An update on nuclear PDFs

Ingo Schienbein UGA/LPSC Grenoble





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Gluod namics



Plan

- Introduction/Motivation
- Global analyses of nuclear PDFs
 - Updates from EPPS
 - Updates from nCTEQ
 - Updates from nNNPDF
- Conclusions

Introduction/Motivation

Nuclear PDFs



- Fundamental quest:
 - Hadron Structure:
 x,Q,A dependence
 - Nuclear modifications
- Necessary tool:
 - Cross sections for hard processes in IA, vA, pA, AA
 - Fixed target, colliders, atmosphere

nCTEQ15, arXiv:1509.00792 $xf_i^{p/A}(x,Q_0) = x^{c_1}(1-x)^{c_2}e^{c_3x}(1+e^{c_4}x)^{c_5} \quad c_k(A) = c_{k,0} + c_{k,1}(1-A^{-c_{k,2}})$



Theoretical Framework (pQCD formalism)

Collinear Factorization Theorems:

- Provide (field theoretical) **definitions** of the **universal** PDFs
- Make the formalism **predictive**!
- Make a statement about the **error** of the factorization formula

PDFs and predictions for observables+uncertainties refer to this standard pQCD framework

Need a solid understanding of the standard framework!

- For pp and ep collisions there a **rigorous factorization proofs**
- For pA and AA factorization is a **working assumption** to be tested phenomenologically

There might be breaking of collinear factorization, deviations from DGLAP evolution, other nuclear matter effects to be included (higher twist)

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Example: Factorization for pp collisions



Parton Distribution Functions (PDFs) $f_{P \rightarrow a, b}(x, \mu^2)$

🖈 Universal

Describe the structure of hadrons

Obey DGLAP evolution equations

The hard part $\hat{\sigma}_{ab \rightarrow c}(\mu^2)$

★ Free of short distance scales

- Calculable in perturbation theory
- ★ Depends on the process

 Similar factorisation formulae for inclusive IA, nuA processes and one-particle inclusive processes (involving also fragmentation functions)

Predictive Power

Universality: <u>same</u> PDFs/FFs enter different processes:

- **DIS:** $F_2^A(x,Q^2) = \sum_i [f_i^A \otimes C_{2,i}](x,Q^2)$
- DY: $\sigma_{A+B\to\ell^++\ell^-+X} = \sum_{i,j} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to\ell^++\ell^-+X}$
- A+B-> H + X: $\sigma_{A+B\to H+X} = \sum_{i,j,k} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{i+j\to k+X} \otimes D_k^H$
- Predictions for unexplored kinematic regions and for your favorite new physics process

Flavor separation of PDFs

NC charged lepton DIS: 2 structure functions (y-exchange)

$$F_2^{\gamma}(x) \sim \frac{1}{9} [4(u + \bar{u} + c + \bar{c}) + d + \bar{d} + s + \bar{s}](x)$$

$$F_2^{\gamma}(x) = 2x F_1^{\gamma}(x)$$

CC Neutrino DIS: 6 additional structure functions $F_{1,2,3}$ ^{W+}, $F_{1,2,3}$ ^{W-}

$$F_2^{W^+} \sim [d + s + \bar{u} + \bar{c}] \qquad F_3^{W^+} \sim 2[d + s - \bar{u} - \bar{c}]$$
$$F_2^{W^-} \sim [\bar{d} + \bar{s} + u + c] \qquad F_3^{W^-} \sim 2[u + c - \bar{d} - \bar{s}]$$

Useful/needed to disentangle different quark parton flavors in a **global analysis** of proton or nuclear PDFs

Scale dependence predicted by QCD

- ► *x*-dependence of PDFs is NOT calculable in pQCD
- µ²-dependence is calculable in pQCD given by DGLAP
 (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations

DGLAP evolution equations

$$\frac{df_q(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) f_q(y,\mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y,\mu^2) \right] \\ \frac{df_g(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{gg}\left(\frac{x}{y}\right) f_g(y,\mu^2) + P_{gq}\left(\frac{x}{y}\right) f_q(y,\mu^2) \right]$$

- ▶ Different PDFs mix set of $(2n_f + 1)$ coupled integro-differential equations.
- Need boundary conditions f_i(x,Q₀) at some perturbative initial scale Q₀ ≥ I
 GeV
- The x-dependence is not calculable in pQCD, perform global analysis of experimental data [EPPS, nCTEQ, nNNPDF, ...]
- Progress on the lattice: see arXiv:1711.07916, 2006.08636

Sum rules provide constraints

Number sum rules – connect partons to quarks from SU(3) flavour symmetry of hadrons; proton (*uud*), neutron (*udd*). For protons:

$$\int_{0}^{1} dx [\underbrace{f_{u}(x) - f_{\bar{u}}(x)}_{u-\text{valence distr.}}] = 2 \qquad \qquad \int_{0}^{1} dx [\underbrace{f_{d}(x) - f_{\bar{d}}(x)}_{d-\text{valence distr.}}] = 1$$
$$\int_{0}^{1} dx [f_{s}(x) - f_{\bar{s}}(x)] = \int_{0}^{1} dx [f_{c}(x) - f_{\bar{c}}(x)] = 0$$

▶ Momentum sum rule – momentum conservation connecting all flavours

For all scales:

For all

scales:

$$\sum_{i=q,\bar{q},g} \int_0^1 dx \ x f_i(x) = 1$$

Momentum carried by up and down quarks is only around half of the total proton momentum the rest of the momentum is carried by gluons and small amount by sea quarks. In case of CT14NLO PDFs ($\mu = 1.3$ GeV):

At 1.3 GeV:

$$\int_0^1 dx \, x [f_u(x) + f_d(x)] \simeq 0.51$$
$$\int_0^1 dx \, x f_g(x) \simeq 0.40$$

Nuclear modifications

 $F_2^A(x) \neq ZF_2^p(x) + NF_2^n(x)$



 Nuclear modifications can be incorporated/parameterized inside nPDFs but underlying dynamics remains to be fully theoretically understood

Global analyses of nuclear PDFs

Global analysis of nuclear PDFs

Same approach as for proton PDF determinations

Boundary conditions:
 Parameterize x-dependence of PDFs at initial scale Q₀

 $f(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} P(x; A_3, ...); f = u_v, d_v, g, \overline{u}, \overline{d}, s, \overline{s}$

- - 3. Define suitable χ^2 function and miniphize T_{w_r} . If $X_{global}^2 [A_i] = \sum_{n=1}^{\infty} w_n X_n^2; X_n^2 = \sum_{I} (\frac{IP_n IZe}{\sigma_{nI}})^2$.





nPDFs ca. 2017

	nNNPDF1.0 EPJC79(2019471	EPPS16 EPJC77(2017)163	nCTEQ15 PRD93(2016)085037	KA15 PRD93(2016)014036	DSSZ12 PRD85(2012)074028	EPS09 JHEP0904(2009)065
IA DIS	✓	✓	✓	✓	✓	~
DY in p+A	×	~	✓	✓	~	~
RHIC π d+Au	×	~	 Image: A set of the set of the	×	✓	~
vA DIS	×	~	×	×	✓	×
DY in π+A	×	~	×	×	×	×
LHC p+Pb dijets	×	 ✓ 	×	×	×	×
LHC p+Pb W,Z	×	 Image: A start of the start of	×	×	×	×

Order in a_s	NNLO	NLO	NLO	NNLO	NLO	NLO
Q-cut in DIS	1.87 GeV	1.3 GeV	2 GeV	1 GeV	1 GeV	1.3 GeV
W-cut	3.53 GeV	-	3.5 GeV	-	-	-
Data points	451	1811	708	1479	1579	929
Free parameters	Neural Net	20	16	16	25	15
Error tolerance	MC replica	52	35	N.N.	30	50
Proton baseline	NNPDF3.1	CT14NLO	~CTEQ6.1	JR09	MSTW08	CTEQ6.1
Mass scheme	FONLL-B	GM-VFNS	GM-VFNS	ZM-VFNS	GM-VFNS	ZM-VFNS
Flavour sep.	-	val.+sea	valence	-	-	-

Global analyses of nPDFs: 2022

• EPPS

- EKS98: hep-ph/9807297
- EKPS07: hep-ph/0703104
- EPS08: 0802.0139
- EPS09: 0902.4154
- EPPS16: 1612.05741
- EPPS21: 2112.12462
- nCTEQ
 - nCTEQ09: 0907.2357
 - nCTEQ15: 1509.00792
 - nCTEQ15WZ: 2007.09100
 - nCTEQ15HiX: 2012.11566
 - nCTEQ15WZSIH: 2105.09873
 - nCTEQ15HQ: 2204.09982
 - nCTEQ15WZSIHdeut: 2204.13157
 - BaseDimuChorus: 2204.13157

• nNNPDF

- nNNPDF1.0: 1904.00018
- nNNPDF2.0: 2006.14629
- nNNPDF3.0: 2201.12363
- TUJU (open source XFitter, fit of proton baseline)
 - TUJU19: 1908.03355
 - TUJU21: 2112.11904
- KA
 - KAI5: 1601.00939
 - KSASG20: 2010.00555
- nDS
 - nDS03: hep-ph/0311227
 - DSSZ12: 1112.6324
- HKM/HKN
 - HKM01: hep-ph/0103208
 - HKN04: hep-ph/0404093
 - HKN07: 0709.3038

Similarities and Differences

Similarities

- All use the same twist-2 pQCD formalism based on collinear factorisation: DGLAP evolution, sum rules, pQCD observables, χ^2 minimisation
- Isospin symmetry, region x>1 is neglected in all analyses
- Main differences
 - **Parametrisation** of the boundary conditions at the initial scale Q_0 : different functional forms or neural network
 - **Choice of analysed data**: which processes, kinematic cuts, treatment of correlations, normalisation uncertainties
 - Analysis of PDF errors: MC replica, Hessian error analysis, Tolerance criterion for 90% CL
- Other differences
 - parameters $Q_0, m_c, m_b, \alpha_s(M_Z)$, heavy flavour scheme, perturbative order (NLO, NNLO)
 - Deuteron corrections, Target mass corrections, Higher twist contributions

Used data sets

- **IA DIS:** backbone of all global analyses
 - Data from SLAC, NMC, EMC, BCDMS, FNAL: all groups (but different cuts)
 - Data from JLAB (CLAS, Hall-C): nCTEQ15HiX, EPPS2, KSASG20
- nuA DIS: quark flay Single pion production PDF
 - CHORUS nu-Pb c nNNPDF2.0, nNN KSASG20, TUJUI S

ion production

Deep Inelastic Scattering $\nu \longrightarrow l$ $V \longrightarrow l$ $N \longrightarrow l + X$ $\nu(\bar{\nu}) + N \rightarrow l + X$

- NuTeV, CCFR, CDHSW nu-Fe data: Tensions (see 2204.13157), used by KSASG20, TUJU19, TUJU21
- nuA SIDIS charm production (dimuon data): strange PDF
 - NuTeV, CCFR nu-Fe: nNNPDF2.0, BaseDimuCHORUS





Used data sets II

- **pA DY:** disentangle valence and sea quarks
 - E772, E866 data: EPPS16, EPPS21,, nCTEQ DSSZ12, nNNPDF3.0
 - π -A DY data: EPPS16, EPPS21



 SIH data: gluon distribution (weaker impact compared to HQ and dijet data)

Single pion production



 CMS, ATLAS (ALICE, LHCb) Run I (5 TeV), CMS Run II (8 TeV): EPPS16, EPPS21, nCTEQ15WZ, nCTEQ15WZSIH, nCTEQ15WZSIHdeut, nNNPDF2.0, nNNPDF3.0, TUJU21

Used data sets III

- LHC Heavy Quark data: strong constraints on gluon at small-x
 - EPPS21 (D-mesons), nCTEQ15HQ (Heavy quarks and quarkonia, Crystal Ball fit), nNNPDF3.0 (D-mesons), Bayesian reweighting)
- LHC dijet data: strong constraint on gluon distribution in shadowing and antishadowing region (medium x, medium-small x)
 - CMS 5 TeV dijet p-Pb data: EPPS16, EPPS21, nNNPDF3.0
- LHC prompt photon data: gluon distribution (medium x, medium-small x) nNNPDF3.0

Updates from EPPS

EPPS21 [2121.12462]

Bound proton PDF:

 $f_i^{\rm p/A}(x,Q^2) = R_i^{\rm p/A}(x,Q^2) f_i^{\rm p}(x,Q^2)$

- $Q_0 = m_c = 1.3 \text{ GeV}$
- Proton baseline: CT18ANLO
- $m_c = 1.3, m_b = 4.75, \alpha_s(Q)$ as in CT18ANLO
- Isospin symmetry
- Parametrization is a piecewise defined function
- Some changes w.r.t. EPPS16
- Deuteron taken to be free
- 24 free parameters

$$R_{i}^{A}(x, Q_{0}^{2}) = \begin{cases} a_{0} + a_{1}(x - x_{a}) \left[e^{-xa_{2}/x_{a}} - e^{-a_{2}} \right], & x \leq x_{a} \\ b_{0}x^{b_{1}}(1 - x)^{b_{2}}e^{xb_{3}}, & x_{a} \leq x \leq x_{e} \\ c_{0} + c_{1}(c_{2} - x)(1 - x)^{-\beta}, & x_{e} \leq x \leq 1. \end{cases}$$

- Full nuclear PDF $f_i^A(x,Q^2) = Z f_i^{\mathrm{p}/A}(x,Q^2) + N f_i^{\mathrm{n}/A}(x,Q^2)$

• Nuclear modification

$$R_i^A(x,Q^2) = \frac{Zf_i^{p/A}(x,Q^2) + Nf_i^{n/A}(x,Q^2)}{Zf_i^p(x,Q^2) + Nf_i^n(x,Q^2)}$$



EPPS21: Details on Parametrization [2121.12462]

Table 1 Values of parameters that define the central EPPS21 nuclear PDFs at $Q_0^2 = 1.69 \,\text{GeV}^2$. The 24 parameters that were kept free in the fit are indicated in bold.

Parameter	$ $ $u_{\rm V}$	$d_{ m V}$	\overline{u}
$y_0(A_{ m ref})$	sum rule	sum rule	0.870
$\gamma_{oldsymbol{y}_0}$	sum rule	sum rule	0.401
a_2	0, fixed	0, fixed	0, fixed
x_a	0.0577	as $u_{\rm V}$	0.110
x_e	0.700	as $u_{\rm V}$	as $u_{\rm V}$
$y_a(A_{ m ref})$	1.07	1.04	0.992
γ_{y_a}	0.221	as $u_{\rm V}$	0, as $u_{\rm V}$
$y_e(A_{ m ref})$	0.877	0.968	0.956
γ_{y_e}	0.176	as $u_{\rm V}$	as $u_{\rm V}$
c_0	1.8, fixed	1.8, fixed	1.8, fixed
eta	2.20	as $u_{\rm V}$	1.3, fixed
f_3	0.291	as $u_{\rm V}$	as $u_{\rm V}$
f_6	0.495	as $u_{\rm V}$	as $u_{\rm V}$
Parameter	\overline{d}	s	g
$\frac{\text{Parameter}}{y_0(A_{\text{ref}})}$	<i>d</i> 0.921	<i>s</i> 0.403	<i>g</i> sum rule
$\frac{\text{Parameter}}{y_0(A_{\text{ref}})}$ $\frac{\gamma_{y_0}}{\gamma_{y_0}}$	$\begin{array}{ c c }\hline \overline{d} \\ \hline 0.921 \\ \text{as } \overline{u} \end{array}$	s 0.403 as \overline{u}	<i>g</i> sum rule sum rule
$\frac{\text{Parameter}}{\begin{array}{c} y_0(A_{\text{ref}}) \\ \gamma_{y_0} \\ a_2 \end{array}}$	$\begin{array}{ c c }\hline \overline{d} \\ \hline 0.921 \\ \text{as } \overline{u} \\ 0, \text{ fixed} \end{array}$	s 0.403 as \overline{u} 0, fixed	<i>g</i> sum rule sum rule 3.66
$\begin{array}{c} \label{eq:parameter} \hline \\ \hline y_0(A_{\mathrm{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \end{array}$	$\begin{array}{ c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ \text{as } \overline{u} \\ 0, \text{ fixed} \\ \text{as } \overline{u} \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u}	<i>g</i> sum rule 3.66 0.0975
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\text{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \end{array}$	$\begin{array}{ c c }\hline \overline{d} \\ \textbf{0.921} \\ \text{as } \overline{u} \\ 0, \text{fixed} \\ \text{as } \overline{u} \\ \text{as } u_{\text{V}} \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as $u_{\rm V}$	$\begin{array}{c} g \\ {\rm sum \ rule} \\ {\rm sum \ rule} \\ {\rm 3.66} \\ {\rm 0.0975} \\ {\rm as \ } u_{\rm V} \end{array}$
$egin{array}{c} { m Parameter} \\ y_0(A_{ m ref}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{ m ref}) \end{array}$	$\begin{array}{ c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ \text{as } \overline{u} \\ 0, \text{fixed} \\ \text{as } \overline{u} \\ \text{as } u_{\text{V}} \\ \textbf{0.971} \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09	$g \\ sum rule \\ sum rule \\ 3.66 \\ 0.0975 \\ as u_V \\ 1.10$
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\text{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\text{ref}}) \\ \gamma_{y_a} \end{array}$	$\begin{array}{ c c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ as \ \overline{u} \\ 0, fixed \\ as \ \overline{u} \\ as \ u_V \\ \textbf{0.971} \\ u_V \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09 u_V	$\begin{array}{c} g \\ \text{sum rule} \\ \textbf{3.66} \\ \textbf{0.0975} \\ \text{as } u_{\text{V}} \\ \textbf{1.10} \\ \text{as } u_{\text{V}} \end{array}$
$\begin{array}{c} \begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\mathrm{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\mathrm{ref}}) \\ \gamma_{y_a} \\ y_e(A_{\mathrm{ref}}) \end{array}$	$\begin{array}{ c c c }\hline \overline{d} \\ \textbf{0.921} \\ \text{as } \overline{u} \\ 0, \text{fixed} \\ \text{as } \overline{u} \\ \text{as } u_{\text{V}} \\ \textbf{0.971} \\ u_{\text{V}} \\ \text{as } \overline{u} \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09 u_V as \overline{u}	<i>g</i> sum rule sum rule 3.66 0.0975 as <i>u</i> _V 1.10 as <i>u</i> _V 0.852
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\text{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\text{ref}}) \\ \gamma_{y_a} \\ y_e(A_{\text{ref}}) \\ \gamma_{y_e} \end{array}$	$\begin{array}{ c c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ \text{as } \overline{u} \\ 0, \text{fixed} \\ \text{as } \overline{u} \\ \text{as } \overline{u} \\ \textbf{as } u_{\text{V}} \\ \hline \textbf{0.971} \\ u_{\text{V}} \\ \text{as } \overline{u} \\ \text{as } u_{\text{V}} \\ \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09 u_V as \overline{u} as u_V	$\begin{array}{c} g \\ \text{sum rule} \\ \textbf{3.66} \\ \textbf{0.0975} \\ \text{as } u_{\text{V}} \\ \textbf{1.10} \\ \text{as } u_{\text{V}} \\ \textbf{0.852} \\ \text{as } u_{\text{V}} \end{array}$
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\mathrm{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\mathrm{ref}}) \\ \gamma_{y_a} \\ y_e(A_{\mathrm{ref}}) \\ \gamma_{y_e} \\ c_0 \end{array}$	$\begin{array}{ c c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ as \ \overline{u} \\ 0, fixed \\ as \ \overline{u} \\ as \ \overline{u} \\ as \ u_V \\ \textbf{0.971} \\ u_V \\ as \ \overline{u} \\ as \ u_V \\ 1.8, fixed \\ \hline \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09 u_V as \overline{u} as u_V 1.8, fixed	g sum rule sum rule 3.66 0.0975 as u_V 1.10 as u_V 0.852 as u_V 1.8 , fixed
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\mathrm{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\mathrm{ref}}) \\ \gamma_{y_a} \\ y_e(A_{\mathrm{ref}}) \\ \gamma_{y_e} \\ c_0 \\ \beta \end{array}$	$\begin{array}{ c c c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ as \ \overline{u} \\ 0, fixed \\ as \ \overline{u} \\ as \ \overline{u} \\ as \ u_V \\ \textbf{0.971} \\ u_V \\ as \ \overline{u} \\ as \ u_V \\ 1.8, fixed \\ 1.3, fixed \\ \end{array}$	s 0.403 as \overline{u} 0, fixed as \overline{u} as u_V 1.09 u_V as \overline{u} as u_V 1.8, fixed 1.3, fixed	g sum rule 3.66 0.0975 $\operatorname{as} u_{\mathrm{V}}$ 1.10 $\operatorname{as} u_{\mathrm{V}}$ 0.852 $\operatorname{as} u_{\mathrm{V}}$ 1.8 , fixed 1.3 , fixed
$\begin{array}{c} \text{Parameter} \\ \hline y_0(A_{\text{ref}}) \\ \gamma_{y_0} \\ a_2 \\ x_a \\ x_e \\ y_a(A_{\text{ref}}) \\ \gamma_{y_a} \\ y_e(A_{\text{ref}}) \\ \gamma_{y_e} \\ c_0 \\ \beta \\ f_3 \end{array}$	$\begin{array}{ c c c c }\hline \overline{d} \\ \hline \textbf{0.921} \\ as \ \overline{u} \\ 0, fixed \\ as \ \overline{u} \\ as \ \overline{u} \\ as \ u_V \\ \textbf{0.971} \\ u_V \\ as \ \overline{u} \\ as \ u_V \\ 1.8, fixed \\ 1.3, fixed \\ as \ u_V \\ \end{array}$	s 0.403 as \overline{u} 0 , fixedas \overline{u} as u_V 1.09 u_V as \overline{u} as u_V 1.8 , fixed 1.3 , fixedas u_V	g sum rule 3.66 0.0975 as u_V 1.10 as u_V 0.852 as u_V 1.8 , fixed 1.3 , fixed as u_V

- Parametrization is a piecewise defined function
- Some changes w.r.t. EPPS16 $R_i^A(x,Q_0^2) =$

$$\begin{cases} a_0 + a_1 (x - x_a) \left[e^{-xa_2/x_a} - e^{-a_2} \right], & x \le x_a \\ b_0 x^{b_1} (1 - x)^{b_2} e^{xb_3}, & x_a \le x \le x_e \\ c_0 + c_1 (c_2 - x) (1 - x)^{-\beta}, & x_e \le x \le 1. \end{cases}$$

• Params: $a_0, a_1, a_2, b_0, b_1, b_2, c_0, c_1, c_2, x_a, x_e, \beta$ (first derivatives zero at x_a, x_e : fixes 4 para)

•
$$a_i, b_i, c_i = a_i, b_i, c_i[y_a, y_e, y_0];$$

- sum rules: y_0^g , $y_0^{u_v}$, $y_0^{a_v}$ fixed for each A
- A-dep: $y_i(A) = 1 + [y_i(A_{ref}) 1](A/A_{ref})^{\gamma_i}$ with $A_{ref} = 12$
- Strange quarks: $\gamma_{y_0} \rightarrow \gamma_{y_0} y_0 \theta (1 y_0)$
- Extra modification for Li-6 and He-3: parameters f_3, f_6 ; deuteron taken to be free

EPPS21 vs EPPS16

- more LHC p-Pb data
 - 5 TeV CMS dijet data from (run I)
 - 5 TeV LHCb D-meson data from (run I)
 - 8 TeV CMS W^{\pm} data (run II)
- JLAB DIS data
- Uncertainties due to baseline proton PDF uncertainties
- EPPS16: no W-cut, EPPS21:W>1.8 GeV
- EPPSI6: $\Delta \chi^2 \sim 50$, EPPS2I: $\Delta \chi^2 \sim 33$
- EPPS16: 20 free parameters, EPPS21: 24 free parameters



EPPS21 vs EPPS 6



 Largest difference for strange quarks and gluons: much better constrained in EPPS21. Gluon due to D-meson and dijet data (gluon). Strange quark due to W,Z data and the more precise gluon.

$E^{1} + S^{2} I_{x}^{0^{-1}} v S^{0^{-1}} n CTE^{1} Q I^{0} S V_{x}^{0^{-1}} Z^{10^{-1}} and n N P D_{x}F2^{10^{-1}} O^{-1}$



- General agreement within the shown 90% CL uncertainties
- \bar{u}, \bar{d} : nCTEQ no flavour separation; nNNPDF no DY fixed target data
- Strange quark uncertainty large in nCTEQ15WZ: no neutrino DIS data

Updates from nCTEQ

Towards the next nCTEQ global analysis

nCTEQ nuclear PDFs:

- Preparation of next global release (nCTEQ2023)
 - Performed detailed analysis of neutrino DIS data [2204.13157]
 Next global analysis use (CHORUS+Dimuon data)
- LHC heavy quark data (gluon)
 [2204.09982]
 - Inclusive hadron production data (gluon) [2105.09873]

[2012.11566]

[2007.09100]

[Nov/Dec 2022]

- Explored lower W and Q-cuts using JLAB data
- LHC W/Z production data
- New review of Target Mass Corrections

nCTEQI5 framework PRD93(2016)085037

• Functional form of the bound proton PDF same as for the free proton (CTEQ6M, x restricted to 0 < x < 1)

$$xf_i^{p/A}(x,Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4} x)^{c_5}, \qquad i = u_v, d_v, g, \dots$$

$$\bar{d}(x,Q_0)/\bar{u}(x,Q_0) = c_0 x^{c_1} (1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

• A-dependent fit parameters (reduces to free proton for A = 1)

$$c_k \to c_k(A) \equiv c_{k,0} + c_{k,1} \left(1 - A^{-c_{k,2}} \right), \quad k = \{1, \dots, 5\}$$

• PDFs for nucleus (A, Z)

$$f_i^{(A,Z)}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

(bound neutron PDF $f_i^{n/A}$ by isospin symmetry)

nCTEQ15 data sets

Fit properties:

- fit @NLO
- $Q_0 = 1.3 \text{GeV}$
- using ACOT heavy quark scheme
- kinematic cuts: Q > 2 GeV, W > 3.5 GeV $p_T > 1.7 \text{ GeV}$
- 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- 16+2 free parameters
 - 7 gluon
 - 7 valence
 - 2 sea
 - 2 pion data normalizations

•
$$\chi^2 = 587$$
, giving $\chi^2/dof = 0.81$
N = Au

Error analysis:

• use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0) (a_j - a_j^0)$$
$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- tolerance $\Delta \chi^2 = 35$ (every nuclear target within 90% C.L.)
- eigenvalues span 10 orders of magnitude \rightarrow require numerical precision
- use noise reducing derivatives

nCTEQHiX nPDFs with lower W-cut and JLAB data



arXiv:2012.11566

Standard cuts: Q>2 GeV, W>3.5 GeV

This analysis: Q>1.3 GeV, W> 1.7 GeV

Good fit $\chi^2/dof \sim 0.84$ Extension to even smaller W possible

Number of data depending on cuts

		Wcut	Wcut	Wcut	Wcut	W_{cut}
Q_{cut}^2	Q_{cut}	No Cut	1.3	1.7	2.2	3.5
1.3	$\sqrt{1.3}$	1906	1839	1697	1430	1109
1.69	1.3	1773	1706	1564	1307	1024
2	$\sqrt{2}$	1606	1539	1402	1161	943
4	2	1088	1042	952	817	708

Several effects included

- Higher Twist
- TMC
- Deuteron corrections
- Shape of the parametrisation at large x

See also the reweighing analysis arXiv:2003.02195

nCTEQHiX nPDFs with lower W-cut and JLAB data



• u, d, g increased at large x; sea quarks $\bar{u}, \bar{d}, s = \bar{s}$ are suppressed at large x

nCTEQHQ nPDFs

arXiv:2204.09982



Heavy quark(-onium) data cover a wide kinematic range down to $x \lesssim 10^{-5}$

puts strong constraints on gluon distribution

FIG. 1: Coverage of the kinematic (p_T, y_{cms}) -plane of the quarkonium and open heavy quark production data sets from proton-lead collisions. ALICE data is shown in red, ATLAS in blue, CMS in orange and LHCb in green. The dashed and solid contours show the estimated x-dependence for $\sqrt{s} = 5$ and 8 TeV, respectively.

> See also 2012.11462 and 1712.07024

nCTEQ15HQ nPDFs

• Data:

- IA DIS + pA DY
- LHC W,Z
- RHIC/LHC SIH
- LHC Heavy quark(-onium)
- 19 fit parameters (3 strange parameters open)
- Heavy quark(-onium) data: Data-driven approach relying on the following assumptions
 - gg-channel dominates
 - 2->2 kinematics

$$\sigma(AB \to \mathcal{Q} + X) = \int \mathrm{d}x_1 \, \mathrm{d}x_2 f_{1,g}(x_1, \mu) \, f_{2,g}(x_2, \mu) \, \frac{1}{2\hat{s}} \overline{|\mathcal{A}_{gg \to \mathcal{Q} + X}|^2} \mathrm{dPS}.$$

The effective scattering ME is parameterised with the Crystal Ball function:

$$\overline{\left|\mathcal{A}_{gg\to\mathcal{Q}+X}\right|^{2}} = \frac{\lambda^{2}\kappa\hat{s}}{M_{\mathcal{Q}}^{2}}e^{a|y|}$$

$$\times \begin{cases} e^{-\kappa\frac{p_{T}^{2}}{M_{\mathcal{Q}}^{2}}} & \text{if } p_{T} \leq \langle p_{T} \rangle \\ e^{-\kappa\frac{\langle p_{T} \rangle^{2}}{M_{\mathcal{Q}}^{2}}} \left(1 + \frac{\kappa}{n}\frac{p_{T}^{2} - \langle p_{T} \rangle^{2}}{M_{\mathcal{Q}}^{2}}\right)^{-n} & \text{if } p_{T} > \langle p_{T} \rangle \end{cases}$$

arXiv:2204.09982

nCTEQ15HQ nPDFs



FIG. 2: Comparison between prompt J/ψ production in pp collisions for LHCb[87], ALICE[88] and ATLAS[89] kinematics as predicted by NRQCD and with the data-driven approach. The uncertainties of the NRQCD predictions come from scale variation $1/2 < \mu_r/\mu_{r,0} = \mu_f/\mu_{f,0} = \mu_{\rm NRQCD}/\mu_{\rm NRQCD,0} < 2$ around the base scale $\mu_{r,0} = \mu_{f,0} = \sqrt{p_T^2 + 4m_c^2}$ and $m_{\rm NRQCD,0} = m_c$. Different rapidity bins are separated by multiplying the cross sections by powers of ten for visual clarity.

nCTEQ15HQ nPDFs

arXiv:2204.09982



FIG. 3: Comparison between prompt D^0 production as predicted in the GMVFNS (red) and with the data-driven approach (blue). The uncertainties of the GMVFNS predictions come from varying the scales individually by a factor of 2, such that there is never a factor 4 between two scales. Different rapidity bins are separated by multiplying the cross sections by powers of ten for visual clarity.

nCTEQ15HQ nPDFs

TABLE XI: χ^2/N_{dof} values for the individual heavy-quark final states, the individual processes DIS, DY, WZ, SIH, HQ, and the total. The shown χ^2 is the sum of regular χ^2 and normalization penalty. Excluded processes are shown in parentheses. Note that both nCTEQ15 AND nCTEQ15WZ included the neutral pions from STAR and PHENIX.

	D^0	J/ψ	$\Upsilon(1S)$	$\psi(2S)$	DIS	DY	WZ	SIH	HQ	Total
nCTEQ15	(0.56)	(2.50)	(0.82)	(1.06)	0.86	0.78	(2.19)	(0.78)	(1.96)	1.23
nCTEQ15WZ	(0.32)	(1.04)	(0.76)	(1.02)	0.91	0.77	0.63	(0.47)	(0.92)	0.90
nCTEQ15WZ+SIH	(0.46)	(0.84)	(0.90)	(1.07)	0.91	0.77	0.72	0.40	(0.93)	0.92
nCTEQ15HQ	0.35	0.79	0.79	1.06	0.93	0.77	0.78	0.40	0.77	0.86



FIG. 4: Lead PDFs from different nCTEQ15 versions. The baseline nCTEQ15 fit is shown in black, nCTEQ15WZ in blue, nCTEQ15WZSIH in green, and the new fit in red.

nCTEQ and neutrino data

Neutrino deep inelastic scattering:

- Neutrino data important for many reasons: flavour separation of PDFs, ew precision physics, ...
- Are nuclear corrections in neutrino DIS the same as in charged lepton DIS?
- Several studies have been performed:
 - "iron PDFs: PRD77(2008)054013
 - nCTEQ analysis of nuA+IA+DY data: PRL106(2011)122301
 - Differences independent of the proton baseline: Kalantarians, Keppel, PRC96(2017)032201



Neutrino DIS vs Charged lepton DIS

Ultimate analysis: "Compatibility of Neutrino DIS data and Its Impact on Nuclear Parton Distribution Functions", arXiv:2204.13157

Data set	Nucleus	$E_{\nu/\bar{\nu}}(\text{GeV})$	# pts	Corr.sys.	Ref.
CDHSW ν	Fo	22 - 188	465	No	[48]
CDHSW $\bar{\nu}$	10	20 - 100	464	110	
CCFR ν	Fe	35 - 340	1109	No	[50]
CCFR $\bar{\nu}$	1.6	<u> 55 - 540</u>	1098	NO	
NuTeV ν	Fe	35 - 340	1170	Ves	[22]
NuTeV $\bar{\nu}$	1.6	<u> </u>	966	165	[20]
Chorus ν	Ph	25 - 170	412	Ves	[27]
Chorus $\bar{\nu}$	10	20 - 110	412	105	[2]]
CCFR dimuon ν	Fe	110 - 333	40	No	[10]
CCFR dimuon $\bar{\nu}$	10	87 - 266	38	110	[19]
NuTeV dimuon ν	Fe	90 - 245	38	No	[19]
NuTeV dimuon $\bar{\nu}$	10	79 - 222	34	110	[10]

- Most thorough analysis so far (thesis K. F. Muzak, U Münster): different tools to analyse compatibility of data
- Neutrino data creates significant tensions between key data sets: neutrino vs charged lepton+DY+LHC
- Tensions among different neutrino data sets: iron (CDHSW, NuTeV, CCFR) vs lead (CHORUS)?
- Next global analysis will include CHORUS and Dimuon data but not NuTeV, CCFR, CDHSW data



Updates from nNNPDF

New data in nNNPDF3.0 w.r.t. nNNPDF2.0

Process	Dataset	Ref.	$n_{ m dat}$	Nucl. spec.	Theory
	NMC 96	[53]	123/260	$^{2}\mathrm{D/p}$	APFEL
NC DIS	SLAC 91	[54]	38/211	$^{2}\mathrm{D}$	APFEL
	BCDMS 89	[55]	250/254	$^{2}\mathrm{D}$	APFEL
Fined terret DV	FNAL E866	[56]	15/15	$^{2}\mathrm{D/p}$	APFEL
Fixed-target DY	FNAL E605	[57]	85/119	64 Cu	APFEL
	ALICE W^{\pm} , Z (5.02 TeV)	[58]	6/6	²⁰⁸ Pb	MCFM
Collidor DV	LHCb Z (5.02 TeV)	[28]	2/2	²⁰⁸ Pb	MCFM
Collider DY	ALICE Z (8.16 TeV)	[60]	2/2	²⁰⁸ Pb	MCFM
	CMS Z (8.16 TeV)	[61]	36/36	²⁰⁸ Pb	MCFM
Dijet production	CMS p–Pb/pp (5.02 TeV)	[27]	84/84	²⁰⁸ Pb	NLOjet++
Prompt photon production	ATLAS p–Pb/pp (8.16 TeV)	[62]	43/43	²⁰⁸ Pb	MCFM
Prompt D^0 production	LHCb p–Pb/pp (5.02 TeV)	[28]	37/37	²⁰⁸ Pb	POWHEG

Table 2.1. The new measurements included in nNNPDF3.0 with respect to nNNPDF2.0. For each dataset, we indicate the name used throughout the paper, the reference, the number of data points n_{dat} after/before kinematic cuts, the nuclear species involved, and the codes used to compute the corresponding theoretical predictions. The datasets in the upper (lower) part of the table correspond to the first (second) group described in the text.

LHCb prompt D-meson production data included via Bayesian reweighting (no fit)

Kinematic coverage significantly expanded



DIS-Cuts: • $Q^2 > 3.5 \text{ GeV}^2$ • $W^2 > 12.5 \text{ GeV}^2$

Cuts to FNAL-E605 p-Cu DY to remove points close to the production threshold

After cuts:

- 2188 points (3.0)
- 1467 points (2.0)

Figure 2.1. The kinematic coverage in the (x, Q^2) plane of the nNNPDF3.0 dataset. The evaluation of x and Q^2 for the hadronic processes assumes LO kinematics. Data points are classified by process. Data points new in nNNPDF3.0 in comparison to nNNPDF2.0 are marked with a grey edge. Data points excluded by kinematic cuts are filled grey.

- General settings
 - $m_c = 1.51 \text{ GeV}, m_b = 4.92 \text{ GeV}, \alpha_s(M_Z) = 0.118$
 - Input scale $Q_0 = 1 \text{ GeV}$
 - PDFs at NLO MSbar
 - Isospin symmetry
 - Heavy quarks in DIS: FONLL GM-VFNS
 - pA collisions: ZM-VFNS
 - Various methodological improvements w.r.t. nNNPDF2.0

Impact of LHCb D-meson data: large uncertainty reduction at small-x, more shadowing



Figure 4.5. Comparison of the nPDFs of lead nuclei at Q = 10 GeV between nNNPDF3.0 (no LHCb D) and nNNPDF3.0, normalised to the central value of the former.

Comparison with EPPS16 and nCTEQ15WZSIH



Figure 4.13. The nNNPDF3.0 predictions for the nuclear modification ratios in lead at Q = 10 GeV, compared to the corresponding results from the EPPS16 and nCTEQWZ+SIH global analyses. The PDF uncertainty bands correspond in all cases to 68% CL intervals.

Methodological improvements



Figure 5.1. Comparison of nNNPDF2.0 with the nNNPDF2.0r variant. Results are shown for the lead PDFs at Q = 10 GeV normalised to the central value of nNNPDF2.0, and the uncertainty bands represent the 68% CL intervals.

Summary/Conclusions/Outlook

Conclusions

- A lot of progress in recent years, more to come!
 - HQ-data, di-jet data: much improved gluon
 - LHC W,Z data: gluon, strange PDF
 - JLAB data: improved determination of valence distributions
 - Neutrino data:
 - quark flavour separation
 - But tensions with neutrino-iron data, not with neutrino-lead data
- Different groups: EPPS, nCTEQ, nNNPDF, TUJU, KA, ... Important to test systematics, new ideas, driving improvements!

Conclusions

- Future:
 - More data, more truly global fits, improved precision
 - Combined proton PDF and nPDF fits ↔ Lead-only fits
 - Better understanding of nuclear (A,Z)-dependence, xdependence:
 - Test of nuclear models
 - Test of collinear factorisation
 - Competitive lattice calculations (also for nuclei)

Backup

Nuclear modifications

- Neutrino experiments use heavy nuclear targets: Pb, Fe, Ar, H₂O, C
- As discovered more than 30 years ago by the European Muon Collaboration, nucleon structure functions are modified by the nuclear medium (EMC effect)
- Studies of nucleon structure: need to correct for nuclear effects
- Nuclear effects interesting in its own right!
 - Many models exist.
 - However, charged lepton nuclear effects still not fully explained, in particular the EMC effect (0.3 < x < 0.7)

The EMC effect

 $F_2^A(x) \neq ZF_2^p(x) + NF_2^n(x)$



DIS on a nuclear target

Consider deep inelastic lepton–nucleon collisions: $I(k) + A(p_A) \rightarrow I'(k') + X$

Introduce the usual DIS variables: $q \equiv k - k'$, $Q^2 \equiv -q^2$, $x_A \equiv \frac{Q^2}{2p_A \cdot q}$

Hadronic tensor: $W^A_{\mu\nu} \propto \langle A(p_A) | J_\mu J^{\dagger}_\nu | A(p_A) \rangle = \sum_i a^{(i)}_{\mu\nu} \tilde{F}^A_i(x_A, Q^2)$,

where $a_{\mu\nu}^{(i)}$ are Lorentz-tensors composed out of the 4-vectors q and p_A and the metric $g_{\mu\nu}$

Express structure functions in the QCD improved parton model in terms of NPDFs

$$\tilde{\mathcal{F}}_k^A(x_A, Q^2) = \int_{x_A}^1 \frac{\mathrm{d}y_A}{y_A} \tilde{f}_i^A(y_A, Q^2) C_{k,i}(x_A/y_A) + \tilde{\mathcal{F}}_k^{A,\tau \ge 4}(x_A, Q^2)$$

NPDFs: Fourier transforms of matrix elements of twist-two operators composed out of the quark and gluon fields:

$$\widetilde{f}_i^A(x_A, Q^2) \propto \langle A(p_A) | O_i | A(p_A) \rangle$$

Definitions of $\tilde{F}_i^A(x_A, Q^2)$, $\tilde{f}_i^A(x_A, Q^2)$, and the varibale $0 < x_A < 1$ carry over one-to-one from the well-known free nucleon case

Evolution Equations and Sum Rules

DGLAP as usual:

$$\frac{d\tilde{f}_{i}^{A}(x_{A}, Q^{2})}{d \ln Q^{2}} = \frac{\alpha_{s}(Q^{2})}{2\pi} \int_{x_{A}}^{1} \frac{dy_{A}}{y_{A}} P_{ij}(y_{A}) \tilde{f}_{j}^{A}(x_{A}/y_{A}, Q^{2}) ,$$
$$= \frac{\alpha_{s}(Q^{2})}{2\pi} \int_{x_{A}}^{1} \frac{dy_{A}}{y_{A}} P_{ij}(x_{A}/y_{A}) \tilde{f}_{j}^{A}(y_{A}, Q^{2}) ,$$

Sum rules:

$$\int_{0}^{1} dx_{A} \tilde{u}_{v}^{A}(x_{A}, Q^{2}) = 2Z + N, \qquad \text{B cons.: } 1/3 < u_{v} > + < d_{v} > = A$$
$$\int_{0}^{1} dx_{A} \tilde{d}_{v}^{A}(x_{A}, Q^{2}) = Z + 2N, \qquad \text{C cons.: } 2/3 < u_{v} > - 1/3 < d_{v} > = Z$$

and the momentum sum rule

$$\int_0^1 dx_A x_A \left[\tilde{\Sigma}^A(x_A, Q^2) + \tilde{g}^A(x_A, Q^2) \right] = 1 ,$$

where N = A - Z and $\tilde{\Sigma}^A(x_A) = \sum_i (\tilde{q}_i^A(x_A) + \tilde{\bar{q}}_i^A(x_A))$ is the quark singlet combination

Rescaled definitions!

Problem: average momentum fraction carried by a parton $\propto A^{-1}$ since there are 'A-times more partons' which have to share the momentum

- Different nuclei (A, Z) not directly comparable
- Functional form for *x*-shape would change drastically with *A*
- Need to rescale!

PDFs are number densities: $\tilde{f}_i^A(x_A) dx_A$ is the number of partons carrying a momentum fraction in the interval $[x_A, x_A + dx_A]$

Define rescaled NPDFs $f^{A}(x_{N})$ with $0 < x_{N} := Ax_{A} < A$:

$$f_i^A(x_N) dx_N := \tilde{f}_i^A(x_A) dx_A$$

The variable x_N can be interpreted as parton momentum fraction w.r.t. the **average** nucleon momentum $\bar{p}_N := p_A/A$

Rescaled evolution equations and sum rules

Evolution:

$$\frac{\mathrm{d}f_i^A(x_N, Q^2)}{\mathrm{d}\ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N/A}^1 \frac{\mathrm{d}y_A}{y_A} P(y_A) f_i^A(x_N/y_A, Q^2) ,$$
$$= \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^A \frac{\mathrm{d}y_N}{y_N} P(x_N/y_N) f_i^A(y_N, Q^2) .$$

Assume that $f_i^A(x_N) = 0$ for $x_N > 1$, then **original**, symmetrical form recovered:

$$\frac{\mathrm{d}f_i^A(x_N, Q^2)}{\mathrm{d}\ln Q^2} = \begin{cases} \frac{\alpha_s(Q^2)}{2\pi} \int_{x_N}^1 \frac{\mathrm{d}y_N}{y_N} P(y_N) f_i^A(x_N/y_N, Q^2) &: 0 < x_N \le 1\\ 0 &: 1 < x_N < A, \end{cases}$$

Sum rules for the rescaled PDFs:

$$\int_0^A dx_N \, u_v^A(x_N) = 2Z + N \,,$$
$$\int_0^A dx_N \, d_v^A(x_N) = Z + 2N \,,$$

and

$$\int_0^A \mathrm{d}x_N x_N \left[\Sigma^A(x_N) + g^A(x_N) \right] = A \,,$$

Rescaled structure functions

The rescaled structure functions can be defined as

 $\mathbf{x}_{N}\mathcal{F}_{i}^{A}(\mathbf{x}_{N}) := \mathbf{x}_{A}\tilde{\mathcal{F}}_{i}^{A}(\mathbf{x}_{A})$,

with $\mathcal{F}_{1,2,3}(x) = \{F_1(x), F_2(x)/x, F_3(x)\}.$

More explicitly:

$$\begin{array}{rcl} F_2^A(x_N) & := & \tilde{F}_2^A(x_A) \; , \\ x_N F_1^A(x_N) & := & x_A \tilde{F}_1^A(x_A) \; , \\ x_N F_3^A(x_N) & := & x_A \tilde{F}_3^A(x_A) \; . \end{array}$$

This leads to consistent results in the parton model using the rescaled PDFs.

Consistent also for the target mass corrected structure functions!

Effective PDFs of bound nucleons

Further decompose the NPDFs $f_i^A(x_N)$ in terms of effective parton densities for **bound** protons, $f_i^{p/A}(x_N)$, and neutrons, $f_i^{n/A}(x_N)$, inside a nucleus *A*:

$$f_i^A(x_N, Q^2) = Z f_i^{p/A}(x_N, Q^2) + N f_i^{n/A}(x_N, Q^2)$$

- The bound proton PDFs have the **same** evolution equations and sum rules as the free proton PDFs **provided** we neglect any contributions from the region $x_N > 1$
- Neglecting the region $x_N > 1$, is consistent with the DGLAP evolution
- The region $x_N > 1$ is expected to have a minor influence on the sum rules of less than one or two percent (see also [PRC73(2006)045206])
- Isospin symmetry: $u^{n/A}(x_N) = d^{p/A}(x_N)$, $d^{n/A}(x_N) = u^{p/A}(x_N)$

An observable \mathcal{O}^A is then given by:

$$\mathcal{O}^{A} = Z \mathcal{O}^{p/A} + N \mathcal{O}^{n/A}$$

In conclusion: the free proton framework can be used to analyse nuclear data