ÉCOLE POLYTECHNIQUE

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Gluodynamics

P2IO
Physique des 2 Infinis et des Origines

From initial gluons to hydrodynamics: Gluons inside hadrons and their thermalization

Oct. 24-25, Institut Pascal

Generalized parton distributions and energy-momentum tensor

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Energy-momentum tensor (EMT)

EMT is a key fundamental object

It is the conserved current associated with invariance under spacetime translations

$$\boxed{P^\mu = \int \mathrm{d}^3x \, T^{0\mu}(x)}$$

$$\phi(x+a) = e^{iP\cdot a}\phi(x)e^{-iP\cdot a} \quad \Leftrightarrow \quad i[P^\mu, \phi(x)] = \partial^\mu \phi(x)$$

It also plays the role of source for gravitation in the Einstein equations of GR

$$\boxed{R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} = \kappa \, T_{\mu\nu}}$$

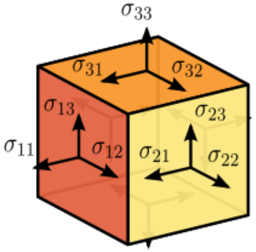
QCD EMT operator

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$\begin{aligned} T_q^{\mu\nu} &= \overline{\psi}\gamma^\mu \tfrac{i}{2} \overset{\leftrightarrow}{D}{}^\nu \psi \\ T_g^{\mu\nu} &= -G^{\mu\lambda} G^\nu{}_\lambda + \tfrac{1}{4} \, g^{\mu\nu} \, G^2 \end{aligned}$$

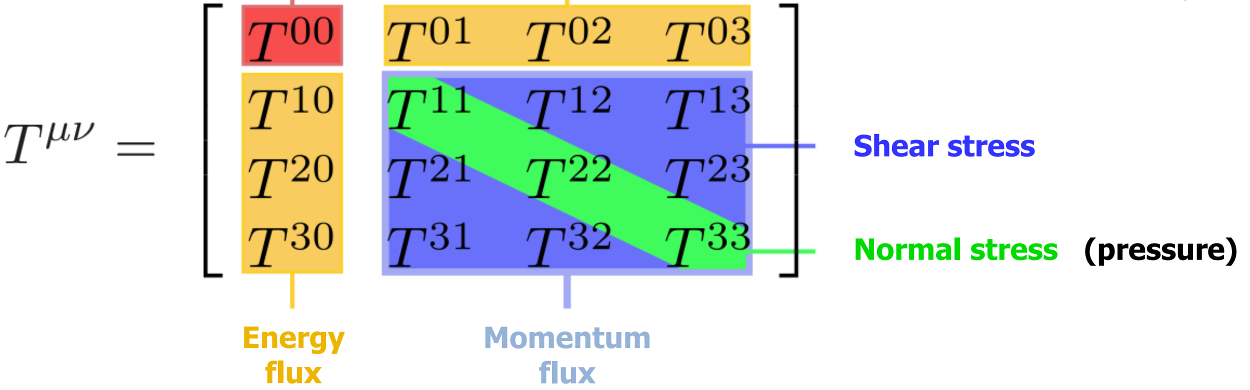
Energy-momentum tensor (EMT)

Mass, spin and pressure are all encoded in the EMT



Energy density

Momentum density



$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

Shear stress

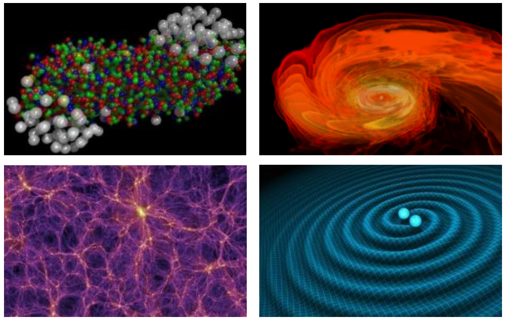
Normal stress (pressure)

Energy flux

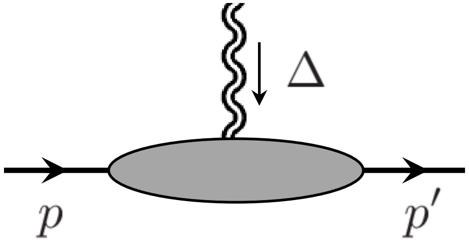
Momentum flux

Central object for

- **Nucleon mechanical properties**
- **Quark-gluon plasma**
- **Relativistic hydrodynamics**
- **Stellar structure and dynamics**
- **Cosmology**
- **Gravitational waves**
- **Modified theories of gravitation**
- **…**



Gravitational form factors (GFFs)



Poincaré symmetry constrains the form of the EMT matrix elements

Symmetrized variables $P = \frac{p'+p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$

$$p'^2 = p^2 = M^2 \rightarrow \boxed{P \cdot \Delta = 0}$$

Spin-0 target $T^{\mu\nu} = \sum_a T_a^{\mu\nu}$

$$\langle p'|T_a^{\mu\nu}(0)|p\rangle = 2M\left[\frac{P^\mu P^\nu}{M}A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a(t) + Mg^{\mu\nu}\bar{C}_a(t)\right]$$

Non-conserved

$$0 = \langle p'|\partial_\mu T^{\mu\nu}(x)|p\rangle = i\Delta_\mu\langle p'|T^{\mu\nu}(x)|p\rangle \rightarrow \sum_a \bar{C}_a(t) = 0$$

Translation symmetry $\phi(x+a) = e^{iP\cdot a}\phi(x)e^{-iP\cdot a}$

Gravitational form factors (GFFs)

Spin-1/2 target

$$\langle p', s'|T_a^{\mu\nu}(0)|p, s\rangle = \overline{u}(p', s')\Gamma_a^{\mu\nu}(P, \Delta)u(p, s)$$

$$\Gamma_a^{\mu\nu}(P, \Delta) = \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + M g^{\mu\nu}\bar{C}_a(t)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M} J_a(t) - \frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{2M} S_a(t)$$

$$x^{\{\mu}y^{\nu\}} = x^\mu y^\nu + x^\nu y^\mu$$
$$x^{[\mu}y^{\nu]} = x^\mu y^\nu - x^\nu y^\mu$$

NB: Because of the Dirac equation, alternative but equivalent parametrizations may *look* quite different !

Gordon identity

$$\overline{u}(p', s')\gamma^\mu u(p, s) = \overline{u}(p', s')\left[\frac{P^\mu}{M} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}\right] u(p, s)$$

Gravitational form factors (GFFs)

Spin-*j* target

$$\langle p', s'|T_a^{\mu\nu}(0)|p, s\rangle = \overline{\eta}(p', s')\Gamma_a^{\mu\nu}(P,\Delta)\eta(p,s)$$

$$\begin{aligned}\Gamma_a^{\mu\nu}(P,\Delta) = {} & \frac{P^\mu P^\nu}{M}A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a(t) + Mg^{\mu\nu}\bar{C}_a(t) \\ & + \frac{iP^{\{\mu}\Sigma^{\nu\}\lambda}\Delta_\lambda}{2M}J_a(t) - \frac{iP^{[\mu}\Sigma^{\nu]\lambda}\Delta_\lambda}{2M}S_a(t) \\ & + \text{higher spin multipoles}\end{aligned}$$

Integer *j*>0	$9j+2$ GFFs
Half-integer *j*>1/2	$9j-\frac{1}{2}$ GFFs

Same parametrization holds for hadrons, nuclei, …

Poincaré sum rules

Four-momentum conservation

$$p^\mu = \sum_a \frac{\langle p,s|P^\mu_a|p,s\rangle}{\langle p,s|p,s\rangle} \quad \Rightarrow \quad \boxed{\begin{array}{l} \sum\limits_a A_a(0) = 1 \\[1em] \sum\limits_a \bar{C}_a(0) = 0 \end{array}}$$

LF momentum fraction $\langle x\rangle_a = \frac{\langle P_a^+\rangle}{p^+} = A_a(0)$

Mechanical equilibrium !

$$\langle \int \mathrm{d}^3r\, T_a^{\mu\nu}\rangle\big|_{\text{rest}} = M \left(\begin{array}{c|ccc} A_a(0)+\bar{C}_a(0) & 0 & 0 & 0 \\ \hline 0 & -\bar{C}_a(0) & 0 & 0 \\ 0 & 0 & -\bar{C}_a(0) & 0 \\ 0 & 0 & 0 & -\bar{C}_a(0) \end{array}\right) \quad \Leftrightarrow \quad \left(\begin{array}{c|ccc} \varepsilon_a & 0 & 0 & 0 \\ \hline 0 & p_a & 0 & 0 \\ 0 & 0 & p_a & 0 \\ 0 & 0 & 0 & p_a \end{array}\right) V$$

Partial pressure

Angular momentum conservation

$$\langle J^i\rangle = \sum_a \frac{\langle p,s|\int \mathrm{d}^3r\, \epsilon^{ijk} r^j \frac{1}{2} T_a^{\{0k\}}|p,s\rangle}{\langle p,s|p,s\rangle} \quad \Rightarrow \quad \boxed{\sum_a J_a(0) = j}$$

$$\partial_\mu(x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + S^{\mu\alpha\beta}) = 0 \quad \Rightarrow \quad \boxed{\begin{array}{l} S_q(t) = j\, G_A^q(t) \\ S_g(t) = 0 \end{array}}$$

Orbital AM **Intrinsic AM**

Let us recap

Spin-1/2 target

$$\langle p', s'|T_a^{\mu\nu}(0)|p,s\rangle = \overline{u}(p',s')\Gamma_a^{\mu\nu}(P,\Delta)u(p,s)$$

$$\Gamma_a^{\mu\nu}(P,\Delta) = \frac{P^\mu P^\nu}{M}A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M}C_a(t) + Mg^{\mu\nu}\bar{C}_a(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M}J_a(t) - \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{2M}S_a(t)$$

$A_a(0) \leftrightarrow$ **Momentum**

$\bar{C}_a(0) \leftrightarrow$ **Pressure**

$J_a(0) \leftrightarrow$ **Total angular momentum**

$S_q(0) \leftrightarrow$ **Intrinsic angular momentum**

$C_a(0) \leftrightarrow$ **?**

Also, what do we learn from the t-dependence ?

Spatial distributions

Phase-space approach

$$\langle\psi|O(x)|\psi\rangle = \int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\mathrm{d}^3R\,\rho_\psi(\vec{R},\vec{P})\,\langle O\rangle_{\vec{R},\vec{P}}(x)$$

Wigner distribution

$$\rho_\psi(\vec{R},\vec{P}) = \int \mathrm{d}^3z\,e^{-i\vec{P}\cdot\vec{z}}\,\psi^*(\vec{R}-\tfrac{\vec{z}}{2})\psi(\vec{R}+\tfrac{\vec{z}}{2})$$

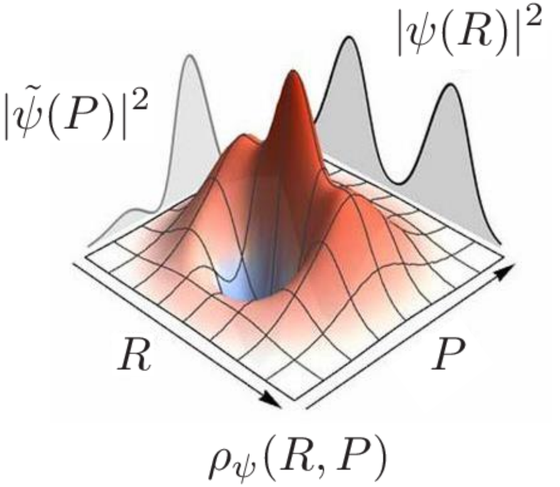
$$= \int \frac{\mathrm{d}^3q}{(2\pi)^3}\,e^{-i\vec{q}\cdot\vec{R}}\,\tilde{\psi}^*(\vec{P}+\tfrac{\vec{q}}{2})\tilde{\psi}(\vec{P}-\tfrac{\vec{q}}{2})$$

$$\psi(\vec{r}) = \int \frac{\mathrm{d}^3p}{(2\pi)^3}\,e^{i\vec{p}\cdot\vec{r}}\,\tilde{\psi}(\vec{p})$$

Quasi-probabilistic interpretation

$$\int \mathrm{d}^3R\,\rho_\psi(\vec{R},\vec{P}) = |\tilde{\psi}(\vec{P})|^2$$

$$\int \frac{\mathrm{d}^3P}{(2\pi)^3}\,\rho_\psi(\vec{R},\vec{P}) = |\psi(\vec{R})|^2$$



Spatial distributions

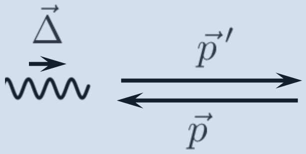
Internal distribution (for a state localized in phase-space)

$$\langle O\rangle_{\vec{R},\vec{P}}(\vec{x}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot(\vec{x}-\vec{R})}\, \frac{\langle P+\frac{\Delta}{2}|O(0)|P-\frac{\Delta}{2}\rangle}{2P^0}$$

$$= \langle O\rangle_{\vec{0},\vec{P}}(\vec{r}), \qquad \vec{r} = \vec{x} - \vec{R}$$

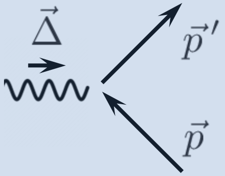
Elastic frames $\Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} \overset{!}{=} 0$ (no energy transfer → same initial and final boost factor)

$|\vec{P}| = 0$

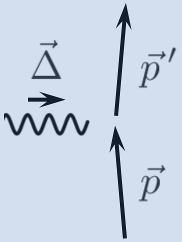


Breit (or brick-wall or rest) frame

$|\vec{P}| \neq 0$



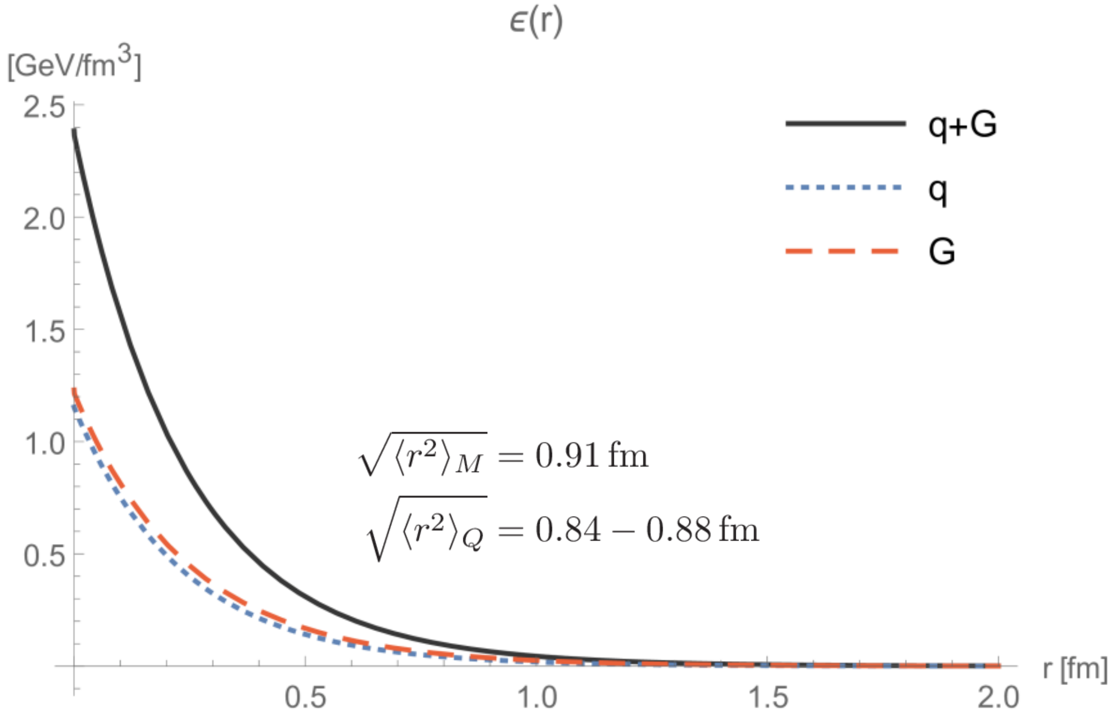
$|\vec{P}| \gg M$



Infinite-momentum frame (IMF)

Energy distribution (3D Breit frame)

$$\langle T^{00}\rangle_{\vec{0},\vec{0}}(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{\langle P+\frac{\Delta}{2}|T^{00}(0)|P-\frac{\Delta}{2}\rangle}{2P^0}\bigg|_{\vec{P}=\vec{0}}$$



$$\sqrt{\langle r^2\rangle_M} = 0.91\ \mathrm{fm}$$

$$\sqrt{\langle r^2\rangle_Q} = 0.84 - 0.88\ \mathrm{fm}$$

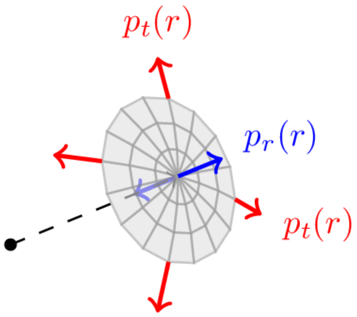
Multipole model for the gravitational form factors

$$F(t) = \frac{F(0)}{(1+t/\Lambda^2)^n}$$

Pressure distributions (3D Breit frame)

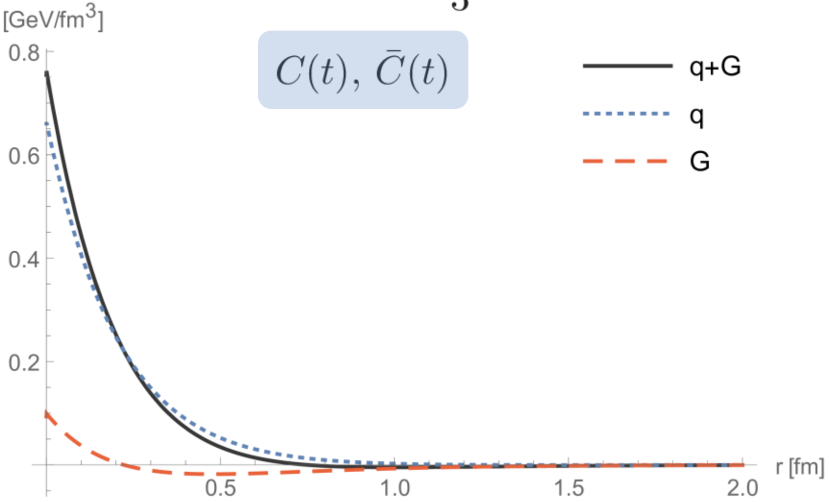
$$\langle T^{ij}\rangle_{\vec{0},\vec{0}}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \frac{\langle P+\frac{\Delta}{2}|T^{ij}(0)|P-\frac{\Delta}{2}\rangle}{2P^0}\bigg|_{\vec{P}=\vec{0}}$$

$$= \delta^{ij}\, p(r) + \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\,\delta^{ij}\right) s(r)$$



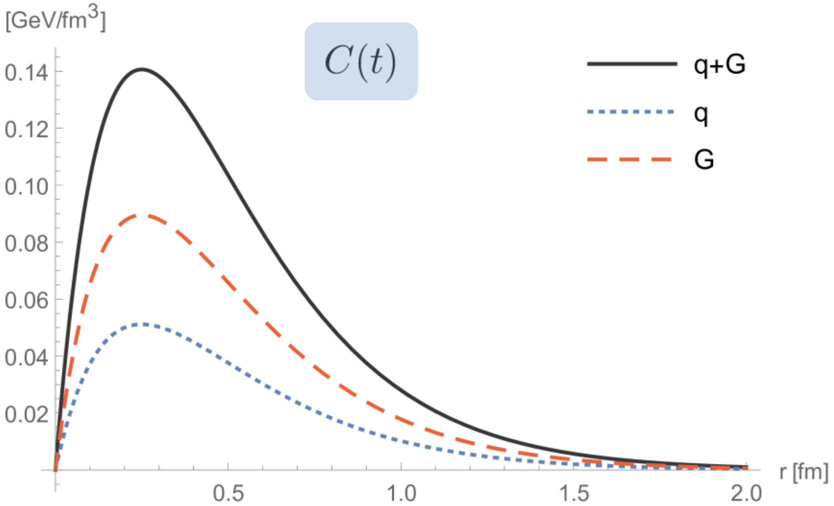
Isotropic pressure

$$p(r) = \frac{p_r(r) + 2p_t(r)}{3}$$



Pressure anisotropy

$$s(r) = p_r(r) - p_t(r)$$



How to access GFFs ?

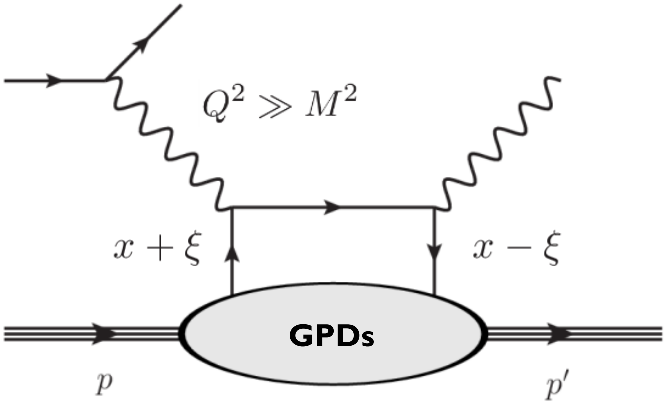
Measuring *directly* GFFs is currently not realistic …

… but a spin-2 exchange can be mimicked by two spin-1 exchanges !

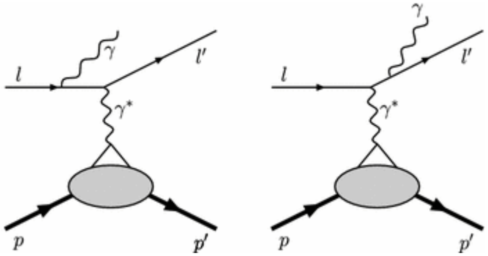
Target must remain intact → exclusive reaction

Generalized PDFs

Deeply virtual Compton scattering (DVCS)



interferes with



Bethe-Heitler

Correlator (in $A^+ = 0$ gauge)

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \langle p', s'|\overline{\psi}(-\tfrac{z}{2})\gamma^+\psi(\tfrac{z}{2})|p, s\rangle\Big|_{z^+=z_\perp=0}$$

$$= \frac{1}{2P^+}\,\overline{u}(p', s')\left[\gamma^+ H_q(x,\xi,t) + \frac{i\sigma^{+\lambda}\Delta_\lambda}{2M}\, E_q(x,\xi,t)\right] u(p, s)$$

Momentum transfer

$$\xi = -\frac{\Delta^+}{2P^+} = \frac{p^+ - p'^+}{p^+ + p'^+}, \qquad t = \Delta^2 = (p' - p)^2$$

Generalized PDFs

Link with other non-perturbative functions

$$\boxed{H_q(x,0,0) = f_q(x)}$$

PDF

$$\boxed{\begin{aligned} \int \mathrm{d}x\, H_q(x,\xi,t) &= F_1^q(t) \\ \int \mathrm{d}x\, E_q(x,\xi,t) &= F_2^q(t) \end{aligned}}$$

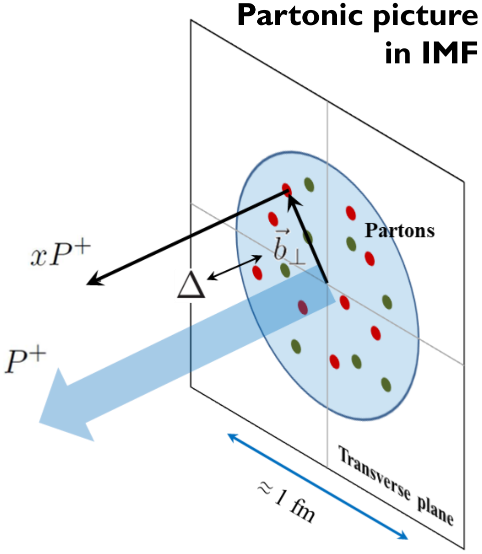
Electromagnetic form factors

2+1D imaging (in $A^+ = 0$ gauge)

$$\rho(x,\vec{b}_\perp) = P^+ \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2}\, e^{-i\vec{\Delta}_\perp\cdot\vec{b}_\perp}\, \frac{\langle p'|j^+(xP^+)|p\rangle}{2P^+}\Big|_{\Delta^+=0}$$

$$j^\mu(k^+) = \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ik^+z^-}\, \overline{\psi}(-\tfrac{z}{2})\gamma^\mu\psi(\tfrac{z}{2})\Big|_{z^+=z_\perp=0}$$

Partonic picture in IMF



Generalized PDFs

Link with gravitational form factors

$$T_q^{++}(0) = (\overline{\psi}\gamma^+ \tfrac{i}{2}\overset{\leftrightarrow}{D}{}^+\psi)(0)$$

$$= 2(P^+)^2 \int \mathrm{d}x\, x\, \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-}\, \overline{\psi}(-\tfrac{z}{2})\gamma^+\psi(\tfrac{z}{2})\Big|_{z^+=z_\perp=0}$$

Operator entering GPD correlator !

$$\boxed{\begin{aligned} \int \mathrm{d}x\, x\, H_q(x,\xi,t) &= A_q(t) + 4\xi^2\, C_q(t) \\ \int \mathrm{d}x\, x\, \frac{1}{2}\,[H_q + E_q](x,\xi,t) &= J_q(t) \end{aligned}}$$

A similar reasoning applies to the gluonic sector

Conclusions

- The **energy-momentum tensor** contains key information about the system internal structure

- **Poincaré symmetry** constrains the structure of the EMT matrix elements in terms of gravitational form factors

- **Spatial distributions** of energy, pressure, … can be expressed in terms Fourier transforms of the gravitational form factors

- **Generalized parton distributions** are off-forward extensions of the usual PDFs and can be measured in e.g. DVCS

- **x-moments** of GPDs give access to electromagnetic form factors and to some of the gravitational form factors

Some references

- **Diehl, Phys. Rep. 388 (2003) 41**
- **Leader, Lorcé, Phys. Rep. 541 (2014) 3, 163**
- **Polyakov, Schweitzer, IJMPA33 (2018) 26, 1830025**
- **Lorcé, Moutarde, Trawinski, Eur. Phys. J. C79 (2019) 89**

… and references therein !