Machine Learning Techniques for HEP Data Analysis with 7MVA

Andreas Hoecker^(*) (CERN)

Seminar, LPNHE Paris, June 20, 2007

(*) On behalf of the author team: A. Hoecker, P. Speckmayer, J. Stelzer, F. Tegenfeldt, H. Voss, K. Voss

(home),

And the contributors: A. Christov, S. Henrot-Versillé, M. Jachowski, A. Krasznahorkay Jr., Y. Mahalalel, R. Ospanov, X. Prudent, M. Wolter, A. Zemla

See acknowledgments on page 43

wiki/bir

LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with TMVA

On the web:

1/62

(tutorial)

<u>advert isement</u>

We (finally) have a Users Guide !

Available on http://tmva.sf.net

TMVA Users Guide 97pp, incl. code examples arXiv physics/0703039 arXiv physics/0703039 CERN-OPEN-2007-007 Document version 4 TMVA version 3.8 June 19, 2007 http://tmva.sf.net

TMVA

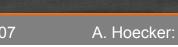
Toolkit for Multivariate Data Analysis with ROOT

Users Guide

A. Höcker, P. Speckmayer, J. Stelzer, F. Tegenfeldt, H. Voss, K. Voss

With contributions from

A. Christov, S. Henrot-Versillé, M. Jachowski, A. Krasznahorkay Jr., Y. Mahalalel, R. Ospanov, X. Prudent, M. Wolter, A. Zemla



LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with TMVA

2/62

Event Classification

Suppose data sample with two types of events: H_0 , H_1

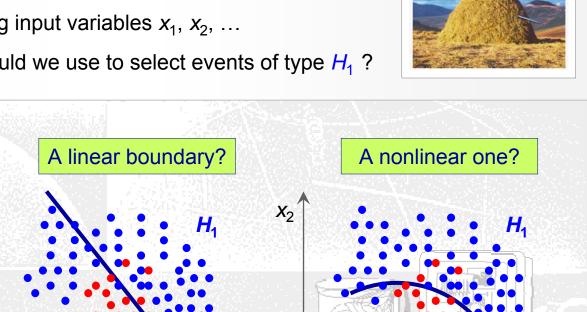
We have found discriminating input variables $x_1, x_2, ...$

x₂

H₄

x₁

What decision boundary should we use to select events of type H_1 ?





H

H

Rectangular cuts?

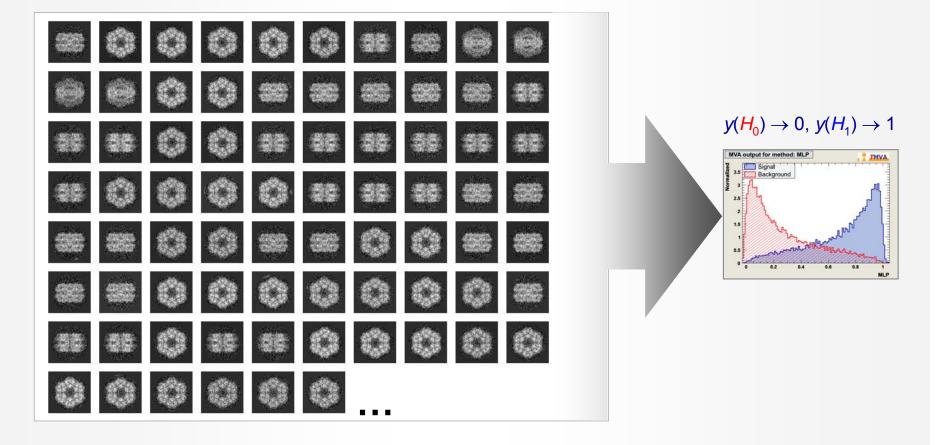
x₂

X₁

X₁

Multivariate Event Classification

- All multivariate classifiers have in common to condense (correlated) multi-variable input information in a single scalar output variable
 - It is a $R^n \rightarrow R$ regression problem; classification is in fact a *discretised regression*



Event Classification in High-Energy Physics (HEP)

Most HEP analyses require discrimination of signal from background:

- Event level (Higgs searches, …)
- Cone level (Tau-vs-jet reconstruction, ...)
- Track level (particle identification, ...)
- Lifetime and flavour tagging (*b*-tagging, …)
- Parameter estimation (*CP* violation in *B* system, ...)
- etc.

The multivariate input information used for this has various sources

- Kinematic variables (masses, momenta, decay angles, ...)
- Event properties (jet/lepton multiplicity, sum of charges, ...)
- Event shape (sphericity, Fox-Wolfram moments, ...)
- Detector response (silicon hits, *dE*/*dx*, Cherenkov angle, shower profiles, muon hits, ...)
- etc.
- Traditionally few powerful input variables were combined; new methods allow to use up to 100 and more variables w/o loss of classification power

Multivariate Classification Algorithms

A large variety of multivariate classifiers (MVAs) exists

- Rectangular cuts (optimisation often "by hand")
- Projective likelihood (up to 2D)
- Linear discriminants (χ^2 estimators, Fisher, ...)
- Nonlinear discriminants (Neural nets, ...)
- Prior decorrelation of input variables (input to cuts and likelihood)
- Function discriminants
- Multidimensional likelihood (k-nearest neighbor methods)

Decision trees with boosting and bagging, Random forests

- Rule-based learning machines
- Support vector machines

Bayesian neural nets, and more general *Committee* classifiers

na

Multivariate Classification Algorithms

How to dissipate (often diffuse) skepticism against the use of MVAs

black boxes !

Certainly, cuts are transparent, so

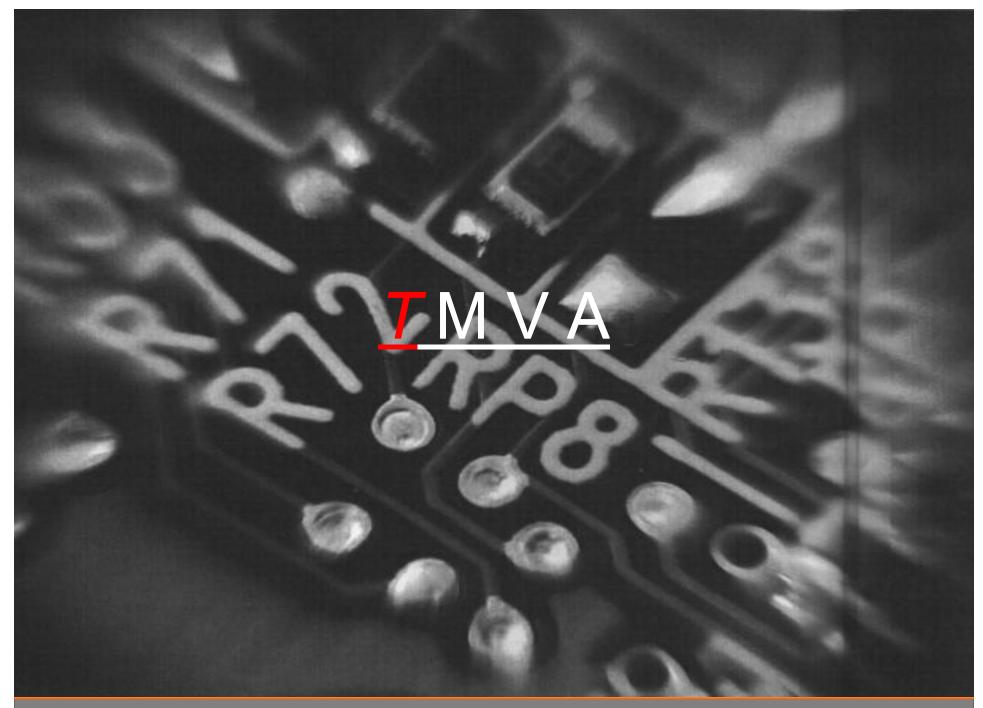
- if cuts are competitive (rarely the case) → use them
- in presence of correlations, cuts loose transparency
- "we should stop calling MVAs black boxes and understand how they behave"

what if the training samples incorrectly describe the data ? Not good, but not necessarily a huge problem:

- performance on real data will be worse than training results
- however: bad training does not create a bias !
- only if the training efficiencies are used in data analysis \rightarrow bias
- optimized cuts are <u>not</u> in general less vulnerable to systematics (on the contrary !)

how can one evaluate systematics ? There is no principle difference in systematics evaluation between single variables and MVAs

need control sample for MVA output (not necessarily for each input variable)



LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with **T**MVA

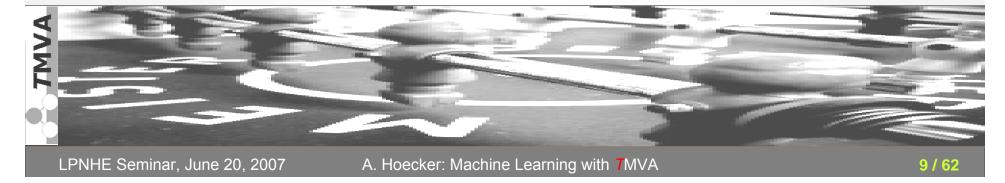
What is **T**MVA

The various classifiers have very different properties

- Ideally, all should be tested for a given problem
- Systematically choose the best performing and simplest classifier
- Comparisons between classifiers improves the understanding and takes away mysticism

TMVA — Toolkit for multivariate data analysis

- Framework for *parallel* training, testing, evaluation and application of MV classifiers
- Training events can have weights
- A large number of linear, nonlinear, likelihood and rule-based classifiers implemented
- The classifiers rank the input variables
- The input variables can be decorrelated or projected upon their principal components
- Training results and full configuration are written to weight files
- Application to data classification using a **Reader** or standalone C++ classes



TMVA Development and Distribution

TMVA is a sourceforge (SF) package for world-wide access

- Home page<u>http://tmva.sf.net</u>/
- SF project page <u>http://sf.net/projects/tmva</u>
- View CVS<u>http://tmva.cvs.sf.net/tmva/TMVA</u>/
- Mailing list<u>http://sf.net/mail/?group_id=152074</u>

Active project \rightarrow fast response time on feature requests

- Currently 6 main developers, and 27 registered contributors at SF
- >1200 downloads since March 2006 (not accounting cvs checkouts and ROOT users)
- Written in C++, relying on core ROOT functionality
 - Full examples distributed with **7**MVA, including analysis macros and GUI
 - Scripts are provided for *T*MVA use in ROOT macro, as C++ executable or with python
- Integrated and distributed with ROOT since ROOT v5.11/03

The TMVA Classifiers

Currently implemented classifiers :

- Rectangular cut optimisation
- Projective and multidimensional likelihood estimator
- k-Nearest Neighbor algorithm
- Fisher and H-Matrix discriminants
- Function discriminant
- Artificial neural networks (3 different multilayer perceptrons)
- Boosted/bagged decision trees with automatic node pruning
- RuleFit
- Support Vector Machine

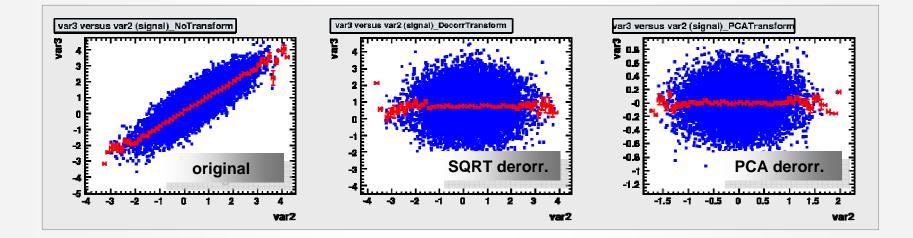
Data Preprocessing: Decorrelation

Commonly realised for all methods in TMVA (centrally in DataSet class)

Removal of linear correlations by rotating input variables

- Determine square-root C' of covariance matrix C, i.e., C = C'C'
- Transform original (x) into decorrelated variable space (x') by: $x' = C'^{-1}x$

Various ways to choose basis for decorrelation (also implemented PCA)



Rectangular Cut Optimisation

Simplest method: cut in rectangular variable volume

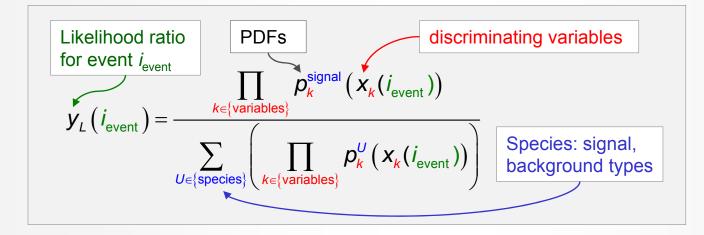
$$\boldsymbol{X}_{\text{cut}}(\boldsymbol{i}_{\text{event}}) \in \{0,1\} = \bigcap_{\boldsymbol{v} \in \{\text{variables}\}} \left(\boldsymbol{X}_{\boldsymbol{v}}(\boldsymbol{i}_{\text{event}}) \subset \left[\boldsymbol{X}_{\boldsymbol{v},\text{min}}, \boldsymbol{X}_{\boldsymbol{v},\text{max}} \right] \right)$$

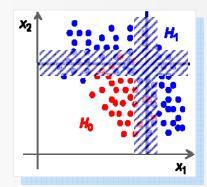


- MINUIT fails due to non-unique solution space
- TMVA uses: Monte Carlo sampling, <u>Genetic Algorithm</u>, Simulated Annealing
- Huge speed improvement of volume search by sorting events in binary tree
- Cuts usually benefit from prior decorrelation of cut variables

Projective Likelihood Estimator (PDE Approach)

Much liked in HEP: probability density estimators for each input variable combined in likelihood estimator



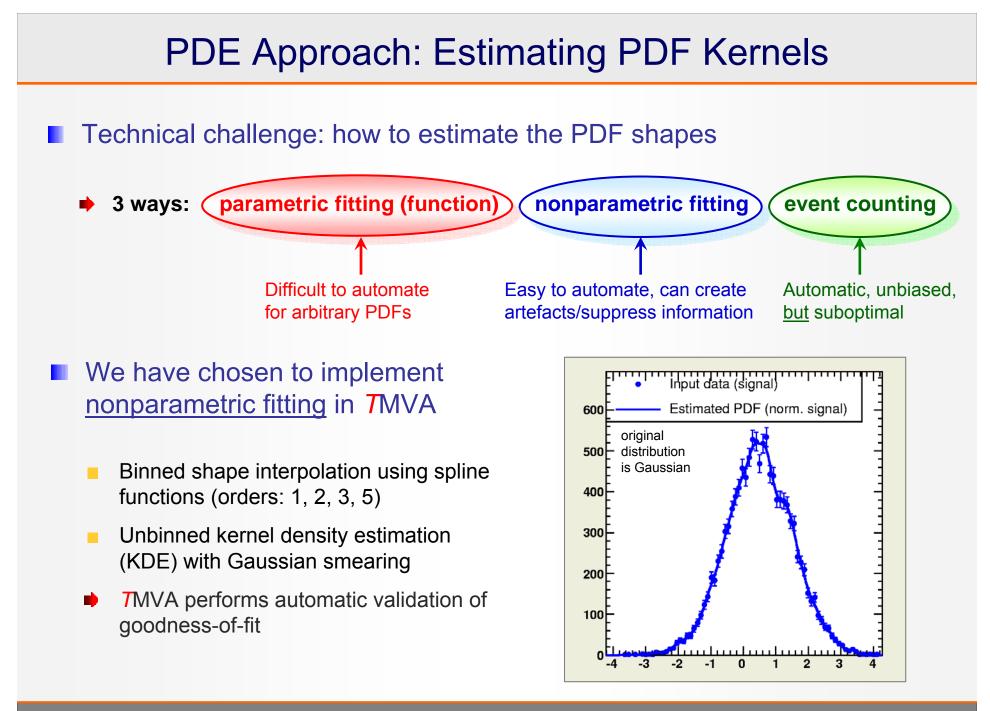


PDE introduces fuzzy logic

Ignores correlations between input variables

- Optimal approach if correlations are zero (or linear \rightarrow decorrelation)
- Otherwise: significant performance loss

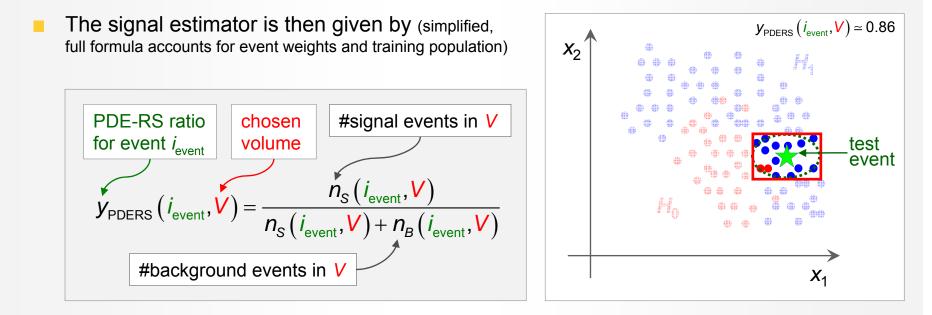




Multidimensional PDE Approach

■ Use a single PDF per event class (sig, bkg), which spans *N*_{var} dimensions

■ PDE Range-Search: count number of signal and background events in "vicinity" of test event → preset or adaptive volume defines "vicinity"



Improve y_{PDERS} estimate within V by using various N_{var} -D kernel estimators

Enhance speed of event counting in volume by binary tree search

Carli-Koblitz, NIM A501, 576 (2003)

Multidimensional PDE Approach Use a single PDF per event class (sig, bkg), which spans N_{var} dimensions PDE Range-Search: count number of signal and background events in Carli-Koblitz, NIM A501, 576 (2003) "vicinity" of test event \rightarrow preset or **adaptive** volume defines "vicinity" Rew classifier: k-Nearest Neighbor – implemented by R. Ospanov (Texas U.): Better than searching within a volume (fixed or floating), count adjacent reference events till statistically significant number reached Method intrinsically adaptive Very fast search with kd-tree event sorting #background events in V X_1

Improve y_{PDERS} estimate within V by using various N_{var} -D kernel estimators

Enhance speed of event counting in volume by binary tree search

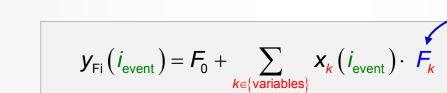
Fisher's Linear Discriminant Analysis (LDA)

Well known, simple and elegant classifier

Classifier response couldn't be simpler:

- LDA determines axis in the input variable hyperspace such that a projection of events onto this axis pushes signal and background as far away from each other as possible

"Fisher coefficients"

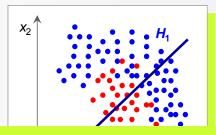


- Compute Fisher coefficients from signal and background covariance matrices
- Fisher requires distinct sample means between signal and background
- Optimal classifier for linearly correlated Gaussian-distributed variables

Fisher's Linear Discriminant Analysis (LDA)

Well known, simple and elegant classifier

LDA determines axis in the input variable hyperspace such that a projection of events onto this axis pushes signal and background as far away from each other as possible.



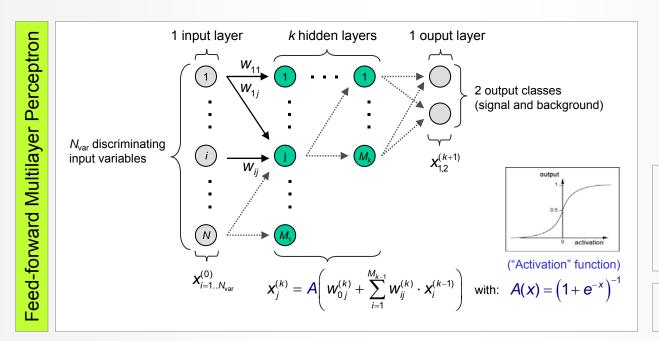
NeW classifier: Function discriminant analysis (FDA)

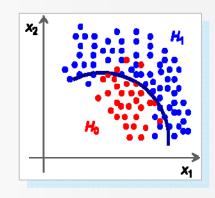
Fit any user-defined function of input variables requiring that signal events return $\rightarrow 1$ and background $\rightarrow 0$

- Parameter fitting: Genetics Alg., MINUIT, MC and combinations
- Easy reproduction of Fisher result, but can add nonlinearities
- Very transparent discriminator
- Compute Fisher coefficients from signal and background covariance matrices
- Fisher requires distinct sample means between signal and background
- Optimal classifier for linearly correlated Gaussian-distributed variables

Nonlinear Analysis: Artificial Neural Networks

- Achieve nonlinear classifier response by "activating" output nodes using nonlinear weights
- Call nodes "neurons" and arrange them in series:





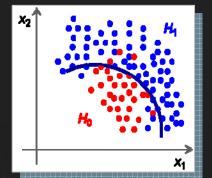
<u>Weierstrass theorem</u>: can approximate any continuous functions to arbitrary precision with a single hidden layer and an infinite number of neurons

Three different multilayer perceptrons available in *T*MVA

Adjust weights (=training) using "back-propagation"

Decision Trees

Sequential application of cuts splits the data into nodes, where the final nodes (leafs) classify an event as signal or background



LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with TMVA

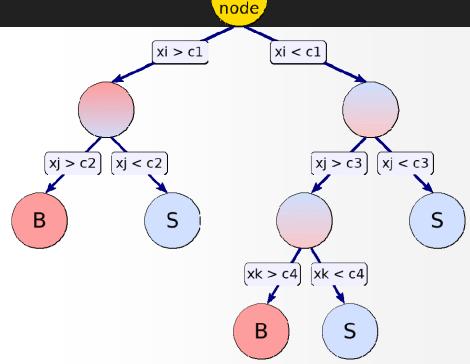
21/62

Decision Trees

Sequential application of cuts splits the data into nodes, where the final nodes (leafs) classify an event as signal or background

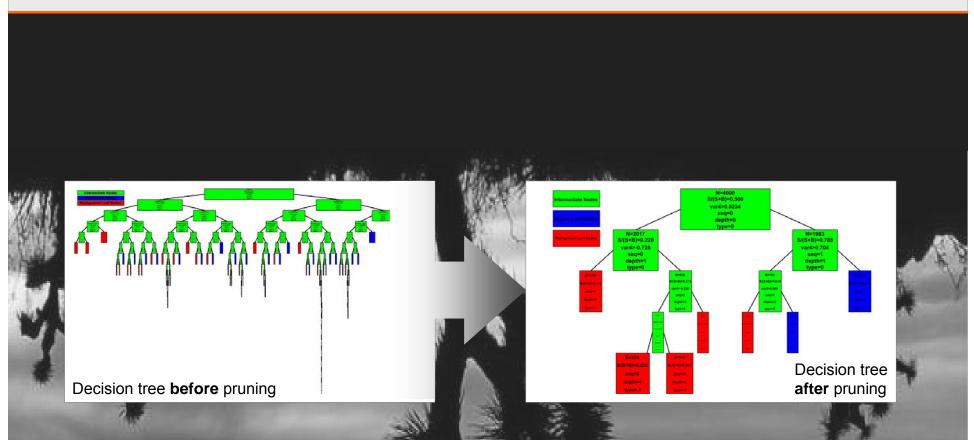
Growing a decision tree:

- Start with Root node
- Split training sample according to cut on best variable at this node
- Splitting criterion: e.g., maximum
 "Gini-index": purity × (1– purity)
- Continue splitting until min. number of events or max. purity reached
- Classify leaf node according to majority of events, or give weight; unknown test events are classified accordingly



Root

Decision Trees



Bottom-up "pruning" of a decision tree

Remove statistically insignificant nodes to reduce tree overtraining \rightarrow automatic in TMVA

23/62

Boosted Decision Trees (BDT)

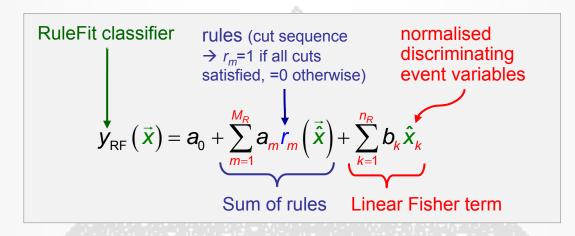
- **Data mining with decision trees is popular in science** (so far mostly outside of HEP)
 - Advantages:
 - Easy interpretation can always be represented in 2D tree
 - Independent of monotonous variable transformations, immune against outliers
 - Weak variables are ignored (and don't (much) deteriorate performance)
 - Shortcomings:
 - Instability: small changes in training sample can dramatically alter the tree structure
 - Sensitivity to overtraining (\rightarrow requires pruning)
- Boosted decision trees: combine forest of decision trees, with differently weighted events in each tree (trees can also be weighted), by majority vote
 - e.g., "AdaBoost": incorrectly classified events receive larger weight in next decision tree
 - "Bagging" (instead of boosting): random event weights, resampling with replacement
 - Boosting or bagging are means to create set of "basis functions": the final classifier is linear combination (*expansion*) of these functions → improves stability !

Predictive Learning via Rule Ensembles (RuleFit)

Following RuleFit approach by <u>Friedman-Popescu</u>

Friedman-Popescu, Tech Rep, Stat. Dpt, Stanford U., 2003

Model is linear combination of *rules*, where a rule is a sequence of cuts



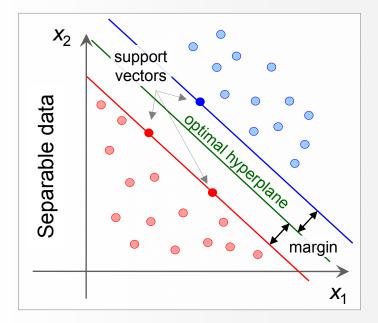
- The problem to solve is
 - Create rule ensemble: use forest of decision trees
 - Fit coefficients a_m , b_k : gradient direct regularization minimising Risk (Friedman et al.)
- Pruning removes topologically equal rules" (same variables in cut sequence)

One of the elementary cellular automaton rules (Wolfram 1983, 2002). It specifies the next color in a cell, depending on its color and its immediate neighbors. Its rule outcomes are encoded in the binary representation 30=00011110₂.

25/62

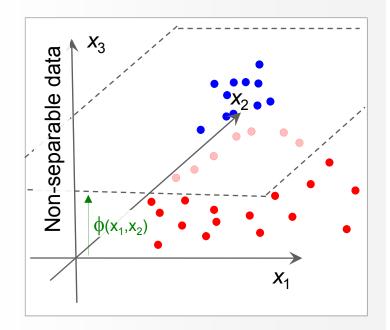
Support Vector Machine (SVM)

- Find hyperplane that best separates signal from background
 - Best separation: maximum distance (margin) between closest events (*support*) to hyperplane
 - Linear decision boundary
 - If data non-separable add *misclassification cost* parameter to minimisation function



Support Vector Machine (SVM)

- Find hyperplane that best separates signal from background
 - Best separation: maximum distance (margin) between closest events (*support*) to hyperplane
 - Linear decision boundary
 - If data non-separable add *misclassification cost* parameter to minimisation function



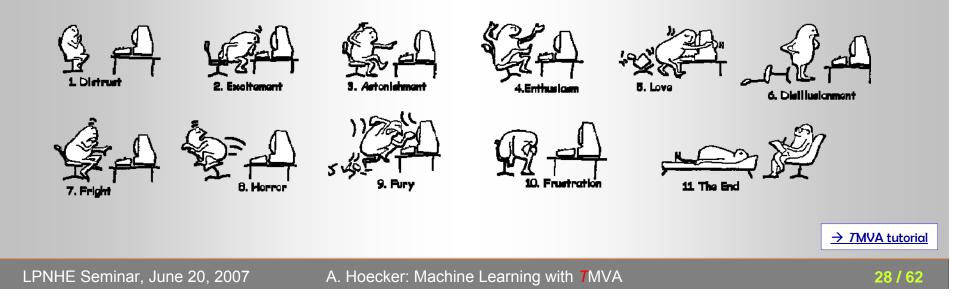
Non-linear cases:

- Transform variables into higher dimensional space where again a linear boundary (hyperplane) can separate the data
- Explicit transformation form not required: use Kernel Functions to approximate scalar products between transformed vectors in the higher dimensional space
- Choose Kernel and fit the hyperplane using the linear techniques developed above
- Available Kernels: Gaussian, Polynomial, Sigmoid

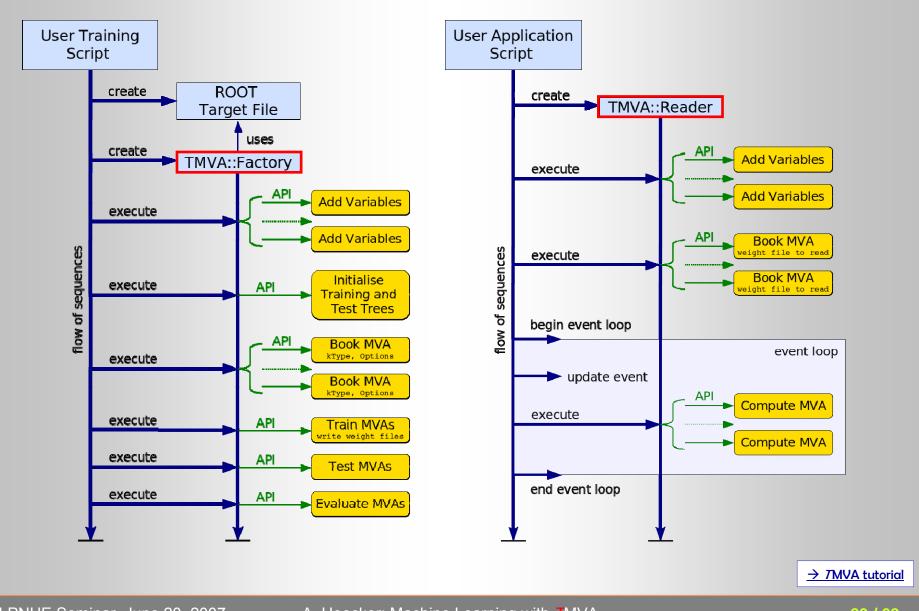
Using TMVA

A typical **T**MVA analysis consists of two main steps:

- 1. Training phase: training, testing and evaluation of classifiers using data samples with known signal and background composition
- 2. Application phase: using selected trained classifiers to classify unknown data samples
- Illustration of these steps with toy data samples



Code Flow for *Training* and *Application* Phases

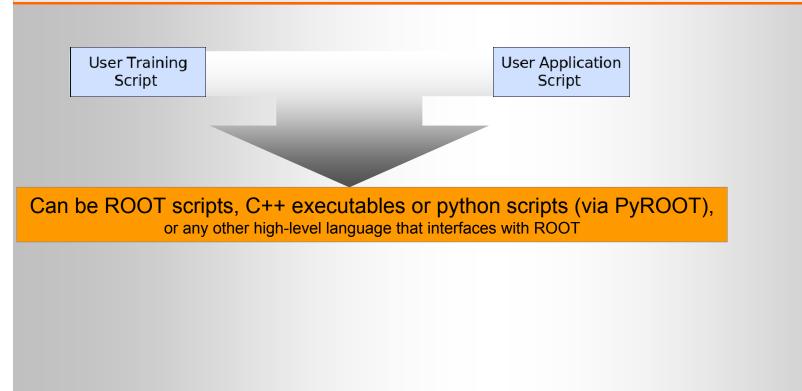


LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with TMVA

29/62

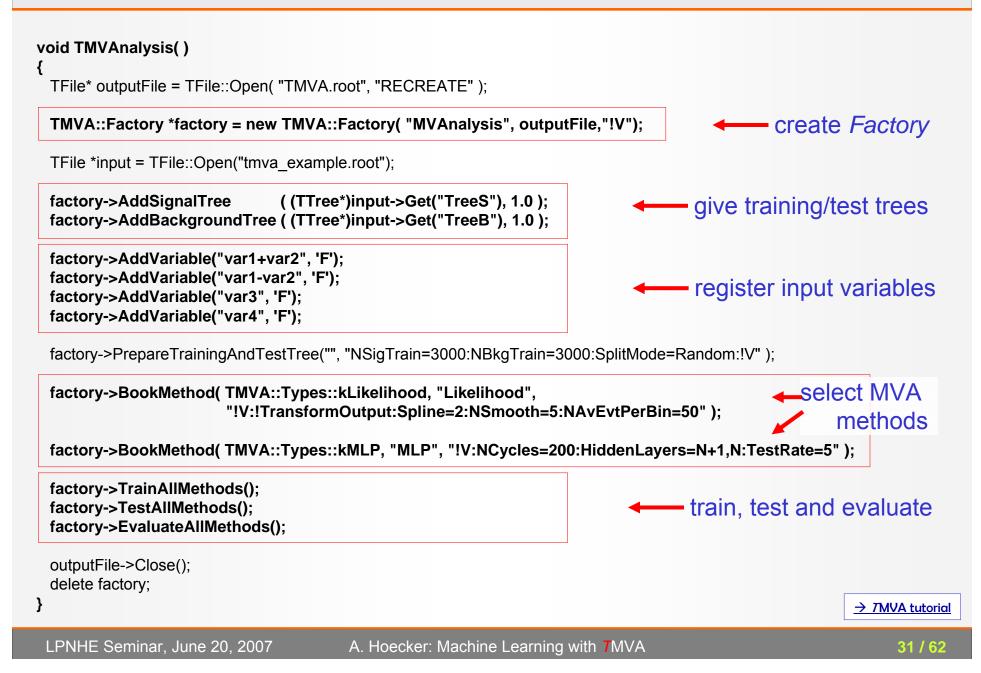




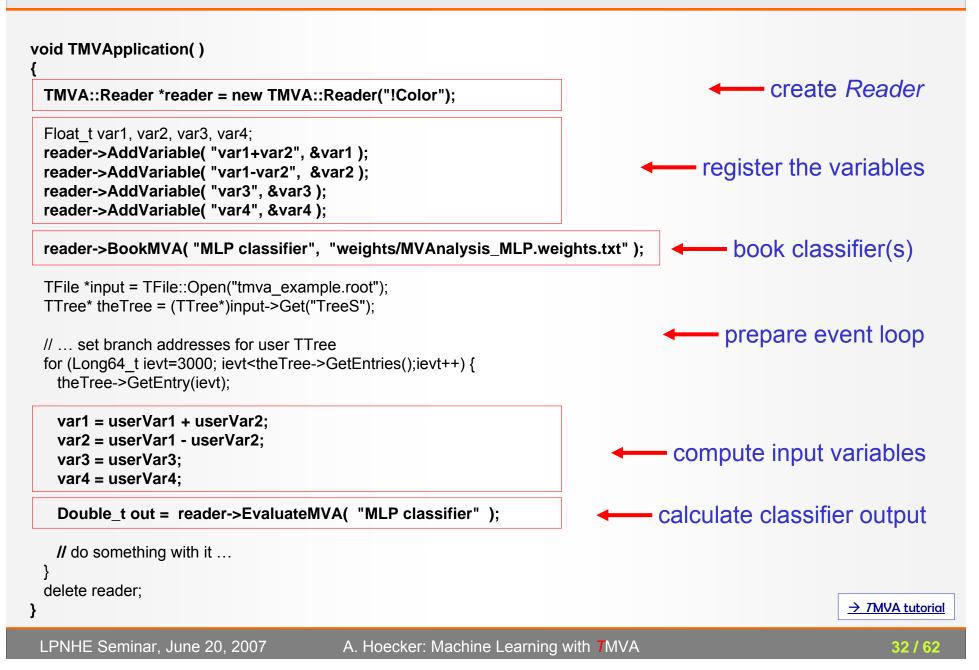
→ 7MVA tutorial

30/62

A Simple Example for *Training*

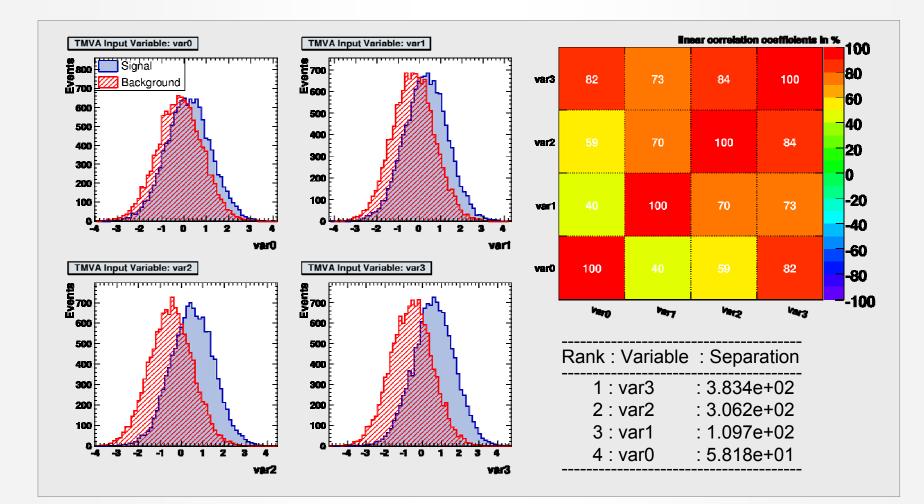


A Simple Example for an Application



A Toy Example (idealized)

Use data set with 4 linearly correlated Gaussian distributed variables:

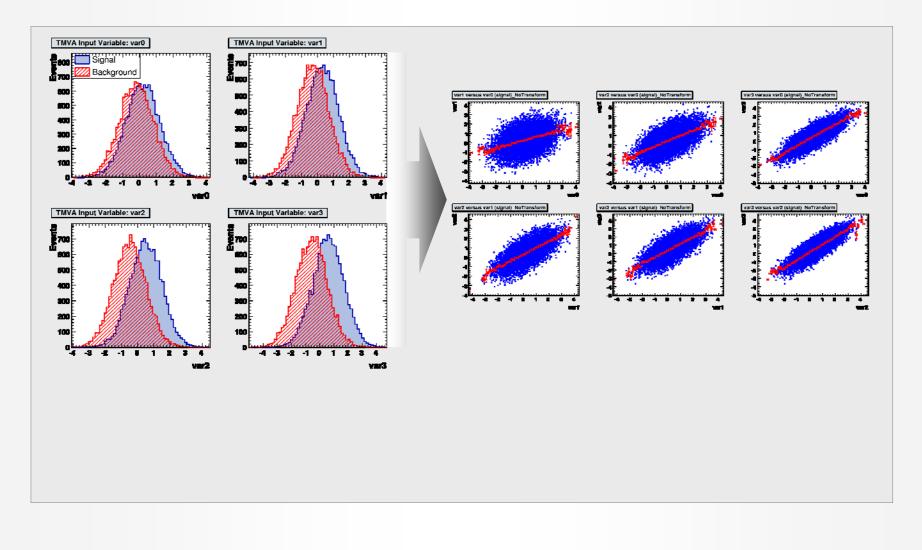


LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with **T**MVA

Preprocessing the Input Variables

Decorrelation of variables before training is useful for this example

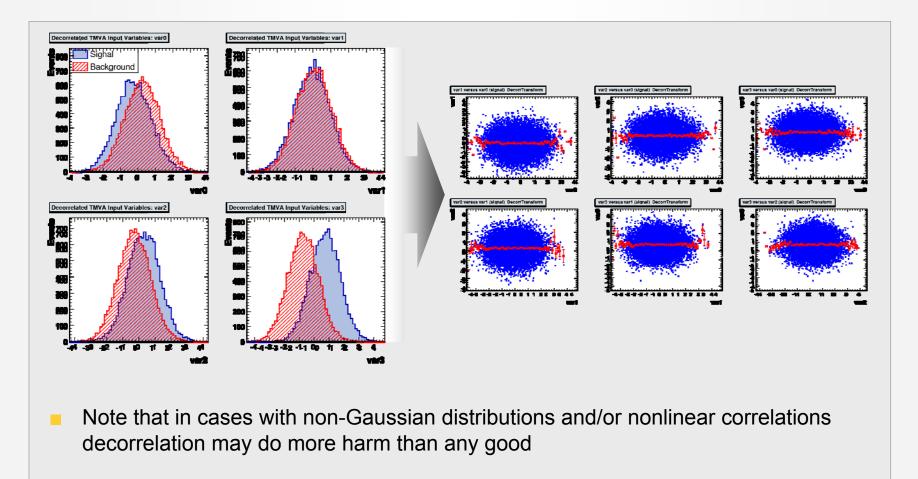


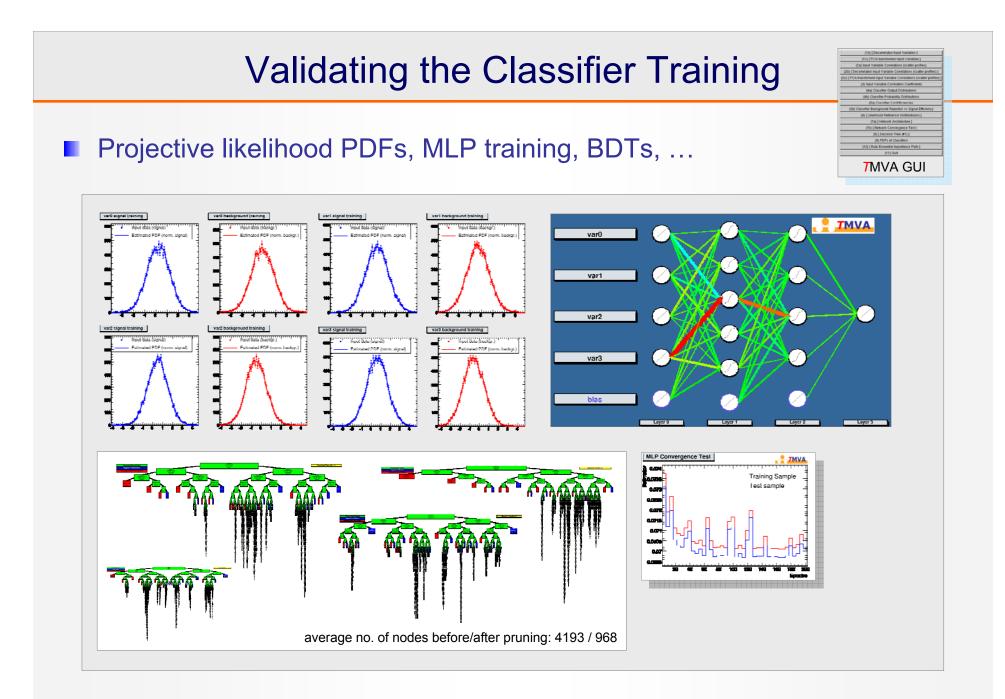
LPNHE Seminar, June 20, 2007

A. Hoecker: Machine Learning with TMVA

Preprocessing the Input Variables

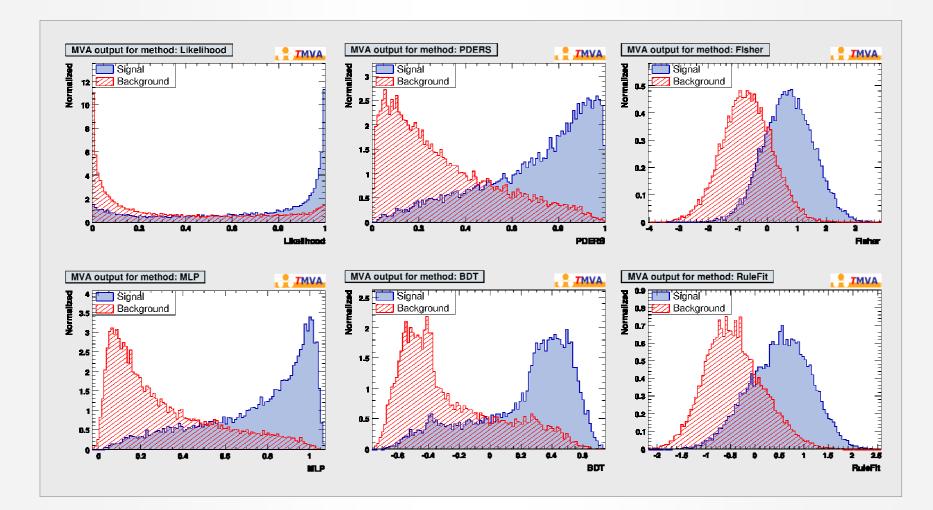
Decorrelation of variables before training is useful for *this* example





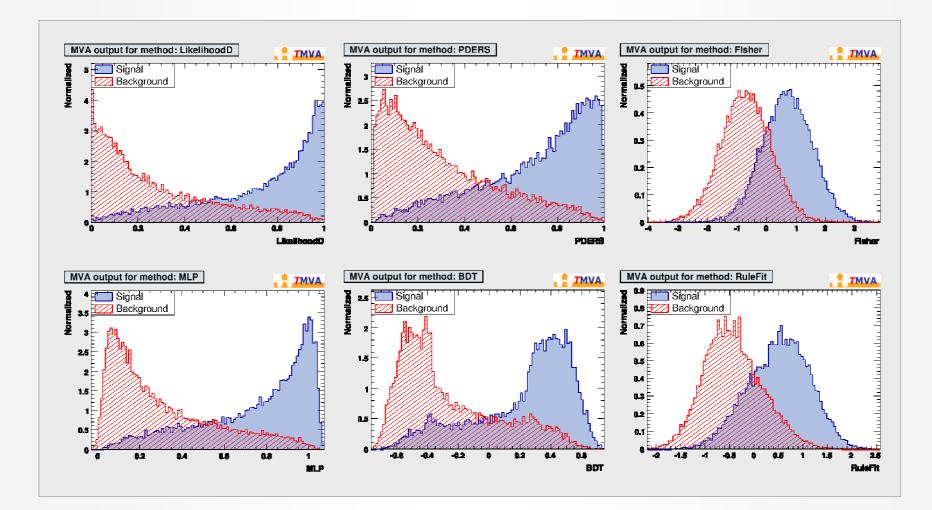
Testing the Classifiers

Classifier output distributions for independent test sample:



Testing the Classifiers

Classifier output distributions for independent test sample:



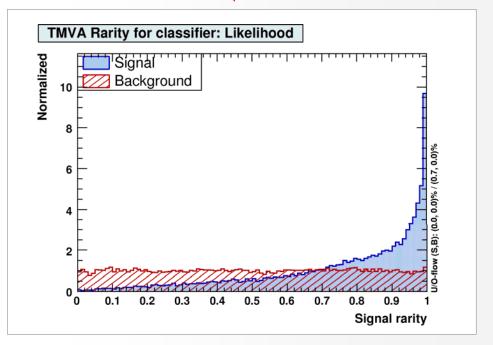
Evaluating the Classifiers

- There is no unique way to express the performance of a classifier → several benchmark quantities computed by *T*MVA
 - Signal eff. at various background effs. (= 1 rejection) when cutting on classifier output

The Separation:
$$\frac{1}{2}\int \frac{(\hat{y}_{S}(y) - \hat{y}_{B}(y))^{2}}{\hat{y}_{S}(y) + \hat{y}_{B}(y)} dy$$

• "Rarity" implemented (background flat): $R(y) = \int \hat{y}(y') dy'$

Other quantities ... see <u>Users Guide</u>

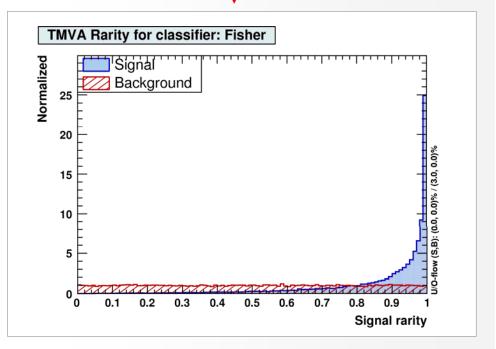


Evaluating the Classifiers

- There is no unique way to express the performance of a classifier → several benchmark quantities computed by *T*MVA
 - Signal eff. at various background effs. (= 1 rejection) when cutting on classifier output

The Separation:
$$\frac{1}{2}\int \frac{(\hat{y}_{S}(y) - \hat{y}_{B}(y))^{2}}{\hat{y}_{S}(y) + \hat{y}_{B}(y)} dy$$

- "Rarity" implemented (background flat): $R(y) = \int \hat{y}(y') dy'$
- Other quantities ... see <u>Users Guide</u>



Evaluating the Classifiers

- There is no unique way to express the performance of a classifier \rightarrow several benchmark quantities computed by *T*MVA
 - Signal eff. at various background effs. (= 1 rejection) when cutting on classifier output

The Separation:
$$\frac{1}{2} \int \frac{(\hat{y}_{S}(y) - \hat{y}_{B}(y))^{2}}{\hat{y}_{S}(y) + \hat{y}_{B}(y)} dy$$

• "Rarity" implemented (background flat): $R(y) = \int \hat{y}(y') dy'$

Other quantities ... see <u>Users Guide</u>

Remark on overtraining

- Occurs when classifier training has too few degrees of freedom because the classifier has too many adjustable parameters for too few training events
- Sensitivity to overtraining depends on classifier: e.g., Fisher weak, BDT strong
- Compare performance between training and test sample to detect overtraining
- Actively counteract overtraining: e.g., smooth likelihood PDFs, prune decision trees, …

Evaluating the Classifiers (taken from TMVA output...)

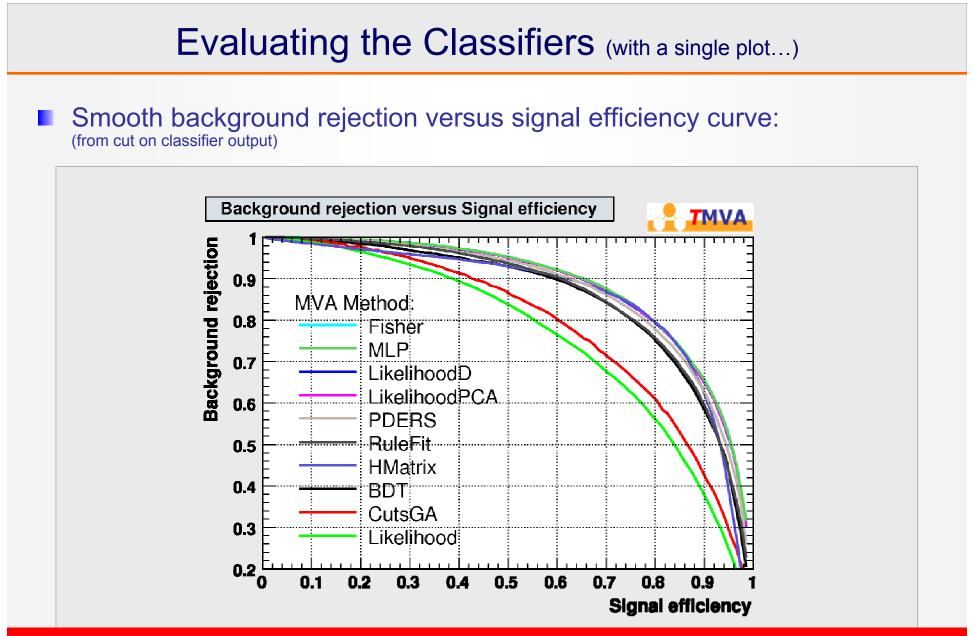
Evaluation results ranked by best signal efficiency and purity (area)

MVA Methods:	Signal eff @B=0.01	iciency at @B=0.10	bkg eff. (e @B=0.30	rror): Area	Sepa- ration:	Signifi- cance:
Fisher	· 0.268(03)	0.653(03)	0.873(02)	0.882	0.444	 1.189
MLP	: 0.266(03)	0.656(03)	0.873(02)	0.882	0.444	1.260
LikelihoodD	: 0.259(03)	0.649(03)	0.871(02)	0.880	0.441	1.251
PDERS	: 0.223(03)	0.628(03)	0.861(02)	0.870	0.417	1.192
RuleFit	: 0.196(03)	0.607(03)	0.845(02)	0.859	0.390	1.092
HMatrix	: 0.058(01)	0.622(03)	0.868(02)	0.855	0.410	1.093
BDT	: 0.154(02)	0.594(04)	0.838(03)	0.852	0.380	1.099
CutsGA	: 0.109(02)	1.000(00)	0.717(03)	0.784	0.000	0.000
Likelihood	: 0.086(02)	0.387(03)	0.677(03)	0.757	0.199	0.682

Testing efficiency compared to training efficiency (overtraining check)

	MVA	Signal efficiency:	from test sample	(from traing sample)		
	Methods:	@B=0.01	@B=0.10	@B=0.30		
Check for over- training	Fisher MLP LikelihoodD PDERS RuleFit HMatrix BDT CutsGA Likelihood	<pre>: 0.268 (0.275) : 0.266 (0.278) : 0.259 (0.273) : 0.223 (0.389) : 0.196 (0.198) : 0.058 (0.060) : 0.154 (0.268) : 0.109 (0.123) : 0.086 (0.092)</pre>	0.653 (0.658) 0.656 (0.658) 0.649 (0.657) 0.628 (0.691) 0.607 (0.616) 0.622 (0.623) 0.594 (0.736) 1.000 (0.424) 0.387 (0.379)	0.873 (0.873) 0.873 (0.873) 0.871 (0.872) 0.861 (0.881) 0.845 (0.848) 0.868 (0.868) 0.838 (0.911) 0.717 (0.715) 0.677 (0.677)		

LPNHE Seminar, June 20, 2007



Note: Nearly All Realistic Use Cases are Much More Difficult Than This One

LPNHE Seminar, June 20, 2007

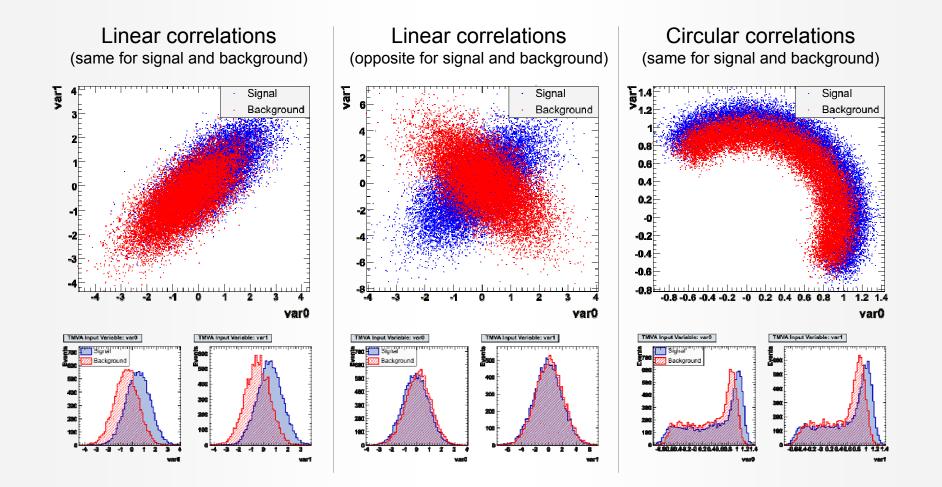
A. Hoecker: Machine Learning with TMVA

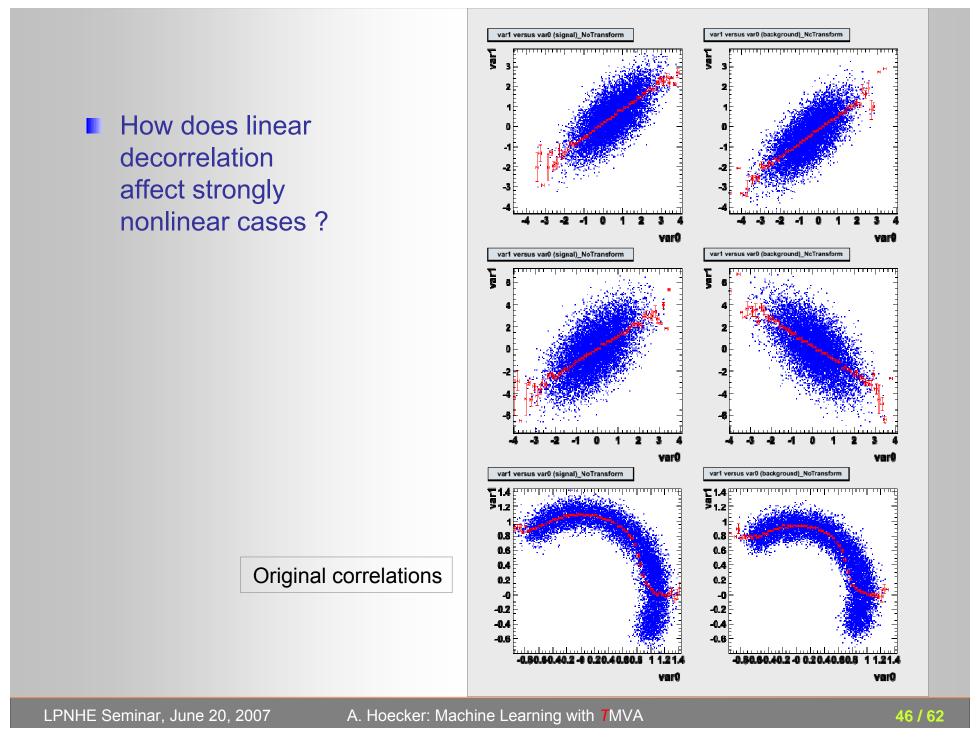
43 / 62

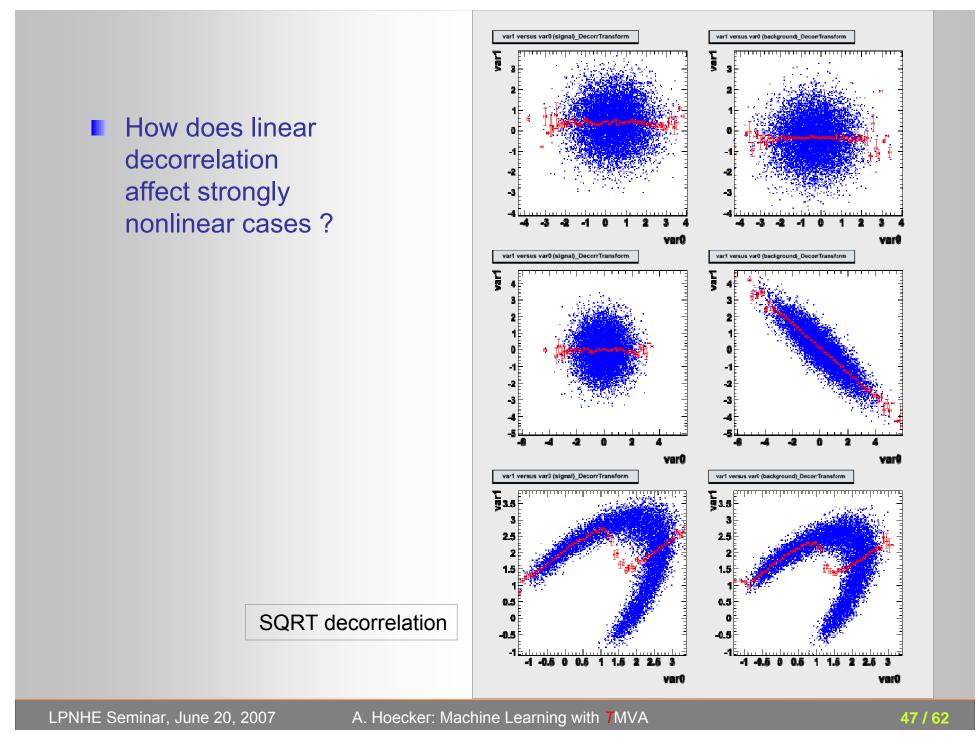
More Toy Examples

More Toys: Linear-, Cross-, Circular Correlations

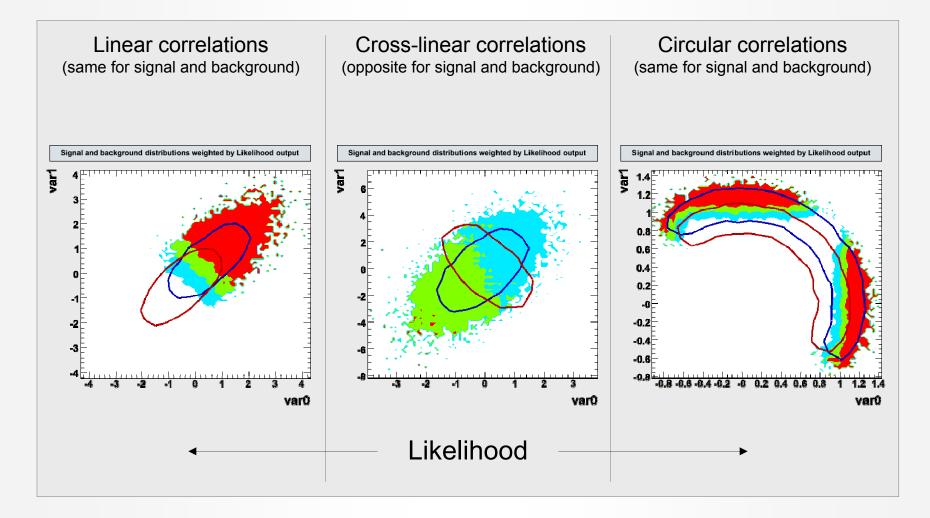
Illustrate the behaviour of linear and nonlinear classifiers





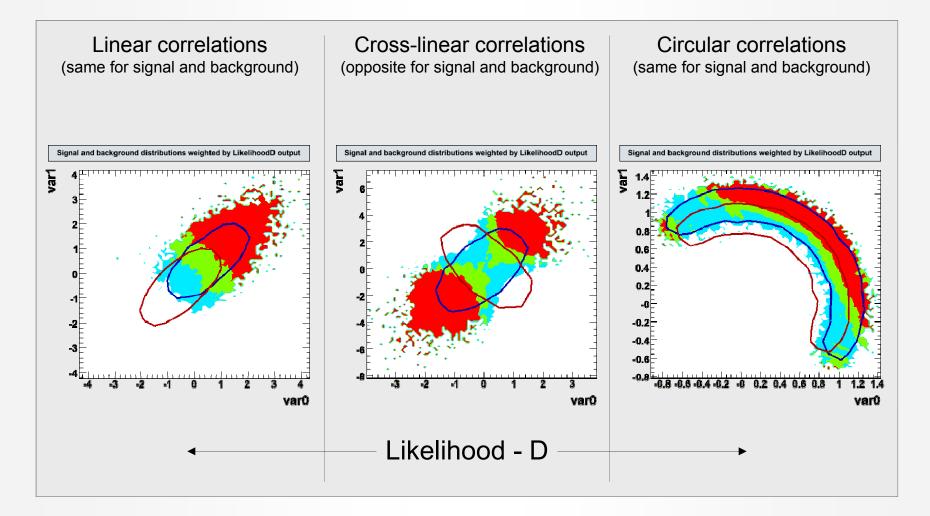


How well do the classifier resolve the various correlation patterns ?

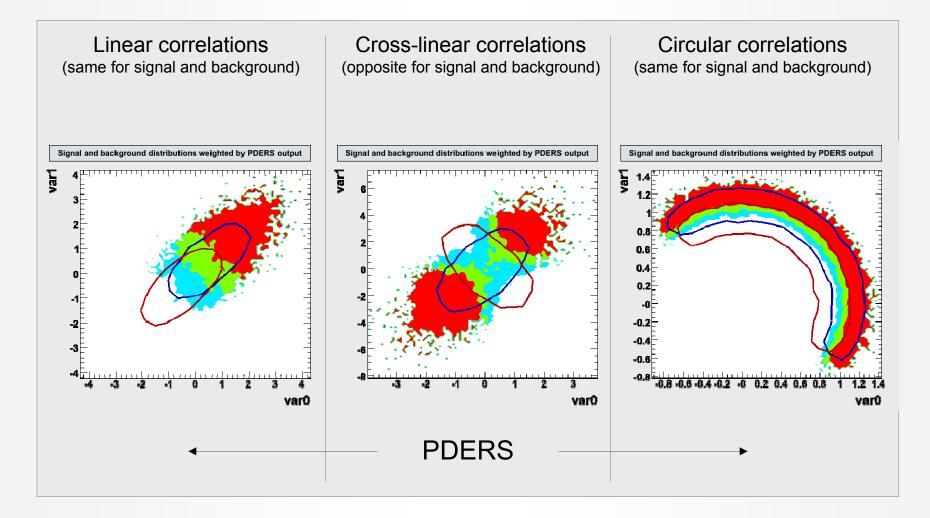


LPNHE Seminar, June 20, 2007

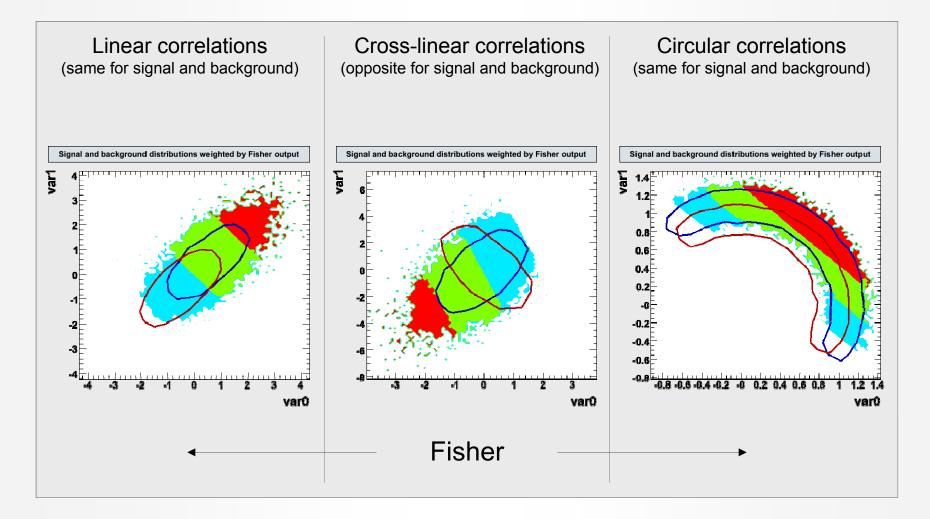
How well do the classifier resolve the various correlation patterns ?



How well do the classifier resolve the various correlation patterns ?

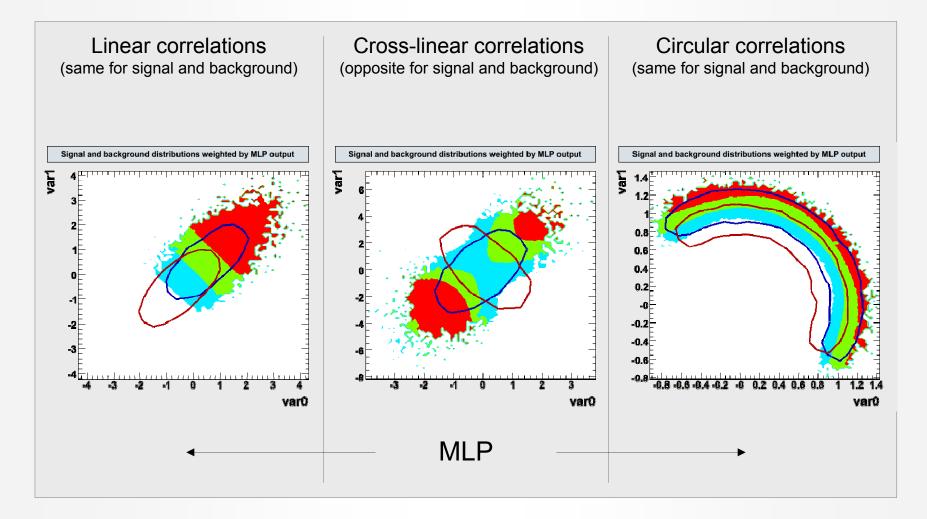


How well do the classifier resolve the various correlation patterns ?



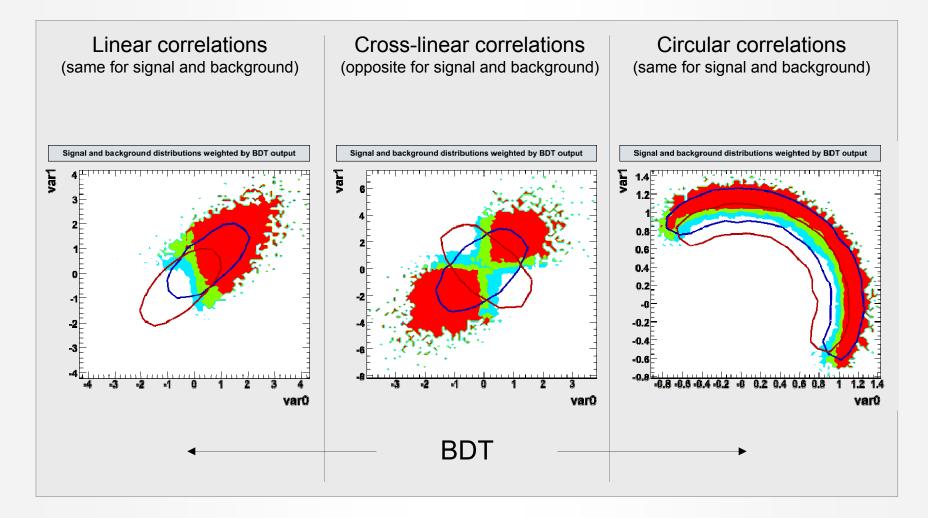
LPNHE Seminar, June 20, 2007

How well do the classifier resolve the various correlation patterns ?



LPNHE Seminar, June 20, 2007

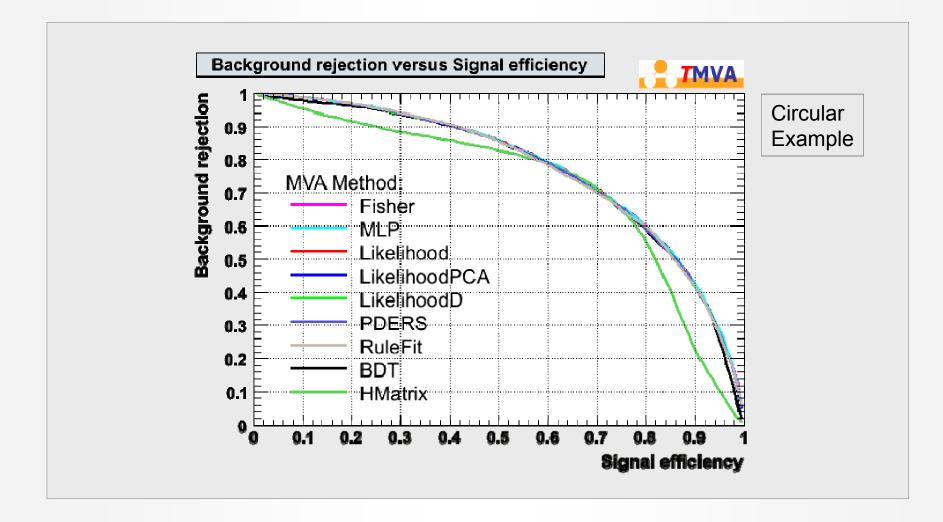
How well do the classifier resolve the various correlation patterns ?



LPNHE Seminar, June 20, 2007

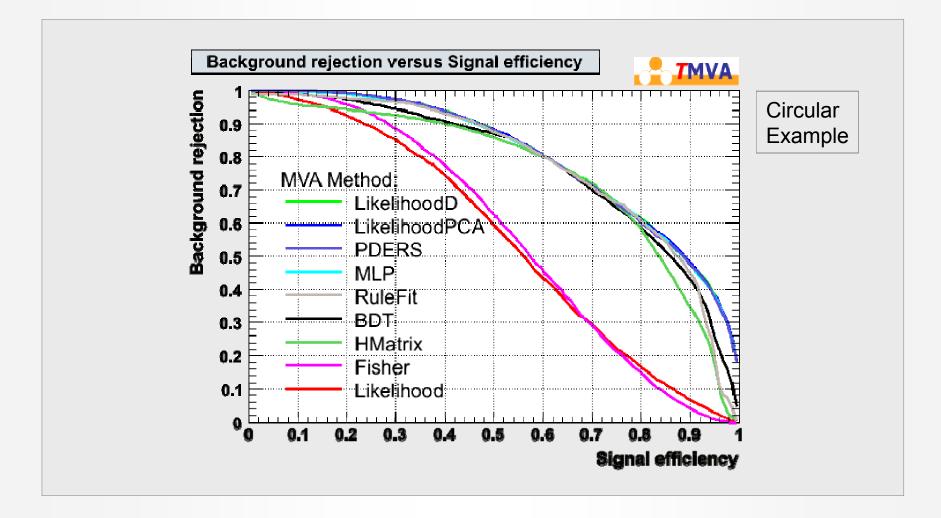
Final Classifier Performance

Background rejection versus signal efficiency curve:



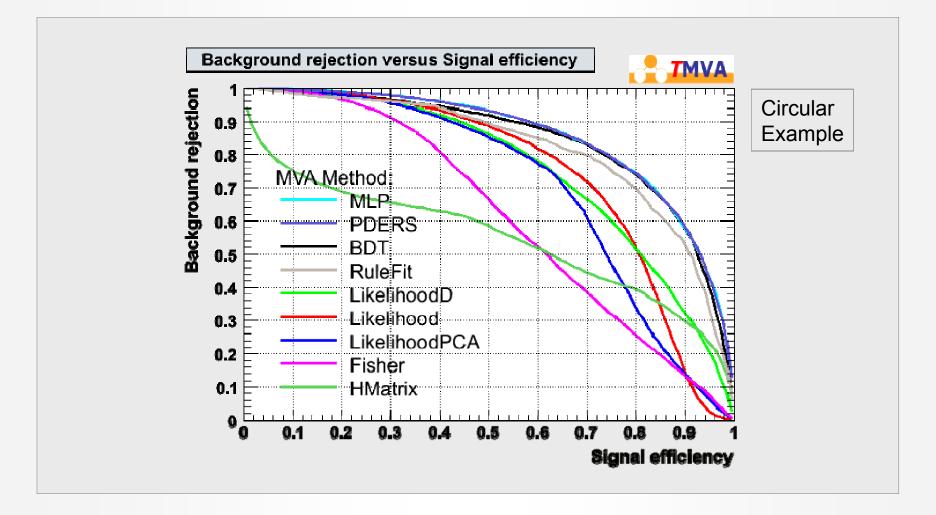
Final Classifier Performance

Background rejection versus signal efficiency curve:

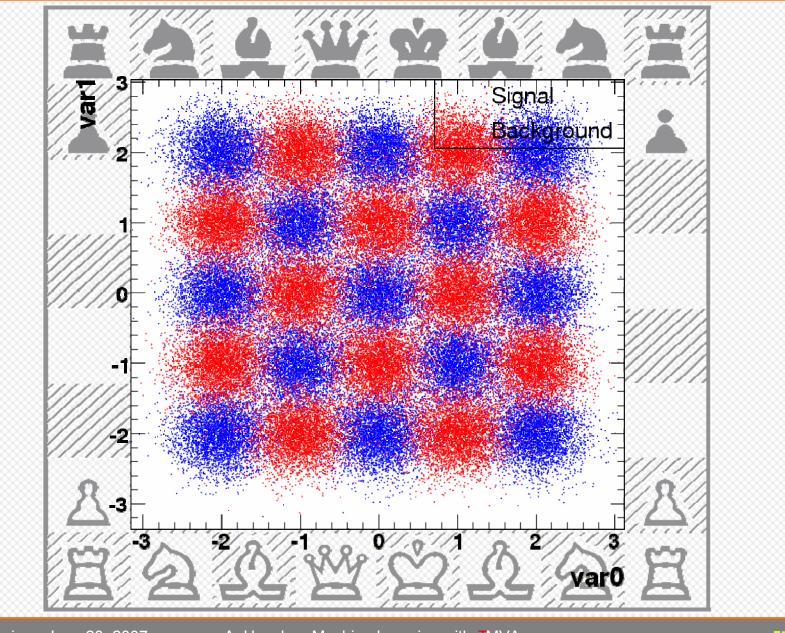


Final Classifier Performance

Background rejection versus signal efficiency curve:

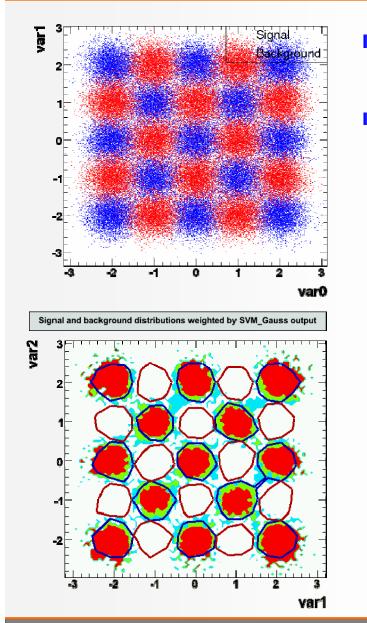


The "Schachbrett" Toy

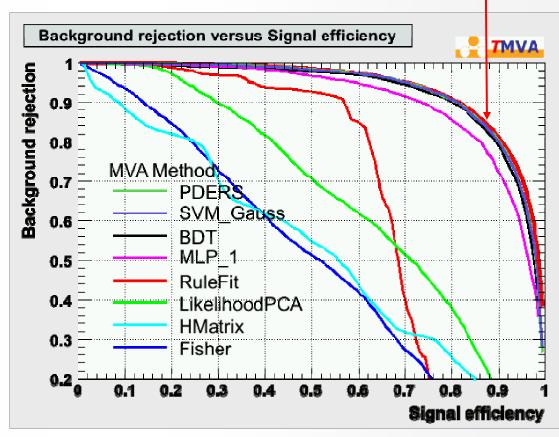


LPNHE Seminar, June 20, 2007

The "Schachbrett" Toy



- Performance achieved without parameter tuning: PDERS and BDT best "out of the box" classifiers
 - After specific tuning, also SVM und MLP perform well



Theoretical maximum

LPNHE Seminar, June 20, 2007

Summary & Plans

LPNHE Seminar, June 20, 2007

TMVA

A. Hoecker: Machine Learning with TMVA

59/62

Summary of the Classifiers and their Properties

Criteria		Classifiers								
		Cuts	Likeli- hood	PDERS / k-NN	H-Matrix	Fisher	MLP	BDT	RuleFit	SVM
Perfor- mance	no / linear correlations		\odot	\odot	÷	\odot	\odot		\odot	\odot
	nonlinear correlations	:	8	\odot	8	$\overline{\mathbf{S}}$	\odot	\odot		\odot
Speed	Training	\odot			\odot	\odot		$\overline{\mathbf{S}}$		$\overline{\mathbf{O}}$
	Response	\odot		8/9	\odot	\odot	\odot			(
Robust -ness	Overtraining	\odot		æ	\odot	\odot	$\overline{\mathbf{S}}$	8	æ	
	Weak input variables	(1)		\bigotimes	\odot	\odot			æ	()
Curse of dimensionality		(\mathbf{i})	\odot	\bigotimes	\odot	\odot			((
Clarity		3	\odot		\odot	\odot	8	8	8	8
The properties of the Function discriminant (FDA) depend on the chosen function										
LPNHE Seminar, June 20, 2007 A. Hoecker: Machine Learning with 7 MVA 60 / 62							60 / 62			

Plans

Primary goal for this Summer: Generalised Committee classifier

Combine any classifier with any other classifier using any combination of input variables in any phase space region

Backup slides on: (i) treatment of systematic uncertainties

(ii) sensitivity to weak input variables

IMVA

61 / 62

Copyrights & Credits

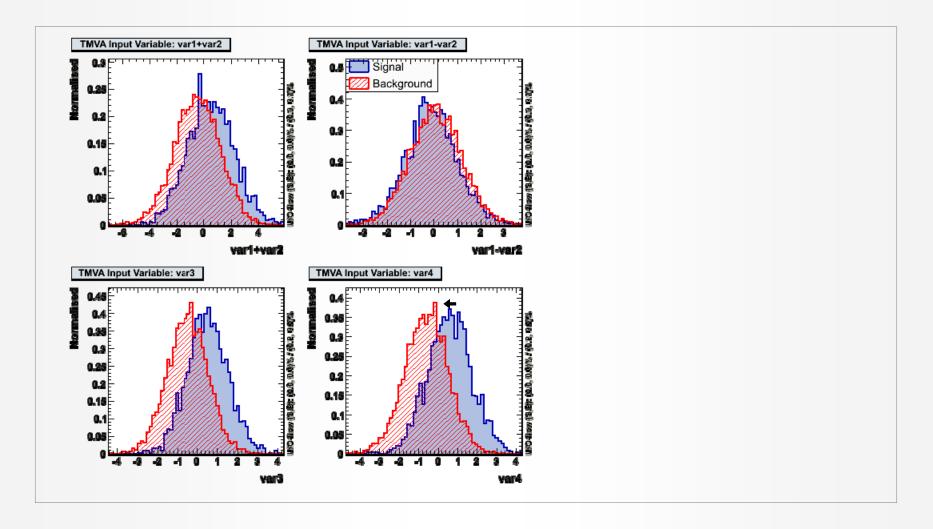
- TMVA is open source software
- Use & redistribution of source permitted according to terms in <u>BSD license</u>
- Several similar data mining efforts with rising importance in most fields of science and industry
- Important for HEP:
 - Parallelised MVA training and evaluation pioneered by Cornelius package (BABAR)
 - Also frequently used: StatPatternRecognition package by I. Narsky
 - Many implementations of individual classifiers exist

Acknowledgments: The fast development of TMVA would not have been possible without the contribution and feedback from many developers and users to whom we are indebted. We thank in particular the CERN Summer students Matt Jachowski (Stanford) for the implementation of TMVA's new MLP neural network, and Yair Mahalalel (Tel Aviv) for a significant improvement of PDERS, the Krakow student Andrzej Zemla and his supervisor Marcin Wolter for programming a powerful Support Vector Machine, as well as Rustem Ospanov for the development of a fast k-NN algorithm. We are grateful to Doug Applegate, Kregg Arms, René Brun and the ROOT team, Tancredi Carli, Zhiyi Liu, Elzbieta Richter-Was, Vincent Tisserand and Alexei Volk for helpful conversations.



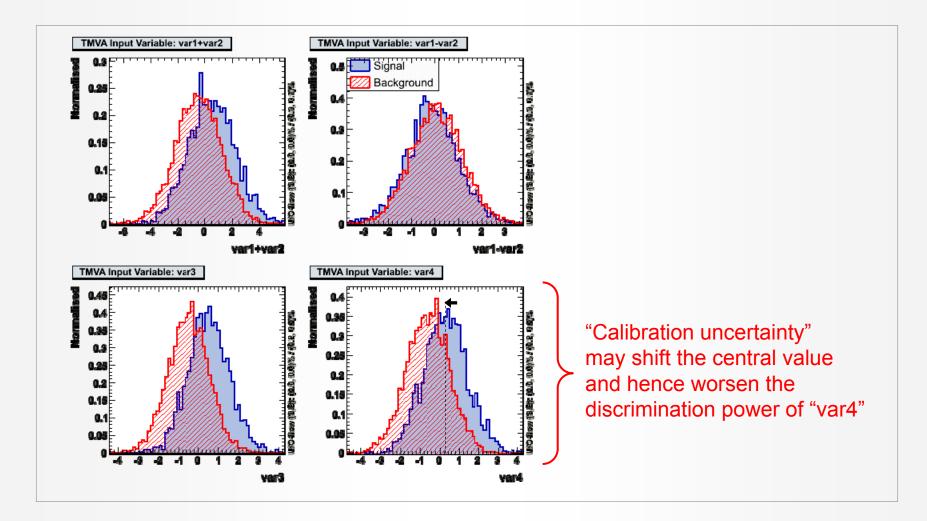
backup slides

Assume strongest variable "var4" suffers from systematic uncertainty

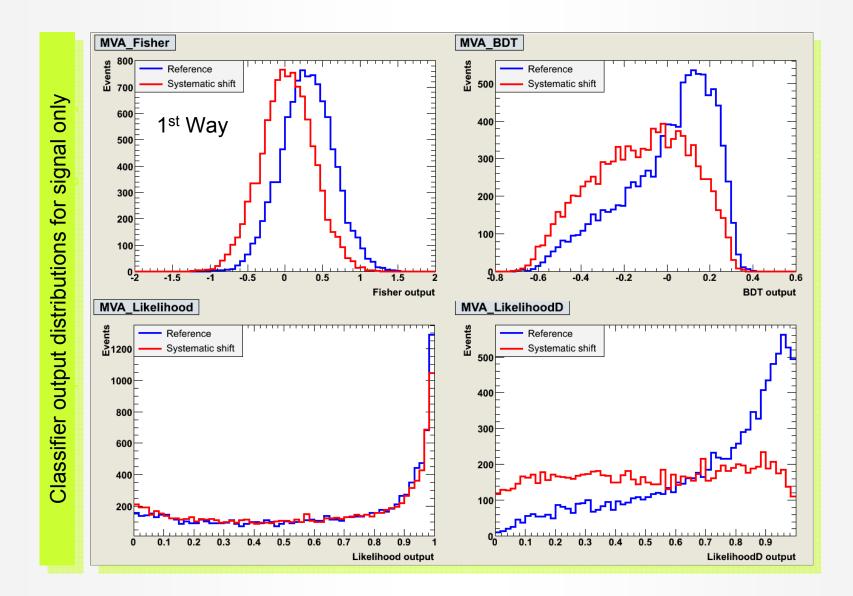


LPNHE Seminar, June 20, 2007

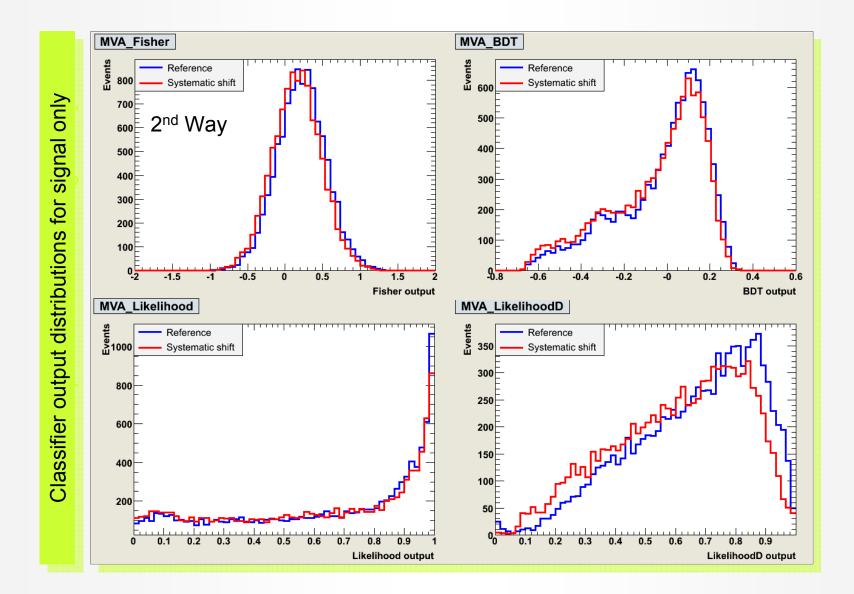
Assume strongest variable "var4" suffers from systematic uncertainty



- Assume strongest variable "var4" suffers from systematic uncertainty
 (at least) Two ways to deal with it:
 - 1. Ignore the systematic in the training, and evaluate systematic error on classifier output
 - Drawbacks:
 - "var4" appears stronger in training than it might be \rightarrow suboptimal performance
 - Classifier response will strongly depend on "var4"
 - 2. Train with shifted (= weakened) "var4", and evaluate systematic error on classifier output
 - Cures previous drawbacks
 - If classifier output distributions can be validated with data control samples, the second drawback is mitigated, but not the first one (the performance loss) !



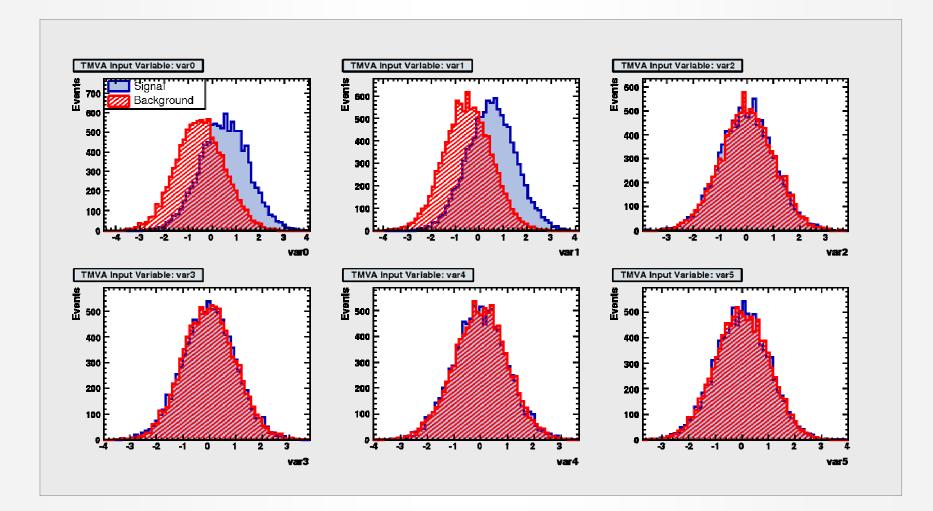
LPNHE Seminar, June 20, 2007



LPNHE Seminar, June 20, 2007

Stability with Respect to Irrelevant Variables

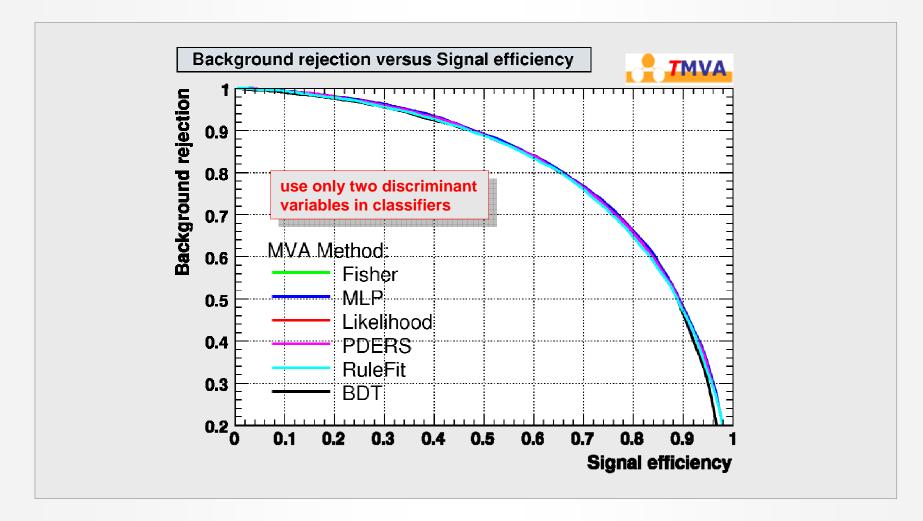
Toy example with 2 discriminating and 4 non-discriminating variables ?



LPNHE Seminar, June 20, 2007

Stability with Respect to Irrelevant Variables

Toy example with 2 discriminating and 4 non-discriminating variables ?



Stability with Respect to Irrelevant Variables

Toy example with 2 discriminating and 4 non-discriminating variables ?

