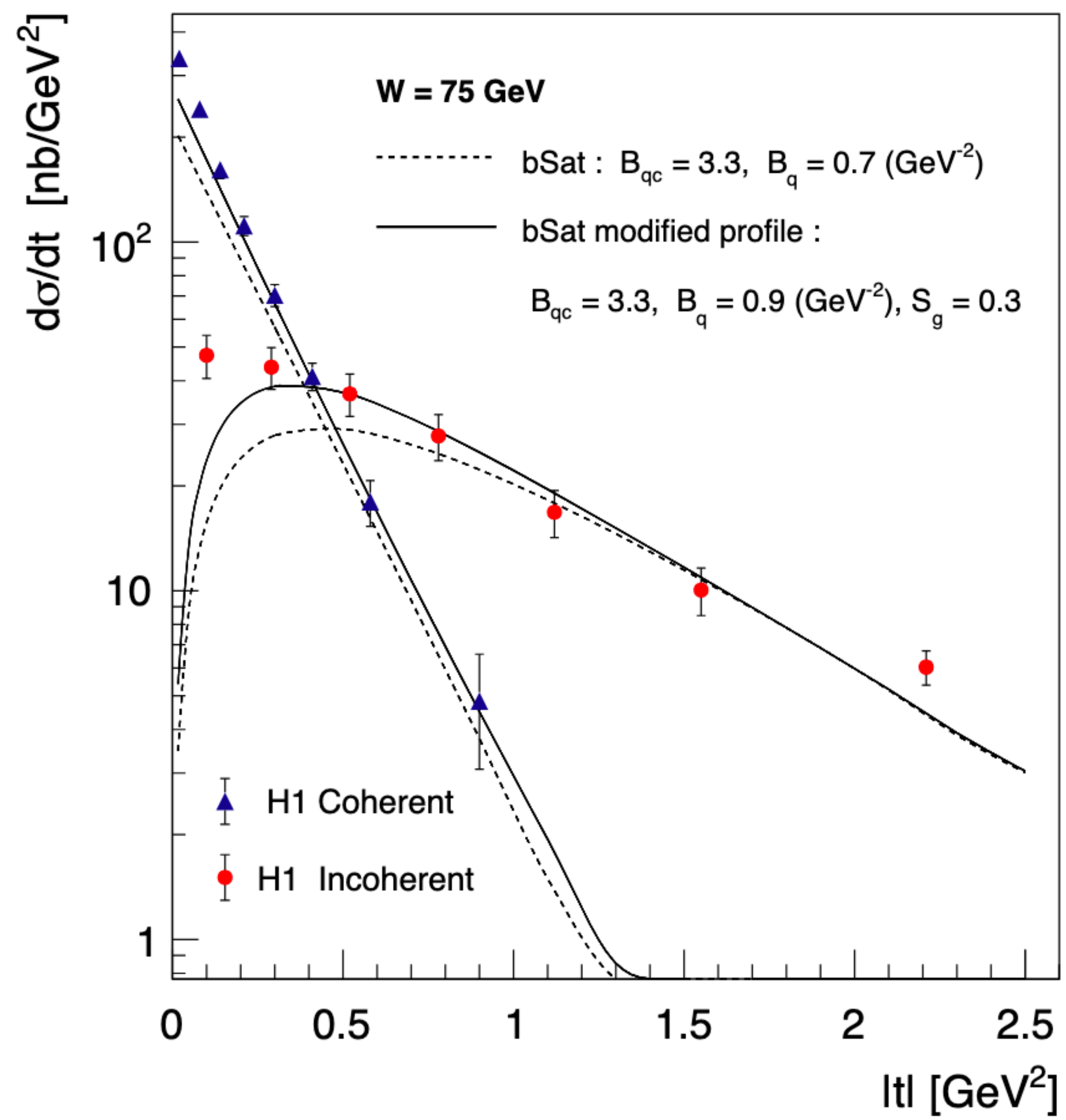

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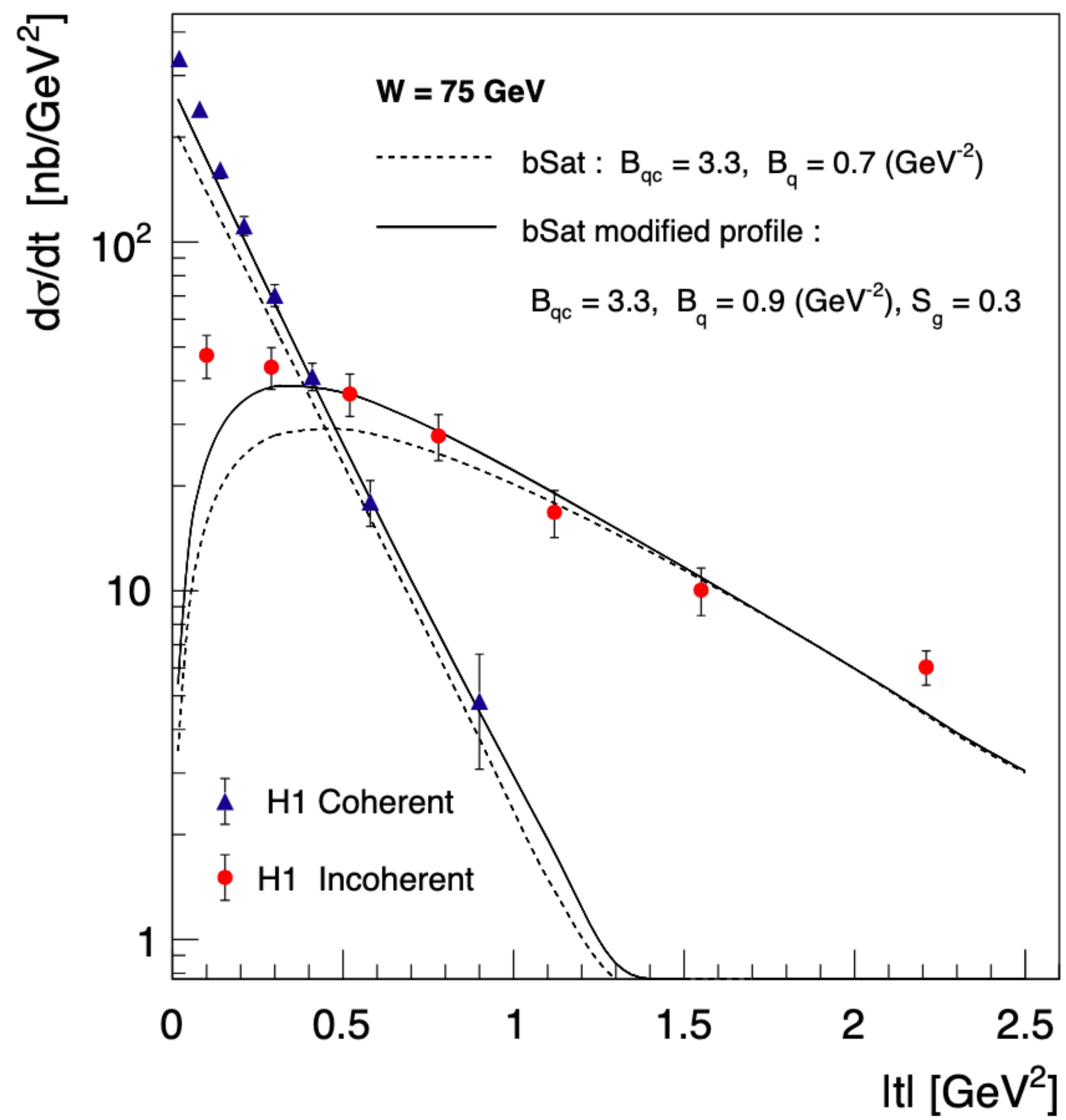
$$\frac{d\sigma^{\text{nosat}}_{\text{q}\bar{\text{q}}}}{d\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)$$

For bNonSat, $\langle \mathscr{A} \rangle \propto \langle T(b) \rangle$

For bSat this is not the case.
Therefore the coherent cross-section gets affected

$\overrightarrow{b}_i$ with a Gaussian distribution of width B_{qc}



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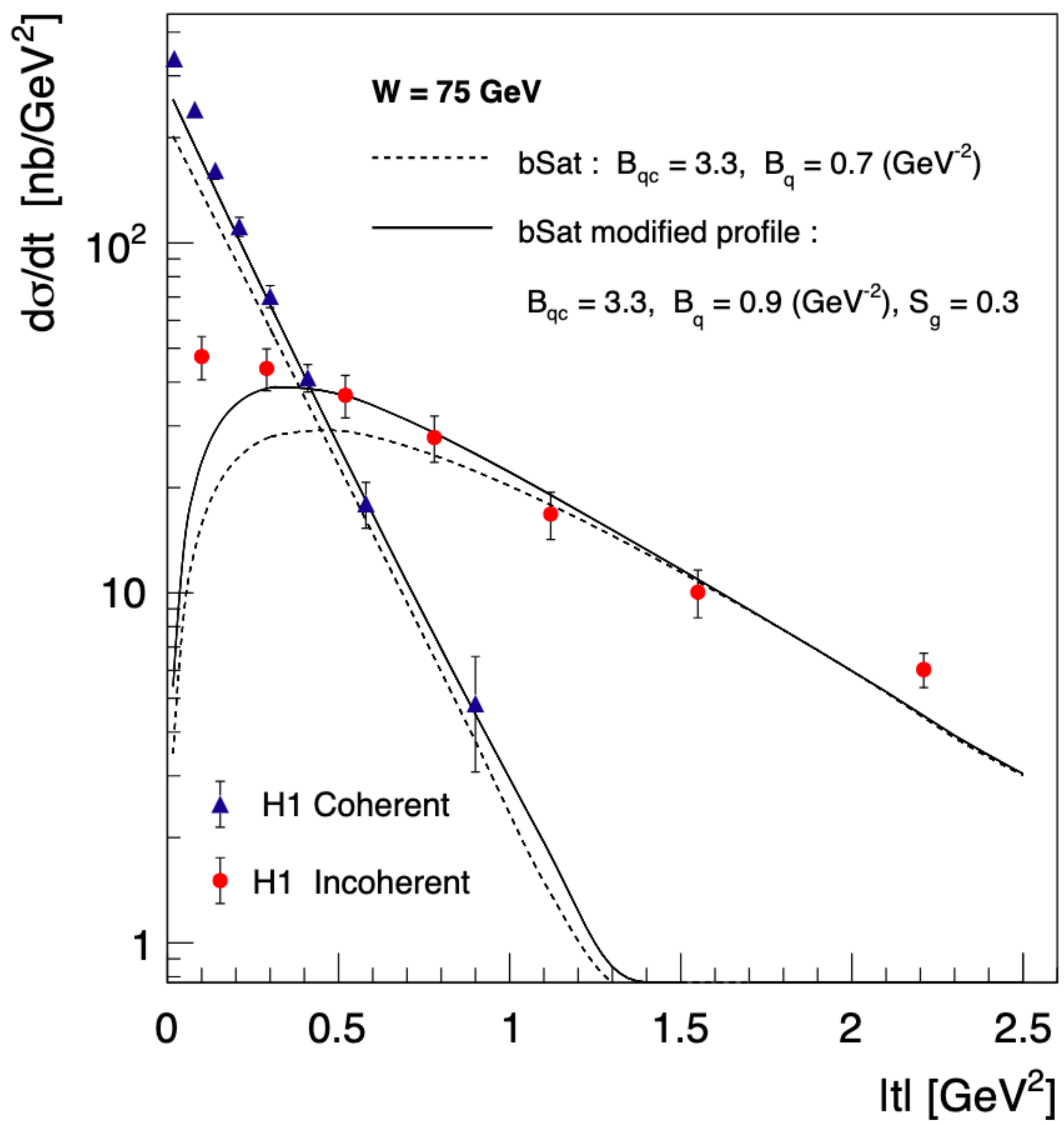
$$xg(x,\mu_0^2) = A_g x^{-\lambda_g}(1-x)^6$$

$$\mu^2 = \mu_0^2 + \frac{C}{r^2}$$

Modified profile:

$$T_q(b) = \frac{1}{2\pi B_q} \frac{1}{\exp\left(\frac{b^2}{2B_q}\right) - S_g}$$

$\vec{b}_i$ with a Gaussian distribution of width B_{qc}



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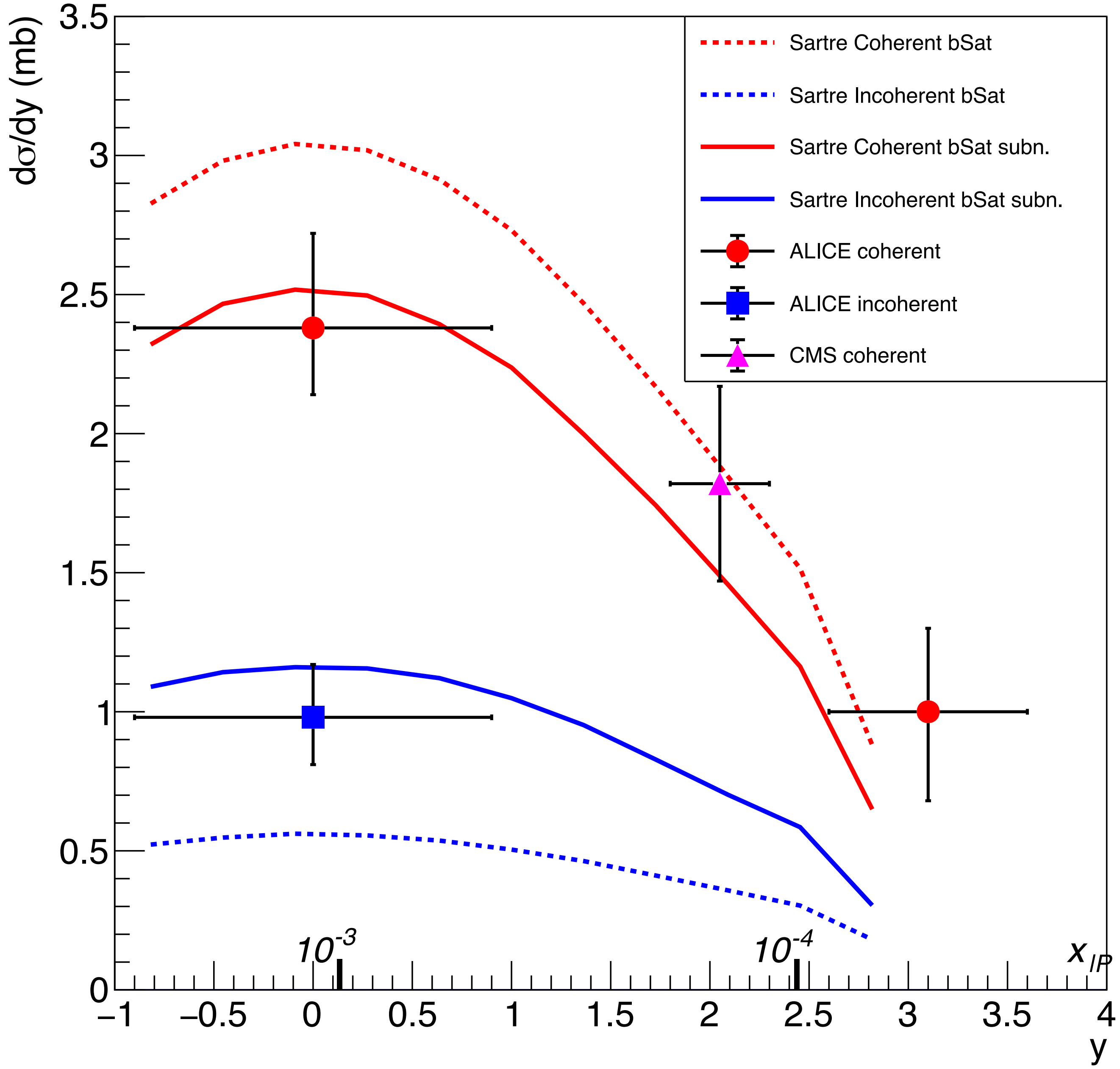
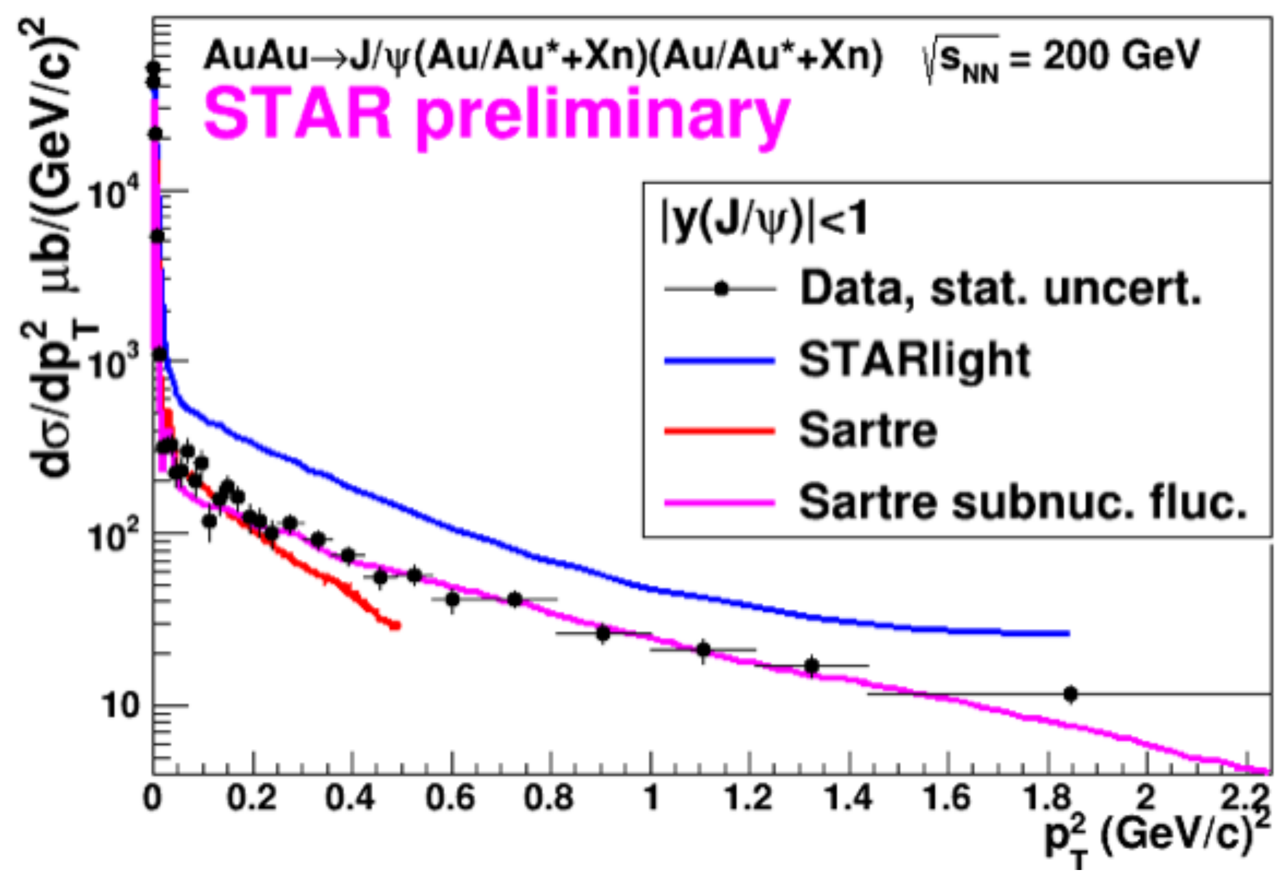
A-A UPC at the LHC & RHIC



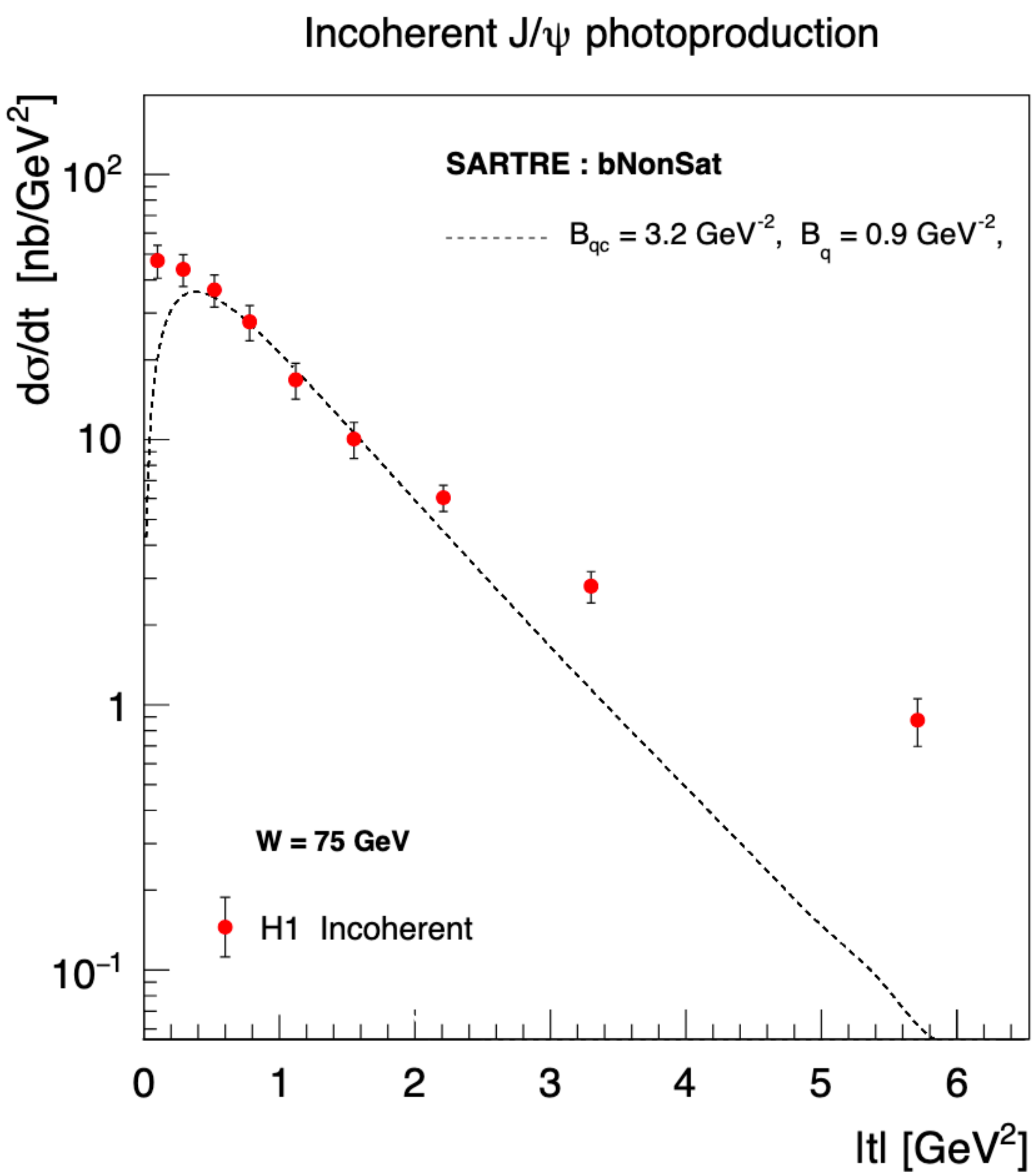
TT: SciPost Phys.Proc. 8 (2022) 148



Eventhough coherent events dominate, the large $|t|$ tails have a significant effect on the cross sections!

Subnucleon structure becomes important for $|t| > 0.2$ GeV^2

Larger $|t|$?



Appears to be two slopes in the data:
One for $0.5 \leq |t| \leq 2$ GeV^2
Another for $|t| > 2$ GeV^2

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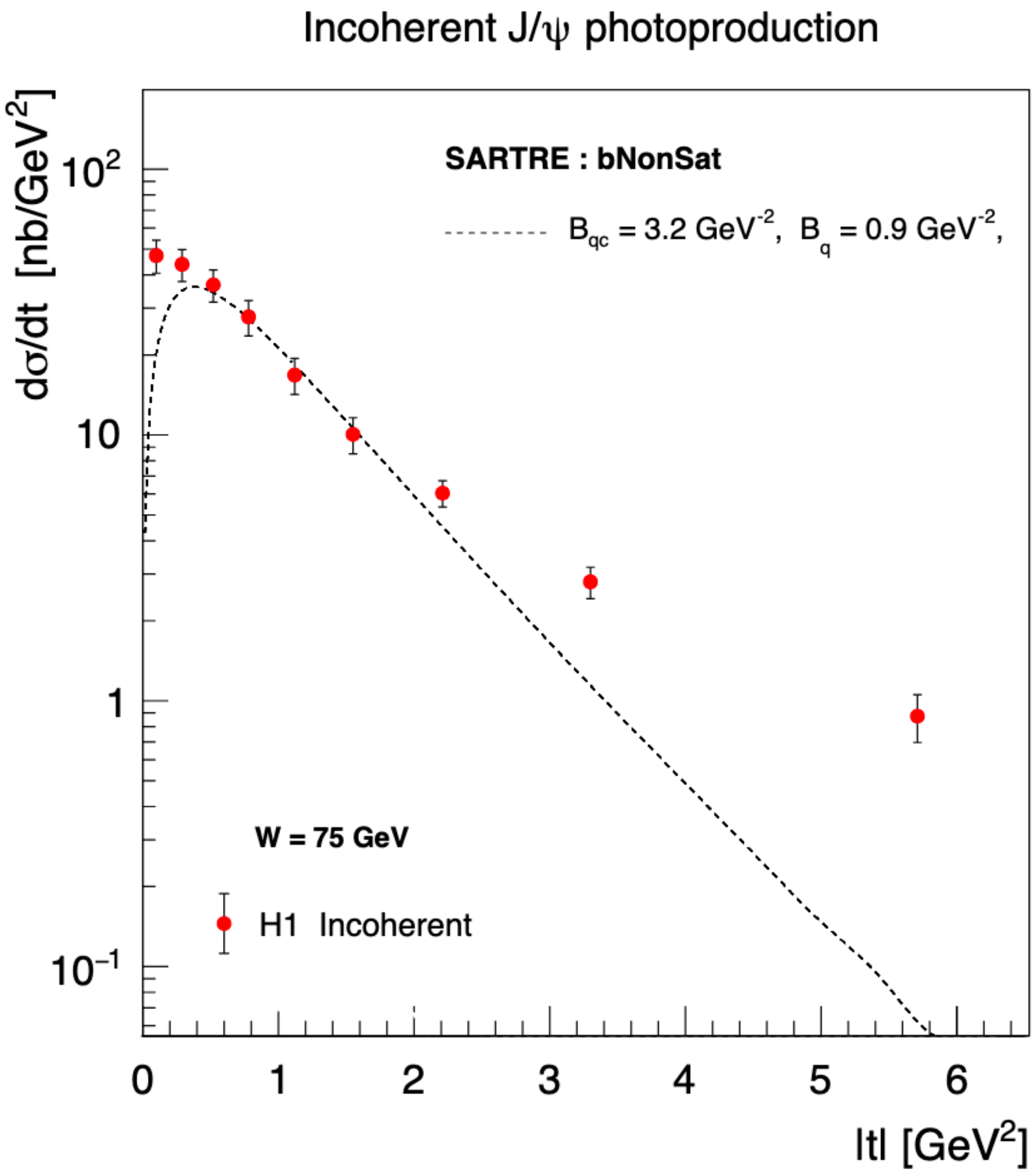
Larger $|t|$?



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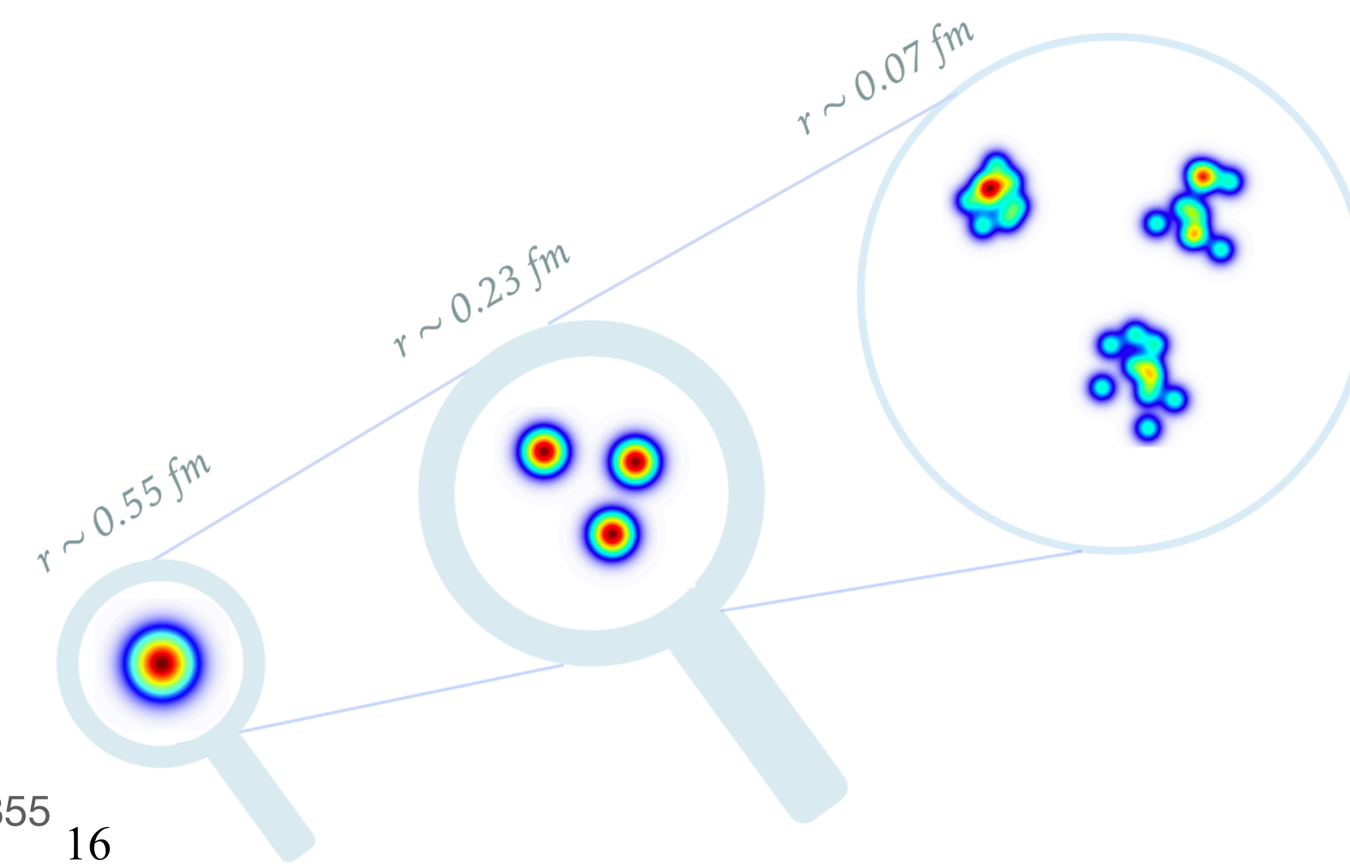
Hotspots within hotspots!



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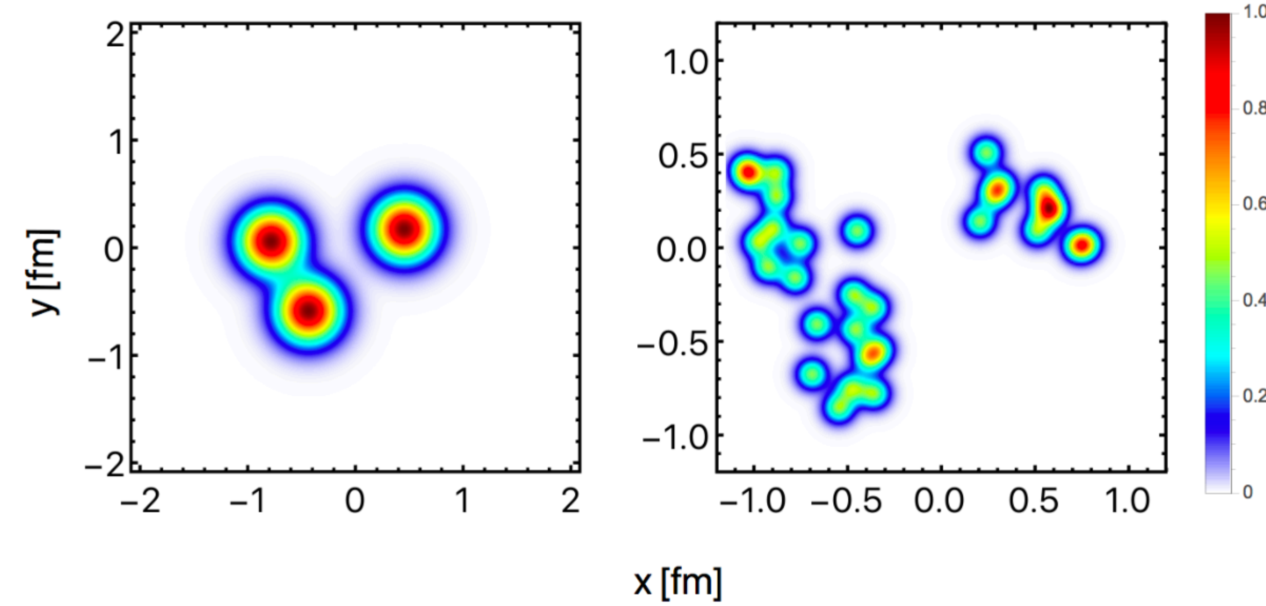
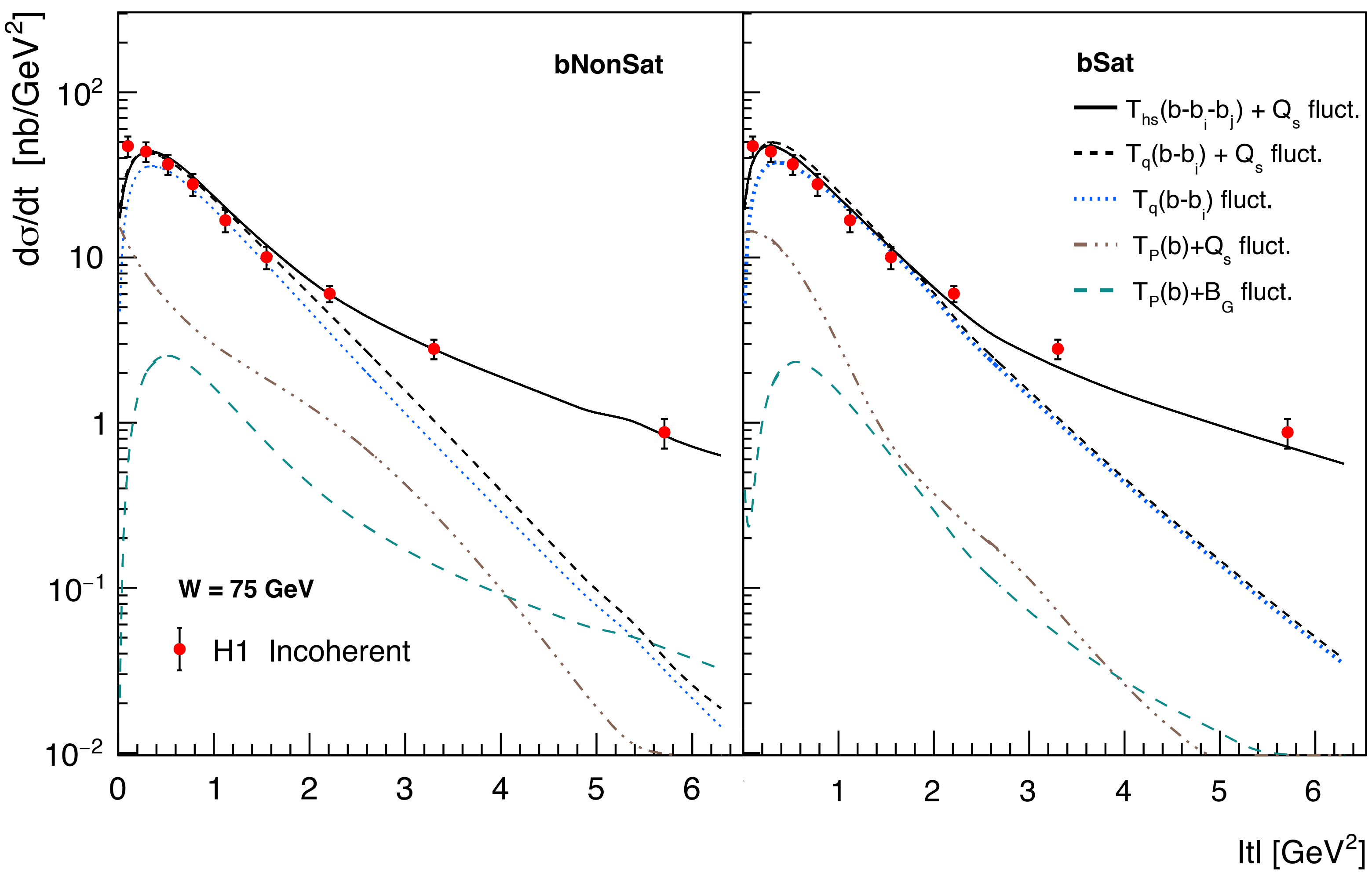
Hotspots within Hotspots

Model	B_{qc}	B_q	B_{hs}	S_g	N_{hs}	σ
bNonSat hotspot	3.2	0.9	–	–	–	0.4
bSat hotspot	3.3	0.7	–	–	–	0.5
modified bSat hotspot	3.3	0.9	–	0.3	–	0.4
bNonSat refined hotspot	**3.2**	**1.15**	**0.05**	–	**10**	**0.4**
bSat refined hotspot	**3.3**	**1.08**	**0.09**	**0.4**	**10**	**0.5**





$$T_p(b) = \frac{1}{N_q}\sum_{i=1}^{N_q} T_q(\vec{b} - \vec{b}_i)$$

$$T_q(b) = \frac{1}{N_{hs}}\sum_{i=1}^{N_{hs}} T_{hs}(\vec{b} - \vec{b}_i)$$

$$T_{hs}(b) = \frac{1}{2\pi B_{hs}} e^{-\frac{b^2}{2B_{hs}}}$$

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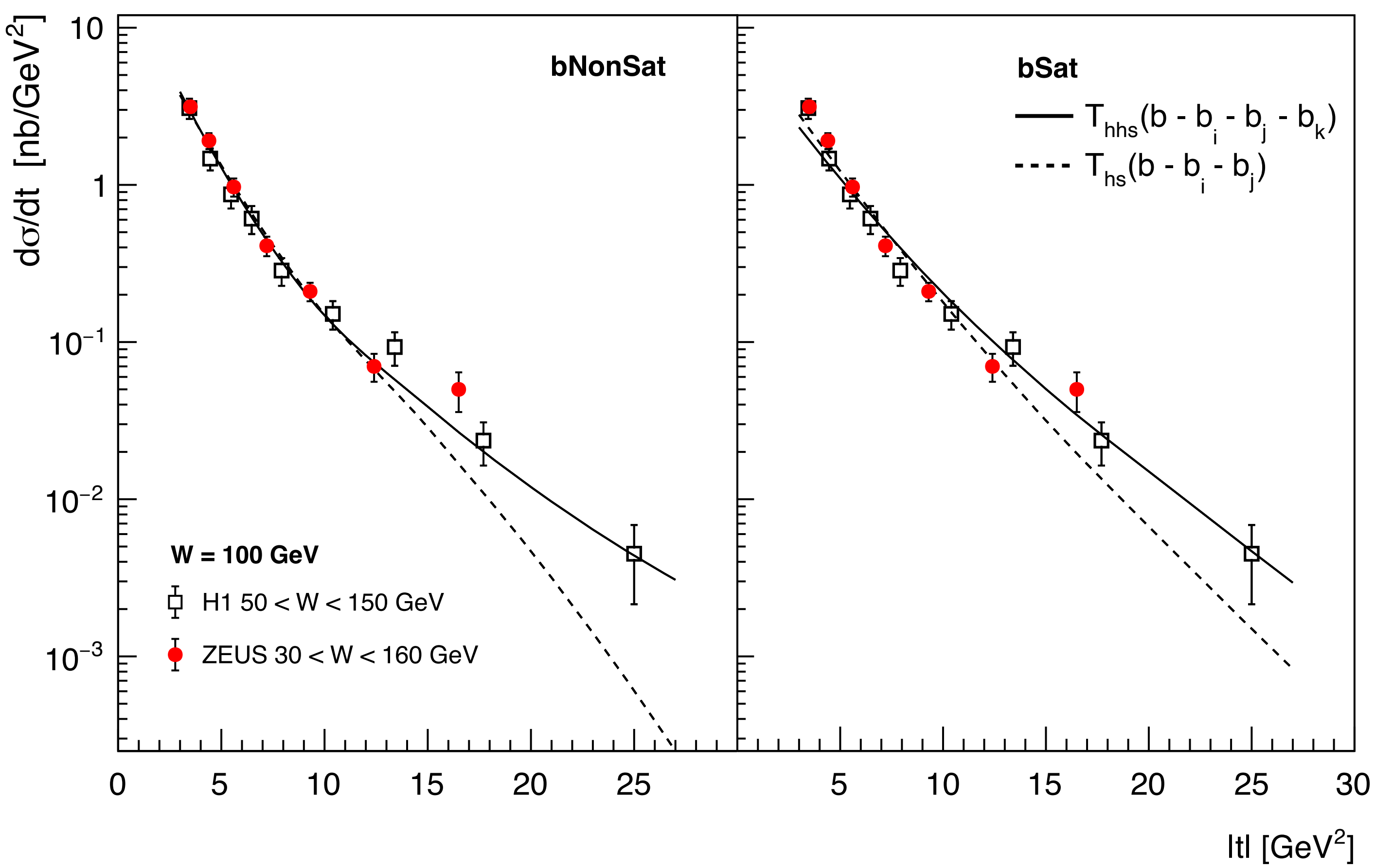
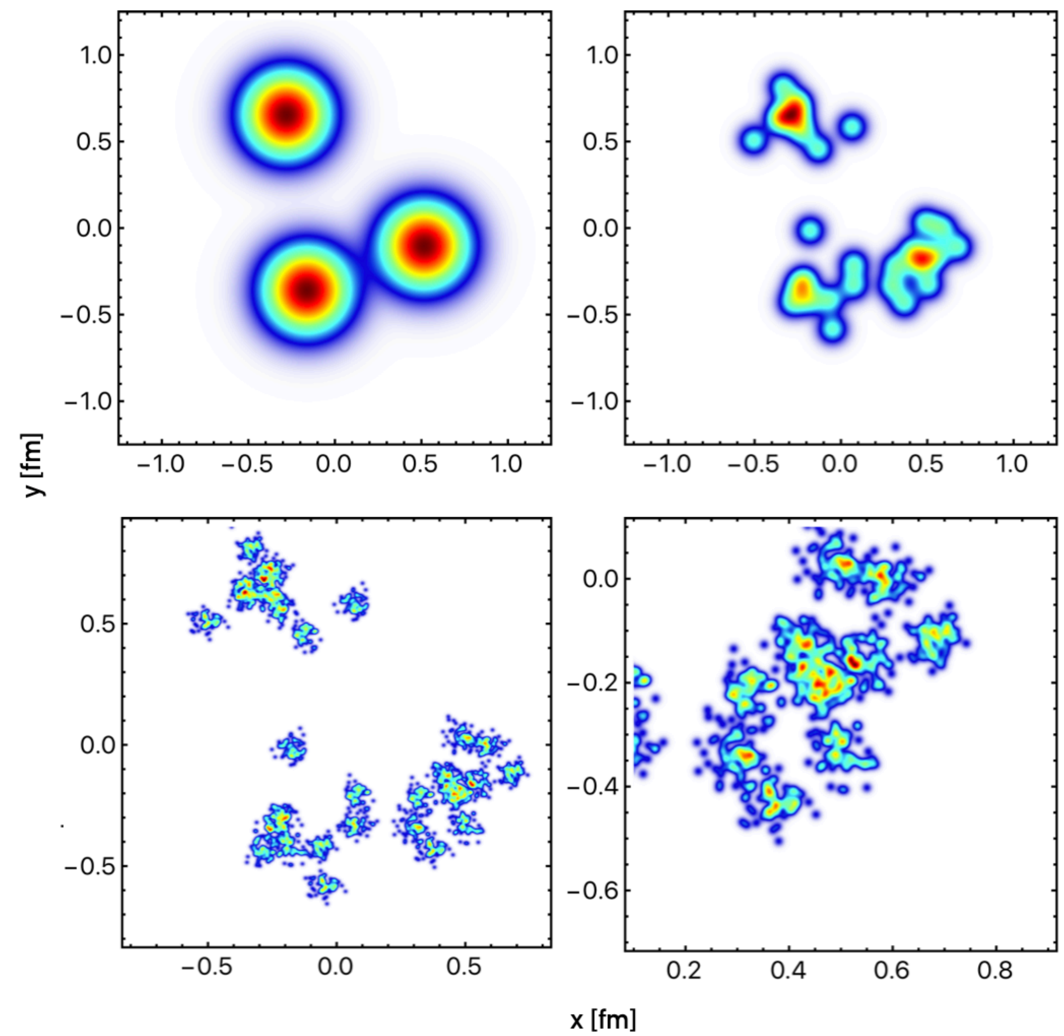
Even larger $|t|$

Hotspots withing hotspots within hotspots

Model	B_{qc}	B_q	N_q	B_{hs}	N_{hs}	B_{hhs}	N_{hhs}	S_g	σ
bNonSat further refined hotspot	3.2	1.15	3	0.05	10	0.0006	65	–	0.4
bSat further refined hotspot	3.3	1.08	3	0.09	10	0.0006	60	0.4	0.5





$$T_P(\vec{b}) = \frac{1}{2\pi N_q N_{hs} N_{hhs} B_{hhs}} \sum_i^{N_q} \sum_j^{N_{hs}} \sum_k^{N_{hhs}} e^{-\frac{(\vec{b} - \vec{b}_i - \vec{b}_j - \vec{b}_k)^2}{2B_{hhs}}}$$

Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837, arXiv: 2106.12855

Even larger $|t|$

Hotspots withing hotspots within hotspots

Model	B_{qc}	B_q	N_q	B_{hs}	N_{hs}	B_{hhs}	N_{hhs}	S_g	σ
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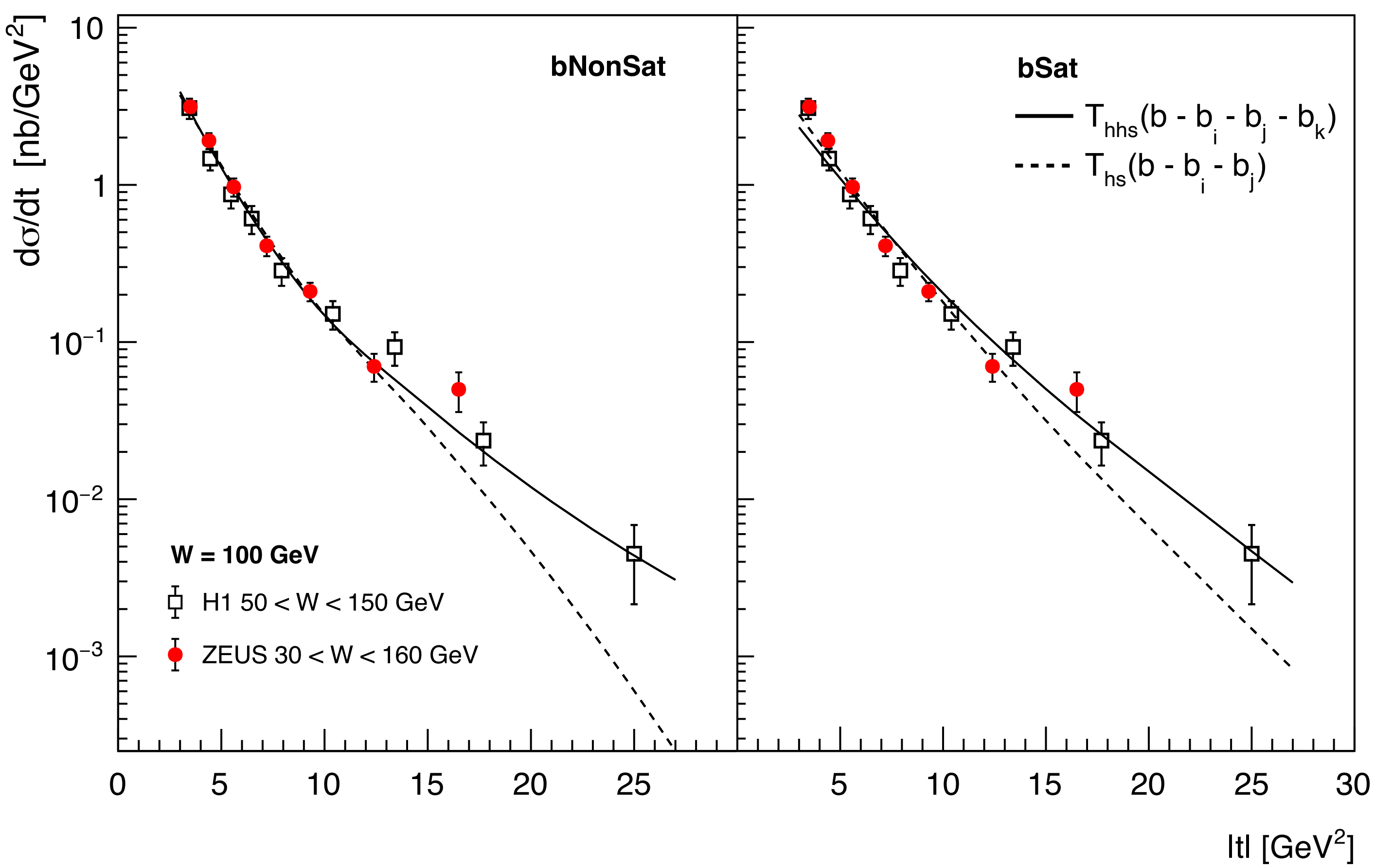

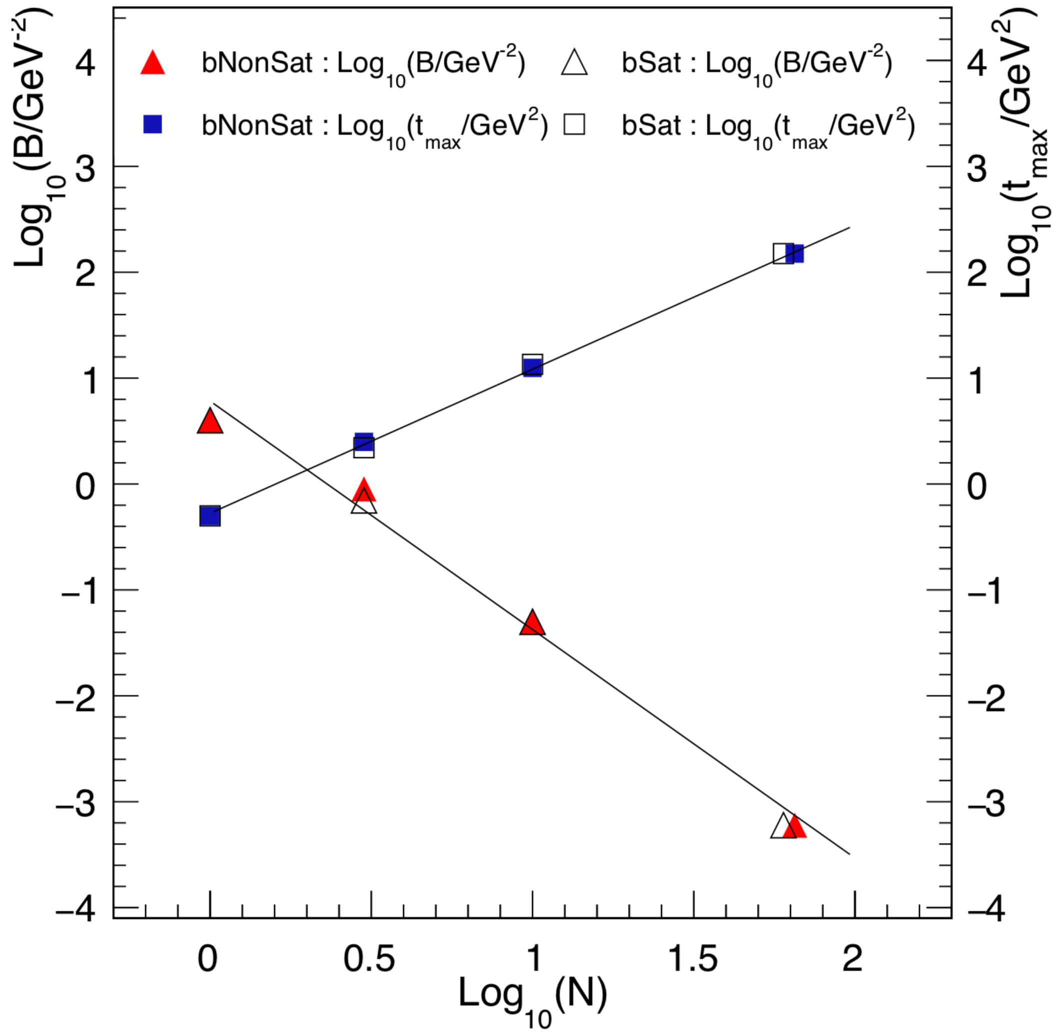

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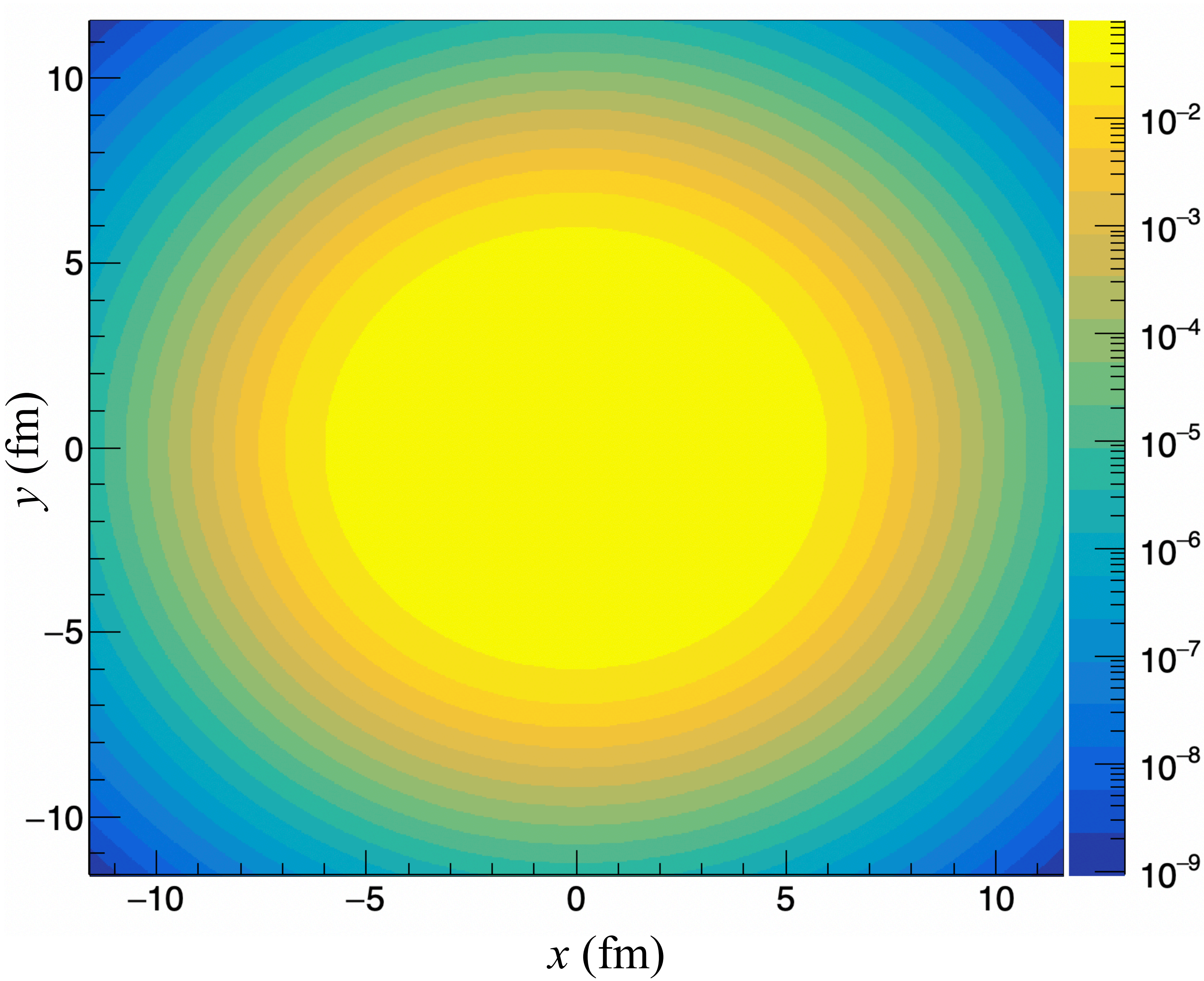
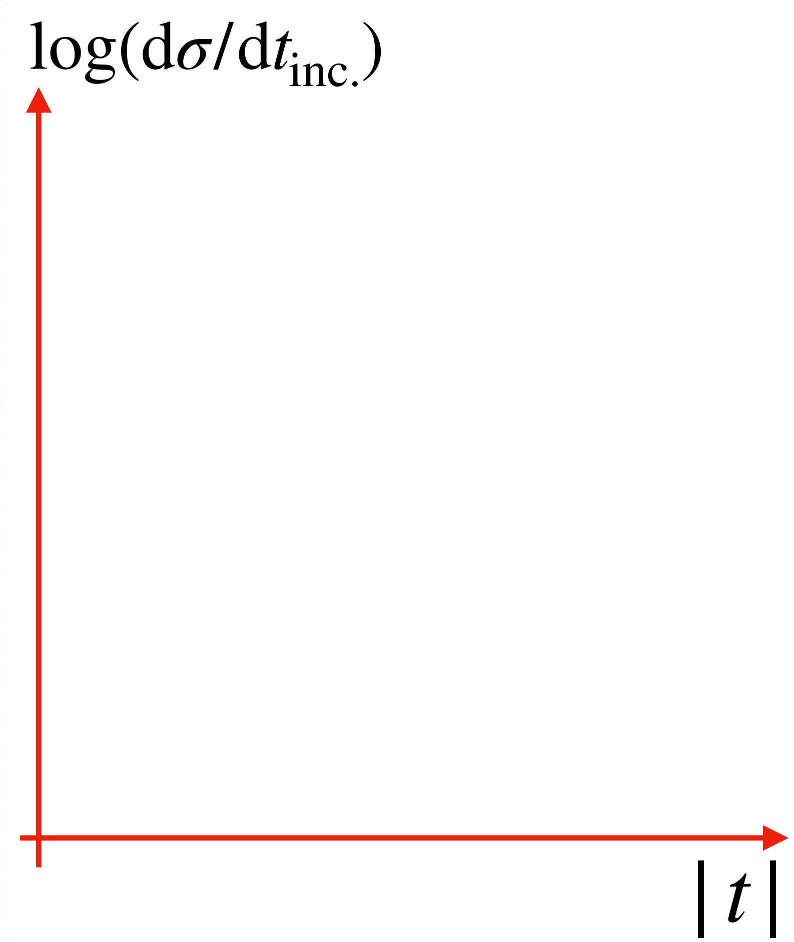
Into the heavy nucleus



$|t| = 0$



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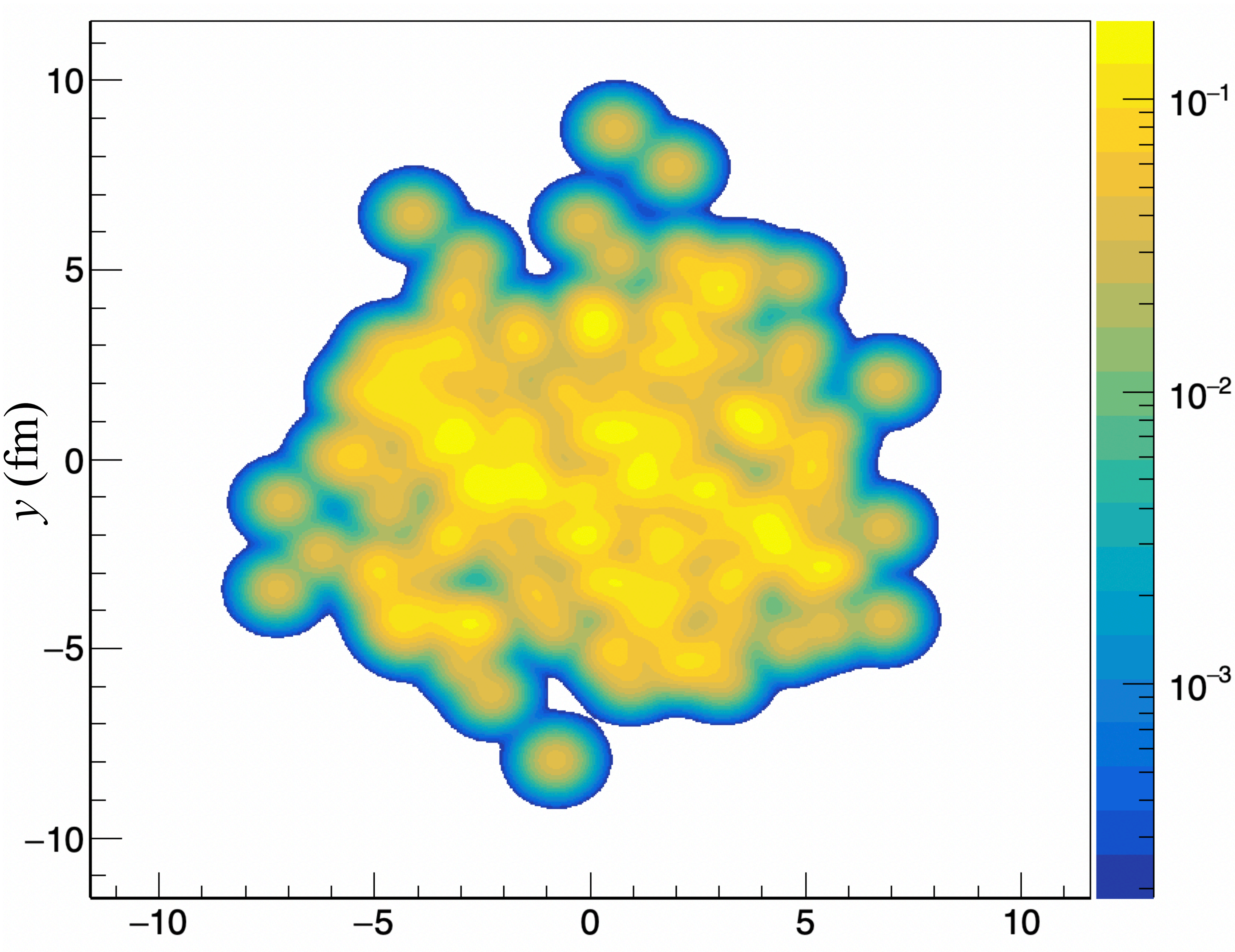
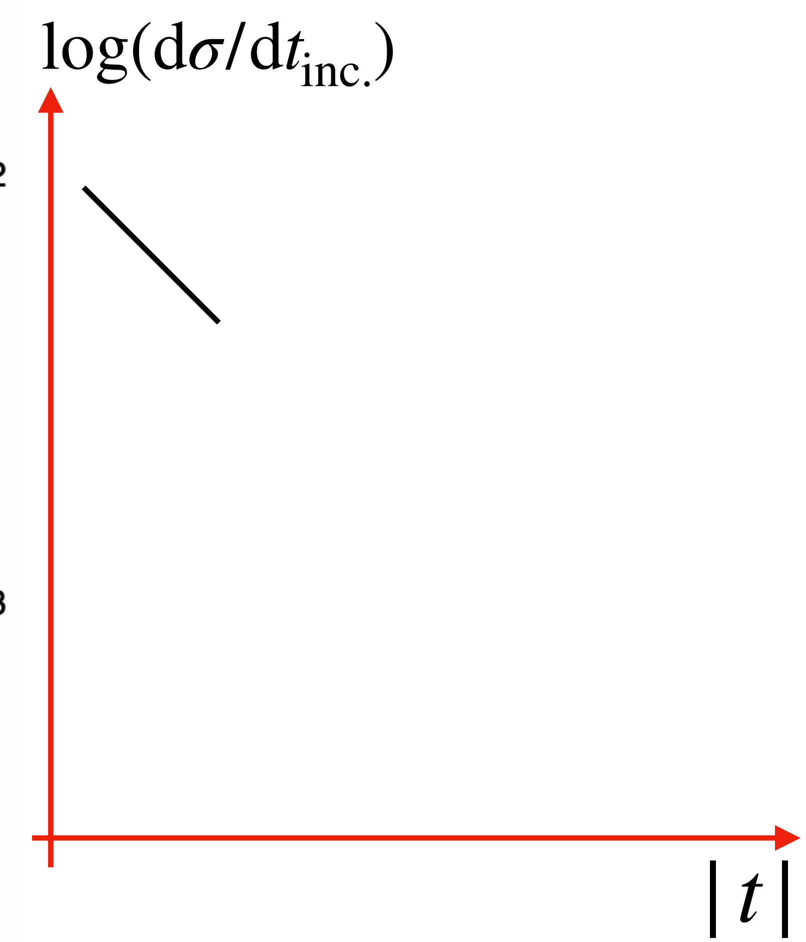
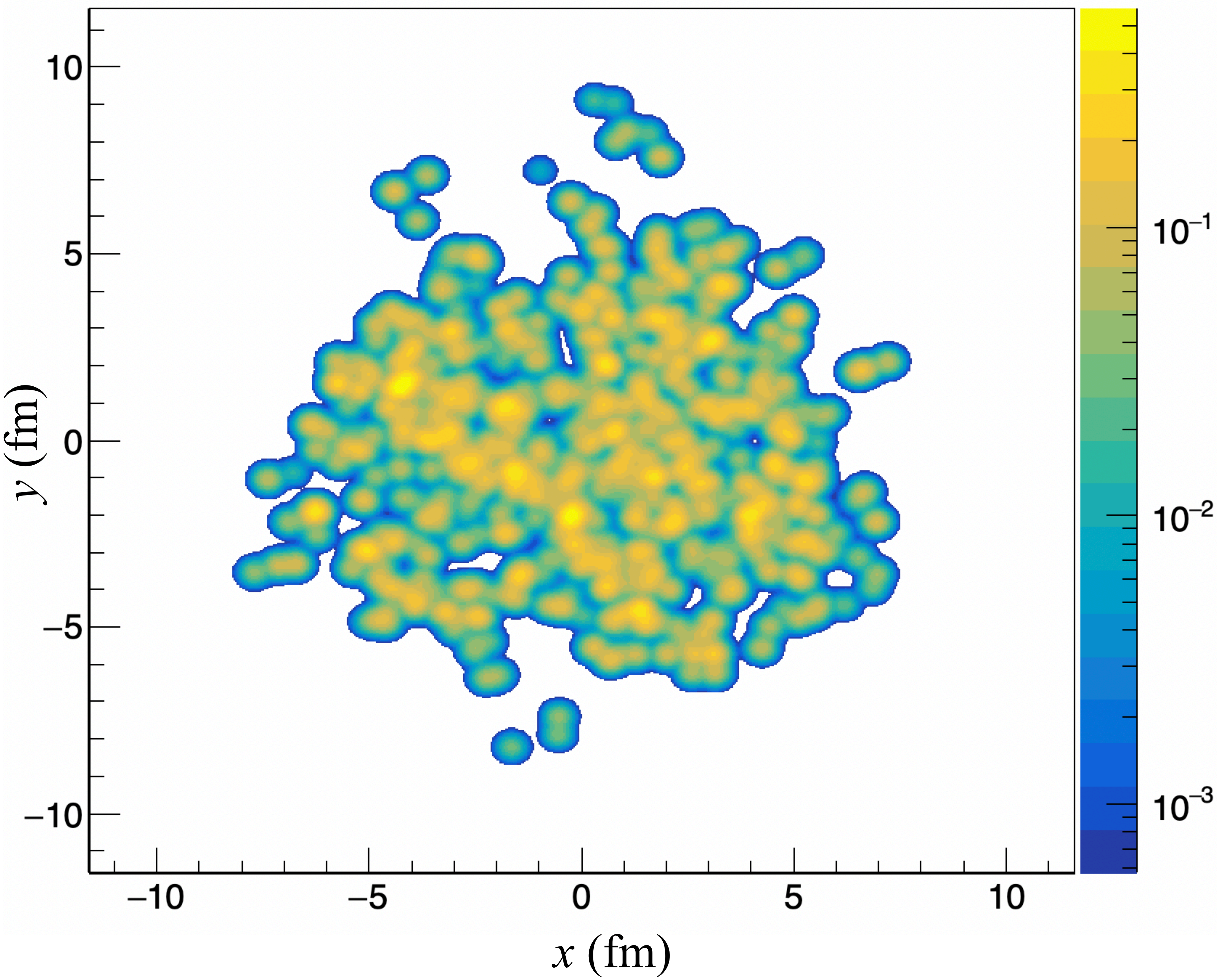


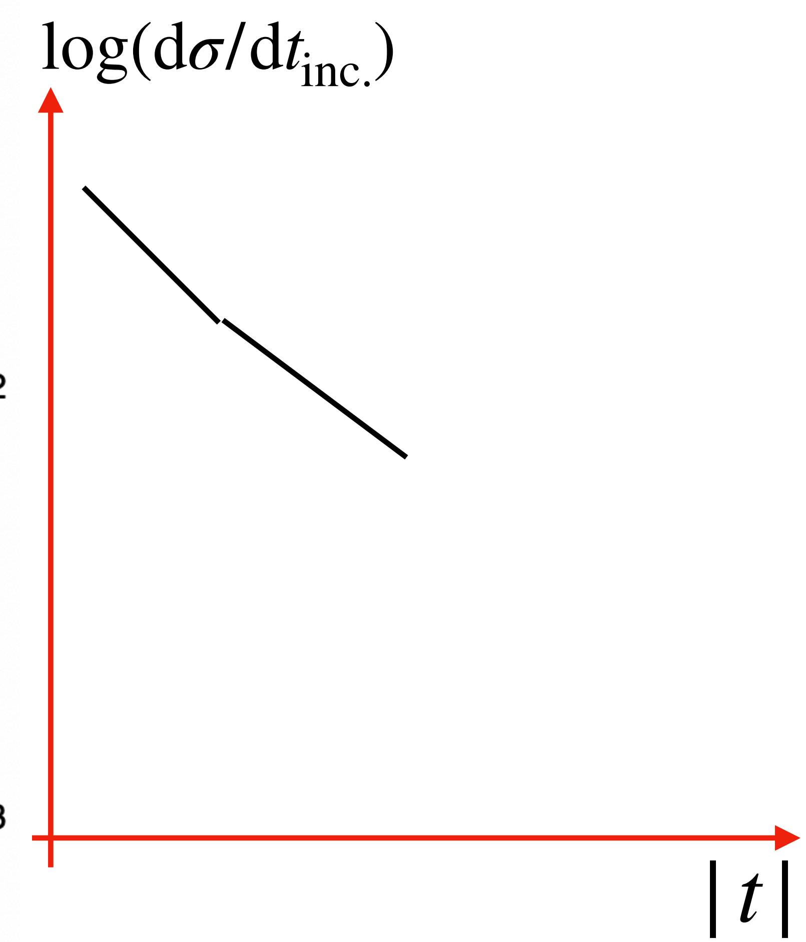
$|t| \lesssim 0.2\ \mathrm{GeV}^2$



Into the heavy nucleus



$0.2 \lesssim |t| \lesssim 2\ \mathrm{GeV}^2$



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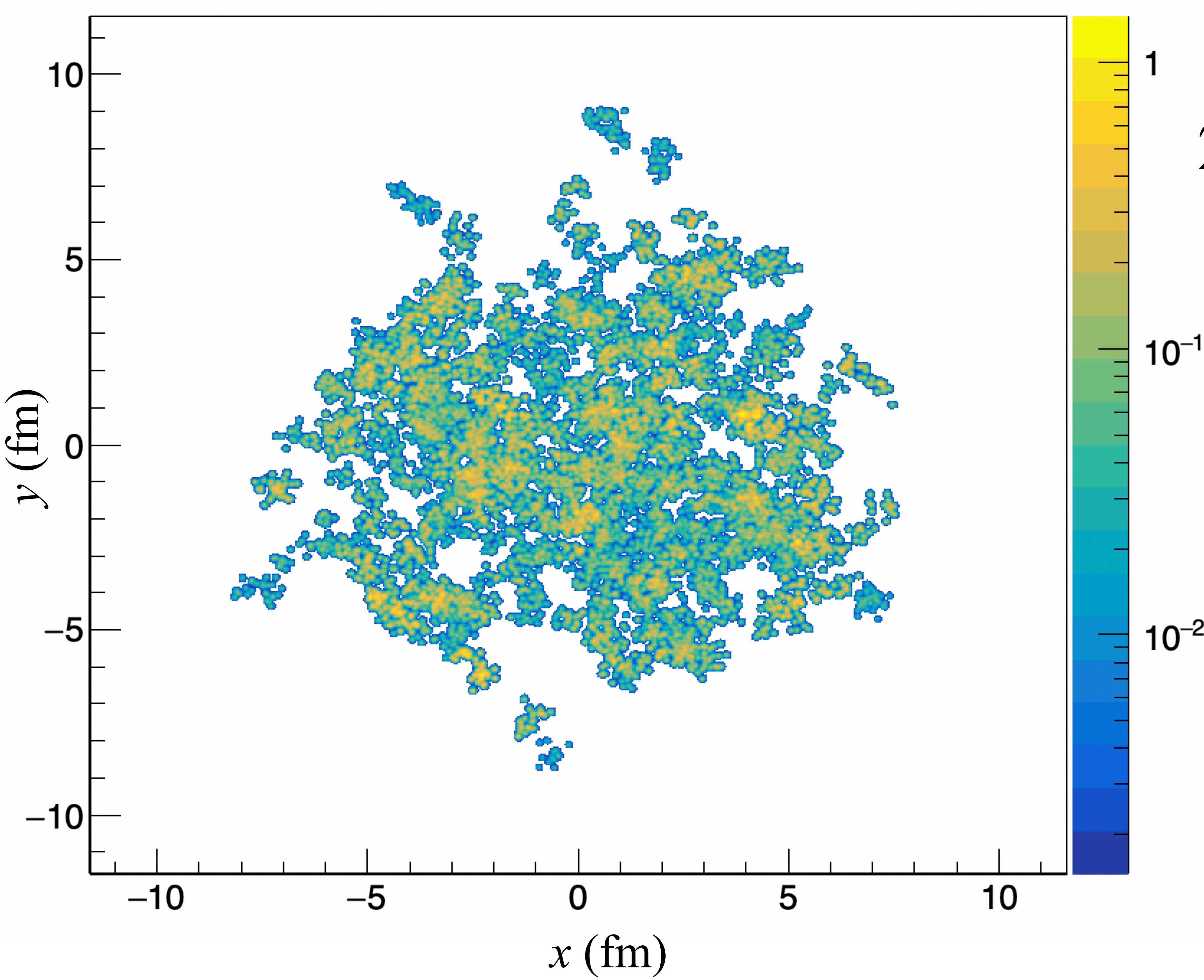
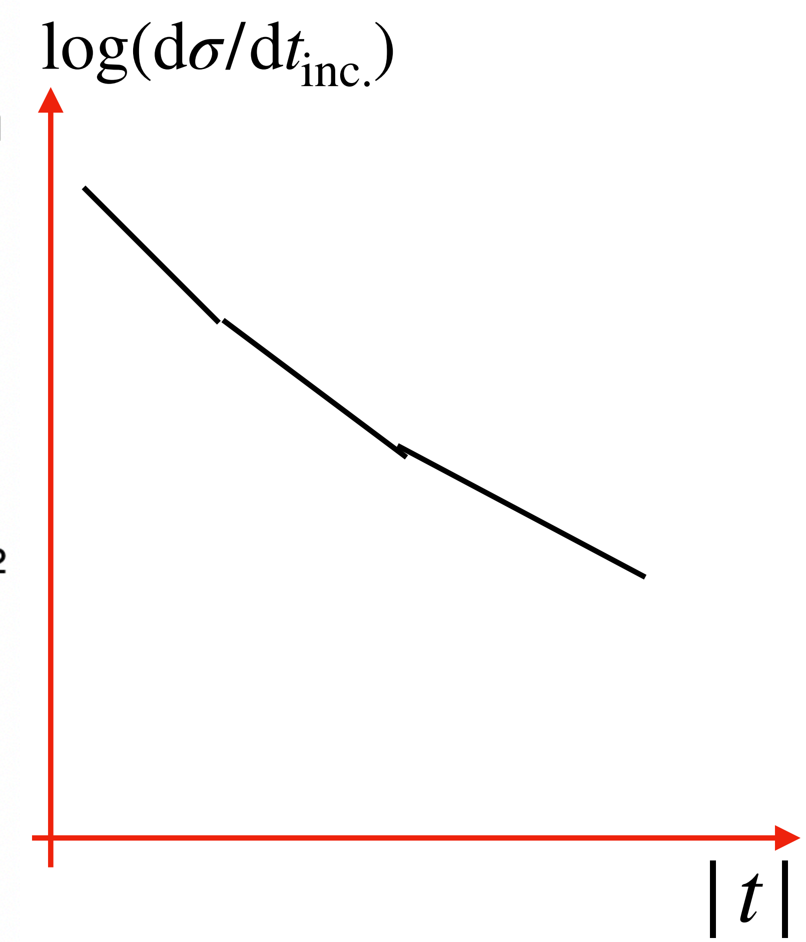
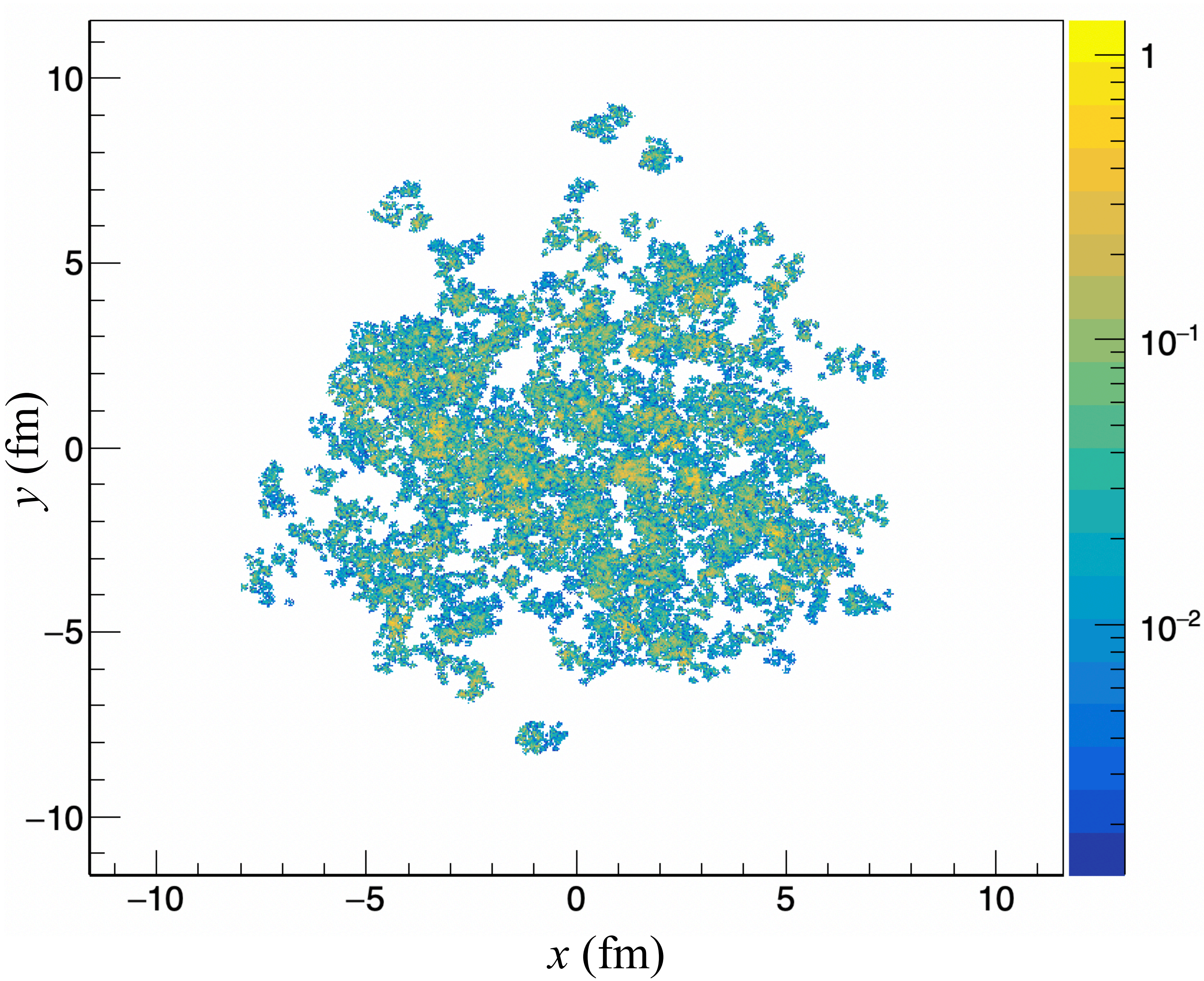


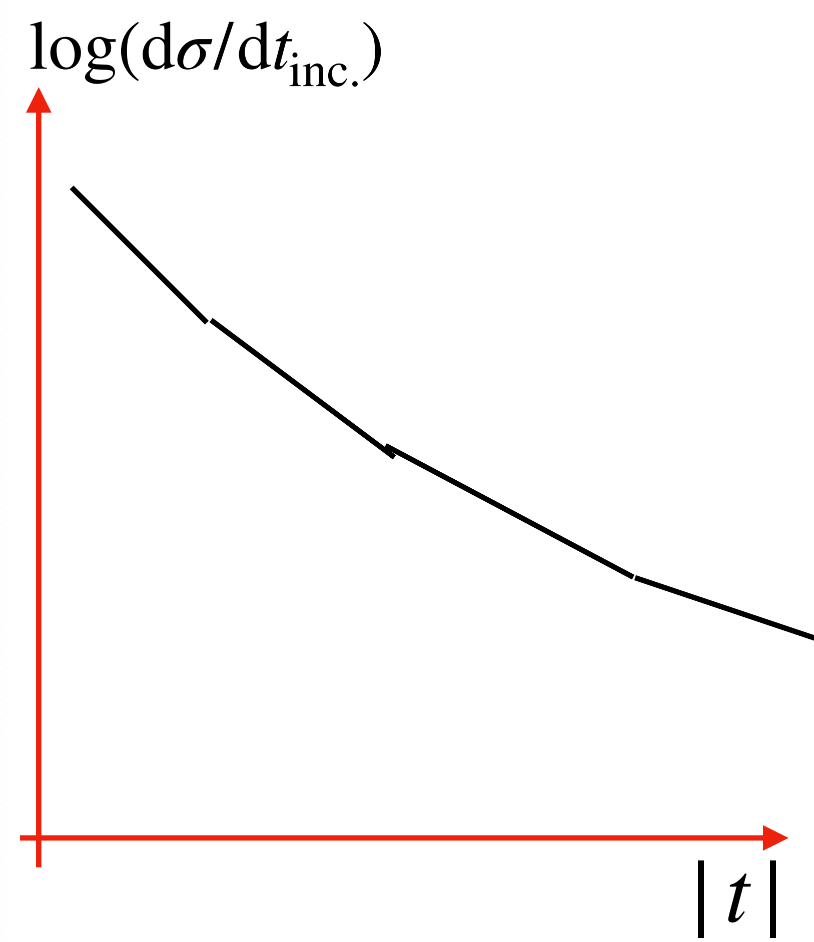
$2 \lesssim |t| \lesssim 12 \text{ GeV}^2$



Into the heavy nucleus



$$12 \lesssim |t| \ \mathrm{GeV}^2$$



Shortcomings of this approach

Hotspot model is non-perturbative. Should not be extended further than $|t| \gtrsim 1\ \mathrm{GeV}^2$
I lieu of a perturbative mechanism for substructure at large $|t|$ we put it in by hand

Introduced many parameters to describe a few data points.

But: It has given us insights into the how the substructure should look in the Good-Walker mechanism to describe incoherent diffraction in *ep* at large $|t|$.

Towards a model for hotspot evolution

We consider a "DGLAP parton shower-like" approach based on resolution, where a hotspot may split into two as the resolution increases.

Initial State at $t = t_0$:

$$T_p(\vec{b}) = \frac{1}{N_q}\sum_{i=1}^{N_q} T_q(|\vec{b} - \vec{b}_i|)$$

$$T_q(\vec{b}) = \frac{1}{2\pi B_q} e^{-\frac{b^2}{2B_q}}$$

Inital State Parameters:
B_q, B_{qc}, N_q with $B_q + B_{qc} \approx 4\ \mathrm{GeV}^{-2}$
Evolution parameter: α, t_0

Probability of a hotspot created at t_0 splitting at $t > t_0$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\alpha}{|t|}\left(\frac{t_0}{t}\right)^{\alpha}$$

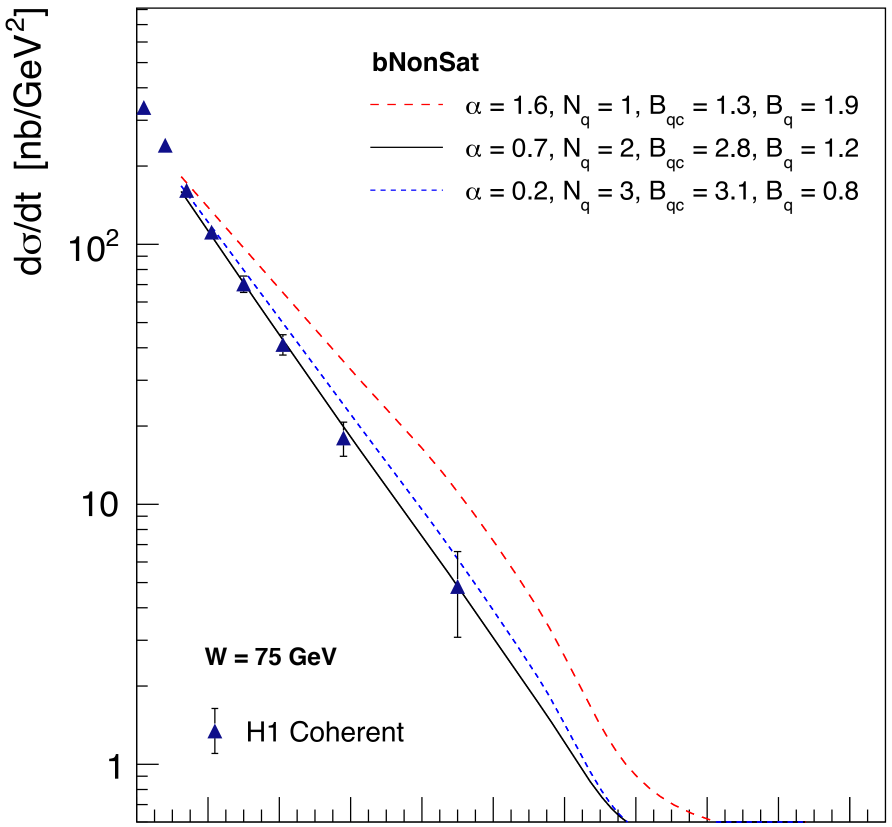
Two offspring hotspots i, j created at distance $d_{ij} = |\vec{b}_i - \vec{b}_j|$, with widths $B_{i,j} = \frac{1}{|t|}$

Conditions for resolution:

Probe resolution: $d_{ij} > \frac{2}{|\vec{\Delta}|}$ Geometry: $d_{ij} > 2\sqrt{B_{i,j}}$

Generate offspring $\vec{b}_{i,j}$ from parent $T_{\mathrm{parent}}(\vec{b}_{i,j})$.
Reject if not resolved.

Hotspot Evolution

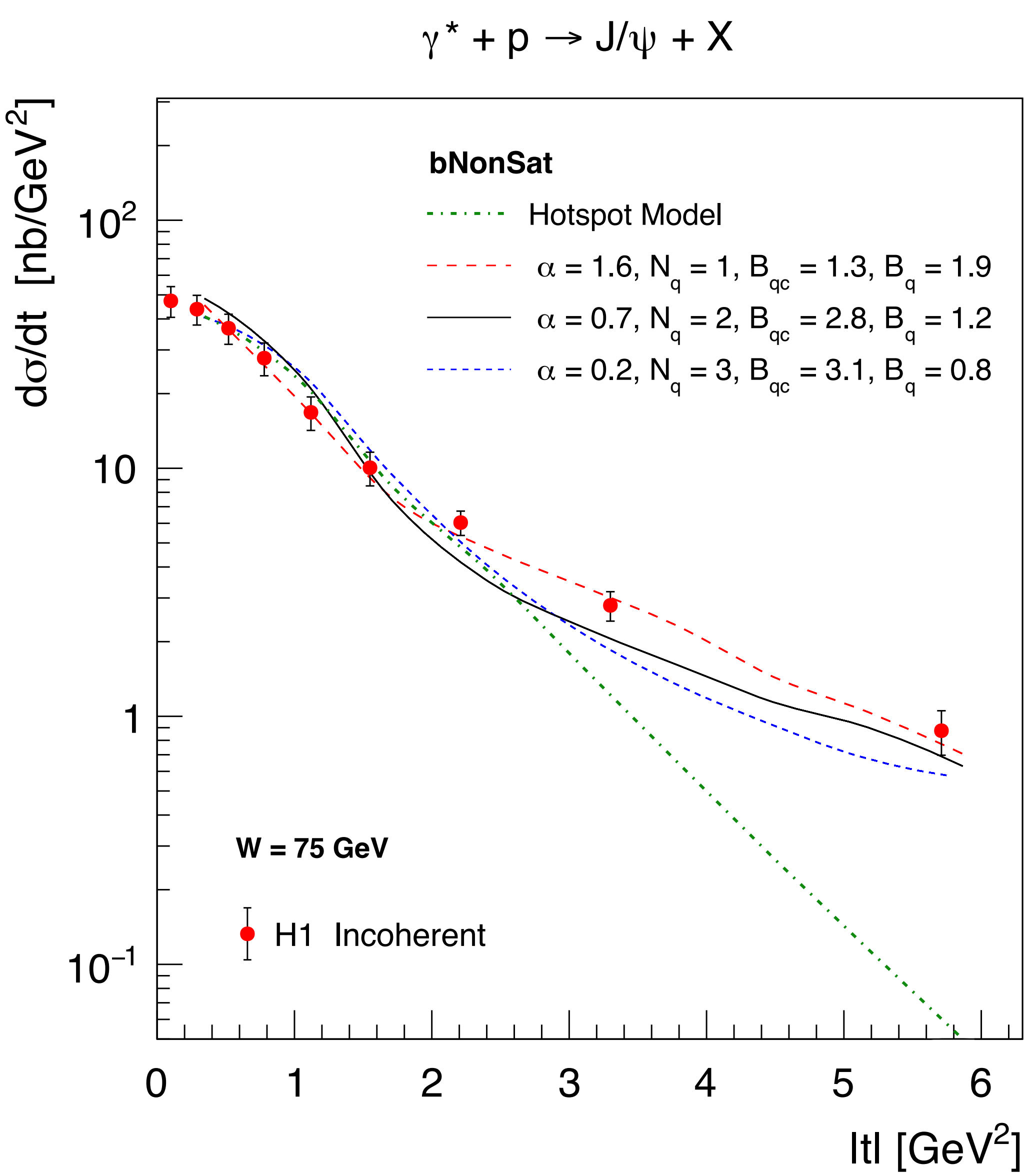


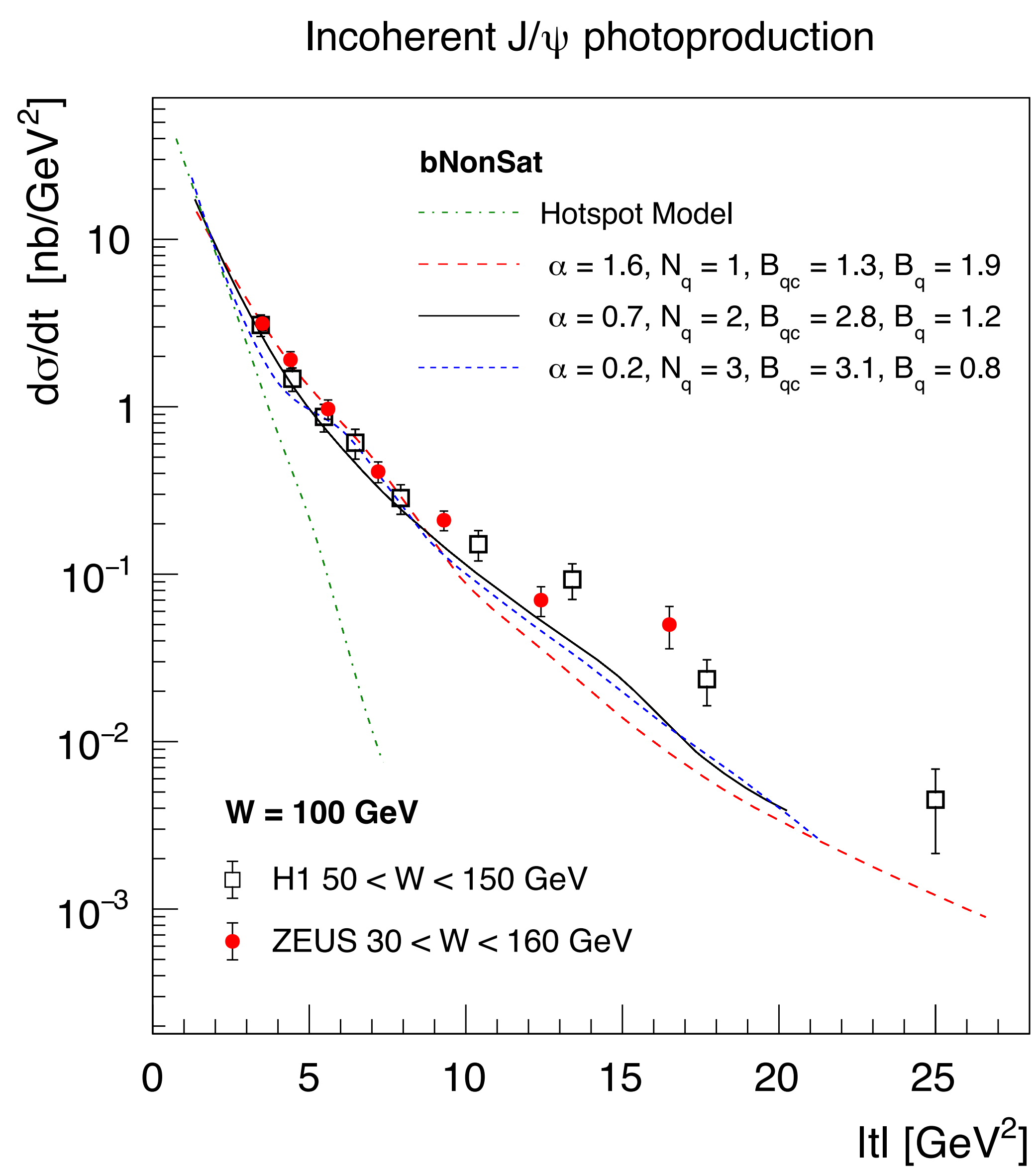
Initial State:

$$T_p(\vec{b}) = \frac{1}{N_q}\sum_{i=1}^{N_q} T_q(|\vec{b} - \vec{b}_i|)$$

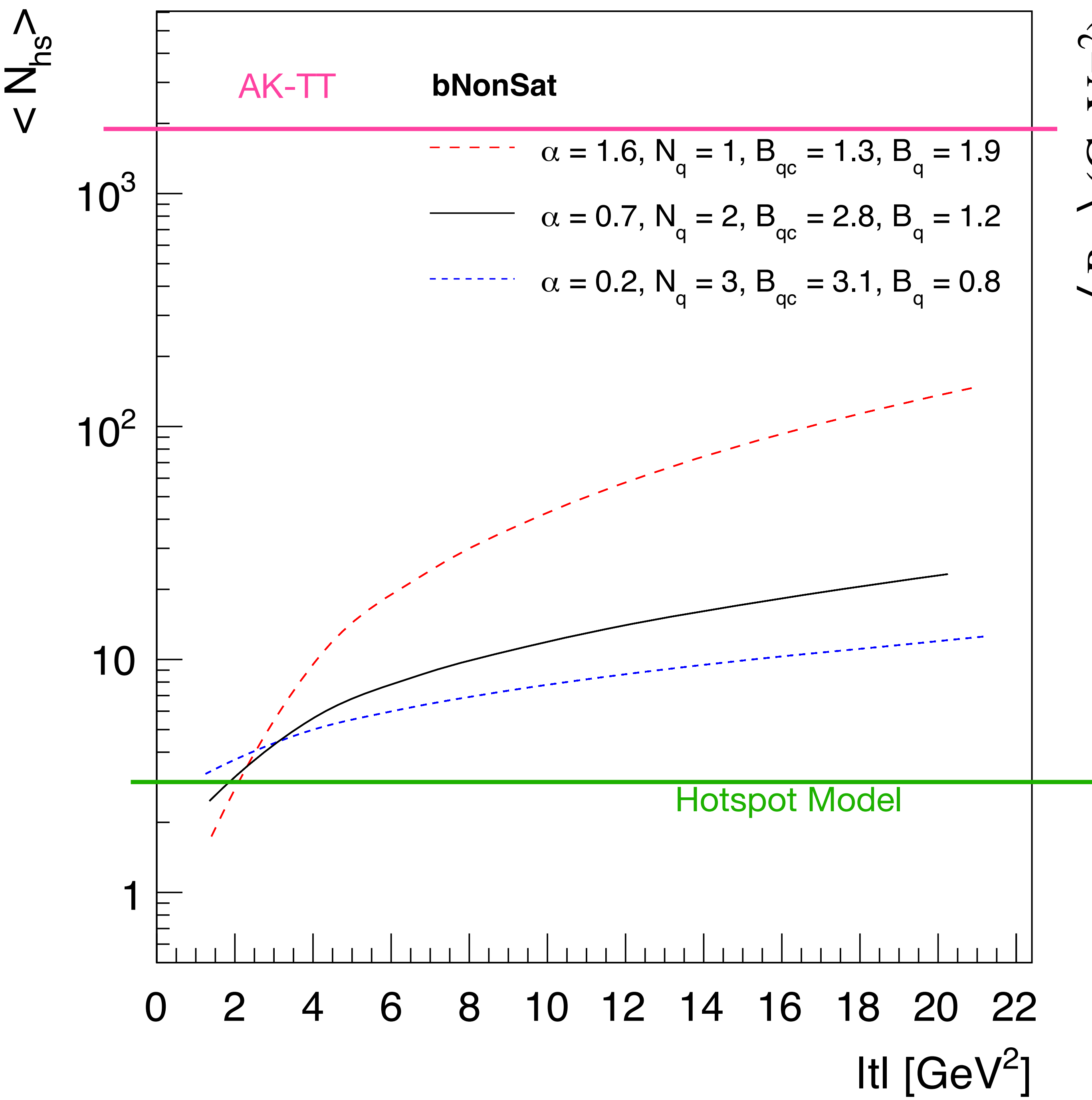
$$T_q(\vec{b}) = \frac{1}{2\pi B_q} e^{-\frac{b^2}{2B_q}}$$

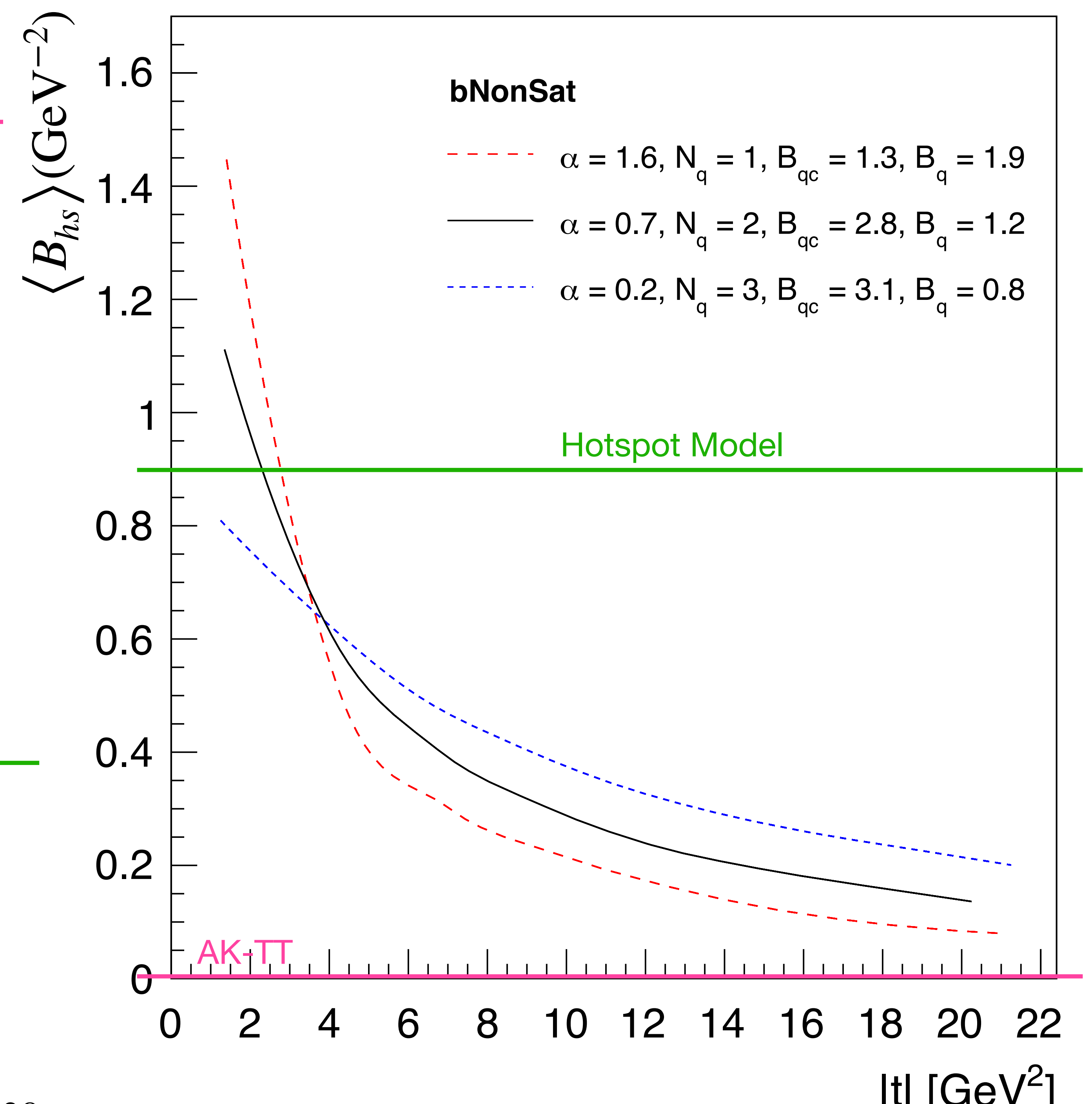
$$\frac{dP}{dt} = \frac{\alpha}{|t|}\left(\frac{t_0}{t}\right)^{\alpha}$$



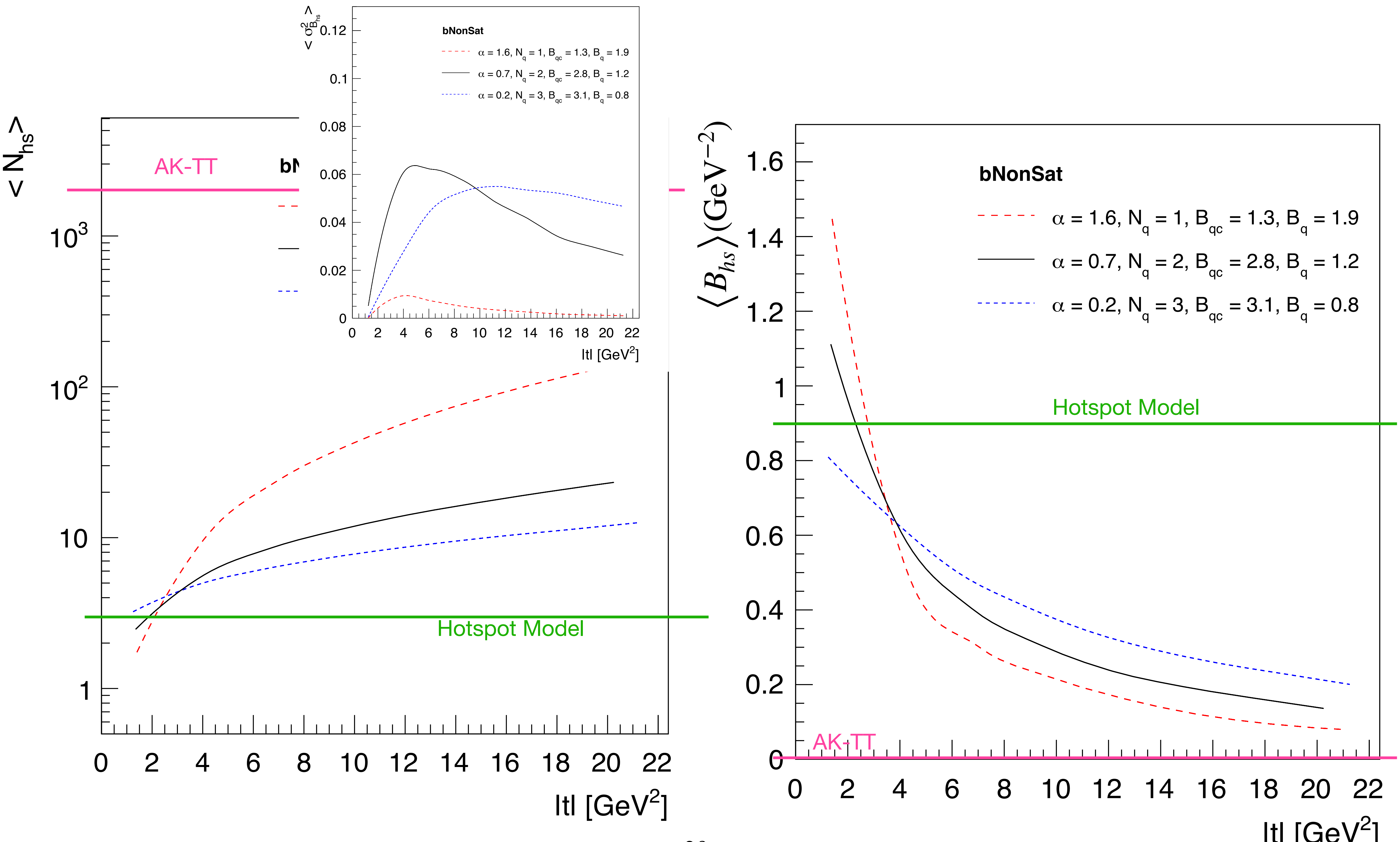


Towards a model for hotspot evolution



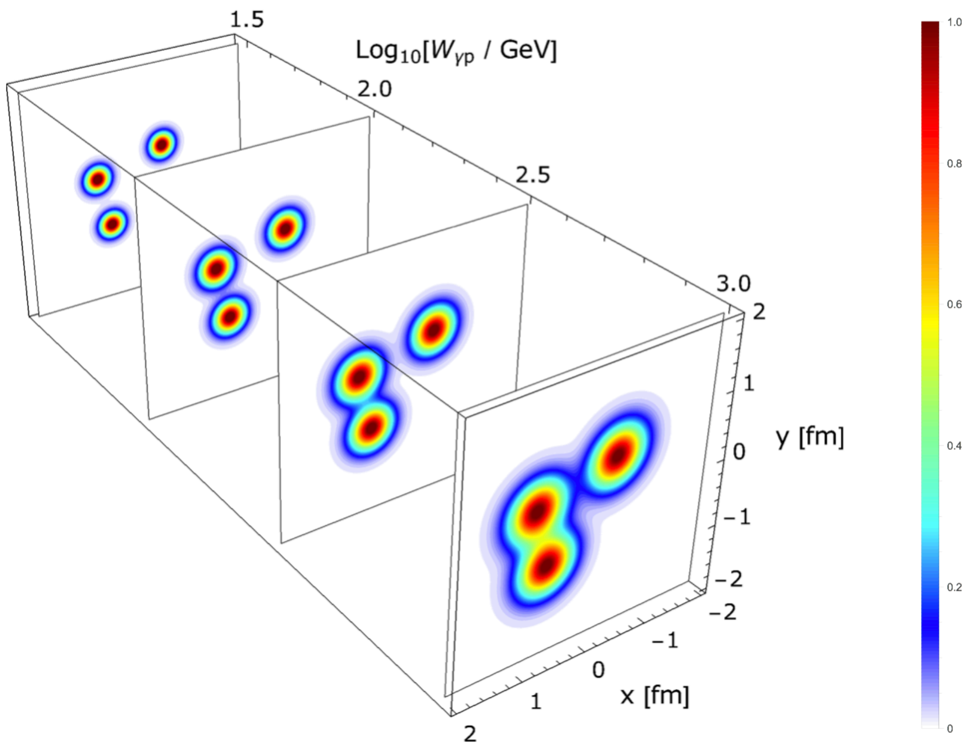


Towards a model for hotspot evolution

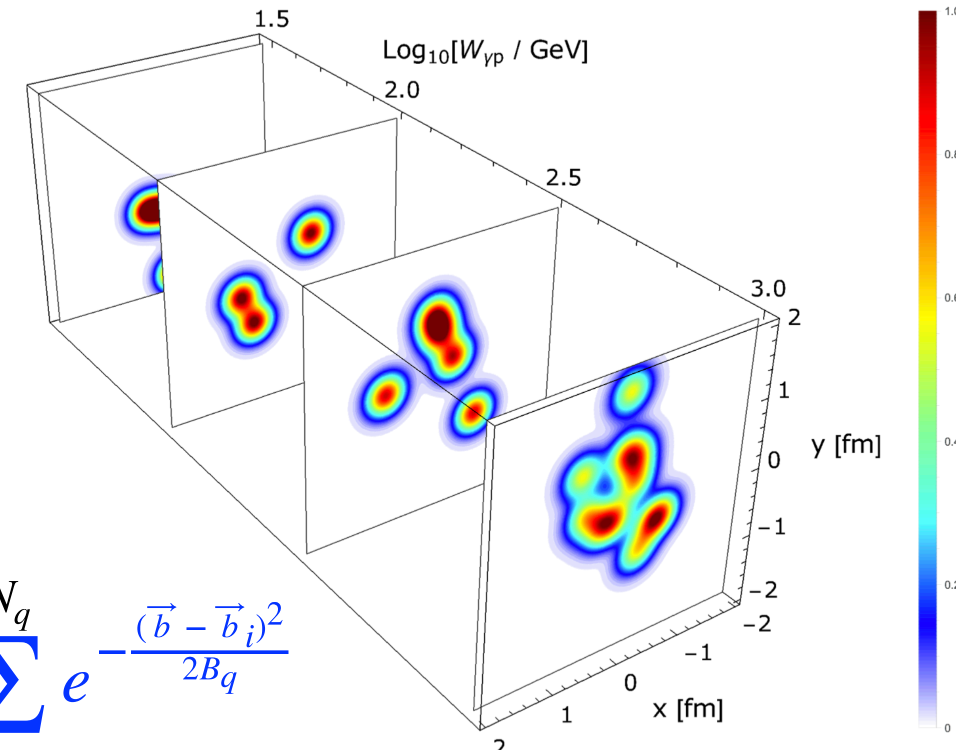


Inital distribution $x_{I\!P}$-dependence

Arjun Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631



$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b} - \vec{b}_i)^2}{2B_q}}$$



$$B_q(x_{I\!P}) = b_0 \ln^2 \frac{x_0}{x_{I\!P}}$$

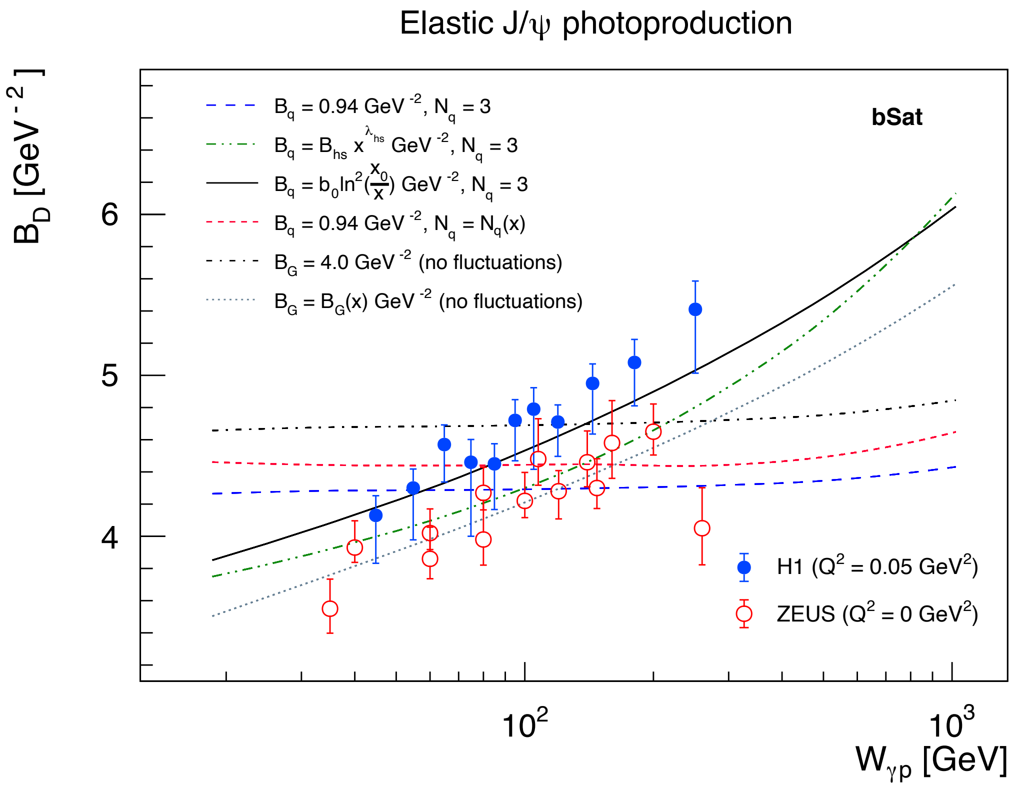
$$r_{\text{rms}} = \sqrt{2(B_{qc} + B_q(x_{I\!P}))}$$

$$N_q \to N_q(x_P) = p_0 \; x_{I\!P}^{p_1}(1+p_2\sqrt{x_{I\!P}})$$

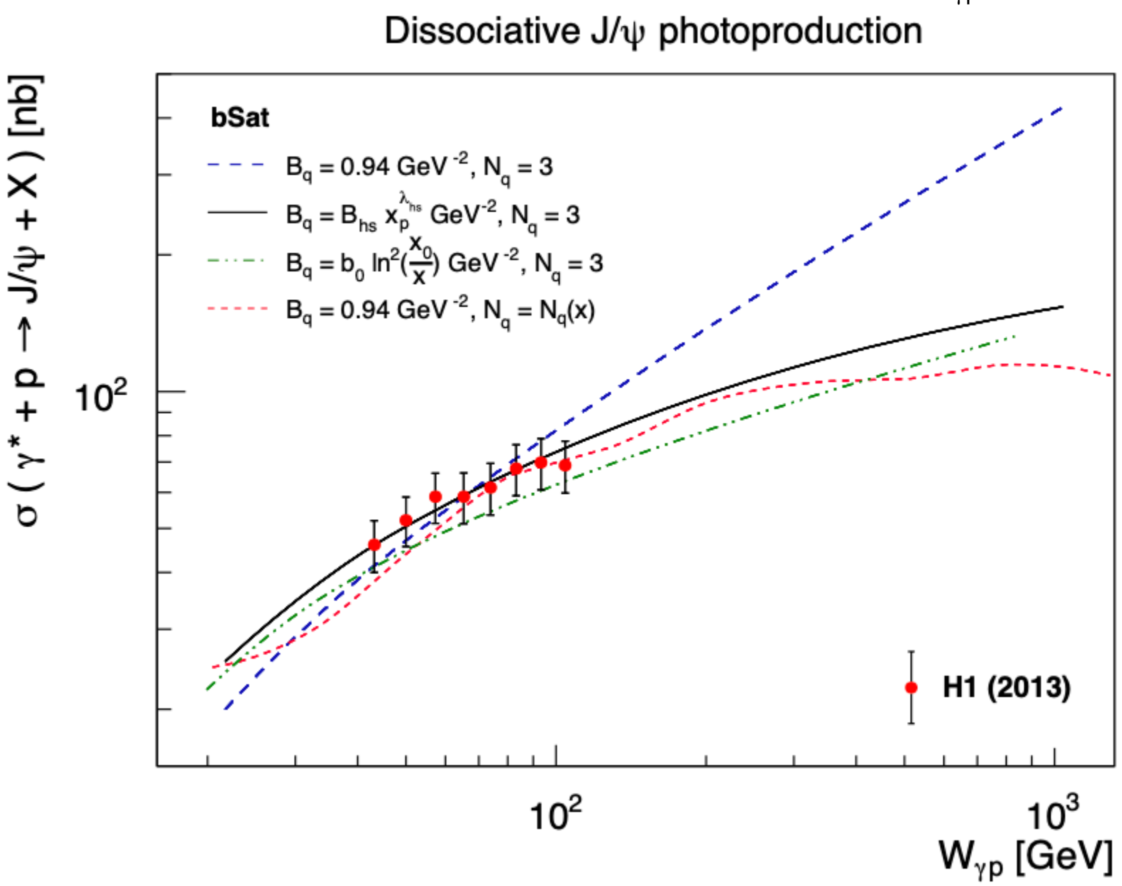
$$p_0 = 0.011, \; p_1 = -0.56, \; p_2 = 165$$

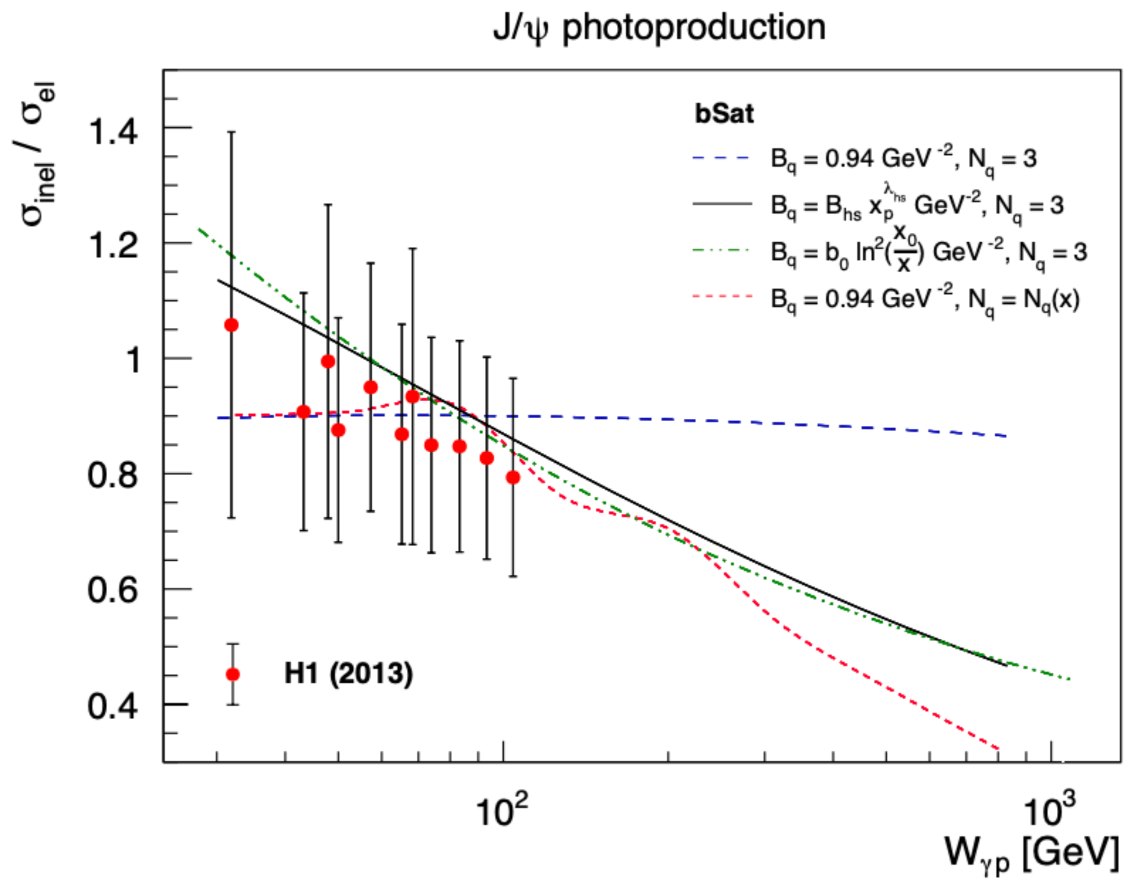
J. Cepila, J. G. Contreras, J. D. Tapia Takaki, *Energy dependence of dissociative J/ψ photoproduction as a signature of gluon saturation at the LHC*, Phys. Lett. B 766 (2017) 186–191.

Inital distribution $x_{I\!P}$-dependence



Arjun Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631





Conclusions and Outlook

I have shown how to implement nuclear and nucleon substructures in Sar*t*re.
These are encoded in the *gluon thickness profile* $T(\vec{b})$ and manifest as the Fourier transform of the *t*-spectrum.

We have seen that we can understand the full *t*-spectrum for $|t| < 30\ \mathrm{GeV}^2$ in the Good-Walker picture, by extending the hotspot model

However, in doing so we extend the hotspot model beyond its applicability and introduce many parameters.

We can regain the description of the full *t*-spectrum with a hotspot evolution model

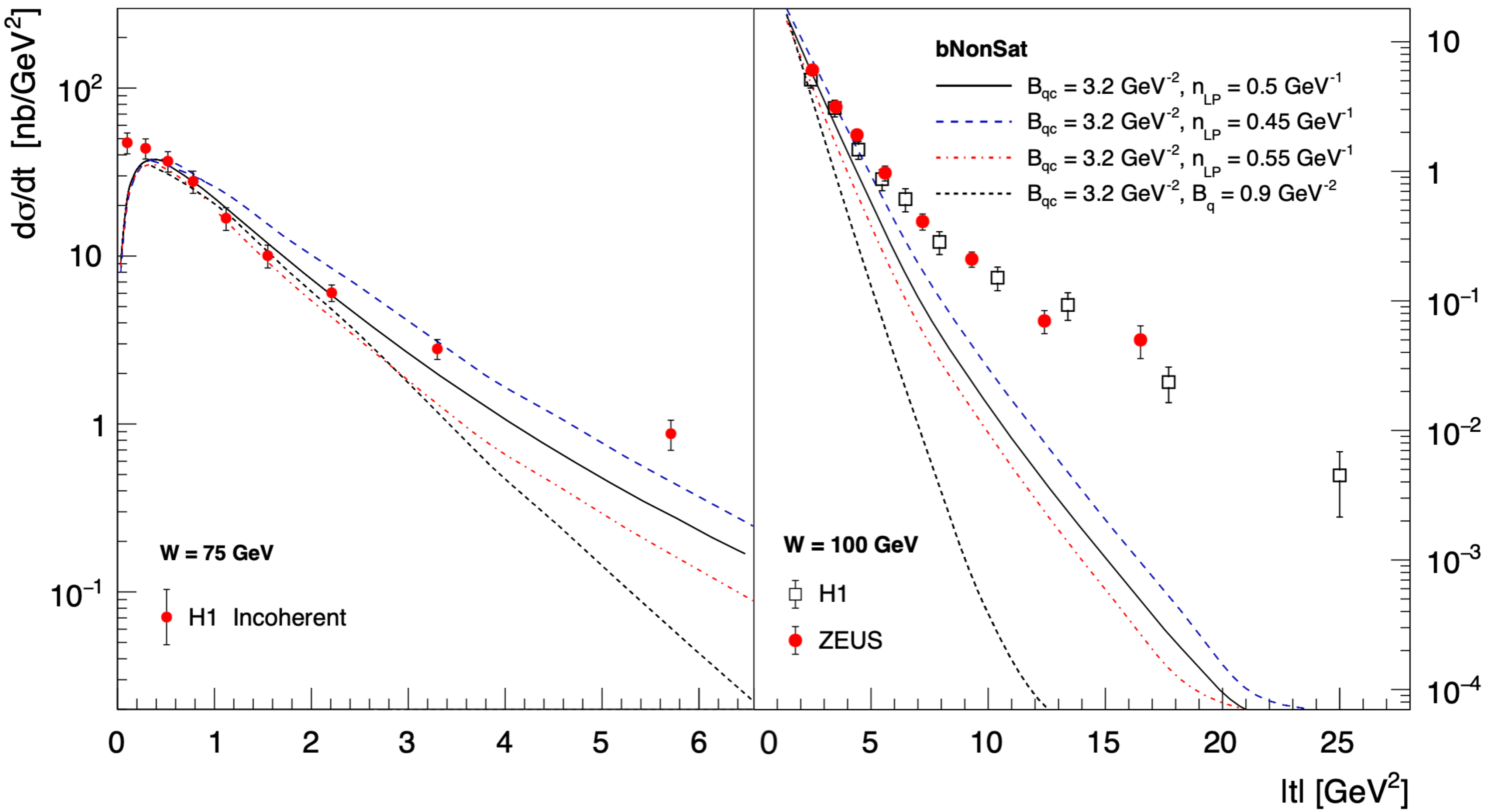
Want to extend this approach to *W*-dependence as well.

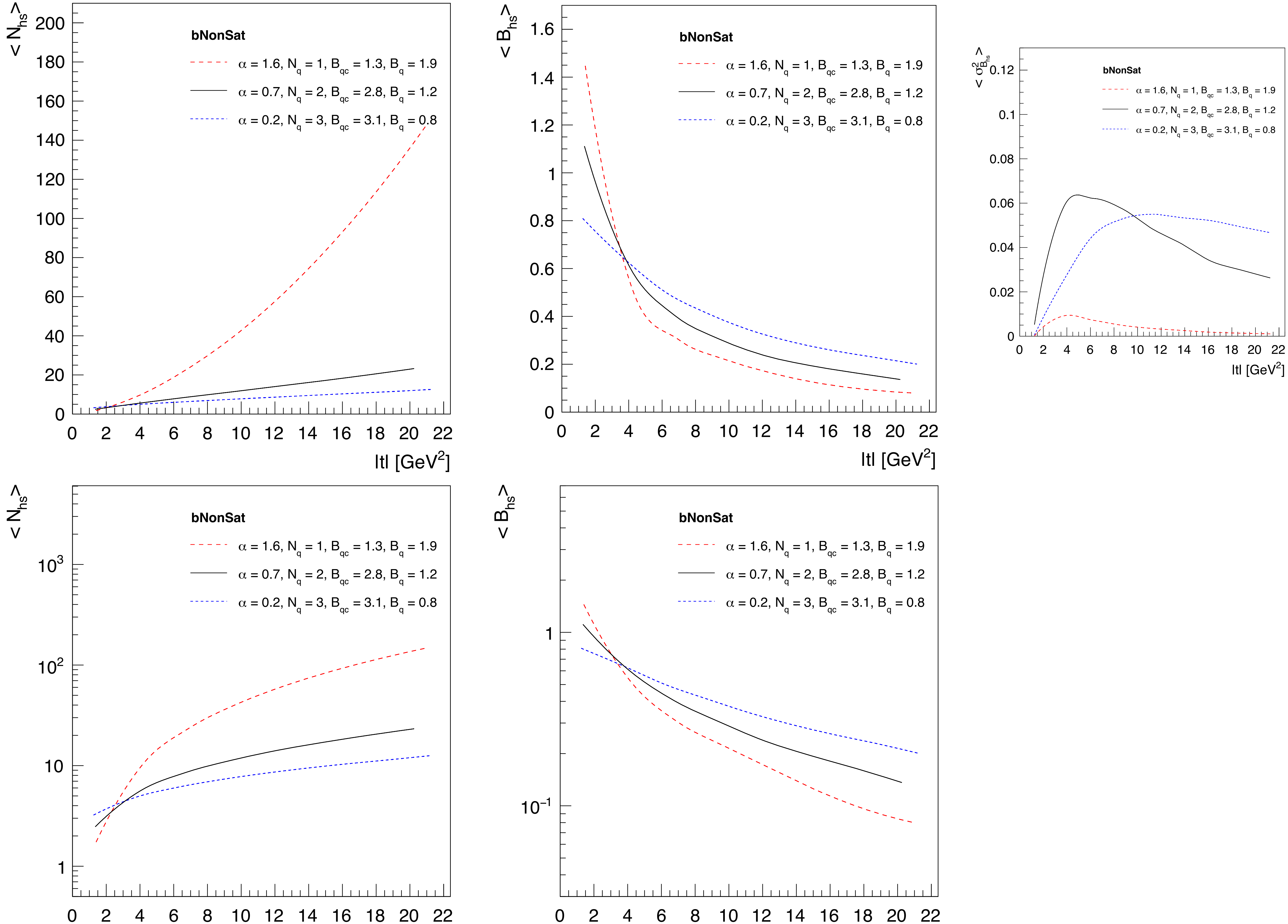
Highly interesting to study the *W*- and *t*- spectra with big level arms and resolution in future experiments such as the EIC.

Back Up

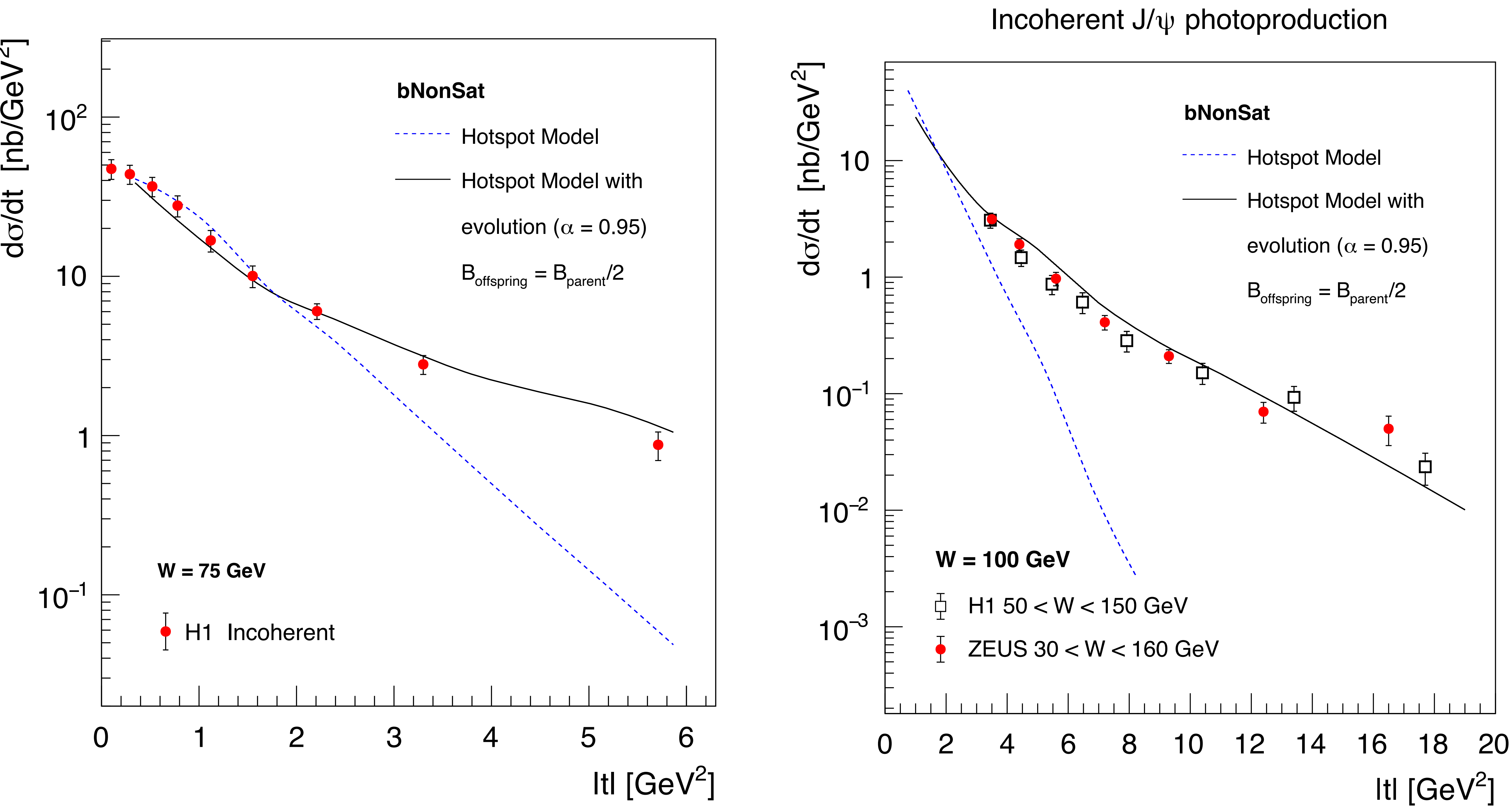
Laplacian hotspots

$$T_q(\mathbf{b}) = \frac{1}{4\pi n_{LP}^3} bK_1\left[-\frac{b}{n_{LP}}\right]$$





< N_hs >
bNonSat
α = 1.6, N_q = 1, B_qc = 1.3, B_q = 1.9
α = 0.7, N_q = 2, B_qc = 2.8, B_q = 1.2
α = 0.2, N_q = 3, B_qc = 3.1, B_q = 0.8
|t| [GeV^2]
< B_hs >
bNonSat
α = 1.6, N_q = 1, B_qc = 1.3, B_q = 1.9
α = 0.7, N_q = 2, B_qc = 2.8, B_q = 1.2
α = 0.2, N_q = 3, B_qc = 3.1, B_q = 0.8
|t| [GeV^2]
< σ^2_B_hs >
bNonSat
α = 1.6, N_q = 1, B_qc = 1.3, B_q = 1.9
α = 0.7, N_q = 2, B_qc = 2.8, B_q = 1.2
α = 0.2, N_q = 3, B_qc = 3.1, B_q = 0.8
|t| [GeV^2]
< N_hs >
bNonSat
α = 1.6, N_q = 1, B_qc = 1.3, B_q = 1.9
α = 0.7, N_q = 2, B_qc = 2.8, B_q = 1.2
α = 0.2, N_q = 3, B_qc = 3.1, B_q = 0.8
|t| [GeV^2]
< B_hs >
bNonSat
α = 1.6, N_q = 1, B_qc = 1.3, B_q = 1.9
α = 0.7, N_q = 2, B_qc = 2.8, B_q = 1.2
α = 0.2, N_q = 3, B_qc = 3.1, B_q = 0.8
|t| [GeV^2]



Incoherent J/ψ photoproduction
dσ/dt [nb/GeV²]
|t| [GeV²]
bNonSat
Hotspot Model
Hotspot Model with
evolution (α = 0.95)
B_offspring = B_parent/2
W = 75 GeV
H1 Incoherent
W = 100 GeV
H1 50 < W < 150 GeV
ZEUS 30 < W < 160 GeV